MODERN'S

abc OF PHYSICS

(VOL-II)

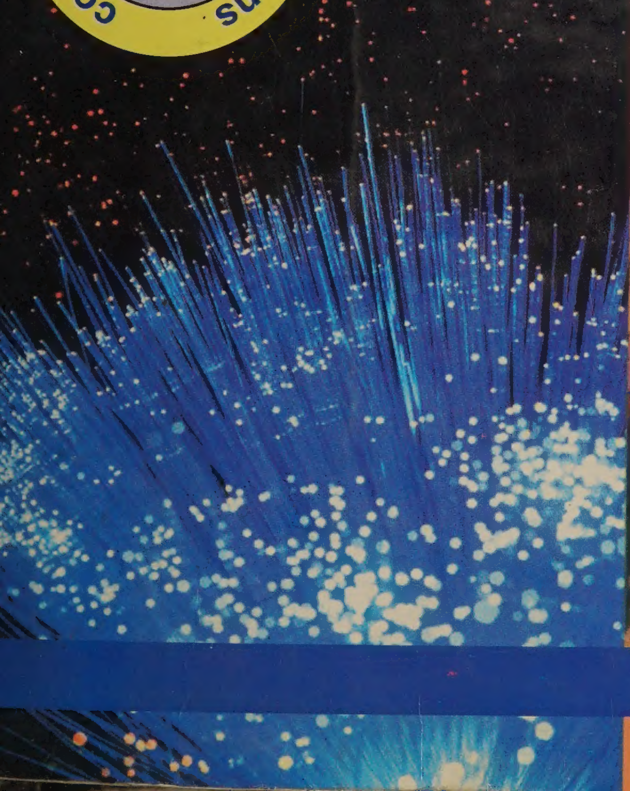
AN ISO 9001 : 2000 CERTIFIED COMPANY

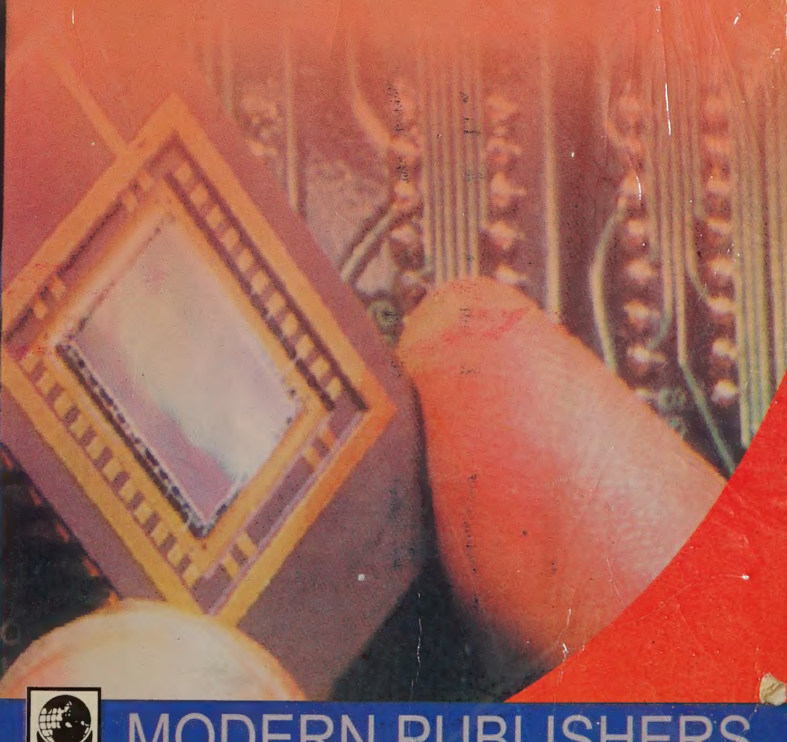
FOR CLASS - XII

SATISH K. GUPTA

covers all the new specifications

UNDOUBTEDLY NO. 1





MODERN PUBLISHERS

MODERN'S

abc OF PHYSICS

(VOL. II)

For Class XII (10+2 System under New Education Policy) of Central Board of Secondary Education, New Delhi and Boards of other Indian States

By

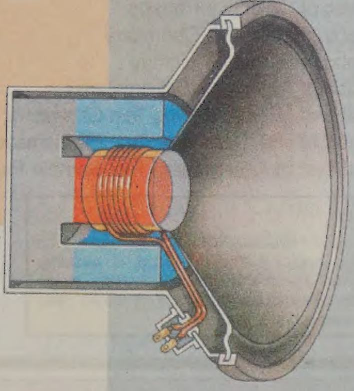
Satish K. Gupta

B.Sc. (Hons.), M.Sc., M.Phil.
Head, Department of Physics,
R.S.D. College, Ferozepur (Pb.)

Edited by

Anirudh Gupta

B.E., M.A.S.C.E.
Director, D.C. Model Group of Schools,
Ferozepur (Pb.)



2005

STRICTLY IN ACCORDANCE WITH THE LATEST GUIDELINES & SYLLABUS

MODERN PUBLISHERS
(Producers of Quality Text Books)

Price : Rs 500.00

OUR ADDRESSES IN INDIA

☐ **New Delhi** : Gulab Bhawan, 6, Bahadur Shah Zafar Marg *Ph. 23317931, 23318301, 23357723*
☐ **Jalandhar City** : MBD House, Railway Road *Ph. 2458388, 2459046, 2455663*
☐ **Delhi** : 1675, Nai Sarak *Ph. 23264887*
☐ **Delhi (Shakarpur)** : MB 161, Street No. 4 *Ph. 22546557, 22518122*
☐ **Delhi (Daryaganj)** : MBD House, 4587/15, Opp. Times of India *Ph. 23245676*
☐ **Delhi (Patparganj)** : Plot No. 225, Industrial Area *Ph. 22149691, 22147073*
☐ **Mumbai** : A-683, T.T.C. Industrial Area, M.I.D.C. Off. Thane-Belapur Road, Navi Mumbai *Ph. 55906739, 55906740*
☐ **Pune** : Kaul Building No. 2, IInd Floor, Flat 'C', Guru Nanak Nagar, Shankar Sheth Road, *Ph. 26431413, 26435071*
☐ **Kolkata** : Satyam Building, 46-D, Rafi Ahmed Kidwai Marg *Ph. 22296863, 22161670*
☐ **Chennai** : Plot No. 26 B/2 SIDCO Estate, North Phase, Near and Parallel Road of Pattravakkam Railway Station Road, Ambattur Industrial Estate *Ph. 26356350, 26359376*
☐ **Chennai** : 25, Motilal Street, T. Nagar *Ph. 24343171, 24332564*
☐ **Hyderabad** : 3-4-829/A, 1st Floor, Barkathpura *Ph. 26781808, 27564788, 27560086*
☐ **Bangalore** : 124/31, 1st Main, Industrial Town (Near Chowdeshwari Kalyan Mantap), West of Chord Road, Rajajinagar *Ph. 23103329, 23104667*
☐ **Ernakulam** : Surabhi Building, South Janatha Road, Palarivattom *Ph. 2338107, 2347371*
☐ **Patna** : Ist Floor, Annapurna Complex, Naya Tola *Ph. 2672732, 2672478, 2686994*
☐ **Bhopal** : MBD House, 13, Hamidia Road *Ph. 2741540, 2741103, 2240662*
☐ **Jabalpur** : 840, Palash Chamber, Malviya Chowk *Ph. 2405854*
☐ **Lucknow** : 173/15, Dr. B. N. Verma Road, Old 30 Kutchery Road *Ph. 2228062, 2759178*
☐ **Agra (U.P.)** : 8/13 I, Kaushal Pur, Bye-pass Road *Ph. 2527420*
☐ **Ahmedabad** : 119, Golden Plaza, Kankaria Road, Raipur *Ph. 5469145, 5471831*
☐ **Guwahati** : Chancellor Commercial, Hem Baruah Road, Paan Bazar *Ph. 2510492, 2511635*
☐ **Goa** : H. No. 932, Plot No. 66, Kranti Nagar (Behind Azad Bhawan), Alto Porvorim, Bardez *Ph. 2413982, 2414394*
☐ **Cuttack** : Badam Bari, Link Road *Ph. 2312795, 2314767*
☐ **Sahibabad (U.P.)** : B-9 & 10, Site IV, Industrial Area *Ph. 2896933, 2896939*
☐ **Nagpur** : Govind Bhavan, 49/C, Bhagwagar Layout, Dharampeth *Ph. 2536610, 2550587, 2550592*
☐ **Jaipur** : D-181, Uday Apartment, Bhrigu Marg, Bani Park *Ph. 2204889, 2207038*
☐ **Raipur** : 9/16, Govind Nagar, Near New Bus Stand *Ph. 2251370, 5038006*
☐ **Karnal** : Opposite Ramgarhia Bhawan, Railway Road
☐ **Shimla** : Sector-I, Phase-I 24/25-D, Below B.C.S. New Shimla *Ph. 2870221*
☐ **Dehradun** ☐ **Ranchi**

MODERN'S OUTSTANDING TEXT BOOKS FOR CLASS XII

- ☐ Modern's abc of Physics
- ☐ Modern's abc of Practical Physics
- ☐ Modern's abc of Chemistry
- ☐ Modern's abc of Solution Manual in Chemistry
- ☐ Modern's abc of Practical Chemistry
- ☐ Modern's abc of Numerical Problems in Chemistry
- ☐ Modern's abc of Biology
- ☐ Modern's abc of Practical Biology
- ☐ Modern's abc of Mathematics
- ☐ Modern's abc of Solution Manual in Mathematics
- ☐ Modern's abc of Computer Science

FOR MEDICAL AND ENGINEERING ENTRANCE EXAMINATIONS

- ☐ Modern's abc of Objective Physics
- ☐ Modern's abc of Objective Chemistry
- ☐ Modern's abc of Objective Biology
- ☐ Modern's abc of Objective Mathematics
- ☐ Modern's abc of I.I.T. Screening Test Chemistry
- ☐ Modern's abc of I.I.T. Screening Test Mathematics
- ☐ Modern's abc of I.I.T. Screening Test Sample Papers

KIND ATTENTION : DEAR CUSTOMER

To avoid duplicacy, a HOLOGRAM has been pasted on the cover of this book. If this hologram is missing, please do not purchase the book as it is likely to be a duplicate book. Any such duplicacy if noted may please be intimated to the Publishers.

© All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise without the prior written permission of the publisher. Any breach will entail legal action and prosecution without further notice.

Published by : **MODERN PUBLISHERS**
Gulab Bhawan, 6, Bahadur Shah Zafar Marg, New Delhi – 110 002.

Printed at : **HOLY FAITH INTERNATIONAL (P) LTD.**
B-9 & 10, Site IV, Industrial Area, Sahibabad (U.P.).

PREFACE & Acknowledgements

The twelfth thoroughly revised and enlarged edition of **Modern's abc of Physics** was brought out in March, 2002 for Class XII of Senior Secondary Certificate Examination of C.B.S.E. and School Education Boards of other Indian States. It is really heartening and satisfying for the author that the new features of the book earned a great appreciation from both the students and the learned colleagues of various schools and colleges. In the present edition, the text has been thoroughly revised and recast to make it more systematic and uptodate.

Salient Features of the Book

- The whole text has been arranged strictly according to N.C.E.R.T. pattern.
- To provide clarity of the subject, the whole text is studded with **The jargon** (to define and introduce a new physical term), **Key point** (to highlight the important concept in an article), **Watch out** (to make distinction between the physical meaning of a term and its meaning in everyday life) and **Self-test Question** (to enable the students to test their grip on the article).
- The article work in each chapter of a unit is coupled with well graded and carefully selected **Solved Numerical Problems**.
- **Very Short Answer Questions** and **Short Answer Questions** set in various Board Examinations have been throughly scanned and incorporated in the book with proper Answers/Hints.
- Under the heading **Techie-Stuff**, a good number of both **Conceptual Numerical Problems** and **Conceptual Short Answer Questions** have been provided for ambitious, brilliant and curious students with an aim to inculcate the much needed urge and desire in such student to probe the subject more deeply.
- **Unsolved Numerical Problems for Self-practice** have been categorised into various types, so as to enable the students to choose the appropriate formula with ease.
- **Competitive Examination File** at the end of each unit forms another special feature of the book.

  Revision at a Glance provides an overview of the contents of a unit for easy and handy reference of various physical laws, principles, terms and formulae dealt in the unit.

  Numerical Problems from Competitive Examinations, such as I.I.T., Roorkee and I.S.M., Dhanbad have been provided with solutions by adopting a novel technique in the form of **Thought Process**. Armed with this technique, the students will be able to attack the otherwise brainteasing and seemingly incomprehensible numerical peoblems with great ease.

  Multiple Choice Questions set in various competitive examinations, such as C.B.S.E., A.I.I.M.S., A.F.M.C., M.N.R., C.P.M.T., I.I.T., etc have been thoroughly covered in the book. So that students can prepare them easily, these questions have been categorised into **Text-Based** and **Thought-Based** Multiple Choice Questions.

Acknowledgements

The author owes his sincere thanks to Sh. Ashok Malhotra, Managing Director and Sh. S.K. Sikka, Manager, Modern Publishers for the keen interest they have taken to give the book its present shape. The author gratefully acknowledges the tremendous efforts on the part of Ishara Laser Graphics for doing a neat laser setting job. Mr. Rakesh Rana has done a great job of doing a neat diagram work. The author wishes to thank them for their help.

Suggestions for further improvement of the book will be gratefully acknowledged and incorporated in the next edition.

Satish K. Gupta

18-Professors' Colony,
Ferozepore city – 152 002
e-mail : satishkgupta@seekindia.com

FACE TO FACE

It is a general feeling among both the teachers and the taught that the numerical problems that are set in the Competitive Examinations for admission to I.I.T., Roorkee and some other Engineering Colleges are so intricate in nature that they become seemingly incomprehensible. But clue to the problem is always somewhere in the cacoon of it. Therefore, in order to train the students to dig out that much vital clue, **Thought Process** has been given just before the formal solution of such a menacing problem. Thought Process will enable the students to prepare a proper line of attack for the problem. The same approach has been followed in case of a few Conceptual Numerical Problems also. The author firmly believes that armed with this technique, the students will be able to attack the otherwise much mind-boggling problems with great ease and rare confidence and the problem will simply surrender before him/her.

Wishing you a Happy Numerical Problems Solving.

To keep pace with the fast changing intellectual climate, the author invites the readers to visit the book site on the Internet. The on line addition of this book is an attempt to reach out to the students so as to keep them abreast with the latest questions set in the various examinations. With this factility the readers can have a direct interaction with the author through e-mails. It is a direct outcome of the deliberations between Mr. Sameer Jain, my son-in-law and Sh A.K. Malhotra, the dynamic publisher.

– Author

SYLLABUS

Central Board of Secondary Education, Delhi

CLASS XII (2003 – 04)

One Paper **Time : 3 Hours** **70 Marks**

I.	Electrostatics	8
II.	Current Electricity	8
III.	Magnetic Effects of Current and Magnetism	8
IV.	Electromagnetic Induction and Alternating Current	8
V.	Electromagnetic waves	3
VI.	Optics	10
VII.	Dual Nature of Matter and Radiation	4
VIII.	Atomic Nucleus	6
IX.	Solids and Semiconductor Devices	8
X.	Principles of Communication	7
	Total	70

Unit I : Electrostatics (Periods 22)

- Frictional electricity, charges and their conservation; Coulomb's law - Forces between two point electric charges, Forces between multiple electric charges; Superposition principle and continuous charge distribution.

Electric field and its physical significance, electric field due to a point charge, electric field lines; Electric dipole, electric field due to a dipole and behaviour of dipole in a uniform electric field.

Electric potential-physical meaning, potential difference, electric potential due to a point charge, a dipole and system of charges; Equipotential surfaces, Electrical potential energy of a system of two point charges and of electric dipoles in an electrostatic field.

Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Conductors and insulators, presence of free charges and bound charges inside a conductor; Dielectrics and electric polarisation, general concept of a capacitor and capacitance, combination of capacitors in series and in parallel, energy stored in a capacitor, capacitance of a parallel plate capacitor with and without dielectric medium between the plates; Van de Graff generator.

Unit II : Current Electricity (Periods 22)

- Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility, and their relation with electric current; Ohm's law, electrical resistance, V–I characteristics, Exceptions of Ohm's law (Non-linear V–I characteristics), electrical resistivity and conductivity, classification of materials in terms of conductivity; Superconductivity (elementary idea); Carbon resistors, colour code for carbon resistors; combination of resistances - series and parallel. Temperature dependence of resistance. Internal resistance of a cell, Potential difference and e.m.f. of a cell, combination of cells in series and in parallel.

Kirchhoff's law–Illustration by simple applications, Wheatstone bridge and its applications for temperature measurements, Metre bridge–special case of Wheatstone bridge.

Potentiometer–principle and applications to measure potential difference and for comparing e.m.fs. of two cells.

Electric power, thermal effects of current and Joule's law; Chemical effects of current–Faraday's laws of electrolysis; Electro-chemical cells–Primary and secondary cells, solid state cells.

Thermoelectricity–origin, elementary ideas of Seebeck effect, Thermocouple, Thermo-emf, neutral and inversion temperatures. Measurement of temperature using a thermocouple.

Unit III : Magnetic Effect of Current and Magnetism (Periods 22)

- Concept of magnetic field. Oersted's experiment, Biot-Savart law, magnetic field due to an infinitely long current carrying straight wire and a circular loop ; Ampere's circuital law and its applications to straight and toroidal solenoids ; Force on a moving charge in uniform magnetic and electric fields, Cyclotron ; Force on current–carrying conductor in a uniform magnetic field. Forces between two parallel current carrying conductors–definition of ampere ; Torque experienced by a current loop in a uniform magnetic field, moving coil galvanometer–its current sensitivity and conversion to ammeter and voltmeter.

Current loop as a magnetic dipole and its magnetic dipole moment ; Magnetic dipole moment of a revolving electron; Magnetic field intensity due to magnetic dipole (bar magnet) along the axis and perpendicular to the axis ; Torque on a magnetic dipole (bar magnet) in a uniform magnetic field ; Bar magnet as an equivalent solenoid, Magnetic field lines ; Earth's magnetic field and magnetic elements ; Para, dia and ferromagnetic substances with examples, Electromagnets and permanent magnets.

Unit IV : Electromagnetic Induction and Alternating Current (Periods 18)

- Electromagnetic induction, Faraday's laws, Induced e.m.f. and current, Lenz's law, Eddy currents, self and mutual inductance.

Alternating current, peak and r.m.s. value of alternating current/voltage, reactance and impedance ; LC-oscillations, LCR series circuit (Phasor diagram) – Resonant circuits and Q-factor; Power in a.c. circuits, wattless current.

a.c. generator and transformer.

Unit V : Electromagnetic Waves (Periods 7)

- Electromagnetic waves and their characteristics (qualitative ideas only); Transverse nature of electromagnetic waves.

Electromagnetic spectrum (Radio, microwaves, infra-red, optical, ultra-violet, X-rays, gamma rays) including elementary facts about their uses; Propagation of electromagnetic waves in atmosphere.

Unit VI : Optics (Periods 26)

- Refraction of light, total internal reflection and its applications, spherical lenses, thin lens formula, lens maker's formula; Magnification, Power of a lens, combination of thin lenses in contact; Refraction and dispersion of light due to a prism, Scattering of light – Blue colour of the sky and reddish appearance of the sun at sun-rise and sun-set.

Optical instruments–Compound microscope, astronomical telescope (refraction and reflection type) and their magnifying powers.

Wave front and Huygens' principles ; Reflection and refraction of plane wave at a plane surface using wave fronts (qualitative idea) ; Interference – Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light; Diffraction – diffraction due to a single slit, width of central maximum, difference between interference and diffraction ; Resolving power of microscope and telescope; Polarisation, Plane polarised light, Brewster's law ; Use of plane polarised light and polaroids.

Unit VII : Dual Nature of Matter and Radiation (Periods 6)

- Photo-electric effect, Einstein's Photo-electric equation—particle nature of light, photo-cell, Matter waves – wave nature of particles, de-Broglie relation, Davisson and Germer experiment.

UNIT VIII: Atomic Nucleus (Periods 12)

- Alpha-particle scattering experiment, size of the nucleus, composition of the nucleus – protons and neutrons. Nuclear instability – radioactivity – alpha, beta and gamma particles/rays and their properties, radioactive decay law, simple explanation of α-decay, β-decay and γ-decay. Mass-energy relation, mass defect, Binding energy per nucleon, its variation with mass number, Nature of nuclear forces, nuclear reaction – Nuclear fission and Nuclear fusion.

UNIT IX : Solids and Semiconductor Devices (Periods 18)

- Energy bands in solids (qualitative ideas only), difference between metals, insulators and semi-conductors using band theory; Intrinsic and extrinsic semi-conductors, *p-n* junction, Semiconductor diode – characteristics in forward and reverse bias, diode as a rectifier, solar cell, photo-diode, LED, zener diode as a voltage regulator; Junction transistor, transistor action, characteristics of a transistor; Transistor as an amplifier (common-emitter configuration) and oscillator; Logic gates (OR, AND, NOT, NAND and NOR); Elementary ideas about I.C.

UNIT X : Principles of Communication (Periods 20)

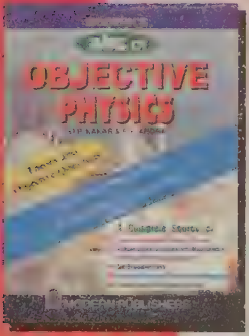
- Elementary idea of analog and digital communication ; Need for modulation; Modulation – amplitude, frequency and pulse modulation; Elementary idea about demodulation, Data transmission and retrieval – Fax and Modem.

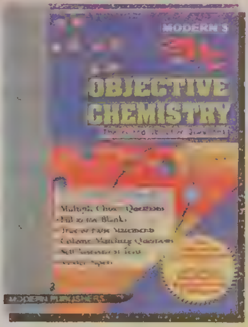
Space Communication : Propagation of e.m. waves in atmosphere. Sky and space wave propagation. Satellite communication. Applications in remote sensing.

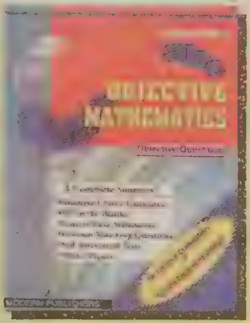
Line Communication : 2-wire lines, cables, telephone links; optical communication (optical fibre, Lasers), elementary principle of light modulation.

UNDOUBTEDLY NO.1

TO ENSURE YOUR ENTRANCE
in Engineering, IIT and Medical







Modern's abc of

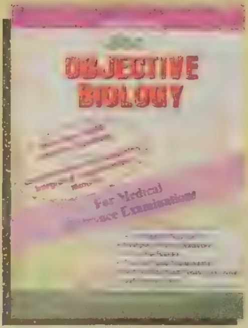
- Objective Physics
- Objective Chemistry
- Objective Mathematics
- Objective Biology

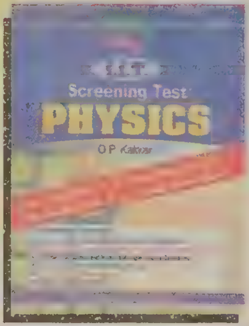
IIT Screening Test

- Physics
- Chemistry
- Sample Papers

A Complete Source of

- Precise discussion of chapters, emphasising all basic principles, definitions, diagrams, terms and analytical relations.
- Exclusive Numerical Problems (Solved).
- Multiple Choice Questions with miscellaneous MCQ's (with Hints and Solutions).
- Fill in the Blanks and True or False Statements.
- Self Evaluation Tests.
- Practical Model Test Papers.
- Coverage of previous years' objective and subjective questions of various examinations.





- All questions have been picked up from the various entrance exams.
- Also cover the most tricky and difficult questions.
- The most exhaustive books ever published.
- Modern's abc Series Always First with the Best

Unmistakably and undoubtedly the best books on the subjects. You can 100% depend on these books to ensure your admission in Medical and Engineering Colleges. This is not a self praise. A thorough study of the books will definitely convince you about their true worth.

MODERN PUBLISHERS
(A Unit of MBD Group)
Gulab Bhawan, 6, Bahadur Shah Zafar Marg, New Delhi–110 002
Ph. : 23317931, 23318301, Fax : 011-23319471
E-mail : mbdgroup@vsnl.net, website : mbdgroup.com

THIS BOOK CONTAINS....

SOLVED NUMERICAL PROBLEMS	....	678
CONCEPTUAL QUESTIONS	....	457
VERY SHORT ANSWER QUESTIONS	....	1028
SHORT ANSWER QUESTIONS	....	856
UNSOLVED NUMERICAL PROBLEMS	....	868
NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS	....	172
MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS	....	1522

CONTENTS

UNIT I. ELECTROSTATICS 1—173

1. Coulomb's Law 3—28
2. Electric Field 29—52
3. Electric Potential 53—81
4. Gauss's Theorem 82—96
5. Capacitor 97—139

Competitive Examination File (Unit I) 140—173

UNIT II. CURRENT ELECTRICITY 175—337

1. Electric Current, Resistance and E.M.F. 177—223
2. Electrical Measurements 224—254
3. Thermal Effect of Current 255—273
4. Chemical Effect of Current 274—296
5. Thermoelectric Effect 297—308

Competitive Examination File (Unit II) 309—337

UNIT III. MAGNETIC EFFECTS OF CURRENT AND MAGNETISM 339—494

1. Magnetic Effect of Current 341—365
2. Motion of Charged Particle 366—406
3. Magnetic dipole 407—426
4. Earth's magnetism 427—447
5. Classification of Magnetic Materials 448—465

Competitive Examination File (Unit III) 466—494

UNIT IV. ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT 495—608

1. Electromagnetic Induction 497—525
2. Transient Current 526—542
3. Alternating Current 543—573
4. Electrical Machines and Devices 574—589

Competitive Examination File (Unit IV) 590—608

UNIT V. ELECTROMAGNETICS WAVES 609—627

1. Electromagnetic Waves 611—623

Competitive Examination File (Unit V) 624—627

UNIT VI. OPTICS 629—794

1. Refraction of Light 631—678
2. Dispersion 679—694
3. Optical Instruments 695—713
4. Huygens' Principle 714—720
5. Interference of Light 721—741
6. Diffraction of Light 742—753
7. Polarisation o Light 754—762

Competitive Examination File (Unit VI) 763—794

UNIT VII. DUAL NATURE OF MATTER AND RADIATIONS 795 — 828
1. Particle Nature of Radiation 797 — 812
2. Wave nature of Matter 813 — 820
Competitive Examination File (Unit VII) 821 — 828

UNIT VIII. ATOMIC NUCLEUS 829 — 892
1. Structure of Nucleus 831 — 845
2. Radioactivity 846 — 865
3. Nuclear Reactions 866 — 880
Competitive Examination File (Unit VIII) 881 — 892

UNIT IX. SOLIDS AND SEMICONDUCTOR DEVICES 893 — 961
1. Solids 895 — 910
2. Semiconductor Devices 911 — 950
Competitive Examination File (Unit IX) 951 — 961

UNIT X. PRINCIPLES OF COMMUNICATION 963 — 1015
1. Analogue and Digital Communication 965 — 982
2. Space Communication 983 — 992
3. Line Communication 993 — 1010
Competitive Examination File (Unit X) 1011 — 1015

REVISION TECHNIQUES 1016 — 1017

LOGARITHMIC AND OTHER TABLES (*i*) — (*viii*)

UNIT

I

ELECTROSTATICS

1. Coulomb's Law
2. Electric Field
3. Electric Potential
4. Gauss's Theorem
5. Capacitor

> "Charge is that certain something possessed by material objects that makes it possible for them to exert electrical force and to respond to electrical force."
>
> – William Gilbert

CHAPTER 1

Coulomb's Law

1.01. FRICTIONAL ELECTRICITY

In 600 B.C., the Greek philosopher Thales observed that amber, when rubbed vith wool, acquires the property of attracting objects like small bits of paper, dry eaves, dust particles, etc. There are many other substances like amber, which can lso acquire the property of attracting such light objects on being rubbed with uitable substances. For example, a glass rod on being rubbed with silk, an ebonite od on being rubbed with fur, a plastic comb on being run through dry hair, etc. The amber, glass rod, ebonite rod or plastic comb are said to be ***electrified*** or ***charged vith electricity***. Wool (in case of amber), silk (in case of glass rod) and fur (in case f ebonite rod) are also found to be electrically charged.

The electricity developed on objects, when they are rubbed with each other, is called rictional electricity.

The electric charges so developed cannot move from one part of the object to he other part. For this reason, frictional electricity is also known as ***static electricity***.

The jargon . . . . .

Amber is a yellow resinous substance. It is a hardened sap of a tree – similar to a pine tree. In Greek, the meaning of amber is ***electrum*** and probably the words like, electricity, electric charge, electric field, electric potential and finally electron owe their origin to electrum.

1.02. TWO KINDS OF ELECTRIC CHARGES

To show that electric charges are of two types, take a glass rod and suspend it vith a thread after rubbing it with silk. If another glass rod rubbed with silk is rought near it, the two rods are found to repel each other. In the same fashion, two bonite rods each rubbed with fur are also found to repel each other. But when an bonite rod rubbed with fur is brought near a glass rod rubbed with silk, the two ods are found to attract each other. The above observations lead to the following wo conclusions :

1. The electric charge developed on glass rod (rubbed with silk) is different from he charge developed on ebonite rod (rubbed with fur).

2. The electric charges of same kind (charges developed on two glass rods or on two ebonite rods) repel each other, while those of different kinds (charges leveloped on a glass rod and an ebonite rod) attract each other.

Key point ✓

Like charges repel each other and unlike charges attract each other.

Historically, the electric charge developed on ebonite rod (or amber) was called ***esinous,*** while the charge developed on glass rod was ***viterous***. The American cientist Benjamin Franklin introduced the concept of positive and negative charges n order to distinguish the two kinds of charges. He named viterous as positive harge and resinous as negative charge.

Self-test Question ?

Consider three objects A, B and C. A and B repel each other, while A and C attract. What is the nature of force between B and C ?

As said earlier, when glass rod is rubbed with silk, silk is also found to be lectricially charged. It is found that the nature of charge on silk is opposite to that on glass rod. Thus, when glass rod is rubbed with silk, the glass rod becomes ositively charged and silk negatively charged. In the same manner, when an ebonite od is rubbed with fur, ebonite rod becomes negatively charged and fur positively harged.

The jargon . . . . .

The charge developed on amber was called ***resinous***, as amber is a resin.

In the table given below, if an object in first column is rubbed against the object iven in second column, the object in first column will acquire positive charge, while hat in second column will acquire negative charge :

The name of the object, which acquires	
positive charge	**negative charge**
Glass rod	Silk cloth
Fur or catskin	Ebonite rod
Woollen cloth	Amber
Woollen cloth	Rubber shoes
Woollen cloth	Plastic object

Obviously, if the two charged objects are from the same column, they will repel each other and if the two charged objects are from two different columns, they will attract each other.

1.03. ELECTRIC CHARGE

All the substances are made of small particles called atoms, which are made of three elementary particles namely electrons, protons and neutrons. Because of their masses, these particles exert ***gravitational forces*** on each other. However, in addition to gravitational force, protons and electrons exert an extra force on each other, which is quite large as compared to gravitational force. This extra force is called ***electric force***. Therefore, apart from their masses, protons and electrons must possess some additional intrinsic property.

The additional property of protons and electrons, which gives rise to electric force between them is called electric charge.

Electric charge is a scalar quantity. A proton possesses positive charge $+e$, while an electron possesses an equal negative charge $-e$, where

$$e = 1{\cdot}6 \times 10^{-19} \text{ coulomb}$$

Normally, an object contains equal number of protons and electrons and as such, it is electrically neutral. When the numbers of protons and electrons are not equal, the object possesses a net positive or negative charge.

Key point ✓

Electric charge is a scalar quantity. In SI, its unit is **coulomb (C)**.

1.04. ORIGIN OF ELECTRIC CHARGE IN FRICTIONAL ELECTRICITY

When two different objects (such as a glass rod and silk) are rubbed together, they get electrified. It happens due to transfer of electrons from one object to the other. The object which gains electrons becomes negatively charged, while the object which loses electrons becomes positively charged. The appearance of charges on the two bodies can be explained on the basis of atomic model.

According to atomic model, an atom consists of a central positive core, called ***nucleus***. The nucleus is made of protons and neutrons, which are held inside it with strong ***nuclear forces***. The electrons revolve around the nucleus in circular orbits. The number of electrons in an atom is equal to the number of protons in the nucleus. Since magnitude of charge on a proton and an electron is same, an atom is always electrically neutral. The electrostatic force of attraction between electrons and the nucleus is much weaker as compared to nuclear force between protons and neutrons constituting the nucleus. Because of this, whereas it is practically impossible to remove protons from the nucleus, electrons can be easily removed from an atom by supplying energy to them. The energy* required to remove an electron from the atom depends upon how tightly it is bound to the atom.

Let us now explain the appearance of charge on a glass rod and silk, when they are rubbed together. When a glass rod is rubbed with silk, energy is provided to overcome friction between them. The energy so provided tends to remove electrons from both the glass rod and silk. In a glass rod, electrons are less tightly bound as compared to those in silk. As a result, the electrons get removed from glass rod and are transferred to silk. As the glass rod loses electrons, it becomes positively charged. On the other hand, silk acquires an equal negative charge on gaining electrons. It, therefore, follows that in the charging process due to rubbing, the appearance of charges is due to actual transfer of electrons.

It may be pointed out that when two objects are charged by rubbing them together, the masses of both the objects change. It is because, electrons are material particles having a small finite mass and in the rubbing process, electrons are transferred from one object to the other. The positive charge appears on the object, which loses electrons. Hence, the mass of positively charged object is less than its original mass. On the other hand, the mass of negatively charged object is more than its original mass as the negative charge appears on it due to gain of electrons.

Key point

In frictional electricity, the charges on two objects rubbed together appear due to actual transfer of electrons (material particles) from one object to the other.

1.05. ADDITIVE NATURE OF CHARGE

We know that mass of an object is equal to the sum of the masses of its constituent particles. Owing to this, mass is said to be ***additive*** in nature. Like mass, electric charge also possesses additive property.

The total electric charge on an object is equal to the algebraic sum of all the electric charges distributed on the different parts of the object.

If q_1, q_2, q_3, ... are electric charges present on different parts of an object, then total electric charge on the object,

$$q = q_1 + q_2 + q_3 + \ldots\ldots$$

It may be pointed out that while taking the algebraic sum, the sign (positive or negative) of the electric charges must be taken into account.

Self-test Question

An object contains n_1 protons and n electrons. What is the net charge on th object ?

The additive property of mass and electric charge differ on following two accounts :

1. The masses of the particles constituting an object are always positive, whereas the charges distributed on the different parts of the object may be positive or negative.

2. The total mass of an object is always non-zero, whereas the total charge on the object may be positive, zero or negative.

1.06. QUANTIZATION OF CHARGE

The magnitude of charge on a proton or an electron ($e = 1{\cdot}6 \times 10^{-19}$ C) is called elementary charge. Since protons and electrons are the only charged particles constituting the matter, the charge on an object must be integral multiple of $\pm e$. In other words, the charge possessed by an object cannot be a fraction of $\pm e$. Mathematically, the charge on any object must always be equal to

$$q = \pm n e,$$

where n is an integer. Further, the charge on an object can be increased or decreased in steps of e. It is known as ***quantization of charge*** or ***discrete nature of charge***.

The fact that all observable charges are always some integral multiple of elementary charge e (= $\pm 1{\cdot}6 \times 10^{-19}$ *C) is known as quantization of charge.*

The grainness or discrete nature of electric charge is usually not observable in actual partice. It is because, step size e of electric charge is very small. Due to this, number of elementary charges involved becomes extremely large, even when the electric charge on an object is changed by a very small amount. For example, when a glass rod is rubbed with silk, a charge of the order of one microculomb ($= 10^{-6}$ C) appears on the glass rod or silk. Since one elementary charge is equal to $1{\cdot}6 \times 10^{-19}$ C, the number of elementary charges on glass rod (or silk) is given by

$$n = \frac{10^{-6}\ \text{C}}{1{\cdot}6 \times 10^{-19}\ \text{C}} = 6{\cdot}25 \times 10^{12}$$

Key point ✓

The quantization effect of charge can be observed only at microscopic level.

Since it is a very large number, the quantization of charge is not observed. Contrary to it, one observes continuous variation in charge. The quantization of charge cannot be explained on the basis of classical theory.

Note. According to recent advancements in physics, it is now believed that protons and neutrons are themselves built out of more elementary units called ***quarks***. They are of two types. The up quark (u) carries charge $+2e/3$ and the down quark (d) has charge $-e/3$. A proton and neutron consist of three quarks each and likewise quark composition of a proton is ***u u d***, while that of neutron is ***u d d***. It may be clearly understood that even on quark model, the quantization of charge is not affected. It is only the step size of charge that decreases from e to $e/3$.

1.07. CONSERVATION OF CHARGE

Just as in mechanics, the total linear momentum of an isolated system always remains constant, the electric charge also obeys a similar law. It is called ***law of conservation of charge.***

It states that *for an isolated system, the net charge always remains constant.* In any physical pocess, the charge it may get transferred from one part of the system to another, but net charge will always remain the same. In other words, *charge can neither be created nor destroyed.*

The following examples explain the law of conservation of charge :

1. We know that when a glass rod is rubbed with silk, glass rod becomes positively charged and silk becomes negatively charged. The amount of positive charge on glass rod is found to be exactly the same as negative charge on silk. Thus, the system of glass rod and silk, which had zero net charge before rubbing, still possesses zero net charge after rubbing.

2. In all nuclear transformations, the proton number is found to remain unchanged. For example, in nuclear fission of uranium (${}_{92}\text{U}^{235}$) by a neutron (${}_{0}n^{1}$), barium (${}_{56}\text{Ba}^{141}$), krypton (${}_{36}\text{Kr}^{92}$) and three neutrons are produced along with liberation of energy. The nuclear reaction may be represented as below :

$$ {}_{0}n^{1} + {}_{92}\text{U}^{235} \longrightarrow {}_{56}\text{Ba}^{141} + {}_{36}\text{Kr}^{92} + 3\,{}_{0}n^{1} + \text{energy}$$

Proton number before fission $= 0 + 92 = 92$

Proton number after fission $= 56 + 36 + 3(0) = 92$

Thus, net charge (proton number) is same before and after the nuclear fission of U^{235}.

3. Annihilation of matter (a positron and an electron combine to produce a γ-ray photon) and pair production (a γ-ray photon materialises into an electron and positron pair) are two important nuclear phenomena. Charge is conserved in both the two processes.

1.08. PROPERTIES OF ELECTRIC CHARGE

The following are a few important properties of electric charge :

1. Like charges repel each other and unlike charges attract each other.

2. The magnitude of elementary negative or positive charge is same and is equal to $1{\cdot}6 \times 10^{-19}$ C.

3. The electric charge is ***additive*** in nature. It implies that total charge on an object is algebraic sum of the charges located at different points in the object.

4. The charge is ***quantized*** *i.e.* charge carried by a charged object is equal to $\pm\, n\, e$, where n is an integer.

5. The electric charge of a system is always ***conserved***.

6. Unlike mass, the electric charge on an object is not affected by the motion of the object.

Key point

The charge on an object is not affected by its motion.

1.09. COULOMB'S LAW IN ELECTROSTATICS

Coulomb's law in electrostatics is electrical analogue of Newton's law of gravitation. In 1785, Coulomb measured the force of attraction or repulsion between two electrical charges by using a torsion balance. His observations are known as ***Coulomb's law in electrostatics.***

It states that *two point charges attract or repel each other with a force which is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.*

The jargon

A force, which acts along the line joining the centres of two interacting objects, is called a central force.

The force is repulsive, if the charges are alike and attractive in case of unlike charges. Further, electrostatic force between two charges is ***central*** in nature. It may be pointed out that Coloumb's law in electrostatic holds for ***stationary*** charges. Further, the two charges should be ***points*** in size.

Key point

Coulomb's law in electrostatics holds for two point charges at rest.

Consider that two charges q_1 and q_2 are present at points A and B at a distance r apart as shown in Fig. 1.01. Then, force between the two charges *i.e.*

$$F \propto q_1 q_2 \qquad \text{...(1.01)}$$

$$\propto \frac{1}{r^2} \qquad \text{...(1.02)}$$

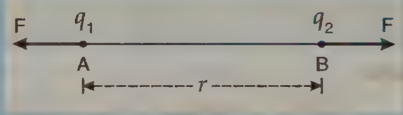


Fig. 1.01

Combining equations (1.01) and (1.02), we have

$$F \propto \frac{q_1 q_2}{r^2}$$

or

$$F = k\,\frac{q_1 q_2}{r^2}, \qquad \text{...(1.03)}$$

where k is constant of proportionality. Its value depends upon the nature of the medium in which the two charges are located and also the system of units adopted to measure F, q_1, q_2 and r.

In SI, when the two charges are located in vacuum or air,

$$k = \frac{1}{4\pi\varepsilon_0}$$

where ε_0 is called ***absolute permittivity of free space****. Therefore, from equation (1.03), the force between two charges located in air or vacuum is given by

$$F_{vac} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \qquad \text{...(1.04)}$$

Self-test Question

Using equation (1.04), show that units of ε_0 are $C^2\,N^{-1}\,m^{-2}$.

The absolute permittivity of free space is measured to be

$$\varepsilon_0 = 8{\cdot}854 \times 10^{-12}\ C^2\,N^{-1}\,m^{-2}$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8{\cdot}854 \times 10^{-12}\ C^2\,N^{-1}\,m^{-2}} = 9 \times 10^9\ N\,m^2\,C^{-2}$$

Hence, the equation (1.04) may be written as

$$F_{vac} = 9 \times 10^9 \times \frac{q_1 q_2}{r^2} \quad \text{(in newton)} \qquad \text{...(1.05)}$$

Key point

Another unit[†] of ε_0 is farad metre^{-1} (F m^{-1}).

---

* The absolute permittivity of air is only 1·005 times as that of free space (vacuum). For this reason, ε_0 is usually taken as absolute permittivity of both free space and air.

[†] Refer to section 5.04 of the chapter 5 on 'Capacitor.'

1.10. UNIT OF CHARGE

In SI, force between two charges q_1 and q_2 held in vacuum at a distance r apart is given by

$$F_{vac} = 9 \times 10^9 \times \frac{q_1 q_2}{r^2}$$

Suppose that $q_1 = q_2 = q$ (say) ; $r = 1$ m and $F_{vac} = 9 \times 10^9$ N

Then, $9 \times 10^9 = 9 \times 10^9 \times \dfrac{q \times q}{1^2}$

or $q^2 = 1$

or $q = \pm 1$ coulomb (C)

Therefore, *one coulomb is that charge which repels an equal and similar charge with a force of 9×10^9 N, when placed in vacuum (or air) at a distance of one metre from it.*

Note. 1. In electrostatic cgs system, the unit of charge is known as ***electrostatic unit of charge (e.s.u. of charge)***. It is also called *statcoulomb (stat C)*

$$\mathbf{1\ C = 3 \times 10^9\ stat\ C}$$

2. In electromagnetic cgs system, the unit of charge is known as ***electromagnetic unit of charge (e.m.u. of charge)***.

$$\mathbf{1\ C = \frac{1}{10}\ e.m.u.\ of\ charge}$$

Self-test Question

Prove that
1 coulomb = 3×10^9 statcoulomb

1.11. RELATIVE PERMITTIVITY (DIELECTRIC CONSTANT)

When two charges are situated in some other medium in place of air, the force between the charges may be greatly affected. For example, when the two charges are placed at the same distance in water, the force between the two charges becomes about $\frac{1}{80}$ th of the force between them, when the same two charges are located in air. It is because, absolute permittivity of water is about 80 times as large as the absolute permittivity of free space.

The force between two charges q_1 and q_2 located at a distance r in a medium may be expressed as

$$F_{med} = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2}, \qquad \text{...(1.06)}$$

where ε is called ***absolute permittivity*** of the medium.

The equation (1.04) gives force between the charges q_1 and q_2, when they are placed in vacuum at a distance r apart. Dividing equation (1.04) by (1.06), we have

$$\frac{F_{vac}}{F_{med}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \Big/ \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2} = \frac{\varepsilon}{\varepsilon_0} \qquad \text{...(1.07)}$$

Self-test Question

Do relative permittivity and dielectric constant of a medium imply different physical quantities ?

The ratio $\frac{\varepsilon}{\varepsilon_0}$ is denoted by ε_r, called ***relative permittivity*** of the medium w.r.t. vacuum. It is also denoted by K, called ***dielectric constant*** of the medium. Therefore, equation (1.07) gives

$$\varepsilon_r \text{ (or K)} = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{vac}}{F_{med}} \qquad \text{...(1.08)}$$

Thus, *relative permittivity or dielectric constant of a medium may be defined as the ratio of the force between two charges placed at a certain distance apart in air to the force between the same two charges placed the same distance apart in that medium.*

Setting $\varepsilon = \varepsilon_0 \varepsilon_r$ or $\varepsilon = \varepsilon_0 K$ in equation (1.06), we have

$$F_{med} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{q_1 q_2}{r^2}$$

or

$$F_{med} = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q_1 q_2}{r^2} \qquad \text{...(1.09)}$$

Key point ✓

Two charges situated at a certain distance apart exert maximum force on each other, when they are placed in vacuum (or air). In a dielectric medium, the force between two charges always decreases.

1.12. COULOMB'S LAW IN VECTOR FORM

Consider that two like charges q_1 and q_2 are present at points A and B in vacuum at a distance r apart. The two charges will exert equal repulsive forces on each other. Let $\vec{F}_{12}$ be the force on charge q_1 due to charge q_2 and $\vec{F}_{21}$ be force on charge q_2 due to charge q_1[Fig. 1.02].

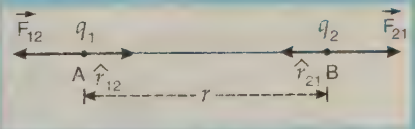


Fig. 1.02

According to Coulomb's law, the magnitude of force on charge q_1 due to q_2 (or on charge q_2 due to q_1) is given by

$$|\vec{F}_{12}| = |\vec{F}_{21}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \qquad ...(1.10)$$

Let $\hat{r}_{12}$ be a unit vector pointing from charge q_1 to q_2 and $\hat{r}_{21}$, a unit vector pointing from charge q_2 to q_1. As the force vector $\vec{F}_{12}$ is along the direction of unit vector $\hat{r}_{21}$, we have

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21} \qquad ...(1.11)$$

Also, as the force vector $\vec{F}_{21}$ is along the direction of unit vector $\hat{r}_{12}$, it follows that

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \qquad ...(1.12)$$

The equations (1.11) and (1.12) express Coulomb's law in vector form.

Importance of Coulomb's law in vector form. Coulomb's law in vector form is more informative than in its scalar form for the following reasons :

1. In vector form, Coulomb's law straightway shows that the forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal and opposite.

Since $\hat{r}_{12}$ and $\hat{r}_{21}$ are unit vectors pointing in opposite direction, we have

$$\hat{r}_{21} = -\hat{r}_{12}$$

Therefore, equation (1.11) becomes,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} (-\hat{r}_{12}) = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \qquad ...(1.13)$$

From equations (1.12) and (1.13), it follows that

$$\vec{F}_{12} = -\vec{F}_{21} \qquad ...(1.14)$$

Key point

From Coulomb's law in vector form, it follows that

(*i*) the two charges exert equal and opposite forces on each other.

(*ii*) the electrostatic force is central in nature.

2. From Coulomb's law in vector form, it follows that the electrostatic force between the two charges is a *central force i.e.* it acts along the line joining the two charges.

From equations (1.12) and (1.13), it follows that force experienced by one charge due to the other acts in the direction of unit vector $\hat{r}_{12}$ or in the direction opposite to that of $\hat{r}_{12}$. Since $\hat{r}_{12}$ is a unit vector along the line joining the two charges, the electrostatic force between two charges is a central force.

Note. In case the two charges are unlike, they will exert equal attractive forces on each other as shown in Fig. 1.03. Proceeding as above, it can be obtained that

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

and

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

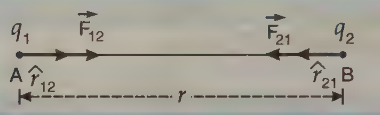


Fig. 1.03

1.13 FORCE BETWEEN TWO CHARGES IN TERMS OF THEIR POSITION VECTORS

To express force between two point charges in terms of their position vectors, consider a co-ordinate frame of reference OXYZ. Suppose that the two point charges q_1 and q_2 are present in space at the points A and B, whose position vectors are $\vec{OA} = \vec{r}_1$ and $\vec{OB} = \vec{r}_2$ as shown in Fig. 1.04. Join the two charges q_1 and q_2 by a straight line. If the distance between the two charges is denoted by r and $\hat{r}_{21}$ is unit vector from charge q_2 to q_1, then force on charge q_1 due to q_2 is given by

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

The above, equation can be rewritten as

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^3} (r\,\hat{r}_{21}) \qquad ...(1.15)$$

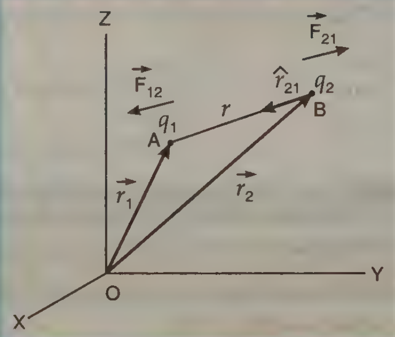


Fig. 1.04

From triangle law of addition of vectors, we have

$$\vec{OB} + \vec{BA} = \vec{OA} \qquad \text{or} \qquad \vec{BA} = \vec{OA} - \vec{OB}$$

or $$r\,\hat{r}_{21} = \vec{r}_1 - \vec{r}_2 \qquad (\because \vec{BA} = r\,\hat{r}_{21})$$

and $$r = |\vec{r}_1 - \vec{r}_2|$$

In equation (1.15), setting $r\,\hat{r}_{21} = \vec{r}_1 - \vec{r}_2$ and $r = |\vec{r}_1 - \vec{r}_2|$, we have

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) \qquad \text{...(1.16)}$$

Similarly, it can be obtained that the force on charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1) \qquad \text{...(1.17)}$$

1.14. PRINCIPLE OF SUPERPOSITION

Coulomb's law gives the force between two point electric charges. The principle of superposition gives a method to find force on a charge, when a group of charges are interacting.

The principle of superposition states that *when a number of charges are interacting, the total force on a given charge is the vector sum of the individual forces exerted on the given charge by all the other charges.*

The force between any two charges is not affected by the presence of other charges.

Key point ✓

The force between any two charges is not affected by the presence of other charges.

Consider that n point charges $q_1, q_2, q_3, \ldots q_n$ are distributed in space in a discrete manner. The charges are interacting with each other. Let us find the total force on charge say q_1 due to all other remaining charges. If the charges $q_2, q_3, \ldots q_n$ exert forces $\vec{F}_{12}, \vec{F}_{13}, \ldots \vec{F}_{1n}$ on the charge q_1 [Fig. 1.05], then according to principle of superposition, the total force on charge q_1 is given by

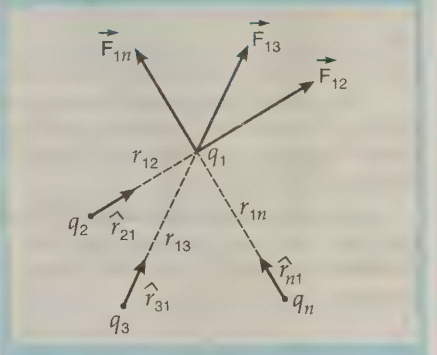


Fig. 1.05

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1n} \qquad \text{...(1.18)}$$

If the distance between the charges q_1 and q_2 is denoted as r_{12} ; and $\hat{r}_{21}$ is unit vector from charge q_2 to q_1, then

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{21}$$

Similarly, the force on charge q_1 due to other charges is given by

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{r_{13}^2}\,\hat{r}_{31}$$

....

....

and $$\vec{F}_{1n} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_n}{r_{1n}^2}\,\hat{r}_{n1}$$

Hence, in equation (1.18), substituting for $\vec{F}_{12}, \vec{F}_{13}, \ldots\ldots\ldots\ldots \vec{F}_{1n}$; the total force on charge q_1 due to all other charges is given by

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{21} + \frac{q_1 q_3}{r_{13}^2}\,\hat{r}_{31} + \ldots\ldots\ldots\ldots + \frac{q_1 q_n}{r_{1n}^2}\,\hat{r}_{n1}\right) \qquad \text{...(1.19)}$$

The same procedure can be adopted for finding the force on any other charge due to the remaining charges of the group of n charges. For example, force on charge q_2 due to all other charges is given by

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \ldots\ldots + \vec{F}_{2n}$$

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2 q_1}{r_{21}^2}\,\hat{r}_{12} + \frac{q_2 q_3}{r_{23}^2}\,\hat{r}_{32} + \ldots\ldots\ldots\ldots + \frac{q_2 q_n}{r_{2n}^2}\,\hat{r}_{n2}\right)$$

Superposition principle in terms of position vectors of the charges. Suppose that the point charges $q_1, q_2, q_3, \ldots\ldots q_n$ are located at points in space, whose position vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots\ldots \vec{r}_n$ respectively. Let $\hat{r}_{21}, \hat{r}_{31}, \ldots\ldots \hat{r}_{n1}$ be unit vectors from charge q_2 to q_1, from q_3 to q_1, ... from q_n to q_1 as shown in Fig. 1.06. Then, by proceeding as in section 1.13, the force on charge q_1 due to q_2 is given by

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

Similarly, force on charge q_1 due to q_3

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|^3} (\vec{r}_1 - \vec{r}_3)$$

....

....

and
$$\vec{F}_{1n} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_n|^3} (\vec{r}_1 - \vec{r}_n)$$

Therefore, total force on charge q_1 due to all other charges is given by

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) + \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|^3} (\vec{r}_1 - \vec{r}_3) + ... + \frac{q_1 q_n}{|\vec{r}_1 - \vec{r}_n|^3} (\vec{r}_1 - \vec{r}_n) \right) \quad ...(1.20)$$

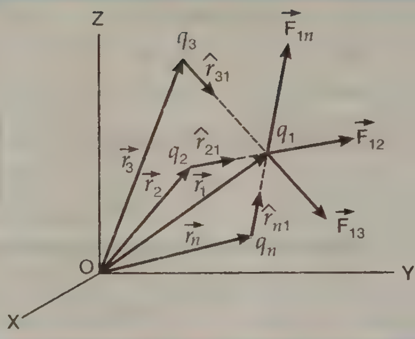


Fig. 1.06

1.15. CONTINUOUS DISTRIBUTION OF CHARGE

As said earlier, the electric charge is quantized and the charge on a charged body is always an integral multiple of the charge on a single electron or a proton. In most of the practical situations, charge on a charged body is so large as compared to the magnitude of charge on an electron or a proton that the quantization of charge may be ignored. In other words, we can assume that on a charged body of reasonable size, charge has continuous distribution.

The continuous distribution of charge may be one dimensional, two dimensional or three dimensional. Accordingly, the distribution of charge is called ***linear charge distribution, surface charge distribution*** and ***volume charge distribution*** respectively.

1. Linear charge density. When charge is distributed along a line (straight or curved), the charge distribution is called linear. Fig. 1.07 shows the uniform distribution of charge q over the length L of a straight rod. Then, linear charge density is defined as

$$\lambda = \frac{q}{L}$$

Its unit is **coulomb metre^{-1} (C m^{-1}).**

For example, if charge q is uniformly distributed over a circular ring of radius R, then its linear charge density is $\frac{q}{2\pi R}$.

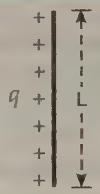


Fig. 1.07

2. Surface charge density. When charge is distributed over a surface (plane or curved), the charge distribution is called surface charge distribution. Fig. 1.08 shows the uniform distribution of charge q over a plane surface of area A. Then, surface charge distribution is defined as

$$\sigma = \frac{q}{A}$$

Its unit is **coulomb metre^{-2} (C m^{-2}).**

For example, if charge q is uniformly distributed over the surface of a spherical conductor of radius R, then its surface charge density is $\frac{q}{4\pi R^2}$.

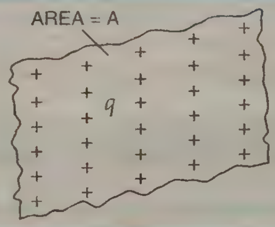


Fig. 1.08

3. Volume charge density. When charge is distributed over the volume of an object, it is called volume charge distribution. Fig. 1.09 shows the uniform distribution of charge q over the volume V of a spherical object. Then, volume charge density is defined as

$$\rho = \frac{q}{V}$$

Its unit is **coulomb metre^{-3} (C m^{-3}).**

For example, if charge q is uniformly distributed over the entire volume of a sphere, then its volume charge density is $\frac{q}{\frac{4}{3}\pi R^3}$.

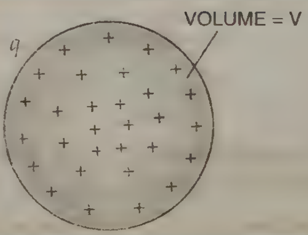


Fig. 1.09

1.16. FORCE DUE TO A CONTINUOUS DISTRIBUTION OF CHARGES

Now, we calculate the force experienced by a charge q_0 due to a continuous charge distribution in the following cases :

1. When charge is distributed continuously along a line. Consider a line charge L (straight or curved) having a uniform continuous distribution of charge along its length. Suppose that we have to find force due to the line charge on a point charge q_0 placed at a point, whose position vector is $\vec{r}$ [Fig. 1.10].

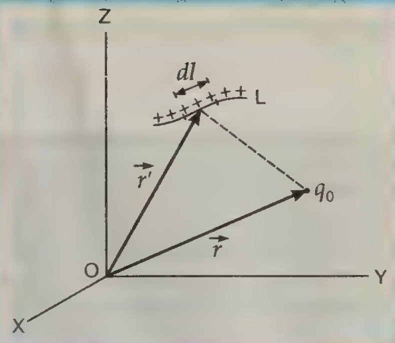


Fig. 1.10

Let *dl* be an infinitesimally small length of the line charge L. If λ is linear charge density of the line L charge, then charge on the length element *dl* is given by

$$dq = \lambda \, dl$$

Let $\vec{r}'$ be position vector of this elementary portion of the line charge having charge *dq*.

Then, force on charge q_0 due to charge *dq* (= λ *dl*) on length element *dl* is given by

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 \cdot dq}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

Substituting for *dq*, we have

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 \, \lambda \, dl}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

The force on charge q_0 due to the whole line charge distribution can be found by integrating the above over its total length *i.e.*

$$\vec{F} = \int_L \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 \, \lambda \, dl}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

or

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \int_L \frac{\lambda \, dl}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \qquad \text{...(1.21)}$$

2. When charge is distributed continuously over a surface. Consider a surface S having a uniform continuous distribution of charge along its surface. Suppose that we have to find force due to surface charge S on a point charge q_0 placed at a point, whose position vector is $\vec{r}$ [Fig. 1.11].

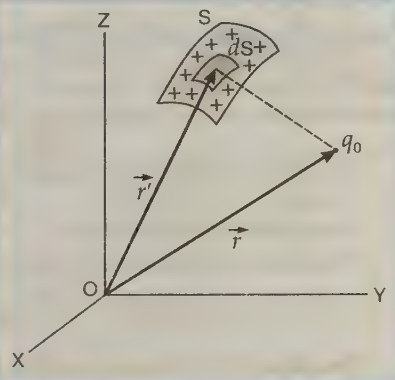


Fig. 1.11

Let *d*S be an infinitesimally small area of the surface charge S. If σ is surface charge density of surface S, then charge on the surface element *d*S is given by

$$dq = \sigma \, dS$$

Let $\vec{r}'$ be position vector of this elementary portion of the surface charge having charge *dq*.

Then, force on charge q_0 due to charge *dq* (= σ *d*S) on the surface element *d*S is given by

$$d\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 \, \sigma \, dS}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

The force on charge q_0 due to whole surface charge distribution can be found by integrating the above over its total surface area *i.e.*

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \int_S \frac{\sigma \, dl}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \qquad \text{...(1.22)}$$

3. When charge is distributed continuously over a volume. Consider a volume V having a uniform continuous distribution of charge. Suppose that we have to find force due to volume charge V on a point charge placed at a point, whose position vector is $\vec{r}$ [Fig. 1.12].

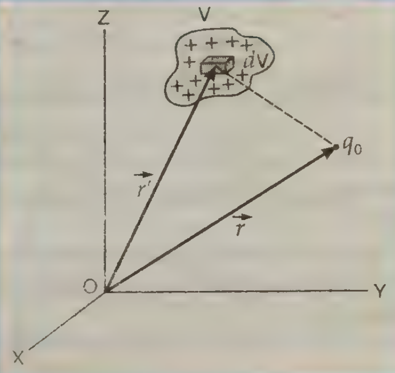


Fig. 1.12

Let *d*V be an infinitesimally small volume of the volume charge V. If ρ is volume charge density of volume V, then charge on the volume element *d*V is given by

$$dq = \rho \, dV$$

Let $\vec{r}'$ be position vector of this elementary portion of the volume charge having charge *dq*.

The force on the charge q_0 due to the charge dq $(= \rho\, dV)$ on the volume element dV is given by

$$\overrightarrow{dF} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0\, \rho\, dV}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

The force on charge q_0 due to whole volume charge distribution can be found by integrating the above over its total volume *i.e.*

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \qquad \text{...(1.23)}$$

Note. 1. In case charges are distributed in a discrete manner, the force on a charge due to the discrete distribution of charges is found by adding the forces exerted by the individual charges. Accordingly, the summation sign is used to add such forces. However, when charges are distributed in a continuous manner, the effect can only be summed up by the method of integration.

2. If charge is distributed continuously over a line L, over a surface S, over a volume V and also in a discrete manner in the space, then total force on charge q_0 at location $\vec{r}$ will be

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \left(\int_L \frac{\lambda\, dl}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') + \int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') + \int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') + \sum_{\substack{\text{all} \\ \text{points charges}}} \frac{q'}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \right) \qquad \text{...(1.24)}$$

In equation (1.24), $\vec{r}'$ has been used to denote the position vector of elementary ortion of line charge L, surface charge S, volume charge V and any one charge q' f the discrete distribution of charges in space.

SOLVED NUMERICAL PROBLEMS

Type A. On Quantization of Charge

Problem 1.01. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C. Estimate the number of electrons transferred from wool to polythene. *(C.B.S.E. 1990 ; Text Problem)*

Sol. Here, total charge transferred, $q = -3 \times 10^{-7}$ C

Charge on an electron, $e = -1 \cdot 6 \times 10^{-19}$ C

From quantization of charge, $q = n\, e$

Therefore, number of electrons transferred,

$$n = \frac{q}{e} = \frac{-3 \times 10^{-7}}{-1 \cdot 6 \times 10^{-19}} = \mathbf{1 \cdot 875 \times 10^{12}}$$

Problem 1.02. Calculate the total positive or negative charge on a $3 \cdot 11$ g copper penny. Given Avogadro number $= 6 \cdot 02 \times 10^{23}$, atomic number of copper = 29 and atomic mass of copper = $63 \cdot 5$.

Sol. Here, mass of copper penny, $m = 3 \cdot 11$ g

$Z = 29$; $A = 63 \cdot 5$ and $N = 6 \cdot 02 \times 10^{23}$

Number of atoms in the copper penny, $n = \frac{N}{A} \times m$

or $$n = \frac{6 \cdot 02 \times 10^{23} \times 3 \cdot 11}{63 \cdot 5} = 2 \cdot 95 \times 10^{22}$$

Number of protons or electrons in the copper penny

$= n \times Z$

$= 2 \cdot 95 \times 10^{22} \times 29$

Therefore, total positive or negative charge in the copper penny,

$q = 2 \cdot 95 \times 10^{22} \times 29 \times 1 \cdot 6 \times 10^{-19}$

$= \mathbf{1 \cdot 37 \times 10^5}$ **C**

Type B. On Coulomb's law in electrostatics

Problem 1.03. What is the force between two sma charged spheres having charges of 2×10^{-7} C and 3×10^{-7} placed 30 cm in air ? *(Text Probler*

Sol. Here, $q_1 = 2 \times 10^{-7}$ C ; $q_2 = 3 \times 10^{-7}$ C;

$r = 30$ cm $= 0 \cdot 3$ m

In air, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$= 9 \times 10^9 \times \frac{(2 \times 10^{-7}) \times (3 \times 10^{-7})}{(0 \cdot 3)^2}$

$= \mathbf{6 \times 10^{-3}}$ **N (repulsive)**

Problem 1.04. The electrostatic force of repulsic between two equal positively charged ions is $3 \cdot 7 \times 10^{-9}$ I when they are separated by a distance of 5 Å. How mar electrons are missing from each other ? *(C.B.S.E. 200*

Sol. Here, $F = 3 \cdot 7 \times 10^{-9}$ N ; $r = 5$ Å $= 5 \times 10^{-10}$ m

Suppose that n electrons have been removed to for each positive ion. Then, charge on each positive ion,

$q_1 = q_2 = n \times 1 \cdot 6 \times 10^{-19}$ C

Now, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$\therefore\ 3 \cdot 7 \times 10^{-9} = \frac{9 \times 10^9 \times (n \times 1 \cdot 6 \times 10^{-19})^2}{(5 \times 10^{-10})^2}$

or $n = \mathbf{2}$

**Problem 1.05. The electrostatic force on a small sphe of charge $0 \cdot 4\ \mu$C due to another small sphere of charg $-0 \cdot 8\ \mu$C in air is 0·2 N. (*a*) What is the distance betwe

the two spheres ? (b) What is the force on the second sphere due to the first ? (*Text Problem*)

Sol. Here, $q_1 = 0 \cdot 4\ \mu C = 0 \cdot 4 \times 10^{-6}\ C$;

$q_2 = -0 \cdot 8\ \mu C = -0 \cdot 8 \times 10^{-6}\ C$

(*a*) Force on charge q_1 due to q_2, $F_{12} = 0 \cdot 2\ N$

In air, $F_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{q_1 q_2}{r^2}$

or $0.2 = 9 \times 10^9 \times \frac{(0 \cdot 4 \times 10^{-6}) \times (0 \cdot 8 \times 10^{-6})}{r^2}$

or $r = \mathbf{0 \cdot 12\ m}$

(*b*) Force on charge q_2 due to q_1,

$F_{21} = F_{12} = \mathbf{0 \cdot 2\ N}$ **(attractive)**

Problem 1.06. A free pith-ball of 8 g carries a positive charge of 5 × 10⁻⁸ C. What must ball be nature and magnitude of charge that should be given to a second pith-ball fixed 5 cm vertically below the former pith ball so that the upper pith ball is stationary ? (*H.S.S.C.E. 2001*)

Sol. Here, charge on pith-ball A, $q_1 = 5 \times 10^{-8}\ C$

mass of pith-ball A, $m_1 = 8\ g = 8 \times 10^{-3}\ kg$

The weight $m_1 g$ of the pith-ball A acts vertically downwards.

Let q_2 be charge on the pith-ball B held 5 cm below the pith-ball A, so that the pith-ball A remains stationary. It can be possible only, if the charges on two pith-balls are of same signs *i.e.* if charge on pith-ball A is positive, charge on B should also be **positive.** Then, the force on pith-ball A due to B *i.e.* F_{AB} will act vertically upwards [Fig. 1.13].

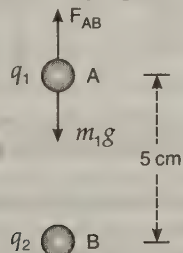


Fig. 1.13.

For charge q_1 to remain stationary,

$$F_{AB} = m_1 g$$

or $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{AB^2} = m_1 g$

Here, $AB = 5\ cm = 0 \cdot 05\ m$

$\therefore\ 9 \times 10^9 \times \frac{5 \times 10^{-8} \times q_2}{(0 \cdot 05)^2} = 8 \times 10^{-3} \times 9 \cdot 8$

or $q_2 = \mathbf{4 \cdot 36 \times 10^{-7}\ C}$ **(positive)**

Problem 1.07. (a) Two insulated charged copper, spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion, if charge on each is 6 · 5 × 10⁻⁷ C ? The radii of A and B are negligible as compared to the distance of separation.

(b) What is the force of repulsion, if (i) each sphere is charged to double the above amount and the distance between them is halved, (ii) the two spheres are placed in water ? (Dielectric constant of water = 80). (*Text Problem*)

Sol. (*a*) Here, charge on sphere A, $q_1 = 6 \cdot 5 \times 10^{-7}\ C$

Charge on sphere B, $q_2 = 6 \cdot 5 \times 10^{-7}\ C$

Distance between the spheres A and B,

$r = 50\ cm = 0 \cdot 5\ m$

Now, $F_{air} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{(6 \cdot 5 \times 10^{-7})^2}{(0 \cdot 5)^2}$

$= \mathbf{1 \cdot 521 \times 10^{-2}\ N}$

***(b) (i) When each sphere is charged to double the amount*:**

Here, $q_1 = q_2 = 2 \times 6 \cdot 5 \times 10^{-7}\ C$; $r = \frac{0 \cdot 5}{2} = 0 \cdot 25\ m$

Now, $F_{air} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$

$= 9 \times 10^9 \times \frac{(2 \times 6 \cdot 5 \times 10^{-7})^2}{(0 \cdot 25)^2} = \mathbf{0 \cdot 2434\ N}$

***(ii) When charges are placed in water* :**

$F_{water} = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q_1 q_2}{r^2} = \frac{F_{air}}{K}$

Here, $K = 80$

$\therefore\ F_{water} = \frac{1 \cdot 521 \times 10^{-2}}{80} = \mathbf{1 \cdot 9 \times 10^{-4}\ N}$

Problem 1.08. Suppose in the problem 1.07 (a), the spheres A and B have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between A and B ? (*Text Problem*)

Sol. Original charge on sphere A or B,

$q_1 = q_2 = 6 \cdot 5 \times 10^{-7}\ C$

Distance between the two spheres, $r = 0{\cdot}05\ m$

Since all the spheres are of same size, they will possess equal charges on being brought in contact.

When uncharged sphere C ($q_3 = 0$) is brought in contact with A, charge left on sphere A,

$q_1' = \frac{q_1 + q_3}{2} = \frac{6 \cdot 5 \times 10^{-7} + 0}{2} = 3 \cdot 25 \times 10^{-7}\ C$

and charge on sphere C, $q_3' = 3 \cdot 25 \times 10^{-7}\ C$

Therefore, when sphere C is brought in contact with B, charge left on sphere B,

$q_2' = \frac{q_2 + q_3'}{2} = \frac{6 \cdot 5 \times 10^{-7} + 3 \cdot 25 \times 10^{-7}}{2}$

$= 4 \cdot 875 \times 10^{-7}\ C$

Therefore, new force of repulsion between spheres A and B,

$F_{air} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{3}$

$= 9 \times 10^9 \times \frac{3 \cdot 25 \times 10^{-7} \times 4 \cdot 875 \times 10^{-7}}{(0 \cdot 5)^2}$

$= \mathbf{5 \cdot 704 \times 10^{-3}\ N}$

Problem 1.09. Two point charges of charge values Q and *q* are placed at a distance of *x* and *x*/2 respectively from a third charge of charge value 4 *q*, all charges being in the same straight line. Calculate the magnitude and nature of charge Q, such that the net force experienced by the charge *q* is zero. (*C.B.S.E. 1998*)

Sol. Suppose that the charge 4 *q* is located at point A. The charges Q and *q* are placed at points B and C, such that AB = *x* and AC = *x*/2. Also, all the charges lie on the same straight line [Fig. 1.14]. We assume that charges of 4 *q* and *q* are of same nature, say positive.

Then, force on charge *q* due to 4 *q*,

$$F_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q.q}{(x/2)^2} \qquad \text{(along CB)}$$

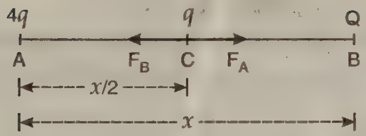


Fig. 1.14.

The net force experienced by charge q will be zero only if charge Q exerts force on charge q equal and opposite to that exerted by charge 4 q. Thus, charge Q should exert force F_B on charge q equal to F_A (in magnitude) and along CA. For this, charge Q has to be **positive** (*i.e* of the nature, same as that of 4 q or q).

Now, force on charge q due to charge Q,

$$F_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q.q}{(BC)^2}$$

or $$F_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q.q}{(x/2)^2} \qquad \text{(along CA)}$$

For net force on charge q to be zero, $F_B = F_A$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q.q}{(x/2)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q.q}{(x/2)^2}$$

or $$Q = 4q$$

Problem 1.10. Two pith-balls each weighing 10^{-3} kg are suspended from the same point by means of silk threads 0·5 long. On charging the pith-balls equally, they are found to repel each other to a distance of 0·2 m. Calculate the charge on each ball. (*H.S.S.C.E. 2002*)

Sol. Consider two pith-balls A and B each having charge q and mass 10^{-3} kg.

When the pith-balls are suspended from point S by two threads each 0 · 5 m long, they repel each other to distance AB = 0·2 m as shown in Fig. 1.15.

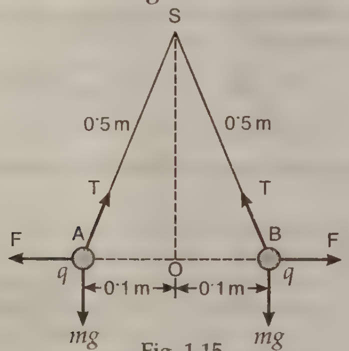


Fig. 1.15

Each of the two pith-balls is in equilibrium under action of the following three forces :

(*i*) The electrostatic repulsive force F.

(*ii*) The weight mg acting vertically downwards.

(*iii*) The tension T in the string directed towards point S.

The three forces mg, F and T can be represented by the three sides, SO, OA and AS of the Δ AOS taken in order. Therefore, according to triangle law of forces,

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS} \qquad \text{...}(i)$$

Here, $F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{(AB)^2} = 9\times10^9\times\frac{q^2}{(0\cdot2)^2}$ N,

$mg = 10^{-3}\times9\cdot8 = 9\cdot8\times10^{-3}$ N

From equation (*i*), we have

$$F = mg\times\frac{OA}{SO}$$

or $$9\times10^9\times\frac{q^2}{(0\cdot2)^2} = 9\cdot8\times10^{-3}\times\frac{0\cdot1}{\sqrt{(0\cdot5)^2-(0\cdot1)}}$$

or $$q = 2\cdot357\times10^{-6}\text{ C}$$

Problem 1.11. Two small spheres each having ma m kg and charge q coulomb are suspended from a point insulating threads each l metre long but of negligible ma If θ is the angle, each string makes with the vertical wh equilibrium has been attained, show that

$$q^2 = (4\,m\,g\,l^2\sin^2\theta\tan\theta)\,4\pi\varepsilon_0 \qquad \text{(P.S.S.C.E. 199}$$

Sol. Consider two small spheres A and B each of ma m kg and charge q coulomb. When the two spheres a suspended from point S by two threads each of length l, th repel each other and when equilibrium is attained, each stri makes an angle θ with the vertical [Fig. 1.16].

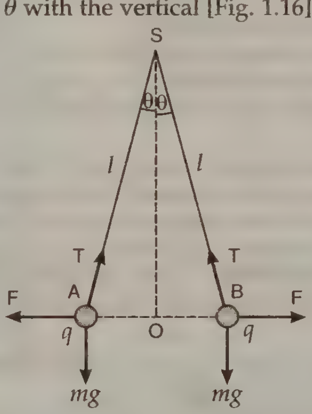


Fig. 1.16

Each of the two spheres is acted upon by the followi three forces :

(*i*) The electrostatic force of repulsion F directed aw from each other.

(*ii*) The weight mg of the sphere acting vertica downwards.

(*iii*) The tension T in the string directed towards poin

Since the two spheres are in equilibrium, the three for acting on a sphere can be represented by the three sides of Δ AOS taken in order. For sphere A, we have

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS}$$

Here, OA = $l\sin\theta$; SO = $l\cos\theta$

and AB = 2 AO = $2\,l\sin\theta$

and $$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times q}{AB^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{4l^2\sin^2\theta}$$

From equation (*i*), we have

$$F = mg\times\frac{OA}{SO}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{4l^2\sin\theta} = mg\times\frac{l\sin\theta}{l\cos\theta}$$

or $$q^2 = (4\,m\,g\,l^2\sin^2\theta\tan\theta)\,4\pi\varepsilon_0$$

Type C. On Superposition Principle

Problem 1.12. Three charges 10 μC, 5 μC and – 5 μC a plaed in air at the three corners A, B and C of an equilate triangle of side 0 · 1 m. Find the resultant force experienc by charge placed at corner A.

Sol. Here, $q_A = 10\ \mu C = 10^{-5}\ C$; $q_B = 5 \times 10^{-6}\ C$;
$q_C = -5 \times 10^{-6}\ C$ and $AB = BC = AC = 0 \cdot 1$ m

$$\text{Now, } F_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A q_B}{AB^2} = \frac{9 \times 10^9 \times 10^{-5} \times 5 \times 10^{-6}}{(0 \cdot 1)^2}$$

$$\text{or} \quad F_{AB} = 45\ N \quad \text{(repulsive)}$$

$$\text{Also, } F_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A q_C}{AC^2} = \frac{9 \times 10^9 \times 10^{-5} \times 5 \times 10^{-6}}{(0 \cdot 1)^2}$$

$$\text{or} \quad F_{AC} = 45\ N \quad \text{(attractive)}$$

The forces F_{AB} and F_{AC} are inclined at an angle of 120° as shown in Fig. 1.17.

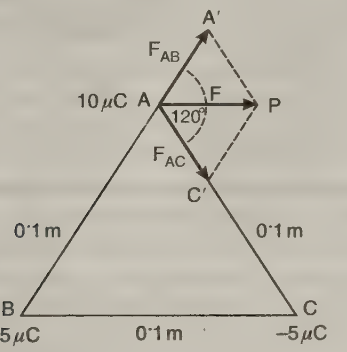


Fig. 1.17

If F is the resultant force on charge q_A, then

$$F = \sqrt{F_{AB}^2 + F_{AC}^2 + 2\, F_{AB} \cdot F_{AC} \cos 120°}$$
$$= \sqrt{45^2 + 45^2 + 2 \times 45 \times 45 \times (-0 \cdot 5)}$$
$$= 45\sqrt{1 + 1 - 1} = \mathbf{45\ N}$$

The resultant force acts on charge 10 μC charge along AP *i.e.* **parallel to side BC of the Δ ABC.**

Problem 1.13. ABC is an equilateral triangle of side 10 m and D is the middle point of BC. Charges of +100, – 100 and + 75 coulomb are placed at B, C and D respectively. What is the force experienced by 1 coulomb positive charge placed at A ?

Sol. Consider an equilateral Δ ABC of each side 10 m. The point D is middle point of side BC. The charges $q_B = +100$ C, $q_C = -100$ C and $q_D = +75$ C are placed at the points B, C and D and a charge $q_A = 1$ C is placed at point A [Fig. 1.18].

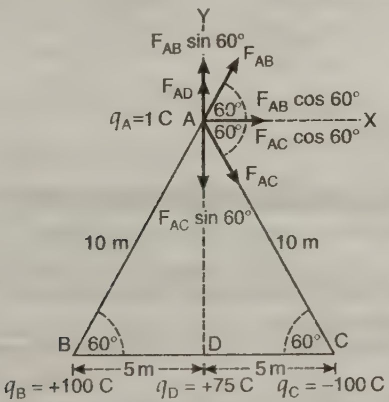


Fig. 1.18

Due to the charges q_B, q_C and q_D, the charge q_A experiences forces F_{AB}, F_{AC} and F_{AD} in the directions as shown in the figure.

$$\text{Now, } F_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A q_B}{AB^2} = 9 \times 10^9 \times \frac{1 \times 100}{10^2}$$
$$= 9 \times 10^9\ N \quad \text{(along BA)}$$

$$F_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A q_C}{AC^2} = 9 \times 10^9 \times \frac{1 \times 100}{10^2}$$
$$= 9 \times 10^9\ N \quad \text{(along AC)}$$

$$F_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A q_D}{AC^2} = 9 \times 10^9 \times \frac{1 \times 75}{(10^2 - 5^2)}$$
$$= 9 \times 10^9\ N \quad \text{(along DA)}$$

To find the resultant force on charge q_A, through point A, draw a line AX parallel to side BC as X-axis. Let us regard the line DA, the right bisector of line BC as Y-axis.

Resolve both the forces F_{AB} and F_{AC} into components along X-axis and Y-axis as shown in Fig. 1.18. Then, total force on charge q_A along X-axis,

$$F_x = F_{AB} \cos 60° + F_{AC} \cos 60°$$
$$= 9 \times 10^9 \times 0 \cdot 5 + 9 \times 10^9 \times 0 \cdot 5 = 9 \times 10^9\ N$$

The components $F_{AB} \sin 60°$ and $F_{AC} \sin 60°$ being equal and opposite, just cancel each other. Therefore,

force on charge q_A along Y-axis, ,

$$F_y = F_{AD} = 9 \times 10^9\ N$$

The two forces F_x and F_y acting on the charge q_A have been represented separately in Fig. 1.19.

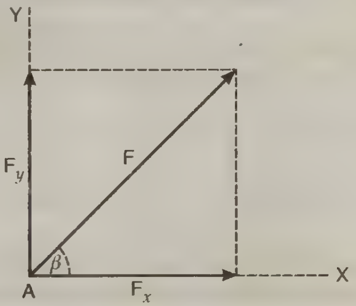


Fig. 1.19

If F is the resultant force on charge q_A, then

$$F = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{(9 \times 10^9)^2 + (9 \times 10^9)^2}$$
$$= \mathbf{9\sqrt{2} \times 10^9\ N}$$

If the resultant force makes an angle β with X-axis, then

$$\tan\beta = \frac{F_y}{F_x} = \frac{9 \times 10^9}{9 \times 10^9} = 1 \quad \text{or} \quad \boldsymbol{\beta = 45°}$$

i.e. the resultant force on charge q_A is inclined at 45° with X-axis *i.e.* with the side BC of the triangle.

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 1.14. Two charges each of + Q units are placed along a line. A third charge – *q* is placed between them. At what position and for what value of *q*, will the system be in equilibrium ?

Sol. Let the two charges + Q each be placed at points A and B at a distance *r* between them and charge – *q* be placed at point O, such that AO = x and OB = $r-x$ [Fig. 1.20].

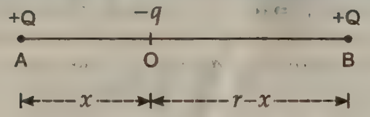


Fig. 1.20

The system will be in equilibrium, if charge $-q$ as well as either of the two charges $+Q$ each placed at points A and B experience *zero* net force.

(i) For charge − q to be in equilibrium : For this, force on charge $-q$ at point O due to charge $+Q$ at point A should be equal and opposite to that due to charge $+Q$ at point B *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{x^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(r-x)^2}$$

$$\text{or}\quad x^2=(r-x)^2\quad\text{or}\quad x=\pm(r-x)\quad\text{or}\quad x=\frac{r}{2}$$

(ii) For charge + Q at A (or at B) to be in equilibrium : For this, force on charge $+Q$ at point A due to charge $-q$ at O should be equal and opposite to that due to charge $+Q$ at B *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(r/2)^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r^2}\quad\text{or}\quad q=\frac{Q}{4}$$

Therefore, the system of three charges will be in equilibrium, if charge $-q$ is equal to $-\frac{Q}{4}$ and is placed at the **mid-point** of the distance between the two charges of $+Q$ each.

Problem 1.15. Two identical point charges of charge Q are kept at a distance *r* from each other. A third point charge is placed on the line joining the above two charges, such that all the three charges are in equilibrium. Calculate the magnitude and location of the third charge.

(C.B.S.E. 1998)

Ans. Let the two charges of $+Q$ each be placed at points A and B at a distance r apart as shown in Fig. 1.21.

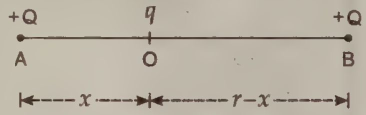


Fig. 1.21

Suppose that the third charge q (in magnitude) is placed at point O on the line joining the other two charges, such that AO = x and OB = $r-x$. For the system of the three charges to be in equilibrium, net force on each of the three charges must be zero.

At first instance, if we assume that charge q placed a point O is positive in nature, then it will experience forces du to other two charges in opposite directions and hence ne forces on the charge q may be zero. But as such (when q i taken positive), the force on the charge Q at point A or at poin B will not be zero. It is because, the forces on a charge Q du to the other two charges will act in same direction. Howeve if the charge q placed at point O is taken negative, then on charge Q, forces due to the other two charges will act i opposite directions. Hence, the third charge q placed at poin O must be **negative** in nature.

(i) For charge − q to be in equilibrium : For this, force o charge $-q$ at point O due to charge $+Q$ at point A should b equal and opposite to that due to charge $+Q$ at point B *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{x^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(r-x)^2}$$

$$\text{or}\quad x^2=(r-x)\quad\text{or}\quad x=\pm(r-x)\quad\text{or}\quad x=\frac{r}{2}$$

(ii) For charge + Q at A (or at B) to be in equilibrium : Fo this, force on charge $+Q$ at point A due to charge $-q$ at O shoul be equal and opposite to that due to charge $+Q$ at B *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(r/2)^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{r^2}\quad\text{or}\quad q=\frac{Q}{4}$$

Therefore, the system of three charges will be i equilibrium, if charge $-q$ is equal to $-\frac{Q}{4}$ and is placed at th **mid-point** of the distance between the two charges of + each.

Problem 1.16. A charge having magnitude Q is divide into two parts *q* and (Q − *q*). If the two parts exert maximum force of repulsion on each other, then find th ratio Q/*q*.

Sol. Suppose that the parts q and $(Q-q)$ are placed at distance r apart. Then,

$$F=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}$$

Now, for force F to be maximum, $\frac{\partial F}{\partial q}=0$

$$i.e.\qquad \frac{\partial}{\partial q}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}\right)=0$$

$$\text{or}\qquad \frac{\partial}{\partial q}[q(Q-q)]=0$$

$$\text{or}\quad 1(Q-q)+q(0-1)=0\quad\text{or}\quad Q-2q=0\quad\text{or}\quad \frac{Q}{q}=2$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What kind of charges are produced on each, when (*i*) a glass rod is rubbed with silk and (*ii*) an ebonite rod is rubbed with wool ? *(C.B.S.E. 1990)*

Ans. (*i*) On glass rod : positive charge and
on silk : negative charge
(*ii*) On ebonite rod : negative charge and
on wool : positive charge.

Q. 1.02. An ebonite rod held in hand can be charged by rubbing with flannel but a copper rod cannot be charged like this. Why ? *(H.P.S.S.C.E. 1997)*

Ans. Both the human body and the copper rod conduct electricity. When it is attempted to charge a metal rod by rubbing, the charge flows from the rod to the earth through the hand. However, when ebonite rod is charged by rubbin the charges so produced stay on the ebonite rod, as it is ba conductor of electricity.

Q. 1.03. What does $q_1+q_2=0$ signify in electrostatics ? *(C.B.S.E. 2001 S*

Ans. The charges q_1 and q_2 are equal and opposite.

Q. 1.04. When a polythene piece is rubbed with woo it acquires negative charge. Is there a transfer of mass fro wool to polythene ?

Ans. The polythene piece acquires negative charge du to transfer of electrons from wool to it. Since electrons a material particles, there is a transfer of mass from wool t polythene.

Q. 1.05. A glass rod, when rubbed with silk cloth, acquires a charge $1 \cdot 6 \times 10^{-13}$ coulomb. What is the charge on the silk cloth ? *(H.P.S.S.C.E. 1999 ; H.S.S.C.E. 1999)*

Ans. Silk cloth will also acquire a charge $1 \cdot 6 \times 10^{-13}$ coulomb. However, it will be negative in nature.

Q. 1.06. How much is the charge on an electron in SI units ? *(H.S.S.C.E. 1996)*

Ans. In SI, charge on electron is $1 \cdot 6 \times 10^{-19}$ C (negative).

Q. 1.07. What is the least possible value of charge ? *(H.S.S.C.E. 2002)*

Ans. A positively charged particle can possess positive charge equal to the charge on a proton ($+ 1 \cdot 6 \times 10^{-19}$ C) and a negatively charged particle equal to the charge on an electron ($- 1 \cdot 6 \times 10^{-19}$ C). A particle can not have a fractional part of this elementary amount of charge *i.e.* $1 \cdot 6 \times 10^{-19}$ C.

Q. 1.08. Can a body have a charge of $0 \cdot 8 \times 10^{-19}$ C ? Justify your answer by comment ? *(H.P.S.S.C.E. 1999 S)*

Ans. No. For details, refer to VSQ 1.07.

Q. 1.09. What is meant by quantization of charge ? *(P.S.S.C.E. 2000 ; H.S.S.C.E. 1999 S)*

Ans. Refer to section 1.06.

Q. 1.10. Name the experiment, which established quantum nature of electric charge. *(C.B.S.E. 1998)*

Ans. Millikan's oil drop experiment for measuring charge on an electron.

Q. 1.11. State two properties of electric charge. *(P.S.S.C.E. 1998 S)*

Ans. 1. Electric charge is quantized. **2.** Electric charge is always conserved in a physical process.

Q. 1.12. Given two point charges q_1 and q_2, such that $q_1 q_2 < 0$. What is the nature of force between them ? *(H.P.S.S.C.E. 2002)*

Ans. When $q_1 q_2 < 0$, it implies that if one charge is positive, the other must be negative. Hence, when $q_1 q_2 < 0$, the force between the two charges will be attractive in nature.

Q. 1.13. State Coulomb's law in electrostatics. *(C.B.S.E. 1994 ; I.S.C.E. 1993)*

Ans. Refer to section 1.09.

Q. 1.14. Give mathematical expression for Coulomb's law for electric charges. *(H.S.S.C.E. 2002)*

Ans. $F = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r^2}$

Q. 1.15. Give the unit of electrical permittivity of free space. *(H.S.S.C.E. 2002)*

Ans. $C^2 N^{-1} m^{-2}$.

Q. 1.16. Write down the value of absolute permittivity of free space (vacuum). *(P.S.S.C.E. 1996)*

Ans. $\varepsilon_0 = 8 \cdot 854 \times 10^{-12} C^2 N^{-1} m^{-2}$.

Q. 1.17. What is the value of constant of proportionality $\dfrac{1}{4\pi\varepsilon_0}$ involved in expression for Coulomb's force between two charges ?

Ans.
$$\frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8 \cdot 854 \times 10^{-12} C^2 N^{-1} m^{-2}}$$
$$= 9 \times 10^9 \text{ N m}^2 \text{C}^{-2}$$

Q. 1.18. In the relation $F = k \dfrac{q_1 q_2}{r^2}$, what is the value of k in free space ? *(H.S.S.C.E. 2002)*

Ans. In SI, the value of constant k is 9×10^9 N m² C⁻².

Q. 1.19. Electrostatic force between two charges is called central force. Why ?

Ans. The force between two electric charges always acts along the line joining the two charges. For this reason, it is called central force.

Q. 1.20. Define coulomb as a unit of charge. *(P.S.S.C.E. 2002 ; H.P.S.S.C.E. 1999)*

Or

Define the unit of electric charge. *(P.S.S.C.E. 2000, 1999)*

Ans. Refer to section 1.10.

Q. 1.21. In Coulomb's law, on what factors the value of electrostatic force constant k depends ? *(H.P.S.S.C.E. 2002)*

Ans. On the system of units adopted and the nature of medium, in which two charges are placed.

Q. 1.22. Define dielectric constant of a medium. *(P.S.S.C.E. 2000, 1999 ; H.S.S.C.E. 1995)*

Ans. Refer to section 1.11.

Q. 1.23. Dielectric constant of water is 80. What is its permittivity. *(H.S.S.C.E. 1997 S)*

Ans. Since dielectric constant and relative permittivity of a medium represent the same quantity, it implies that the relative permittivity (ε_r) of water is 80. The absolute permittivity (ε) of water will be $\varepsilon_0 \varepsilon_r$ *i.e.* $8 \cdot 854 \times 10^{-12} \times 80$ $C^2 N^{-1} m^{-2}$.

Q. 1.24. Force between two point electric charges kept at a distance d apart in air is F. If these charges are kept at the same distance in water, how does the force between them change ? *(C.B.S.E. 1997)*

Ans. It will become $\dfrac{F}{K}$, where K is dielectric constant of water.

Q. 1.25. Calculate the Coulomb's force between two α-particles separated by a distance of $3{\cdot}2 \times 10^{-15}$ m. *(C.B.S.E. 1992)*

Ans. Here, $r = 3 \cdot 2 \times 10^{-15}$ m ;

charge on α-particles, $q_1 = q_2 = 2 \times 1 \cdot 6 \times 10^{-19}$ C

$$\text{Now,} \quad F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$
$$= 9 \times 10^9 \times \frac{(2 \times 1{\cdot}6 \times 10^{-19})^2}{(3{\cdot}2 \times 10^{-15})^2} = \mathbf{90\ N}$$

Q. 1.26. Calculate the Coulomb's force between a proton and electron separated by $0 \cdot 8 \times 10^{-15}$ m. *(P.S.S.C.E. 1998 S ; C.B.S.E. 1992)*

Ans. Proceed as in VSQ 1.25.

Q. 1.27. If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them ? *(I.S.C.E. 1995)*

Ans. Originally, force between the two charges,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2}$$

When the individual charges and the distance between the charges are doubled, the force is given by

$$F' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q \times 2q}{(2r)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \cdot q}{r^2} = F$$

i.e. force will remain the same.

Q. 1.28. State the principle of superposition of forces in electrostatics. *(H.S.S.C.E. 2001)*

Ans. Refer to section 1.14.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. A comb run through one's dry hair attracts small bits of paper. Why ? What happens, if the hair is wet or if it is a rainy day ? *(Text Question)*

Ans. When a comb is run through dry hair, it gets charged. As such, it starts attracting small bits of paper. If the hair is wet or it is a rainy day, then in absence of friction, the comb will not get charged when run through hair. In absence of charge on the comb, the bits of paper will not be attracted.

Q. 1.02. Ordinary rubber is an insulator. But the special rubber tyres of aircrafts are made slightly conducting. Why is this necessary ? *(Text Question)*

Ans. During landing, the tyres of a space-craft get charged due to friction between tyres and ground. In case, the tyres are slightly conducting, the charge developed on the tyres will not stay on them and it will find its way (leak) to the earth.

Q. 1.03. Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why ?

(P.S.S.C.E. 1999 ; H.P.S.S.C.E. 1998 ; Text Question)

Ans. Due to air friction, body of the vehicle also gets charged in addition to charge developed on tyres. In case, charge accumulation becomes large, it may prove hazardous to the vehicle carrying inflammable materials. So that it does not happen, metallic ropes from the vehicle are mde to touch the ground. Due to this, the charge on the vehicle leak to earth.

Q. 1.04. When you touch a metal object like a door handle on a dry winter day, you are likely to see a spark and feel a definite shock. We usually explain this by saying that we have built up a static charge. Why does this not happen on a humid day in the summer ?

Ans. When we build up a sizeable static charge and touch a metal object (door handle), the transfer of charge is in the form of a spark.

The static charge can build up on a substance, when it is well insulated from its surroundings. On a humid day in summer, the static charge can not build up. In fact, the charge leaks away as fast as it builds up. As on a humid day in the summer, we do not have static charge built on us ; no spark occurs, when we touch a metal object.

Q. 1.05. How many electrons are present in one coulomb of charge ? *(P.S.S.C.E. 1999 ; H.P.S.S.C.E. 1992)*

Ans. Let there be n electrons in one coulomb. From quantization of charge,

$$q = n\,e$$

$$\therefore \qquad n = \frac{q}{e} = \frac{1}{1 \cdot 6 \times 10^{-19}} = \mathbf{6 \cdot 25 \times 10^{18}}$$

Q. 1.06. Suppose we have a large number of identical particles, very small in size. Any of them at 10 cm separation repel with a force of 3×10^{-10} N.

(*a*) If one of them is at 10 cm from a group of *n* others, how strongly do you expect it to be repelled ?

(*b*) Suppose you measure the repulsion and find it 6×10^{-6} N ; how many particles were there in the group ?

Ans. **(*a*)** The group of n small charged particles must behave as a single point charge, so that it can have a separation of 10 cm from the one particle in question. Obviously, force on this particle due to the group of n particles is n times the force due to a single particle. Hence, force due to grou of n particles,

$$F = n \times 3 \times 10^{-10}\ \text{N}$$

(*b*) Here, $\quad F = 6 \times 10^{-6}$ N

$$\therefore \qquad n = \frac{F}{3 \times 10^{-10}} = \frac{6 \times 10^{-6}}{3 \times 10^{-10}} = \mathbf{2 \times 10^4}$$

Q. 1.07. Why can one ignore quantization of electri charge, when dealing with macroscopic *i.e.* large scal charges ? *(Text Question*

Ans. In practice, the charge on a charged body is ver large. On the other hand, the charge on an electron is ver small. When electrons are added to a body (negativel charged body) or removed from a body (positively charge body), the change taking place in the total charge on the bod is so small that the charge seems to be varying in a continuou manner. Therefore, quantization of electric charge can b ignored, when dealing with a large scale charged body.

Q. 1.08. Force of attraction between two point electri charges placed at a distance *d* in a medium is F. Wha distance apart should these be kept in the same mediun so that force between them becomes F / 3 ?

(C.B.S.E. 2001 S, 1998

Ans. Let q_1 and q_2 be the two point charges. Then, forc between the charges, when kept at a distance d apart,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{d^2}$$

Suppose that force between the two charges become F/3, when the charges are kept at a distance x apart. Then,

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{x^2} = \frac{F}{3} \quad \text{or} \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{x^2} = \frac{1}{3}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{d^2}\right.$$

$$\text{or} \qquad x = \sqrt{3}\, d$$

Q. 1.09. Bring out a comparison between the nature o electrostatic and gravitational forces.

Ans. The gravitational force between two bodies i because of their masses, while the electrostatic force is du to charge on them.

Points of similarities. 1. Both the forces are central i nature.

2. Both the forces obey inverse square law.

3. Both the forces are conservative in nature *i.e.* wor done in moving a body (mass in case of gravitational forc and charge in case of electrostatic force) between any tw points does not depend upon the path followed.

4. Both the forces are long range forces.

Points of dissimilarities. 1. Whereas the electrostati force may be attractive or repulsive, the gravitational forc is always attractive in nature.

2. The electrostatic force between two charged bodies i greatly affected by the medium, in which the two bodies ar located. There is no effect of medium on the gravitationa force.

3. Electrostatic force is very strong as compared t gravitational force. It can be easily shown that electrostati force between an electron and a proton is about 10^{38} times a large as the gravitational force between them.

Q. 1.10. What is the importance of Coulomb's law i vector form ? *(H.P.S.S.C.E. 1998 S*

Ans. From Coulomb's law in vector form, it directly follows that the forces exerted by two charges on each other are equal and opposite. Further, the force, which one charge exerts on the other, acts along the line joining the two charges *i.e.* electrostatic force is a central force.

Q. 1.11. Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\ \mu C$ and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on charge of 1 μC placed at the centre of the square ? *(Text Question)*

Ans. Consider the square ABCD of each side 10 cm and centre O. The charge of 1 μC is placed at the point O. Obviously, OA = OB = OC = OD [Fig. 1.22].

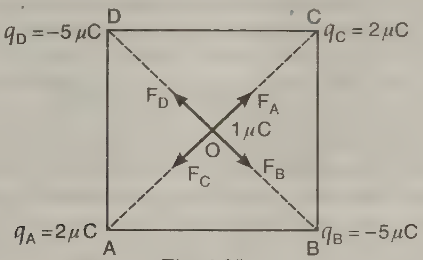


Fig. 1.22

Since $q_A = q_C$, the charge of 1 μC experiences equal and opposite forces due to charges q_A and q_C. Similarly, the charge of 1 μC experiences equal and opposite forces F_B and F_D due to charges q_B and q_D. Hence, the net force on charge of 1 μC due to the given arrangement of charges is zero.

Q. 1.12. State the principle of superposition. What is its use ?

Ans. The principle of superposition states that *when a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges.*

It is used to find force on a charge, when a group of charges are interacting.

Q. 1.13. Two identical point charges Q are kept at a distance *r* from each other. A third point charge *q* is placed on the line joining the above two charges, such that all the three charges are in equilibrium. What is the magnitude, sign and position of the third charge ? *(C.B.S.E. 1994)*

Ans. The charge q should be equal to $-\dfrac{Q}{4}$.

For detailed solution, refer to solved problem 1.14 or 1.15.

Q. 1.14. A charge *q* is placed at the centre of line joining two equal charges Q. Show that the system of three charges will be in equilibrium, if $q = -\dfrac{Q}{4}$. *(H.P.S.S.C.E. 2002)*

Ans. Refer to solved numerical problem 1.14 or 1.15.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulomb and the other an equal negative charge by friction. Are their masses after charging equal ? *(I.I.T. 1983)*

Ans. When two bodies are rubbed together, electrons get removed from one body and these electrons so removed get transferred to the other body. The body from which electrons get removed becomes positively charged and the body to which electrons get transferred becomes negatively charged. Since an electron is a material particle, the mass of the positively charged body decreases and that of the negatively charged body increases.

Hence, when one of the two identical metallic spheres is charged positive and the other negative, their masses after charging will **not be equal.**

Q. 1.02. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain, how this observation is consistent with the law of conservation of charge. *(Text Question)*

Ans. Before rubbing, both the glass rod and the silk cloth are electrically neutral. In other words, net charge on the glass rod and the silk cloth is zero. When the glass rod is rubbed with silk cloth, a few electrons from glass rod get transferred to the silk cloth. As a result, the glass rod becomes positively charged and the silk cloth negatively charged. Since the magnitude of the positive charge on the glass is same as that of negative charge on the silk cloth, the net charge on the system is still zero. Thus, the appearance of charges on glass rod and silk cloth is in accordance with the law of conservation of charge.

Q. 1.03. In filling the gasoline tank of an aeroplane, the metal nozzle of the hose from the gasoline truck is always carefully connected to the metal of the aeroplane by a wire, before the nozzle is inserted in the tank. Explain, why.

Ans. Since the aeroplane and the gasoline truck usually have wheels with rubber tyres, they are insulated from the ground. Further, the service ramps are usually made of concrete and are not necessarily good conductors to the earth. Therefore, inspite of grounding metallic ropes, the aeroplane and the truck could remain charged. A spark could jump and ignite the explosive gasoline, when the metal nozzle was brought near the aeroplane. The connection of metal of the aeroplane and the nozzle of the hose with a wire avoids any unbalance of charge and hence the risk of gasoline catching fire due to the spark.

Also, the stream of gasoline flowing out of the nozzle may become charged by friction. The charged gasoline would go into the aeroplane tank and charge it oppositely to the nozzle. Unless a good conducting path is provided so that the charges on the aeroplane and nozzle could recombine, a spark could result with disastrous consequences.

Q. 1.04. (*a*) If you accumulate a static charge and then touch a wooden frame of a door, you often find no spark or shock, although there would be, you touched the metal handle. Why ?

(*b*) Sometimes, if you touch the wooden frame and then touch the metal handle, there is no spark or shock in either case, although there would have been, if you had touched the handle first. Suggest an explanation.

Ans. (*a*) Metals are good conductors. When we are carrying a static charge and touch the metal handle of the door, the charge transfers to it immediately in the form of a spark and distributes itself quickly throughout the conductor.

When the wooden frame (insulator) of the door is touched, the charge is unable to flow through. As such, only a little charge may get transferred and there is no spark.

(*b*) When the wooden frame of the door is touched, it does not accept charge fast enough to cause a spark. However, if the contact lasts long, the charge may slowly flow away from us. When we subsequently touch the metal handle, there is no spark. It is because, there is no charge left on us, which may transfer to the handle and may cause a spark.

Q. 1.05. 'Automobile ignition failure occurs in damp weather.' Explain, why.

Ans. The insulating porcelain of the spark plugs accumulates a film of dirt. The surface dirt is hygroscopic and picks up moisture from air. Therefore, in humid weather, the insulating porcelain of the plugs becomes quasi-conductor. This allows an appreciable proportion of the spark to leak across the surface of the plug instead of discharging across the gap.

Q. 1.06. It is now believed that protons and neutrons are themselves built out of more elementary units called quarks. A proton and neutron consist of three quarks each. Two types of quarks, so called 'up quark' denoted by *u* of charge + 2 *e*/3 and the 'down quark' denoted by *d* of charge – *e*/3, together with electrons build up ordinary matter. Suggest a possible quark composition of a proton and neutron. *(Text Question)*

Ans. The charge on up quark (u) = $+2\,e/3$

and the charge on down quark (d) = $-e/3$

The charge on the proton is + *e* and it is made of three quarks. Therefore, the possible quark composition of proton is ***u u d***.

On the other hand, neutron is a neutral particle but it is also made of three quarks. For this, the possible composition of neutron is ***u d d***.

Q. 1.07. A glass rod rubbed with silk is brought close to two uncharged metallic spheres in contact with each other inducing charges on them as shown in Fig. 1.23. Describe what happens, when (*i*) the spheres are slightly separated, (*ii*) the glass rod is subsequently removed and finally (*iii*) the spheres are separated far apart. *(Text Question)*

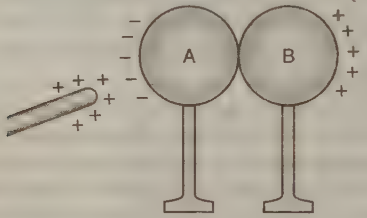


Fig. 1.23

Ans. (*i*) On separating the two spheres, the right face of sphere A will acquire positive induced charge and the left face of B will acquire negative induced charge.

(*ii*) If the glass rod is removed, the charges on the two spheres will disappear.

(*iii*) In this case also, charges on spheres will disappear.

Q. 1.08. Can a charged body attract another uncharged body ? Explain.

Or

Why does a charged glass rod attract a piece of paper ?

Ans. Yes, a charged body can attract another uncharged body. It is because, when the charged body is placed in front of the uncharged body, the induced charges of opposite kind are produced on the uncharged body. Due to this, the charged body attracts the uncharged body.

Q. 1.09. A positively charged glass rod attracts suspended pith-ball. Does it imply that the pith-b is negatively charged ?

Ans. The pith-ball can be uncharged also. The positive charged glass rod can attract the pith-ball due to induc charges of opposite kind produced on the pith-ball.

Q. 1.10. Can two balls having same kind of charge them attract each other ? Explain.

Ans. Yes, two balls having same kind of charge can attr each other, if one ball possesses charge quite large as compar to that on the other ball.

The reason is that when two charged balls are placed ne each other, they induce opposite kinds of charges on the fac of each other. Since on the ball having a small amount charge, quite a large amount of induced charge is produce two balls get attracted towards each other due to for between the large inducing charge of one ball and lar induced charge (opposite in nature) on the other ball.

Q. 1.11. The graph in Fig. 1.24 shows the electric for of repulsion on a tiny charged conducting sphere A a function of its separation from a large conducting sphe B. The sphere B has 10 times the charge on the sphere Explain the behaviour of the force between separations 2 and 1 cm.

Ans. When the spheres are at large separation, the for between the spheres varies as $\frac{1}{r^2}$ *i.e.* varies in the sar manner as between two point charges.

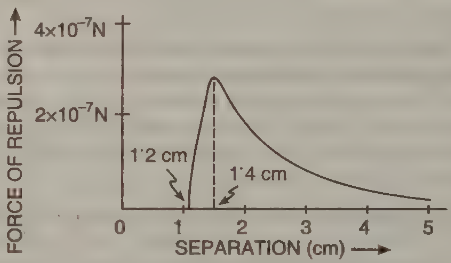


Fig. 1.24

As the spheres move closer, the charge on large sphe B is redistributed as shown in Fig. 1.25. As the spheres mo still more closely, the part of the large sphere B, which nearest to the small sphere A becomes negatively (opposite charged [Fig. 1.26] and it results in force of attraction betwe them.

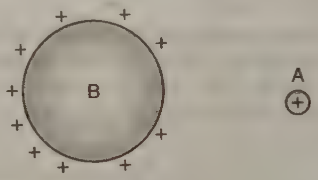


Fig. 1.25

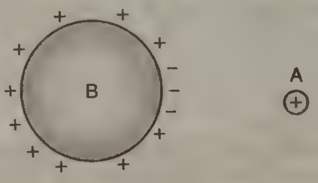


Fig. 1.26

Behaviour of the force between separations 2 cm and 1 cm : Refer to Fig. 1.24. At 2 cm, the force of repulsion between the spheres increases ; till at $1 \cdot 4$ cm, the force of repulsion becomes maximum. As the spheres move closer ($1 \cdot 2 \text{ cm} < r < 1 \cdot 4$ cm), the force of repulsion begins to fall due to the attractive force between the sphere A and the negative charge induced on sphere B. At $1 \cdot 2$ cm, the attractive force due to inductive effect equals the repulsion between the two positively charged spheres. As the spheres approach still more closely ($r < 1 \cdot 2$ cm), the net force between the spheres becomes strongly attractive.

Q. 1.12. Two large conducting spheres carrying charges q_1 and q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $\frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$, where r is the distance ?

Ans. The expression for electrostatic force between two charges q_1 and q_2 *i.e.*

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

holds, when charges q_1 and q_2 are point charges only. The expression will not be true in case of large conducting spheres. It is because, when large conducting spheres are brought together, charge distribution will not remain uniform.

Q. 1.13. Check that the ratio $k\,e^2/G\,m_e\,m_p$ is dimensionless. Look up a table of physical constants and determine the value of this ratio. What does the ratio signify ?

Ans. $$\left[\frac{k\,e^2}{G m_e m_p}\right] = \frac{[\text{N m}^2\,\text{C}^{-2}]\times[\text{C}]^2}{[\text{N m}^2\,\text{kg}^{-2}]\times[\text{kg}]\times[\text{kg}]} = \text{no unit}$$

Since the ration $k\,e^2/G\,m_e\,m_p$ has no unit, it is **dimensionless.**

Now, $k = 9 \times 10^9\,\text{N m}^2\text{C}^{-2}$; $G = 6 \cdot 67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$;

$e = 1 \cdot 6 \times 10^{-19}$ C ; $m_e = 9 \cdot 1 \times 10^{-31}$ kg

and $m_p = 1 \cdot 66 \times 10^{-27}$ kg

$$\therefore \frac{k\,e^2}{G\,m_e\,m_p} = \frac{9\times10^9\times(1{\cdot}6\times10^{-19})^2}{6{\cdot}67\times10^{-11}\times9{\cdot}1\times10^{-31}\times1{\cdot}66\times10^{-27}}$$

$$= 2 \cdot 287 \times 10^{39}$$

The factor $\frac{k\,e^2}{G\,m_e\,m_p}$ represents the ratio of electrostatic force to the gravitational force between an electron and proton.

Q. 1.14. Is Coulomb's law in electrostatics applicable in all situations ?

Or

State the limitations of coulomb's law in electro-statics.

(P.S.S.C.E. 1999)

Ans. No, Coulomb's law in electrostatics does not hold in all the situations. It is applicable only in the following situations :

1. The electric charges must be ***stationary.***
2. The electric charges must be ***points in size.*** Coulomb's law does not apply to two charged bodies of finite sizes, say two charged spheres. It is because, the distribution of charge does not remain uniform, when the two bodies are brought together.

Q. 1.15. A bird perches on a bare high-power line and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why ?

(Text Problem)

Ans. When a bird is perched on a bare high power line, the circuit does not get completed between the bird and the earth. Therefore, nothing happens to the bird.

When a man standing on ground touches the same line, the circuit between man and the earth gets completed. As a result, he gets a fatal shock.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain, what is meant by quantization of charge. *(H.P.S.S.C.E. 1996 ; H.S.S.C.E. 1994 ; C.B.S.E. 1992 ; P.S.S.C.E. 1991)*
2. Explain conservation of charge giving two examples. *(H.S.S.C.E. 2000, 1998 ; P.S.S.C.E. 1994, 1991)*
3. Explain the following : (*i*) Conservation of electric charge (*ii*) Quantization of electric charge. *(P.S.S.C.E. 2001)*
4. Give six properties of electric charge. *(P.S.S.C.E. 1999 S)*
5. State Coulomb's law and derive definition of a unit charge (coulomb). *(P.S.S.C.E. 1999 ; H.S.S.C.E. 1996)*
6. State Coulomb's law of electrostatic force between two charges and state their limitations. Also deduce unit charge from these laws. *(P.S.S.C.E. 2000)*
7. Write Coulomb's law in vector form. Derive the definition of one coulomb. *(H.S.S.C.E. 1992)*
8. Write the vector form of force acting between two charges q_1 and q_2 having $\vec{r}_1$ and $\vec{r}_2$ as their position vectors respectively. *(H.P.S.S.C.E. 2000)*
9. State Coulomb's law of electrostatic force and express this law in vector form. *(P.S.S.C.E. 2000)*
10. State Coulomb's law in vector form and prove that $\vec{F}_{21} = -\vec{F}_{12}$, where letters have their usual meanings. *(H.S.S.C.E. 1997)*
11. What is meant by continuous charge distribution ? Define linear charge density. Obtain expression for force on a charge q due to a continuous distribution of charges along a line.
12. Define surface charge density. Obtain expression for force on a charge q due to a continuous distribution of charges over a surface.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Explain the terms : (*i*) quantization of charge (*ii*) conservation of charge (*iii*) additive nature of charge.
2. (*a*) Define dielectric constant of a medium in terms of force between two electric charges. *(P.S.S.C.E. 1997 S ; C.B.S.E. 1997)*
   (*b*) Explain, what is meant by dielectric constant. Give some example. *(P.S.S.C.E. 1994)*
3. State Coulomb's law in electrostatics. Express the same in SI units. Mention two similarities and two dissimilarities between electrostatic and gravitational interactions. *(C.B.S.E. Sample Q. Paper)*

4. State Coulomb's law in electrostatics. Express it in vector form. What is importance of expressing it in vector form ?
5. State the principle of superposition and use it to obtain the expression for the total electric force exerted at a point charge due to an assembly of n discrete point charges. *(H.S.S.C.E. 2002)*
6. State principle of superposition of charges. Hence, expres the force on a charge q due to a discrete distribution of charges in terms of their position vectors.
7. Distinguish between linear, surface and volume charg density. Obtain expression for force on a charge q due t continuous distribution of charge over a volume.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Quantization of charge

1. Which is bigger, a coulomb or charge on an electron ? How many electronic charges form one coulomb of charge ? [**Ans.** One coulomb, $6{\cdot}25 \times 10^{18}$]
2. How many electrons must be removed from a conductor, so that it acquires a charge of 3·5 nC ?[**Ans.** $2{\cdot}1875 \times 10^{10}$]
3. What is the amount of charge possessed by 1 kg of electrons ? Given that mass of an electron is $9{\cdot}1 \times 10^{-31}$ kg. [**Ans.** $1{\cdot}76 \times 10^{11}$ C]
4. A polythene piece rubbed with wool is found to have a negative charge of $3{\cdot}6 \times 10^{-7}$ C. Calculate the number of electrons transferred from wool to polythene. If an electron has a mass $9{\cdot}1 \times 10^{-31}$ kg, find the mass transferred to polythene. [**Ans.** $2{\cdot}25 \times 10^{12}$; $2{\cdot}05 \times 10^{-18}$ kg]
5. Assume that each of copper has one free electron. Estimate the number of free electrons in 1 mg of copper. Given that atomic weight of copper = 63·5 and Avogadro number = $6{\cdot}02 \times 10^{23}$. What is the charge possessed by these free electrons ? [**Ans.** $9{\cdot}48 \times 10^{18}$; 1·517 C]
6. Estimate the number of free electrons in 1 g of water and the negative charge possessed by them. Given that Avogadro number = $6{\cdot}02 \times 10^{23}$ and molecular weight of water = 18. [**Ans.** $3{\cdot}344 \times 10^{23}$; $5{\cdot}35 \times 10^{4}$ C]

Type B. On Coulomb's law in electrostatics

7. Two equal charges placed in air separated by a distance 3 m repel each other with force 0· 1gf. Calculate the magnitude of either of the charges. *(P.S.S.C.E. 1999)* [**Ans.** $9{\cdot}9 \times 10^{-7}$ C]
8. Two electrons have been removed from each atom. Find the distance between two such atoms, if they repel each other with a force of $8{\cdot}8 \times 10^{-9}$ N, when placed in free space. *(P.S.S.C.E. 2002)* [**Ans.** $3{\cdot}24 \times 10^{-10}$ m]
9. How far apart should the two electrons be, if the force each exerts on the other is equal to the weight of the electron ? Given that $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ SI unit, $e = 1{\cdot}6 \times 10^{-19}$ C and $m_e = 9{\cdot}1 \times 10^{-31}$ kg. *(H.S.S.C.E. 2002)* [**Ans.** 5·08 m]
10. Calculate the magnitude of the electrostatic force exerted by the proton on the electron in the hydrogen atom and compare it with the weight of the electron. Given that mass of electron is $9{\cdot}1 \times 10^{-31}$ kg, charge on electron is $1{\cdot}6 \times 10^{-19}$ C and radius of hydrogen atom is 0·53 Å. [**Ans.** $8{\cdot}2 \times 10^{-8}$ N ; $9{\cdot}2 \times 10^{21}$]
11. Two identical balls each have a mass of 10 g. What charges should these balls be given so that their interaction equalizes the force of universal gravitation acting between them ? The radii of the balls may be ignored in comparison to distance between them. [**Ans.** $8{\cdot}56 \times 10^{-13}$ C]
12. What equal charges would have to be placed on earth and moon to neutralize their gravitational force of attraction ? Given that mass of earth $=10^{25}$ kg and mass of moon $= 10^{23}$ kg. [**Ans.** $8{\cdot}56 \times 10^{13}$ C]
13. Calculate the ratio of electrostatic to gravitational force between two electrons placed at certain distance in air. Given that $m_e = 9{\cdot}1 \times 10^{-31}$ kg, $e = 1{\cdot}6 \times 10^{-19}$ C and $G = 6{\cdot}6 \times 10^{-11}$ N m² kg⁻². [**Ans.** $4{\cdot}2 \times 10^{42}$]
14. A pith-ball A of mass 9×10^{-5} kg carries a charge of + 5 μ What must be the magnitude and sign of the charge on pith-ball B held 2 cm directly above the pith-ball A suc that the pith-ball A remains stationary ? [**Ans.** $7{\cdot}84 \times 10^{-12}$ C (negative
15. Two point charges q_1 and q_2 are 3 m apart and the combined charge is 20 μC. If one repels the other with force of 0·075 N, what are the two charges ? *(H.S.S.C.E. 2001 ; P.S.S.C.E. 1991)* [**Ans.** 15 μC, 5 μ
16. The force between two equal charges placed in a mediu at a distance of 9 cm from each other is 16 dyne. O increasing one of the charges by 56 statC, it is found th the distance between the charges must be changed by 3 c in order to keep the force between them the sam Calculate the magnitude of the charges and the dielectr constant of the medium. Given, 1 C = 3×10^{9} statC. [**Ans.** 72 statC ;
17. The distance between two equal balls having unlik charges is 2 cm. The radii of the balls are much less tha the distance between them. The balls attract each oth with a force of 36×10^{-5} N. After, the balls have bee connected by a wire and the latter has been removed, th balls repel each other with a force of $20{\cdot}25 \times 10^{-5}$ Determine the original charges on the balls. [**Ans.** 8×10^{-9} C, -2×10^{-9}
18. Two charged particles having charge 5 μC each are joine by an insulating string of length 1 m and the system placed on a smooth horizontal table. Find the tension i the string. [**Ans.** 0·225 N
19. Two small spheres each of mass 10 mg are suspended fro a point by threads 0·5 m long. They are equally charge and repel each other to a distance of 0·28 m. If $g = 10$ m s what is the charge on each ? *(Karnataka, 199* [Ans. $1{\cdot}594 \times 10^{-8}$
20. Two small spheres each of mass 10^{-6} kg are suspende from a point by silk threads 50 cm long. These are equall charged and repel each other to a distance of 20 cm apa Calculate the charge on each sphere. Take $g = 10$ m s^{-2} *(P.S.S.C.E. 1998 S)* [**Ans.** $3{\cdot}01 \times 10^{-9}$
21. Two pith-balls each of mass 5×10^{-4} kg are suspended fro the same point by silk threads 0·2 m long. Equal charg are given to the balls, which separate, until the threa enclose an angle of 30°. Calculate the charge on each pit ball. [**Ans.** $3{\cdot}954 \times 10^{-8}$
22. Two similar small metallic spheres are suspended by equ threads from the same point. When either sphere is give a similar charge of 36 stat C, the distance between them 6 cm. If the length of each thread is 5 cm, find the mass each sphere and the tension in each thread. [**Ans.** 0·049 g ; 60 dyn
23. Two equal small spheres each weighing 1 g are hung b equal silk threads attached to the same point. The spher are charged when in contact and come to rest with the centres 2 cm apart and 20 cm vertically below the point support. Find the charge on each sphere, if $g = 980$ cm s How will the force of repulsion be affected, if the plate glass of K = 8 is placed between the spheres ? [Ans. 14 statC ; 6·125 dyn

24. Three equally charged objects are located as shown in Fig. 1.27. The electric force exerted by A and B is $3{\cdot}0 \times 10^{-6}$ N.

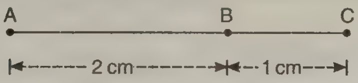


Fig. 1.27

(*a*) What electric force C exert upon B ?
(*b*) What is the net electric force on B ?

[**Ans.** (*a*)$12{\cdot}0 \times 10^{-6}$ N (along BA) ; (*b*) $9{\cdot}0 \times 10^{-6}$ N (along BA)]

25. Three small charged spheres, with equal charges on them are placed as shown in Fig. 1.28. A and C are fixed in position and B can move. C exerts a force of 4×10^{-6} N on B.

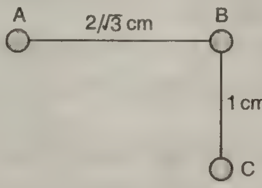


Fig. 1.28

(*a*) What force A exerts on B ?
(*b*) What is the net force on B ?

[**Ans.** (*a*) 3×10^{-6} N ; (*b*) 5×10^{-6} N in a direction 36·9° east of north]

26. Two equal positive charges, each of 5 μC interact with a third positive charge of 10 μC situated as shown in Fig. 1.29. Find the magnitude and direction of the force experienced by the charge of 10 μC.

[**Ans.** 0·161 N (along CO)]

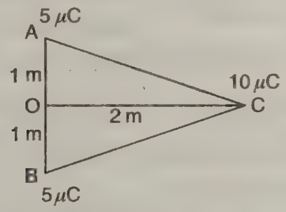


Fig. 1.29

27. Three equal charges, each having a charge of $+2 \times 10^{-6}$ C are placed at the three corners of a right angled triangle of sides 3 cm, 4 cm and 5 cm. Find the force on the charge at the right-angled corner.

[**Ans.** 45·9 N at an angle of 60·64° with the side of length 4 cm]

28. Two charges of $+2\times 10^{-8}$ C and -10^{-8} C are placed at points A and B respectively. Find the resultant force on a charge of 1 μC held at point P, such that AP = 10 cm, BP = 5 cm and $\angle$APB = 90°.

[**Ans.** $4{\cdot}0 \times 10^{-2}$ N making an angle of 63·4° with AP]

Type C. On Superposition principle

29. Charges of $+5\,\mu$C, $+10\,\mu$C and $-10\,\mu$C are placed in air at the corners A, B and C of an equilateral triangle ABC, having each side equal to 5 cm . Determine the resultant force on the charge at A. [**Ans.** 180 N (parallel to BC)]

30. Four charges $+q, +q, -q$ and $-q$ are placed respectively at the four corners A, B, C and D respectively of a square of side a. Calculate the force on a charge Q placed at the centre of the square.

$$\left[\textbf{Ans.}\ \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\sqrt{2}\,Qq}{a^2},\ \text{(parallel to AD or BC)}\right]$$

31. Two opposite corners of a square carry Q charge each and the other two opposite corners of the square carry q charge each. If the resultant force on Q is zero, how are Q and q related ? [**Ans.** $Q = -2\sqrt{2}\,q$]

MISCELLANEOUS PROBLEMS

32. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63·5 g mole^{-1}. Let us now take two pieces of copper each weighing 10 g. Let us transfer one electron from one piece to another for every 100 atoms in that piece. What will be the coulomb force between the two pieces after the transfer of electrons, if they are 1 cm apart ? Avogadro number $= 6 \times 10^{23}$ mole^{-1}, charge on an electron $= -1{\cdot}6 \times 10^{-19}$ C. (*Roorkee, 1980*)

[**Ans.** $2{\cdot}06 \times 10^{18}$ N (attractive)]

33. A small cork ball with a mass 0·58 g is suspended from a thread 10 cm long. Another ball is fixed at a distance 10 cm from the point of suspension and at a distance of 5 cm from the thread as shown in Fig. 1.30. What should the magnitude of the like and equal charges on the balls be to deflect the thread through 30° ? [**Ans.** 181·16 statC]

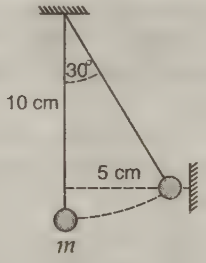


Fig. 1.30

34. A particle of mass m and carrying charge $-q_1$ is moving around a charge $+q_2$ along a circular path of radius r. Prove that period of revolution of the charge $-q_1$ about $+q_2$ is given by

$$T = \sqrt{\frac{16\pi^3\varepsilon_0 m r^3}{q_1 q_2}}$$

35. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Calculate the position of charge Q in terms of q and d. (*C.B.S.E. 1998*)

[**Ans.** $x = d/(\sqrt{2}+1)$ or $-d/(\sqrt{2}-1)$, where x is distance of charge Q from q]

36. Two fixed point charges 4 Q and 2 Q are separated by a distance x. Where should a third point charge q be placed for it to be in equilibrium ? (*P.S.S.C.E. 1999*)

[**Ans.** At a distance 0·586 x or 3·414 x from charge 4 Q]

37. Two free point charges $+4\,q$ and $+q$ are at a distance a apart. Where should a third point charge Q be placed between them so that the entire system is in equilibrium ? What will be the magnitude and sign of the charge Q ?

[**Ans.** At a distance $2\,a/3$ from the charge $+4\,q$; $Q = 4\,q/9$ (negative)]

38. It is required to hold four equal point charges $+q$ in equilibrium at the corners of a square. Find the point charge that will do this, if placed at the centre of the square.

$$\left[\textbf{Ans.}\ \frac{1+2\sqrt{2}}{4}\,q\ \text{(negative)}\right]$$

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $n = \frac{q}{e} = \frac{1\,\text{C}}{1\cdot 6 \times 10^{-19}\,\text{C}} = \mathbf{6{\cdot}25 \times 10^{18}}$

2. $n = \frac{q}{e} = \frac{3\cdot 5 \times 10^{-9}\,\text{C}}{1\cdot 6 \times 10^{-19}\,\text{C}} = \mathbf{2{\cdot}1875 \times 10^{10}}$

3. $n = \frac{\text{M}}{m_e} = \frac{1\,\text{kg}}{9\cdot 1 \times 10^{-31}\,\text{kg}} = 1{\cdot}1 \times 10^{30}$

   Therefore, charge on 1 kg of electrons,

   $q = n\,e = 1{\cdot}1 \times 10^{30} \times 1{\cdot}6 \times 10^{-19} = \mathbf{1{\cdot}76 \times 10^{11}\ C}$

4. $n = \frac{q}{e} = \frac{-3\cdot 6 \times 10^{-7}\,\text{C}}{-1\cdot 6 \times 10^{-19}\,\text{C}} = \mathbf{2{\cdot}25 \times 10^{12}}$

   Therefore, mass transferred from wool to polythene,

   $\text{M} = n \times m_e = 2{\cdot}25 \times 10^{12} \times 9{\cdot}1 \times 10^{-31}$

   $= \mathbf{2{\cdot}05 \times 10^{-18}\ kg}$

5. Number of atoms in 1 g of copper,

   $n = \frac{\text{Avogadro number}}{\text{atomic weight}} = \frac{6\cdot 02 \times 10^{23}}{63\cdot 5} = 9{\cdot}48 \times 10^{21}$

   Therefore, number of atoms *i.e.* number of free electrons in 1 mg of copper,

   $n' = 9{\cdot}48 \times 10^{21} \times 10^{-3} = \mathbf{9{\cdot}48 \times 10^{18}}$

   Charge possessed by these free electrons,

   $q = n'\,e = 9{\cdot}48 \times 10^{18} \times 1{\cdot}6 \times 10^{-19} = \mathbf{1{\cdot}517\ C}$

6. One molecule of water (H_2O) contains 2 + 8 *i.e.* 10 electrons.

   Number of molecules in 1 g of water,

   $n = \frac{\text{Avogadro number}}{\text{atomic weight}} = \frac{6\cdot 02 \times 10^{23}}{18} = 3{\cdot}344 \times 10^{22}$

   Therefore, number of electrons in 1 g of water,

   $n' = n \times 10 = 3{\cdot}344 \times 10^{22} \times 10 = \mathbf{3{\cdot}344 \times 10^{23}}$

   The negative charge possessed by 1 g of water,

   $q = n'\,e = 3{\cdot}344 \times 10^{23} \times 1{\cdot}6 \times 10^{-19} = \mathbf{5{\cdot}35 \times 10^{4}\ C}$

7. For $q_1 = q_2 = q$ (say),

   $\text{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2}$ or $q = \sqrt{4\pi\varepsilon_0\,\text{F}} \times r$

   Setting $\text{F} = 0{\cdot}1\ \text{gf} = 0{\cdot}1 \times 10^{-3}\ \text{kg f} = 0{\cdot}1 \times 10^{-3} \times 9{\cdot}8\ \text{N}$

   and $r = 3$ m, we get

   $q = \mathbf{9{\cdot}9 \times 10^{-7}\ C}$

8. When two electrons are removed from an atom, charge on atom

   $q = +2 \times 1{\cdot}6 \times 10^{-19} = +3{\cdot}2 \times 10^{-19}\ \text{C}$

   For $q_1 = q_2 = q$, we have

   $\text{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2}$ or $r = \left(\frac{1}{4\pi\varepsilon_0 \text{F}}\right)^{1/2} \times q$

   Setting $\text{F} = 8{\cdot}8 \times 10^{-9}$ N, we get

   $r = \mathbf{3{\cdot}24 \times 10^{-10}\ m}$

9. $\frac{1}{4\pi\varepsilon_0} \cdot \frac{e \times e}{r^2} = m_e\,g$ or $r = \left(\frac{1}{4\pi\varepsilon_0\,m_e\,g}\right)^{1/2} \times e$

   Setting $m_e = 9{\cdot}1 \times 10^{-31}$ kg ; $e = 1{\cdot}6 \times 10^{-19}$ C ;

   $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ SI units, we get

   $r = \mathbf{5{\cdot}08\ m}$

10. If e is magnitude of charge on an electron or a proton, then

    $\text{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \times e}{r^2}$

    Setting $e = 1{\cdot}6 \times 10^{-19}$ C and $r = 0{\cdot}53$ Å $= 0{\cdot}53 \times 10^{-10}$ m, w get

    $\text{F} = \mathbf{8{\cdot}2 \times 10^{-8}\ N}$

    Also, $\frac{\text{F}}{m_e g} = \frac{8\cdot 2 \times 10^{-8}}{9\cdot 1 \times 10^{-31} \times 9\cdot 8} = \mathbf{9 \cdot 2 \times 10^{21}}$

11. If q is charge given to each ball, then

    $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2} = \text{G}\frac{m \times m}{r^2}$ or $q = \sqrt{4\pi\varepsilon_0\,\text{G}} \times m$

    Setting $\text{G} = 6{\cdot}6 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$ and

    $m = 10\ \text{g} = 10 \times 10^{-3}$ kg, we get

    $q = \mathbf{8{\cdot}56 \times 10^{-13}\ C}$

12. If q is charge placed on earth and moon, then

    $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2} = \text{G}\frac{\text{M}_{earth} \times \text{M}_{moon}}{r^2}$

    or $q = \sqrt{4\pi\varepsilon_0\,\text{G}\,\text{M}_{earth} \times \text{M}_{moon}}$

    Setting $\text{G} = 6{\cdot}6 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$; $\text{M}_{earth} = 10^{25}$ kg an $\text{M}_{moon} = 10^{23}$ kg, we get

    $q = \mathbf{8{\cdot}56 \times 10^{13}\ C}$

13. $\frac{\text{F}_{elec}}{\text{F}_{grav}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \times e}{r^2} \Big/ \text{G}\frac{m_e \times m_e}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{\text{G}m_e^{\,2}}$

    Setting $\text{G} = 6{\cdot}6 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$; $m_e = 9{\cdot}1 \times 10^{-31}$ kg an $e = 1{\cdot}6 \times 10^{-19}$ C, we get

    $\frac{\text{F}_{elec}}{\text{F}_{grav}} = \mathbf{4{\cdot}2 \times 10^{42}}$

14. Proceed as in solved problem no. 1.06.

15. $\text{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ or $q_1 q_2 = 4\pi\varepsilon_0 \text{F} r^2$

    Setting F = 0·075 N and $r = 3$ m, we get

    $q_1 q_2 = 7{\cdot}5 \times 10^{-11}$ ...(i

    Also, $q_1 + q_2 = 20\ \mu\text{C} = 20 \times 10^{-6}\ \text{C}$ ...(

    From equation (*i*) and (*ii*), it can be obtained that

    $q_1 = \mathbf{15 \times 10^{-6}\ C}$ and $q_2 = \mathbf{5 \times 10^{-6}\ C}$

16. $\text{F} = \frac{1}{\text{K}} \cdot \frac{q_1 q_2}{r^2}$ (in cgs system)

    In first case **:** Setting $q_1 = q_2 = q$; F = 16 dyne and $r = 9$ cn we have

    $16 = \frac{1}{\text{K}}\frac{q^2}{81}$ ...(

    In second case **:** Setting $q_1 = q$, $q_2 = q + 56$; F = 16 dyne an $r = 9 + 3 = 12$ cm, we have

    $16 = \frac{1}{\text{K}}\frac{q \times (q + 56)}{144}$ ...(i

    From equations (*i*) and (*ii*), we get

    $q = \mathbf{72\ statC}$

    Using equation (*i*), we get

    $\text{K} = \mathbf{4}$

17. Let q_1 and $-q_2$ be the charges on two balls.

    Before connecting with a wire :

    $\text{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ or $q_1 q_2 = 4\pi\varepsilon_0 \text{F} r^2$

    Setting $\text{F} = 36 \times 10^{-5}$ N and $r = 2\ \text{cm} = 0 \cdot 02$ m, we get

    $q_1 q_2 = 16 \times 10^{-18}$ ...(

After connecting with a wire : Let q' be charge on each ball. Then,

$$q' = \frac{q_1 + (-q_2)}{2} = \frac{q_1 - q_2}{2}$$

$$\therefore \quad F' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q_1 - q_2)/2 \times (q_1 - q_2)/2}{r^2}$$

or $\quad q_1 - q_2 = 4 \times 4\pi\varepsilon_0 F' r^2$

Setting $F' = 20·25 \times 10^{-5}$ N and $r = 0·02$ m, we get

$q_1 - q_2 = 6 \times 10^{-9}$ ...(*ii*)

Solving equations (*i*) and (*ii*), we obtain

$q_1 = \mathbf{8 \times 10^{-9}}$ **C** and $q_2 = \mathbf{2 \times 10^{-9}}$ **C (negative)**

18. Tension in the string,

T = force of repulsion between the charges

or $\quad T = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2}$

Setting $q = 5\ \mu C = 5 \times 10^{-6}$ C and $r = 1$ m, we get

T = 0·225 N

19. Proceed as in solved problem no. 1.10.

20. Proceed as in solved problem no. 10.

21. Refer to Fig. 1.16 :

Here, AS = 0 · 2 m and $\theta = 30°/2 = 15°$

According to triangle law of forces,

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS} \quad \text{or} \quad F = mg \times \frac{OA}{SO}$$

or $\quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{(AB)^2} = mg \times \frac{OA}{SO} = mg \tan\theta$

Setting $m = 5 \times 10^{-4}$ kg, $g = 9·8$ m s^{-2} and

AB = 2 × OA = 2 × AS sin 15°, we get

$q = \mathbf{3·954 \times 10^{-8}}$ **C**

22. Refer to Fig. 1.15. Here, AB = 6 cm and AS = 5 cm

Now, $\quad F = \frac{q \times q}{(AB)^2}$ (in cgs system)

Setting $q = 36$ stat C, we get

F = 36 dyne

As obtained in problem no. 21,

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS}$$

$$\therefore \quad mg = F \times \frac{SO}{OA} \quad \text{and} \quad T = F \times \frac{AS}{OA}$$

Now, OA = AB/2 = 3 cm and SO = $\sqrt{AS^2 - OA^2}$ = 4 cm

Taking $g = 980$ cm s^{-2}, we get

$m = \mathbf{0·049}$ **g** and **T = 60 dyne**

23. Refer to Fig 1.15. Here, AB = 2 cm and SO = 20 cm

Proceeding as in problem no. 21, it can be obtained that

$$\frac{q \times q}{(AB)^2} = mg \times \frac{OA}{SO} \quad \text{(in cgs system)}$$

Setting $m = 1$ g, $g = 980$ cm s^{-2} and OA = AB/2 = 1 m, we get

$q = \mathbf{14}$ **statC**

On introducing dielectric : $F = \frac{1}{K} \cdot \frac{q \times q}{(AB)^2}$

Setting K = 8, we get

F = 6·125 dyne

24. Here, AB = 2 cm = 0·02 m and BC = 1 cm = 0·01 m

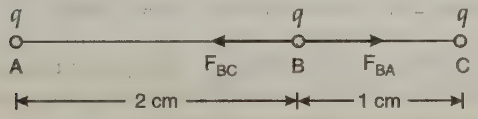


Fig. 1.31

Let charge on each object be q.

Now, $\quad F_{BA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{(AB)^2}$

Setting $F_{BA} = 3·0 \times 10^{-6}$ N, we get

$q = 3·65 \times 10^{-8}$ N

(*a*) $F_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{(BC)^2}$ **= 12·0 × 10⁻⁶ N (along BA)**

(*b*) Net force on charge at B,

$F = F_{BC} - F_{BA} = 12·0 \times 10^{-6} - 3·0 \times 10^{-6}$

$= \mathbf{9·0 \times 10^{-6}}$ **N (along BA)**

25. Here, AB = $\frac{2}{\sqrt{3}}$ cm = $\frac{0 · 02}{\sqrt{3}}$ m ; BC = 1 cm = 0·01 m

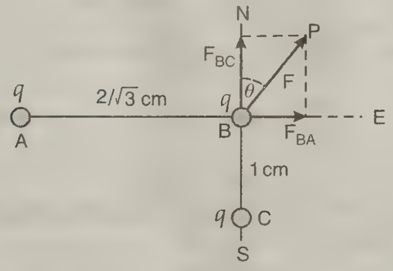


Fig. 1.32

Let q be charge on each object.

Now, $\quad F_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{(BC)^2}$

Setting $F_{BC} = 4 \times 10^{-6}$ N, we get

$q = 2·1 \times 10^{-10}$ C

(*a*) $F_{BA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{(AB)^2}$ **= 3 × 10⁻⁶ N (along BE)**

(*b*) Net force on charge at B,

$F = \sqrt{(F_{BA})^2 + (F_{BC})^2}$ **= 5 × 10⁻⁶ N (along BP)**

If the direction of F makes an angle θ with NS-line, then

$\tan\theta = \frac{F_{BA}}{F_{BC}} = 0·7500$ or $\theta = \mathbf{36·9°}$ **(east of north)**

26. Here, $q_A = q_B = 5\ \mu C = 5 \times 10^{-6}$ C ; $q_C = 10\ \mu C = 10 \times 10^{-6}$ C

AC = OC = $\sqrt{1^2 + 2^2} = \sqrt{5}$ m

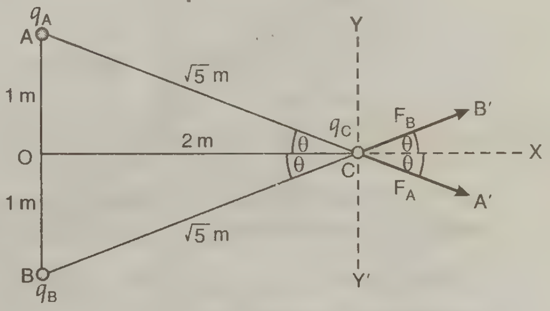


Fig. 1.33

Let F_A and F_B be the forces exerted by charges q_A and q_B on q_C. Then,

$F_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A \times q_B}{(AC)^2}$ m = 0·09 N (along CA')

Similarly, $F_B = 0·09$ N (along CB')

Resolve the forces F_A and F_B into rectangular components along X-axis and Y-axis. The components along Y-axis are equal and opposite and hence cancel each other, while the components along X-axis are equal and in the same direction. Therefore, net force on charge at C,

$$F = 2\,F_A \cos\theta = 2 \times 0{\cdot}09 \times \frac{2}{\sqrt{5}} = \mathbf{0{\cdot}161\ N\ (along\ CX)}$$

27. Here, $q_A = q_B = q_C = +2 \times 10^{-6}$ C ;
AB = 3 cm = 0·03 m, BC = 4 cm = 0·04 m and
AC = 5 cm = 0·05 m

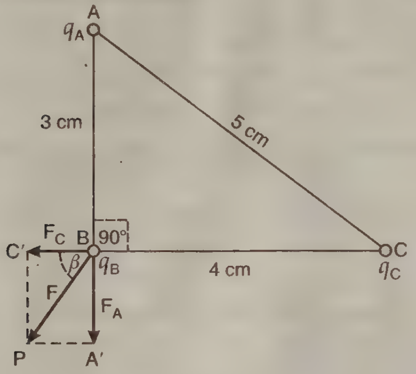


Fig. 1.34

Let F_A and F_B be the forces exerted by charges q_A and q_C on charge q_B at point B. Then,

$$F_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A \times q_B}{(AB)^2} = 40\ \text{N (along BA')}$$

and $$F_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C \times q_B}{(BC)^2} = 22{\cdot}5\ \text{N (along BC')}$$

The resultant force on charge at B,

$$F = \sqrt{F_A^{\,2} + F_C^{\,2}} = \mathbf{45{\cdot}9\ N\ (along\ BP)}$$

If the direction of F makes an angle β with side BC, then

$$\tan\beta = \frac{F_A}{F_C} = 1{\cdot}7778 \quad \text{or} \quad \boldsymbol{\beta = 60{\cdot}64°}\ \textbf{(with side BC)}$$

28. Proceed as in problem no. 27.

29. Proceed as in solved problem no. 1.12.

30. Here, AB = BC = CD = DA = a

Obviously, AO = BO = CO = DO = $a \cos 45° = a/\sqrt{2}$

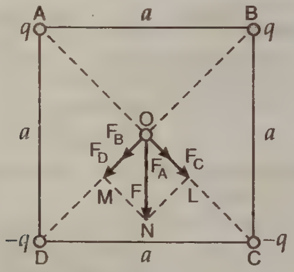


Fig. 1.35

Let F_A, F_B, F_C and F_D be the forces exerted by charges at points A, B, C and D on charge Q at point O. It follows that

$$F_A = F_B = F_C = F_D = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q \times Q}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a^2}$$

The resultant of the forces F_A and F_C,

$$F_1 = F_A + F_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a^2}$$

or $$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4qQ}{a^2}\ \text{(along OL)}$$

Similarly, resultant of the forces F_B and F_D,

$$F_2 = F_B + F_D = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4qQ}{a^2}\ \text{(along OM)}$$

Therefore, resultant force on charge Q,

$$F = \sqrt{F_1^{\,2} + F_2^{\,2}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\sqrt{2}\,q\,Q}{a^2}\ \textbf{(along ON)}$$

Since forces F_1 and F_2 are equal in magnitude, the resultan force F will act along the bisector of ∠COD *i.e.* **parallel t side AD or BC.**

31. Let each side of the square be of length a.

Then, $AC = \sqrt{a^2 + a^2} = \sqrt{2}\,a$

It follows that resultant force on charge at point C can b zero, if charges at points B and D are negative in natur Then, the forces F_A, F_B and F_D will act on charge at poin C as shown in the Fig. 1.36.

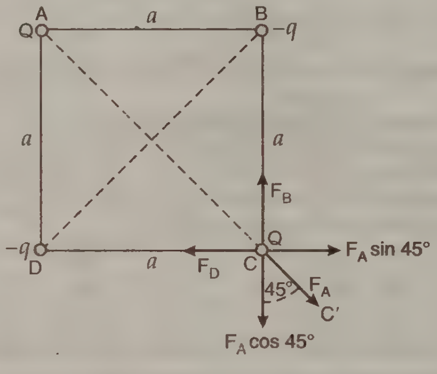


Fig. 1.36

So that resultant force on charge at point C is zero,

$F_B = F_A \cos 45°$ ...(i

and $F_D = F_A \sin 45°$ ...(i

From equation (*i*), we have

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)\times Q}{BC^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q \times Q}{AC^2} = \cos 45°$$

or $$= -\frac{q}{a^2} = \frac{Q}{(\sqrt{2}\,a)^2} \times \frac{1}{\sqrt{2}} \quad \text{or} \quad \mathbf{Q = -2\sqrt{2}\,q}$$

32. Here, Avogadro number, N = 6×10^{23} mole^{-1} and atomic weight of copper, A = 63·5 g mole^{-1}.

Therefore, number of atoms in 10 g of copper,

$$n = \frac{N}{A} \times 10 = \frac{6 \times 10^{23} \times 10}{63{\cdot}5} = 9{\cdot}45 \times 10^{22}$$

Since one electron per 100 atoms is transferred from coppe piece A to B, the charge on copper piece A,

$$q_A = +\frac{n}{100} \times e = +151{\cdot}2\ \text{C}$$

Obviously, charge on copper piece B,

$$q_B = -151{\cdot}2\ \text{C}$$

Force between two copper pieces,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A \times q_B}{r^2}$$

Setting r = 1 cm = 0·01 m, we get

$$\mathbf{F = 2{\cdot}06 \times 10^{18}\ N\ (attractive)}$$

33. Let q be charge on each ball. Suppose that cork ball A i deflected to position A′, so that ∠A′SO = 30°.

Then, OA′ = A′S sin 30° = 10 × 0·5 = 5 cm

∴ A′B = 10 cm = 0·1 m

It follows that SA′B is an equilateral triangle. Th electrostatic force of repulsion F and weight $m\,g$ of the cor ball will act as shown in Fig. 1.37.

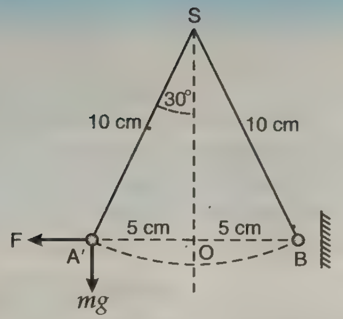


Fig. 1.37

Since cork ball is in equilibrium,

$$\frac{F}{OA'} = \frac{m g}{SO}$$

or $$F = m g \times \frac{OA'}{SO} \qquad ...(i)$$

Here, $F = \dfrac{q \times q}{(A'B)^2}$ (in cgs system),

$$SO = 10\sqrt{10^2 - 5^2} = 5\sqrt{3}\ \text{cm},$$

$m = 0{\cdot}58$ g and $g = 980$ cm s^{-2}

Setting for F, m, g, OA′ and SO in equation (*i*), we get

$$q = \mathbf{181{\cdot}16\ statC}$$

34. Suppose that the charge $-q_1$ moves around charge q_2 along circular path of radius r with speed v. The necessary centripetal force is provided by the electrostatic force of attraction between them *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 \times q_2}{r^2} = \frac{m v^2}{r}$$

or $$v = \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 \times q_2}{m r}\right)^{1/2}$$

If T is period of revolution of the charge $-q_1$ about q_2, then

$$T = \frac{2\pi r}{v}$$

Substituting for v, we get

$$T = \sqrt{\frac{16\pi^3\varepsilon_0 m r^3}{q_1 q_2}}$$

35. Suppose that the charge q, $2q$ and Q are placed as shown in the Fig. 1.38.

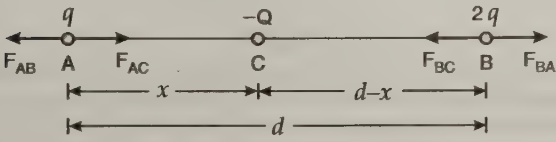


Fig. 1.38

It follows that net force on charge q and $2q$ can be zero, only if the charge Q is of opposite sign to those of charges q and $2q$. Therefore, if charges q and $2q$ are considered positive, then charge Q is negative in nature. Let the distance of charge Q (negative) from q be equal to x.

For force on charge q to be zero, $F_{AB} = F_{AC}$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2q)}{d^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{x^2}$$

or $$\frac{Q}{q} = \frac{2x^2}{d^2} \qquad ...(i)$$

For force on charge $2q$ to be zero, $F_{BA} = F_{BC}$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2q)}{d^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2q)Q}{(d-x)^2}$$

or $$\frac{Q}{q} = \frac{(d-x)^2}{d^2} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$x = \frac{d}{\sqrt{2}+1} \quad \text{or} \quad -\frac{d}{\sqrt{2}-1}$$

36. Suppose that charge q is placed at a distance AC = r from the charge 4 Q as shown in Fig. 1.39.

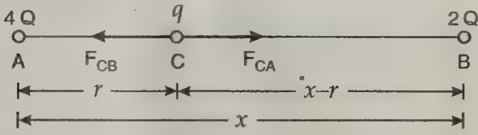


Fig. 1.39

Then, $BC = x - r$

For charge q to be in equilibrium,

$$F_{CA} = F_{CB}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(4Q)q}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2Q)}{(x-r)^2}$$

or $r = \mathbf{0{\cdot}586\ x}$ or $\mathbf{3{\cdot}414\ x}$

37. As explained in solved problem no. 1.15, for the system to be in equilibrium, the third charge Q has to be negative in nature.

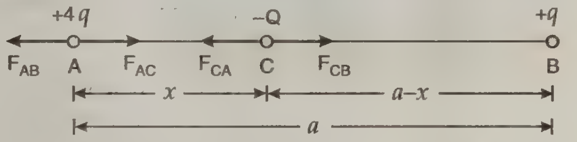


Fig. 1.40

For charge $-Q$ to be in equilibrium, $F_{CA} = F_{CB}$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(4q)Q}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(a-x)^2} \quad \text{or} \quad x = \frac{2a}{3} \quad \text{or} \quad 2a$$

Since third charge is placed between the charges $+4q$ and $+q$,

$$x = \frac{2a}{3}$$

For charge $+4q$ to be in equilibrium, $F_{AC} = F_{AB}$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(4q)Q}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(4q)q}{a^2} \quad \text{or} \quad Q = q\,\frac{x^2}{a^2}$$

Setting $x = 2a/3$, we get

$$Q = \frac{4}{9}q \qquad \textbf{(negative)}$$

38. Suppose that the charge Q placed at the centre of the square keeps all the charges in equilibrium [Fig. 1.41]. It follows that the charge Q has to be negative in nature.

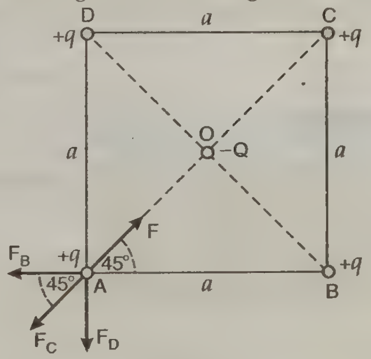


Fig. 1.41

If the net force on a charge at any corner (say A) is zero, then by symmetry it follows that the net force experienced by charges at other corners will also be zero. The charge at corner A will experience force F_B, F_C and F_D due to charges at corners B, C and D and a force F due to charge at the centre of the square. If each side of the square is of length a, then

$$AC = \sqrt{a^2 + a^2} = \sqrt{2}\, a \quad \text{and} \quad OA = \frac{1}{2}\, AC = a/\sqrt{2}$$

$$\text{Now, } F_B = F_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2},$$

$$F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(\sqrt{2}\,a)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{2a^2},$$

$$\text{and} \quad F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Qq}{a^2}$$

For net force on charge at A to be zero,

$$F_B + F_C \cos 45° = F \cos 45° \qquad \text{...}(i)$$

$$\text{and} \quad F_D + F_C \sin 45° = F \sin 45° \qquad \text{...}(ii)$$

Substituting for F_B, F_C and F in equation (*i*), we get

$$Q = \left(\frac{1 + 2\sqrt{2}}{4}\right) q \quad \textbf{(negative)}$$

CHAPTER

2

Electric Field

2.01. ELECTRIC FIELD

When two charges are placed at a distance apart, they exert force on each other. Such a force is called ***action at a distance.***

Coulomb's law in electrostatics provides a method to find the magnitude of force between two point charges kept at a distance. It cannot explain the mechanism as to how two charges exert force on each other at a distance. The concept of electric field provides an explanation as to how a charge can exert force on another charge kept at a distance from it. According to field concept, the force between two charges is a two-step process.

Key point ✓

The concept of electric field provides an explanation to action at a distance.

In order to explain, consider that a charge q is present at some point in space. In the first step, it is assumed that the charge q creates something called ***electric field*** in the space around it. Likewise, charge q is called ***source charge***. In the second step, it is assumed that when another charge q_0, called ***test charge***, is placed in the electric field produced by the source charge q, the electric field exerts force on it.

Thus, *electric field due to a charge is the space around the charge, in which any other charge is acted upon by an electrostatic force.*

The electric field of a charge is the space property by virtue of which the charge modifies the space around itself. As a result, if any other charge is brought in the space around the charge, it experiences electrostatic force. It may be pointed out that the source charge does not experience any force due to electric field produced by it. The electric field due to a source charge has its own existence and is present even if there is no test charge to experience the force. Further, the electric field takes finite time to propagate. If a charge is displaced from its position, the change in electric field at a distance r will be noticed after a time r/c, where c is the speed of light.

Key point ✓

A charge does not experience any force due to electric field produced by it.

2.02. ELECTRIC FIELD INTENSITY

The electric field intensity at a point due to a source charge may be defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge. The electric field intensity is also called ***strength of electric field*** or simply ***electric field.***

Key point

Usually, electric field intensity or strength of electric field is called simply electric field.

Consider that a positive test charge q_0 experiences a force $\vec{F}$, when placed at the observation point (the point, where electric field is to be determined). Then, electric field at the observation point is given by

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{...(2.01)}$$

In the above expression for electric field, it is assumed that when the test charge q_0 is placed at the observation point, it does not disturb the source charge. One way to ensure that test charge q_0 does not disturb the source charge is to keep its magnitude vanishingly small. If the test charge q_0 is not small, it will disturb the source charge and hence electric field at the observation point will be different from the one, had the source charge remained in its original position.

Key point ✓

To determine electric field due to a source charge, the magnitude of test charge placed at the observation point should be vanishingly small so that it does not disturb the source charge.

Therefore, *electric field intensily at a point due to a source charge may be defined as the force experienced per unit positive charge on a vanishingly small positive test charge placed at that point.*

If $\vec{F}$ is force experienced by positive test charge q_0 placed at the observation point, then electric field at that point is given by

$$\vec{E} = \underset{q_0 \to 0}{\text{Lt}} \ \frac{\vec{F}}{q_0} \qquad \text{...(2.02)}$$

Self-study Question ?

In equation (2.02), the limit $q_0 \to 0$ is against the fact that in nature, the charge less than that on an electron or a proton does not exist. How, then, this condition is justified ?

The idea of taking the limit $q_0 \to 0$ is that on placing the test charge at the observation point, the source charge will not be disturbed.

The electric field at a point is a vector quantity. From equation (2.01) or (2.02), it follows that unit of electric field in SI is **newton coulomb^{-1} (N C^{-1})**.

Key point

Another unit * of electric field is vol
metre^{-1} (V m^{-1}).

Force on a charge inside electric field. If $\vec{E}$ is electric field at a point, then by definition, a charge q placed at that point will experience a force $\vec{F}$ given by

$$\vec{F} = q\,\vec{E} \qquad ...(2.03)$$

2.03. PHYSICAL SIGNIFICANCE OF ELECTRIC FIELD

The concept of electric field is of a great significance. Two charges kept at a distance exert force on each other, which is termed as action at a distance. The concept of electric field helps us to understand the mechanism by which two charges kept at a distance exert force on each other. Further, as an alternative to Coulomb's law in electrostatics, it gives a method to find the force experienced by a charge due to another in terms of the electric field produced by it.

2.04. ELECTRIC FIELD DUE TO A POINT CHARGE

Consider that a point charge $+q$ is placed at the origin O of the co-ordinate frame. Let P be the point, where electric field due to the point charge $+q$ is to be determined [Fig. 2.01]. Let $\overrightarrow{OP} = \vec{r}$ be the position vector of the point P.

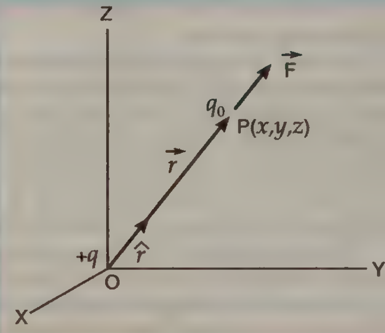


Fig. 2.01

To find electric field at point P, place a vanishingly small positive test charge q_0 at point P. According to Coulomb's law, force on the test charge q_0 due to charge q is given by

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,q_0}{r^2}\,\hat{r},$$

where $\hat{r}$ is unit vector along OP. If $\vec{E}$ is the electric field at point P, then

$$\vec{E} = \underset{q_0 \to 0}{\text{Lt}} \frac{\vec{F}}{q_0} = \underset{q_0 \to 0}{\text{Lt}} \left(\frac{1}{q_0} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,q_0}{r^2}\,\hat{r} \right)$$

or

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^3}\,\vec{r}, \qquad ...(2.04)$$

Key point

The electric field due to a point charge i
spherically symmetric about the charge

The magnitude of the electric field at point P is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad ...(2.05)$$

From the above expression for electric field, it follows that electric field is ***spherically symmetric*** as seen from the charge q and its magnitude decreases inversely as the square of the distance from the charge.

Representation of electric field. Let us represent the electric field due to a positive charge by associating a vector (in order to represent magnitude and direction). The magnitude and direction of the electric field at various points may be represented by arrows of suitable lengths. The points, where the electric field has large magnitude, longer arrows may be used and at the points, where the field has small magnitude, smaller arrows may be used. To be accurate, the lengths of the arrows will vary inversely as the square of the distance of the observation point from the charge. Further, the arrows will point away from the charge, in case it is a positive charge and will point into the charge, in case it is a negative charge. According to this scheme, Fig. 2.02 represents the electric field due to a positive charge, while Fig. 2.03 represents the electric field due to a negative charge.

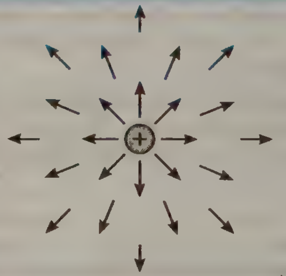


Fig. 2.02

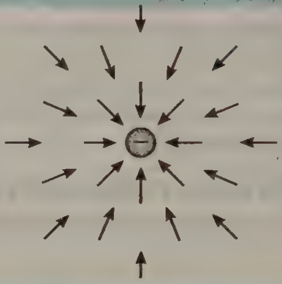


Fig. 2.03

Three components of electric field. Refer to Fig. 2.01. If (x, y, z) are the co-ordinates of point P, then

$$\vec{r} = \hat{i}\,x + \hat{j}\,y + \hat{k}\,z$$

and

$$r = |\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$$

Substituting for $\vec{r}$ and r in equation (2.04), we have

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(\hat{i}\,x + \hat{j}\,y + \hat{k}\,z)}{(x^2 + y^2 + z^2)^{3/2}} \qquad ...(2.06)$$

---

† Refer to section 3.08 of the chapter 3 on 'Electric Potential.'

Therefore, at point P, the components of electric field (magnitude only) along the three axes are given by

$$E_x = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qx}{(x^2+y^2+z^2)^{3/2}}$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qy}{(x^2+y^2+z^2)^{3/2}}$$

and $$E_z = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qz}{(x^2+y^2+z^2)^{3/2}}$$

Electric field in terms of position vectors of source charge and the observation point. Consider that the point charge q is present at point A, whose position vector is $\overrightarrow{OA} = \vec{r}\,'$ and the position vector of the observation point P is $\overrightarrow{OP} = \vec{r}$ [Fig. 2.04]. A vanishingly small positive test charge q_0 placed at point P experiences force due to charge q, which is given by

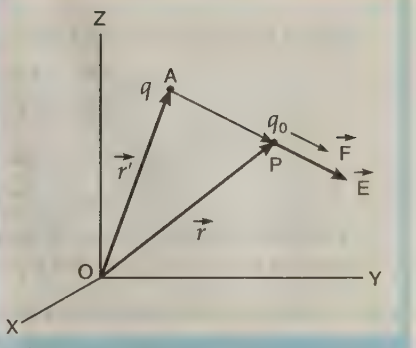


Fig. 2.04

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{(AP)^2}\,\frac{\overrightarrow{AP}}{AP}$$

or $$\vec{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{(AP)^3}\,\overrightarrow{AP} \qquad ...(2.07)$$

Here, $\overrightarrow{AP}$ is vector from charge q to the test charge q_0.

From Fig. 2.04, it follows that

$$\overrightarrow{OA} + \overrightarrow{AP} = \overrightarrow{OP}$$

or $$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \vec{r} - \vec{r}\,'$$

Therefore, equation (2.07) becomes

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{|\vec{r} - \vec{r}\,'|^3}(\vec{r} - \vec{r}\,')$$

Hence, the electric field at point P is given by

$$\vec{E} = \underset{q_0 \to 0}{\mathrm{Lt}}\ \frac{\vec{F}}{q_0}$$

or $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|\vec{r} - \vec{r}\,'|^3}(\vec{r} - \vec{r}\,') \qquad ...(2.08)$$

2.05. ELECTRIC FIELD DUE TO A DISCRETE DISTRIBUTION OF CHARGES

Electric field at a point due to discrete distribution of charges can be found by making use of ***principle of superposition of electric fields.***

Principle of superposition of electric field states that *electric field at any point due to a group of charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.*

Consider that n point charges $q_1, q_2, q_3, ..., q_n$ are distributed in space in a discrete manner. Suppose that we have to find electric field at point P due to the discrete distribution of charges. For this, place a vanishingly small positive test charge q_0 at the observation point P [Fig. 2.05]. Suppose that the charges $q_1, q_2, q_3, ... q_n$ exert forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, ... \vec{F}_n$ on the test charges q_0.

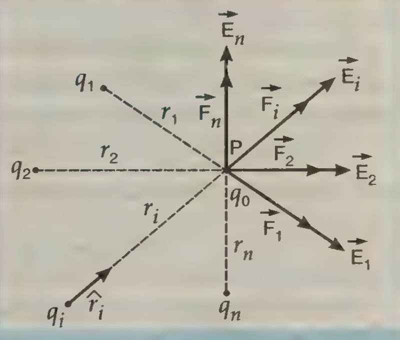


Fig. 2.05

If $\vec{F}_i$ is force due to the ith charge q_i on the test charge q_0, then

$$\vec{F}_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i q_0}{r_i^2}\,\hat{r}_i,$$

where r_i is the distance of the test charge q_0 from the ith charge q_i and $\hat{r}_i$ is unit vector from charge q_i to q_0.

The electric field due to the ith charge q_i at the observation point is given by

$$\vec{E}_i = \underset{q_0 \to 0}{\mathrm{Lt}}\ \frac{\vec{F}_i}{q_0}\ \underset{q_0 \to 0}{\mathrm{Lt}}\left(\frac{1}{q_0} \times \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i q_0}{r_i^2}\,\hat{r}_i\right)$$

or $$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\,\hat{r}_i \qquad ...(2.09)$$

The electric field $\vec{E}_i$ due to the ith charge q_i at the observation point acts along the direction of the force on charge q_i. Likewise, the electric field $\vec{E}_1, \vec{E}_2, \ldots\ldots \vec{E}_n$ due to the charges $q_1, q_2, \ldots\ldots q_n$ act along the direction of the forces on the test charge due to them as shown in Fig. 2.05.

If $\vec{E}$ is the electric field due to the discrete distribution of charges, then by principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots\ldots + \vec{E}_n = \sum_{i=1}^{n} \vec{E}_i$$

Using equation (2.09), we have

$$\vec{E} = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_i}{r_i^2} \hat{r}_i$$

or

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i \qquad \ldots(2.10)$$

Electric field in terms of position vectors of source charges and the observation point. Suppose that n point charges $q_1, q_2, q_3, \ldots\ldots q_n$ are placed in space at the points, whose position vectors are $\vec{r}_1, \vec{r}_2, \ldots\ldots \vec{r}_n$ respectively. Let $\vec{r}_i$ be the position vector of the ith charge q_i and $\vec{r}$, the position vector of the observation point P [Fig. 2.06]. As obtained in section 2.04, the electric due to a point charge at an observation point in terms of position vectors of the charge and the observation point is given by equation (2.08). Using this equation, electric field due to charge q_i at point P is given by

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

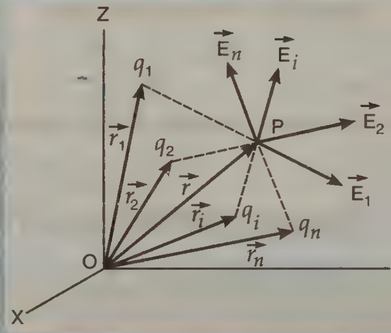


Fig. 2.06

Hence, electric field at point P due to the discrete distribution of n point charges is given by

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i$$

or

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i) \qquad \ldots(2.11)$$

[illegible] ELECTRIC FIELD DUE TO A CONTINUOUS DISTRIBUTION OF CHARGES

Now, we calculate electric field due to a continuous distribution of charges in the following cases :

1. Electric field due to a line charge distribution. Refer to Fig. 1.10 of the previous chapter. Let $\vec{E}$ be electric field at location $\vec{r}$ due to the line charge distribution L having linear charge density λ.

From equation (1.21), the force on a vanishingly small test charge q_0 at location $\vec{r}$ due to the line charge distribution is given by

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \int_L \frac{\lambda\, dl}{|\vec{r} - \vec{r}\,'|^3} (\vec{r} - \vec{r}\,')$$

Here, $\vec{r}\,'$ is position vector of the elementary portion dl of the line charge L having charge $\lambda\, dl$. Therefore, electric field at location $\vec{r}$ due to line charge distribution is given by

$$\vec{E} = \underset{q_0 \to 0}{\text{Lt}} \frac{\vec{F}}{q_0}$$

or

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda\, dl}{|\vec{r} - \vec{r}\,'|^3} (\vec{r} - \vec{r}\,') \qquad \ldots(2.12)$$

2. Electric field due to a surface charge distribution. Refer to Fig. 1.11 of the previous chapter. Let $\vec{E}$ be electric field at location $\vec{r}$ due to the surface charge distribution S having surface charge density σ. From the equation (1.22), the force on a vanishingly small test charge q_0 at location $\vec{r}$ due to surface charge distribution S is given by

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

Here, $\vec{r}'$ is position vector of the elementary portion dS of the surface charge distribution S having charge σ dS. Therefore, electric field at location $\vec{r}$ due to the surface charge distribution is given by

$$\vec{E} = \underset{q_0 \to 0}{\text{Lt}} \frac{\vec{F}}{q_0}$$

or
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \qquad ...(2.13)$$

3. Electric field due to a volume charge distribution. Refer to Fig. 1.12 of the previous chapter. Let $\vec{E}$ be electric field at location $\vec{r}$ due to volume charge distribution V having volume charge density ρ. From the equation (1.23), the force on a vanishingly small test charge q_0 at location $\vec{r}$ due to volume charge distribution V is given by

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \int_V \frac{\rho\, dS}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}')$$

Here, $\vec{r}'$ is position vector of the elementary portion dV of the volume charge distribution V having charge ρ dV. Therefore, electric field at location $\vec{r}$ due to the volume charge distribution is given by

$$\vec{E} = \underset{q_0 \to 0}{\text{Lt}} \frac{\vec{F}}{q_0}$$

or
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \qquad ...(2.14)$$

Note. If charge is distributed in space continuously over a line L, over a surface S, over a volume V and also in a discrete manner in space, then total electric field at location $\vec{r}$ will be

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \left(\int_L \frac{\lambda\, dl}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') + \int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \right.$$
$$\left. + \int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') + \sum_{\substack{\text{all} \\ \text{point charges}}} \frac{q''}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \right) \qquad ...(2.15)$$

2.07. ELECTRIC FIELD DUE TO A CIRCULAR LOOP OF CHARGE

Consider a circular loop of a wire of negligible thickness having radius a. Suppose that the total charge q is distributed uniformly over its circumference. Suppose the loop is placed in YZ plane, such that its centre coincides with the origin O [Fig. 2.07]. Let P be a point at a distance x from the centre of the loop on its axis OX, where electric field due to the loop is to be determined.

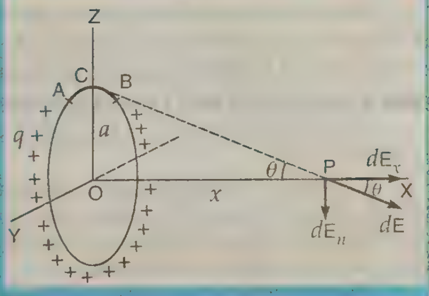


Fig. 2.07

Consider an elementary length AB of the loop of length dl. Then, charge on the elementary length AB,

$$dq = \frac{q}{2\pi a} dl$$

Let $d\vec{E}$ be electric field at point P due to charge dq on the elementary portion AB of the ring. Then, magnitude of electric field due to the elementary portion is given by

$$dE = |\vec{dE}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{CP^2} \text{ (along CP)}$$

Now, $CP = (x^2 + a^2)^{1/2}$

Substituting for dq and CP, we have

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,dl}{2\pi a\,(x^2 + a^2)}$$

The electric field dE due to elementary portion AB can be resolved into two components :

(*i*) dE_x along PX *i.e.* along X-axis and

(*ii*) dE_n along normal to PX. For element AB, the component dE_n will be along Z-axis and in the downward direction.

The component dE_n of dE due to various elementary portions of the circular loop will not contribute to the electric field at point P due to the whole loop. It is because of the fact that the components dE_n due to any two elementary portions of the loop located opposite to each other are equal and opposite and hence cancel out. In other words, net electric field at point P due to the whole loop of charge will be only along X-axis.

If $\vec{dE}$ makes angle θ with X-axis, then

$$dE_x = dE \cos\theta$$

In right angled triangle POC, ∠OPC = θ. Therefore,

$$\cos\theta = \frac{OP}{PC} = \frac{x}{(x^2 + a^2)^{1/2}}$$

Hence, $dE_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,dl}{2\pi a\,(x^2 + a^2)} \cdot \frac{x}{(x^2 + a^2)^{1/2}}$

or $$dE_x = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,x\,dl}{2\pi a\,(x^2 + a^2)^{3/2}}$$

Electric field due to whole of the circular loop is given by

$$E = \int_{\text{total loop}} dE_x = \int_{\text{total loop}} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,x\,dl}{2\pi a\,(x^2 + a^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,x}{2\pi a\,(x^2 + a^2)^{3/2}} \int_{\text{total loop}} dl = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,x\,(2\pi a)}{2\pi a\,(x^2 + a^2)^{3/2}}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,x}{(x^2 + a^2)^{3/2}} \text{ (along PX)} \qquad ...(2.16)$$

Special cases. Let us evaluate the electric field due to the circular loop of the charge in the following cases :

1. ***When point P lies at centre of the loop*** : In such a case, $x = 0$. Then, from equation (2.16), it follows that

$$E = 0$$

2. ***When point P lies at a distance*** $x >> a$: In such a case, in equation (2.16), a^2 can be neglected in comparison to x^2. Then,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2} \qquad \text{(along PX)}$$

It is an expression for electric field due to point charge q at a distance x from it. Therefore, *a circular loop of charge behaves as a point charge, in case observation point on its axis is at a distance quite large as compared to the dimensions (radius) of the loop.*

Key point

The electric fields due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius.

2.08. ELECTRIC LINES OF FORCE

An electric line of force is the path along which a unit positive charge would move, if it is free to do so.

The electric line of force points in the direction of electric field. The electric line of force may be straight or curved. In case, electric line of force is a curve, then direction of electric field at any point is given by the tangent to the line of force at that point as shown in Fig. 2.08.

The strength of the electric field at any point is measured as the number of lines of force crossing a unit area held normal to the lines of force at that point.

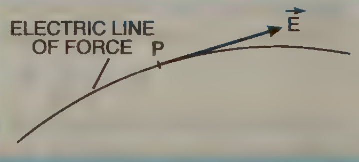


Fig. 2.08

Properties of electric lines of force. The electric lines of force have the following properties :

1. *The lines of force start from the positive charge and end at the negative charge.*

2. *The lines of force originate from or terminate at a surface always at right angles to the surface.*

3. *The lines of force do not pass through a conductor.*

It indicates that the electric field inside a conductor is always zero.

Key point ✓

Electric field inside a conductor is always zero.

4. *The lines of force never intersect each other.*

In case, two lines of force intersect at a point, then it will mean that at that point, electric field has two directions. As it is not possible, the lines of force can never ntersect each other.

5. *The relative closeness of lines of force in different regions of space gives the idea about the relative strengths of the electric field in the different regions.*

The regions, where lines of force are closer, the electric field is strong and the egions, where the lines of forces are farther apart, the field is weak.

6. *The lines of force contract longitudinally i.e. lengthwise.*

This property of lines of force leads to explain attraction between two unlike charges.

7. *The lines of force exert a lateral pressure on each other.*

This property of lines of force leads to explain the repulsion between two like charges.

Key point ✓

The property that electric lines of force contract longitudinally leads to explain the attraction between opposite charges. On the other hand, the property that electric lines of force exert lateral pressure on each other leads to explain repulsion between like charges.

Let us represent the electric field lines in the following cases :

1. ***Field lines in case of isolated point charges.*** Figs. 2.09 (*a*) and 2.09 (*b*) represent the electric field due to an isolated positive and a negative charge respectively. From the positive charge, the lines of force originate ; while in the case of negative charge, the lines of force terminate at it.

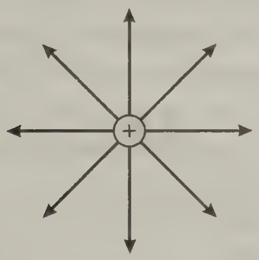


Fig. 2.09 (*a*)

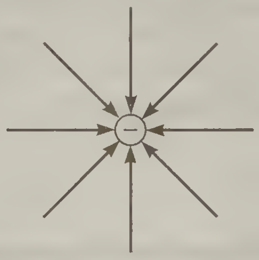


Fig. 2.09 (*b*)

2. ***Field lines in case of a system of two charges.*** Fig. 2.10 (*a*) represents field lines in case of a system of two equal and opposite charges. The field lines behave as rubber bands between the two charges and tend to contract lengthwise. Therefore, the lines of force tend to bring the two charges close to each other, which shows that opposite charges attract each other.

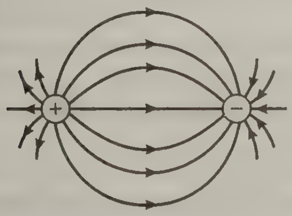


Fig. 2.10 (*a*)

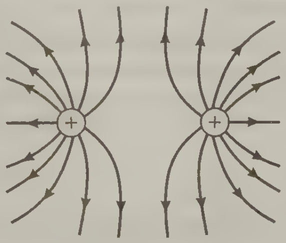


Fig. 2.10 (*b*)

Fig. 2.10 (*b*) represents field lines between two equal positive charges. The field lines seem to exert sideways pressure and hence it shows that the two like charges repel each other.

Importance of electric lines of force. The electric lines of force can be used to represent the electric field due to a point charge or a system of point charges. It may be noted that though the lines of force are imaginary, the electric field they represent s real.

2.09. ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a certain distance is called an electric dipole.

Fig. 2.11 represents an electric dipole consisting of two charges $-q$ and $+q$ and separated by distance AB = $2a$. The distance AB is called length of the dipole and is a vector $\overrightarrow{2a}$, whose direction is from the charge $-q$ to charge $+q$.

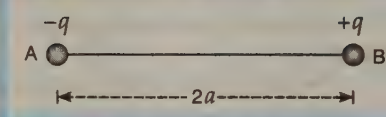


Fig. 2.11

The molecules of water, ammonia, etc behave as electric dipoles. It is because, the centres of positive and negative charges in these molecules lie at a small distance from each other.

2.10. ELECTRIC DIPOLE MOMENT

It is defined as the product of either charge and the length of the electric dipole.

It is denoted by vector $\vec{p}$, which has the same direction as that of $\overrightarrow{2a}$. Thus,

$$\vec{p} = q(\overrightarrow{2a}) \qquad \text{...(2.17)}$$

In SI, unit of electric dipole moment is **coulomb metre (C m).**

Note. If an electric dipole having electric dipole moment p (= $q \times 2a$) is an ideal one, then the charge q on either of the two poles is very very large and length $2a$ of the dipole is negligibly small, so that the product $q \times 2a$ equals the electric dipole moment of the dipole.

2.11. ELECTRIC FIELD ON AXIAL LINE OF AN ELECTRIC DIPOLE

Consider an electric dipole consisting of charges $-q$ and $+q$, separated by a distance $2a$ and placed in free space. Let P be a point on the line joining the two charges (axial line) at a distance r from the centre O of the dipole [Fig. 2.12].

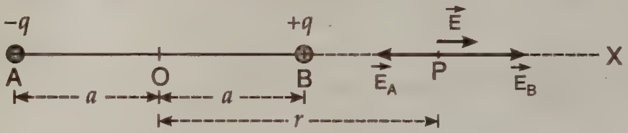


Fig. 2.12.

The electric field $\vec{E}$ at point P due to the dipole will be the resultant of the electric field $\vec{E}_A$ (due to charge $-q$ at point A) and $\vec{E}_B$ (due to charge $+q$ at point B) *i.e.*

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

Now,
$$|\vec{E}_A| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r+a)^2} \qquad \text{(along PA)}$$

and
$$|\vec{E}_B| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r-a)^2} \qquad \text{(along PX)}$$

Obviously, $|\vec{E}_B|$ is greater than $|\vec{E}_A|$.

Since $\vec{E}_A$ and $\vec{E}_B$ act along the same line but in opposite direction, the magnitude of the electric field at point P is given by

$$E = |\vec{E}| = |\vec{E}_B| - |\vec{E}_A| \qquad \text{(along PX)}$$

or
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r+a)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q \frac{(r+a)^2 - (r-a)^2}{(r^2 - a^2)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(4ra)}{(r^2 - a^2)^2}$$

Now, $q(2a) = p$, the magnitude of the electric dipole moment of the dipole.

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2} \text{ (along PX)} \qquad \text{...(2.18)}$$

It may be noted that direction of electric field at a point on axial line of the dipole is from charge $-q$ to $+q$ *i.e.* same as that of electric dipole moment of the dipole.

Key point

The direction of electric field due to a electric dipole at a point on its axial lin is same as that of the electric dipo moment.

When dipole is of very small length. If the dipole is of small length, such that $<< r$, then in equation (2.18), a^2 can be neglected as compared to r^2. Therefore, for n electric dipole of very small length

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \text{ (along PX)} \qquad ...(2.19)$$

2.12. ELECTRIC FIELD ON EQUITORIAL LINE OF AN ELECTRIC DIPOLE

Consider an electric dipole consisting of charges $-q$ and $+q$ separated by a istance $2a$ and placed in free space. Let P be a point on equitorial line of the dipole ight bisector of the length of dipole) at a distance r from the centre of the dipole ig. 2.13].

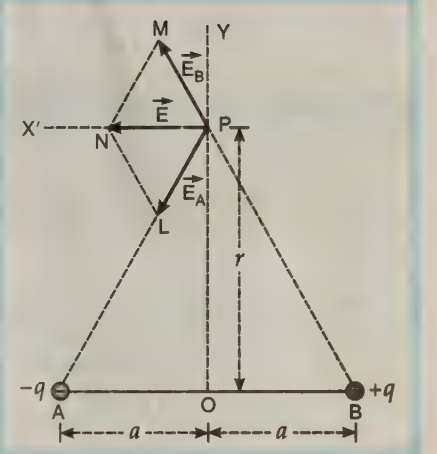


Fig. 2.13.

Let $\vec{E}_A$ and $\vec{E}_B$ be electric fields at point P due to charge $-q$ at point A and harge $+q$ at point B. Then, resultant electric field at point P is given by

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

Now, $$|\vec{E}_A| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r^2+a^2)} \text{ (along PA)}$$

and $$|\vec{E}_B| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r^2+a^2)} \text{ (along BP)}$$

It may be noted that $\vec{E}_A$ and $\vec{E}_B$ have same magnitude. To find the resultant ectric field at point P due to the dipole, represent $\vec{E}_A$ and $\vec{E}_B$ by the two adjacent des PL and PM of a parallelogram. Then, diagonal PN of the parallelogram presents the resultant electric field $\vec{E}$ due to the dipole, which acts along PX′. The sultant electric field can also be found by using triangle law of addition of vectors. Δ PAB, $\overrightarrow{PA}$, $\overrightarrow{BP}$ and $\overrightarrow{BA}$ represent $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}$ respectively. Therefore, by iangle law of addition of vectors,

$$\frac{|\vec{E}|}{BA} = \frac{|\vec{E}_A|}{PA} = \frac{|\vec{E}_B|}{BP}$$

or $$|\vec{E}| = |\vec{E}_A| \times \frac{BA}{PA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r^2+a^2)} \times \frac{2a}{(r^2+a^2)^{1/2}}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(2a)}{(r^2+a^2)^{3/2}}$$

Now $q(2a) = p$, the magnitude of the elecric dipole moment of the dipole. herefore,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2+a^2)^{3/2}} \text{ (along PX')} \qquad ...(2.20)$$

It may be noted that direction of electric field at a point on the equitorial line the dipole is from charge $+q$ to $-q$ *i.e.* opposite to the direction of electric dipole oment of the dipole.

Key point ✓

The direction of electric field due to an electric dipole at a point on its equitorial line is opposite to that of electric dipole moment.

When dipole is of very small length. If the dipole is of small length such that $<< r$, then in equation (2.20), a^2 can be neglected as compared to r^2. Therefore, for n electric dipole of very small length,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3} \text{ (along PX')} \qquad (2.21)$$

Key point ✓

The strength of electric field at a point on axial line of an electric dipole of very small length is twice at the point at same distance on its equitorial line.

2.13. ELECTRIC FIELD AT ANY POINT DUETO AN ELECTRIC DIPOLE

Consider an electric dipole AB of small length having charge $-q$ at point A and harge $+q$ at point B. Let O be the centre of the dipole and P be any point at a istance r from its centre, where electric intensity due to the dipole is to be etermined. Let $\angle POB = \theta$ [Fig. 2.14].

The dipole moment $\vec{p}$ of the dipole can be resolved into two components :

(*i*) The component $p \cos\theta$ along OP and

(*ii*) the component $p \sin\theta$ along a direction perpendicular to OP.

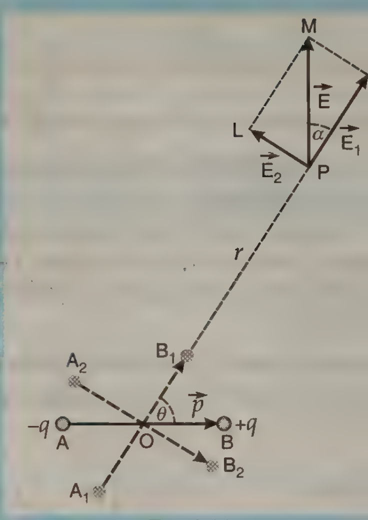


Fig. 2.14

The electric dipole AB of dipole moment *p* can be considered as to be equivalent to the combination of two electric dipoles *i.e.* one dipole $A_1 B_1$ having dipole moment $p \cos \theta$ and another electric dipole $A_2 B_2$ (placed perpendicular to $A_1 B_1$) having electric dipole moment $p \sin \theta$.

The point P lies on the axial line of dipole $A_1 B_1$. Therefore, electric field intensity at point P due to dipole $A_1 B_1$,

$$|\vec{E}_1| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p\cos\theta}{r^3} \quad \text{(along PK)}$$

Further, point P lies on the equitorial line of dipole $A_2 B_2$. Therefore, electric field intensity at point P due to dipole $A_2 B_2$,

$$|\vec{E}_2| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\sin\theta}{r^3} \quad \text{(along PL, perpendicular to PK)}$$

The resultant electric field intensity at point P,

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

or
$$E = \sqrt{|\vec{E}_1| + |\vec{E}_2|}$$

$$= \sqrt{\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p\cos\theta}{r^3}\right) + \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{p\sin\theta}{r^3}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3} \sqrt{4\cos^2\theta + \sin^2\theta}$$

or
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\sqrt{3\cos^2\theta + 1}}{r^3} \qquad \text{...(2.22)}$$

The equation (2.22) gives the magnitude of electric field intensity due to a short electric dipole at a distance *r* from its centre in a direction making an angle θ with the dipole.

To find the direction of electric field intensity due to the dipole, suppose that it makes an angle α with the direction of $\vec{E}_1$ *i.e.* with the line OK. Then, from right angled Δ PKM, we have

$$\tan\alpha = \frac{MK}{PK} = \frac{|\vec{E}_2|}{|\vec{E}_1|} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\sin\theta}{r^3} \Big/ \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p\cos\theta}{r^3}$$

or
$$\tan\alpha = \frac{1}{2}\tan\theta \qquad \text{...(2.23)}$$

The equation (2.23) can be used to find the value of angle α and hence the direction of electric field intensity due to the dipole.

2.14. ELECTRIC DIPOLE IN UNIFORM ELECTRIC FIELD

Consider an electric dipole consisting of charges $-q$ and $+q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with the direction of the field as shown in Fig. 2.15.

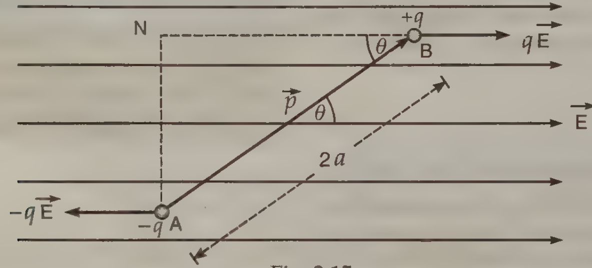


Fig. 2.15

Force on charge $-q$ at A $= -q\vec{E}$ (opposite to $\vec{E}$)

and force on charge $+q$ at B $= q\vec{E}$ (along $\vec{E}$)

Thus, electric dipole is under the action of two equal and unlike parallel forces, which give rise to a torque on the dipole. The magnitude of the torque is given by

τ = either force × perpendicular distance between the two forces

$$= q\,E\,(AN) = q\,E\,(2\,a \sin\theta) = q\,(2\,a)\,E \sin\theta$$

or $$\tau = p\,E \sin\theta \qquad ...(2.24)$$

Here, $p = q\,(2\,a)$ is electric dipole moment of the electric dipole.

The torque on the dipole tends to align it along the direction of the electric field.

Since electric dipole moment vector $\vec{p}$ is a vector from the charge $-q$ to $+q$, e equation (2.24) may be expressed as

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad ...(2.25)$$

Note. 1. It may be pointed out that when dipole is placed in uniform electric eld, it experiences only a torque. Net force on the dipole is zero.

2. Torque on the dipole becomes zero, when it aligns itself parallel to the electric eld.

3. Torque on the dipole is maximum, when dipole is placed at right angles to e direction of the electric field.

$$\therefore \qquad \tau_{max} = p\,E \sin 90° = p\,E$$

Key point ✓

In a uniform electric field, an electric dipole experiences only a torque. But when the electric field is not uniform, it experiences both a torque and force.

Key point ✓

The orientation of the electric dipole along the electric field $(\vec{p} \parallel \vec{E})$ corresponds to the stable equilibrium.

SOLVED NUMERICAL PROBLEMS

ype A. On Force experienced by a charge in an electric field

Problem 2.01. Calculate the electric field strength equired to just support a water drop of mass 10^{-7} kg and aving a charge $1{\cdot}6 \times 10^{-19}$ C. *(C.B.S.E. 1999)*

Sol. Here, $m = 10^{-7}$ kg ; $q = 1{\cdot}6 \times 10^{-19}$ C

Let E be the sterngth of the electric field required to just pport the water drop. Then,

$$E\,e = m\,g$$

or $$E = \frac{m\,g}{e} = \frac{10^{-7} \times 9{\cdot}8}{1{\cdot}6 \times 10^{-19}} = \mathbf{6{\cdot}125 \times 10^{12}\ N\ C^{-1}}$$

Problem 2.02. A particle of mass 10^{-3} kg and charge 5 C is thrown at a speed 20 m s^{-1} against a uniform electric eld of strength 2×10^5 N C^{-1}. How much distance will it avel before coming to rest momentarily ?

Sol. Here, $m = 10^{-3}$ kg ; $q = 5\,\mu C = 5 \times 10^{-6}$ C ;

$$u = 20 \text{ m s}^{-1} \text{and } E = 2 \times 10^5 \text{ N C}^{-1}$$

Force on the charged particle,

$$F = q\,E = 5 \times 10^{-6} \times 2 \times 10^5 = 1 \text{ N}$$

Since the particle is thrown against the electric field, cceleration of the particle,

$$a = -\frac{F}{m} = -\frac{1}{10^{-3}} = -10^3 \text{ m s}^{-1}$$

From the relation : $v^2 - u^2 = 2\,a\,S$, we have

$$S = \frac{v^2 - u^2}{2\,a} = \frac{0^2 - 20^2}{2 \times (-10^3)} = \mathbf{0{\cdot}2\ m}$$

Problem 2.03. An oil drop of 12 excess electrons is held ationary under a constant electric field of $2{\cdot}55 \times 10^4$ V m^{-1} Millikan's oil drop experiment. The density of the oil $1{\cdot}26$ g cm^{-3}. Estimate the radius of the drop. Given that $= 9{\cdot}81$ m s^{-2}and $e = 1{\cdot}6 \times 10^{-19}$ C. *(Text Problem)*

Sol. Here, $E = 2{\cdot}55 \times 10^4$ V m^{-1}

Charge on the drop,

$$q = 12\,e = 12 \times 1{\cdot}6 \times 10^{-19}$$
$$= 1{\cdot}92 \times 10^{-18} \text{ C}$$

Force on oil drop due to electric field,

$$F_e = q\,E = 1{\cdot}92 \times 10^{-18} \times 2{\cdot}55 \times 10^4 \text{ N}$$

Force on drop due to gravity,

$$F_g = m\,g = \frac{4}{3}\pi r^3 r \rho g$$

Here, $\rho = 1{\cdot}26 \text{ g cm}^{-3} = 1{\cdot}26 \times (10^{-3} \text{ kg}) \times (10^{-2} \text{ m})^{-3}$

$$= 1{\cdot}26 \times 10^3 \text{ kg m}^{-4}$$

and $$g = 9{\cdot}81 \text{ m s}^{-2}$$

$$\therefore \qquad F_g = \frac{4}{3}\pi r^3 \times 1{\cdot}26 \times 10^3 \times 9{\cdot}81 \text{ N}$$

The drop will be held stationary, if F_g and F_e are equal and opposite *i.e.*

$$\frac{4}{3}\pi r^3 \times 1{\cdot}26 \times 10^3 \times 9{\cdot}81 = 1{\cdot}92 \times 10^{-18} \times 2{\cdot}25 \times 10^4$$

or $$r = \left(\frac{3 \times 1{\cdot}92 \times 10^{-18} \times 2{\cdot}25 \times 10^4}{4\,\pi \times 1{\cdot}26 \times 10^3 \times 9{\cdot}81}\right)^{\frac{1}{3}}$$

$$= (0{\cdot}9456 \times 10^{-18})^{1/3}$$
$$= 9{\cdot}81 \times 10^{-7} \text{ m} = \mathbf{9{\cdot}81 \times 10^{-4}\ mm}$$

Problem 2.04. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} coulomb is at rest in a horizontal uniform electric field of 20,000 V s^{-1}. Find the tension in thread of the pendulum and the angle it makes with the vertical. *(I.I.T. 1979)*

Sol. Consider that a pendulum bob of weight $m\,g$ and carrying a charge q is suspended in the electric field E. Suppose that it comes to rest at point A, so that it makes an angle θ with the vertical as shown in Fig. 2.16.

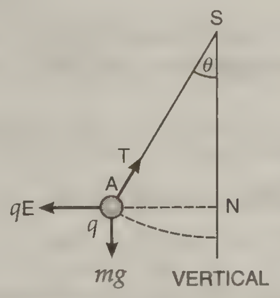


Fig. 2.16

At point A, the bob is acted upon by the following three forces :

(*i*) Weight mg acting vertically downward.

(*ii*) Tension T in the string along AS.

(*iii*) Electrostatic force q E on the bob along horizontal.

Since the bob is in equilibrium under the action of the forces mg, T and q E, these forces can be represented by the sides SN, AS and NA of the triangle ANS. Therefore,

$$\frac{mg}{SN}=\frac{qE}{NA}=\frac{T}{AS} \qquad ...(i)$$

Here, $m = 80\text{ mg} = 80 \times 10^{-6}\text{ kg}$;

$q = 2 \times 10^{-8}$ C and E = 20,000 V m^{-1}

From equation (*i*), we have

$$\frac{NA}{SN}=\frac{qE}{mg}$$

or $$\tan\theta=\frac{qE}{mg}=\frac{2\times10^{-8}\times20{,}000}{80\times10^{-6}\times9\cdot8}=0\cdot51$$

or $$\theta = 27\cdot02°$$

Again from equation (*i*), we have

$$\frac{T}{AS}=\frac{qE}{NA}$$

or $$T=\frac{qE}{NA/AS}=\frac{qE}{\sin\theta}=\frac{2\times10^{-8}\times20{,}000}{\sin(27\cdot02°)}$$

$$=\frac{2\times10^{-8}\times20{,}000}{0\cdot4543}=8\cdot8\times10^{-4}\text{ N}$$

Type B. On Electric field due to a point charge

Problem 2.05. Two point charges of 2 × 10^{-7} C and 1·0 × 10^{-7} are 1·0 cm apart. What is the magnitude of the field produced by either charge at the site of the other ? Use standard value of 1/4 π ε$_0$. (*P.S.S.C.E. 1998*)

Ans. Here, $q_1 = 2\cdot0 \times 10^{-7}$ C ; $q_2 = 1\cdot0 \times 10^{-7}$ C

and $r = 1\cdot0\text{ cm} = 10^{-2}$ m

The electric field at the site of a charge will be the electric field produced by the other charge.

Therefore, electric field due to charge q_1 at the site of q_2,

$$E_1=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r^2}=\frac{9\times10^9\times2\cdot0\times10^{-7}}{(10^{-2})^2}=1\cdot8\times10^7\text{ N C}^{-1}$$

Also, electric field due to charge q_2 at the site of q_1,

$$E_2=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r^2}=\frac{9\times10^9\times1\cdot0\times10^{-7}}{(10^{-2})^2}=9\times10^6\text{ N C}^{-1}$$

Type C. On Electric field due to a group of charges

Problem 2.06. Two point charges of + 5× 10^{-19} C and +20 × 10^{-19} C are separated by a distance 2 m. Find the point on the line joining them at which electric field intensity is zero. (*C.B.S.E. 2001 S*)

Sol. Here, $q_1 = +5 \times 10^{-19}$ C ; $q_2 = 20 \times 10^{-19}$ C and $r = 2$ m

Suppose that the resultant electric field is zero at point O [Fig. 2.17].

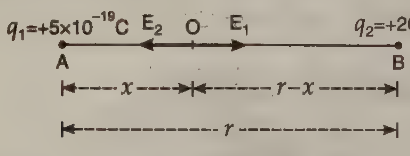


Fig. 2.17

If AO = x, then BO = $r - x = 2 - x$

Let E$_1$ and E$_2$ be electric fields at point O due to charges q_1 and q_2 respectively. Then, electric field will be zero at point O, if E$_1$ and E$_2$ are equal in magnitude and opposite in direction *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{(AO)^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{(BO)^2}$$

or $$\frac{5\times10^{-19}}{x^2}=\frac{20\times10^{-19}}{(2-x)^2}$$

or $$4x^2=(2-x)^2$$

or $$2x=\pm(2-x)^2$$

or $$x = 2/3\text{ m} \quad\text{or}\quad -2\text{ m}$$

Thus, electric intensity will be zero at a distanc 2/3 m from charge q_1 (towards charge q_2) or at a dista of 2 m on the other side of charge q_1.

Problem 2.07. Two point charges q_A = 3 μC q_B = – 3 μC are located 20 cm apart in vacuum. (*a*) W is the electric field at the mid-point O of the line joining the two charges ? (*b*) If a negative test charg magnitude 1·5 × 10^{-9} is placed at this point, what is force experienced by the test charge ? (*Text Probl*

Sol. Here, $q_A = 3\ \mu C = 3 \times 10^{-6}$ C ;

$q_B = -3\ \mu C = -3 \times 10^{-6}$ C and r = 20 cm = 0·2 m

Let O be the mid-point of the line AB [Fig. 2.18].

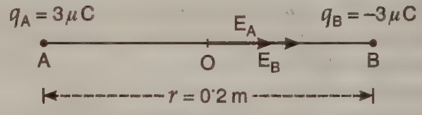


Fig. 2.18

Then, $$OA = OB = \frac{r}{2}=\frac{0\cdot2}{2}=0\cdot1\text{ m}$$

(*a*) Electric field at point O due to q_A,

$$E_A=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{(OA)^2}=9\times10^9\times\frac{3\times10^{-6}}{(0\cdot1)^2}$$

$= 2\cdot7 \times 10^6$ N C^{-1} (along OB)

Electric field at point O due to q_B,

$$E_B=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{(OB)^2}=9\times10^9\times\frac{3\times10^{-6}}{(0\cdot1)^2}$$

$= 2\cdot7 \times 10^6$ N C^{-1} (along OB)

Therefore, net electric field at point O due to charge and q_B,

$$E = E_A + E_B = 2\cdot7\times10^6 + 2\cdot7\times10^6$$

$= \mathbf{5\cdot4 \times 10^6\ N\ C^{-1}}$ (along OB)

(*b*) Force on a negative charge of magnitude 1·5 × C placed at point O,

$F = qE = 1\cdot5\times10^{-9}\times5\cdot4\times10^6 = \mathbf{8\cdot1\times10^{-3}}$ **N (along**

The force on negative charge acts in a direction oppo to that of the electric field.

Problem 2.08. Two point charges of +16μC and – are placed 8 cm apart in air. Determine the position of point at which the resultant electric field is zero.

(*P.S.S.C.E. 1*

Sol. Here, $q_A = +16\ \mu C = +16 \times 10^{-6}$ C ;

$q_B = -9\ \mu C = -9 \times 10^{-6}$ C and r = 8 cm = 0·08 m

The electric field cannot be zero at a point between two charges. It is because, the charges are of opposite si Also, the electric field cannot be zero at a point to the le charge q_A. It is because the magnitude of charge q_A is gre than that of q_B.

Suppose that the resultant electric field due to the two charges is zero at point O located to the right of charge q_B as shown in Fig. 2.19.

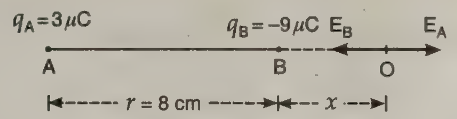


Fig. 2.19

If $OB = x$, then $OA = x + 0.08$.

The electric fields E_A and E_B due to the two charges at the point O are in opposite directions. Since resultant electric field is zero at point O, E_A and E_B are equal in magnitude *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{OA^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{OB^2}$$

$$\text{or}\quad \frac{16\times10^{-6}}{(x+0.08)^2} = \frac{9\times10^{-6}}{x^2} \quad\text{or}\quad \frac{(x+0.08)}{4} = \pm\frac{x}{3}$$

$$\text{or}\quad x = 0.24\text{ m} \quad\text{or}\quad -\frac{0.24}{7}\text{ m}$$

At $x = -\frac{0.24}{7}$ m (at point to the left of point B), the magnitudes of E_A and E_B are equal but the two fields will not cancel each other. It is because at this point, both the fields will be in the same direction. Hence, electric field is zero at a point at distance **0·24 m** from the charge of – 9 μC as shown in Fig. 2.18.

Problem 2.09. Point charges 4 × 10⁻⁶ C and 2 × 10⁻⁶ C are placed at the vertices A and B of a right angled triangle ABC respectively. B is the right angle, AC = 2 × 10⁻² m and BC = 10⁻² m. Find the magnitude and direction of the resultant electric intensity at C. *(Karnataka, 1990)*

Sol. Fig. 2.20 shows the right angled triangle ABC, such

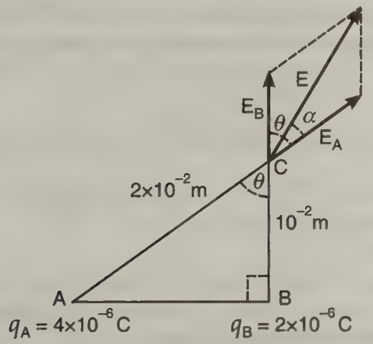


Fig. 2.20

that AC = 2 × 10⁻² m and BC = 10⁻² m. The charges of $q_A = 4\times10^{-6}$ and $q_B = 2\times10^{-6}$ C are placed at the vertices A and B respectively. Let E_A and E_B be electric intensity at point C due to charges q_A and q_B respectively. Then,

$$E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{AC^2} = 9\times10^9\times\frac{4\times10^{-6}}{(2\times10^{-2})^2}$$

$$= 9\times10^7\text{ N C}^{-1} \qquad \text{(along AC)}$$

$$E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{BC^2} = 9\times10^9\times\frac{2\times10^{-6}}{(10^{-2})^2}$$

$$= 18\times10^7\text{ N C}^{-1} \qquad \text{(along BC)}$$

If θ is angle between the directions of E_A and E_B, then

$$E = \sqrt{E_A^2 + E_B^2 + 2E_AE_B\cos\theta} \qquad ...(i)$$

In right angled Δ ABC, $\angle ACB = \theta$.

$$\therefore\quad \cos\theta = \frac{BC}{AC} = \frac{10^{-2}}{2\times10^{-2}} = 0.5 \quad\text{or}\quad \theta = 60°$$

Therefore, from equation (*i*), we have

$$E = \sqrt{(9\times10^7)^2 + (18\times10^7)^2 + 2\times9\times10^7\times18\times10^7\times0.5}$$

$$= 9\times10^7\sqrt{1+4+2} = \mathbf{2.38\times10^8\ N\ C^{-1}}$$

Suppose that the resultant electric intensity E makes an angle α with the line AC. Then,

$$\tan\alpha = \frac{E_A\sin\theta}{E_A+E_B\cos\theta} = \frac{18\times10^7\sin60°}{9\times10^7+18\times10^7\times\cos60°}$$

$$= \frac{18\times10^7\times0.866}{9\times10^7+18\times10^7\times0.5} = 0.866$$

or $\alpha = \mathbf{40.9°}$

Type D. On Electric field due to an electric dipole

Problem 2.10. A dipole of length 0·1 m consists of two charges of ± 500 μC. What is its electric dipole moment ? Calculate the electric field due to the dipole at a point on the axis distant 0·2 m from one of the charges in air.

Sol. Here, $q = 500\,\mu C = 500\times10^{-6}$ C ; $2a = 0.1$ m

Therefore, electric dipole moment of the dipole,

$$p = q\,(2a) = 500\times10^{-6}\times0.1 = \mathbf{50\times10^{-6}\ C\ m}$$

The electric field at a point on axial line at a distance r from the centre of the electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2pr}{(r^2-a^2)^2}$$

Here, $p = 50\times10^{-6}$ C m ; $a = 0.1/2 = 0.05$ m

and $r = 0.2 + 0.05 = 0.25$ m

$$\therefore\quad E = 9\times10^9\times\frac{2\times50\times10^{-6}\times0.25}{(0.25^2-0.05)^2}$$

$$= \mathbf{6.25\times10^7\ N\ C^{-1}}$$

Problem 2.11. Two charges of ± 0·2 $\mu\mu$C and – 0·2 $\mu\mu$C are placed 10⁻⁶ cm apart. Calculate the electric field at an axial point at a distance of 10 cm from their mid point. Use the standard value of ε_0. *(P.S.S.C.E. 1997)*

Sol. The two charges of + 0·2 $\mu\mu$C and – 0·2 $\mu\mu$C placed 10⁻⁶ cm apart constitute an electric dipole.

Thus, $q = 0.2\ \mu\mu C = 0.2\times10^{-12}$ C ;

$2a = 10^{-6}$ cm $= 10^{-8}$ m and $r = 10$ cm $= 0.1$ m

Since $a \ll r$, the electric field on the axial line of an electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}$$

Here, $p = q\,(2a) = 0.2\times10^{-12}\times10^{-8} = 0.2\times10^{-20}$ C m

Also, $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ N m² C⁻²

$$\therefore\quad E = \frac{9\times10^9\times2\times0.2\times10^{-20}}{(0.1)^3}$$

$$= \mathbf{3.6\times10^{-8}\ N\ C^{-1}}$$

Problem 2.12. Calculate the field due to an electric dipole of length 10 cm and consisting of charges of ± 100 μC at a point 20 cm from each charge.

Sol. Here, $q = 100\ \mu C = 100\times10^{-6}\text{ C} = 10^{-4}$ C

and $2a = 10$ cm $= 0.1$ m

Let P be the point at a distance of 20 cm from either charge as shown in Fig. 2.21.

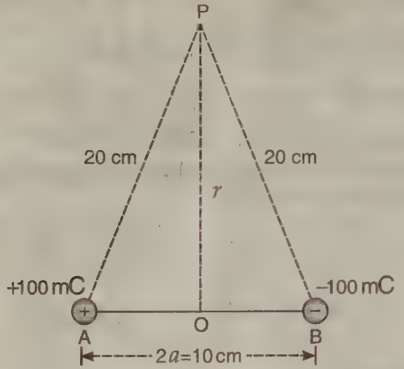


Fig. 2.21

Then, AP = BP = 20 cm = 0·2 m

Let E be electric field at point P due to the electric dipole. The point P lies at a distance OP = r on the equitorial line of the dipole. Therefore,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}} \quad ...(i)$$

Here, $p = q\,(2\,a) = 10^{-4} \times 0{\cdot}1$ C m

Also, $(OP^2 + OA^2)^{1/2} = AP$

or $(r^2 + a^2)^{1/2} = 0{\cdot}2$ m

Therefore from equation (*i*), we have

$$E = \frac{9 \times 10^9 \times 10^{-4} \times 0{\cdot}1}{(0{\cdot}2)^3} = \frac{9}{8} \times 10^7$$

$$= \mathbf{1{\cdot}125 \times 10^7\ N\ C^{-1}}$$

Type E. On Torque on an electric dipole

Problem 2.13. An electric dipole, when held at 30° wi respect to a uniform electric field of 10^4 N C^{-1} experienc a torque of 9 × 10^{26} N m. Calculate dipole moment of tl dipole. (*C.B.S.E. 199*

Sol. Here, E = 10^4 N C^{-1} ; $\tau = 9 \times 10^{-26}$ N m and $\theta = 3$

When electric dipole is placed at an angle θ with tl direction of the electric field, torque acting on the dipole,

$$\tau = p\,E \sin\theta$$

Therefore, electric moment of the dipole,

$$p = \frac{\tau}{E \sin\theta} = \frac{9 \times 10^{-26}}{10^4 \times \sin 30°} = \frac{9 \times 10^{-26}}{10^4 \times 0{\cdot}5}$$

$$= \mathbf{1{\cdot}8 \times 10^{-19}\ C\ m}$$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 2.14. A uniform electric field of strength 2 × 10^3 N C^{-1} is established between two parallel plates of length 0·1 m held horizontally at a distance 0·02 m apart. An electron is projected at a speed of 6 × 10^6 m s^{-1} making an angle 45° as shown in Fig. 2.22. The field is directed vertically upwards. Will the electron strike the either plate ? If it strikes the plate, where does it do so ?

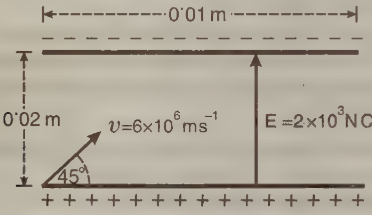


Fig. 2.22

Sol. The speed of electron can be resolved into the following two rectangular components :

(*i*) $v_x = v \cos 45° = 6 \times 10^6 \times 0{\cdot}707$

$= 4{\cdot}24 \times 10^6$ m s^{-1} (along horizontal)

As there is no force on electron along horizontal, the velocity component v_x along horizontal remains unchanged.

(*ii*) $v_y = v \sin 45° = 6 \times 10^6 \times 0{\cdot}707$

$= 4{\cdot}24 \times 10^6$ m s^{-1} (along vertical)

Since due to electric field, the electron experiences force in downward direction (opposite to its motion), the velocity component v_y along vertical goes on decreasing.

Force on electron, F = q E (downwards)

Acceleration of electron, $a = \frac{F}{m} = \frac{q\,E}{m}$

Here, $q = 1{\cdot}6 \times 10^{-19}$ C ; $m = 9{\cdot}1 \times 10^{-31}$ kg and E = 2 × 10^3 N C^{-1}

$$\therefore \quad a = \frac{1{\cdot}6 \times 10^{-19} \times 2 \times 10^3}{9{\cdot}1 \times 10^{-31}}$$

$= 3{\cdot}516 \times 10^{14}$ m s^{-2} (downwards)

For motion along vertical :

Initial velocity, $v_y = 4{\cdot}24 \times 10^{-6}$ m s^{-1} ;

$a = -\,3{\cdot}516 \times 10^{14}$ m s^{-2} and S = 0·02 m

If t is time taken by electron to reach the upper plate, th

$$S = v_y\,t + \frac{1}{2}\,a\,t^2$$

or $0{\cdot}02 = 4{\cdot}24 \times 10^6 \times t + \frac{1}{2}\,(-\,3{\cdot}516 \times 10^{14})\,t^2$

or $1{\cdot}758 \times 10^{14}\,t^2 - 4{\cdot}24 \times 10^6\,t + 0{\cdot}02 = 0$

On solving for t, we have

$t = 6{\cdot}4 \times 10^{-9}$ s

Since t has a finite real value, the electron will strike t upper plate. Suppose that it strikes the upper plate at distance x from the left end. Then,

$x = v_x \times t$

$= 4{\cdot}24 \times 10^6 \times 6{\cdot}4 \times 10^{-19} = 2{\cdot}71 \times 10^{-2}$ m = **2·71 c**

Problem 2.15. An infinite number of charges, ea equal to q are placed along X-axis at x = 1, x = 2, x = x = 8, ... and so on.

(*a*) Find electric field at the point x = 0 due to this s of charges.

(*b*) What will be the electric field, if in the above s up, the consecutive charges have opposite sign ?

(*I.I.T. 1995, 197*

Sol. Suppose that the charges $q_1, q_2, q_3, q_4, ...$ are plac at distances $r_1, r_2, r_3, r_4, ...$ from the origin. Then, electric fie at the origin due to the system of charges,

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \frac{q_3}{r_3^2} + \frac{q_4}{r_4^2} + ...\right]$$

(*a*) Here, $q_1 = q_2 = q_3 = q_4 = ... = q$

and $r_1 = 1, r_2 = 2, r_3 = 4, r_4 = 8, ...$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\frac{q}{8^2}+.....\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+.....\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\cdot S_\infty,$$

where $S_\infty = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+.....$

i.e. sum of a geometrical progression, whose first term $a = 1$ and the common ratio $r = 1/4$. Therefore,

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-1/4} = \frac{4}{3}$$

Hence, $E = \frac{1}{4\pi\varepsilon_0}\cdot q \times \frac{4}{3} = \frac{q}{3\pi\varepsilon_0}$

(*b*) When the consecutive charges have opposite sign,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q}{1^2}-\frac{q}{2^2}+\frac{q}{4^2}-\frac{q}{8^2}+.....\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{4^2}-\frac{1}{8^2}+.....\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\,\frac{1}{1-(1/4)} = \frac{q}{5\pi\varepsilon_0}$$

Proplem 2.16. Four points charges, each having a charge *q* are placed on the four corners A, B, C and D of a regular pentagon ABCDE. The distance of each corner from the centre is *a*. Find the electric field at the centre of the pentagon.

Sol. Fig. 2.23 shows a regular pentagon ABCDE with O as its centre. Four point charges, each of charge *q* are placed at the corners A, B, C and D of the pentagon.

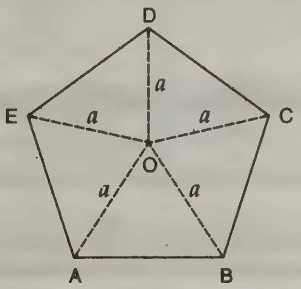


Fig. 2.23

If we put a charge *q* at the corner E also, then due to symmetric arrangement of the charges about the point O, the electric field O will be zero. It follows that electric field at point O due to the charges at four corners A, B, C and D must be equal and opposite to the field due to the charge *q* placed at corner E alone.

The electric field at point O due to the charge *q* at corner E,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2} \text{ (along EO)}$$

Therefore, electric field due to the four charges placed at corners A, B, C and D is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2} \textbf{ (along OE)}$$

Problem 2.17. In a certain region of space, electric field is along the Z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive Z-direction at the rate of 10^5 N C^{-1} m^{-1}. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} C m in the negative Z-direction ? (*Text Problem*)

Sol. Suppose that a dipole consisting of charges $-q$ at A and $+q$ at B is placed along Z-axis, such that it has dipole moment along negative Z-direction [Fig. 2.24]

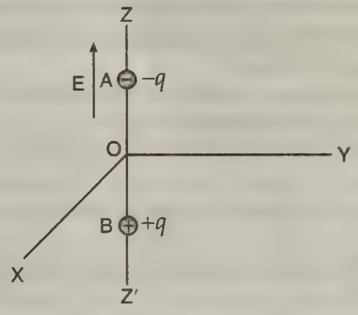


Fig. 2.24

The dipole moment of the dipole along Z-axis,

$$p_z = -10^{-7} \text{ C m}$$

The negative sign has been taken for the reason that dipole moment is along negative Z-axis.

The electric field is applied along the positive direction of Z-axis, such that $\frac{\partial E}{\partial z} = 10^5$ N C^{-1} m^{-1} (along Z-axis)

In a non-uniform electric field, the force on dipole is given by

$$F = p_x\frac{\partial E}{\partial x} + p_y\frac{\partial E}{\partial y} + p_z\frac{\partial E}{\partial z}$$

Here, $p_x = p_y = 0$ Also, $\frac{\partial E}{\partial x} = \frac{\partial E}{\partial y} = 0$

$$\therefore \quad F = p_z\frac{\partial E}{\partial z} = -10^{-7}\times 10^5 = \mathbf{-10^{-2}\ N}$$

The negative sign shows that force on the dipole is along negative Z-axis.

Since the electric dipole moment is along negative Z-axis and the electric field is along positive Z-axis, $\theta = 180°$.

Hence, torque on the dipole, $\tau = p\,E\,\sin 180° = 0$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Is electric field intensity a scalar or a vector quantity. Give its SI units. (*C.B.S.E. 1999 S*)

Ans. Electric field intensity is a vector quantity. Its SI unit is **N C^{-1} or V m^{-1}.**

Q. 2.02. The test charge used to measure electric field at a point should be vanishingly small. Why ?

Ans. In case, test charge is not vanishingly small, it will produce its own electric field and the measured value of electric field will be different from the actual value of electric field at that point.

Q. 2.03. Define electric field intensity at a point. *(P.S.S.C.E. 2000 ; C.B.S.E 1991)*

Ans. Refer to section 2.02.

Q. 2.04. Does an electric charge experience a force due to the field, it produces itself ?

Ans. No, an electric charge does not experience any force due to the electric field it produces itself.

Q. 2.05. Name the physical quantity, whose SI unit is newton coulomb^{-1}. *(C.B.S.E 1998)*

Ans. It is unit of electric field intensity.

Q. 2.06. A point charge q is placed at the origin. How does the electric field due to the charge vary with distance r from the origin ? *(Text Question)*

Ans. The electric field varies inversely as the square of the distance from the point charge.

Q. 2.07. Define electric dipole and electric dipole moment. *(H.S.S.C.E. 1996)*

Ans. For definitions, refer to sections 2.07 and 2.08.

Q. 2.08. What is the direction of electric dipole moment vector of an electric dipole ?

Ans. From charge $-q$ to $+q$ *i.e.* same as that of vector representing length of the electric dipole.

Q. 2.09. Give the SI unit of electric dipole moment. *(C.B.S.E. 1992)*

Ans. The unit of electric dipole moment is coulomb metre (C m).

Q. 2.10. Is it correct to write the unit of electric dipole moment as mC ?

Ans. No, it is not correct to write the unit of electric dipole moment as mC. The symbol mC represents millicoulomb *i.e.* unit of electric charge.

Q. 2.11. What is the direction of electric field at a point on axial line of an electric dipole ?

Ans. Same as that of electric dipole moment vector.

Q. 2.12. What is the direction of electric field at a point on the equitorial line of an electric dipole ?

Ans. Opposite to that of electric dipole moment vecto

Q. 2.13. What do electric lines of force represent ? *(P.S.S.C.E. 1997 S*

Ans. Electric lines of force represent the direction of th electric field.

Q. 2.14. Draw the lines of force to represent a uniforr electric field. *(C.B.S.E. 1995, 1992*

Ans. A uniform electric field is represented by paralle arrows drawn in the same direction.

Q. 2.15. Why two electric lines of force cannot intersec each other ? *(H.P.S.S.C.E. 2002 ; P.S.S.C.E. 2002, 2001 H.S.S.C.E. 2001 ; C.B.S.E. 1992*

Ans. The tangent at a point on the line of force give the direction of electric field at that point. If two lines of forc intersect each other at a point, then electric field at that poir will have two directions. As the same cannot be ture, two line of force can never intersect each other.

Q. 2.16. Does an electric dipole always experience torque, when placed in a uniform electric field ?

Ans. No. It does not experience a torque, when it i placed along the direction of electric field.

Q. 2.17. What is the net force on an electric dipol placed in a uniform electric field ?

Ans. An electric dipole does not experience any net forc in a uniform electric field.

Q. 2.18. When is the torque acting on an electric dipol maximum, when placed in uniform electric field ?

Ans. The torque is maximum, when the electric dipol is placed perpendicular to the direction of electric field.

Q. 2.19. What orientation of an electric dipole in uniform electric field corresponds to its stable equilibrium *(C.B.S.E. 2001*

Ans. When the electric dipole aligns itself along th direction of electric field ($\vec{p}$ parallel to $\vec{E}$), the torque on th dipole becomes zero. Obviously, this orientation of the electri dipole (length parallel to the electric field) corresponds to it stable equilibrium.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Determine the magnitude of an electric field that will balance the weight of an electron. *(H.S.S.C.E. 2002, 1995)*

Ans. Let E be the electric field that will balance the weight of electron. Then,

$$q\,E = m\,g$$

Here, $q = 1{\cdot}6 \times 10^{-19}$ C ; $m = 9{\cdot}1 \times 10^{-31}$ kg

$$\therefore \quad E = \frac{m\,g}{q} = \frac{9{\cdot}1 \times 10^{-31} \times 9{\cdot}8}{1{\cdot}6 \times 10^{-19}}$$

$$= \mathbf{5{\cdot}574 \times 10^{-11}\ N\ C^{-1}} \text{ (vertically downwards)}$$

Q. 2.02. Two point electric charges of unknown magnitude and sign are placed a distance apart. The electric field intensity is zero at a point not between the charges but on the line joining them. Write two essential conditions for this to happen. *(C.B.S.E. 1997)*

Ans. 1. Both the charges can not be of same sign.

2. The observation point (where electric field intensity is zero) has to be closer to the smaller charge than to the bigge charge.

Q. 2.03. An electric dipole is formed by + 6 μC and – μC charges at 5 mm distance. Calculate the dipole momen *(H.S.S.C.E. 2001*

Ans. Here, $q = +\,6\ \mu\text{C} = \pm\,6 \times 10^{-6}$ C,

$$2\,a = 5 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

Therefore, electron dipole moment,

$$p = q\,(2\,a) = 6 \times 10^{-6} \times 5 \times 10^{-3} = \mathbf{3 \times 10^{-8}\ C\ m}$$

Q. 2.04. What is an ideal electric dipole ? *(C.B.S.E. 1992*

Ans. Refer to section 2.08.

Q. 2.05. What is the ratio of the strength of electric fiel at a point on axial line and at a point at same distance o equitorial line of an electric dipole of very small length ?

Ans. Electric fields at distance r on axial line an equitorial line due to an electric dipole of very small lengt are given by

$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \quad \text{and} \quad E_{equi} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$$

$$\therefore \frac{E_{axial}}{E_{equi}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \Big/ \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3} = 2$$

Q. 2.06. Define intensity of electric field at a point. At what points is the electric dipole field intensity parallel to the line joining the charges ? *(C.B.S.E. 1997)*

Or

Two point charges + q and – q are placed at a distance d apart. What are points at which the resultant field is parallel to the line joining the two charges ? *(I.I.T. 1978)*

Ans. For definition of intensity of electric field, refer to section 2.02.

Electric field intensity due to an electric dipole is parallel to the line joining the charges at points

(*i*) on axial line of the dipole and

(*ii*) on the plane, which is right bisector of the dipole.

Q. 2.07. A system has two charges $q_A = 2{\cdot}5 \times 10^{-7}$ C and $q_B = -2{\cdot}5 \times 10^{-7}$ C located at the points A (0, 0, –15 cm) and B (0, 0, +15 cm). What is the total charge and electric dipole moment ? *(Text Question)*

Ans. The charges q_A and q_B are located at points A (0, 0, –15 cm) and B (0, 0,+15 cm). The points A and B lie on Z-axis as shown in Fig. 2.25.

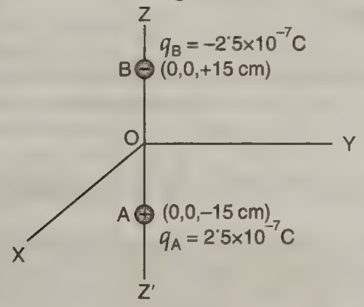

Z
$q_B = -2{\cdot}5 \times 10^{-7}$ C
B
(0,0,+15 cm)
O
Y
A
(0,0,–15 cm)
$q_A = 2{\cdot}5 \times 10^{-7}$ C
X
Z′

Fig. 2.25

It follows that

AB = OA + OB = 15 + 15 = 30 cm = 0·3 m

Total charge, $q = q_A + q_B$

$= 2{\cdot}5 \times 10^{-7} + (-2{\cdot}5 \times 10^{-7}) = 0$

Electric dipole moment, p = either charge × AB

$= 2{\cdot}5 \times 10^{-7} \times 0{\cdot}3 = \mathbf{7{\cdot}5 \times 10^{-8}\ C\ m}$

The electric dipole moment is directed from B to A along Z-axis.

Q. 2.08. What is meant by the statement that the electric field of a point charge has spherical symmetry, whereas that of an electric dipole is cylindrically symmetrical ? *(H.S.S.C.E. 1996)*

Ans. The electric field due to a point charge q at a distance r from it is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

All such points, where electric field is equal to E (as given above) lie on the surface of a sphere of radius r, with charge q at its centre. For this reason, the electric field due to a point charge is said to be spherically symmetric.

The electric field at a distance r on the equitorial line of an electric dipole of dipole moment p and length $2a$ is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(r^2+a^2)^{3/2}}$$

All such points lie on the surface of a cylinder of radius r drawn with electric dipole as its axis. For this reason, the electric field due to an electric dipole is said to be cylindrically symmetric.

Q. 2.09. What is electric line of force ? What is its importance ? *(P.S.S.C.E. 1998 S)*

Ans. An electric line of force is the path along which a unit positive charge will move inside the electric field, if it is free to do so.

The importance of electric line of force is that the tangent at any point to it gives the direction of electric field at that point. Further, the relative closeness of the electric lines of force in different regions of an electric field provides an estimate of the strength of the electric field in those regions.

Q. 2.10. Give two properties of electric lines of force. Sketch them for an isolated positive charge. *(C.B.S.E. 1992)*

Ans. For properties of electric lines of force, refer to section 2.08 and for sketch of electric lines of force for an isolated positive charge, refer to Fig. 2.09 (*a*).

Q. 2.11. Sketch the electric lines of force due to point charges (*i*) $q > 0$ and (*ii*) $q < 0$. *(C.B.S.E. 1995)*

Ans. (*i*) For a charge $q > 0$ *i.e.* for a positive charge, the electric lines of force are as shown in Fig. 2.09 (*a*).

(*ii*) For a charge $q < 0$ *i.e.* for a negative charge, the electric lines of force are is as shown in Fig. 2.09 (*b*).

It may be noted that in case of a negative charge, the lines of force terminate at the charge ; while in case of a positive charge, the lines of force originate from it.

Q. 2.12. Electric field intensity within a conductor is always zero. Why ?

Ans. The electric lines of force cannot pass through a conductor. As a result, there can be no electric field inside a conductor. Hence, electric field intensity inside a conductor is zero.

Q. 2.13. Are the field lines a reality ?

Ans. The electric field lines are purely geometrical constructions, which are used to represent electric field graphically. In other words, the electric field lines themselves are imaginary, but the electric field they represent is real.

Q. 2.14. An electric dipole with electric dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 N C^{-1}. Calculate the magnitude of the torque acting on the dipole. *(Text Question)*

Ans. Here, $p = 4 \times 10^{-9}$ C m ; $E = 5 \times 10^4$ N C^{-1} ; $\theta = 30°$

Now, magnitude of torque acting on the dipole,

$$\tau = p\,E \sin\theta = 4 \times 10^{-9} \times 5 \times 10^4 \sin 30°$$

$$= 2 \times 10^{-4} \times \frac{1}{2} = \mathbf{10^{-4}\ N\ m}$$

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 2.01. In defining electric field due to a point charge, the test charge has to be vanishingly small. How this condition can be justified, when we know that charge less than that on an electron or a proton is not possible ? *(Text Question)*

Ans. In the macroscopic situation, the source charges are always very large as compared to charge on an electron or a proton. Therefore, for a test charge having charge equal to that of an electron or a proton, the condition that the test charge $q_0 \to 0$ is justified for source charges of macroscopic sizes.

Q. 2.02. A ball of charge q is placed in a hollow conductive uncharged sphere. After this, the sphere is connected with earth for a short time and the ball is then removed from the sphere. The ball has not been brought into contact with the sphere.

(*a*) What charge will the sphere have after these operations ? Where and how will this charge be distributed ? (*b*) What will be the nature of the field and how will it be located ?

Ans. (*a*) The charge q will be distributed on the external surface of the sphere.

(*b*) The intensity of the electric field inside the sphere will be zero. An electric field similar to the field of the point charge q situated at the centre of the sphere will be set up outside the sphere.

Q. 2.03. Fig. 2.26 shows tracks of three charged particles crossing a uniform electrostatic field with same velocities along horizontal. Give the signs of the three charges. Which particle has the highest charge to mass ratio ? (*C.B.S.E. 2001 S ; Text Question*)

Ans. Charges 1 and 2 are deflected towards the positive plate, hence they are negatively charged particles. On the other hand, charge 3 is positively charged particle, as it gets deflected towards negative plate [Fig. 2.26].

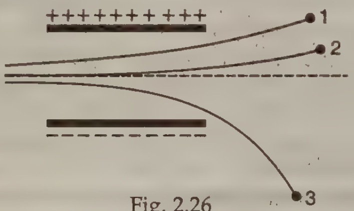


Fig. 2.26

Since all the three charged particles are crossing the electric field with same velocities, they remain under the action of electric field E (say) for the same time t. The, deflection produced in the path of a charged particle along the vertical,

$$y = u\,t + \frac{1}{2}a\,t^2$$

Now, the initial velocity of the charged particle along vertical is zero. If e is charge on a particle and m is its mass, then acceleration produced in its path,

$$a = \frac{e\,E}{m}$$

$$\therefore \qquad y = 0 \times t + \frac{1}{2}\frac{e\,E}{m}t^2 = \frac{1}{2}\frac{e\,E}{m}t^2$$

As E and t are same for all the charged particles, it follows that

$$\frac{e}{m} \propto y$$

Since the charged particle 3 suffers maximum deflection along vertical, its charge to mass ratio is highest.

Q. 2.04. A small metal ball of mass m is suspended from a thread of length l between the plates of a large plane capacitor. How will the period of oscillations of such a pendulum change, if a charge $+q$ is placed on the ball and the upper plate is (*a*) positively charged (*b*) negatively charged ?

Ans. (*a*) Fig. 2.27 shows the forces on the metal ball, whe the upper plate of the capacitor is positively charged. If E i strength of the electric field between the plates, then apparen weight of the ball

$$m\,g' = m\,g + q\,E$$

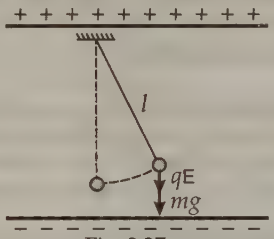


Fig. 2.27

or $\qquad g' = g + \dfrac{q\,E}{m}$

The period of oscillation of the ball is given by

$$t = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g + q\,E/m}}$$

Since $(g + q\,E/m) > g$, the time period of the ball wi decrease.

(*b*) When the upper plate of the capacitor is mad negative, then

$$m\,g' = m\,g - q\,E$$

Obviously, the period of oscillation of the ball will be

$$t = 2\pi\sqrt{\frac{l}{g - q\,E/m}}$$

Since $(g - q\,E/m) < g$, the time period of the ball wi increase.

Q. 2.05. A metal ball suspended from a long threa is placed between the plates of a capacitor as show in Fig. 2.28 (*a*). How will the oscillations of this pendulu change [Fig. 2.28 (*b*)], if the ball and the plates of th capacitor are charged ?

Ans. Suppose that the ball is given charge $+q$ and th plate A of the capacitor is made positive, while the plate B made negative [Fig. 2.28 (*b*)]. The ball will experience a forc q E (along horizontal) and force mg (along vertical).

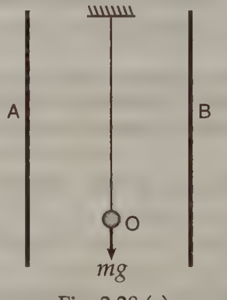


Fig. 2.28 (*a*)

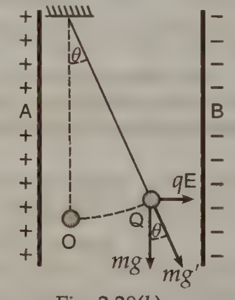


Fig. 2.28(*b*)

The equilibrium position of the ball will change from to Q. If θ is the angle, the string makes with vertical, then

$$\tan\theta = \frac{q\,E}{mg}$$

The apparent weight of the ball is given by

$$mg' = \sqrt{(mg)^2 + (q\,E)^2}$$

or $\qquad g' = \sqrt{g^2 + (q\,E/m)^2}$

The period of the oscillation of the ball is given by

$$t = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + (q\,E/m)^2}}}$$

Since $\sqrt{g^2 + (q\,E/m)^2} > g$, the time period of the ball **will decrease.**

Q. 2.06. A charged particle is free to move in an electric field. Will it always move along an electric line of force ? (*I.I.T. 1979*)

Ans. The charged particle may or may not move along an electric line of force. If the charged particle was initially at rest, it will move along an electric line of force. In case the charged particle had some initial velocity making certain angle with a line of force, then its resultant path will not be along the line of force.

Q. 2.07. A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the line of force passing through that point ? (*Text Question*)

Ans. The small test charge will travel along the line of force only, if the line of force is a straight line. In case the line of force is curved, it will not move along the line of force. The reason is that line of force does not give the direction of velocity. It gives the direction, in which charge is accelerated.

Q. 2.08. An electrostatic line of force is a continuous curve *i.e.* a line of force cannot have sudden breaks. Why not ? (*Text Question*)

Ans. The tangent at a point on the electric line of force tells the direction of electric field at that point. Generally, the direction of electric field changes from point to point. Therefore, the lines of force are, generally, curved lines. Further, they are continuous curves and cannot have sudden breaks. For if it is so, it will indicate the absence of electric field at the break points.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain the concept of electric field. (*P.S.S.C.E. 1990*)
2. Establish the relation between electric field strength and force. (*H.S.S.C.E. 1994 ; P.S.S.C.E. 1990*)
3. Derive an expression for electric field intensity at a point at distance *r* from a point electric charge. (*H.S.S.C.E. 1999, 1995 ; C.B.S.E. 1993*)
4. Derive an expression for the electric field intensity at any point on the axis of a uniformly charged ring. (*P.S.S.C.E. 2002*)
5. What is an electric dipole ? Define electric dipole moment and give its units. (*H.S.S.C.E. 2002*)
6. Define the term electric dipole moment. Give its units. Derive an expression for the maximum torque acting on an electric dipole, when held in a uniform electric field. (*C.B.S.E. 2002*)
7. Derive an expression for the electric field intensity at any point along the axial line of an electric dipole. (*H.P.S.S.C.E. 2002 ; P.S.S.C.E. 1999 S ; C.B.S.E. 1995*)
8. What is an electric dipole ? Calculate electric field intensity at a point on the axial line of a dipole. (*H.S.S.C.E. 2002*)
9. Define electric dipole moment. Calculate electric field intesnity at a point on the equitorial line of a dipole. (*H.S.S.C.E. 2002*)
10. Derive a relation for the intensity of electric field at an equitorial point of an electric dipole. (*H.P.S.S.C.E. 2000 ; P.S.S.C.E. 1999 S, 1997 S ; H.S.S.C.E. 1999 S*)
11. Find the electric field at a point on the right bisector line of the electric dipole. (*H.S.S.C.E. 1998 S*)
12. Find an expression for electric intensity due to a short electric dipole at any point situated along a line inclined at an angle θ to the dipole axis. (*H.S.S.C.E. 2002*)
13. Find out electric field intensity at any far point due to an electric dipole. (*H.S.S.C.E. 2001*)
14. Explain the properties of electric lines of force. (*H.S.S.C.E. 1999 S*)
15. What is meant by electric lines of force ? Give their important properties. (*H.S.S.C.E. 1992*)
16. What do electric lines of force represent ? Explain repulsion between two like charges on their basis. (*P.S.S.C.E. 1997 S*)
17. Derive an expression for the torque experienced by an electric dipole placed in a uniform electric field. (*H.P.S.S.C.E. 2002 ; H.S.S.C.E. 2001, 1998 S*)
18. Draw a labelled diagram showing an electric dipole making an angle θ with a uniform electric field E. Derive an expression for the torque experienced by the dipole. (*I.S.C.E. 1995*)
19. An electric dipole is placed in uniform field $\vec{E}$. Prove that torque $\vec{\tau}$ acting on the dipole is given by
$$\vec{\tau} = \vec{p} \times \vec{E},$$
where $\vec{p}$ is dipole strength of the dipole. (*P.S.S.C.E. 1995*)
20. Derive an expression for the torque experienced by an electric dipole placed in a uniform electric field. What is the net force acting on this electric dipole ? (*P.S.S.C.E. 2001*)
21. Find an expression for the torque experienced by an electric dipole placed in a uniform electric field. Hence, define electric dipole moment. (*H.S.S.C.E. 2002*)
22. Derive an expression for the torque acting on an electric dipole suspended freely in a uniform electric field. How will you determine the direction of torque ? (*P.S.S.C.E. 2000*)
23. An electric dipole free to move is placed in an uniform electric field. Explain with a diagram, its motion, when it is placed (*a*) parallel to the field (*b*) perpendicular to the field. (*C.B.S.E. Sample Q. Paper*)

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Obtain expressions for electric field due to (*i*) a point charge and (*ii*) a continuous distribution of charges along a line.
2. Obtain expression for electric field due to a continuous distribution of charge over (*i*) a surface and (*ii*) a volume.
3. What is an electric dipole ? Obtain an expression for the electric field due to an electric dipole at a point on the line passing through the centre of the dipole and perpendicular to the dipole axis. (*I.S.C.E. 1994*)

4. Define electric field intensity and derive an expression for it at a point on the neutral axis of a dipole. Also determine its direction. *(P.S.S.C.E. 2000)*
5. Define electric intensity at a point and find its value at a point due to an electric dipole, when the point is on the equitorial line. *(H.S.S.C.E. 1992)*
6. Prove that for a short dipole, the intensity at a point on the axial line is twice that on the equitorial line.
7. Define electric field at a point. Two point charges q and $-q$ are placed at a distance $2\,a$ apart. Calculate the electric field at a point P situated at a distance r along the perpendicular bisector of the line joining the charges. What is the field, when $r >> a$? *(C.B.S.E. 1990)*
8. A circular loop of charge is placed in YZ-plane with it centre at origin. Find expression for electric field at a poir on X-axis.
9. A charge is distributed uniformly over a ring of radius Obtain an expression for the electric intensity $\vec{E}$ at a poir on the axis of the ring. Hence, show that for points at larg distances, the ring behaves as a point charge. *(C.B.S.E. Sample Q. Paper*
10. Show that in a uniform electric field, a dipole experience only a torque but no net force. Derive expression for th torque. *(C.B.S.E. 1999 S, 1992*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Force experienced by a charge in an electric field

1. In the electric field shown in Fig. 2.29, the electric field lines on the left have twice the separation as that between those on the right. If the magnitude of the field at point A is 40 N C^{-1}, calculate the force experienced by a proton placed at point A. Also find the magnitude of electric field at point B. **[Ans.** $6{\cdot}4 \times 10^{-18}$ N, 20 N C^{-1}]

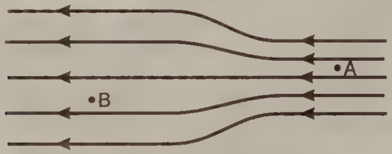


Fig. 2.29

2. Calculate the magnitude and direction of the electric field, which keeps a proton just floating. Given that mass of proton $1{\cdot}67 \times 10^{-27}$ kg, charge on proton = $1{\cdot}6 \times 10^{-19}$ C and $g = 9{\cdot}8$ m s^{-2}.
   [Ans. $1{\cdot}023 \times 10^{-7}$ N C^{-1} (vertically upward)]
3. Calculate the magnitude of electric field, which can just balance a deutron of mass $3{\cdot}2 \times 10^{-27}$ kg. Take $g = 10$ m s^{-2}. *(P.S.S.C.E. 1999)* **[Ans.** $2{\cdot}0 \times 10^{-7}$ N C^{-1}]
4. An electron above the earth is balanced by the gravitational force and the electric field of the earth. Find the electric field of the earth. **[Ans.** $5{\cdot}57 \times 10^{-11}$ N C^{-1}]
5. How many electrons should be removed from a coin of mass 1·6 g, so that it may float in an electric field of intensity 10^9 N C^{-1} directed upward. *(P.S.S.C.E. 1998 S)* **[Ans.** $9{\cdot}8 \times 10^7$]
6. An electron is released with a velocity of 5×10^6 m s^{-1} in an electric field of 10^3 N C^{-1} which has been applied so as to oppose its motion. What distance would the electron travel and how much time could it take before it is brought to rest ? **[Ans.** $7{\cdot}11 \times 10^{-2}$ m, $2{\cdot}844 \times 10^{-8}$ s]
7. An electron falls through a distance of 1·5 cm in a uniform electric field of magnitude $2{\cdot}0 \times 10^4$ N C^{-1} N [Fig. 2.30 (*a*)]. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [Fig. 2.30 (*b*)]. Compute the time of fall in each case. Given that mass of electron = $9{\cdot}1 \times 10^{-31}$ kg and mass of proto = $1{\cdot}67 \times 10^{-27}$ kg. *(Text Problem*
   [Ans. (*a*) $2{\cdot}92 \times 10^{-9}$ s (*b*) $1{\cdot}566 \times 10^{-7}$

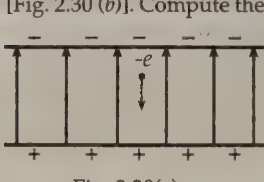


Fig. 2.30(*a*)

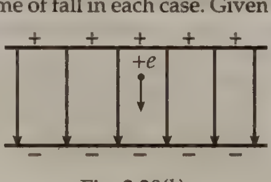


Fig. 2.30(*b*)

8. A charged particle of mass 1 g is suspended through silk thread of length 40 cm in a horizonal electric fiel of $4{\cdot}0 \times 10^4$ N C^{-1}. If the particle stays at a distance 24 cm from the wall in equilibrium, find the charge o the particle. **[Ans.** $1{\cdot}838 \times 10^{-7}$ C

Type B. On Electric field due to a group of point charges

9. Two point charges of + 20 μC and + 80 μC are place 18 cm apart. Find the position of the point, where electri field is zero.
   [Ans. at a distance 0·06 m from 20 μC charg (between the two changes) or at a distance 0·18 from 20 μC charge (on the other side of it or at 18 cm away from 20 μC on the other side of 80 μC
10. Three charges, each equal to q, are placed at the thre corners of a square of side a. Find the electric field at th fourth corner of the square.
    $$\left[\textbf{Ans. } \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sqrt{9+4\sqrt{2}}\,q}{2a^2}\right.$$
11. Fig. 2.31 shows four point charges at the corners of a squar of side 2 cm. Find the magnitude and direction of th electric field at the centre O of the square, if Q = 0·02 μC
    Use $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}.

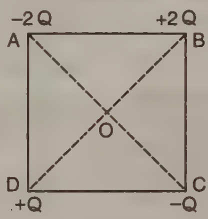


Fig. 2.31

[Ans. $9\sqrt{2} \times 10^5$ N C^{-1}, (parallel to side BA

12. Four charges $+q, +q, -q, -q$ are placed respectively at th four corners of a square of side a. Find the magnitude an direction of the electric field at the centre of the square. *(P.S.S.C.E. 1996 S*
    [Ans. $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{4\sqrt{2}\,q}{a^2}$, parallel to the side having charge $+q$ and $-q$ at its two ends

13. ABC is an equilateral triangle of side 5 cm. Charges of + 60 statC and – 30 statC are placed at points A and B respectively. Calculate completely the electric field at point C. Given, 1 C = 3 × 10^9 statC. [**Ans.** 6·24 × 10^4 N C^{-1} making 90° with CB and 30° with AC (when produced)]

14. ABC is a right-angled triangle, the right angle being at point B. Charges of –10·24, 11·52, + 3·24 $\mu\mu$C are placed at points A, B and C respectively. If AB = 4 cm and BC = 3 cm, calculate the magnitude and direction of the resultant electric field at the foot D of the perpendicular drawn from point B on the side AC.

[**Ans.** $180\sqrt{2}$ N C^{-1}, inclined equally with both DA and BD (when produced)]

ype C. On Electric field due to an electric dipole

15. Two point charges of 2 μC but opposite in sign are placed 10 cm apart. Calculate the electric field at a point distant 10 cm from the mid point on the axial line of the dipole. (*P.S.S.C.E. 2002*) [**Ans.** 6·4 × 10^6 N C^{-1}]

16. Two charges each of 0·1 μC but opposite in sign are 1 mm apart. What is the electric field at a point on the line joining them at a distance of 10 cm from the mid-point ? [**Ans.** 1800 N C^{-1}]

17. Two charges of + 25 × 10^{-9} coulomb and – 25 × 10^{-9} coulomb are placed 6 m apart. Find the electric field at a point 4 m from the centre of the electric dipole (*i*) on axial line (*ii*) on equitorial line.

[**Ans.** (*i*) 220·4 N C^{-1} (*ii*) 10·8 N C^{-1}]

18. Two charges ± 10 μC are placed 5·0 mm apart. Determine the electric field at (*a*) a point on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, (*b*) a point 15 cm away from point O on a line passing through O and normal to the axis of the dipole.

[**Ans.** (*a*) 2·67 × 10^5 N C^{-1} ; (*b*) 1·33 × 10^5 N C^{-1}]

19. The electric field at a point on the axial line at a distance of 10 cm from the centre of an electric dipole is 3·75 × 10^5 N C^{-1} in air, while at a distance of 20 cm, the electric field is 3 × 10^4 N C^{-1}. Calculate the length of the electric dipole. [**Ans.** 0·1 m]

ype D. On Torque on an electric dipole

20. An electric dipole of dipole moment 4 × 10^{-5} C m is placed in a uniform electric field of 10^{-3} N C^{-1} making an angle of 30° from direction of field. Determine the torque exerted by the electric field on the dipole. (*H.S.S.C.E. 2002*) [**Ans.** 2 × 10^{-8} N m]

21. A dipole consisting of an electron and a proton separated by a distance 4 × 10^{-10} m is situated in an electric field of intensity 3 × 10^5 N C^{-1} at an angle of 30° with the field. Calculate the dipole moment and the torque acting on it. (Charge on an electron = 1·602 × 10^{-19} coulomb). (*Pre-degree Kerala, 1994*)

[**Ans.** 6·41 × 10^{-29} C m, 9·615 × 10^{-24} N m]

22. An electric dipole, when held at 30° with respect to an uniform electric field of 3 × 10^4 N C^{-1} experiences a torque of 27 × 10^{26} N m. Calculate the dipole moment of the dipole. (*C.B.S.E. 1996*) [**Ans.** 1·8 × 10^{-29} C m]

23. An electric dipole is placed at angle of 60° with an electric field of strength 4 × 10^{-5} N C^{-1}. It experiences a torque equal to $8\sqrt{3}$ N m. Calculate the charge on the dipole, if dipole is of length 4 cm. [**Ans.** 10^{-3} C]

MISCELLANEOUS PROBLEMS

24. A copper ball of density 8·6 g cm^{-3}, 1 cm in diameter is immersed in oil of density 0·8 g cm^{-3}. If the ball remains suspended in oil in a uniform electric field of intensity 36000 N C^{-1} acting in upward direction, what is the charge on the ball ? [**Ans.** 1·11 × 10^{-6} C]

25. A charge of 4 × 10^{-9} C is distributed uniformly over the circumference of a conducting ring of radius 0·3 m. Calculate the field intensity at a point on the axis of the ring at 0·4 m from its centre. Also, calculate the electric field at the centre of the ring. (*Roorkee 1990*)

[**Ans.** 115·2 N C^{-1}, zero]

26. A dipole consists of two charges +10 μC and –10 μC separated by a certain distance. Let they be located at x = 6·0 cm, y = 0 and x = –6·0 cm, y = 0 respectively. Calculate the field strength at a point x = 0, y = 8 cm.

[**Ans.** 1·08 × 10^7 N C^{-1}, parallel to the line joining the two charges]

27. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field of 100 V m^{-1} as shown in Fig. 2.32. A particle of mass 1 kg and charge 0·01 C is allowed to slide down from rest from a height of 1 m. If the coefficient of friction is 0·2, find the time it will take the particle to reach the bottom. (*Roorkee, 1987*) [**Ans.** 1·337 s]

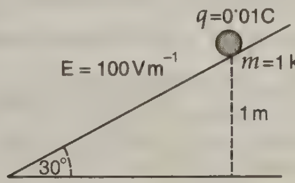


Fig. 2.32

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Force on proton at point A,

$$F = q\,E_A = 1{\cdot}6 \times 10^{-19} \times 40 = \mathbf{6{\cdot}4 \times 10^{-18}\ N}$$

Since separation between electric field lines at point B is twice as that point A, electric field at point B,

$$E_B = 40 \times \frac{1}{2} = \mathbf{20\ N\ C^{-1}}$$

2. Proceed as in solved problem no. 2.01.

3. The direction of electric field is vertically upwards and charge on deutron, $q = e = 1{\cdot}6 \times 10^{-19}$ C

Now, proceed as in solved problem no. 2.01.

4. Proceed as in solved problem no. 2.01.

5. Let n be the number of electrons removed from the coin. Then, charge on coin, $q = +\,n\,e$

Now, $q\,E = m\,g$ or $(n\,e)\,E = m\,g$

or $$n = \frac{m\,g}{e\,E} = \frac{1{\cdot}6 \times 10^{-3} \times 9{\cdot}8}{1{\cdot}6 \times 10^{-19} \times 10^{9}} = \mathbf{9{\cdot}8 \times 10^7}$$

6. Since electric field is applied so as to oppose the motion of electron,

$$a = -\frac{e\,E}{m} \text{ (retardation)}$$

$$= -\frac{1{\cdot}6 \times 10^{-19} \times 10^3}{9{\cdot}1 \times 10^{-31}} = -1{\cdot}758 \times 10^{14}\ \text{m s}^{-2}$$

Now, $u = 5 \times 10^6$ m s^{-1}, $v = 0$

Using the relation : $v^2 - u^2 = 2\,a\,S$, we get

$$S = 7{\cdot}11 \times 10^{-2}\ \mathrm{m}$$

Using the relation : $v = u + a\,t$, we get

$$t = 2{\cdot}844 \times 10^{-8}\ \mathrm{s}$$

7. Refer to Fig. 2.30 (*a*) :

Here, $a = \dfrac{eE}{m} = \dfrac{1{\cdot}6\times10^{-19}\times2{\cdot}0\times10^{4}}{9{\cdot}1\times10^{-31}}$

$$= 3{\cdot}516 \times 10^{-15}\ \mathrm{m\,s^{-2}}$$

Now, $u = 0$ and $S = 1{\cdot}5\ \mathrm{cm} = 1{\cdot}5 \times 10^{-2}\ \mathrm{m}$

Using the relation : $S = u\,t + \frac{1}{2}\,a\,t^2$, we get

$$t = 2{\cdot}92 \times 10^{-9}\ \mathrm{s}$$

Refer to Fig. 2.30 (*b*) :

Here, $a = \dfrac{eE}{m} = \dfrac{1{\cdot}6\times10^{-19}\times2{\cdot}0\times10^{4}}{1{\cdot}67\times10^{-27}}$

$$= 1{\cdot}916 \times 10^{12}\ \mathrm{m\,s^{-2}}$$

Using the relation : $S = u\,t + \frac{1}{2}\,a\,t^2$, we get

$$t = 1{\cdot}566 \times 10^{-7}\ \mathrm{s}$$

8. Let q be charge on the particle. The charged particle experiences a force q E along horizontal, which takes it away from the wall as shown in Fig. 2.33.

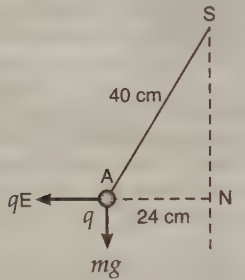


Fig. 2.33

Since the charged particle is in equilibrium,

$$\frac{qE}{AN} = \frac{mg}{SN} \quad \text{or} \quad q = \frac{mg}{E} \times \frac{AN}{SN}$$

Here, $m = 1\ \mathrm{g} = 10^{-3}\ \mathrm{kg}$, $E = 4{\cdot}0 \times 10^4\ \mathrm{N\,C^{-1}}$,

$g = 9{\cdot}8\ \mathrm{m\,s^{-2}}$, $AN = 24\ \mathrm{cm} = 0{\cdot}24\ \mathrm{m}$

and $SN = \sqrt{40^2 - 24^2} = 32\ \mathrm{cm} = 0{\cdot}32\ \mathrm{m}$

$$\therefore \quad q = \frac{10^{-3}\times9{\cdot}8\times0{\cdot}24}{4{\cdot}0\times10^{4}\times0{\cdot}32} = 1{\cdot}838 \times 10^{-7}\ \mathrm{C}$$

9. Proceed as in solved problem no. 2.06.

10. Here, AB = BC = CD = AD = a

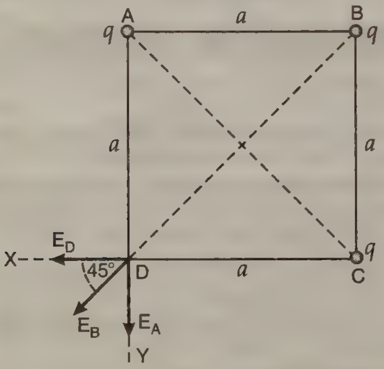


Fig. 2.34

$$\therefore \quad BD = \sqrt{a^2 + a^2} = \sqrt{2}\,a$$

Now, $E_A = E_C = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{a^2}$

and $E_B = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{(\sqrt{2}\,a)^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{2a^2}$

Net electric field along DX,

$$E_1 = E_C + E_B\cos 45^\circ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{2a^2}\times$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

Similarly, net electric field along DY,

$$E_2 = E_A + E_B\sin 45^\circ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

Hence, resultant electric field at point D

$$E = \sqrt{E_1^{\,2} + E_2^{\,2}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{9 + 4\sqrt{2}}\;q}{2a^2} \quad \textbf{(along BD)}$$

11. Here, AB = BC = CD = AD = 2 cm

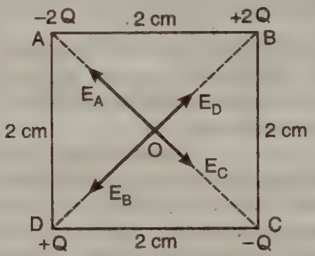


Fig. 2.35

$$\therefore AO = BO = CO = DO = \frac{\sqrt{2^2 + 2^2}}{4^2} = \sqrt{2}\ \mathrm{cm}$$

$$= \sqrt{2} \times 10^{-2}\,\mathrm{m}$$

Now, $E_A = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{2Q}{(OA)^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot Q\times10^4$ (along O

$E_B = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{2Q}{(OB)^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot Q\times10^4$ (along OI

$E_C = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{(OC)^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{2}\times10^4$ (along O

and $E_D = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{(OB)^2} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{2}\times10^4$ (along OB

Net electric field along OA,

$$E_1 = E_A - E_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2}\times10^4$$

Also, net electric field along OD,

$$E_2 = E_B - E_D = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2}\times10^4$$

Hence, resultant electric field at point O,

$$E = \sqrt{E_1^{\,2} + E_2^{\,2}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{\sqrt{2}}\times10^4$$

(parallel to side I

Now, $Q = 0{\cdot}02\ \mu\mathrm{C} = 0{\cdot}02 \times 10^6$

$$\therefore \quad E = 9\times10^9\cdot\frac{0{\cdot}02\times10^{-6}\times10^4}{\sqrt{2}}$$

$$= \mathbf{9\sqrt{2}\times10^5\ N\,C^{-1}}\ \textbf{(parallel to side BA)}$$

12. Proceed as in problem no. 11.

13. Here, AB = BC = CA = 5 cm = 5×10^{-2} m

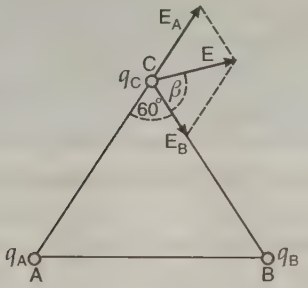


Fig. 2.36

$$q_A = 60 \text{ statC} = \frac{60}{3\times 10^9} = 2 \times 10^{-8}\text{ C},$$

$$q_B = -30 \text{ statC} = -10^{-8}\text{ C}$$

Now, $E_A = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_A}{AC^2} = 7.2 \times 10^4 \text{ N C}^{-1}$

(along AC, when produced)

and $E_B = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_B}{BC^2} = 3.6 \times 10^4 \text{ N C}^{-1}$ (along CB)

Electric fields E_A and E_B are inclined at angle 180° – 60° *i.e.* 120°.

Therefore, resultant electric field at point C,

$$E = \sqrt{{E_A}^2 + {E_B}^2 + 2\,E_A\,E_B \cos 120^\circ} = \mathbf{6.24 \times 10^4\ N\ C^{-1}}$$

If resultant electric field makes an angle β with CB, then

$$\tan\beta = \frac{E_A \sin 120^\circ}{E_B + E_A \cos 120^\circ} = \infty \quad \text{or} \quad \beta = 90^\circ$$

14. The charges $q_A = -10.24 \times 10^{-12}$ C, $q_B = 11.52 \times 10^{-12}$ C and $q_C = 3.24 \times 10^{-12}$ C are placed at corners A, B and C of right angled Δ ABC as shown in Fig. 2.37. The point D is foot of the perpendicular drawn from point B on the side AC.

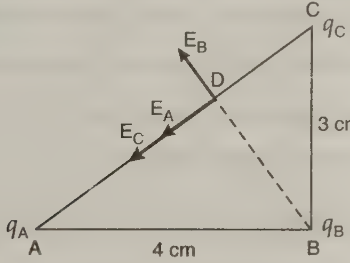


Fig. 2.37

Since AB = 4 cm and BC = 3 cm, it can be obtained that

AC = 5 cm, CD = 1.8 cm,

AD = 3.2 cm and BD = 2.4 cm

Now, $E_A = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_A}{AD^2} = 90 \text{ N C}^{-1}$ (along DA)

$E_B = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_B}{BD^2} = 180 \text{ N C}^{-1}$

(along BD, when produced)

and $E_C = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_C}{CD^2} = 90 \text{ N C}^{-1}$ (along DA)

The resultant electric field at point D,

$$E = \sqrt{(E_A + E_C)^2 + {E_B}^2} = \mathbf{180\sqrt{2}\ N\ C^{-1}}$$

Since $(E_A + E_C)$ and E_B are equal, the resultant electric field will be inclined equally with DA and BD (when produced).

15. Proceed as in solved problem no. 2.10.

16. Proceed as in solved problem no. 2.11.

17. Here, $q = 25 \times 10^{-9}$ C, $2a = 6$ m, $r = 4$ m

$\therefore \quad p = q\,(2a) = 25 \times 10^{-9} \times 6 = 1.5 \times 10^{-7}$ C m

Now, $E_{axial} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{2pr}{(r^2 - a^2)^2} = \mathbf{220.4\ N\ C^{-1}}$

and $E_{equi} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{p}{(r^2 + a^2)^{3/2}} = \mathbf{10.8\ N\ C^{-1}}$

18. Here, $q = 10\ \mu$C $= 10^{-5}$ C, $2a = 5.0$ mm $= 5 \times 10^{-3}$ m, $r = 15$ cm $= 0.15$ m,

$\therefore \quad p = q\,(2a) = 10^{-5} \times 5 \times 10^{-3} = 5 \times 10^{-8}$ C m

For an electric dipole of short length ($2a << r$),

$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} = \frac{9\times 10^9 \times 2 \times 5 \times 10^{-8}}{(0.15)^3}$$

$$= \mathbf{2.67 \times 10^5\ N\ C^{-1}}$$

and $E_{equi} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{p}{r^3} = \dfrac{9\times 10^9 \times 5 \times 10^{-8}}{(0.15)^3}$

$$= \mathbf{1.33 \times 10^5\ N\ C^{-1}}$$

19. $E_{axial} = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{2pr}{(r^2 - a^2)^2}$

In first case :

When $r = 10$ cm $= 0.1$ m, $E_{axial} = 3.75 \times 10^5$ N C^{-1}

$$\therefore \quad 3.75 \times 10^5 = 9 \times 10^9 \times \frac{2p \times 0.1}{[(0.1)^2 - a^2]^2} \qquad ...(i)$$

In second case :

When $r = 20$ cm $= 0.2$ m, $E_{axial} = 3 \times 10^4$ N C^{-1}

$$\therefore \quad 3 \times 10^4 = 9 \times 10^9 \times \frac{2p \times 0.2}{[(0.2)^2 - a^2]^2} \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we obtain

$a = 0.5$ m

Therefore, length of dipole, $2a = 0.5 \times 2 = \mathbf{0.1\ m}$

20. $\tau = p\,E \sin\theta = 4 \times 10^{-5} \times 10^{-3} \sin 30^\circ = \mathbf{2 \times 10^{-8}\ N\ m}$

21. $p = q\,(2a) = 1.602 \times 10^{-19} \times 4 \times 10^{-10} = \mathbf{6.41 \times 10^{-29}\ C\ m}$

and $\tau = p\,E \sin\theta = 6.41 \times 10^{-29} \times 3 \times 10^5 \times \sin 30^\circ$

$= \mathbf{9.615 \times 10^{-24}\ N\ m}$

22. Proceed as in solved problem no. 2.13.

23. To find p, proceed as in solved problem no. 2.13.

Then, $q = \dfrac{p}{2a}$

24. Apparent weight of the ball = $m\,g$ – upthrust

$$= \frac{4}{3}\pi r^3 (\rho - \sigma)\,g = \frac{4}{3}\pi \times (0.5)^3 \times (8.6 - 0.8) \times 980$$

$= 4002.4$ dyne $= 4.0024 \times 10^{-2}$ N

Let q be charge on the ball. As the ball remains suspended,

$q\,E$ = apparent weight of the ball

or $q = \dfrac{\text{apparent weight}}{E} = \dfrac{4.0024 \times 10^{-2}}{36000} = \mathbf{1.11 \times 10^{-6}\ C}$

25. Now, $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q\,x}{(x^2 + a^2)^{3/2}}$

Here, $a = 0.3$ m, $x = 0.4$ m and $q = 4 \times 10^{-9}$ C

$$\therefore \quad E = \frac{9 \times 10^9 \times 4 \times 10^{-9} \times 0.4}{[(0.4)^2 + (0.3)^2]^{3/2}} = \mathbf{115.2\ N\ C^{-1}}$$

Electric field at the centre of the ring is zero.

26. Here, $q = 10\ \mu$C $= 10^{-5}$ C, $2a = 6 - (-6) = 12$ cm $= 0.12$ m,

$r = 8$ cm $= 0.08$ m

Now, $E_{equi} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2+a^2)^{3/2}}$

$$= \frac{9\times10^9\times1\cdot2\times10^{-6}}{[(0\cdot08)^2+(0\cdot06)^2]^{3/2}} = \mathbf{1{\cdot}08\times10^7\ N\ C^{-1}}$$

27. The various forces act on the charged particle as shown in Fig. 2.38.

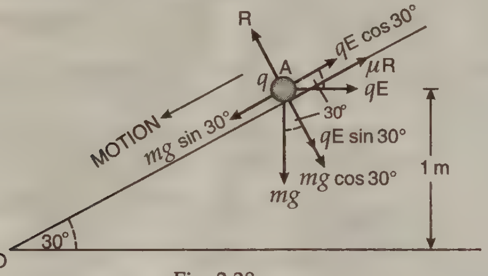


Fig. 2.38

Net force on the charged particle along the inclined pla in downward direction,

$F = m\,g \sin 30° - \mu\,R - q\,E \cos 30°$

Now, $R = m\,g \cos 30° + q\,E\ \sin 30°$

$\therefore\quad F = m\,g\,(\sin 30° - \mu \cos 30°) - q\,E\,(\cos 30° + \mu \sin 3$

$= 1 \times 9{\cdot}8\ (0{\cdot}5 - 0{\cdot}2 \times 0{\cdot}866)$

$- 0{\cdot}01 \times 100\ (0{\cdot}866 + 0{\cdot}2 \times 0$

$= 2{\cdot}237\ N$

Acceleration of the particle along the inclined plane,

$$a = \frac{F}{m} = \frac{2\cdot237}{1} = 2\cdot237\ m\,s^{-2}$$

Let OA = S. Then, $\sin 30° = \frac{AQ}{OA}$

or $\quad S = OA = \frac{AQ}{\sin 30°} = 2\ m$

Initial velocity of charged particle, $u = 0$

Using the relation : $S = u\,t + \frac{1}{2}\,a\,t^2$, we have

$t = \mathbf{1{\cdot}337\ s}$

CHAPTER

3

Electric Potential

3.01. CONSERVATIVE NATURE OF ELECTROSTATIC FORCE

A force is said to be conservative, if work done by the force between two points is ndependent of the path followed between the two points. Further, work done by a onservative force along a closed path is always zero.

All the forces, which obey inverse square law are conservative in nature. Therefore, gravitational and electrostatic forces are conservative forces. Hence, work done by electrostatic force between two points is same for all the paths between the wo points. Also, the work done by electrostatic force along a closed path is zero.

Key point

The electrostatic and gravitational forces are conservative in nature.

3.02. INDEPENDENCE OF LINE INTEGRAL OF ELECTRIC FIELD ON THE NATURE OF PATH BETWEENTWO POINTS

The electric field possesses one basic property that *the line integral of electric field etween two points is independent of the path followed between the two points.*

This property of electric field leads to the concept of electric potential, an lectrical parameter of great importance. As we shall learn, electric field can also e represented in terms of electric potential. Before doing so, let us first prove that ine integeral of electric field between two points is independent of the path followed etween the two points.

Consider a point charge + q located at the origin O of the co-ordinate frame OXYZ as shown in Fig. 3.01. The electric field due to the point charge will be directed long the dotted arrows *i.e.* radially outwards from the point charge + q. Suppose hat AB is any path (straight or curved) between two points A and B in the electric ield of the point charge + q, such that OA = r_A and OB = r_B. Further, suppose that is any point on the path AB and $\vec{E}$ is electric field due to the point charge + q at his point. Suppose that distance of point P from the point charge + q is OP = r. Let $\overrightarrow{PQ} = \vec{dl}$ be infinitesimally small displacement along the path AB. Since electric ield $\vec{E}$ gives measure of force on a unit positive charge placed at point P,

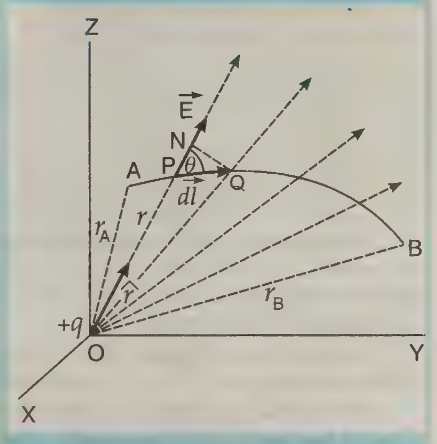


Fig. 3.01

$\vec{E} \cdot \vec{dl}$ = work done by the electric field $\vec{E}$ in moving a unit positive charge from point P to Q *i.e.* through $\overrightarrow{PQ}\ (= \vec{dl})$

Hence, $\int_A^B \vec{E} \cdot \vec{dl}$ = work done by the electric field $\vec{E}$ in moving a unit positive charge from the point A to B

Here, $\int_A^B \vec{E} \cdot \vec{dl}$ is called *the line integral of the electric field $\vec{E}$ between points A and B.*

Let us now find the value of integral $\int_A^B \vec{E} \cdot \vec{dl}$.

The electric field at point P due to point charge + q located at the point O is iven by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,\hat{r},$$

vhere $\hat{r}$ is unit vector along the line OP. If θ is angle between the directions of $\vec{E}$ and $\vec{dl}$, then

$$\vec{E}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,\hat{r}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,(1)\,(dl)\cos\theta$$

or $$\vec{E}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,dl\cos\theta \qquad \text{...(3.01)}$$

From point Q, drop QN perpendicular to OP. Then, from right angled Δ PNQ, we have

$$PN = PQ\cos\theta = dl\cos\theta$$

Further, as PN (the projection of $\vec{dl}$ along the line OP) is in the direction of increasing r, the distance PN is taken as $+dr$. Therefore,

$$dl\cos\theta = dr$$

Setting $dl\cos\theta = dr$ in equation (3.01), we have

$$\vec{E}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,dr$$

Hence, $$\int_A^B \vec{E}\cdot\vec{dl} = \int_A^B \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0}\cdot q\int_A^B r^{-2}\,dr \qquad \text{...(3.02)}$$

The values of r corresponding to points A and B are r_A and r_B respectively. Therefore, in equation (3.02), corresponding to the limits from A to B, the proper limits of integration are from r_A to r_B. Likewise, equation (3.02) becomes

$$\int_A^B \vec{E}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\cdot q\int_{r_A}^{r_B} r^{-2}\,dr = \frac{1}{4\pi\varepsilon_0}\,q\left|\frac{r^{-1}}{-1}\right|_{r_A}^{r_B} = \frac{1}{4\pi\varepsilon_0}\cdot q\left|-\frac{1}{r_B}-\left(-\frac{1}{r_A}\right)\right|$$

or $$\int_A^B \vec{E}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r_A}-\frac{1}{r_B}\right) \qquad \text{...(3.03)}$$

Key point

Line integral of electric field between tw
points is independent of the path followe
between the points.

The equation (3.03) gives the value of line integral of the electric field between two points A and B along the path AB shown in Fig. 3.01. It follows that the expression depends only on the distances of the end points A and B (*i.e.* r_A and r_B) and not upon the path AB.

Hence, *the line integral of the electric field between two points is same for all the paths between the two points i.e. it is independent of the path followed between the two points.*

The following points may be noted about the line integral of an electric field :

1. Line integral of an electric field along a closed path is zero. Consider a closed path AP_1BP_2A in the electric field of point charge $+q$ [Fig. 3.02]. Then, line integral of electric field between points A and B along path P_1 is given by

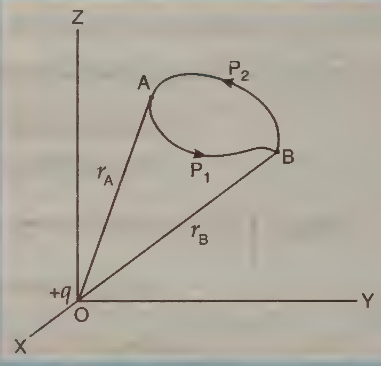


Fig. 3.02

$$\underset{\text{along path } P_1}{\int_A^B \vec{E}\cdot\vec{dl}} = \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r_A}-\frac{1}{r_B}\right)$$

Similarly, line integral of electric field between points B and A along path P_2 is given by

$$\underset{\text{along path } P_2}{\int_B^A \vec{E}\cdot\vec{dl}} = \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r_B}-\frac{1}{r_A}\right)$$

$$\therefore \quad \underset{\text{along path } P_1}{\int_A^B \vec{E}\cdot\vec{dl}} + \underset{\text{along path } P_2}{\int_B^A \vec{E}\cdot\vec{dl}} = \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r_A}-\frac{1}{r_B}\right) + \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r_B}-\frac{1}{r_A}\right)$$

or $$\underset{\text{along path } P_1}{\int_A^B \vec{E}\cdot\vec{dl}} + \underset{\text{along path } P_2}{\int_B^A \vec{E}\cdot\vec{dl}} = 0$$

Key point

Line integral of electric field along
closed path is zero.

The L.H.S. of the above equation represents sum of the line integrals of electric field from point A to B along path P_1 and then from point B to A along path P_2. Mathematically, this sum of two integrals is denoted as $\oint \vec{E}\cdot\vec{dl}$ *i.e.* line integral of the electric field along the closed path AP_1BP_2A. Therefore, the above equation may be expressed as

$$\oint \vec{E} \cdot \vec{dl} = 0 \qquad \text{...(3.04)}$$

Hence, *the line integral of an electric field along a closed path in an electric field is always zero.*

2. Line integral of electric field between two points is equal to work done by electric field in moving a unit positive test charge between the two points in the electric field. The electric field at a point measures the force experienced by a unit positive test charge placed at that point. Therefore,

$$\vec{E} \cdot \vec{dl} = \text{work done by electric field in moving a unit positive test charge through } \vec{dl}$$

Hence, work done by the electric field in moving a unit positive test charge from point A to B,

$$W = \int_A^B \vec{E} \cdot \vec{dl} \qquad \text{...(3.05)}$$

Key point ✓

$\int_A^B \vec{E} \cdot \vec{dl}$ represents work done by electric field $\vec{E}$ in moving a unit positive cnarge from point A to B.

Obviously, work done in moving a unit positive test charge against the electric field from point A to B

$$W = -\int_A^B \vec{E} \cdot \vec{dl} \qquad \text{...(3.06)}$$

Key point ✓

$-\int_A^B \vec{E} \cdot \vec{dl}$ represents work done by an external agent to move a unit positive charge against the electrostatic force exerted on the charge due to electric field $\vec{E}$.

3. Line integral of electric field is same for all the paths between two points only in case of static electric fields. The equation (3.03) does not hold for electric fields due to moving charges.

3.03. CONCEPT OF ELECTROSTATIC POTENTIAL DIFFERENCE AND POTENTIAL FROM THE LINE INTEGRAL OF ELECTRIC FIELD

Consider a point charge $+q$ located at the origin O of the co-ordinate frame OXYZ as shown in Fig. 3.03. The electric field due to charge $+q$ will be directed along the dotted arrows *i.e.* radially outwards from the charge $+q$. Let A and B be two points in the electric field of charge $+q$ and AB be any arbitrary path between the two points.

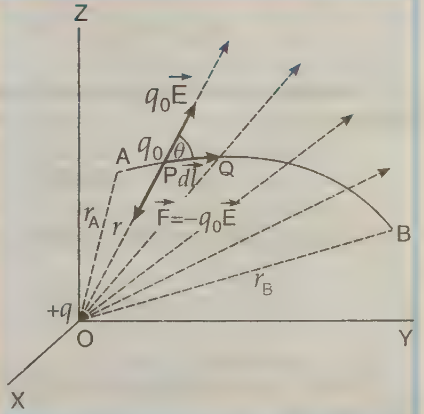


Fig. 3.03

The electrostatic potential difference between two points B and A in an electric field is defined as the amount of work done per unit positive test charge in moving it from point A to B against the electrostatic force due to the electric field.

If V_A and V_B are electrostatic potentials at points A and B in the electric field of point charge $+q$ and W_{AB} is work done in moving a vanishingly small positive test charge q_0, then by definition

$$V_B - V_A = \frac{W_{AB}}{q_0} \qquad \text{...(3.07)}$$

Suppose that at any instant, the test charge q_0 is at point P, such that $OP = r$. Let $\vec{E}$ be electric field at point P due to the source charge $+q$ placed at point O.

The force on test charge q_0 due to electric field of source charge $= q_0 \vec{E}$

To keep the charge in equilibrium, an external force $\vec{F} = -q_0 \vec{E}$ has to be applied on the test charge so as to balance the electrostatic force due to the source charge. Further, suppose that the test charge q_0 is moved through an infinitesimally small displacement $\vec{PQ} = \vec{dl}$. Then, small work done in moving the test charge from point P to Q,

$$dW = \vec{F} \cdot \vec{dl} = (-q_0 \vec{E}) \cdot \vec{dl}$$

Therefore, work done in moving the test charge q_0 from point A to B is given by

$$W_{AB} = \int_A^B dW = \int_A^B (-q_0 \vec{E}) \cdot \vec{dl} = -q_0 \int_A^B \vec{E} \cdot \vec{dl}$$

The work done per unit positive test charge in moving it from point A to B,

$$\frac{W_{AB}}{q_0} = -\int_A^B \vec{E} \cdot \vec{dl} \qquad ...(3.08)$$

From equations (3.07) and (3.08), we have

$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dl} \qquad ...(3.09)$$

Hence, *the electrostatic potential difference between two points B and A in an electric field is equal to the negative of the line integral of the electric field between the points A and B.*

Key point

Potential difference between two points B and A in an electric field $\vec{E}$ is equal to negative of line integral of $\vec{E}$ from point A to B.

To define and express electrostatic potential at a point in an electric field, consider that point A is located at infinity. The electrostatic potential at a point at infinite distance from charge $+q$ is zero. As such, the electrostatic potential difference between point B and point A (at infinity) will be just the electrostatic potential at point B. Likewise, electrostatic potential at a point is defined as below :

The electrostatic potential at a point in an electric field is defined as the amount of work done per unit positive test charge in moving it from infinity to that point against the electrostatic force due to the electric field.

If V_B is electrostatic potential at point B in the electric field of point charge $+q$ and $W_{\infty B}$ is work done in moving a vanishingly small positive charge q_0 from infinity to point B, then by definition

$$V_B = \frac{W_{\infty B}}{q_0} \qquad ...(3.10)$$

By proceeding as above, it can be obtained that work done in moving the positive test charge q_0 from infinity to point B,

$$W_{\infty B} = -q_0 \int_\infty^B \vec{E} \cdot \vec{dl}$$

Therefore, work done per unit positive test charge in moving it from infinity to point B,

$$\frac{W_{\infty B}}{q_0} = -\int_\infty^B \vec{E} \cdot \vec{dl} \qquad ...(3.11)$$

From equations (3.10) and (3.11), we have

$$V_B = -\int_\infty^B \vec{E} \cdot \vec{dl} \qquad ...(3.12)$$

Thus, *electrostatic potential at a point in an electric field is equal to the negative of the line integral of the electric field between infinity and that point.*

Key point

Potential at a point in an electric field $\vec{E}$ is equal to line integral of $\vec{E}$ between infinity and that point.

3.04. EXPRESSION FOR ELECTRIC POTENTIAL DIFFERENCE

The electrostatic potential difference between two points in an electric field may be defined as the amount of work done per unit positive test charge in moving it from one point to the other (without acceleration) against the electrostatic force due to the electric field.

Since potential difference is defined as the work done per unit positive charge between two points, it is a ***scalar quantity.***

Consider that a point charge $+q$ is placed at point O and points A and B are any two points in the electric field of the point charge $+q$. Let r_A and r_B be distances of points A and B from the point charge $+q$ [Fig. 3.04]. Suppose that V_A and V_B are electric potentials at the points A and B respectively.

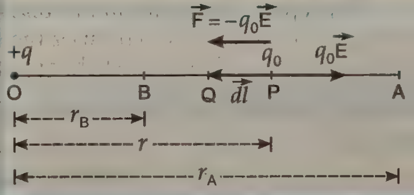


Fig. 3.04

If W_{AB} is the amount of work done in moving a vanishingly small positive test charge q_0 from point A to B, then

$$V_B - V_A = \frac{W_{AB}}{q_0} \qquad ...(3.13)$$

Suppose that at any instant, the test charge is at point P. If $\vec{E}$ is electric field at point P due to charge $+q$ placed at point O, then

force on the test charge q_0 due to the electric field of the source charge $= q_0 \vec{E}$

To move the test charge against this electrostatic force without acceleration, the external force to be applied is given by

$$\vec{F} = -q_0\,\vec{E}$$

Suppose the test charge is moved through infinitesimally small displacement $\overrightarrow{PQ} = \vec{dl}$, then small work done to move the test charge from point P to Q,

$$dW = \vec{F}\,.\,\vec{dl} = (-q_0\,\vec{E})\,.\,\vec{dl} = -q_0\,E\,dl\cos 180° = q_0\,E\,dl$$

As the test charge is moved from point A to B, the distance r decreases in the direction of displacement $\vec{dl}$. For this reason, the distance dl is taken as $-dr$. Therefore,

$$dW = q_0 E\,(-dr) = -q_0\,E\,dr$$

Now, electric field at point P due to the charge $+q$,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

$$\therefore \qquad dW = -q_0\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\right)dr = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,q_0}{r^2}\,dr$$

Hence, work done in moving the test charge q_0 from point A to B is given by

$$W_{AB} = \int_A^B dW = \int_{r_A}^{r_B} -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,q_0}{r^2}\,dr = -\frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\int_{r_A}^{r_B} r^{-2}\,dr$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left|\frac{r^{-1}}{-1}\right|_{r_A}^{r_B} = \frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left|\frac{1}{r}\right|_{r_A}^{r_B}$$

or

$$W_{AB} = \frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left(\frac{1}{r_B}-\frac{1}{r_A}\right) \qquad ...(3.14)$$

From equations (3.13) and (3.14), we have

$$V_B - V_A = \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r_B}-\frac{1}{r_A}\right) \qquad ...(3.15)$$

Unit of electrostatic potential difference. The SI unit of potential difference is **volt (V).** In SI, work and charge are measured in joule and coulomb respectively. Therefore, from equation (3.13), the SI unit of potential difference is also given by

$$\mathbf{1\ volt\ (V)} = \frac{\mathbf{1\ joule}}{\mathbf{1\ coulomb}} = \mathbf{1\ joule\ coulomb^{-1}\ (J\,C^{-1})}$$

The electrostatic potential difference between two points is said to be 1 volt, if 1 joule of work is done in moving a charge of 1 coulomb from one point to the other (without acceleration) against the electrostatic force due to electric field.

Note. If the source charge is located at a point, whose position vector is $\vec{r}\,'$, then

$$V_B - V_A = \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{|\vec{r}_B - \vec{r}\,'|}-\frac{1}{|\vec{r}_A - \vec{r}\,'|}\right) \qquad ...(3.16)$$

Key point

Electric potential is a characteristic of electric field. It does not matter, whether a charge is placed in the electric field or not.

3.05. ELECTRICAL POTENTIAL DUE TO A POINT CHARGE

The electrostatic potential at a point in an electric field may be defined as the amount of work done per unit positive test charge in moving it from infinity to that point (without acceleration) against the electrostatic force due to the electric field.

It is a ***scalar quantity.***

Consider a point charge $+q$ placed at point O. Suppose that V_A is electric potential at point A, whose distance from the source charge $+q$ is r_A[Fig. 3.05].

If $W_{\infty A}$ is work done in moving a vanishingly small positive test charge q_0 from infinity to point A, then

$$V_A = \frac{W_{\infty A}}{q_0} \qquad ...(3.17)$$

Suppose that at any instant, the test charge is at point P. The electric field $\vec{E}$ due to the charge $+q$ exerts force $q_0\,\vec{E}$ on the test charge placed at point P. The test

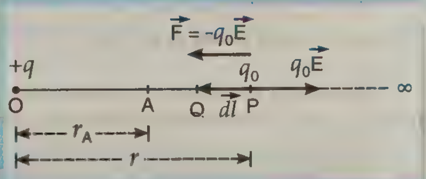


Fig. 3.05.

charge can be moved against this force without acceleration by applying an external force $\vec{F} = -q_0\,\vec{E}$ on it. If the test charge q_0 is moved through infinitesimally small displacement $\overrightarrow{PQ} = \vec{dl}$, then small work done is given by

$$dW = \vec{F}\,.\,\vec{dl} = (-q_0\,\vec{E})\,.\,\vec{dl} = -q_0\,E\,dl\cos 180^\circ = q_0\,E\,dl$$

or
$$dW = -q_0\,E\,dr$$

Here, as discussed in last section, dl has been taken as $-dr$, as the distance r decreases in the direction of $\vec{dl}$.

Since
$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\text{, we have}$$

$$dW = -q_0\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\right)dr = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,q_0}{r^2}\,dr$$

Hence, work done in moving the test charge from infinity to point A is given by

$$W_{\infty A} = \int_{\infty}^{A} dW = \int_{\infty}^{r_A} -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,q_0}{r^2}\,dr = -\frac{1}{4\pi\varepsilon_0}\,q\,q_0\int_{\infty}^{r_A} r^{-2}\,dr$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left|\frac{r^{-1}}{-1}\right| = \frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left|\frac{1}{r}\right|_{\infty}^{r_A}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left(\frac{1}{r_A}-\frac{1}{\infty}\right) = \frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left(\frac{1}{r_A}-0\right)$$

or
$$W_{\infty A} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_0}{r_A} \qquad ...(3.18)$$

From equations (3.17) and (3.18), we have

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_A}$$

In case, the distance of point A is from the charge $+q$ is denoted by r (in place of r_A), then subscript A may be dropped. If V denotes the electrostatic potential at any point at distance r from the charge $+q$, then

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \qquad ...(3.19)$$

Unit of electrostatic potential. The unit of electric potential is also **volt (V).**

1 volt (V) = 1 joule coulomb^{-1} (J C^{-1})

The electrostatic potential at a point is said to be 1 volt, if 1 joule of work is done in moving a charge of 1 coulomb from infinity to that point (without acceleration) against the electrostatic force due to electric field.

Note. If the source charge is located at a point, whose position vector is $\vec{r}\,'$, then

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|\vec{r}-\vec{r}\,'|} \qquad ...(3.20)$$

3.06. ELECTRIC POTENTIAL DUE TO A SYSTEM OF CHARGES

Let us now find electrostatic potential at a point P due to a group of point charges $q_1, q_2, q_3 .... q_n$ lying at distances $r_1, r_2, r_3 .... r_n$ from point P [Fig. 3.06]. The electrostatic potential at point P due to these charges is found by calculating electrostatic potential at point P due to each individual charge, considering the other charges to be absent and then adding up these electrostatic potentials algebraically.*

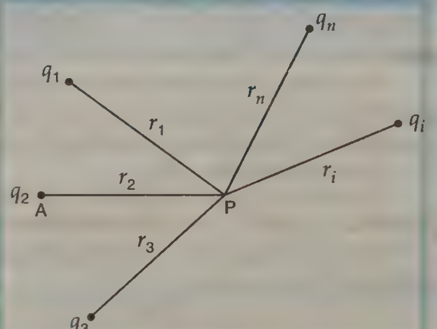


Fig. 306.

The electrostatic potential at point P due to charge q_1, when other charges are considered absent,

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1}$$

Similarly, electrostatic potentials at point P due to the individual charges $q_2, q_3, ... q_n$ (when other charges are absent) are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2}\,;\ V_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_3}{r_3}\,;\,.........;\ V_n = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_n}{r_n}$$

---

*Because, electrostatic potential is a scalar quantity.

Hence, electrostatic potential at point P due to the group of n point charges,

$$V = V_1 + V_2 + V_3 + \ldots\ldots + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_3}{r_3} + \ldots\ldots + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_n}{r_n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \ldots\ldots + \frac{q_n}{r_n}\right)$$

or

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad \ldots(3.21)$$

Key point

Electric potential at a point due to a group of charges is equal to algebraic sum of the potentials at that point due to individual charges.

Note. 1. In case the electric charges $q_1, q_2, q_3, \ldots\ldots q_n$, are located at points, whose sition vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots\ldots \vec{r}_n$, then electrostatic potential at a point at ation $\vec{r}$ can be found by making use of equation (3.20). Therefore, electrostatic tential due to the discrete distribution of charges is given by

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\vec{r} - \vec{r}_i|} \qquad \ldots(3.22)$$

Here, $\vec{r}_i$ is position vector of the ith charge q_i.

2. It can be easily deduced that electrostatic potential at location $\vec{r}$

(*a*) due to the line charge distribution L is given by

$$V = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\lambda\, dl}{|\vec{r} - \vec{r}\,'|} \qquad \ldots(3.23)$$

(*b*) due to the surface charge distribution S is given by

$$V = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}\,'|} \qquad \ldots(3.24)$$

(*c*) and due to the volume charge distribution V is given by

$$V = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}\,'|} \qquad \ldots(3.25)$$

Here, in equations (3.23), (3.24) and (3.25), λ, σ and ρ are linear charge density, face charge density and volume charge density respectively and $\vec{r}\,'$ is position ctor of the elementary portion dl, dS or dV of the linear, surface or volume charge tribution.

3.07. ELECTRIC POTENTIAL AT ANY POINT DUE TO AN ELECTRIC DIPOLE

Consider an electric dipole AB having charge $-q$ at point A and charge $+q$ point B. Let O be the centre of the dipole and P be any point at a distance r m its centre, where electric potential due to the dipole is to be determined. Let POB = θ [Fig. 3.07].

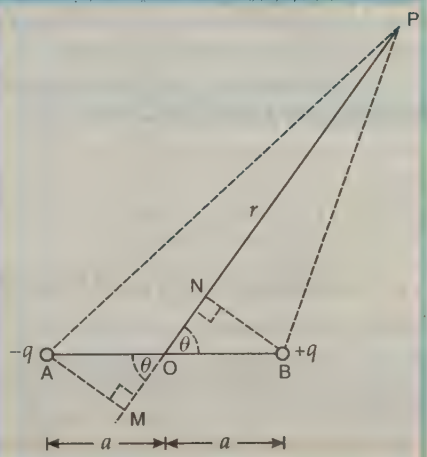


Fig. 3.07

The potential at point P due to charge $-q$,

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PA}$$

and the potential at point P due to charge $+q$,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PB}$$

Therefore, net potential at point P due to the dipole,

$$V = V_1 + V_2 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PA} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PB}$$

or

$$V = \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{PB} - \frac{1}{PA}\right] \qquad \ldots(3.26)$$

To find PB and PA, draw BN perpendicular to OP and AM perpendicular to PO after producing it. From right angled Δ AMO, we have

$$\cos\theta = \frac{\text{OM}}{\text{OA}} = \frac{\text{OM}}{a} \quad \text{or} \quad \text{OM} = a\cos\theta$$

In case the length of the dipole is very small as compared to distance r, then

$$\text{PA} \approx \text{PM} = \text{PO} + \text{OM} = r + a\cos\theta$$

Similarly, it can be obtained that

$$\text{PB} = r - a\cos\theta$$

Substituting for PA and PB in equation (3.26), we have

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right] = \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{r + a\cos\theta - r + a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q \cdot \frac{2a\cos\theta}{(r^2 - a^2\cos^2\theta)}$$

Since $q\,(2\,a) = p$, the electric dipole moment of the dipole, the above equation becomes

$$\text{V} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{(r^2 - a^2\cos^2\theta)} \qquad \text{...(3.27)}$$

The equation (3.27) gives electric potential due to the dipole at a distance r from its centre in a direction making an angle θ with the dipole.

Special cases. Let us find the electric potential due to the dipole in the following two cases :

1. *When point P lies on the axial line of the dipole.* In such a case, $\theta = 0°$ and $\cos\theta = \cos 0° = 1$

Therefore, equation (3.27) gives

$$\text{V}_{axial} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(r^2 - a^2)} \qquad \text{...(3.28)}$$

In case $a << r$, then

$$\text{V}_{axial} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} \qquad \text{...(3.29)}$$

2. *When point P lies on the equatorial line of the dipole.* In such a case, $\theta = 90°$ and $\cos\theta = \cos 90° = 0$

Therefore, equation (3.27) gives

$$\text{V}_{equi} = 0 \qquad \text{....(3.30)}$$

Key point

Electric potential at a point on equitoria line of an electric dipole is always zero.

Note. For derivation of expressions for electric potential due to a dipole at a point on its axial and equitorial lines independently, refer to SAQ 3.10 and SAQ 3.11 respectively.

3.08. ELECTRIC FIELD AS GRADIENT OF POTENTIAL

Consider a point charge $+q$ placed at point O. Suppose that V and V + dV are electrostatic potentials at points P and Q, whose distances from the charge $+q$ are r and $r - dr$ respectively [Fig. 3.08].

If dW is the amount of work done in moving a vanishingly small positive test charge q_0 from point P to Q, then

$$(\text{V} + d\text{V}) - \text{V} = \frac{d\text{W}}{q_0}$$

$$\text{or} \qquad d\text{V} = \frac{d\text{W}}{q_0} \qquad \text{...(3.31)}$$

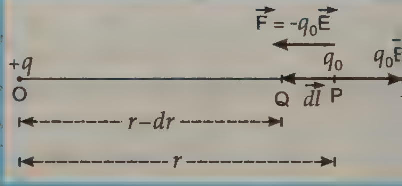


Fig. 3.08

If $\vec{E}$ is electric field at point P due to charge $+q$ placed at point O, then the test charge q_0 experiences a force equal to $q_0\vec{E}$ and the external force required to move the test charge (without acceleration) against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0\vec{E}$$

Therefore, work done to move the test charge through an infinitesimally small displacement $\overrightarrow{PQ} = \vec{dl}$ is given by

$$d\text{W} = \vec{F}\cdot\vec{dl} = (-q_0\vec{E})\cdot\vec{dl} = -q_0\,\text{E}\,dl\cos 180° = q_0\,\text{E}\,dl$$

As the distance r decreases in the direction of $\vec{dl}$, the distance dl is taken as $-dr$. erefore, the above equation becomes

$$dW = -q_0 E\, dr$$

or $$\frac{dW}{q_0} = -E\, dr \qquad ...(3.32)$$

From equations (3.31) and (3.32), we have

$$dV = -E\, dr$$

or $$E = -\frac{dV}{dr} \qquad ...(3.33)$$

Therefore, *electric field at a point is equal to the negative gradient of the electrostatic tential at that point.* The negative sign indicates that the direction of $\vec{E}$ is always the direction of decrease of electric potential.

Unit of potential gradient. In SI, the unit of potential gradient is **volt etre^{-1} (V m^{-1}).**

3.09. EQUIPOTENTIAL SURFACES

Any surface, which has same electrostatic potential at every point, is called an uipotential surface.

The equipotential surfaces can be drawn through any region, in which there electric field. If all the points at same potential in the electric field are joined, an uipotential surface is obtained.

Since all points of an equipotential surface are at same potential, no work is ne in moving a test charge between any two points on the equipotential surface. us, if $\vec{E}$ is electric field at a point on the equipotential surface and a test charge is moved through a displacement $\vec{dl}$ on the surface, then

$$q_0\, \vec{E} \cdot \vec{dl} = 0 \qquad \text{or} \qquad \vec{E} \cdot \vec{dl} = 0$$

. $\vec{E}$ is perpendicular to $\vec{dl}$. Since $\vec{dl}$ is along the equipotential surface, electric eld $\vec{E}$ must be perpendicular to it. In other words, electric lines of force are always rpendicular to the equipotential surface.

3.10. PROPERTIES OF EQUIPOTENTIAL SURFACES

The following properties are associated with the equipotential surfaces :

1. No work is done in moving a test charge over an equipotential surface. t A and B be two points on an equipotential surface [Fig. 3.09]. If a positive test arge q_0 is moved from point A to B, then work done in moving the test charge is lated to electrostatic potential difference between the two points as

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

Since the two points A and B are on the same equipotential surface,

$$V_B - V_A = 0$$

$$\therefore \qquad \frac{W_{AB}}{q_0} = 0 \qquad \text{or} \qquad W_{AB} = 0$$

Hence, no work is done in moving a test charge between two points on an uipotential surface.

2. The electric field is always at right angles to the equipotential surface. Since ork done in moving a test charge between two points on an equipotential surface zero, the displacement of the test charge and the force applied on it must be rpendicular to each other. Since displacement is along the equipotential surface, d force on test charge if $q_0 \vec{E}$, the electric field ($\vec{E}$) must be at right angles to the uipotential surface.

3. The equipotential surfaces help to distinguish regions of strong field from ose of weak field. We know that

$$E = -\frac{dV}{dr} \qquad \text{or} \qquad dr = -\frac{dV}{E}$$

Key point

Electric field at a point is equal to the negative gradient of potential at that point and is directed in the direction of decrease of potential.

Watch out !

Electric potential is a scalar quantity but electric potential gradient is a vector quantity !

Self-test Question

Show that electric field may also be measured as **volt metre^{-1} (V m^{-1})**

Key point

Electric field is always perpendicular to an equipotential surface and as a result, work done in moving a charge between two points on an equipotential surface is zero.

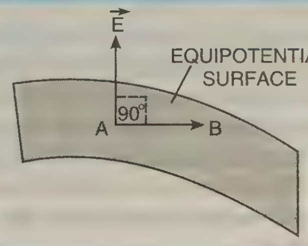


Fig. 3.09

Key point

In the regions, where equipotential surfaces are relatively closer, the electric field is stronger and vice-versa.

For same change in value of dV *i.e.* dV=constant, we have

$$dr \propto \frac{1}{E}$$

i.e. the spacing between the equipotential surfaces will be lesser in the regions, where the electric field is stronger and vice-versa. Therefore, *the equipotential surfaces are closer together, where the electric field is stronger and farther apart, where the field is weaker.*

4. The equipotential surfaces tell the direction of the electric field. Again,

$$E = -\frac{dV}{dr}$$

The negative sign tells that electric field is directed in the direction of electric potential with distance. Therefore, direction of electric field is from the equipotential surfaces which are closer to each other to those which are more and more away from each other, provided such surfaces having been drawn for same change in value of dV.

5. No two equipotential surfaces can intersect each other. In case, two equipotential surfaces intersect each other, then at their point of intersection, there will be two values of electric potential. As it is not possible, the two equipotential surfaces can not intersect each other.

3.11. TO DRAW EQUIPOTENTIAL SURFACES

The various properties of equipotential surfaces discussed in last section can be used to draw and study the equipotential surfaces in different situations. Let us now study the equipotential surfaces in the following cases :

1. For a uniform electric field. In a uniform electric field, the strength and direction of the field is same at every point inside it. A uniform electric field is represented by a set of parallel arrows. For a uniform electric field (E = constant), we have

$$\frac{dV}{dr} = \text{constant} \qquad \text{or} \qquad dV \propto dr$$

Therefore, in a uniform electric field, equipotential surfaces differing by same amount of potential difference will be equidistant from each other. Since electric field is perpendicular to the equipotential surface, such surfaces will be a set of parallel planes. Fig. 3.10 shows equipotential surfaces I, II and III at potentials V_1, V_2 and V_3 respectively. Since electric field is in the direction of decrease of potential, $V_1 > V_2 > V_3$. Further, as the equipotential surfaces are equidistant from each other, $V_1 - V_2$ is equal to $V_2 - V_3$.

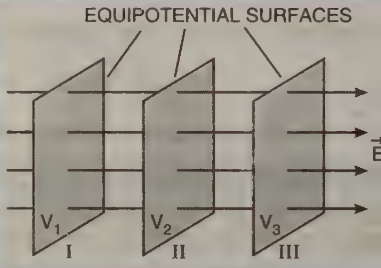


Fig. 3.10

2. For an isolated point charge. The electric field due to an isolated point charge is radial in nature and varies inversely as the square of the distance from the charge. The potential at all the points equidistant from the charge is same. All such points lie on the surface of a spherical shell, such that the charge lies at its centre. Therefore, for a point charge, equipotential surfaces will be a series of concentric spherical shells with the charge at the centre. Since for same change dV in potential,

$$dr \propto \frac{1}{E},$$

the separation dr between the equipotential surfaces will go on increasing with decrease of electric field. Fig. 3.11 shows equipotential surfaces I, II and III due to an isolated positive charge. The potential of each of the three surfaces is different, but it has the same value for all points on the surface. As the separation between surfaces II and III is more than that between I and II, it follows that electric field is stronger at points near the charge and weaker at points away from it. The electric lines of force are perpendicular to equipotential surfaces and point radially outwards.

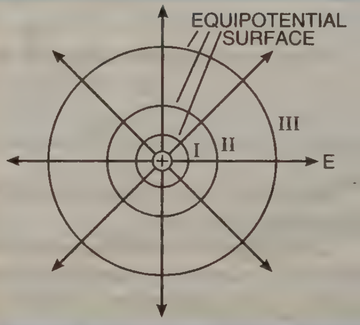


Fig. 3.11

3. For a system of two point charges. Fig. 3.12 and Fig. 3.13 represent the equipotential surfaces and electric lines of force for two equal positive charges and a pair of equal and opposite charges respectively. The dotted lines represent electric lines of force. The thick circles around the charges represent equipotential surfaces due to individual charges. In the two cases, the equipotential surfaces due to the wo charges are obtained by adding potentials at a point due to two charges lgebraically. They have been represented by thick curves or lines.

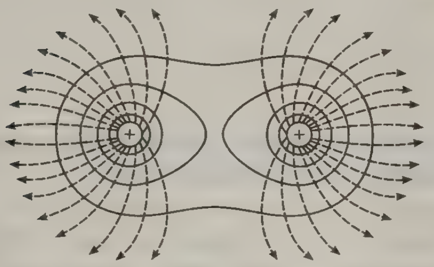


Fig. 3.12

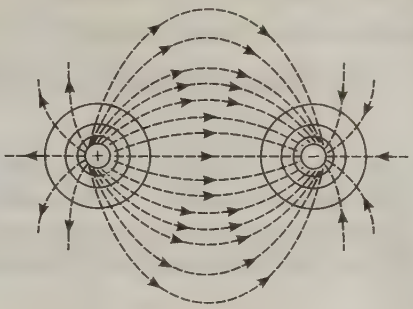


Fig. 3.13.

3.12. ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

The electrostatic potential energy of a system of point charges is defined as the work quired to be done to bring the charges constituting the system to their respective locations om infinity.

We first obtain expression for electrostatic potential energy of a system of two oint charges, then for a system of three point charges and finally we shall generalise e result for a system of n point charges.

(*a*) Potential energy of a system of two point charges. Consider two point harges q_1 and q_2 lying at points A and B, whose locations are $\vec{r_1}$ and $\vec{r_2}$ respectively ig. 3.14].

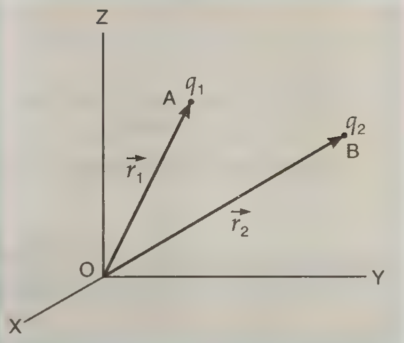


Fig. 3.14

To calculate the electric potential energy of the two charges, remove the two harges to positions, such that they are at infinite distance from each other. First of l, bring the charge q_1 from infinity to its original position A. For this, no work is equired. It is because, when charge q_1 is moved, no electrostatic force due to any ther charge opposes it.

Now, move charge q_2 to its original position B. When charge q_2 is moved, the ectric field due to the charge q_1 lying at point A, opposes it. Hence, work has to e done. The work done in moving charge q_2 from infinity to point B in the electric eld of charge q_1 is given by

$$W = (\text{electric potential due to charge } q_1 \text{ at B}) \times q_2 = \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{AB}\right)q_2$$

Since $AB = |\vec{r_1} - \vec{r_2}|$, we have

$$W = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|}$$

The work donw in bringing the two charges to their respective positions is ored as the potential energy of the configuration of two charges *i.e.*

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|} \qquad ...(3.34)$$

If distance $|\vec{r_1} - \vec{r_2}|$ is denoted as r_{12}, then

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}} \qquad ...(3.35)$$

The equations (3.34) and (3.35) give electric potential energy of a system of two oint charges.

Key point

Potential energy of a configuration of two charges is due to work done in bringing the charges to their respective positions after having been put infinite distance apart.

(*b*) Potential energy of a system of three point charges. Consider that three oint charges q_1, q_2 and q_3 are lying at locations $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ respectively. Let us nd the electric potential energy of the system of the three charges.

(*i*) First of all, remove all the three charges to infinite distance from each other nd then move the charge q_1 to its location $\vec{r_1}$. For this, no work has to be done.

(*ii*) Now, move the charge q_2 from infinity to its location $\vec{r_2}$. The work has to be done for moving charge q_2 in the electric field produced by charge q_1 (or due to electric potential of charge q_1) at the location $\vec{r_2}$. The work done for moving the charge q_2 from infinity to location $\vec{r_2}$, when charge q_1 is present at $\vec{r_1}$ is given by

$$W_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|}$$

(*iii*) Now, move charge q_3 from infinity to its location $\vec{r_3}$. The work, say W_{13} has to be done for moving charge q_3 in the electric field of charge q_1 and work, say W_{23} has to be done for moving it in the electric field of charge q_2. The values of W_{13} and W_{23} are given by

$$W_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\vec{r_1} - \vec{r_3}|} \quad \text{and} \quad W_{23} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 q_3}{|\vec{r_2} - \vec{r_3}|}$$

Therefore, total work done in order to bring all the three charges at their respective locations and hence the electric potential energy of the system of the three point charges is given by

$$U = W_{12} + W_{13} + W_{23}$$

or
$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\vec{r_1} - \vec{r_3}|} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 q_3}{|\vec{r_2} - \vec{r_3}|}$$

or
$$U = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|} + \frac{q_1 q_3}{|\vec{r_1} - \vec{r_3}|} + \frac{q_2 q_3}{|\vec{r_2} - \vec{r_3}|} \right) \qquad \text{...(3.36)}$$

The result (3.36) can also be expressed as

$$U = \frac{1}{2} \left(\frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{3} \sum_{\substack{i=1 \\ j \neq i}}^{3} \frac{q_i q_j}{|\vec{r_i} - \vec{r_j}|} \right) \qquad \text{...(3.37)}$$

The factor $\frac{1}{2}$ has been included for the reason that when potential energy of the three charges is expressed in summation form, each term gets counted twice. For example, if in equation (3.37), a term is obtained for $i = 1$ and $j = 2$, then the same term comes in again for $i = 2$ and $j = 1$.

If we denote the distance $|\vec{r_1} - \vec{r_2}|$, $|\vec{r_1} - \vec{r_3}|$ and $|\vec{r_2} - \vec{r_3}|$ as r_{12}, r_{13} and r_{23} respectively, then

$$U = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \qquad \text{...(3.38)}$$

The result (3.38) can also be expressed as

$$U = \frac{1}{2} \left[\frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{3} \sum_{\substack{i=1 \\ j \neq i}}^{3} \frac{q_i q_j}{r_{ij}} \right] \qquad \text{...(3.39)}$$

(*c*) **Potential energy of a system of *n* point charges.** The expression obtained for the potential energy of a system of three point charges can be easily generalised to a system of *n* point charges. The potential energy of a system of *n* point charges $q_1, q_2, q_3, \ldots, q_n$ present at locations $\vec{r_1}, \vec{r_2}, \vec{r_3}, \ldots, \vec{r_n}$ respectively is given by

$$U = \frac{1}{2} \left(\frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{n} \sum_{\substack{i=1 \\ j \neq i}}^{n} \frac{q_i q_j}{|\vec{r_i} - \vec{r_j}|} \right) \qquad \text{...(3.40)}$$

If we write the distance $|\vec{r_i} - \vec{r_j}|$ as r_{ij}, the above equation may be expressed as

$$U = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n}\sum_{\substack{i=1\\ j\neq i}}^{n}\frac{q_i\, q_j}{r_{ij}}\right) \qquad ...(3.41)$$

3.13. POTENTIAL ENERGY OF AN ELECTRIC DIPOLE, WHEN PLACED IN UNIFORM ELECTRIC FIELD

Suppose that an electric dipole having dipole moment p is placed along a rection making an angle θ with the direction of an external uniform electric field Then, torque acting on the dipole is given by

$$\tau = p\, E \sin\theta$$

If the dipole is rotated through an infinitesimally small angle $d\theta$ against the rque acting on it, then small work done

$$dW = \tau\, d\theta = p\, E \sin\theta\, d\theta$$

Therefore, work done in rotating the dipole from its orientation making an angle with the direction of electric field to θ_2 is given by

$$W = \int_{\theta_1}^{\theta_2} p\, E \sin\theta\, d\theta = p\, E \left| -\cos\theta \right|_{\theta_1}^{\theta_2}$$

or $$W = p\, E\, (\cos\theta_1 - \cos\theta_2)$$

This work done is stored in the dipole in the form of its potential energy.

Therefore, when an electric dipole is rotated inside the electric field, potential ergy stored in the electric dipole and it is given by

$$U = p\, E\, (\cos\theta_1 - \cos\theta_2) \qquad ...(3.42)$$

Let us assume that dipole is initially oriented perpendicular to the electric field d is then brought to the orientation making an angle θ with the field direction. en, potential energy of the dipole, when it makes angle θ with the direction of e electric field can be obtained from equation (3.42) by setting $\theta_1 = 90°$ and $\theta_2 =$ Therefore,

$$U = p\, E\, (\cos 90° - \cos\theta)$$

or $$U = -p\, E \cos\theta \qquad ...(3.43)$$

In vector notation,

$$U = -\vec{p} \cdot \vec{E} \qquad ...(3.44)$$

Key point

The work done in rotating an electric dipole against the torque acting on it due to the electric field is stored in the dipole in the form of its potential energy.

Key point

In a uniform electric field, an electric dipole experiences a torque only. Whereas in a non-uniform field, it experiences torque as well as a net force.

Key point

The potential energy of an electric dipole in an electric field is zero, when electric dipole moment vector is parallel to the field $(\vec{p} \parallel \vec{E})$ and maximum, when antiparallel.

SOLVED NUMERICAL PROBLEMS

pe A. On Potential difference and work done in moving a charge

Problem 3.01. If 100 joule of work must be done to move ectric charge equal to 4 C from a place, where potential –10 volt to another place, where potential is V volt, find e value of V. *(H.P.S.S.C.E. 1998, 1990)*

Sol. Here, $q_0 = 4\,C$; $V_A = -10$ volt; $V_B = V =$ volt;

$$W_{AB} = 100 \text{ joule}$$

Now, $$V_B - V_A = \frac{W_{AB}}{q_0}$$

or $$V - (-10) = \frac{100}{4} = 25 \quad \text{or} \quad V = \mathbf{15\ volt}$$

pe B. On Potential due to a point charge

Problem 3.02. Calculate the electric potential at the rface of a gold nucleus. Given, the radius of nucleus 6·6 × 10–15 m and atomic number of gold = 79. *(H.P.S.S.C.E. 1996)*

Sol. Here, charge on gold nucleus,

$$q = Z\,e = 79 \times 1{\cdot}6 \times 10^{-19}\, C;$$

radius of gold nucleus, $r = 6{\cdot}6 \times 10^{-15}$ m

Therefore, electric potential at the surface of the nucleus,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = \frac{9\times10^9\times79\times1{\cdot}6\times10^{-19}}{6{\cdot}6\times10^{-15}}$$

$$= \mathbf{1{\cdot}724 \times 10^7\ V}$$

Problem 3.03. (*a*) Calculate the potential at a point P due to a charge of 4 × 10–7 C located 9 cm away [Fig. 3.15].

(*b*) Hence, obtain the work done in bringing a charge of 2 × 10–9 C from infinity to the point P. Does the answer depend on the path along which the charge is brought ? *(Text Problem)*

Sol. Here, $q = 4 \times 10^{-7}$ C; $r = 9$ cm $= 0{\cdot}09$ m

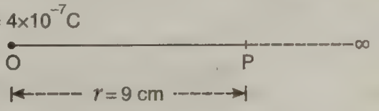


Fig. 3.15

(*a*) Potential at point P due to charge *q*,

$$V_P = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

$$= 9\times10^9 \times \frac{4\times10^{-7}}{0\cdot09} = \mathbf{4\times10^4\ V}$$

(*b*) From the definition, the potential at point P equals the work done in bringing a unit positive charge from infinity to point P. Therefore, the work done in bringing a charge of 2×10^{-9} C from infinity to point P,

$$W = 2\times10^{-9}\times \text{potential at point P}$$

$$= 2\times10^{-9}\times4\times10^{4}$$

$$= \mathbf{8\times10^{-5}\ J}$$

The work done in bringing the charge from infinity to point P **does not depend** on the path along which the charge is brought.

Problem 3.04. A charge of 20 μC produces an electric field. Two points are 10 cm and 5 cm from this charge. Find the values of potentials at these points and also find the amount of work done to take an electron from one point to the other.

Sol. Here, $q = 20\,\mu C = 20\times10^{-6}$ C

Let A and B be two points at distances 10 cm and 5 cm from the charge. Thus,

$$r_1 = 10\text{ cm} = 0\cdot1\text{ m}\ ;\ r_2 = 5\text{ cm} = 0\cdot05\text{ m}$$

Now, $$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_1} = 9\times10^9\times\frac{20\times10^{-6}}{0\cdot1}$$

$$= \mathbf{1\cdot8\times10^6\ V}$$

Also, $$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_2} = 9\times10^9\times\frac{20\times10^{-6}}{0\cdot05}$$

$$= \mathbf{3\cdot6\times10^6\ V}$$

$$\therefore\ V_B - V_A = 3\cdot6\times10^6 - 1\cdot8\times10^6 = 1\cdot8\times10^6\text{ V}$$

Work done to take an electron from A to B,

$$W_{AB} = (V_B - V_A)\times\text{charge on electron}$$

$$= 1\cdot8\times10^6\times1\cdot6\times10^{-19} = \mathbf{2\cdot88\times10^{-13}\ J}$$

Problem 3.05. At a point due to a point charge, the values of electric field intensity and potential are 32 N C^{-1} and 16 J C^{-1} respectively. Calculate magnitude of charge and distance of the charge from the point of observation. *(C.B.S.E. 2002)*

Sol. Let the observation point be at a distance *r* from the given point charge *q*. Since at the observation point, the electric field is 32 N C^{-1},

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} = 32 \qquad ...(i)$$

Also, as the electric potential at the observation point is 16 J C^{-1},

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = 16 \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$r = \frac{16}{32} = \mathbf{0\cdot5\ m}$$

Substituting for *r* in equation (*ii*), we have

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{0\cdot5} = 16 \quad\text{or}\quad 9\times10^9\times\frac{q}{0\cdot5} = 16$$

or $$q = \frac{16\times0\cdot5}{9\times10^9} = \mathbf{8\cdot89\times10^{-10}\ C}$$

Type C. On potential due to a group of Charges

Problem 3.06. Calculate the potential at the centre a square ABCD of each side $\sqrt{2}$ m due to charges 2, –2, and 6 μC at four corners of it. *(H.S.S.C.E. 199*

Sol. Four charges q_1, q_2, q_3 and q_4 are placed at the fo corners of the square ABCD as shown in Fig. 3.16.

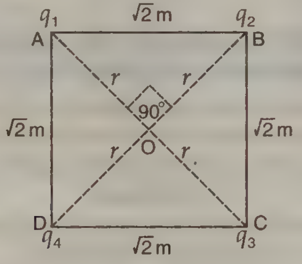


Fig. 3.16

Here, $$q_1 = 2\,\mu C = 2\times10^{-6}\,C\,;$$
$$q_2 = -2\,\mu C = -2\times10^{-6}\,C\,;$$
$$q_3 = -3\,\mu C = -3\times10^{-6}\,C\,;$$
$$q_4 = 6\,\mu C = 6\times10^{-6}\,C$$

and $$AB = BC = CD = AD = \sqrt{2}\text{ m}$$

Let *r* be the distance of each charge from the centre O the square.

Then, $$\sqrt{r^2+r^2} = \sqrt{2} \quad\text{or}\quad r = 1\text{ m}$$

Potential at point O due to charges at the four corne

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{r}+\frac{q_3}{r}+\frac{q_4}{r}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{r}(q_1+q_2+q_3+q_4)$$

$$= \frac{9\times10^9}{1}\left(2\times10^{-6}+(-2\times10^{-6})+(-3\times10^{-6})+6\times10^{-}\right.$$

$$= \mathbf{2\cdot7\times10^4\ V}$$

Q. 3.07. A regular hexagon of side 10 cm has a char 5 μC at each of its vertices. Calculate the potential at t centre of the hexagon. *(Text Proble*

Ans. Let O be the centre of the hexagon [Fig. 3.17]. follows that the point O, when joined to the two ends of a si of the hexagon forms an equilateral triangle. Therefore,

$$AO = BO = CO = DO = EO = FO = 10\text{ cm} = 0\cdot1\text{ m}$$

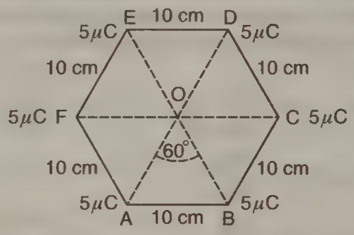


Fig. 3.17

Since at the each corner of the hexagon, a charge of 5 μ *i.e.* 5×10^{-6} C is placed, total electric potential at point O d to the charges at the six corners,

$$V = 6\times\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}\right)$$

$$= 6\times9\times10^9\times\frac{5\times10^{-6}}{0\cdot1} = \mathbf{2\cdot7\times10^6\ V}$$

Problem 3.08. A cube of side b has a charge q at each of its eight vertices. Determine the potential and electric field due to this charge array at the centre of the cube. *(Text Problem)*

Sol. The charge on each vertex of the cube = q ;
side of the cube = b

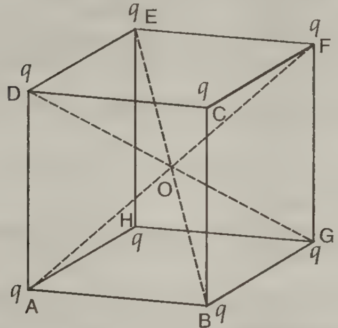


Fig. 3.18

Now, distance r of the centre O of the cube from any vertex of the cube is equal to half of its diagonal of the cube *i.e.*

$$r = \frac{1}{2}\sqrt{b^2 + b^2 + b^2} = \sqrt{3}\,b/2$$

The potential at point O due to charge q at any vertex,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

Therefore, potential at the centre of the cube due to charges at its eight vertices,

$$V' = 8\,V = 8 \times \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{3}\,b/2}$$

or $\quad V' = \dfrac{4\,q}{\sqrt{3}\,\pi\,\varepsilon_0\,b}$

From the symmetry, it follows that electric field at point O due to charges at the eight vertices of the cube will be **zero.**

Problem 3.09. A metal wire is bent into a circle of radius 10 cm. It is given a charge of 200 μC, which spreads on it uniformly. Calculate the electric potential at its centre. *(C.B.S.E. 1995)*

Sol. Here r = 10 cm = 0·1 m

and $\quad q = 200\ \mu C = 200 \times 10^{-6}$ C

Consider an elementary portion of the circular wire. Suppose that it possesses a small amount of charge dq over it. If we assume that the charge on elementary portion behave as point charge, then potential due to elementary portion at the centre,

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r}$$

The potential due to the whole circular wire,

$$V = \Sigma\, dV = \Sigma\,\frac{1}{4\pi\varepsilon_0}\,\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0\, r}\,\Sigma\, dq = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r}$$

$$= 9 \times 10^9 \times \frac{200 \times 10^{-6}}{0{\cdot}1} = \mathbf{1{\cdot}8 \times 10^7\ V}$$

Problem 3.10. Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electrical potential zero ? Take the potential at infinity to be zero. *(C.B.S.E. 2000 ; Text Problem)*

Sol. Here, $q_A = 3 \times 10^{-8}$ C ; $q_B = -2 \times 10^{-8}$ C ;
r = 15 cm = = 0·15 m

Let O be the point, where the electric potential is zero due to the two charges [Fig. 3.19].

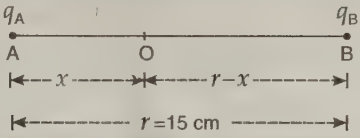


Fig. 3.19

Suppose that the distance AO = x. Then,

$$BO = r - x = 0{\cdot}15 - x$$

Electric potential at point O due to q_A,

$$V_A = \frac{1}{4\pi\varepsilon_0}\,\frac{q_A}{AO}$$

$$= 9 \times 10^9 \times \frac{3 \times 10^{-8}}{x} = \frac{270}{x}$$

Electric potential at point O due to q_B,

$$V_B = \frac{1}{4\pi\varepsilon_0}\,\frac{q_B}{BO}$$

$$= 9 \times 10^9 \times \frac{(-2 \times 10^{-8})}{0{\cdot}15 - x} = -\frac{180}{0{\cdot}15 - x}$$

Since the electric potential at point O is zero, we have

$$V_A + V_B = 0$$

or $\quad \dfrac{270}{x} + \left(-\dfrac{180}{0{\cdot}15 - x}\right) = 0 \quad$ or $\quad \dfrac{270}{x} = \dfrac{180}{0{\cdot}15 - x}$

or $\quad x = 0{\cdot}09$ m = **9 cm (from charge of 3×10^{-8} C)**

Problem 3.11. ABCD is a square of side 0·2 m. Charges of 2×10^{-9}, 4×10^{-9}, 8×10^{-9} coulomb are placed at the corners A, B and C respectively. Calculate the work required to transfer a charge of 2×10^{-9} coulomb from corner D to the centre of the square. *(Karnataka, 1988)*

Sol. The charges of $q_A = 2 \times 10^{-9}$ C ; $q_B = 4 \times 10^{-9}$ C and $q_C = 8 \times 10^{-9}$ C are placed at the corners A, B and C of the square ABCD of each side of length 0·2 m [Fig. 3.20].

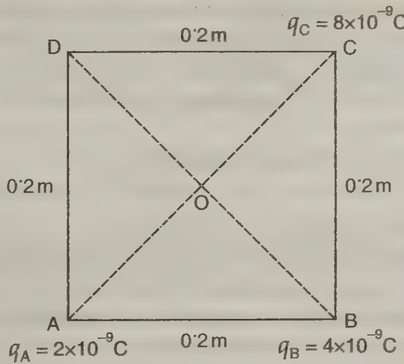


Fig. 3.20

Let V_D be potential at point D due to charges placed at points A, B and C. Then,

V_D = sum of the potentials due the charges q_A, q_B and q_C

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{AD} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{BD} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C}{CD}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AD} + \frac{q_B}{BD} + \frac{q_C}{CD}\right)$$

Here, AD = CD = 0 · 2 m

and $\quad BD = \sqrt{(0{\cdot}2)^2 + (0{\cdot}2)^2} = 0{\cdot}2\sqrt{2}$ m

$$\therefore \quad V_D = 9\times10^{9}\left(\frac{2\times10^{-9}}{0\cdot2}+\frac{4\times10^{-9}}{0\cdot2\sqrt{2}}+\frac{8\times10^{-9}}{0\cdot2}\right)$$

$$= \frac{9\times10^{9}\times2\times10^{-9}}{0\cdot2}(1+\sqrt{2}+4)$$

$$= 577{\cdot}26 \text{ V}$$

Now, potential at point O due to charges q_A, q_B and q_C,

$$V_O = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AO}+\frac{q_B}{BO}+\frac{q_C}{CO}\right)$$

Now, $AO = BO = CO = \frac{1}{2}\times$ diagonal of square

$$= \frac{1}{2}\times0\cdot2\sqrt{2} = 0\cdot1\sqrt{2}\text{ m} \quad (\because\ BD = 0{\cdot}2\sqrt{2}\text{ m})$$

$$\therefore \quad V_O = 9\times10^{-9}\times\left(\frac{2\times10^{-9}}{0\cdot1\sqrt{2}}+\frac{4\times10^{-9}}{0\cdot1\sqrt{2}}+\frac{8\times10^{-9}}{0\cdot1\sqrt{2}}\right)$$

$$= \frac{9\times10^{9}\times2\times10^{-9}}{0\cdot1\sqrt{2}}(1+2+4) = 890{\cdot}82\text{ V}$$

Potential difference between the points O and D,

$$V_O - V_D = 890{\cdot}82 - 577{\cdot}26 = 313{\cdot}56\text{ V}$$

Therefore, work done to transfer a charge $q = 2\times10^{-9}$ C from point D to O is given by

$$W = q\times(V_O - V_D) = 2\times10^{-9}\times313{\cdot}56$$

$$= \mathbf{6{\cdot}27\times10^{-7}\ J}$$

Type D. On Potential gradient and electric field

Problem 3.12. Two parallel plates are 5 cm apart and a potential difference of 60 V is set up across them. Find the electric field intensily between the two plates.

Sol. Here, $dV = 60$ V ; $dr = 5$ cm $= 0\cdot05$ m

Now, $$E = -\frac{dV}{dr} = -\frac{60}{0\cdot05} = \mathbf{-1200\ V\ m^{-1}}$$

The negative sign indicates that electric field is directed in the direction of decrease of potential.

Problem 3.13. Calculate the voltage needed to balance an oil drop carrying 10 electrons, when located between plates of a capacitor, which are 5 mm apart. Given, mass of the drop = 3×10^{-16} kg, charge on electron = $1{\cdot}6\times10^{-19}$ C and $g = 9.8$ m s^{-2}. *(C.B.S.E. 1995 S)*

Sol. Here, charge on oil drop,

$$q = 10\,e = 10\times1{\cdot}6\times10^{-19} = 1{\cdot}6\times10^{-18}\text{ C}$$

Mass of oil drop, $m = 3\times10^{-16}$ kg

Distance between the plates of the capacitor,

$$r = 5\text{ mm} = 5\times10^{-3}\text{ m}$$

The electric field at a point is numerically equal to the gradient of electric potential at that point. Since electric field between the plates of a capacitor is uniform, it is given by

$$E = \frac{V}{r}$$

Here, V is potential difference between the plates of the capacitor held at distance r apart.

So that the oil drop is balanced,

$$qE = mg \quad\text{or}\quad q\left(\frac{V}{r}\right) = mg$$

or $$V = \frac{mgr}{q} = \frac{3\times10^{-16}\times9\cdot8\times5\times10^{-3}}{1\cdot6\times10^{-18}} = \mathbf{9{\cdot}19\ V}$$

Type E. On Potential energy of a group of charges

Problem 3.14. (*a*) Determine the electrostatic energy o a system containing two charges 7 μC and – 2 μC separated by a distance of 18 cm.

(*b*) How much work is required to separate the tw charges infinitely away from each other ?

Sol. Here, $q_1 = 7\,\mu C = 7\times10^{-6}$ C ;

$$q_2 = -2\,\mu C = -2\times10^{-6}\text{ C};$$

$$r_{12} = 18\text{ cm} = 0{\cdot}18\text{ m}$$

(*a*) Electrostatic potential energy of the system of tw charges,

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}}$$

$$= 9\times10^{9}\times\frac{(7\times10^{-6})\times(-2\times10^{-6})}{0\cdot18} = \mathbf{-0{\cdot}7\ J}$$

(*b*) When the two charges are separated infinitely awa ($r_{12} = \infty$) from each other, the electrostatic potential energ of the system becomes zero. Therefore,

work required to separate the two charges infinitel away from each other,

$$W = \text{final P.E.} - \text{initial P.E.} = 0 - (-0{\cdot}7) = \mathbf{0{\cdot}7\ J}$$

Problem 3.15. An electron is circulating around th nucleus of a hydrogen atom in a circular orbit of radiu $5{\cdot}3\times10^{-11}$ m. Calculate (*a*) the electric potential at thi radius. (*b*) the electric potential energy of the atom in eV What would be the electric potential due to a heliun nucleus at the same radius. Given, $(4\pi\varepsilon_0)^{-1} = 9\times10^{9}$ m F^{-1} and $e = 1{\cdot}6\times10^{-19}$ C. *(I.S.C.E. 1997*

Ans. Here, $\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}$ m F^{-1} ; $e = 1{\cdot}6\times10^{-19}$ C

(*a*) Radius of electron orbit in hydrogen atom,

$$r = 5{\cdot}3\times10^{-11}\text{ m}$$

The nucleus of hydrogen atom contains one proton Therefore, charge on the nucleus of hydrogen atom,

$$q_1 = +e = 1{\cdot}6\times10^{-19}\text{ C}$$

Electric potential at electron orbit of hydrogen atom,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r} = \frac{9\times10^{9}\times1\cdot6\times10^{-19}}{5\cdot3\times10^{-11}} = \mathbf{27{\cdot}17\ V}$$

(*b*) Charge on electron, $q_2 = -e = -1{\cdot}6\times10^{-19}$ C

Therefore, potential energy of H-atom (proton-electror system),

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$$

$$= \frac{9\times10^{9}\times1\cdot6\times10^{-19}\times(-1\cdot6\times10^{-19})}{5\cdot3\times10^{-11}}$$

$$= -27\cdot17\times1\cdot6\times10^{-19}\text{ J}$$

Now, 1 eV $= 1{\cdot}6\times10^{-19}$ J

$$\therefore \quad U = -\frac{27\cdot17\times1\cdot6\times10^{-19}}{1\cdot6\times10^{-19}} = \mathbf{-27{\cdot}17\ eV}$$

A helium nucleus contains two protons. Therefore, charge on helium nucleus,

$$q = +2e = 2\times1{\cdot}6\times10^{-19} = 3{\cdot}2\times10^{-19}\text{ C}$$

Hence, electric potential at distance r (= $5{\cdot}3\times10^{-11}$ m)

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = \frac{9\times10^{9}\times3\cdot2\times10^{-19}}{5\cdot3\times10^{-11}} = \mathbf{54{\cdot}34\ V}$$

Problem 3.16. If one of the two electrons of a H_2 olecule is removed, we get a hydrogen molecular ion H_2^+). In the ground state of a H_2^+, the two protons are eparated by roughly 1·5 Å and the electron is roughly 1 Å rom each proton. Determine the potential energy of the ystem in eV. (1 eV = $1\cdot6 \times 10^{-19}$ J).

Sol. Let r_{12} be the distance between the two protons $_1$and p_2. Further, let r_{13} and r_{23} be the distances of the electron from the protons p_1 and p_2 respectively [Fig. 3.21].

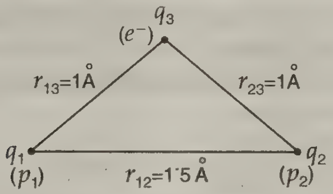


Fig. 3.21

Here, $r_{12} = 1\cdot5$ Å $= 1\cdot5 \times 10^{-10}$ m ;

$r_{13} = r_{23} = 1$ Å $= 10^{-10}$ m,

$q_1 = q_2 = +\,1\cdot6 \times 10^{-19}$ C and $q_3 = -\,1\cdot6 \times 10^{-19}$ C

Therefore, potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= 9 \times 10^{-19}\left(\frac{1\cdot6 \times 10^{-19} \times 1\cdot6 \times 10^{-19}}{1\cdot5 \times 10^{-10}} + \frac{1\cdot6 \times 10^{-19} \times (-\,1\cdot6 \times 10^{-19})}{10^{-10}} + \frac{1\cdot6 \times 10^{-19} \times (-\,1\cdot6 \times 10^{-19})}{10^{-10}}\right)$$

$$= 9 \times 10^{9}\,(1\cdot707 \times 10^{-28} - 2\cdot56 \times 10^{-28} - 2\cdot56 \times 10^{-28})$$

$$= 9 \times 10^{9} \times (-\,3\cdot413 \times 10^{-28})$$

$$= -\,30\cdot72 \times 10^{-19}\text{ J}$$

$$= -\frac{30\cdot72 \times 10^{-19}}{1\cdot6 \times 10^{-19}} = \mathbf{-\,19\cdot2\ eV}$$

Type F. On Potential energy of an electric dipole

Problem 3.17. An electric dipole consists of two opposite charges each of 1 μC separated by 2 cm. The dipole is placed in an external uniform field of 10^5 N C^{-1} intensity. Find (*a*) maximum torque exerted by the field on the dipole and (*b*) the work done in rotating the dipole through 180° starting from the position $\theta = 0°$.

Sol. Here, $E = 10^5$ N C^{-1} ; $q = 1\,\mu$C $= 10^{-6}$ C ;

$2a = 2$ cm $= 0\cdot02$ m

$\therefore \quad p = q\,(2a) = 10^{-6} \times 0\cdot02 = 2 \times 10^{-8}$ C m

(*a*) The torque acting on a dipole is given by

$$\tau = p\,E \sin\theta$$

Torque is maximum, when $\sin\theta = 1$

$$\therefore \quad \tau_{max} = 2 \times 10^{-8} \times 10^5 \times 1$$

$$= \mathbf{0\cdot002\ N\ m}$$

(*ii*) Now, work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is given by

$$W = p\,E\,(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0°,\ \theta_2 = 180°$

$$\therefore \quad W = 2 \times 10^{-8} \times 10^5\,(\cos 0° - \cos 180°)$$

$$= 0\cdot002\,(1 - (-1))$$

$$= \mathbf{0\cdot004\ J}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 3.18. A point charge of 8 mC is located at the origin [Fig. 3.22]. Calculate the work done in taking a nall charge of $-\,2 \times 10^{-9}$ C from a point A (0, 0, 3 cm) to a oint B (0, 4 cm, 0) via a point C (0, 6 cm, 9 cm).

(*Text Problem*)

Ans. Fig. 3.22 shows the charge q (= 8 mC or 8×10^{-3} C) ocated at the origin. A charge q_0 (= -2×10^{-9} C) is moved from oint A (0, 0, 3 cm) to point B (0, 4 cm, 0) via point C (0, 6 cm, 9 cm).

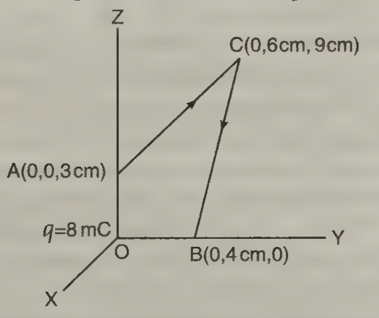


Fig. 3.22

The initial distance of charge q_0 from q,

r_A = OA = 3 cm = 0·03 m

The final distance of charge q_0 from q,

r_B = OB = 4 cm = 0·04 m

Work done in moving the charge q_0 from point A to B is ndependent of the path followed and depends only upon the initial and final distances *i.e.* r_A and r_B. Therefore, work done to move charge q_0 from point A to B is given by

$$W = \frac{1}{4\pi\varepsilon_0}\cdot q\,q_0\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$= 9 \times 10^9 \times 8 \times 10^{-3} \times (-\,2 \times 10^{-9}) \times \left(\frac{1}{0\cdot04} - \frac{1}{0\cdot03}\right)$$

$$= \mathbf{1\cdot2\,J}$$

Problem 3.19. Two tiny spheres carrying charges 1·5 μC and 2·5 μC are located 30 cm apart. Find the potential and electric field (*a*) at the mid-point of the line joining the two charges and (*b*) at a point 10 cm from this mid-point in a plane normal to the line and passing through the mid-point. (*Text Problem*)

Sol. Here, $q_A = 1\cdot5\,\mu$C $= 1\cdot5 \times 10^{-6}$ C ;

$q_B = 2\cdot5\,\mu$C $= 2\cdot5 \times 10^{-6}$ C

(*a*) The point P is mid-point of the line joining the two charges [Fig. 3.23].

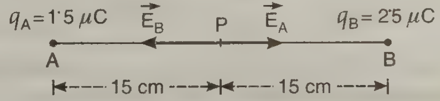


Fig. 3.23

Therefore, AP = BP = 15 cm = 0·15 m

Potential at point P due to the two charges,

$$V = V_A + V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AP} + \frac{q_B}{BP}\right)$$

$$= 9 \times 10^9 \left(\frac{1{\cdot}5 \times 10^{-6}}{0{\cdot}15} + \frac{2{\cdot}5 \times 10^{-6}}{0{\cdot}15}\right) = \mathbf{2{\cdot}4 \times 10^5\ V}$$

Electric field at point P due to charge q_A,

$$E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{(AP)^2} = 9 \times 10^9 \times \frac{1{\cdot}5 \times 10^{-6}}{(0{\cdot}15)^2}$$

$$= 6 \times 10^5\ N\ C^{-1} \qquad \text{(along PB)}$$

Electric field at point P due to charge q_B,

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{(BP)^2} = 9 \times 10^9 \times \frac{2{\cdot}5 \times 10^{-6}}{(0{\cdot}15)^2}$$

$$= 10^6\ N\ C^{-1} \qquad \text{(along PA)}$$

Therefore, net electric field at point P due to the two charges,

$$E = E_B - E_A = 10^6 - 6 \times 10^5$$

$$= \mathbf{4 \times 10^5\ N\ C^{-1}} \qquad \textbf{(along PA)}$$

(*b*) The point Q is at a distance of 10 cm from the mid-point P on a plane normal to the line AB and passing through the point P [Fig. 3.24].

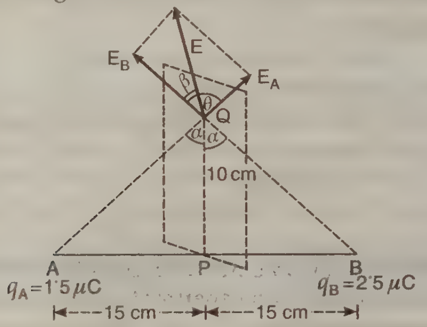


Fig. 3.24

Now, $AQ = BQ = \sqrt{15^2 + 10^2} = 18\ cm = 0 \cdot 18\ m$

Potential at point Q due to the two charges,

$$V = V_A + V_B$$

or $$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{AQ} + \frac{q_B}{BQ}\right)$$

$$= 9 \times 10^9 \left(\frac{1{\cdot}5 \times 10^{-6}}{0{\cdot}18} + \frac{2{\cdot}5 \times 10^{-6}}{0{\cdot}18}\right)$$

$$= \mathbf{2 \times 10^5\ V}$$

Electric field at point Q due to charge q_A,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{(AQ)^2} = 9 \times 10^9 \times \frac{1{\cdot}5 \times 10^{-6}}{(0{\cdot}18)^2}$$

$$= 4{\cdot}167 \times 10^5\ N\ C^{-1} \qquad \text{(along AQ)}$$

Electric field at point Q due to charge q_B,

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{(BQ)^2} = 9 \times 10^9 \times \frac{2{\cdot}5 \times 10^{-6}}{(0{\cdot}18)^2}$$

$$= 6{\cdot}944 \times 10^5\ N\ C^{-1} \qquad \text{(along BQ)}$$

The net electric field at point Q is given by

$$E = (E_A{}^2 + E_B{}^2 + 2\,E_A\,E_B \cos\theta)^{1/2},$$

where θ is angle between the directions of E_A and E_B.

If $\angle AQP = \angle BQP = \alpha$, then $\theta = 2\,\alpha$

In right angled Δ APQ,

$$\tan\alpha = \frac{15}{10} = 1{\cdot}5 \qquad \text{or} \qquad \alpha = 56{\cdot}3°$$

$$\therefore \quad \theta = 56{\cdot}3° \times 2 = 112{\cdot}6°$$

and $$E = [(4{\cdot}167 \times 10^5)^2 + (6{\cdot}944 \times 10^5)^2 + 2 \times 4{\cdot}167 \times 10^5 \times 6{\cdot}944 \times 10^5 \cos 112{\cdot}6°]^{\frac{1}{2}}$$

$$= 10^5 \times [17{\cdot}364 + 48{\cdot}219 + 57{\cdot}871 \times (-0{\cdot}3843)]^{\frac{1}{2}}$$

$$= \mathbf{6{\cdot}58 \times 10^5\ N\ C^{-1}}$$

Suppose that the direction of E makes an angle β wit the direction of E_B. Then,

$$\tan\beta = \frac{E_A \sin\theta}{E_B + E_A \cos\theta}$$

$$= \frac{4{\cdot}167 \times 10^5 \sin 112{\cdot}6°}{6{\cdot}944 \times 10^5 + 4{\cdot}167 \times 10^5 \cos 112{\cdot}6°}$$

$$= \frac{4{\cdot}167 \times 10^5 \times 0{\cdot}9232}{6{\cdot}944 \times 10^5 + 4{\cdot}167 \times 10^5 \times (-0{\cdot}3843)}$$

$$= \frac{3{\cdot}847}{6{\cdot}944 - 1{\cdot}601} = 0{\cdot}7193$$

or $\beta = \mathbf{35{\cdot}7°}$ **(with the direction of E_B)**

Note. To find the direction of E w.r.t. the line AB, suppos that $\angle QBP = \gamma$. Then,

$$\gamma = 90° - \alpha = 90° - 56{\cdot}3° = 33{\cdot}7°$$

The resultant electric field is inclined at an angle $\gamma +$ *i.e.* $33{\cdot}7° + 35{\cdot}7°$ = **69·4° with the line AB joining the tw charges.**

Problem 3.20. An infinite number of charges eac equal to q are placed along the X-axis at $x = 1$, $x = 2$ $x = 4$, $x = 8$..... and so on.

(*a*) Find potential at the point X = 0 due to this set o charges.

(*b*) What will be the potential, if in the above set up the consecutive charges have opposite sign ? (*I.I.T. 1974*

Sol. Suppose that charges q_1, q_2, q_3, q_4, ... are place at distances r_1, r_2, r_3, r_4, ... from the origin. Then, potentia at the origin ($x = 0$) due to the system of charges,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4} + ...\right]$$

(*a*) Here, $q_1 = q_2 = q_3 = ... = q$

and $r_1 = 1,\ r_2 = 2,\ r_3 = 3, r_4 = 4,$ .....

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1} + \frac{q}{2} + \frac{q}{4} + \frac{q}{8} + ...\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right] = \frac{1}{4\pi\varepsilon_0} \cdot q \, . S_\infty$$

where $$S_\infty = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$$

i.e. sum of a geometrical progression, whose first term $a =$ and the common ratio $r = 1/2$. Therefore,

$$S_\infty = \frac{a}{1-r} = \frac{1}{1 - 1/2} = 2$$

Hence, $$V = \frac{1}{4\pi\varepsilon_0} q \times 2 = \frac{q}{2\pi\varepsilon_0}$$

(*b*) When the consecutive charges have opposite sign,

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+.....\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{1}-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+.....\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot q\times\frac{1}{1-(-1/2)}=\frac{q}{6\pi\varepsilon_0}$$

Problem 3.21. Fig. 3.25 shows a charge array known as an electric quadrupole. For a point P on the axis of the uadrupole, obtain the dependence of potential on distance for $r/a>>1$. (*Text Problem*)

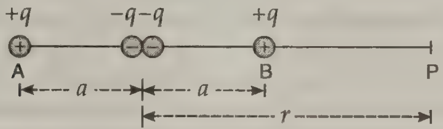


Fig. 3.25

Sol. The electric potential V at the point P due to electric uadrupole can be found by just adding the potentials at point due to the four charges forming the quadrupole. Thus,

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{AP}+\frac{(-q)}{BP}+\frac{(-q)}{BP}+\frac{q}{CP}\right)$$

or
$$V=\frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{AP}-\frac{2}{BP}+\frac{1}{CP}\right)$$

Here, $AP=r+a$; $BP=r$ and $CP=r-a$

$$\therefore\quad V=\frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r+a}-\frac{2}{r}+\frac{1}{r-a}\right)$$

$$=\frac{q}{4\pi\varepsilon_0}\times\frac{r(r-a)-2(r^2-a^2)+r(r+a)}{r(r^2-a^2)}$$

$$=\frac{q}{4\pi\varepsilon_0}\cdot\frac{2qa^2}{r(r^2-a^2)}$$

Since $r/a>>1$ or $r>>a$; in the above expressin, a^2 can e neglected in comparison to r^2.

$$\therefore\quad V=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2qa^2}{r(r^2)}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2qa^2}{r^3}$$

Thus, for $r/a>>1$, potential due to an electric uadrupole falls off as $1/r^3$.

Problem 3.22. Three point charges q, $2q$, $8q$ are to be placed on a 0·09 m long straight line. Find the ositions, where the charges should be placed such that the otential energy of this system is minimum. In this onfiguration, what is the electric field at the charge q due o the other two charges ?

Sol. The potential energy of the system of three charges , $2q$ and $8q$ is contributed by the three pairs, which these harges form with one another. So that the potential energy f the system is minimum, the first condition is that the harges constituting the strongest pair *i.e.* charges $2q$ and $8q$ hould be at the largest separation and the smallest charge hould be between them. Therefore, the charges should be placed as shown in Fig. 3.26.

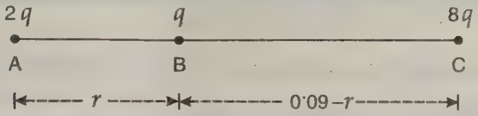


Fig. 3.26

Thus, the distance between the charges $2q$ and $8q$,

$$AC=0{\cdot}09\text{ m}$$

Let $AB=r$. Then, $BC=(0{\cdot}09-r)$

The potential energy of the system

$$U=\frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{(2q)q}{AB}+\frac{q(8q)}{BC}+\frac{(2q)(8q)}{AC}\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot 2q^2\left[\frac{1}{r}+\frac{4}{(0{\cdot}09-r)}+\frac{8}{0{\cdot}09}\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot 2q^2\cdot x,$$

where
$$x=\frac{1}{r}+\frac{4}{(0{\cdot}09-r)}+\frac{8}{0{\cdot}09}$$

The potential energy will be minimum, if x is minimum

i.e.
$$\frac{dx}{dr}=0\quad\text{or}\quad\frac{d}{dr}\left[\frac{1}{r}+\frac{4}{(0{\cdot}09-r)}+\frac{8}{0{\cdot}09}\right]=0$$

$$\text{or}\quad-\frac{1}{r^2}+\frac{4}{(0{\cdot}09-r)^2}+0=0\quad\text{or}\quad\frac{1}{r^2}=\frac{4}{(0{\cdot}09-r)^2}$$

or $r=$ **0·03 m**

The electric field at point B (the position of charge q)

$$E=E_A-E_B=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2q)}{(0{\cdot}03)^2}-\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(8q)}{(0{\cdot}06)^2}=0$$

Problem 3.23. In a hydrogen atom, the electron and proton are bound at a distance of about 0·53 Å.

(*a*) Estimate the potential energy of the system in eV, taking the zero of potential energy at infinite separation of the electron from proton.

(*b*) What is the minimum work required to free electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (*a*) ?

(*c*) What is the answer to (*a*) above, if the zero of potential energy is taken at 1·06 Å separation ? (*Text Problem*)

Ans. Here, charge on electron, $q_1=-1{\cdot}6\times10^{-19}$ C ;

charge on proton $=q_2=1{\cdot}6\times10^{-19}$ C

distance between electron and proton,

$$r_{12}=0{\cdot}53\text{ Å}=0{\cdot}53\times10^{-10}\text{ m}$$

(*a*) Potential energy of system,

$$U=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_{12}}$$

$$=9\times10^9\times\frac{(-1{\cdot}6\times10^{-19})\times1{\cdot}6\times10^{-19}}{0{\cdot}53\times10^{-10}}$$

$$=-43{\cdot}47\times10^{-19}\text{ J}$$

$$=-\frac{43{\cdot}47\times10^{-19}}{1{\cdot}6\times10^{-19}}=-27{\cdot}17\text{ eV}$$

At infinite separation, potential energy is zero. Therefore, if zero of potential energy is taken at infinite separation,

potential energy of the system $=-27{\cdot}17-0=$ **−27·17 eV** (*b*) The magnitude of kinetic energy of electron (always positive) is half of the potential energy.

Therefore, K.E. of electron $= \frac{27\cdot17}{2} = 13\cdot585$ eV

Therefore, total energy of electron

$= -27\cdot17 + 13\cdot585 = \mathbf{-13\cdot585\ eV}$

Therefore, work required to free the electron

$= 0 - (-13\cdot585) = \mathbf{13\cdot585\ eV}$

(*c*) Potential energy of system at 1·06 Å (= $1\cdot06 \times 10^{-10}$ m) separation,

$$U' = 9 \times 10^9 \times \frac{(-1\cdot6 \times 10^{-19}) \times 1\cdot6 \times 10^{-19}}{1\cdot06 \times 10^{-10}}$$

$$= -21\cdot74 \times 10^{-19}\ \text{J}$$

$$= -\frac{21\cdot74 \times 10^{-19}}{1\cdot6 \times 10^{-19}}\ \text{eV} = -13\cdot585\ \text{e V}$$

If – 13·585 eV is taken as zero of potential energy, then potential energy of the system

$= -27\cdot17 - (-13\cdot585) = \mathbf{-13\cdot585\ eV}$

Problem 3.24. A molecule of a substance has permanent electric dipole moment equal to 10^{-29} C m. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude 10^6 V m^{-1}. The direction of the field is suddenly changed by an angle of 60°. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarization of the sample.

(*Text Problem*)

Sol. Here, strength of electrostatic field, $E = 10^6$ V m^{-1}

Electric dipole moment of a molecule, $p = 10^{-29}$ C m

In one mole, there are molecules equal to Avogadro number N (= $6\cdot023 \times 10^{23}$ mole^{-1}).

Therefore, electric dipole moment of one mole of substance,

$$N\,p = 6\cdot023 \times 10^{23} \times 10^{-29} = 6\cdot023 \times 10^{-6}\ \text{C m}$$

When the substance is completely polarised, the electric dipole moment is along the direction of the electric field *i.e.* $\theta = 0°$.

Therefore, initial potential energy of the molecular dipoles,

$$U_1 = (N\,p)\,E \cos\theta = 6\cdot023 \times 10^{-6} \times 10^6 \times 1 = 6\cdot023\ \text{J}$$

When the direction of the electric field is changed by 60°, then final potential energy of the molecular dipoles,

$$U_2 = (N\,p)\,E \cos 60° = 6\cdot023 \times 10^{-6} \times 10^6 \times 0\cdot5$$
$$= 3\cdot012\ \text{J}$$

The loss in potential energy of the dipoles is released as the heat.

Therefore, heat released,

$U_1 - U_2 = 6\cdot023 - 3\cdot012 = \mathbf{3\cdot011\ J}$

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. What is the work done by the field of a nucleus in a complete circular orbit of the electron ? What, if the orbit is elliptical ?

Ans. In either case, work done by the field is zero. It is because,

$$\oint \vec{E}\,.\,\vec{dl} = 0$$

Q. 3.02. A charge 5 μC is placed at a point. What is the work required to carry 1 C of charge once round it in a circle of 12 cm radius ?

Ans. It is zero. It is because, all the points on the circular path are at same potential.

Q. 3.03. What would be the work done, if a point charge +*q* is taken from a point A to a point B on the circumference of a circle drawn with another point charge +*q* at the centre ? (*C.B.S.E. 2001 S*)

Ans. Since points A and B are at the same distance from the charge + *q* placed at centre of the circle [Fig. 3.27], the points A and B are at same potential.

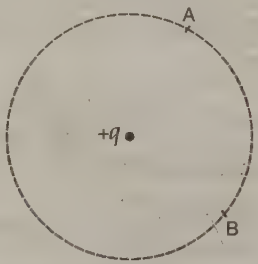


Fig. 3.27

Hence, work done in taking a charge + *q* from point A to B is zero.

Q. 3.04. What is the work done in moving a 2 μC point charge from corner A to corner B of a square ABCD [Fig. 3.28], when a 10 μC charge exists at the centre of the square ? (*C.B.S.E. 1998 S*)

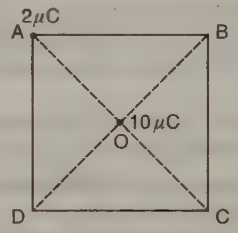


Fig. 3.28

Ans. The points A and B are at the same distance from 10 μC charge. Since $V_A = V_B$, no work will be done in moving a 2 μC charge from point A to B.

Q. 3.05. A uniform field E exists between two charged plates as shown in Fig. 3.29. What would be the work done in moving a charge *q* along the closed rectangular path ABCDA ? (*C.B.S.E. 2001 S*)

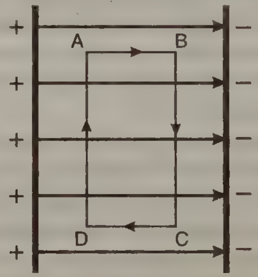


Fig. 3.29

Ans. It is zero.

Also refer to SAQ 3.04.

Q. 3.06. Define electric potential difference between two points. Is it scalar or vector ? *(P.S.S.C.E. 2001 ; H.S.S.C.E. 1996)*

Ans. For definition, refer to section 3.03.

Electric potential difference is a scalar quantity.

Q. 3.07. 5 J work is done in moving a positive charge of 0·5 C between two points. What is the potential difference between these points ? *(I.S.C.E. 1995)*

Ans. Now, $V_B - V_A = \frac{W_{AB}}{q_0} = \frac{5}{0 \cdot 5} = 10$ **volt**

Q. 3.08. A charge of 2 C moves between two plates maintained at a potential difference of 1 volt. What is the energy acquired by the charge ? *(I.S.C.E. 1994)*

Ans. Energy gained by charge = $V.\, q = 1 \times 2 = 2$ **J**

Q. 3.09 If a point charge + q is taken first from A to C and then from C to B of a circle drawn with another point charge + q as centre, then along which path more work will be done ? *(C.B.S.E. 2001 S)*

Ans. As explained in VSAQ 3.03, points A and B are at same potential [Fig. 3.30]. It follows that

$$V_C - V_A = V_C - V_B$$

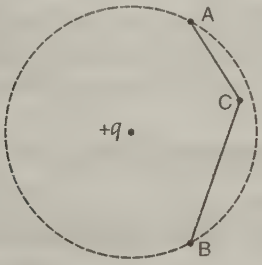


Fig. 3.30

Hence, work done in taking a point charge from A to C or from C to B will be the same.

Q. 3.10. Define electric potential at a point in an electric field. *(P.S.S.C.E. 2001 ; I.S.C.E. 1998)*

Ans. Refer to section 3.04.

Q. 3.11. Is electric potential a scalar or a vector quantity ?

Ans. It is scalar quantity.

Q. 3.12. Define the unit of electric potential. *(P.S.S.C.E. 2002, 2000, 1999, 1998 S)*

Ans. It is volt. For definition, refer to section 3.04.

Q. 3.13. Name the physical quantity, whose SI unit is joule coulomb^{-1}. *(C.B.S.E. 1998)*

Ans. Electric potential or potential difference.

Q. 3.14. How is electric field at a point related to potential gradient ?

Ans. Electric field at any point is equal to the negative of potential gradient at that point *i.e.*

$$E = -\frac{dV}{dr}$$

Q. 3.15. In a certain 0·1 m^3 of space, electric potential is found to be 5 V throughout. What is the electric field in this region ? *(I.S.S.C.E. 1997)*

Ans. Now, $E = -\frac{dV}{dr}$

As electric potential is 5 V throughout *i.e.* constant, electric field in the region will be zero.

Q. 3.16. The electric potential is constant in a region. What can you say about electric field there ? *(H.S.S.C.E. 2001 ; H.P.S.S.C.E. 1998 S)*

Ans. We know, $E = -\frac{dV}{dr}$

Since V is constant, electric field is zero.

Q. 3.17. If the electrostatic field at a given point is zero, must the electrostatic potential be also zero at that point ?

Ans. No. Electric potential may be zero or constant at that point.

Q. 3.18. Two protons A and B are placed between two parallel plates having a potential difference V as shown in the Fig. 3.31. Will these protons experience equal or unequal force ? *(C.B.S.E. 1998)*

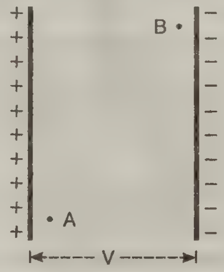


Fig. 3.31

Ans. The electric field due to the potential difference V applied across the two parallel plates of the capacitor is same everywhere. Hence, two protons A and B will experience equal force.

Q. 3.19. We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there ? *(Text Question)*

Ans. No, electric potential is continuous.

Q. 3.20. What is an equipotential surface ? *(P.S.S.C.E. 2002, 2000, 1999, 1998 S, 1997 ; H.S.S.C.E. 1997 S)*

Ans. *Any surface, which has same electrostatic potential everywhere, is called an equipotential surface.*

Q. 3.21. What is the shape of equipotential surfaces for a given point charge ? *(P.S.S.C.E. 2001 ; C.B.S.E. 1995)*

Ans. For a point charge, the equipotential surfaces are concentric spherical shells, whose centres are located at the given point charge.

Q. 3.22. What is the shape of equipotential surfaces for a uniform electric field ? *(P.S.S.C.E. 2001)*

Ans. For a uniform electric field, equipotential surfaces are planes at right angle to the direction of electric field.

Q. 3.23. How much work is done in moving a charge 500 μC charge between two points on an equipotential surface ? *(C.B.S.E. 2002, 2000, 1996 ; H.P.S.S.C.E. 2002)*

Or

No work is done in moving a test charge over an equipotential surface. Explain, why. *(P.S.S.C.E. 2001)*

Ans. As the two points on an equipotential surface are at the same potential, no work is done in moving a charge between two points.

Q. 3.24. A charge of +1 C is placed at the centre of a spherical shell of radius 10 cm. What will be the work done in moving a charge of +1 μC on its surface through a distance of 5 cm ? *(H.S.S.C.E. 1998 S)*

Ans. It is zero.

Q. 3.25. What is the direction of electric field w.r.t. an equipotential surface ?

Ans. It is along the normal to the equipotential surface.

Q. 3.26. Why does a configuration of charges possess potential energy ?

Ans. Work done in putting the charges at their present locations gets stored in the configuration as its potential energy.

Q. 3.27. When is the potential energy of an electric dipole maximum, when placed in uniform electric field ?

Ans. The potential energy of an electric dipole is maximum, when it is aligned antiparallel to electric field.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. Work done in moving a test charge between two points in an electric field is independent of the path followed. Comment.

Ans. The work done in moving a test charge between two points in an electric field is independent of path *i.e.* the work done depends only on the location of initial and final positions of the charge. In fact, it is true for all the fields obeying inverse square law.

Q. 3.02. A positive charge + q is located at a point. What is the work done, if a unit positive charge is carried once around this charge along a circle of radius r about this point ? (*I.I.T. 1974*)

Ans. The potential at each point on the circular path around the charge are at same potential *i.e.* potential difference between the initial and final position is zero.

Therefore, work done, $W = V \times q = 0 \times 1 = 0$

Q. 3.03. If a point charge be rotated in a circle of radius r around a charge q, what will be work done and why ? (*P.S.S.C.E. 1996*)

Ans. Refer to SAQ 3.02.

Q. 3.04. The intensity of an electric field inside a capacitor is $\vec{E}$. Calculate the work needed to move a charge q along a closed rectangular path ABCD [Fig. 3.32].

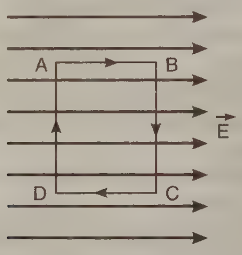


Fig. 3.32

Ans. Force on charge q in electric field $\vec{E}$, $\vec{F} = q\vec{E}$

Work done to move charge q along path ABCD,

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= q\vec{E}.\overrightarrow{AB} + q\vec{E}.\overrightarrow{BC} + q\vec{E}.\overrightarrow{CD} + q\vec{E}.\overrightarrow{DA}$$

$$= qE\,(AB)\cos 0^\circ + qE\,(BC)\cos 90^\circ + qE\,(CD)\cos 180^\circ + qE\,(DA)\cos 90^\circ$$

$$= qE\,(AB) + 0 + qE\,(CD)\,(-1) + 0$$

$$= qE\,(AB) - qE\,(CD) = 0$$

Q. 3.05. Fig. 3.33 (*a*) and 3.33 (*b*) show the field lines of a single positive and negative charge respectively :

(*i*) Give the sign of the potential difference $V_P - V_Q$ and $V_B - V_A$.

(*ii*) Give the sign of the potential energy difference of a small negative charge between the points Q and P ; A and B.

(*iii*) Give the sign of the work done by the field in moving a small positive charge from point Q to P.

(*iv*) Give the sign of the work done by an external agency in moving a small negative charge from point B to A.

(*v*) Does the kinetic energy of a small negative charge increase or decrease in going from point B to A ? (*Text Question*)

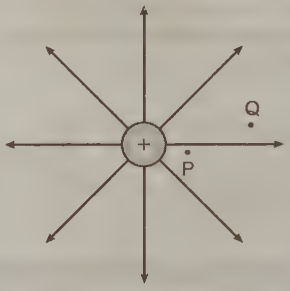


Fig. 3.33 (*a*)

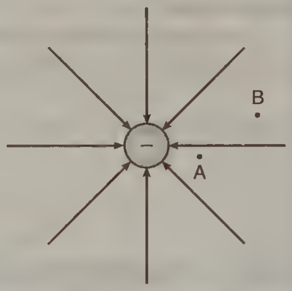


Fig. 3.33 (*b*)

Ans. (*i*) In Fig. 3.33 (*a*), as the field is due to positive charge, $V_P - V_Q > 0$.

In Fig. 3.33 (*b*), as the field is due to negative charge, $V_A - V_B < 0$ or $V_B - V_A > 0$.

(*ii*) The potential energy of a negative charge at point Q will be negative and at point P, it will be still more negative. Therefore,

$$(P.E.)_Q - (P.E.)_P > 0$$

For similar reasons,

$$(P.E.)_A - (P.E.)_B > 0$$

(*iii*) A small positive charge will tend to move from point P to Q and the work done by the field in moving the charge from point P to Q will be positive. Therefore, work done by electric field in moving a small positive charge from point Q to P will be negative.

(*iv*) For the reasons as given in (*iii*), work done by external agency in moving small negative charge from B to A will be positive.

(*v*) As the potential energy of the negative charge increases, kinetic energy of the negative charge decreases in going from point B to A.

Q. 3.06. A point charge q is placed at O as shown in Fig. 3.34. Is $V_A - V_B$ positive, negative or zero, if q is a (*i*) positive, (*ii*) negative charge ? (*C.B.S.E. 2001 S*)

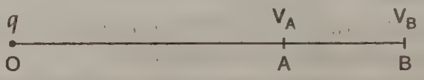


Fig. 3.34

Ans. Let $OA = r_A$ and $OB = r_B$.

Then, $$V_A - V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_A} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_B}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

Since $r_A < r_B$, $\frac{1}{r_A} - \frac{1}{r_B}$ = a positive quantity. It follows

(*i*) if q is a positive charge, $V_A - V_B$ = **positive**
(*ii*) if q is a negative charge, $V_A - V_B$ = **negative**

Q. 3.07. The electric potential at 0·1 m from a point ge is 50 V. What is the magnitude of the charge ?

(P.S.S.C.E. 1998 S ; H.S.S.C.E. 1992)

Ans. Here, V = 50 volt ; r = 0·1 m

Now, $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

$$\therefore \quad q = 4\pi\varepsilon_0 V r = \frac{1}{9\times10^9}\times 50\times 0{\cdot}1$$

$$= \frac{5}{9}\times10^{-9}\ \text{C}$$

Q. 3.08. Define electric potential at a point. When kept n electric field does a proton move from lower to higher ential or from higher to lower potential region ?

(C.B.S.E. 1997)

Ans. For definition of electric potential, refer to section .

In an electric field, a proton (positive charged particle) move in the direction of electric field *i.e.* from the higher wer potential region.

Q. 3.09. Is the electrostatic potential necessarily zero point, where the electric field strength is zero ? Give xample to illustrate your answer.

Ans. If the electric field strength is zero at a point, the trostatic potential is not necessarily zero at that point. For nple, at a point midway between two equal and similar ges, the electric field strength is zero but the electrostatic ential is twice that due to a single charge.

Q. 3.10. Derive an expression for the potential at a nt along the axial line of a short electric dipole.

(C.B.S.E. 2001 S)

Ans. Fig. 3.35 shows an electric dipole consisting of ges – q at point A and + q at point B. Let P be any point he axial line of the dipole at a distance r from its centre.

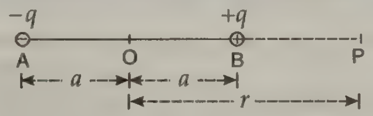


Fig. 3.35

If $2a$ is length of the dipole, then

PA = $r + a$ and PB = $r - a$

Electric potential at point P due to the dipole,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{PA} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PB}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r+a)} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(r-a)}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\left(\frac{1}{r-a} - \frac{1}{r+a}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\,\frac{(r+a)-(r-a)}{(r^2-a^2)}$$

or $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(r^2-a^2)}$$

For a dipole of short length, a^2 can be neglected as compared to r^2. Then,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$$

Q. 3.11. Show mathematically that the potential at a point on the equitorial line of an electric dipole is zero.

(C.B.S.E. 2001 S)

Ans. Fig 3.36 shows an electric dipole consisting of charges – q at point A and + q at point B.

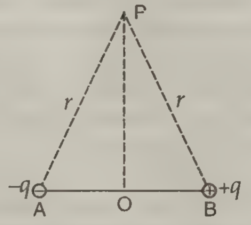


Fig. 3.36

Let P be at point on the equitorial line of the dipole.

Then, PA = PB = r (say)

Electrical potential at point P due to the dipole,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{PA} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PB}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = 0$$

Q. 3.12. What is an equipotential surface ? Give two properties of equipotential surface. *(H.S.S.C.E. 2001)*

Ans. Equipotential surface. Refer to section 3.09

Properties of equipotential surface. Refer to section 3.10.

Q. 3.13. What is an equipotential surface ? Show that the electric field is always directed perpendicular to an equipotential surface. *(C.B.S.E. 2001 S)*

Ans. Equipotential surface. Refer to section 3.09.

Let $\vec{E}$ be electric field at a point on equipotential surface. Then, small work done in moving a test charge q_0 through a small displacement $\vec{dr}$ along the surface,

$$dW = \vec{F}\cdot d\vec{r} = (-q_0\vec{E})\cdot d\vec{r}$$

Since work done in moving a test charge along an equipotential surface is always zero,

$$(-q_0\vec{E})\cdot d\vec{r} = 0 \quad \text{or} \quad \vec{E}\cdot d\vec{r} = 0$$

Hence, electric field is directed perpendicular to the surface.

Q. 3.14. Show that the amount of work done in moving a test charge along the equipotential surface is zero.

(H.P.S.S.C.E. 1997)

Ans. Refer to section 3.10.

Q. 3.15. What the electric field is always at right angles to the equipotential surface ? Give reasons. (*P.S.S.C.E. 2001*)

Ans. The work done in moving a test charge between two points on an equipotential surface can be zero, only if electric field is at right angles to the equipotential surface.

Q. 3.16. No two equipotential surfaces intersect each other. Why ? Explain. (*P.S.S.C.E. 2001*)

Ans. Refer to section 3.10.

Q. 3.17. Draw one equipotential surface (*i*) in a uniform electric field and (*ii*) for a point charge (Q < 0). (*P.S.S.C.E. 2001*)

Ans. (*i*) In a uniform electric field, the equipotential surface is a plane at right angle to direction field [Fig. 3.10].

(*ii*) For a point charge, equipotential surface is a spherical shell with the charge lying at the centre of the shell [Fig. 3.11].

Q. 3.18. What will be the equipotential surfaces corresponding to

(*i*) a constant electric field in the Z-direction ?

(*ii*) a field that uniformly increases in magnitude but remains in the same direction (say Z-axis) ?

(*iii*) a single positive charge at the origin ?

(*iv*) a uniform grid consisting of long equally spaced parallel charged wires in a plane ?

Ans. (*i*) For a constant electric field in the Z-direction, equipotential surfaces will be planes parallel to XY- plane.

(*ii*) In this case also, the equipotential surfaces will be planes parallel to XY-plane. However, as field increases, such planes will get closer.

(*iii*) For a single positive charge at the origin, the equipotential surfaces will be concentric spherical shells having origin as their common centre.

(*iv*) Near the grid consisting of long equally spaced parallel charged wires, the equipotential surfaces will have varying shapes. At far off distances, the equipotential surfaces will be planes parallel to the grid.

Q. 3.19. The equipotential surfaces of a certain electric field are shown in Fig. 3.37. It is known that $V_1 > V_2$. Use this pattern to reproduce approximately the lines of force of this field. Also indicate the region in which the intensity of the electric field is highest.

Ans. The lines of force are everywhere perpendicular to the equipotential surfaces and are directed in the direction of decreasing potential. Likewise, for the equipotential surfaces shown in Fig. 3.37, the lines of fo will be as shown in Fig. 3.38.

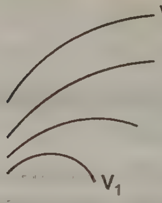


Fig. 3.37

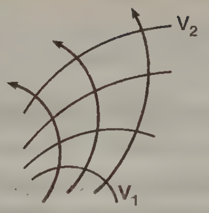


Fig. 3.38

The intensity of electric field is highest in the regi where the equipotential surfaces are closest to each othe

Q. 3.20. Distinguish between electric potential a potential energy and state the relation between these. (*P.S.S.C.E. 1996*

Ans. For definitions of electric potential and potent energy, refer to sections 3.04 and 3.12.

Electric potential due to a charge q_1 at a point P distance r is given by

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r}$$

If another charge q_2 is placed at observation point P, th potential energy of the system of the two charges is given

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r}\times q_2$$

= electric potential due to charge q_1 at the location of q_2 ×

Q. 3.21. What do you mean by potential energy of electric dipole, when placed in electric field ?

Ans. An electric dipole always tends to orient itself alo the direction of electric field. Work has to be done in rotati the dipole to some other orientation and this work done rotating the dipole gets stored in the dipole in the form of potential energy.

Q. 3.22. What does the negative sign in the expressi for potential energy ($U = -p\,E\cos\theta$) signify ?

Ans. While deriving the expression for potential ener the potential energy of the dipole is taken as zero, when i oriented perpendicular to the direction of the electric fie In any other orientation between $\theta = 0°$ to $\theta = 90°$, the potent energy of the dipole is less than zero and hence the negat sign.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 3.01. 'Work done to move a charge along a closed path inside an electric field is always zero.' Use this fact to prove that it is impossible to produce an electric field in which all the lines of force would be parallel straight lines and the density of their distribution would constantly increase in a direction perpendicular to the lines of force [Fig. 3.39].

Ans. Suppose that a charge q is moved along a closed path ABCD inside the electric field [Fig. 3.40]. Since work done to move a charge q along a closed path inside an electric field is zero,

$$W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0$$

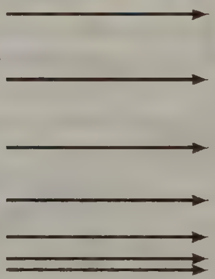


Fig. 3.39

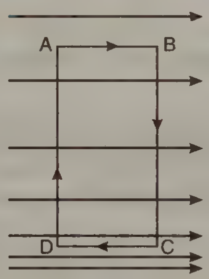


Fig. 3.40

Since electric field is perpendicular to paths BC and D $W_{BC} = W_{DA} = 0$.

$\therefore \qquad W_{AB} + 0 + W_{CD} + 0 = 0$

or $\qquad W_{AB} - = - W_{CD}$

But for the electric field shown in Fig. 3.40, work done to ve a charge along paths AB and CD can not be equal in gnitude. As the lines of force are closer to each other near path CD, intensity of electric field is more along path CD n along path AB. Therefore, work done to move the nge q along the path CD will be more than that along the h AB. Thus, work done to move a charge along a closed h cannot be zero for the electric field as shown in Fig. 3.39 l hence, it is impossible to produce an electric field shown he figure.

Q. 3.02. Prove that if the lines of force are shaped like arcs of concentric circles with their centre at point O certain section of an electric field [Fig. 3.41], the intensity the field in this section at each point is inversely portional to its distance from point O.

Ans. Consider a closed path ABCD in the electric field, h that paths AB and CD are circular arcs of radii r_1 and r_2 pectively as shown in Fig. 3.42.

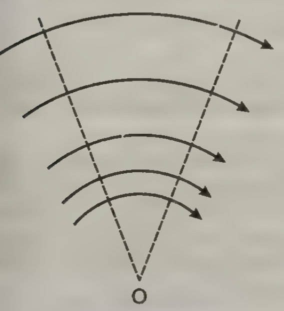


Fig. 3.41

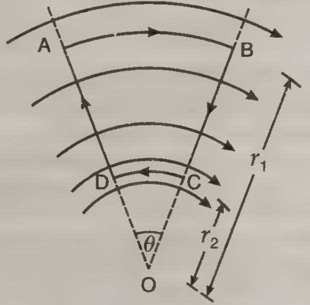


Fig. 3.42

Suppose that a charge q is moved along the path ABCD. en,

$$W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0 \qquad ...(i)$$

As electric field is perpendicular to paths BC and DA,

$$W_{BC} = W_{DA} = 0$$

Therefore, equation (*i*) becomes

$$W_{AB} + W_{CD} = 0 \qquad ...(ii)$$

Let $\vec{E_1}$ and $\vec{E_2}$ be electric fields along paths AB and CD pectively. Then,

$$W_{AB} = q\,\vec{E_1} \cdot \vec{(AB)} = q\,E_1\,(AB) \cos 0°$$

$$= q\,E_1\,(AB) = q\,E_1\,(r_1\,\theta)$$

$$W_{CD} = q\,\vec{E_2} \cdot \vec{(CD)} = q\,E_2\,(CD) \cos 180°$$

$$= -q\,E_2\,(CD) = -q\,E_2\,(r_2\,\theta)$$

Substituting for W_{AB} and W_{CD} in equation (*ii*), we have

$$q\,E_1\,(r_1\,\theta) - q\,E_2\,(r_2\,\theta) = 0$$

or $\qquad \dfrac{E_1}{E_2} = \dfrac{r_2}{r_1} \quad$ or $\quad E \propto \dfrac{1}{r}$

Q. 3.03. Two point charges $+q$ and $-q$ are separated by a distance d. Where besides at infinity is the electric potential zero ?

Ans. Suppose that electric potential is zero at the point, which is at a distance r_1 from charge $+q$ and r_2 from charge $-q$. Then,

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{r_2} = 0 \quad \text{or} \quad r_1 = r_2$$

All such points for which $r_1 = r_2$ lie on a plane, which is right bisector of the line joining the two charges.

Q. 3.04. Two small charged bodies ineract in air with a force F. What will the force of interaction between these bodies be after they are placed in a dielectric of permitivity K, if their potentials are kept the same as they were in air ?

Ans. Suppose that the two charged bodies possess charges q_1 and q_2 and are placed in air at a distance r apart. The force between the two charged bodies in air is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \qquad ...(i)$$

When the charges are placed in dielectric of permitivity K, the potential due to each charged body will decrease by a factor K. In order to keep their potentials same, the charges on the two bodies should be increased K times *i.e.* the two bodies should possess charges K q_1 and K q_2 respectively. Then, the force between the two charged bodies, when placed in dielectric is given by

$$F' = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{(K q_1)(K q_2)}{r^2} = K\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}\right) \qquad ...(ii)$$

From equations (*i*) and (*ii*), it follows that

$$F' = K F$$

Q. 3.05. Two charges 2μC and −2μC are placed at points A and B 6 cm apart. (*a*) Identify an equipotential surface of the system. (*b*) What is the direction of the electric field at every point on this surface ? (*Text Question*)

Ans. Fig. 3.43 shows two charges of 2 μC and − 2 μC located at points A and B, such that AB = 6 cm.

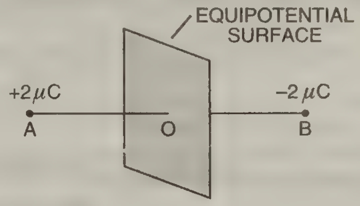


Fig. 3.43

(*a*) For the given system of two charges, the equipotential surface will be a plane normal to the line AB joining the two charges and passing through its mid-point O. On any point on this plane, potential is zero.

(*b*) The electric field is in a direction from point A to point B *i.e.* from positive charge to negative charge and normal to the equipotential surface.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Find an expression for line integral of electric intensity. (*H.S.S.C.E. 2001*)
2. Derive a relation for the work done in moving a charge in an electric field. (*P.S.S.C.E. 1998*)
3. Show that the work done in moving a unit charge along a closed path is zero. (*H.P.S.S.C.E. 2000*)
4. Derive an expression for electric potential at a point due to a point charge. (*P.S.S.C.E. 1999 S ; C.B.S.E. 1993*)

5. Define electric potential at a point. Is it a scalar or a vector quantity ? Derive an expression for the potential at a point due to a point charge.
6. Derive an expression for the electric potential at a point along the axial line of an electric dipole. *(C.B.S.E. 2002, 2000 ; P.S.S.C.E. 2001)*
7. Define electric potential. Deduce an expression for the electric potential at a point at distance *r* from a point charge ($q > 0$). *(C.B.S.E. 2001)*
8. Find an expression for electric potential due to an electric dipole. *(H.S.S.C.E. 2001)*
9. Define potential gradient. Obtain the relation between electric field and potential gradient at a point in the electric field.
10. What do you mean by an equipotential surface ? What w be the shapes of equipotential surfaces in case of a unifor field and a point charge ? Discuss.
11. What do you mean by potential energy of the config ration of two electric charges ? Derive expression fo system of two point charges lying at distance *r*.
12. Derive expression for potential energy of a system of tv point charges and find its relation with electric potent of a charge. *(P.S.S.C.E. 2000, 199*
13. Derive an expression for the total work done in rotati an electric dipole through an angle θ in a uniform elect field. *(C.B.S.E. 199*
14. Derive an expression for the potential energy of an elect dipole in uniform electric field. *(H.S.S.E.C. 2000, 1994 ; P.S.S.C.E. 1995*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Show that the line integral of electric field intensity between any two points depends only on the position of these points and is independent of the path followed between these points. *(H.P.S.S.C.E. 2002 ; P.S.S.C.E. 1993)*
2. Show that work done in moving an electric charge between two points in the electric field of a point charge is independent of the path along which the charge is moved. *(P.S.S.C.E. 1992)*
3. Define potential difference between two points in an electric field. Hence, obtain expression for it.
4. Define the potential difference between two points in an electric field and derive an expression for it. Also give SI unit of potential difference. *(P.S.S.C.E. 1999 S)*
5. Define potential difference between two points in an electric field. Derive the relationship between electric field and the potential difference.
6. Show that electric potential can be represented as li integral of an electric field. *(H.S.S.C.E. 199*
7. Obtain an expression for the electrostatic potential a point in a space due to an electric dipole consisting charges $+q$ at $\left(0, 0, \frac{a}{2}\right)$ and $-q$ at $\left(0, 0, -\frac{a}{2}\right)$.
8. Obtain expression for potential energy of the configurati of three charges. Hence, generalise the result for a syste of *n* point charges.
9. Derive expression for potential energy of an electric dipo placed in a uniform electric field. *(H.P.S.S.C.E. 200*
10. What is meant by potential energy of an electric dipo when placed in an external electric field ? Show that t potential energy U of an electric dipole of dipo moment $\vec{p}$ in a uniform field $\vec{E}$ is given by

$$U = -\vec{p} \cdot \vec{E}$$

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Potential difference and Work done in moving a charge

1. 60 J of work must be done to move electric charge equal to 5 C from a point, where potential is +20 V to another point, where potential is V volt. Find the value of V. **[Ans.** 32 V]
2. 10 joule of work must be done to move a charge of – 200 C from the point A to point B.
   (*i*) Which of the two points is at higher potential ?
   (*ii*) What is the potential difference ?
   [Ans. (*i*) A ; (*ii*) 0·05 V]

Type B. On Potential due to point charges

3. The potential at a point 0·1 m from an isolated point charge is +100 volt. Find the nature and magnitude of the point charge. **[Ans.** $1{\cdot}11 \times 10^{-9}$ C (positive)]
4. What is the electric potential at the surface of an iron nucleus ? The radius of the nucleus is $4{\cdot}2 \times 10^{-15}$ m and the atomic number is 26. **[Ans.** $8{\cdot}91 \times 10^{6}$ V]
5. The electric field at a point due to a point charge is 20 N C^{-1} and the electric potential at that point is 10 J C^{-1}. Calculate the distance of the point from the charge and the mangitude of the charge. *(C.B.S.E. 1996)* **[Ans.** 0·5 m ; $5{\cdot}56 \times 10^{-10}$ C]
6. The potential at a certain distance from a point charge is 600 volt and the electric field is 200 N C^{-1}. Find
   (*i*) the distance of the point from the charge ;
   (*ii*) the magnitude of the charge. **[Ans.** 3 m ; 2×10^{-7}
7. A spherical oil drop of raidus 10^{-4} cm has on it at a certa time a charge of 40 electrons. Calculate the energy th would be required to place an additional electron on t drop. Charge on an electron = $1{\cdot}6 \times 10^{-19}$ C. **[Ans.** $9{\cdot}22 \times 10^{-21}$

Type C. On Potential due to group of charges

8. Two charges equal to + 20 μC and – 10 μC are placed points 6 cm apart. Find the value of the potential at a poi distant 4 cm on the right bisector of the line joining the tv charges. **[Ans.** $1{\cdot}8 \times 10^{6}$
9. Two tiny spheres carrying charges 1·5 μC and 2·5 μC a located 30 cm apart. Find the potential and electric field the mid-point of the line joining the two charges. **[Ans.** $2{\cdot}4 \times 10^{5}$ V, $4{\cdot}0 \times 10^{5}$ N C^{-1} (from 2·5 μC to 1·5 μC
10. Two charges of values 50 μC and 100 μC are placed a distance of 6 cm apart. Find the field and potential a point (between two charges) 2 cm from the charge of val 50 μC. **[Ans.** $5{\cdot}625 \times 10^{8}$ N C^{-1} (from 50 μC to 100 μC), $4{\cdot}5 \times 10^{7}$
11. Two point charges, each of 3×10^{-9} C located at the tv vertices of an equilateral triangle of side 20 cm. How mu work must be done to bring a charge of 10^{-9} C upto t third corner of the triangle from infinity ? **[Ans.** $2{\cdot}7 \times 10^{-7}$

12. The sides of a rectangle ABCD are 15 cm and 5 cm. Two point charges of $+2\mu$C and -5μC are placed at the corners A and C respectively [Fig. 3.44]. Calculate the work done in carrying a charge of 3 μC from point B to D. [**Ans.** 2·52 J]

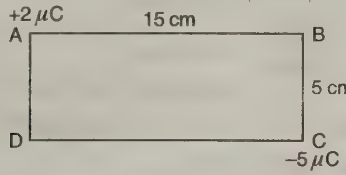


Fig. 3.44

pe D. On Potential gradient and electric field

13. A small particle carrying a negative charge of $1{\cdot}6 \times 10^{-19}$ C is suspended in equilibrium between the horizontal metal plates 5 cm apart, having a potential difference of 3000 volt across them. Find the mass of the particle. [**Ans.** $9{\cdot}8 \times 10^{-16}$ kg]

14. What is the potential gradient (in V m^{-1}) at a distance of 10^{-12} m from the centre of the platinum nucleus ? What is the potential gradient at the surface of the nucleus ? Atomic number of platinum is 78 and the radius of platinum nucleus may be taken as 5×10^{-15} m. [**Ans.** $1{\cdot}123 \times 10^{17}$ V m^{-1} ; $4{\cdot}493 \times 10^{21}$ V m^{-1}]

15. The electric potential V (x) in a region along the X-axis varies with the distance x (in metre) according to the relation $V(x) = 4x^2$. Calculate the force experienced by a 1 μC charge placed at point $x = 1$ m. (*P.S.S.C.E. 1999 S*) [**Ans.** -8×10^{-6} N]

16. In problem no. 2, if the points are 4 cm apart, what is the component of the electric field parallel to the line joining the points A and B. [**Ans.** 1·25 V m^{-1}]

pe E. On Potential energy of a system of charges

17. In the nucleus of ${}_{92}U^{238}$, two protons are at a distance of 6×10^{-15} m. Calcualte their electrostatic potential energy. [**Ans.** $3{\cdot}84 \times 10^{-14}$ J]

18. Two point charges equal to + 4 μC and + 2 μC are 12 cm apart. What is the amount of work done to bring them closer through a distance of 3 cm ? [**Ans.** 0·2 J]

19. There charges $-q$, Q and $-q$ are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, then what is the ratio of Q : q ? [**Ans.** 1 : 4]

20. Three points charges $+q$, $+2q$ and Q are placed at the three vertices of an equilateral triangle. Find the value of charge Q (in terms of q), so that electric potential energy of the system is zero. [**Ans.** $-2q/3$]

21. Find the electrostatic potential energy of the configuration of four charges $+q, -q, +q$ and $-q$ placed at the four corners A, B, C and D of a square of side r.

$$\left[\textbf{Ans. } \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2(\sqrt{2}-4)}{r}\right]$$

22. Two isolated metallic solid spheres of radii R and 2 R are charged, such that both of these have same charge density σ. The spheres are located far away from each other and connected by a thin conducting wire. Find the new charge density on the bigger sphere. (*I.I.T. 1996*) [**Ans.** $\frac{5}{6}\sigma$]

23. Two electrons are moving towards each other, each with a velocity of 10^6 m s^{-1}. What will be closest distance of approach between them. [**Ans.** $2{\cdot}53 \times 10^{-10}$ m]

Type F : On potential energy of an electric dipole

24. An electric dipole of length 2 cm is placed with its axis making an angle of 30° to a uniform electric field of 10^5 N C^{-1}. If it experiences a torque of $10\sqrt{3}$ N m, calculate (*i*) the magnitude of the charge on the dipole and (*ii*) potential energy of the dipole. (*C.B.S.E. 2000*) [**Ans.** (*i*) $\sqrt{3} \times 10^{-2}$ C (*ii*) -30 J]

25. An electric dipole consists of two opposite charges each of magnitude $6 \times 10^{-\mathcal{E}}$ coulomb separated by 6·0 cm. The dipole is placed in an external electric field of 5×10^{-5} N C^{-1}. (*i*) What maximum torque will the field exert on the dipole ? (*ii*) How much work will an external agent have to do in turning the dipole through 180°, starting from the position $\theta = 0°$? [**Ans.** (*i*) $1{\cdot}8 \times 10^{-13}$ N m ; (*ii*) $3{\cdot}6 \times 10^{-13}$ J]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Proceed as in solved problem 3.01.
2. (*i*) Since work is done in moving a negative charge from point A to B, **point A is at higher potential.**

(*ii*) $V_A - V_B = \frac{10}{200} = \mathbf{0{\cdot}05\ V}$

3. Now, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

or $q = 4\pi\varepsilon_0 r V = \frac{0 \cdot 1 \times 100}{9 \times 10^9} = \mathbf{1{\cdot}11 \times 10^{-9}\ C\ (positive)}$

4. Proceed as in solved problem no. 3.02.
5. Proceed as in solved problem no. 3.05.
6. Proceed as in solved problem no. 3.05.
7. Charge on drop, $q = 40\,e = 40 \times 1{\cdot}6 \times 10^{-19} = 6{\cdot}4 \times 10^{-18}$ C

radius of drop, $r = 10^{-4}$ cm $= 10^{-6}$ m

Potential at the surface of the drop,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 6 \cdot 4 \times 10^{-18}}{10^{-6}} = 5{\cdot}76 \times 10^{-2}\ \text{V}$$

Energy required to place an additional electron on the drop,

$W = V \times e = 5{\cdot}76 \times 10^{-2} \times 1{\cdot}6 \times 10^{-19} = \mathbf{9{\cdot}22 \times 10^{-21}\ J}$

8. Here, $q_A = +20\ \mu\text{C} = 20 \times 10^{-6}$ C,

$q_B = -10\ \mu\text{C} = -10 \times 10^{-6}$ C,

AB = 6 cm, OP = 4 cm

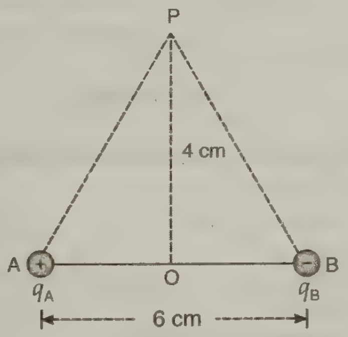


Fig. 3.45

$\therefore$ $AP = BP = \sqrt{3^2 + 4^2} = 5\ \text{cm} = 0{\cdot}05$ m

Potential at point P, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{AP} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{BP}$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A + q_B}{AP} = \frac{9 \times 10^9 \times (20 \times 10^{-6} - 10 \times 10^{-6})}{0 \cdot 05}$$

$= \mathbf{1{\cdot}8 \times 10^6\ V}$

9. Here, $q_A = 1{\cdot}5\ \mu C = 1{\cdot}5 \times 10^{-6}\ C$, $q_B = 2{\cdot}5\ \mu C = 2{\cdot}5 \times 10^{-6}\ C$

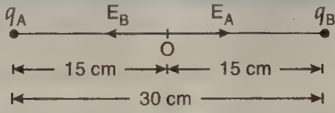


Fig. 3.46

Potential at point O, $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\left(\dfrac{q_A}{OA}+\dfrac{q_B}{OB}\right)$

$= \mathbf{2{\cdot}4 \times 10^5\ V}$

Electric field at point O, $E = E_B - E_A$ $(\because E_B > E_A)$

or $E = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_B}{OB^2}-\dfrac{q_A}{OA^2}\right) = \mathbf{4\times10^5\ N\ C^{-1}}$ **(along OA)**

10. Proceed as in problem no. 9.

11. Suppose that charges q_A and q_B are placed at corners A and B of an equilateral Δ ABC of side r.
Here, $q_A = q_B = 3 \times 10^{-9}$ C and $r = 20$ cm $= 0{\cdot}2$ m
Potential at the corner C due to charges q_A and q_B,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{AC} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{BC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A+q_B}{r}$$

$$= 9\times10^9 \times \frac{3\times10^{-9}+3\times10^{-9}}{0{\cdot}2} = 270\ \text{V}$$

Therefore, work done to bring charge of 10^{-9} C from infinity to corner C,

$W = V \times q = 270 \times 10^{-9} = \mathbf{2{\cdot}7 \times 10^{-7}\ J}$

12. $$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A}{AB}+\frac{q_C}{CB}\right)$$

$$= 9\times10^9\left(\frac{2\times10^{-6}}{0{\cdot}15}+\frac{-5\times10^{6}}{0{\cdot}05}\right) = -78\times10^4\ \text{V}$$

$$V_D = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A}{AD}+\frac{q_C}{CD}\right)$$

$$= 9\times10^9\left(\frac{2\times10^{-6}}{0{\cdot}05}+\frac{-5\times10^{-6}}{0{\cdot}15}\right) = +6\times10^4\ \text{V}$$

Work done to move 3 μC charge from point B to D,
$W = (V_D - V_B) \times q = [6 \times 10^4 - (-78 \times 10^4)] \times 3 \times 10^{-6}$
$= \mathbf{2{\cdot}52\ J}$

13. $E = \dfrac{V}{d} = \dfrac{3000}{5\times10^{-2}} = 6\times10^4\ \text{V m}^{-1}$

As the charged particle remains suspended,

$qE = mg$ or $m = \dfrac{qE}{g} = \dfrac{1{\cdot}6\times10^{-19}\times6\times10^4}{9{\cdot}8}$

$= \mathbf{9{\cdot}8 \times 10^{-16}\ kg}$

14. Here, $q = +Ze = 78 \times 1{\cdot}6 \times 10^{-19}$
Potential gradient at a point is numerically equal to electric field at that point *i.e.*

$$\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

$$\therefore \quad \frac{dV}{dr}\ (\text{at } r = 10^{-12}\ \text{m}) = \frac{9\times10^9\times78\times1{\cdot}6\times10^{-19}}{(10^{-12})^2}$$

$= \mathbf{1{\cdot}123 \times 10^{17}\ V\ m^{-1}}$

Similarly, $\dfrac{dV}{dr}$ (at $r = 5 \times 10^{-15}$ m) $= \mathbf{4{\cdot}493 \times 10^{21}\ V\ m^{-1}}$

15. $V(x) = 4x^2$

Now, $E = -\dfrac{dV}{dx} = -\dfrac{d}{dx}(4x^2) = -8x$

$\therefore$ E (at $x = 1$ m) $= -8 \times 1 = -8$ N C^{-1} (or V m^{-1})

Force experienced by the 1 μC charge,
$F = qE = 10^{-6} \times (-8) = \mathbf{-8 \times 10^{-6}\ N}$

16. Here, $dV = V_A - V_B = 0{\cdot}05$ V and $dr = 4$ cm $= 0{\cdot}04$ m

Now, $E = \dfrac{dV}{dr}$ (in magnitude)

$= \dfrac{0{\cdot}05}{0{\cdot}04} = \mathbf{1{\cdot}25\ V\ m^{-1}}$

17. $$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r} = \frac{9\times10^9\times1{\cdot}6\times10^{-19}\times1{\cdot}6\times10^{-19}}{6\times10^{-15}}$$

$= \mathbf{3{\cdot}84 \times 10^{-14}\ J}$

18. Here, $q_1 = 4\ \mu C = 4 \times 10^{-6}$ C and $q_2 = 2\ \mu C = 2 \times 10^{-6}$ C

Now, $U = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1 q_2}{r}$

When at r = 12 cm :

$$U_1 = \frac{9\times10^9\times4\times10^{-6}\times2\times10^{-6}}{0{\cdot}12} = 0{\cdot}6\ \text{J}$$

When at r = 12 − 3 i.e. 9 cm :

$$U_2 = \frac{9\times10^9\times4\times10^{-6}\times2\times10^{-6}}{0{\cdot}09} = 0{\cdot}8\ \text{J}$$

Therefore, work done,
$W = U_2 - U_1 = 0{\cdot}8 - 0{\cdot}6 = \mathbf{0{\cdot}2\ J}$

19. Suppose that charges $-q$, Q and $-q$ are placed at poin A, B and C on a straight line, such that AB = BC = r.

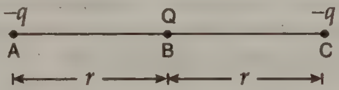


Fig. 3.47

Then, $\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{(-q)Q}{AB}+\dfrac{(-q)(-q)}{AC}+\dfrac{Q(-q)}{BC}\right] = 0$

or $-\dfrac{Q}{r}+\dfrac{q}{2r}-\dfrac{Q}{r} = 0$ or $\dfrac{2Q}{r} = \dfrac{q}{2r}$

or $\dfrac{Q}{q} = \dfrac{1}{4}$

20. Suppose that charges $+q$, $+2q$ and Q are placed at corne A, B and C of an equilateral ΔABC of side r. Then,

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q(2q)}{r}+\frac{qQ}{r}+\frac{(2q)Q}{r}\right] = 0$$

or $2q + Q + 2Q = 0$ or $Q = -2q/3$

21. Suppose that charges q_A, q_B, q_C and q_D are placed at t four corners A, B, C and D of a square of each side r.
Here, $q_A = +q$, $q_B = -q$, $q_C = +q$ and $q_D = -q$

$$U = \frac{1}{4\pi\varepsilon_0}$$

$$\times\left[\frac{q_A q_B}{AB}+\frac{q_A q_C}{AC}+\frac{q_A q_D}{AD}+\frac{q_B q_C}{BC}+\frac{q_B q_D}{BD}+\frac{q_C q_D}{CD}\right.$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{r}+\frac{q^2}{\sqrt{2}r}-\frac{q^2}{r}-\frac{q^2}{r}+\frac{q^2}{\sqrt{2}r}-\frac{q^2}{r}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2(\sqrt{2}-4)}{r}$$

22. Let σ be charge density of the two spheres. Then,

$q_1 = 4\pi R^2\sigma$ and $q_2 = 4\pi(2R)^2\sigma = 16\pi R^2\sigma$

Let q_1' and q_2' be charges on the two spheres, when th are connected with a wire. Then,

$q_1' + q_2' = q_1 + q_2$ ...

Since the two spheres are at same potential,

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2'}{2R} \quad \text{and} \quad q_1' = \frac{q_2'}{2}$$

Substituting for q_2' in equation (*i*), we have

$$\frac{q_2'}{2} + q_2' = q_1 + q_2 \quad \text{or} \quad q_2' = \frac{2}{3}(q_1 + q_2)$$

or $$q_2' = \frac{2}{3}(4\pi R^2\sigma + 16\pi R^2\sigma) = \frac{40\pi R^2\sigma}{3}$$

Therefore, new charge density of the bigger sphere,

$$\sigma' = \frac{q_2'}{4\pi(2R)^2} = \frac{40\pi R^2\sigma}{3\times16\pi R^2} = \frac{5}{6}\sigma$$

23. The electrons will approach each other to a distance r_0 (closest distance), where their kinetic energy will become zero. It will appear as their potential energy. Therefore,

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e\times e}{r_0} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

or $$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{mv^2} = \frac{9\times10^9\times(1\cdot6\times10^{-19})^2}{9\cdot1\times10^{-31}\times(10^6)^2}$$

$$= \mathbf{2{\cdot}53\times10^{-10}\ m}$$

24. **(*i*)** $\tau = pE\sin\theta$

or $$p = \frac{\tau}{E\sin\theta} = \frac{10\sqrt{3}}{10^5\sin30^\circ} = 2\sqrt{3}\times10^{-4}\ \text{C m}$$

Now, $$q = \frac{p}{2a} = \frac{2\sqrt{3}\times10^{-4}}{2\times10^{-2}} = \sqrt{3}\times10^{-2}\ \text{C}$$

(*ii*) $U = -pE\cos\theta = -2\sqrt{3}\times10^{-4}\times10^5\times\cos30^\circ = \mathbf{-30\ J}$

25. Proceed as in solved problem no. 3.17.

CHAPTER
4

Gauss's Theorem

4.01. AREA VECTOR – A MATHEMATICAL NOTE

In many problems, area of a surface is treated as a vector. An area element dS is represented by vector $\vec{dS}$, such that the arrow respresenting the area vector $\vec{dS}$ is perpendicular to the area element [Fig. 4.01.]. The length of the area vector $\vec{dS}$ represents the magnitude of the area element dS. In case $\hat{n}$ is a unit vector along normal to the area element dS, then

$$\vec{dS} = dS\,\hat{n} \qquad \text{...(4.01)}$$

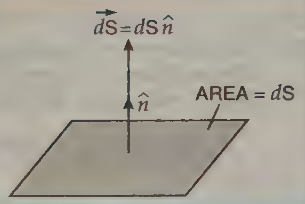


Fig. 4.01

Note. Conventionally, the area vector representing an area element of a closed surface enclosing a volume is assigned direction, such that the area vector is directed along the normal pointing outwards from the closed surface.

4.02. ELECTRIC FLUX

The electric flux through a surface held inside an electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.

Electric flux is a ***scalar quantity*** and is denoted by ϕ*.

In fact, flux is property of a vector field and likewise electric flux is associated with electric field.

Relation between electric field intensity and electric flux. Suppose that a surface having area S is placed inside electric field of intensity $\vec{E}$ as shown in Fig. 4.02. In order to find the electric flux through the surface of area S, consider a small area element dS of the surface S. The elementary area dS can be represented by a vector $\vec{dS}$, which is directed along normal to the area element dS. Suppose that electric field $\vec{E}$ makes an angle θ with the area vector $\vec{dS}$. Then, component of electric field along normal to the area element dS *i.e.* along area vector $\vec{dS}$ is given by

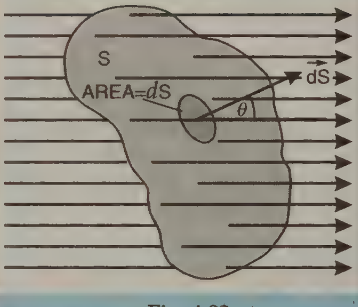


Fig. 4.02

$$E_n = E\cos\theta$$

Hence, electric flux crossing the area element dS in a direction along normal to it given by

$$d\phi = E_n\, dS = (E\cos\theta)\, dS$$

or
$$d\phi = \vec{E}\cdot\vec{dS} \qquad \text{...(4.02)}$$

The electric flux through the whole surface S can be found by integrating the above over the whole surface S. Therefore, total electric flux through the surface S is given by

$$\phi = \int_S \vec{E}\cdot\vec{dS} = \int_S E_n\, dS \qquad \text{...(4.03)}$$

Thus, *electric flux linked with a surface in an electric field may be defined as the surface integral of the electric field over that surface.*

The unit of electric flux is **N m^{-2} C^{-1}**.

Self-test Question

Show that the unit of electric flux
N m^{-2} C^{-1}.

Note. In case the surface S is a closed surface, then the total electric flux through the closed surface is given by

$$\Phi = \oint_S \vec{E}\cdot\vec{dS} = \oint_S E_n\, dS \qquad \text{...(4.04)}$$

---

* The electric flux through a surface is denoted by ϕ, while the electric flux through a closed surface is denoted by Φ.

4.03. GAUSS'S THEOREM

It states that *the total electric flux through a closed surface enclosing a charge is equal* $\frac{1}{\varepsilon_0}$ *times the magnitude of the charge enclosed.*

Here, ε_0 is the absolute permittivity of the free space.

If a closed surface S encloses an electric charge q, then according to Gauss's eorem, the total electric flux through the closed surface is given by

$$\Phi = \frac{q}{\varepsilon_0} \qquad \text{...(4.05)}$$

By definition, the total electric flux through the closed surface S is given by

$$\Phi = \oint_S \vec{E} \cdot \vec{dS},$$

here $\vec{E}$ is electric field at the area element $\vec{dS}$.

Therefore, Gauss's theorem may be expressed as

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0} \qquad \text{...(4.06)}$$

Hence, Gauss's theorem may also be stated as below :

If a closed surface encloses a charge, then surface integral of the electric field (due to e enclosed charge) over the closed surface is equal to $\frac{1}{\varepsilon_0}$ *times the charge enclosed.*

4.04. TO PROVE GAUSS'S THEOREM (For a spherically symmetric closed surface)

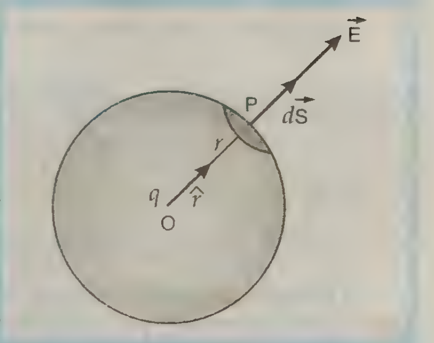


Fig. 4.03

Consider that a point electric charge q is situated at the centre of a sphere of dius r. Let $\vec{E}$ be the electric field at any point P on the surface of the sphere [Fig. 03]. Then, according to Coulomb's law,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,\hat{r},$$

here $\hat{r}$ is unit vector along the line OP.

Consider a small area element $\vec{dS}$ (shown shaded) around the point P. Since e small area element is located on the surface of the sphere, the area vector $\vec{dS}$ will so be along OP *i.e.* in the direction of unit vector $\hat{r}$. Therefore, electric flux through ea element $\vec{dS}$ is given by

$$d\phi = \vec{E} \cdot \vec{dS} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,\hat{r} \cdot \vec{dS} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,(1)\,(dS)\cos 0^\circ$$

or

$$d\phi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,dS$$

Therefore, electric flux through the closed surface of the sphere,

$$\Phi = \oint_S d\phi = \oint_S \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,dS = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \oint_S dS$$

Now, $\oint_S dS$ = surface area of the sphere of radius $r = 4\pi r^2$

$$\therefore \qquad \Phi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

It proves the Gauss's theorem in electrostatics.

Note. Gauss's theorem has been proved above in a special case, when the charge ies at the centre of a sphere. In fact, the theorem holds for a closed surface of any ape.

4.05. GAUSSIAN SURFACE

The expression for electric field intensity can be obtained by applying oulomb's law only in simple cases. In the situations, where Coulomb's law or inciple of superposition cannot be applied to calculate the electric field, the same achieved by using Gauss's law. For this, one has to evaluate the surface integral.

So that surface integral can be evaluated easily, a closed surface is chosen cleverly around the charge distribution. The surface so chosen is called the ***gaussian surface***.

Thus, *gaussian surface around a charge distribution (may be a point charge, a line charge, a surface charge or a volume change) is a closed surface, such that electric field intensity at all the points on the surface is same and the electric flux through the surface is along the normal to the surface.*

4.06. TO DEDUCE COULOMB'S LAW FROM GAUSS'S THEOREM

Consider that two point charges q_1 and q_2 are placed at points A and B at a distance r apart in vacuum. Let E be the magnitude of electric field at the location of point charge q_2 due to the charge q_1. Then, force experienced by the point charge q_2 due to the electric field of the charge q_1,

$$F = q_2 E \qquad ...(4.07)$$

To find E, draw a spherical surface of radius r with point A as the centre as shown in Fig. 4.04. According to Gauss's theorem, the total electric flux through the spherical gaussian surface is given by

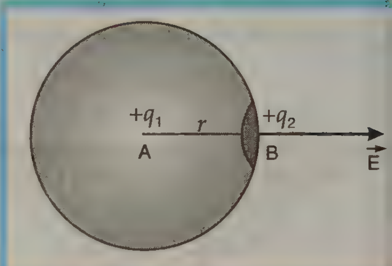


Fig. 4.04

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{q_1}{\varepsilon_0}$$

$$\therefore \quad \frac{q_1}{\varepsilon_0} = \oint_S \vec{E} \cdot \vec{dS} = \oint E\, dS \qquad (\because \vec{E} \parallel \vec{dS})$$

Now, the value of E (magnitude of electric field) is same at all the points on the spherical surface *i.e.* E is constant. Therefore,

$$\frac{q_1}{\varepsilon_0} = E \oint dS$$

Now, $\oint dS = 4\pi r^2$ (surface area of the spherical guassian surface of radius r)

$$\therefore \quad \frac{q_1}{\varepsilon_0} = E \times 4\pi r^2$$

or

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r^2} \qquad ...(4.08)$$

From equations (4.07) and (4.08), we have

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

It is the mathematical form of Coulomb's law in electrostatics.

4.07. ELECTRIC FIELD DUE TO A LINE CHARGE

Consider a thin infinitely long straight line charge having a uniform linear charge density λ placed along YY′. By symmetry, it follows that electric field due to line charge at a distance r in any plane at right angles to the line charge is of the same magnitude and is directed radially outward. To find electric field due to line charge at point P distant r from it, draw a cylindrical surface of radius r and length l about the line charge as its axis [Fig. 4.05]. This cylindrical surface may be treated as the gaussian surface for the line charge.

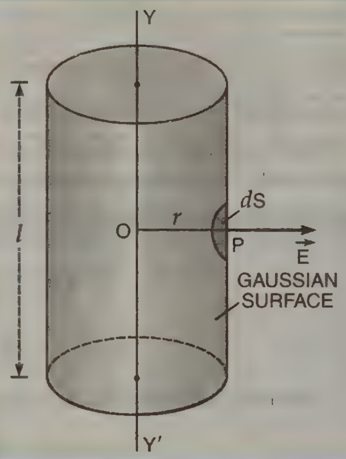


Fig. 4.05

Let us now calculate the electric flux that crosses the Gaussian surface from the charge enclosed by the gaussian surface. Since electric lines of force are parallel to end faces (circular caps) of the cylinder, there is no component of field along the normal to the end faces. The electric flux crosses only through the curved surface of the cylinder, as the electric field due to the line charge is normal to the curved surface. If E is the magnitude of electric field at point P, then electric flux through the gaussian surface is given by

$\Phi = E \times$ area of the curved surface of a cylinder of radius r and length l

or

$$\Phi = E \times 2\pi r l \qquad ...(4.09)$$

According to Gauss's theorem, we have

$$\Phi = \frac{q}{\varepsilon_0}$$

Now, charge enclosed by the gaussian surface, $q = \lambda l$

$$\therefore \quad \Phi = \frac{\lambda l}{\varepsilon_0} \qquad ...(4.10)$$

Key point

The electric field due to a line charge fall off with distance as $1/r$.

From equations (4.09) and (4.10), we have

$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

or $$E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r} \qquad ...(4.11)$$

In contrast to electric field due to a point charge (which decreases inversely as he square of the distance from the charge), the field due to a line charge falls off as $\frac{1}{r}$.

Note. The above discussion can be easily extended to find electric field due to uniformly charged cylindrical shell. The same has been deduced in section 5.08 cylindrical capacitor) in the next chapter.

4.08. ELECTRIC FIELD DUE TO AN INFINITE PLANE SHEET OF CHARGE

Consider an infinite thin plane sheet* of positive charge having a uniform urface charge density σ on both sides of the sheet. By symmetry, it follows that the lectric field is perpendicular to the plane sheet of charge and is directed in outward irection. To find electric field due to the plane sheet of charge at any point P distant from it, draw a cylinder of area of cross-section A through the point P as the aussian surface [Fig. 4.06]. Since electric lines of force are parallel to the curved urface of the cylinder, the flux due to electric field of the plane sheet of charge passes nly through the two circular caps of the cylinder. If E is the magnitude of electric eld at point P, then electric flux crossing through the gaussian surface is given by

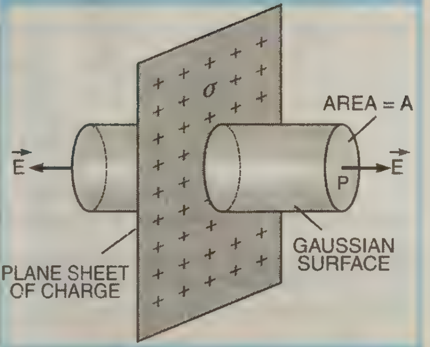


Fig. 4.06

$$\Phi = E \times \text{area of the end faces (circular caps) of the cylinder}$$

or $$\Phi = E \times 2A \qquad ...(4.12)$$

According to Gauss's theorem, we have

$$\Phi = \frac{q}{\varepsilon_0}$$

Here, the charge enclosed by the gaussian surface, $q = \sigma A$

∴ $$\Phi = \frac{\sigma A}{\varepsilon_0} \qquad ...(4.13)$$

From equations (4.12) and (4.13), we have

$$E \times 2A = \frac{\sigma A}{\varepsilon_0}$$

or $$E = \frac{\sigma}{2\varepsilon_0} \qquad ...(4.14)$$

Thus, we find that the magnitude of the electric field at a point due to an infinite lane sheet of charge is independent of its distance from the sheet of charge.

Key point

The magnitude of electric field due to an infinite plane sheet of charge is independent of the distance of the observation point from the sheet of charge.

Note. In case of an infinite plane conductor having finite thickness, the gaussian urface will include charge σ × 2A (σ A on each side of the conductor). Therefore, n such a case,

$$E \times 2A = \frac{\sigma \times 2A}{\varepsilon_0}$$

or $$E = \frac{\sigma}{\varepsilon_0} \qquad ...(4.15)$$

Watch out !

In a sheet of charge, the same charge shows up on its two sides. Since a plane charge conductor has a finite thickness, different charges show up on its two sides !

4.09. ELECTRIC FIELD DUE TO TWO INFINITE PLANE PARALLEL SHEETS OF CHARGE

Consider two infinite plane parallel sheets of charge A and B, having surface harge densities equal to σ_A and σ_B respectively. The two sheets divide the space nto three regions namely region I lying to the left of sheet A, region II between the heets A and B and region III to the right of sheet B as shown in Fig. 4.07.

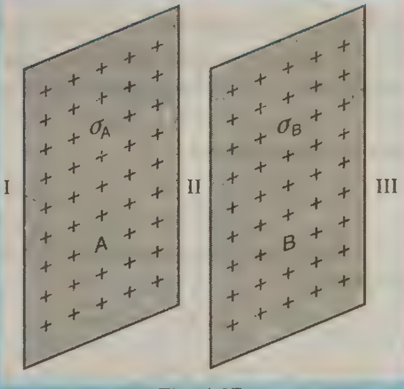


Fig. 4.07

(*i*) ***In region I.*** The electric fields due to both the sheets of charge will be from ight to left (opposite to the direction, in which distances are measured as positive). ince electric field due to a sheet of charge is given by equation (4.14), the electric ield due to sheets A and B in region I will be

$$E = \left(-\frac{\sigma_A}{2\varepsilon_0}\right) + \left(-\frac{\sigma_B}{2\varepsilon_0}\right)$$

---

*In a sheet of charge, the same charge shows up on its two sides.

or $$E = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B) \qquad ...(4.16)$$

(ii) In region II. The electric field due to sheet of charge A will be from left to right (along positive direction) and that due to sheet of charge B will be from right to left (along negative direction). Therefore, in region II,

$$E = \frac{\sigma_A}{2\varepsilon_0} + \left(-\frac{\sigma_B}{2\varepsilon_0}\right)$$

or $$E = -\frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B) \qquad ...(4.17)$$

(iii) In region III. The electric fields due to both the sheets of charge will be from left to right *i.e.* along positive direction. Therefore, in region III,

$$E = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0}$$

$$E = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B) \qquad ...(4.18)$$

Special case. If $\sigma_A = \sigma$ and $\sigma_B = -\sigma$, then it follows that electric field is zero in regions I and III, while in the region II, the electric field is given by

$$E = \frac{1}{2\varepsilon_0} + [\sigma - (-\sigma)]$$

or $$E = \frac{\sigma}{\varepsilon_0} \qquad ...(4.19)$$

Thus, in case of two infinite plane sheets of charge having equal and opposite surface charge densities, the field is non-zero only in the space between the two sheets and it is constant *i.e.* uniform in this region. Further, the field is independent of the distance between the infinite plane sheets of charge.

Note. In case two infinite plane charged conductors of finite thickness are placed parallel to each other, the equations (4.16), (4.17) and (4.18) will modify to :

$$E = -\frac{1}{\varepsilon_0}(\sigma_A + \sigma_B) \qquad ...\text{in region I}$$

$$E = \frac{1}{\varepsilon_0}(\sigma_A + \sigma_B) \qquad ...\text{in region II}$$

and $$E = \frac{1}{\varepsilon_0}(\sigma_A + \sigma_B) \qquad ...\text{in region III}$$

4.10. ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

Consider a thin spherical shell of radius R and centre O. Let q be the charge on the spherical shell. Let us find electric field at point P distant r from the centre of the spherical shell.

(a) When point P lies outside the spherical shell. Draw the gaussian surface through point P. It will be a spherical shell of radius r and centre O [Fig. 4.08].

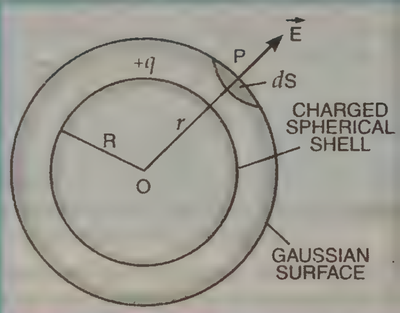


Fig. 4.08

Let $\vec{E}$ be the electric field at point P due to charge q on the spherical shell.

It is evident that the field due to charged spherical shell is radial and spherically symmetric. At every point on the surface of shell, the field has same magnitude and is along normal to the surface. Therefore, total flux through the gaussian surface is given by

$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \oint_S E\, dS = E \oint_S dS = E \cdot 4\pi r^2$$

Since the charge enclosed by the gaussian surface is q, according to the Gauss's theorem,

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \quad (\text{for } r > R) \qquad ...(4.20)$$

From equation (4.20), it follows that for the points outside the charged spherical shell, the shell behaves as if the charge on the shell were concentrated at its centre.

Key point

For points outside a charged spherical shell, the shell behaves as if the charge on it were concentrated at its centre.

The above result for electric field due to a charged spherical shall can also be xpressed in terms of its surface density as explained below :

If σ is uniform surface charge density of the spherical shell, then

$$q = 4\pi R^2 \sigma$$

Substituting for q in equation (4.20), we have

$$E = \frac{\sigma}{\varepsilon_0} \cdot \frac{R^2}{r^2} \qquad (\text{for } r > R) \qquad \text{...(4.21)}$$

Watch out !

When a conducting sphere is charged, the charge appears on its outer surface. A solid sphere of charge is built of charges, which are distributed uniformly inside it and stay there as such !

(b) When point P lies on the surface of spherical shell. The gaussian surface hrough point P will just enclose the charged spherical shell. Therefore, according o Gauss's theorem,

$$E \,.\, 4\pi R^2 = \frac{q}{\varepsilon_0}$$

or
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} \qquad (\text{for } r = R) \qquad \text{...(4.22)}$$

Since $q = 4\pi R^2 \sigma$, the equation (4.22) becomes

$$E = \frac{\sigma}{\varepsilon_0} \qquad (\text{for } r = R) \qquad \text{...(4.23)}$$

(c) When point P lies inside the spherical shell. In such a case, the gaussian urface through point P will not enclose any charge and hence according to the Gauss's theorem,

$$E \,.\, 4\pi r^2 = \frac{0}{\varepsilon_0}$$

or
$$E = 0 \qquad (\text{for } r < R) \qquad \text{...(4.24)}$$

Thus, at a point inside the charged spherical shell, electric field is zero.

Key point

At a point inside a charged spherical shell, electric field is zero.

4.11. ELECTRIC FIELD DUE TO A SOLID SPHERE OF CHARGE

Consider an isolated sphere of charge q having radius R. Suppose we have to alculate electric field at point P at a distance r from its centre O.

(a) When point P lies outside the sphere of charge : Draw the gaussian surface hrough point P so as to enclose the sphere of charge. The gaussian surface is a pherical shell of radius r and centre O [Fig. 4.09].

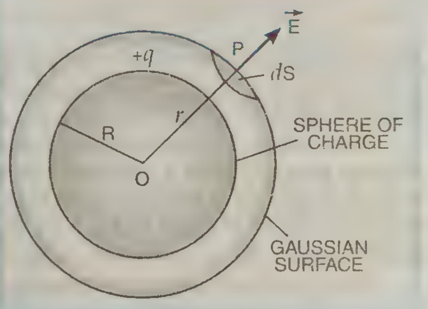


Fig. 4.09

Let $\vec{E}$ be the electric field at point P due to the sphere of charge q. It is evident hat the field due to the sphere charge is spherically symmetric. At every point on he surface of the sphere, the field has same magnitude and is along normal to the urface.

Therefore, total flux through the gaussian surface is given by

$$\Phi = \oint_S \vec{E} \,.\, \vec{dS} = \oint_S E \,.\, \vec{dS} = E \oint_S dS = E \,.\, 4\pi r^2$$

According to Gauss's theorem,

$$E \,.\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad (\text{for } r > R) \qquad \text{...(4.25)}$$

It is the same as that at a distance r from a point charge q. It implies that for points outside the sphere of charge, the sphere behaves as if all the charge on the sphere were concentrated at its centre. If ρ is the uniform volume charge density of the sphere of charge, then

Key point

For points outside the sphere of charge, the sphere also behaves as if charge on the sphere were concentrated at its centre.

$$q = \frac{4}{3}\pi R^3 \rho$$

Substituting for q in equation (4.25), we have

$$E = \frac{\rho}{\varepsilon_0} \cdot \frac{R^3}{3r^2} \qquad (\text{for } r > R) \qquad \text{...(4.26)}$$

(b) When point P lies at the surface of the sphere of charge : In such a case, the rea of the gaussian surface is equal to the surface area of the sphere of charge *i.e.* equal to $4\pi R^2$. If E is the electric field at point P on the surface of the sphere of charge, hen according to Gauss's theorem,

$$E \,.\, 4\pi R^2 = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} \qquad \text{(for } r = R\text{)} \qquad ...(4.27)$$

Since $q = \frac{4}{3}\pi R^3 \rho$, equation (4.27) becomes

$$E = \frac{\rho R}{3\varepsilon_0} \qquad \text{(for } r = R\text{)} \qquad ...(4.28)$$

(c) When point P lies inside the sphere of charge : In such a case, the gaussian surface is *a* spherical shell, whose centre is O and radius is r (= OP) as shown in Fig. 4.10. If q' is charge enclosed by the gaussian surface, then

$$E \,.\, 4\pi r^2 = \frac{q'}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q'}{r^2} \qquad \text{(for } r < R\text{)} \qquad ...(4.29)$$

Now, $$q' = \frac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = q\frac{r^3}{R^3}$$

Substituting for q' in equation (4.29), we have

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,r}{R^3} \qquad \text{(for } r < R\text{)} \qquad ...(4.30)$$

Since $q = \frac{4}{3}\pi R^3 \rho$, the equation (4.30) becomes

$$E = \frac{\rho\, r}{3\varepsilon_0} \qquad \text{(for } r < R\text{)} \qquad ...(4.31)$$

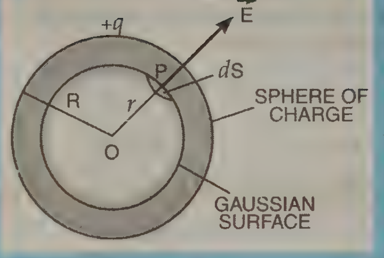


Fig. 4.10

SOLVED NUMERICAL PROBLEMS

Type A. On Electric Flux

Problem 4.01. A rectangular surface of sides 10 cm and 15 cm is placed inside a uniform electric field of 25 V m^{-1}, such that normal to the surface makes an angle of 60° with the direction of electric field. Find the flux of the electric field through the rectangular surface.

Sol. The flux through the rectangular surface is given by

$$\phi = \vec{E} \,.\, \vec{\Delta S} = E\, \Delta S \cos\theta$$

Here, $E = 25 \text{ V m}^{-1}$;

$\Delta S = 10 \times 15 = 150 \text{ cm}^2 = 150 \times 10^{-4} \text{ m}^2$

and $\theta = 60°$

$\therefore$ $\phi = 25 \times 150 \times 10^{-4} \cos 60°$

$= \mathbf{0 \cdot 1875 \text{ N m}^2 \text{ C}^{-1}}$

Problem 4.02. Consider a uniform electric field $\vec{E} = 3 \times 10^3 \,\hat{i}\, \text{N C}^{-1}$. (*a*) What is the flux of this field through a square of 10 cm on a side, whose plane is parallel to the YZ-plane ? (*b*) What is the flux through the same square, if the normal to its plane makes a 60° angle with the X-axis ? (*Text Problem*)

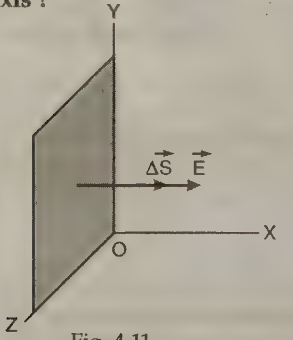


Fig. 4.11

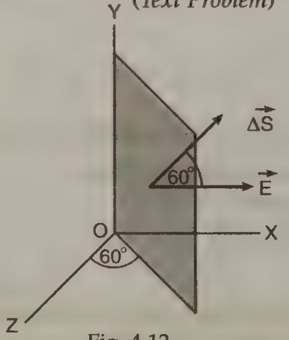


Fig. 4.12

Sol. (*a*) The area of a surface can be represented as a vector along normal to the surface [Fig. 4.11].

Here, $\vec{E} = 3 \times 10^3 \,\hat{i}\, \text{N C}^{-1}$

Area of the square, $\Delta S = 10 \times 10 = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$

Since normal to the square is along X-axis, we have

$$\vec{\Delta S} = 10^{-2}\, \hat{i} \text{ m}^2$$

Electric flux through the square,

$$\phi = \vec{E} \,.\, \vec{\Delta S} = (3 \times 10^3\, \hat{i}) \,.\, (10^{-2}\, \hat{i})$$

$$= \mathbf{30 \text{ N m}^2 \text{ C}^{-1}}$$

(*b*) Here, angle between normal to the square *i.e.* area vector and the electric field is 60° [Fig. 4.12]. Therefore,

$$\phi = \vec{E} \,.\, \vec{\Delta S} = E\, \Delta S \cos 60° = 3 \times 10^3 \times 10^{-2} \times 0{\cdot}5$$

$$= \mathbf{15 \text{ N m}^2 \text{ C}^{-1}}$$

Type B. On Gauss's Theorem

Problem 4.03. Calculate the number of electric lines of force originating from a charge of 1 C.

Given, $\varepsilon_0 = 8 \cdot 854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$.

Sol. Here, $q = 1 \text{ C}$; $\varepsilon_0 = 8 \cdot 854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

From Gauss's theorem, electric flux *i.e.* electric lines of force through a surface enclosing a charge q is given by

$$\Phi = \frac{q}{\varepsilon_0}$$

Therefore, electric lines of force originating from a charge of 1 C,

$$\Phi = \frac{1}{8{\cdot}854 \times 10^{-12}} = \mathbf{1 \cdot 129 \times 10^{11}}$$

Problem 4.04. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8 · 0 × 10^3 N m^2 C^{-1}.

) What is the net charge inside the box ? (*b*) If the net itward flux through the surface of the box were zero, uld you conclude that there were no charges inside the x ? Why or why not ? (*Text Problem*)

Sol. Here, $\Phi = 8 \cdot 0 \times 10^3 \text{ N m}^2 \text{ C}^{-1}$

(*a*) Suppose that the net charge inside the box is q. Then, cording to Gauss's theorem,

$$\Phi = \frac{q}{\varepsilon_0} \quad \text{or} \quad q = \varepsilon_0 \Phi$$

Now, $\varepsilon_0 = 8 \cdot 854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

$\therefore$ $q = 8 \cdot 854 \times 10^{-12} \times 8{\cdot}0 \times 10^3$

$= \mathbf{70 \cdot 832 \times 10^{-9}}$ **C**

(*b*) If the net outward flux through the surface of the box zero, it cannot be concluded that there is no charge inside e box. There may be equal amounts of positive and negative arges inside the box. Therefore, if the net outward flux is ro, we cannot conclude that the charge inside the box is ro. One can only say that the net charge inside the box is ro.

Problem 4.05. A point charge + 10 μC is at a distance cm directly above the centre of a square of side 10 cm as own in Fig. 4.13. What is the magnitude of the electric flux rough the square ? (*Text Problem*)

Sol. Here, $q = +10 \ \mu\text{C} = 10^{-5}$ C

Consider that the charge q is at a distance of 5 cm from e square ABCD of each side 10 cm [Fig. 4.13]. The square

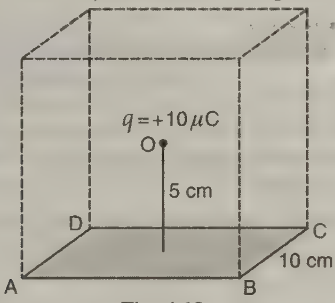


Fig. 4.13

BCD can be considered as one of the six faces of a cube of ch side 10 cm. Then, according to Gauss's theorem, total ectric flux through all the six faces of the cube,

$$\Phi = \frac{q}{\varepsilon_0}$$

Obviously, the flux through the square ABCD will be

$$\phi = \frac{1}{6} \times \Phi = \frac{1}{6} \times \frac{q}{\varepsilon_0}$$

$$= \frac{1}{6} \times \frac{10^{-5}}{8{\cdot}854 \times 10^{-12}}$$

$$= \mathbf{1 \cdot 88 \times 10^5 \text{ N m}^2 \text{ C}^{-1}}$$

pe C. On Calculation of Electric field

Problem 4.06. An infinite line charge produces a field 9 × 10⁴ N C⁻¹ at a distance of 4 cm. Calculate the linear arge density. (*H.S.S.C.E. 2001*)

Sol. Here, $E = 9 \times 10^4 \text{ N C}^{-1}$; $r = 4 \text{ cm} = 0 \cdot 04$ m

Electric field at a distance r from a line charge having ear charge density λ is given by

$$E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r}$$

$$\therefore \quad \lambda = 2\pi\varepsilon_0 E r = 4\pi\varepsilon_0 \cdot \frac{E r}{2}$$

$$= \frac{1}{9 \times 10^9} \times \frac{9 \times 10^4 \times 0{\cdot}04}{2}$$

$$= \mathbf{2 \times 10^{-7} \text{ C m}^{-1}}$$

Problem 4.07. A large plane sheet of charge having surface charge density $5 \cdot 0 \times 10^{-6}$ C m⁻² lies in the XY-plane. Find the electric flux through a circular area of radius 0 · 1 m, if the normal to the circular area makes an angle of 60° with the Z-axis. Given that $\varepsilon_0 = 8 \cdot 85 \times 10^{-12}$ C² N⁻¹ m⁻².

Sol. The electric flux through a surface,

$$\phi = \vec{E} \cdot \vec{\Delta S} = E \, \Delta S \cos\theta$$

We know that electric field near the plane sheet of charge,

$$E = \frac{\sigma}{2\varepsilon_0}$$

and circular area, $\Delta S = \pi r^2$

$$\therefore \quad \phi = \frac{\sigma}{2\varepsilon_0} \times \pi r^2 \cos\theta$$

Here, $\sigma = 5 \cdot 0 \times 10^{-6} \text{ C m}^{-2}$;

$\varepsilon_0 = 8 \cdot 85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^2$; $r = 0 \cdot 1$ m

and $\theta = 60°$

$$\text{Hence, } \phi = \frac{5{\cdot}0 \times 10^{-16} \times \pi \times (0{\cdot}1)^2 \cos 60°}{2 \times 8{\cdot}85 \times 10^{-12}}$$

$$= \mathbf{4 \cdot 44 \times 10^3 \text{ N m}^2 \text{ C}^{-1}}$$

Problem 4.08. Two large thin metal plates are parallel and close to each other as shown in the Fig. 4.14. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17 \cdot 0 \times 10^{-22}$ C m⁻². What is E (*a*) to the left of the plates, (*b*) to the right of the plates and (*c*) between the plates ? (*Text Problem*)

Sol. Here, $\sigma = 17 \cdot 0 \times 10^{-22} \text{ C m}^{-2}$

(*a*) ***To the left of plates*** : The region I is to the left of the plates. Therefore, as discussed in section 4.09, the electric field to the left of plates is **zero.**

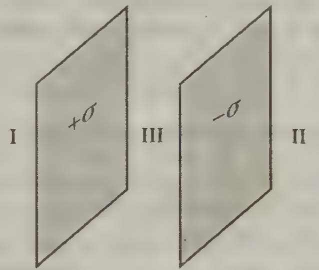


Fig. 4.14

(*b*) ***To the right of plates*** : The region II is to the right of the plates. Again, the electric field in the region II is **zero.**

(*c*) ***Between the two plates*** : In the region III between the two plates, the electric field given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{17{\cdot}0 \times 10^{-22}}{8{\cdot}854 \times 10^{-12}} = \mathbf{1 \cdot 92 \times 10^{-10} \text{ N C}^{-1}}$$

Problem 4.09. A spherical conductor of radius 12 cm has a charge of $1 \cdot 6 \times 10^{-7}$ C distributed uniformly on its surface. What is the electric field (*a*) inside the sphere (*b*) just outside the sphere (*c*) at a point 18 cm from the centre of the sphere ? (*Text Problem*)

Sol. Here, $q = 1 \cdot 6 \times 10^{-7}$ C ;

radius of the sphere ; R = 12 cm = 0 · 12 cm

(*a*) ***Inside the sphere*** : The charge on a conductor resides on its outer surface. Therefore, electric field inside the sphere is **zero.**

84-0(

(*b*) *Just outside the sphere* : For a point on the charged spherical conductor or outside it, the charge may be assumed to be concentrated at its centre. Therefore,

electric field just outside the sphere

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = 9\times10^9\times\frac{1\cdot6\times10^{-7}}{(0\cdot12)^2} = \mathbf{10^5\ N\ C^{-1}}$$

(*c*) *At a point 18 cm from the centre* : Here, $r = 18$ cm

Electric field at a point 18 cm from the centre

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = 9\times10^9\times\frac{1\cdot6\times10^{-7}}{(0\cdot18)^2}$$

$$= \mathbf{4\cdot44\times10^4\ N\ C^{-1}}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem. 4.10. A charge q is disturbed uniformly on a ring of radius r. A sphere of equal radius r is centred at the circumference of the ring [Fig. 4.15]. Find the flux of the electric field through the surface of the sphere.

Sol. Let C be the centre of the ring and O be the centre of the sphere. The sphere encloses charge present on the part AOB of the ring.

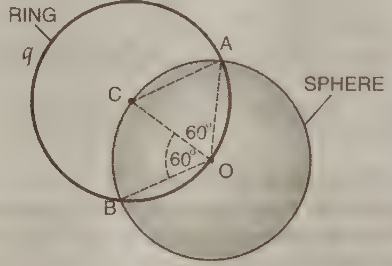


Fig. 4.15

Let us first find the length of part AOB of the ring. It follows that

CA = CO = r (radius of the ring)

and OA = r (radius of the sphere)

Therefore, Δ AOC is an equilaterial triangle and hence ∠ AOC = 60°.

Similarly, it can be proved that ∠ BOC = 60°.

∴ ∠ AOB = ∠ AOC + ∠ BOC = 60° + 60° = 120°

Since the part AOB of the ring subtends an ∠ 120° at the centre of the ring, its one third part is inside the sphere.

Hence, charge enclosed by the sphere, $q' = \frac{q}{3}$

From Gauss's theorem, the flux of the electric field through the surface of the sphere,

$$\phi = \frac{q'}{\varepsilon_0} = \frac{q}{3\varepsilon_0}$$

Problem 4.11. The electric field in a region is radially outward and varies with distance r as

$$\mathbf{E = 250\,r\ V\ m^{-2}}$$

Calculate the charge contained in a sphere of radius 0 · 2 m centred at the origin.

Sol. Here, $E = 250\,r$ V m^{-2} and radius of sphere, $a = 0{\cdot}2$ m

Let $E_{surface}$ be electric field at the surface of the sphere.

Then, $E_{surface} = 250\,a$ V m^2 = 250 × 0 · 2 = 50 V m^{-2}

Electric flux crossing through the sphere,

$$\phi = \oint \vec{E}_{surface}\cdot\vec{dS} = E_{surface}\times4\pi a^2$$

or $\phi = 50\times4\ \pi\times(0\cdot2)^2$ N m^2 C^{-1}

If q is charge contained in the sphere, then according to Gauss's theorem,

$$\phi = \frac{q}{\varepsilon_0}$$

or $q = \varepsilon_0\phi = \varepsilon_0\times50\times4\ \pi\times(0\cdot2)^2$

$$= 4\pi\ \varepsilon_0\times50\times(0\cdot2)^2$$

$$= \frac{1}{9\times10^9}\times50\times(0\cdot2)^2 = \mathbf{2\cdot22\times10^{-10}\ C}$$

Problem 4.12. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q [Fig. 4.16]. A charge q is placed at the centre of the shell. What is the surface charge density of the inner and outer surfaces of the shell ? (*Text Question*)

Sol. When charge Q is given to the conducting shell, it will appear on its outer surface. When charge q is placed at the centre of the shell, it must induce charge $-q$ on the inner surface and $+q$ on the outer surface of the shell. Otherwise, there will be electric field inside the conducting shell. Thus, there is charge $-q$ on the inner surface and Q + q on the outer surface of the shell.

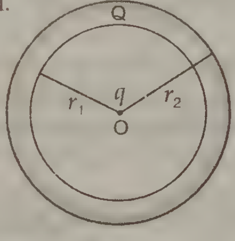


Fig. 4.16

Therefore, charge density of inner surface,

$$\sigma_1 = -\frac{q}{4\pi r_1^2}$$

and charge density of outer surface,

$$\sigma_2 = \frac{Q+q}{4\pi r_2^2}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. What do you understand by electric flux ? (*P.S.S.C.E. 2001, 2000 ; H.S.S.C.E. 1995*)

Ans. Electric flux is associated with electric field.

The electric flux through a surface held inside an electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.

Q. 4.02. Is electric flux a scalar or a vector ? Give the SI unit of electric flux.

Ans. Electric flux is a scalar quantity and its unit is N m^2 C^{-1}.

Q. 4.03. An area S is held inside an electric field E, such that normal to the area S subtends angle θ with the direction

electric field. What is the electric flux linked with the rface area ?

Ans. The electric flux linked with the surface is given by

$$\phi = E\,S\cos\theta$$

Q. 4.04. What is the relation between electric intensity d flux ? *(P.S.S.C.E. 1999, 1998, 1997)*

Ans. If an elementary surface area dS is held inside the ctric field E and E_n is the component of electric field normal he surface area dS, then electric flux through the surface

$$d\phi = E_n\,dS$$

In vector notation, $d\phi = \vec{E}\,.\,\vec{dS}$

Q. 4.05. State Gauss's theorem in electrostatics. *(P.S.S.C.C.E. 2002)*

Ans. Refer to section 4.03.

Q. 4.06. A box encloses an electrical dipole consisting charge 5 μC and – 5 μC and of length 10 cm. What is the al electric flux through the box ?

Ans. Since net charge enclosed by the box is zero, electric x through the box is also zero.

Q. 4.07. What is the use of gaussian surface ?

Ans. For finding electric field due to a charge tribution by apply Gauss's theorem, one is required to aluate a surface integral. The surface integral can be luated easily by choosing a suitable gaussian surface for charge distribution.

Q. 4.08. If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true ?

Ans. If Coulomb's law involved $1/r^3$ dependence, Gauss's law would not be true.

Q. 4.09. Does the strength of electric field due to an infinitely long line charge depend upon the distance of the observation point from the line charge ?

Ans. Yes, the electric field due to an infinitely long line charge depends upon the distance of the observation point from the line charge.

Q. 4.10. Does the strength of electric field due to an infinite plane sheet of charge depend upon the distance of the observation point from the sheet of charge ?

Ans. No, the electric field due to an infinite plane sheet of charge does not depend upon the distance of the observation point from the plane sheet of charge.

Q. 4.11. What is the difference between a sheet of charge and a plane conductor having charge ?

Ans. On a sheet of charge, the same charge shows up on its two sides ; whereas in case of a charged plane conductor, charges showing up on the two surfaces are not the same.

Q. 4.12. What is the strength of the electric field inside a charged spherical shell ?

Ans. The strength of electric field inside a charged spherical shell is zero.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

Q. 4.01. If the electric field is given by $6\hat{i}+3\hat{j}+4\hat{k}$, culate the electric flux through a surface of area 20 units ng in YZ-plane. *(H.S.S.C.E. 1997 S)*

Ans. Here, $\vec{E} = 6\hat{i}+3\hat{j}+4\hat{k}$

The area vector representing the surface of area 20 units YZ-plane is given by

$$\vec{S} = 20\,\hat{i}$$

Therefore, electric flux through the surface,

$$\phi = \vec{E}\,.\,\vec{S} = (6\hat{i}+3\hat{j}+4\hat{k})\,.\,20\,\hat{i}$$

$$= \mathbf{120\ units}$$

Q. 4.02. What is the importance of Gauss's theorem ?

Ans. Gauss's theorem is of great importance. Those ations, in which calculation of electric field by applying ulomb's law or the principle of superposition of electric ds becomes very difficult, the results can be obtained by plying Gauss's theorem with great ease.

Q. 4.03. What is a gaussian surface ? Explain in brief.

Ans. Refer to section 4.05.

Q. 4.04. Suppose a gaussian surface does not include net charge. Does it necessarily mean that E is equal to o for all points on the surface ? *(Text Question)*

Ans. If a gaussian surface does not include any net rge, there can be electric field on the surface of the gaussian face but parallel to the surface. However, it will require t the source charge should lie outside the gaussian surface.

Q. 4.05. A spherical Gaussian surface encloses a charge 8·85 × 10^{-8} C. (*i*) Calculate the electric flux passing ough the surface, (*ii*) If the radius of the Gaussian surface doubled, how would the flux change ? *(C.B.S.E. 2001)*

Ans. (*i*) Electric flux through the surface,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{8{\cdot}85\times10^{-8}}{8{\cdot}85\times10^{-12}} = \mathbf{10^4\ N\ m^2\ C^{-1}}$$

$$(\because \varepsilon_0 = 8\cdot85\times10^{-12}\ C^2\,N^{-1}\,m^{-2})$$

(*ii*) There will be no effect on electric flux, when the radius of the Gaussian surface is doubled. It is because, the charge enclosed by the Gaussian surface remains the same.

Q. 4.06. A point charge causes an electric flux of $-1\cdot0\times10^3$ N m^2 C^{-1} to pass through a spherical Gaussian surface of 10 · 0 cm radius centred on the charge. (*a*) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface ? (*b*) What is the value of the point charge ? *(Text Question)*

Ans. Here, $\Phi = -1{\cdot}0\times10^3$ N m^2 C^{-1} ;

radius of gaussian surface, r = 10 · 0 cm = 0·1 m

(*a*) If the radius of the gaussian surface is doubled, the electric flux through the new surface will still remain same *i.e.* **$-1\cdot0\times10^3$ N m^2 C^{-1}.**

It is because the same charge is enclosed by the gaussian surface in the two cases.

(*b*) Let q be the charge inside the gaussian surface. Then,

$$\Phi = \frac{q}{\varepsilon_0}$$

$$\text{or}\qquad q = \varepsilon_0\,\phi = 8\cdot854\times10^{-12}\times(-1{\cdot}0\times10^3)$$

$$= \mathbf{-8\cdot854\times10^{9}\ C}$$

Q. 4.07. An electric flux of – 5 × 10^3 N m^2 C^{-1} passes through a spherical Gaussian surface of radius 20 cm due to the charge placed at its centre. (*i*) Calculate the charge enclosed by the Gaussian surface. (*ii*) If the radius of the

Gaussian surface is doubled, how much flux would pass through the surface ? (*C.B.S.E. 1998 S*)

Ans. Proceed as in SAQ 4.06.

Q. 4.08. A uniformly charged conducting sphere of 2·4 m diameter has a surface charge density of 80·0 μC m^{-2}. (*a*) Find the charge on the sphere. (*b*) What is the total electric flux leaving the surface of the sphere ? (*Text Question*)

Ans. Here, $\sigma = 80{\cdot}0\ \mu C\ m^{-2} = 80 \times 10^{-6}\ C\ m^{-2}$;

radius of the charged sphere, $R = \dfrac{2{\cdot}4}{2} = 1{\cdot}2$ m

(*a*) Charge on the sphere $= \sigma \times 4\,\pi R^2$
$= 80 \times 10^{-6} \times 4\,\pi \times (1{\cdot}2)^2 = \mathbf{1 \cdot 45 \times 10^{-3}\ C}$

(*b*) Total electric flux leaving the surface of the sphere,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1{\cdot}45 \times 10^{-3}}{8{\cdot}854 \times 10^{-12}}$$
$$= \mathbf{1 \cdot 64 \times 10^{8}\ N\ m^2\ C^{-1}}$$

Q. 4.09. Two infinite parallel planes have uniform charge densities ± σ. What is the electric field in (*a*) the region between the planes and (*b*) outside ? In what way does the infinite extension of the planes simplify your derivation ?

Ans. (*a*) In the region between the two infinite parallel planes having uniform charge densities ± σ, the electric field is equal to $\dfrac{\sigma}{\varepsilon_0}$.

(*b*) The electric field is zero in the region outside the plates.

The problem has symmetry due to infinite extension of planes.

Q. 4.10. A and B are two conducting spheres of sam radius, A being solid and B hollow. Both are charged to th same potential. What will be the relation between th charges on the two spheres ? (*C.B.S.E. 2001 S*

Ans. Let Q_A and Q_B be the charges on the solid an hollow conducting spheres respectively and R be the radiu of each sphere. When charge is given to a solid conductin sphere, it appears on the outer surface. However, for th calculation of electric field or potential due to a spher (whether hollow or solid) at a point on or outside th sphere, the charge behaves as if it is concentrated at th centre of the sphere. If V_A and V_B are potentials of the tw spheres, then

$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_A}{R} \quad \text{and} \quad V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_B}{R}$$

Since $V_A = V_B$, it follows that
$$Q_A = Q_B$$

Q. 4.11. What is the principle of electrostatic shielding
(*P.S.S.C.E. 2002, 200*

Ans. The electric field inside a hollow conductor is zer Therefore, if an object located inside a hollow conductor i subjected to electric field, there will be no effect of electric fiel on the object. In other words, the hollow conductor shield the object from external electric field.

This concept is made use of to shield the wires carryin audio signals (low frequency signal) from the external electr fields, such as due to atmospheric electricity or due to electr sparks. If such wires are not shielded, the audio signal wi give rise to a noisy sound on reproduction.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For **a**mbitious, **b**rilliant & **c**urious Students

Q. 4.01. Is it possible to transfer all the charge from a conductor to another insulated conductor ?

Ans. Yes, it is possible. For this to happen, place the charged conductor inside a hollow conductor so that it touches the inside wall of the hollow conductor. Now, whatsoever charge is given to the charged conductor, it gets immediately transferred to the hollow conductor. It is because of the fact that in case of a charged conductor, the charge resides on its surface.

In fact, it forms the principle of Van de Graaff generator.

Q. 4.02. A conductor A with a cavity as shown in Fig. 4.17 is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor. (*Text Question*)

Ans. Consider the gaussian surface (shown dotted) so as to enclose the cavity but lying wholly inside the conductor A as shown in Fig. 4.17.

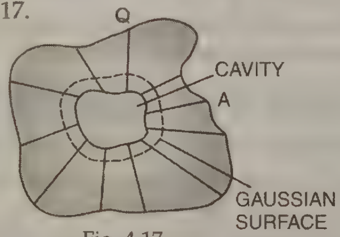


Fig. 4.17

Since the electric field inside a conductor is zero, the part of the conductor within the cavity does not contain any net charge. By extending the same argument, we can say that no charge can be present inside the gaussian surface, which lies just within the conductor.Hence, the entire charge Q mus appear on the outer surface of the conductor A.

Q. 4.03. If in the Q. 4.02, another conductor B wit charge *q* is inserted into the cavity keeping B insulate from A [Fig. 4.18], show that the total charge on the outsid surface of the conductor A is Q + *q*. (*Text Questio*

Ans. Let us again consider the Gaussian surface (show dotted) so that it encloses the cavity and the conductor B wit charge *q* inserted in the cavity [Fig. 4.18].

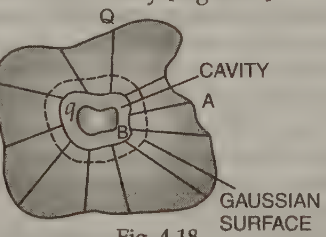


Fig. 4.18

As such, the electric flux will cross the gaussian surfac and it would imply that electric field exists inside th conductor A. But the electric field inside the conductor must be zero. It will be true only if the charge *q* on conduct B induces charge – *q* on the inner surface and + *q* on the oute surface of the conductor A. As there is already a charge + on the outer surface of the conductor A, the total charge o its outer surface becomes Q + *q*.

Q. 4.04. S$_1$ and S$_2$ are two parallel concentric sphere enclosing charges Q and 2 Q respectively as shown i Fig. 4.19.

Vhat is the ratio of the electric flux through S_1 and S_2 ? Iow will the electric flux through the sphere S_1 change, medium of dielectric consant 5 is introduced in the e inside S_1 in place of air ? (C.B.S.E. 2002)

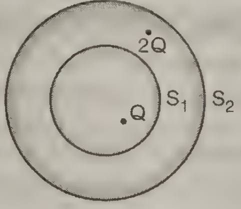


Fig. 4.19

Ans. (*a*) Let Φ_1 and Φ_2 be the electric flux through eres S_1 and S_2 respectively. Then,

$$\Phi_1 = \frac{Q}{\varepsilon_0} \quad \text{and} \quad \Phi_2 = \frac{Q+2Q}{\varepsilon_0} = \frac{3Q}{\varepsilon_0}$$

$$\therefore \quad \frac{\Phi_1}{\Phi_2} = \frac{Q}{\varepsilon_0} \Big/ \frac{3Q}{\varepsilon_0} = \frac{1}{3}$$

(*b*) Let $\vec{E}$ be electric field intensity on the surface of ere S_1 due to charge Q placed inside the sphere. Then, rding to Gauss's theorem,

$$\Phi_1 = \oint \vec{E} \cdot \vec{dS} = \frac{Q}{\varepsilon_0} \qquad ...(i)$$

On introducing medium of dielectric constant K inside the sphere S_1, suppose that electric field on the surface of sphere becomes $\vec{E}'$. Then

$$\vec{E}' = \frac{1}{K}\vec{E} \qquad ...(ii)$$

If the electric flux through the sphere is now Φ_1', then

$$\Phi_1' = \oint \vec{E}' \cdot \vec{dS} = \oint \frac{1}{K}\vec{E} \cdot \vec{dS} = \frac{1}{K}\frac{Q}{\varepsilon_0}$$

[Using equations (*i*) and (*ii*)]

Here, $K = 5$

$$\therefore \quad \Phi_1' = \frac{Q}{5\varepsilon_0}$$

Q. 4.05. A sensitive instrument is to be shielded from the strong electrostatic field in its environment. Suggest a possible way. (*Text Question*)

Ans. The electric field inside a metallic conductor is zero. Therefore, the instrument can be shielded from the strong electrostatic fields by enclosing it with a hollow metallic structure.

Q. 4.06. A man inside an insulated metallic cage does not receive a shock, when the cage is highly charged. Why ?

Ans. It is because, the potential at each point inside the cage is same as that of the cage itself. Since there is no potential difference between the man and the highly charged cage, the man does not receive any shock.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is electric flux ? Explain, how the electric flux through a surface is related to electric field intensity, when the surface is held inside the electric field.
2. State Gauss's theorem in electrostatics. How will you prove it for spherically symmetric surfaces ? (*C.B.S.E . 1996, 1995, 1992, 1991 ; H.S.S.C.E. 1992*)
3. State Gauss's theorem. How Coulomb's law can be derived from it ? (*H.P.S.S.C.E. 1999, 1995 ; H.S.S.C.E. 1996 ; P.S.S.C.E. 1995, 1991*)
4. Deduce Coulomb's law from Gauss's theorem in electrostatics. State Gauss's theorem in electrostatics. Use this to find the electric field inside the spherical shell. (*H.P.S.S.C.E. 2002*)
5. State Gauss's theorem in electrostatics. Hence obtain expression for the force between two point charges. (*C.B.S.E. 1991*)
6. Derive, using Gauss's theorem, an expression for the electric field of an infinitely long line of charge having a uniform charge density λ C m^{-1}. (*C.B.S.E. 1990*)
7. State Gauss's theorem in electrostatic and express it mathematically. Using it, derive an expression for electric field intensity at a point near a thin infinite plane sheet of electric charge. (*C.B.S.E. 2001 S*)
8. State Gauss's theorem in electrostatics. Apply this theorem to calculate the electric field due to an infinite plane sheet of charge. (*C.B.S.E. 1995*)
9. State Gauss's theorem in electrostatics. Using it, derive the expression for the electric field at any point due to an infinite plane sheet of charge of uniform charge density σ. (*C.B.S.E. 1996*)
10. State Gauss's theorem. Derive expression for electric field between two infinite plane parallel sheets of charge at a point in the region between the two plates.
11. State Gauss's theorem in electrostatics and express it mathematically. Using the theorem, derive an expression for electric field intensity at a point inside a hollow charged conducting sphere. (*C.B.S.E. 2001 S*)
12. State Gauss's theorem in electrostatics. Prove that no electric field exists inside a hollow charged sphere. (*C.B.S.E. 2002*)
13. Using Gauss's law, show that no electric field intensity exists inside a hollow charged conductor. (*C.B.S.E. 2000*)
14. (*a*) Define the term electric flux. State its units.
    (*b*) A sphere S_1 of radius r_1 encloses a charge Q. If there is another concentric sphere S_2 of the radius r_2 ($r_2 > r_1$) and there be no additional charges between S_1 and S_2, find the ratio of electric flux through S_1 and S_2. (*C.B.S.E. 2001 S*)

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. State and prove Gauss's theorem in electrostatics. Deduce Coulomb's law from Gauss's theorem ? (*H.P.S.S.C.E. 1999 S*)
2. State and prove Gauss's law in electrostatics. Find electric field intensity due to a uniformly charged wire. (*H.S.S.C.E. 1992*)
3. State Gauss's theorem. Give its mathematical expression. Derive an expression for the electric field intensity at any point due to an infinite plane sheet of charge density σ C m^{-2}. (*C.B.S.E. 2001*)
4. State Gauss's law in electrostatics. Apply this law to show that for a spherical shell, the electric field inside the shell

vanishes, whereas outside it, the field is as if all the charge has been concentrated at the centre. (*C.B.S.E. 1992*)

5. Derive expression for the electric field due to a uniformly charged spherical shell at a point (*i*) inside and (*ii*) outside the shell. (*C.B.S.E. 1991*)

6. State Gauss's theorem in electrostatics. Apply this theore to calculate the electric field due to a solid sphere of char at a point (*a*) outside the sphere (*b*) on the sphere and inside the sphere. (*P.S.S.C.E. 1998 S ; C.B.S.E. 199*

FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Electric flux

1. (*a*) A rectangular frame of wire of 25 cm × 15 cm is placed in a uniform electric field of strength 2×10^4 N C^{-1}, such that the plane of the coil is normal to field. Find the electric flux linked with the rectangular frame.
(*b*) Calculate the electric flux linked with the frame, when it is converted into (*i*) a square and (*ii*) a circular frame.
(*c*) In which case is the electric flux maximum ?
[**Ans.** (*a*) 750 N m^2 C^{-1} ; (*b*) (*i*) 800 N m^2 C^{-1} ; (*ii*) 1018·6 N m^2 C^{-1} ; (*c*) circular frame]

2. A plane surface having area of 0·5 m^2 is placed inside a uniform electric field of strength 5×10^3 N C^{-1}. Calculate the electric flux through the surface, when the normal to its plane makes an angle of (*i*) 30°, (*ii*) 60°, (*iii*) 90° with the direction of the electric field.
[**Ans.** (*i*) 2165 · 1 N m^2 C^{-1} ; (*ii*) 1250 N m^2 C^{-1} ; (*iii*) 0]

3. There is uniform electric field of $3 \times 10^3 \, \hat{i}$ N C^{-1}. What is the net flux of the uniform electric field through a cube of side 20 cm oriented so that its faces are parallel to the co-ordinate planes ? [**Ans.** 0]

Type B. On Gauss's theorem

4. A point charge of 10^{-7} coulomb is situated at the centre of a cube of 1 m side. Calculate the electric flux through its surface. (*P.S.S.C.E. 1992 S*) [**Ans.** $1{\cdot}13 \times 10^4$ N m^2 C^{-1}]

5. A point charge of 5 μC is situated at the centre of a sphere of radius 0 · 2 m. Calculate the electric flux through its surface. Given, $\varepsilon_0 = 8 \cdot 854 \times 10^{-12}$ C^2 N^{-1} m^{-2}.
[**Ans.** $5{\cdot}65 \times 10^5$ N m^2 C^{-1}]

6. Find the electric flux through each face of a hollow cube of side 10 cm, if a charge of 8·854 μC is placed at its centre.
[**Ans.** $1{\cdot}67 \times 10^5$ N m^2 C^{-1}]

7. A point charge of 12 μC is located at the centre of a cube of side 1 m. Calculate the electric flux through each face of the cube. (*H.S.S.C.E. 1999 S*) [**Ans.** $2{\cdot}26 \times 10^5$ N m^2 C^{-1}]

8. If the number of electric lines of force emerging out of a closed surface are 10^3, calculate the charge enclosed by the surface. [**Ans.** $8{\cdot}854 \times 10^{-9}$ C]

9. An electric flux of -6×10^3 N m^2 C^{-1} passes through a spherical Gaussian surface of radius 10 cm due to a point charge placed at the centre. (*i*) What is the charge enclosed by the Gaussian surface ? (*ii*) If the radius of the Gussian surface is doubled, how much flux would pass through the surface ? (*C.B.S.E. 1999*)
[**Ans.** (*i*) $-5{\cdot}31 \times 10^{-8}$ C ; (*ii*) unchanged]

Type C. On calculation of Electric field on the basis of Gauss's theorem

10. A charged particle having a charge of -3 μC is placed close to a sheet of charge having a surface charge density 5×10^{-6} C m^{-2}. Find the force of attraction between the particle and the sheet of charge.
[**Ans.** 0·847 N (attractive)]

11. In problem no. 10, if the sheet of charge is replaced by a conducting charged plate, what will be the force of attraction between the particle and the plate.
[**Ans.** 1·694 N (attractiv

12. A particle of mass 9×10^{-5} g is kept over a large horizon sheet of charge having surface charge density 5×10^{-5} C m What charge should be given to this particle so tha released, it does not fall down? [**Ans.** $3{\cdot}12 \times 10^{-13}$

13. In problem no. 12, if the sheet of charge is replaced by conducting charged plate, what charge would be given the particle, so that if released, it does not fall ?
[**Ans.** $1{\cdot}56 \times 10^{-13}$

14. Two large conducting plates are placed parallel to ea other with a separation of 2 cm between them. An electr starting from rest near one of the plates reaches the oth plate in 2 μs. Find the surface charge density of the plat
[**Ans.** $2{\cdot}52 \times 10^{-13}$ C m

15. In problem no. 14, if the two conducting plates are replac by two sheets of charge, what will be the charge density the sheets of charge ? [**Ans.** $5{\cdot}04 \times 10^{-13}$ C m

16. Assume that earth has a charge of surface density electron per metre^2. Calculate the earth's (*i*) electric field electric potential just outside the earth's surface.
[**Ans.** (*i*) $1{\cdot}81 \times 10^{-8}$ N C^{-1} (radially inwwar
(*ii*) – 0·1157

17. The radius of gold nucleus (Z = 79) is about $7{\cdot}0 \times 10^{-15}$ Assuming that the positive charge is distributed uniform throughout the nuclear volume, find the strength of electric field (*a*) at the surface of the nucleus and (*b*) at middle point of the radius.
[**Ans.** (*a*) $2{\cdot}32 \times 10^{21}$ N C^{-1} ; (*b*) $1{\cdot}16 \times 10^{21}$ N C

MICELLANEOUS PROBLEMS

18. A 1 mg ball carrying a charge of 2×10^{-8} C hangs fror thread. When a large conducting plate of charge brought near the ball, the thread makes an angle of with the plate. What is the surface charge density the plate ? [**Ans.** $2{\cdot}505 \times 10^{-9}$ C m

19. If a small sphere of mass m and charge q hangs from a thread at angle θ with a charged conducting plate, sh that for equilibrium of the sphere, the surface cha density for the plate is $\varepsilon_0 \dfrac{mg}{q} \tan\theta$.

20. An electric dipole AB consists of charges ± 5 nC a separated by a distance of 2×10^{-3} m. As shown in F 4.20. The dipole is placed near a long line charge hav linear charge density $4{\cdot}5 \times 10^{-4}$ C m^{-1}, such that negative charge is at a distance OA = 2·5 cm from line charge [Fig. 4.20]. Find the force acting on dipole. [**Ans.** 0·12 N (towards the line char

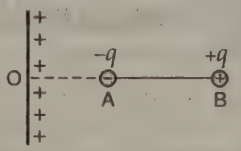


Fig.4.20.

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Here, $A = 25 \times 15 = 375\ \text{cm}^2 = 375 \times 10^{-4}\ \text{m}^2$;
$E = 2 \times 10^4\ \text{N C}^{-1}$

(a) $\phi = EA = 2 \times 10^4 \times 375 \times 10^{-4} = \mathbf{750\ N\ m^2\ C^{-1}}$

(b) (i) *When rectangular frame is converted into a square frame :*

$$\text{One side of the square} = \frac{2(25+15)}{4} = 20\ \text{cm}$$

Area of square, $A_1 = 20 \times 20 = 400\ \text{cm}^2 = 400 \times 10^{-4}\ \text{cm}^2$

Now, $\phi_1 = E A_1 = 2 \times 10^4 \times 400 \times 10^{-4} = \mathbf{800\ N\ m^2\ C^{-1}}$

(ii) When rectangular frame is converted into a circular frame :

$$\text{Radius of circle, } r = \frac{2(25+15)}{2\pi} = \frac{40}{\pi}\ \text{cm}$$

$$\text{Area of circle, } A_2 = \pi r^2 = \pi \left(\frac{40}{\pi}\right)^2$$

$$= 509{\cdot}3\ \text{cm}^2 = 509{\cdot}3 \times 10^{-4}\ \text{m}^2$$

Now, $\phi_2 = E A_2 = 2 \times 10^4 \times 509{\cdot}3 \times 10^{-4} = \mathbf{1018{\cdot}6\ N\ m^2\ C^{-1}}$

(c) Since ϕ_2 is greater than both ϕ and ϕ_1, the electric flux is maximum in case of circular frame.

2. Proceed as in solved problem no. 4.02.

3. Here, $\vec{E} = 3 \times 10^3\ \hat{i}\ \text{N C}^{-1}$;

$A = 20 \times 20 = 400\ \text{cm}^2 = 4 \times 10^{-2}\ \text{m}^2$

Through a face parallel to XY-plane :

$$\vec{A}_1 = 4 \times 10^{-2}\ \hat{k}\ \text{m}^2$$

$$\therefore \quad \phi_1 = \vec{E} \cdot \vec{A}_1 = (3 \times 10^3\ \hat{i}) \cdot (4 \times 10^{-2}\ \hat{k}) = 0$$

Through a face parallel to YZ-plane :

$$\vec{A}_2 = 4 \times 10^{-2}\ \hat{i}\ \text{m}^2$$

$$\therefore \quad \phi_2 = \vec{E} \cdot \vec{A}_2 = (3 \times 10^3\ \hat{i}) \cdot (4 \times 10^{-2}\ \hat{i}) = 120\ \text{N m}^2\ \text{C}^{-1}$$

Through a face parallel to ZX-plane :

$$\vec{A}_3 = 4 \times 10^{-2}\ \hat{j}\ \text{m}^2$$

$$\therefore \quad \phi_3 = \vec{E} \cdot \vec{A}_3 = (3 \times 10^3\ \hat{i}) \cdot (4 \times 10^{-2}\ \hat{j}) = 0$$

In each plane, there is a set of two faces of the cube. Through one face, electric flux enters and through the other face, an equal flux leaves. Therefore, net flux through the cube,

$$\phi = (\phi_1 - \phi_1) + (\phi_2 - \phi_2) + (\phi_3 - \phi_3) = 0$$

4. $$\Phi = \frac{q}{\varepsilon_0} = \frac{10^{-7}}{8{\cdot}854 \times 10^{-12}} = \mathbf{1{\cdot}13 \times 10^4\ N\ m^2\ C^{-1}}$$

5. Proceed as in problem no. 4.

6. Total electric flux through the cube (all the six faces),

$$\Phi = \frac{q}{\varepsilon_0} = \frac{8{\cdot}854 \times 10^{-6}}{8{\cdot}854 \times 10^{-12}} = 10^6\ \text{N m}^2\ \text{C}^{-1}$$

Therefore, electric flux through each face of the cube,

$$\phi = \frac{1}{6}\Phi = \frac{10^6}{6} = \mathbf{1{\cdot}67 \times 10^5\ N\ m^2\ C^{-1}}$$

7. Proceed as in problem no. 5.

8. $q = \varepsilon_0 \Phi = 8{\cdot}854 \times 10^{-12} \times 10^3 = \mathbf{8{\cdot}854 \times 10^{-9}\ C}$

9. *(i)* $q = \varepsilon_0 \Phi = 8{\cdot}854 \times 10^{-12} \times (-6 \times 10^3) = \mathbf{-5{\cdot}31 \times 10^{-8}\ C}$

(ii) There will be **no change** in flux, when radius of Gaussian surface is doubled.

10. $$F = qE = q\left(\frac{\sigma}{2\varepsilon_0}\right) = 3 \times 10^{-6} \times \frac{5 \times 10^{-6}}{2 \times 8{\cdot}854 \times 10^{-12}}$$

$$= \mathbf{0{\cdot}847\ N\ (attractive)}$$

11. $$F = qE = q\left(\frac{\sigma}{\varepsilon_0}\right) = 3 \times 10^{-6} \times \frac{5 \times 10^{-6}}{8{\cdot}854 \times 10^{-12}}$$

$$= \mathbf{1{\cdot}694\ N\ (attractive)}$$

12. Let q be charge given to the particle. As the particle does not fall on being released,

$$qE = mg \quad \text{or} \quad q\left(\frac{\sigma}{2\varepsilon_0}\right) = mg$$

$$\text{or} \quad q = \frac{2\varepsilon_0 mg}{\sigma} = \frac{2 \times 8{\cdot}854 \times 10^{-12} \times 9 \times 10^{-8} \times 9{\cdot}8}{5 \times 10^{-5}}$$

$$= \mathbf{3{\cdot}12 \times 10^{-13}\ C}$$

13. $$\text{Now, } q = \frac{\varepsilon_0 mg}{\sigma} = \frac{8{\cdot}854 \times 10^{-12} \times 9 \times 10^{-8} \times 9{\cdot}8}{5 \times 10^{-5}}$$

$$= \mathbf{1{\cdot}56 \times 10^{-13}\ C}$$

14. Let σ be surface charge density of the conducting plates.

Then, electric field between conducting plates, $E = \dfrac{2\sigma}{\varepsilon_0}$

Force on electron, $F = e \times \dfrac{2\sigma}{\varepsilon_0}$

If m is mass of electron, then acceleration produced,

$$a = \frac{F}{m} = \frac{e}{m} \times \frac{2\sigma}{\varepsilon_0}$$

Since the electron is initially at rest

$$S = 0 \times t + \frac{1}{2} a t^2 = \frac{1}{2} \times \frac{e}{m} \times \frac{2\sigma}{\varepsilon_0} \times t^2$$

$$\text{or} \quad \sigma = \frac{\varepsilon_0 m S}{e t^2}$$

Here, $m = 9{\cdot}1 \times 10^{-31}\ \text{kg}$; $S = 2\ \text{cm} = 0{\cdot}02\ \text{m}$;
$e = 1{\cdot}6 \times 10^{-19}\ \text{C}$ and $t = 2\ \mu s = 2 \times 10^{-6}\ \text{s}$

$$\therefore \quad \sigma = \frac{8{\cdot}854 \times 10^{-12} \times 9{\cdot}1 \times 10^{-31} \times 0{\cdot}02}{1{\cdot}6 \times 10^{-19} \times (2 \times 10^{-6})^2}$$

$$= \mathbf{2{\cdot}52 \times 10^{-13}\ C\ m^{-2}}$$

15. Electric field between two large sheets of charge,

$$E = \frac{\sigma}{\varepsilon_0}$$

Proceeding as in problem no. 14, it can be obtained that

$$\sigma = \frac{2\varepsilon_0 m S}{e t^2} = \frac{2 \times 8{\cdot}854 \times 10^{-12} \times 9{\cdot}1 \times 10^{-31} \times 0{\cdot}02}{1{\cdot}6 \times 10^{-19} \times (2 \times 10^{-6})^2}$$

$$= \mathbf{5{\cdot}04 \times 10^{-13}\ C\ m^{-2}}$$

16. Here, $\sigma = 1$ electron $\text{m}^{-2} = -1{\cdot}6 \times 10^{-19}\ \text{C m}^{-2}$,
radius of earth, $R = 6400\ \text{km} = 6{\cdot}4 \times 10^6\ \text{m}$

Therefore, total charge on earth, $q = 4\pi R^2 \sigma$

(i) Electric field just outside the surface of earth,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\pi R^2 \sigma}{R^2} = \frac{\sigma}{\varepsilon_0}$$

$$= -\frac{1{\cdot}6 \times 10^{-19}}{8{\cdot}854 \times 10^{-12}} = -1{\cdot}81 \times 10^{-8}\ \text{N C}^{-1}$$

$$= \mathbf{1{\cdot}81 \times 10^{-8}\ N\ C^{-1}\ (radially\ inward)}$$

(ii) Electric potential just outside the surface of earth,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\pi R^2 \sigma}{\varepsilon_0} = \frac{R\sigma}{\varepsilon_0}$$

$$= \frac{6{\cdot}4 \times 10^6 \times (-1{\cdot}6 \times 10^{-19})}{8{\cdot}854 \times 10^{-12}} = \mathbf{-0{\cdot}1157\ V}$$

17. Here, $q = Z e = 79 \times 1{\cdot}6 \times 10^{-19}$ C, $R = 7{\cdot}0 \times 10^{-15}$ m

(*a*) Electric field at the surface of gold nucleus,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = \frac{9\times10^9\times79\times1\cdot6\times10^{-19}}{(7\cdot0\times10^{-15})^2}$$

$$= \mathbf{2{\cdot}32 \times 10^{21}\ N\ C^{-1}}$$

(*b*) Despite the fact that gold is a conductor, it has been assumed that the positive charge is uniformly distributed over the entire nuclear volume. To find electric field at the middle point of the radius, draw a spherical shell of radius R/2 as the Gaussian surface. Therefore,
charge enclosed by Gaussian surface,

$$q' = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \times \rho = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \times \frac{q}{4/3\times\pi R^3} = \frac{1}{8}q$$

Therefore, electric field at the middle point of the radius

$$E' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q'}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{1/8\,q}{(R/2)^2}$$

$$= \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}\right) = \frac{1}{2}E$$

$$= \frac{1}{2}\times 2{\cdot}32\times10^{21} = \mathbf{1{\cdot}16\times10^{21}\ N\ C^{-1}}$$

18. Here, $m = 1$ mg $= 10^{-6}$ kg ; $q = 2 \times 10^{-8}$ C

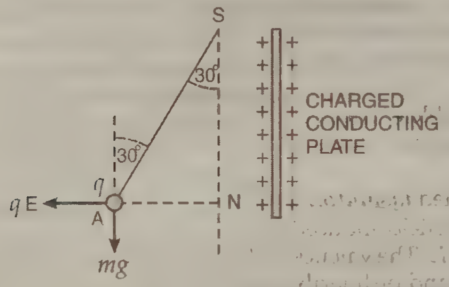


Fig. 4.21

Let σ be charge density of the conducting plate of charge. Then, electric field due to the plate,

$$E = \frac{\sigma}{\varepsilon_0}$$

Force on the charged ball, $F = qE$
Since the charged ball is in equilibrium

$$\frac{qE}{AN} = \frac{mg}{SN} \quad\text{or}\quad \frac{q(\sigma/\varepsilon_0)}{AN} = \frac{mg}{SN}$$

$$\text{or}\quad \sigma = \frac{\varepsilon_0\, mg}{q}\times\frac{AN}{SN} = \frac{\varepsilon_0\, mg}{q}\tan 30°$$

$$= \frac{8\cdot854\times10^{-12}\times10^{-6}\times9\cdot8\times0\cdot5774}{2\times10^{-8}}$$

$$= \mathbf{2{\cdot}505\times10^{-9}\ C\ m^{-2}}$$

19. Proceed as in problem no. 13.

20. Electric field at a distance r from the line charge,

$$E = \frac{1}{2\pi\varepsilon_0}\cdot\frac{\lambda}{r}$$

To calculate force on charge $-q$ at point A :

Here, OA = 2·5 cm = $2{\cdot}5\times10^{-2}$ m

Electric field at point A,

$$E_1 = \frac{1}{2\pi\times8\cdot854\times10^{-12}}\times\frac{4\cdot5\times10^{-4}}{2\cdot5\times10^{-2}}$$

$$= 3{\cdot}24\times10^{8}\ N\ C^{-1}$$

Force on charge $-q$ at point A,

$$F_1 = qE_1 = 5\times10^{-9}\times3{\cdot}24\times10^{8}$$

$= 1{\cdot}62$ N (towards line charg

To calculate force on charge $+q$ at point B :

Here, OB = $2{\cdot}5\times10^{-2} + 2\times10^{-3} = 2{\cdot}7\times10^{-2}$ m

Electric field at point B,

$$E_2 = \frac{1}{2\pi\times8\cdot854\times10^{-12}}\times\frac{4\cdot5\times10^{-4}}{2\cdot7\times10^{-2}}$$

$$= 3{\cdot}0\times10^{8}\ N\ C^{-1}$$

Force on charge $+q$ at point B,

$$F_2 = qE_2 = 5\times10^{-9}\times3{\cdot}0\times10^{-8}$$

$= 1{\cdot}5$ N (away from line charg

Hence, net force on electric dipole,

$$F = F_1 - F_2 = 1{\cdot}62 - 1{\cdot}5$$

$= \mathbf{0{\cdot}12}$ **N (towards line charge)**

CHAPTER
5

Capacitor

5.01. CONDUCTORS, INSULATORS AND DIELECTRICS

The carriers of current in metals are free electrons. The free electron model for ne metal can explain some of the observed properties of the metals in a qualitative nanner.

The elements, in which the valence shell is filled less than half, are found to e good conductors. For example, in metals such as copper, aluminium, silver, etc, ne valence shell contains three or less electrons. Since an atom has a tendency to ave a filled valence shell, the valence electrons in the atoms of a metal leave the toms and are free to move through the metal lattice in a random manner. They onstantly collide among themselves and with the positive ions located in the metal ttice. They have practically no affinity to their parent atoms.

The average velocity of free electrons in a metal is zero. But when an external lectric field is applied across the two ends of a metal, the free electrons experience orce and get accelerated. There is a net flow of electrons through the metal. It is ound that as the strength of the applied electric field is increased, more and more ee electrons cross through a section of the metal. As such, the metals are termed s conductor for electricity.

The materials which do not have free electrons in them are unable to conduct lectricity and are termed as ***insulators***. An insulator may behave in the following vo ways :

1. It may not conduct electricity through it and as such it is called ***insulator***.

2. It may not conduct electricity through it but on applying electric field, induced harges are produced on its faces. Such an insulator is called ***dielectric***. The valence lectrons in atoms of a dielectric are tightly bound to their nuclei and ordinarily annot detach themselves.

Key point

The dielectrics do not conduct electricity. On applying electric field, induced charges of opposite kinds develop on their opposite faces.

5.02. BEHAVIOUR OF A CONDUCTOR IN ELECTROSTATIC FIELD

We know that a conductor possesses free electrons inside it. When a conductor placed inside an electrostatic field, the free electrons in the conductor move in a irection opposite to that of the applied field [Fig. 5.01]. Thus, rearrangement of lectrons takes place. The final distribution of charges at the surface of the conductor such that field set up due to charges on the surface of conductor just cancels the pplied field and hence net field inside the conductor is zero. In case the electric eld had been non-zero inside the conductor, it would lead to flow of current in ne interior of the conductor. Further, the electrostatic field at the surface of the onductor or just outside it cannot have tangential component. Had it been so, it vould lead to the flow of surface current. Thus, when the conductor is placed in ne electric field, the field will be of the form as shown in Fig. 5.02 *i.e.* the electric eld lines will enter or leave the conductor at right angles to the surface of the onductor.

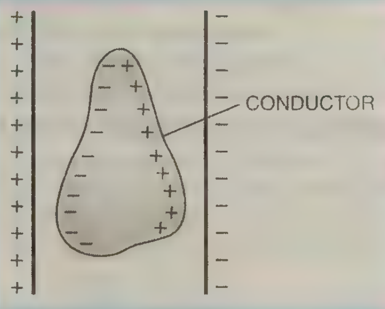


Fig. 5.01

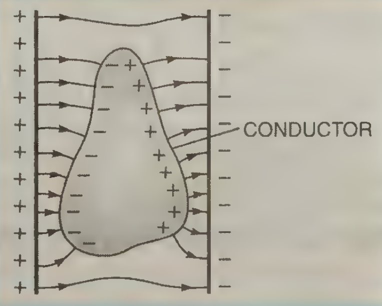


Fig. 5.02

The fact that net electric field inside the conductor is zero implies that there an be no net charge anywhere in the interior of the conductor. It can be proved by sing Gauss's theorem as follows :

Consider a gaussian surface just inside the surface of the conductor. Since there no electric field inside the conductor, the electric flux through the gaussian surface hould be zero. According to Gauss's theorem, the charge inside the gaussian surface iust be zero *i.e.* there is no charge in the interior of the conductor.

Therefore, when a conductor is placed in an electric field, the following onditions apply to it :

1. *Inside the conductor, both the electric field and the charge are zero.*

2. *Charges reside only at the surface of the conductor.*

3. *Immediately outside the surface of the conductor, the electric field is perpendicular to its surface.*

4. *The entire body of the conductor (including its surface) is at constant potential. It is because, electric field inside the conductor can be zero, only if potential inside the conductor is constant.*

5.03. ELECTRICAL CAPACITANCE

Electrical capacitance of a conductor is related to its ability to store the electric charge.

When a conductor is given some electric charge, it is raised to some potential. As more and more charge is given, its potential increases accordingly. If charge q is given to the conductor so that its potential increases by an amount V, then it is found that

$$q \propto V$$

or

$$q = CV \qquad \text{...(5.01)}$$

where the constant of proportionality C is called the *capacitance* of the conductor. Its value depends upon the *shape* and *dimensions* of the conductor. It may be pointed out that the capacitance of a conductor does not depend upon the material of the conductor but depends upon the nature of the medium (its relative permittivity or dielectric constant) in which the conductor is located.

From equation (5.01), we have

$$C = \frac{q}{V}$$

Thus, *capacitance of a conductor may be defined as the ratio of the electric charge on it to its electric potential due to that charge.*

Further, if V=1, then

$$C = q$$

Therefore, *the capacitance of a conductor is numerically equal to the electric charge required to raise its electric potential through one unit.*

In SI, the unit of capacitance is **farad.**

$$\textbf{1 farad (F)} = \frac{\textbf{1 coulomb (C)}}{\textbf{1 volt (V)}} = \textbf{1 coulomb volt}^{-1}(\textbf{C V}^{-1})$$

A conductor is said to have a capacitance of one farad, if 1 coulomb of charge raises its electric potential through 1 volt.

In practice, the following smaller units of capacitance are also used :

1 microfarad (μF) = 10^{-6} farad

1 microfarad ($\mu\mu$F) = 10^{-12} farad

1 $\mu\mu$F is also known as **1 picofarad (pF).**

5.04. CAPACITANCE OF AN ISOLATED SPHERE

Consider an isolated spherical conductor of radius r having charge q. Suppose the conductor is situated in vacuum or air [Fig. 5.03]. The charge on the spherical conductor may be assumed to be present at its centre O. Then, potential at any point on the surface of the conductor is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r},$$

where $\varepsilon_0 = 8 \cdot 854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$ is absolute permittivity of the free space.

Therefore, capacitance of the isolated spherical conductor situated in vacuum (or air) is given by

$$C = \frac{q}{V} = q \Big/ \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

or

$$C = 4\pi\varepsilon_0 r \qquad \text{...(5.02)}$$

Hence, the capacitance of an isolated spherical conductor is $4\pi\varepsilon_0$ times its radius. It may be pointed out that in the above expression for capacitance, r is in **metre** and C is in **farad.**

5.05. PRINCIPLE OF A CAPACITOR

A capacitor is a device for storing a large quantity of electric charge.

A conductor can store a small amount of charge and hence it possesses a small capacitance. To store a large amount of charge, two (or more) conductors are arranged to form a capacitor. Thus, a capacitor posseses a large capacitance to store charge and that too in a small space.

Key point

1. When a conductor is charged, charge resides on its outer surface.

2. When a conductor is placed in electric field, the field inside the conductor is zero.

Key point

The capacitance of a conductor depends upon its shape and size. It does not depend upon the nature of the material of the conductor but depends upon the nature of medium in which it is placed.

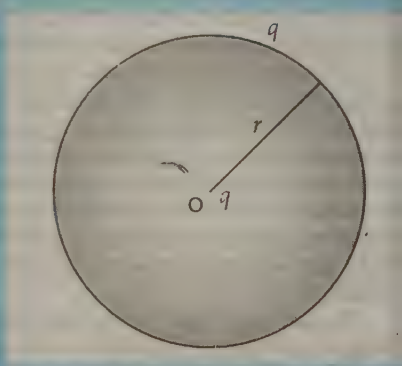


Fig. 5.03

Self-test Question

From equation (5.02), show that units of absolute permittivity are **farad metre^{-1} (F m^{-1}).**

Self-test Question

Using equation (5.02), show that one farad is a huge capacitance.

Principle. To understand the principle of a capacitor, consider an insulated al plate A. Suppose that it is given positive charge, till its potential becomes ximum. The metal plate will not hold any more charge over it. If charge is given he metal plate A, it will leak to the surroundings. Now, place another metallic e B near plate A. Due to induction, negative charge will be induced on the nearer of plate B and positive charge on its farther face as shown in Fig. 5.04. The ential of plate A gets lowered due to induced negative charge on plate B and a raised due to the induced positive charge. Since the metal plates have finite kness, induced negative charge is nearer to the plate A than the induced positive rge on plate B is. Therefore, on the whole, the potential of plate A gets lowered, ugh by a very small amount. In order to make the potential of the metal plate A in the same, an additional small amount of charge has to be given to plate A. It icates that the capacitance of a conductor increases by a small amount, when ther uncharged conductor is placed near it.

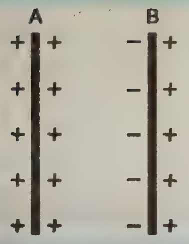


Fig. 5.04

Now, connect the metal plate B to the earth as shown in Fig. 5.05. The induced itive charge on plate B will immediately flow to the earth as it is repelled by itive charge on plate A. However, the induced negative charge on plate B will on it. It does not flow to earth as it is attracted by the positive charge on plate hus, the metal plate B will have only negative charge on it after being connected arth. Due to the induced negative charge on plate B, the potential of A will get ered by a large amount. In order to raise the metal plate A again to the same ential, a large amount of charge has to be given to it. Thus, it follows that the acitance of a conductor is increased greatly, when an earth connected conductor laced near it. It forms the principle of a capacitor and such an arrangement of metal plates is called a **capacitor.**

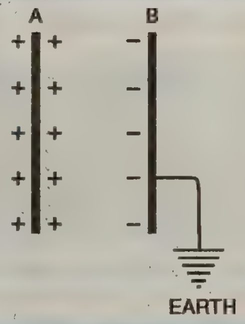


Fig. 5.05

An arrangement of two metallic conductors, so that when one conductor is connected e earth ; the other conductor has the ability to store a large amount of charge on it, is d a capacitor.

The two metallic conductors are called *coatings* of the capacitor. If the coatings spherical, cylindrical or plane ; the capacitor is accordingly called a ***spherical acitor, cylindrical capacitor*** or ***parallel plate capacitor.*** The insulating medium ween the two plates is called *dielectric.* If the insulating medium between the two es is air, it is called air capacitor. If dielectric has dielectric constant K, then acitance of the capacitor becomes K times its capacitance as air capacitor.

5.06. PARALLEL PLATE CAPACITOR

It is most commonly used capacitor. It consists of two conducting plates placed allel to each other [Fig. 5.06]. The separation d between the plates is very small ompared to the area of the plates. Due to small separation between the plates, fringing of electric field at the boundaries is negligible.

If charge $+q$ is given to plate A, then charge $-q$ is induced on the left face of e B and charge $+q$ on its right face. When plate B is earthed, the charge $+q$ on ight face flows to earth. Due to charge $+q$ on plate A and $-q$ on plate B, electric l is set up between the two plates.

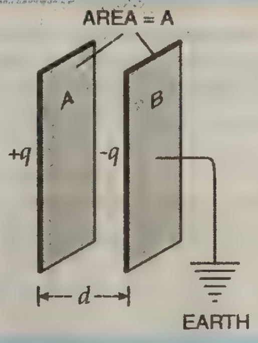


Fig. 5.06

The electric field between the two plates is also given by

$$E = \frac{dV}{dr} \text{ (in magnitude)}$$

Between the two parallel plates, the electric field is uniform* and perpendicular he plates. Therefore, if V is potential difference between the two plates, then

$$E = \frac{V}{d} \qquad \left(\text{For uniform field, } \frac{dV}{dr} = \frac{V}{d}\right)$$

or $$V = E\,d \qquad \text{...(5.03)}$$

Substituting for E in equation (5.03), we have

If σ is surface charge density of the plates, then the two electric field between plates is given by

$$E = \frac{\sigma}{\varepsilon_0},$$

ere ε_0 is absolute permittivity of the free space. (It is assumed that medium ween the plates is vacuum or air).

Key point

In a parallel plate capacitor, the fringing of electric field takes place near the edges of the two plates. In order to minimise the edge effects, the separation between the plates is kept small.

---

* However, near the edges of the plates, the electric field is not uniform. But the *edge effects* become small, if the area of plates is very e as compared to the square of the separation between the plates. To avoid edge effect, the capacitors with *guard rings* are used.

Substituting for E in equation (5.03), we have

$$V = \frac{\sigma}{\varepsilon_0} d$$

Since $\sigma = \frac{q}{A}$, we have

$$V = \frac{q\,d}{\varepsilon_0\,A}$$

If C is the capacitance of the parallel plate capacitor, then

$$C = \frac{q}{V} = \frac{q}{q\,d/\varepsilon_0\,A}$$

or $$C = \frac{\varepsilon_0\,A}{d} \qquad \text{...(5.04)}$$

It gives the capacitance of a parallel plate capacitor, when its plates are held in air or vacuum.

Key point

Electric field between the two plates o parallel plate capacitor is uniform.

Note. 1. If K is dielectric constant of the medium between the two plates, then

$$C = \frac{\varepsilon_0\,K\,A}{d}$$

2. In electrical circuit, a capacitor of fixed capacitance is represented by the symbol as shown in Fig. 5.07 (*a*). In case, if its capacitance is variable, it is represented by the symbol shown in Fig. 5.07 (*b*).

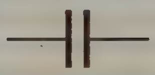


Fig. 5.07 (*a*)

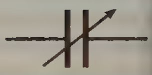


Fig. 5.07(*b*)

5.07. SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric spherical shells, such that one shell is earthed and the other is used to store charge on it. The two spherical shells have a very small spacing between them, which is filled with some dielectric medium, such as wax, paper, etc. As such, a spherical capacitor possesses a very large electrical capacitance.

Expression for capacitance of a spherical capacitor. Fig. 5.08 shows a spherical capacitor consisting of two spherical shells A and B. Let *a* and *b* be their respective radii. The outer spherical shell B is connected to earth. When charge + *q* is given to inner spherical shell A, it induces charge – *q* on the inner surface of the shell B and + *q* on its outer surface. As the shell B is earthed, the charge + *q* on its outer surface flows to earth. If V is potential difference between the spherical shells A and B, then capacitance of the spherical capacitor is given by

$$C = \frac{q}{V}$$

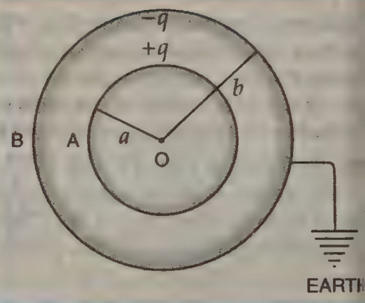


Fig. 5.08

Let us now find the potential difference between the two spherical shells. In case of parallel plate capacitor, the electric field between the two plates was uniform and the potential difference between the two plates was simply equal to E *d*. Here, in the space between the two spherical shells, the electric field is not uniform. It varies with distance and is given by

$$E = \frac{dV}{dr} \qquad \text{(in magnitude)}$$

$$\therefore \quad dV = E\,dr$$

Key point

Electric field between the two coatings a spherical capacitor is not uniform.

The potential difference V between the two spherical shells A and B can be found by integrating both sides between the proper limits. Therefore,

$$V = \int_A^B dV = \int_a^b E\,dr \qquad \text{...(5.05)}$$

To find the value of V, let us find electric field E at any point, say P between the two spherical shells. Let *r* be the distance of point P from the centre O of the spherical shells. Consider the spherical surface of radius *r* (shown dotted) through point P as the gaussian surface [Fig. 5.09]. The gaussian surface encloses the spherical shell A and hence the charge enclosed by the gaussian surface is *q*. According to Gauss's theorem, the total electric flux through the gaussian surface,

$$\Phi = \frac{q}{\varepsilon_0} \qquad \text{...(5.06)}$$

As the magnitude of electric field at every point on the gaussian surface is same, we have

$$\Phi = E \times \text{surface area of gaussian surface}$$

or $$\Phi = E \times 4\pi r^2 \qquad \text{...(5.07)}$$

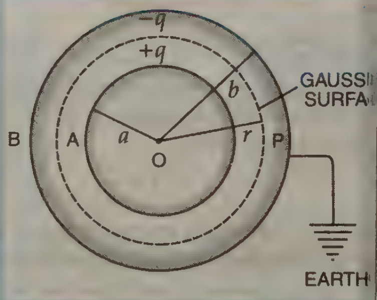


Fig. 5.09

From equations (5.06) and (5.07), we have

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Substituting for E in equation (5.05), we have

$$V = \int_a^b \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\, dr = \frac{q}{4\pi\varepsilon_0} \int_a^b r^{-2}\, dr = \frac{q}{4\pi\varepsilon_0} \left| \frac{r^{-1}}{-1} \right|_a^b = \frac{q}{4\pi\varepsilon_0} \left| -\frac{1}{r} \right|_a^b$$

$$= \frac{q}{4\pi\varepsilon_0} \left| -\frac{1}{b} + \frac{1}{a} \right|_a^b = \frac{q}{4\pi\varepsilon_0} \cdot \frac{b-a}{ab}$$

Therefore, capacitance of the spherical capacitor is given by

$$C = \frac{q}{V} = q \bigg/ \frac{q}{4\pi\varepsilon_0} \cdot \frac{b-a}{ab}$$

or $$C = 4\pi\varepsilon_0 \cdot \frac{ab}{b-a} \qquad ...(5.08)$$

It is the expression for the capacitance of a spherical capacitor.

We know that the capacitance of spherical conductor of radius a is given by

$$C' = 4\pi\varepsilon_0 \cdot a \qquad ...(5.09)$$

Since $\frac{ab}{b-a} > a$, from equations (5.08) and (5.09), it follows that $C > C'$.

Thus, the arrangement of the two spherical shells as explained above leads to crease the capacitance of a spherical conductor.

Key point

The smaller the difference between the radii of the two coatings of a spherical capacitor, greater is its capacitance.

Self-test Question

Show that the capacitance of a spherical capacitor having coating of radii r and R (R > r) is quite large as compared to the capacitance of a conductor of radius R.

5.08. CYLINDRICAL CAPACITOR

A cylindrical capacitor consists of two coaxial cylindrical shells, such that one all is earthed and the other is used to store charge on it. The two cylinderical shells ave a very small spacing between them, which is filled with some dielectric edium.

Expression for capacitance of a cylindrical capacitor. Fig. 5.10 shows a linder capacitor consisting of two cylindrical shells A and B. Let a and b be their spective radii and l be length of the cylindrical shells. When charge + q is given the inner cylindrical shell A, charge equal to – q is induced on the inner surface the cylindrical shell B and charge + q on its outer surface. As the shell B is earthed, e induced charge + q on the outer surface flows to earth.

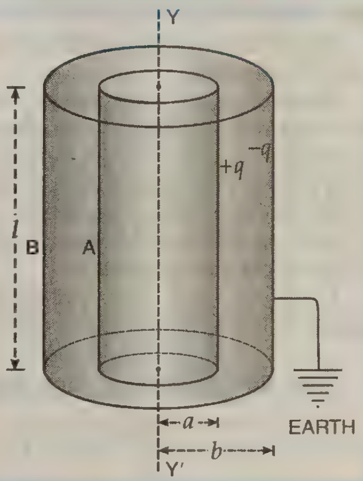


Fig. 5.10

If V is potential difference between the cylindrical shells A and B, then pacitance of the cylindrical capacitor is given by

$$C = \frac{q}{V}$$

As in case of a spherical capacitor, electric field in the space between the two linder shells is also not uniform. Therefore, if electric field at any point P at a stance r from the axis of the cylindrical shells is E, then potential difference between e two shells is given by

$$V = \int_a^b E\, dr \qquad ...(5.10)$$

To find the value of E, draw a cylindrical shell of radius r through point P as e gaussian surface [Fig. 5.11]. It encloses charge q on the inner cylindrical shell A. ccording to Gauss's theorem, the total electric flux through the gaussian surface,

$$\Phi = \frac{q}{\varepsilon_0} \qquad ...(5.11)$$

As the magnitude of electric field at every point on the gaussian surface is same, e have

$$\Phi = E \times \text{surface area of gaussian surface}$$

or $$\Phi = E \times 2\pi r l \qquad ...(5.12)$$

From equations (5.11) and (5.12), we have

$$E \times 2\pi r l = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{q}{2\pi\varepsilon_0 r l}$$

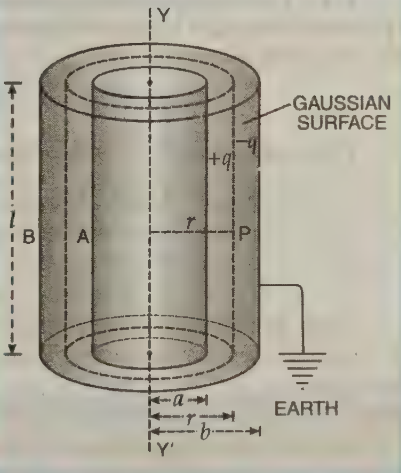


Fig. 5.11

Substituting the value of E in equation (5.10), we have

$$V = \int_a^b E\,dr = \int_a^b \frac{q}{2\pi\varepsilon_0 r l}\,dr = \frac{q}{2\pi\varepsilon_0 l}\int_a^b \frac{1}{r}\,dr = \frac{q}{2\pi\varepsilon_0 l}\Big|\log_e r\Big|_a^b$$

$$= \frac{q}{2\pi\varepsilon_0 l}(\log_e b - \log_e a) = \frac{q}{2\pi\varepsilon_0 l}\log_e\frac{b}{a}$$

Therefore, capacitance of the cylindrical capacitor is given by

$$C = \frac{q}{V} = q \Big/ \frac{q}{2\pi\varepsilon_0 l}\log_e\frac{b}{a}$$

or $$C = \frac{2\pi\varepsilon_0 l}{\log_e\frac{b}{a}} = \frac{2\pi\varepsilon_0 l}{2\cdot 303\log_{10}\frac{b}{a}} \qquad \text{...(5.13)}$$

Also, capacitance of the cylindrical capacitor per unit length is given by

$$C' = \frac{2\pi\varepsilon_0}{\log_e\frac{b}{a}} = \frac{2\pi\varepsilon_0}{2\cdot 303\log_{10}\frac{b}{a}} \qquad \text{...(5.14)}$$

5.09. GROUPING OF CAPACITORS

In practice, the capacitors are grouped in two ways *i.e. in series* and *in parallel.* When the capacitors are connected in series, the net capacitance decreases, while when connected in parallel, the net capacitance increases.

5.10. CAPACITORS IN SERIES

Fig. 5.12 shows the method of connecting three capacitors of capacitances C_1, C_2 and C_3 in series.

When charge $+q$ is given to plate A of the capacitor C_1, it induces charge $-q$ on left face and $+q$ on right face of the plate B. Thus, plate C of the capacitor C_2 also gets charge $+q$ on it. Then, it induces charge $-q$ on left face of plate D and charge $+q$ on plate E of the capacitor C_3.The charge $+q$ on plate E induces charge $-q$ on left face of plate F, while the charge $+q$ induced on its right face flows to earth. Thus, the three capacitors get charged with same amount of charge $+q$, when charge $+q$ is given to the first capacitor.

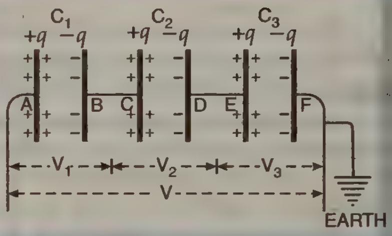


Fig. 5.12

Let V_1, V_2 and V_3 be potential differences across the plates of the three capacitors. If V is the potential difference across the series combination, then

$$V = V_1 + V_2 + V_3$$

From the definition of capacitance,

$$V_1 = \frac{q}{C_1}\ ;\ V_2 = \frac{q}{C_2} \text{ and } V_3 = \frac{q}{C_3}$$

$$\therefore \qquad V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \qquad \text{...(5.15)}$$

Let C be the equivalent capacitance of the series combination. Therefore, when a potential difference V is applied across a capacitor of capacitance C, it will also sotre the same amount of charge.

Hence, $$V = \frac{q}{C} \qquad \text{...(5.16)}$$

From equations (5.15) and (5.16), we have

$$\frac{q}{C} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad \text{...(5.17)}$$

Thus, *the reciprocal of the resultant (equivalent) capacitance of the series combination of a number of capacitors is equal to the sum of the reciprocals of the capacitances of individual capacitors.*

Key point

In series combination of capacitors,
(*i*) charge on each capacitor is same,
(*ii*) potential difference across a capacitor is inversely proportioal to its capacitance and
(*iii*) the equivalent capacitance is less than the capacitance of the smallest capacitor.

5.11. CAPACITORS IN PARALLEL

Fig. 5.13 shows the method of connecting three capacitors of capacitances C_1, C_2 and C_3 in parallel. One plate of the each capacitor is connected to the source of e.m.f. and the other to the earth. In this manner, all the capacitors are directly

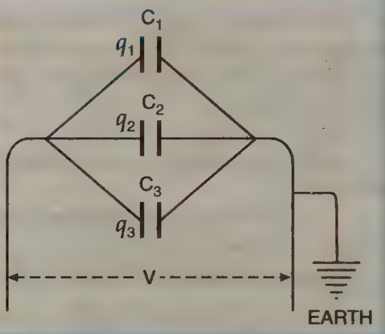


Fig. 5.13

onnected between the source of e.m.f. and the earth and thus possess the same otential difference across their plates. As the capacitors have different capacitances, ey will draw different amounts of charge from the source of e.m.f. If V is potential fference across the parallel combination and q_1, q_2 and q_3 are the values of the arges on the capacitors C_1, C_2 and C_3 respectively, then

$$q_1 = C_1 V; \quad q_2 = C_2 V \quad \text{and} \quad q_3 = C_3 V$$

Total charge on the parallel combination of the capacitors,

$$q = q_1 + q_2 + q_3 = C_1 V + C_2 V + C_3 V$$

or
$$q = (C_1 + C_2 + C_3) V \qquad \text{...(5.18)}$$

If C is the capacitance of the parallel combination, then

$$q = C V \qquad \text{...(5.19)}$$

From equations (5.18) and (5.19), we have

$$C V = (C_1 + C_2 + C_3) V$$

or
$$C = C_1 + C_2 + C_3 \qquad \text{...(5.20)}$$

Thus, *the resultant (equivalent) capacitance of the parallel combination of a number capacitors is equal to the sum of the capacitances of the individual capacitors.*

Key point

In parallel combination of capacitors,
(*i*) potential differene across each capacitor is same,
(*ii*) charge on a capacitor is proportional to its capacitance and
(*iii*) the equivalent capacitance is greater than the capacitance of the largest capacitor.

5.12. ENERGY STORED IN A CHARGED CAPACITOR

If we connect a battery across the two plates of the capacitor, the work is done r energy is supplied) by the battery in charging the capacitor. As the capacitor arges, potential difference across its plates increases. More and more work has be done by the battery in delivering the same amount of charge to the capacitor ue to continuously increasing potential difference across its plates.

The work done in charging a capacitor is stored in the capacitor in the form of electric ergy.

Consider a capacitor of capacitance C. Suppose that the capacitor does not ossess any charge, initially. On being connected to a battery, suppose that it charges a potential V after some time. If q is charge on the plate of the capacitor at that me, then

$$q = C V$$

Suppose battery supplies an infinitesimally small amount of charge dq to the pacitor at constant potential V. Then, according to the definition of potential fference, the small amount of work done by the battery is given by

$$dW = V\, dq = \frac{q}{C}\, dq$$

Therefore, amount of work done in delivering charge q to the capacitor is given

$$W = \int_0^q \frac{q}{C}\, dq = \frac{1}{C}\int_0^q q\, dq = \frac{1}{C}\left|\frac{q^2}{2}\right|_0^q = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored inside the capacitor in the form of the electric potential ergy. Therefore, energy stored in the capacitor,

$$U = \frac{1}{2}\frac{q^2}{C} \qquad \text{...(5.21)}$$

Substituting for q (= C V), equation (5.21) becomes

$$U = \frac{1}{2}\frac{(C V)^2}{C}$$

or
$$U = \frac{1}{2} C V^2 \qquad \text{...(5.22)}$$

Substituting for C $\left(= \frac{q}{V}\right)$, equation (5.22) becomes

$$U = \frac{1}{2}\frac{q}{V} V^2$$

or
$$U = \frac{1}{2} q V \qquad \text{...(5.23)}$$

Therefore, energy stored in a capacitor is given by

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} C V^2 = \frac{1}{2} q V$$

Key point

The energy stored in a charged capacitor resides inside it in the form of electric field.

5.13. LOSS OF ENERGY ON SHARING CHARGES BYTWO CAPACITORS

Consider two capacitors having capacitances C_1, C_2 and potentials V_1, V_2 respectively.

Then, the total energy of the two capacitors, before they are connected,

$$U = U_1 + U_2 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

When the two capacitors are connected together, total charge on the capacitor,

$$q = q_1 + q_2 = C_1 V_1 + C_2 V_2$$

Total capacitance of the two capacitors, $C = C_1 + C_2$

Therefore, total energy of the two capacitors, after they are connected,

$$U' = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

Now, $$U - U' = \left(\frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2\right) - \frac{1}{2}\frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

$$= \frac{C_1^2 V_1^2 + C_1 C_2 V_1^2 + C_1 C_2 V_2^2 + C_2^2 V_2^2 - (C_1 V_1 + C_2 V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{C_1 C_2 (V_1^2 + V_2^2 - 2 V_1 V_2)}{2(C_1 + C_2)} = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}$$

= a positive quantity

Since $U - U'$ is positive, there is always a loss of energy on connecting the two charged capacitors.

Key point

When two charged capacitors are connected to each other, they share charges, till they acquire a common potential. On sharing charges, there is always some loss of energy. However, total charge of the system remains conserved.

5.14. POLAR AND NON-POLAR DIELECTRICS

We know that in an atom, the negative charge in the form of electron cloud is distributed around the nucleus having equal positive charge. As such, in an atom or molecule of a dielectric, the centres of gravity of the positive and negative charges may or may not coincide and likewise the atoms or molecules of the dielectric are termed as ***non-polar*** or ***polar***.

Non-polar dielectric. *A dielectric, in the atoms or molecules of which, the centres of gravity of positive and negative charges coincide, is called a non-polar dielectric.*

Thus, in atoms or molecules of a non-polar dielectric, the positive and negative charges have a symmetrical charge distribution about their centres [Fig. 5.14]. Due to zero separation between the positive and negative charges, the electric dipole moment of such atoms or molecules is zero.

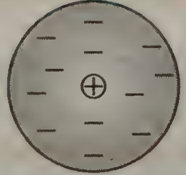


Fig. 5.14

Polar dielectric. *A dielectric, in the atoms or molecules of which, the centres of gravity of positive and negative charges do not coincide, is called a polar dielectric.*

Therefore, in the atoms and molecules of a polar dielectric, the positive and negative charges possess an asymmetrical charge distribution about their centres [Fig. 5.15]. Due to finite separation between the positive and negative charges, such atoms or molecules possess a finite electric dipole moment. The molecules, such as NH_3, HCl, H_2O, CO_2, etc are examples of polar molecules. Water molecule has a permanent dipole moment of the order of 6×10^{-28} C m.

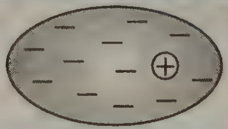


Fig. 5.15

In fact, molecules have normally an asymmetric distribution of charge. For example, in an ionic molecule, the electrons are transferred from one atom to some other atom in the molecule. The resulting molecule is, therefore, made of positive and negative ions in different location and must possess electric dipole moment.

5.15. NON-POLAR ATOM PLACED IN ELECTRIC FIELD (POLARISATION)

When a non-polar atom is subjected to an electric field, the positive nucleus gets pulled along the direction of electric field, while electrons get pulled in the opposite direction [Fig. 5.16]. Due to this, the restoring force is set up so as to bring the electron cloud back to its original state w.r.t. the nucleus. The stretching of atom continues, till the restoring force becomes just equal and opposite to the force exerted by the applied electric field on the charges.

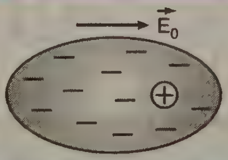


Fig. 5.16

The stretching of dielectric atoms due to displacement of charges in the atoms under the action of the applied electric field is called ***polarisation****.*

As a result of stretching of the atom, centre of gravity of positive nucleus does coincide with that of negative electrons. Due to this, the atom acquires a dipole ment.

5.16. POLARISATION OF A DIELECTRIC SLAB

Consider a parallel plate capacitor having vacuum between the plates. Suppose t the capacitor is charged with a battery so that electric field of strength E_0 is set between the plates [Fig. 5.17]. If σ and $-\sigma$ are surface charge densities of the two tes of the capacitor, then the electric field between two plates is given by

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

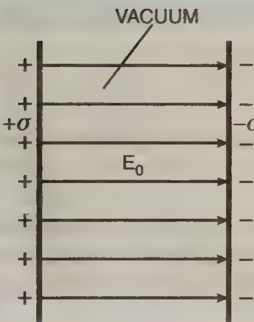


Fig. 5.17

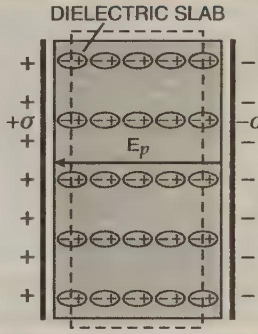


Fig. 5.18

Suppose that a dielectric slab of non-polar atoms is introduced between the two tes of the capacitor [Fig. 5.18]. As soon as the dielectric slab is introduced, each lecule of the dielectric gets polarised *i.e.* centres of gravity of positive and negative rges get displaced from each other. On the left face, a net negative charge $-q_i$ and he right face, a positive charge $+q_i$ appears. There is no net charge in the interior he dielectric marked by the dotted boundary. The charges $-q_i$ and $+q_i$ on the two faces of the dielectric slab are called ***induced charges***. The induced charges set up electric field E_p inside the dielectric. It is called ***electric field due to polarisation***. e direction of electric field due to polarisation is from positive to the negative uced charges developed onthe dielectric slab *i.e.* along a direction opposite to direction of applied electric field E_0. Therefore, the resultant electric field in the lectric is given by

$$E = E_0 - E_p \qquad \text{...(5.24)}$$

In other words, on placing a dielectric slab inside an electric field, the strength he electric field gets reduced and likewise E is called the ***reduced value of the tric field.***

Dielectric constant. *The ratio of the strength of the applied electric field to the strength ne reduced value of the electric field on placing the dielectric between the plates of the acitor is called the dielectric constant of the dielectric medium.*

It is also known as ***relative permittivity*** or ***specific inductive capacity*** and is oted by K (or ε_r). Therefore, dielectric constant of a dielectric medium is given by

$$K = \frac{E_0}{E} \qquad \text{...(5.25)}$$

The value of K is always greater than 1.

Polarisation density. *The induced dipole moment developed per unit volume in a ectric slab on placing it inside the electric field is called polarisation density.* It is denoted P.

If p is induced dipole moment acquired by an atom of the dielectric and N is number of atoms per unit volume, then polarisation density is given by

$$P = N p \qquad \text{...(5.26)}$$

The induced dipole moment acquired by the atom is found to be

$$p = \alpha \, \varepsilon_0 \, E \qquad \text{...(5.27)}$$

Here, constant of proportionality α is called ***atomic polarizability***. For atoms h as hydrogen, lithium, sodium, potassium, etc, the value of α is of the order of ^{29} m^3, while for atoms such as helium, neon, argon, etc, α has a value of the order 0^{-30} m^3. Knowing that $\varepsilon_0 = 8 \cdot 854 \times 10^{-12}$ C^2 N^{-1} m^{-2}, a field of the order of V m^{-1} will produce an induced dipole moment of the order of 10^{-34} C m in case i, Na, K, etc or 10^{-35} C m in case of He, Ne, Ar, etc.

From equations (5.26) and (5.27), we have

$$P = N \alpha \, \varepsilon_0 \, E \qquad \text{...(5.28)}$$

Let A be area of each plate of the capacitor and d, separation between them.

Then, the volume of the dielectric slab = A d

Since induced charges $-q_i$ and $+q_i$ are developed on the two faces of the lectric, the slab possesses total dipole moment equal to $q_i d$. Therefore, from inition of polarisation density,

$$P = \frac{q_i d}{A d} = \frac{q_i}{A}$$

Since $\frac{q_i}{A} = \sigma_p$, polarisation surface charge density, we have

$$P = \sigma_p \qquad \text{...(5.29)}$$

Therefore, on placing dielectric between the two plates of a capacitor, the reduced value of electric field may be expressed as

$$E = \frac{\sigma - \sigma_p}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$$

or $$E = E_0 - \frac{P}{\varepsilon_0} \qquad \text{...(5.30)}$$

Electric susceptibility. The polarisation density of a dielectric slab is directly proportional to the reduced value of the electric field and may be expressed as

$$P = \chi\, \varepsilon_0\, E \qquad \text{...(5.31)}$$

The constant of proportionality χ is called ***electric susceptibility*** of the dielectric slab. It is a dimensionless constant.

Setting the value of P in equation (5.30), we have

$$E = E_0 - \frac{\chi\, \varepsilon_0 E}{\varepsilon_0} = E_0 - \chi E$$

or $$E_0 = E\,(1 + \chi) \qquad \text{...(5.32)}$$

or $$\frac{E_0}{E} = 1 + \chi \qquad \text{...(5.33)}$$

Using equation (5.25), we have

$$K = 1 + \chi \qquad \text{...(5.34)}$$

Also, from equations (5.28) and (5.31), we have

$$\chi\, \varepsilon_0\, E = N \alpha\, \varepsilon_0\, E$$

or $$\chi = N\alpha \qquad \text{...(5.35)}$$

so that $$K = 1 + N\alpha \qquad \text{...(5.36)}$$

5.17. DIELECTRIC STRENGTH

Consider a dielectric slab placed between the parallel plates of a capacitor. As the potential difference across the plates of the capacitor is increased, the strength of applied electric field increases and the molecules of the dielectric undergo more and more stretching. If potential difference is increased gradually, a stage will come, when the electrons break up from the molecules of the dielectric. As a result of this electric break down, the dielectric becomes conducting.

The dielectric strength of a dielectric is defined as the maximum value of electric field (or potential gradient) that can be applied to the dielectric without its electric break down.

The unit of dielectric strength is same as that of electric field **V m^{-1}**. However, the practical unit of dielectric strength is **kV mm^{-1}**.

The table given ahead gives the value of dielectric constants and strengths of some common materials :

Material	Dielectric constant or relative permittivity	Dielectric strength (kV mm^{-1})
Vacuum	1 · 00000	∞
Air	1 · 00054	0 · 8
Water	78	–
Paper	3 · 5	14
Mica	5 · 4	160
Porcelain	6 · 6	4

5.18. CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH DIELECTRIC SLAB BETWEEN PLATES

Consider a parallel plate capacitor having each plate of area A and separated by a distance d. When there is vacuum between the two plates, the capacitance of the parallel plate capacitor is given by

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad \text{...(5.37)}$$

Suppose that when the capacitor is connected to a battery, electric field of strength E_0 is produced between the two plates of the capacitor. Further, suppose that when dielectric slab of thickness t ($t < d$) is introduced between the two plates of the capacitor [Fig. 5.19], the electric field reduces to E due to polarisation of the dielectric.

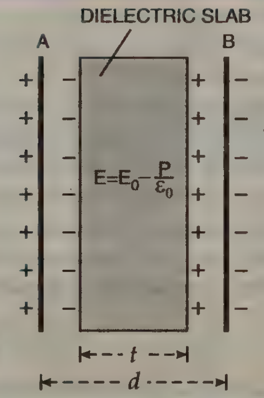


Fig. 5.19

Now, between the two plates of the capacitor ; over a distance t, the strength f electric field is E and over the remaining distance $(d-t)$, the strength of electric eld is E_0. If V is a potential difference between the plates of the capacitor, then

$$V = E\,t + E_0\,(d-t)$$

Since $\frac{E_0}{E} = K$ or $E = \frac{E_0}{K}$, we have

$$V = \frac{E_0}{K}\,t + E_0\,(d-t) = E_0\left(d - t + \frac{t}{K}\right)$$

Now, $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$

∴ $V = \frac{q}{\varepsilon_0 A}\left(d - t + \frac{t}{K}\right)$

Hence, capacitance of a parallel plate capacitor on introduction of dielectric ab is given by

$$C = \frac{q}{V} = q \Big/ \frac{q}{\varepsilon_0 A}\,(d - t + t/K) = \frac{\varepsilon_0 A}{d - t - t/K}$$

or
$$C = \frac{\varepsilon_0 A}{d - t\,(1 - 1/K)} \qquad \text{...(5.38)}$$

If the dielectric slab wholly fills the space between the two plates, then $t = d$. n that case,

$$C = \frac{\varepsilon_0 A}{d - d\,(1 - 1/K)}$$

or
$$C = \frac{\varepsilon_0 K A}{d} \qquad \text{...(5.39)}$$

From equations (5.37) and (5.39), it follows that

$$C = K\,C_0$$

Since $K > 1$, it follows that capacitance of a capacitor increases on introducing dielectric slab between its plates.

Key point

When a dielectric slab is introduced between the plates of a capacitor, its
(*i*) capacitance increases,
(*ii*) potential difference between plates decreases,
(*iii*) charge on the plates remains unchanged and
(*iv*) energy stored in the capacitor decreases.

Key point

When a dielectric slab is introducced between the plates of a capacitor, which is kept connected to a battery, its
(*i*) capacitance increses,
(*ii*) potential difference between the plates remains unchanged,
(*iii*) charge on plates increases and
(*iv*) energy stored in the capacitor increases.

5.19. CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH CONDUCTING SLAB BETWEEN PLATES

Consider a parallel plate capacitor having each plate of area A and separated y a distance d [Fig. 5.20]. The capacitance of the capacitor is given by

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad \text{...(5.40)}$$

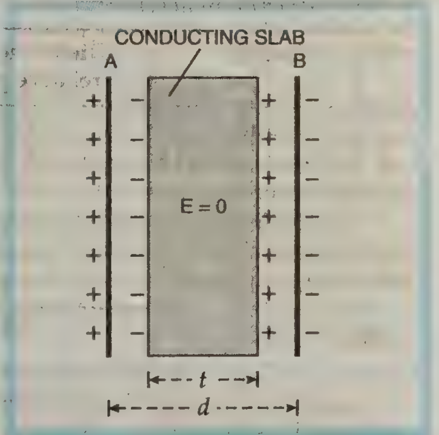


Fig. 5.20

Suppose that when the capacitor is connected to a battery , electric field of rength E_0 is produced between the two plates of the capacitor. Further, suppose that conducting slab of thickness t $(t < d)$ is introduced between the two plates of the apacitor. The electric field inside the conducting slab is zero. Thus, out of the total eparation d between the two plates, the electric field exists only over the distance $l-t)$. As a result, the potential difference will be reduced to

$$V = E_0\,(d-t)$$

or
$$V = \frac{q}{\varepsilon_0 A}\,(d-t) \qquad \left(\because E_0 = \frac{q}{\varepsilon_0 A}\right)$$

Therefore, capacitance of a parallel plate capacitor on introduction of the onducting slab is given by

$$C = \frac{q}{V} = q \Big/ \frac{q}{\varepsilon_0 A}\,(d-t) = \frac{\varepsilon_0 A}{d-t}$$

or
$$C = \frac{\varepsilon_0 A}{d\,(1 - t/d)} \qquad \text{...(5.41)}$$

Using equation (5.40), we have

$$C = \frac{C_0}{(1 - t/d)} \qquad \text{...(5.42)}$$

Thus, on introduction of a conducting slab, the capacitance of a parallel plate apacitor increases.

If the conducting slab wholly fills the space between the two plates, then $t = d$. In that case,

$$C = \frac{C_0}{(1 - d/d)} = \frac{C_0}{0} = \infty$$

i.e. the capacitance of the capacitor will become infinite. In other words, whatsoever amount of charge may be delivered A of the capacitor, it will go on accepting it. The reason is that when the conducting slab wholly fills the space between the two plates, plate A also gets connected to the earth. As such, when charge is delivered to A, it simply flows to the earth through the earth connected plate B. To the observer, it appears as if the capacitance of the capacitor has become infinite.

Self-Test Question

When a conducting slab wholly fills the space between the two plates of a capacitor, its capacitance becomes infinite. Explain, why.

5.20. VAN DE GRAAFF GENERATOR (High Voltage Generator)

In 1931, R. J. Van de Graaff designed an electrostatic generator capable of generating very high potential of the order of 5×10^6 V, which was then made use of in accelerating charged particles so as to carry out nuclear reactions.

Principle. It is based on the following two electrostatic phenomena :

1. The electric discharge takes place in air or gases readily at pointed conductors.
2. If a hollow conductor is in contact with an other conductor, then as charge is supplied to the conductor, the hollow conductor continues accepting the charge irrespective of the fact, howsoever large its potential may grow. The charge immediately shifts to outer surface of the hollow conductor.

Construction and working. Van de Graaff generator consists of a large hollow metallic sphere S mounted on two insulating columns C C′ as shown in Fig. 5.21. An endless belt of an insulating material is made to run on two pulleys P_1 and P_2 with the help of an electric motor. The metal comb C_1, called ***spray comb*** is held near the lower end of the belt. When the spray comb is maintained at high positive potential with the help of E.H.T. source ($\approx 10^4$ V), it produces ions in its vicinity. The positive ions get sprayed on the belt due to the repulsive action of comb C_1, which are carried upward by the moving belt. A comb C_2 called ***collection comb*** is positioned near the upper end of the belt, such that the pointed ends touch the belt and the other end is in contact with the inner surface of the metallic sphere S. The comb C_2 collects the positive ions and transfers them to the metallic sphere. The charge transferred by the comb C_2 immediately moves on to the outer surface of the hollow sphere. As the belt goes on moving, the accumulation of positive charge on the sphere also keeps on taking place continuously and its potential rises considerably. With the increase of charge on the sphere, its leakage due to ionisation of surrounding air also becomes faster. The maximum potential to which the sphere can be raised is reached, when the rate of loss of charge due to leakage becomes equal to the rate at which the charge is transferred to the sphere. To prevent the leakage of charge from the sphere, the generator is completely enclosed inside an earth-connected steel tank, which is filled with air under pressure.

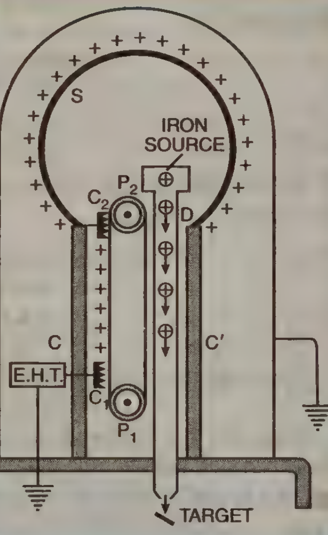


Fig. 5.21

If the projectiles, such as protons, deutrons, etc are now generated in the discharge tube D with lower end earthed and upper end inside the hollow sphere, they get accelerated in downward direction along the length of the tube. At the other end, they come to hit the target with large kinetic energy and bring about nuclear disintegration.

Van de Graaff generator of this type was installed at the Carnegie Institute in Washington in 1937. One such generator was installed at Indian Institute of Technology, Kanpur in 1970 and it accelerates particles to 2 MeV energy.

SOLVED NUMERICAL PROBLEMS

Type A. On Capacitance of a Spherical Conductor

Problem 5.01. Assuming the earth to be a spherical conductor of radius 6400 km, calculate its capacitance. Given,

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}.$$

(H.P.S.S.C.E. 1998 S ; H.S.S.C.E. 1998 S)

Sol. Here, $r = 6400 \text{ km} = 6 \cdot 4 \times 10^6 \text{ m}$;

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

In SI, the capacitance of a spherical conductor is given by

$$C = 4\pi\varepsilon_0 r = \frac{1}{9 \times 10^9} \times 6 \cdot 4 \times 10^6$$

$$= 7 \cdot 11 \times 10^{-4} \text{ F} = 711\ \mu\text{F}$$

Problem 5.02. Can a metal sphere of radius 1 cm hold a charge of 1 coulomb ? *(I.I.T. 1975)*

Sol. Here, $r = 1 \text{ cm} = 0{\cdot}01 \text{ m}$; $q = 1$ C

Capacitance of metal sphere,

$$C = 4\pi\varepsilon_0 r = \frac{1}{9 \times 10^9} \times 0 \cdot 01 \text{ F}$$

Therefore, potential of metal sphere, when a charge of is given to it,

$$V = \frac{q}{C} = \frac{1 \times 9 \times 10^9}{0 \cdot 01} = 9 \times 10^{11} \text{ V}$$

Therefore, for metal sphere to hold a charge of 1 C, it l have to be raised to a potential of 9 × 10^{11} V. But even ore such a high potential is reached, the surrounding air omes ionised and the charge leaks to the surroundings. erefore, the given metal sphere **will not be able to hold rge of 1 C.**

Problem 5.03. 27 drops of same size are charged at 220 ach. They coalesce to form a bigger drop. Calculate the ential of the bigger drop. *(P.S.S.C.E 2001)*

Sol. Let r be the radius of each small drop. Then,

capacitance of each small drop, $C = 4\pi\varepsilon_0 r$

Charge on each small drop, $q = CV = 4\pi\varepsilon_0 r \times 220$

Total charge on 27 small drops,

$$Q = 27\,q = 27 \times 4\pi\varepsilon_0 r \times 220$$

When 27 small drops coalesec, let R be the radius of the ger drop formed. Since the volume of the bigger drop must equal to that of 27 small drops,

$$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3 \quad \text{or} \quad R = 3r$$

Therefore, capacitance of the bigger drop

$$= 4\pi\varepsilon_0 R = 4\pi\varepsilon_0 \times 3r$$

The charge on bigger drop will be same as that on 27 all drops. Therefore,

$$\text{potential of bigger drop} = \frac{27 \times 4\pi\varepsilon_0 r \times 220}{4\pi\varepsilon_0 \times 3r} = \mathbf{1980\ V}$$

e B. On Capacitance of a parallel plate capacitor

Problem 5.04. What is the area of the plates of a 2 F allel plate capacitor, given that the separation between plates is 0·5 cm ? *(Text Problem ; C.B.S.E. 1990 S)*

Sol. Here, $C = 2$ F ; $d = 0{\cdot}5$ cm $= 0{\cdot}5 \times 10^{-2}$ m

We know, $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}\ \text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$

$$\text{Now,} \quad C = \frac{\varepsilon_0 A}{d}$$

$$\text{or} \quad A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0 \cdot 5 \times 10^{-2}}{8 \cdot 854 \times 10^{-12}} = \mathbf{1{\cdot}13 \times 10^9\ m^2}$$

Problem 5.05. A parallel plate capacitor having plate a 100 cm² and plate separation 1 mm holds a charge of 2 μ C, when connected to a 120 V battery. Find the lectric constant of the material between the plates.

Sol. Here, $A = 100\ \text{cm}^2 = 10^{-2}\ \text{m}^2$; $d = 1$ mm $= 10^{-3}$ m ;

$$q = 0{\cdot}12\ \mu\text{C} = 0{\cdot}12 \times 10^{-6}\ \text{C} ; V = 120\ \text{V}$$

Let C be the capacitance of the capacitor. Then

$$C = \frac{q}{V} = \frac{0 \cdot 12 \times 10^{-6}}{120} = 10^{-9}\ \text{F}$$

If K is dielectric constant of the material between the tes, then

$$C = \frac{\varepsilon_0 KA}{d}$$

$$\text{or} \quad K = \frac{Cd}{\varepsilon_0 A} = \frac{10^{-19} \times 10^{-3}}{8 \cdot 854 \times 10^{-12} \times 10^{-2}} = \mathbf{11{\cdot}3}$$

Problem 5.06. Two parallel plate air capacitors have ir plate areas 100 and 500 cm² respectively. If they have the same charge and potential and the distance between the plates of the first capacitor is 0·5 mm, what is the distance between the plates of the second capacitor ? *(P.S.S.C.E. 1997 S)*

Sol. Let A_1 and d_1 be area of plates and distance between the plates of first capacitor ; and A_2 and d_2 be corresponding values in case of the second capacitors. If C_1 and C_2 are the capacitances of the two capacitors, then

$$C_1 = \frac{\varepsilon_0 A_1}{d_1} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 A_2}{d_2}$$

$$\text{We know, } C = \frac{q}{V}$$

Since the two capacitors have same charge and potential, their capacitances must be equal *i.e.* $C_1 = C_2$.

$$\text{or} \quad \frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 A_2}{d_2}$$

$$\text{or} \quad d_2 = \frac{A_2}{A_1} d_1$$

Here, $A_1 = 100\ \text{cm}^2$; $A_2 = 500\ \text{cm}^2$

and $d_1 = 0{\cdot}5$ mm $= 0{\cdot}05$ cm

$$\therefore \quad d_2 = \frac{500 \times 0 \cdot 05}{100} = \mathbf{0{\cdot}25\ cm}$$

Type C. On Capacitance of a spherical capacitor

Problem 5.07. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Sol. We know that the capacitance of a spherical capacitor,

$$C = 4\pi\varepsilon_0 \cdot \frac{ab}{b-a}$$

Here, b = radius of the top of stratosphere layer

$= 6400$ km $+ 50$ km $= 6450$ km $= 6{\cdot}45 \times 10^6$ m

a = radius of earth $= 6400$ km $= 6{\cdot}4 \times 10^6$ m

$$\therefore \quad C = \frac{1}{9 \times 10^9} \times \frac{6 \cdot 45 \times 10^6 \times 6 \cdot 4 \times 10^6}{6 \cdot 45 \times 10^6 - 6 \cdot 4 \times 10^6}$$

$$= \mathbf{0{\cdot}092\ F}$$

Problem 5.08. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2·5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32. (*a*) Determine the capacitance of the capacitor. (*b*) What is the potential of the inner sphere ? (*c*) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. *(Text Problem)*

Sol. Here, $a = 12$ cm $= 12 \times 10^{-2}$ m ;

$b = 13$ cm $= 13 \times 10^{-2}$ m

$q = 2{\cdot}5\ \mu\text{C} = 2{\cdot}5 \times 10^{-6}$ C and K = 32

$$(a) \quad C = 4\pi\varepsilon_0 \cdot \frac{Kab}{b-a}$$

$$= \frac{1}{9 \times 10^9} \times \frac{32 \times 12 \times 10^{-2} \times 13 \times 10^{-2}}{13 \times 10^{-2} - 12 \times 10^{-2}}$$

$$= \mathbf{5{\cdot}547 \times 10^{-9}\ F}$$

(*b*) As the outer sphere is earthed, the potential of the inner sphere is equal to the potential difference between the two spheres of the capacitors. Therefore,

$$V = \frac{q}{C} = \frac{2 \cdot 5 \times 10^{-6}}{5 \cdot 547 \times 10^{-9}} = \mathbf{450 \cdot 7\ V}$$

(*c*) Radius of sphere, $r = 12$ cm $= 12 \times 10^{-2}$ m

Now, $$C' = 4\pi\varepsilon_0 r = \frac{1}{9 \times 10^9} \times 12 \times 10^{-2}$$

$$= 1 \cdot 333 \times 10^{-11}\ \text{F}$$

$$\therefore \quad \frac{C}{C'} = \frac{5 \cdot 547 \times 10^{-9}}{1 \cdot 333 \times 10^{11}} \approx \mathbf{416}$$

Type D. On Capacitance of a cylindrical capacitor

Problem 5.09. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1·5 cm and 1·4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3·5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (*i.e.* bending of field lines at the ends). (*Text Problem*)

Sol. Here, $l = 15$ cm $= 15 \times 10^{-2}$ m ;
$a = 1{\cdot}4$ cm $= 1{\cdot}4 \times 10^{-2}$ m ; $b = 1{\cdot}5$ cm $= 1{\cdot}5 \times 10^{-2}$ m ;
$q = 3{\cdot}5\ \mu\text{C} = 3{\cdot}5 \times 10^{-6}$ C

Now, $$C = \frac{2\pi\varepsilon_0 l}{2 \cdot 303 \log_{10}\left(\frac{b}{a}\right)}$$

$$= \frac{2\pi \times 8 \cdot 854 \times 10^{-12} \times 15 \times 10^{-2}}{2 \cdot 303 \log_{10} \frac{1 \cdot 5 \times 10^{-2}}{1 \cdot 4 \times 10^{-2}}}$$

$$= \mathbf{1{\cdot}21 \times 10^{-8}\ F}$$

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them.

Therefore, potential of inner cylinder,

$$V = \frac{q}{C} = \frac{3 \cdot 5 \times 10^{-6}}{1 \cdot 21 \times 10^{-10}} = \mathbf{2{\cdot}89 \times 10^4\ V}$$

Type E. On Grouping of capacitors

Problem 5.10. Three capacitors each of capacitance 9 pF are connected in series as shown in Fig. 5.22. (*a*) What is the total capacitance of the combination ? (*b*) What is the potential difference across each capacitor, if the combination is connected to a 120 volt supply ?

(*Text Problem*)

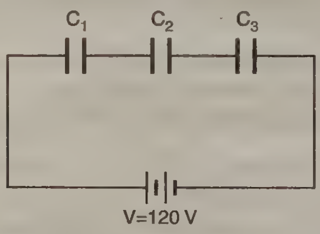


Fig. 5.22

Sol. Here, $C_1 = C_2 = C_3 = 9$ pF $= 9 \times 10^{-12}$ F ; V = 120 volt

(*a*) Total capacitance of the series combination is give
by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = 3 \times \frac{1}{9 \times 10^{-12}} = \frac{1}{3 \times 10^{-12}}$$

$$\therefore \quad C = 3 \times 10^{-12}\ \text{F} = \mathbf{3\ pF}$$

(*b*) Let q be charge on each capacitor. Then, sum of th
potential differences across the plates of three capacitors mus
be equal to 120 V *i.e.*

$$V_1 + V_2 + V_3 = 120$$

$$\text{or} \quad \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = 120$$

$$\text{or} \quad \frac{q}{9 \times 10^{-12}} + \frac{q}{9 \times 10^{-12}} + \frac{q}{9 \times 10^{-12}} = 120$$

$$\text{or} \quad \frac{3q}{9 \times 10^{-12}} = 120$$

$$\text{or} \quad q = 360 \times 10^{-12}\ \text{C}$$

Therefore, potential difference across a capacitor

$$= \frac{q}{\text{capacitance}} = \frac{360 \times 10^{-12}}{9 \times 10^{-12}}\ \text{s} = \mathbf{40\ V}$$

Alternative method. Since the three capacitors are of th
same capacitances,

potential difference across each capacitor $= \frac{120}{3} = \mathbf{40}$

**Problem. 5.11. Three capacitors of capacitances 2 p
3 pF and 4 pF are connected in parallel as shown in Fig 5.2
(*a*) What is the total capacitance of the combination ? (
Determine the charge on each capacitor, if the combinatio
is connected to a 100 V supply.** (*Text Problem*

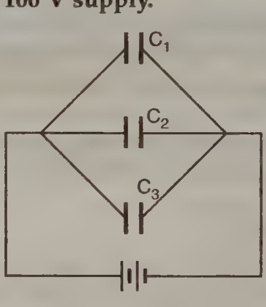


Fig. 5.23

Ans. Here, $C_1 = 2$ pF ; $C_2 = 3$ pF ; $C_3 = 4$ pF ;
V = 100 volt

(*a*) Total capacitance of the parallel combination is give
by

$$C = C_1 + C_2 = C_3 = 2 + 3 + 4 = \mathbf{9\ pF}$$

(*b*) Let q_1, q_2 and q_3 be the charges on C_1, C_2 and C
respectively. In the parallel combination, the potentia
difference across each capacitor will be equal to the suppl
voltage *i.e.* 100 V.

$$\therefore \quad q_1 = C_1 V = 2 \times 10^{-12} \times 100 = \mathbf{2 \times 10^{-10}\ C},$$

$$q_2 = C_2 V = 3 \times 10^{-12} \times 100 = \mathbf{3 \times 10^{-10}\ C}$$

$$q_3 = C_3 V = 4 \times 10^{-12} \times 100 = \mathbf{4 \times 10^{-12}\ C}$$

**Problem. 5.12. Three capacitors of capacitances 5, 4 an
3 μF respectively are connected with the first and secon
in series and the third in parallel with them. Find th
capacitance of the combination.** (*Karnataka, 1983*

Ans. Here, $C_1 = 5\ \mu$F ; $C_2 = 4\ \mu$F ; $C_3 = 3\ \mu$F

The three capacitors have been connected as shown in . 5.24.

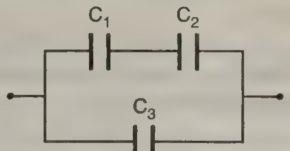


Fig. 5.24

Let C′ be capacitance of the series combination of C_1 and Then,

$$\frac{1}{C'}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{5}+\frac{1}{4}=\frac{4+5}{20}=\frac{9}{20}\quad\text{or}\quad C'=2{\cdot}22\,\mu\text{F}$$

The series combination of C_1 and C_2 have been put in rallel with C_3.

If C is the effective capacitance of the arrangement, then

$$C = C' + C_3 = 2{\cdot}22 + 3 = \mathbf{5{\cdot}22\ \mu F}$$

Problem 5.13. The effective capacitances of two pacitors are 3 μF and 16 μF, when they are connected in ies and parallel respectively. Compute the capacitance each capacitor. *(Karnataka, 1991 S)*

Sol. Let C_1 and C_2 be capacitances of the two capacitors.

Here, $C_s = 3\ \mu F$; $C_p = 16\ \mu F$

The effective capacitance in parallel, $C_p = C_1 + C_2$

$$\therefore \qquad C_1 + C_2 = 16 \qquad \text{...}(i)$$

The effective capacitance in series is given by

$$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}\quad\text{or}\quad C_s=\frac{C_1C_2}{C_1+C_2}$$

$$\therefore \qquad \frac{C_1C_2}{C_1+C_2}=3\quad\text{or}\quad\frac{C_1C_2}{16}=3$$

$$\text{or} \qquad C_1C_2 = 48$$

Now, $(C_1-C_2)^2 = (C_1+C_2)^2 - 4C_1C_2$

$$= (16)^2 - 4\times 48 = 256 - 192 = 64$$

$$\text{or} \qquad C_1 - C_2 = 8 \qquad \text{...}(ii)$$

Adding the equations (*i*) and (*ii*), we have

$$(C_1+C_2)+(C_1-C_2) = 16+8$$

$$\text{or} \qquad \mathbf{C_1 = 12\ \mu F}$$

From equation (*i*), it follows that

$$C_2 = 16 - C_1 = 16 - 12 = \mathbf{4\ \mu F}$$

Problem 5.14. Find the resultant capacitance of the pacitor connected as shown in the Fig. 5.25. *(H.P.S.S.C.E. 1997)*

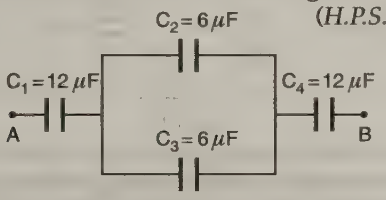


Fig. 5.25

Sol. Let C_{23} be the equivalent capacitance of the pacitors C_2 and C_3 connected in parallel. Then,

$$C_{23} = C_2 + C_3 = 6 + 6 = 12\ \mu F$$

The given network will be equivalent to series angement of the capacitors as shown below in Fig. 5.26.

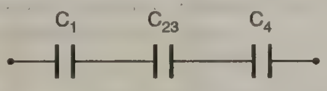


Fig. 5.26

If C is the equivalent capacitance of the series com-
ation, then

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_{23}}+\frac{1}{C_4}=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{1}{4}$$

$$\text{or} \qquad C = 4\ \mu F$$

Problem 5.15. If C_1 = 3 pF and C_2 = 2 pF, calculate the equivalent capacitance of the network shown in Fig. 5.27 between points A and B ? *(C.B.S.E. Sample Q. Paper)*

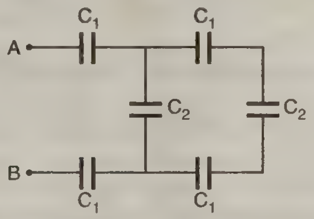


Fig. 5.27

Sol. The given network shown in Fig. 5.27 is equivalent to the arrangement shown in Fig. 5.28, which is further equivalent to arrangement shown in Fig. 5.29.

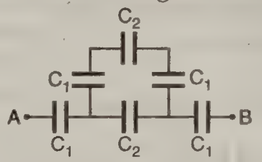


Fig. 5.28

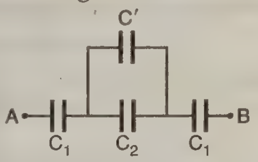


Fig. 5.29

Here, C′ is capacitance of the series combination of C_1, C_2 and C_1. Therefore,

$$\frac{1}{C'}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_1}=\frac{1}{3}+\frac{1}{2}+\frac{1}{3}=\frac{7}{6}\quad\text{or } C'=\frac{6}{7}\text{ pF}$$

The arrangement shown in Fig. 5.29 is equivalent to the network drawn in Fig. 5.30.

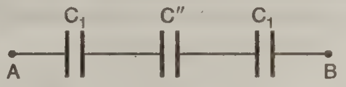


Fig. 5.30

Here, C″ is capacitance of the parallel combination of C′ and C_2. Therefore,

$$C'' = C' + C_2 = \frac{6}{7}+2=\frac{20}{7}\text{pF}$$

If C_{equi} is capacitance of the series combination of C_1, C″ and C_1, then

$$\frac{1}{C_{equi}}=\frac{1}{C_1}+\frac{1}{C''}+\frac{1}{C_1}=\frac{1}{3}+\frac{7}{20}+\frac{1}{3}=\frac{61}{60}$$

$$\text{or} \qquad C_{equi}=\frac{60}{61}\approx \mathbf{1\ pF}$$

Problem 5.16. Five capacitors of capacitances $C_1 = C_5$ = 1 μF , $C_2 = C_3 = C_4$ = 2 μF are connected as shown in Fig. 5.31. Find the equivalent capacitance of the system between points X and Y.

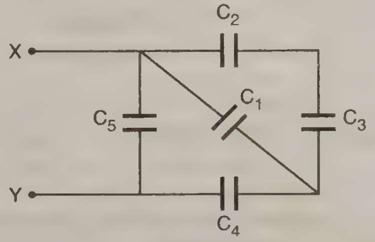


Fig. 5.31

Ans. The capacitors C_2 and C_3 are in series. Their effective capacitance,

$$C_{23} = \frac{C_2 \times C_3}{C_2 + C_3} = \frac{2 \times 2}{2 + 2} = 1\ \mu F$$

The given arrangement of capacitor is equivalent to the arrangement as shown in Fig. 5.32.

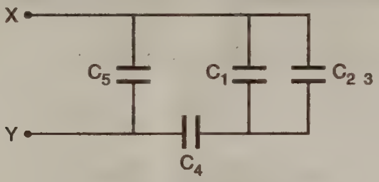


Fig. 5.32

Now, capacitors C_1 and C_{23} are in parallel. Let C_{123} be their effective capacitance. Then,

$$C_{123} = C_1 + C_{23} = 1 + 1 = 2\ \mu F$$

The arrangement of capacitors shown in Fig. 5.32 is now equivalent to the arrangement shown in Fig. 5.33.

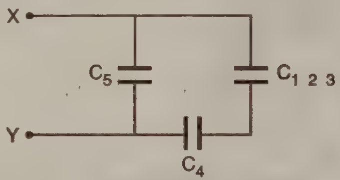


Fig. 5.33

Since the capacitors C_{123} and C_4 are in series, their equivalent capacitance is given by

$$C_{1234} = \frac{C_{123} \times C_4}{C_{123} + C_4} = \frac{2 \times 2}{2 + 2} = 1\ \mu F$$

The arrangement of capacitors shown in Fig. 5.33 is equivalent to the arrangement shown in Fig. 5.34.

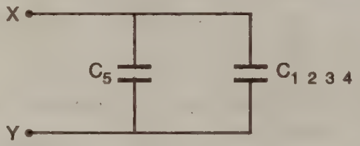


Fig. 5.34

Since capacitors C_{123} and C_5 are in parallel, the equivalent capacitance between points X and Y is given by

$$C = C_{1234} + C_5 = 1 + 1 = 2\ \mu F$$

Problem 5.17. Find the equivalent capacitance of the combination of capacitors between the points A and B as shown in Fig. 5.35. Also calculate the total charge that flows in the circuit, when a 100 V battery is connected between the points A and B. (*C.B.S.E. 2002*)

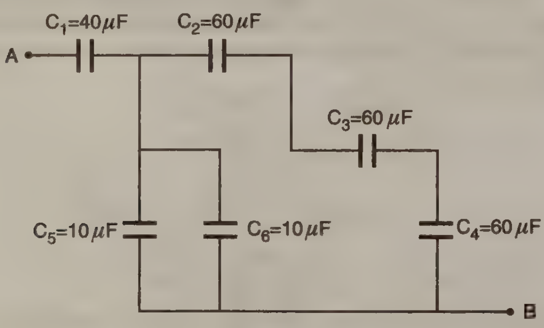


Fig. 5.35

Ans. Refer to the arrangement of capacitors shown i Fig. 5.35. The capacitors C_5 and C_6 are in parallel. Thei effective capacitance,

$$C_{56} = C_5 + C_6 = 10 + 10 = 20\ \mu F$$

Also, capacitors C_2, C_3 and C_4 are in series, so that thei effective capacitance is given by

$$\frac{1}{C_{234}} = \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{1}{20}$$

or $\quad C_{234} = 20\ \mu F$

The given arrangement of capacitance is equivalent t the arrangement as shown in Fig. 5.36.

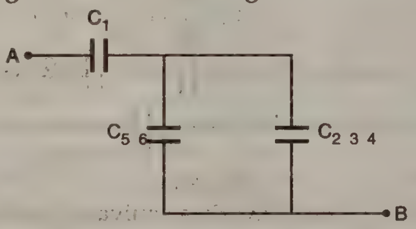


Fig. 5.36

Now, capacitors C_{56} and C_{234} are in parallel. Let C′ b their effective capacitance. Then,

$$C' = C_{56} + C_{234} = 20 + 20 = 40\ \mu F$$

The arrangement of capacitors shown in Fig. 5.36 is no equivalent to the arrangement shown in Fig. 5.37.

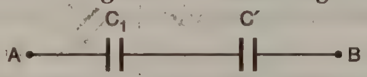


Fig. 5.37

Since the capacitors C_1 and C′ are in series, th equivalent capacitance of the combination between points and B is given by

$$C = \frac{C_1\, C'}{C_1 + C'} = \frac{40 \times 40}{40 + 40} = 20\ \mu F$$

Therefore, total charge that flows in the circuit, when 100 V battery is connected between points A and B is given b

$$q = CV = 20 \times 10^{-6} \times 100 = 2 \times 10^{-3}\ C$$

Problem 5.18. Find the capacitance of the infinit ladder between points X and Y [Fig. 5.38].

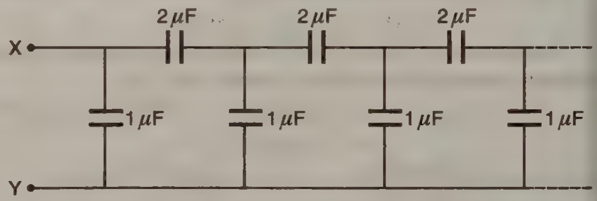


Fig. 5.38

Ans. Let the equivalent capacitance of the entire ladde be C. Since the ladder is infinitely long, adding one more s of two capacitors (1μF and 2μF) across the points X and Y wi not affect the total capacitance *i.e.* it should still remain equ to C. The ladder (whose equivalent capacitance is C) on addin such two capacitors will appear as shown in Fig. 5.39.

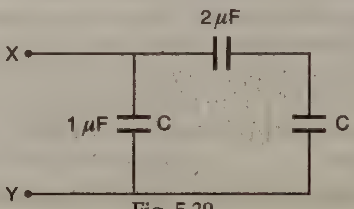


Fig. 5.39

The equivalent capacitance of the arrangement shown bove should again be equal to C *i.e.*

$$C = 1 + \frac{2\times C}{2+C} \quad \text{or} \quad C^2 - C - 2 = 0$$

$$C = 4\,\mu F \quad \text{or} \quad -1\,\mu F$$

Since capacitance can not be negative, the equivalent apacitance of the ladder is **4 μF.**

Problem 5.19. Find the effective capacitance between he terminals X and Y of the network shown in Fig. 5.40. iven that $C_1 = 5\ \mu F$, $C_2 = 10\ \mu F$, $C_3 = 2\ \mu F$, $C_4 = 4\ \mu F$ and $_5 = 10\ \mu F$.

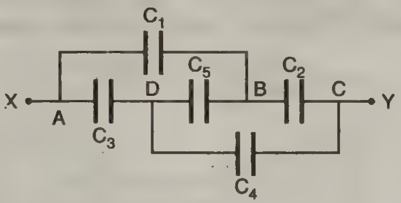


Fig. 5.40

Sol. The given network is equivalent to the network hown in Fig. 5.41.

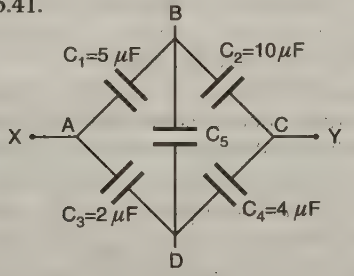


Fig. 5.41

We find that $\frac{C_1}{C_2} = \frac{C_3}{C_4}$

Therefore, the bridge is balanced and hence when some otential difference is applied across terminals X and Y, no urrent will flow across the path BD. Thus, the path BD ehaves as open path and the given network is equivalent to he network as shown in Fig. 5.42.

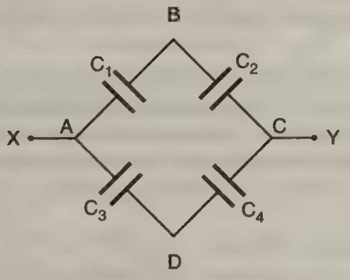


Fig. 5.42

Let C_{12} and C_{34} be capacitances of the series combination f C_1, C_2 and C_3, C_4 respectively.

Then, $\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$ or $C_{12} = \frac{10}{3}\ \mu F$

Also, $\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ or $C_{34} = \frac{4}{3}\ \mu F$

Now, C_{12} and C_{34} form a parallel combination. If C is net apacitance of the network, then

$$C = C_{12} + C_{34} = \frac{10}{3} + \frac{4}{3} = \frac{14}{3} = \mathbf{4{\cdot}67\ \mu F}$$

Problem 5.20. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him, each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires a minimum number of capacitors. (*Text Problem*)

Sol. Suppose that the technician makes use of N capacitors and connects them in *m* rows, each row having *n* capacitors. Then, $N = m\,n$

Capacitance of each capacitor = 1 μF ;
required capacitance of the combination, C = 2 μF
voltage rating of each capacitor = 400 V ;
required voltage rating of the combination = 10^3 V

When capacitors are connected in series, potential differences across their plates get added. Therefore, *n* capacitors connected in a row will stand a voltage equal to 400 × *n* V.

Therefore, number of capacitors to be connected in a row is given by

$$400\,n = 10^3$$

$$\text{or} \quad n = \frac{10^3}{400} = 2{\cdot}5 \quad \text{or} \quad n = 3$$

(It can not be a fraction*)

The total capacitance of the capacitors in a row is given by

$$\frac{1}{C'} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3 \quad \text{or} \quad C' = \frac{1}{3}\ \mu F$$

The total capacitance of *m* rows is given by

$$C = m\,C' \quad \text{or} \quad m = \frac{C}{C'} = \frac{2}{1/3} = 6$$

Therefore, he should make ***three rows of such capacitors, each row containing six capacitors.***

Problem 5.21. Two capacitors C_1 and C_2 are connected to a battery of 6 V as shown in Fig. 5.43. Find the charge on each capacitor.

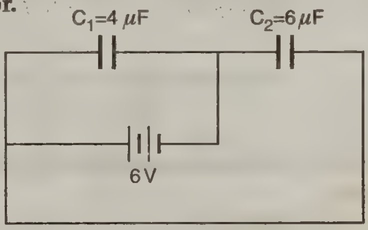


Fig. 5.43

Ans. The given arrangement of two capacitors connected to a battery is equivalent to the arrangement shown in Fig. 5.44

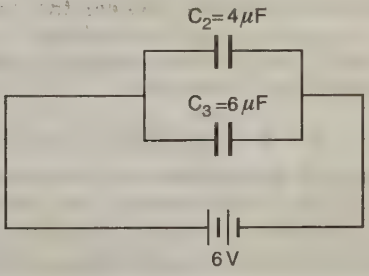


Fig. 5.44

Therefore, charge on capacitor C_1,

$$q_1 = C_1\,V = 4 \times 6 = \mathbf{24\ \mu C}$$

and charge on capacitor C_2,

$$q_2 = C_2\,V = 6 \times 6 = \mathbf{36\ \mu C}$$

---

*n cannot be equal to 2. Otherwise, potential difference across a capacitor will be 500 V *i.e.* more than its voltage rating.

Problem 5.22. Three capacitors C_1, C_2 and C_3 are connected to a battery of 6 V as shown in Fig. 5.45. Find the charges on the three capacitors.

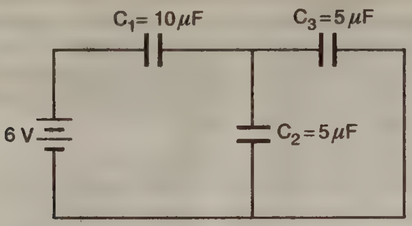


Fig. 5.45

Sol. The given arrangement of three capacitors connected to a battery is equivalent to the arrangement shown in Fig. 5.46.

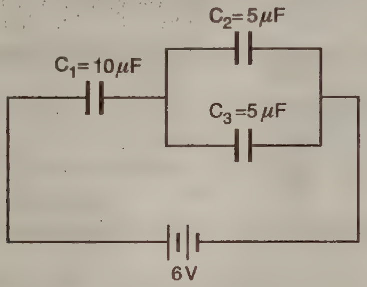


Fig. 5.46

Since capacitors C_2 and C_3 are in parallel, their equivalent capacitance,

$$C_{23} = 5 + 5 = 10\ \mu F$$

The arrangement of capacitors shown in Fig. 5.46 is now equivalent to the network shown in Fig. 5.47.

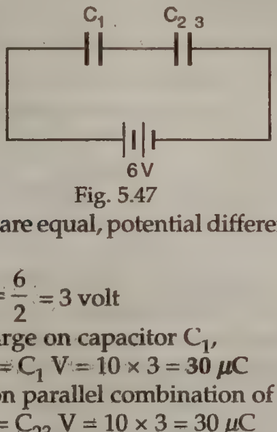


Fig. 5.47

As C_1 and C_{23} are equal, potential difference across each of them will be

$$V = \frac{6}{2} = 3 \text{ volt}$$

Therefore, charge on capacitor C_1,

$$q_1 = C_1 V = 10 \times 3 = 30\ \mu C$$

Also, charge on parallel combination of C_2 and C_3,

$$q_{23} = C_{23} V = 10 \times 3 = 30\ \mu C$$

As C_2 and C_3 are equal, the charge q_{23} is shared equally by the two capacitors. Therefore,

$$q_2 = q_3 = \frac{30}{2} = 15\ \mu C$$

Problem 5.23. A capacitor of 4 μF is connected to 400 V supply. It is then disconnected and connected to an uncharged capacitor of 2 μF. Calculate the common potential after the capacitors are connected together.

(Pre-degree Kerala, 1992)

Sol. Here, $C_1 = 4\ \mu F = 4 \times 10^{-6}$ F ; $V_1 = 400$ V ;

$$C_2 = 2\ \mu F = 2 \times 10^{-6}\ F$$

Therefore, charge on capacitor C_1,

$$q_1 = C_1 V_1 = 4 \times 10^{-6} \times 400 = 1{\cdot}6 \times 10^{-3}\ C$$

Charge on capacitor C_2, $q_2 = 0$

When C_1 (after being charged) is connected to C_2, the two capacitors will form a parallel combination. If C is the capacitance of the combination, then

$$C = C_1 + C_2 = 4 \times 10^{-6} + 2 \times 10^{-6} = 6 \times 10^{-6}\ C$$

Total charge on combination,

$$q = q_1 + q_2 = 1{\cdot}6 \times 10^{-13} + 0 = 1{\cdot}6 \times 10^{-3}\ C$$

If V is common potential, then

$$V = \frac{q}{C} = \frac{1{\cdot}6 \times 10^{-3}}{6 \times 10^{-6}} = \mathbf{266{\cdot}67\ V}$$

Problem 5.24. Four capacitors C_1, C_2, C_3 and C_4 are connected to a battery of 12 V as shown in Fig. 5.48. Find the potential difference between points A and B.

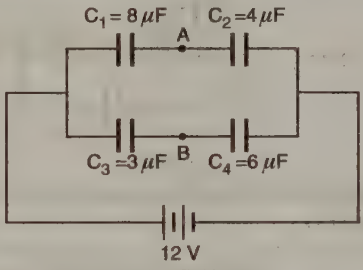


Fig. 5.48

Sol. Here, $C_1 = 8\ \mu F$; $C_2 = 4\ \mu F$; $C_3 = 3\ \mu F$ and $C_4 = 6\ \mu F$

Let V_A and V_B be potentials of points A and B respectively.

Then, potential difference across $C_1 = 12 - V_A$

potential difference across $C_2 = V_A - 0 = V_A$

potential difference across $C_3 = 12 - V_B$

and potential difference acorss $C_4 = V_B - 0 = V_B$

Since capacitors C_1 and C_2 are connected in series,

$$q_1 = q_2$$

i.e. $C_1(12 - V_A) = C_2 V_A$ or $8(12 - V_A) = 4 V_A$

or $V_A = 8$ V

Also, as capacitors C_3 and C_4 are in series,

$$q_3 = q_4$$

i.e. $C_3(12 - V_B) = C_4 V_B$ or $3(12 - V_B) = 6 V_B$

or $V_B = 4$ V

Therefore, potential difference between points A and B,

$$V_A - V_B = 8 - 4 = 4\ V$$

Type F. On Energy stored in a capacitor

Problem 5.25. The plates of a parallel plate capacitor have an area of 90 cm² each and are separated by 2·5 mm. The capacitor is charged by connecting it to a 400 volt supply. How much electrostatic energy is stored by the capacitor ? *(Text Problem)*

Sol. Here, $A = 90\ cm^2 = 90 \times 10^{-4}\ m^2$;

$$d = 2{\cdot}5\ mm = 2{\cdot}5 \times 10^{-3}\ m \ ;\ V = 400 \text{ volt}$$

Now, $$C = \frac{\varepsilon_0 A}{d} = \frac{8{\cdot}854 \times 10^{-12} \times 90 \times 10^{-4}}{2{\cdot}5 \times 10^{-3}}$$

$$= 3{\cdot}187 \times 10^{-11}\ F$$

Now, $$W = \frac{1}{2} C V^2 = \frac{1}{2} \times 3{\cdot}187 \times 10^{-11} \times (400)^2$$

$$= \mathbf{2{\cdot}55 \times 10^{-6}\ J}$$

Problem 5.26. Fig. 5.49 gives a circuit for a camera flash. A 2000 μF capacitor is charged by a 1·5 V cell. When a flash is required, the energy stored in the capacitor is made to discharge (actuated by an electronic trigger) through the discharge tube in 0·1 millisecond giving a powerful flash. Calculate the energy stored in the capacitor and the power of the flash. *(I.S.C.E. 1997)*

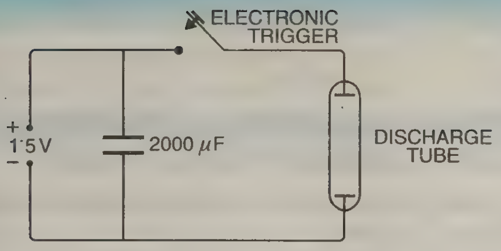


Fig. 5.49

ol. Here, $V = 1\cdot5$ volt ; $C = 2000\ \mu F = 2000 \times 10^{-6}$ F

herefore, energy stored in the capacitor,

$$U = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \times 2000 \times 10^{-6} \times (1\cdot5)^2$$

$$= \mathbf{2\cdot25 \times 10^{-3}\ J}$$

he time in which the capacitor discharges in order to ce the flash,

$$t = 0\cdot1 \text{ millisecond} = 0\cdot1 \times 10^{-3} = 10^{-4} \text{ s}$$

lence, power of the flash, $P = \frac{U}{t} = \frac{2\cdot25 \times 10^{-3}}{10^{-4}} = \mathbf{22\cdot5\ W}$

G. On Loss of energy on sharing charges

roblem 5.27. A 4 μF capacitor is charged by a 200 V y. It is then disconnected from the supply and is cted to another uncharged 2 μF capacitor. How much ostatic energy of the first capacitor is lost in the form t and electromagnetic radiation ? *(Text Problem)*

ol. Here, $C_1 = 4\ \mu F = 4 \times 10^{-6}$ F ; $V_1 = 200$ V

nergy stored in the capacitor,

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2$$

$$= 8 \times 10^{-2} \text{ J}$$

Vhen this charged capacitor is connected to another rged capacitor C_2 (= 2 μF), they will share charges, till tential differences across their plates become equal.

otal charge on the two capacitors,

$$q = C_1V_1 + C_2V_2 = 4 \times 10^{-6} \times 200 + 0 = 8 \times 10^{-4} \text{ C}$$

otal capacitance of the two capacitors,

$$C = C_1 + C_2 = 4\ \mu F + 2\ \mu F = 6\ \mu F = 6 \times 10^{-6} \text{ F}$$

herefore, common potential of the two capacitors,

$$V = \frac{q}{C} = \frac{8 \times 10^{-4}}{6 \times 10^{-6}} = \frac{400}{3} \text{ V}$$

nergy stored in the combination,

$$U_2 = \frac{1}{2} C V^2 = \frac{1}{2} \times 6 \times 10^{-6} \times \left(\frac{400}{3}\right)^2 = 5\cdot33 \times 10^{-2} \text{ J}$$

herefore, energy lost by capacitor C_1 (in the form of heat ectromagnetic radiation),

$$U_1 - U_2 = 8 \times 10^{-2} - 5\cdot33 \times 10^{-2} = \mathbf{2\cdot67 \times 10^{-2}\ J}$$

H. On Effect of dielectric

roblem 5.28. A parallel plate capacitor having plate ation of 3 mm possesses a capacitance of 17·7 pF. The tor is connected to a 100 V supply. Explain what would n, if a 3 mm thick mica sheet of dielectric constant 6 inserted between the plates (*a*) while the voltage y remains connected, (*b*) after the supply was nected. *(Text Problem)*

Sol. Here, dielectric constant of mica sheet, K = 6 ; capacitance of the capacitor, C = 17·7 pF

(a) When the voltage supply remains connected. The capacitance of the capacitor will become K times.

Therefore, $C' = K\,C = 6 \times 17\cdot7 = \mathbf{106\cdot2\ pF}$

The potential difference across the two plates of the capacitor will remain equal to supply voltage *i.e.* **100 V.**

The charge on capacitor,

$$q' = C'\,V = 106\cdot2 \times 10^{-12} \times 100$$

$$= \mathbf{1\cdot06 \times 10^{-8}\ C}$$

(b) After the voltage supply is disconnected. As calculated above, the capacitance of the capacitor,

$$C' = \mathbf{106\cdot2\ pF}$$

The potential difference will decrease on introducing mica sheet by a factor K, so that

$$V' = \frac{100}{6} = \mathbf{16\cdot67\ V}$$

The charge on the capacitor,

$$q' = C'V' = 106\cdot2 \times 10^{-12} \times 16\cdot67 = \mathbf{1\cdot77 \times 10^{-9}\ C}$$

Problem 5.29. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength of about 10^7 V m^{-1}. (Dielectric strength is the maximum electric field, a material can tolerate without breakdown *i.e.* without starting to conduct electricity through partial ionization). For safety, we should like the field never to exceed say 10% of the dielectric strength. What minimum area of the plate is required to have a capacitance of 50 pF ?

(P.S.S.C.E. 1995)

Sol. Here, K = 3 ; $\varepsilon_0 = 8\cdot854 \times 10^{-12}$ C^2 N^{-1} m^{-2}, dielectric strength = 10^7 V m^{-1} ; C = 50 pF = 50×10^{-12} F

Since electric field should not exceed 10% of the dielectric strength, we have

$$E = \frac{10^7 \times 10}{100} = 10^6 \text{ V m}^{-1},$$

Voltage rating = 1 kV = 10^3 V

$$\therefore \quad q = C \times \text{voltage rating} = 50 \times 10^{-12} \times 10^3$$

$$= 50 \times 10^{-9} \text{ C}$$

Now, $\quad C = \frac{\varepsilon_0 K A}{d} \quad$ or $\quad \frac{q}{V} = \frac{\varepsilon_0 K A}{d}$

or $\quad q = \varepsilon_0 K A \left(\frac{V}{d}\right) = \varepsilon_0 K A E$

Therefore, required area of the plates of the capacitor,

$$A = \frac{q}{\varepsilon_0 K E} = \frac{50 \times 10^{-9}}{8\cdot854 \times 10^{-12} \times 3 \times 10^6}$$

$$= 18\cdot8 \times 10^{-4} \text{ m}^2 = \mathbf{18\cdot8\ cm^2}$$

Type I. On Van de Graaff generator

Problem 5.30. In a Van de Graaff type generator, a spherical metal shell is to be a 15 × 10^5 V electrode. The dielectric strength of the gas surrounding the electrode is 5 × 10^7 V m^{-1}. What is the minimum radius of the spherical shell required ? *(Text Problem)*

Sol. The electric field and potential due to a charged shell of radius *r* are given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad ...(i)$$

and $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$r = \frac{V}{E}$$

Here, $V = 15 \times 10^5$ V ; $E = 5 \times 10^7$ V m^{-1}

$$\therefore \quad r = \frac{15 \times 10^5}{5 \times 10^7} = 3 \times 10^{-2} \text{ m} = \mathbf{3 \text{ cm}}$$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 5.31. A plane capacitor is placed in a glass vessel and connected to a storage battery of e.m.f. 12 V as shown in Fig. 5.50. The area of the capacitor plates is 100 cm^2 and the distance between the plates is 1 mm. The switch S is closed. Determine the magnitude of the charge on the plate of the capacitor in the following cases :

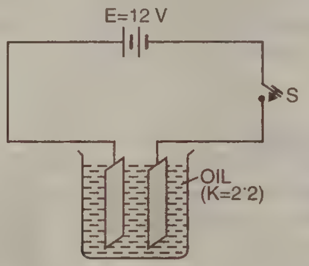


Fig. 5.50

(*a*) Switch S is opened and then the vessel is filled to the top with an oil of permittivity 2·2.

(*b*) The vessel is first filled with oil and then switch S is opened.

Also indicate as to how the intensity of electric field will change in the capacitor in these two cases, when the oil is poured in.

Sol. Here, $A = 100 \text{ cm}^2 = 100 \times 10^{-4} = 10^{-2} \text{ m}^2$; $K = 2{\cdot}2$; $d = 1 \text{ mm} = 10^{-3}$ m ; e.m.f. of battery, E = 12 V

Now, capacitance of the capacitor,

$$C = \frac{\varepsilon_0 A}{d} = \frac{8{\cdot}854 \times 10^{-12} \times 10^{-2}}{10^{-3}} = 8{\cdot}854 \times 10^{-11} \text{ F}$$

Therefore, charge on capacitor,

$$q = CE = 8{\cdot}854 \times 10^{-11} \times 12 = 1{\cdot}06 \times 10^{-19} \text{ C}$$

(*a*) When the switch is opened (battery disconnected) and the vessel is filled with oil, the charge on the plate of the capacitor will not change *i.e.* will remain **$1{\cdot}06 \times 10^{-9}$ C.**

The capacitance of the capacitor will become K (= 2·2) times *i.e.* **K C.**

It will result in the decrease of potential difference between the plates by a factor K. As a result, the intensity of electric field will also **decrease** by a factor K (= 2·2).

(*b*) When the oil is poured in the vessel with the switch closed (battery remains connected), the capacitance of the capacitor becomes K (= 2·2) times *i.e.* **K C** and the potential difference remains equal to e.m.f. of the battery *i.e.* E (= 12 V). Therefore, charge on the capacitor,

$$q' = C'E = (KC)E = 2{\cdot}2 \times 8{\cdot}854 \times 10^{-11} \times 12$$
$$= \mathbf{2{\cdot}34 \times 10^{-9} \text{ C}}$$

Since the potential difference between the plates remains unchanged, the intensity of electric field will also remain unchanged.

Problem 5.32. You are given three capacitors each of capacitance 9 μF. In how many ways can they be combined ? What will be the effective capacitance in each case ? *(Karnataka, 1988 S)*

Sol. Let the capacitance of each of the three capacitors C. Then, C = 9 μF

The three capacitors of equal capacitances can connected between two points A and B in the four ways shown in Fig. 5.51 :

Arrangement in Fig. 5.51 (a) : All the three capacitors connected in parallel. Therefore, effective capacitance,

$$C_1 = C + C + C = 3\,C = 3 \times 9 = \mathbf{27\ \mu F}$$

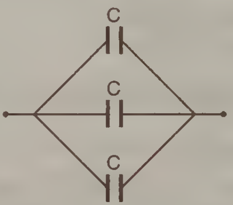


Fig. 5.51 (*a*)

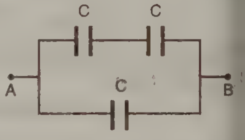


Fig. 5.51 (*b*)

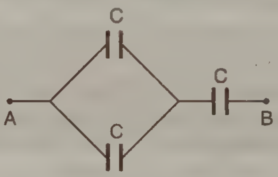


Fig. 5.51 (*c*)

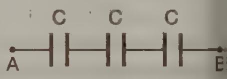


Fig. 5.51 (*d*)

Arrangement in Fig. 5.51 (b) : A series combination of capacitors have been put in parallel with the third capac If C′ is capacitance of the two capacitors in series, then

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \quad \text{or} \quad C' = \frac{C}{2} = \frac{9}{2} = 4{\cdot}5$$

Therefore, effective capacitance of the combinatio given by

$$C_2 = C' + C = 4{\cdot}5 + 9 = \mathbf{13{\cdot}5\ \mu F}$$

Arrangement in Fig. 5.51 (c) : A parallel combinatio two capacitors has been put in series with the third capac If C″ is capacitance of the two capacitors in parallel, the

$$C'' = C + C = 9 + 9 = 18\ \mu F$$

Therefore, effective capacitance of the combinatio given by

$$\frac{1}{C_3} = \frac{1}{C''} + \frac{1}{C} = \frac{1}{18} + \frac{1}{9} = \frac{3}{18} \quad \text{or} \quad C_2 = \mathbf{6\ \mu F}$$

Arrangement in Fig. 5.51 (d) : All the three capacitors in series. The effective capacitance is given by

$$\frac{1}{C_4} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} \quad \text{or} \quad C_4 = \frac{C}{3} = \frac{9}{3} =$$

Problem 5.33. There are infinite number of capacit each of capacitance 1 μF. They are connected in ro such that number of capacitors in the first row, second r third row, fourth row, ... are respectively 1, 2, 4, 8 ..... , rows of these capacitors are then connected between po X and Y [Fig. 5.52]. Determine the equivalent capacitanc the network between points X and Y.

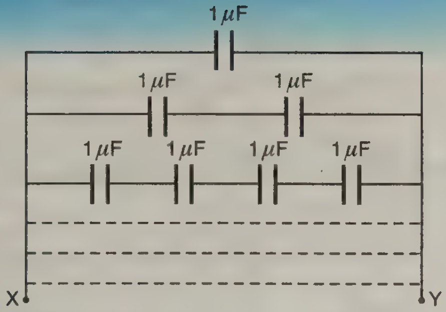


Fig. 5.52

Sol. Let C_1, C_2, C_3, C_4 ..... be capacitances of the capacitors tained in first row, second row, third row, fourth row, ... pectively. Since the rows of capacitances are connected in allel between points X and Y, the equivalent capacitance he network,

$$C = C_1 + C_2 + C_3 + C_4 + \text{.....} \qquad \text{...}(i)$$

Now, $C_1 = 1\,\mu F$; $C_2 = \frac{1 \times 1}{1+1} = \frac{1}{2}\,\mu F$;

$$\frac{1}{C_3} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 4 \quad \text{or} \quad C_3 = \frac{1}{4}\,\mu F$$

Similarly, $C_4 = \frac{1}{8}\,\mu F$; .....

Substituting for C_1, C_2, C_3, C_4, ...in equation (*i*), we get

$$C = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \text{.....}$$

It is a geometric progression, whose first term $a = 1$ and nmon ratio $r = 1/2$.

The sum of infinite terms of a geometric progression is en by

$$S_\infty = \frac{a}{1-r}$$

$$\therefore \quad C = \frac{1}{1 - 1/2} = 2\,\mu F$$

Problem 5.34. Obtain the equivalent capacitance of the following network shown in Fig. 5.53. Given that = C_4 = 100 pF and $C_2 = C_3$ = 200 pF. For a 300 V supply, ermine the charge and voltage across each capacitor. (*Text Problem*)

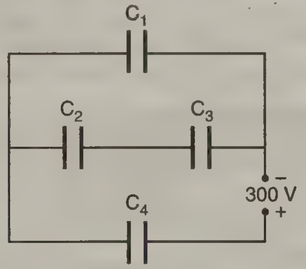


Fig. 5.53

Sol. The given network can be redrawn as shown in Fig. 4.

Here, $C_1 = C_4 = 100$ pF ; $C_2 = C_3 = 200$ pF

Let C_{23} be the capacitance of the series combination of and C_3. Then,

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{200} + \frac{1}{200} = \frac{1}{100}$$

or $\quad C_{23} = 100$ pF

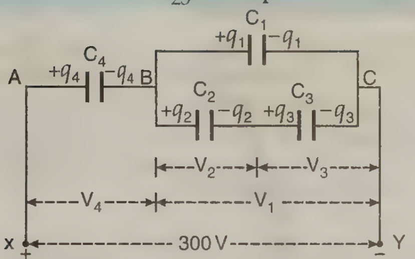


Fig. 5.54

Let C′ be the capacitance of the parallel combination of C_1 and C_{23}. Then,

$$C' = C_1 + C_{23} = 100 + 100 = 200 \text{ pF}$$

Let C be the capacitance of the series combination of C_4 and C′. Then,

$$\frac{1}{C} = \frac{1}{C_4} + \frac{1}{C'} = \frac{1}{100} + \frac{1}{200} = \frac{3}{200}$$

$$\text{or} \quad C = \frac{200}{3} \text{ pF} = \mathbf{66{\cdot}67 \text{ pF}}$$

Let q_4 be charge on the capacitor C_4. As C′ is in series with C_4, charge on C′ will also be equal to q_4.

$$\therefore \quad \text{P.D. across } C_4 = \frac{q_4}{C_4} = \frac{q_4}{100 \times 10^{-12}} \text{ V}$$

$$\text{and P.D. across } C' = \frac{q_4}{C'} = \frac{q_4}{200 \times 10^{-12}} \text{ V}$$

Now, P.D. across C_4 + P.D. across C′ = 300 V

$$\therefore \quad \frac{q_4}{100 \times 10^{-12}} + \frac{q_4}{200 \times 10^{-12}} = 300$$

$$\text{or} \quad \frac{3q_4}{200 \times 10^{-12}} = 300$$

$$\text{or} \quad q_4 = \frac{300 \times 200 \times 10^{-12}}{3} = \mathbf{2 \times 10^{-8}\ C}$$

$$\therefore \text{ P.D. across } C_4,\ V_4 = \frac{q_4}{C_4} = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = \mathbf{200\ V}$$

As C_1 and C_{23} are connected in parallel, potential difference across either of them is same.

$$\therefore \quad V_1 = \text{P.D. across } C_1 - \text{P.D. across } C_{23}$$
$$= 300 - 200 = \mathbf{100\ V}$$

Now, $q_1 = C_1 V_1 = 100 \times 10^{-12} \times 100 = \mathbf{10^{-8} C}$

As C_2 and C_3 have equal capacitances,

$$V_2 = V_3 = \frac{100}{2} = \mathbf{50\ V}$$

and $\quad q_2 = q_3 = 50 \times 200 \times 10^{-12} = \mathbf{10^{-8}\ C}$

Alternative method. Applying the principle of conservation of charge to the conductors meeting at point B, we have

$$q_1 + q_2 - q_4 = 0 \qquad \text{...}(i)$$

Also, applying the principle of conservation of charge to the conductors joining the negative plate of C_2 and the positive plate of C_3, we have

$$-q_2 + q_3 = 0$$

$$\text{or} \quad q_3 = q_2 \qquad \text{...}(ii)$$

The potential difference across the terminals A and B is same across the paths $XABC_1CY$ or $XABC_2C_3CY$ and is equal to 300 V.

Along path $XABC_1CY$:

$$\frac{q_4}{C_4}+\frac{q_1}{C_1}=300\text{ V} \quad\text{or}\quad \frac{q_4}{100\times10^{-12}}+\frac{q_1}{100\times10^{-12}}=300$$

or $\quad q_4+q_1=3\times10^{-8}\text{ C}$ ...(*iii*)

Along path $XABC_2C_3CY$:

$$\frac{q_4}{C_4}+\frac{q_2}{C_2}+\frac{q_3}{C_3}=300\text{ V}$$

or $$\frac{q_4}{100\times10^{-12}}+\frac{q_2}{200\times10^{-12}}+\frac{q_3}{200\times10^{-12}}=300$$

or $\quad 2\,q_4+q_2+q_3=6\times10^{-8}$ ...(*iv*)

From equations (*ii*) and (*iv*), we have

$$2\,q_4+q_2+q_2=6\times10^{-8}$$

or $\quad q_2+q_4=3\times10^{-8}$ ...(*v*)

Subtracting equation (*i*) from (*iii*), we have

$$q_4+q_1-(q_1+q_2-q_4)=3\times10^{-8}$$

or $\quad 2\,q_4-q_2=3\times10^{-8}$ ...(*vi*)

Adding equations (*v*) and (*vi*), we have

$$q_2+q_4+2\,q_4-q_2=3\times10^{-8}+3\times10^{-8}$$

or $\quad 3\,q_4=6\times10^{-8}$

or $\quad \mathbf{q_4=2\times10^{-8}}$

Substituting for q_4 in equation (*iii*), we have

$$q_1=3\times10^{-8}-q_4=3\times10^{-8}-2\times10^{-8}=\mathbf{10^{-8}\text{ C}}$$

Substituting for q_1 and q_4 in equation (*i*), we have

$$q_2=q_4-q_2=2\times10^{-8}-10^{-8}=\mathbf{10^{-8}\text{ C}}$$

From equation (*ii*), $q_3=q_2=\mathbf{10^{-8}\text{ C}}$

Now, $$V_1=\frac{q_1}{C_1}=\frac{10^{-8}}{100\times10^{-12}}=\mathbf{100\text{ V}},$$

$$V_2=\frac{q_2}{C_2}=\frac{10^{-8}}{200\times10^{-12}}=\mathbf{50\text{ V}},$$

$$V_3=\frac{q_3}{C_3}=\frac{10^{-8}}{200\times10^{-12}}=\mathbf{50\text{ V}}$$

and $$V_4=\frac{q_4}{C_4}=\frac{2\times10^{-8}}{100\times10^{-12}}=\mathbf{200\text{ V}}$$

If C is equivalent capacitance of the network, then C is equal to charge on the either terminal divided by the potential difference between the terminals. Hence,

$$C=\frac{q_4}{V} \qquad \left(\text{or take } C=\frac{q_1+q_3}{V}\right)$$

$$\therefore \quad C=\frac{2\times10^{-8}}{300}=\frac{200}{3}\times10^{-12}\text{ F}=\mathbf{\frac{200}{3}\text{ pF}}$$

Note. Students are advised to solve this numerical by both the methods so as to acquaint themselves with some new principles in electrostatics. It will help them to solve those problems, which otherwise cannot be solved by conventional methods.

Problem 5.35. Find the effective capacitance between points X and Y of the network of capacitors shown in Fig. 5.55. Given that $C_1=1\,\mu F$, $C_2=2\,\mu F$, $C_3=2\,\mu F$, $C_4=1\,\mu F$ and $C_5=1\,\mu F$. If a potential difference of 10^6 V is applied across X and Y, then calculate (*a*) charges on various capacitors, (*b*) voltages across the various capacitors and (*c*) the equivalent capacitance of the network.

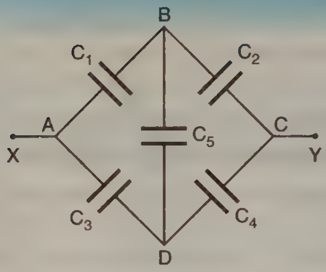


Fig. 5.55

Sol. Let q_1, q_2, q_3, q_4 and q_5 be charges on the capacitor C_1, C_2, C_3, C_4 and C_5 respectively, when a potential differenc of 10^6 V is applied across X and Y. Fig. 5.56 shows th distribution of charges on the various capacitors.

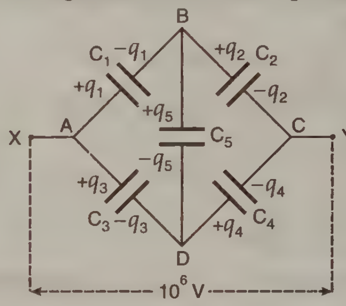


Fig. 5.56

(*a*) Applying law of conservation of charges for th conductors meeting at points B and D, we have

$-q_1+q_2+q_5=0$ ...(*i*

$-q_3+q_4-q_5=0$ ...(*i*

Now, the potential difference across the terminals X an Y is the same across paths XABCY, XADCY, XABDCY an XADBCY.

Across path XABCY:

$$\frac{q_1}{C_2}+\frac{q_2}{C_2}=10^6 \quad\text{or}\quad \frac{q_1}{10^{-6}}+\frac{q_2}{2\times10^{-6}}=10^6$$

or $\quad 2\,q_1+q_2=2$ ...(*ii*

Across path XADCY:

$$\frac{q_3}{C_3}+\frac{q_4}{C_4}=10^6 \quad\text{or}\quad \frac{q_3}{2\times10^{-6}}+\frac{q_4}{10^{-6}}=10^6$$

or $\quad q_3+2\,q_2=2$ ...(*i*

Across path XABDCY:

$$\frac{q_1}{C_1}+\frac{q_5}{C_5}+\frac{q_4}{C_4}=10^6 \text{ or } \frac{q_1}{10^{-6}}+\frac{q_5}{10^{-6}}+\frac{q_4}{10^{-6}}=10^6$$

or $\quad q_1+q_4+q_5=1$ ...(*v*

Across path XADBCY:

$$\frac{q_3}{C_1}+\left(-\frac{q_5}{C_5}\right)+\frac{q_2}{C_2}=10^6\text{ V}$$

or $$\frac{q_3}{2\times10^{-6}}-\frac{q_5}{10^{-6}}+\frac{q_2}{2\times10^{-6}}=10^6$$

or $\quad q_2+q_3-2\,q_5=2$ ...(*v*

From equation (*iii*), we have

$q_2=2-2\,q_1$ ...(*vi*

From equation (*i*), we have

$q_5=q_1-q_2=q_1-(2-2\,q_1)=3\,q_1-2$ ...(*vii*

From equation (*iv*), we have

$$q_3 = 2 - 2\,q_4 \qquad ...(ix)$$

From equations (*ii*) and (*ix*), we have

$$-2\,(2 - 2\,q_4) + q_4 - q_5 = 0$$

or

$$3\,q_4 = q_5 + 2$$

From equation (*viii*), substituting the value of q_5, we have

$$3\,q_4 = 3\,q_1 - 2 + 2 = 3\,q_1$$

or

$$q_4 = q_1$$

Therefore, from equation (*ix*), we have

$$q_3 = 2 - 2\,q_1 \qquad ...(x)$$

Substituting for q_4 and q_5 in equation (*v*), we have

$$q_1 + q_1 + (3\,q_1 - 2) = 1 \quad \text{or} \quad 5\,q_1 = 3$$

or

$$q_1 = \mathbf{0{\cdot}6\ C}$$

From equation (*vii*), we have

$$q_2 = 2 - 2\,q_1 = 2 - 2 \times 0{\cdot}6 = \mathbf{0{\cdot}8\ C}$$

From equation (*x*), we have

$$q_3 = 2 - 2\,q_1 = 2 - 2 \times 0{\cdot}6 = \mathbf{0{\cdot}8\ C}$$

Further, $q_4 = q_1 = \mathbf{0{\cdot}6\ C}$

From equation (*viii*), we have

$$q_5 = 3 \times 0{\cdot}6 - 2 = \mathbf{-0{\cdot}2\ C}$$

(*b*) Now,

$$V_1 = \frac{q_1}{C_1} = \frac{0{\cdot}6}{10^{-6}} = \mathbf{0{\cdot}6 \times 10^6\ V},$$

$$V_2 = \frac{q_2}{C_2} = \frac{0{\cdot}8}{2 \times 10^{-6}} = \mathbf{0{\cdot}4 \times 10^6\ V},$$

$$V_3 = \frac{q_3}{C_3} = \frac{0{\cdot}8}{2 \times 10^{-6}} = \mathbf{0{\cdot}4 \times 10^6\ V},$$

$$V_4 = \frac{q_4}{C_4} = \frac{0{\cdot}6}{10^{-6}} = \mathbf{0{\cdot}6 \times 10^6\ V}$$

and

$$V_5 = \frac{q_5}{C_5} = \frac{-0{\cdot}2}{10^{-6}} = \mathbf{-0{\cdot}2 \times 10^6\ V}$$

(*c*) If C is equivalent capacitance of the given network, en C is equal to charge on the either terminal divided by e potential difference between the terminals. Hence,

$$C = \frac{q_1 + q_3}{V} \qquad \left(\text{or take } C = \frac{q_2 + q_4}{V}\right)$$

$$\therefore \qquad C = \frac{0{\cdot}6 \times 0{\cdot}8}{10^6} = 1{\cdot}4 \times 10^{-6}\ \text{F} = \mathbf{1{\cdot}4\ \mu F}$$

Problem 5.36. The capacitance of a parallel plate capacitor with plate area A and separation *d* is C. The pace between the plates is filled with two wedges of ielectric constants K_1 and K_2 respectively [Fig. 5.57]. Find e capacitance of the resulting capacitor.

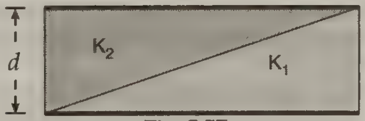


Fig. 5.57

Sol. Let each plate of the capacitor be of length *l*, eadth *b* and area A (= *l* × *b*). Fig. 5.58 shows the elementary portion of the capacitor between two vertical planes at distances *x* and *d* + *dx* from the left end of the plates. This elementary portion forms a series combination of two elementary capacitors of capacitances, say dC_1 and dC_2 :

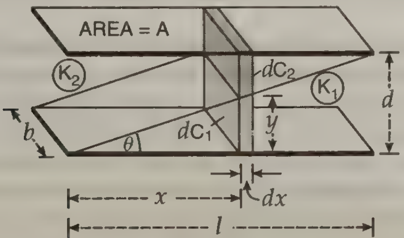


Fig. 5.58

(*i*) The two plates of the capacitor dC_1 are of area *b dx* and filled with a dielectric medium of dielectric constant K_1 and thickness *y*. Therefore,

$$dC_1 = \frac{\varepsilon_0\, K_1\, (b\, dx)}{y}$$

(*ii*) The two plates of the capacitor dC_2 are of area *b dx* and filled with a dielectric medium of dielectric constant K_2 and thickness (*d* − *y*). Therefore,

$$dC_2 = \frac{\varepsilon_0\, K_2\, (b\, dx)}{(d - y)}$$

Since the two elementary capacitors are in series, their resultant capacitance is given by

$$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2} = \frac{y}{\varepsilon_0\, K_1\, (b\, dx)} + \frac{(d - y)}{\varepsilon_0\, K_2\, (b\, dx)}$$

$$= \frac{1}{\varepsilon_0\, (b\, dx)} \left[\frac{y}{K_1} + \frac{d - y}{K_2}\right]$$

$$= \frac{1}{\varepsilon_0\, (b\, dx)} \left[\frac{K_2\, y + K_1\, (d - y)}{K_1\, K_2}\right]$$

$$= \frac{K_1\, d + (K_2 - K_1)\, y}{\varepsilon_0\, K_1\, K_2\, b\, dx}$$

or

$$dC = \frac{\varepsilon_0\, K_1\, K_2\, b}{K_1\, d + (K_2 - K_1)\, y}\, dx$$

Therefore, capacitance of the whole capacitor is given by

$$C = \int_0^l dC = \int_0^l \frac{\varepsilon_0\, K_1\, K_2\, b}{K_1\, d + (K_2 - K_1)\, y}\, dx \qquad ...(i)$$

Now, *y* is function of *x* and is related to *x* as below :

$$\tan\theta = \frac{y}{x} = \frac{d}{l} \qquad \text{or} \qquad y = \frac{d}{l}\, x$$

Substituting for *y* in equation (*i*), we have

$$C = \varepsilon_0\, K_1\, K_2\, b \int_0^l \frac{1}{K_1\, d + (K_2 - K_1)\, \frac{d}{l}\, x}\, dx$$

$$= \frac{\varepsilon_0\, K_1\, K_2\, l \times b}{d} \int_0^l \frac{dx}{K_1\, l + (K_2 - K_1)\, x}$$

$$= \frac{\varepsilon_0\, K_1\, K_2\, A}{d} \int_0^l \frac{dx}{K_1\, l + (K_2 - K_1)\, x}$$

$$= \frac{\varepsilon_0\, K_1\, K_2\, A}{d} \left| \frac{\log_e\, [K_1\, l + (K_2 - K_1)\, x}{K_2 - K_1} \right|_0^l$$

$$= \frac{\varepsilon_0\, K_1\, K_2\, A}{(K_2 - K_1)\, d} \left| \log_e\, [K_1\, l + (K_2 - K_1)\, x \right|_0^l$$

$$= \frac{\varepsilon_0\, K_1\, K_2\, A}{(K_2 - K_1)\, d} \Big| \log_e\, [K_1\, l + (K_2 - K_1)\, l] - \log_e\, [K_1\, l + (K_2 - K_1)\, 0] \Big|$$

$$= \frac{\varepsilon_0 K_1 K_2 A}{(K_2 - K_1) d} \left| \log_e K_2 l - \log_e K_1 l \right| \qquad = \frac{\varepsilon_0 K_1 K_2 A}{(K_2 - K_1) d} \log_e \frac{K_2}{K_1}$$

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. Is there any electric field inside a charged conductor ?

Ans. Inside a charged conductor, electric field is always zero.

Q. 5.02. What is meant by capacitance ? Give its SI unit. *(I.S.C.E. 1998)*

Or

Define capacitance. Give its SI unit.
(C.B.S.E. 1999 ; H.P.S.S.C.E. 1998 ; P.S.S.C.E. 1993)

Ans. *The capacitance of a conductor may be defined as the ratio of the electric charge on it to its electric potential due to the charge.*

SI unit of capacitance is **farad.**

Q. 5.03. What is one picofarad ?

Ans. 1 picofarad (pF) = 10^{-12} F

Q. 5.04. Name the physical quantity, whose SI unit is coulomb volt^{-1}. *(C.B.S.E. 1998)*

Ans. It is electrical capacitance.

Q. 5.05. What is a capacitor ? *(P.S.S.C.E. 1996 S)*

Ans. It is an arrangement of increasing the capacity of a conductor so as to store a large quantity of charge.

Q. 5.06. What meaning would you give to the capacitance of a single conductor ?

Ans. A single conductor also possesses capacitance. It is a capacitor, whose one plate is at infinity.

Q. 5.07. Can we give any desired amount of charge to a capacitor ?

Ans. No, the maximum charge that can be given to a capacitor is limited by the dielectric strength of the medium between the two plates of the capacitor.

Q. 5.08. On what factors does the capacity of a parallel plate capacitor with dielectric depend ? *(P.S.S.C.E. 1993)*

Ans. The capacitance of a parallel plate capacitor depends upon the area of its plates, separation between the plates and the dielectric constant of the dielectric between the plates.

Q. 5.09. Why is a parallel plate capacitor named so ? *(H.S.S.C.E. 1998)*

Ans. The plates of a parallel plate capacitor are plane and are placed parallel to each other. For this reason, it is called a parallel plate capacitor.

Q. 5.10. Three capacitors each of capacitance 2 μF are connected in series. Find the resultant capacitance in farad. *(H.S.S.C.E. 1996)*

Ans. Here, $C_1 = C_2 = C_3 = 2\ \mu F$

If C is capacitance, when the three capacitors are connected in series, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

or $$C = \frac{2}{3}\ \mu F = 0{\cdot}067 \times 10^{-6}\ F$$

Q. 5.11. A capacitor of capacitance 20 μF is charged to a potential of 300 V. Calculate the energy stored in the capacitor. *(C.B.S.E. 1995)*

Ans. Here, $C = 20\ \mu F = 20 \times 10^{-6}$ F ; V = 300 volt

Energy stored,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \times 20 \times 10^{-6} \times (300)^2 = \mathbf{0{\cdot}9\ J}$$

Q. 5.12. How much work must be done to charge a μF capacitor, when the potential difference between th plate is 500 V ? *(H.S.S.C.E. 200*

Ans. Here, $C = 24\ \mu F = 24 \times 10^{-6}$ F and V = 500 V

$$\text{Therefore, work done} = \frac{1}{2} C V^2$$
$$= \frac{1}{2} \times 24 \times 10^{-6} \times (500)^2 = \mathbf{3\ J}$$

Q. 5.13. A capacitor is charged through a potenti difference of 200 V, when 0·1 C charge is stored in it. Ho much energy will it release, when it is discharged ? *(I.S.C.E. 199*

Ans. Here, V = 200 volt ; q = 0·1 C

$$\text{The energy stored, } U = \frac{1}{2} q V \times \frac{1}{2} \times 0{\cdot}1 \times 200 = 10\ J$$

On being discharged, the capacitor will release the san amount of energy *i.e.* **10 J.**

Q. 5.14. What should be the capacitance of a capacit capable of storing one joule of energy, when used with 100 V d.c. supply ? *(I.S.C.E. 199*

Ans. Here, U = 1 joule ; V = 100 volt

If C is capacitance of the capacitor, then

$$U = \frac{1}{2} C V^2$$

or $$C = \frac{2U}{V^2} = \frac{2 \times 1}{(100)^2} = \mathbf{2 \times 10^{-4}\ F}$$

Q. 5.15. In what form is the energy stored in a charg capacitor ? *(P.S.S.C.E. 1997*

Ans. In a charged capacitor energy is stored in the for of electric field between the two plates *i.e.* in the form electric energy.

Q. 5.16. Distinguish between polar and non-pol dielectrics. *(H.S.S.C.E. 199*

Ans. A dielectric, the molecules of which possess electr dipole moment even when electric field is not applied is call polar dielectric.

On the other hand, a dielectric, whose molecules do n possess permanent dipole moment is called non-polar.

Q. 5.17. What is a dielectric ? *(P.S.S.C.E. 200*

Ans. A material that does not conduct electricity but applying electric field, induced charges are produced on faces.

Q. 5.18. Guess a possible reason, why water has a mu greater dielectric constant (= 80) than say mica (= 6).

Ans. It is because, water molecules have permane dipole moment, while the molecules of mica do not have.

Q. 5.19. The dielectric constant of a conductor can taken to be infinitely large, infinitely small or optimu Which of the three alternatives is correct ?

Ans. When a conductor is placed inside an electric field, eld inside the conductor becomes zero. The dielectric ant, which is ratio of the strength of applied electric field e reduced value of electric field will be infinite for the uctor.

Q. 5.20. Write down the relation between dielectric tant and electric susceptibility.

Ans. The dielectric constant (K) and susceptibility of a ctric are related to each other by the relation

$$K = 1 + \chi$$

Q. 5.21. Define dielectric strength of the medium ?

(*H.S.S.C.E. 2001; H.P.S.S.C.E. 1999 S*)

Ans. The maximum value of electric field that a ectric can withstand, without the break down of the ctric, gives the measure of the dielectric strength.

Q. 5.22. Define dielectric constant in terms of the citance of a capacitor. (*C.B.S.E. 1995*)

Ans. *The dielectric constant of a dielectric medium is defined e ratio of the capacitance of the capacitor, when the dielectric he space between the plates to the capacitance, when there is um between the two plates of the capacitor.*

Q. 5.23. On inserting a dielectric between the plates of acitor, its capacitance is found to increase 5 times. What e relative permittivity of the dielectric ?(*I.S.C.E. 1995*)

Ans. When dielectric of relative permittivity ε_r is ted, the capacitance of the capacitor increases ε_r times. Since on inserting dielectric, capacitance increases 5 times, the relative permittivity of dielectric is 5.

Q. 5.24. In a parallel plate capacitor, the capacitance increases from 4 μF to 80 μF on introducing a dielectric medium between the plates. What is the dielectric constant of the medium ? (*C.B.S.E. 1996*)

Ans. Here, capacitance without dielectric, $C = 4\ \mu F$

and capacitance with dielectric, $C' = 80\ \mu F$

If K is dielectric constant of the dielectric medium, then

$$K = \frac{C'}{C} = \frac{80}{4} = 20$$

Q. 5.25. An air capacitor is given a charge of 2 μC raising its potential to 200 V. If on inserting a dielectric medium, its potential falls to 50 V, what is the dielectric constant of the medium ?

Ans. When dielectric is introduced, the potential between the plates of the capacitor decreases by a factor equal to dielectric constant. Therefore,

$$K = \frac{V}{V'} = \frac{200}{50} = 4$$

Q. 5.26. Why the Van de Graaff generator is enclosed inside an earth connected steel tank filled with air under pressure ?

Ans. It prevents the leakage of charge due to ionisation. The reason is that in air under pressure, as soon as free ions are produced, they recombine to form neutral air molecules.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. A metal of radius of 1 cm can not hold a charge columb. Why ? (*P.S.S.C.E. 1995 S*)

Ans. Refer to solved problem 5.02.

Q. 5.02. Two metallic spheres of same radii, one solid the other hollow are charged to the same potential. ch of the two will hold more charge ? Give reasons.

(*H.S.S.C.E. 1997 S ; I.I.T. 1974*)

Ans. Both the spheres will hold the same amount of ge. It is because, the two spheres possess equal citance. The capacitance of a sphere depends only on its is. It does not matter, whether a sphere is hollow or solid.

Q. 5.03. Why is not possible to make a spherical con- or of capacity one farad ? Explain. (*P.S.S.C.E. 2002*)

Ans. The capacitance of a spherical conductor of radius iven by

$$C = 4\pi\varepsilon_0 r$$

The radius of spherical conductor having capacitance given by

$$r = \frac{C}{4\pi\varepsilon_0} = 1 \times 9 \times 10^9 = 9 \times 10^9 \text{ m} = 9 \times 10^6 \text{ km}$$

Since one cannot have a spherical conductor of such a adius, it is not possible to make a spherical conductor of citance 1 F.

Q. 5.04. n small drops of same size are charged to V volt . They coalsece to form a bigger drop. Calculate the ntial of the bigger drop. (*P.S.S.C.E. 2001*)

Ans. Let r be the radius of each small drop. Then,

capacitance of each small drop, $C = 4\pi\varepsilon_0 r$

charge on each small drop, $q = CV = 4\pi\varepsilon_0 r \times V$

Total charge on n small drops, $Q = nq = n \times 4\pi\varepsilon_0 r \times V$

When n small drops coalesce, let R be the radius of the bigger drop formed. Since the volume of the bigger drop must be equal to that of n small drops,

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \quad \text{or} \quad R = n^{1/3} r$$

Therefore, capacitance of the bigger drop

$$= 4\pi\varepsilon_0 R = 4\pi\varepsilon_0 \times n^{1/3} r$$

Since the bigger drop will possess the same charge as that on n small drops, potential of bigger drop

$$= \frac{n \times 4\pi\varepsilon_0 r \times V}{4\pi\varepsilon_0 n^{1/3} r} = n^{2/3} V$$

Q. 5.05. An uncharged insulated conductor A is brought near a charged insulated conductor B. What happens to charge and potential of B ? (*C.B.S.E. 2001 S*)

Ans. The charge on conductor B remains the same, but its potential gets lowered.

For details, refer to section 5.05–Principle of a capacitor.

Q. 5.06. In a parallel plate capacitor, how is the capacity affected, when without changing the charge :

(*i*) the distance between the plates is doubled ;

(*ii*) area of the plate is halved.

(*Pre-degree Kerala, 1992*)

Ans. (*i*) When the distance between the plates is doubled, the capacitance of the capacitor becomes half. It is because,

$$C \propto \frac{1}{d}$$

(*ii*) When the area of the plates is halved, the capacitance of the capacitor becomes half. It is because,

$$C \propto A$$

Q. 5.07. A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1 pF = 10^{-12} F). What will be the capacitance, if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant 6 ? *(Text Question)*

Ans. Let A be the area of each plate and *d* be the distance between the two plates of the parallel plate capacitor. When air is between the two plates of the capacitor, then

$$C = \frac{\varepsilon_0 A}{d}$$

Here, $C = 8 \text{ pF} = 8 \times 10^{-12}$ F

$$\therefore \quad \frac{\varepsilon_0 A}{d} = 8 \times 10^{-12} \qquad \text{...}(i)$$

Suppose that the capacitance of the capacitor becomes C′, when distance between the plates is reduced to half *i.e.* $d' = \frac{d}{2}$ and the space between them is filled with a substance of dielectric constant K = 6. Then,

$$C' = \frac{\varepsilon_0 K A}{d'} = \frac{\varepsilon_0 \times 6 \times A}{d/2} = 12 \frac{\varepsilon_0 A}{d} \qquad \text{...}(ii)$$

From the equations (*i*) and (*ii*), we have

$$C' = 12 \times 8 \times 10^{-12} = 96 \times 10^{-12} \textbf{ F = 96 pF}$$

Q. 5.08. Two identical metal plates are given charges q_1 and $q_2 < q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, what will be the potential difference between the plates ? *(I.I.T. 1999)*

Ans. Let each of the two plates be of area A. When they are placed at a distance *d*, the capacitance of the parallel plate capacitor formed,

$$C = \frac{\varepsilon_0 A}{d}$$

If E_1 and E_2 are electric fields due to the two plates, then net electric field between the two plates of the capacitor,

$$E = E_1 - E_2 = \frac{q_1/A}{2\varepsilon_0} - \frac{q_2/A}{2\varepsilon_0} = \frac{1}{2\varepsilon_0 A}(q_1 - q_2)$$

Potential difference between the two plates of the capacitor,

$$V = E\,d = \frac{1}{2\varepsilon_0 A}(q_1 - q_2) \times d = \frac{d}{2\varepsilon_0 A}(q_1 - q_2)$$

$$= \frac{q_1 - q_2}{2C}$$

Q. 5.09. Five identical capacitors, each of capacitance C, are connected between points X and Y as shown in Fig. 5.59. If the equivalent capacitance of the combination between points X and Y is 5 μF. calculate the capacitance of each capacitor.

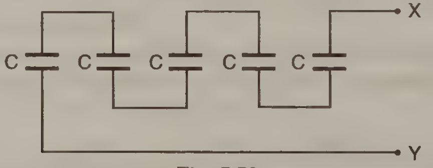


Fig. 5.59

Ans. It follows that the given arrangement of the five capacitors is simply their series arrangement.

Therefore, equivalent capacitance is given by

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{5}{C}$$

or $\quad C' = \frac{C}{5}$

Q. 5.10. What is the effective capacitance of two ductors of capacitance 3 μF and 4 μF, when connected series ; (*ii*) in parallel ? *(Pre-degree Kerala,*

Ans. Here, $\quad C_1 = 3\,\mu\text{F}\,;\, C_2 = 4\,\mu\text{F}$

(*i*) In series, $\quad \frac{1}{C_s} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} \quad$ or $\quad C_s = \frac{12}{7}\,\mu\text{F}$

(*ii*) In parallel, $C_p = 3 + 4 = 7\,\mu\text{F}$

Q. 5.11. Three capacitors of equal capacitance, connected in series, have a net capacitance of C_1 and connected in parallel, have a capacitance of C_2. What be the value of C_1/C_2 ?

(H.S.S.C.E. 2002, 1998 ; C.B.S.E. 1995 ; P.S.S.C.E.

Ans. Let C be the capacitance of each of the capacitors and C_1 and C_2 be the capacitances of their and parallel combinations respectively.

Then, $\quad \frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} \quad$ or $\quad C_1 =$

Also, $\quad C_2 = C + C + C = 3C$

$$\therefore \quad \frac{C_1}{C_2} = \frac{C/3}{3C} = \frac{1}{9}$$

Q. 5.12. Two capacitors of capacitances C_1 and C joined in series and this combination is joined in pa with a capacitor of capacitance C_3. Find the resu capacitance.

Ans. The three capacitors of capacitances C_1, C_2 ar are joined as shown in Fig. 5.60.

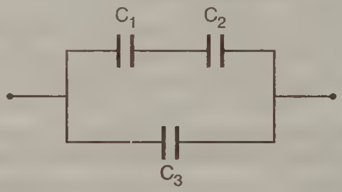


Fig. 5.60

The capacitance of series combination of C_1 and C

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C' = \frac{C_2\,C_2}{C_1 + C_2}$$

Therefore, resultant capacitance,

$$C = C' + C_3 = \frac{C_2\,C_2}{C_1 + C_2} + C_3$$

Q. 5.13. Two capacitors of capacitances 3 μF and respectively are joined in parallel and the combinati connected in series with a third capacitor of capacit 2 μF. Find the resultant capacitance. *(H.S.S.C.E.*

Ans. The three capacitors of capacitances 3 μF, 5 μ 2 μF are joined as shown in Fig. 5.61.

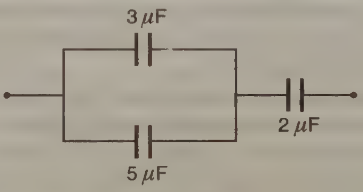


Fig. 5.61

The capacitance of parallel combination of 3 μF and 5 μF capacitors,

$$C' = 3 + 5 = 8\ \mu\text{F}$$

Therefore, resultant capacitance, $C = \dfrac{8 \times 2}{8 + 2} = 1{\cdot}6\ \mu\text{F}$

Q. 5.14. Three capacitors C_1, C_2 and C_3 are connected as shown in Fig. 5.62. Find the equivalent capacitance of the combination between points X and Y.

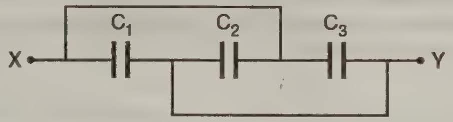


Fig. 5.62

Ans. The given arrangement of three capacitors is equivalent to their parallel combination as shown in Fig. 5.63.

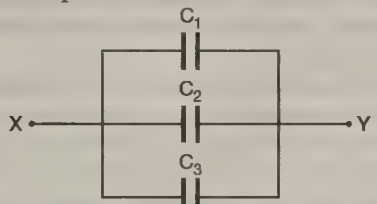


Fig. 5.63

Therefore, equivalent capacitance,

$$C = C_1 + C_2 + C_3$$

Q. 5.15. Find the equivalent capacitance of the arrangement of three capacitors shown in Fig. 5.64 between points X and Y.

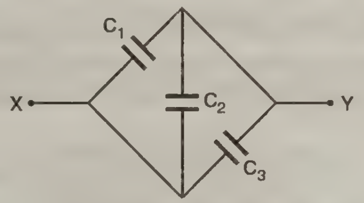


Fig. 5.64

Ans. The given arrangement of the three capacitors is equivalent to their parallel combination as shown in Fig. 5.65.

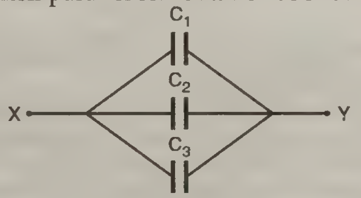


Fig. 5.65

Therefore, equivalent capacitance,

$$C = C_1 + C_2 + C_3$$

Q. 5.16. Four metallic plates, each with a surface area A are placed at a distance d apart from each other. The two other plates are connected to point X and the two inner plates to point Y as shown in Fig. 5.66. What is the equivalent capacitance of the system of plates ?

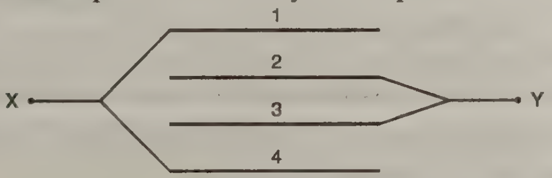


Fig. 5.66

Ans. The given arrangement of four plates is equivalent to the parallel combination of two capacitors C_1 and C_2. The capacitor C_1 is formed between plates 1 and 2, while the capacitor C_2 is formed between plates 4 and 3 [Fig. 5.67].

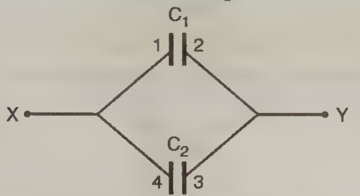


Fig. 5.67

Since area of each plate is A and separation between any two plates is d,

$$C_1 = C_2 = \frac{\varepsilon_0 A}{d}$$

Therefore, equivalent capacitance,

$$C = C_1 + C_2 = \frac{2\,\varepsilon_0 A}{d}$$

Q. 5.17. Three metallic plates, each with a surface area A, are placed at a distance d apart from each other. The two other plates are connected to each other as shown in Fig. 5.68. What is the equivalent capacitance of the given system of plates between points X and Y ?

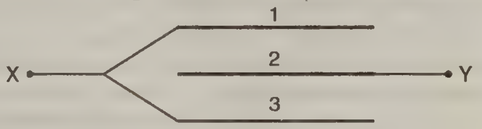


Fig. 5.68

Ans. The given arrangement of three plates is equivalent to parallel combination of two capacitors C_1 and C_2. The capacitor C_1 is formed between plates 1 and 2, while the capacitor C_2 is formed between plates 3 and 2 [Fig. 5.69].

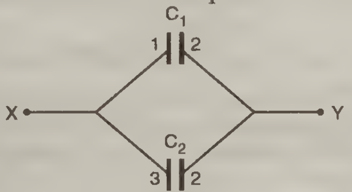


Fig. 5.69

Since area of each plate is A and separation between any two plates is d,

$$C_1 = C_2 = \frac{\varepsilon_0 A}{d}$$

Therefore, equivalent capacitance,

$$C = C_1 + C_2 = \frac{2\,\varepsilon_0 A}{d}$$

Q. 5.18. Four metallic plates, each with a surface area A, are placed at a distance d from each other. The alternate plates are connected to point X and Y as shown in Fig. 5.70. What is the equivalent capacitance of the system ?

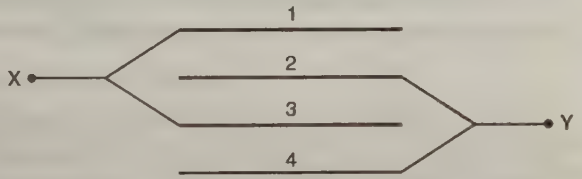


Fig. 5.70

Ans. The given arrangement of four plates is equivalent to the parallel combination of three capacitors C_1, C_2 and C_3.

The capacitor C_1 is formed between plates 1 and 2, C_2 between plates 3 and 2 and C_3 between plates 3 and 4 [Fig. 5.71].

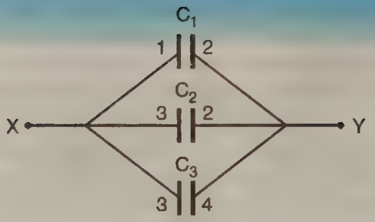


Fig. 5.71

Since area of each plate is A and separation between any two plates is d,

$$C_1 = C_2 = C_3 = \frac{\varepsilon_0 A}{d}$$

Therefore, equivalent capacitance,

$$C = C_1 + C_2 + C_3 = \frac{3\varepsilon_0 A}{d}$$

Q. 5.19. Two capacitors of capacitances 5 μF and 10 μF are charged to 16 volt and 13 volt respectively. What is the common potential, when they are connected in parallel ? (*Karnataka, 1985*)

Ans. Here, $C_1 = 5\ \mu F = 5 \times 10^{-6}$ F ;

$C_2 = 10\ \mu F = 10 \times 10^{-6}$ F ; $V_1 = 16$ volt ; $V_2 = 13$ volt

Therefore, charge on capacitor C_1,

$$q_1 = C_1 V_1 = 5 \times 10^{-6} \times 16 = 80 \times 10^{-6}\ C$$

Charge on capacitor C_2,

$$q_2 = C_2 V_2 = 10 \times 10^{-6} \times 13 = 130 \times 10^{-6}\ C$$

Let V be the common potential, when the two capacitors re connected in parallel. Then,

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{80 \times 10^{-6} + 130 \times 10^{-6}}{5 \times 10^{-6} + 10 \times 10^{-6}}$$

$$= \frac{210 \times 10^{-6}}{15 \times 10^{-6}} = \mathbf{14\ V}$$

Q. 5.20. Three capacitors of capacitances 10 μF, 20 μF and 30 μF are connected in parallel to a 100 V battery as shown in Fig. 5.72. Calculate the total energy stored in the capacitors. (*I.S.S.C.E. 1994*)

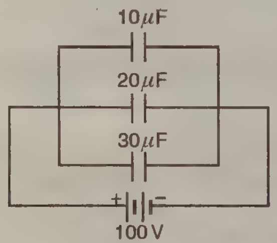


Fig. 5.72

Ans. Here, $C_1 = 10\ \mu F$; $C_2 = 20\ \mu F$; $C_3 = 30\ \mu F$

If C is their equivalent capacitance, then

$$C = C_1 + C_2 + C_3 = 10 + 20 + 30 = 60\ \mu F$$
$$= 60 \times 10^{-6}\ F$$

The total energy stored in the combination,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \times 60 \times 10^{-6} \times (100)^2 = \mathbf{0{\cdot}3\ J}$$

Q. 5.21. Capacitors P, Q and R have each a capacitance C. A battery can charge the capacitor P to a potential difference V. If after charging P, the battery is disconnected from it and the charged capacitor P is connected in following separate instances to Q and R (*i*) to Q in parallel, and (*ii*) to R in series, then what will be potential differences between the plates of P in the two instances ? (*C.B.S.E. 1997*)

Ans. When capacitor P is connected to a potential difference V, then

charge acquired by capacitor, $q = C V$

(i) When capacitor P is connected to Q in parallel : When the capacitors P and Q (each of capacitance C) are connected in parallel,

total capacitance = C + C = 2 C

total charge on the two capacitors = $q + 0 = q = C V$

Therefore, potential difference across each capacitor or capacitor P will become

$$V' = \frac{C V}{2 C} = \frac{V}{2}$$

(ii) When capacitor P is connected to R in series : When the capacitor P is connected to R in series, the circuit will not get completed and likewise sharing of charges does not take place. Therefore, potential difference across capacitor P will remain unchanged *i.e.* **V.**

Q. 5.22. How does a dielectric differ from an insulator?

Ans. Both the dielectrics and insulators cannot conduct electricity. However, in case of a dielectric, when an external field is applied ; induced charges appear on the faces of the dielectric. In other words, dielectrics have the property of transmitting electric effects without conducting.

Q. 5.23. Prove that atomic polarizability has the dimensions of volume.

Ans. The induced dipole moment of an atom is given by

$$p = \alpha \varepsilon_0 E$$

Therefore, atomic polarizability, $\alpha = \frac{p}{\varepsilon_0 E}$

Since the units of p, E and ε_0 are C m, N C^{-1} and C^2 N^{-1} m^2 respectively, the units of α are given by

$$[\alpha] = \left[\frac{p}{\varepsilon_0 E}\right] = \frac{C\,m}{C^2 N^{-1} m^{-2} \times N C^{-1}} = m^3$$

Hence, α has dimensions of volume.

Q. 5.24. What is the function of a dielectric in a capacitor ? (*P.S.S.C.E. 2002, 1999*)

Or

Explain, why presence of a dielectric increases capacitance of the capacitor. (*P.S.S.C.E. 1995*)

Ans. When a dielectric slab is introduced between the two plates of a capacitor, the electric field between the plates gets reduced due to polarisation of the dielectric. The reduced value of electric field is equivalent to a decreased value of potential difference between the plates. In order to make the potential difference again same, more charge has to be given to the capacitor. *i.e.* the capacitance of the capacitor increases.

Q. 5.25. For a given potential difference, a capacitor stores charge q, when dielectric of dielectric constant (K) is inserted between the two plates of capacitor. What is the new value of charge stored in the capacitor ? (*H.P.S.S.C.E. 2001*)

Ans. When a dielectric of dielectric constant (K) is inserted between the two plates of a capacitor, its capacitance increases K times, while the potential difference between the two plates decreases to $\frac{1}{K}$ of the original value. Therefore, new value of charge on the capacitor,

$$q' = C' \times V' = K C \times \frac{V}{K} = C V = q$$

i.e there is no effect on change on the capacitor.

Q. 5.26. Two equal dielectric slabs of dielectric stants K_1 and K_2 have been put between the plates of a acitor as shown in Fig. 5.73. If the area of each plate of capacitor is A and separation between them is *d*, find capacitance of the capacitor.

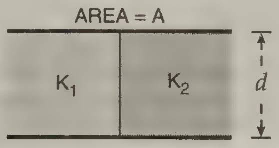


Fig. 5.73

Ans. The given arrangement is equivalent to parallel nbination of two capacitors, each of plate area A/2, plate aration *d*, such that one has medium of dielectric constant and other of dielectric constant K_2. If C_1 and C_2 are acitances of the two capacitors so formed, then

$$C_1 = \frac{\varepsilon_0 K_1 A/2}{d} = \frac{\varepsilon_0 K_1 A}{2d}$$

and $$C_2 = \frac{\varepsilon_0 K_2 A/2}{d} = \frac{\varepsilon_0 K_2 A}{2d}$$

If C is the capacitance of the capacitor, then

$$C = C_1 + C_2 = \frac{\varepsilon_0 K_1 A}{2d} + \frac{\varepsilon_0 K_2 A}{2d}$$

$$= \frac{\varepsilon_0 (K_1 + K_2) A}{2d}$$

Q. 5.27. Two dielectric slabs of dielectric constants K_1 l K_2 have been put in between the plates of a capacitor hown in Fig. 5.74. What would be the capacitance of the acitor ? *(H.S.S.C.E. 2001)*

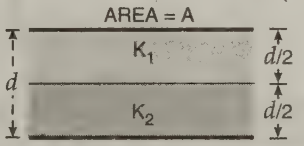


Fig. 5.74

Ans. The arrangement is equivalent to a series nbination of two capacitors, each of plate area of A, plate aration $d/2$, such that one has medium of dielectric stant K_1 and other of dielectric constant K_2. If C_1 and C_2 capacitances of the two capacitors so formed, then

$$C_1 = \frac{\varepsilon_0 K_1 A}{d/2} = \frac{2\varepsilon_0 K_1 A}{d}$$

and $$C_2 = \frac{\varepsilon_0 K_2 A}{d/2} = \frac{2\varepsilon_0 K_2 A}{d}$$

If C is the capacitance of the capacitor, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2\varepsilon_0 K_1 A} + \frac{d}{2\varepsilon_0 K_2 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right) = \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

or $$C = \frac{2\varepsilon_0 K_1 K_2 A}{(K_1 + K_2) d}$$

Q. 5.28. Is there any kind of material that when inserted between the plates of a capacitor reduces its capacitance ?

Ans. The dielectric constant K of a material is always greater than 1 and is defined as the ratio of the capacitance (C′) of the capacitor with dielectric between its plates to its capacitance (C) without the dielectric between the plates, Thus,

$$K = \frac{C'}{C}$$

As $K > 1, C' > C$ *i.e.* capacitance of a capacitor on placing the dielectric between the plates is always greater than the capacitance without the dielectric. Hence, there is no such material which when placed between the plates of a capacitor would reduce its capacitance.

Q. 5.29. If instead of a dielectric slab, we put a slab of metal between the plates of a capacitor keeping it insulated from them, what effect does it have on capacitance ?

Ans. When a slab of metal is put between the plates of a capacitor, its capacitance increases, provided the slab does not touch the plates of the capacitor. In case, the slab of metal touches the two plates, both plates become at the same potential and as a result, the capacitance of the capacitor becomes infinite.

Q. 5.30. Write the principle of Van de Graaff generator. *(H.S.S.C.E. 1997)*

Ans. Refer to section 5.20.

Q. 5.31. What limits maximum potential to which the hollow sphere in Van de Graaff generator can be raised ?

Ans. Because of leakage due to ionisation of surrounding air, when the rate of loss of charge becomes equal to the rate at which the charge is transferred to the sphere, the maximum potential of the sphere is reached.

Q. 5.32. Why should circuits containing capacitor be handled cautiously, even when there is no current ?

Ans. A capacitor does not discharge itself. In case the capacitor is connected in a circuit containing a source of high voltage, the capacitor charges itself to a very high potential. If some person handles such a capacitor without dicharging it first, he may get a severe shock.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 5.01. Two isolated metallic solid spheres of radii R and 2 R are charged, such that both of these have same face charge density σ. The spheres are located away from h other and connected by a thin conducting wire. Find new charge density on the two spheres. *(I.I.T. 1996)*

Ans. Let q_1 and q_2 be the initial charges on the spheres adii R and 2 R respectively. Then, total charge on the two eres,

$$q = q_1 + q_2 = 4\pi R^2 \sigma + 4\pi (2R)^2 \sigma$$

or $$q = 20\pi R^2 \sigma$$

When the two spheres are connected by a thin conducting wire, they will share charges, till their potential becomes the same. On reaching that stage, the charges on the two capacitors will be in the direct ratio of their capacitances. Therefore, if q'_1 and q'_2 are new values of charge on the two spheres, then

$$q_1' = q \times \frac{C_1}{C_1 + C_2} \quad \text{and} \quad q_2' = q \times \frac{C_2}{C_1 + C_2}$$

Since capacitance of a sphere is directly proportional to its radius,

$$q_1' = q \times \frac{R}{R+2R} = \frac{q}{3} = \frac{20}{3}\pi R^2 \sigma$$

and $$q_2' = q \times \frac{2R}{R+2R} = \frac{2q}{3} = \frac{40}{3}\pi R^2 \sigma$$

Therefore, new charge density of the two spheres,

$$\sigma_1' = \frac{q_1'}{4\pi R^2} = \frac{20\pi R^2 \sigma/3}{4\pi R^2} = \frac{5}{3}\sigma$$

and $$\sigma_2' = \frac{q_2'}{4\pi (2R)^2} = \frac{40\pi R^2 \sigma/3}{16\pi R^2} = \frac{5}{6}\sigma$$

Q. 5.02. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in Fig. 5.75. If the width of each stair is A/3 and the height is *d*, find the capacitance of the arrangement.

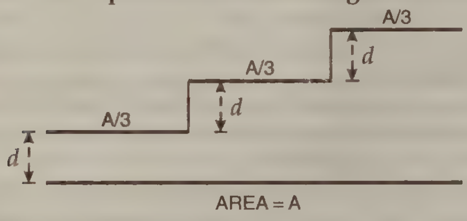


Fig. 5.75

Ans. The given arrangement of flat plate and the stair-like structure is equivalent to the parallel combination of three capacitors C_1, C_2 and C_3. The plate area of each of the three capacitors is A/3, while the separation between the plates of the three capacitors is *d*, 2 *d* and 3 *d* respectively.

$$\therefore \quad C_1 = \frac{\varepsilon_0 A/3}{d} = \frac{\varepsilon_0 A}{3d}\,;\; C_2 = \frac{\varepsilon_0 A/3}{2d} = \frac{\varepsilon_0 A}{6d}$$

and $$C_3 = \frac{\varepsilon_0 A/3}{3d} = \frac{\varepsilon_0 A}{9d}$$

Threfore, equivalent capacitance of the arrangement,

$$C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{6d} + \frac{\varepsilon_0 A}{9d}$$

$$= \frac{\varepsilon_0 A}{3d}\left(1 + \frac{1}{2} + \frac{1}{3}\right)$$

or $$C = \frac{11\varepsilon_0 A}{18d}$$

Q. 5.03. Five identical capacitor plates each of area A are arranged, such that adjacent are at a distance *d* apart. The plates are connected to a battery of e.m.f V volt [Fig. 5.76]. What is the magnitude and nature of charge on plates 1 and 4 ?

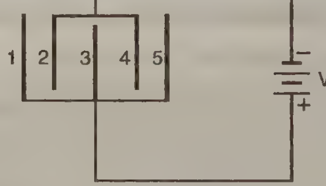


Fig. 5.76

Ans. The given arrangement of five plates is equivalent to the parallel combination of four capacitors namely C_1 (between plates 1 and 2), C_2 (between plates 2 and 3), C_3 (between plates 3 and 4) and C_4 (between plates 4 and 5). T positive plates of these capacitors are plates 1, 3, 3 an respectively.

Since area of each plate is A and separation between a two plates is *d*,

$$C_1 = C_2 = C_3 = C_4 = \frac{\varepsilon_0 A}{d}$$

The plate 1 is positive plate of capactor C_1. Therefo

charge on plate 1, $q = +C_1 V = +\frac{\varepsilon_0 A}{d} V$

The plate 4 acts as the negative plate of both capacitors C_3 and C_4. Therefore charge on plate 4,

$$q' = -(C_3 + C_4)V = -\frac{2\varepsilon_0 A}{d} V$$

Q. 5.04. Two parallel metal plates P and Q are inser at equal distances into a plane capacitor as shown Fig. 5.77. Plates A and B of the capacitor are connected battery of e.m.f. V.

(*a*) What are the potentials of each of the four plate

(*b*) How will the potentials of plates P and Q and intensities of the fields in each of the three spaces cha after plates P and Q have been connected by a wire ?

(*c*) What will happen to the charges on plates A B, when plates P and Q are connected with a wire.

(*d*) Will there be charges on the plates P and Q af connecting them with a wire ?

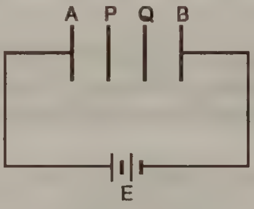


Fig. 5.77

Ans. (*a*) The plates P and Q divide the space betw plates A and B in three equal parts. Since V = E*d*, the potent of plates A, P, Q and B will be respectively V, 2 V/3, and 0.

(*b*) When the plates P and Q are connected with a w the space between plates A and B gets divided into two eq parts. Hence, potentials of plates A, P, Q and B will respectively V, V/2, V/2 and 0.

Since potential difference between plates A and P ; between plates Q and B has increased from V/3 to V/2, electric field between these plates will increase. As potential difference between plates P and Q is zero, the elec field will also be zero.

(*c*) Since potential difference between plates A and P between plates Q and B has increased, the charge on pla A and B will increase.

(*d*) Yes. Plate P will have a positive charge and plat will have a negative charge.

Q. 5.05. Prove that energy stored per unit volume i capacitor is given by $\frac{1}{2}\varepsilon_0 E^2$, where E is electric fi of the capacitor.

Ans. Consider a parallel plate capacitor having ea plate of area A and separated by a distance *d*. Then,

capacitance of parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

If σ is surface charge density of the plates of the capacitor, n electric field between the plates of capacitor,

$$E = \frac{\sigma}{\varepsilon_0}$$

or $$\sigma = \varepsilon_0 E$$

Also, charge on the plate of the capacitor,

$$q = \sigma A = \varepsilon_0 E A$$

Now, energy stored in the capacitor, $U = \frac{1}{2}\frac{q^2}{C}$

Substituting for q and C, we have

$$U = \frac{1}{2}(\varepsilon_0 E A)^2 \Big/ \frac{\varepsilon_0 A}{d} = \frac{1}{2}\varepsilon_0 E^2 A d$$

Now, $A d$ = volume of the capacitor

Therefore, energy stored per unit volume in the capacitor

$$= \frac{1}{2}\varepsilon_0 E^2$$

Q. 5.06. Prove that the total energy stored in a series combination of capacitors is equal to the sum of ergies stored in the individual capacitors.

Ans. Suppose that three capacitors of capacitances C_1, C_2 d C_3 are connected in series to a battery. The equivalent pacitance of the series combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Further, when a series combination of capacitors is arged through a battery, the charge on the left plate of each pacitor is q, while induced charges $-q$ and $+q$ are produced the right plate of each capacitor. Thus, net charge stored the series combination is just q.

Therefore, total energy stored in series combination,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}q^2\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$= \frac{1}{2}\frac{q^2}{C_1} + \frac{1}{2}\frac{q^2}{C_2} + \frac{1}{2}\frac{q^2}{C_3}$$

or $$U = U_1 + U_2 + U_3$$

Thus, energy stored in series combination of the pacitors is equal to the sum of the energies stored in the dividual capacitors.

Q. 5.07. Prove that the total energy stored in a parallel combination of capacitors is equal to the sum of the ergies stored in the individual capacitors.

Ans. When three capacitors of capacitances C_1, C_2 and are connected in parallel, the resultant capacitance of the rallel combination is given by

$$C = C_1 + C_2 + C_3$$

Further, when a parallel combination of capacitors is arged through a battery, the potential difference across the o plates of each capacitor is equal to the applied voltage Therefore, total energy stored in the parallel combination,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(C_1 + C_2 + C_3)V^2$$

$$= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2$$

or $$U = U_2 + U_2 + U_3$$

Therefore, total energy stored in parallel combination of e capacitors is also equal to the sum of the energies stored individual capacitors.

Q. 5.08. The plate A of a parallel plate capacitor is connected to a spring of force constant k and can move, while the plate B is fixed. The arrangement is held between two rigid supports as shown in Fig. 5.78. If a charge $+q$ is placed on plate A and $-q$ on plate B, by how much does the spring elongate ?

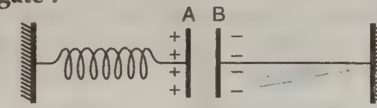


Fig. 5.78

Ans. Suppose that the two plates of the capacitors attract each other with a force F and as a result, the spring elongates by an amount l. Then,

$$F = -k\,l \qquad ...(i)$$

If the area of each of the two plates of the capacitor is A and separation between the plates is x, then capacitance of the capacitor,

$$C = \frac{\varepsilon_0 A}{x} \qquad ...(ii)$$

The energy stored in the capacitor,

$$U = \frac{1}{2}\frac{q^2}{C}$$

Using equation (*ii*), we have

$$U = \frac{q^2 x}{2\varepsilon_0 A}$$

As the electrostatic force (F) is a conservative force, it can be expressed as negative gradient of corresponding potential *i.e.*

$$F = -\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{q^2 x}{2\varepsilon_0 A}\right)$$

or $$F = -\frac{q^2}{2\varepsilon_0 A} \qquad ...(iii)$$

The negative sign indicates that force between the plates of the capacitor is attractive in nature.

From equations (*i*) and (*ii*), we have

$$-k\,l = -\frac{q^2}{2\varepsilon_0 A} \quad \text{or} \quad l = \frac{q^2}{2\varepsilon_0 A}$$

Q. 5.09. If a parallel plate capacitor of capacitance C is kept connected to a supply voltage V to just fill the space and then a dielectric slab is inserted between the plates. What will be the change in the capacitance, potential difference, the charge, electric field and the energy stored ?

Ans. Let K be dielectric constant of the dielectric slab and q, E and U be charge on capacitor, electric field between plates and energy stored in the capacitor before inserting the slab.

On inserting dielectric slab :

The capacitance of the capacitor will become

$C' = K C$ **(increases)**

Since the capacitor is kept connected to the supply voltage, potential difference will remain **unchanged** *i.e.* **V.**

The charge on capacitor will become,

$q' = C'V = KCV = Kq$ **(increases)**

It may be pointed out that as battery remains connected to the capacitor, it can draw more charge from the battery.

Since potential difference between plates does not change, electric field will also remain **unchanged.**

The energy stored in the capacitor will become

$$U' = \frac{1}{2} C' V^2 = \frac{1}{2} K C V^2 = \mathbf{K\,U} \textbf{ (increases)}$$

Q. 5.10. A parallel plate capacitor of capacitance C is charged to a potential difference V and then the battery is disconnected. Now a dielectric slab of the dimensions equal to the spacing between the plates is inserted between the plates. What are the changes, if any, in the capacitance, charge, potential difference, electric field and the energy stored ?

Ans. Let K be dielectric constant of the slab and q, E and U be charge on the plates of the capacitor, electric field between the plates and energy stored in the capacitor before inserting the slab.

On inserting dielectric slab :

The capacitance of capacitor will become,

$$C' = KC \qquad \textbf{(increases)}$$

Since battery has been disconnected, the charge c capacitor will remain the same.

P.D. between the plates, $V' = \frac{q}{C'} = \frac{q}{KC} = \frac{V}{K}$ **(decrease**

Electric field between the plates,

$$E' = \frac{V'}{d} = \frac{V}{K} d = \frac{E}{K} \qquad \textbf{(decreases)}$$

Energy stored in the capacitor,

$$U' = \frac{1}{2} C' V'^2 = \frac{1}{2} (KC) \left(\frac{V}{K}\right)^2 = \frac{1}{K}\left(\frac{1}{2} C V^2\right)$$

$$= \frac{U}{K} \qquad \textbf{(decreases)}$$

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Define capacitance. Give its units. Show that the capacitance of an insulated spherical conductor is $4\pi\varepsilon_0$ times the radius of the spherical conductor.
2. What is a capacitor ? Explain its principle. *(P.S.S.C.E. 2002, 2001, 2000, 1999)*
3. Explain the principle of a capacitor. *(P.S.S.C.E. 1997 ; C.B.S.E. 1994)*
4. Explain the action of a capacitor. *(P.S.S.C.E. 1994)*
5. What are the factors on which capacitance of a parallel plate capacitor depends ? Give the corresponding relation. *(C.B.S.E. 1991)*
6. Obtain a relation for the capacitance of a parallel plate capacitor. *(P.S.S.C.E. 1999 S, 1999, 1994 ; C.B.S.E. 1994 ; H.S.S.C.E. 1994)*
7. Define capacitance of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor. *(P.S.S.C.E. 2001 ; C.B.S.E. 1992)*
8. What is the resultant capacitance, when a number of capacitors are connected in series ? Derive the expression for it. *(H.P.S.S.C.E. 1997)*
9. Three capacitors C_1, C_2 and C_3 are connected in series. Derive an expression for the equivalent capacitance. *(H.P.S.S.C.E. 1995 ; H.S.S.C.E. 1994 ; C.B.S.E. 1991)*
10. What is the resultant capacitance, when a number of capacitors are connected in series ? Derive the expression for it. *(H.P.S.S.C.E. 1999)*
11. Three capacitors of capacitances C_1, C_2 and C_3 are connected in parallel. Calculate the equivalent capacitance. *(C.B.S.E. 1994 ; P.S.S.C.E. 1992)*
12. How will you connect a number of capacitors to increase the capacitance ? Find the capacitance of the combination.
13. Describe series and parallel combinations of capacitors. What combination do you use, if the capacitance is to be (*i*) increased (*ii*) reduced ?
14. Prove that energy stored in a parallel plate capacitor is $\frac{1}{2}CV^2$. *(C.B.S.E. 2001)*
15. Prove that energy stored in a parallel plate capacitor can be expressed by the relation $\frac{Q^2}{2C}$. *(C.B.S.E. 2001 S)*
16. What is a capacitor ? Derive an expression for the electrostatic energy stored in the charged capacitor. *(P.S.S.C.E. 2001 ; H.S.S.C.E. 2001)*
17. Derive an expression for the energy stored in a parallel plate capacitor with air as the medium between its plates. *(P.S.S.C.E. 2000, 1999 ; C.B.S.E. 2000, 1998 ; H.S.S.C.E. 1997 S)*
18. Derive an expression for the energy stored by a capacit of capacitance C, carrying a charge q at a potenti difference V. *(H.P.S.S.C.E. 2002 ;P.S.S.C.E. 2000 ; I.S.C.E. 199*
19. When two charged conductors having differe capacitances and different potentials are joined togeth show that there is always a loss of energy. *(P.S.S.C.E. 2001, 1999 S, 199*
20. What are polar and non-polar molecules ? What is physic importance of dielectric constant ?
21. Distinguish between polar and non-polar dielectrics. *(H.S.S.C.E. 199*
22. What is meant by a dielectric ? Explain how it reduces t electric field between two oppositely charged plates, if t space between them be filled with it ?
23. What do you understand by polarisation of dielectric Define dielectric constant in terms of electric fields. *(H.S.S.C.E. 199*
24. Explain what is meant by dielectric polarisation. Henc establish the relation $K = 1 + \chi$. *(H.S.S.C.E. 2001 ; C.B.S.E. 199*
25. Derive a relation for the capacitance of a parallel pla capacitor with dielectric slab of thickness t filled betwe the plates. *(H.P.S.S.C.E. 200*
26. Derive an expression for the capacitance of a parallel pla capacitor with dielectric as the medium between the plat *(C.B.S.E. 1999, 199*
27. Explain why the capacitance of a parallel plate capacit increases, when a dielectric slab is introduced between t plates. Define relative permittivity of a dielectric. *(C.B.S.E. 199*
28. Explain the effect of introducing a dielectric slab betwe the plates of a parallel plate capacitor on its capacitanc *(C.B.S.E. 199*
29. What happens to the capacitance of a capacitor, when dielectric is introduced between its plates ? Expla qualitatively. *(P.S.S.C.E. 199*
30. A conducting slab of thickness t is introduced witho touching between the plates of a parallel plate capacit separated by a distance d ($t < d$). Derive an expression f the capacitance of the capacitor. *(C.B.S.E. 2001*
31. Derive expression for the capacitance of a parallel pla capacitor, when a conducting slab is partially introduc between the plates of the capacitor.
32. What happens to the capacitance of the capacitor, when t conducting slab wholly fills the space between the tw plates ? Justify the result.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What is a capacitor ? Explain its principle. Obtain a relation for the capacitance of a parallel plate capacitor. (P.S.S.C.E. 2000, 1996 S)
2. Explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor. (H.S.S.C.E. 2002)
3. What is a spherical capacitor ? Derive expression for its capacitance.
4. Define capacitance. Derive an expression for the capacitance of a cylindrical capacitor. (C.B.S.E. 1992)
5. What is a capacitor ? Derive expression for the total capacitance, when three capacitors of capacitances C_1, C_2 and C_3 are connected in (*i*) series (*ii*) parallel. (H.S.S.C.E. 2002)
6. Define capacitance of a capacitor. Give its SI unit. Prove that the total electrostatic energy stored in a parallel plate capacitor is $\frac{1}{2} C V^2$. (C.B.S.E. 2001 S, 1996)
7. Derive the expression for the capacitance of a parallel plate capacitor with a dielectric medium of dielectric contant K between its plates. Obtain also the expression for the energy stored in the above case. (C.B.S.E. 2001 S)
8. Define capacitance of a capacitor. Give its SI unit. Derive an expression for the electrostatic energy stored in a charged capacitor. (P.S.S.C.E. 1999 S)
9. How does the total energy stored by the capacitor change, when the medium of air is replaced by a medium of dielectric constant K ? Explain. (C.B.S.E. 1998)
10. Derive the expression for the energy stored in a charged capacitor. Show that there is always a loss of energy on connecting the two charged capacitors having capacitances C_1, C_2 and potentials V_1, V_2. (C.B.S.E. 1998 S)
11. Derive an expression for the energy stored in a parallel plate capacitor. Assuming that the capacitor is disconnected from the battery, explain how the (*i*) capacitance (*ii*) potential difference across the plates and (*iii*) energy stored in the parallel plate capacitor change, when a medium of dielectric constant K is introduced between the plates. (C.B.S.E. 2001, 2000)
12. Derive an expression for the energy stored in a parallel plate capacitor. A parallel plate capacitor with air as dielectric is charged by a d.c. source to a potential V. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant 10. State with reasons, how does (*i*) electric field between the plates and (*ii*) energy stored in the capacitor change. (C.B.S.E. 2002)
13. Deduce an expression for the capacitance of a parallel plate capacitor, when dielectric slab is inserted between its plates. Assume the slab thickness to be less than the plate separation. (H.P.S.S.C.E. 2002)
14. Draw a labelled diagram of a Van de Graaff generator. Give its principle and explain its working. (C.B.S.E. 1996)
15. Give the principle and explain the working of a Van de Graaff generator with the help of a labelled diagram. (C.B.S.E. 1998 S, 1997)
16. Discuss the principle, construction and working of Van de Graaff generator. (H.S.S.C.E. 2002, 20021, 1993 ; P.S.S.C.E. 2001, 1991 ; C.B.S.E. 1999, 1990)

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

ype A. On Capacitance of a conductor

1. When electrons equal to Avogadro number are transferred from one conductor to another, a potential difference of 10^6 V appears between them. Calculate the capacitance of the system of two conductors. [**Ans.** $9{\cdot}63 \times 10^{-2}$ F]
2. A charged spherical conductor has a surface density of $0{\cdot}07$ C cm^{-2}. When the charge is increased by 4·4 C, the surface density changes by $0{\cdot}084$ C cm^{-1}. Find the initial charge and capacitance of the spherical conductor. [**Ans.** 22 C, $5{\cdot}56 \times 10^{-12}$ F]
3. If 64 small mercury drops each charged to 220 V coalesce, what will be the potential of bigger drop ? [**Ans.** 3520 V]
4. Twenty seven spherical droplets of radius 3 mm and each carrying 10^{-12} C of charge are combined to form a bigger drop. Find the capacitance of the bigger drop. (H.S.C.C.E 2001) [**Ans.** 10^{-12} F]
5. N drops of mercury of equal radii and possessing equal charges combine to form a big drop. Compare the charge, capacitance and potential of bigger drop with the corresponding quantities of individual drop. [**Ans.** N, $N^{1/3}$, $N^{2/3}$]
6. The radii of two charged metallic spheres are 5 cm and 10 cm. Each has a charge of 75 μC. They are connected by a conducting wire. Calculate (*a*) common potential of the spheres after connecting and (*b*) amount of charge transferred. [**Ans.** (*a*) 9×10^6 V ; (*b*) 25 μC]

Type B. On Capacitance of a capacitor

7. What distance should the two plates each of area 0·2 m × 0·1 m of an air capacitor be placed in order to have the same capacitance as a spherical conductor of radius 0·5 m ? [**Ans.** $3{\cdot}183 \times 10^{-3}$ m]
8. An air capacitor has plates of 6 cm diameter. At what distance should the plates be placed so as to have the same capacitance as sphere of diameter of 90 cm ? [**Ans.** 0·25 mm]
9. A parallel plate capacitor has each plate of 0·06 m diameter separated by 0·05 cm of air. What is the capacitance of the capacitor ? What would be the radius of a sphere having the same capacitance ? [**Ans.** 50 pF, 0·45 m]
10. In a parallel plate capacitor with air between the plates, each plate has an area 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 volt supply, what is the charge on each plate of the capacitor ? (*Text Problem*) [**Ans.** $1{\cdot}77 \times 10^{-9}$ V]
11. A parallel plate capacitor has two plates of sides 0·055 m and 0·04 m of air. Their distance apart is 0·7 mm. The dielectric constant of the medium in between is 4. Find the capacitance of the capacitor. (*Karnataka, 1985 S*) [**Ans.** $1{\cdot}1 \times 10^{-10}$ F]
12. A parallel plate capacitor has plates of area 0·02 m^2 and separation between the plates 1 mm. What potential difference will be developed, if a charge of 1 *n*C is given

to the capacitor ? If the plate separation is now increased to 2 mm, what will be the new potential difference ? **[Ans.** 5·65 V, 11·3 V]

Type C. On Capacitance of a spherical capacitor

13. Calculate the capacitance of a spherical capacitor, if the diameter of inner sphere is 0 · 2 m and that of the outer sphere is 0·3 m, the space between them being filled with a liquid having dielectric constant 20. **[Ans.** 1333 pF]

14. A spherical capacitor has 10 cm and 12 cm as the radii of inner and outer spheres. The space between the two is filled with a dielectric of dielectric constant 5. Find its capacitance, when the inner sphere is earthed. **[Ans.** 333 pF]

15. A spherical capacitor has an outer sphere of radius 0 · 15 m and the inner sphere of radius 0·1 m. The outer sphere is earthed and the inner sphere is given a charge of 6 μC. The space between the concentric spheres is filled with a material of dielectric constant 18. Calculate the capacitance and potential of the inner sphere. **[Ans.** 6×10^{-10} F, 10^4 V]

Type D. On capacitance of cylindrical capacitors

16. The inner and outer radii of a coaxial cable are 0 · 1 mm and 0·6 mm respectively. If the core material has dielectric constant 12, calculate the capacitance per metre of the cable. **[Ans.** $3{\cdot}725 \times 10^{-10}$ F m^{-1}]

17. A cable consisting of a wire 3 mm in diameter and insulated with 3 mm thick dielectric of relative permittivity 7 is placed in water. Calculate the capacitance of 1 km length of the cable. **[Ans.** 0·354 μF]

Type E. On Grouping of capacitors

18. Two spheres of radii 2 cm and 7 cm are connected together. What is the total capacitance of the combination ? **[Ans.** 1·73 pF (in series), 10·0 pF (in parallel)]

19. Three capacitors of capacitances 2 μF, 3 μF and 4 μF are connected (*i*) in series ; (*ii*) in parallel. Compare the effective capacitances in the two cases. **[Ans.** 4 : 39]

20. The capacitances of three capacitors are in the ratio 1 : 2 : 3. Their equivalent capacitance in parallel is gerater than their equivalent capacitance in series by 60/11 μF. Calculate their individual capacitances. **[Ans.** 1 μF, 2 μF ; 3 μF]

21. Three capacitors of capacitances 1, 3 and 6 microfarad and connected so that the second and the third are in series and the first is in parallel with them. Calculate the capacitance of the combination. **[Ans.** 3 μF]

22. Three capacitors, each of capacitance 6 μF are connected together in series and are also connected in series with three capacitors of values 2 μF, 4 μF and 2 μF, which are grouped together in parallel. Calculate the total combined capacitance. **[Ans.** 1·6 μF]

23. Calculate the capacitance of the capacitor C in the network shown in Fig. 5.79. The equivalent capacitance of the combination between P and Q is 30 μF. (*C.B.S.E. 1995*) **[Ans.** 60 μF]

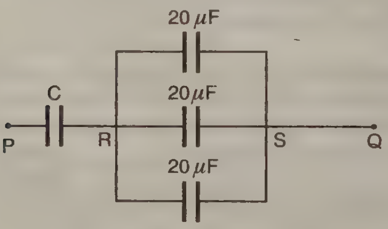


Fig. 5.79

24. Find the equivalent capacitance of the system shown in Fig. 5.80 between the points X and Y.

$$\left[\textbf{Ans.}\ C + \frac{2C_1C_2}{C_1+C_2}\right]$$

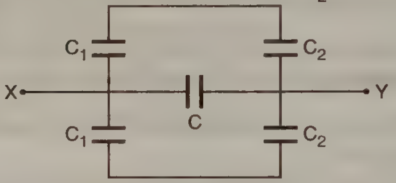


Fig. 5.80

25. Four capacitors are connected as shown in Fig. 5.81. Calculate the equivalent capacitance between the points X and Y. (*C.B.S.E. 2000*) **[Ans.** 15 μF]

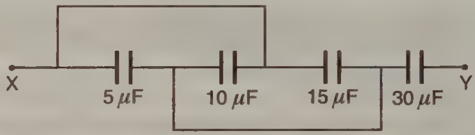


Fig. 5.81

26. Find the resultant capacitance between points X and Y of the combination of capacitors shown in Fig. 5.82. (*H.S.S.C.E. 2001*) **[Ans.** 2·5 μF]

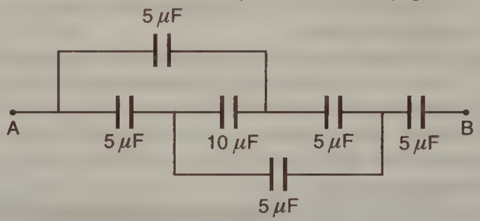


Fig. 5.82

27. The outer cylinders of two cylindrical capacitors of capacitance 2·2 μF each are kept in contact and the inner cylinders are connected through a wire. A battery of e.m.f 10 V is connected as shown in Fig. 5.83. Find the total charge supplied by the battery to the inner cylinders. **[Ans.** 44 μC]

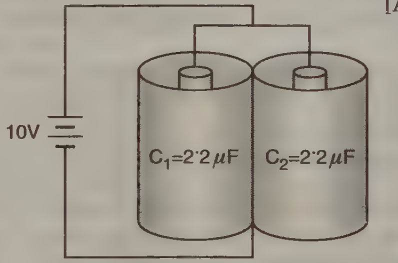


Fig. 5.83

28. Find the equivalent capacitance between the points X and Y of the network of capacitors shown in Fig. 5.84. **[Ans.** 1 μF]

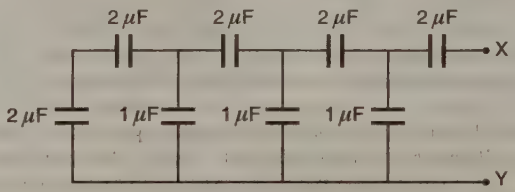


Fig. 5.84

29. A 0·5 μF capacitor connected in parallel to 0·75 μF capacitor and this combination is joined to a 100 volt d.c.

source. Calculate charge taken from the source and charge on each capacitor. (*H.S.S.C.E. 1997*)
[**Ans.** 125 μC, 50 μC, 75 μC]

0. Calculate the capacitance of a combination of three capacitors of capacitances 8, 12 and 24 μC, when they are joined in (*i*) series (*ii*) parallel. Find the charge on each plate in the series arrangement, if the potential difference across the system is 100 volt.
[**Ans.** (*i*) 4 μF, (*ii*) 44 μF ; 4×10^{-4} C]

1. In the network of capacitors shown in Fig. 5.85, find (*i*) equivalent capacitance and (*ii*) total charge.
(*P.S.S.C.E. 1993*) [**Ans.** (*i*) 66·67 pF ; (*ii*) 2×10^{-8} C]

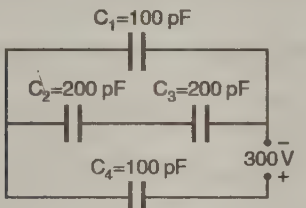


Fig. 5.85

2. A network of four 10 μF capacitors is connected to a 500 V supply as shown in Fig. 5.86. Determine the (*a*) equivalent capacitance of the network and (*b*) charge on each capacitor.
[**Ans.** (*a*) 13·33 μF (*b*) $q_1 = q_2 = q_3 = 1{\cdot}67 \times 10^{-3}$ C, $q_4 = 5{\cdot}0 \times 10^{-3}$ C]

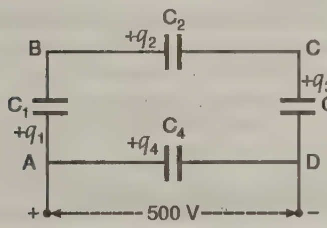


Fig. 5.86

3. The capacitors C_1, C_2 and C_3 are connected as shown in Fig. 5.87 to a 100 V d.c. supply. If $C_1 = 2$ μF, $C_2 = 3$ μF and $C_3 = 5$ μF, find potential difference across each capacitor [**Ans.** $V_1 = 80$ V, $V_2 = V_3 = 20$ V]

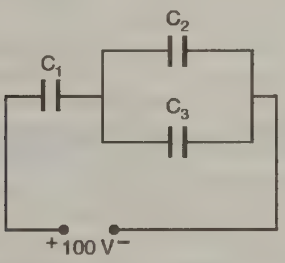


Fig. 5.87

4. Three capacitors have capacitances 10 μF, 50 μF and 25 μF respectively. Calculate (*i*) charge on each, when connected in parallel to a 250 V supply, (*ii*) total capacitance and (*iii*) potential difference across each, when connected in series.
[**Ans.** (*i*) $q_1 = 2500$ μC, $q_2 = 12500$ μC, $q_3 = 6250$ μC ; (*ii*) 85 μF ; (*iii*) $V_1 = 156{\cdot}25$ V, $V_2 = 31{\cdot}25$ V, $V_3 = 62{\cdot}5$ V]

5. The capacitance of all the capacitors shown in Fig. 5.88 are as below :
$C_1 = C_5 = 3$ μF, $C_2 = 5$ μF, $C_3 = C_4 = C_6 = 4$ μF and $C_7 = 2$ μF.
(*a*) What is the equivalent capacitance between the points A and B ? (*b*) If the charge on 5 μF capacitor is 120 μC, what is the potential difference between the two points A and B ? [**Ans.** (*a*) 5 μF ; (*b*) 96 V]

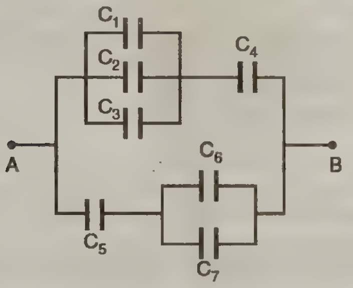


Fig. 5.88

36. Two capacitors of capacitances 2 μF and 4 μF are separately charged to potentials 100 V and 75 V respectively. What are the charges carried by each ? What would be the value of the potential, when they are connected together in parallel and what would be charges on them after contact ?
[**Ans.** 200 μC, 300 μC, 83·33 V ; 166·67 μC, 333·33 μC]

37. Two capacitors A and B of capacitances 4 μF and 2 μF respectively are connected in series with a battery of e.m.f. 100 V. The connections are broken and the like terminals of the capacitors are then joined. Find the final charge on each capacitor. [**Ans.** 177·8 μC, 88·9 μC]

38. A capacitor of 3 μF is charged to a potential difference of 300 V and a capacitor of 2 μF to 200 V. After charging the capacitors, their plates charged with unlike charges are connected. What quantity of electricity will be transferred during connection and from what capacitor will this take place ? [**Ans.** 6×10^{-4} C from capacitor of 3 μF]

39. A conductor charged to a potential of 40 V is connected to an uncharged conductor of 10 F capacitance. The common potential developed is 25 V. Calculate the capacitance of the first conductor. [**Ans.** 16·67 F]

40. A conductor has a charge of 0·5 μC and at potential of 20 V. Another uncharged conductor, whose capacity is 0·015 μF is momentarily connected to the first and then separated. Calculate the charges carried by each after contact and what is the value of the common potential.
[**Ans.** 0·3125 μC, 0·1875 μC ; 12·5 V]

41. An air capacitor has an area of 1000 cm^2 and the plates being 4 mm apart. It is connected in series with another capacitor having a plate area of 20 cm^2, the plates being separated by a dielectric 0·1 mm thick having dielectric constant of 2·5. If the potential difference across the second capacitor is 100 V, find the voltage applied across the series combination. [**Ans.** 300 V]

Type F. On Energy stored in a capacitor

42. Two insulated spherical conductors of radii 5·0 cm and 10·0 cm are charged to potentials of 600 V and 300 V respectively. Calculate the total energy of the system. Also calculate the energy after the spheres have been connected by a fine wire. [**Ans.** $1{\cdot}5 \times 10^{-6}$ J, $1{\cdot}33 \times 10^{-6}$ J]

43. A capacitor charged from a 50 V d.c. supply is found to have a charge of 10 μC. What is the capacitance of the capacitor and how much energy is stored in it ?
(*I.S.C.E. 1993*) [**Ans.** 0·2 μF ; $2{\cdot}5 \times 10^{-4}$ J]

44. A capacitor with a capacitance of 50 μF is connected to a battery of 10 V. Find the charge on it in coulomb and its energy in joule. [**Ans.** 500 μC, $2{\cdot}5 \times 10^{-3}$ J]

45. A parallel plate capacitor of 300 μF is charged to 200 V. If the distance between its plates is halved, what will be the potential between the plates and what will be the change in energy stored in it ? (*P.S.S.C.E. 1999*)

[**Ans.** 100 V, 3 J (decrease)]

46. The plates of a parallel plate capacitor are 50 cm^2 and one mm apart. (*a*) What is its capacitance ? (*b*) When it is connected to a 45 V battery, what is the charge on either plate and the energy stored in it ? (*c*) What is the electric field between the plates ?

[**Ans.** (*a*) 44·27 pF ; (*b*) $1{\cdot}99 \times 10^{-9}$ C ; $4{\cdot}48 \times 10^{-8}$ J ; (*c*) $4{\cdot}5 \times 10^{4}$ V m^{-1}]

47. A 400 pF capacitor is charged by a 100 V battery. How much electrostatic energy is stored by the capacitor ? The capacitor is disconnected from the battery and connected to another 400 pF capacitor. What is the electrostatic energy stored by the system ? (*H.S.S.C.E. 1999 S ; P.S.S.C.E. 1991*)

[**Ans.** $2{\cdot}0 \times 10^{-6}$ J, 10^{-6} J]

48. A capacitor carrying a charge equal to 2 μC up to a potential 100 V is put into contact with another capacitor carrying a charge of 1·5 μC up to a potential of 60 V. What is the resulting potential, the charge carried by each, the total energy before and after connection ?

[**Ans.** 77·78 V ; $1{\cdot}56 \times 10^{-6}$ C ; $1{\cdot}94 \times 10^{-6}$ C ; $1{\cdot}45 \times 10^{-4}$ J ; $1{\cdot}36 \times 10^{-4}$ J]

49. Two capacitors are in parallel and the energy stored is 45 J, when the combination is raised to a potential of 3000 V. With the same two capacitors in series, the energy is 4·05 J for the same potential. What are their capacitances ? [**Ans.** 9 μF, 1 μF]

Type G. On Loss of energy on sharing charges

50. A variable capacitor is kept connected to a 10 V battery. If the capacitance of the capacitor is changed from 7 μF to 3 μF, what is the change in the energy ? What happens to this energy ? (*I.S.C.E. 1996*)

[**Ans.** 2×10^{-4} J (decreases), lost as heat and electromagnetic radiation]

51. A capacitor of capacitance 6 μF is charged to a potential equal to 150 V. Its potential falls to 90 V, when another capacitor is connected to it. Find the capacitance of the second capacitor and the amount of energy lost due to the connection. [**Ans.** 4 μF ; 0·027 J]

52. A 5 μF capacitor is charged by a 220 V supply. It is then disconnected from the supply and is connected to another uncharged 2·5 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation ? (*P.S.S.C.E. 1998 S*)

[**Ans.** 0·04 J]

53. A 12 mF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 8 mF capacitor. How much electrostatic energy of first capacitor is lost in the form of heat and electromagnetic radiation ? [**Ans.** 96 J]

54. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process ? (*Text Problem*)

[**Ans.** 6×10^{-6} J]

55. Two capacitors of 25 μF and 100 μF are connected in series to a source of 120 V. Keeping their charges unchanged, they are separated and connected in parallel to each other. Find out (*a*) potential difference between the plates of each capacitor and (*b*) energy loss in the process.

[**Ans.** 38·4 V ; 0·052 J]

Type H. On Effect of dielectric

56. An air capacitor of capacitance 2 μF is charged to a potential 200 V. What is the energy stored in it ? If a medium of dielectric constant 2 is inserted between its plates and the potential is raised again to the same valu
what is the amount of energy, it would store ?

[**Ans.** 0·04 J ; 0·08

57. A parallel plate capacitor has capacitance 20 μF. A potent
difference of 220 V is applied across it. Calculate the char
on the plates and energy stored in capacitor. If a dielect
slab of dielectric constant 55 is introduced between
plates, calculate the new value of potential difference a
the energy stored.

[**Ans.** $4{\cdot}4 \times 10^{-3}$ C, 0·0484 J ; 4 V, $8{\cdot}8 \times 10^{-3}$

58. When a slab of insulating material 4 mm thick
introduced between the plates of a parallel plate capacit
it is found that the distance between the plates has to
increased by 3·2 mm to restore the capacitance to
original value. Calculate dielectric constant of the materi

[**Ans.**

59. A parallel plate capacitor with plate separation 5 mm
charged by a battery. It is found that on introducting a mi
sheet 2 mm thick, while keeping the battery connectio
intact, the capacitor draws 25% more charge from t
battery than before. Find the dielectric constant of mic

[**Ans.**

60. An air parallel plate capacitor having plates of area
6×10^{-3} m^2 and plate separation 3 mm is connected t
200 V supply. Calculate charge on each plate of t
capacitor. Explain what would happen, when a 3 mm thi
mica sheet (K = 8) is inserted between the plates (*a*) wh
supply voltage remains connected and (*b*) after the supp
is disconnected.

[**Ans.** $3{\cdot}54 \times 10^{-9}$ C ; (*a*) $2{\cdot}83 \times 10^{-8}$ C ; (*b*) $3{\cdot}54 \times 10^{-9}$

61. A parallel plate capacitor has a capacitance of 2 μF. A sl
of dielectric constant 5 is inserted between the plates a
the capacitor is charged to 100 V and then isolated. (*a*) Wh
is the new potential difference, if the dielectric slab
removed ? (*b*) How much work is required to remove t
dielectric slab ? [**Ans.** (*a*) 500 V ; (*b*) 0 · 2

Type I. On Van de Graaff generator

62. In a Van de Graaff generator, the spherical metal shell
to be kept at 18×10^{5} V. The dielectric strength of the g
surrounding the metal shell is $4 \cdot 5 \times 10^{7}$ V m^{-1}. What
the minimum radius of the spherical shell required ?

[**Ans.** 4 c

63. A spark passes in air, when the potential gradient at t
surface of a charged conductor is 3×10^{6} V m^{-1}. What m
be the radius of an insulated metal sphere, which can
raised to a potential of 3×10^{6} V, before sparking into t
air ? What will be the energy stored immediately bef
sparking occurs ? (*Text Problem*) [**Ans.** 1 m ; 500

MISCELLANEOUS PROBLEMS

64. A capacitor is made of a flat plate of area A and a seco
plate having a stair-like structure. The height of first st
of the stair is *d*, while those of subsequent steps is *b* [F
5.89]. If all the steps are of same width, calculate t
capacitance of the structure.

$$\left[\textbf{Ans. } \frac{\varepsilon_0 A (3d^2 + 6bd + 2b^2}{3d(d+b)(d+2b)}\right.$$

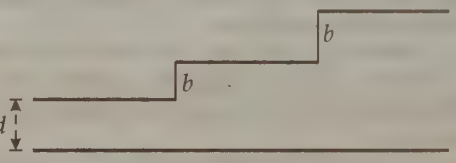


Fig. 5.89

65. Find the capacitance of a system of three parallel plates each of area A metre^2 separated by distances d_1 and d_2 metre respectively. The space between them is filled with dielectrics of relative dielectric constants ε_1 and ε_2. The dielectric constant of free space is ε_0. *(Roorkee 1987)*

$$\left[\textbf{Ans. } \frac{\varepsilon_0\,\varepsilon_1\,\varepsilon_2\,A}{d_1\,\varepsilon_2+d_2\,\varepsilon_1}\right]$$

66. A parallel plate capacitor is constructed using three different dielectric materials as shown in Fig. 5.90. The parallel plates across which a potential difference is applied are of area A = 1 cm^2 and are separated by a distance d = 2 mm. If $K_1 = 4$, $K_2 = 6$ and $K_3 = 2$, find capacitance across points A and B. **[Ans. 1·56 pF]**

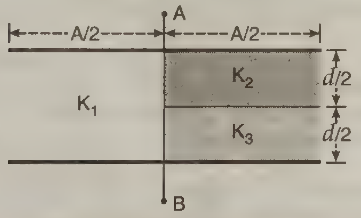


Fig. 5.90

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Charge on each conductor,

$$q = N\,e = 6{\cdot}02\times10^{23}\times1{\cdot}6\times10^{-19}\text{ C}$$

Now, $$C=\frac{q}{V}=\frac{6\cdot02\times10^{23}\times1\cdot6\times10^{-19}}{10^6}=\mathbf{9{\cdot}63\times10^{-2}\ F}$$

2. Let q be charge on the spherical conductor and r be its radius. Then,

$$\frac{q}{4\pi r^2}=0\cdot07\text{ C m}^{-2} \qquad ...(i)$$

When charge is increased by 4·4 C, we have

$$\frac{q+4\cdot4}{4\pi r^2}=0\cdot084\text{ C m}^{-2} \qquad ...(ii)$$

From equation (*i*) and (*ii*), we get

$$\frac{q+4\cdot4}{q}=\frac{0\cdot084}{0\cdot07} \quad\text{or}\quad q=\mathbf{22\ C}$$

From equation (*i*), we have

$$r=\sqrt{\frac{q}{4\pi\times0\cdot07}}=\sqrt{\frac{22}{4\pi\times0\cdot07}}=5\text{ cm}=0\cdot05\text{ m}$$

Therefore, capacitance of spherical conductor,

$$C=4\pi\,\varepsilon_0\,r=\frac{0\cdot05}{9\times10^9}=\mathbf{5{\cdot}56\times10^{-12}\ F}$$

3. Proceed as in solved problem no. 5.03.

4. By proceeding as in solved problem no. 5.03, find the radius of bigger drop. It will be found that

$$R=3\,r=3\times3=9\text{ mm}=9\times10^{-3}\text{ m}$$

$$\therefore\quad C=4\pi\,\varepsilon_0\,R\ \frac{9\times10^{-3}}{9\times10^9}=\mathbf{10^{-12}\ F}$$

5. Let q be charge on each small drop and r be its radius. Then, capacitance and potential of each small drop,

$$C=4\pi\,\varepsilon_0\,r \quad\text{and}\quad V=\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{r}$$

Let R is radius of the big drop formed. Then,

$$\frac{4}{3}\pi\,r^3\times N=\frac{4}{3}\pi\,R^3 \quad\text{or}\quad R=N^{1/3}\,r$$

Charge on big drop, $q'=N\,q$ or $\dfrac{q'}{q}=N$

Capacitance of big drop,

$$C'=4\pi\,\varepsilon_0\,R=4\pi\,\varepsilon_0\,N^{1/3}\,r=N^{1/3}\,C$$

or $$\frac{C'}{C}=\mathbf{N^{1/3}}$$

Potential of big drop, $$V'=\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q'}{R}$$

$$=\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{N\,q}{N^{1/3}r}=N^{2/3}\times\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{r}$$

or $V'=N^{2/3}\,V$ or $\dfrac{V'}{V}=\mathbf{N^{2/3}}$

6. Here, $r_1=5$ cm $=0{\cdot}05$ m ; $r_2=10$ cm $=0{\cdot}1$ m,

$$q_1=q_2=75\ \mu C=75\times10^{-6}\text{ C}$$

(*a*) Now, $C_1=4\pi\,\varepsilon_0\,r_1=4\pi\,\varepsilon_0\times0{\cdot}05$

and $C_2=4\pi\,\varepsilon_0\,r_2=4\pi\,\varepsilon_0\times0{\cdot}1$

Common potential,

$$V=\frac{q_1+q_2}{C_1+C_2}=\frac{75\times10^{-6}+75\times10^{-6}}{4\pi\,\varepsilon_0\times0\cdot05+4\pi\,\varepsilon_0\times0\cdot1}$$

$$=\frac{9\times10^9\times150\times10^{-6}}{0\cdot15}$$

or $$V=\mathbf{9\times10^6\ V}$$

(*b*) Now, $q_1'=C_1V=4\pi\,\varepsilon_0\times0{\cdot}05\times9\times10^6$

$$=50\times10^{-6}\text{ C}=50\ \mu C$$

Therefore, charge transferred $=q_1-q_1'=75-50=\mathbf{25\ \mu C}$

7. Capacitance of spherical conductor, $C=4\pi\,\varepsilon_0\,R$

or $$C=4\pi\,\varepsilon_0\times0{\cdot}5 \qquad ...(i)$$

Let d be distance between the two plates of air capacitor.

Area of each plate, $A=0{\cdot}2\times0{\cdot}1=0{\cdot}02\text{ m}^2$

$$\therefore\quad C=\frac{\varepsilon_0\,A}{d}=\frac{\varepsilon_0\times0\cdot02}{d} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{\varepsilon_0\times0\cdot02}{d}=4\pi\,\varepsilon_0\times0\cdot5$$

or $$d=\frac{0\cdot02}{4\pi\times0\cdot5}=\mathbf{3{\cdot}183\times10^{-3}\ m}$$

8. Proceed as in problem no. 7.

9. Proceed as in problem no. 7.

10. $$C=\frac{\varepsilon_0\,A}{d}=\frac{8\cdot854\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}=1\cdot77\times10^{-11}\text{F}$$

Now, $q=C\,V=1{\cdot}77\times10^{-11}\times100=\mathbf{1{\cdot}77\times10^{-9}\ V}$

11. Now, $C=\dfrac{\varepsilon_0\,K\,A}{d}$

Here, $K=4$, $A=0{\cdot}055\times0{\cdot}04\text{ m}^2$,

$d=0{\cdot}7$ mm $=0{\cdot}7\times10^{-3}$ m

$$\therefore\quad C=\frac{8\cdot854\times10^{-12}\times4\times0\cdot055\times0\cdot04}{0\cdot7\times10^{-3}}$$

$$=\mathbf{1{\cdot}1\times10^{-10}\ F}$$

12. $$C=\frac{\varepsilon_0\,A}{d}=\frac{8\cdot854\times10^{-12}\times0\cdot02}{1\times10^{-3}}=1\cdot77\times10^{-10}\text{ F}$$

Now, $$V=\frac{q}{C}=\frac{10^{-9}}{1\cdot77\times10^{-10}}=\mathbf{5{\cdot}65\ V}$$

On increasing the separation :

$$C=\frac{\varepsilon_0\,A}{d}=\frac{8\cdot854\times10^{-12}\times0\cdot02}{2\times10^{-3}}=8\cdot854\times10^{-11}\text{ F}$$

Now, $V = \frac{q}{C} = \frac{10^{-9}}{8\cdot854\times10^{-11}} = \mathbf{11{\cdot}3\ V}$

13. $C = 4\pi\varepsilon_0 K\cdot\frac{ab}{b-a} = \frac{1}{9\times10^{-9}}\times20\times\frac{0\cdot2\times0\cdot3}{0\cdot3-0\cdot2}$

$= 1{\cdot}333\times10^{-9}\ \text{F} = \mathbf{1333\ pF}$

14. Proceed as in problem no. 13.
15. Proceed as in solved problem no. 5.08.
16. Capacitance per metre,

$$C = \frac{2\pi\varepsilon_0 K}{2\cdot303\log_{10}\frac{b}{a}} = \frac{2\pi\times8\cdot854\times10^{-12}\times12}{2\cdot303\times\log_{10}\frac{0\cdot6\times10^{-3}}{0\cdot1\times10^{-3}}}$$

$= \mathbf{3{\cdot}725\times10^{-10}\ F\ m^{-1}}$

17. Now, $C = \frac{2\pi\varepsilon_0 K l}{2\cdot303\log_{10}\frac{b}{a}}$

Here, $K = 7$, $l = 1$ km $= 10^3$ m, $a = \frac{3}{2} = \mathbf{1{\cdot}5\ mm}$

$b = a$ + thickness of dielectric $= 1{\cdot}5 + 3 = 4{\cdot}5$ mm

$$\therefore\quad C = \frac{2\pi\times8\cdot854\times10^{-12}\times7\times10^3}{2\cdot303\log_{10}\frac{4\cdot5\times10^{-3}}{1\cdot5\times10^{-3}}} = 0{\cdot}354\times10^{-6}\ \text{F}$$

$= \mathbf{0{\cdot}354\ \mu F}$

18. $C_1 = 4\pi\varepsilon_0 r_1 = \frac{2\times10^{-2}}{9\times10^9} = 2\cdot22\times10^{-12}\ \text{F} = 2\cdot22\ \text{pF}$

$C_2 = 4\pi\varepsilon_0 r_2 = \frac{7\times10^{-2}}{9\times10^9} = 7\cdot78\times10^{-12}\ \text{F} = 7\cdot78\ \text{pF}$

When the two spheres are connected in series :

$C = \frac{C_1C_2}{C_1+C_2} = \frac{2\cdot22\times7\cdot78}{2\cdot22+7\cdot78} = \mathbf{1{\cdot}73\ pF}$

When the two spheres are connected in parallel :

$C' = C_1 + C_2 = 2{\cdot}22 + 7{\cdot}78 = \mathbf{10{\cdot}0\ pF}$

19. Proceed as in SAQ 5.11.
20. Let the capacitances of the three capacitors be C, 2 C and 3 C.

When connected in parallel :

$C' = C + 2C + 3C = 6C$

When connected in series :

$\frac{1}{C''} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$ or $C'' = \frac{6C}{11}$

According to the statement of the problem,

$6C = \frac{6C}{11} + \frac{60}{11}$ or $C = 1\ \mu F$

Therefore, capacitances of the three capacitors are **1 μF, 2 μF**, and **3 μF**.

21. Proceed as in solved problem no. 5.12.
22. The capacitors are connected as shown in Fig. 5.91.

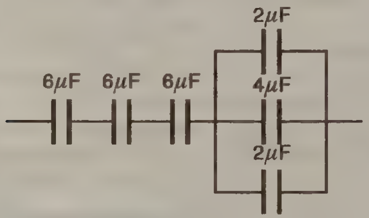


Fig. 5.91

The capacitance of the parallel combination of 2 μF, 4 μF and 2 μF capacitors,

$C' = 2 + 4 + 2 = 8\ \mu F$

The capacitance of the whole combination is given by

$$\frac{1}{C} = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{C'} = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{8} = \frac{15}{24}$$

or $C = \frac{24}{15} = \mathbf{1{\cdot}6\ \mu F}$

23. The capacitance of parallel combination of three capacito each of 20 μF,

$C' = 20 + 20 + 20 = 60\ \mu F$

Now, $\frac{1}{C}+\frac{1}{C'} = \frac{1}{30}$ or $\frac{1}{C} = \frac{1}{30} - \frac{1}{60} = \frac{1}{60}$ or $C = 60$

24. Let C′ be capacitance of the series combination of C_1 a C_2. Then,

$C' = \frac{C_1C_2}{C_1+C_2}$.

Therefore, equivalent capacitance of the system betwe points X and Y,

$C_{equi} = C' + C + C' = C + 2C' = C + \frac{2C_1C_2}{C_1+C_2}$

25. Refer to SAQ 5.14 : The three capacitors connect between points A and B are in parallel. If C′ is capacitance of their combination, then

$C' = 5 + 10 + 15 = 30\ \mu F$

Now, capacitor C′ and 30 μF capacitor are in seri Therefore, equivalent capacitance between points X anc

$C = \frac{C'\times30}{C'+30} = \frac{30\times30}{30+30} = \mathbf{15\ \mu F}$

26. By proceeding as in solved problem 5.19, find equivalent capacitance C′ between points A and B of arrangement. It can be obtained that

$C' = 5\ \mu F$

Now, capacitor C′ and the left out 5 μF capacitor are series. Therefore, equivalent capacitance between poi X and Y,

$C = \frac{C'\times5}{C'+5} = \frac{5\times5}{5+5} = \mathbf{2{\cdot}5\ \mu F}$

27. The two capacitors are connected in parallel.

$\therefore\quad C = 2{\cdot}2 + 2{\cdot}2 = 4{\cdot}4\ \mu F$

Now, $q = CV = 4{\cdot}4\times10 = \mathbf{44\ \mu C}$

28. It follows that the two 2 μF capacitors connected at extreme left end of the network are in series. The equival capacitance of these two capacitors,

$C_s = \frac{2\times2}{2+2} = 1\ \mu F$

This equivalent capacitor C_s and the next capacitor of 1 are in parallel. Therefore, equivalent capacitance of C_s a this 1 μF capacitor,

$C_p = 1 + 1 = 2\ \mu F$

Now, the series combination of C_p and the next 2 μ connected in parallel to the next 1 μF capacitor. If continue finding equivalent capacitance in such ste finally the series combination of two 2 μF capacitors be left. Therefore, equivalent capacitance of the netwo

$C = \frac{2\times2}{2+2} = \mathbf{1\ \mu F}$

29. Let q_1 and q_2 be charges on the two capacitors.

Then, $q_1 = C_1V = 0{\cdot}5\times10^{-6}\times100 = 50\times10^{-6}\ \text{C} = \mathbf{50\ \mu}$

and $q_2 = C_2V = 0{\cdot}75\times10^{-6}\times100 = 75\times10^{-6} = \mathbf{75\ \mu}$

Therefore, charge taken from the source,

$q = q_1 + q_2 = 50 + 75 = \mathbf{125\ \mu C}$

30. (*i*) Now, $\frac{1}{C_s} = \frac{1}{8}+\frac{1}{12}+\frac{1}{24} = \frac{3+2+1}{24} = \frac{6}{24}$

or $C_s = \mathbf{4\ \mu F}$

(ii) Also, $C_p = 8 + 12 + 24 = \mathbf{44\ \mu F}$

When capacitors are connected in series, charge on each capacitor is same. Let q be charge on each capacitor. If V_1, V_2 and V_3 are potential differences across these capacitors, then

$$V_1 + V_2 + V_3 = 100$$

or $$\frac{q}{8\times10^{-6}} + \frac{q}{12\times10^{-6}} + \frac{q}{24\times10^{-6}} = 100$$

or $$\frac{q}{24\times10^{-6}}(3+2+1) = 100 \quad \text{or} \quad q = \mathbf{4\times10^{-4}\ C}$$

Note. Charge on each capacitor of the series combination,

$$q = C_s \times V = 4\times10^{-6}\times100 = \mathbf{4\times10^{-4}\ C}$$

31. *(i)* By proceeding as in solved problem no. 5.34, it can be obtained that equivalent capacitance,

$$C = \frac{200}{3} = \mathbf{66{\cdot}67\ pF}$$

(ii) Now, $q = C\,V = \frac{200}{3}\times10^{-12}\times300 = \mathbf{2\times10^{-8}\ C}$

32. *(a)* The capacitance of series combination of C_1, C_2 and C_3 is given by

$$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10} \quad \text{or} \quad C' = \frac{10}{3} = 3{\cdot}33\ \mu F$$

Therefore, equivalent capacitance of the combination between points A and D,

$$C = C' + C_4 = 3{\cdot}33 + 10 = \mathbf{13{\cdot}33\ \mu F}$$

(b) Let q_1, q_2, q_3 and q_4 be charge on capacitors C_1, C_2, C_3 and C_4 respectively.

As, C_1, C_2 and C_3 are in series,

$$q_1 = q_2 = q_3 = C' \times V = \frac{10}{3}\times10^{-6}\times500$$
$$= \mathbf{1{\cdot}67\times10^{-3}\ C}$$

Also, $q_4 = C_4 \times V = 10\times10^{-6}\times500 = \mathbf{5{\cdot}0\times10^{-3}\ C}$

33. Equivalent capacitance of parallel combination of C_2 and C_3,

$$C_{23} = C_2 + C_3 = 3 + 5 = 8\ \mu F$$

Equivalent capacitance of the whole network,

$$C = \frac{C_1 \times C_{23}}{C_1 + C_{23}} = \frac{2\times8}{2+8} = 1{\cdot}6\ \mu F$$

The charge acquired by the network,

$$q = C\,V = 1{\cdot}6\times10^{-6}\times100 = 1{\cdot}6\times10^{-4}\ C$$

Since C_1 and C_{23} are in series, charge on each of them is same as on the network *i.e.* equal to $1{\cdot}6\times10^{-4}$ C.

Potential difference across C_1,

$$V_1 = \frac{q}{C_1} = \frac{1{\cdot}6\times10^{-4}}{2\times10^{-6}} = \mathbf{80\ V}$$

As C_2 and C_3 are in parallel,

$$V_2 = V_3 = \frac{q}{C_{23}} = \frac{1{\cdot}6\times10^{-4}}{8\times10^{-6}} = \mathbf{20\ V}$$

34. *(i)* $q_1 = C_1\,V = 10\times10^{-6}\times250 = \mathbf{2500\ \mu C}$,

$q_2 = C_2\,V = 50\times10^{-6}\times250 = \mathbf{12500\ \mu C}$

and $q_3 = C_3\,V = 25\times10^{-6}\times250 = \mathbf{6250\ \mu C}$

(ii) $C_p = C_1 + C_2 + C_3 = 10 + 50 + 25 = \mathbf{85\ \mu F}$

(iii) $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{50} + \frac{1}{25} = \frac{8}{50}$$

or $$C_s = \frac{50}{8} = 6{\cdot}25\ \mu F$$

Charge on each capacitor is equal to charge on the series combination. Therefore,

$$q_1 = q_2 = q_3 = C_s \times V = 6{\cdot}25\times10^{-6}\times250$$
$$= 1{\cdot}5625\times10^{-3}\ C$$

$$\therefore \quad V_1 = \frac{q_1}{C_1} = \frac{1{\cdot}5625\times10^{-3}}{10\times10^{-6}} = \mathbf{156{\cdot}25\ V},$$

$$V_2 = \frac{q_2}{C_2} = \frac{1{\cdot}5625\times10^{-3}}{50\times10^{-6}} = \mathbf{31{\cdot}25\ V}$$

and $$V_3 = \frac{q_3}{C_3} = \frac{1{\cdot}5625\times10^{-3}}{25\times10^{-6}} = \mathbf{62{\cdot}5\ V}$$

35. *(a)* Equivalent capacitance of parallel combination of C_1, C_2 and C_3,

$$C_{123} = C_1 + C_2 + C_3 = 3 + 5 + 4 = 12\ \mu F$$

Equivalent capacitance of series combination of C_{123} and C_4,

$$C' = \frac{C_{123}\times C_4}{C_{123} + C_4} = \frac{12\times4}{12+4} = 3\ \mu F$$

Equivalent capacitance of parallel combination of C_6 and C_7,

$$C_{67} = C_6 + C_7 = 4 + 2 = 6\ \mu F$$

Equivalent capacitance of series combination of C_5 and C_{67},

$$C'' = \frac{C_5 \times C_{67}}{C_5 + C_{67}} = \frac{3\times6}{3+6} = 2\ \mu F$$

Therefore, capacitance of network between points A and B,

$$C = C' + C'' = 3 + 2 = \mathbf{5\ \mu F}$$

(b) Charge on 5 μF capacitor *i.e.* C_2,

$$q_2 = 120\ \mu C = 120\times10^{-6}\ C$$

$$\therefore \quad V_2 = \frac{q_2}{C_2} = \frac{120\times10^{-6}}{5\times10^{-6}} = 24\ V$$

Therefore, charge on parallel combination of C_1, C_2 and C_3,

$$q_{123} = C_{123} \times V_2 = 12\times10^{-6}\times24 = 2{\cdot}88\times10^{-4}\ C$$

Since C_{123} and C_4 are in series,

$$q_4 = q_{123} = 2{\cdot}88\times10^{-4}\ C$$

and $$V_4 = \frac{q_4}{C_4} = \frac{2{\cdot}88\times10^{-4}}{4\times10^{-6}} = 72\ V$$

Hence, potential difference between points A and B,

$$V = V_2 + V_4 = 24 + 72 = \mathbf{96\ V}$$

36. Here, $C_1 = 2\ \mu F$; $C_2 = 4\ \mu F$; $V_1 = 100$ V ; $V_2 = 75$ V

$$q_1 = C_1\,V_1 = 2\times100 = \mathbf{200\ \mu C}$$

and $$q_2 = C_2\,V_2 = 4\times75 = \mathbf{300\ \mu C}$$

When capacitors are connected in parallel :

Total capacitance, $C = C_1 + C_2 = 2 + 4 = 6\ \mu C$

Total charge, $q = q_1 + q_2 = 200 + 300 = 500\ \mu C$

Therefore, potential difference across the combination (or individual capacitor),

$$V = \frac{q}{C} = \frac{500}{6} = \mathbf{83{\cdot}33\ V}$$

Now, $$q_1' = C_1\,V = 2\times\frac{500}{6} = \mathbf{166{\cdot}67\ \mu C}$$

and $$q_2' = C_2\,V = 4\times\frac{500}{6} = \mathbf{333{\cdot}33\ \mu C}$$

37. ***When capacitors are connected in series with a battery of 100 V :***

Total capacitance, $$C = \frac{C_1\,C_2}{C_1 + C_2} = \frac{4\times2}{4+2} = \frac{4}{3}\ \mu F$$

Charge on capacitors C_1 and C_2,

$$q_1 = q_2 = C\,V = \frac{4}{3}\times100 = \frac{400}{3}\ \mu C$$

When charged capacitors are connected in parallel :

Total charge on the two capacitors,

$$q' = q_1 + q_2 = \frac{400}{3} + \frac{400}{3} = \frac{800}{3}\ \mu C$$

Total capacitance of the combination,

$C' = 4 + 2 = 6\ \mu F$

Common potential of the two capacitors,

$$V = \frac{q'}{C'} = \frac{800/3}{6} = \frac{400}{9}\ V$$

$\therefore \quad q_1' = C_1 V = 4 \times \frac{400}{9} = \mathbf{177{\cdot}8\ \mu C}$

and $\quad q_2' = C_2 V = 2 \times \frac{400}{9} = \mathbf{88{\cdot}9\ \mu C}$

38. Here, $C_1 = 3\ \mu F$; $V_1 = 300$ V ; $C_2 = 2\ \mu F$; $V_2 = 200$ V

Now, $q_1 = C_1 V_1 = 3 \times 300 = 900\ \mu C$

and $\quad q_2 = C_2 V_2 = 2 \times 200 = 400\ \mu C$

When plates of two capacitors charged with unlike charges are connected :

Total charge on the combination,

$q = 900 + (-400) = 500\ \mu C$

Total capacitance of the combination,

$C = C_1 + C_2 = 3 + 2 = 5\ \mu F$

Common potential, $V = \frac{q}{C} = \frac{500}{5} = 100$ V

$\therefore \quad q_1' = 3 \times 100 = 300\ \mu C$

Hence, charge transferred from capacitor C_1,

$q_1 - q_1' = 900 - 300 = 600\ \mu C = \mathbf{6 \times 10^{-4}\ C}$

39. Let C_1 be capacitance of the first conductor,

Here, $V_1 = 40$ V ; $C_2 = 10$ F, $q_2 = 0$

Now, $q_1 = C_1 V_1 = C_1 \times 40$

Common potential,

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 \times 40 + 0}{C_1 + 10} = \frac{C_1 \times 40}{C_1 + 10}$$

Thus, $\frac{C_1 \times 40}{C_1 + 10} = 25$ or $C_1 = \mathbf{16{\cdot}67\ F}$

40. Here, $q_1 = 0{\cdot}5\ \mu C$, $V_1 = 20$ V, $C_2 = 0{\cdot}015\ \mu F$, $q_2 = 0$

Now, $C_1 = \frac{q_1}{V_1} = \frac{0{\cdot}5}{20} = 0{\cdot}025\ \mu F$

When capacitor C_2 is connected momentarily to C_1 (in parallel),

total capacitance, $C = C_1 + C_2 = 0{\cdot}025 + 0{\cdot}015 = 0{\cdot}04\ \mu F$

total charge, $\quad q = q_1 + q_2 = 0{\cdot}5 + 0 = 0{\cdot}5\ \mu C$

common potential, $V = \frac{q}{C} = \frac{0{\cdot}5}{0{\cdot}04} = \mathbf{12{\cdot}5\ V}$

Now, $q_1' = C_1 V = 0{\cdot}025 \times 12{\cdot}5 = \mathbf{0{\cdot}3125\ \mu C}$

and $\quad q_2' = C_2 V = 0{\cdot}015 \times 12{\cdot}5 = \mathbf{0{\cdot}1875\ \mu C}$

41. Here, $A_1 = 1000\ cm^2 = 0{\cdot}1\ m^2$; $d_1 = 4$ mm $= 4 \times 10^{-3}$ m ;

$K_1 = 1$

$A_2 = 20\ cm^2 = 20 \times 10^{-4}\ m^2$; $d_2 = 0{\cdot}1$ mm $= 10^{-4}$ m ;

$K_2 = 2{\cdot}5$

$$\therefore \quad C_1 = \frac{\varepsilon_0 K_1 A_1}{d_1} = \frac{8{\cdot}854 \times 10^{-12} \times 1 \times 0{\cdot}1}{4 \times 10^{-3}}$$

$$= 221{\cdot}35 \times 10^{-12}\ F$$

$$\text{and} \quad C_2 = \frac{\varepsilon_0 K_2 A_2}{d_2} = \frac{8{\cdot}854 \times 10^{-12} \times 2{\cdot}5 \times 20 \times 10^{-4}}{10^{-4}}$$

$$= 442{\cdot}7 \times 10^{-12}\ F$$

Now, potential difference across second capacitor,

$V_2 = 100$ V

$\therefore \quad q_2 = C_2 V_2 = 442{\cdot}7 \times 10^{-12} \times 100 = 442{\cdot}7 \times 10^{-10}$ C

As the two capacitors are in series, charge on first capacitor,

$q_1 = q_2 = 442{\cdot}7 \times 10^{-10}$ C

Therefore, potential difference across first capacitor,

$$V_1 = \frac{q_1}{C_1} = \frac{442{\cdot}7 \times 10^{-10}}{221{\cdot}35 \times 10^{-12}} = 200\ V$$

Hence, potential difference across series combinatio

$V = V_1 + V_2 = 200 + 100 = \mathbf{300\ V}$

42. Here, $r_1 = 5{\cdot}0$ cm $= 0{\cdot}05$ m, $r_2 = 10{\cdot}0$ cm $= 0{\cdot}1$ m ;

$V_1 = 600$ V ; $V_2 = 300$ V

Now, $C_1 = 4\pi\varepsilon_0 r_1 = \frac{0{\cdot}05}{9 \times 10^9} = \frac{5}{9} \times 10^{-11}$ F ;

$$C_2 = 4\pi\varepsilon_0 r_2 = \frac{0{\cdot}1}{9 \times 10^9} = \frac{10}{9} \times 10^{-11}\ F$$

Now, total energy, $U = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$

$$= \frac{1}{2} \times \frac{5}{9} \times 10^{-11} \times (600)^2 + \frac{1}{2} \times \frac{10}{9} \times 10^{11} \times (3$$

$$= 10^{-6} + 0{\cdot}5 \times 10^{-6} = \mathbf{1{\cdot}5 \times 10^{-6}\ J}$$

On connecting with wire :

$C = C_1 + C_2 = \frac{5}{9} \times 10^{-11} + \frac{10}{9} \times 10^{-11} = \frac{5}{3} \times 10^{-11}$ F

Total charge, $q = q_1 + q_2 = C_1 V_1 + C_2 V_2$

$$= \frac{5}{9} \times 10^{-11} \times 600 + \frac{10}{9} \times 10^{-11} \times 300 = \frac{20}{3} \times 10$$

$$\text{Total energy,} \quad U' = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(\frac{20}{3} \times 10^{-9})^2}{\frac{5}{3} \times 10^{-11}}$$

$$= \mathbf{1{\cdot}33 \times 10^{-6}\ J}$$

43. Here, V = 50 V and $q = 10\ \mu C$

$\therefore \quad C = \frac{q}{V} = \frac{10}{50} = \mathbf{0{\cdot}2\ \mu F}$

Energy stored, $U = \frac{1}{2} C V^2 = \frac{1}{2} \times 0{\cdot}2 \times 10^{-6} \times (50)^2$

$= \mathbf{2{\cdot}5 \times 10^{-4}\ J}$

44. Here, $C = 50\ \mu F$; V = 10 V

$\therefore \quad q = C V = 50 \times 10 = \mathbf{500\ \mu C}$

Energy stored, $U = \frac{1}{2} C V^2 = \frac{1}{2} \times 50 \times 10^{-6} \times (10)^2$

$= \mathbf{2{\cdot}5 \times 10^{-3}\ J}$

45. Here, $C = 300\ \mu F$; V = 200 V

$\therefore \quad q = C V = 300 \times 10^{-6} \times 200 = 6 \times 10^{-2}$ C

When the distance between plates of a capacitor is hal its capacitance inreases by a factor 2 *i.e.*

$C' = 2C = 2 \times 300 = 600\ \mu F$

Therefore, potential between the plates of capacitor,

$$V' = \frac{q}{C'} = \frac{6 \times 10^{-2}}{600 \times 10^{-6}} = \mathbf{100\ V}$$

Initial energy stored in the capacitor,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \times 300 \times 10^{-6} \times (200)^2 = 6\ J$$

Final energy stored in the capacitor,

$$U' = \frac{1}{2} C' V'^2 = \frac{1}{2} \times 600 \times 10^{-6} \times (100)^2 = 3\ J$$

Therefore, decrease in energy stored, $U - U' = 6 - 3 =$

46. Here, $A = 50\ cm^2 = 50 \times 10^{-4}\ m^2$; $d = 1$ mm $= 10^{-3}$ m

$$(a) \quad C = \frac{\varepsilon_0 A}{d} = \frac{8{\cdot}854 \times 10^{-12} \times 50 \times 10^{-4}}{10^{-3}}$$

$$= 44{\cdot}27 \times 10^{-12}\ F = \mathbf{44{\cdot}27\ pF}$$

(*b*) Now, $q = CV = 44{\cdot}27 \times 10^{-12} \times 45 = \mathbf{1{\cdot}99 \times 10^{-9}\ C}$

Also, $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 44{\cdot}27 \times 10^{-12} \times (45)^2$

$= \mathbf{4{\cdot}48 \times 10^{-8}\ J}$

(*c*) Electric field between the plates of capacitor,

$$E = \frac{V}{d} = \frac{45}{10^{-3}} = \mathbf{4{\cdot}5 \times 10^4\ V\ m^{-1}}$$

47. Here, $C_1 = 400$ pF, $V_1 = 100$ V

$\therefore\ U = \frac{1}{2}C_1 V_1^2 = \frac{1}{2} \times 400 \times 10^{-12} \times (100)^2$

$= \mathbf{2 \times 10^{-6}\ J}$

On connecting the charged capacitor to another capacitor :

Here, $C_2 = 400$ pF

Total capacitance, $C = C_1 + C_2 = 400 + 400 = 800$ pF

Total charge, $q = q_1 + q_2 = C_1 V_1 + 0 = 400 \times 10^{-12} \times 100$

$= 4 \times 10^{-8}$ C

Therefore, common potential,

$$V = \frac{q}{C} = \frac{4 \times 10^{-8}}{800 \times 10^{-12}} = 50\ \text{V}$$

Therefore, energy stored in the system,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 800 \times 10^{-12} \times (50)^2 = \mathbf{10^{-6}\ J}$$

48. Here, $q_1 = 2\ \mu C$; $V_1 = 100$ V ; $q_2 = 1{\cdot}5\ \mu C$ and $V_2 = 60$ V

$\therefore\quad C_1 = \frac{q_1}{V_1} = \frac{2 \times 10^{-6}}{100} = 2 \times 10^{-8}$ F

and $C_2 = \frac{q_2}{V_2} = \frac{1{\cdot}5 \times 10^{-6}}{60} = 2{\cdot}5 \times 10^{-8}$ F

When two capacitors are put into connect :

$C = C_1 + C_2 = 2 \times 10^{-8} + 2{\cdot}5 \times 10^{-8} = 4{\cdot}5 \times 10^{-8}$ F

$q = q_1 + q_2 = 2 + 1{\cdot}5 = 3{\cdot}5\ \mu C = 3{\cdot}5 \times 10^{-6}$ C

Common potential, $V = \frac{q}{C} = \frac{3{\cdot}5 \times 10^{-6}}{4{\cdot}5 \times 10^{-8}} = \mathbf{77{\cdot}78\ V}$

$\therefore\quad q_1' = C_1 V = 2 \times 10^{-8} \times 77{\cdot}78 = \mathbf{1{\cdot}56 \times 10^{-6}\ C}$

and $\quad q_2' = C_2 V = 2{\cdot}5 \times 10^{-8} \times 77{\cdot}78 = \mathbf{1{\cdot}94 \times 10^{-6}\ C}$

Energy stored in the capacitors before connection,

$U = \frac{1}{2}C_1 V_1^2 + \frac{1}{2}C_2 V_2^2$

$= \frac{1}{2} \times 2 \times 10^{-8} \times (100)^2 + \frac{1}{2} \times 2{\cdot}5 \times 10^{-8} \times (60)^2$

$= \mathbf{1{\cdot}45 \times 10^{-4}\ J}$

Energy stored in the capacitors after connection,

$U' = \frac{1}{2}CV^2 = \frac{1}{2} \times 4{\cdot}5 \times 10^{-8} \times (77{\cdot}78)^2$

$= 1{\cdot}36 \times 10^{-4}$ J

49. ***When capacitors are connected in parallel :***

Here, $\frac{1}{2}(C_1 + C_2) \times (3000)^2 = 45$

or $\qquad C_1 + C_2 = 10^{-5}$ ...(*i*)

When capacitors are connected in series :

Now, $\frac{1}{2}\left(\frac{C_1 C_2}{C_1 + C_2}\right) \times (3000)^2 = 4{\cdot}05$

or $\qquad \frac{1}{2}\left(\frac{C_1 C_2}{10^{-5}}\right) \times (3000)^2 = 4{\cdot}05$

or $\qquad C_1 C_2 = 9 \times 10^{-12}$ ...(*ii*)

Solving equations (*i*) and (*ii*), we get

$C_1 = 9 \times 10^{-6}$ F $= \mathbf{9\ \mu F}$

and $\quad C_2 = 10^{-6}$ F $= \mathbf{1\ \mu F}$

50. Here, $C_1 = 7\ \mu$F, V = 10 V

$\therefore\quad U_1 = \frac{1}{2}C_1 V^2 = \frac{1}{2} \times 7 \times 10^6 \times (10)^{-2} = 3{\cdot}5 \times 10^{-4}$ J

Now, $C_2 = 3\ \mu$F, $\quad V = 10$ V

$\therefore\ U_2 = \frac{1}{2}C_2 V^2 = \frac{1}{2} \times 3 \times 10^{-6} \times (10)^2 = 1{\cdot}5 \times 10^{-4}$ J

Therefore, change in energy stored,

$U = U_1 - U_2 = 3{\cdot}5 \times 10^{-4} - 1{\cdot}5 \times 10^{-4}$

$= \mathbf{2{\cdot}0 \times 10^{-4}\ J\ (decrease)}$

The energy is lost as heat and electromagnetic radiation.

51. Here, $C_1 = 6\ \mu$F ; $\quad V_1 = 150$ V

Now, $U_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2} \times 6 \times 10^{-6} \times (150)^2 = 6{\cdot}75 \times 10^{-2}$ J

On connecting the two capacitors :

Let C_2 be capacitance of the other capacitor.

Total capacitance, $C = C_1 + C_2 = 6 \times 10^{-6} + C_2$

Total charge, $q = q_1 + q_2 = C_1 V_1 + 0$

$= 6 \times 10^{-6} \times 150 = 9 \times 10^{-4}$ C

Therefore, common potential,

$$V = \frac{q}{C} = \frac{9 \times 10^{-4}}{6 \times 10^{-6} + C_2}$$

Since common potential is 90 V, we have

$\frac{9 \times 10^{-4}}{6 \times 10^{-6} + C_2} = 90 \quad$ or $\quad C_2 = 4 \times 10^{-6}$ F $= \mathbf{4\ \mu F}$

Total energy stored in the system,

$U = \frac{1}{2}CV^2 = \frac{1}{2}(6 \times 10^{-6} + 4 \times 10^{-6}) \times (90)^2$

$= 4{\cdot}05 \times 10^{-2}$ J

Therefore, loss of energy on connecting the two capacitors,

$U_1 - U = 6{\cdot}75 \times 10^{-2} - 4{\cdot}05 \times 10^{-2} = 2{\cdot}7 \times 10^{-2}$

$= \mathbf{0{\cdot}027\ J}$

52. Proceed as in solved problem no. 5.27.

53. Proceed as in solved problem no. 5.27.

54. Proceed as in solved problem no. 5.27.

55. Here, $C_1 = 25\ \mu F = 25 \times 10^{-6}$ F ; $C_2 = 100\ \mu F = 100 \times 10^{-6}$ F

When capacitors are connected in series :

$$C_s = \frac{C_1 C_2}{C_1 + C_2} = \frac{25 \times 10^{-6} \times 100 \times 10^{-6}}{25 \times 10^{-6} + 100 \times 10^{-6}} = 20 \times 10^{-6}\ \text{F}$$

When connected to 120 V battery, charge on the two capacitors,

$q_1 = q_2 = C_s V = 20 \times 10^{-6} \times 120 = 2{\cdot}4 \times 10^{-3}$ C

On connecting the capacitors in parallel :

Total charge, $q = q_1 + q_2 = 2{\cdot}4 \times 10^{-3} + 2{\cdot}4 \times 10^{-3}$

$= 4{\cdot}8 \times 10^{-3}$ C

Total capacitance, $C_p = C_1 + C_2 = 25 \times 10^{-6} + 100 \times 10^{-6}$

$= 125 \times 10^{-6}$ F

(*a*) Potential difference between the plates of each capacitor,

$$V' = \frac{q}{C_p} = \frac{4{\cdot}8 \times 10^{-3}}{125 \times 10^{-6}} = \mathbf{38{\cdot}4\ V}$$

(*b*) Energy stored in the series combination,

$$U = \frac{1}{2}C_s V^2 = \frac{1}{2} \times 20 \times 10^{-6} \times (120)^2 = 0{\cdot}144\ \text{J}$$

Energy stored in parallel combination,

$$U' = \frac{1}{2} C_p V'^2 = \frac{1}{2} \times 125 \times 10^{-6} \times (38 \cdot 4)^2 = 0 \cdot 092 \text{ J}$$

Therefore, energy lost in the process,

$$U - U' = 0{\cdot}144 - 0{\cdot}092 = \mathbf{0{\cdot}052\ J}$$

56. $U = \frac{1}{2} C V^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (200)^2 = \mathbf{0{\cdot}04\ J}$

On inserting dielectric :

$$C' = K C = 2 \times 2 \times 10^{-6} = 4 \times 10^{-6} \text{ F}$$

$$\therefore \quad U' = \frac{1}{2} C' V^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2 = \mathbf{0{\cdot}08\ J}$$

57. Now, $q = C V = 20 \times 10^{-6} \times 220 = \mathbf{4{\cdot}4 \times 10^{-3}\ C}$

Also, $U = \frac{1}{2} C V^2 = \frac{1}{2} \times 20 \times 10^{-6} \times (220)^2 = \mathbf{0{\cdot}484\ J}$

On inserting dielectric :

$$C' = K C = 55 \times 20 \times 10^{-6} = 1{\cdot}1 \times 10^{-3} \text{ C}$$

$$\therefore \quad V' = \frac{q}{C'} = \frac{4 \cdot 4 \times 10^{-3}}{1 \cdot 1 \times 10^{-3}} = \mathbf{4\ V}$$

and $U' = \frac{1}{2} C' V'^2 = \frac{1}{2} \times 1 \cdot 1 \times 10^{-3} \times (4)^2$

$$= \mathbf{8{\cdot}8 \times 10^{-3}\ J}$$

58. Let A be area of each plate and d be distance between them. Then,

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{...}(i)$$

On introducing dielectric slab :

Let K be dielectric constant of the slab and d', the distance between the plates. Then,

$$C = \frac{\varepsilon_0 A}{d' - t(1 - 1/K)} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$d' - t\left(1 - \frac{1}{K}\right) = d$$

Here, $d' = d + 3{\cdot}2 \times 10^{-3}$ m and $t = 4$ mm $= 4 \times 10^{-3}$ m

$$\therefore \quad d + 3{\cdot}2 \times 10^{-3} - 4 \times 10^{-3}\left(1 - \frac{1}{K}\right) = d$$

or $\quad 1 - \frac{1}{K} = \frac{3 \cdot 2 \times 10^{-3}}{4 \times 10^{-3}} = 0 \cdot 8$

or $\quad K = 5$

59. ***Before inserting dielectric slab :***

$$C = \frac{\varepsilon_0 A}{d}; q = C V = \frac{\varepsilon_0 A V}{d}$$

or $\quad q = \frac{\varepsilon_0 A V}{5 \times 10^{-3}} \qquad \text{...}(i)$

After inserting dielectric slab :

$$C' = \frac{\varepsilon_0 A}{d - t(1 - 1/K)}, \quad q' = C' V = \frac{\varepsilon_0 A V}{d - t(1 - 1/K)}$$

or $\quad q' = \frac{\varepsilon_0 A V}{5 \times 10^{-3} - 2 \times 10^{-3}(1 - 1/K)} \qquad \text{...}(ii)$

Now, $q' = \frac{125}{100} q$

Using equations (*i*) and (*ii*), we have

$$\frac{\varepsilon_0 A V}{5 \times 10^{-3} - 2 \times 10^{-3}(1 - 1/K)} = \frac{125}{100} \times \frac{\varepsilon_0 A V}{5 \times 10^{3}}$$

or $\quad 5 \times 10^{-3} - 2 \times 10^{-3}\left(1 - \frac{1}{K}\right) = 4 \times 10^{-3}$

or $\quad K = 2$

60. ***Before inserting dielectric slab :***

$$C = \frac{\varepsilon_0 A}{d} = \frac{8 \cdot 854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}} = 17 \cdot 7 \times 10^{-12} \text{ F}$$

$$\therefore \quad q = C V = 17{\cdot}7 \times 10^{-12} \times 200 = \mathbf{3{\cdot}54 \times 10^{-9}\ C}$$

Now, proceed as in solved problem no. 5.28.

61. The capacitance of capacitor on introducing dielectric sl

$$C = 5 \times 2 = 10\ \mu\text{F}$$

$$\therefore \quad q = C V = 10 \times 10^{-6} \times 100 = 10^{-3} \text{ C}$$

On removing the dielectric slab :

(*a*) The capacitance of capacitor, $C' = 2\ \mu$F

$$\therefore \quad V' = \frac{q}{C'} = \frac{10^{-3}}{2 \times 10^{-6}} = \mathbf{500\ V}$$

(*b*) Initial energy stored in the capacitor,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \times 10 \times 10^{-6} \times (100)^2 = 0 \cdot 05 \text{ J}$$

Final energy stored in the capacitor,

$$U' = \frac{1}{2} C' V'^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (500)^2 = 0 \cdot 25 \text{ J}$$

Therefore, work done to remove the dielectric slab,

$$W = U' - U = 0{\cdot}25 - 0{\cdot}05 = \mathbf{0{\cdot}2\ J}$$

62. Proceed as in solved problem no. 5.30.

63. Let r be radius of the metal sphere.

Here, $\frac{dV}{dr} = E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = 3 \times 10^6 \text{ V m}^{-1}$

and $\quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = 3 \times 10^6 \text{ V}$

Now, $r = \frac{V}{E} = \frac{3 \times 10^6}{3 \times 10^6} = \mathbf{1\ m}$

and $U = \frac{1}{2} C V^2 = \frac{1}{2} \times (4\pi\varepsilon_0 r) \times V^2 = \frac{1}{2} \times \frac{1 \times (3 \times 10^6}{9 \times 10^9}$

$$= \mathbf{500\ J}$$

64. The given arrangement is equivalent to the para combination of three capacitors C_1, C_2 and C_3. The pl area of each of the three capacitors is A/3, while the pl separations in the three capacitors are d, $d + b$ and $d +$ respectively.

$$\therefore \quad C_1 = \frac{\varepsilon_0 A/3}{d}, \quad C_2 = \frac{\varepsilon_0 A/3}{d + b} \quad \text{and} \quad C_3 = \frac{\varepsilon_0 A/}{d + 2}$$

Hence, equivalent capacitance of the arrangement,

$$C = C_1 + C_2 + C_3$$

On substituting for C_1, C_2 and C_3, it can be obtained t

$$C = \frac{\varepsilon_0 A(3d^2 + 6bd + 2b^2)}{3d(d + b)(d + 2b)}$$

65. The system of three plates is equivalent to the ser combination of two capacitors C_1 and C_2. It follows t

$$C_1 = \frac{\varepsilon_0 \varepsilon_1 A}{d_1} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 \varepsilon_2 A}{d_2}$$

The equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{\varepsilon_0 \varepsilon_1 A} + \frac{d_2}{\varepsilon_0 \varepsilon_2 A}$$

$$= \frac{1}{\varepsilon_0 A}\left(\frac{d_1 \varepsilon_2 + d_2 \varepsilon_1}{\varepsilon_1 \varepsilon_2}\right)$$

or $\quad C = \frac{\varepsilon_0 \varepsilon_1 \varepsilon_2 A}{d_1 \varepsilon_2 + d_2 \varepsilon_1}$

6. The arrangement is equivalent to three capacitors C_1, C_2 and C_3, such that C_1 is parallel to the series combination of C_2 and C_3.

Now, $C_1 = \frac{\varepsilon_0 K_1 A/2}{d} = \frac{\varepsilon_0 K_1 A}{2d}$;

$$C_2 = \frac{\varepsilon_0 K_2 A/2}{d/2} = \frac{\varepsilon_0 K_2 A}{d}$$

and $C_3 = \frac{\varepsilon_0 K_3 A/2}{d/2} = \frac{\varepsilon_0 K_3 A}{d}$

The equivalent capacitance is given by

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

$$= \frac{\varepsilon_0 K_1 A}{2d} + \frac{(\varepsilon_0 K_2 A/d) \times (\varepsilon_0 K_3 A/d)}{\varepsilon_0 K_2 A/d + \varepsilon_0 K_3 A/d}$$

$$= \frac{\varepsilon_0 K_1 A}{2d} + \frac{\varepsilon_0 K_2 K_3 A}{(K_2 + K_3) d} = \frac{\varepsilon_0 A}{d}\left(\frac{K_1}{2} + \frac{K_2 K_3}{K_2 + K_3}\right)$$

Here, $K_1 = 4$; $K_2 = 6$; $K_3 = 2$, $A = 1\ \text{cm}^2 = 10^{-4}\ \text{m}^2$

and $d = 2\ \text{mm} = 2 \times 10^{-3}\ \text{m}$

$$\therefore \quad C = \frac{8 \cdot 854 \times 10^{-12} \times 10^{-4}}{2 \times 10^{-3}}\left(\frac{4}{2} + \frac{6 \times 2}{6 + 2}\right)$$

$= 1{\cdot}55 \times 10^{-12}$ F = **1·55 pF**

Competitive Examination File (Unit I)

REVISION AT A GLANCE

Chapter 1. Coulomb's Law

☐ **Frictional electricity.** *Electricity developed on bodies, when they are rubbed against each other.*

The bodies get charged due to transfer of electrons from one body to the other.

☐ **Smallest amount of charge.** Charge (e) on an electron or a proton ($e = \pm 1{\cdot}6 \times 10^{-19}$ C)

☐ **Additive nature of charge.** *The total charge of a system is equal to the algebraic sum of all individual charges of the sys*

☐ **Quantization of charge.** *Charge (q) on a body is always integral multiple of e i.e.* $q = \pm n\,e$.

This fact is also known as ***discrete nature of charge.***

☐ **Conservation of charge.** *Net charge on an isolated system always remains constant.* In other words, *charge can neith created nor destroyed.*

☐ **Coulomb's law in electrostatics.** *Two electrical charges always attract or repel each other with a force which is dir proportional to the product of the charges and inversely proportional to the square of the distance between them.*

Mathematical form **: 1.** In vacuum (or air), $F = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r^2}$

Here, $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^2$ is ***absolute permittivity*** of vacuum.

2. In a dielectric medium, $F = \dfrac{1}{4\pi\varepsilon_0 K} \cdot \dfrac{q_1 q_2}{r^2}$

Here, K is called ***dielectric constant.*** It is also called ***relative permittivity*** (ε_r) of the medium.

☐ **Coulomb's law in vector form. 1.** Force on charge q_2 due to q_1,

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}\,\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^3}\,\vec{r}_{12}$$

Here, r is distance between the two charges, $\hat{r}_{12}$ is unit vector and $\vec{r}_{12} = r\,\hat{r}_{12}$ is the vector from charge q_1 to q_2.

2. If charges q_1 and q_2 are located at the points, whose position vectors are $\vec{r}_1$ and $\vec{r}_2$, then

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}\,(\vec{r}_2 - \vec{r}_1)$$

☐ **Unit of charge.** It is **coulomb (C)** in SI.

One coulomb is that charge, which will repel an equal and similar charge with a force of 9 × 10^9 N, when placed at a distan 1 m from it in vacuum (or air).

☐ **Dielectric constant.** *The dielectric constant of a medium is the ratio of the force between two charges placed at a certain dist apart in vacuum (or air) to the force between the same two charges placed at the same distance apart in that medium.*

☐ **Principle of superposition.** *When a number of charges are interacting, the total force on a particular charge is equal t vector sum of the forces exerted on it by all other charges.*

Some Useful Facts :

1. The charges developed on the bodies during the process of rubbing are due to the transfer of *electrons from one to the other.*

2. Coulomb's law in electrostatics holds only for stationary and point charges.

3. When the same two charges located in air are placed in a dielectric medium without altering the distance betw them, electrostatic force always decreases.

4. A system of charge is said to be in equilibrium, if the net force experienced by each charge of the system is zer

5. Electrostatic force between two charges is not altered due to the presence of a third charge in their vicinity.

Chapter 2. Electric Field

☐ **Electric field.** *The space around an electric charge, in which its effect can be experienced.*

Quantitatively : *Electric field at a point is equal to the force per unit test charge acting on a vanishingly small positive test ch placed at that point.*

Mathematically : $\vec{E} = \underset{q_0 \to 0}{\mathrm{Lt}} \dfrac{\vec{F}}{q_0}$, where q_0 is vanishingly small test charge.

Unit. In SI, the unit of electric field is **newton coulomb^{-1} (N C^{-1}).**
It is also expressed as **volt metre^{-1} (V m^{-1}).**

☐ **Electric field due to a point charge.** $E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r^2}$, where r is distance of the observation point from the source
ge q.

☐ **Electric dipole.** *A system of two equal and opposite charges separated by a certain distance.*

☐ **Electric dipole moment.** *The product of the magnitude of either charge and the length of the electric dipole.*

Mathematically : $\vec{p} = q(2\,\vec{a})$

Here, $2\,\vec{a}$ is length of the electric dipole and is a vector from charge $-\,q$ to $+\,q$ constituting the dipole.

Unit. Its unit in SI is **coulomb metre (C m).**

☐ **Electric field on axial line of the electric dipole.** 1. At the point at a distance r from the centre of the electric dipole,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

2. For dipole of small length $(2a << r)$, $E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2p}{r^3}$

☐ **Electric field on equitorial line of the electric dipole. 1.** At the point at a distance r from the centre of the electric
le,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$$

2. For dipole of small length $(2a << r)$, $E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{p}{r^3}$

☐ **Electric field due to a circular loop of charge. 1.** For a circular loop of radius a and total charge q, the electric field at
oint on its axis at a distance x from its centre,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qx}{(a^2 + x^2)^{3/2}}$$

2. When the distance of observation point is very large $(x >> a)$, then

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}$$

3. At the centre of the circular loop of charge, $E = 0$

☐ **Torque on an electric dipole placed in a uniform electric field.**

1. For an electric dipole of dipole moment $\vec{p}$ placed in electric field $\vec{E}$,

$$\vec{\tau} = \vec{p} \times \vec{E}$$

2. If θ is angle between the directions of $\vec{p}$ and $\vec{E}$, then

$$\tau = p\,E \sin\theta$$

☐ **Potential energy of an electric dipole in a uniform electric field.**

1. If the electric dipole is rotated from initial orientation making angle θ_1 with the electric field to the final orientation
ing angle θ_2 with the field, then

$$U = p\,E\,(\cos\theta_1 - \cos\theta_2)$$

2. If the electric dipole is rotated from its initial orientation perpendicular to the field to the final orientation so as to
e an angle θ with the field, then

$$U - p\,E\cos\theta = -\,\vec{p}\,.\,\vec{E}$$

☐ **Electric line of force.** *It is the path straight or curved, such that tangent to it at any point gives the direction of electric field*
at point.

ne Useful Facts :

1. The test charge used to measure electric field at a point has to be vanishingly small. If the test charge is not vanishingly
ll, then the test charge may bring about a change in the electric field at the observation point.

For this, one is advised to understand the difference between the statement `*the force experienced by a unit positive test*
ge' and `*the force experienced per unit positive test charge.*' A unit positive charge is quite large and if it is used as the test
ge, it will cause a change in the value of electric field at the observation point.

2. The unit of electric dipole moment is **C m** and not **mC**. It is because, **mC** is used as millicoulomb.

3. The direction of electric field at a point on the axial line of an electric dipole is same as that of $\vec{p}$.

4. The direction of electric field at a point on equitorial line of an electric dipole is opposite to that of $\vec{p}$.

5. For the observation point at very large distance from the charged circular loop ($x >> a$), the loop behaves as a p charge.

6. No torque acts on a dipole, when it is aligned along the direction of electric field.

7. The electrostatic potential energy of the dipole is maximum, when it is aligned antiparallel to the direction of the elec field.

8. The electric lines of force do not exist but what they represent is a reality.

It gives the path along which positive test charge would move, when free to do so.

In a uniform electric field, the lines of force are parallel lines. The electric lines of force crowd near each other in regi of strong electric field. The relative closeness of the lines of force at different points in the electric field gives an estimat the strength of the electric field at these points.

Chapter 3. Electric Potential

☐ **Electric potential difference.** *The electric potential difference between two points in an electric field is defined as the amo of work done per unit positive test charge in moving the test charge from one point to the other against the electrostatic force due to field.*

Mathematically : If W_{AB} is work done in moving a small positive test charge q_0 from point A to B in the electrost field of charge q, then potential difference between points A and B,

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot q\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Here, r_A and r_B are distances of points A and B from the source charge q.

Unit. Its unit in SI is volt (V)

1 volt (V) = 1 joule coulomb^{-1} (J C^{-1})

☐ **Electric potential.** *The electric potential at a point in an electric field is defined as the amount of work done per unit posi test charge in moving the test charge from infinity to that point against the electrostatic force due to the field.*

Mathematically : If $W_{\infty A}$ is work done in moving a small positive test charge from infinity to point A in the electrost field of charge q, then potential at point A,

$$V = \frac{W_{\infty A}}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Here, r is the distance of the point A from the source charge q.

Unit. Its unit is also volt. **1 V = 1 J s^{-1}**

☐ **Electric potential due to group of charges.** The electric potential at a point due to a group of charges is equal to algebraic sum of the electric potentials due to individual charges at that point. It is because of the reason that electric poter is a scalar quantity.

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\ldots \frac{q_n}{r_n}\right) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

☐ **Potential gradient.** *The rate of change of potential with distance at a point is called potential gradient at that point.*

The electric field at a point is equal to negative potential gradient at that point.

Mathematically : $E = -\dfrac{dV}{dr}$

Unit. Its unit in SI is **volt metre^{-1} (V m^{-1}).**

☐ **Equipotential surface.** *The surface at every point of which, the electric potential is same, is called equipotential surface.*

Two equipotential surfaces never intersect each other.

☐ **Electrostatic potential energy of a system of charges.** *It is defined as the work done to put the charges constituting system at their respective locations after having been removed to infinity.*

Mathematically : **1.** Potential energy of the system of two charges q_1 and q_2,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}},$$

2. Potential energy of the system of three charges q_1, q_2 and q_3,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right),$$

where r_{12}, r_{23} and r_{31} have their usual meanings.

Unit. Its unit in SI is **joule (J) or electron volt (eV).**

1 eV = 1·6 × 10^{-19} J

me Useful Facts :

1. The work done in moving a test charge between two points in an electric field is independent of the path followed veen the two points.

2. The work done in moving a test charge over a closed path in an electric field is always zero.

The above two results hold for the reason that electrostatic force between two charges obeys inverse law *i.e.* electrostatic d is inverse square field.

3. The work done per unit positive test charge from a point A to point B is equal to $V_B - V_A$ *i.e.* potential difference between points B and A (and not between points A and B).

4. Electric potential at any point inside a charged conductor is always equal to that at a point on its surface *i.e.* it is same stant) every where.

As a result of it, the electric field inside a charged conductor is zero.

5. For a point outside the charged spherical conductor, the whole charge appears to be concentrated at its centre.

6. The electrostatic potential increases steadily as one moves against the direction of electric field and it decreases in direction of electric field.

The above result is a consequence of negative sign in the relation : $E = -\frac{dV}{dr}$

7. The electric field is always at right angle to the equipotential surface.

8. No work is done in moving a test charge between two points on an equipotential surface.

9. If work has to be done in moving one of the charges of a system from one point to some other, then electrostatic potential rgy of the system increases by an amount equal to the work done in moving the charge.

Chapter 4. Gauss's Theorem

☐ **Electric flux.** *The electric flux through a small surface is defined as the electric lines of force passing through that area, when normally to the lines of force.*

Mathematically : $\phi = \vec{E} \cdot \Delta \vec{S}$,

ere $\vec{E}$ is the electric field and $\Delta \vec{S}$ is the area vector representing the elementary surface area.

Unit. In SI, unit of electric flux is **newton metre^2 coulomb^{-1} (N m^2 C^{-1}).**

☐ **Gauss's theorem.** *It states that the total outward electric flux through a closed surface is* $\frac{1}{\varepsilon_0}$ *times the charge enclosed by the ed surface.*

Mathematically: $\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$,

ere q is charge enclosed by the closed surface.

☐ **Gaussian surface.** *Any closed surface around the charge distribution (may be a point charge, a line charge, a surface charge or lume charge) so that Gauss's theorem can be conveniently applied to find electrical field due to it is called the gaussian surface.*

☐ **Electric field due to infinitely long straight wire of linear charge density λ :**

$$E = \frac{\lambda}{2\pi \varepsilon_0 r},$$

ere r is perpendicular distance of the observation point from the wire.

☐ **Electric field between two infinite thin plane parallel sheets of charge density σ and $-\sigma$:**

$$E = \frac{\sigma}{\varepsilon_0}$$

☐ **Electric field due to infinite thin plane sheet of charge density σ :**

$$E = \frac{\sigma}{2\varepsilon_0}$$

☐ **Electric field due to spherical shell of charge density σ and radius R :**

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$, for $r > R$ (outside the shell)

$E = 0$, for $r < R$ (inside the shell)

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$, for $r > R$ (at the surface)

Here, $q = 4\pi R^2 \sigma$

☐ **Electric field due to a solid sphere of charge density ρ and radius R :**

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$, for $r > R$ (outside the sphere)

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qr}{R^3}, \quad \text{for } r < R \quad \text{(inside the sphere)}$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}, \quad \text{for } r = R \quad \text{(at the surface)}$$

Here, $q = \frac{4}{3}\pi R^3 \rho$

Some Useful Facts :

1. The total electrical flux through a closed surface is not affected by the charge present outside the surface. It depen only upon the charges enclosed by the closed surface. The electric flux through a small portion of the closed surface is affect by the charges present outside the surface.

2. The electric flux through a small portion of the closed surface may change, if the charges inside the closed surfa are moved to the new positions.

3. The total electric flux through a closed surface is not affected, if the charges inside the closed surface are moved new positions.

4. For both the spherical shell and the sphere, electric field at a point outside is same as if the charge is concentrated their centre.

5. Inside a spherical shell, electric potential is zero. It is zero inside a sphere also, provided sphere is conducting.

Chapter 5. Capacitors

☐ **Behaviour of a charged conductor. 1.** Charges reside only at the surface of the charged conductor.
2. The electric potential is constant at the surface and inside the conductor.
3. The electric field is zero inside the conductor and just outside it, the electric field is normal to the surface.

☐ **Electrical capacitance.** *The ability of a conductor to store charge is called its electrical capacitance.*

Mathematically : $C = \frac{q}{V}$

Unit. Its unit in SI is **farad (F).**

1 farad (F) = 1 coulomb volt^{-1} (C V^{-1})

☐ **Capacitance of a spherical conductor.** $C = 4\pi\varepsilon_0 r$
Here, r is radius (in metre) of the spherical conductor.

☐ **Capacitor.** *An arrangement for storing a very large amount of charge.*
Principle. The capacitance of a conductor gets increased greatly, when an earth connected conductor is placed near

☐ **Parallel plate capacitor.** $C = \frac{\varepsilon_0 A}{d}$ (when air is between the plates)

$C = \frac{\varepsilon_0 K A}{d}$ (when dielectric is between the plates)

Here, A is area of each plate and d is separation between the two plates.

☐ **Spherical capacitor.** $C = 4\pi\varepsilon_0 \frac{ab}{b-a}$

Here, a and b are the radii of inner and outer coatings (spherical shells) of the spherical capacitor.

☐ **Cylindrical capacitor.** $C = 2\pi\varepsilon_0 \frac{l}{\log_e \frac{b}{a}} = 2\pi\varepsilon_0 \cdot \frac{l}{2{\cdot}303 \log_{10}\frac{b}{a}}$

Here, a and b are the radii of the inner and outer coatings (cylindrical shells) and l is the length of the either coating the cylindrical capacitor.

☐ **Capacitors in series.** $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

☐ **Capacitors in parallel.** $C = C_1 + C_2 + C_3$

☐ **Energy stored in a capacitor.** Work done in charging a capacitor gets stored in the capacitor in the form of its electrosta potential energy.

Mathematically : $U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C}$

☐ **Polar atom.** *The atom (or molecule) in which the positive and negative charges possess asymmetric charge distribution abo its centre.*

☐ **Polarisation.** *The stretching of the atoms of a dielectric slab (displacement of the charges in the atoms) under the action of applied electric field.*

Due to polarisation, equal and opposite induced charges are developed on the two faces of the slab and an electric field alled electric field due to polarisation is produced. It opposes the applied electric field (E_0). As a result, due to the risation of the dielectric, the strength of the electric field gets reduced inside it.

The reduced value of electric field in a polarised dielectric is given by

$$E = E_0 = E_p$$

☐ **Polarisation density.** *The induced dipole moment developed per unit volume in a dielectric slab on placing it inside the electric is called polarisation density.*

☐ **Dielectric constant.** *The ratio of the strength of applied electric field to the strength of reduced value of electric field on inserting ielectric slab between the plates of a capacitor is called the dielectric constant of the slab.*

☐ **Dielectric strength.** *The maximum value of electric field (or potential gradient) that can be applied to the dielectric without its ric breakdown is called dielectric strength of the dielectric.*

Unit. Its unit in SI is **V m^{-1}** (same as that of electric field).

☐ **Effect of dielectric slab on the capacitance of a parallel plate capacitor.**

1. When a dielectric slab of dielectric constant K and thickness t ($t << d$) is introduced between the plates, then

$$C = \frac{\varepsilon_0 A}{d - t(1 - 1/K)}$$

2. When a conducting slab of thickness t ($t < d$) is introduced, then

$$C = \frac{\varepsilon_0 A}{d(1 - t/d)}$$

☐ **Principle of Van de Graaff generator.** *Electric discharge takes place in air or gases easily at pointed conductors and that a w conductor continues accepting charge through its inner surface irrespective of the fact, howsoever large its potential may grow.*

☐ **Atmospheric electricity. 1.** Earth has an electric field of about 100 V m^{-1} at its surface in downward direction.

2. The surface of earth has a negative surface charge density of about 10^{-9} C m^{-2}.

3. Atmosphere has a slight conductivity up to 50 km. Due to this, about + 1800 C of charge is pumped every second the earth. The earth does not get discharged, as an equal amount of negative charge is continuously pumped into earth to thunder storms and lightning occurring over the earth.

4. Beyond 50 km, the atmosphere is a good conductor.

ne Useful Facts :

1. The capacitors are connected in parallel to increase the capacitance.

2. The capacitors are connected in series to decrease the capacitance.

3. When a dielectric slab of dielectric osnstant K is introduced between the plates of a charged air-capacitor (battery nnected after charging), then

(*a*) capacitance increases by a factor K.
(*b*) potential difference decreases by a factor K.
(*c*) charge on the capacitor remains unaffected.
(*d*) electrostatic potential energy stored in the capacitor decreases by a factor K.

4. When a dielectric slab of dielectric constant K is introduced between the plates of an air capacitor (battery remains connected across the capacitor), then

(*a*) capacitance of the capacitor increases by a factor K.
(*b*) potential difference across capacitor remains unaffected.
(*c*) charge on the capacitor increases by a factor K.
(*d*) electrostatic potential energy stored in the capacitor increases by a factor K.

5. The dielectric slab can be removed from the space between the plates of a charged capacitor only by performing work. ever, the work done appears as increase in the potential energy of the capacitor.

6. When the space between the two plates of a capacitor is filled with two dielectric slabs, then the two parts of the capacitor be looked upon as the combination of two capacitors.

The capacitor behaves as a parallel combination of the two subcapacitors, if each slab is of thickness equal to separation the other hand, it behaves as a series combination of the two subcapacitors, if the total thickness of the two slabs put her is equal to *d*.

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Coulomb's Law

Problem 1.01. Two equally charged identical metal res A and B repel each other with a force 2·0 × 10^{-5} N. her identical uncharged sphere C is touched to A and then placed at the mid point between A and B. What is the net electric force on C ? (*Roorkee, 1981*)

Sol. Consider that two identical spheres A and B each having charge q are placed at a distance r apart as shown in

Fig. 1.01. The two spheres repel each other with a force of $2{\cdot}0 \times 10^{-5}$ N.

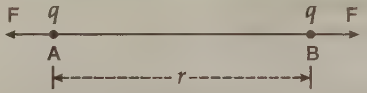


Fig. 1.01.

Now, $$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times q}{r^2}$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2} = 2{\cdot}0\times10^{-5} \qquad ...(i)$$

When an identical uncharged sphere C is touched to A, both will share the charge on A equally *i.e.* each of the spheres A and C will possess charge $q/2$. The sphere C placed at the mid point between A and B experiences forces F_{CA} (due to sphere A) and F_{CB}(due to sphere B) in the directions as shown in Fig 1.02.

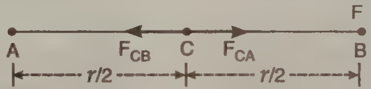


Fig. 1.02.

Now, F $$F_{CA} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q/2\times q/2}{(r/2)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2} \qquad \text{(along CB)}$$

and $$F_{CB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q/2\times q}{(r/2)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q^2}{r^2} \qquad \text{(along CA)}$$

Since $F_{CA} >> F_{CA}$, the resultant force on the sphere C is given by

$$F' = F_{CB} - F_{CA}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q^2}{r^2} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2} \qquad \text{(along CA)}$$

Using equation (*i*), we have

$$F' = 2{\cdot}0\times10^{-5}\ \text{N} \qquad \text{(along CA)}$$

Problem 1.02. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle 30° with each other. When suspended in a liquid of density 800 kg m^{-3}, the angle remains the same. What is the dielectric constant of the liquid ? The density of the material of the sphere is 1600 kg m^{-3}.

(*Roorkee, 1988 ; I.I.T. 1976*)

Sol. The two spheres A and B each of weight W and each carrying charge q, when suspended by strings of equal lengths, come to rest so that the strings make an angle of 30° with each other. Therefore, each string will make an angle of 15° with the vertical [Fig 1.03].

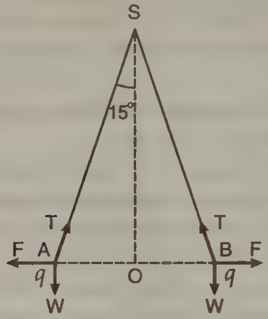


Fig. 1.03

Following forces act on a charged sphere :

(*i*) Weight W acting vertically downwards.

(*ii*) Tension T in the string along AS.

(*iii*) Electrostatic force F on one sphere due to the oth

As the sphere A is in equilibrium, the three forces W and T acting on it, can be represented by the the three sic SO, OA and AS respectively of the triangle AOS. Therefo

$$\frac{W}{SO} = \frac{F}{OA} = \frac{T}{AS}$$

Here, density of the material of the sphere,

$$\rho = 1600\ \text{kg m}^{-3}$$

If V is volume of the sphere, then

$$W = V\rho g = V\times1600\times9{\cdot}8\ \text{N}$$

Also, $$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{(AB)^2}$$

From equation (*i*), we have

$$F = W\times\frac{OA}{SO}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{(AB)^2} = V\times1600\times9{\cdot}8\ \tan 15° \qquad ...$$

When spheres are suspended in a liquid. Suppose the t spheres are suspended in a liquid of dielectric constant K shown in Fig. 1.04. The weight of the sphere decreases to (say) due to upthrust and electrostatic force between t spheres decreases to F′ (say) due to liquid as the dielect medium. However, the angle between the two strings remai the same.

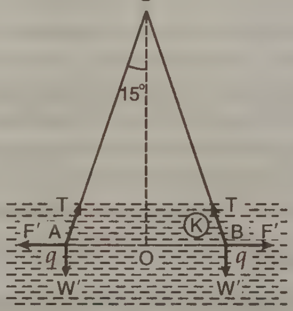


Fig. 1.04

The electrostatic force between the two spheres,

$$F' = \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q^2}{AB^2}$$

The apparent weight of the sphere,

$$W' = W - \text{upthrust}$$

If σ is density of the liquid, then

$$W' = V\rho g - V\sigma g = V(\rho - \sigma) g$$

Since density of the liquid, $\sigma = 800 \text{ kg m}^{-3}$, we have

$$W' = V \times (1600 - 800) \times 9{\cdot}8$$
$$= V \times 800 \times 9{\cdot}8$$

As the sphere is in equilibrium, the three forces <$E ıan W prime by the three sides SO, AO and AS ɔectively of the triangle AOS. Therefore,

$$\frac{W'}{SO} = \frac{F'}{OA} = \frac{T'}{AS}$$

From the above equation, we have

$$F' = W' \times \frac{OA}{SO}$$

or $$\frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q^2}{(AB)^2} = V \times 800 \times 9{\cdot}8 \tan 15° \qquad ...(iii)$$

Dividing the equation (*ii*) by (*iii*), we have

$$K = \frac{V \times 1600 \times 9{\cdot}8 \times \tan 15°}{V \times 800 \times 9{\cdot}8 \tan 15°} = 2$$

Problem 1.03. Three particles each of mass 1 g and ying a charge *q* are suspended from a common point by ılated massless strings, each 100 cm long. If the particles in equilibrium and are located at the corners of an ilateral triangle of sides of length 3 cm, calculate the rge *q* on each particle. Take $g = 10 \text{ m s}^{-2}$. (*I.I.T. 1988*)

ɔught Process

1. *Each particle is under the action of three forces namely its weight, tension in the string and the resultant electrostatic force on it due to the other two particles.*
2. *For the three particles to be in equilibrium, the net force on each particle should be zero.*

Sol. Three particles each of mass 1 g and charge *q* are ɔended from point S with the help of three threads SA, SB SC each of length 100 cm. In equilibrium, the three rges are located at the corners of an equilateral triangle C of sides of length 3 cm as shown in Fig. 1.05.

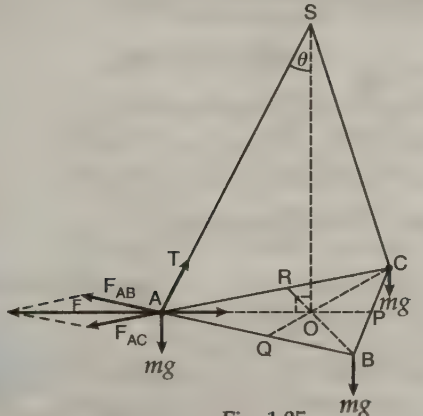


Fig. 1.05.

The point O is vertically below the point of suspension ıd is centroid of the triangle ABC.

Consider the particle A. It is under the effect of the ɔwing three forces :

(*i*) Weight *m g* acting vertically downwards.

(*ii*) Tension T in the string along AS.

(*iii*) Electrostatic force F, which is resultant of the forces, F_{AB} and F_{AC} acting on it A due to particles B and C respectively.

Now, AB = AC = 3 cm = 03 m

Therefore, $$F_{AB} = F_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(0{\cdot}03)^2}$$

$$= 9 \times 10^9 \times \frac{q^2}{9 \times 10^{-4}}$$

$$= 10^{13} q^2$$

The forces F_{AB} and F_{AC} are inclined at 60°. Therefore,

$$F = (F_{AB}^2 + F_{AC}^2 + 2 F_{AB} F_{AC} \cos 60°)^{1/2}$$

$$= \left[(10^{13} q^2)^2 + (10^{13} q^2)^2 + 2 \times 10^{13} q^2 \times 10^{13} q^2 \times 0{\cdot}5\right]^{1/2}$$

$$= 10^{13} q^2 \sqrt{1+1+1}$$

$$= \sqrt{3} \times 10^{13} q^2 \qquad \text{(along OA)}$$

Since $m = 1 \text{ g} = 10^{-3} \text{ kg}$ and $g = 10 \text{ m s}^{-2}$,

$$m g = 10^{-3} \times 10 = 10^{-2} \text{ N} \quad \text{(vertically downwards)}$$

Since the particle A is in equilibrium, the forces F, *m g* and T acting on the particle A can be represented respectively by the sides OA, SO and AS of the triangle OAS. Therefore,

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS}$$

or $$F = mg \frac{OA}{SO} = mg \tan\theta, \qquad ...(i)$$

where θ is the angle the string AS makes with the vertical.

To find tan θ : For the sake of clarity ; in Fig. 1.06, the equilateral Δ ABC of Fig. 1.05 has been reproduced.

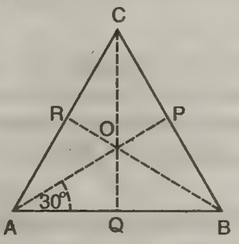


Fig. 1.06

Obviously, ∠ OAQ = 30° and from right angled triangle AQO, we have

$$\cos 30° = \frac{AQ}{AO} = \frac{\frac{1}{2} AB}{AO}$$

or $$AO = \frac{AB}{2\cos 30°} = \frac{3}{2\sqrt{3}/2} = \sqrt{3} \text{ cm}$$

Since AO < < SO, the angle θ is also very small and therefore

$$\tan\theta \approx \sin\theta = \frac{AO}{AS} = \frac{\sqrt{3}}{100}$$

Substituting for F, *mg* and tan θ in equation (*i*), we have

$$\sqrt{3} \times 10^{13} q^2 = 10^{-2} \frac{\sqrt{3}}{100}$$

or $\quad q^2 = 10^{-17} = 10 \times 10^{-18}$

or $\quad q = 3{\cdot}162 \times 10^{-9}$ C

Problem 1.04. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0·8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity, which should be imparted to the lower ball so that it can make complete revolution. *(I.I.T. 2001)*

Thought Process

1. *The suspended charged ball will be just able to complete the revolution, if the necessary centripetal force required at the highest point of the vertical circular path is provided by resultant of its weight and electrostatic repulsion between the two charged balls.*
2. *When the suspended charged ball goes from lowest to the highest point, electrostatic potential energy remains the same. Therefore, increase in gravitational potential energy of the ball is equal to decrease in its kinetic energy.*

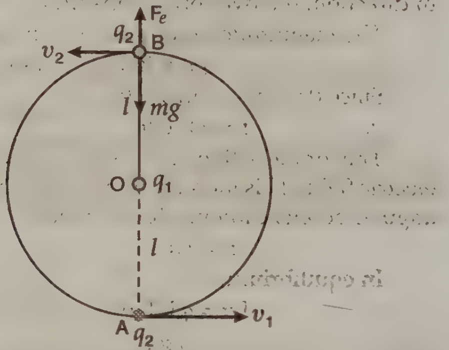


Fig. 1.07

Sol. Here, $q_1 = q_2 = 1\mu\text{C} = 10^{-6}\,\text{C}\,; l = 0{\cdot}8\,\text{m}\,; m = 2 \times 10^{-3}\,\text{kg}$

Suppose that the charged ball A is imparted horizo velocity v_1 at the lowest point A, so that it can just complete revolution [Fig. 1.07]. It requires that tension i string at the highest point is zero.

At the highest point B : Let v_2 be velocity of the b the highest point. The required centripetal force $\frac{m v_2}{l}$ provided by the resultant of weight ($m\,g$) of the suspen ball and the electrostatic repulsion (F_e) between the charged balls *i.e.*

$$\frac{m v_2^2}{l} = m\,g - F_e \quad \text{or} \quad \frac{m v_2^2}{l} = m\,g - \frac{1}{4\pi\varepsilon_0} \cdot q$$

$$\text{or} \quad v_2^2 = g\,l - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1\,q_2}{m\,l}$$

$$= 9{\cdot}8 \times 0{\cdot}8 - \frac{9 \times 10^9 \times 10^{-6} \times 10^{-6}}{2 \times 10^{-3} \times 0{\cdot}8}$$

$$= 7{\cdot}84 - 5{\cdot}625 = 2{\cdot}215\ \text{m}^2\,\text{s}^{-2}$$

When the suspended ball goes from point A to B :

Increase in gravitational potential energy

$$= m\,g \times 2\,l = 2 \times 10^{-3} \times 9{\cdot}8 \times 2 \times 0{\cdot}8 = 31{\cdot}36 \times 10$$

Decrease in kinetic energy

$$= \frac{1}{2} m\,v_1^2 - \frac{1}{2} m\,v_2^2 = \frac{1}{2} m\,(v_1^2 - v_2^2)$$

$$= \frac{1}{2} \times 2 \times 10^{-3}\,(v_1^2 - 2{\cdot}215)$$

From conservation of mechanical energy,

$$\frac{1}{2} \times 2 \times 10^{-3}\,(v_1^2 - 2{\cdot}215) = 31{\cdot}36 \times 10^{-3}$$

$$\text{or} \quad v_1^2 = \frac{2 \times 31{\cdot}36 \times 10^{-3}}{2 \times 10^{-3}} + 2{\cdot}215$$

$$\text{or} \quad v_1 = \sqrt{31{\cdot}36 + 2{\cdot}215} = 5{\cdot}79\ \text{m s}^{-1}$$

Chapter 2. Electric Field

Problem 2.01. Four charges of + 4, − 3, + 2 and + 3 coulomb are placed at the corners of a square of each side 1 m. Find the electric field at the centre of the square.

Given, $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}\ \text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$

(I.S.M. Dhanbad, 1989)

Sol. Fig. 2.01 shows the four charges A, B, C and D of a square of each side 1 m. The electric field at the centre O of the square will be in the directions as shown.

Let AO = BO = CO = DO = r.

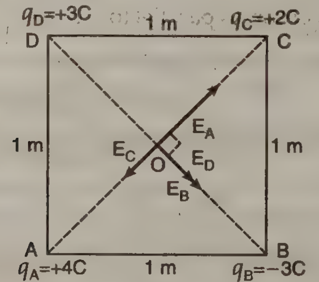


Fig. 2.01

Then, $\sqrt{r^2 + r^2} = 1 \quad \text{or} \quad r = \frac{1}{\sqrt{2}}$ m

Also, $\frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8{\cdot}85 \times 10^{-12}}$

$$= 9 \times 10^9\ \text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$$

Now, $E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{r^2} = 9 \times 10^9\ \frac{4}{(1/\sqrt{2})^2}$

$= 72 \times 10^9$ N (along OC)

$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{r^2} = 9 \times 10^9\ \frac{3}{(1/\sqrt{2})^2}$

$= 54 \times 10^9$ N (along OB)

$E_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_C}{r^2} = 9 \times 10^9\ \frac{2}{(1/\sqrt{2})^2}$

$= 36 \times 10^9$ N (along OA)

$E_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_D}{r^2} = 9 \times 10^9\ \frac{3}{(1/\sqrt{2})^2}$

$= 54 \times 10^9$ N (along OB)

Since $E_A > E_C$, the resultant E_A and E_C acts along OC and iven by

$$E_1 = E_A - E_C = 72 \times 10^9 - 36 \times 10^9 = 36 \times 10^9 \text{ N} \quad \text{(along OC)}$$

The resultant of E_B and E_D is given by

$$E_2 = E_B + E_D = 54 \times 10^9 + 54 \times 10^9 = 108 \times 10^9 \text{ N} \quad \text{(along OB)}$$

The two electric fields E_1 and E_2 have been separately wn in Fig. 2.02.

The two fields are inclined at $\theta = 90°$. Therefore, their ultant is given by

$$E = \sqrt{{E_1}^2 + {E_2}^2}$$

$$= \sqrt{(36 \times 10^9)^2 + (108 \times 10^9)^2} \quad \text{(along OB)}$$

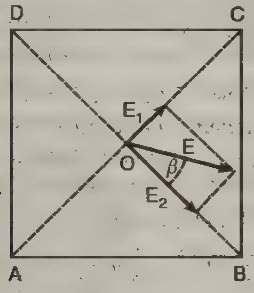


Fig. 2.02.

$$= 36 \times 10^9 \sqrt{1+9} = 1{\cdot}138 \times 10^{11} \text{ N C}^{-1}$$

Suppose that the resultant electric field E makes an angle ith theline OB. Then,

$$\tan\beta = \frac{E_1}{E_2} = \frac{36 \times 10^9}{108 \times 10^9} = 0{\cdot}3333$$

or $\beta = 18{\cdot}43$ (with OB)

Problem 2.02. A point particle of mass M is attached ne end of a massless rigid non conducting rod of length Another point particle of the same mass is attached to the er end of the rod. The two particles carry charges $+q$ and respectively. This arrangement is held in the region of iform electric field E, such that the rod makes a small gle θ (say about 5°) with the field direction [Fig. 2.03]. d an expression for the minimum time needed for the to become parallel to the field after it is set free. *(I.I.T. 1989)*

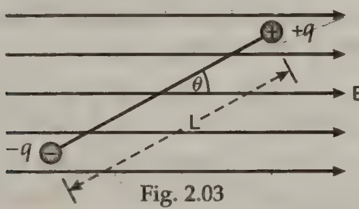


Fig. 2.03

ought Process

1. *The two equal and opposite charges at its two ends of the rod form an electric dipole.*
2. *The system executes S.H.M. under the effect of the torque due to electric field.*
3. *Required time* $= \frac{1}{4} \times$ *time period of S.H.M.*

Sol. The rod AB carrying two particles of charges q and $-q$ at the ends A and B and of mass M each has been placed in electric field, such that it makes a small angle θ with the direction of the field.

The force q E acts on the two particles at points A and B along and opposite to the direction of the electric field as shown in Fig. 2.04.

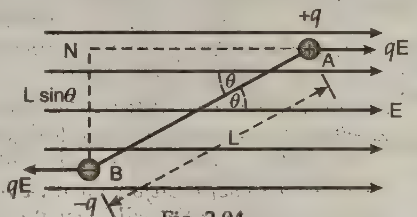


Fig. 2.04

Since the two forces are equal in magnitude and opposite in direction, the two forces consitiute a torque.

The magnitude of torque is given by

$$\tau = q\,\text{E} \times \text{BN} = q\,\text{E L} \sin\theta$$

Since θ is small, $\sin\theta \approx \theta$

$$\therefore \quad \tau = q\,\text{E L}\,\theta$$

The torque acting on the system tends to rotate it in the electric field. If I is moment of inertia of the system and α, the angular acceleration produced, then

$$\tau = \text{I}\alpha$$

In equilibrium,

$$\text{I}\alpha = q\,\text{E L}\,\theta$$

$$\text{or} \quad \alpha = \frac{q\,\text{E L}}{\text{I}}\,\theta \quad \text{or} \quad \alpha \propto \theta$$

Therefore, the motion executed by the system is S.H.M. and its time period is given by

$$\text{T} = 2\pi\sqrt{\frac{\theta}{\alpha}} = 2\pi\sqrt{\frac{\text{I}}{q\,\text{E L}}}$$

Moment of inertia of the system about an axis, which is right bisector of the rod AB is given by

$$\text{I} = 2\pi\left(\frac{\text{L}}{2}\right)^2 + \text{M}\left(\frac{\text{L}}{2}\right)^2 = \frac{1}{2}\text{M L}^2$$

$$\therefore \quad \text{T} = 2\pi\sqrt{\frac{\frac{1}{2}\text{M L}^2}{q\,\text{E L}}} = 2\pi\sqrt{\frac{\text{M L}}{2q\,\text{E}}}$$

The rod becomes parallel to the direction of electric field after executing one fourth of its one vibration *i.e.* in a time $\frac{\text{T}}{4}$.

$$\text{Therefore, required time, } \frac{\text{T}}{4} = \frac{\pi}{2}\sqrt{\frac{\text{M L}}{2q\,\text{E}}}$$

Chapter 3. Electric Potential

Problem 3.01. A charge Q is distributed over two ncentric hollow spheres of radii r and R (when (R > r), such that the surface densities are equal. Find the potential at the common centre. *(I.I.T. 1981)*

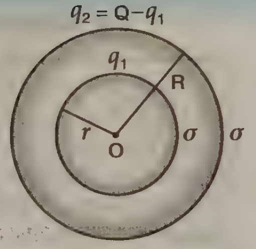


Fig. 3.01

Sol. Fig 3.01 shows two concentric hollow spheres of radii r and R, such that $R > r$.

A total charge Ω is distributed over the two hollow spheres, so that the surface densities of the two spheres are equal. If charge q_1 is distributed over the smaller sphere and q_2 over the larger sphere, then

$$q_1 + q_2 = Q \quad \text{...}(i)$$

If σ is the surface charge density of the two spheres, then

$$\sigma = \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

or $\quad q_1 = 4\pi r^2 \sigma \quad$ or $\quad q_2 = 4\pi R^2 \sigma$

Substituting for q_1 and q_2 in equation (*i*), we have

$$4\pi r^2 \sigma + 4\pi R^2 \sigma = E$$

or $$\sigma = \frac{R}{4\pi (r^2 + R^2)}$$

$\therefore$ $$q_1 = 4\pi r^2 \times \frac{Q}{4\pi (r^2 + R^2)} = \frac{Q r^2}{r^2 + R^2}$$

and $$q_2 = 4\pi R^2 \times \frac{Q}{4\pi (r^2 + R^2)} = \frac{Q R^2}{r^2 + R^2}$$

The potential at a point inside the charged sphere is equal to the potential at its surface.

Therefore, the potential due to the smaller sphere at the common centre,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q r}{(r^2 + R^2)}$$

Also, the potential due to the larger sphere at the common centre,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q R}{(r^2 + R^2)}$$

Therefore, total potential at the common centre,

$$V = V_1 + V_2$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q r}{(r^2 + R^2)} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q R}{(r^2 + R^2)}$$
$$= \frac{1}{4\pi\varepsilon_0} \frac{Q (r + R)}{(r^2 + R^2)}$$

Probelm 3.02 Three concentric spherical metal shells A, B and C of radii a, b and c ($a < b < c$)have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively.

(*i*) Find the potentials of three shells A , B and C.

(*ii*) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c. (*I.I. T. 1990*)

Thought Process

1. The potential of any shell is due to its own charge and the charge on the other two shells.

2. In case of a charged metallic spherical shell or a sphere, (a) for a point outside the conductor, the charge appears to be concentrated at its centre and (b) for all the points inside the conductor, the potential is same every where and is equal to that at a point on its surface.

Sol. Fig 3.02 shows three concentric spherical metal shells A, B and C of radii a, b and c ($a < b < c$).

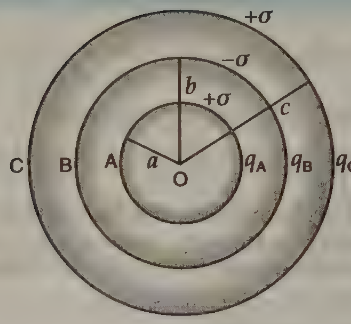


Fig. 3.02

Further, the surface charge densities of the shells A and C are $+\sigma$, $-\sigma$ and $+\sigma$ respectively. If q_A, q_B and q_C charges on the respective shells, then

$$q_A = 4\pi a^2 \sigma, \; q_B = -4\pi b^2 \sigma$$

and $$q_C = 4\pi c^2 \sigma$$

(*i*) Let V_A, V_B and V_C be potentials of the shells A and C.

Potential of shell A : A point on the surface of shell lies inside the shells B and C. Therefore,

$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{a} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{b} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_C}{c}$$
$$= \frac{1}{4\pi\varepsilon_0} \left(\frac{4\pi a^2 \sigma}{a} - \frac{4\pi b^2 \sigma}{b} + \frac{4\pi c^2 \sigma}{c} \right)$$

or $$V_A = \frac{\sigma}{\varepsilon_0} (a - b - c)$$

Potential of shell B : A point on shell B lies outside shell A and inside the shell C. Therefore,

$$V_B = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_A}{b} + \frac{q_B}{b} + \frac{q_C}{c} \right)$$
$$= \frac{1}{4\pi\varepsilon_0} \left(\frac{4\pi a^2 \sigma}{b} - \frac{4\pi b^2 \sigma}{b} + \frac{4\pi c^2 \sigma}{c} \right)$$
$$= \frac{\sigma}{\varepsilon_0} \left(\frac{a^2 - b^2}{b} + c \right)$$

Potential of shell C : A point on shell C lies outside the shells A and B. Therefore,

$$V_C = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_A}{c} + \frac{q_B}{c} + \frac{q_C}{c} \right)$$
$$= \frac{1}{4\pi\varepsilon_0} \left(\frac{4\pi a^2 \sigma}{c} - \frac{4\pi b^2 \sigma}{c} + \frac{4\pi c^2 \sigma}{c} \right)$$

or $$V_C = \frac{\sigma}{\varepsilon_0} \left(\frac{a^2 - b^2 + c}{c} \right)$$

(*ii*) If the shells A and C are at the same potential,

$$V_A = V_C$$

or $$\frac{\sigma}{\varepsilon_0} (a - b + c) = \frac{\sigma}{\varepsilon_0} \left(\frac{a^2 - b^2 + c}{c} \right)$$

or $$(a-b+c)=\frac{a^2-b^2}{c}+c$$

or $$c\ (x-b)=a^2-b^2$$

or $$c=a+b$$

Problem 3.03. A drop of water of mass 18 × 10^{-3} falls ay from the bottom of charged conducting sphere of ius 20 cm, carrying with it a charge of 10^{-9} C and leaving the sphere a uniformly distributed charge of 2·5 × 10^{-6} C. at is the speed of the drop after it has fallen 30 cm ? $\varepsilon_0)^{-1} = 9 \times 10^9$ J m C^{-2}. *(Roorkee, 1985)*

Sol. The drop of water mass m and charge q leaves the ducting sphere of charge Q at point A and falls through istance of 30 cm up to point B [Fig. 3.03]. The drop acquires etic energy at point B at the expense of gravitational ential energy and the electrostatic potential energy. Let v the speed of the drop on reaching the point B. Since, rease in K.E. of drop is equal to decrease in electrostatic l gravitational potential energies, we have

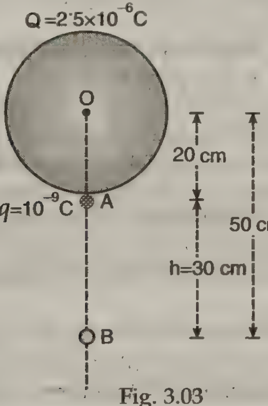


Fig. 3.03

$$\frac{1}{2}mv^2=\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{OA}-\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{OB}\right)+mgh$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot Qq\left(\frac{1}{OA}-\frac{1}{OB}\right)+mgh$$

Here, $h = 30$ cm $= 0{\cdot}3$ m ;

$m = 18 \times 10^{-3}$ g $= 18 \times 10^{-6}$ kg ;

$Q = 2{\cdot}5 \times 10^{-6}$ C ; $q = 10^{-9}$ C,

OA = radius of sphere = 20 cm = 0·2 m ;

OB = 20 + 30 = 50 cm = 0·5 m

$$\therefore\ \frac{1}{2}\times 18\times 10^{-6}\ v^2$$

$$=9\times 10^9\times 2{\cdot}5\times 10^{-6}\times 10^{-9}\times\left(\frac{1}{0{\cdot}2}-\frac{1}{0{\cdot}5}\right)$$

$$+\ 18\times 10^{-6}\times 9{\cdot}8\times 0{\cdot}3$$

$$9\times 10^{-6}\ v^2$$

$$=9\times 2{\cdot}5\times 10^{-6}\ (5-2)+18\times 10^{-6}\times 9{\cdot}8\times 0{\cdot}3$$

$$=67{\cdot}5\times 10^{-6}+52{\cdot}92\times 10^{-6}$$

or $$v^2=\frac{67{\cdot}5+52{\cdot}92}{9}=13{\cdot}38$$

or $$v = 3{\cdot}66\ \text{m s}^{-1}$$

Problem 3.04. Three charges of + 0·1 C each are placed he vertices of an equilateral triangle of each side 1 m. If the energy is supplied at the rate of 1·0 kW, how many hours would be required to move one of the charges on to the mid point of the line joining the other two ?

Given, $$\frac{1}{4\pi\varepsilon_0}=9\times 10^9\ \text{N m}^2\ \text{C}^{-2}.$$

(I.S.M. Dhanbad, 1991)

Thought Process

Work done in moving one of the charges on to the mid point of the line joining the other two charges is equal to the increase in electrostatic potential energy of the system.

Sol. Three charges q_A and q_C, each of 0·1 C are placed at the vertices of the equilateral triangle ABC of each side 1 m as shown in Fig 3.04.

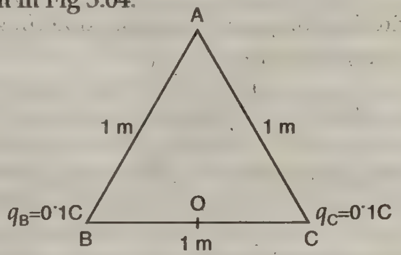


Fig. 3.04

The initial potential energy of the configuration of three charges,

$$U_1=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_A\,q_B}{AB}+\frac{q_B\,q_C}{BC}+\frac{q_C\,q_A}{CA}\right)$$

Since $q_A = q_B = q_C = 0{\cdot}1$ C

and AB = BC = CA = 1 m, we have

$$U_1=9\times 10^9\left(\frac{0{\cdot}1\times 0{\cdot}1}{1}+\frac{0{\cdot}1\times 0{\cdot}1}{1}+\frac{0{\cdot}1\times 0{\cdot}1}{1}\right)$$

$$=27\times 10^7\ \text{J}$$

Suppose the charge q_A is moved from the vertex A to the mid point O of the line AB. Then, the final potential energy of the configuration of three charges,

$$U_2=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_A\,q_B}{OB}+\frac{q_B\,q_C}{OC}+\frac{q_B\,q_C}{BC}\right)$$

Now, $$OB = OC = \frac{AB}{2}=0{\cdot}5\ \text{m}$$

$$U_2=9\times 10^9\left(\frac{0{\cdot}1\times 0{\cdot}1}{0{\cdot}5}+\frac{0{\cdot}1\times 0{\cdot}1}{0{\cdot}5}+\frac{0{\cdot}1\times 0{\cdot}1}{0{\cdot}5}\right)$$

$$=45\times 10^7\ \text{J}$$

Work done to move the charge q_A from point A to O,

$$W = U_2 - U_1 = 45\times 10^7 - 27\times 10^7 = 18\times 10^7\ \text{J}$$

Since energy is supplied at the rate of 1·0 kW *i.e.* 10^3 J s^{-1}, the time required to move the charge q_A from point A to O,

$$t=\frac{18\times 10^7}{10^3}=18\times 10^4\ \text{s}=\frac{18\times 10^4}{3600}=50\ \text{h}$$

Problem 3.05. Three charges each of value q are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle.

(*i*) If Q = – q, will the charges at the corners move towards the centre or fly away from it ?

(ii) For what value of Q, will the charges remain stationary ? In this situation, how much work is done in removing the charges to infinity ? (*I.I.T. 1978*)

Sol. (*i*) Consider that three charges each of value q are placed at the corners of an equilateral triangle ABC of each side of length r as shown in Fig. 3.05.

When the charge Q placed at the centre of the triangle is $-q$, the three charges at the corners of the triangle experience attractive forces due to the charge $-q$ placed at centre. Therefore, ***all the charges will move towards the centre of the triangle.***

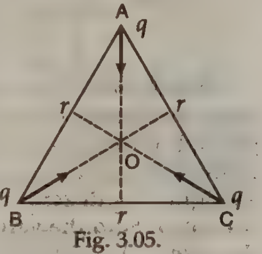


Fig. 3.05.

It may be pointed out that it is the charge at the centre, which experiences zero net force due to charges at the corners. The charge at each corner experiences an unbalanced force due to the charge at centre.

(*ii*) The charge at the centre of the triangle will be in equilibrium irrespective of its magnitude and sign. But a charge at the corner can be in equilibrium only if charge at the centre is negative and has a definite magnitude. Suppose that when a charge $-Q$ is placed at the centre of the triangle, all the charges are in equilibrium. Consider the charge q at point A. It experiences forces F_B, F_C and F_O due to charges lying at points B, C and O as shown in Fig. 3.06.

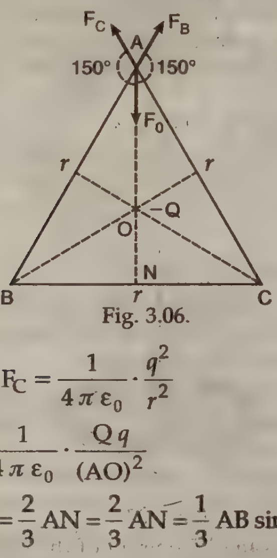


Fig. 3.06.

Now, $$F_B = F_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

and $$F_O = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(AO)^2}$$

Now, $$AO = \frac{2}{3}AN = \frac{2}{3}AN = \frac{1}{3}AB\sin 60°$$

$$= \frac{2}{3}\times r\times\frac{\sqrt{3}}{2}\;\frac{r}{\sqrt{3}}$$

$$\therefore \quad F_O = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Qq}{r^2}$$

For the charge q at point A to be in equilibrium,

$$\frac{F_B}{\sin 150°} = \frac{F_C}{\sin 150°} = \frac{F_O}{\sin 60°} \quad \text{(Lami's theorem)}$$

or $$F_O = F_C\times\frac{\sin 60°}{\sin 30°} = \sqrt{3}\,F_C$$

$(\because \sin 150° = \sin 30°)$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{3Qq}{r^2} = \sqrt{3} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

or $$Q = \frac{q}{\sqrt{3}} \qquad \text{(magnitud}$$

As said above, the charge at the centre of the triangle ha to be negative.

Therefore, $Q = -\dfrac{q}{\sqrt{3}}$

The potential energy of the system in its present stat

$$U_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq}{AB}+\frac{qq}{BC}+\frac{qq}{CA}+\frac{q(-q/\sqrt{3})}{AO}\right.$$
$$\left.+\frac{q(-q/\sqrt{3})}{BO}+\frac{q(-q/\sqrt{3})}{CO}\right.$$

$$= \frac{1}{4\pi\varepsilon_0}q^2\left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}-\frac{1}{\sqrt{3}(r/\sqrt{3})}\right.$$
$$\left.-\frac{1}{\sqrt{3}(r/\sqrt{3})}-\frac{1}{\sqrt{3}(r/\sqrt{3})}\right) =$$

When all the charges are removed to infinity, th potential energy of the system, $U_2 = 0$

Therefore, work done to remove the charges to infinit

$$W = U_1 - U_2 = 0 - 0 = 0$$

Problem 3.06. A conducting sphere S_1 of radius attached to an insulating handle. Another conductin sphere S_2 of radius R is mounted on an insulating stan S_2 is initially uncharged. S_1 is given a charge Q, broug into contact with S_2, and removed. S_1 is recharged such th the charge on it is again Q ; and it is again brought in contact with S_2 and removed. This procedure is repeate *n* times.

(*a*) Find the electrostatic energy of S_2 after *n* suc contacts with S_1.

(*b*) What is the limiting value of this energy as $n \to \infty$

(*I.I.T. 199*

Sol. (*a*) ***After first contact*** : Suppose that the sphere acquires charge q_1 after it is brought in contact with sphe having charge Q. Then, the charge left on sphere S_1 will $Q - q_1$. As the two spheres are conducting, the charg redistribute on them so that their potentials become th same *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q-q_1}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{R}$$

or $$\frac{Q}{r} = \frac{q_1}{r}+\frac{q_1}{R} = \frac{q_1(R+r)}{rR}$$

or $$q_1 = \frac{QR}{R+r}$$

After second contact : The sphere S_1 is recharged, so to have charge Q again. The sphere S_2 is having charge q_1, that the total charge on the two spheres is $Q+q_1$. Suppose th when the sphere S_2 is removed, the charge on S_2 becomes Then,

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q+q_1)-q_2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{R}$$

or $$\frac{Q}{r}+\frac{q_1}{r} = \frac{q_2}{r}+\frac{q_2}{R}$$

or $$\frac{1}{r}(Q+q_1) = q_2\left(\frac{1}{r}+\frac{1}{R}\right) = \frac{q_2(R+r)}{rR}$$

or
$$q_2 = \frac{R}{R+r}(Q+q_1)$$

Substituting the value of q_1, we have

or
$$q_2 = \frac{R}{R+r}\left(Q+\frac{QR}{R+r}\right)$$
$$= \frac{QR}{R+r}\left(1+\frac{R}{R+r}\right)$$

After third contact: Charge q_3 on the sphere S_2 after third act is given by

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q+q_2)-q_3}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_3}{R}$$

or
$$\frac{Q}{r}+\frac{q_2}{r} = \frac{q_3}{r}+\frac{q_3}{R}$$

or
$$\frac{1}{r}(Q+q_2) = q_3\left(\frac{1}{r}+\frac{1}{R}\right) = \frac{q_3(R+r)}{rR}$$

or
$$q_3 = \frac{R}{R+r}(Q+q_2)$$

Substituting the value of q_2, we obtain

$$q_3 = \frac{R}{R+r}\left[Q+\frac{QR}{R+r}\left(1+\frac{R}{R+r}\right)\right]$$

or
$$= \frac{QR}{R+r}\left[1+\left(\frac{QR}{R+r}\right)+\left(\frac{R}{R+r}\right)^2\right]$$

It follows that charge on the sphere S_2 after nth contact be

$$q_n = \frac{QR}{R+r}\left[1+\left(\frac{R}{R+r}\right)+\left(\frac{R}{R+r}\right)^2 + \dots\dots\dots + \left(\frac{R}{R+r}\right)^{n-1}\right]$$

It is a G.P. having first term = 1, common ratio $\frac{R}{R+r}$ and ber of terms equal to n.

∴
$$q_n = \frac{QR}{R+r}\left[\frac{1-\left(\frac{R}{R+r}\right)^n}{1-\left(\frac{R}{R+r}\right)^n}\right]$$
$$= \frac{QR}{r}\left[1-\left(\frac{R}{R+r}\right)^n\right]$$

The energy stored in the sphere S_2 after n contacts,

$$U = \frac{1}{2}\frac{q_n^2}{C},$$

re $C = 4\pi\varepsilon_0 R$ is the capacitance of the sphere S_2.

$$\therefore\quad U = \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0 R}\times\left(\frac{QR}{r}\right)^2\left[1-\left(\frac{R}{R+r}\right)^n\right]^2$$
$$= \frac{Q^2 R}{8\pi\varepsilon_0 r^2}\left[1-\left(\frac{R}{R+r}\right)^n\right]^2$$

(b) When $n \to \infty$:

$$\therefore\quad \underset{n\to\infty}{\text{Lt}}\ U = \underset{n\to\infty}{\text{Lt}}\ \frac{Q^2 R}{8\pi\varepsilon_0 r^2}\left[1-\left(\frac{R}{R+r}\right)^n\right]^2$$
$$= \frac{Q^2 R}{8\pi\varepsilon_0 r^2}[1-0]^2 = \frac{Q^2 R}{8\pi\varepsilon_0 r^2}$$

Problem 3.07. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped along the axis of the disc from a height H with zero initial velocity. The particle has $q/m = 4\pi\varepsilon_0/\sigma$. Find the value of H, if the particle just reaches the disc. (I.I.T. 1999)

Thought Process

1. When the charge is at a height H above the disc, it possesses both electrostatic potential energy and gravitational potential energy.

2. When the charge just reaches the disc, it possesses only electrostatic potential energy.

From the principle of conservation of energy, potential energy (electrostatic + gravitational) of the particle at height H = electrostatic potential energy of the particle at the disc.

Sol. Fig. 3.07 shows the non conducting disc of radius a and centre O having surface charge density σ. The particle of mass m and charge q is dropped from point P.

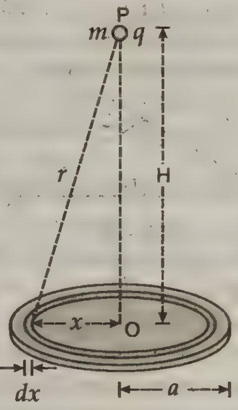


Fig. 3.07

Gravitational potential energy of the particle, when at point P,

$$U_1 = m\,g\,H \qquad \dots(i)$$

Let U_2 be electrostatic potential energy of the particle, when at point P. If V is electrostatic potential due to the charged disc at point P, then

$$U_2 = V\,q$$

To find V. In order to find V, consider a small portion of the disc between two concentric circles of radii x and $x + dx$. If dq is charge on the elementary portion, then potential at point P due to the elementary portion,

$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{r}$$

Now, $dq = (2\pi x\,dx)\,\sigma = 2\pi\sigma x\,dx$;

$r = (H^2 + x^2)^{1/2}$

$$\therefore \quad dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi\sigma x}{(H^2+x^2)^{1/2}}\,dx$$

$$= \frac{\sigma}{2\varepsilon_0}\cdot\frac{x}{(H^2+x^2)^{1/2}}\,dx$$

The potential at point P due to the whole disc can be found by integrating the above within the limits $x = 0$ to $x = a$ *i.e.*

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^a \frac{x}{(H^2+x^2)^{1/2}}\,dx$$

Put $H^2 + x^2 = z$

Then, $2x\,dx = dz$ or $x\,dx = \frac{1}{2}dz$

The limits of integration corresponding to $x = 0$ and $x = a$ now become $z = H^2$ and $z = H^2 + a^2$ respectively.

$$\therefore \ V = \frac{\sigma}{2\varepsilon_0}\int_{H^2}^{H^2+a^2}\frac{\frac{1}{2}dz}{z^{1/2}} = \frac{\sigma}{4\varepsilon_0}\int_{H^2}^{H^2+a^2} z^{-1/2}\,dz$$

$$= \frac{\sigma}{4\varepsilon_0}\left|\frac{z^{1/2}}{\frac{1}{2}}\right|_{H^2}^{H^2+a^2} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{H^2+a^2} - H\right)$$

Therefore, total energy of the particle, when at point P,

$$U = U_1 + U_2 = m\,g\,H + V\,q$$

$$= m\,g\,H + \frac{q\sigma}{2\varepsilon_0}\left(\sqrt{H^2+a^2} - H\right) \qquad ...(i)$$

The energy of the particle, when it reaches the disc, can be found by setting $H = 0$ in the above equation. Thus, energy of the particle, when it reaches the disc,

$$U' = 0 + \frac{q\sigma}{2\varepsilon_0}\left(\sqrt{0^2+a^2} - 0\right) = \frac{q\sigma a}{2\varepsilon_0} \qquad ...(ii)$$

From the principle of conservation of energy, we have

$$U = U'$$

$$\text{or} \quad m\,g\,H = \frac{q\sigma}{2\varepsilon_0}\left(\sqrt{H^2+a^2} - H\right) = \frac{q\sigma a}{2\varepsilon_0} \qquad ...(iii)$$

Now, $\frac{q}{m} = \frac{4\varepsilon_0\,g}{\sigma}$ or $m = \frac{q\sigma}{4\varepsilon_0\,g}$

Substituting for m in equation (*iii*), we have

$$\frac{q\sigma}{4\varepsilon_0\,g}\,g\,H + \frac{q\sigma}{2\varepsilon_0}\left(\sqrt{H^2+a^2} - H\right) = \frac{q\sigma a}{2\varepsilon_0}$$

$$\text{or} \quad \frac{H}{2} + \left(\sqrt{H^2+a^2} - H\right) = a$$

$$\text{or} \quad \sqrt{H^2+a^2} = \left(a + \frac{H}{2}\right)$$

$$\text{or} \quad H^2 + a^2 = \left(a + \frac{H}{2}\right)^2$$

$$\text{or} \quad H = \frac{4}{3}a$$

Problem 3.08. Three charges q, 2 q and 8 q are to be placed on a 9 cm long straight line. Find the positions, where the charges should be placed, such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to th other two charges ? (*I.I.T. 198*

Sol. Consider that the three charges q, 2 q and 8 q ar placed at points A, B and C on a line AC = 9 cm = 0·09 m a shown in Fig. 3.08.

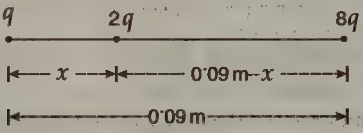


Fig. 3.08.

Suppose that AB = x.

Then, BC = 0·09 − x

The electrostatic potential energy of the system of thre charges is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q\times 2q}{AB} + \frac{q\times 8q}{AC} + \frac{2q\times 8q}{BC}\right)$$

$$\text{or} \quad U = \frac{q^2}{2\pi\varepsilon_0}\left(\frac{1}{x} + \frac{4}{0\cdot 09} + \frac{8}{(0\cdot 09 - x)}\right) \qquad ...(i)$$

For the positions of the three charges shown in the figur potential energy of the system will be minimum, if $\frac{dU}{dx} = 0$

Differentiating both sides of equation (*i*) *w.r.t.* x, we hav

$$\frac{dU}{dx} = \frac{q^2}{2\pi\varepsilon_0}\left(-\frac{1}{x^2} + 0 - \frac{8}{(0\cdot 09 - x)^2}(-1)\right)$$

$$= \frac{q^2}{2\pi\varepsilon_0}\left(\frac{8}{(0\cdot 09 - x)^2} - \frac{1}{x^2}\right)$$

Since U is minimum, $\frac{dU}{dx} = 0$

$$\text{i.e.} \quad \frac{q^2}{2\pi\varepsilon_0}\left(\frac{8}{(0\cdot 09 - x)^2} - \frac{1}{x^2}\right) = 0$$

$$\text{or} \quad \frac{8}{(0\cdot 09 - x)^2} = \frac{1}{x^2}$$

$$\text{or} \quad (0\cdot 09 - x)^2 = 8x^2$$

$$\text{or} \quad 0\cdot 09 - x = 2\sqrt{2}\,x$$

$$\text{or} \quad (1 + 2\sqrt{2})\,x = 0\cdot 09$$

$$\text{or} \quad x = 0\cdot 0235 \text{ m} = 2\cdot 35 \text{ cm}$$

Therefore, potential energy of the system of three charg will be minimum, if the charge 2 q is placed at a distance 2·35 cm from the charge q and 9 − 2·35 *i.e.* 6·65 cm from th charge 8 q.

The electric field at the location of charge q will be th resultant of the electric fields E_B and E_C due to charges 2 q an 8 q located at points B and C respectively. Since both the field will be along CA, the resultant electric field at point A is give by

$$E = E_B = E_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{AB^2} + \frac{8q}{AC^2}\right)$$

$$= 9\times 10^9 \times 2q\left(\frac{1}{(0\cdot 0235)^2} + \frac{4}{(0\cdot 09)^2}\right)$$

$$= 9 \times 10^9 \times 2\,q\,(1810{\cdot}8 + 493{\cdot}8)$$

$$= 4{\cdot}15 \times 10^{13}\,q\ \text{N C}^{-1}$$

roblem 3.09. Two fixed equal positive charges each agnitude 5×10^{-5} are located at points A and B rated by a distance of 6 m. An equal and opposite e moves towards them along the line COD, the endicular bisector of the line AB [Fig. 3.09]. The ng charge, when it reaches the point C at a distance of rom O, has a kinetic energy of 4 J. Calculate the nce of the farthest point D, which the negative charge each before returning towards C. (*I.I.T. 1985*)

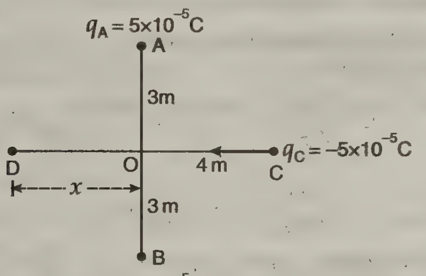


Fig. 3.09.

ught Process

At point C, the charge – q possesses electrostatic potential energy and kinetic energy. It will go up to that farthest point D, where its total energy is in the form of potential energy.

ol. The two charges q_A and q_B each of 5×10^{-5} C are d at the points A and B, such that AB = 6 m. A charge -5×10^{-5} C moves along the bisector COD of the line

At point C, the charge q_C possesses potential energy in ion to the kinetic energy of 4 J.

Now, potential energy of charge of -5×10^{-5} C at the C,

$$U_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C\,q_A}{AC} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C\,q_B}{BC}$$

ince $q_A = q_B = 5 \times 10^{-5}$ C ; $q_C = -5 \times 10^{-5}$ C

nd $AC = BC\sqrt{3^2 + 4^2} = 5$ m, we have

$$U = 9\times10^9 \times \frac{(-5\times10^{-5})\times(5\times10^{-5})}{5}$$

$$+9\times10^9 \times \frac{(-5\times10^{-5})\times(5\times10^{5})}{5}$$

$$= -4{\cdot}5 - 4{\cdot}5 = -9\ \text{J}$$

otal energy of the charge q_C at point C,

E = U + kinetic energy of the charge at point C

$$= -9 + 4 = -5\ \text{J}$$

he charge q_C can move upto a farthest point D, where tential energy will become equal to the total energy of t point C.

Potential energy of charge q_C, when at point D,

$$U' = 9\times10^9 \times \frac{(-5\times10^{-5})\times(5\times10^{5})}{DA}$$

$$+9\times10^9 \times \frac{(-5\times10^{-5})\times(5\times10^{5})}{DB}$$

$$= -22{\cdot}5\left(\frac{1}{DA} + \frac{1}{DB}\right)$$

Suppose that the distance of point D from O is x. Then,

$$DA = DB = \sqrt{3^2 + x^2} = \sqrt{9 + x^2}$$

$$\therefore\ U' = -22{\cdot}5\left(\frac{1}{\sqrt{9+x^2}} + \frac{1}{\sqrt{9+x^2}}\right) = -\frac{45}{\sqrt{9+x^2}}$$

Since U′ = E, we have

$$\text{or}\quad -\frac{45}{\sqrt{9+x^2}} = -5 \qquad \text{or}\quad \sqrt{9+x^2} = 9$$

$$\text{or}\quad x = 8{\cdot}49\ \text{m}$$

Problem 3.10. A particle of mass 40 mg and carrying a charge 5×10^{-9} C is moving directly towards a fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed positive point, it has a velocity of 50 cm s^{-1}. At what distance from the fixed point charge, will the particle come momentarily to rest ? Is the acceleration constant during motion ? (*I.I.T. 1975*)

Hint. This problem is similar to the problem 3.09.

Suppose that the charged particle momentarily comes to rest, when its distance from the fixed charge is r.

Then, P.E. of the system at separation r is equal to the sum of the P.E. of the system at 10 cm separation and the kinetic energy of the moving charge.

Problem 3.11. (*a*) A charge Q coulomb is uniformly distributed over a sphere volume of radius R metres. Obtain an expression for the energy of the system.

(*b*) What will be the corresponding expressions for the energy needed to completely deassemble the planet each against the gravitational pull amongst its constituent particles ? Assume the earth to be a sphere of uniform mass density. Calculate this energy, given that the product of mass and the radius of the earth to be $2{\cdot}5 \times 10^{-31}$ kg m.

(*c*) If the same charge of Q coulomb as in part (*a*) above is given to a spherical conductor of the same radius R, what will be the energy of the system ? (*I.I.T. 1992*)

Sol. (*a*) Consider a sphere of radius R. The charge Q is uniformly distributed over its volume [Fig. 3.10]. If ρ is volume charge density, then

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

The sphere of volume charge density ρ may be assumed to be built by putting the spherical layers of increasing radii around the initial spherical core, till a sphere of radius R is formed.

Suppose that at any instant, there is a spherical core of radius x as shown in the figure.

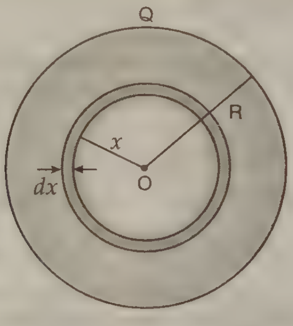


Fig. 3.10.

Then, charge on the spherical core, $q = \frac{4}{3}\pi x^3 \rho$

Suppose a spherical layer of infinitesimally small thickness dx having charge density ρ is put around the spherical core. The small amount of charge on the spherical layer,

dq = volume of the layer $\times \rho = (4\pi x^2 dx)\rho$

The system of the charged spherical core (of radius x) and the charged spherical layer (of thickness dx) possess electrostatic potential energy, which is given by

$$dU = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,dq}{x} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\frac{4}{3}\pi x^3\rho\times(4\pi x^2\,dx)\rho}{x}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{16}{3}\pi^2\rho^2 x^4\,dx$$

Therefore, electrostatic energy of the sphere of radius R is given by

$$U = \int_0^R dU = \frac{1}{4\pi\varepsilon_0}\cdot\frac{16}{3}\pi^2\rho^2\int_0^R x^4\,dx$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{16}{3}\pi^2\rho^2\left|\frac{x^5}{5}\right|_0^R = \frac{1}{4\pi\varepsilon_0}\cdot\frac{16}{3}\pi^2\rho^2\frac{R^5}{5}$$

Substituting for ρ, we have

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{16}{3}\pi^2\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)^2 = \frac{R^5}{5} = \frac{3}{5}\cdot\frac{Q}{4\pi\varepsilon_0 R}$$

(*b*) In expression for gravitational potential energy,

constant $\frac{1}{4\pi\varepsilon_0}$ and Q get replaced by gravitational consta

G and mass M of the earth. In addition to it, the gravitatio potential energy is negative (gravitational force is attract in nature). Therefore,

Gravitational energy of the earth, $U = -\frac{3}{5}\cdot\frac{GM^2}{R}$

Here, R is radius of the earth.

The energy needed to completely deassemble the ea amongst its constituent particles will be

$$W = 0 - U = -U = -\frac{3}{5}\cdot\frac{GM^2}{R}$$

Now, $g = \frac{GM}{R}$

or $GM = gR^2$

$$\therefore \quad W = \frac{3}{5}\cdot\frac{gR^2\times M}{R} = \frac{3}{5}gMR$$

Here, $MR = 2.5\times10^{31}$ kg m and $g = 9.8\times$ m s^{-2}

$$\therefore \quad W = \frac{3}{5}\times 9.8\times 2.5\times 10^{31} = 1.47\times 10^{32}\ \text{J}$$

(*c*) In case, the same charge Q is given to a spheri conductor of radius R, the whole of the charge will app on its outer surface. Then,

Potential of the spherical conductor, $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$

Further, the capacitance of the spherical conductor,

$C = 4\pi\varepsilon_0 R$

Therefore, energy stored in the spherical conductor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\times 4\pi\varepsilon_0 R\times\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}\right)^2 = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R}$$

Chapter 4. Gauss's Theorem

Problem 4.01. It has been experimentally observed that the electric field in a large region of earth's atmospher directed vertically down. At an altitude of 300 m, the electric field is 60 V m^{-1} and at an altitude of 200 m, the field is 100 V m^{-1}. Calculate the net amount of charge contained in a cube of each side of 100 m located between 200 m and 300 m altitude.

Sol. Fig. 4.01 shows a cube of each side 100 m located between 200 m and 300 m altitude. Therefore, area of each of the six faces of the cube,

$$A = 100\times100 = 10^4\ \text{m}^2$$

The electric fields $\vec{E}_1$ and $\vec{E}_2$ at the top and bottom face of the cube are directed along negative Z-axis. Therefore,

$$\vec{E}_1 = -60\,\hat{k}\ \text{V m}^{-1} \quad \text{and} \quad \vec{E}_2 = -100\,\hat{k}\ \text{V m}^{-1}$$

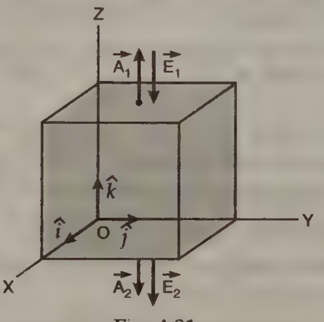


Fig. 4.01

The electric field $\vec{E}_1$ or $\vec{E}_2$ is parallel to the four verti faces of the cube. Therefore, no electric flux crosses throu four faces. If q is charge enclosed by the cube, then ing to Gauss's theorem, electric flux through the cube,

$$\Phi = \vec{E_1}\cdot\vec{A_1} + \vec{E_2}\cdot\vec{A_2}$$

r $\frac{q}{\varepsilon_0} = \vec{E_1}\cdot\vec{A_1} + \vec{E_2}\cdot\vec{A_2}$ ...(*i*)

$\vec{A_1}$ and $\vec{A_2}$ are the area vectors representing the area and bottom face of the cube respectively as shown in ure.

Iow, $\vec{A_1} = 10^4\,\hat{k}\,m^2$ and $\vec{A_2} = -10^4\,\hat{k}\,m^2$

ubstituting for $\vec{E_1}$, $\vec{E_2}$, $\vec{A_1}$ and $\vec{A_2}$ in equation (*i*), we

$$\frac{q}{\varepsilon_0} = (-60\,\hat{k})\cdot 10^4\,\hat{k} + (-100\,\hat{k})\cdot(-10^4\,\hat{k})$$

$$= -6\times10^5 + 10\times10^5 = 4\times10^5$$

r $q = \varepsilon_0 \times 4\times10^5 = 8{\cdot}854\times10^{-12}\times4\times10^5$

$= \mathbf{3{\cdot}542\times10^{-6}\ C}$

roblem 4.02. A conducting sphere of radius 10 cm has known charge. If the electric field 20 cm from the of the sphere is $1{\cdot}5\times10^3$ N C^{-1} and points radially ds, what is the net charge on the sphere ?

ol. Consider a sphere of radius R (= 10 cm) and centre charge on the conductor be q. Further, take a point P listance r = 20 cm from the centre of the sphere [Fig.

he electric field E (= $1{\cdot}5\times10^3$ N C^{-1}) is directed radially ds as shown in the figure. So as to enclose the charged sphere, draw a sphere (gaussian surface) with O as centre and r (= OP = 20 cm) as radius. As the electric field is radial, it is normal to the gaussian surface everywhere. Therefore,

electric flux through the gaussian surface,

$\Phi = -\,\mathrm{E}\times$ area of the gaussian surface

The negative sign shows that the flux is *entering into* the gaussian surface.

$\therefore\ \Phi = -\,\mathrm{E}\times4\pi r^2 = -\,1{\cdot}5\times10^{-2}\times4\pi\,(0{\cdot}2)^2$ ...(*i*)

According to Gauss's theorem,

$$\Phi = \frac{q}{\varepsilon_0} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{q}{\varepsilon_0} = -\,1{\cdot}5\times10^3\times4\pi\times(0{\cdot}2)^2$$

or $q = -\,1{\cdot}5\times10^3\times4\pi\,\varepsilon_0\times(0{\cdot}2)^2$

$$= \frac{-\,1{\cdot}5\times10^3\times(0{\cdot}2)^2}{9\times10^9} = \mathbf{-\,6{\cdot}67\times10^{-9}\ C}$$

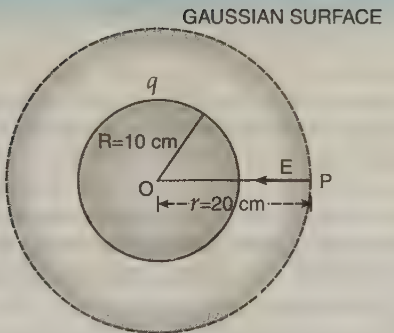


Fig. 4.02.

Chapter 5. Capacitors

roblem 5.01. A radioactive source in the form of a sphere of diameter 10^{-3} m emits β-particles at a nt rate of $6{\cdot}25\times10^{10}$ particles per second. If the source ctrically insulated, how long will it take for its ial to rise by 1·0 V, assuming that 80% of emitted β-es escape from the surface ? *(Roorkee, 1989)*

ol. Here, radius of the metal sphere,

$$r = \frac{10^{-2}}{2} = 0{\cdot}5\times10^{-3}\ \mathrm{m}$$

herefore, capacitance of the metal sphere,

$$C = 4\pi\,\varepsilon_0\,r = \frac{1}{9\times10^9}\times0{\cdot}5\times10^{-7} = \frac{1}{18}\times10^{-12}\ \mathrm{F}$$

equired electric potential of the metal sphere,

V = 1·0 V

q is the charge that should escape from the metal e (in the form of β-particles), so as to raise it to the ed potential, then

$$q = \mathrm{C\,V} = \frac{1}{18}\times10^{-12}\times1 = \frac{1}{18}\times10^{-12}\ \mathrm{C}$$

he number of β-particles emitted per second by the , $n = 6{\cdot}25\times10^{10}$ s^{-1}

ince 80% of emitted β-particles escape from the metal e, the number of β-particles escaping per second

$$= \frac{6{\cdot}25\times10^{10}\times80}{100} = 5\times10^{10}\ \mathrm{s}^{-1}$$

Since charge on a β-particle is $1{\cdot}6\times10^{-19}$ C, the amount of charge escaping per second

$= 5\times10^{10}\times1{\cdot}6\times10^{-19} = 8\times10^{-9}$ C s^{-1}

Then, required time, $t = \frac{q}{8\times10^{-9}}$

$$= \frac{\frac{8}{9}\times10^{-12}}{8\times10^{-9}} = \mathbf{6{\cdot}944\times10^{-6}\ s}$$

Problem 5.02. A capacitor of capacitance C_1 = 1·0 μF withstands the maximum voltage V_1 = 6·0 kV, while another capacitor of capacitance C_2 = 2·0 μF withstands the maximum voltage 4·0 kV. What maximum voltage will the system of these two capacitors withstand, if they are connected in series ? *(M.N.R. 1992)*

Sol. Here, $C_1 = 1{\cdot}0\ \mu F = 10^{-6}$ F ;

$C_2 = 2{\cdot}0\ \mu F = 2\times10^{-6}$ F

Maximum voltage, which capacitor C_1 can withstand,

$V_1 = 6{\cdot}0$ kV $= 6\times10^3$ V

Maximum voltage, which capacitor C_2 can withstand,

$V_2 = 4{\cdot}0$ kV $= 4\times10^3$ V

Let q_1 and q_2 be the values of maximum charge that can be stored by the capacitors C_1 and C_2 respectively.

Then, $q_1 = C_1 V_1 = 10^{-6} \times 6 \times 10^3 = 6 \times 10^{-3}$ C

and $q_2 = C_2 V_2 = 2 \times 10^{-6} \times 4 \times 10^3 = 8 \times 10^{-3}$ C

When capacitors are connected in series, charge on the plates of each capacitor is same. Therefore, when the two capacitors are connected in series, the maximum voltage should not charge C_1 or C_2 to a value more than 6×10^{-3} C, (the maximum charge which the capacitor C_1 can withstand). Therefore, when the capacitors are connected in series,

$$q_{max} = 6 \times 10^{-3}\ \text{C}$$

Let C be capacitance of the two capacitors in series. Then,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10^{-6}} + \frac{1}{2 \times 10^{-6}} = \frac{2+1}{2 \times 10^{-6}}$$

or $C = \frac{2}{3} \times 10^{-6}$ F

Therefore, the maximum voltage which the series combination can withstand,

$$V_{max} = \frac{q_{max}}{C} = \frac{6 \times 10^{-3}}{\frac{2}{3} \times 10^{-6}} = 9 \times 10^3\ \text{V}$$

Problem 5.03. From the given Fig. 5.01, find the value of the capacitance C, if the equivalent capacitance between points A and B is to be 1 μF. *(I.I.T. 1997)*

Sol. Refer to the arrangement of capacitors in Fig. 5.01.

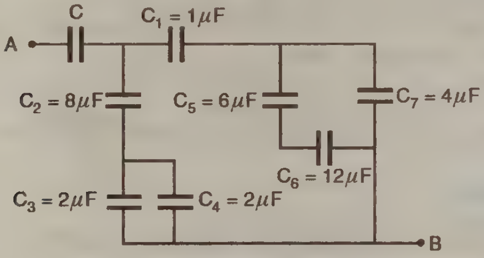


Fig. 5.01.

The two capacitors C_3 and C_4 are in parallel. Their effective capacitance,

$$C_{34} = C_3 + C_4 = 2 + 2 = 4\ \mu F$$

Also, the two capacitors C_5 and C_6 are in series so that their effective capacitance,

$$C_{56} = \frac{C_5\, C_6}{C_5 + C_5} = \frac{6 \times 12}{6 + 12} = 4\ \mu F$$

The given arrangement of capacitors is equivalent to the arrangement as shown in Fig. 5.02.

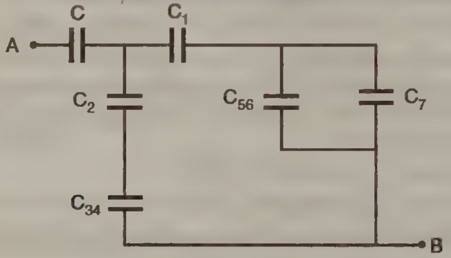


Fig. 5.02.

Since the capacitors C_2 and C_{34} are in series, their effective capacitance,

$$C_{234} = \frac{C_2 \times C_{34}}{C_2 + C_{34}} = \frac{8 \times 4}{8 + 4} = \frac{8}{3}\ \mu F$$

Also, the capacitors C_{56} and C_7 are in parallel so t their effective capacitance,

$$C_{567} = C_{56} + C_7 = 4 + 4 = 8\ \mu F$$

The arrangement shown in Fig. 5.02 is now equiva to the arrangement shown in Fig. 5.03.

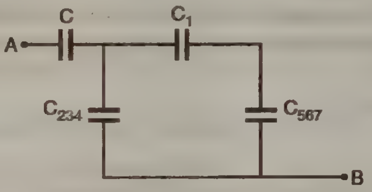


Fig. 5.03.

Since the capacitors C_1 and C_{567} are in series, t effective capacitance,

$$C' = \frac{C_1 \times C_{567}}{C_1 + C_{567}} = \frac{1 \times 8}{1 + 8} = \frac{8}{9}\ \mu F$$

The arrangement shown in Fig. 5.03 can be represen as shown in Fig. 5.04.

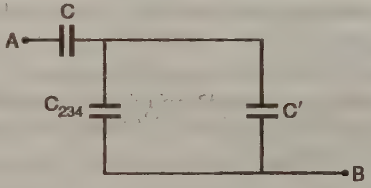


Fig. 5.04

Now, the capacitors C_{234} and C' are in parallel. T effective capacitance,

$$C'' = C_{234} + C' = \frac{8}{9} + \frac{8}{9} = \frac{32}{9}\ \mu F$$

Finally, the given arrangement of the capacitors redu to the one shown in Fig. 5.05.

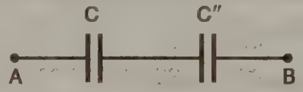


Fig. 5.05.

Since the equivalent capacitance of the gi arrangement between points A and B is 1 μF, it follows

$$\frac{1}{C} + \frac{1}{C''} = \frac{1}{1} \quad \text{or} \quad \frac{1}{C} = 1 - \frac{1}{C''} = 1 - \frac{1}{32/9} = \frac{23}{32}$$

or $C = \frac{23}{32} = 1{\cdot}39\ \mu F$

Problem 5.04. A capacitor is filled with two dielec of same dimensions but of dielectric constant $K_1 = 2$ an $= 3$ respectively. Find the ratio of the capacities in possible arrangements. *(M.N.R. 1*

Thought Process

1. One arrangement of filling the capacitor is to us two dielectric slabs each of area A and thickness d/2 that of the distance between the plates of the capaci As such, the capacitor acts as a series combinatio two capacitors [Fig. 5.06].

2. The other arrangement of filling the capacitor is to take the two dielectric slabs each of thickness d and area A/2. As such, the capacitor acts as a parallel combination of two capacitors [Fig. 5.07].

Sol. A capacitor having plates of area A and separation be filled with two dielectric slabs of dielectric constants d K_2 in the two ways as shown in the Fig. 5.06 and 5.07.

Arrangement in Fig. 5.06 : Let C_1 and C_2 be the ctances of the two parts of the capacitor having dielectric each of area A, thickness $d/2$ and of dielectric constants 2 and $K_2 = 3$.

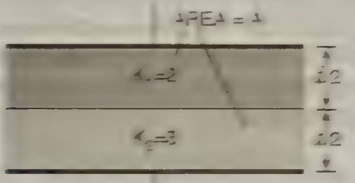


Fig. 5.06.

Then, $C_1 = \frac{\varepsilon_0 K_1 A}{d/2} = \frac{2\varepsilon_0 \times 2 \times A}{d} = \frac{4\varepsilon_0 A}{d}$

and $C_2 = \frac{\varepsilon_0 K_2 A}{d/2} = \frac{2\varepsilon_0 \times 3 \times A}{d} = \frac{6\varepsilon_0 A}{d}$

The capacitor behaves as the parallel combination of C_1 C_2. Therefore, the equivalent capacitance of the gement is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{4\varepsilon_0 A} + \frac{d}{6\varepsilon_0 A} = \frac{15\,d}{12\,\varepsilon_0 A}$$

or $$C = \frac{12\,\varepsilon_0 A}{5\,d}$$

Arrangement in Fig. 5.07 : Let C_1' and C_2' be the citances of the two parts of the capacitor having dielectric each of area A/2, thickness d and dielectric constants 2 and $K_2 = 3$.

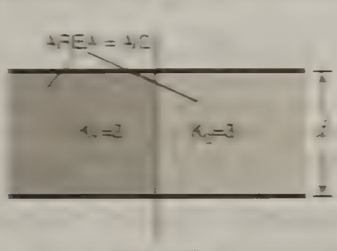


Fig. 5.07

Then, $C_1' = \frac{\varepsilon_0 K_1 . A/2}{d} = \frac{\varepsilon_0 \times 2 \times A}{2d} = \frac{\varepsilon_0 A}{d}$

and $C_2' = \frac{\varepsilon_0 K_2 . A/2}{d} = \frac{\varepsilon_0 \times 3 \times A}{2d} = \frac{3\varepsilon_0 A}{2d}$

The capacitor behaves as the parallel combination of C_1' C_2'.

Therefore, the equivalent capacitance of the arrangement en by

$$C' = C_1' + C_2 = \frac{\varepsilon_0 A}{d} + \frac{3\varepsilon_0 A}{2d} = \frac{5\varepsilon_0 A}{2d}$$

Hence $\frac{C}{C'} = \frac{12\varepsilon_0 A}{5d} \Big/ \frac{5\varepsilon_0 A}{2d} = \frac{24}{25}$

Problem 5.05. A parallel plate capacitor is maintained at a certain potential difference. When a 3 mm thick slab is introduced to keep the potential difference same, the distance between the plates has to be increased by 2·4 mm. Find the dielectric constant of the slab.

(M.N.R. 1982 ; I.S.M. Dhanbad, 1982)

Thought Process

1. It follows that only a part of the space between the two plates has been filled.

2. The charge on the capacitor does not change on introducing dielectric between the plates of a capacitor.

Sol. Let A be the area of each plate and d (in metre) be separation between the two plates of the capacitor.

The capacitance of the parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

When dielectric slab is introduced. The charge on the plates remains unchanged. Therefore, on introducing dielectric slab and increasing the separation between the plates, the capacitance of the capacitor should not alter, so that potential difference remains the same.

Now, the capacitance of the capacitor, when dielectric slab of thickness t is introduced, is given by

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

where d' is the new value of the separation between the two plates of the capacitor.

Here, $d' = d + 2{\cdot}4 \text{ mm} = d + 2{\cdot}4 \times 10^{-3}$ m ;

$t = 3 \text{ mm} = 3 \times 10^{-3}$ m]

$$\therefore \quad C' = \frac{\varepsilon_0 A}{(d + 2{\cdot}4 \times 10^{-3}) - 3 \times 10^{-3}\left(1 - \frac{1}{K}\right)}$$

Equating C and C′, we have

$$\frac{\varepsilon_0 A}{(d + 2{\cdot}4 \times 10^{-3}) - 3 \times 10^{-3}\left(1 - \frac{1}{K}\right)} = \frac{\varepsilon_0 A}{d}$$

or $$(d + 2{\cdot}4 \times 10^{-3}) - 3 \times 10^{-3}\left(1 - \frac{1}{K}\right) = d$$

or $$1 - \frac{1}{K} = \frac{2{\cdot}4 \times 10^{-3}}{3 \times 10^{-3}} = 0{\cdot}8 \quad \text{or} \quad \frac{1}{K} = 0{\cdot}2$$

or $$K = 5$$

Problem 5.06. A 28 μF capacitor is charged to 100 V and another 2 μF capacitor to 200 V. Then they are connected in parallel. Determine the total initial and final energies. Account for the difference in two values.

(I.S.M. Dhanbad, 1983)

Sol. Here, $C_1 = 28\ \mu F = 28 \times 10^{-6}$ F ;

$C_2 = 2\ \mu F = 2 \times 10^{-6}$ F ;

$V_1 = 100$ V V ; $V_2 = 200$ V

Charge on capacitor C_1,

$q_1 = C_1 V_1 = 28 \times 10^{-6} \times 100 = 28 \times 10^{-4}$ C

Charge on capacitor C_2,

$q_2 = C_2 V_2 = 28 \times 10^{-6} \times 100 = 28 \times 10^{-4}$ C

When the two charged capacitors are connected in parallel, they will share charges, till a common potential is attained. If V is common potential, then

$$V = \frac{\text{total charge}}{\text{total capacitor}} = \frac{q_1 + q_2}{C_1 + C_2}$$

$$= \frac{28\times10^{-4} + 6\times10^{-4}}{28\times10^{-6} + 3\times10^{-6}} = 109\cdot68 \text{ V}$$

Total initial energy of the two capacitors,

$$U_1 = \frac{1}{2} C_1 C_1^2 + \frac{1}{2} C_2 V_2^2$$

$$\text{or } U_1 = \frac{1}{2} \times 28\times10^{-6} \times (100)^2 + \frac{12}{2} \times 3\times10^{-6} \times (200)^2$$

$$= 0\cdot14 + 0\cdot06 = 0\cdot2 \text{ J}$$

Total final energy of the two capacitors,

$$U_2 = \frac{1}{2}(C_1 + C_2) V^2$$

$$\text{or } U_2 = \frac{1}{2}(28\times10^{-6} + 3\times10^{-6}) \times (109\cdot68)^2$$

$$= 0\cdot1865 \text{ J}$$

Loss of energy, $U_1 - U_2 = 0\cdot2 \;\; 0\cdot1865 = \mathbf{0\cdot0135 \text{ J}}$

The loss of energy appears as heat and electrical spark.

Problem 5.07. The capacitance of a variable radio capacitor can be changed from 50 pF to 950 pF by turning the dial from 0° to 180°. With the dial set at 180°, the capacitor is connected to a 400 V battery. After charging, the capacitor is disconnected from the battery and the dial is turned at 0°.

(*a*) What is the potential difference across the capacitor, when the dial reads 0° ?

(*b*) How much work is required to turn the dial, if friction is neglected ? (*M.N.R. 1986*)

Thought Process

1. *When the dial is turned at 0°C (from 180°), the capacitance of the capacitor decreases from 950 pF to 50 pF.*
2. *The decrease in capacitance of the capacitor by certain factor brings about increase in potential difference across its plates by the same factor. Since electrostatic energy stored is directly proportional to square of the potential difference, increase in potential energy of the capacitor is quite large.*
3. *The work done to turn the dial at 0° is equal to the increase in the potential energy of the capacitor.*

Sol. Here, the source voltage, V = 400 V

The capacitance of the capacitor, when dial is at 0°,

$$C_1 = 50 \text{ pF} = 50 \times 10^{-12} \text{ F}$$

The capacitance of the capacitor, when dial is at 180°,

$$C_2 = 950 \text{ pF} = 950 \times 10^{-12} \text{ F}$$

The battery is connected to the capacitor, when dial is at 180° *i.e.* when its capacitance is C_2.

Therefore, charge stored in the capacitor,

$$q = C_2 V = 950 \times 10^{-12} \times 400 = 3\cdot8 \times 10^{-7} \text{ C}$$

(*a*) After charging, the dial is turned at 0°, so that its capacitance becomes C_1.

Therefore, potential difference across the capacitor, when the dial reads 0°,

$$V' = \frac{q}{C_1} = \frac{3\cdot8\times10^{-7}}{50\times10^{-12}} = \mathbf{7600 \text{ V}}$$

(*b*) Initial potential energy of the capacitor,

$$U_1 = \frac{1}{2} C_2 V^2 = \frac{1}{2} \times 950 \times 10^{-12} \times (400)^2$$

$$= 7\cdot6 \times 10^{-5} \text{ J}$$

Final potential energy of the capacitor,

$$U_2 = \frac{1}{2} C_1 V'^2 = \frac{1}{2} \times 50 \times 10^{-12} \times (7600)^2$$

$$= 1\cdot444 \times 10^{-3} \text{ J}$$

Therefore, work done to turn the dial,

$$W = U_2 - U_1 = 1\cdot444 \times 10^{-3} - 7\cdot6 \times 10^{-5}$$

$$= \mathbf{1\cdot368 \times 10^{-3} \text{ J}}$$

Problem 5.08. A conductor of capacity 15 uni charged to potential of 40 units and is then made to s its charge with an uncharged sphere, whereby the com potential drops to 30 units. Find the radius of the sphe terms of the loss of energy due to sharing of charge.

(*I.S.M. Dhanbad, 1*

Sol. Here, capacitance of conductor,

$$C_1 = 15 \text{ F (SI units)}$$

Potential of the conductor, V = 40 V (SI u

Therefore, charge on the conductor,

$$q_1 = C_1 V = 15 \times 40 = 600 \text{ C}$$

Let *r* (metre) be the radius of the uncharged spl

Then,

$$\text{Capacitance of the sphere, } C_2 = 4\pi\varepsilon_0 r = \frac{r}{9\times10}$$

Since the sphere is uncharged ($q_2 = 0$), the com potential,

$$V' = \frac{q_1 + q_2}{C_1 + C_2} = \frac{600 + 0}{15 + \frac{r}{9\times10^9}} = \frac{600}{15 + \frac{r}{9\times10^9}}$$

Initial potential energy of the system,

$$U_1 = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 \times 0^2 = \frac{1}{2} \times 15 \times 40^2 = 1\cdot2 \times 10$$

Final potential energy of the system,

$$U_2 = \frac{1}{2}(C_1 + C_2) V'^2$$

$$= \frac{1}{2}\left(15 + \frac{r}{9\times10^9}\right) \times \left(\frac{600}{15 + \frac{r}{9\times10^9}}\right)^2 = \frac{1\cdot8\times}{15 + \frac{}{9\times}}$$

$$= \frac{1\cdot62\times10^{15}}{1\cdot35\times10^{11} + r}$$

On sharing charges , there is always loss of en Therefore,

$$\text{loss of energy, } \Delta U = 1\cdot2\times10^4 - \frac{1\cdot62\times10^{15}}{1\cdot35\times10^{11} + r}$$

$$\text{or} \quad \frac{1\cdot62\times10^{15}}{1\cdot35\times10^{11} + r} = 1\cdot2\times10^4 - \Delta U$$

$$\text{or} \quad 1\cdot35\times10^{11} + r = \frac{1\cdot62\times10^{15}}{1\cdot2\times10^4 - \Delta U}$$

$$\text{or} \quad r = \frac{1\cdot62\times10^{15}}{1\cdot2\times10^4 - \Delta U} - 1\cdot35\times10^{11}$$

$$= \frac{1{\cdot}62\times10^{15} - 1{\cdot}35\times1{\cdot}2\times10^{15} + 1{\cdot}35\times10^{11}\,\Delta U}{1{\cdot}2\times10^{4} - \Delta U}$$

$$= \frac{1{\cdot}62\times10^{15} - 1{\cdot}62\times10^{15} + 1{\cdot}35\times10^{11}\,\Delta U}{1{\cdot}2\times10^{4} - \Delta U}$$

$$= \frac{\mathbf{1{\cdot}35\times10^{11}\,\Delta U}}{\mathbf{1{\cdot}2\times10^{4} - \Delta U}}$$

Problem 5.09. Fig 5.08 shows two identical parallel e capacitors connected to a battery with the switch S ed. The switch is now opened and the free space veen the plates of the capacitors is filled with a ectric of dielectric constant (or relative permittivity) 3. the ratio of the total electrostatic energy stored in both citors before and after the introduction of the dielectric.

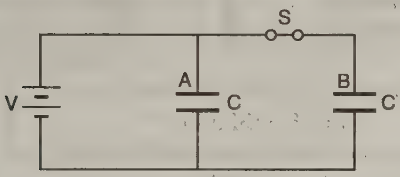


Fig. 5.08.

ought Process

1. *On introduction of dielectric slab, the potential difference between the plates of a capacitor decreases by a factor equal to K (dielectric constant).*
2. *When the switch S is opened, the potential difference will not decrease in case of capacitor A. It is because, even on opening the switch, the capacitor A remains connected to the battery.*

Sol. ***When the switch is closed.*** The potential difference ss the plates of both the capacitors will be equal to the ery voltage V.

Therefore, electrostatic energy stored in the two citors, $U_1 = \frac{1}{2} C V^2 \times 2 = C V^2$

When the switch is opened and dielectric is introduced. en dielectric of dielectric constant K = 3 is introduced een the plates of the two capacitors, the capacitance of of the two capacitors A and B will become K times *i.e.* will become equal to 3 C. Further, whereas the potential erence across the capacitor A will remain V (as it still ains connected to the battery), the potential difference ss the capacitor B will become $\frac{V}{3}$.

Therefore, electrostatic energy stored in the two citors,

$$U_2 = \frac{1}{2}(3\,C)\times V^2 + \frac{1}{2}(3\,C)\times\left(\frac{V}{3}\right)^2$$

$$= \frac{3}{2} C V^2 + \frac{1}{6} C V^2 = \frac{5}{3} C V^2$$

Thus, $\frac{U_1}{U_2} = \frac{C V^2}{\frac{5}{3} C V^2} = \frac{3}{5}$

Problem 5.10. Two parallel plate capacitors A and B e the same separation d = 8·85 × 10^{-4} m between the es. The plate areas of A and B are 0·04 m^2 and 0·02 m^2 respectively. A slab of dielectric constant (relative permittivity) K = 9, has dimensions, such that it can exactly fill the space between the plates of capacitor B.

(*i*) The dielectric slab is placed inside A as shown in Fig. 5.09. A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.

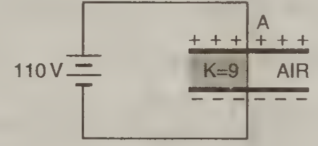


Fig. 5.09

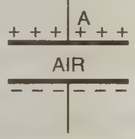


Fig. 5.10

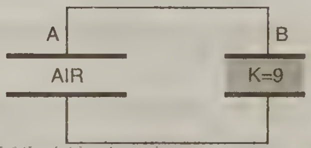


Fig. 5.11

(*ii*) The battery is disconnected and then the dielectric slab is removed from A [Fig. 5.10]. Find the work done by the external agency in removing the slab from A.

(*iii*) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in Fig. 5.11. Calculate the energy stored in the system. (*I.I.T. 1993*)

Thought Process

1. *When the space between the plates of a capacitor is partially filled with a dielectric slab (thickness of slab is equal to separation between the plates), the two parts of the capacitor behave as two separate capacitors and the arrangement may be looked as the parallel combination of the two parts behaving as two separate capacitors.*
2. *When dielectric slab is removed from the space between the plates of a charged capacitor, the capacitance decreases, while the potential difference increases by the factor equal to K (dielectric constant).*

Since $U = \frac{1}{2} C V^2$, *the potential energy increases by a factor* $\frac{1}{K} \times K^2$ *i.e. K. The energy of the capacitor increases at the expense of the work done in removing the dielectric slab.*

Sol. Here, Area of the plates of capacitor A,

$$A_1 = 0{\cdot}04 \text{ m}^2$$

Area of the plates of capacitor B,

$$A_2 = 0{\cdot}02 \text{ m}^2$$

Separation between the plates of the two capacitors,

$$d_1 = d_2 = 8{\cdot}85 \times 10^{-4} \text{ m}$$

Since the dielectric slab can exactly fill the space between the plates of capacitor B,

area of dielectric slab, $A_2 = 0{\cdot}02 \text{ m}^2$;

and dielectric constant of the dielectric slab, K = 9

(*i*) As shown in Fig. 5.09, when the dielectric slab is introduced between the plates of capacitor A, then half of the space between the two plates gets filled with slab and the other half has air. Let C′ and C″ be the capacitances of the two parts of the capacitor A. The area of the plates of capacitor C′ is A_2, same as that of dielectric ; while that of C″ is equal to $A_1 - A_2$. As already said, the capacitor A can be looked as a parallel combination of C′ and C″ and its capacitance is given by

$$C_1 = C' + C'' = \frac{\varepsilon_0 K A_2}{d} + \frac{\varepsilon_0 (A_1 - A_2)}{d}$$

Since $A_1 = 2 A_2$, it follows that

$$C_1 = \frac{\varepsilon_0 K A_2}{d} + \frac{\varepsilon_0 A_2}{d} = \frac{\varepsilon_0 A_2}{d}(K+1)$$

$$= \frac{8 \cdot 85 \times 10^{-12} \times 0 \cdot 02 (9+1)}{8 \cdot 85 \times 10^{-4}} = \mathbf{2 \times 10^{-9}\ F}$$

The energy stored in the capacitor A on applying a potential difference of 110 V,

$$U_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 2 \times 10^{-9} \times (110)^2 = \mathbf{1{\cdot}21 \times 10^{-5}\ J}$$

(ii) As shown in Fig. 5.10, when the battery is disconnected and the dielectric slab is removed, the capacitor A becomes completely an air capacitor and its capacitance becomes

$$C_1{}' = \frac{\varepsilon_0 A_1}{d} = \frac{8 \cdot 85 \times 10^{-2} \times 0 \cdot 04}{8 \cdot 85 \times 10^{-4}} = 4 \times 10^{-10}\ F$$

Let V′ be the value of potential difference across the capacitor A on removing the dielectric slab. Since the charge on the capacitor remains same, we have

$$C_1{}' V' = C_1 V$$

or $$V' = \frac{C_1 V}{C_1{}'} = \frac{2 \times 10^{-9} \times 110}{4 \times 10^{-10}} = 550\ V$$

Therefore, electrostatic energy stored in the capacitor on removing the dielectric slab [Fig. 5.08 (*c*)],

$$U_2 = \frac{1}{2} C_1{}' V'^2 = \frac{1}{2} \times 40 \times 10^{-10} \times (550)^2$$

$$= 6{\cdot}05 \times 10^{-5}\ J$$

The energy of the capacitor increases at the expense of the work done by the external agency in removing the slab. Therefore, work done by the external agency

$$W = U_2 - U_1 = 6{\cdot}05 \times 10^{-5} - 1{\cdot}21 \times 10^{-5}$$

$$= \mathbf{4{\cdot}84 \times 10^{-5}\ J}$$

(iii) Refer to Fig. 5.11 : The capacitance of the capacitor B, when dielectric slab is placed between its plates,

$$C_2 = \frac{4 \varepsilon_0 K A_2}{d} = \frac{8 \cdot 85 \times 10^{-12} \times 9 \times 0 \cdot 02}{8 \cdot 85 \times 10^{-4}}$$

$$= 1{\cdot}8 \times 10^{-9}\ F$$

When the charged capacitor A (without slab) is connected to the capacitor B, the capacitance of the parallel combination becomes

$$C = C_1{}' + C_2 = 4 \times 10^{-10} + 1{\cdot}8 \times 10^{-9}$$

$$= 2{\cdot}2 \times 10^{-9}\ F$$

Charge on the parallel combination of the capacitors,

$$q = C_1{}' V' = 4 \times 10^{-10} \times 550\ C$$

Therefore, energy stored in the combination,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{(4 \times 10^{-10} \times 550)^2}{2 \times 2 \cdot 2 \times 10^{-9}} = \mathbf{1{\cdot}1 \times 10^{-5}\ J}$$

Problem 5.11. Two square metallic plates of 1 m side kept 0·01 m apart like a parallel plate capacitor in air in s a way that one of their edges is perpendicular to an surface in a tank filled with an insulating oil. The plates connected to a battery of e.m.f. 500 V. The plates are t lowered vertically into the oil at a speed of 0·001 m Calculate the current drawn from the battery during process. Dielectric constant of oil = 11, ε_0 = 8·85 $C^2 N^{-1}$

(I.I.T. 1

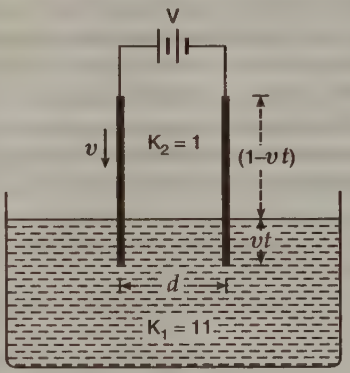


Fig. 5.12

Thought Process

1. *As the plates of the capacitor are lowered into th the capacitance of the capacitor goes on increasing to continuous increase in the area of the diele (insulating oil) between the two plates.*
2. *Since battery remains connected to the capac potential difference across the plates remains equ the battery voltage.*
3. *As capacitance is continuously increasing electrostatic energy also goes on increasing. For capacitor draws current from the battery.*

Sol. Suppose that at $t = 0$, the edges of the two squ plates of the parallel plate capacitor just touch the oil surf If the plates are lowered with velocity v, then at any tin length $v\,t$ of the vertical side of the plates will be inside oil [Fig. 5.12]. Since the plates are square of side 1 m, the le $(1 - v\,t)$ of the vertical side will be in air.

Thus, the capacitor is partially filled with oil partially with air. Let C_1 and C_2 be capacitances of the parts of the capacitor having oil and air respectively as t dielectrics. If A_1 is area of the portion of the plates havin inbetween, then

$$C_1 = \frac{\varepsilon_0 K_1 A_1}{d}$$

Here, $A_1 = v\,t \times 1\,1 = 11$

$$\therefore \quad C_1 = \frac{\varepsilon_0 \times 11 \times v\,t}{d} = \frac{11\,\varepsilon_0\,v\,t}{d}$$

On the other hand, if A_2 is area of the portion of plates having air inbetween, then

$$C_2 = \frac{\varepsilon_0 \times K_2 A_2}{d}$$

Here, $A_2 = (1 - v\,t) \times 1 = (1 - v\,t)\ m^2$

and $K_2 = 1$

$$\therefore \quad C_2 = \frac{\varepsilon_0 \times 1 \times (1 - v)}{d} = \frac{\varepsilon_0 (1 - v\,t)}{d}$$

The given capacitor is equivalent to a parallel bination of C_1 and C_2. If C is capacitance of the given citor at any time t, then

$$C = C_1 + C_2 = \frac{11\,\varepsilon_0\, v\,t}{d} + \frac{\varepsilon_0\,(1 - v\,t)}{d}$$

$$= \frac{\varepsilon_0\,(1 - 10\,v\,t)}{d}$$

Since the capacitor remains connected to the battery, the ntial difference across the capacitor always remains equal e battery voltage V.

Therefore, electrostatic energy stored in the capacitor y time t,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \frac{\varepsilon_0\,(1 + 10\,v\,t)}{d} V^2 \qquad ...(i)$$

If q is charge on the capacitor (supplied by battery) at any t, then

$$U = \frac{1}{2} q V \qquad ...(ii)$$

From the equations (*i*) and (*ii*), we have

$$\frac{1}{2} q V = \frac{1}{2} \frac{\varepsilon_0\,(1 + 10\,v\,t)}{d} V^2$$

$$\text{or} \qquad q = \frac{\varepsilon_0\,(1 + 10\,v\,t)}{d} V$$

Differentiating both sides *w.r.t.* t, we have

$$\frac{dq}{dt} = \frac{d}{dt}\left(\frac{\varepsilon_0\,(1 + 10\,v\,t)\,V}{d}\right)$$

$$= \frac{\varepsilon_0}{d}(0 + 10\,v)\,V = \frac{10\,\varepsilon_0\,v\,V}{d}$$

Now, $\frac{dq}{dt} = I$, the current drawn by the capacitor from the battery.

$$\therefore \qquad I = \frac{10\,\varepsilon_0\,v\,V}{d}$$

Here, $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}\ C^2\,N^{-1}\,m^{-2}$;
$d = 0{\cdot}01$ m ; $v = 0{\cdot}001\ m\,s^{-1}$; V = 500 volt

$$\therefore\ I = \frac{10 \times 8{\cdot}85 \times 10^{-12} \times 0{\cdot}001 \times 500}{0{\cdot}01}$$

$$= \mathbf{4{\cdot}425 \times 10^{-9}\ A}$$

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Coulomb's Law

PART I. TEXT-BASED QUESTIONS

1. Static electricity is produced due to

(A) friction (B) conduction
(C) induction (D) both A and C. (*C.E.T. 1999*)

2. A polythene piece rubbed with wool is found to have a negative charge of 1·6 × 10^{-6} C. (The charge on electron is 1·6 × 10^{-19} C). Then, the number of electrons transferred is

(A) 10^{13} from wool to polythene.
(B) 10^{13} from polythene to wool.
(C) 2·56 × 10^{15} from wool to polythene.
(D) 10^{-13} from wool to polythene.

(*Karnataka Entrance, 1991*)

3. A positively charged glass rod is brought near an uncharged pith ball pendulum. The pith ball is

(A) attracted towards the rod.
(B) repelled away from the rod.
(C) not affected by the rod.
(D) attracted towards the rod, touches it and is then thrown away from it.

4. In SI, unit of permittivity is :

(A) N m^2 C^{-2} (B) N m^{-2} C^{-1}
(C) C^2 N^{-1} m^{-2} (D) A m^{-1}

5. Two point charges of +3 μC and − 8 μC attract each other with a force of 1 N. A charge of + 5 μC is added to each of them. Now, the force will be :

(A) 1 N (attractive) (B) 1 N (repulstive)
(C) zero (D) cannot be found.

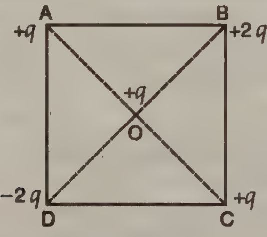
6. When air is replaced by a dielectric medium of dielectric constant K, the maximum force of attraction between two charges separated by a distance :

(A) decreases K times
(B) remains unchanged
(C) increases K times
(D) decreases K^2 times (*C.B.S.E. 1999*)

7. Force between two stationary charges, when placed in free space is 10 N. If they are placed in a medium of relative permittivity 5, the force between them is

(A) 50 N (B) 2 N
(C) 0·5 N (D) 10 N (*C.E.T. 1999*)

8. Four charges are arranged at the corners of a square ABCD as shown in figure. The force on the charge +*q* kept at the centre O is



(A) along the diagonal AC.
(B) along the diagonal BD.
(C) perpendicular to side AB.
(D) zero. (*N.C.E.R.T. 1983*)

9. Three charges +4 *q*, Q and *q* are placed in a straight line of length *l* at points at distances 0, *l* / 2 and *l* respectively. What should be Q in order to make the net force on *q* to zero ?

(A) − *q* (B) − 2 *q*
(C) − *q*/2 (D) 4 *q* (*A.I.I.M.S. 19*

10. A charge q_1 exerts some force on a second charge q_2. A thi charge q_3 is brought near. Then, force exerted by q_1 on will

(A) decrease in magnitude.
(B) increase in magnitude.
(C) remain unchanged.
(D) increase, if q_2 is of the same sign as q_1 and will decrea if q_2 is of opposite sign. (*N.C.E.R.T. 19*

PART II. THOUGHT-BASED QUESTIONS

11. Choose the correct answer :

(A) Total charge present in the universe is constant.
(B) Total positive charge present in the universe is consta
(C) Total negative charge present in the universe is consta
(D) Total number of charged particles present in t universe is constant.

12. Five balls numbered 1 to 5 are suspended using separ threads. Pairs (1, 2), (2, 4) and (4, 1) show electrosta attraction, while pairs (2, 3) and (4, 5) show repulsi Therefore, ball 1 must be

(A) positively charged. (B) negatively charged.
(C) neutral. (D) made of metal.
(*N.C.E.R.T. 19*

13. Two small spheres each having the charge *q* are suspend by insulating threads of length *l* from a hook . Th arrangement is taken in space, where there is gravitational effect. Then, the angle between the t suspensions and the tension in each will be

(A) 180° ; $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2l)^2}$ (B) 90° ; $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}$
(C) 180° ; $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2l^2}$ (D) 180° ; $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}$ (*I.I.T. 19*

14. A soap bubble is given a negative charge. Then, its radi

(A) decreases
(B) increases
(C) remains unchanged
(D) nothing can be predicted. (*M.N.R. 19*

15. A charge *q* is divided into two parts *i.e.* *q*' and *q* − *q*'. If t two parts exert a maximum force of repulsion on each oth then the ratio $\frac{q'}{q}$ is given by

(A) 1 : 2 (B) 2 : 1
(C) 1 : 4 (D) 4 : 1

16. A charge *q* is placed at the centre of the line joining t equal positive charges Q. The system of the three charg will be in equilibrium, if *q* is equal to

(A) − Q/2 (B) − Q/4
(C) + Q/4 (D) + Q/2
(*C.B.S.E. 1995 ; I.I.T. 19*

17. Two charges each of +*q* units are placed along a line. A th charge of − *q* / 4 is placed midway between them. T system will

(A) be in equilibrium (B) not be in equilibrium
(C) lose charge (D) oscillate

18. Two equal and similar charges are placed along a straig line. A third equal and similar charge is placed midw between the two charges. Then, system will

(A) be in equilibrium (B) not be in equilibrium
(C) lose charge (D) oscillate

Two equal and similar charges are placed along a straight line. A third charge equal but opposite to the other two charges is placed midway between them. The system will :
(A) be in equilibrium (B) not be in equilibrium
(C) lose charge (D) oscillate

Point charges $+4\,q, -q$ and $+4\,q$ are kept on the X-axis at points $x = 0, x = a$ and $x = 2\,a$ respectively.
(A) Only $-q$ is in stable equilibrium.
(B) None of the charges is in equilibrium.
(C) All the charges are in unstable equilibrium.
(D) All the charges are in stable equilibrium.

(C.B.S.E. 1992)

21. Two equal negative charges $-q$ are fixed at the points $(0, a)$ and $(0, -a)$ on the Y-axis. A positive charge Q is released from rest at the point $(2\,a, 0)$ on the X-axis. The charge Q will
(A) execute simple harmonic motion above the origin.
(B) move to the origin and remain at rest.
(C) move to infinity.
(D) execute oscillatory but not simple hamonic motion.

(I.I.T. 1983)

Chapter 2. Electric Field

RT I. TEXT-BASED QUESTIONS

In SI, unit of electric field is
(A) A m^{-1} (B) N C^{-1}
(C) C m^{-1} (D) C m^{-2}

The strength of the electric field is such that an electron of charge e placed inside it experiences an electric force equal to its own weight mg. The strength of the field is given by
(A) e/mg (B) mg/e
(C) $mg\,e$ (D) cannot be found.

A cube of side b has a charge q at each of its vertices. The electric field due to this charge distribution at the centre of the cube will be
(A) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{b^2}$ (B) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{2b^2}$
(C) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{32\,q}{b^2}$ (D) zero

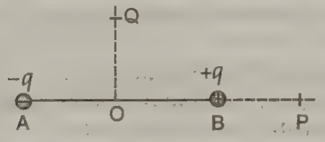
(Karnataka Entrance, 1994)

Three charges each of $+q$ are placed at the three vertices of an equilateral triangle. If the distance of centroid from each vertex of the triangle is r, then strength of electric field at the centroid will be
(A) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{3q}{r}$ (B) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{3q}{r^2}$
(C) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$ (D) zero *(C.P.M.T. 1985)*

A given charge situated at a certain distance from an electric dipole of very small length along its axial line experiences a force F. If the distance of the charge is doubled, the force on charge will become :
(A) 2 F (B) F/2
(C) F/4 (D) F/8 *(M.N.R. 1998)*

The number of electrons to be put on a spherical conductor of radius 0·1 m to produce electric field of $0{\cdot}36\text{ N C}^{-1}$ just above the surface is
(A) $2{\cdot}7 \times 10^6$ (B) $2{\cdot}6 \times 10^6$
(C) $2{\cdot}5 \times 10^6$ (D) $2{\cdot}4 \times 10^6$ *(A.F.M.C. 1994)*

Due to the dipole shown in the figure, the electric intensity will be parallel to the dipole at the point



(A) Q (B) P
(C) Both P and Q (D) neither P, nor Q

(Roorkee, 1976)

A point Q lies on the perpendicular bisector of an electric dipole of dipole moment p. If the distance of Q from the dipole is r (much larger than the size of the dipole), then the electric field at Q is proportional to
(A) p^{-1} and r^{-2} (B) p and r^{-2}
(C) p^2 and r^{-3} (D) p and r^{-3} *(C.B.S.E. 1998)*

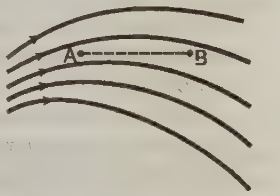
9. An electric dipole of moment p is placed in the position of stable equilibrium in a uniform electric field of intensity E. The torque required to rotate, when the dipole makes an angle θ with the initial position is
(A) $p\,E\cos\theta$ (B) $p\,E\sin\theta$
(C) $p\tan\theta$ (D) $p\cos\theta$ *(C.E.T. 1999)*

10. Torque acting on electric dipole of dipole moment $\vec{p}$ placed in uniform electric field $\vec{E}$ is
(A) $\vec{p}\times\vec{E}$ (B) $\vec{p}\,.\,\vec{E}$
(C) $\vec{p}\times(\vec{E}\times\vec{p})$ (D) $\vec{E}\,.\,\vec{p}\,/\,p^2$ *(C.B.S.E. 2001)*

11. When an electric dipole is placed in a uniform electric field, it experiences
(A) force only. (B) torque only.
(C) both force and torque.
(D) neither a force, nor a torque.

12. A dipole of electric dipole moment $\vec{p}$ is placed in a uniform electric field of strength $\vec{E}$. If θ is angle between $\vec{p}$ and $\vec{E}$, then potential energy of the dipole becomes largest, when θ is
(A) zero (B) $\pi/4$
(C) $\pi/2$ (D) π *(C.E.T. 1998)*

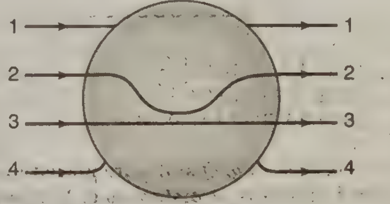
13. The figure shows electric lines of force emerging from a charged body. If the electric fields at points A and B are E_A and E_B respectively and the distance between points A and B is r, then



(A) $E_A > E_B$
(B) $E_A < E_B$
(C) $E_A = E_B$
(D) cannot be predicted.

(C.P.M.T. 1980)

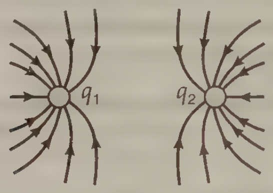
14. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as :



(A) 1 (B) 2

(C) 3 (D) 4 (*I.I.T. 1996*)

15. Figure gives electric lines of force due to two charges q_1 and q_2. What are the signs of the two charges ?



(A) Both are negative.

(B) Both are positive.

(C) q_1 is positive but q_2 is negative.

(D) q_1 is negative but q_2 is positive. (*C.B.S.E. 1994*)

PART II. THOUGHT-BASED QUESTIONS

16. A positively charged ball hangs from a silk thread. We put a positive test charge q_0 at a point and measure $\frac{F}{q_0}$. Then, it can be predicted that the electric field strength E

(A) $> \frac{F}{q_0}$ (B) $= \frac{F}{q_0}$

(C) $< \frac{F}{q_0}$ (D) cannot be predicted.

(*C.P.M.T. 1989*)

17. A stationary charge produces :

(A) an electric field only.

(B) a magnetic field only.

(C) both electric and magnetic fields.

(D) an electromagnetic wave.

18. A proton and an electron are placed in a uniform electric field. Then,

(A) the electric forces acting on them will be equal.

(B) the magnitudes of the forces acting on them will be equal.

(C) their accelerations will be equal.

(D) the magnitudes of their accelerations will be equal.

19. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If high energy X-ray beam falls on it,

(A) the ball will be deflected in the direction of field.

(B) the ball will be deflected opposite to the direction of field.

(C) the ball will not deflect at all.

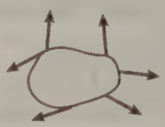
(D) the ball will fly to infinity. (*I.I.T. 1983*)

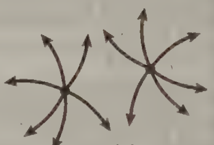
20. A charged wire is bent in the form of a semicircular arc of radius a. If charge per unit length is λ C m^{-1}, the electric field at the centre O is :

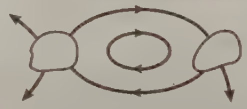
(A) $\frac{\lambda}{2\pi\varepsilon_0 a^2}$ (B) $\frac{\lambda}{4\pi^2\varepsilon_0 a}$

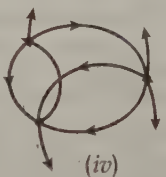
(C) $\frac{\lambda}{2\pi\varepsilon_0 a}$ (D) zero. (*C.B.S.E. 2000*)

21. Which of the following figures cannot possibly represent electrostatic field lines ?


(*i*)


(*ii*)


(*iii*)


(*iv*)

(A) (*i*), (*ii*), (*iii*), (*iv*) (B) (*i*), (*ii*), (*iii*) only

(C) (*i*), (*iii*), (*iv*) only (D) (*ii*), (*iii*), (*iv*) only

(*Karnataka Entrance, 1991*)

Chapter 3. Electric Potential

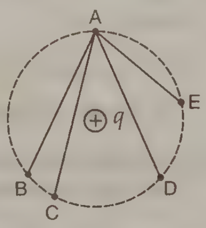
PART I. TEXT-BASED QUESTIONS

1. The work done in carrying a charge Q once round the circle of radius r with charge Q at the centre of the circle is

(A) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$ (B) $\frac{Q.Q}{4\pi\varepsilon_0 r}$

(C) zero (D) $\frac{Q.Q}{2r}$ (*A.I.I.M.S. 1982*)

2. In the electric field of a point charge q, a certain charge is carried from point A to B, C, D and E. Then, the work done



(A) is least along the path AB.

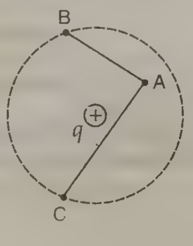
(B) is least along the path AD.

(C) is zero along any one of the paths AB, AC, AD and AE.

(D) is least along AE.

(*N.C.E.R.T. 1987*)

3. In the electric field of a point charge q shown, a charge is carried from A to B and from A to C. Compare the work done.



(A) Work done is greater along the path AC than along AB.

(B) Work done is the same in both the cases.

(C) Work done is greater along the path AB than along AC.

(D) Work done is zero in both the cases. (*N.C.E.R.T. 1986*)

4. Two small spheres each carrying a charge q placed r metres apart repel each other with a force F. If one of the spheres is taken around the other one in a circular path of radius r, the work done will be

(A) F × r (B) $F \times 2\pi r$

(C) $\frac{F}{2\pi r}$ (D) zero

(*C.P.M.T. 1991, 1975 ; N.C.E.R.T. 1983*)

5. The electric potential at the surface of an atomic nucleus (Z = 50) of radius $9{\cdot}0 \times 10^{-15}$ m is

(A) 80 V (B) 8×10^6 V

(C) 9 V (D) 9×10^5 V (*C.P.M.T. 1990*)

6. If a charge is moved against the Coulomb force of an electric field,

(A) work is done by the electric field.

(B) energy is used from some outside source.

(C) the strength of field is decreased.

(D) the energy of the system is decreased.

(*N.C.E.R.T. 1978*)

7. A helium ion and a hydrogen ion are accelerated from rest through a potential difference V to velocities v_1 and v_2 respectively. If the helium has lost one electron, the ratio v_1/v_2 is

(A) 1 (B) $\frac{1}{2}$

(B) $\frac{1}{4}$ (D) $\sqrt{2}$

8. A particle A has charge $+q$ and particle B has charge $+4q$ with each of them having the same mass m. When allowed to fall from rest through the same electrical potential difference, the ratio of their speeds v_A/v_B will become
(A) 2 : 1 (B) 1 : 2
(C) 1 : 4 (D) 4 : 1 *(M.N.R. 1991)*

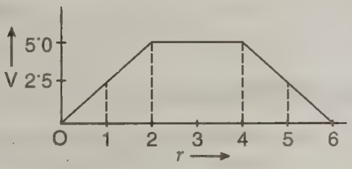
9. A ball of mass 1 g carrying a charge 10^{-8} C moves from a point A at potential 600 V to a point B at zero potential. The change in its K.E. is
(A) -6×10^{6} erg (B) -6×10^{-6} J
(C) 6×10^{-6} J (D) 6×10^{-6} erg *(Roorkee, 1988)*

10. A hollow metallic sphere of radius 10 cm is charged, such that potential of its surface is 70 V. The potential at the centre of the sphere would be
(A) 0 V (B) 7 V
(C) 70 V (D) 700 V
(A.F.M.C. 1995 ; similar I.I.T. 1993 ; M.N.R. 1990)

11. In SI, unit of electric field is :
(A) C m^{-2} (B) A m^{-1}
(C) V m^{-1} (D) C m^{-1}

12. A charge of 5 μC experiences a force of 2 N in an electric field. The potential gradient at that point is
(A) 0·4 N C^{-1} (B) 4×10^{5} N C^{-1}
(C) $2{\cdot}5 \times 10^{-6}$ N C^{-1} (D) cannot be found.

13. The variation of potential (in volt) with distance (in metre) from a fixed point is as shown in the figure. The electric field at $r = 5$ m is equal to
(A) 2·5 V m^{-1}
(B) – 2·5 V m^{-1}
(C) 0·6 V m^{-1}
(D) – 0·6 V m^{-1}



(N.C.E.R.T. 1973)

14. Let V be the electric potential at a given point. Then, the electric field E_x along X-direction at that point is given by
(A) $\int_0^\infty V\,dx$ (B) $\frac{dV}{dx}$
(C) $-\frac{dV}{dx}$ (D) $-V\frac{dV}{dx}$
(Karnataka Entrance, 1986)

15. If the electric field in a given region is zero, the potential in that region
(A) can only be zero throughout the region.
(B) can only be uniform throughout the region.
(C) can be zero or uniform throughout the region.
(D) cannot be uniform throughout the region.

16. In the direction of electric field, the electric potential
(A) decreases. (B) increases.
(C) remains unchanged. (D) becomes zero.

17. There is an electric field E in X-direction. If the work done in moving a charge of 0·2 C through a distance of 2 m along a line making an angle 60° with X-axis is 4 J, then what is the value of E ?
(A) $\sqrt{3}$ N C^{-1} (B) 4 N C^{-1}
(C) 5 N C^{-1} (D) 20 N C^{-1}
(C.E.T. 1999 ; C.B.S.E. 1995)

18. In bringing an electron towards another electron, the electrostatic potential energy of the system :
(A) increases. (B) decreases.
(C) remains unchanged. (D) becomes zero.
(C.B.S.E. 1999)

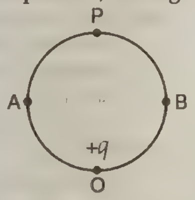
19. Identical charges ($-q$) and placed at each corner of a cube of side b. Then the electrostatic potential energy of charge ($+q$) placed at the centre of the cube will be
(A) $-\frac{4\sqrt{2}\,q^2}{\pi\,\varepsilon_0}$ (B) $\frac{8\sqrt{2}\,q^2}{\pi\,\varepsilon_0\,b}$
(C) $-\frac{4q^2}{\sqrt{3}\,\pi\,\varepsilon_0\,b}$ (D) $\frac{8\sqrt{2}\,q^2}{4\pi\,\varepsilon_0\,b}$ *(C.B.S.E. 2002)*

PART II. THOUGHT-BASED QUESTIONS

20. Can a body have charge and still be at zero potential ?
(A) Yes, always. (B) Yes, but not always.
(C) Never
(D) Depends upon the nature of the charge.

21. Can a body have electric potential and still be uncharged ?
(A) Yes, always. (B) Yes, but not always.
(C) Never
(D) Depends upon the type of the potential.

22. In the electric field of a point charge $+q$ at point O, a charge is carried from point A to B along the diameter and then along the curved path APB. Then work done
(A) along path APB is more.
(B) along path AB is more.
(C) along two paths is equal.
(D) is zero in both cases.



23. Some charge is being given to a conductor. Then its potential is
(A) maximum at surface
(B) maximum at centre
(C) same throughout the condutor
(D) maximum some where between the surface and centre.
(C.B.S.E. 2002)

24. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surfaces of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge – 3 Q, the new potential difference between the same two surfaces is
(A) V (B) 2 V
(C) 4 V (D) – 2 V *(I.I.T. 1989)*

25. The electric potential V at any point x, y, z (all the coordinates are in metres) in space is given by $V = 4x^2$ volt. The electric field at the point (1 m, 0, 2 m) in volt metre^{-1} is
(A) 8 along negative X-axis.
(B) 8 along positive X-axis.
(C) 16 along negative X-axis.
(D) 16 along positive X-axis. *(I.I.T. 1989)*

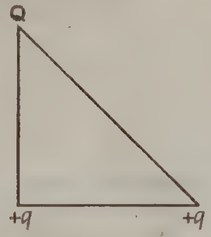
26. A uniform electric field pointing in positive X-direction exists in a region. Let A be the origin, B be the point on the X-axis at $x = +1$ cm. Then the potentials at points A, B and C satisfy :
(A) $V_A < V_B$ (B) $V_A > V_B$
(C) $V_A < V_C$ (D) $V_A > V_C$ *(I.I.T. 2001)*

27. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the X-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the X-axis, is approximately proportional to
(A) x (B) x^2
(C) x^3 (D) $1/x$ *(I.I.T. 2001)*

28. When the separation between two charges is increased, the electric potential energy of the system

(A) increases (B) decreases
(C) remains the same (D) may increase or decrease.

29. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero, if Q is equal to

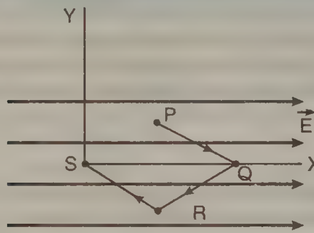


(A) $\frac{-q}{1+\sqrt{2}}$
(B) $\frac{-2q}{2+\sqrt{2}}$
(C) $-2q$
(D) $+q$ (I.I.T. 2000)

30. Which of the following quantities do not depend upon the choice of the zero of electric potential or zero of electric potential energy ?
(A) potential at a point.
(B) potential difference between two points.
(C) potential energy of a system of two point charges.
(D) change in potential energy of a system of two point charges.

31. Two identical thin rings, each of radius R metre are coaxially placed at distance R metre apart. If q_1 and q_2 coulomb are respectively the charges uniformly spread on two rings, the work done in moving a charge q from the centre of one ring to that of the other is
(A) zero
(B) $(\sqrt{2}-1)\,q\,(q_1-q_2)/\sqrt{2}\,4\pi\varepsilon_0 R$
(C) $\sqrt{2}\,q\,(q_1+q_2)/4\pi\varepsilon_0 R$
(D) $(\sqrt{2}+1)\,q\,(q_1+q_2)/\sqrt{2}\,4\pi\varepsilon_0 R$ (I.I.T. 1992)

32. Point charge q moves from point P to point S along th path PQRS (as shown in figure) in a uniform electric fiel



E pointing co-parallel to the positive direction of X-axi The cordinates of the points P, Q, R and S are $(a, b,$ $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$ respectively. The work don by the field in the above process is given by the expressio
(A) qEa (B) $-qEa$
(C) $qEa\sqrt{2}$ (D) $qE\sqrt{(2a)^2+b^2}$
(I.I.T. 198

33. A charge $+q$ is fixed at each of the points $x = x_0$, $x_0 = 3x$ $x = 5x_0$, ... to ∞ on the X-axis and a charge $-q$ is fixed each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$........ to ∞. Her x_0 is a positive constant. Take the electric potential at point due to a charge Q at a distance r from it to b $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$. Then, the potential at the origin due to th above system of the charges is :
(A) 0 (B) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{2x_0\log_e 2}$
(C) ∞ (D) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q\log_e 2}{x_0}$
(I.I.T. 199

Chapter 4. Gauss's Theorem

PART I. TEXT-BASED QUESTIONS

1. The electric flux through a hemispherical surface of radius R placed in a uniform electric field E parallel to the axis of the circular plane is :
(A) $(2\pi R)E$ (B) $(\pi R^2)E$
(C) $\left(\frac{4}{3}\pi R^3\right)E$ (D) $\left(\frac{2}{3}\pi R^3\right)E$

2. A cylinder of length L and radius R is placed in a uniform electric field E parallel to the axis of the cylinder. The total electric flux for the surface of the cylinder is given by
(A) $(2\pi R^2)E$ (B) $(2\pi RL)E$
(C) $2\pi R(R+L)E$ (D) zero.

3. A point charge $+q$ is placed at the mid point of a cube of side L. The electric flux emerging from the cube is
(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{q}{6L^2\varepsilon_0}$
(C) $\frac{6qL^2}{\varepsilon_0}$ (D) zero. (C.B.S.E. 1996, 1993)

4. A charge $q\ \mu$C is palced at the centre of a cube of side 0·1 m. Then the electric flux diverging from each face of this cube is
(A) $\frac{q\times10^{-6}}{24\varepsilon_0}$ (B) $\frac{q\times10^{-4}}{\varepsilon_0}$
(C) $\frac{q\times10^{-6}}{\varepsilon_0}$ (D) $\frac{q\times10^{-4}}{12\varepsilon_0}$ (C.B.S.E. 2001)

5. An electric charge q is placed at one of the corner of a cub of side a. The electric flux on one of its faces will be
(A) $q/a\varepsilon_0$ (B) $q/\varepsilon_0 a^2$
(C) $q/4\pi\varepsilon_0 a^2$ (D) $\frac{q}{24\varepsilon_0}$ (C.B.S.E. 2000, 199

6. Gauss's theorem :
(A) does not hold, if the closed surface encloses a discre distribution of charges.
(B) does not hold, if the closed surface encloses a line, surface or a volume charge distribution.
(C) holds, if the surface encloses a point charge only.
(D) holds, irrespective of the form in which charges a enclosed by the closed surface.

7. Electric field varies as r^{-3} due to :
(A) a point charge. (B) an infinite line charge.
(C) an electric dipole.
(D) an infinite plane sheet of charge.

8. Electric field varies as r^{-1} due to
(A) a point charge. (B) a quadrupole.
(C) an infinite line charge.
(D) an infinite plane sheet of charge.

9. The electric field inside a spherical shell of unifor surface charge density is
(A) zero. (B) uniform.
(C) non-uniform.
(D) proportional to distance from the centre.

10. A spherical shell of radius R has a charge $+q$ units. Th electric field due to the shell at a point :

(A) inside is zero and varies as r^{-1} outside it.
(B) inside is constant and varies as r^{-2} outside it.
(C) inside is zero and varies as r^{-2} outside it.
(D) inside is constant and varies as r^{-1} outside it.

A sphere of charge of radius R has uniform volume charge density. The electric field due to the sphere of charge at a point :
(A) inside varies as r^{-2} and as r outside it.
(B) inside varies as r and as r^{-2} outside it.
(C) inside is zero and varies as r outside it.
(D) inside is constant and varies as r^{-2} outside it

A hollow insulated conducting sphere is given a positive charge of 10 μC. What will be the electric field at the centre of the sphere, if its radius is 2 m ?
(A) zero (B) 5 μC m $^{-2}$
(C) 20 μC m $^{-2}$ (D) 32 μC m $^{-2}$
(C.B.S.E. 1998)

T II. THOUGHT-BASED QUESTIONS

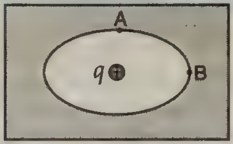
A charge q is placed at the corner of a cube of side a. The electric flux through the cube is :
(A) q/ε_0 (B) $q/3\varepsilon_0$
(C) $q/6\varepsilon_0$ (D) $q/8\varepsilon_0$

If the eletric flux through a closed surface is zero, then
(A) the electric field must be zero everywhere on the surface.
(B) the electric field may be zero everywhere on the surface.
(C) the charge inside the surface must be zero.
(D) the charge in the vicinity of the surface must be zero.

An electric dipole is placed inside a hollow sphere at its centre. Then,
(A) the electric flux through the sphere is zero.
(B) the electric field is zero at every point on the surface of the sphere.
(C) the electric field is non-zero at every point on the surface of the sphere.
(D) the electric field is non-zero at every point on the surface of the sphere.

A capacitor of capacitance C is charged to a potential V and is placed inside a closed surface. The electric flux through the closed surface is
(A) CV/ε_0 (B) $2CV/\varepsilon_0$
(C) $CV/2\varepsilon_0$ (D) zero

An ellipsoidal cavity is carved within a perfect conductor as shown in figure. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface. Then,
(A) electric field near A in the cavity = electric field near B in the cavity
(B) charge density at A = charge density at B



(C) potential at A = potential at B
(D) total electric field flux through the surface of the cavity is q/ε_0. *(I.I.T. 1999)*

18 **A metallic particle having no net charge is placed near a positively charged metal plate having finite size. The electric force on the particle will be**
(A) towards the plate. (B) away from the plate.
(C) parallel to the plate. (D) zero.

19. **If one penerates a uniformly charged sphere, the electric field strength E**
(A) increases. (B) decreases.
(C) remains the same as at the surface.
(D) is zero at all points. *(C.P.M.T. 1991, 1980)*

20. **A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre**
(A) increases as r increases, for $r << R$.
(B) decreases as r increases, for $0 < r < \infty$
(C) decreases as r increases, for $R < r < \infty$.
(D) is discontinuous at $r = R$. *(I.I.T. 1998)*

21. **Potential at any point inside a charged hollow sphere**
(A) increases with distance.
(B) is a constant.
(C) decreases with distance from centre.
(D) is zero. *(Karnataka Entrance, 1990)*

22. **A hollow metal sphere of radius 5 cm is charged, such that the potential on its surface is 10 V. The potential at the centre of the sphere is**
(A) 0 V (B) 10 V
(C) same as at a point 5 cm away from the surface.
(D) same as at a point 25 cm away from the surface.
(M.N.R. 1990 ; I.I.T. 1983)

23. **A hollow sphere of charge does not produce an electric field at any**
(A) interior point (B) outer point
(C) beyond 2 m (D) beyond 10 m. *(M.N.R. 1985)*

24. **The force between the plates of a parallel plate capacitor of area A, capacitance C and having charge q will be**
(A) $q^2/2\varepsilon_0 A$ (B) $-q^2/2\varepsilon_0 A$
(C) $q^2/4\varepsilon_0 A$ (D) $-q^2/4\varepsilon_0 A$
(A.F.M.C. 1998)

Chapter 5. Capacitors

T I. TEXT-BASED QUESTIONS

In SI, unit of permittivity is :
(A) F m^{-1} (B) N m^2 C^{-2}
(C) N m^{-2} C^{-1} (D) A m^{-1}

The capacitance of the earth viewed as a spherical conductor of radius 6408 km is
(A) 980 μF (B) 1424 μF
(C) 712 μF (D) 600 μF *(C.P.M.T. 1992)*

N identical spherical drops charged to the same potential V are combined to from a big drop. The potential of the new drop will be
(A) V (B) V/N
(C) V N (D) V N$^{2/3}$ *(Karnataka, 2000)*

4. **A capacitor having capactiance of 2 μF has a charge of 1 μC on its plates. If the charge on the plates is changed to 2 μC, the new capacitance of the capacitor is**
(A) 4 μF (B) 2 μF
(C) 1 μF (D) 0· μF *(C.E.T. 1998)*

5. **A parallel plate capacitor is charged. If the plates are pulled apart,**
(A) the potential difference increases.
(B) the capacitance increases.
(C) the total charge increases.
(D) the charge and the potential difference remain the same.
(Karnataka Entrance , 1987)

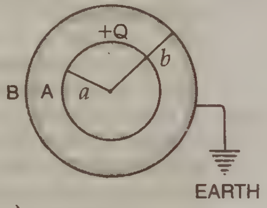
6. **The capacitance of a parallel plate capacitor increases with**
(A) decrease of its area.
(B) increase of its distance.

(C) increase of its area.
(D) decrease of its distance. *(A.F.M.C. 1995)*

7. **When air is replaced by a dielectric medium of constant K, the maximum capacitance of the capacitor**
(A) increases K times (B) remains unchanged
(C) increases K^2 times (D) decreases K times
(Karnataka Entrance ; 1994 ; N.C.E.R.T. 1990)

8. **A parallel plate capacitor with oil between the plates (dielectric constant of oil, K = 2) has a capacitance C. If the oil is removed, then capacitance of the capacitor becomes :**
(A) $\sqrt{2}\,C$ (B) 2 C
(C) $C/\sqrt{2}$ (D) C/2 *(C.B.S.E. 1999)*

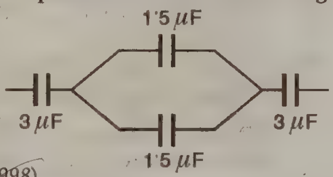
9. **Two spherical conductors A and B of radii *a* and *b* (*b* > *a*) are placed concentrically in air. A is given a charge + Q, while B is earthed. Then, the equivalent capacitance of the system is :**
(A) $4\pi\varepsilon_0 \frac{ab}{b-a}$
(B) $4\pi\varepsilon_0 (a+b)$
(C) $4\pi\varepsilon_0 b$
(D) $4\pi\varepsilon_0 \frac{b^2}{b-a}$



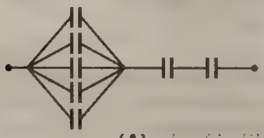
10. The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor
(A) is same throughout.
(B) is higher near the outer cylinder than near the inner cylinder.
(C) varies as $\frac{1}{r}$, where r is the distance from the axis.
(D) varies as $\frac{1}{r^2}$, where r is the distance from the axis.
(I.I.T. 1996)

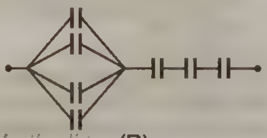
11. **There are 10 capacitors each of capacitance 5 μF. The ratio between maximum and minimum capacitance that can be obtained from these capacitors is :**
(A) 5 : 1 (B) 10 : 1
(C) 50 : 1 (D) 100 : 1

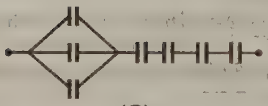
12. **What is the equivalent capacitance of the following circuit ?**
(A) 1 μF
(B) 2 μF
(C) 1·5 μF
(D) 3 μF *(A.F.M.C. 1998)*

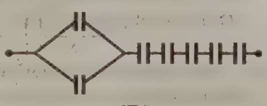


13. **Seven capacitors each of capacitance 2 μF are connected in a configuration to obtain an effective capacitance of $\frac{10}{11}$ μF. Which of the combination(s) shown in the figure will achieve the desired result ?**


(A)

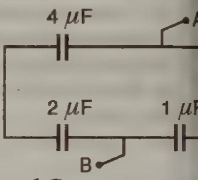

(B)


(C)


(D)

14. **Three identical capacitors (of capacitance C each)
connected in series and this connection is connecte
parallel with one such more identical capacitor. Then,
capacitance of the whole combination is**
(A) 3 C (B) $\frac{4}{3}$C
(C) $\frac{3}{4}$C (D) 2 C
(Karnataka Entrance, 1992, 1

15. **For the arrangement of the capacitors as shown in figure,
net capacitance between points
A and B is :**
(A) 1·7 μF
(B) 2·33 μF
(C) 2·8 μF
(D) 7 μF

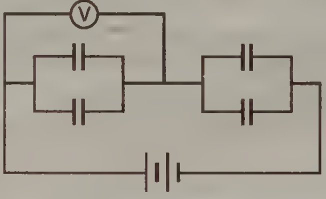


16. **Two capacitors of capacitances C_1 and C_2 are connecte
parallel. If a charge *q* is given to the assembly, the ch
gets shared. The ratio of the charge on the capacitor C
the charge that on C_2 is :**
(A) C_1/C_2 (B) C_2/C_1
(C) $C_1 C_2$ (D) $1/C_1 C_2$ *(I.I.T. 1*

17. **A capacitor of capacitance C_1 is charged upto potenti
and then connected in parallel to an uncharged capa
of capacitance C_2. The final potentail difference ac
each capacitor will be :**
(A) $\frac{C_2 V}{C_1 + C_2}$ (B) $\frac{C_1 V}{C_1 + C_2}$
(C) $\left(1+\frac{C_2}{C_1}\right)V$ (D) $\left(1-\frac{C_2}{C_1}\right)V$ *(C.B.S.E. 2*

18. **An isolated capacitor of unknown capacitance has b
charged to a potential difference of 100 V. When
charged capacitor is connected in parallel to an uncha
10 μF capacitor, the voltage across the combination is 3
The unknown capacitance is closer to**
(A) 1· 6 μF (B) 2 μF
(C) 4· 3 μF (D) 6· 2 μF *(C.E.T. 1*

19. **Four capacitors of 25 μF each are connected as show
figure. If the d.c. voltmeter reads 200 V, charge on each p
of the capacitor is :**



(A) 2×10^{-3} C (B) 5×10^{-3} C
(C) 2×10^{-2} C (D) 5×10^{-2} C *(C.B.S.E. 1*

20. **In a charged capacitor, the energy resides**
(A) on the positive plate.
(B) on both the positive and negative charged plates.
(C) in the field between the plates.
(D) around the edge of the capacitor plates.
(Karnataka 2000 ; J.I.P.M.E.R. 1

21. **The energy stored in a capacitor of capacitance C
potential V is given by**
(A) $\frac{1}{2}C^2 V$ (B) $\frac{1}{2}C V^2$
(C) $\frac{1}{2}C V$ (D) $\frac{1}{2}C^2 V^2$ *(C.B.S.E. 1*

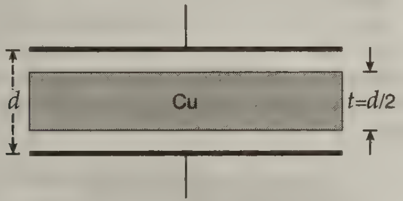
2. **700 pF capacitor is charged by a 50 V battery. How much electrostatic energy is stored by it ?**
(A) $6{\cdot}7 \times 10^{-7}$ J (B) $8{\cdot}7 \times 10^{-7}$ J
(C) $13{\cdot}6 \times 10^{-9}$ J (D) $17{\cdot}0 \times 10^{-8}$ J
(A.F.M.C. 1997)

3. **A capacitor of capacitance 10 μF was originally charged to 10 V. Now, the potential difference is increased to 20 V. The increase in potential energy is**
(A) 4×10^{-4} J (B) 10×10^{-4} J
(C) 15×10^{-4} J (D) 5×10^{-4} J *(C.E.T. 1999)*

4. **Two identical capacitors are joined in parallel, charged to a potential V, separated and then are connected in series *i.e.* the positive plate of one is connected to the negative of the other. Then,**
(A) the charges on the free plates connected together are destroyed.
(B) the charges on the free plates are enhanced.
(C) the energy stored in the system increases.
(D) the potential difference between the free plates becomes 2 V.
(Karnataka Entrance, 1993 ; N.C.E.R.T. 1973)

5. **A 4 μF capacitor is charged to 400 V. If its plates are joined through a resistance of 2 kΩ, then heat produced in the resistance is :**
(A) $0{\cdot}16$ J (B) $0{\cdot}32$ J
(C) $0{\cdot}64$ J (D) $1{\cdot}28$ J *(C.B.S.E. 1995, 1994)*

6. **A copper slab of thickness t is inserted between the plates of a parallel plate capacitor, such that t is half of the separation between the plates. The ratio of the capacitances of the capacitor before and after the introduction of the copper slab is :**



(A) 1 : 1 (B) $\sqrt{2}:1$
(C) $1:\sqrt{2}$ (D) 1 : 2

7. **A metal foil of negligible thickness is introduced between two plates of a capacitor at the centre. The capacitance of capacitor will be :**
(A) same (B) double
(C) half (D) K times *(I.I.T. 1982)*

8. **A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles,**
(A) the charge on the capacitor increases.
(B) the voltage across the plates increases.
(C) the capacitance increases.
(D) the electrostatic energy stored in the capacitor increases. *(I.I.T. 1987)*

9. **A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with capacitor are given by Q_0, V_0, E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous ones as :**
(A) $Q > Q_0$ (B) $V > V_0$
(C) $E > E_0$ (D) $U > U_0$ *(I.I.T. 1985)*

0. **Van de Graaff generator is used to**
(A) measure high potential difference.
(B) produce high d.c. potential.
(C) produce high a.c. potential.
(D) compare high d.c. potential.
(Karnataka Entrance, 1992, 1989)

31. **The accumulation of charge on clouds, which produces lightning, is caused by**
(A) rain drops changing into electrons.
(B) the electric field of the earth.
(C) ionisation by the sun.
(D) electrification due to motion of water molecules.
(Karnataka Entrance 1985 ; similar 1987)

PART II. THOUGHT-BASED QUESTIONS

32. **In a guard ring capacitor, the purpose of the guard ring is**
(A) to increase the capacitance.
(B) to decrease the capacitance.
(C) to increase the effective area of the capacitor.
(D) to avoid the variation of the electric field intensity at the edges. *(Karnataka Entrance, 1992, 1989)*

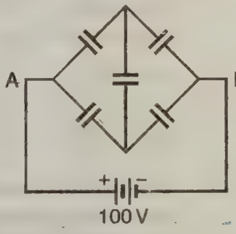
33. **Two identical metal plates are given positive charges q_1 and q_2 ($< q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is**
(A) $(q_1 + q_2)/2\,C$ (B) $(q_1 + q_2)/C$
(C) $(q_1 - q_2)/C$ (D) $(q_1 - q_2)/2\,C$ *(I.I.T. 1999)*

34. **Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is**
(A) $\frac{1}{4}C(V_1^2 - V_2^2)$ (B) $\frac{1}{4}C(V_1^2 + V_2^2)$
(C) $\frac{1}{4}C(V_1 - V_2)^2$ (D) $\frac{1}{4}C(V_1 + V_2)^2$ *(I.I.T. 2002)*

35. **Two capacitors with capacitances C_1 and C_2 are charged to potentials V_1 and V_2 respectively. When they are connected in parallel, the ratio of their respective charges is**
(A) C_1/C_2 (B) V_1/V_2
(C) V_1^2/V_2^2 (D) C_1^2/C_2^2 *(Karnataka, 2000)*

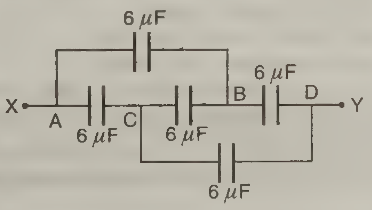
36. **A dielectric slab of thickness d is inserted in a parallel plate capacitor, whose negative plate is at $x = 0$ and positive plate is at $x = 3\,d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3\,d$,**
(A) the magnitude of the electric field remains the same.
(B) the direction of the electric field remains the same.
(C) the electric potential increases continuously.
(D) the electric potential increases at first, then decreases and again increases. *(I.I.T. 1998)*

37. **Five capacitors each of 10 μF capacitance are connected as shown to a source of e.m.f. The equivalent capacitance between points A and B is :**



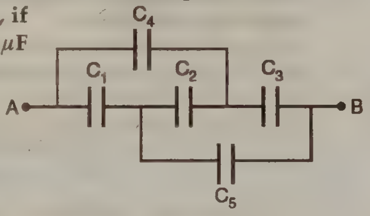
(A) 40 μF
(B) 20 μF
(C) 30 μF
(D) 10 μF
(C.E.T. 1999 ; C.P.M.T. 1991, 1988)

38. **What is the effective capacitance between points X and Y ?**



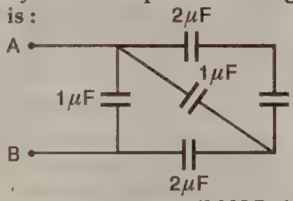
(A) 24 μF (B) 18 μF
(C) 12 μF (D) 6 μF (*C.B.S.E. 1999*)

39. Find the effective capacitance between points A and B in the following figure, if $C_1 = C_3$, $C_0 = C_5 = 4\ \mu F$ and $C_2 = 10\ \mu F$:



(A) 8 μF
(B) 6 μF
(C) 2·5 μF
(D) 4 μF

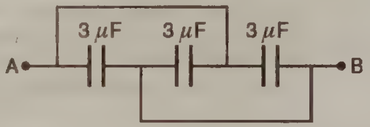
40. The net capacitance of system of capacitors in figure between points A and B is :



(A) 1 μF
(B) 2 μF
(C) 3 μF
(D) 4 μF

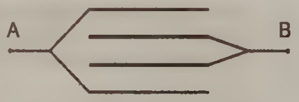
(*M.N.R. 1987*)

41. The equivalent capacitance between the points A and B in the following figure will be :



(A) 9 μF
(B) 1 μF
(C) 4·5 μF
(D) 6 μF (*C.P.M.T. 1985*)

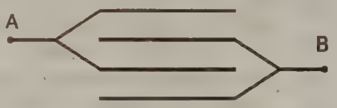
42. Four metallic plates each with a surface area of one side A are placed at a distance *d* apart from each other. The two inner plates are connected to point B and the other two plates to another point A as shown in figure. Then, the capacitance of the system is :



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{2\varepsilon_0 A}{d}$

(C) $\frac{3\varepsilon_0 A}{d}$ (D) $\frac{4\varepsilon_0 A}{d}$

43. Four metallic plates, each with a surface area of one side A are placed at a distance *d* from each other. The alternate plates are connected to points A and B as shown in figure. The equivalent capacitance of the system will be :



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{2\varepsilon_0 A}{d}$

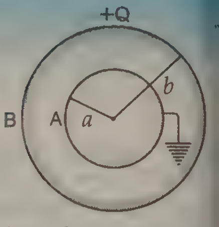
(C) $\frac{3\varepsilon_0 A}{d}$ (D) $\frac{4\varepsilon_0 A}{d}$

44. A solid sphere and a hollow sphere of equal diameters are raised to the same potential. Then,

(A) hollow sphere has more charge.
(B) both have equal charge.
(C) only hollow sphere has charge.
(D) solid sphere has more charge.

(*Karnataka Entrance, 1994 ; similar I.I.T. 1974*)

45. Two spherical conductors A and B of radii *a* and *b* (*b* > *a*) are placed concentrically in air. B is given a charge + Q and A is earthed. The equivalent capacitance of the system is :



(A) $4\pi\varepsilon_0 \frac{ab}{b-a}$

(B) $4\pi\varepsilon_0 (a+b)$

(C) $4\pi\varepsilon_0 b$

(D) $4\pi\varepsilon_0 \frac{b^2}{b-a}$

46. A capacitor is charged by connecting a battery across i plates. It stores energy U. Now the batery is disconnecte and another identical capacitor is connected across it. The the energy stored by both capacitors of the system will b

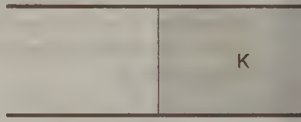
(A) U (B) U/2
(C) 2 U (D) 3 U/2 (*C.B.S.E. 200*

47. In a parallel plate capacitor, the distance between the plate is *d* and potential difference across plates is V. Energ stored per unit volume between the plates of capacitor i

(A) $\frac{Q^2}{2V^2}$ (B) $\frac{1}{2}\varepsilon_0 \frac{V^2}{d^2}$

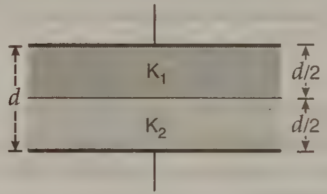
(C) $\frac{1}{2}\frac{V^2}{\varepsilon_0 d^2}$ (D) $\frac{1}{2}\varepsilon_0 \frac{V^2}{d}$ (*C.B.S.E. 200*

48. A dielectric is placed inbetween the two parallel plates c a capacitor as shown in figure. The dielectric constant c the dielectric is K. If the initial capacitance is C, then modified capacitance will be



(A) K C
(B) (K+1) C
(C) (K+1) C/2
(D) (K – 1) C

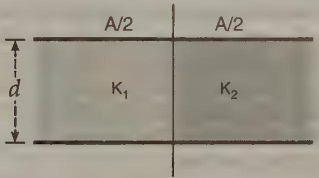
49. A parallel plate capacitor with plate area A and separatio *d* is filled with dielectrics as shown. Its capacitance wi become



(A) $\frac{\varepsilon_0 A}{d}(K_1 + K_2)$ (B) $\frac{\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$

(C) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$ (D) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$

50. A parallel plate capacitor is filled with two dielectrics a shown. Area of each plate is A metre^2 and the separatio is *d* metre. The dielectric constants are K_1 and K respectively. Its capacitance in farad will be :



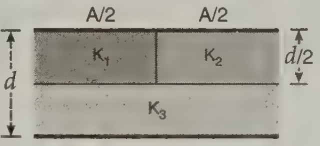
(A) $\frac{\varepsilon_0 A}{d}(K_1 + K_2)$ (B) $\frac{\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$

(C) $\frac{\varepsilon_0 A}{d}\frac{K_1 K_2}{K_1 + K_2}$ (D) $\frac{\varepsilon_0 A}{d}2(K_1 + K_2)$

A parallel plate capacitor with air as medium between the plates has a capacitance of 10 μF. The area of the capacitor is divided into two equal halves and filled with two media having dielectric constants 2 and 4 respectively. The modified capacitance is :

(A) 10 μF (B) 20 μF
(C) 30 μF (D) 40 μF

A parallel plate capacitor of area A , plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant K is given by

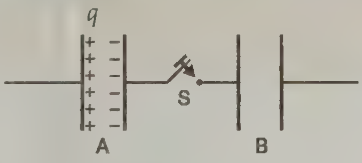


(A) $\frac{1}{K}=\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{2K_3}$ (B) $\frac{1}{K}=\frac{1}{K_1+K_2}+\frac{1}{2K_3}$

(C) $K=\frac{K_1K_2}{K_1+K_2}+2K_3$ (D) $K=K_1+K_2+2K_3$

(I.I.T. 2000)

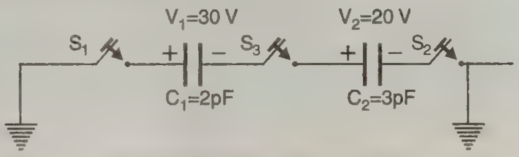
53. Consider the situation shown in the figure. The capacitor A has charge q on it, whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed, is :



(A) zero
(B) $q/2$
(C) q
(D) $2q$

(I.I.T. 2002)

54. For the circuit shown in figure, which of the following statements is true ?



(A) with S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V.
(B) with S_3 closed, $V_1 = V_2 = 25$ V.
(C) with S_1 and S_2 closed, $V_1 = V_2 = 0$.
(D) with S_1 and S_3 closed, $V_1 = 30$ V, $V_2 = 20$ V.

(I.I.T. 1999)

ANSWERS TO MULTIPLE CHOICE QUESTIONS

ter 1. Coulomb's Law

(A) 2. (A) 3. (D) 4. (C) 5. (A) 6. (A) 7. (B) 8. (B) 9. (A) 10. (C)
(A) 12. (C) 13. (A) 14. (B) 15. (A) 16. (B) 17. (A) 18. (B) 19. (B) 20. (D)
(D)

ter 2. Electric Field

(B) 2. (B) 3. (D) 4. (D) 5. (D) 6. (C) 7. (C) 8. (D) 9. (B) 10. (A)
(B) 12. (D) 13. (A) 14. (D) 15. (A) 16. (A) 17. (A) 18. (B) 19. (A) 20. (C)
(C)

ter 3. Electric Potential

(C) 2. (C) 3. (B) 4. (D) 5. (B) 6. (B) 7. (B) 8. (B) 9. (B) 10. (C)
(C) 12. (B) 13. (A) 14. (C) 15. (C) 16. (A) 17. (D) 18. (A) 19. (C) 20. (B)
(B) 22. (D) 23. (C) 24 (A) 25 (A) 26 (B) 27 (B) 28. (D) 29. (B) 30. (B & D)
(B) 32. (B) 33. (D)

ter 4. Gauss's Law

(B) 2. (D) 3. (A) 4. (C) 5. (D) 6. (D) 7. (C) 8. (C) 9. (A) 10. (C)
(B) 12. (A) 13. (D) 14. (B & C) 15. (A & D) 16. (D) 17. (C & D) 18. (A) 19. (B) 20. (A & C)
(B) 22. (B) 23. (A) 24. (C)

ter 5. Capacitor

(A) 2. (C) 3. (D) 4. (B) 5. (A) 6. (C & D) 7. (A) 8. (D) 9. (A) 10. (C)
(D) 12. (A) 13. (A) 14. (B) 15. (B) 16. (A) 17. (B) 18. (C) 19. (B) 20. (C)
(B) 22. (B) 23. (C) 24. (D) 25. (B) 26. (D) 27. (A) 28. (B & D) 29. (A & D) 30. (B)
(D) 32. (D) 33. (D) 34. (C) 35. (A) 36. (B & C) 37. (D) 38. (D) 39. (D) 40. (B)
(A) 42. (B) 43. (C) 44. (B) 45. (D) 46. (B) 47. (B) 48. (C) 49. (C) 50. (B)
(C) 52. (B) 53. (A) 54. (B)

UNIT

II

CURRENT ELECTRICITY

- **Electric Current, Resistance and E.M.F.**
- **Electrical Measurements**
- **Thermal Effects of Current**
- **Chemical Effects of Current**
- **Thermoelectric Effect**

> “The cell disturbs the electric field, or gives it a certain impulse. Do not ask in what manner : it is enough that it is a principle, and a general principle.”
>
> – **Alessandro Volta**

CHAPTER 1

Electric Current, Resistance and E.M.F.

1.01. ELECTRIC CURRENT

It is defined as the rate of flow of electric charge through any section of a wire. It is noted by I. Thus,

$$I = \frac{\text{total charge flowing}}{\text{time taken}}$$

If charge q flows in a time t through any section of a wire, then

$$I = \frac{q}{t} \qquad ...(1.01)$$

If charge dq flows through a wire in small time dt, then

$$I = \frac{dq}{dt}$$

If n carriers of electricity, each having charge e cross any section of the conductor ime t, then

$$I = \frac{n\,e}{t} \qquad ...(1.02)$$

The direction in which the positive charge will flow gives the direction of *ventional current*. However, in metallic conductors, free electrons are carriers electricity and hence electrons constitute the electric current. Therefore, the ection of electronic current is opposite to that of conventional current.

Although, a direction is associated with electric current, yet it is a scalar antity. The reason is that the laws of ordinary algebra are used to add electric rent. The laws of vector algebra do not apply to the addition of electric currents.

Unit of electric current. The SI unit of electric current is **ampere.** It is also the ctical unit of current. It is denoted by **A.**

$$\textbf{1 ampere (A)} = \frac{\textbf{1 coulomb (C)}}{\textbf{1 second (s)}} = \textbf{1 coulomb second}^{-1}\ (\mathbf{C\ s^{-1}})$$

The current through a wire is called one ampere, if one coulomb of charge flows through wire in one second.

The jargon .....

The current that flows from a point at higher (positive) potential to a point at lower (negative) potential is called conventional current.

Watch out !

The electric current has both magnitude and direction but it is not a vector quantity !

Self-test question ?

How many electrons crossing per second through any section of a conductor constitute a current of one ampere ?

1.02. DRIFT VELOCITY

In a conductor, the atoms are quite close to each other and are strongly bound one another. However, valence electrons in an atom of a conductor donot remain ached to a particular atom. These electrons are free to move through the lattice ositive ions in the conductor. Such valence electrons in a conductor are called *e electrons* or *conduction electrons*. The free electrons in a conductor are always a continuous random motion [Fig. 1.01]. This random motion of free electrons is e to thermal energy of the conductor. Likewise, the velocity of a free electron due hermal energy of the conductor is termed as its thermal velocity and is usually y large.

Further, the number of free electrons in a conductor is very large. Due to their dom motion, the free electrons keep on sufferring collisions with the positive s in the conductor so frequently that the net flow of electrons in any particular ection is zero. In other words, average thermal velocity of free electrons in a nductor is zero.

If $\vec{u}_1, \vec{u}_2, \vec{u}_3, \ldots, \vec{u}_n$ are random thermal velocities of n electrons in a nductor, then their average thermal velocity *i.e.*

$$\frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots.. \vec{u}_n}{n} = 0 \qquad ...(1.03)$$

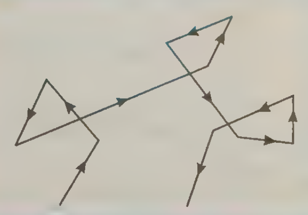


Fig. 1.01

Key point ✓

The thermal velocities of free electrons are of the order of 10^5 m s^{-1}.

Key point ✓

If we consider one free electron per atom, then number of free electrons in 1 m^3 of a conductor is about 10^{29}.

However, when some potential difference is applied across the two ends of a conductor, an electric field is set up. If V is potential difference applied and l, the length of the conductor [Fig. 1.02], then magnitude of the electric field set up is given by

$$E = \frac{V}{l}$$

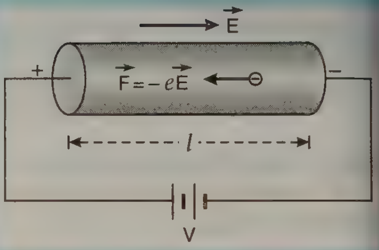


Fig. 1.02

The direction of electric field is from positive end of the conductor to its negative end as shown in the figure. Since charge on an electron is $-e$, each free electron in the conductor experiences a force

$$\vec{F} = -e\,\vec{E} \qquad \text{...(1.04)}$$

in a direction opposite to the direction of electric field. If m is mass of the electron, then acceleration produced is given by

$$\vec{a} = -\frac{e\,\vec{E}}{m} \qquad \text{...(1.05)}$$

Under the effect of electric field, the free electrons accelerate and acquire a velocity component in a direction opposite to the direction of electric field in addition to their thermal velocities. However, the gain in velocity of an electron due to electric field takes place only for a very short time. It is because, as the electron accelerates, it gets scattered or deflected on suffering collision against the positive ion in the conductor. The electron, then starts afresh with a random thermal velocity.

The short time, for which a free electron accelerates before it undergoes a collision with the positive ion in the conductor, is called ***relaxation time****.*

Therefore, if an electron having random thermal velocity $\vec{u}_1$ accelerates for a time t_1 (before it suffers a collision), then it will attain a velocity

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\,\tau_1$$

where the acceleration $\vec{a}$ is given by equation (1.05). Similarly, velocities acquired by the other electrons in the conductor will be

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\,\tau_2,\ \vec{v}_3 = \vec{u}_3 + \vec{a}\,\tau_3, \ldots\ldots, \vec{v}_n = \vec{u}_n + \vec{a}\,\tau_n$$

The drift velocity is defined as the average velocity with which free electrons in a conductor get drifted under the influence of an external electric field applied across the conductor. It is denoted by $\vec{v}_d$. Thus,

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots\ldots + \vec{v}_n}{n}$$

$$= \frac{(\vec{u}_1 + \vec{a}\,\tau_1) + (\vec{u}_2 + \vec{a}\,\tau_2) + (\vec{u}_3 + \vec{a}\,\tau_3) + \ldots\ldots + (\vec{u}_n + \vec{a}\,\tau_n)}{n}$$

or $$\vec{v}_d = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots\ldots + \vec{u}_n}{n} + \vec{a}\,\frac{\tau_1 + \tau_2 + \tau_3 + \ldots\ldots + \tau_n}{n}$$

Now, $\frac{\tau_1 + \tau_2 + \tau_3 + \ldots\ldots + \tau_n}{n}$ is called ***average relaxation time*** and is denoted by τ. As obtained in equation (1.03), $\frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \ldots\ldots + \vec{u}_n}{n}$ is equal to zero. Therefore, the above equation becomes

Key point

The random motion of free electrons does not contribtue to their drift velocity.

$$\vec{v}_d = 0 + \vec{a}\,\tau = \vec{a}\,\tau$$

Using equation (1.05), we have

$$\vec{v}_d = -\frac{e\,\vec{E}}{m}\,\tau \qquad \text{...(1.06)}$$

The equation (1.06) gives drift velocity of free electrons in a conductor in terms of electric field and average relaxation time.

1.03. RELATION BETWEEN DRIFT VELOCITY AND ELECTRIC CURRENT

Consider a conductor of length l and uniform area of cross-section A. Then,

volume of conductor = A l

Suppose that the conductor contains n free electrons per unit volume. Then, number of free electrons in the conductor = $n\,A\,l$

If e is charge on an electron, then total charge on all the free electrons in the onductor,

$$q = n\,A\,l\,e \qquad ...(1.07)$$

When a battery is connected across the two ends of the conductor, an electric ield $\vec{E}$ is set up across the two ends of the conductor as shown in Fig. 1.03. If the lectrons drift towards the positive end of the conductor with drift velocity v_d, then ime taken by the free electrons to cross the length of the conductor,

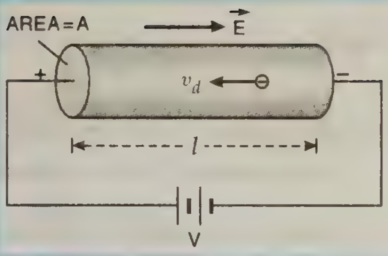


Fig. 1.03

$$t = \frac{l}{v_d} \qquad ...(1.08)$$

Since current is defined as the rate of flow of charge through the conductor,

$$I = \frac{q}{t}$$

Using equations (1.07) and (1.08), we have

$$I = \frac{n\,A\,l\,e}{l/v_d}$$

or

$$I = n\,A\,v_d\,e \qquad ...(1.09)$$

The equation (1.09) gives the relation between drift velocity of free electrons nd the current flowing through the conductor.

Note. From equation (1.06), the magnitude of drift velocity is given by

$$v_d = \frac{e\,E}{m}\,\tau$$

Substituting for v_d in equation (1.09), we get

$$I = \frac{n\,A\,e^2\,\tau}{m}\,E \qquad ...(1.10)$$

1.04. ELECTRON MOBILITY

The mobility of free electrons in a conductor is defined as the drift velocity acquired er unit strength of the electric field applied across the conductor. It is denoted by μ.

If v_d is drift velocity attained by free electrons on applying electric field E, then lectron mobility is given by

$$\mu = \frac{v_d}{E} \qquad ...(1.11)$$

Substituting for v_d (= μE) in equation (1.09), we have

$$I = n\,A\,\mu\,E\,e \qquad ...(1.12)$$

The equation (1.12) gives the relation between electron mobility and the current hrough the conductor.

From equation (1.11), it follows that in SI, the unit of electron mobility is **metre^2 olt^{-1} second^{-1}** ($\mathbf{m^2\,V^{-1}\,s^{-1}}$).

1.05. ELECTRIC CURRENT AND CURRENT DENSITY

Consider that a current I is flowing through a conductor of area of cross-section A. The current through the conductor can be defined in terms of a vector quantity $\vec{j}$ alled ***current density.*** If the current flowing through the conductor is uniform over ts cross section, then current may be defined as

$$I = \vec{j}\,.\,\vec{A}, \qquad ...(1.13)$$

here $\vec{A}$ is area vector representing the area of cross-section of the conductor. In case he current flowing through the conducor is not uniform over its cross-section, then he current through a small area $d\vec{A}$ of the conductor is given by

$$dI = \vec{j}\,.\,d\vec{A}$$

Hence, the current through the whole cross-section of the conductor is given y

$$I = \int \vec{j}\,.\,d\vec{A} \qquad ...(1.14)$$

Further, in case current density $\vec{j}$ is normal to the cross-sectional area of the conductor *i.e.* is $\hat{j}$ is parallel to $\vec{A}$, then

$$I = j A$$

or $$j = \frac{I}{A} \qquad ...(1.15)$$

From equation (1.09), substituting for I, we have

$$j = n v_d e \qquad ...(1.16)$$

Also, from equation (1.10) and (1.15), we have

$$j = \frac{n e^2 \tau}{m} E \qquad ...(1.17)$$

1.06. OHM'S LAW

It is the most fundamental law of electricity and was given by George Simon Ohm in 1828.

It states that *the physical conditions (temperature, mechanical strain, etc) remaining unchanged, the current flowing through a conductor is always directly proportional to the potential difference across its two ends.* Mathematically,

$$V \propto I$$

or $$V = R I, \qquad ...(1.18)$$

where the constant of proportionality R is called the ***electrical resistance*** or simply ***resistance*** of the conductor. Its value depends upon the nature of conductor, its dimensions and the physical conditions. It is independent of the values of V and I. The graph between potential difference (V) applied across a conductor and the current (I) flowing through it is a straight line as shown in Fig. 1.04.

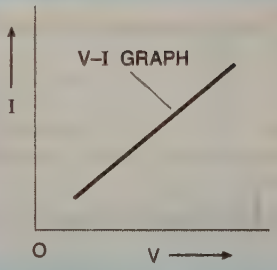


Fig. 1.04

1.07. RESISTANCE OF A CONDUCTOR

Resistance of a conductor is defined as the ratio of the potential difference applied across the conductor to the current flowing through it. Mathematically,

$$R = \frac{V}{I}$$

Unit of resistance. The SI unit of resistance is **ohm.** It is denoted by Ω.

$$\mathbf{1\ ohm\ (\Omega) = \frac{1\ volt\ (V)}{1\ ampere\ (A)}\ 1\ volt\ ampere^{-1}\ (V\ A^{-1})}$$

The resistance of a conductor is said to be one ohm, if one ampere of current flows through it, when a potential difference of one volt is applied across it.

Cause of resistance. The resistance of a conductor implies the opposition, which the conductor offers to the flow of charge through it. When a potential difference is applied across a conductor, an electric field is set up across its two ends. Due to this, the free electrons get accelerated. As the electrons move, they collide against the ions and atoms and their motion is thus opposed. This opposition offered by the ions and atoms is termed as the resistance of the conductor.

The jargon ......

A potential difference may be taken as the energy lost by 1 C of charge, as it moves through a resistor,

$$1\ V = 1\ J\ C^{-1}$$

The jargon ......

Ohmic conductors are those, which obey Ohm's law.

1.08. FAILURE OF OHM'S LAW

Ohm's law is not a fundamental law of nature. In many cases, the relation

$$V = R I$$

is not strictly obeyed and they lead to the failure of Ohm's law. A few cases are as explained below :

1. ***Current may vary non-linearly with potential difference.*** For a metallic conductor, graph between V and I is expected to be a straight line. However, the graph does not remain so, when the current is continuously increased through the conductor [Fig. 1.05]. The reason is that as current is increased, the temperature of the conductor rises and its resistance increases. The dotted straight line represents the theoretical curve, while the thick curve represents the actual V – I graph for a metallic conductor.

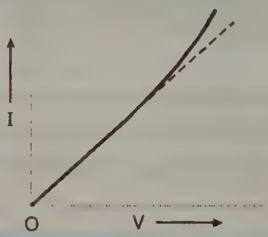


Fig. 1.05

2. ***The variation of current with potential difference may depend upon sign of the potential difference applied.*** For a semiconductor diode or *p-n* junction*, the variation of current is different, when the sign of potential difference applied across it is changed. When positive terminal of a battery is connected to *p*-section and negative terminal to the *n*-section (forward bias), the variation of current with

---

* For details about $p-n$ junction, refer to Unit IX.

ential difference is much more rapid than what it is when applied in opposite nner (reverse bias). It is as shown in Fig. 1.06.

It may be noted that in case of a semiconductor, the variation of current with ential difference is non-linear in addition to the fact that magnitude of variation ends upon the sign of potential difference applied across it.

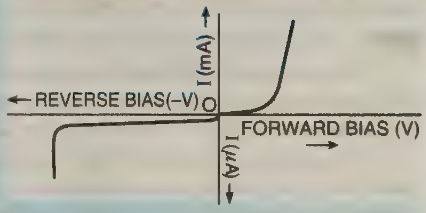


Fig. 1.06

3. ***The current may decrease on increasing the potential difference.*** A thyristor sists of four alternate layers of p and n-type semiconductors. The V – I graph th for forward and reverse bias) of a thyristor is of the type as shown in Fig. 1.07. portion AB of the graph shows that current increases on decreasing the potential erence. It may be noted that in addition to the fact that current increases with rease in potential difference for a thyristor, the current varies non-linearly with ential difference and also the magnitude of the variation of current depends upon sign of the potential difference applied across it.

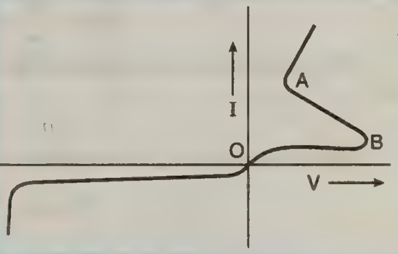


Fig. 1.07

1.09. RESISTIVITY

The resistance of a conductor depends upon the following factors :

(*i*) *It is directly proportional to the length of the conductor i.e.*

$$R \propto l$$

(*ii*) *It is inversely proportional to the area of cross-section of the conductor i.e.*

$$R \propto \frac{1}{A}$$

Combining the above two factors, we have

$$R \propto \frac{l}{A}$$

or

$$R = \rho \frac{l}{A} \qquad ...(1.19)$$

ere the constant of proportionality ρ is called ***electrical resistivity*** or ***specific stance*** of the conductor. Its value depends upon the nature of the material of conductor and its temperature. The equation (1.19) gives the resistance of a ductor in terms of its length, area of cross-section and resistivity of the material.

If $l = 1$, $A = 1$, then $R = \rho \frac{1}{1}$ or $\rho = R$

Hence, *resistivity or specific resistance of the material of a conductor is the resistance red by a wire of this material of unit length and unit area of cross-section.*

Unit of resistivity. From equation (1.19), we have

$$\rho = R \frac{A}{l}$$

In SI, unit of resistivity $= \text{ohm} \times \frac{\text{metre}^2}{\text{metre}} = \text{ohm metre}$

Therefore, SI unit of resistivity is **ohm metre (Ω m).**

The electrical resistivity of substances varies over a very wide range as given he following table :

MATERIAL	RESISTIVITY AT 0°C (Ω m)	MATERIAL	RESISTIVITY AT 0°C (Ω m)
(*a*) **Conductors :**		(*c*) **Semiconductors :**	
Silver	1.6×10^{-8}	**Carbon**	3.5×10^{-5}
Copper	1.7×10^{-8}	**Germanium**	0.46
Aluminium	2.7×10^{-8}	**Silicon**	2300
Tungsten	5.6×10^{-8}		
Iron	10×10^{-8}	(*d*) **Insulators**	
Platinum	11×10^{-8}	**Wood**	$10^8 - 10^{11}$
Mercury	98×10^{-8}	**Glass**	$10^{10} - 10^{14}$
(*b*) **Alloys :**		**Amber**	$5 - 10^{14}$
Manganin	44×10^{-8}	**Mica**	$10^{11} - 10^{15}$
Constantan	49×10^{-8}		
Nichrome	100×10^{-8}		

The first column lists the electrical resistivity of metals. It follows that metals ve low resistivities and they are called *conductors* of electricity. In the second umn, materials such as glass and hard rubber are about 10^{22} to 10^{24} times more istive than silver and copper. These are called *insulators*. Materials like carbon, germanium and silicon having resistivity in between those of conductors and insulators are called *semiconductors*.

1.10. FACTORS AFFECTING ELECTRICAL RESISTIVITY

The concept of average relaxation time helps in a great deal to understand the nature of electrical resistance. In order to do so, let us derive expression for resistance of a conductor in terms of average relaxation time (τ), electron density (n), etc.

Consider a conductor having length l and area of cross-section A. Let n be number of electrons per unit volume in the conductor. If an electric field E is applied across the two ends of the conductor, then drift velocity (in magnitude) of electrons is given by

$$v_d = \frac{e\,\mathrm{E}}{m}\tau$$

The current flowing through the conductor due to drift of electrons is given by

$$\mathrm{I} = n\,\mathrm{A}\,v_d\;e$$

Substituting for v_d in the above equation, we have

$$\mathrm{I} = n\,\mathrm{A}\left(\frac{e\,\mathrm{E}}{m}\tau\right)e$$

or

$$\mathrm{I} = \frac{n\,\mathrm{A}\,e^2\,\mathrm{E}\,\tau}{m} \qquad \text{...(1.20)}$$

If V is potential difference applied across the two ends of the conductor, then

$$\mathrm{E} = \frac{\mathrm{V}}{l}$$

Substituting for E in equation (1.20), we have

$$\mathrm{I} = \frac{n\,\mathrm{A}\,e^2\,\mathrm{V}\,\tau}{m\,l}$$

or

$$\frac{\mathrm{V}}{\mathrm{I}} = \frac{m\,l}{n\,e^2\,\tau\,\mathrm{A}}$$

But according to Ohm's law, $\frac{\mathrm{V}}{\mathrm{I}} = \mathrm{R}$, the resistance of the conductor.

$$\therefore \qquad \mathrm{R} = \frac{m}{n\,e^2\,\tau}\cdot\frac{l}{\mathrm{A}} \qquad \text{...(1.21)}$$

Comparing the above result with the expression for resistance obtained earlier *i.e.*

$$\mathrm{R} = \rho\frac{l}{\mathrm{A}},$$

it follows that resistivity of the material of a conductor is given by

$$\rho = \frac{m}{n\,e^2\,\tau} \qquad \text{...(1.22)}$$

It follows that resistivity of the material of a conductor depends on the following factors :

1. *It is inversely proportional to the number of free electrons per unit volume* (n) *of the conductor.* Since the value of n depends upon nature of the material, *the resistivity of a conductor depends upon the nature of the material.*

2. *It is inversely proportional to the average relaxation time* (τ) *of the free electrons in the conductor.* As we shall study in next section, the value of τ decreases with increase in temperature of the conductor. Since $\rho \propto \frac{1}{\tau}$, the *resistivity of conductor depends upon its temperature and it increases with increase in temperature of the conductor.*

Resistivity in terms of current density. From equation (1.17), we have

$$\frac{m}{n\,e^2\tau} = \frac{\mathrm{E}}{j}$$

Therefore, equation (1.22) becomes

$$\rho = \frac{\mathrm{E}}{j} \qquad \text{...(1.23)}$$

The equation (1.23) gives the relation between current density and resistivity of the material of the conductor.

Resistivity in terms of electron mobility. From equation (1.16), substituting in equation (1.23), we have

$$\rho = \frac{E}{n\,e\,v_d} \qquad ...(1.24)$$

Substituting for v_d (= μ E) in equation (1.24), we have

$$\rho = \frac{1}{n\,e\,\mu} \qquad ...(1.25)$$

The equation (1.25) gives the relation between resistivity of the material of a ductor and the electron mobility.

Note. From equation (1.24), we have

$$v_d = \frac{E}{n\,e\,\rho}$$

Since $E = \frac{V}{l}$, the above equation becomes

$$v_d = \frac{V}{n\,e\,l\,\rho} \qquad ...(1.26)$$

The equation (1.26) can be used to predict the effect of temperature and length e conductor on drift velocity of electrons.

(a) Effect of temperature. On increasing the temperature of a conductor, the e of resistivity of its material increases. Therefore, from equation (1.26), it follows *drift velocity of electrons decreases on increasing the temperature of the conductor.*

(b) Effect of length of conductor. From equation (1.26) it follows that *drift velocity ectrons decreases, when length of the conductor is increased.*

1.11. CONDUCTANCE AND CONDUCTIVITY

Conductance. *The reciprocal of resistance of a conductor is called its conductance.* It is denoted by G. Thus, conductance of a conductor having resistance R is n by

$$G = \frac{1}{R} \qquad ...(1.27)$$

The SI unit of conductance is **ohm^{-1}** (Ω^{-1}), which is also called **mho.** In SI, the of conductance is also called **siemen** and is denoted by the symbol **S.**

Conductivity. *The reciprocal of resistivity of the material of a conductor is called its uctivity.* It is denoted by σ. Thus,

$$\sigma = \frac{1}{\rho} \qquad ...(1.28)$$

From equations (1.23) and (1.28), it follows that

$$\sigma = \frac{j}{E} \qquad ...(1.29)$$

The equation (1.29) gives the relation beween conductivity and current density.

From equations (1.25), and (1.28), we have

$$\sigma = n\,e\,\mu \qquad ...(1.30)$$

The equation (1.30) gives the relation between conductivity of the material of nductor and the electron mobility.

From equation (1.28), it follows that unit of conductivity is reciprocal of the of resistivity. Therefore, in SI, the unit of conductivity is **ohm^{-1} metre^{-1} m^{-1})** or **mho metre^{-1}** or **siemen metre^{-1} (S m^{-1}).**

1.12. TEMPERATURE DEPENDENCE OF RESISTIVITY

The variation of resistivity of a metallic conductor, an alloy, a semi-conductor n insulator with increase in temperature is not same in all the cases.

Temperature dependence of the resistivity of a metallic conductor. In terms elaxation time, the resistivity of the material of a conductor is given by

$$\rho = \frac{m}{n\,e^2\,\tau},$$

re the letters have their usual meanings.

It follows that the resistivity of a conductor is inversely proportional to the age relaxation time of the free electrons in the conductor. The average relaxation is not a constant parameter. It changes with change of temperature of the ductor.

If the temperature of the conductor increases, the amplitude of the vibrations of the positive ions in the conductor also increases. Due to this, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases. Since $\rho \propto \frac{1}{\tau}$, the resistivity of the material of a conductor increases with increase of temperature.

For pure conductors such as copper, the resistivity increases linearly with temperature in the temperature range around and above the room temperature. If ρ_0 and ρ are resistivity of the material of a conductor at 0°C and θ°C respectively, then it is found that

$$\rho = \rho_0 (1 + \alpha \theta) \qquad ...(1.31)$$

Here, α is called ***temperature coefficient of resistivity***. It may be pointed out that at low temperature, the resistivity of pure conductors increases as a higher power of temperature.

Since $R = \rho \frac{l}{A}$, from the equation (1.31), it follows that

$$R = R_0 (1 + \alpha \theta) \qquad ...(1.32)$$

or

$$\alpha = \frac{R - R_0}{R_0 \times \theta} \qquad ...(1.33)$$

Here, R_0 and R_θ represent resistance of the conductor at 0°C and θ°C respectively. As such, α may also be called as ***temperature coefficient of resistance*** of the conductor.

Thus, resistivity of a conductor increases linearly with increase of temperature and vice versa. As temperature of a pure metallic conductor approaches 0 K, its resistivity also approaches zero. For metals, value of α lies between 10^{-2} to 10^{-4} °C^{-1}.

2. Temperature dependence of the resistivity of an alloy. In the case of an alloy, such as nichrome, the resistivity is very large and it has a very weak dependence on temperature. For example, manganin, which is an alloy of copper, nickel, iron and manganese has a resistance which is thirty to forty times as that of pure copper for the same dimensions but its temperature coefficient is very small *i.e.* 0·00001 °C^{-1}. For this reason, it is an excellent substance to be used for the construction of a standard resistance coil.

Key point

The resistivity of conductors increase that of semiconductors decreases increase of temperature.

3. Temperature dependence of the resistivity of a semi-conductor and insulator. The variation of resistivity with temperature in case of a semi-conductor or an insulator is different from that in case of a conductor. Whereas, in case of a conductor, the resistivity varies linearly with temperature of the conductor, the resistivity of a semi-conductor and an insulator has an exponential dependence on temperature. In case of a semi-conductor and an insulator, the resistivity at temperature T (in kelvin) is given by

$$\rho = \rho_0 \, e^{E_g/kT} \qquad ...(1.34)$$

Here, k is Boltzmann's constant and E_g is called energy gap between valence and conduction bands in the atoms of such materials. It may be noted that both semiconductors and insulators are non-conducting materials at low temperatures. The division of non-conducting materials into semiconductors and insulators is purely on the basis of the size of the value of E_g in two cases. In case of a semi-conductor, $E_g \le 1$ eV or so and likewise the resistivity of a semiconductor at room temperature is not very high. On the other hand, in case of an insulator, E_g is of the order of several eV and hence at room temperature, resistivity of an insulator is extremely high. At 0 K, both the semiconductor and an insulator have infinite value of the resistivity.

1.13. THERMISTORS

A thermistor is a heat sensitive resistor usually made of a semi conducting material, such that its resistance varies appreciably with change in temperature.

A thermistor differs from an ordinary resistance in the following ways :

1. The resistance of a thermistor changes very rapidly with change of temperature.
2. The temperature coefficient of a thermistor is very high.
3. The temperature coefficient of a thermistor can be both positive and negative.

The thermistors having negative temperature coefficients find a variety of plications in practice.

Thermistors are made from the oxides of semi-conductors. They are generally he form of beads, discs or rods. A pair of platinum leads are attached at the two ds for electric connections. The arrangement is enclosed in a small glass bulb and led. The size of a thermistor is very small.

Depending upon the composition, a thermistor may have resistance in the range ·1 Ω to 10^7 Ω. The thermistors can be used over a wide range of temperature *i.e.* m a very low temperature to about 1100°C. The change in the value of a thermistor be caused either due to change in temperature of its surroundings or by the wer dissipated in it on the passage of current.

Applications. Some of the important applications of thermistors are as given ow :

1. Due to their rapid response to temperature changes, thermistors are used detect small temperature change and to measure very low temperature (≈ 10 K).

2. Thermistors are used in temperature control units and for detecting excessive nperature within an industrial equipments.

3. Thermistors are used in the protection of windings of motors, transformers d generators.

4. Thermistors are used to safeguard the heater of the picture tube of a television against variation in current.

5. Thermistors are used for voltage stabilisation, temperature control and note sensing.

.14. SUPER-CONDUCTIVITY

The phenomenon, due to which a substance loses all signs of its resistance, when cooled ts critical temperature, is called ***superconductivity.***

The substances, which offer superconductivity (no resistance) are called ***perconductors.***

Explanation. When the temperature of a conductor is lowered, its resistance creases. In other words, a conductor becomes a better conductor as the nperature is lowered. There was speculation that resistance of a conductor will imately become zero at 0 K.

The boiling points of liquid oxygen, liquid nitrogen and liquid hydrogen are pectively –183° C, –196° C and –263° C. Therefore, the temperature up to which decrease in resistance with decrease of temperature could be studied was –263° e. 10 K. Ultimately, K. Onnes succeeded in liquefying helium gas and its boiling int was found to be 0·7 K. In 1911, K. Onnes studied the variation of resistivity solid mercury (it solidifies at–39° C) and he found that its resistivity kept on creasing in a normal manner with decrease in temperature but at 4·2 K, its istivity suddenly became zero [Fig. 1.08]. Thus, at 4·2 K, all signs of resistance appear in case of mercury *i.e.* it becomes superconductor. In case of mercury, this nperature of 4·2 K is called its ***critical temperature.***

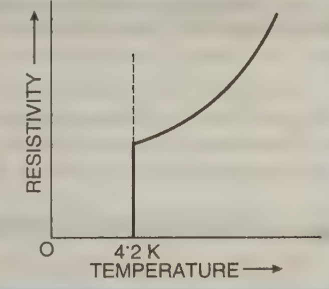


Fig. 1.08

Superconductivity is exhibited by a large number of metals and alloys. It has en discovered that alloys of niobium and tin, and of niobium and zirconium show perconductivity up to a temperature of 18 K. Today, there are thousands of perconductors with ***critical temperature*** (the temperature at which the resistivity ddenly drops to zero) as high as 23 K. Scientists all over the world are trying to nieve superconductivity at room temperature.

The cause of superconductivity is that electrons in a superconductor are tually coherent. The ionic vibrations, which could deflect free electrons in metals, unable to deflect the coherent cloud of electrons in superconductors.

Key point ✓

The conductivity of metals and alloys increases, as they are cooled. At critical temperature, their resistivity becomes zero and they behave as superconductors.

Applications (or uses) of superconductors. At present, superconductivity has en achieved in case of a few metals and alloys only at very low temperature. As ch, the superconductors so developed find their use in research work only. The y superconductivity is achieved at room temperature, the superconductors will come of very striking and interesting uses. A few of them are as below :

1. They will help to produce superfast computers.

2. It will be possible to transmit electrical power without any loss of electrical ergy by using superconducting cables.

3. Superconductors will enable to produce very high magnetic field without y expenditure of electrical power.

1.15. COLOUR CODE FOR CARBON RESISTANCES

The value of resistances used in electric and electronic circuits vary over a very wide range. Such high resistances used are usually carbon resistances and the values of such resistances are marked on them according to a colour code.

A carbon resistance has usually four rings or bands A, B, C and D of different colours [Fig. 1.09]. The colours of first two bands A and B indicate the first two significant figures of the resistance in ohm, while the colour of third band C indicates the decimal multiplier. The colour of fourth band D (which is either silver or gold) tells the *tolerance* of the resistance. If the colour of fourth band is silver or gold, then tolerance of the resistance is 10% and 5% respectively. Sometimes, there are only three colour bands A, B and C to indicate the value of the resistance. Since there is no fourth colour band, the tolerance of such a resistance is not indicated. The tolerance of such a resistance is 20%.

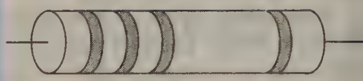


Fig. 1.09

To read the value of a carbon resistance, the following sentence serves as an aid to memory :

B B R O Y Great Britain Very Good Wife

The bold face letters in the above sentence (B, B, R, O, Y, G, B, V, G and W) correspond to the colours Black, Brown, Red, Orange, Yellow, Green, Blue, Violet, Grey and White respectively. The colours of first two bands A and B correspond to figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and the colour of the third band C corresponds to multipliers 10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8 and 10^9 respectively. If the colour of the fourth band is gold, the tolerance is 5% and in case the colour is silver, the tolerance is 10%. In case, there is no fourth band, then its tolerance is 20%.

The following table gives the colour code for carbon resistance :

Letter as an aid to memory	Colour	Figure	Multiplier	Colour	Tolerance
B	Black	0	10^0	Gold	5%
B	Brown	1	10^1	Silver	10%
R	Red	2	10^2	No colour	20%
O	Orange	3	10^3		
Y	Yellow	4	10^4		
G	Green	5	10^5		
B	Blue	6	10^6		
V	Violet	7	10^7		
G	Grey	8	10^8		
W	White	9	10^9		

For example, consider a carbon resistance, for which the bands A, B and C are of colours brown, green and orange respectively and the last band D is of silver in colour. The value of the resistance can be found from the table as explained below :

Corresponding to colours of bands A and B, which are brown and green, the figures are 1 and 5. Corresponding to the third band of orange colour, the multiplier is 10^3. Therefore, resistance is of the value $15 \times 10^3\ \Omega$. Since silver colour of the fourth band corresponds to a tolerance of 10%, the value of the given resistance is

$$R = 15 \times 10^3\ \Omega \ \pm 10\%$$

1.16. RESISTORS IN SERIES

Two or more resistors are said to be connected in series, if same current passes through each of them, when some potential difference is applied across the combination.

The jargon . . . . .

An electric component that offers ohm
resistance to the flow of current is calle
a resistor.

Fig. 1.10 shows the series combination of three resistors having resistances R_1, R_2 and R_3. If a battery of e.m.f. E is connected across the series combination, the same current I will pass through each of them. If V_1, V_2 and V_3 are the values of potential difference across R_1, R_2 and R_3 respectively, then according to Ohm's law,

$$V_1 = I R_1 ;\ V_2 = I R_2 \text{ and } V_3 = I R_3$$

Also, $$E = V_1 + V_2 + V_3$$

$\therefore$ $$E = I R_1 + I R_2 + I R_3$$

or $$E = I (R_1 + R_2 + R_3) \qquad ...(1.35)$$

If R_s is resistance equivalent to the series combination of R_1, R_2 and R_3, then resistance R_s will allow the same current I to flow through it, when the battery of e.m.f. E is connected across it. Therefore,

$$E = I R_s \qquad ...(1.36)$$

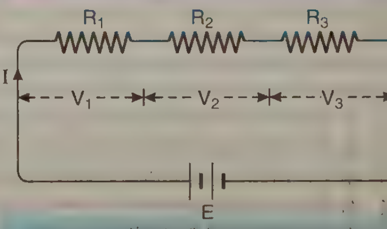


Fig. 1.10

om equations (1.35) and (1.36), we have

$$I R_s = I (R_1 + R_2 + R_3)$$

$$R_s = R_1 + R_2 + R_3 \qquad (1.37)$$

us, *when the resistors are connected in series, equivalent resistance of the series tion is equal to the sum of individual resistances.*

s obvious that the value of equivalent resistance of the series combination ys greater than the value of the largest individual resistance. Therefore, s are connected in series, if the resistance in the circuit is to be increased.

series resistance circuit, it should be remembered that

he current through all the resistors is same.

he total resistance of the circuit is equal to the sum of individual resistances including resistance of the cell, if any.

he potential difference across any resistor is proportional to its resistance.

he current in the circuit is independent of the relative positions of the various resistors rcuit.

Key point

In series combination of resistors,
(*i*) current through all the resistors is same and
(*ii*) potential difference across a resistor is proportional to its resistance.

Key point

The equivalent resistance of a series combination of resistors is always greater than the largest resistance in the combination. Therefore, resistors are connected in series in order to increase the resistance of the circuit.

17. RESISTORS IN PARALLEL

o or more resistors are said to be connected in parallel, if potential difference across hem is equal to the applied potential difference.

. 1.11 shows the parallel combination of three resistors having resistances nd R_3. If a battery of e.m.f. E is connected across the parallel combination, ntial difference across each resistor will be equal to E, the e.m.f. of the battery. and I_3 are values of the current through the resistances R_1, R_2 and R_3 vely, then current in main circuit is given by

$$I = I_1 + I_2 + I_3 \qquad ...(1.38)$$

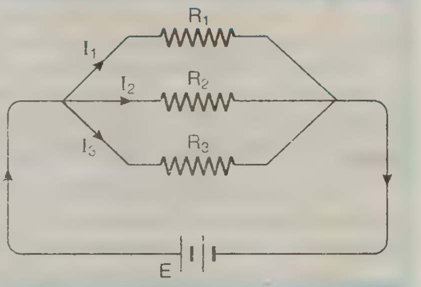


Fig. 1.11

w, the potential difference across each resistor is E. Therefore, applying aw, we have

$$E = I_1 R_1 = I_2 R_2 = I_3 R_3$$

$$I_1 = \frac{E}{R_1}\ ;\ I_2 = \frac{E}{R_2} \text{ and } I_3 = \frac{E}{R_3}$$

erefore, equation (1.38) becomes

$$I = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

$$I = R\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad ...(1.39)$$

Key point

In parallel combination of resistors,
(*i*) potential difference across all of them is same and
(*ii*) current through any resistor is inversely proportional to its resistance.

R_p is resistance equivalent to the parallel combination of R_1, R_2 and R_3 *i.e.* if stance R_p allows the same current I to flow through it, when the battery of is connected across it, then

$$E = I R_p$$

$$I = \frac{E}{R_p} \qquad ...(1.40)$$

om equations (1.39) and (1.40), we have

$$\frac{E}{R_p} = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad ...(1.41)$$

us, *when resistors are connected in parallel, reciprocal of equivalent resistance of llel combination is equal to the sum of the reciprocals of individual resistances.*

may be pointed out that the value of equivalent resistance is always less than e of the smallest individual resistance. Therefore, resistors are connected lel, if the resistance in the circuit is to be decreased.

a parallel resistance circuit, it should be remembered that

otal current through the parallel combination is equal to the sum of individual through the various resistors.

he potential difference across all the resistors is same.

the current through any resistor is inversely proportional to its resistance.

he reciprocal of the total resistance of the combination is equal to the sum of the ls of individual resistances.

Key point

The equivalent resistance of a parallel combination of resistors is always less than the smallest resistance in the combination. Therefore, resistors are connected in parallel in order to decrease the resistance of the circuit.

1.18. INTERNAL RESISTANCE AND ELECTROMOTIVE FORCE OF THE CELL

Internal resistance of a cell. When a cell is connected in a circuit, it drives the charge in the circuit. The rate of flow of charge is termed as current. When the current flows through the cell, its electrolyte offers resistance to the flow of current. It is because of the fact that ions have to move against the background of other ions and the neutral atoms.

The resistance offered by the electrolyte of the cell, when the electric current flows through it, is known as internal resistance of the cell. It is denoted by r.

For a freshly prepared cell, the value of internal resistance is very low, but as the cell is put to more and more use, its internal resistance goes on increasing.

Electromotive force (e.m.f.). *The potential difference between the two poles of the cell in an open circuit (when no current is drawn from the cell) is called the electromotive force (e.m.f.) of the cell.*

Due to its e.m.f., a cell drives the charge round the circuit. Therefore, *e.m.f. of a cell is also defined as the energy supplied by the cell to drive a unit charge round the complete circuit.* It is denoted by E.

In SI, the unit of e.m.f. is **volt (V)** or **joule coulomb^{-1} (J C^{-1}).**

The e.m.f. of a cell is called one volt, if 1 joule of work is performed by the cell to drive one coulomb of charge round the circuit.

Note. It may be pointed out that the word electromotive force (e.m.f.) is misnomer. It does not represent force on the carriers of electricity. Instead, it represents the work done per unit charge to drive the carriers of electricity.

1.19. TERMINAL POTENTIAL DIFFERENCE OF A CELL

Terminal potential difference. *The potential difference between the two poles of a cell in a closed circuit (when current is drawn from the cell) is called the terminal potential difference of the cell.* It is denoted by V. Its unit is same as that of e.m.f.

When current flows through the circuit, there is fall of potential across the internal resistance of the cell. The potential difference between the two poles of the cell is less than the e.m.f. of the cell by an amount equal to potential drop across the internal resistance. Therefore, the terminal potential difference of a cell is always less than its e.m.f.

Expression for internal resistance. Let us consider a cell of e.m.f. E and internal resistance r connected to an external resistance R and a one way key K as shown in Fig. 1.12. A voltmeter V of high resistance is connected across the poles of the cell.

When the circuit is open (plug not inserted in the key), no current flows through the circuit. Therefore, from the definition of the e.m.f., the reading of the voltmeter is equal to E, e.m.f. of the cell.

Now, close the key [Fig. 1.13]. The current will flow through the circuit. Since internal resistance (r) of the cell and the external resistance (R) are in series, the total resistance in the circuit is (R + r).

According to Ohm's law, the current in the circuit is given by

$$I = \frac{E}{R + r} \qquad \text{...(1.42)}$$

Since current is drawn from the cell on closing the key, according to the definition of terminal potential difference, the reading of the voltmeter connected across the two poles of the cell is equal to the terminal potential difference between its two poles. The terminal potential difference V is less than e.m.f. E of the cell by an amount equal to potential drop across the internal resistance of the cell *i.e.*

$$V = E - I r \qquad (1.43)$$

Here, I r is the potential drop across internal resistance of the cell. It is also called ***lost volt***.

If no current is drawn from the cell *i.e.* I = 0, then from equation (1.43), we have

$$V = E - 0\,(r) = E$$

Hence, the terminal potential difference between the two poles of a cell is equal to its e.m.f., if no current is drawn from it.

Also, terminal potential difference is equal to potential differences across external resistance R *i.e.*

$$V = I R \qquad \text{...(1.44)}$$

Watch out !

Though called force, electromotiv
does not represent force. It repr
work done per unit charge !

The jargon . . . . .

E.M.F. of a cell (in volt) is energy
by 1 C of charge as it passes thro

Self-test Question

Why is there no way to separate th
nal resistance from the e.m.f. of a

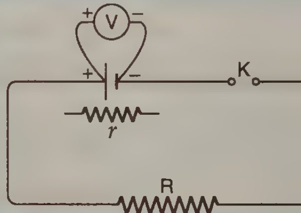


Fig. 1.12

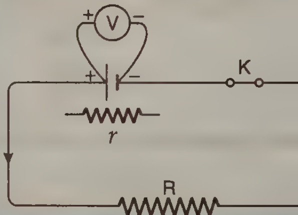


Fig. 1.13

Key point

(*i*) When a current I flows ***out*** of
terminal potential difference,

$$V = E - I r$$

i.e. potential drop across interna
tance ***opposes*** the e.m.f.

(*ii*) When a current I flows ***into***
terminal potential difference,

$$V = E + I r$$

i.e. potential drop across interna
tance ***aids*** the e.m.f.

From equations (1.42) and (1.44), we have

$$V = \frac{E}{R+r} R \quad \text{or} \quad r = \frac{R-V}{V} R$$

or $$r = \left(\frac{E}{V} - 1\right) R \qquad \text{...(1.45)}$$

It is expression for internal resistance of the cell in terms of e.m.f. and terminal ntial difference.

Key point

The potential drop across internal resistance of the cell is called ***lost volt***.

1.20. GROUPING OF CELLS

From a single cell, only a limited amount of current can be drawn. In many riments, the current required may be much more than that can be supplied by gle cell. In order to have a large value of current, the cells are usually connected ries or in parallel or the cells are mixed grouped.

1.21. CELLS IN SERIES

Fig. 1.14. shows a combination of n cells each of e.m.f. E and internal resistance nected in series so as to send current through an external resistance R. For series bination, the negative terminal of the first cell is connected to the positive inal of the second, the negative terminal of the second cell to the positive inal of the third and so on.

When the cells are connected in series, the total e.m.f. of the series combination ual to the sum of the e.m.fs. of the individual cells. Thus,

the total e.m.f. of the cells = nE

As the internal resistances of the cells also come in series,

the total internal resistance of the cells = $r + r + r + \ldots$ n times = nr

Therefore, total resistance of the circuit = $R + nr$

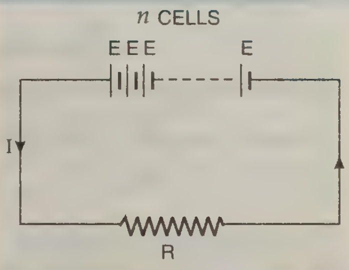


Fig. 1.14

Fig. 1.15 shows the effective e.m.f. and the internal resistance in the circuit, when ells are connected in series.

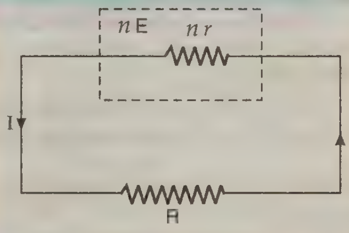


Fig. 1.15

If I is current flowing through the circuit, then

$$I = \frac{\text{total e.m.f.}}{\text{total resistance}}$$

or $$I = \frac{nE}{R + nr} \qquad \text{...(1.46)}$$

1. ***When R < < n r.*** In such a case,

$$R + nr \approx nr$$

Therefore, equation (1.46) becomes

$$I = \frac{nE}{nr} = \frac{E}{r} = \text{current due to single cell}$$

2. ***When R > > n r.*** In such a case,

$$R + nr \approx R$$

Therefore, equation (1.46) becomes

$$I = \frac{nE}{R} = n \text{ times the current due to a single cell}$$

From the above discussion, it may be concluded that *in order to have maximum nt, the cells should be connected in series, when the total internal resistance of the cells gligible as compared to the external resistance in the circuit.*

Key point

When n indentical cells are connected in series,

(*i*) e.m.f. of the combination becomes n times the e.m.f. of a single cell.

(*ii*) internal resistance of the combination becomes n times the internal resistance of a single cell.

Therefore, when external resistance of the circuit is very large, the cells should be connected in series so as to obtain maximum current.

1.22. CELLS IN PARALLEL

Consider that m cells each of e.m.f. E and internal resistance r are connected rallel so as to send current through the external resistance R as shown in Fig.

The cells are connected in parallel by connecting the positive terminals of all ells to one end of the external resistance and the negative terminals of all the to the other end of the external resistance.

When the cells are connected in parallel, the total e.m.f. of the parallel bination remains equal to the e.m.f. of a single cell. Therefore,

the total e.m.f. of the cells = E

As the internal resistance of the cells also come in parallel, the total internal tance r' of the cells is given by

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \ldots m \text{ times}$$

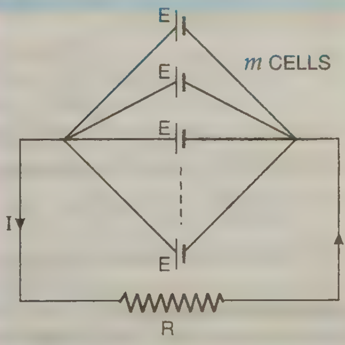


Fig. 1.16

or $$r' = \frac{r}{m}$$

Fig. 1.17 shows the effective e.m.f. and the internal resistance in the circuit, when the cells are connected in parallel.

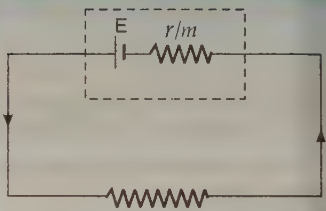


Fig. 1.17

If I is current flowing through the circuit, then

$$I = \frac{\text{total e.m.f.}}{\text{total resistance}} = \frac{R}{R + r'}$$

or $$I = \frac{E}{R + r/m} \qquad \text{...(1.47)}$$

1. ***When R > > r/m.*** In such a case,

$$R + r/m \approx R$$

Therefore, equation (1.47) becomes

$$I = \frac{E}{R} = \text{current due to single cell}$$

2. ***When R < < r/m.*** In such a case,

$$R + r/m \approx r/m$$

Therefore, equation (1.47) becomes

$$I = \frac{E}{r/m} = m\frac{E}{r} = m \text{ times current due to a single cell}$$

From the above discussion, it may be concluded that *in order to have maximum current, the cells should be connected in parallel, when the external resistance in the circuit is negligible as compared to the total internal resistance of the cells.*

Key point

When m identical cells are conencte
parallel,
(*i*) e.m.f. of the combination is equa
that of a single cell.
(*ii*) internal resistance of the combina
becomes $\frac{1}{m}$ th of the internal resistanc
a single cell.
Therefore, when external resistance in
circuit is negligible, the cells shoul
connected in parallel so as to obtain m
mum current.

1.23. MIXED GROUPING OF CELLS

When the external resistance of the circuit is of the same order as the internal resistance of the cells, then a large value of current is obtained by mixed grouping of the cells.

Fig. 1.18 shows N cells each of e.m.f. E and internal resistance r arranged in m rows, each row having n cells. Obviously,

$$N = m\,n$$

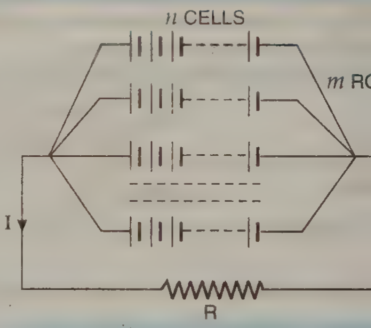


Fig. 1.18

This series-parallel combination of the cell sends current through an external resistance R.

Now, total e.m.f. of the cells = total e.m.f. of the cells connected in a row = n E

The internal resistance of the cells connected in a row = $n\,r$

Since there are such m rows, the total internal resistance of the cells is given by

$$\frac{1}{r'} = \frac{1}{n\,r} + \frac{1}{n\,r} + \frac{1}{n\,r} + \ldots.. + m \text{ times } = \frac{m}{n\,r}$$

or $$r' = \frac{n\,r}{m}$$

Fig. 1.19 shows the effective e.m.f. and the internal resistance in circuit, when the cells are mixed grouped as said above.

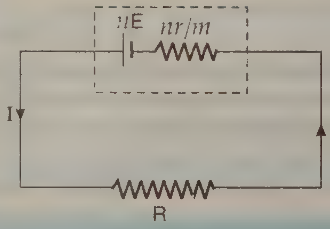


Fig. 1.19

$$\text{Total resistance of the circuit} = R + r' = R + \frac{n\,r}{m}$$

If I is current flowing through the circuit, then

$$I = \frac{n\,E}{R + n\,r/m} = \frac{m\,n\,E}{m\,R + n\,r} = \frac{N\,E}{m\,R + n\,r} \qquad (\because m\,n = N)$$

The current in the circuit will be maximum, if the factor in the denominator *i.e.* $m\,R + n\,r$ is minimum.

Now, $$m\,R + n\,r = (\sqrt{m\,R})^2 + (\sqrt{n\,r})^2 - 2\sqrt{m\,R}\,\sqrt{n\,r} + 2\sqrt{m\,R}\,\sqrt{n\,r}$$
$$= (\sqrt{m\,R} - \sqrt{n\,r})^2 + 2\sqrt{m\,n\,R\,r}$$

The R.H.S. of the above expression is sum of the two positive factors. Therefore, the factor $m\,R + n\,r$ will be minimum, if

$$\sqrt{m\,R} - \sqrt{n\,r} = 0 \quad \text{or} \quad m\,R = n\,r \quad \text{or} \quad R = \frac{n\,r}{m}$$

i.e. when external resistance in the circuit is equal to the total internal resistance of the cells.

From the above discussion, it follows that *in order to obtain maximum current, the cells should be mixed grouped in such a manner that the external resistance in the circuit is equal to the total internal resistance of the cells in mixed grouping.*

SOLVED NUMERICAL PROBLEMS

e A. On Definition of Current

Problem 1.01. How many electrons pass through a in one minute, if the current is 300 mA ? Given, charge ectron = 1.6×10^{-19} C.

(*P.S.S.C.E. 2002 ; H.P.S.S.C.E. 1995*)

Sol. Here, $I = 300 \text{ mA} = 300 \times 10^{-3}$; $e = 1.6 \times 10^{-19}$ C ;

$t = 1$ minute $= 60$ s

Charge passing through lamp in one minute,

$$q = I \times t = 300 \times 10^{-3} \times 60 \text{ C}$$

Suppose that n electrons pass through the lamp in one te. If e is charge on an electron, then

$$q = n\,e$$

or
$$q = \frac{n}{e} = \frac{300 \times 10^{-3} \times 60}{1.6 \times 10^{-19}} = 1.125 \times 10^{20}$$

Problem 1.02. In hydrogen atom, electron revolves nd the nucleus along a path of radius 0·51 Å making 10^{15} revolutions per second. Calculate the equivalent nt. Given charge on electron = 1.6×10^{-19} C.

Sol. Here, $e = 1.6 \times 10^{-19}$ C ; $\nu = 6.8 \times 10^{15}$ revolutions s^{-1} ;

$r = 0.51 \text{ Å} = 0.51 \times 10^{-10}$ m

f T is time period of electron, then it crosses a point on cular path after T seconds. Hence, equivalent current,

$$I = \frac{e}{T} = e\,\nu = 1.6 \times 10^{-19} \times 6.8 \times 10^{15}$$
$$= 1.088 \times 10^{-3} \text{ A}$$

e B. On Drift velocity

Problem 1.03. A current of 10 A is maintained in a uctor of cross-section 10^{-4} m^2. If the number density ee electrons be 9×10^{28} m^{-3}, calculate the drift ity of free electrons. Given that charge on electron, × 10^{-9} C.

Sol. Here $I = 10$ A ; $n = 9 \times 10^{28}$ m^{-3} ; $A = 10^{-4}$ m^2

and $e = 1.6 \times 10^{-19}$C

Now,
$$v_d = \frac{I}{n\,e\,A} = \frac{10}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-4}}$$
$$= 6.94 \times 10^{-6} \text{ m s}^{-1}$$

Problem 1.04. The number density of conduction rons in a copper conductor is 8.5×10^{28} m^{-3}. How long an electron take to drift from one end of a wire 3·0 m to its other end ? The area of cross-section of the wire × 10^{-6}m^2 and it is carrying a current of 3·0 A.

(*Text Problem*)

Sol. Here, $n = 8.5 \times 10^{28}$ m^{-3} ; $I = 3.0$ A ;

$A = 2.0 \times 10^{-6}$ m^2 ; $l = 3.0$ m

We know that charge on electron, $e = 1.6 \times 10^{-19}$ C

Now, drift velocity of electrons,

$$v_d = \frac{I}{n\,e\,A} = \frac{3.0}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.0 \times 10^{-6}}$$
$$= 1.103 \times 10^{-4} \text{ m s}^{-1}$$

Therefore, time taken by electron to drift from one end e other,

$$t = \frac{l}{v_d} = \frac{3.0}{1.103 \times 10^{-4}}$$
$$= 2.72 \times 10^4 \text{ s} = 7 \text{ hours and } 33 \text{ minutes}$$

Problem 1.05. Calculate the average drift velocity of conduction electrons in a copper wire of cross-section 10^{-7} m^2 carrying a current of 1 A. Assume that each copper atom contributes one conduction electron. Given that density of copper = 9×10^3 kg m^{-3} and its atomic mass = 63·5.

Sol. Here $A = 10^{-7}$ m^2 ; $I = 1$ A

Now, 63·5 g of copper contains atoms equal to Avogadro number *i.e.* 6.02×10^{23}.

Therefore, number of atoms in 1 kg of copper

$$= \frac{6.02 \times 10^{23}}{63.5} \times 1000$$

Since density of copper is 9×10^3 kg m^{-3}, number of atoms in 1 m^3 of copper

$$= \frac{6.02 \times 10^{23} \times 1000}{63.5} \times 9 \times 10^3 = 8.53 \times 10^{28}$$

As one copper atom contributes one conduction electron, number density of electron in copper is equal to number of atoms in 1 m^3 of copper *i.e.*

$$n = 8.53 \times 10^{28}$$

Hence, drift velocity of electrons,

$$v_d = \frac{I}{n\,e\,A} = \frac{1}{8.53 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-7}}$$
$$= 7.33 \times 10^{-4} \text{ m s}^{-1}$$

Type C. On Electron mobility

Problem 1.06. A potential difference of 5 V is applied across a conductor of length 0·1 m. If drift velocity of electrons is 2.5×10^{-4} m s^{-1}, calculate the electron mobility.

Sol. Here, $V = 5$ volt ; $l = 0.1$ m ; $v_d = 2.5 \times 10^{-4}$ m s^{-1}

Electric field set up across the conductor,

$$E = \frac{V}{l} = \frac{5}{0.1} = 50 \text{ V m}^{-1}$$

Now, electron mobility,

$$\mu = \frac{v_d}{E} = \frac{2.5 \times 10^{-4}}{50} = 5 \times 10^{-6} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

Problem 1.07. The number density of electrons in copper is 8.5×10^{28} m^{-3}. Find the current flowing through a copper wire of length 0·2 m, area of cross-section 1 mm^2, when connected to a battery of 3 V. Given that electron mobility = 4.5×10^{-6} m^2 V^{-1} s^{-1} and charge on electron = 1.6×10^{-19} C.

Sol. Here, $V = 3$ volt ; $l = 0.2$ m ; $A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$;

$n = 8.5 \times 10^{28}$ m^{-3} ; $\mu = 4.5 \times 10^{-6}$ m^2 V^{-1} s^{-1}

and $e = 1.6 \times 10^{-19}$ C

Electric field set up across the conductor,

$$E = \frac{V}{l} = \frac{3}{0.2} = 15 \text{ V m}^{-1}$$

Now, current through the wire,

$$I = n\,A\,\mu\,E\,e = 8.5 \times 10^{28} \times 10^{-6} \times 15 \times 1.6 \times 10^{-19}$$
$$= 0.92 \text{ A}$$

Type D. On Current density

Problem 1.08. A current of 1·8 A flows through a wire of area of cross-section 0·5 mm^2. Find the current density

in the wire. If the number of density of electrons in the wire is $8{\cdot}8 \times 10^{28}$ m^{-3}, find the drift velocity of electrons.

Sol. Here, $I = 1{\cdot}8$ A ; $A = 1{\cdot}2$ mm^2 $= 0{\cdot}5 \times 10^{-6}$ m^2

and $n = 8{\cdot}8 \times 10^{28}$ m^{-3}

Now, current density,

$$j = \frac{I}{A} = \frac{1\cdot 8}{0\cdot 5 \times 10^{-6}} = \mathbf{3{\cdot}6 \times 10^6\ A\ m^{-2}}$$

Also, $j = n\, v_d\, e$

or $$v_d = \frac{j}{n\,e} = \frac{3\cdot 6 \times 10^6}{8\cdot 8 \times 10^{28} \times 1\cdot 6 \times 10^{-19}}$$

$$= \mathbf{2{\cdot}56 \times 10^{-4}\ m\ s^{-1}}$$

Problem 1.09. When a potential difference of 1·5 V is applied across a wire of length 0·2 m and area of cross-section 0·33 mm^2, a current of 2·4 A flows through the wire. If the number density of free electrons in the wire is $8{\cdot}4 \times 10^{28}$ m^{-3}, calculate the average relaxation time. Given that mass of electron = $9{\cdot}1 \times 10^{-31}$ kg and charge on electron = $1{\cdot}6 \times 10^{-19}$ C.

Sol. Here, $V = 1{\cdot}5$ V ; $n = 8{\cdot}4 \times 10^{28}$ m^{-3} ; $l = 0{\cdot}2$ m ;
$A = 0{\cdot}3$ mm^2 $= 0{\cdot}3 \times 10^{-6}$ m^2, $I = 2{\cdot}4$ A ;
$m = 9{\cdot}1 \times 10^{-31}$ kg and $e = 1{\cdot}6 \times 10^{-19}$ C

Electric field set up across the conductor,

$$E = \frac{V}{l} = \frac{1\cdot 5}{0\cdot 2} = 7\cdot 5\ \text{V m}^{-1}$$

The current density in the wire,

$$j = \frac{I}{A} = \frac{2\cdot 4}{0\cdot 3 \times 10^{-6}} = 8 \times 10^6\ \text{A m}^{-2}$$

Now, $$j = \frac{n\,e^2\,\tau}{m}\,E$$

Therefore, average relaxation time,

$$\tau = \frac{m\,j}{n\,e^2\,E} = \frac{9\cdot 1 \times 10^{-31} \times 8 \times 10^6}{8\cdot 4 \times 10^{28} \times (1\cdot 6 \times 10^{-19})^2 \times 7\cdot 5}$$

$$= \mathbf{4{\cdot}51 \times 10^{-16}\ s}$$

Type E. On Ohm's law

Problem 1.10. A potential difference of 200 volt is maintained across a conductor of resistance 100 ohm. Calculate the number of electrons flowing through it in one second. Charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C.

Sol. Here, V = 200 volt ; R = 100 ohm ; $e = 1{\cdot}6 \times 10^{-19}$ C

Now, $$I = \frac{V}{R} = \frac{200}{100} = 2\ \text{ampere}$$

Charge flowing in 1 s, $q = I\,t = 2 \times 1 = 2$ C

Therefore, number of electrons flowing through the conductor in 1 s,

$$n = \frac{q}{e} = \frac{2}{1\cdot 6 \times 10^{-19}} = \mathbf{1{\cdot}25 \times 10^{19}}$$

Type F. On Resistivity

Problem 1.11. Calculate the resistivity of the material of a wire 1·0 m long, 0·4 mm in diameter and having a resistance of 2·0 ohm. *(H.S.S.C.E. 2002 ; P.S.S.C.E. 1994)*

Sol. Here, $R = 2{\cdot}0\,\Omega$; $l = 1{\cdot}0$ m ;

diameter of wire, $d = 0{\cdot}4$ mm $= 4 \times 10^{-4}$ m

Therefore, area of cross-section of wire,

$$A = \frac{1}{4}\pi d^2 = \frac{1}{4}\pi \times (4 \times 10^{-4})^2 = 4\pi \times 10^{-8}\ \text{m}^2$$

If ρ is resistivity of the material of the wire, then

$$R = \rho\,\frac{l}{A}$$

or $$\rho = \frac{R\,A}{l} = \frac{2\cdot 0 \times 4\pi \times 10^{-8}}{1\cdot 0} = \mathbf{2{\cdot}153 \times 10^{-7}\ \Omega}$$

Problem 1.12. A wire having a mass of 0·4 possesses a resistance of 0·14 Ω. If the resistivity o material of wire is $1{\cdot}78 \times 10^{-8}$ Ω m, calculate its length radius. Given that density of the material of wire is 8 10^3 kg m^{-3}.

Sol. Here, $m = 0{\cdot}45$ kg ; $R = 0{\cdot}14\,\Omega$; $\rho = 1{\cdot}78 \times 10^{-8}$
and density of the material of wire, $d = 8{\cdot}93 \times 10^3$ kg

Let l be length and r be the radius of the wire.

Now, $$R = \rho\,\frac{l}{A} = \rho\,\frac{l}{\pi r^2}$$

Also, mass of the wire, m = volume × density

or $$m = \pi r^2\, l \times d$$

Multiplying equation (*i*) and (*ii*), we have

$$R \times m = \rho\,\frac{l}{\pi r^2} \times \pi r^2\, l \times d$$

or $$l = \sqrt{\frac{R\,m}{\rho\,d}} = \sqrt{\frac{0\cdot 14 \times 0\cdot 45}{1\cdot 78 \times 10^{-8} \times 8\cdot 93 \times 10^3}} = \mathbf{1{\cdot}}$$

From equation (*ii*), we have

$$r = \sqrt{\frac{m}{\pi\,l\,d}} = \sqrt{\frac{0\cdot 45}{\pi \times 1\cdot 99 \times 8\cdot 93 \times 10^3}}$$

$$= 2{\cdot}84 \times 10^{-3}\ \text{m} = \mathbf{2{\cdot}84\ mm}$$

Problem 1.13. A wire of resistance 5 ohm is draw so that its length is increased to twice its original le Calculate its new resistance.

(C.B.S.E. 1998 S ; H.P.S.S.C.E. 1

Sol. Let l be the length and A be the area of cross-sec of the wire. If ρ is the resistivity of its material, then

$$R = \rho\,\frac{l}{A}$$

Suppose that when length of the wire is increased (= 2 l), its area of cross-section becomes A′. As the volu the wire must remain the same, we have

$$A\,l = A'\,l' \qquad \text{or} \qquad A' = \frac{A\,l}{2\,l} = \frac{A}{2}$$

Let R′ be new resistance of the wire, then,

$$R' = \rho\,\frac{l'}{A'} = \rho\,\frac{2\,l}{A/2} = 4\,\rho\,\frac{l}{A}$$

From equation (*i*) and (*ii*), we have

$$R' = 4\,R = 4 \times 5 = \mathbf{20\ \Omega} \qquad (\because R =$$

Type G. On Conductance and conductivity

Problem 1.14. Calculate the electrical conductivi the material of a conductor of length 3 m, area of c section 0·2 mm^2 having a resistance of 2 ohm.

(C.B.S.E. 1

Sol. Here, $l = 3$; $A = 0{\cdot}02$ mm^2 $= 0{\cdot}02 \times 10^{-6}$ m^2 ; R =

Now, $$R = \rho\,\frac{l}{A} \qquad \text{or} \qquad \rho = \frac{R\,A}{l}$$

Also, conductivity of the wire, $\sigma = \dfrac{1}{\rho}$

or $$\sigma = \frac{l}{R\,A} = \frac{3}{2 \times 0\cdot 02 \times 10^{-6}} = \mathbf{7{\cdot}5 \times 10^7\ S}$$

blem 1.15. A wire carries a current of 1·2 A, when ial difference of 1·8 V is applied across it. What is uctance ? If the wire is of length 3 m and area of ction 5·4 × 10^{-6} m^2, calculate its conductivity.

. Here, $V = 1{\cdot}8\ V$; $I = 1{\cdot}2\ A$;

$l = 3$ m and $A = 5{\cdot}4 \times 10^{-6}\ m^2$

e resistance of wire, $R = \frac{V}{I} = \frac{1\cdot8}{1\cdot2} = 1\cdot5\ \Omega$

erefore, conductance of wire, $G = \frac{1}{R} = \frac{1}{1\cdot5} = \mathbf{0{\cdot}67\ S}$

w, $R = \rho\frac{l}{A}$ or $\rho = \frac{RA}{l}$

o, conductivity of wire, $\sigma = \frac{1}{\rho}$

$$\sigma = \frac{l}{RA} = \frac{3}{1\cdot5\times5\cdot4\times10^{-6}}$$

$= \mathbf{3{\cdot}7 \times 10^5\ S\ m^{-1}}$

. On Variation of resistance with temperature

blem 1.16. A wire has a resistance of 2·1 Ω at 300 a resistance of 2·7 Ω at 373 K. Determine the ture coefficient of resistance of the material.

(Pre-degree Kerala, 1991 ; Text Problem)

. Here, $R_1 = 2{\cdot}1\ \Omega$; $\theta_1 = 300$ K ; $R_2 = 2{\cdot}7\ \Omega$;

$\theta_2 = 373$ K

w, $$\alpha = \frac{R_2 - R_1}{R_1(\theta_2 - \theta_1)} = \frac{2\cdot7 - 2\cdot1}{2\cdot1\times(373-300)}$$

$= \mathbf{3{\cdot}194 \times 10^{-3}\ K^{-1}}$

blem 1.17. The resistance of a conductor is 6 Ω at d 7 Ω at 100°C. Calculate the mean temperature ent of resistance of the material. Find the resistance onductor at 0°C. *(Karnataka, 1983 S)*

. Here, $\theta_1 = 50°C$; $R_1 = 6\ \Omega$; $\theta_2 = 100°C$; $R_2 = 7\ \Omega$

an temperature coefficient of resistance of the

,

$$\alpha = \frac{R_2 - R_1}{R_1\times(\theta_2 - \theta_1)} = \frac{7-6}{6\times(100-50)}$$

$$= \frac{1}{6\times50} = \mathbf{\frac{1}{300}}\ °C^{-1}$$

$_0$ is resistance of the conductor at 0°C, then

$R_1 = R_0(1 + \alpha\,\theta)$

$$R_0 = \frac{R_1}{1+\alpha\,\theta} = \frac{6}{1+\frac{1}{300}\times50} = \frac{36}{7} = 5{\cdot}143\ W$$

blem 1.18. A heating element using nichrome ed to a 230 V supply draws an initial current of hich settles after a few seconds to a steady value of What is the steady temperature of the heating t, if the room temperature is 27°C ? Temperature ent of resistance of nichrome averaged over the ture range involved is 1·70 × 10^{-4} $°C^{-1}$.

(Text Problem)

.. Here, V = 230 volt ;

ial room temperature, $\theta_1 = 27°C$

ial current, $I_1 = 3{\cdot}2$ A; steady current, $I_2 = 2{\cdot}8$ A

erefore, resistance of wire at room temperature (θ_1),

$$R_1 = \frac{V}{I_1} = \frac{230}{3\cdot2} = 71{\cdot}875\ \Omega$$

Resistance of wire at steady temperature (θ_2),

$$R_2 = \frac{V}{I_2} = \frac{230}{2\cdot8} = 82{\cdot}143\ \Omega$$

Temperature coefficient of the wire, $\alpha = 1{\cdot}70 \times 10^{-4}\ °C^{-1}$

Now, $$\alpha = \frac{R_2 - R_1}{R_1(\theta_2 - \theta_1)}$$

$$\therefore\ 1{\cdot}70\times10^{-4} = \frac{82\cdot143 - 71\cdot875}{71\cdot875\times(\theta_2 - 27)}$$

or $$\theta_2 = 27 + \frac{82\cdot143 - 71\cdot875}{71\cdot875\times1\cdot70\times10^{-4}}$$

$= 27 + 840{\cdot}35 = \mathbf{867{\cdot}35\ °C}$

Type I. On colour code of carbon resistance

Problem 1.19. A carbon resistor has coloured strips as shown in Fig. 1.20. What is its resistance ?

(H.S.S.C.E. 1996 ; H.P.S.S.C.E. 1995 ; C.B.S.E. 1990)

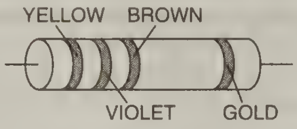


Fig. 1.20

Sol. Corresponding to first two colour bands of yellow and violet colours, the figures are 4 and 7. Corresponding to the third band of brown colour, the multiplier is 10^1 *i.e.* 10. Therefore, the given carbon resistor is of value 47 × 10 *i.e.* 470 Ω. Since the band showing the tolerance is golden, the value of resistor,

$$R = \mathbf{470\ \Omega \pm 5\%}$$

Problem 1.20. How will you represent a resistance of 3700 Ω ± 10% by colour code ? *(H.S.S.C.E. 2001)*

Sol. Here, $R = 3700\ \Omega \pm 10\% = 37 \times 10^2\ \Omega \pm 10\%$

According to colour code of carbon resistance, the colour of bands corresponding to figures 3 and 7 are orange and violet respectively and corresponding to multiplier 10^2, the colour of band is red. Finally, corresponding to tolerance of 10%, the colour of band is silver.

Therefore, the resistance of 3700 Ω ± 10% will be represented by the bands of colour **orange, violet, red and silver.**

Type J. On series and parallel combination of resistances

Problem 1.21. (*a*) **Three resistors, 1 Ω, 2 Ω and 3 Ω are combined in series. What is the total resistance of the combination ?**

(*b*) **If the combination is connected to a battery of e.m.f. 12 V and negligible internal resistance, obtain the potential drop across each resistor.** *(Text Problem)*

Sol.(*a*) Here, $R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$; $R_3 = 3\ \Omega$

Now, $R_s = R_1 + R_2 + R_3 = 1 + 2 + 3 = 6\ \Omega$

(*b*) Here, $E = 12\ V$; $r = 0$

Therefore, current in the circuit, $I = \frac{E}{R_s} = \frac{12}{6} = 2\ A$

P.D. across $R_1 = I\,R_1 = 2 \times 1 = \mathbf{2\ V}$;

P.D. across $R_2 = I\,R_2 = 2 \times 2 = \mathbf{4\ V}$;

P.D. across $R_3 = I\,R_3 = 2 \times 3 = \mathbf{6\ V}$

Problem 1.22. (*a*) **Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination ?**

(b) If the combination is connected to a battery of e.m.f. 20 V and negligible internal resistance, determine the current through each resistor and the total current drawn from the battery. (*Text Problem*)

Sol. (*a*) Here, $R_1 = 2\,\Omega$; $R_2 = 4\,\Omega$; $R_3 = 5\,\Omega$

The resistance of the parallel combination of the resistors given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$

$$= \frac{10+5+4}{20} = \frac{19}{20}$$

or $\quad R_p = \frac{20}{19}\,\Omega$

(*b*) Here, $\quad E = 20\text{ V}$; $r = 0$

The current drawn from the battery,

$$I = \frac{E}{R_p} = \frac{20}{20/19} = \mathbf{19\ A}$$

Let I_1, I_2 and I_3 be current through resistors R_1, R_2 and R_3 respectively.

Then, $\quad I_1 = \frac{E}{R_1} = \frac{20}{2} = \mathbf{10\ A}$; $I_2 = \frac{E}{R_2} = \frac{20}{4} = \mathbf{5\ A}$;

$$I_3 = \frac{E}{R_3} = \frac{20}{5} = \mathbf{4\ A}$$

Problem 1.23. The resistance of two conductors in series is 40 ohm and their resistance becomes 7·5 ohm, when connected in parallel. Find the resistance of individual conductors.

Sol. Let the resistance of the two conductors be R_1 and R_2.

Then, $\quad R_s = R_1 + R_2 = 40\,\Omega \quad$ ...(*i*)

and $\quad R_p = \frac{R_1 R_2}{R_1 + R_2} = 7{\cdot}5\,\Omega \quad$ ...(*ii*)

From equation (*ii*), we have

$$R_1 R_2 = 7{\cdot}5 \times (R_1 + R_2)$$

Using equation (*i*), we have

$$R_1 R_2 = 7{\cdot}5 \times 40 = 300$$

Now, $(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4 R_1 R_2 = 40^2 - 4 \times 300 = 400$

$\therefore \quad R_1 - R_2 = 20\,\Omega \quad$ ...(*iii*)

Adding equations, (*i*) and (*iii*), we have

$$2 R_1 = 40 + 20 = 60 \quad \text{or} \quad \mathbf{R_1 = 30\,\Omega}$$

Subtracting equation (*iii*) from (*i*), we have

$$2 R_2 = 40 - 20 = 20 \quad \text{or} \quad \mathbf{R_2 = 10\,\Omega}$$

Problem 1.24. A resistor of 5 Ω resistance is connected in series with a parallel combination of a number of resistors each of 6 Ω. If the total resistance of the combination is 7 Ω, how many resistors are in parallel ?

Sol. Let *n* resistors each of 6 Ω be connected in parallel and then the combination be connected in series with resistor of 6 Ω. The resistance of the parallel combination of *n* resistors each of 6 Ω is given by

$$\frac{1}{R_p} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \ldots\ldots n \text{ times} = \frac{n}{6}$$

or $\quad R_p = \frac{6}{n}\,\Omega$

As this parallel combination is connected in series with the resistor of 5 Ω, the total resistance of the combination is given by

$$R = R_p + 5 = \frac{6}{n} + 5$$

Since resistance of the combination is 7 Ω, it follo

$$\frac{6}{n} + 5 = 7 \quad \text{or} \quad \frac{6}{n} = 2 \quad \text{or} \quad n = 3$$

Problem 1.25. Five resistors are connected as sh Fig. 1.21. Find the equivalent resistance between the B and C. (*P.S.S.C.E*

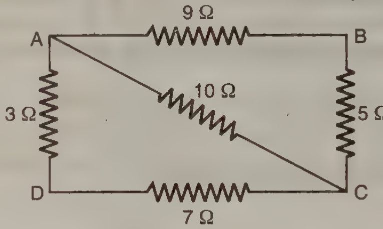


Fig. 1.21

Sol. The resistances of segments AB and BC are in If R_1 is effective resistance of the part ABC of the circu

$$R_1 = 9 + 5 = 14\,\Omega$$

Also, the resistances of segments AD and DC series. If R_2 is effective resistance of the part ADC circuit, then

$$R_2 = 3 + 7 = 10\,\Omega$$

The resistances R_1, 10 Ω (of segment AC) and R parallel. If R is the effective resistance of the circuit,

$$\frac{1}{R} = \frac{1}{14} + \frac{1}{10} + \frac{1}{10} = \frac{5+7+7}{70} = \frac{19}{70}$$

or $\quad R = \frac{70}{19}\,\Omega$

Problem. 1.26. For the combination of resis shown in the Fig. 1.22, find the effective resistance b the points A and B. (*I.I.T*

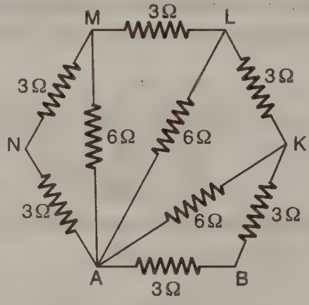


Fig. 1.22

Sol. Consider the part AMNA of the network. Th tances AN (= 3 Ω) and NM (= 3 Ω) are in series and the their effective resistance,

$$R_1 = 3 + 3 = 6\ \Omega$$

The effective resistance R_1 and resistance AM (= in parallel. Therefore, effective resistance of part A given by

$$R_2 = \frac{6 \times 6}{6+6} = 3\,\Omega$$

Now, effective resistance R_2 and resistance ML are in series and this series combination having res 3 + 3 *i.e.* 6 Ω is in parallel to resistance AL (= 6 Ω). If R_3 effective resistance, then

$$R_3 = \frac{6 \times 6}{6+6} = 3\,\Omega$$

Similarly, resistance of part AKLMNA is given by

$$R_4 = \frac{6\times6}{6+6} = 3\ \Omega$$

Finally, the effective resistance R_4 and the resistance KB Ω) are in series and this series combination having stance 3 + 3 *i.e.*6 Ω is connected in paralled across tance AB (= 3 Ω).

Therefore, equivalent resistance of the network is given

$$R = \frac{6\times3}{6+3} = 2\ \Omega$$

Problem 1.27. Three resistances of 30 Ω each are ected to form a triangle. A cell of e.m.f. 2 V and negli- e internal resistance is connected between any two ces [Fig. 1.23]. Find the current through the cell.

(I.I.T. 1983)

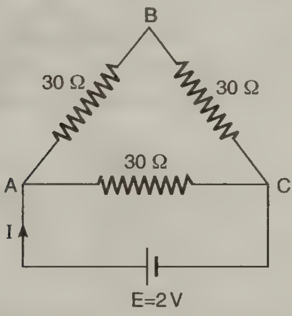


Fig. 1.23

Sol. The resistances AB and BC are in series and their tive resistance,

$$R_1 = 30 + 30 = 60\ \Omega$$

The effective resistance R_1 (= 60 Ω) and the resistance = 30 Ω) are in parallel. The equivalent resistance of the ork is given by

$$R = \frac{60\times30}{60+30} = 20\ \Omega$$

If I is current through the cell, then

$$I = \frac{E}{R} = \frac{2}{20} = \mathbf{0{\cdot}1\ A}$$

Problem 1.28. As shown in Fig. 1.24, a variable rheostat kΩ is used to control the potential difference across a Ω load.

(*a*) If the resistance AB is 500 Ω, what is the potential erence across the load ?

(*b*) If the load is removed, what should be the tance at BC to get 40 V between B and C ?

(I S.C.E. 1996)

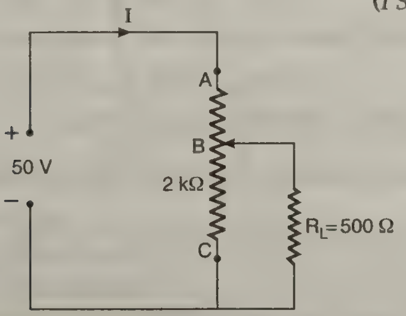


Fig. 1.24

Sol. Here, load resistance, $R_L = 500\ \Omega$;

and e.m.f. of the source, E = 50 V

Total resistance of the rheostat between points A and C,

$$R_{AC} = 2\ k\Omega = 2000\ \Omega$$

Resistance of the rheostat between points A and B,

$$R_{AB} = 500\ \Omega$$

Therefore, resistance between points B and C,

$$R_{BC} = R_{AC} - R_{AB} = 2000 - 500 = 1500\ \ \Omega$$

Now, R_{BC} and R_L are connected in parallel. If R′ is resistance of their combination, then

$$\frac{1}{R'} = \frac{1}{R_{BC}} + \frac{1}{R_L} = \frac{1}{1500} + \frac{1}{500} = \frac{4}{1500}$$

or $R' = 375\ \Omega$

Further, R_{AB} and R′ are in series. If I is current in the circuit, then

$$I = \frac{E}{R_{AB}+R'} = \frac{50}{500+375} = \frac{2}{35}\ A$$

(*a*) It follows that potential drop across R_L is same as the potential drop across the parallel combination of R_{BC} and R_L.

Therefore, potential drop across R_L

= applied potential difference

−potential drop across R_{AB}

$$= 50 - I\,R_{AB} = 50 - \frac{2}{35} \times 500 = \mathbf{21{\cdot}43\ V}$$

(*b*) When the load is removed, the source of e.m.f. will send current through the rheostat wire AB having a total resistance of 2000 Ω. Therefore, current through the rheostat,

$$I' = \frac{50}{2000} = \frac{1}{40}\ A$$

If R_{BC}' is the value of the resistance of the rheostat between points B and C, which will give a potential difference of 40 V across these two points, then

$$I'\,R_{BC}' = 40$$

or $$R_{BC}' = \frac{40}{I'} = \frac{40}{1/40} = \mathbf{1600\ \Omega}$$

Problem 1.29. Four resistors of 12 Ω each are connected in parallel. Three such combinations are then connected in series. What is the total resistance ? If a battery of 9 V e.m.f. and negligible internal resistance is connected across the network of resistors, find the current flowing through each resistor.

(H.S.S.C.E. 2002)

Sol. The network of resistors is connected to the battery of e.m.f. 9 V as shown in Fig. 1.25. Let I be the total current in the circuit. If R′ is effective resistance of the four resistors of 12 Ω each connected in parallel, then

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{12} \quad \text{or} \quad R' = 3\ \Omega$$

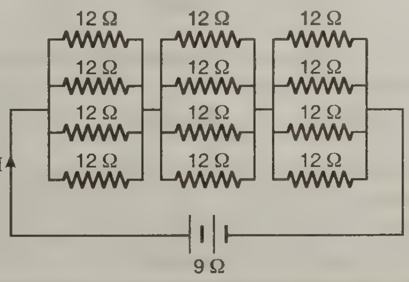


Fig. 1.25

Therefore, effective resistance of the network of resistors,

$$R = R' + R' + R' = 3 + 3 + 3 = \mathbf{9\ \Omega}$$

The current in the circuit,

$$I=\frac{E}{R}=\frac{9}{9}=\mathbf{1\,A}$$

Since all the four resistors are of same resistance, equal current will pass through each resistor. Therefore,

$$\text{current through each resistor}=\frac{1}{4}I=\frac{1}{4}\times 1=\mathbf{0{\cdot}25\ A}$$

Problem. 1.30. Find the current drawn from a cell of e.m.f. 1 V and internal resistance 2/3 Ω connected to the network shown in Fig. 1.26. *(C.B.S.E. 2001 S)*

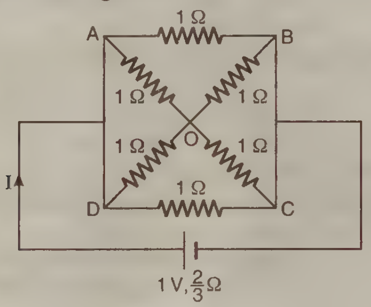


Fig. 1.26

Sol. Here, E = 1 and $r = 2/3\ \Omega$

Let I be the current drawn by the network from the cell. The given network can be respresented as shown in Fig. 1.27.

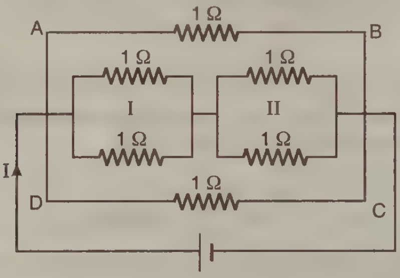


Fig. 1.27

Each of the sub-networks I and II is a parallel combination of two resistances of 1 Ω each. Further, the two sub-networks I and II are in series with each other. If R′ is the effective resistance of I and II, then

$$R'=\frac{1\times 1}{1+1}+\frac{1\times 1}{1+1}=\frac{1}{2}+\frac{1}{2}=1\,\Omega$$

The network shown in Fig. 1.27 is now, equivalent to the arrangement as shown in Fig. 1.28.

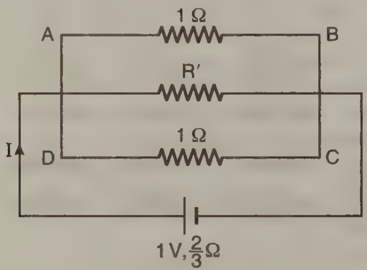


Fig. 1.28

If R is equivalent resistance to the network ABCD, then

$$\frac{1}{R}=\frac{1}{1}+\frac{1}{1}+\frac{1}{1}=\frac{3}{1}\quad\text{or}\quad R=\frac{1}{3}\,\Omega$$

Therefore, current in the circuit,

$$I=\frac{E}{R+r}=\frac{1}{1/3+2/3}=\mathbf{1\,A}$$

Problem 1.31. A letter A is constructed of a unifo wire of resistance 1 Ω cm^{-1}. The sides of the letter are cm long and the cross-piece in the middle is 10 cm lor while the apex angle is 60° [Fig. 1.29]. Find the curr flowing through the different parts, when a battery of e.n 15 V and internal resistance 3·33 Ω is connected across two ends of the legs.

Sol. The letter A is formed of a uniform wire as sho in Fig 1.29.

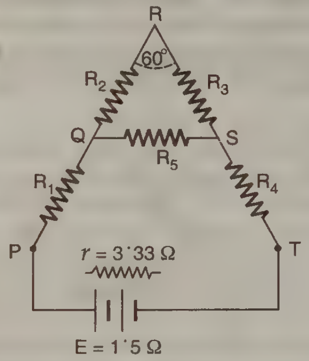


Fig. 1.29

As the apex angle is 60° and the cross-piece QS is in middle of the two legs PR and RT,

$$PQ = QR = RS = ST = QS = 10\text{ cm}$$

Now, the wire has a resistance of 1 Ω cm^{-1}.

$$\therefore\qquad R_1=R_2=R_3=R_4=R_5=10\,\Omega$$

Let R_p be the resistance of the triangular part QRS of letter A. It is series combination of R_2 and R_3 connectec parallel with R_5. Then, It is

$$\frac{1}{R_p}=\frac{1}{R_5}+\frac{1}{R_2+R_3}=\frac{1}{10}+\frac{1}{10+10}=\frac{1}{10}+$$

$$\text{or}\qquad R_p=\frac{20}{3}\,\Omega$$

If R is the resistance of the letter between the two e of the legs *i.e.* P and T, then

$$R=R_1+R_p+R_4=10+\frac{20}{3}+10=\frac{80}{3}$$

$$=\mathbf{26{\cdot}67\ W}$$

Let I be the current flowing through the main circu

$$\text{Then,}\qquad I=\frac{E}{R+r}=\frac{15}{26{\cdot}67+3{\cdot}33}=\frac{15}{30}=0{\cdot}5\,A$$

Potential difference across the triangular part QRS,

$$V=I\times R_p=0{\cdot}5\times\frac{20}{3}=\frac{10}{3}\,V$$

Therefore, current through part QRS

$$=\frac{V}{R_2+R_3}=\frac{10/3}{10+10}=\frac{1}{6}\,A=\mathbf{0{\cdot}167\ A}$$

$$\text{and current through part QS}=\frac{V}{R_5}=\frac{10/3}{10}=\frac{1}{3}\,A$$

$$=\mathbf{0{\cdot}333\ A}$$

The current through parts PQ or ST is same as t flowing through the main circuit *i.e.* **0·5 A.**

Problem 1.32. Calculate the steady state curr through the circuit shown in Fig. 1.30. The internal re

of the battery is negligible and the capacitance of the itor is 0·5 μF. (*H.S.S.C.E. 2001*)

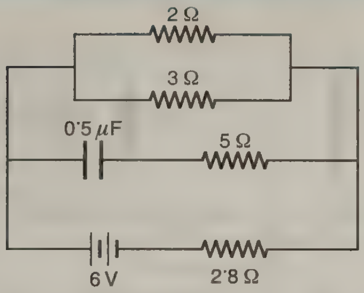


Fig. 1.30

Sol. Let I be the steady state current through the circuit. When steady state is reached *i.e* when the current in the it becomes constant, the capacitor offers infinite ance. As a result, it behaves as an open path and the ance of 5 Ω in the circuit becomes ineffective. Therefore, ven circuit may be represented as shown in Fig. 1.31.

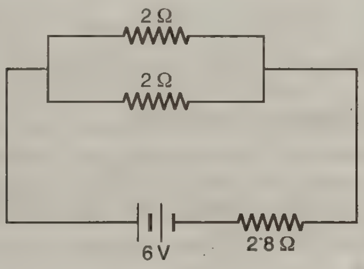


Fig. 1.31

The effective resistance of the circuit,

$$R = \frac{2 \times 3}{2+3} + 2{\cdot}8 = 1{\cdot}2 + 2{\cdot}8 = 4\ \Omega$$

Therefore, current in steady state of the circuit,

$$I = \frac{E}{R} = \frac{6}{4} = 1{\cdot}5\ \text{A}$$

K. On internal resistance of a cell

Problem 1.33. A battery of e.m.f. 10 V and internal tance 3 Ω is connected to a resistor. If the current gh the circuit is 0·5 A, what is the resistance of the or ? What is the terminal voltage of the battery, when rcuit is closed ? (*Text Problem*)

Sol. Here, E = 10 V ; $r = 3\ \Omega$; I = 0·5 A

Let R be the resistance of the resistor. Then,

$$I = \frac{E}{R + r}$$

or $$R = \frac{E}{I} - r - \frac{10}{0{\cdot}5} - 3 = 17\ \Omega$$

Now, terminal voltage of the battery

= potential difference across R

= I R = 0·5 × 17 = **8·5 V**

Note. The terminal voltage of the battery is also given

$$E - I\,r = 10 - 0{\cdot}5 \times 3 = \mathbf{8{\cdot}5\ V}$$

Problem 1.34. A battery of e.m.f. E and internal ance *r* gives a current of 0·5 A with an external resistor ohm and a current of 0·25 A with an external resistor ohm. Calculate (*a*) internal resistance of the cell and m.f. of the cell. (*C.B.S.E. 2002*)

Sol. When an external resistance R is connected to a battery of e.m.f. E and internal resistance *r*, current in the circuit,

$$I = \frac{E}{R + r}$$

In first case : Here, R = 12 Ω ; I = 0·5 A

$$\therefore \qquad 0{\cdot}5 = \frac{E}{(12 + r)}$$

or $$E = (12 + r) \times 0{\cdot}5 \qquad \text{...(}i\text{)}$$

In second case : Here, R = 25 Ω ; I = 0·25 A

$$\therefore \qquad 0{\cdot}25 = \frac{E}{(25 + r)}$$

or $$E = (25 + r) \times 0{\cdot}25 \qquad \text{...(}ii\text{)}$$

From equation (*i*) and (*ii*), we have

$$(12 + r) \times 0{\cdot}5 = (25 + r) \times 0{\cdot}25$$

or $$r = \mathbf{1\ \Omega}$$

Substituting for *r* in equation (*i*), we have

$$E = (12 + 1) \times 0{\cdot}5 = \mathbf{6{\cdot}5\ V}$$

Problem 1.35. Three resistors of 3 Ω, 4 Ω and 6 Ω are connected in parallel. The combination is connected to a cell of e.m.f. 2 V and internal resistance 2/3 Ω. Find the current drawn from the cell and the current through 3 Ω resistance. (*Karnataka, 1989*)

Sol. The resistors of 3 Ω, 4 Ω and 6 Ω are connected in parallel to the cell of e.m.f. 2 V as shown in Fig. 1.32.

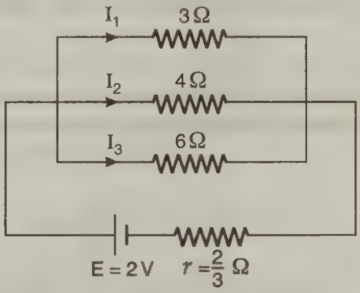


Fig. 1.32

Let R_p be the effective resistance of the parallel combination of 3 Ω, 4 Ω and 6 Ω resistances. Then,

$$\frac{1}{R_p} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{9}{12} \quad \text{or} \quad R_p = \frac{12}{9} = \frac{4}{3}$$

Total resistance of the circuit,

$$R = R_p + r = \frac{4}{3} + \frac{2}{3} = 2\ \Omega$$

Therefore, current is the circuit, $I = \frac{E}{R} = \frac{2}{2} = \mathbf{1\ A}$

Now, potential difference across the parallel combination

$$= I \times R_p = 1 \times \frac{4}{3} = \frac{4}{3}\ \text{V}$$

As potential difference across each of the three resistances is 4/3 V, the current through 3 Ω resistance,

$$I = \frac{4/3}{3} = \frac{4}{9}\ \mathbf{A}$$

Problem 1.36. A uniform wire of resistance 12 Ω is cut into three pieces in the ratio 1 : 2 : 3 and the three pieces are connected to form a triangle. A cell of e.m.f. 8 V and internal resistance 1 Ω is connected across the highest of the three resistors. Calculate the current through each part of the circuit. (*Karnataka, 1994*)

Sol. Suppose that the 12 Ω wire is cut into three pieces AB, BC and AC in the ratio 1 : 2 : 3. Then, resistances of parts

AB, BC and AC are 2 Ω, 4 Ω, and 6 Ω respectively. The cell of e.m.f. 8 V and internal resistance 1 Ω is connected across AC (highest resistor) as shown in Fig. 1.33.

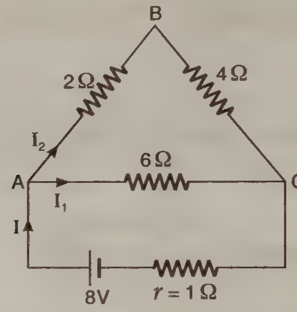


Fig. 1.33

Since the series combination of resistors AB and BC is in parallel with AC, it can be obtained that the resistance of the triangular part ABC of the network is 3 Ω. Therefore,

$$\text{current from the battery, } I = \frac{E}{R+r} = \frac{8}{3+1} = 2\text{ A}$$

Suppose that a part I_1 of the total current I flows along path AC and remaining part I_2 flows along path ABC. Since resistances of the paths AC and ABC are equal,

$$I_1 = I_2 = I/2 = \mathbf{1\ A}$$

Problem 1.37. In the circuit given in Fig. 1.34, a potential difference of 3 V is required across the points A and B. Calculate the value of the resistance R_1.

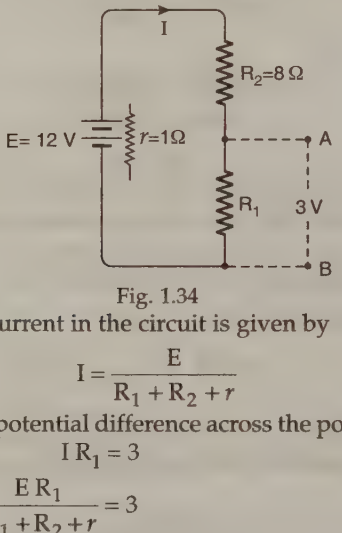


Fig. 1.34

Sol. The current in the circuit is given by

$$I = \frac{E}{R_1 + R_2 + r}$$

Required potential difference across the points A and B,

$$I R_1 = 3$$

$$\therefore \quad \frac{E R_1}{R_1 + R_2 + r} = 3$$

Here, $E = 12\text{ V}$; $R_2 = 8\ \Omega$ and $r = 1\ \Omega$

$$\therefore \quad \frac{12 R_1}{R_1 + 8 + 1} = 3 \quad \text{or} \quad 4 R_1 = R_1 + 9$$

$$\text{or} \quad \mathbf{R_1 = 3\ \Omega}$$

Problem 1.38. A 20 V battery of internal resistance 1 Ω is connected to three coils of 12 Ω, 6 Ω and 4 Ω in parallel, a resistor of 5 Ω and a reversed battery (e.m.f. 8 V and internal resistance 2 Ω) as shown in Fig. 1.35. Calculate (*a*) the current in the circuit, (*b*) current in resistor of 12 Ω coil and (*c*) potential difference acorss each battery. (*C.B.S.E .2001 S*)

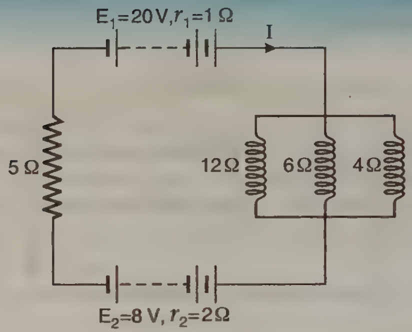


Fig. 1.35

Sol. Here, $E_1 = 20\text{ V}$; $r_1 = 1\ \Omega$; $E_2 = 8\text{ V}$ and $r_2 = 2$

(*a*) Let I be current in the circuit.

Total e.m.f of the circuit,

$$E = E_1 - E_2 = 20 - 8 = 12\text{ V}$$

Let R′ be the resistance of parallel combiantion o coils of resistances 12 Ω, 6 Ω and 4 Ω. Then,

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2} \quad \text{or} \quad R' = 2\ \Omega$$

Therefore, total resistance of the circuit,

$$R = R' + r_1 + r_2 + 5 = 2 + 1 + 2 + 5 = 10$$

Hence, current in the circuit,

$$I = \frac{E}{R} = \frac{12}{10} = \mathbf{1{\cdot}2\ A}$$

(*b*) Let current through 12 Ω, 6 Ω and 4 Ω resista be I_1, I_2 and I_3 respectively.

Then, $I_1 + I_2 + I_3 = 1{\cdot}2$

Also $\quad I_1 \times 12 = I_2 \times 6 = I_3 \times 4$

From equation (*ii*), we have

$$I_2 = 2 I_1 \quad \text{and} \quad I_3 = 3 I_1$$

Therefore, equation (*i*) becomes

$$I_1 + 2 I_1 + 3 I_1 = 1{\cdot}2 \quad \text{or} \quad I_1 = \mathbf{0{\cdot}2\ A}$$

(*c*) The terminal potential difference across 12 V bat

$$V_1 = E_1 - I r_1 = 20 - 1{\cdot}2 \times 1 = \mathbf{18{\cdot}8\ V}$$

Since potential drop across internal resistance of battery aids its e.m.f., the terminal potential difference ac 8 V battery,

$$V_2 = E_2 + I r_2 = 8 + 1{\cdot}2 \times 2 = \mathbf{10{\cdot}4\ V}$$

Type L. On Grouping of cells

Problem 1.39. Six lead-acid type of secondary c each of e.m.f. 2·0 V and internal resistance 0·015 Ω are joi in series to provide a supply to a resistance of 8·5 Ω. W is the current drawn from the supply and its termi voltage ? (*Text Prob*

Sol. When cells are connected in series, the total e. is equal to sum of their e.m.fs.

Therefore, total e.m.f. of the cells, $E = 2{\cdot}0 \times 6 = 12{\cdot}$

Also, total internal resistance, $r = 0{\cdot}015 \times 6 = 0{\cdot}09$

External resistance, $R = 8{\cdot}5\ \Omega$

Therefore, current drawn from the supply,

$$I = \frac{E}{R+r} = \frac{12{\cdot}0}{8{\cdot}5 + 0{\cdot}09} = \mathbf{1{\cdot}397\ A}$$

Terminal voltage, $V = I R = 1{\cdot}397 \times 8{\cdot}5 = \mathbf{11{\cdot}857\ V}$

Problem 1.40. Three identical cells each of e.m.f. and internal resistance *r* are connected in series to a resistor. If the current flowing in the circuit is 1· calculate (*a*) internal resistance of each cell and (*b*) terminal voltage across the cells. (*C.B.S.E. 2*

Sol. The three cells are connected in series to the external tance of 6 Ω as shown in Fig. 1.36.

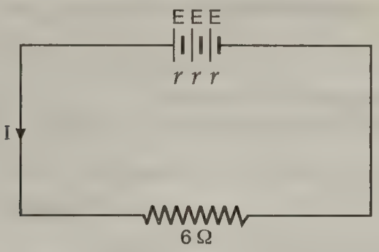


Fig. 1.36

(*a*) Here E = 4 V ; I = 1·5 A and R = 6 Ω

The total e.m.f. in the circuit, $E' = 3E = 3 \times 4 = 12$ V

Total resistance of the circuit, $R' = R + 3r = 6 + 3r$

Therefore, current in the circuit,

$$I = \frac{E'}{R'} \quad \text{or} \quad 1{\cdot}5 = \frac{12}{6+3r}$$

or $\quad r = \frac{2}{3}\,\Omega$

(*b*) Terminal voltage across each cell,

$$V = E - Ir = 4 - 1{\cdot}5 \times \frac{2}{3} = 3\text{ V}$$

Problem 1.41. Four identical cells, each of e.m.f. 2 V, are ed in parallel providing supply of current to external it consisting of two 15 Ω resistors joined in parallel. terminal voltage of the cells as read by an ideal neter is 1·6 V. Calculate the internal resistance of each (*C.B.S.E. 2002*)

Sol. Four cells are connected in parallel to the parallel bination of two 15 Ω resistors as shown in Fig. 1.37.

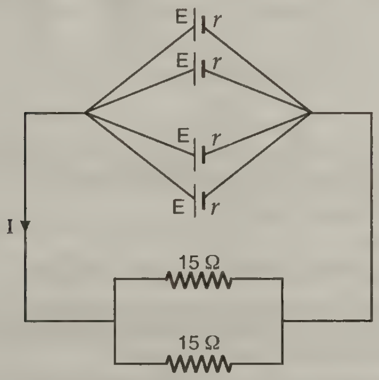


Fig. 1.37

Let *r* be internal resistance of each cell and I be the ent in the circuit. Since the cells are connected in parallel, total e.m.f. in the circuit = e.m.f. of one cell = 2 V

Further, total internal resistance of the cells is given by

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{4}{r} \quad \text{or} \quad r' = r/4$$

Let R be resistance of the parallel combination of two resistors. Then, total external resistance,

$$R = \frac{15 \times 15}{15 + 15} = 7{\cdot}5\ \Omega$$

Now, internal resistance of the parallel combination of is given by

$$r' = \left(\frac{E}{V} - 1\right)R \quad \text{or} \quad \frac{r}{4} = \left(\frac{2}{1{\cdot}6} - 1\right) \times 7{\cdot}5$$

or $\quad r = 7{\cdot}5\ \Omega$

Problem 1.42. Two identical cells, whether joined together in series or in parallel give the same current, when connected to an external resistance of 1 Ω. Find the internal resistance of each cell. (*I.S.C.E. 1995*)

Sol. Let each cell be of e.m.f. E and internal resistance *r*.

When the two cells are connected in series : Fig. 1.38 shows the series combination of the two cells connected to an external resistance R = 1 Ω.

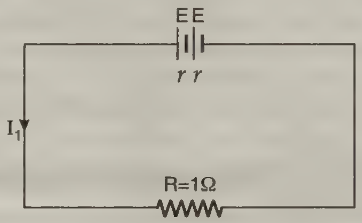


Fig. 1.38

Total resistance of the circuit, $R_1 = R + r + r$

or $\quad R_1 = 1 + 2r$

Total e.m.f. in the circuit, $E_1 = E + E = 2E$

If I_1 is current in the circuit, then

$$I_1 = \frac{E_1}{R_1} = \frac{2E}{1+2r} \qquad ...(i)$$

When the two cells are connected in parallel : Fig. 1.39 shows the parallel combination of the two cells connected to an external resistance R = 1 Ω.

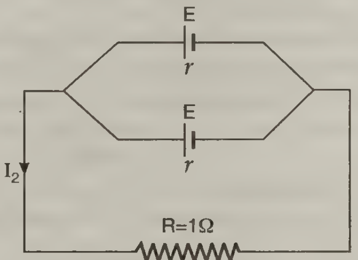


Fig. 1.39

Total internal resistance of two cells = $r/2$

Total resistance of the circuit, $R_2 = R + r/2$

or $\quad R_2 = 1 + r/2$

Total e.m.f. in the circuit = e.m.f. of a single cell = E

If I_2 is current in the circuit, then

$$I_2 = \frac{E_2}{R_2} = \frac{E}{1 + r/2} \qquad ...(ii)$$

Since the two cells give the same current in two cases, we have

$$I_1 = I_2$$

or $\quad \frac{2E}{1+2r} = \frac{E}{1+r/2} \quad$ or $\quad 1 + 2r = 2(1 + r/2)$

or $\quad r = 1\ \Omega$

Problem 1.43. A set of 4 cells, each of e.m.f. 2 V and internal resistance 1·2 Ω, are connected across an external load of 10 Ω with 2 rows, 2 cells in each branch. Calculate the current in each branch and potential difference across 10 Ω. (*Karnataka, 1991 S*)

Sol. The four cells, each of e.m.f. E and internal resistance are connected to the external resistance R as shown in Fig. 1.40.

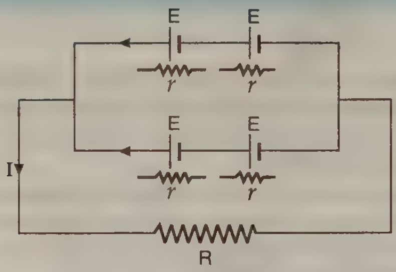


Fig. 1.40

Here, $E = 2\text{ V}$; $r = 1{\cdot}5\ \Omega$ and $R = 10\ \Omega$

The net e.m.f. of the two cells in a row

$$= 2\text{E} = 2 \times 2 = 4\text{ V}$$

The total e.m.f. E′ of the 4 cells is equal to that of a row *i.e.*

$$\text{E}' = 4\text{ V}$$

The internal resistance of the two cells in a row

$$= 1{\cdot}5 + 1{\cdot}5 = 3\ \Omega$$

If r' is internal resistance of the 4 cells, then

$$\frac{1}{r'} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \quad \text{or} \quad r' = \frac{3}{2} = 1{\cdot}5\ \Omega$$

Total resistance of the circuit,

$$\text{R}' = \text{R} + r' = 10 + 1{\cdot}5 = 11{\cdot}5\ \Omega$$

Therefore, current in the circuit, $\text{I} = \dfrac{\text{E}'}{\text{R}'} = \dfrac{4}{11{\cdot}5}$

$$= 0{\cdot}348\text{ A}$$

As the two branches are identical,

current in each branch $= \dfrac{\text{I}}{2} = \dfrac{0{\cdot}348}{2}$ = **0·174 A**

Potential difference across external resistance

$$= \text{I R} = 0{\cdot}348 \times 10 = \mathbf{3{\cdot}48\ V}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 1.44. In a discharge tube, the number of hydrogen ions (*i.e.* protons) drifting across a section er second is 1·0 × 10^{18}, while the number of electrons rifting in the opposite direction across another cross-ection is 2·7 × 10^{18} per second. If the supply voltage is 230 , what is the effective resistance of the tube ?

(*P.S.S.C.E. 1995 ; Text Problem*)

Sol. Here, number of electrons moving per second,

$$n_e = 2{\cdot}7 \times 10^{18}$$

and number of protons moving per second,

$$n_p = 1{\cdot}0 \times 10^{18}$$

Since electrons and protons move in opposite directions, otal charge crossing per second *i.e.*

current in the discharge tube, $\text{I} = (n_e + n_p)\, e$

$$= (2{\cdot}7 \times 10^{18} + 1{\cdot}0 \times 10^{18}) \times 1{\cdot}6 \times 10^{-19}$$

$$= 3{\cdot}7 \times 1{\cdot}6 \times 10^{-1} = 0{\cdot}592\text{ A}$$

Now, $\quad$ V = 230 volt

Therefore, effective resistance of the discharge tube,

$$\text{R} = \frac{\text{V}}{\text{I}} = \frac{230}{0{\cdot}592} = \mathbf{388{\cdot}5\ \Omega}$$

Problem 1.45. Two wires of equal length, one of aluminium and the other of copper have the same resisance. Which of the two wires is lighter ? Hence, explain vhy aluminium wires are preferred for overhead power ables. Given that $\rho_{al} = 2{\cdot}63 \times 10^{-8}\ \Omega$ m, $\rho_{cu} = 1{\cdot}72 \times 10^{-8}\ \Omega$ m, elative density of Al = 2·7 and of Cu = 8·9.

(*Text Problem*)

Sol. Let A_1 and A_2 be area of cross-sections of the wires nade from aluminium and copper respectively. Let R_1 and $_2$ be resistances of the same length l of the wires made of luminium and copper respectively.

For aluminium wire :

Density* of aluminium, $d_1 = 2{\cdot}7 \times 10^3\text{ kg m}^{-3}$

and resistivity of aluminium, $\rho_{al} = 2{\cdot}63 \times 10^{-8}\ \Omega\text{ m}$

Then, $\text{R}_1 = \rho_{al}\dfrac{l}{\text{A}_1} = \dfrac{2{\cdot}63 \times 10^{-8} \times l}{\text{A}_1}$

Mass of the aluminium wire, $m_1 = \text{A}_1\, l\, d_1$

or $\quad m_1 = \text{A}_1\, l \times 2{\cdot}7 \times 10^3$

For copper wire :

Density of copper, $d_2 = 8{\cdot}9 \times 10^3\text{ kg m}^{-3}$

and resistivity of copper, $\rho_{cu} = 1{\cdot}72 \times 10^{-8}\ \Omega\text{ m}$

Then, $\text{R}_2 = \rho_{cu}\dfrac{l}{\text{A}_2} = \dfrac{1{\cdot}72 \times 10^{-8} \times l}{\text{A}_2}$

Mass of the copper wire, $m_2 = \text{A}_2\, l\, d_2$

or $\quad m_2 = \text{A}_2\, l \times 8{\cdot}9 \times 10^3$

$$\therefore \quad \frac{m_2}{m_1} = \frac{\text{A}_2\, l \times 8{\cdot}9 \times 10^3}{\text{A}_1\, l \times 2{\cdot}7 \times 10^3} = \frac{8{\cdot}9}{2{\cdot}7} \times \frac{\text{A}_2}{\text{A}_1} \qquad \text{...(}i\text{)}$$

Since two wires are of equal resistances, we have

$$\frac{2{\cdot}63 \times 10^{-8} \times l}{\text{A}_1} = \frac{1{\cdot}72 \times 10^{-8} \times l}{\text{A}_2}$$

$$\text{or} \quad \frac{\text{A}_2}{\text{A}_1} = \frac{1{\cdot}72}{2{\cdot}63} \qquad \text{...(}ii\text{)}$$

From equations (*i*) and (*ii*), we have

$$\frac{m_2}{m_1} = \frac{8{\cdot}9}{2{\cdot}7} \times \frac{1{\cdot}72}{2{\cdot}63} = 2{\cdot}16 \quad \text{or} \quad m_1 = \frac{m_2}{2{\cdot}16}$$

It follows that **aluminium wire** is lighter.

Since for the same value of resistance and length, aluminium wire has lesser mass, aluminium wires are preferred for overhead power cables.

Problem 1.46. It is desired to make a coil of resistance 20 Ω having a zero temperature coefficient of resistance. To achieve this, a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportion of carbon and iron resistances are so chosen that $\text{R}_1 + \text{R}_2 = 20\ \Omega$ for all temperatures near the room temperature. If the temperature coefficient of resistance for carbon and iron are $\alpha_1 = -0{\cdot}5 \times 10^{-3}\ {}^\circ\text{C}^{-1}$ and $\alpha_2 = 5 \times 10^{-3}\ {}^\circ\text{C}^{-1}$ respectively, calculate the values of R_1 and R_2.

Sol. Here, $\text{R}_1 + \text{R}_2 = 20\ \Omega$;

$$\alpha_1 = -0{\cdot}5 \times 10^{-3}\ {}^\circ\text{C}^{-1}\ ; \alpha_2 = 5 \times 10^{-3}\ {}^\circ\text{C}^{-1}$$

Let θ be a small rise in temperature near the room temperature.

Then, new resistance of the carbon and iron resistances will be

$$\text{R}_1' = \text{R}_1(1 + \alpha_1\theta) \quad \text{and} \quad \text{R}_2' = \text{R}_2(1 + \alpha_2\theta)$$

---

*If relative density of aluminium is 2·7, its density in SI will e 2·7 × 10^3 kg m^{-3}.

Since the resistance coil formed should have zero emperature coefficient,

$$R_1' + R_2' = R_1 + R_2$$

or $R_1(1+\alpha_1\theta) + R_2(1+\alpha_2\theta) = R_1 + R_2$

or $$R_1\alpha\,\theta + R_2\,\alpha_2\,\theta = 0$$

or $R_1 = -\dfrac{\alpha_2}{\alpha_1}R_2 = -\dfrac{5\times10^{-3}}{-0.5\times10^{-3}}R_2 = 10\,R_2$

Now, $R_1 + R_2 = 20$

$\therefore$ $10\,R_2 + R_2 = 20$ or $R_2 = \mathbf{1.82\ \Omega}$

Also, $R_1 = 20 - R_2 = 20 - 1.82 = \mathbf{18.18\ \Omega}$

Problem 1.47. Determine the current drawn from a 12 V supply with internal resistance 0·5 Ω by the infinite etwork shown in Fig. 1.41. Each resistor has 1 Ω resistance.

(H.S.S.C.E. 2002 ;Text Problem)

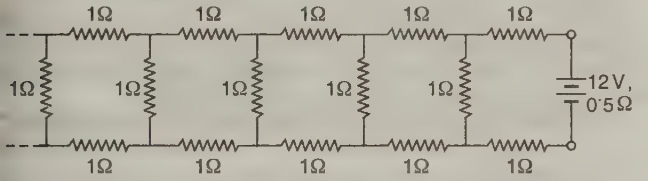


Fig. 1.41

Sol. Let the resistance equivalent to the entire network e X. Since the network is infinite, adding one more set of hree resistances each of value R = 1 Ω across the terminals vill not affect the total resistance *i.e.* it should still remain qual to X. Therefore, the network on adding one set of three esistances would appear as shown in Fig. 1.42.

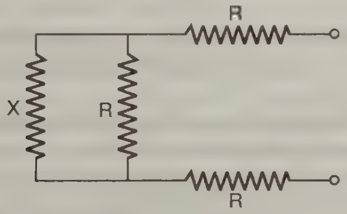


Fig. 1.42

If R′ is equivalent resistance of this new network, then

R′ = R + (resistance equivalent to parallel combination of X and R) + R

$$= R + \frac{XR}{X+R} + R = 2R + \frac{XR}{X+R}$$

As said above, the addition of one set of such three esistances should not alter the total resistance of the infinite etwork. Therefore,

$$R' = X \quad \text{or} \quad 2R + \frac{XR}{X+R} = X$$

Since R = 1 Ω, we have

$$2\times1 + \frac{X\times1}{X+1} = X \quad \text{or} \quad 2 + \frac{X}{X+1} = X$$

or $X^2 - 2X - 2 = 0$

or $$X = \frac{-(-2)\pm\sqrt{(-2)^2 - 4\times1\times(-2)}}{2}$$

or $$X = 1 \pm \sqrt{3}$$

Since the value of resistance cannot be negative, we have

$$X = 1+\sqrt{3} = 2.732\ \Omega$$

Now, $E = 12\text{ V}\,;\, r = 0.5\ \Omega$

If I is current drawn by the network, then

$$I = \frac{E}{X+r} = \frac{12}{2.732+0.5} = \mathbf{3.713\ A}$$

Problem 1.48. A battery of e.m.f. 24 V is used to supply current through the combination of three resistors R_1, R_2 and R_3 as shown in Fig. 1.43. When the key K is closed, the current through 5 Ω resistor is found to be I = 3 A.

(*a*) Find the internal resistance (*r*) of the cell and the currents I_1 and I_2 through the resistors R_1 and R_2. (*b*) What current will flow in the circuit, if the key is opened ?

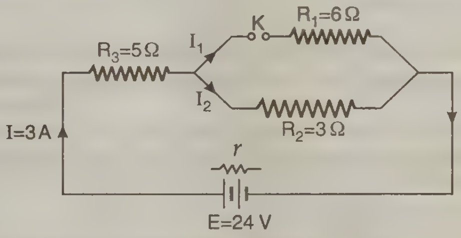


Fig. 1.43

Sol. (*a*) ***On closing the key*** : Let R_p be the resistance of the parallel combination of R_1 and R_2.

Then, $$R_p = \frac{R_1R_2}{R_1+R_2} = \frac{6\times3}{6+3} = 2\ \Omega$$

If *r* is internal resistance of the battery, then total resistance of the circuit,

$$R = R_3 + R_p + r$$

or $$R = 5 + 2 + r = 7 + r$$

Therefore, current in the main circuit,

$$I = \frac{E}{R}$$

Here, E = 24 V and I = 3 A

$\therefore$ $3 = \dfrac{24}{7+r}$ or $7 + r = 8$

or $r = 1\ \Omega$

Now, potential difference across parallel combination of R_1 and R_2,

$$V = I\times R_p = 3\times2 = 6\text{ volt}$$

$\therefore$ $I_1 = \dfrac{V}{R_1} = \dfrac{6}{6} = 1\text{ A}$ and $I_2 = \dfrac{V}{R_2} = \dfrac{6}{3} = 2\text{ A}$

(*b*) ***On opening the key*** : Total resistance of circuit,

$$R' = R_2 + R_3 + r = 3 + 5 + 1 = 9\ \Omega$$

Therefore, current in the circuit, $I' = \dfrac{E}{R'} = \dfrac{24}{9} = \dfrac{8}{3}$ **A**

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What is the number of electrons that consti-utes a current of one ampere ? *(H.S.S.C.E. 2001)*

Ans. Number of electrons constituting a current of A,

$$n = \frac{I}{e} = \frac{1\,\text{C s}^{-1}}{1.6\times10^{-19}\ \text{C}}$$

$$= \mathbf{6.25\times10^{18}\ s^{-1}}$$

Q. 1.02. It is found that 10^{20} electrons, each having a charge of $1{\cdot}6 \times 10^{-19}$ C, pass from a point X towards another point Y in 0·1 s. What is the current and its direction ?

Ans. Here, $q = n\,e = 10^{20} \times 1{\cdot}6 \times 10^{-19} = 16\text{ C}\,;\, t = 0{\cdot}1\text{ s}$

$$\therefore \qquad I = \frac{q}{t} = \frac{16}{0{\cdot}1} = \mathbf{160\ A}$$

The direction of conventional current is **from point Y to X.**

Q. 1.03. What is conventional current ?

Ans. The current that flows from positive pole to negative pole of a cell in the external circuit is called conventional current.

Q. 1.04. Define the unit of current. (*P.S.S.C.E. 1999, 1997 S*)

Ans. *The current through a conductor is called one ampere, if one coulomb of charge flows through the conductor in one second.*

Q. 1.05. Write down the relation between a coulomb and an ampere. (*I.S.C.E. 1996*)

Ans. $1\text{ ampere} = \dfrac{1\text{ coloumb}}{1\text{ second}}$

Q. 1.06. How does the drift velocity of electrons in a metallic conductor vary with increase in temperature ? (*C.B.S.E. 2002*)

Ans. The drift velocity of electrons in a metallic conductor decreases with increase in temperature of the conductor.

For details, refer to note of section 1.10.

Q. 1.07 A steady current flows in a metallic conductor of non-uniform cross-section. Explain which of these quantities is constant along the conductor : current, current density, electric field and drift speed ? (*H.S.S.C.E 2001*)

Ans. Except current, the value of all the quantities depends upon the area of cross-section of the conductor. Hence, only current remains constant, when it flows through a conductor of uniform area of cross-section.

Q. 1.08. The potential difference across a given copper wire is increased. What happens to the drift velocity of the charge carriers ? (*C.B.S.E. 1999 S*)

Ans. We know, $v_d = \dfrac{e\,E}{m}\,\tau$

If l is length of the copper wire and V the potential difference across it, then

$$v_d = \frac{e}{m}\left(\frac{V}{l}\right)\tau = \frac{e\,V}{m\,l}\,\tau$$

Thus, $v_d \propto V$ *i.e.* if potential difference is increased, drift velocity of the electrons will ***increase***.

Q. 1.09. When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metals are moving in the same direction ? (*Text Question*)

Ans. It is not so. The drift velocity is superposed over the random thermal velocities of the electrons.

Q. 1.10. If the electron drift velocity is so small and the charge on electron is small, how can we still obtain large amount of current ? (*Text Question*)

Ans. It is because, number of electrons per unit volume is very large ($\approx 10^{28}\ \text{m}^{-3}$).

Q. 1.11. State Ohm's law. (*H.S.S.C.E. 1996 ; I.S.C.E. 1995 ; C.B.S.E. 1990*)

Ans. For statement of Ohm's law, refer to section 1.06.

Q. 1.12. Does the value of resistance of a conductor depend upon the potential difference applied across it or the current passed through it ?

Ans. No, the resistance of a conductor does not deper upon the potential difference applied across it or the curre passed through it.

Q. 1.13. What is non-ohmic device ? State one examp of it. (*P.S.S.C.E. 200*

Ans. A device, which does not obey ohm's law is calle a non-ohmic device. The resistance offered by a sem conductor diode is an example of non-ohmic device.

Q. 1.14. Define resistivity of the material of a wir State its SI units. (*H.S.S.C.E. 1996, 1995 ; I.S.C.E. 199*

Ans. For definition of resistivity and its SI unit, refer section 1.09.

Q. 1.15. Resistivities of copper, silver and mangani are $1{\cdot}7 \times 10^{-8}\ \Omega$ m, $1{\cdot}0 \times 10^{-8}\ \Omega$ m and $44 \times 10^{-8}\ \Omega$ respectively. Which of these is the best conductor ? (*C.B.S.E. 1999*

Ans. For given length and area of cross-section, tl resistance of a material is directly proportional to its specif resistance. Since the specific resistance is least for silver, it the best conductor of the three given materials.

Q. 1.16. What is the order of resistivity of an insulato (*P.S.S.C.E. 1997*

Ans. The resistivity of an insulator, such as glass or ha rubber is of the order 10^{10}–$10^{14}\ \Omega$ m.

Q. 1.17. If a wire is stretched to double its origin length without loss of mass, how will the resistivity of tl wire be influenced ? (*C.B.S.E. 199*

Or

A wire of resistivity ρ is stretched to double its lengt What will be its new resistivity ? (*H.S.S.C.E. 1998 ; C.B.S.E. 199*

Ans. The resistivity of a wire depends on the nature its material. The increase in length of the wire will not affe its resistivity.

Q. 1.18. Two wires A and B are of the same metal ar of the same length have their areas of cross-section in tl ratio of 2 : 1. If the same potential difference is applie across each wire in turn, what will be the ratio of tl currents flowing in A and B ? (*C.B.S.E. 199*

Ans. The two wires, having areas of cross-section in tl ratio 2 : 1, possess resistances in the ratio 1 : 2. It is becaus resistance of a wire is inversely proportional to its area cross-section. Since $I = \dfrac{V}{R}$ *i.e.* $I \propto \dfrac{1}{R}$ the currents in the tw wires will be in ratio 2 : 1, when the same potential differen is applied across the two wires.

Q. 1.19. Two wires A and B of same metal have tl same area of cross-section and have their lengths in the rat 2 : 1. What will be the ratio of currents flowing throug them, when the same potential difference is applied acro length of each of them ? (*C.B.S.E 199*

Ans. The two wires having their lengths in the ratio 1, possess resistances also in the ratio 2 : 1. It is becaus resistance of a wire is directly proportional to its length. Sin $I = \dfrac{V}{R}$ *i.e.* $I \propto \dfrac{1}{R}$ the currents in the two wires will be in tl ratio 1 : 2, when same potential difference is applied acro the two wires.

Q. 1.20. Draw a graph to show the variation of resi tance of a metal wire as a function of its diameter, keepi length and temperature constant. (*C.B.S.E. 1999*

Ans. We know, $R = \rho\frac{l}{A} = \rho\frac{l}{\pi D^2/4}$

or $R \propto \frac{1}{D^2}$

The graph between R and D will be as shown in Fig. 1.44.

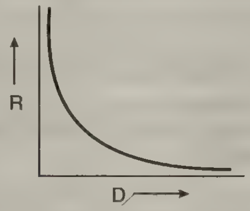


Fig. 1.44

Q. 1.21. Name the units of conductance and conduc- rity. (*I.S.C.E. 1996 ; H.P.S.S.C.E. 1995*)

Ans. The unit of conductance is **simen** and that of nductivity is **simen metre^{-1}**.

Q. 1.22. How does the conductance of a semiconduc- ig material change with rise in temperature ? (*C.B.S.E. 1998*)

Ans. The resistance of a semiconducting material creases on increasing temperature. Likewise, the conduc- nce of such a material increases due to rise in temperature.

Q. 1.23. What do you mean by temperature coefficient resistance ?

Ans. *The change in resistance per unit resistance per degree e or fall in temperature is called temperature coefficient of the istance.*

Q. 1.24. Of metals and alloys, which has greater value temperature coefficient ?

Ans. The value of α is more for metals than that for oys. For example, for copper, $\alpha = 4 \times 10^{-3}$ °C^{-1} and for nganin, $\alpha = 15 \times 10^{-6}$ °C^{-1}.

Q. 1.25. Why is manganin used for making standard sistors ? Give two reasons.

(*P.S.S.C.E. 1999 S ; H.S.S.C.E. 1999 ; C.B.S.E. 1995*)

Ans. Being an alloy, its resistivity is very high and nperature coefficient of resistance is very low.

Q. 1.26. Name the materials used for making standard sistance. Give reasons for this choice.

Ans. The alloys, such as manganin, constantan or chrome are used for making standard resistance. The alloys e used for this purpose for the reason that they possess high sistivity and low temperature coefficient of resistance.

Q. 1.27. Name any one material having a small value temperature coefficient of resistance. Write one use of is material. (*C.B.S.E. 1997*)

Ans. An alloy, such as nichrome, has a very small value temperature coefficient of resistance 4×10^{-4} °C^{-1}. Because this, it is used to make standard resistance.

Q. 1.28. Is the value of temperature coefficient of sistance always positive ?

Ans. No, it is positive only for metals and alloys. For miconductors and insulators, the value of α is negative.

Q. 1.29. What is a thermistor ?

(*P.S.S.C.E. 1999 , 2000 ; H.S.S.C.E. 1994*)

Ans. *A heat sensitive resistor, whose resistance varies preciably with temperature, is called a thermistor.*

Q. 1.30. What is a superconductor ?

(*P.S.S.C.E. 2000, 1999, 1998 S*)

Ans. *A conductor, which loses all signs of resistance on being cooled to its critical temperature, is called a super-conductor.*

Q. 1.31. What is meant by critical temperature in reference to superconductivity ?

Ans. The temperature of the conductor, at which all signs of its resistance disappear, is called critical temperature.

Q. 1.32. What are the carriers of current in a super-conductor ?

Ans. The carriers of current in a superconductor are mutually coherent electrons.

Q. 1.33. State two applications of superconductors.

(*P.S.S.C.E. 2002, 2000, 1995*)

Ans. 1. Superconductors can be used to transmit electric power without any loss of electric energy.

2. They can be used to produce very intense magnetic field without any expenditure of electric energy.

Q. 1.34. Why are resistances connected in series and in parallel.

Ans. The resistances are connected in series to increase the resistance of the circuit ; while they are connected in parallel, when resistance of the circuit is to be decreased.

Q. 1.35. A wire has a resistance of 90 Ω and it is cut into three pieces having equal lengths. If these are now connected in parallel, find the resistance of the combination so formed. (*P.S.S.C.E. 1997 ; H.S.S.C.E. 1997*)

Ans. When the wire having resistance of 90 Ω is cut into three equal pieces, each piece will be of resistance $\frac{90}{3}$ *i.e.* 30 Ω. If R_P is resistance of their parallel combination, then

$$\frac{1}{R_P} = \frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{1}{10} \quad \text{or} \quad R_p = 10\ \Omega$$

Q. 1.36. Given the resistances of 1 Ω, 2 Ω and 3 Ω. How will you combine them to get an equivalent resistance of 6 Ω and 6/11 Ω ?

Ans. In order to obtain an equivalent resistance of 6 Ω, the given resistances should be connected in series and to get 6/11 Ω, these resistances should be connected in parallel.

Q. 1.37. A student obtains resistances of 3, 4, 12 and 16 ohm using two metallic resistance wires, either separately or joined together. What is the value of resistance of each of these wires ? (*C.B.S.E. 1997*)

Ans. When two resistances are connected in series, the equivalent resistance is always greater than the individual resistances. On the other hand, when they are connected in parallel, the equivalent resistance is always less than either of them. Hence, of the measurements 3, 4, 12 and 16 Ω ; 16 Ω represent R_s and 3 Ω represents R_P. Obviously, the resistances of the two wires are 4 Ω and 12 Ω.

Q. 1.38. Define electromotive force (e.m.f.) of a cell.

(*P.S.S.C.E. 2002, 2000, 1999, 1998 S ; H.S.S.C.E. 1997 S*)

Ans. *The potential difference between two poles of a cell, when no current is drawn from it, is called electromotive force (e.m.f.) of the cell.*

1.39. On what factors does the internal resistance of a cell depend ? (*P.S.S.C.E. 2001*)

Ans. Internal resistance of a cell depends on the following factors :

1. The nature of electrolyte

2. The nature of electrodes
3. The distance between electrodes
4. The area of electrodes inside the electrolyte.

Q. 1.40. A cell of e.m.f. 2 V and internal resistance 0.1 Ω is connected to a 3·9 Ω external resistance. What will be the potential difference across the terminals of the cell ? *(C.B.S.E. 2001 S)*

Ans. Here, $E = 2\text{ V}$; $r = 0.1\ \Omega$ and $R = 3.9\ \Omega$

The terminal potential difference is given by

$$V = \frac{E}{R+r}R = \frac{2 \times 3{\cdot}9}{3{\cdot}9 + 0{\cdot}1} = 1{\cdot}95\text{ V}$$

Q. 1.41. A low voltage supply, from which one needs high currents, must have very low resistance. Why ? *(Text Question)*

Ans. The maximum current that can be drawn from a voltage supply is given by

$$I_{max} = \frac{E}{r}$$

Obviously, I_{max} will be large, if r is small.

Q. 1.42. A high voltage supply (H.T.) must have a very large internal resistance. Why ? *(Text Question)*

Ans. If the circuit containing the H.T. supply gets short circuited accidentally, the current in the circuit will not excee the safe limit, in case the internal resistance of H.T. supply very large.

Q. 1.43. In what respect, does a nearly discharged lea acid secondary cell differ mainly from a freshly charged ce in its e.m.f. or in its internal resistance ? *(Text Questio*

Ans. It mainly differs in internal resistance. A nearl discharged lead acid secondary cell possesses a very hig internal resistance in addition to a lower value of e.m.f.

Q. 1.44. A car battery is of 12 V. Eight dry cells of 1.5 connected in series can give 12 V. But such cells are not use in starting a car. Why ? *(C.B.S.E. 2001*

Ans. To start a car, a very high current is required. A ca battery can provide the required high current due to very lo value of its internal resistance. When eight dry cells are joine in series, the internal resistance of the combination of the cel becomes very high. Due to high value of internal resistanc the current that can be drawn from the cells is very smal Hence, such cells cannot be used to start a car.

Q. 1.45. It is easier to start a car engine on a warm da than on a chilly day. Why ? *(Text Questio*

Ans. With increase in temperature (on a warm day), th internal resistance of the car battery decreases.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. A large number of free electrons are present metals. Why is there no current in the absence of electric field across it ? *(P.S.S.C.E. 1998 S)*

Ans. The motion of free electrons is random *i.e.* the electrons move in all directions haphazardly. When electric field is applied, each free electron experiences a force in a direction opposite to that of the field. Due to this, the electrons acquire drift velocity. There is a net flow of charge, which constitutes the current. In the absence of electric field, it does not happen so and hence there is no current.

Q. 1.02. The current I flows through a wire of radius r and the free electrons drift with a velocity v_d. What is the drift velocity of electrons through a wire of same material but having double the radius, when a current 2 I flows through it ?

Ans. Let n be the electron density of the material of the wire.

Then, $$v_d = \frac{I}{nAe} = \frac{I}{n \times \pi r^2 \times e} \qquad ...(i)$$

Let v_d' be drift velocity of electrons in the second wire.

Then, $$v_d' = \frac{I'}{nA'e}$$

Here, $I' = 2I$ and $A' = \pi(2r)^2 = 4\pi r^2$

$$\therefore \quad v_d' = \frac{2I}{n \times 4\pi r^2 \times e} = \frac{1}{2}\frac{I}{n \times \pi r^2 e} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$v_d' = v_d/2$$

Q. 1.03. What happens to the drift velocity (v_d) of electrons and to the resistance (R), if length of a conductor is doubled (keeping potential difference unchanged) ? *(C.B.S.E. 2002)*

Ans. Drift velocity of electrons is given by

$$v_d = \frac{eE}{m}\tau$$

If V is potential difference across the conductor of lengt l, then

$$E = \frac{V}{l}$$

Therefore, $$v_d = \frac{eV}{ml}\tau$$

As l is doubled (keeping potential difference unchar ged), the drift velocity will become **half** of the initial valu

As R is directly proportional to length, the resistance w become **double** of initial value, when length is doubled.

Q. 1.04. If the current supplied to a variable resistor constant, draw a graph between voltage and resistance. *(I.S.C.E. 199*

Ans. From Ohm's law, $V = IR$

If a constant current is passed, then the above relatio represents a straight line passing through origin. Hence, whe a constant current is passed through a variable resistor, th graph will be as shown in Fig. 1.45.

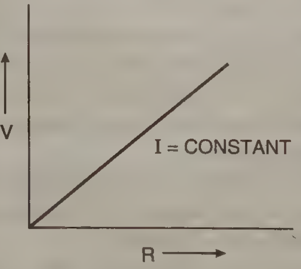


Fig. 1.45

Q. 1.05. Deduce dimensional formula for potenti difference. *(H.P.S.S.C.E. 199*

Ans. Now, potential difference V,

$$= \text{work done per unit charge}$$

$$\therefore \quad [V] = \frac{\text{work}}{\text{charge}} = \frac{\text{work}}{\text{current} \times \text{time}}$$

$$= \frac{[M\,L^2\,T^{-2}]}{[A] \times [T]} = [M\,L^2\,T^{-3}\,A^{-1}]$$

Q. 1.06. What are ohmic and non-ohmic resistors ? Give ne example of each. *(H.S.S.C.E. 2002)*

Ans. Ohmic resistor. *A resistor, which obeys ohm's law is lled an ohmic resistor.* For example, resistance possessed by metallic conductor is ohmic in nature.

Non-ohmic resistor. *A resistor, which does not obey ohm's w, is called a non-ohmic resistor.* For example, resistance ssessed by a semiconductor diode is non-ohmic in nature.

Q. 1.07. Define resistivity of a material and discuss the ctors on which it depends. *(H.S.S.C.E. 1997 S)*

Ans. *The resistivity of the material of a conductor is defined the resistance offered by a wire of this material of unit length and it area of cross-section.*

The resistivity of a conductor is given by

$$\rho = \frac{m}{n\,e^2\,\tau}$$

Thus, the resistivity of the material of a conductor varies

(*i*) inversely as the value of number of electrons per unit volume in the conductor.

(*ii*) inversely as the value of average relaxation time of the free electrons inside the conductor.

The value of n depends upon the nature of the material the conductor, while the average relaxation time depends on the temperature of the material.

Q. 1.08. Define resistivity of a material. Does it depend temperature ? *(C.B.S.E. 1992)*

Ans. For definition of resistivity, refer to section 1.09.

Yes, resistivity of a material depends on temperature.

Q. 1.09. Find out the resistivity of a conductor in which current density $2{\cdot}5 \times 10^6$ A m^{-2} is found to exist, when an ectric field of 15 V m^{-1} is applied on it. *(I.S.C.E. 1998)*

Ans. Let l and A be length and area of cross-section of e conductor respectively.

Here, current density, $\frac{I}{A} = 2{\cdot}5 \text{ A m}^{-2}$;

electric field, $\frac{V}{l} = 15 \text{ V m}^{-1}$

Now, resistivity, $\rho = R \cdot \frac{A}{l} = \frac{V}{I} \cdot \frac{A}{l} = \frac{V/l}{I/A}$

$$= \frac{15}{2{\cdot}5 \times 10^6} = 6 \times 10^{-6}\ \Omega\text{ m}$$

Q. 1.10. A wire of resistance 1 ohm is stretched to uble its length. What is the new resistance ?

(H.P.S.S.C.E. 2002, 1998 S ; P.S.S.C.E. 1995 ; C.B.S..E. 1994)

Ans. Let the wire be of length l and area of cross-section If resistivity of its material is ρ, then

original resistance of the wire, $R_1 = \rho\,\frac{l}{A}$

When the wire is stretched to double its length, then its ea of cross-section reduces to half of the initial area of cross-ction. Therefore, new resistance of the wire,

$$R_2 = \rho\,\frac{2l}{A/2} = 4\,\pi\,\frac{l}{A} = 4\,R_1$$

Since $R_1 = 1\,\Omega$, we have

$$R_2 = 4 \times 1 = 4\,\Omega$$

Q. 1.11. Two wires of equal lengths, one of copper and the other of manganin have the same resistance. Which wire is thicker ? *(C.B.S.E. 1995, 1991)*

Ans. Let ρ_1 and ρ_2 be resistivities of copper and manganin and A_1 and A_2 be the areas of cross-section of the wires made of copper and manganin respectively. Since two wires have equal lengths and have the same resistance,

$$\rho_1\,\frac{l}{A_1} = \rho_2\,\frac{l}{A_2} \quad \text{or} \quad \frac{A_1}{A_2} = \frac{\rho_1}{\rho_2}$$

Since resistivity of copper (ρ_1)is less than that of manganin (ρ_2), it follows that area of cross-section of copper wire(A_1) is less than that of manganin wire (A_2). Thus, **manganin** wire is thicker.

Q. 1.12. Compare the resistance of two wires of same material. Their lengths are in the ratio 2 : 3 and their diameters are in the ratio 1 : 2.

Ans. Let the lengths of the two wires be $2\,l$ and $3\,l$ and their diameters be D and 2 D respectively. Let ρ be the resistivity of the material of the two wires. Now,

$$R = \rho\,\frac{l}{A} = \rho\,\frac{l}{\pi\,D^2/4} = 4\,\rho\,\frac{l}{\pi\,D^2}$$

Therefore, $R_1 = 4\,\rho\,\frac{2l}{\pi\,D^2} = 8\,\rho\,\frac{l}{\pi\,D^2}$

and $R_2 = 4\,\rho\,\frac{3l}{\pi\,(2\,D)^2} = 3\,\rho\,\frac{l}{\pi\,D^2}$

Hence, $\frac{R_1}{R_2} = \left(8\,\rho\,\frac{l}{\pi\,D^2}\right) \Big/ \left(3\,\rho\,\frac{l}{\pi\,D^2}\right) = \frac{8}{3}$

Q. 1.13. A given wire having resistance R is stretched so as to reduce its diameter to half of its previous value. What will be its new resistance ? *(H.S.S.C.E. 2002)*

Ans. Let l be length and A be area of cross-section of the wire. Then,

$$R = \rho\,\frac{l}{A} \qquad \text{...}(i)$$

When the diameter of the wire is reduced to half of its previous value, its new area of cross-section will become one fourth *i.e.*

$$A' = A/4$$

If l' is new length of the wire, then

$$l'\,A' = l\,A \quad \text{or} \quad l' \times A/4 = l\,A \quad \text{or} \quad l' = 4\,l$$

Therefore, new resistance of the wire,

$$R' = \rho\,\frac{l'}{A'} = \rho\,\frac{4l}{A/4}$$

or $$R' = 16\,\rho\,\frac{l}{A} \qquad \text{...}(ii)$$

From equation (*i*) and (*ii*), we have

$$R' = 16\,A$$

Q. 1.14. A potential difference V is applied across a conductor of length L and diameter D. How are the electric field E and the resistance R of conductor affected, when in turn (*a*) V is halved (*b*) L is halved and (*c*) D is doubled ? Justify your answer in each case. *(C.B.S.E. 1997)*

Ans. Fig. 1.46 shows a wire of length L and diameter D, across the two ends of which, a potential difference V has been applied.

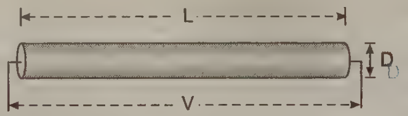


Fig. 1.46

If ρ is resistivity of the material of the wire, then resistance of the wire,

$$R = \rho \frac{L}{A} = \rho \frac{L}{\pi D^2/4}$$

or $$R = \rho \frac{4L}{\pi D^2}$$

Also, electric field along the wire, $E = \frac{V}{L}$

(a) When V is halved : There is **no effect** on resistance of the wire.

When potential difference is made $V' = V/2$, the electric field will become

$$E' = \frac{V'}{L} = \frac{V/2}{L} = \frac{1}{2}\left(\frac{V}{L}\right) = \frac{E}{2}$$

i.e. electric field will also get **halved.**

(b) When L is halved : When length is made $L' = \frac{L}{2}$, the resistance of wire will become

$$R' = \rho \frac{L/2}{\pi D^2} = \frac{1}{2}\left(\rho \frac{L}{\pi D^2}\right) = \frac{R}{2}$$

i.e. resistance of wire will also get **halved.**

Also, the electric field will become,

$$E' = \frac{V}{L'} = \frac{V}{L/2} = 2\left(\frac{V}{L}\right) = 2E$$

i.e. electric field will become **twice.**

(c) When D is doubled : When diameter of the wire is made $D' = 2D$, the resistance of the wire will become,

$$R' = \rho \frac{L}{\pi D'^2} = \rho \frac{L}{\pi (2D)^2} = \frac{1}{4}\left(\rho \frac{L}{\pi D^2}\right) = \frac{R}{4}$$

i.e. resistance of wire will become **onefourth.**

There is **no effect** on the electric field.

Q. 1.15. Write down a simple relationship to show the variation of resistance with temperature. Define temperature coefficient of resistance. (*I.S.C.E. 1995*)

Ans. The resistance of a metallic conductor at temperature θ °C is given by

$$R = R_0 (1 + \alpha \theta),$$

where R_0 is its resistance at 0°C and α, temperature coefficient of the material of the conductor.

From the above equation, we have

$$\alpha = \frac{R - R_0}{R \times \theta}$$

Hence, *temperature coefficient of the material of a wire is defined as the increase in resistance per unit resistance of the conductor at 0°C per degree rise of temperature.*

Q. 1.16. The variation of resistance of a metallic conductor with temperature is given in Fig. 1.47. (*i*) Calculate the temperature coefficient of resistance from the graph. (*ii*) State why the resistance of the conductor increas with rise in temperature. (*C.B.S.E. 1998*

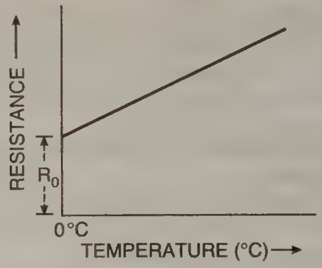


Fig. 1.47

Ans. Consider a point A on the resistance-temperatu graph [Fig. 1.48].

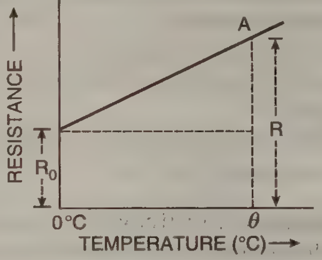


Fig. 1.48

Let R be resistance of the conductor and θ be temperature corresponding to point A on the graph.

(*i*) Temperature coefficient of the conductor,

$$\alpha = \frac{R - R_0}{R \times \theta}$$

(*ii*) The resistance of a wire is given by

$$R = \rho \frac{l}{A} = \frac{m}{ne^2\tau} \cdot \frac{l}{A}$$

When temperature of conductor is increased, its avera relaxation time decreases due to increase in frequency collision of electrons with atoms and ions. Since $R \propto \frac{1}{\tau}$, resistance of the conductor increases with rise in temperatu

Q. 1.17. The voltage-current variation of two metal wires X and Y at constant temperature are as shown in F 1.49. Assuming that the wires have the same length a same diameter, explain which wire will have a larg resistivity and why ? (*C.B.S.E. 1998*

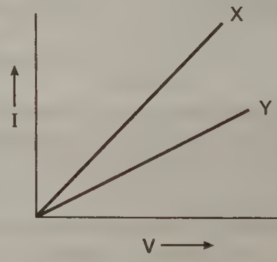


Fig. 1.49

Ans. Suppose that the two wires X and Y poss resistances R_1 and R_2 respectively. Further, suppose that th materials are of resistivity ρ_1 and ρ_2 respectively. Since two wires are of same length l and area A,

$$R_1 = \rho_1 \frac{l}{A} \text{ and } R_2 = \rho_2 \frac{l}{A}$$

Suppose that a potential difference V is applied across e two wires X and Y as shown in Fig. 1.50.

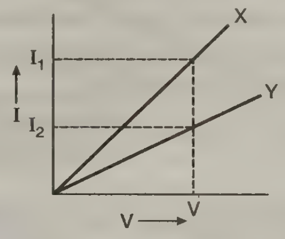


Fig. 1.50

It follows that current through wire X will be greater han that through wire Y *i.e.* $I_1 > I_2$.

The resistances of the wires X and Y are given by

$$R_1 = \frac{V}{I_1} \quad \text{and} \quad R_2 = \frac{V}{I_2}$$

Since $I_1 > I_2$, it follows that $R_1 < R_2$

$$\text{i.e.} \quad \rho_1 \frac{l}{A} < \rho_2 \frac{l}{A} \quad \text{or} \quad \rho_1 < \rho_2$$

e. wire Y has greater resistivity.

Q. 1.18. V-I graph for a metallic wire at two different emperatures T_1 and T_2 is as shown in the Fig. 1.51. Which f the two temperatures T_1 and T_2 is higher and why ? (*C.B.S.E. 1998*)

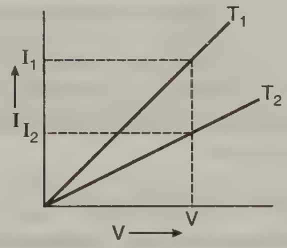


Fig. 1.51

Ans. If the same potential difference V is applied, current qual to I_1 and I_2 will flow through the wire, when its emperature is kept at T_1 and T_2 respectively. Since $R = \frac{V}{I}$, follows that resistance of the wire is more at temperature $_2$. Further, as in case of metals, the resistance increases with ise of temperature, $T_2 > T_1$.

Q. 1.19. A piece of copper and another of germanium re cooled from room temperature to 100 K. What will appen to their conductivities ? Explain. (*P.S.S.C.E. 1993*)

Ans. Copper possesses positive temperature coefficient, vhereas the germanium possesses negative temperature oefficient. Therefore, if they are cooled from room temperaure to 100 K, the resistance of copper will decrease and that f germanium will increase. In other words, the conductivity f copper increases and that of germanium decreases.

Q. 1.20. What is the total value of resistance between oints A and B in the network shown in Fig. 1.52 ?

Ans. If a battery is connected across the points A and B, hen same current will flow through all the resistances and ence the five resistances each of value R are in series.

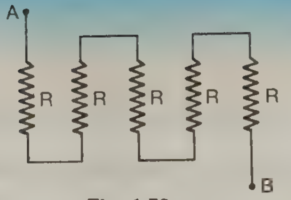


Fig. 1.52

Therefore, total value of resistance between points A and B is equal to 5 **R**.

Q. 1.21. In how many equal parts a wire having resistance of 100 Ω be cut, so that we may obtain a resistance of 1 Ω by connecting them in parallel ?

Ans. Suppose that the resistance of 100 Ω is cut into n equal parts. Then, the resistance of each part will be $\frac{100}{n}\ \Omega$. When these n parts are connected in parallel, the effective resistance becomes 1 Ω.

$$\therefore \quad \frac{n}{100} + \frac{n}{100} + \frac{n}{100} + \ldots\ldots n \text{ times} = \frac{1}{1}$$

$$\text{or} \quad \frac{n^2}{100} = 1 \quad \text{or} \quad n = 10$$

Q. 1.22. When two resistances are in series, they have value 25 Ω and in parallel 4 Ω. Find each. (*H.S.S.C.E. 1997*)

Ans. Let R_1 and R_2 be the two resistances.

Here, $R_s = 25\ \Omega$; $R_p = 4\ \Omega$

$$\therefore \quad R_1 + R_2 = 25 \quad \text{and} \quad \frac{R_1 R_2}{R_1 + R_2} = 4$$

Solving the above two equations, it can be obtained that

$$\mathbf{R_1 = 5\ \Omega \quad \text{and} \quad R_2 = 20\ \Omega}$$

Q. 1.23. A set of n identical resistors, each of resistance R ohm, when connected in series have an effective resistance of X ohm and when the resistors are connected in parallel, the effective resistance is Y ohm. Find the relation between R, X and Y. (*C.B.S.E. 1996*)

Ans. When n resistors, each of resistance, R are connected in series, then

$$X = n R \qquad \ldots(i)$$

When n resistors, each of resistance R are connected in parallel, then

$$Y = \frac{R}{n} \qquad \ldots(ii)$$

Multiplying equations (*i*) and (*ii*), we have

$$X Y = n R \times \frac{R}{n} = R^2$$

$$R = \sqrt{X Y}$$

Q. 1.24. You have three resistances of value R each. How will you connect these for obtaining a total resistance of 3 R/2? Show this connection by a diagram. (*P.S.S.C.E. 1996 S*)

Ans. A parallel combination of two resistors, each of resistance R should be put in series with the third resistor of resistance R as shown in Fig. 1.53.

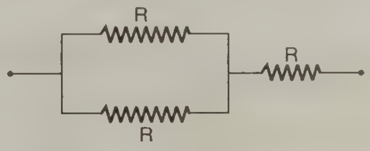


Fig. 1.53

Q. 1.25. Three resistors, each of resistance 4 Ω are connected in parallel. Five resistors, each of resistance 8 Ω are connected in parallel. Another three resistors, each of resistance 10 Ω are in parallel. If these three parallel combinations are in series, find the effective resistance of the circuit. *(Karnataka, 1988)*

Ans. The effective resistance of three resistors each of 4 Ω in parallel, $R_1 = \frac{4}{3} = 1{\cdot}333\ \Omega$

The effective resistance of five resistors each of 8 Ω in parallel, $R_2 = \frac{8}{5} = 1{\cdot}6\ \Omega$

The effective resistance of three resistors each of 10 Ω in parallel, $R_3 = \frac{10}{3} = 3{\cdot}333\ \Omega$

As the three sets are connected in series, the effective resistance is given by

$$R = R_1 + R_2 + R_3 = 1{\cdot}333 + 1{\cdot}6 + 3{\cdot}333$$
$$= 6{\cdot}266\ \Omega$$

Q. 1.26. Given *n* resistors each of resistance R. How will you combine them to get the (*i*) maximum, (*ii*) minimum effective resistance ? What is the ratio of the maximum to the minimum resistance ? *(Text Question)*

Ans. (*i*) The resistance will be maximum, when resistors are connected in series. Therefore, maximum resistance,

$$R_{max} = R + R + R + \ldots\ldots n \text{ times} = nR$$

(*ii*) The resistance will be minimum, when resistors are connected in parallel. Therefore, minimum resistance,

$$\frac{1}{R_{min}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \ldots\ldots n \text{ times} = \frac{n}{R}$$

or $$R_{min} = \frac{R}{n}$$

Now, $$\frac{R_{max}}{R_{min}} = \frac{nR}{R/n} = n^2$$

Q. 1.27. A and B are two poins on a circular ring made of uniform wire of resistance R. If the part AB of the ring subtends an angle θ at the centre C of the ring [Fig. 1.54], find the resistance of the ring between points A and B.

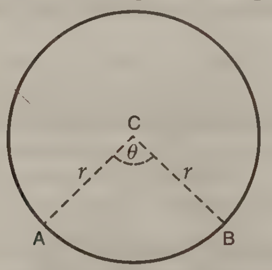


Fig. 1.54

Ans. Let *r* be the radius of the ring. Then,

resistance per unit length of the ring $= \frac{R}{2\pi r}$

Let l_1 be the length of the part of the ring subtending angle θ at the centre of the ring. Then,

$$l_1 = r\theta$$

If R_1 is resistance of the part of the ring subtending angle θ at the centre of the ring, then

$$R_1 = l_1 \times \frac{R}{2\pi r} = r\theta \times \frac{R}{2\pi r} = \frac{R\theta}{2\pi}$$

The other part of the ring subtends an angle $(2\pi - \theta)$ the centre of the ring. Its length will be $r(2\pi - \theta)$ and resistan will be

$$R_2 = r(2\pi - \theta) \times \frac{R}{2\pi r} = \frac{R(2\pi - \theta)}{2\pi}$$

Between the points A and B, the resistances of the tw parts of the ring are in parallel. Therefore, effective resistan of the ring between points A and B,

$$R_{eff} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(R\theta/2\pi) \times [R(2\pi - \theta)/2}{(R\theta/2\pi) + [R(2\pi - \theta)/2}$$
$$= \frac{(2\pi - \theta)\theta}{4\pi^2} R$$

Q. 1.28. A uniform wire of resistance R is shaped in a regular *n* sided polygon, where *n* is even. Find t equivalent resistance between (*a*) opposite corners of t polygon and (*b*) adjacent corners of polygon.

Ans. When a uniform wire of resistance R is shaped in a regular *n* sided polygon,

the resistance of each side of polygon $= \frac{R}{n}$

(*a*) Resistance of polygon between opposite corne Let R_1 and R_2 be resistances of the two parts of the polygo Then,

$$R_1 = R_2 = \frac{n}{2} \times \frac{R}{n} = \frac{R}{2}$$

Since the two parts are in parallel with each other, t resistance of the polygon between opposite corners,

$$R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{R/2 \times R/2}{R/2 + R/2} = \frac{R}{4}$$

(*b*) Resistance of polygon between adjacent corne The polygon is equivalent to the combination of two resi tances R_1 and R_2, where

R_1 = resistance of one side $= \frac{R}{n}$

and R_2 = resistance of $(n - 1)$ sides connected in serie

$$= \frac{(n-1)R}{n}$$

Therefore, resistance of the polygon between adjace corners,

$$R'' = \frac{R/n \times (n-1)R/n}{R/n + (n-1)R/n} = \frac{(n-1)R}{n^2}$$

Q. 1.29. A battery of e.m.f. E and internal resistanc sends a current I_1 and I_2, when connected to an extern resistance of R_1 and R_2 respectively. Find the e.m.f. a internal resistance of the battery.

Ans. Now, $I_1 = \frac{E}{R_1 + r}$

or $E = I_1(R_1 + r)$ ...(

Also, $I_2 = \frac{E}{R_2 + r}$

or $E = I_2(R_2 + r)$ ...(

From equations (*i*) and (*ii*), we have

$$I_1(R_1 + r) = I_2(R_2 + r)$$

or $$r = \frac{I_1 R_1 - I_2 R_2}{I_2 - I_1}$$

From equation (*i*), we have

$$E = I_1 (R_1 + r) = I_1 \left[R_1 + \frac{I_1 R_1 - I_2 R_2}{I_2 - I_1} \right]$$

or $$E = \frac{I_1 I_2 (R_1 - R_2)}{I_2 - I_1}$$

Q. 1.30. The storage battery of a car has an e.m.f. of 12 If the internal resistance of the battery is 0·4 Ω, what is e maximum current that can be drawn from the battery ? (*Text Question*)

Ans. Here, $E = 12\ V ; r = 0{\cdot}4\ \Omega$

The maximum current is drawn from the battery, when external resistance in the circuit is zero *i.e.* R = 0.

$$\therefore \qquad I_{max} = \frac{E}{r} = \frac{12}{0{\cdot}4} = \mathbf{30\ A}$$

Q. 1.31. Is internal resistance a defect of a cell ? Explain.

Ans. The internal resistance of a cell is not a defect of the cell. It is an inherent characteristic of the cell. The cell possesses internal resistance, because the ions have to move against the background of other ions and the neutral atoms of the electrolyte.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. A wire is carrying current. Is it charged ? Why ?

Ans. No. Electric current is the flow of free electrons in e conductor. At any instant, the number of electrons leaving e wire is always equal to the number of electrons flowing om the battery into it. Hence, the net charge on the wire is ro.

Q. 1.02. The electron drift speed is estimated to be very low. How there is current established almost the stant a circuit is closed ? (*Text Question*)

Ans. The information about beginning of the flow of rrent is transmitted through the propagation of ectromagnetic waves (electric impulse) and not with the ift velocity of the electrons. Hence, the electric bulb turns almost instantly, when the switch is closed.

Q. 1.03. Two wires of the same material having lengths in the ratio of 1 : 2 and diameters in the ratio 2 : 3 are connected in series with an accumulator. ompute the ratio of the potential difference across the two ires. (*C.B.S.E. Sample Q. Paper*)

Ans. Let lengths of the two wires 1 and 2 be $l_1 = x$ and $= 2\ x$ and their diameters be $D_1 = 2\ D$ and $D_2 = 3\ D_1$ spectively. Let ρ be resistivity of the material of the two ires.

Then, resistance of wire 1,

$$R_1 = \rho \frac{l_1}{A_1} = \rho_1 \frac{l_2}{\pi r_2^2} = \rho \frac{x}{\pi \left(\frac{2D}{2}\right)^2} = \rho \frac{x}{\pi D^2}$$

The resistance of wire 2,

$$R_2 = \rho \frac{l_2}{A_2} = \rho_1 \frac{l_2}{\pi r_2^2} = \rho \frac{2x}{\pi \left(\frac{3D}{2}\right)^2} = \rho \frac{8x}{9 \pi D^2}$$

Then, total resistance of the series circuit, $R = R_1 + R_2$

or $$R = \rho \frac{x}{\pi D^2} + \rho \frac{8x}{9 \pi D^2} = \rho \frac{17x}{9 \pi D^2}$$

Let E be e.m.f. of the accumulator. Then,

current in the circuit, $I = \frac{E}{R} = \frac{E}{\rho} \cdot \frac{9 \pi D^2}{17 x}$

Now, potential difference across wire 1,

$$V_1 = I R_1 = \frac{E}{\rho} \cdot \frac{9 \pi D^2}{17 x} \cdot \rho \frac{x}{\pi D^2} = \frac{9 E}{17}$$

and potential difference across wire 2,

$$V_2 = I R_2 = \frac{E}{\rho} \cdot \frac{9 \pi D^2}{17 x} \cdot \rho \frac{8x}{9 \pi D^2} = \frac{8 E}{17}$$

$$\therefore \qquad \frac{V_1}{V_2} = \frac{9 E}{17} \times \frac{17}{8 E} = \frac{9}{8}$$

Q. 1.04. Resistivity of the material of a conductor of uniform area of cross-section varies along its length as $\rho = \rho_0 (1 + \alpha x)$. Find the resistance of the conductor, if its length is L and area of cross-section is A.

Sol. Consider a length element *dx* of the conductor at a distance *x* from its one end. Then, small resistance of the length element *dx* of the conductor,

$$dR = \rho \frac{dx}{A} = \rho_0 \ (1 + \alpha x) \frac{dx}{A}$$

The resistance of the whole length of the conductor,

$$R = \int_0^L dR = \int_0^L \rho_0 (1 + \alpha x) \frac{dx}{A} = \frac{\rho_0}{A} \left[\int_0^L dx + \alpha \int_0^L x\, dx \right]$$

$$= \frac{\rho_0}{A} \left[\left| x \right|_0^L + \alpha \left| \frac{x^2}{2} \right|_0^L \right] = \frac{\rho_0}{A} \left[(L - 0) + \frac{1}{2} \alpha (L^2 - 0) \right]$$

$$= \frac{\rho_0}{A} \left(L + \frac{1}{2} \alpha L^2 \right)$$

Q. 1.05. A parallel plate capacitor having plates of area A separated by a distance *d* is first filled with a dielectric of permittivity ε and then with an electrolyte of conductivity σ. Find the ratio between the capacitance of the capacitor in the first case and its conductance in the second.

Ans. ***When the dielectric is filled between the plates of capacitor*** : The capacitance of the capacitor is given by

$$C = \frac{\varepsilon A}{d} \qquad \text{...(}i\text{)}$$

When the electrolyte is filled between the plates of capacitor : The capacitor acts as a conductor of cross-sectional area A, length *d* and resistivity $\frac{1}{\sigma}$. Therefore, resistance of the capacitor is given by

$$R = \frac{1}{\sigma} \cdot \frac{d}{A}$$

If G is conductance of the capacitor, then

$$G = \frac{1}{R} = \frac{\sigma A}{d} \qquad \text{...(}ii\text{)}$$

Dividing equation (*i*) by (*ii*), we have

$$\frac{C}{G} = \frac{\varepsilon A}{d} \times \frac{d}{\sigma A} = \frac{\varepsilon}{\sigma}$$

Q. 1.06. At 0°C, the resistance of a conductor B is *n* times that of the conductor A. The temperature coefficients of resistance of the conductors A and B are α_1 and α_2 respectively. Find the resistance of the series combination of the two conductors at 0°C. Also, find the temperature coefficient of the resistance of the series combination of the two conductors.

Ans. Let R_0 be the resistance of conductor A at 0°C. Then, resistance of conductor B at 0°C, $R_0' = n\,R_0$

Let R and R′ be respectively the resistances of the conductors A and B at θ °C. Then,

$$R = R_0\,(1 + \alpha_1\,\theta)$$

and $$R' = R_0'\,(1 + \alpha_2\,\theta) = n\,R_0\,(1 + \alpha_2\,\theta)$$

If R_s is the resistance of the series combination of the two conductors at θ°C, then

$$R_s = R + R' = R_0\,(1 + \alpha_1\,\theta) + n\,R_0\,(1 + \alpha_2\,\theta)$$

or $$= R_0\,[(1 + n) + (\alpha_1 + n\,\alpha_2)\,\theta]$$

or $$R_s = (1+n)\,R_0\left[1 + \frac{\alpha_1 + n\,\alpha_2}{1+n}\,\theta\right] \qquad ...(i)$$

From equation (*i*) it follows that resistance of series combination of conductors A and B at 0°C is **(1 + *n*) R_0** and their temperature coefficient is equal to $\dfrac{\alpha_1 + n\,\alpha_2}{1+n}$

Q. 1.07. What should the resistances of the sections of a rheostat R_1, R_2 and R_3 be in order to change the current passing through an electrical appliance of resistance R = 30 Ω by 1 A [Fig. 1.55], when the rheostat slide is shifted from one contact to another ? The circuit is powered by a source of E = 120 V.

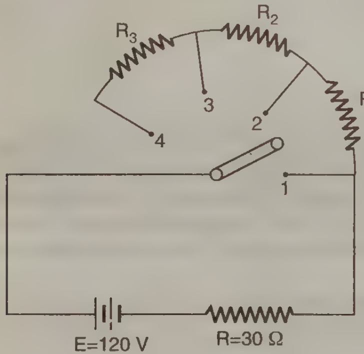


Fig. 1.55

Ans. ***When the rheostat slide is at terminal 1*** **:** When the slide is at terminal 1, the rheostat is completely out of the circuit. Then, current in the circuit,

$$I = \frac{E}{R} = \frac{120}{30} = 4\text{ A}$$

When the rheostat slide is at terminal 2 **:** Now, the resistor R_1 is included in the circuit. This should decrease current in the circuit by 1 A *i.e.* decrease to 4 – 1 *i.e.* 3 A. Thus,

$$\frac{E}{R + R_1} = 3 \quad \text{or} \quad \frac{120}{30 + R_1} = 3$$

or $$30 + R_1 = \frac{120}{3} = 40$$

or $$R_1 = 40 - 30 = \mathbf{10\ \Omega}$$

When the rehostat slide is at terminal 3 **:** As such, resistors R_1 and R_2 are included in the circuit and theref current should decrease to 3 – 1 = 2 A. Thus,

$$\frac{E}{R + R_1 + R_2} = 2 \quad \text{or} \quad \frac{120}{30 + 10 + R_2} =$$

or $$\frac{120}{40 + R_2} = 3 \quad \text{or} \quad 40 + R_2 = \frac{120}{2} =$$

or $$R_2 = 60 - 40 = \mathbf{20\ \Omega}$$

When the rheostat slide is at terminal 4 **:** With inclusion of resistors R_1, R_2 and R_3, the current should n decrease to 2 – 1 = 1 A. Thus,

$$\frac{E}{R + R_1 + R_2 + R_3} = 1 \quad \text{or} \quad \frac{120}{30 + 10 + 20 + R_3} =$$

or $$\frac{120}{60 + R_3} = 1 \quad \text{or} \quad 60 + R_3 =$$

or $$R_3 = 120 - 60 = \mathbf{60\ \Omega}$$

Note. In fact, the above arrangement of resistors for a conventional regulator (such as regulator used with electric fan) for regulating current in an electric circuit.

Q. 1.08. A circular ring having negligible resistanc used to connect four resistors of resistances 6 R, 6 6 R and R as shown in Fig 1.56. Find the equival resistance between points A and B.

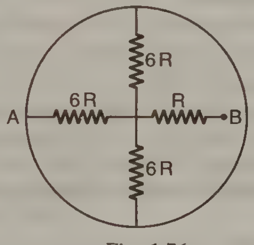


Fig. 1.56

Ans. The given network of resistors is equivalent to arrangement shown in Fig. 1.57.

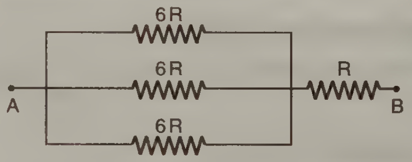


Fig. 1.57

Let R′ be the equivalent of resistance of the para combination of three resistors, each of resistance 6 R. Th

$$\frac{1}{R'} = \frac{1}{6R} + \frac{1}{6R} + \frac{1}{6R} = \frac{3}{6R} \quad \text{or} \quad R' = 2R$$

Therefore, equivalent resistance of the network betw points A and B

$$R_{equi} = 2R + R = \mathbf{3\ R}$$

Q. 1.09. What is the equivalent resistance betw points A and B of the circuit shown in Fig. 1.58 ?

(*I.I.T. 19*

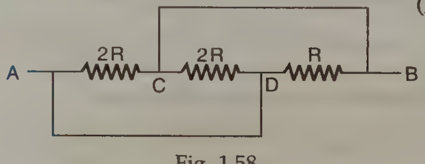


Fig. 1.58

Ans. The given circuit may be redrawn as shown in Fig.

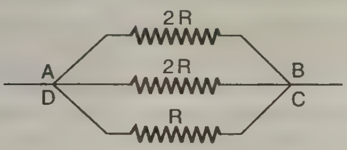


Fig. 1.59

The three resistances form a parallel combination. If R′ e equivalent resistance of the combination, then

$$\frac{1}{R'}=\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}=\frac{1+1+2}{2R}=\frac{2}{R}$$

or $$R'=\frac{R}{2}$$

10. Determine the equivalent resistance of the network wn in Fig. 1.60. *(Text Question)*

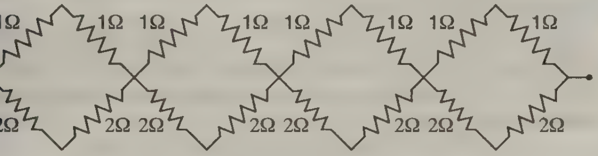


Fig. 1.60.

Sol. The network shown in Fig. 1.60 is a series combi- on of four units. One such unit is shown in the Fig. 1.61.

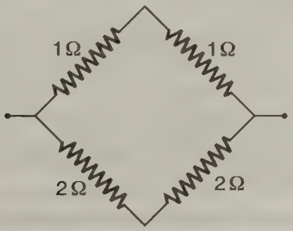


Fig. 1.61

llows that one such unit is equivalent to a parallel bination of two resistances of 2 Ω and 4 Ω [Fig. 1.62].

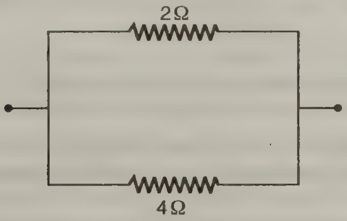


Fig. 1.62

Therefore, resistance of one such unit is given by

$$\frac{1}{R}=\frac{1}{2}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$$

or $$R=\frac{4}{3}\,\Omega$$

Therefore, resistance of the total network (4 such units)

$$=4\times\frac{4}{3}=\frac{16}{3}\,\Omega$$

Q. 1.11. A battery of e.m.f. E and internal resistance r is connected to a variable external resistance R. Find value of R so that (*a*) current in the circuit is maximum (*b*) terminal potential across the battery is maximum. o, find the maximum value of current in case (*a*) and imum terminal potential difference in case (*b*).

Ans. (*a*) The current in the circuit,

$$I=\frac{E}{R+r}$$

It follows that current in the circuit will be maximum, when $R+r$ is minimum *i.e.* **R = 0.**

The maximum value of current,

$$I_{max}=\frac{E}{r+0}=\frac{E}{r}$$

(*b*) The terminal potential difference across the battery,

$$V=E-Ir=E-\frac{Er}{R+r}$$

It follows that terminal potential across the battery will be maximum, when $E\,r/R+r$ is minimum *i..e* **R = ∞.**

The maximum value of terminal potential difference,

$$V_{max}=E-\frac{Er}{\infty+r}=E$$

Q. 1.12. A secondary cell after long use has an e.m.f. 1·9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell ? Could the cell drive the starting motor of a car ? *(Text Question)*

Ans. Here, $E=1{\cdot}9$ V ; $r=380\ \Omega$

The maximum current that can be drawn corresponds to zero external resistance. Therefore,

$$I_{max}=\frac{1{\cdot}9}{380}=\mathbf{0{\cdot}005\ A}$$

A motor needs a current of 100 mA or so for a few seconds. Hence, it cannot be used to drive the starting motor of a car.

Q. 1.13. Two identical storage batteries having e.m.fs. of E and the same internal resistance r are connected as shown in Fig. 1.63. Determine the potential difference, which will be established between points A and B.

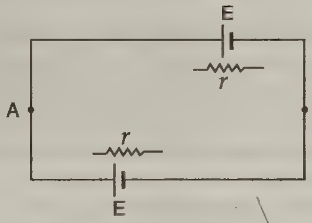


Fig. 1.63

Ans. As the two batteries are connected in series, total e.m.f. of the batteries, $E'=E+E=2E$ and total resistance of the circuit, $R=r+r=2r$

Therefore, current in the circuit,

$$I=\frac{E'}{R}=\frac{2E}{2r}=\frac{E}{r}$$

Now, potential difference between points A and B,

V_A-V_B = e.m.f. of the either cell
– potential drop across its internal resistance

$$=E-r\times\frac{E}{r}=E-E=0$$

Problem 1.14. A storage battery of e.m.f. 8·0 V and internal resistance 0·5 Ω is being charged by a 120 V d.c. supply using a series resistor of 15·5 Ω. What is the terminal voltage of the battery during charging ? *(Text Question)*

Sol. Here, $r=0{\cdot}5\ \Omega$; $R=15{\cdot}5\ \Omega$

When storage battery of e.m.f. 8·0 V is charged with a d.c. supply of 120 V, the net e.m.f. in the circuit,

$$E = 120 - 8{\cdot}0 = 112{\cdot}0 \text{ V}$$

Therefore, current in the circuit during charging,

$$I = \frac{E}{R+r} = \frac{112{\cdot}0}{15{\cdot}5+0{\cdot}5} = 7{\cdot}0 \text{ A}$$

Terminal voltage of the storage battery would be equal to the sum of its e.m.f. and potential difference across its internal resistance *i.e.*

terminal voltage = 8·0 + 0·5 × 7 = **11·5 V**

Q. 1.15. A storage battery is connected to a battery charger supplying V = 12·5 volt [Fig. 1.64]. The internal resistance of the storage battery is 1 Ω. What is the e.m.f. of the storage battery, if a current of 0·5 A flows through it during charging ? (*Text Question*)

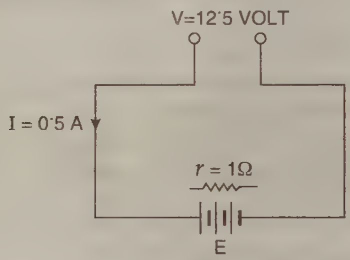


Fig. 1.64

Ans. Let E be the e.m.f. of the battery. The charging current I is opposed by the e.m.f. of the battery. According to Ohm's law,

$$V - E = I\,r$$

or $$E = V - I\,r = 12{\cdot}5 - 0{\cdot}5 \times 1 = \mathbf{12\ V}$$

Q. 1.16. When a secondary cell of e.m.f. 2·0 V is being charged by an external supply, is the terminal voltage of the secondary cell greater or less than 2·0 V ? (*Text Question*)

Ans. When a secondary cell is charged, the current during charging is in the direction of electrostatic field. As a result, the electrostatic field is greater than the non-electrostatic field existing inside the cell and hence the terminal voltage is greater than the e.m.f. of the secondary cell.

Also, refer to Conceptual SAQ. 1.14.

Q. 1.17. A dynamo has internal resistance $r = 0{\cdot}5\,\Omega$ and provides an e.m.f. E_1 = 240 V. It is connected to a storage battery of e.m.f. E_2 = 220 V and an external resistance R as shown in Fig. 1.65. At what maximum value of R will no current pass through the storage battery ? How will the battery operate, when the resistance R is larger or smaller than the above maximum value of R ?

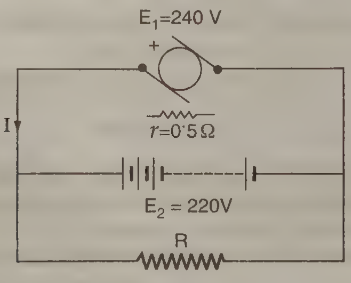


Fig. 1.65

Ans. Let R_{max} be the value of R, so that no current through the battery. As the entire current, say I, produc the dynamo passes through external resistance,

$$I = \frac{E_1}{R_{max}+r} = \frac{240}{R_{max}+0{\cdot}5}$$

Also, for no current to flow through battery, the pot difference between the poles of the dynamo should be to e.m.f. of the battery *i.e.*

$$E_1 - I\,r = E_2$$

or $$I = \frac{E_1 - E_2}{r} = \frac{240-220}{0{\cdot}5} = 40 \text{ A}$$

From equations (*i*) and (*ii*), we have

$$\frac{240}{R_{max}+0{\cdot}5} = 40 \quad \text{or} \quad R_{max} + 0{\cdot}5 = 6$$

or $$R_{max} = \mathbf{5{\cdot}5\ \Omega}$$

If $R > R_{max}$, battery will be charged and when R < battery will be discharged.

Q. 1.18. In the circuit shown in Fig. 1.66, each b is of 5 V and has an internal resistance of 0·2 Ω. will be the reading of an ideal voltmeter connected a battery ? (*I.I.T.*

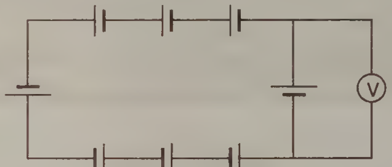


Fig. 1.66

Ans. Here, e.m.f. of each battery, E = 5 V

and internal resistance of each battery, $r = 0{\cdot}2\ \Omega$

If I is current in the circuit, then

reading of the voltmeter, $V = E - I\,r$,

where I is current in the circuit.

Now, $$I = \frac{\text{total e.m.f. of all the eight batterie}}{\text{total internal resistance of all the batt}}$$

$$= \frac{8 \times 5}{8 \times 0{\cdot}2} = 25 \text{ A}$$

Therefore, reading of the voltmeter,

$$V = 5 - 25 \times 0{\cdot}2 = 5 - 5 = 0$$

Problem 1.19. (*a*) Three cells of e.m.f. 2·0 V, 1·8 1·5 V are connected in series. Their internal resist are 0·5 Ω, 0·7 Ω and 1 Ω respectively. If the batt connected to an external resistor of 4 Ω via a ver resistance ammeter, what would be the reading i ammeter ?

(*b*) If the three cells above were joined in par would they be characterized by a definite e.m.f internal resistance ? (Independent of external circuit). how will you obtain currents in different branches circuit ? (*Text Que*

Sol. Here, series resistor, R = 4 Ω

(*a*) When cells are joined in series,

net e.m.f. of cells, E = 2·0 + 1·8 + 1·5 = 5·3 V

The total internal resistance of the cells in series,

$$r = 0{\cdot}05 + 0{\cdot}7 + 1 = 1{\cdot}75\ \Omega$$

$$\therefore \quad I = \frac{E}{R+r} = \frac{5{\cdot}3}{4+1{\cdot}75} = \frac{5{\cdot}3}{5{\cdot}75} = \mathbf{0{\cdot}922\ A}$$

(*b*) No, they will not be characterized by a definite e.m.f. and internal resistance. Since there is no formula for total e.m.f. and total internal resistance of non-similar cells, current may be calculated by using Kirchhoff's laws.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is drift velocity of electrons ? How do you explain the flow of current in a conductor based on this ?
2. What is drift velocity ? Establish the relation between drift velocity and electric current. (*P.S.S.C.E. 2002, 2000, 1999 S, 1999, 1998, 1997, 1996 S ; I.S.C.E. 1997*)
3. Establish the relation between drift velocity of electrons and the electric field applied to the conductor. (*P.S.S.C.E. 2002*)
4. State and explain Ohm's law.
5. Deduce Ohm's law using the concept of drift velocity. (*H.S.S.C.E. 1994*)
6. Is Ohm's law universally applicable to all circuit elements ? Explain. (*Text Question ; Pre-degree Kerala, 1991*)
7. Discuss the various situations, which describe the failure of Ohm's law.
8. Define resistance and give its units. What are the factors on which the resistance of a wire depends ?
9. What are the factors on which the resistance of a conductor depends ? Give the corresponding relation. (*C.B.S.E. 1990*)
10. Define resistivity of a material. Does it depend upon the shape of the cross-section of the wire of the material ? What are its units in cgs and SI system ?
11. A conductor of length l is connected to a d.c. source of potential V. If the length of the conductor is tripled by stretching it, keeping V constant, explain how do the following factors vary in the conductor : (*i*) drift velocity of electrons (*ii*) resistance (*iii*) resistivity. (*C.B.S.E. 2000*)
12. Define resistivity of a conductor. Explain the variation of resistance with temperature in (*a*) metallic conductors and (*b*) semiconductors. (*C.B.S.E. 1999*)
13. Define the term resistivity of a conductor. Give its SI unit. Show that the resistance of a conductor is given by

$$R = \frac{m\,l}{n\,e^2\,\tau\,A},$$

where symbols have their usual meanings. (*C.B.S.E. 2002*)

14. Define relaxation time of electrons in a conductor. Explain how it varies with increase in temperature of a conductor. State the relation between resistivity and relaxation time. (*C.B.S.E. 2001*)
15. Define resistivity and state its SI unit. State and explain how the resistivity of a conductor varies with temperature. (*C.B.S.E. 2001*)
16. Define specific resistance and express it in mass, charge, number density and relaxation time. (*H.S.S.C.E. 2001*)
17. Define the terms : current, ampere, resistance, ohm, resistivity and conductivity.
18. Define conductivity of a conductor. State its SI unit. Explain the variation of conductivity with temperature of a metallic conductor. (*C.B.S.E. 2002*)
19. Define conductivity of a conductor and state its SI unit. State and explain the variation of conductivity of ionic conductor with temperature. (*C.B.S.E. 2001*)
20. Explain the temperature dependence of resistivity in case of a conductor, a semiconductor and insulator.
21. What is a thermistor ? How does it differ from an ordinary resistor ? Explain in brief. List a few applications of a thermistor.
22. What is superconductivity ? Write its applications. (*H.S.S.C.E. 1994*)
23. What are superconductors ? Write their uses. (*P.S.S.C.E. 1995*)
24. Explain the colour code of resistances. (*H.S.S.C.E. 1998, 1995*)
25. How do you connect resistances in series ? Obtain expression for the resistance, when three resistances R_1, R_2 and R_3 are connected in series.
26. Find the total resistance of a circuit, in which three resistances are connected in parallel. (*H.P.S.S.C.E. 1998 S*)
27. Define e.m.f. of a cell. Is it the same as potential difference ? If not, how does it differ from it ?
28. Distinguish between electromotive force and terminal potential difference of a cell. What are their units ? (*H.S.S.C.E. 1998 S*)
29. What is the internal resistance of a cell ? How it can be measured ? (*H.S.S.C.E. 2001*)
30. What do you understand by internal resistance of a cell ? Why does a cell offer internal resistance ? Explain in brief.
31. What is internal resistance of a cell ? Derive an expression for it. (*P.S.S.C.E. 2002, 1999*)
32. Define internal resistance of a cell. Prove that

$$r = \left(\frac{E}{V} - 1\right)R,$$

where R is external resistance used. (*H.P.S.S.C.E. 1999*)

33. When a battery of e.m.f. E and internal resistance r is connected to a resistance R, a current I flows through it. Derive a relation between E, I, r and R. (*C.B.S.E. 1992*)
34. Fig. 1.67 shows a cell with e.m.f. E and internal resistance r, connected to a voltmeter V and a variable resistance R. Deduce the relationship between V, E, R and r. How will the value of V vary, when R is reduced ? (*I.S.C.E. 1998*)

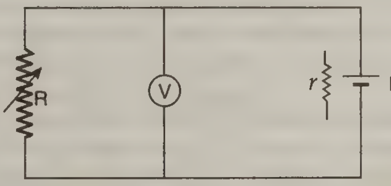


Fig. 1.67

35. Explain, how the cells are grouped in series. Obtain the condition for maximum current through the external resistor.
36. Show with the help of a diagram, how the cells may be connected in parallel. Obtain the condition for the current to be maximum due to a parallel grouping of cells, when connected to an external resistance.

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. State Ohm's law and derive it from the basic ideas of drift velocity of electrons. (*P.S.S.C.E. 2002, 2000*)
2. State Ohm's law. Derive the laws of resistances when they are connected : (*i*) in series ; (*ii*) in parallel. (*Pre-degree Kerala, 1992*)

3. Obtain the condition for maximum current through a resistor, when a number of cells are connected (*i*) in series and (*ii*) in parallel.

4. How the cells are mixed grouped ? Explain. Deduce
condition for obtaining maximum current througl
external resistance, when the cells are mixed groupe

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Definition of current

1. If $2{\cdot}25 \times 10^{20}$ electrons pass through a wire in one minute, find the magnitude of the current flowing through the wire. (*P.S.S.C.E. 2002*) **[Ans.** 0·6 A]
2. How many electrons pass through a lamp in 5 minutes, if the current through it is 0·1 A ? **[Ans.** $1{\cdot}875 \times 10^{20}$]
3. A sulution of sodium chloride discharges $6{\cdot}1 \times 10^{16}$ Na^+ ions and $4{\cdot}6 \times 10^{16}$. Cl^- ions in 2 s. Find the current passing through the solution. **[Ans.** $8{\cdot}56 \times 10^{-3}$ A]
4. An electron moves in a circle of radius 0·15 m with a constant speed of $3{\cdot}6 \times 10^6$ m s^{-1}. What electric current does this correspond to ? **[Ans.** $6{\cdot}1 \times 10^{-13}$ A]
5. The amount of charge passing through the cross-section of a wire in time t is given by

   $$q = a\,t^2 + b\,t + c$$

   (*a*) What are the dimensional formulae of constants a, b, and c ?

   (*b*) If the values of constants a, b, c are 3, 5 and 2 in SI units, find the value of current at $t = 3$ s.

   [Ans. (*a*) $[\mathrm{I\,T^{-1}}]$, [I] and [I T] ; (*b*) 23 A]

Type B. On Drift velocity

6. A current of 5 A is passing through a metallic wire of area of cross-section 4×10^{-6} m^2. If the number density of electrons in the wire is 5×10^{26} m^{-3}, find the drift velocity of electrons. (*Roorkee 1991*) **[Ans.** $1{\cdot}563 \times 10^{-2}$ m s^{-1}]
7. A conductor with a cross-section of 10^{-4} m^2 carries an electric current of 1·2 A. If the number of the free electrons are 5×10^{28} m^{-3}, calculate the electron drift velocity. Charge on the electron, $e = 1{\cdot}6 \times 10^{-6}$ C. **[Ans.** $1{\cdot}5 \times 10^{-6}$ m s^{-1}]
8. A copper wire of diameter 1·0 mm carries a current of 0·2 A. Copper has $8{\cdot}4 \times 10^{28}$ atoms per cubic metre. Find the drift velocity of electrons, assuming that one charge carrier of $1{\cdot}6 \times 10^{-19}$ C is associated with each atom of the metal. (*I.S.C.E. 1997*) **[Ans.** $1{\cdot}895 \times 10^{-5}$ m s^{-1}]
9. A 10 C charge flows through a wire in 5 minutes. The radius of the wire is 1 mm. It contains 5×10^{22} electrons per centimetre^3. Calculate the current and drift velocity. **[Ans.** $3{\cdot}33 \times 10^{-2}$ A, $1{\cdot}325 \times 10^{-6}$ m s^{-1}]
10. What is the drift velocity of electrons in a silver wire of length 1 m, area of cross-section $3{\cdot}14 \times 10^{-6}$ m^2 and carrying a current of 10 A. Given, charge on electron $= 1{\cdot}6 \times 10^{-19}$ C, Avogadro number $= 6{\cdot}02 \times 10^{23}$, atomic weight of silver = 108 and density of silver $= 10{\cdot}5 \times 10^3$ kg m^{-3}. **[Ans.** $3{\cdot}4 \times 10^{-4}$ m s^{-1}]

Type C. On Electron mobility

11. A potential difference of 6 V is applied across a conductor of length 0·12 m. Calculate the drift velocity of electrons, if the electron mobility is $5{\cdot}6 \times 10^{-6}$ m^2 V^{-1} s^{-1}. **[Ans.** $2{\cdot}8 \times 10^{-4}$ m s^{-1}]
12. A current of 1 A flows through a wire of length 0·24 m and area of cross-section 1·2 mm^2, when it is connected to a battery of 3 V. Find the number density of free electrons in the wire, if the electron mobility is $4{\cdot}8 \times 10^{-6}$ m^2 V^{-1} s^{-1}, Given that charge on electron $= 1{\cdot}6 \times 10^{-19}$ C. **[Ans.** $8{\cdot}68 \times 10^{28}$ m^{-3}]

Type D. On Current density

13. A current of 2·4 A flows through a wire of cross-secti
area 1·5 mm^2. Find the current density in the wire. If
wire contains 8×10^{28} free electrons per cubic me
calculate the drift velocity of electrons.
[Ans. $1{\cdot}6 \times 10^6$ A m^{-2} ; $1{\cdot}25 \times 10^{-4}$ m
14. When a potential difference of 1·8 V is applied across a
of length 0·15 m and area of cross-section 0·2 mm
current of 2 A flows through the wire. If the num
density of free electrons in the wire is $6{\cdot}8 \times 10^{28}$
calculate the average relaxation time. Given that charg
electron $= 1{\cdot}6 \times 10^{-19}$ C and mass of electron $= 9{\cdot}1 \times$ 1
kg. **[Ans.** $4{\cdot}36 \times 10$
15. An aluminium wire of diameter 0·24 cm is connecte
series to a copper wire of diameter 0·16 cm. The w
carry an electric current of 10 A. Find (*a*) current densit
free electrons in the aluminium wire and (*b*) drift velo
of electrons in the copper wire. Given that numbe
density of free electrons in copper $= 8{\cdot}4 \times 10^{28}$ m^{-3}.
[Ans. (*a*) $2{\cdot}21 \times 10^6$ A m^{-2} (*b*) $3{\cdot}7 \times 10^{-4}$ m

Type E. On Ohm's law

16. A potential difference of 10 V is applied across a condu
of resistance 1 kΩ. Find the number of electrons flov
through the conductor in 5 minutes. **[Ans.** 1·875 ×1

Type F. On Resistivity

17. A negligibly small current is passed through a wir
length 15 m and uniform cross-section $6{\cdot}0 \times 10^{-7}$ m^2
its resistance is measured to be 5·0 Ω. What is the resisti
of the material at the temperature of the experiment
(*Text Problem*) **[Ans.** 2×10^{-7}
18. The resistance of 100 cm of a thin strip of a metal is fo
to be 2·5 Ω. The cross-section of the strip is a rectang
2 mm × 0·5 mm. Calculate resistivity of the materi
strip. (*Karnataka 1976*)**[Ans.** $2{\cdot}5 \times 10^{-6}$
19. Calculate the resistivity of the material of a wire 2 m l
0·4 mm in diameter and having a resistance of 4 Ω.
(*H.P.S.S.C.E. 1999 S ; P.S.S.C.E. 1*
[Ans. $2{\cdot}513 \times 10^{-7}$
20. A rheostat has 100 turns of a wire of radius 0·4 mm ha
resistivity $4{\cdot}2 \times 10^{-7}$ Ω m. The diameter of each tu
3 cm. What is the maximum value of resistance it
introduce ? **[Ans.** 7·87
21. Given that resistivity of copper is $1{\cdot}68 \times 10^{-8}$
Calculate the amount of copper required to draw a wi
km long having resistance 10 Ω. The density of copp
$8{\cdot}9 \times 10^3$ kg m^{-3}. **[Ans.** 1495·2
22. Find the relaxation time for free electrons in copper, i
density of free electrons is $8{\cdot}4 \times 10^{28}$ m^{-3}. Given
resistivity of copper at room temperature $= 1{\cdot}7 \times 10^{-8}$
mass of electron $= 9{\cdot}1 \times 10^{-31}$ kg and charge on electr
$1{\cdot}6 \times 10^{-19}$ C. **[Ans.** $2{\cdot}49 \times 10$
23. A wire of resistance 2 Ω and resistivity $1{\cdot}1 \times 10^{-6}$ Ω
stretched, so that its length becomes three times o
original length. Determine its new resistance and
resistivity. (*P.S.S.C.E. 1999 S*) **[Ans.** 18 Ω ; unchang

24. A wire has a resistance of 32 Ω. It is melted and drawn into a wire of half of its original length. Calculate the resistance of the new wire. What is the percentage change in resistance ? *(C.B.S.E. 1997)* **[Ans.** 75% (decrease)**]**

25. A wire of stretched by 20%. What is percentage change in its resistance ? **[Ans.** 44%**]**

26. A wire is of resistance R is stretched, so that its radius decreases by factor n. Calculate its new resistance. **[Ans.** n^4 R**]**

27. A wire of uniform cross-section has a resistance of 9 Ω. It is cut into three equal pieces. Each piece is stretched uniformly to three times its length and all the three stretched pieces are connected in parallel. Assuming that stretching of wires does not cause any change in density of their material, calculate total resistance of the combination described. *(C.B.S.E. 1997)* **[Ans.** 9 Ω**]**

28. Two wires is the same material having lengths in the ratio 1 : 2 and diameters in the ratio 2 : 3 are connected in series with an accumulator. Compute the ratio of potential differences across the two wires. *(H.S.S.C .E. 2000)* **[Ans.** 9 : 8**]**

29. A uniform copper wire of mass $2{\cdot}23 \times 10^{-3}$ kg carries a current of 1 A, when a potential difference of 1·7 V is applied across it. Calculate the length and area of cross-section of the wire. If the wire is uniformly stretched to double its length, calculate the new resistance. Given that density of copper is $8{\cdot}92 \times 10^3$ kg m^{-3} and resistivity is $1{\cdot}7 \times 10^{-8}$ Ω m. *(Roorkee 1995)* **[Ans.** 5 m ; 5×10^{-8} m^2 ; 6·8 Ω**]**

pe G. On Conductivity

30. Calculate the conductivity of a wire of length 200 cm, area of cross-section 2 cm^2 and of resistance 5×10^{-4} Ω. *(P.S.S.C.E. 1999)* **[Ans.** 2×10^{-7} S m^{-1}**]**

31. Calculate the conductance and conductivity of a wire of resistance 0·01 Ω, area of cross-section 10^{-4} m^2 and length 0·1 m. *(H.S.S.C.E. 2000, 1999 S)* **[Ans.** 100 S ; 10^5 S m^{-1}**]**

32. Calculate the electric field in a copper wire of cross-sectional area 2·0 mm^2 carrying a current of 1 A. The conductivity of copper = $6{\cdot}25 \times 10^7$ S m^{-1}. **[Ans.** 8×10^{-3} V m^{-1}**]**

pe H. On Variation of resistance with temperature

33. A wire has a resistance of 10·5 Ω at 21°C and 16·4 Ω at 147°C. Find the value of coefficient of resistance. **[Ans.** $4{\cdot}46 \times 10^{-3}$ K^{-1}**]**

34. At room temperature (27·0°C) , the resistance of a heating element is 100 Ω. What is the temperature of the element, if the resistance is found to be 117 Ω ? Given that the temperature coefficient of the material of the resistor is $1{\cdot}70 \times 10^{-4}$ °C^{-1}. *(Text Problem)* **[Ans.** 1027 °C**]**

35. The resistance of a tungsten filament at 150 °C is 133 Ω. What will be its resistance at 500 °C ? The temperature coefficient of resistance of tungsten at 0 °C is 0·0045 °C^{-1}. **[Ans.** 258 Ω**]**

pe I. On Colour code of Carbon resistance

36. The colour code of a carbon resistance is as shown in Fig. 1.68. What is the value of this resistance ? *(H.S.S.C.E. 2001]* **[Ans.** 24×10^7 Ω ± 10%**]**

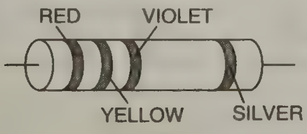


Fig. 1.68

37. A carbon resistor has three colours blue, yellow and red respectively. What will be the resistance ? *(H.S.S.C.E. 2000, 1998)* **[Ans.** 64×10^2 Ω**]**

38. A carbon resistor is marked in coloured bands of red, black, orange and silver. What is the resistance and tolerance value of the resistor ? *(C.B.S.E. 2002)* **[Ans.** 20×10^3 Ω ± 10%**]**

39. A carbon resistor is marked in coloured bands in the sequence blue, green, orange and gold. What is the resistance and tolerance value of the resistor ? *(C.B.S.E. 2002)* **[Ans.** 65×10^3 Ω + 5%**]**

40. A voltage of 200 V is applied across a colour coded carbon resistor with first, second and third rings of blue, black and yellow colours. What is the current flowing through the resistor ? *(H.S.S.C.E. 1999)* **[Ans.** $3{\cdot}33 \times 10^{-4}$ A**]**

41. Draw the colour code scheme of 42 kΩ + 10% carbon resistance. *(H.S.S.C.E. 1998 S)* **[Ans.** Yellow, red, orange and silver**]**

TYPE J. On series and parallel combination of resistances

42. The total resistance of two resistors, when connected in series is 9 Ω and when connected in parallel, their total resistance becomes 2 Ω. Find the value of each resistance. *(P.S.S.C.E. 2000)* **[Ans.** 6 Ω , 3 Ω**]**

43. Two resistances are in the ratio 1 : 4. If these are connected in parallel, their total resistance becomes 20 Ω. Find the value of each resistance. *(P.S.S.C.E. 2000)* **[Ans.** 25 Ω,100 Ω**]**

44. A parallel combination of three resistors takes a current of 5 A from a 20 V supply. If the two resistors are of 10 ohm and 8 ohm, final the value of third one. *(P.S.S.C.E. 1999)* **[Ans.** 40 Ω**]**

45. A parallel combination of three resistors takes a current of 7·5 A from a 30 V supply. If the two resistors are 10 ohm and 12 Ω, find the third one. *(P.S.S.C.E. 1999)* **[Ans.** 15 Ω**]**

46. Three resistances are connected to form the sides of a triangle ABC [Fig. 1.69]. The resistance of the side AB is 40 Ω, of the side BC 60 Ω and of the side CA 100 Ω. Calculate the effective resistance between the points A and B. *(I.I.T. 1975)* **[Ans.** 32 Ω**]**

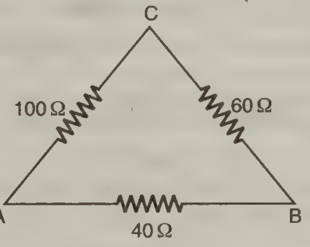


Fig. 1.69

47. The letter A is constructed with resistance wires of resistance 1 Ω each as shown in Fig. 1.70. Find the effective resistance between the points M and N. **[Ans.** 2·67 Ω**]**

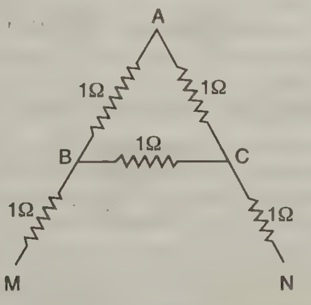


Fig. 1.70

48. The letter A is connected from a uniform wire of resistance 1 Ω cm^{-1}. The sides of the letter are each 20 cm long and cross-piece in the middle is 10 cm long, while the apex angle is 60°. Find the resistance of the letter between two ends of the legs. *(H.P.S.S.C.E. 1998)* **[Ans.** 26·67 Ω**]**

49. The letter A consists of a uniform wire of resistance 1 Ω cm^{-1}. The sides of the letter are 40 cm long and the crosspiece 10 cm long divides the sides in the ratio 1 : 3 from the apex. Find the resistance of the letter between the two ends of the legs. *(P.S.S.C.E. 1998 S)* **[Ans.** 66·67 Ω**]**

50. Calculate the equivalent resistance of the network shown in Fig. 1.71 between points A and B. **[Ans.** 12·73 Ω**]**

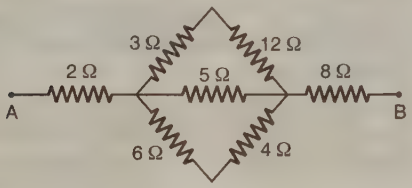


Fig. 1.71

51. What is the net resistance between points A and B in the circuit shown in Fig. 1.72 ? *(P.S.S.C.E. 1993)* **[Ans.** 5 Ω**]**

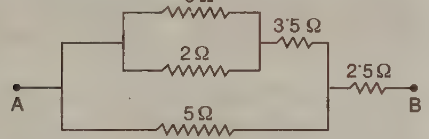


Fig. 1.72

52. Calculate the equivalent resistance of the network shown in Fig. 1.73 between points A and B. **[Ans.** 8 Ω**]**

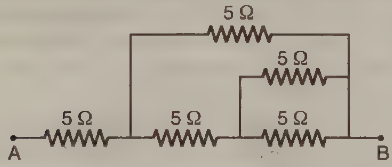


Fig. 1.73

53. Find the equivalent resistance of the network shown in Fig. 1.74 between points A and B. **[Ans.** 4·8 Ω**]**

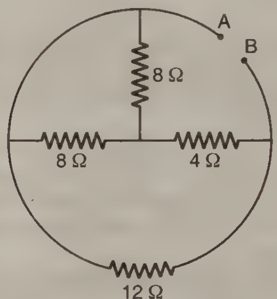


Fig. 1.74

54. Find the equivalent resistance of the network shown in 1.75 between points A and B. **[Ans.** 2·

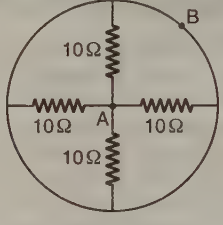


Fig. 1.75

55. Find the charge on the capacitor shown in Fig. 1.76. **[Ans.** 10

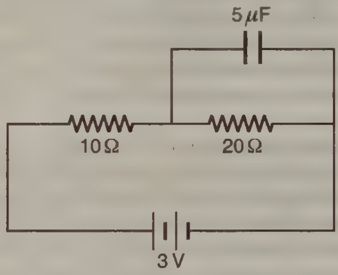


Fig. 1.76

56. Find the potential difference ($V_A - V_B$) between point and B in the network shown in Fig. 1.77. **[Ans.** 1

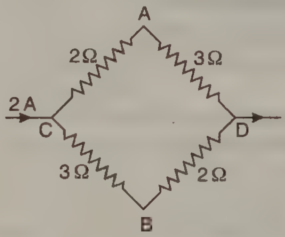


Fig. 1.77

57. Determine the total resistance in the circuit as show Fig. 1.78. Also determine the current in each resistor potential drop it. **[Ans.** 5 Ω, 10 A through 12 Ω, 2 through 6 Ω, 30 A through 4 Ω, 180 V across 3 Ω 120 V across each of 12, 6 and 4 Ω resist

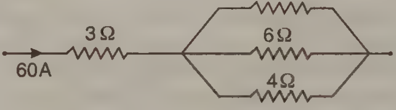


Fig. 1.78

58. In the circuit shown in Fig. 1.79, $R_1 = 100\ \Omega$, $R_2 = R_3 =$ Ω, $R_4 = 75\ \Omega$ and the battery is of e.m.f., E = 4·75 V. F the equivalent resistance of the circuit and current pass through the each resistor.

[Ans. 118·75 Ω ; $I(R_1) = 0{\cdot}04$ A ; $I(R_2) = 0{\cdot}015$
$I(R_3) = 0{\cdot}015$ A, $I(R_4) = 0{\cdot}0$

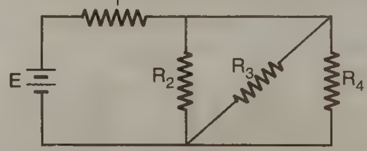


Fig. 1.79

9. Find the resistance of the network between (*a*) points A and B (*b*) points A and C shown in Fig. 1.80.
[Ans. (*a*) 27·5 Ω (*b*) 30 Ω]

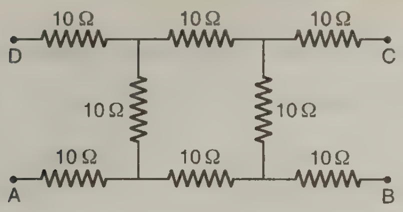


Fig. 1.80

0. Calculate the current shown by the ammeter A in the circuit diagram given in Fig. 1.81. (*C.B.S.E. 2000*)
[Ans. 3·6 A]

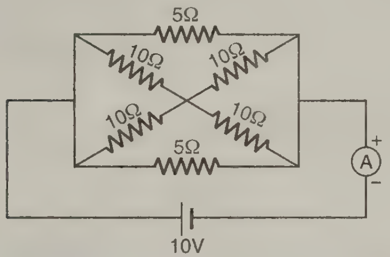


Fig. 1.81

e K. On Internal resistance of a cell

1. The e.m.f. of a cell is 1·5 V. On connecting a 14 Ω resistance across the cell, the terminal potential difference falls to 1·4 V. Calculate the internal resistance of the cell. (*H.S.S.C.E. 2001*) **[Ans.** 1 Ω]

2. Two poles of a cell of e.m.f. 1·5 V are connected to the ends of a 10 Ω coil. If the current in the circuit is 0·1 A, calculate the internal resistance of the coil. (*Karnataka, 1983 S*)
[Ans. 5 Ω]

3. The e.m.f. of Daniel cell is 1·09 V and internal resistance is 2 Ω. If the terminals of the cell are joined by a wire of resistance 18 Ω, find the potential difference recorded by a high resistance voltmeter also connected to the terminals of the cell. **[Ans.** 0·981 V]

4. In the circuit, shown in Fig. 1.82, the voltmeter reads 1·5 V, when the key is open. When the key is closed, the voltmeter reads 1·35 V and ammeter reads 1·5 A. Find the external resistance R and the internal resistance of the cell.
[Ans. 0·9 Ω ; 0·1 Ω]

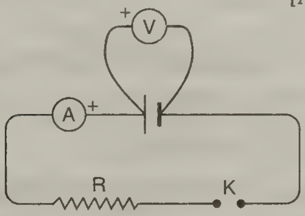


Fig. 1.82

5. If a battery of e.m.f. 6·6 V can supply a current of 3 A through a resistance of 1·8 Ω, what current does it supply through another resistance of 2·9 Ω. Also calculate internal resistance of the battery. **[Ans.** 2 A, 0·4 Ω]

6. Two resistances of 6 Ω and 12 Ω are connected in parallel. Their terminals are connected to a battery of e.m.f. 4·8 V and internal resistance of 2 Ω. Find the current supplied by the battery and the current through each resistance.
[Ans. 0·8 A, 0·533 A, 0·267 A]

7. A cell of e.m.f. 1·5 V and internal resistance 1 Ω sends a current through wires of resistance 6 Ω and 12 Ω connected in parallel. Find the current through each wire.
[Ans. 0·2 A, 0·1 A]

68. The potential difference across the terminals of a battery is 8·5 V, when a current of 3 A flows through it from its negative terminal to the positive terminal. When a current of 2 A flows through it in the opposite direction, the terminal potential difference is 11 V. Find the internal resistance of the battery and its e.m.f. **[Ans.** 0·5 Ω, 10 V]

69. The potential difference across the terminals of a 3 V battery is 3·6 V, when it is being charged by a current of 1·5 A. What is the internal resitance of the battery ?
[Ans. 0·4 Ω]

70. Find the magnitude of the current supplied by the battery in the circuit shown in Fig. 1.83. Also find the potential difference between the points P and Q. **[Ans.** 1 A, 1·5 V]

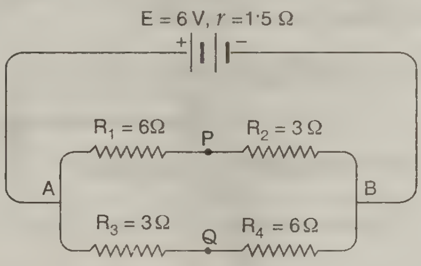


Fig. 1.83

Type L. On Grouping of cells

71. A galvanometer together with an unknown resistance in series is connected across two identical batteries each of 1·5 V. When the batteries are connected in series, the galvanometer records a current of 1 A and when the batteries are in parallel, the current is 0·6 A. What is the internal resistance of the battery ? (*I.I.T. 1973*)
[Ans. 1/3 Ω]

72. Two identical cells, each of e.m.f. 2 V are connected in parallel. This combination is connected to a 5 Ω resistor. If the terminal voltage across the cells in the closed circuit is 1·5 V, calculate the internal resistance of each cell.
(*C.B.S.E. 1998 S*) **[Ans.** 3·33 Ω]

73. Two identical cells of e.m.f. 1·5 V each joined in parallel provide supply to an external circuit consisting of two resistors of 17 Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cells to be 1·4 V. What is the internal resistance of each cell ?
(*Text Problem*) **[Ans.** 1·214 Ω]

74. Three identical cells, each of e.m.f. 2 V and unknown internal resistance are connected in parallel. This combination is connected to a 5 ohm resistor. If the terminal voltage across the cell is 1·5 volt, what is the internal resistance of each cell. (*C.B.S.E. 1999*)
[Ans. 5 Ω]

75. Two cells each of e.m.f. 1·1 V and internal resistance 2 Ω are joined in parallel with a resistance of 10 Ω. Find the current through the resistance and through each cell.
[Ans. 0·1 A, 0·05 A]

MISCELLANEOUS PROBLEMS

76. The earth's surface has a negative surface charge density of 10^{-9} C m^{-2}. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of substaining atmospheric electric field ; how much time would be required to neutralise the earth's surface ? (This never happens in practice, because there is

a mechanism to replenish electric charges namely the continual thunder storms and lightning in different parts of the globe). Radius of the earth = $6{\cdot}37 \times 10^6$ m.

(*Text Problem*) [**Ans.** 283·3 s]

77. Find the equivalent resistance of the network shown in Fig. 1.84 between points A and B, when (*a*) the key K is open and (*b*) the key K is closed. [**Ans.** (*a*) 2·5 Ω (*b*) 2·4 Ω]

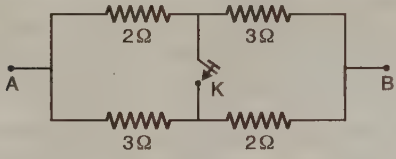


Fig. 1.84

78. In the circuit shown in Fig. 1.85, determine the potential difference across the capacitor. [**Ans.** 12 V]

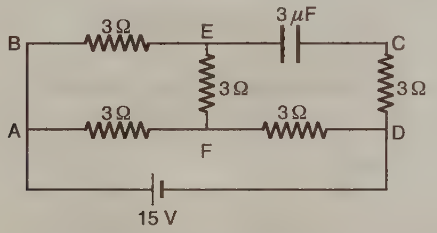


Fig. 1.85

79. In the circuit shown in Fig. 1.86, find the pote difference between points A and B. Given that E_1 = $E_2 = 8$ V, $R_1 = 3\ \Omega$ and $R_2 = 1\ \Omega$ [**Ans.**

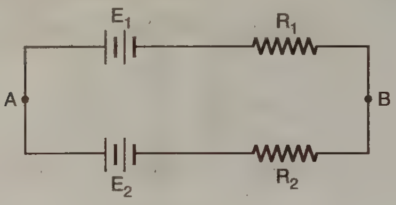


Fig. 1.86

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $$I = \frac{ne}{t} = \frac{2{\cdot}25 \times 10^{20} \times 1{\cdot}6 \times 10^{-19}}{60} = \mathbf{0{\cdot}6\ A}$$

2. Proceed as in solved problem no. 1.01.

3. $$I = \frac{(n^+ + n^-) \times e}{t}$$
$$= \frac{(6{\cdot}1 \times 10^{16} + 4{\cdot}6 \times 10^{16}) \times 1{\cdot}6 \times 10^{-19}}{2}$$
$$= \mathbf{8{\cdot}56 \times 10^{-3}\ A}$$

4. The electron crosses a point on the circle once in every revolution. If v is number of revolutions made by the electron in 1 s, then its motion along circle corresponds to a current,

$$I = e\,v = e \times \frac{1}{T} = \frac{e}{2\pi r / v} = \frac{e\,v}{2\pi r}$$
$$= \frac{1{\cdot}6 \times 10^{-19} \times 3{\cdot}6 \times 10^6}{2\pi \times 0{\cdot}15} = \mathbf{6{\cdot}1 \times 10^{-13}\ A}$$

5. Here, $q = a\,t^2 + b\,t + c$

(*a*) Dimensions of each factor on R.H.S. of the above relation are that of charge *i.e.* current × time or [I T].

$$\therefore [a] = \left[\frac{q}{t^2}\right] = \frac{[\text{I T}]}{[\text{T}^2]} = [\text{I T}^{-1}],\ [b] = \left[\frac{q}{t}\right] = \frac{[\text{I T}]}{[\text{T}]} = [\text{I}]$$

and $[c] = [q] = [\text{I T}]$

(*b*) Here, $a = 3$, $b = 5$ and $c = 2$

Now, $I = \dfrac{dq}{dt} = \dfrac{d}{dt}(a\,t^2 + b\,t + c) = 2\,a\,t + b$

$\therefore$ I (at $t = 3$ s) $= 2 \times 3 \times 3 + 5 =$ **23 A**

6. Proceed as in solved problem no. 1.03,

7. Proceed as in solved problem no. 1.03

8. Diameter of wire, D = 1·0 mm = 10^{-3} m

Therefore, area of cross-section of wire,

$$A = \frac{\pi D^2}{4} = \frac{\pi \times (10^{-3})^2}{4} = 7{\cdot}854 \times 10^{-7}\ \text{m}^2$$

Now, proceed as in solved problem no. 1.03

9. Proceed as in problem no. 8.

10. Mass of silver wire,

$$m = A \times l \times \rho = 3{\cdot}14 \times 10^{-6} \times 1 \times 10{\cdot}5 \times 10^3$$
$$= 3{\cdot}297 \times 10^{-2}\ \text{kg} = 32{\cdot}97\ \text{g}$$

Therefore, number of atoms in silver wire

$$= \frac{6{\cdot}02 \times 10^{23}}{108} \times 32{\cdot}97 = 1{\cdot}838 \times 10^{23}$$

Volume of wire = A l = 1× $3{\cdot}14 \times 10^{-6} = 3{\cdot}14 \times 10^{-6}$ m

Assuming that there is one free electron correspondi each silver atom, number of electrons per unit volur the silver wire,

$$n = \frac{1{\cdot}838 \times 10^{23}}{3{\cdot}14 \times 10^{-6}} = 5{\cdot}854 \times 10^{28}\ \text{m}^{-3}$$

Now, $v_d = \dfrac{I}{n\,e\,A}$

$$= \frac{10}{5{\cdot}854 \times 10^{28} \times 1{\cdot}6 \times 10^{-19} \times 3{\cdot}14 \times 10^{-6}}$$
$$= \mathbf{3{\cdot}4 \times 10^{-4}\ m\ s^{-1}}$$

11. $$E = \frac{V}{l} = \frac{6}{0{\cdot}12} = 50\ \text{V m}^{-1}$$

Now, $v_d = \mu\,E = 5{\cdot}6 \times 10^{-6} \times 50 = \mathbf{2{\cdot}8 \times 10^{-4}\ m\ s^{-1}}$

12. $$E = \frac{V}{l} = \frac{3}{0{\cdot}24} = 12{\cdot}5\ \text{V m}^{-1}$$

Now, $v_d = \mu\,E = 4{\cdot}8 \times 10^{-6} \times 12{\cdot}5 = 6 \times 10^{-5}\ \text{m s}^{-1}$

Also, $v_d = \dfrac{I}{n\,A\,e}$ or $n = \dfrac{I}{A\,e\,v_d}$

or $$n = \frac{1}{1{\cdot}2 \times 10^{-6} \times 1{\cdot}6 \times 10^{-19} \times 6 \times 10^{-5}}$$
$$= \mathbf{8{\cdot}68 \times 10^{28}\ m^{-3}}$$

13. $$j = \frac{I}{A} = \frac{2{\cdot}4}{1{\cdot}5 \times 10^{-6}} = \mathbf{1{\cdot}6 \times 10^6\ A\ m^{-2}}$$

Now, $v_d = \dfrac{j}{ne} = \dfrac{1\cdot6\times10^{6}}{8\times10^{28}\times1\cdot6\times10^{-19}}$

$= \mathbf{1{\cdot}25 \times 10^{-4}\ m\ s^{-1}}$

14. Proceed as in solved problem no. 1.09.

15. (*a*) Area of cross-section of aluminium wire,

$$A = \frac{\pi D^2}{A} = \frac{\pi\times(0\cdot24)^2}{4} = 4{\cdot}524\times10^{-2}\ \text{cm}^2$$

$$= 4{\cdot}524\times10^{-6}\ \text{m}^2$$

Now, $j = \dfrac{I}{A} = \dfrac{10}{4\cdot524\times10^{-6}} = \mathbf{2{\cdot}21\times10^{6}\ A\ m^{-2}}$

(*b*) Area of cross-section of copper wire,

$$A = \frac{\pi D^2}{4} = \frac{\pi\times(0\cdot16)^2}{4} = 2{\cdot}01\times10^{-2}\ \text{cm}^2$$

$$= 2{\cdot}01\times10^{-6}\ \text{m}^2$$

Now, $v_d = \dfrac{I}{neA}$

$$= \frac{10}{8\cdot4\times10^{28}\times1\cdot6\times10^{-19}\times2\cdot01\times10^{-6}}$$

$= \mathbf{3{\cdot}7\times10^{-4}\ m\ s^{-1}}$

16. $I = \dfrac{V}{R} = \dfrac{10}{10^3} = 10^{-2}\ \text{A}$

Now, $q = I\,t$ or $n\,e = I\,t$

or $n = \dfrac{I\,t}{e} = \dfrac{10^{-2}\times5\times60}{1\cdot6\times10^{-19}} = \mathbf{1{\cdot}875\times10^{19}}$

17. Proceed as in solved problem no. 1.11.

18. Proceed as in solved problem no. 1.11.

19. Proceed as in solved problem no. 1.11.

20. Number of turns of rheostat wire, $n = 100$

Diameter of a turn of rheostat wire,

$$D = 3\ \text{cm} = 0{\cdot}03\ \text{m}$$

Therefore, length of rheostat wire,

$$l = n \times \text{circumference of one turn} = n\times\pi D$$
$$= 100\times\pi\times0{\cdot}03\ \text{m}$$

Area of cross-section of wire,

$$A = \pi r^2 = \pi\times(0{\cdot}4\times10^{-3})^2\ \text{m}^2$$

Therefore, maximum resistance, which rheostat can introduce,

$$R = \rho\frac{l}{A} = \frac{4\cdot2\times10^{-7}\times100\times\pi\times0\cdot03}{\pi\times(0\cdot4\times10^{-3})^2} = \mathbf{7{\cdot}875\ \Omega}$$

21. Now, $R = \rho\dfrac{l}{A}$ or $A = \rho\dfrac{l}{R}$

or $A = \dfrac{1\cdot68\times10^{-8}\times10\times10^{3}}{10} = 1\cdot68\times10^{-5}\ \text{m}^2$

Mass of copper required, m = volume × density

or $m = A\,l\times\text{density}$

$= 1{\cdot}68\times10^{-5}\times10\times10^{3}\times8{\cdot}9\times10^{3} = \mathbf{1495{\cdot}2\ kg}$

22. Now, $\rho = \dfrac{m}{ne^2\tau}$ or $\tau = \dfrac{m}{ne^2\rho}$

or $\tau = \dfrac{9\cdot1\times10^{-31}}{8\cdot4\times10^{28}\times(1\cdot6\times10^{-19})^2\times1\cdot7\times10^{-8}}$

$= \mathbf{2{\cdot}49\times10^{-14}\ s}$

23. Proceed as in solved problem no. 1.13.

The resistivity of wire will remain unchanged, as it depends on the nature of material of the wire only.

24. Proceed as in solved problem no. 1.13.

25. Proceed as in solved problem no. 1.13.

26. Now, $R = \rho\dfrac{l}{\pi r^2}$ ...(*i*)

When radius is decreased by a factor n, new resistance of the wire

$$R' = \rho\frac{l'}{\pi(r/n)^2} \qquad ...(ii)$$

As the volume of wire must remain the same,

$$l\times\pi r^2 = l'\times\pi\left(\frac{r}{n}\right)^2 \quad \text{or} \quad l' = n^2 l$$

Substituting for l' in equation (*ii*), we have

$$R' = \rho\frac{n^2 l}{\pi(r/n)^2} = n^4\times\rho\frac{l}{\pi r^2} = \mathbf{n^4 R}$$

27. Resistance of each piece of the wire, $R = \dfrac{9}{3} = 3\,\Omega$

Let l be length and A, the area of cross-section of each piece of the wire. Then,

$$\rho\frac{l}{A} = 3 \qquad ...(i)$$

When wire is stretched to three times to length *i.e.* $3\,l$, its area will decrease to A/3. Therefore,
new resistance of a piece of wire,

$$R' = \rho\frac{3l}{A/3} = 9\times\rho\frac{l}{A} = 9\times3 = 27\,\Omega$$

Let R_p' be the equivalent resistance, when all the three pieces are connected in paralle. Then,

$$\frac{1}{R_p'} = \frac{1}{27}+\frac{1}{27}+\frac{1}{27} = \frac{1}{9} \quad \text{or} \quad R_p' = 9\,\Omega$$

28. Let l and $2\,l$ be length of the two wires and 2 D and 3 D be their diameters. If ρ is resistivity of their material, then

$$R_1 = \rho\frac{l}{\pi(2D)^2/4} = \frac{\rho l}{\pi D^2}$$

and $R_2 = \rho\dfrac{2l}{\pi(3D)^2/4} = \dfrac{8\rho l}{9\pi D^2}$

As the two wires are connected in series with an accumulator, same current will flow through the two wires. Hence, potential difference across the two wires will be proportional to their resistances. Thus,

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \mathbf{\frac{9}{8}}$$

29. Now, $R = \rho\dfrac{l}{A}$ or $\dfrac{V}{I} = \rho\dfrac{l}{A}$

or $\dfrac{l}{A} = \dfrac{V}{I\rho} = \dfrac{1\cdot7}{1\times1\cdot7\times10^{-8}} = 10^{8}$ ...(*i*)

Also, mass of wire = volume × density

or $2{\cdot}23\times10^{-3} = A\,l\times8{\cdot}92\times10^{3}$

or $A\,l = \dfrac{2\cdot23\times10^{-3}}{8\cdot92\times10^{3}} = 2\cdot5\times10^{-7}$ ...(*ii*)

Solving equations (*i*) and (*ii*), we get,

$$l = 5\ \text{m} \quad \text{and} \quad \mathbf{A = 5\times10^{-8}\ m^2}$$

When the wire is stretched to double its length, it can be found that its resistance will become four times.
Therefore, new resistance,

$$R' = \left(\frac{V}{I}\right)\times4 = \frac{1\cdot7}{1}\times4 = \mathbf{6{\cdot}8\ \Omega}$$

30. Proceed as in solved problem no. 1.14.

31. Conductance, $G = \dfrac{1}{R} = \dfrac{1}{0\cdot01} = \mathbf{100\ S}$

For finding conductivity, proceed as in solved problem 1.14.

32. Now, $\rho=\frac{1}{\sigma}=\frac{1}{6\cdot25\times10^{7}}=1\cdot6\times10^{-8}\ \Omega\,\text{m}$

∴ $R=\rho\frac{l}{A}=\frac{1\cdot6\times10^{-8}\times l}{2\cdot0\times10^{-6}}=8\times10^{-3}\,l$

Now, $V=IR=1\times8\times10^{-3}\,l=8\times10^{-3}\,l$

Hence, $E=\frac{V}{l}=\mathbf{8\times10^{-3}\ V\,m^{-1}}$

33. Proceed as in solved problem no. 1.16.

34. Now, $R_2=R_1\,[1+\alpha\,(\theta_2-\theta_1)]$

or $117=100\,[1+1{\cdot}7\times10^{-4}\,(\theta_2-27)]$

or $\theta_2=\frac{117-100}{100\times1\cdot7\times10^{-4}}+27=\mathbf{1027\ ^\circ C}$

35. Now, $R_\theta=R_0\,(1+\alpha\,\theta)$

∴ $R_{150}=R_0\,(1+0{\cdot}0045\times150)$ ...(*i*)

and $R_{500}=R_0\,(1+0{\cdot}0045\times500)$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$$R_{500}=R_{150}\times\frac{1+0\cdot0045\times500}{1+0\cdot0045\times150}=258\ \Omega$$

36. Proceed as in solved problem no. 1.19.

37. Proceed as in solved problem no. 1.19.

38. Proceed as in solved problem no. 1.19.

39. Proceed as in solved problem no. 1.19.

40. To find R, proceed as in solved problem no. 1.19 and then find current through the resistor.

41. Proceed as in solved problem no. 1.20.

42. Proceed as in solved problem no. 1.23.

43. Let R and 4 R be the values of two resistances.

Now, $R_p=\frac{R\times4R}{R+4R}$ or $20=0{\cdot}8\,R$

or $\mathbf{R=25\ \Omega}$ and $4\,R=4\times25=\mathbf{100\ \Omega}$

44. Now, $R_p=\frac{20}{5}=4\,\Omega$

Let R be the unknown resistance. Then,

$\frac{1}{4}=\frac{1}{R}+\frac{1}{10}+\frac{1}{8}$ or $\frac{1}{R}=\frac{1}{4}-\frac{1}{10}-\frac{1}{8}=\frac{1}{40}$

or $\mathbf{R=40\ \Omega}$

45. Proceed as in problem no. 44.

46. Let $R_1=40\ \Omega$, $R_2=60\ \Omega$ and $R_3=100\ \Omega$

Between points A and B, a series combination of R_2 and R_3 is connected in parallel to R_1. Let R_s be resistance of series combination of R_2 and R_3. Then,

$R_s=R_2+R_3=60+100=160\ \Omega$

Hence, the equivalent resistance between points A and B,

$$R=\frac{R_s\times R_1}{R_s+R_1}=\frac{160\times40}{160+40}=32\ \Omega$$

47. By proceeding as in problem 46, find the resistance R′ of the triangular part ABC. It can be obtained that

$R'=0{\cdot}67\ \Omega$

Hence, resistance between points M and N,

$R=1+R'+1=1+0{\cdot}67+1=\mathbf{2{\cdot}67\ \Omega}$

48. Refer to Fig. 1.70. Since apex angle is 60°, Δ ABC is an equilateral triangle *i.e.* AB = BC = AC = 10 cm.

Also, BM = CN = 20 − 10 = 10 cm

Further, as the wire has resistance of 1 Ω cm^{-1}, the resistance of each of the parts AB, BC, AC, BM and CN is 10 Ω.

Now, proceed as in problem no. 47 to find the effective resistance of the letter between two ends of the legs.

49. Refer to Fig 1.70. Since cross-piece divides the sides of the letter in the ratio 1 : 3,

AB = AC = 10 cm and BM = CN = 30 cm

Since the wire has resistance of 1 Ω cm^{-1}, the resistanc of the parts AB, BC, AC, BM and CN are 10 Ω, 10 10 Ω, 30 Ω and 30 Ω respectively.

Now, proceed as in problem no. 47 to find the resistanc of the letter between the two ends of the legs.

50. Let R_1 be resistance of the series combination of 3 Ω an 12 Ω resistances and R_2 that of 6 Ω and 4 Ω resistances.

Then, $R_1=3+12=15\ \Omega$ and $R_2=6+4=10\ \Omega$

If R_p is resistance of parallel combination of R_1, R_2 an 5 Ω resistance, then

$$\frac{1}{R_p}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{5}=\frac{1}{15}+\frac{1}{10}+\frac{1}{5}=\frac{11}{30}$$

or $R_p=\frac{30}{11}=2\cdot73\,\Omega$

Therefore, equivalent resistance of network between points A and B,

$R=2+R_p+8=2+2{\cdot}73+8=\mathbf{12{\cdot}73\ \Omega}$

51. The given network is equivalent to the network shown i Fig. 1.87 :

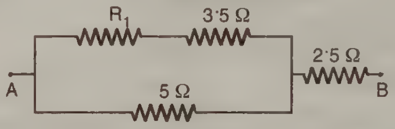


Fig. 1.87

Since R_1 is resistance of the parallel combination o resistors of 6 Ω and 2 Ω,

$$R_1=\frac{6\times2}{6+2}=1\cdot5\,\Omega$$

The network shown above is now equivalent to the network shown in Fig. 1.88 :

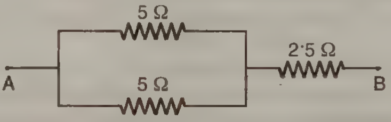


Fig. 1.88

Therefore, net resistance between points A and B,

$$R=\frac{5\times5}{5+5}+2\cdot5=2\cdot5+2\cdot5=5\,\Omega$$

52. The given network is equivalent to the network shown i Fig. 1.89 :

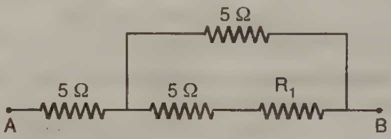


Fig. 1.89

Since R_1 is resistance of the parallel combination of tw resistors of 5 Ω each,

$$R_1=\frac{5\times5}{5+5}=2\cdot5\,\Omega$$

The network shown above is now equivalent to the network shown in Fig. 1.90 :

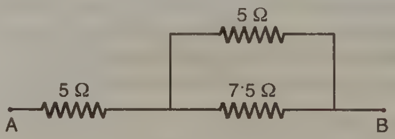


Fig. 1.90

Therefore, equivalent resistance of the network between points A and B,

$$R = 5 + \frac{5 \times 7 \cdot 5}{5 + 7 \cdot 5} = 8\ \Omega$$

53. The given network is equivalent to the network shown in Fig. 1.91 :

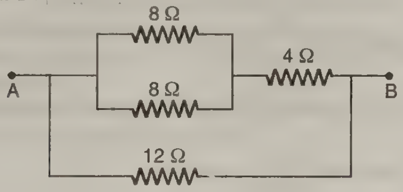


Fig. 1.91

By proceeding as in problem no. 51, it can be obtained that equivalent resistance of the network is **4·8 Ω**.

54. The given network is equivalent to the network shown in Fig. 1.92 :

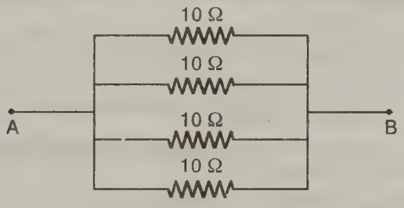


Fig. 1.92

Therefore, equivalent resistance of the network between points A and B is given by

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10} \quad \text{or} \quad R = \mathbf{2{\cdot}5\ \Omega}$$

55. Current in the circuit, $I = \dfrac{3}{10 + 20} = 0 \cdot 1\ A$

Potential difference across 20 Ω resistor,

$V = 20 \times 0{\cdot}1 = 2\ V$

Therefore, charge on capacitor,

$q = C\,V = 5 \times 2 = \mathbf{10\ \mu C}$

56. The resistance of both the branches CAD and CBD of the circuit is same *i.e.* equal to 5 Ω. Therefore, current of 2 A at point C will divide equally through the two branches.

Hence, $V_C - V_A = 2 \times 1 = 2\ V$

and $V_C - V_B = 3 \times 1 = 3\ V$

Now, $V_A - V_B = (V_C - V_B) - (V_C - V_A) = 3 - 2 = \mathbf{1\ V}$

57. Let R_p be resistance of the parallel combination of 12 Ω, 6 Ω and 4 Ω resistors. Then,

$$\frac{1}{R_p} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2} \quad \text{or} \quad R_p = 2\ \Omega$$

Therefore, total resistance of the given circuit,

$R = 3 + R_p = 3 + 2 = 5\ \Omega$

Potential drop across 3 Ω resistor = 3 × 60 = **180 V**

Potential drop across the parallel combination of resistors of 12 Ω, 6 Ω and 4 Ω

$= R_p \times 60 = 2 \times 60 = 120\ V$

Therefore, potential drop across each of the three resistors (*i.e.* 12 Ω, 6 Ω and 4 Ω) is **120 V**.

Current through 12 Ω resistor $= \dfrac{120}{12} = \mathbf{10\ A}$,

current through 6 Ω resistor $= \dfrac{120}{6} = \mathbf{20\ A}$

and current through 4 Ω resistor $= \dfrac{120}{4} = \mathbf{30\ A}$

58. The resistors R_2, R_3 and R_4 are in parallel. Their effective resistance is given by

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{50} + \frac{1}{50} + \frac{1}{75} = \frac{8}{150}$$

or $R_p = 18{\cdot}75\ \Omega$

Therefore, equivalent resistance of the network,

$R = R_1 + R_p = 100 + 18{\cdot}75 = \mathbf{118{\cdot}75\ \Omega}$

Current through R_1

= current through the main circuit from battery

or $I\,(R_1) = \dfrac{E}{R} = \dfrac{4 \cdot 75}{118 \cdot 75} = \mathbf{0{\cdot}04\ A}$

Potential drop across the parallel combination of R_2, R_3 and R_4,

$V = E -$ potential drop across R_1

$= 4{\cdot}75 - 100 \times 0{\cdot}04 = 0{\cdot}75\ V$

$\therefore \quad I\,(R_2) = I\,(R_3) = \dfrac{V}{50} = \dfrac{0 \cdot 75}{50} = \mathbf{0{\cdot}015\ A}$

and $\quad I\,(R_3) = \dfrac{V}{75} = \dfrac{0 \cdot 75}{75} = \mathbf{0{\cdot}01\ A}$

59. (*a*) Between points A and B, the effective part of the network is as shown in Fig. 1.93 :

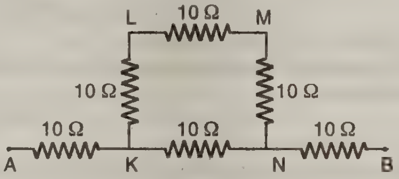


Fig. 1.93

The branch KLMN (having resistance of 10 + 10 + 10 *i.e.* 30 Ω) is parallel to the branch KN (having resistance 10 Ω). Therefore, resistance of the network between points A and B,

$$R = 10 + \frac{30 \times 10}{30 + 10} + 10 = 10 + 7 \cdot 5 + 10$$

$= \mathbf{27{\cdot}5\ \Omega}$

(*b*) Between points A and C, the effective part of the network is as shown in Fig. 1.94 :

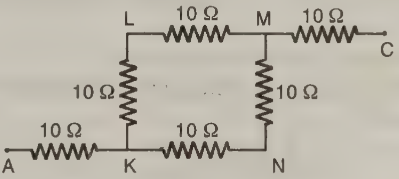


Fig. 1.94

The branch KLM (having resistance of 10 + 10 *i.e.* 20 Ω) is in parallel to the branch KNM (having resistance of 10 + 10 *i.e.* 20 Ω).

Therefore, resistance of the network between points A and C,

$$R = 10 + \frac{20 \times 20}{20 + 20} + 10 = 10 + 10 + 10 = \mathbf{30\ \Omega}$$

60. Proceed as in solved problem no. 1.30.

61. $r = \left(\dfrac{E}{V} - 1\right) R = \left(\dfrac{1 \cdot 5}{1 \cdot 4} - 1\right) \times 14 = \mathbf{1\ \Omega}$

62. $I = \dfrac{E}{R + r} \quad$ or $\quad 0 \cdot 1 = \dfrac{1 \cdot 5}{10 + r} \quad$ or $\quad r = 5\ \Omega$

63. Proceed as in solved problem no. 1.33.

64. $E = 1{\cdot}5\ V$; $V = 1{\cdot}35\ V$; $I = 1{\cdot}5\ A$

Now, $V = I\,R$ or $1{\cdot}35 = 1{\cdot}5 \times R$ or $R = 0{\cdot}9\ \Omega$

Also, $r = \left(\dfrac{E}{V} - 1\right) R = \left(\dfrac{1 \cdot 5}{1 \cdot 35} - 1\right) \times 0 \cdot 9 = \mathbf{0{\cdot}1\ \Omega}$

65. Let r be internal resistance of the cell. Then, current in the circuit,

$$I = \frac{E}{R + r}$$

In first case : $3 = \frac{6 \cdot 6}{1 \cdot 8 + r}$

or $r = \frac{6 \cdot 6}{3} - 1 \cdot 8 = \mathbf{0 \cdot 4\ \Omega}$

In second case : $I = \frac{6 \cdot 6}{2 \cdot 9 + 0 \cdot 4} = \mathbf{2\ A}$

66. $R_p = \frac{6 \times 12}{6 + 12} = 4\,\Omega$

Now, $I = \frac{E}{R_p + r} = \frac{4 \cdot 8}{4 + 2} = \mathbf{0{\cdot}8\ A}$

Let I_1 be current through 6 Ω resistance. Then, current through 12 Ω resistance is $I - I_1$.

Now, $I_1 \times 6 = (I - I_1) \times 12$ or $I_1 \times 6 = (0{\cdot}8 - I_1) \times 12$

or $I_1 = \mathbf{0{\cdot}533\ A}$

Current through 12 Ω resistance

$= I - I_1 = 0{\cdot}8 - 0{\cdot}533 = \mathbf{0{\cdot}267\ A}$

67. Proceed as in problem no. 66.

68. ***When current flows from negative to positive terminal of the battery :***

$V = E - I\,r$ or $8{\cdot}5 = E - 3 \times r$

or $E - 3\,r = 8{\cdot}5$ ...(*i*)

When current flows from positive to negative terminal of the battery :

$V = E + I\,r$ (potential acorss r aids the e.m.f.)

or $11 = E + 2\,r$ ...(*ii*)

Solving equations (*i*) and (*ii*), we get

$r = \mathbf{0{\cdot}5\ \Omega}$ and $E = \mathbf{10\ V}$

69. During charging, potential drop across r aids the e.m.f.

$\therefore$ $V = E + I\,r$ or $3{\cdot}6 = 3 + 1{\cdot}5 \times r$

or $r = \mathbf{0{\cdot}4\ \Omega}$

70. Let R_s be resistance of the series combination of R_1 and R_2.

Then, $R_s = R_1 + R_2 = 6 + 3 = 9\ \Omega$

Further, let R_s' be resistance of the series combination of R_3 and R_4.

Then, $R_s = R_3 + R_4 = 3 + 6 = 9\ \Omega$

If R_p is resistance of the parallel combination of R_s and R_s', then

$$R_p = \frac{R_s \times R_s'}{R_s + R_s'} = \frac{9 \times 9}{9 + 9} = 4 \cdot 5\,\Omega$$

If I is current in the main circuit from the battery, then

$$I = \frac{E}{R_p + r} = \frac{6}{4 \cdot 5 + 1 \cdot 5} = \mathbf{1\ A}$$

Since resistances of the branches APB and AQB are equal, current equal to $I/2$ *i.e.* 0·5 A will flow through each branch.

Now, $V_A - V_P = V_A - 6 \times 0{\cdot}5 = V_A - 3$

and $V_A - V_Q = V_A - 3 \times 0{\cdot}5 = V_A - 1{\cdot}5$

Now, $V_P - V_Q = (V_A - V_Q) - (V_A - V_P)$

$= (V_A - 1{\cdot}5) - (V_A - 3) = \mathbf{1{\cdot}5\ V}$

71. Here, $E_1 = E_2 = 1{\cdot}5\ V$

When batteries are connected in series :

$I = \frac{E_1 + E_2}{R + 2r}$ or $I = \frac{1 \cdot 5 + 1 \cdot 5}{R + 2r}$

or $R + 2\,r = 3$ ...(*i*)

When batteries are connected in parallel :

$I = \frac{E}{R + r/2}$ or $0 \cdot 6 = \frac{1 \cdot 5}{R + r/2}$

or $R + r/2 = 2{\cdot}5$ ...(*ii*)

Solving equations (*i*) and (*ii*), we get

$$r = \frac{1}{3}\,\Omega$$

72. Let r be internal resistance of each cell.

When cells are connected in parallel :

Effective e.m.f. of two cells, E = 2 V

Total internal resistance of the cells, $r' = r/2$

Now, $r' = \left(\frac{E}{V} - 1\right) R$

or $\frac{r}{2} = \left(\frac{2}{1 \cdot 5} - 1\right) \times 5$ or $r = \mathbf{3{\cdot}33\ \Omega}$

73. Here, $R = \frac{17 \times 17}{17 + 17} = 8 \cdot 5\,\Omega$

$\therefore$ $r' = \left(\frac{E}{V} - 1\right) R = \left(\frac{1 \cdot 5}{1 \cdot 4} - 1\right) \times 8 \cdot 5 = \mathbf{1{\cdot}214\ \Omega}$

74. When three cells are connected in parallel,

total internal resistance = $r/3$ and total e.m.f., E = 2 \

$\therefore$ $\frac{r}{3} = \left(\frac{E}{V} - 1\right) R = \left(\frac{2}{1 \cdot 5} - 1\right) \times 5$ or $r = \mathbf{5\ \Omega}$

75. $I = \frac{E}{R + r/2} = \frac{1 \cdot 1}{10 + 2/2} = \mathbf{0{\cdot}1\ \Omega}$

As the two cells are identical, each will supply a curre of 0·1/2 *i.e.* **0·05 A**

76. Here, I = 1800 A, $\sigma = 10^{-9}\ C\ m^{-2}$

Total charge on the surface of earth,

$q = 4\,\pi\,R^2\,\sigma = 4\,\pi \times (6{\cdot}37 \times 10^6)^2 \times 10^{-9}$

If the charge on the surface of earth neutralises in time then resulting current,

$I = \frac{q}{t}$ or $t = \frac{q}{I}$

or $t = \frac{4\,\pi \times (6 \cdot 37 \times 10^6)^2 \times 10^{-9}}{1800} = \mathbf{283{\cdot}3\ s}$

77. ***(a) When the key K is open :***

Each row has a resistance of 2 + 3 *i.e.* 5 Ω.

Since the two rows are in parallel, equivalent resistance the network,

$$R = \frac{5 \times 5}{5 + 5} = \mathbf{2{\cdot}5\ \Omega}$$

(b) When the key K is closed :

The 2 Ω and 3 Ω resistors on the left of the key are parallel. Their equivalent resistance is given by

$$R_p = \frac{2 \times 3}{2 + 3} = \mathbf{1{\cdot}2\ \Omega}$$

Similarly, 2 Ω and 3 Ω resistors on the right of the key a in parallel and equivalent to 1·2 Ω.

Hence, the equivalent resistance of the network,

$R = 1{\cdot}2 + 1{\cdot}2 = \mathbf{2{\cdot}4\ \Omega}$

78. In the steady state (when capacitor is fully charged), r current flows through the branch ECD.

The given network is, therefore, equivalent to the circu shown in Fig. 1.95 :

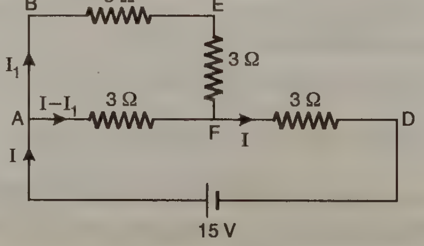


Fig. 1.95

If R is equivalent resistance of the above circuit, then

$$R = \frac{6\times 3}{6+3} + 3 = 5\,\Omega$$

Therefore, current in the main circuit, $I = \frac{15}{5} = 3$ A

Current through the arm BEF, $I_1 = \frac{I\times 3}{6+3} = \frac{3\times 3}{6+3} = 1$ A

Now, a current of 3 – 1 *i.e.* 2 A will flow through the branch AF and a current of 2 + 1 *i.e.* 3 A will flow through the branch FD.

Since no current flows through the branch ECD, potential difference across the capacitor

= potential difference between points E and D

= potential drop across EF + potential drop across FD

$= 3 \times 1 + 3 \times 3 = \mathbf{12\ V}$

79. Net e.m.f., $E = E_1 + (-E_2) = 20 + (-8) = 12$ V

Total resistance, $R = R_1 + R_2 = 3 + 1 = 4\,\Omega$

Net current in the circuit, $I = \frac{E}{R} = \frac{12}{4} = 3$ A

Now, $V_A - V_B = E_1 -$ potential drop across R_1

$= 20 - 3 \times 3 = \mathbf{11\ V}$

Aliter. $V_A - V_B = E_2 +$ potential drop across R_2

$= 8 + 1 \times 3 = \mathbf{11\ V}$

CHAPTER

2

Electrical Measurements

2.01. KIRCHHOFF'S LAWS

Ohm's law is useful in dealing with simple electrical circuits only. The study of the circuits containing more than one source of e.m.f. becomes difficult with the help of Ohm's law. In 1842, Kirchhoff put forward the following two laws to study complicated electrical circuits :

Kirchhoff's first law. *It states that the algebraic sum of the currents meeting at a point in an electrical circuit is always zero.* It is also called ***junction rule.***

This law is based upon the fact that in an electrical circuit, a point can neither act as a source of charge nor the charge can accumulate at that point.

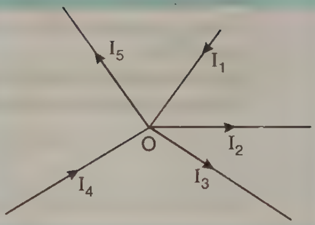
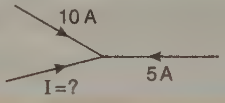


Fig. 2.01

Self-test Questions

Calculate the unknown current in the figure given below :



Explanation. Consider a point O in an electrical circuit at which currents I_1, I_2, I_3, I_4 and I_5 are flowing through five conductors in the directions as shown in Fig. 2.01. Let us adopt the following sign convention :

The currents flowing in a conductor towards the junction of conductors is taken as positive and the current flowing away from the junction is taken as negative.

Likewise, currents I_1 and I_4 flowing towards the point O are positive, while the currents I_2, I_3 and I_5 are negative.

Therefore, according to Kirchhoff's first law,

$$I_1 + (-I_2) + (-I_3) + I_4 + (-I_5) = 0$$

or $$I_1 - I_2 - I_3 + I_4 - I_5 = 0$$

Kirchhoff's second law. *It states that in any closed part of an electrical circuit, the algebraic sum of the e.m.fs. is equal to the algebraic sum of the products of the resistances and currents flowing through them.* It is also called ***loop rule.***

This law is based upon the law of conservation of energy.

Explanation. Consider a closed electrical circuit ABCD containing two cells of e.m.fs. E_1 and E_2 and five resistances R_1, R_2, R_3, R_4 and R_5 as shown in Fig. 2.02.

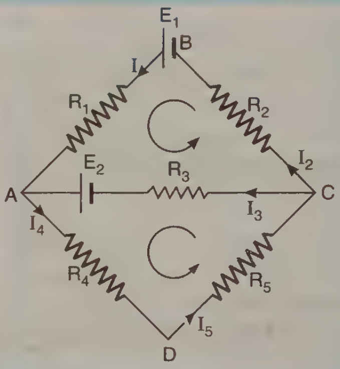


Fig. 2.02

Let us adopt the following sign conventions :

(*i*) The current flowing in anticlockwise direction is positive.

(*ii*) The e.m.f. of a cell is taken positive, if it sends current in the circuit in anticlockwise direction and negative, if the current due to the cell flows in clockwise direction.

Let us consider the closed part ABCA of the electrical circuit. The cell of e.m.f. E_1 will send current in anticlockwise direction and the cell of e.m.f. E_2 will send current in clockwise direction. Thus, e.m.f. E_1 is positive, while E_2 is negative.

Therefore, total e.m.f. of the closed part ABCA $= E_1 + (-E_2) = E_1 - E_2$

Also, in closed part ABCA, currents I_1 and I_2 flow in anticlockwise direction, while the current I_3 flows in clockwise direction. Therefore, in the closed part ABCA,

algebraic sum of the products of resistances and currents in them

$$= I_1R_1 + I_2R_2 + (-I_3)R_3 = I_1R_1 + I_2R_2 - I_3R_3$$

Therefore, for closed part ABCA, according to Kirchhoff's second law, we have

$$E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3$$

Similarly, it can be found that for closed part ACDA,

$$E_2 = I_3R_3 + I_4R_4 + I_5R_5$$

2.02. WHEATSTONE BRIDGE

It is one of the accurate arrangements for measuring an unknown resistance.

Four resistances P, Q, R and X are connected to form a quadrilateral. A galvanometer and a tapping key K_1 (called galvanometer key) are connected between points B and D, while a battery and a one way key K_2 (called battery key) are connected between points A and C [Fig. 2.03]. Usually, P and Q, called *ratio* *arms*, are known resistances ; R is a variable resistance (resistance box) and X is the unknown resistance. The value of resistance R is so adjusted that on pressing key K_1, the galvanometer does not give deflection. It implies that points B and D are at the same potential. In such a case, the bridge is said to be ***balanced***. For a balanced bridge, it can be proved that

$$\frac{P}{Q}=\frac{R}{X}$$

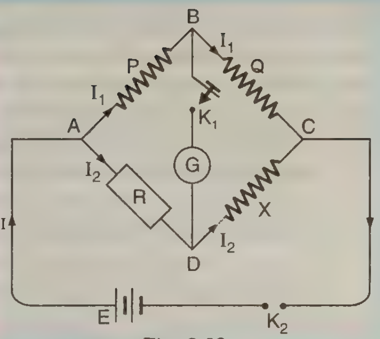


Fig. 2.03

Proof. Let I be the current in the main circuit. At point A, say current equal to I_1 flows through resistance P and the rest $I - I_1 = I_2$ (say) flows through resistance R. When the bridge is in balanced state, points B and D are at the same potential and therefore no current will flow through galvanometer *i.e.* current I_1 will flow as such through Q, while I_2 through X. At point C, the currents through Q and X add up so as to send a total current I through the circuit. Let V_A, V_B, V_C and V_D be electric potentials of points A, B, C and D respectively.

Now, potential difference across P *i.e.* $V_A - V_B = I_1 P$ ...(2.01)

Potential difference across Q *i.e.* $V_B - V_C = I_1 Q$ ...(2.02)

Potential difference across R *i.e.* $V_A - V_D = I_2 R$ ...(2.03)

Potential difference across X *i.e.* $V_D - V_C = I_2 X$ ...(2.04)

When the bridge is in balanced state, no current flows along the path between points B and D. Therefore, points B and D are at same potential *i.e.* $V_B = V_D$. In equations (2.03) and (2.04), replacing V_D by V_B we have

$$V_A - V_B = I_2 R \qquad ...(2.05)$$

and

$$V_B - V_C = I_2 X \qquad ...(2.06)$$

From equations (2.01) and (2.05), we have

$$I_1 P = I_2 R \qquad ...(2.07)$$

From equations (2.02) and (2.06), we have

$$I_1 Q = I_2 X \qquad ...(2.08)$$

Dividing equation (2.07) by (2.08), we have

$$\frac{P}{Q}=\frac{R}{X} \qquad ...(2.09)$$

As we know the values of P, Q and R, the value of unknown resistance X can be found. It may be remembered that the bridge is most sensitive, when all the four resistances are of the same order.

Key point

When the bridge is balanced, the path BD may be considered as an open path. Therefore, even when the galvanometer key (K_1) is kept pressed, no current flows along the path BD.

Key point

Wheatstone bridge is most sensitive, when resistances in the four ratio arms are of the same order.

Merits of Wheatstone bridge. Wheatstone bridge method for measuring an unknown resistance has the following merits over the other methods for measuring the resistance :

1. It is a null method. Therefore, the measurement of resistance made by this method is not affected by the internal resistance of the battery used.

2. As no measurement of current or potential difference is involved, the measurements are not affected because of the fact that the ammeters and voltmeters are not ideal ones.

3. The value of unknown resistance can be measured to a very high degree of accuracy by increasing the ratio of the resistances in arms P and Q.

Key point

The measurement of resistance by Wheatstone bridge is not affected by the internal resistance of the cell.

Watch out !

To avoid inductive effects*, the battery key should be pressed first and the galvanometer key afterwards !

2.03. PRINCIPLE OF WHEATSTONE BRIDGE (FROM KIRCHHOFF'S LAWS)

Four resistances P, Q, R and X are connected across the four arms of Wheatstone bridge as shown in Fig. 2.04. A tapping key K_1 and a galvanometer G are connected between points B and D, whereas a battery of e.m.f. E and a key K_2 are connected between points A and C. The value of resistance R is adjusted, so that on putting plug in key K_2 and pressing the tapping key K_1, the galvanometer shows no deflection. Then, the bridge is said to be balanced. As said earlier, for a balanced bridge,

$$\frac{P}{Q}=\frac{R}{X}$$

It is called condition for balanced wheatstone bridge.

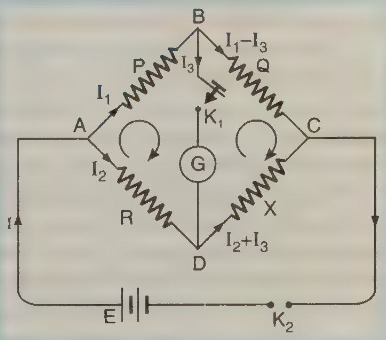


Fig. 2.04

To obtain the condition of balanced bridge using Kirchhoff's laws. When the plug is inserted in the key K_2, the current I enters the network at point A and a part I_1 (say) flows through resistance P and a part I_2 through resistance R. On pressing key K_1, suppose that current I_3 flows through galvanometer G (if we suppose that point B is at higher potential than D). According to Kirchhoff's first law, current $I_1 - I_3$ will flow through resistance Q. At point D, current I_3 flowing through galvanometer G combines with current I_2 flowing through resistance R, so that current $I_2 + I_3$ passes through resistance X. Therefore, at point C, a net current $(I_1 - I_3 + I_2 + I_3)$ *i.e.* $(I_1 + I_2)$ *i.e.* I flows from point C to the negative pole of the battery. Let us consider that current flowing in clockwise direction is positive.

---

* Refer to Chapter on Electromagnetic Induction – Unit IV.

Applying Kirchhoff's second law, for closed part ABDA of the circuit :

$$I_1 P + I_3 G - I_2 R = 0 \qquad ...(2.10)$$

For closed part BCDB of the circuit :

$$(I_1 - I_3) Q - (I_2 + I_3) X - I_3 G = 0 \qquad ...(2.11)$$

The resistance R is adjusted in such a way that points B and D are at the same potential, so that current I_3 passing through the galvanometer is zero. It is indicated by zero deflection in the galvanometer. The bridge is then said to be balanced.

As there is no deflection in galvanometer on pressing key K_1, $I_3 = 0$. In equation (2.10), setting $I_3 = 0$, we get

$$I_1 P - I_2 R = 0$$

or

$$I_1 P = I_2 R \qquad ...(2.12)$$

Also setting $I_3 = 0$ in equation (2.11), we get

$$I_1 Q = I_2 X \qquad ...(2.13)$$

Dividing equation (2.12) by (2.13), we get

$$\frac{P}{Q} = \frac{R}{X}$$

It proves the condition for balanced Wheatstone bridge.

2.04. SLIDEWIRE BRIDGE

A slide wire bridge is also known as **metre bridge.**

Principle. *It is constructed on the principle of balanced Wheatstone bridge, i.e.* when a wheatstone bridge is balanced,

$$\frac{P}{Q} = \frac{R}{X},$$

where the letters have their usual meanings.

Construction. It consists of a wooden board, over which constantan wire AC of one metre in length is stretched between two copper strips. A metre scale is also fitted on the wooden board parallel to the length of the wire. Another central copper strip is fitted on the wooden board to provide two gaps across which a resistance box R and the unknown resistance X are connected. One terminal of a sensitive galvanometer is connected to the terminal D, provided on the central copper strip and the other to a jockey, which can be slided over the wire to balance the bridge. The circuit is now exactly the same as that of the Wheatstone bridge [Fig. 2.05]. The resistance of wire between points A and B (B is the position of jockey in balanced state of the bridge) represents ratio arm P and that of wire between B and C represents Q. Therefore, when bridge is balanced

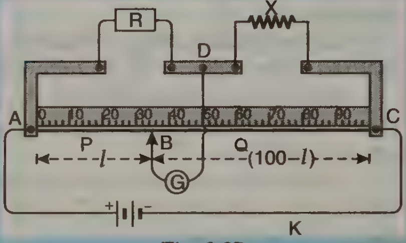


Fig. 2.05

$$\frac{P}{Q} = \frac{R}{X}$$

Applications of metre bridge. A metre bridge can be used to measure an unknown resistance or to compare the values of two unknown resistances.

1. ***To measure an unknown resistance.*** To find the value of unknown resistance X, the circuit as shown in Fig. 2.05 is arranged. The bridge can be balanced by moving the jockey over the wire. The bridge is said to be balanced, when the jockey is at a point B on the wire, such that galvanometer does not give any deflection. For this, resistance in the left gap is introduced with the help of resistance box R and its value is adjusted so as to obtain the balance point. In practice, the value of R is so adjusted that the balance point lies between 35 cm and 65 cm marks. Suppose that when bridge has been balanced, AB $= l$. Then, BC $= 100 - l$. The resistance of wire between points A and B is taken as P and that of the portion between B and C is taken as Q. If wire of the metre bridge is of uniform area of cross-section, then

$$P \propto l \quad \text{and} \quad Q \propto 100 - l$$

$$\therefore \quad \frac{P}{Q} = \frac{l}{(100 - l)}$$

Self-test Questions

Why do we prefer to obtain the balance point between 35 cm and 65 cm marks ?

From the principle of Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{X}$$

$$\therefore \qquad \frac{R}{X} = \frac{l}{(100-l)}$$

$$\text{or} \qquad X = R\,\frac{(100-l)}{l} \qquad \text{...(2.14)}$$

Knowing the values of R and l, the value of X can be found.

2. ***To compare two unknown resistances.*** For comparing two unknown resistances R_1 and R_2, the circuit as shown in Fig. 2.05 is arranged. First, the resistance R_1 is connected in place of unknown resistance X. The jockey is moved over the bridge wire gently and by adjusting the value of R, the balance point is obtained. The bridge is said to be balanced, when the jockey is at a point B on the wire, such that galvanometer does not give any deflection. In such a case, if AB = l_1, then BC = $100 - l_1$. The resistance of the bridge wire between the points A and B is ratio arm P and that between points B and C corresponds to ratio arm Q. Since the bridge wire is of uniform area of cross-section,

$$P \propto l_1 \qquad \text{and} \qquad Q \propto 100 - l_1$$

$$\therefore \qquad \frac{P}{Q} = \frac{l_1}{(100-l)}$$

From the principle of Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{R_1} \qquad (\because X = R_1)$$

$$\therefore \qquad \frac{R}{R_1} = \frac{l_1}{(100-l_1)}$$

$$\text{or} \qquad R_1 = \frac{R\,(100-l_1)}{l_1} \qquad \text{...(2.15)}$$

Now, the experiment is repeated with the other unknown resistance R_2. If l_2 is the balancing length of the bridge wire between the points A and B, then it can be obtained that

$$R_2 = \frac{R\,(100-l_2)}{l_2} \qquad \text{...(2.16)}$$

Dividing equation (2.15) by (2.16), we get

$$\frac{R_1}{R_2} = \frac{R\,(100-l_1)}{l_1} \times \frac{l_2}{R\,(100-l_2)}$$

$$\text{or} \qquad \frac{R_1}{R_2} = \frac{l_2}{l_1} \times \frac{(100-l_1)}{(100-l_2)} \qquad \text{...(2.17)}$$

Knowing the values of l_1 and l_2, the ratio R_1/R_2 can be found.

3. ***To measure the unknown temperature..*** We know that resistance of a conductor increases with increase of temperature. The resistance of a conductor at temperature θ is given by

$$R = R_0\,(1 + \alpha\,\theta), \qquad \text{...(2.18)}$$

where R_0 is resistance of the conductor at 0°C and α is temperature coefficient of resistance of the conductor.

From equation (2.18), we have

$$R - R_0 = R_0\,\alpha\,\theta \qquad \text{...(2.19)}$$

If R_{100} is resistance of the conductor at 100°C, then equation (2.19) may be expressed as

$$R_{100} - R_0 = R_0\,\alpha \times 100 \qquad \text{...(2.20)}$$

Dividing equation (2.20) by (2.19), we obtain

$$\theta = \frac{R - R_0}{R_{100} - R_0} \times 100 \qquad \text{...(2.21)}$$

The equation (2.21) may be used to find the unknown temperature of a hot bath by using the circuit diagram shown in Fig. 2.05. The unknown resitance X (in the form of a metallic wire) is immersed in ice and its resistance R_0 at 0°C is measured. The unknown resistance is then maintained at a temperature of 100°C (by placing the wire in steam) and its resistance R_{100} at 100°C is measured. Finally, the unknown resistance is immersed in hot bath at unknown temperature θ and experiment is repeated to measure its resistance R_θ at θ°C. Then, by substituting the value of R_0, R_{100} and R_θ in equation (2.21), the unknown temperature of the hot bath can be found.

2.05. POTENTIOMETER

A potentiometer is used to compare e.m.fs. of two cells or to measure internal resistance of a cell.

Principle. The principle of potentiometer states that *when a constant current is passed through a wire of uniform area of cross-section, the potential drop across any portion of the wire is directly proportional to the length of that portion.*

Construction. A potentiometer consists of a number of segments of wire of uniform area of cross-section stretched over a wooden board between two thick copper strips. Each segment of wire is 100 cm long. The wire is usually of constantan or manganin. A metre rod is fixed parallel to its length [Fig. 2.06]. A battery connected across the two end terminals sends current through the wire, which is kept constant by using a rheostat.

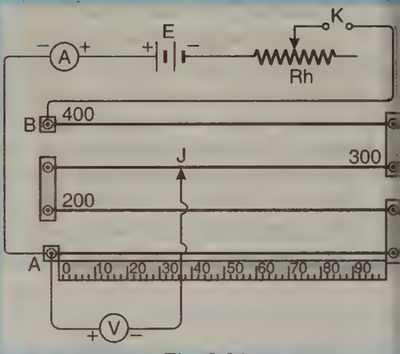


Fig. 2.06

Theory. Let V be potential difference across certain portion of wire, whose resistance is R. If I is the current through the wire, then

$$V = IR$$

We know that $R = \rho \frac{l}{A}$,

where l, A and ρ are length, area of cross-section and resistivity of the material of wire respectively.

$$\therefore \quad V = I\rho\frac{l}{A}$$

If a constant current is passed through the wire of uniform area of cross-section, then I and A are constants. Since, for a given wire, ρ is also constant, we have

$$V = \text{constant} \times l$$

$$\text{or} \quad V \propto l \qquad \text{...(2.22)}$$

Hence, if a constant current flows through a wire of uniform area of cross-section, then potential drop along the wire is directly proportional to the length of the wire.

Key point

The principle of potentiometer requires that

(*i*) potentiometer wire should be o uniform area of cross-section and

(*ii*) current through the wire should remai constant.

Applications of a potentiometer. A potentiometer can be put to the following uses :

1. ***To compare e.m.fs. of two cells.*** Two cells, whose e.m.fs. are E_1 and E_2, can be compared by making use of the circuit as shown in Fig. 2.07. The positive poles of both the cells are connected to the terminal A of the potentiometer. The negative poles of the two cells are connected to terminals 1 and 2 of a two-way key, while its common terminal is connected to a jockey J through a galvanometer G. An auxiliary or driver battery of e.m.f. E, an ammeter A, rheostat Rh and a one way key K are connected between the end terminals A and B of the potentiometer. Thus, the positive poles of the two cells as well as the positive pole of auxiliary battery are connected at the common point A. It may be pointed that e.m.f. of auxiliary battery should always be greater than e.m.f. of either of the two cells.

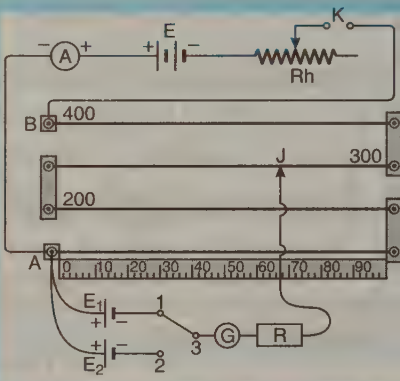


Fig. 2.07

To compare e.m.fs. of the two cells, a constant current is passed through the potentiometer wire between the points A and B. The current is kept constant by using the rheostat.

When the plug is put in the gap between the terminals 1 and 2 of the two-way key, the cell of e.m.f. E_1 will come in the circuit. Suppose that balancing length (length of wire between points A and J) is l_1. If x is the resistance per unit length of the potentiometer wire and I, the constant current flowing through it, then

$$E_1 = (x\,l_1)\,I \qquad \text{...(2.23)}$$

When the key is put in the gap between the terminals 2 and 3 after moving from the gap between 1 and 3, the cell of e.m.f. E_2 will be included in the circuit. Let the balancing length be l_2 in this case. Then,

$$E_2 = (x\,l_2)\,I \qquad \text{...(2.24)}$$

Dividing equation (2.23) by equation (2.24), we get

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \qquad \text{...(2.25)}$$

Key point

The e.m.f. of auxiliary battery mus always be greater than the e.m.f. of eithe of the two cells.

Key point

The positive terminals of the two cells as well as that of auxiliary battery must be connected at the same common terminal.

2. ***To measure internal resistance of a cell.*** The internal resistance of a cell of m.f. E can be found by setting up the circuit as shown in Fig. 2.08. Let E′ be e.m.f. f the auxiliary battery. A constant current I is maintained through the potentiometer ire with the help of the rheostat.

Plug in the key K_2 is kept out and the jockey is moved over the potentiometer ire so as to balance the e.m.f. E of the cell, whose internal resistance is to be found. et l_1 be the balancing length of the potentiometer wire between point A and jockey If x is resistance per unit length of the wire, then

$$E = (x\,l_1)\,I \qquad ...(2.26)$$

With the help of resistance box S, introduce resistance, say S and then put the lug in key K_2. Now, find the balance point for the terminal potential difference V etween the two poles of the cell. If l_2 is the balancing length, then

$$V = (x\,l_2)\,I \qquad ...(2.27)$$

Dividing equation (2.26) by (2.27), we have

$$\frac{E}{V} = \frac{l_1}{l_2} \qquad ...(2.28)$$

The internal resistance* of the cell is given by

$$r = \left(\frac{E}{V} - 1\right)S$$

Using equation (2.28), we have

$$r = \left(\frac{l_1}{l_2} - 1\right)S$$

or

$$r = \frac{l_1 - l_2}{l_2} \times S \qquad ...(2.29)$$

Knowing the values of l_1, l_2 and S, the internal resistance r of the cell can be found.

Note. Apart from these uses, a potentiometer can be used to compare unknown resistances and to calibrate a voltmeter or an ammeter.

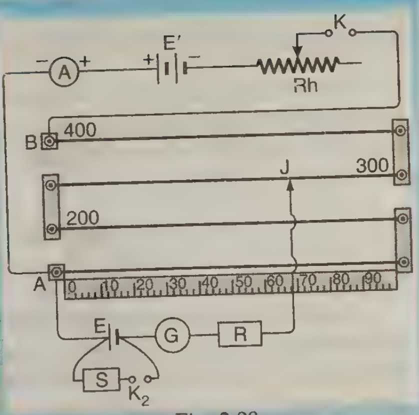


Fig. 2.08

Watch out !

A voltmeter, when connected across a cell, always draws some current and therefore it cannot be used to measure the e.m.f. of a cell. In case of a potentiometer, when the e.m.f. of a cell is balanced against potential drop across a certain length of wire, the wire draws current only from the auxiliary battery. It does not draw any current from the cell and therefore by calibrating the potentiometer wire with a standard cell, a potentiometer can be used to measure e.m.f. of a cell !

SOLVED NUMERICAL PROBLEMS

Type A. On Kirchhoff's laws

Problem 2.01. In the network shown in Fig. 2.09, $E_1 = 6$ V, $E_2 = 4$ V, $R_1 = 2\,\Omega$, $R_2 = 3\,\Omega$ and $R_3 = 5\,\Omega$. Find the currents passing through the resistors R_1, R_2 and R_3.

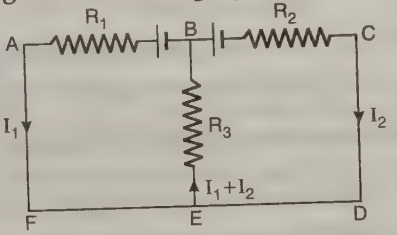


Fig. 2.09

Sol. Suppose that due to battery of e.m.f E_1, current I_1 flows through resistor R_1 and that due to E_2, current I_2 flows through resistor R_2. Then, according to Kirchhoff's first law, current $(I_1 + I_2)$ will flow through resistor R_3 as shown in Fig. 2.09.

In closed part ABEFA of the circuit :

$$I_1 R_1 + (I_1 + I_2) R_3 = E_1$$

or $$I_1 \times 2 + (I_1 + I_2)\,5 = 6$$

or $$7\,I_1 + 5\,I_2 = 6 \qquad ...(i)$$

In closed part BCDEB of the circuit :

$$I_2 R_2 + (I_1 + I_2) R_3 = E_2$$

or $$I_2 \times 3 + (I_1 + I_2)\,5 = 4$$

or $$5\,I_1 + 8\,I_2 = 4 \qquad ...(ii)$$

Multiplying equation (*i*) by 8, equation (*ii*) by 5 and then subtracting, we have

$$(7\,I_1 + 5\,I_2) \times 8 - (5\,I_1 + 8\,I_2) \times 5 = 6 \times 8 - 4 \times 5$$

or $$31\,I_1 = 28 \quad \text{or} \quad I_1 = \frac{28}{31}\,A$$

Substituting for I_1 in equation (*i*), we get

$$7 \times \frac{28}{31} + 5\,I_2 = 6 \quad \text{or} \quad I_2 = -\frac{2}{31}\,A$$

The current through resistor R_3,

$$I_1 + I_2 = \frac{28}{31} + \left(\frac{-2}{31}\right) = \frac{26}{31}\,A$$

Problem 2.02. Two cells of e.m.f. 1·5 V and 2·0 V and internal resistances 1 Ω and 2 Ω respectively are connected in parallel so as to send current in the same direction through an external resistance of 5 Ω. (*a*) Draw the circuit diagram. (*b*) Using Kirchhoff's laws, calculate current through each branch of the circuit and potential across the 5 Ω resistance. (C.B.S.E. 1995)

Sol. (*a*) ***Circuit diagram.*** Let the e.m.f. of two cells be represented by E_1 and E_2 and their internal resistances be r_1 and r_2 respectively. The two cells have been connected in parallel to the external resistance R as shown in Fig. 2.10.

(*b*) Here, $E_1 = 1{\cdot}5\ V\ ;\ E_2 = 2{\cdot}0\ V\ ;$

$$r_1 = 1\,\Omega\ ;\ r_2 = 2\,\Omega\ ;\ R = 5\,\Omega$$

---

*Refer to equation (1.49) of Chapter 1.

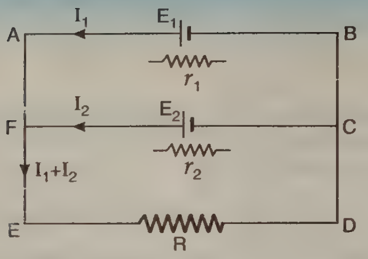


Fig. 2.10

Let I_1 and I_2 be currents due to the cells E_1 and E_2 respectively. As the currents I_1 and I_2 flow into point F ; according to Kirchhoff's first law, current $I_1 + I_2$ will flow out from point F and pass through resistance R.

Applying Kirchhoff's second law to the closed part ABDEA of the circuit, we have

or $$E_1 = I_1 r_1 + (I_1 + I_2) R$$

or $$1{\cdot}5 = I_1 \times 1 + (I_1 + I_2) \times 5$$

or $$6 I_1 + 5 I_2 = 1{\cdot}5 \qquad ...(i)$$

Applying Kirchhoff's second law to the closed part FCDEF of the circuit, we have

$$E_2 = I_2 r_2 + (I_1 + I_2) R$$

or $$2{\cdot}0 = I_2 \times 2 + (I_1 + I_2) \times 5$$

or $$5 I_1 + 7 I_2 = 2{\cdot}0 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we obtain

$$I_1 = 0{\cdot}0294 \text{ A} \quad \text{and} \quad I_2 = 0{\cdot}2647 \text{ A}$$

Potential difference across $R = (I_1 + I_2) R$

$$= (0{\cdot}0294 + 0{\cdot}2647) \times 5 = 1{\cdot}4705 \text{ V}$$

Problem 2.03. Three cells are connected in parallel with their like poles connected together with wires of negligible resistance. If the e.m.fs. of the cells are 2, 1 and 4 V respectively and their internal resistances are 4, 3 and 2 Ω respectively, find the current through each cell.

Sol. Here, $E_1 = 2\text{ V} ; E_2 = 1\text{ V} ; E_3 = 4\text{ V}$

and $r_1 = 4\,\Omega ; r_2 = 3\,\Omega ; r_3 = 2\,\Omega$

Let the currents I_1, I_2 and I_3 flowing from the three cells be as shown in Fig. 2.11.

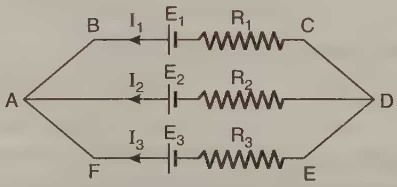


Fig. 2.11

Applying Kirchhoff's first law at point A, we have

$$I_1 + I_2 + I_3 = 0 \qquad ...(i)$$

Applying Kirchhoff's second law to the closed part ABCDA of the circuit, we have

$$I_1 r_1 - I_2 r_2 = E_1 - E_2 \quad \text{or} \quad I_1 \times 4 - I_2 \times 3 = 2 - 1$$

or $$4 I_1 - 3 I_2 = 1 \qquad ...(ii)$$

Applying Kirchhoff's second law to the closed part ADEFA of the circuit, we have

$$I_2 r_2 - I_3 r_3 = E_2 - E_3 \quad \text{or} \quad I_2 \times 3 - I_3 \times 2 = 1 - 4$$

or $$3 I_2 - 2 I_3 = -3 \qquad ...(iii)$$

Solving equations (*i*), (*ii*) and (*iii*), we obtain

$$I_1 = -\frac{2}{13}\text{ A} ; I_2 = -\frac{7}{13}\text{ A and } I_3 = \frac{9}{13}\text{ A}$$

Problem 2.04. Two cells of e.m.f. 2 V and 1 V and internal resistances 1 Ω and 2 Ω respectively have the positive terminals connected by a wire of 10 Ω resistan and their negative terminals by a wire of 4 Ω resistanc Another coil of 10 Ω is connected between the midd points of these wires. Find the potential difference acro the 10 Ω coil. (*Karnataka, 1988*

Sol. The positive terminals of the cells E_1 and E_2 a connected to the wire AE of resistance 10 Ω and negati terminals to the wire BD of resistance 4 Ω. The resistan of 10 Ω is connected between the middle points F and of the wires AE and BD respectively. [Fig. 2.12]

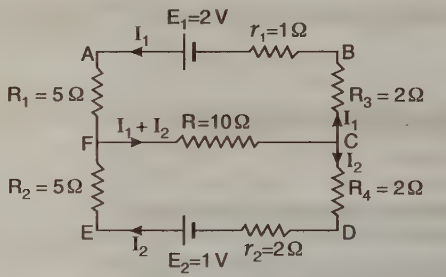


Fig. 2.12

$$\therefore \quad R_1 = R_2 = \frac{10}{2} = 5\,\Omega ; R_3 = R_4 = \frac{4}{2} = 2\,\Omega.$$

The distribution of current in various branches is show in the figure.

In closed part ABCFA of the circuit :

$$I_1 \times r_1 + I_1 \times R_1 + (I_1 + I_2) R + I_1 \times R_3 = E_1$$

or $$I_1 \times 1 + I_1 \times 5 + (I_1 + I_2) \times 10 + I_1 \times 2 = 2$$

or $$9 I_1 + 5 I_2 = 1 \qquad ...($$

In the closed part CDEFC of the circuit :

$$I_2 \times r_2 + I_2 \times R_2 + (I_1 + I_2) R + I_2 \times R_4 = E_2$$

or $$I_2 \times 2 + I_2 \times 5 + (I_1 + I_2) \times 10 + I_2 \times 2 = 1$$

or $$10 I_1 + 19 I_2 = 1 \qquad ...(ii$$

Solving equations (*i*) and (*ii*), we have

$$I_2 = \frac{14}{121}\text{ A} \qquad \text{and } I_2 = -\frac{1}{121}\text{ A}$$

Therefore, current through resistance R,

$$I_1 + I_2 = \frac{14}{121} + \left(-\frac{1}{121}\right) = \frac{13}{121}\text{ A}$$

Potential difference across the resistance R

$$= (I_1 + I_2) R = \frac{13}{121} \times 10 = \mathbf{1{\cdot}0744 \text{ V}}$$

Problem 2.05. Find the equivalent resistance betwee the points X and Y of the network of resistors shown i Fig. 2.13.

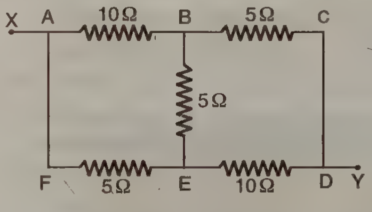


Fig. 2.13

Ans. Let R be equivalent resistance of the network c resistors in Fig. 2.13. In order to find R, connect a battery c n.f. E across the points X and Y as shown in Fig. 2.14.

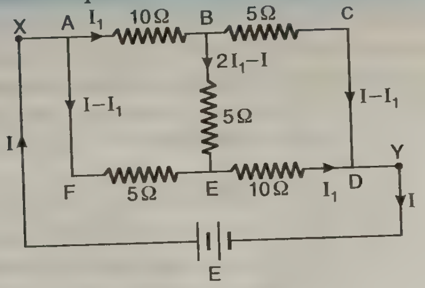


Fig. 2.14

If I is current drawn by the network from the battery, en

$$R=\frac{E}{I} \qquad ...(i)$$

Suppose that when current I enters at point A, a part I_1 ows along path AB. Then, according to Kirchhoff's first law, ne remaining part $(I-I_1)$ will flow along path AFE. By ymmetry, the current I flowing out from point D will be omposed of part I_1 from the path ED and $(I-I_1)$ from the ath BCD. Again, according to Kirchhoff's first law, the urrent along path BE will be $I_1-(I-I_1)$ *i.e.* $2I_1-I$. The istribution of current in various branches of the circuit is as hown in figure.

In closed part ABEFA of the circuit :

$$10I_1+5(2I_1-I)-5(I-I_1)=0$$

or $$25I_1-10I=9 \quad \text{or} \quad I_1=\frac{2}{5}I$$

In closed part XAFEDYX of the circuit :

$$E=5(I-I_1)+10I_1$$

$$=5I+5I_1=5I+5\times\frac{2}{5}I=7I$$

Subsituting for E in equation (*i*), we have

$$R=\frac{7I}{I}=7\,\Omega$$

Type B. On Wheatstone bridge

Problem 2.06. Four resistances of 15 Ω, 12 Ω, 4 Ω and 10 Ω respectively are connected in cyclic order to form a Wheatstone's network. Is the network balanced ? If not, calculate the resistance to be connected in parallel with the resistance of 10 Ω to balance the network.

(*Karnataka, 1974 S*)

Sol. Let P = 15 Ω ; Q = 12 Ω ; S = 4 Ω ; R = 10 Ω

The resistances of 15 Ω, 12 Ω, 4 Ω and 10 Ω are connected in cyclic order as shown in Fig. 2.15.

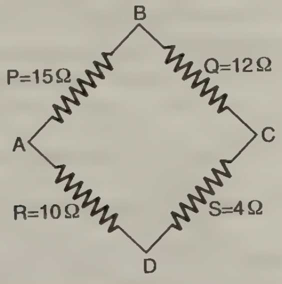


Fig. 2.15

Now, $$\frac{P}{Q}=\frac{15}{12}=1\cdot25\,; \qquad \frac{R}{S}=\frac{10}{4}=2\cdot5$$

Since $\frac{P}{Q}$ is not equal to $\frac{R}{S}$, the bridge is **not balanced**.

To balance the network, suppose a resistance X is connected in parallel to R = 10 Ω, so that the bridge is balanced. If R′ is effective resistance between points A and D, then

$$\frac{1}{R'}=\frac{1}{10}+\frac{1}{X} \qquad ...(i)$$

Since the bridge is now balanced,

$$\frac{P}{Q}=\frac{R'}{S} \quad \text{or} \quad R'=S\times\frac{P}{Q}=4\times\frac{15}{12}=5\,\Omega$$

Substituting for R′ in equation (*i*), we have

$$\text{or} \quad \frac{1}{5}=\frac{1}{10}+\frac{1}{X} \quad \text{or} \quad \frac{1}{X}=\frac{1}{5}-\frac{1}{10}=\frac{1}{10} \quad \text{or } \mathbf{X=10\,\Omega}$$

Problem 2.07. A potential difference of 4 volt is applied between the points A and B shown in the network drawn in the Fig. 2.16. Calculate (*i*) equivalent resistance of the network across the points A and B and (*ii*) the magnitudes of currents flowing in arms AFCEB and AFDEB.

(*H.P.S.S.C.E. 2000, 1998 ; C.B.S.E. 1998 S*)

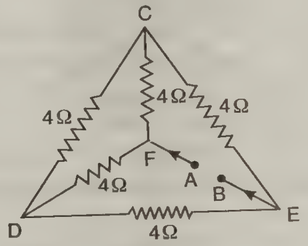


Fig. 2.16

Ans. After connecting a battery of e.m.f. E = 4 V between the points A and B, the given network can be redrawn as shown in Fig. 2.17.

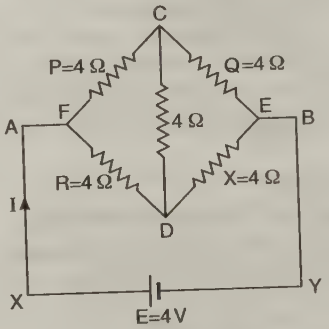


Fig. 2.17

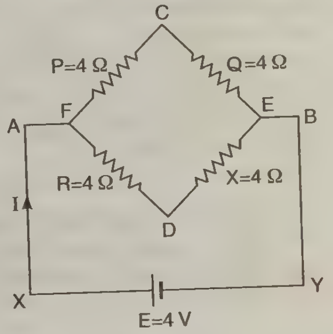


Fig. 2.18

The given network is a Wheatstone bridge arrangement.

Further, as $\frac{P}{Q}=\frac{R}{X}$, the bridge is in balanced slate. As such, no current can flow along the path CD. In other words, CD can be treated as an open path as shown in Fig. 2.18.

(*i*) Now, path FCE contains two resistances, (each of 4 Ω) connected in series. If R_1 is resistance of path FCE, then

$$R_1=P+Q=4+4=8\,\Omega$$

Similarly, if R_2 is resistance of path FDE, then

$$R_2=R+X=4+4=8\,\Omega$$

Now, the path FCE of resistance R_1 and FDE of resistance R_2 are connected in parallel. If R′ is equivalent resistance of the given network between points A and B, then

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

or $R' = 4\ \Omega$

(ii) Let I be the current in the main circuit due to battery.

Then, $I = \frac{E}{R'} = \frac{4}{4} = 1\ A$

Further, suppose that I_1 and I_2 are currents along paths FCE and FDE respectively. Since the two paths FCE and FDE are of equal resistances, the current I in the main circuit will divide equally along the two paths *i.e.* current in the arms AFCEB and AFDEB are given by

$$I_1 = I_2 = I/2 = 1/2 = 0{\cdot}5\ A$$

Problem 2.08. Find the value of unknown resistance X in the circuit shown in Fig. 2.19, if no current flows through the section AO. Also calculate the current drawn by the circuit from the battery of e.m.f. 6 V and negligible internal resistance. *(C.B.S.E. 2002)*

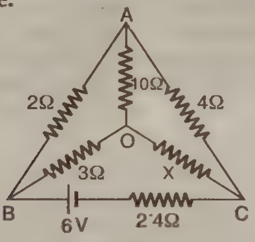


Fig. 2.19

Sol. The given circuit is equivalent to the circuit shown in Fig. 2.20.

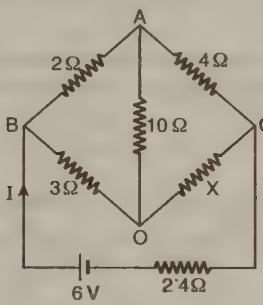


Fig. 2.20

Let I be the current drawn by the circuit from the battery.

As no current flows through branch AO of the circuit, the circuit is a balanced Wheatstone bridge. Therefore,

$$\frac{2}{4} = \frac{3}{X} \quad \text{or} \quad X = \frac{4 \times 3}{2} = 6\ \Omega$$

Since the branch AO acts as open path (no current flows along branch AO), the 10 Ω resistance of the circuit is ineffective. Hence, the network is equivalent to the circuit shown in Fig. 2.21.

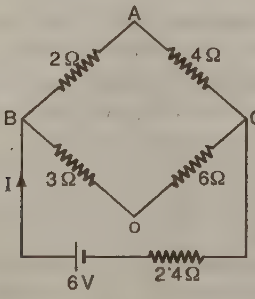


Fig. 2.21

Let R_1 and R_2 be resistances of the paths BAC and BOC of the circuit. Then,

$$R_1 = 2 + 4 = 6\ \Omega \quad \text{and} \quad R_2 = 3 + 6 = 9\ \Omega$$

The given circuit is now equivalent to the circuit shown in Fig. 2.22.

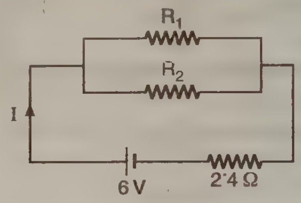


Fig. 2.22

It follows that the effective resistance of the circuit,

$$R = \frac{R_1 R_2}{R_1 + R_2} + 2{\cdot}4 = 3{\cdot}6 + 2{\cdot}4 = 6\ \Omega$$

Hence, current drawn by the circuit from the battery.

$$I = \frac{E}{R} = \frac{6}{6} = 1\ A$$

Problem 2.09. Four resistances 5, 50, 6 and 60 Ω are connected to the corresponding arms of the Wheatstone bridge. If a cell of e.m.f. 1·5 V and negligible internal resistance is connected across the bridge, calculate the current in arms and the cell. It is given that the bridge is in balanced position.

Sol. The four resistances 5, 50, 6, 60 Ω are connected as shown in the Fig. 2.23.

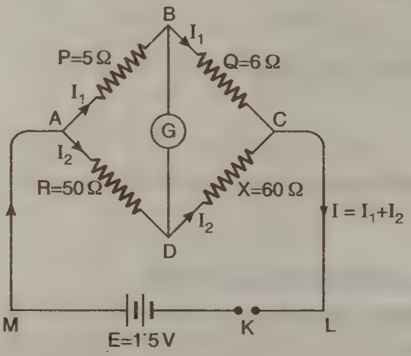


Fig. 2.23

For the given arrangement, it follows that $\frac{P}{Q} = \frac{R}{X}$. Therefore, Wheatstone bridge is in balanced state and hence no current flows through the galvanometer. Therefore, if at point A, current I_1 flows through P, the same will pass through Q and the remaining part of current I_2 that flows through R, will pass through X. From Kirchhoff's first law :

$$I = I_1 + I_2$$

Applying Kirchhoff's second law to the *closed part ABDA* :

$$I_1 \times 5 + (-I_2) \times 50 = 0 \quad \text{or} \quad I_1 - 10\, I_2 = 0$$

or $I_2 = \frac{I_1}{10}$

Applying Kirchhoff's second law to the *closed part ABCLMA :*

$$I_1 \times 5 + 6 \times I_1 = 1{\cdot}5 \quad \text{or} \quad 11\, I_1 = 1{\cdot}5$$

or $\mathbf{I_1 = 0{\cdot}1364\ A}$

∴ $I_2 = \frac{I_1}{10} = \frac{0{\cdot}1364}{10} = \mathbf{0{\cdot}0136\ A}$

Also, $I = I_1 + I_2 = 0{\cdot}1364 + 0{\cdot}0136 = \mathbf{0{\cdot}15\ A}$

ype C. On Slide wire bridge

Problem 2.10. A wire connected in the left gap of a etre bridge balances a 10 Ω resistance in the right gap at point, which divides the bridge wire in the ratio 3 : 2. alculate the resistance of the wire. If the length of the wire s 1 m, calculate the length of one ohm wire. *(Karnataka, 1987)*

Sol. Refer to Fig. 2.05. If R is resistance of the wire, then he wire of unknown resistance R is connected in the left gap nd a resistance X of 10 Ω is connected in the right gap. As he balance point divides the wire in the ratio 3 : 2, we have

$$\frac{P}{Q}=\frac{3}{2}$$

Now, $\frac{P}{Q}=\frac{R}{X}$ or $R = X\frac{P}{Q} = 10\times\frac{3}{2} = \mathbf{15\,\Omega}$

As the length of the wire is 1 m, therefore

$$\text{length of one ohm wire} = \frac{1}{15} = \mathbf{0\cdot067\ m}$$

Problem 2.11. With two resistance wires in the two gaps of a metre bridge, the balance point was found to be 1/3 m from the zero end. When a 6 Ω coil is connected in series with the smaller of the two resistances, the balance point is shifted to2 / 3m from the same end. Find the resistance of the two wires. *(Karnataka, 1986)*

Sol. Refer to Fig. 2.05. Let the two resistance wires be of values R_1 (in left gap) and R_2 (in right gap) respectively.

$$l=\frac{1}{3}\text{ m and } (100-l) = 1-\frac{1}{3}=\frac{2}{3}\text{ m}$$

$$\therefore\quad \frac{R_1}{R_2}=\frac{l}{(100-l)}=\frac{1/3}{2/3}=\frac{1}{2} \quad\text{or}\quad R_2 = 2\,R_1$$

On connecting 6 Ω coil :

Since resistance R_1 is smaller than R_2, 6 Ω coil has been connected in series to R_1. Therefore, resistance in the left gap becomes,

$$R_1' = R_1 + 6$$

Further, $l'=\frac{2}{3}$ and $(100-l')=1-\frac{2}{3}=\frac{1}{3}$ m

Again, $\frac{R'_1}{R_2}=\frac{l'}{(100-l')}$ or $\frac{R_1+6}{R_2}=\frac{2/3}{1/3}=2$

or $\frac{R_1+6}{2R_1}=2$

or $\mathbf{R_1 = 2\,\Omega}$

Also, $\mathbf{R_2 = 2\,R_1 = 2\times 2 = 4\,\Omega}$

Problem 2.12. Find the value of the unknown resistance X and the current drawn by the circuit from the battery, if no current flows through the galvanometer [Fig. 2.24]. Assume the resistance per unit length of the wire AB to be 0·01 Ω cm^{-1}. *(C.B.S.E. 2001)*

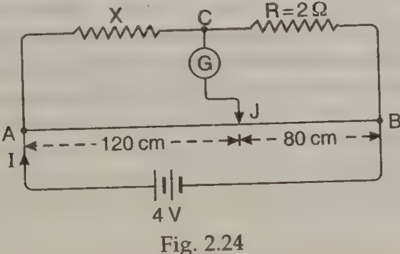


Fig. 2.24

Sol. Let P and Q be the resistances of the portions of wire AJ and JB respectively.

Then, $P = 120\times0{\cdot}01 = 1{\cdot}2\,\Omega$

and $Q = 80\times0{\cdot}01 = 0{\cdot}8\,\Omega$

Since the bridge is balanced,

$$\frac{X}{R}=\frac{P}{Q}\quad\text{or}\quad X = R\times\frac{P}{Q}$$

or $X=\frac{2\times1{\cdot}2}{0{\cdot}8}=\mathbf{3\,\Omega}$

As no current flows throgh galvanometer, the path between points C and J acts as an open path. Therefore, the given circuit is equivalent to the circuit drawn in Fig. 2.25.

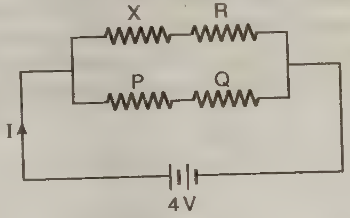


Fig. 2.25

Let R_1 be resistance of series combination of X and R and R_2 of the series combination of P and Q. Then,

$$R_1 = X + R = 3 + 2 = 5\,\Omega$$

and $R_2 = P + Q = 1{\cdot}2 + 0{\cdot}8 = 2\,\Omega$

Therefore, effective resistance of the circuit,

$$R' = \frac{R_1\,R_2}{R_1+R_2}=\frac{5\times2}{5+2}=\frac{10}{7}\,\Omega$$

Hence, current drawn by the circuit from the battery,

$$I=\frac{E}{R'}=\frac{4}{10/7}=\mathbf{2\cdot8\ A} \qquad (\because E = 4\text{ V})$$

Type D. On Potentiometer

Problem 2.13. The resistance of a potentiometer wire of length 10 m is 20 ohm. A resistance box and a 2 volt accumulator are connected in series with it. What resistance should be introduced in the box to have a potential drop of one microvolt per millimetre of the potentiometer wire ? *(Pre-degree Kerala, 1994)*

Sol. Here, resistance of potentiometer wire, R = 20 Ω

Required potential drop along the potentiometer wire

$$= 1\,\mu\text{V mm}^{-1} = (10^{-6}\text{ V})\times(10^{-3}\text{ m})^{-1} = 10^{-3}\text{ V m}^{-1}$$

Length of potentiometer wire, l = 10 m = 10^4 mm

Therefore, potential drop along the potentiometer wire,

$$V = 10\times10^{-3} = 10^{-2}\text{ V}$$

If I is current through the potentiometer wire, then

$$I=\frac{V}{R}=\frac{10^{-2}}{20}=5\times10^{-4}\text{ A}$$

If R′ is resistance that should be introduced in the resistance box, then

$$I=\frac{E}{R+R'}\quad\text{or}\quad 5\times10^{-4}=\frac{2}{20+R'}$$

or $\mathbf{R' = 3980\,\Omega}$

Problem 2.14. With a certain cell, the balance point is obtained at 65 cm from the end of a potentiometer wire. With another cell, whose e.m.f. differs from that of first by 0·1 V, the balance point is obtained at 60 cm. Find the e.m.f. of each cell. *(H.S.S.C.E. 2001)*

Sol. Let E_1 and E_2 be the e.m.fs. of two cells.

Here, balancing length for the first cell, l_1 = 65 cm and balancing length for the second cell, $l_2 = 60$ cm

Since balancing length for first cell is less than that for the second cell, the e.m.f. of second cell is less than that of first cell *i.e.* $E_2 < E_1$. As the e.m.f. of two cells differ by 0·1 V it follows that

$$E_2 = E_1 - 0{\cdot}1$$

Now, $$\frac{E_1}{E_2} = \frac{l_1}{l_2} \quad \text{or} \quad \frac{E_1}{E_1 - 0{\cdot}1} = \frac{65}{60}$$

or $$E_1 = 1{\cdot}3 \text{ V}$$

$\therefore$ $$E_2 = 1{\cdot}3 - 0{\cdot}1 = \mathbf{1{\cdot}2 \text{ V}}$$

Problem 2.15. Fig. 2.26 shows a 2·0 V potentiometer used for the determination of internal resistance of a 1·5 V cell. The balance point of the cell in open circuit is 76·3 cm. When a resistor of 9·5 Ω is used in the external circuit of the cell, the balance point shifts to 64·8 cm length of the potentiometer wire. Determine the internal resistance of the cell. *(Text Problem)*

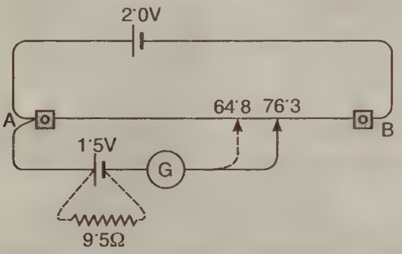


Fig. 2.26

Ans. Here, $S = 9{\cdot}5\ \Omega\ ;\ l_1 = 76{\cdot}3$ cm ; $l_2 = 64{\cdot}8$ cm

Now, $$r = \left(\frac{E}{V} - 1\right) \times S = \left(\frac{l_1}{l_2} - 1\right) \times S$$

$$= \left(\frac{76{\cdot}3}{64{\cdot}8} - 1\right) \times 9{\cdot}5 = \mathbf{1{\cdot}686\ \Omega}$$

Problem 2.16. A potentiometer wire of length 100 cm has a resistance of 10 Ω is connected in series with a resistance and a cell of e.m.f. 2 V of negligible internal resistance. A source of e.m.f. of 10 millivolt is balanced against a length of 40 cm of the potentiometer wire. What is the value of the external resistance ? *(I.I.T. 1976)*

Sol. The potentiometer wire of length 100 cm and resistance 10 Ω is connected to cell of e.m.f. 2 V and resistance R as shown in Fig. 2.27.

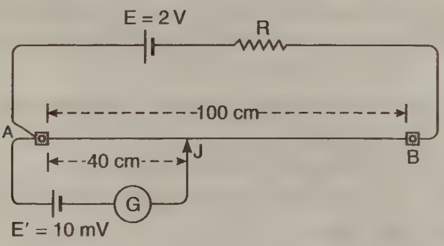


Fig. 2.27

If I is current through the potentiometer wire, then

$$I = \frac{E}{R+10} = \frac{2}{R+10} \qquad ...(i)$$

As the cell of e.m.f. E′ = 10 mV *i.e.* 10×10^{-3} V is balanced against a length of 40 cm of potentiometer wire, it follows that

$$I \times \text{resistance of 40 cm potentiometer wire} = 10 \times 10^{-3} \qquad ...(ii)$$

But resistance of 40 cm potentiometer wire

$$= \frac{10}{100} \times 40 = 4\ \Omega \qquad ...(iii)$$

Using equations (*i*) and (*iii*), equation (*ii*) becomes

$\therefore$ $$\frac{2}{R+10} \times 4 = 10 \times 10^{-3} \quad \text{or} \quad R + 10 = \frac{8}{10 \times 10^{-3}}$$

or $$R = \mathbf{790\ \Omega}$$

Problem 2.17. (*a*) If the galvanometer in the circuit shown in Fig. 2.28 reads zero, calculate the value of the resistor R, assuming that the 12 V source has negligible internal resistance.

(*b*) If cool air is blown across the wire-wound resistor, what effect will be noticed and why ? *(I.S.C.E. 1997)*

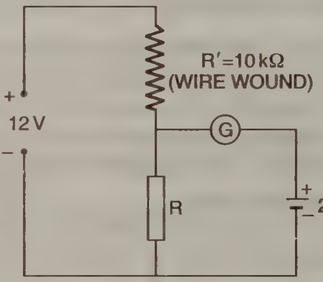


Fig. 2.28

Ans. (*a*) Here, e.m.f. of source battery, E = 12 V

Resistance of the wire wound resistor,

$$R' = 10\ \text{k}\Omega = 10 \times 10^3\ \Omega$$

Since the galvanometer connected in the circuit shows no deflection, the potential difference across the unknown resistance R is just equal to 2 V *i.e.* equal to the e.m.f. of cell connected across it. If I is current through the main circuit *i.e.* through series combination of resistor R and R′ connected to 12 V battery, then,

$$IR = 2 \qquad ...(i)$$

Also, $$I = \frac{E}{R+R'} = \frac{12}{R + 10 \times 10^3} \qquad ...(ii)$$

Substituting for I in equation (*i*), we have

$$\frac{12}{R + 10 \times 10^3} \times R = 2 \quad \text{or} \quad 6R = R + 10 \times 10^3$$

or $$R = 2 \times 10^3\ \Omega = \mathbf{2\ k\Omega}$$

(*b*) When current flows through a resistor, its temperature increases due to heat produced across it. If cool air is blown across the wire-wound resistor, its temperature will get lowered. Due to this, the resistance of the wire-wound resistor will decrease and current in the circuit will increase. As a consequence of it, the potential difference across the unknown resistor R will not remain 2 V. Since the e.m.f. of cell connected across R will no longer balance the potential drop across R, the galvanometer will show deflection.

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

roblem 2.18. 12 wires, each of resistance *r* ohm are onnected in the form of a skeleton cube. Find the ılent resistance of the cube, when the current enters corner and leaves at the diagonally opposite corner.

ol. Consider a skeleton cube ABCDEFGH made of 12 as shown in Fig. 2.29.

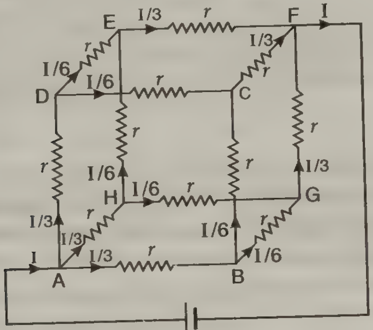


Fig. 2.29

n order to find its resistance, connect a battery of e.m.f. oss the diagonal corners A and F as shown in the figure. Suppose that current I enters the skeleton cube at the er A and leaves at the diagonally opposite corner F. The nt I will divide in three equal parts each equal to I/3 AB, AD and AH.

The current in AB, AD and AH will further divide along rent wires as explained below :

(*i*) The part I/3 flowing along AB will divide in two equal (I/6 each) along BC and BG.

(*ii*) The part I/3 flowing along AD will divide in two l parts (I/6 each) along DC and DE.

(*iii*) The part I/3 flowing along AH will divide in two l parts along HE and HG.

From the symmetry considerations, it follows that ent equal to I/3 should flow through in each of the three s CF, EF and GF, so that a net current equal to I may leave skeleton cube at the corner F.

Applying Kirchhoff's second law to the *closed part GFA of the circuit* :

E = P.D. across AB + P.D. across BG + P.D. across GF

$$E = \frac{I}{3}r + \frac{I}{6}r + r + \frac{I}{3}r = \frac{5I}{6}r \qquad ...(i)$$

If R is effective resistance of the skeleton cube, then

$$E = IR \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$IR = \frac{5I}{6}r \quad \text{or} \quad R = \frac{5}{6}r$$

Problem 2.19. 12 wires each of resistance *r* ohm are connected in the form of a skeleton cube. Find the uivalent resistance of the cube, when a cell is joined oss any one of the 12 wires forming the cube.

Sol. 12 wires each of resistance *r* ohm are connected in form of skeleton cube ABCDEFGH as shown in Fig. 2.30. order to find its resistance, connect a cell of e.m.f. E across wire AB, so that current I enters at corner A and leaves at B as shown in the figure.

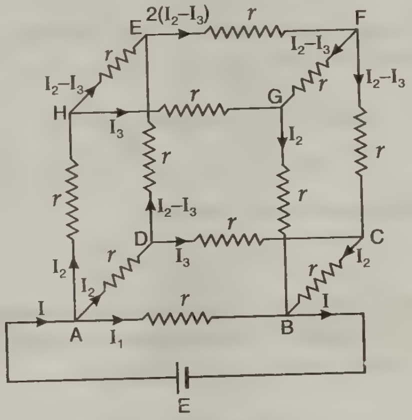


Fig. 2.30

Now, at point A, the current I will divide in three parts along AB, AD and AH and then following three different paths, it reaches point B. It follows that the resistances of the three paths between points A and B are not equal. However, the resistances of the paths followed by the current in reaching the point B through wire AD or AH are equal. Therefore, equal amounts of current will flow through wires AD and AH. Suppose that current I_1 flows along wire AB and current I_2 flows along each of the two wires AD and AH. Then, according to Kirchhoff's first law,

$$I = I_1 + I_2 + I_2$$

or

$$I = I_1 + 2I_2 \qquad ...(i)$$

Now, current I_2 through wire AD will again divide in two parts ; one part flowing along wire DC and then along wire CB ; while the other part reaches point B by flowing through wires DE, EF, FC and then through wire CB. As the above two paths are not of equal resistances, suppose that current I_3 flows through wire DC and the remaining part of the current $(I_2 - I_3)$ through wire DE. From symmetry considerations, it follows that current through wire HG will be I_3 and that through wire HE will be $(I_2 - I_3)$. The current through wire EF will become equal to $2(I_2 - I_3)$, which will then divide equally through wires FC and FG. The current through each of wire GB and CB will become I_2, so that at point B, current equal to $I_1 + I_2 + I_2$ *i.e.* $I_1 + 2I_2$ *i.e.* I will leave the skeleton cube.

For closed part EFGHE :

$$-I_3 r + (I_2 - I_3)r + 2(I_2 - I_3)r + (I_2 - I_3)r = 0$$

or

$$I_3 = \frac{4}{5}I_2$$

For closed part XADCBYX :

$$E = I_2 r + I_3 r + I_2 r = 2I_2 r + I_3 r \qquad ...(ii)$$

Using equation (*ii*), we have

$$E = 2I_2 r + \frac{4}{5}I_2 r = \frac{14}{5}I_2 r \qquad ...(iii)$$

For closed part XABYX :

$$E = I_1 r \qquad ...(iv)$$

From equations (*iii*) and (*iv*), we have

$$\frac{14}{5}I_2 r = I_1 r$$

or $I_2 = \frac{5}{14} I_1$

Substituting for I in equation (*i*), we have

$$I = I_1 + 2 \times \frac{5}{14} I_1 = \frac{12}{7} I_1 \qquad ...(v)$$

If R is equivalent resistance of the skeleton cube between points A and B, then

$$E = I R \qquad ...(vi)$$

From equations (*iv*) and (*vi*), we have

$$I R = I_1\, r \quad \text{or} \quad R = \frac{I_1}{I}\, r$$

Using equation (*v*), we have

$$R = I_1 \times \frac{7}{12 I_1}\, r = \frac{7}{12}\, r$$

Problem 2.20. Find the equivalent resistance betwen points X and Y of the network of resistors shown in Fig. 2.31. Each resistor is of resistance *r*.

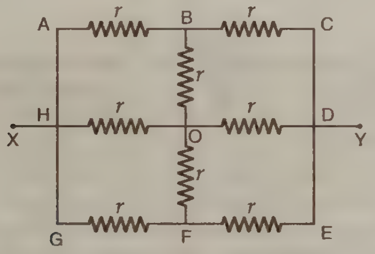


Fig. 2.31

Sol. Let R be equivalent resistance between points X and Y of the network of resistors shown in Fig. 2.31. To find R, connect a battery of e.m.f. E across the points X and Y as shown in Fig. 2.32.

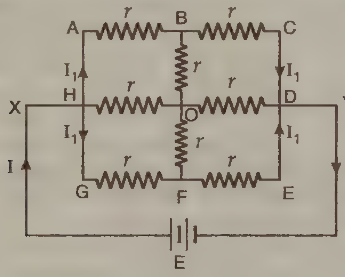


Fig. 2.32

Suppose that the network draws a current I from the battery. When the current I enters at point H, suppose that a part I_1 flows along path HAB. By symmetry, an equal current I_1 will flow along path HGF. According to Kirchhoff's first law, the remaining part $I - 2I_1$ will flow along path HO. Again by symmetry, the current I flowing out of point D will be composed of part I_1 from path BCD, part I_1 from path FED and $(I - 2I_1)$ from path OD. Applying Kirchhoff's first law, it follows that no current will flow along path BO or FO. Thus, resistances connected across paths BO and FO are ineffective and hence the circuit shown in Fig. 2.32 may be represented as shown in Fig. 2.33.

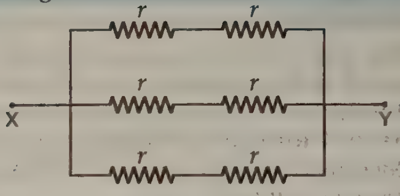


Fig. 2.33

The circuit is, now, equivalent to the pa combination of three equal resistors, each of resistan Therefore, resistance of the network between points X

$$R = \frac{2r}{3}$$

Problem 2.21. Eight equal resistors, each of resis *r* are connected along the edges of a pyramid OA having square base ABCD [Fig 2.34] Calculate the equiv resistance of the network between points A and B.

Sol. Let R be equivalent resistance between poi and B of the network of resistors shown in Fig. 2.34.

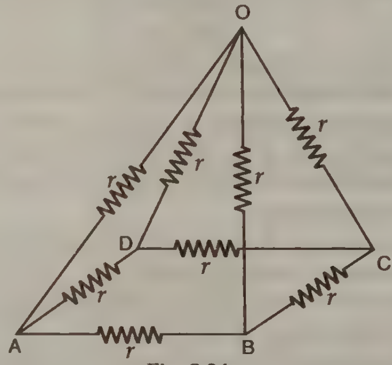


Fig. 2.34

In order to find R, connect a battery of e.m.f. E acro points A and B as shown in Fig. 2.35.

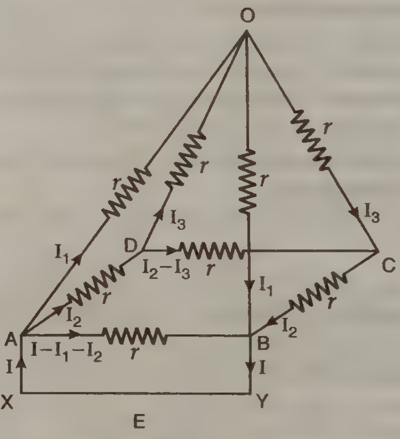


Fig. 2.35

If I is current drawn by the network from the ba then

$$R = \frac{E}{I}$$

ppose that when current I enters at point A, a part ; along path AO and another part I_2 along part AD. ccording to Kirchhoff's first law, the remaining part – I_2) will flow along path AB. By symmetry, the I flowing out from point B will be composed of part path OB, part I_2 from path CB and $(I - I_1 - I_2)$ from B. Also, when current I_2 enters at point D, if a part s along path DO, then remaining part $(I_2 - I_3)$ will ong path DC. Again, by symmetry, the current along C will be equal to I_3. The distribution of current in s branches of the circuit is as shown in the figure.

closed part ABOA of the circuit :

$$I_1 r + I_1 r - (I - I_1 - I_2) r = 0$$

$$3 I_1 + I_2 = I \qquad ...(ii)$$

closed part ADOA of the circuit :

$$I_1 r - I_2 r - I_3 r = 0$$

r $$I_1 - I_2 - I_3 = I \qquad ...(iii)$$

closed part CDOC of the circuit :

$$(I_2 - I_3) r - I_3 r - I_3 r = 0$$

r $$I_2 - 3 I_3 = 0 \quad \text{or} \quad I_3 = \frac{1}{3} I_2$$

ubstituting for I_3 in equation (*iii*), we get

$$I_1 - I_2 - \frac{1}{3} I_2 = 0 \quad \text{or} \quad I_1 = \frac{4}{3} I_2$$

ubstituting for I_1 in equation (*ii*), we get

$$3 \times \frac{4}{3} I_2 + I_2 = I \quad \text{or} \quad I_2 = \frac{1}{5} I$$

$$\therefore \quad I_1 = \frac{4}{3} I_2 = \frac{4}{3} \times \frac{1}{5} I = \frac{4}{15} I$$

n closed part ABYXA of the circuit :

$$E = (I - I_1 - I_2) r$$

Substituting for I_1 and I_2, we have

$$E = \left(I - \frac{4}{15} I - \frac{1}{5} I \right) r = \frac{8}{15} I r$$

or $$\frac{E}{I} = \frac{8}{15} r \qquad ...(iv)$$

From equation (*i*) and (*iv*), we have

$$R = \frac{8}{15} r$$

Problem 2.22. Find the equivalent resistance of the network of resistors shown in Fig. 2.36 between the ts X and Y.

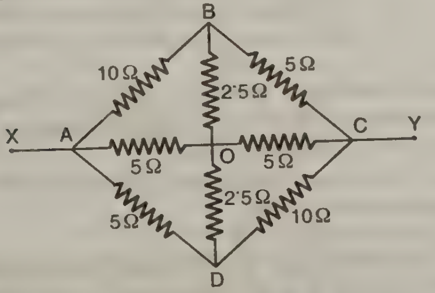


Fig. 2.36

Sol. Let R be equivalent resistance of the network of istors shown in Fig. 2.36. In order to find R, connect a battery of e.m.f E across the points X and Y as shown in Fig. 2.37.

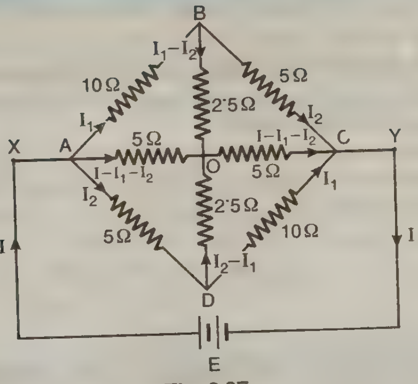


Fig. 2.37

If I is current drawn by the network from the battery, then

$$R = \frac{E}{I} \qquad ...(i)$$

Suppose that when current I enters at point A, a part I_1 flows along path AB and another part I_2 along AD. Then according to Kirchhoff's first law, the remaining part $(I - I_1 - I_2)$ will flow along path AO.

By symmetry, the current I flowing out from point C will be composed of part I_1 from path DC, part I_2 from path BC and $(I - I_1 - I_2)$ from path OC. Also, according to Kirchhoff's first law, the current along path BO will be $I_1 - I_2$ and that along path DO will be $I_2 - I_1$. The distribution of current in various branches of the circuit is as shown in the figure.

In closed part ABOA of the circuit :

$$I_1 \times 10 + (I_1 - I_2)\, 2{\cdot}5 - (I - I_1 - I_2) \times 5 = 0$$

or $$4 I_1 + (I_1 - I_2) - 2 (I - I_1 - I_2) = 0$$

or $$7 I_1 + I_2 = 2 I \qquad ...(ii)$$

In closed part BCOB of the circuit :

$$(I_1 - I_2) \times 2{\cdot}5 + (I - I_1 - I_2) \times 5 - I_2 \times 5 = 0$$

or $$(I_1 - I_2) + 2 (I - I_1 - I_2) - 2 I_2 = 0$$

or $$I_1 + 5 I_2 = 2 I \qquad ...(iii)$$

From equations (*ii*) and (*iii*), we have

$$7 I_1 + I_2 = I_1 + 5 I_2 \quad \text{or} \quad I_2 = \frac{3}{2} I_1$$

Substituting for I_2 in equation (*ii*), we get

$$7 I_1 + \frac{3}{2} I_1 = 2 I \quad \text{or} \quad I_1 = \frac{4}{17} I$$

Substituting for I_1 in equation (*ii*), we get

$$7 \times \frac{4}{17} I + I_2 = 2 I$$

or $$I_2 = \frac{6}{17} I$$

In closed part XADCYX of the circuit :

$$E = I_2 \times 5 + I_1 \times 10 = \frac{6}{17} I \times 5 + \frac{4}{17} I \times 10 = \frac{70}{17} I$$

or $$\frac{E}{I} = \frac{70}{17} \qquad ...(iv)$$

From equations (*i*) and (*iv*), we have

$$R = \frac{70}{17} = 4{\cdot}12\ \Omega$$

Problem 2.23. **(*a*) In a metre bridge, the balance point is found to be at 39·5 cm from the end A, when the resistor R is of 12·5 Ω [Fig. 2.05]. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or metre bridge made of thick copper strips ?**

(*b*) Determine the balance point of the bridge above, if X and R are interchanged.

(*c*) What happens, if the galvanometer and cell are interchanged at the balance point of the bridge ? Would the galvanometer show any current ? (*Text Problem*)

Sol. (*a*) Here, $R = 12{\cdot}5\ \Omega\ ;\ l = 39{\cdot}5$ cm

Now, $$X = R\frac{(100-l)}{l} = \frac{12{\cdot}5\,(100-39{\cdot}5)}{39{\cdot}5}$$

$$= \frac{12{\cdot}5 \times 60{\cdot}5}{39{\cdot}5} = \mathbf{19{\cdot}15\ W}$$

Thick copper strips are used to minimise the resista of the strips, as the resistance of connecting copper strips not been accounted for in the formula.

(*b*) If X and R are interchanged, then

$$R = X\frac{(100-l)}{l}$$

$$\therefore \quad 12{\cdot}5 = 19{\cdot}15 \times \frac{(100-l)}{l}$$

or $\quad 12{\cdot}5\,l = 1915 - 19{\cdot}15\,l \quad$ or $\quad 31{\cdot}65\,l = 1915$

or $\quad \mathbf{l = 60{\cdot}5\ cm}$ (from end

(*c*) The galvanometer will **not** show any current

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. What is the loop rule ?

Ans. Kirchhoff's second law is also known as loop rule. For statement, refer to section 2.01.

Q. 2.02. Draw the circuit diagram of a Wheatstone bridge. (*H.S.S.C.E. 1997 S, 1995*)

Ans. For circuit diagram of Wheatstone bridge, refer to Fig. 2.03.

Q. 2.02. Why is a slide wire bridge also called a metre bridge ?

Ans. It is because, length of wire in a slide wire bridge is one metre.

Q. 2.04. What is principle of a metre bridge ? (*H.S.S.C.E. 1995*)

Ans. A metre bridge is based on Wheatstone bridge.

Q. 2.05. Why are the copper strips in a slide wire bridge made thick ?

Ans. In case, the copper strips are not thick, then their resistances have to be included in the respective ratio arms. In case, the strips are thick, their resistances can be safely ignored.

Q. 2.06. The resistance in the left gap of a metre bridge is 10 Ω and the balance point is 40 cm from the left end. Calculate the value of the unknown resistance.

Ans. Here, $l = 40$ cm ; $R = 10\ \Omega$

Now, $$X = R \times \frac{(100-l)}{l} = 10 \times \frac{100-40}{40}$$

$$= 10 \times \frac{60}{40} = \mathbf{15\ \Omega}$$

Q. 2.07. Name the device used for measuring the e.m.f. of a cell. (*C.B.S.E. 1996*)

Ans. A potentiometer (after calibration) can be used to measure the e.m.f. of a cell.

Q. 2.08. Why is a potentiometer named as potentiometer ?

Ans. Because, it is used to measure potential difference.

Q. 2.09. Describe the principle of a potentiometer. (*H.S.S.C.E. 2000 ; Pre-degree Kerala, 1991*)

Ans. For principle of potentiometer, refer to section 2.07.

Q. 2.10. The wire of a potentiometer should be of uniform area of cross-section. Why ?

Ans. If resistance per unit length of the wire is uniform, then fall of potential per unit length of the wire will also uniform, when constant current is passed through it. C then, it satisfies the requirement of the principle potentiometer.

Q. 2.11. Of which material is a potentiometer w normally made and why ? (*P.S.S.C.E. 19*

Ans. The potentiometer wire is made of an alloy, s as eureka or manganin. It is because, an alloy has h resistivity and a low value of temperature coefficien resistance.

Q. 2.12. What type of cell should be used in the m circuit of the potentiometer and why ?

Ans. A Leclanche cell should be used in the main cir of the potentiometer. This is because of the fact that Leclan cell is useful, when the current is drawn for a short tim

Q. 2.13. In a potentiometer arrangement, a cell of e. 1·25 V gives a balance point at 35·0 cm length of the w If the cell is replaced by another cell and the balance p shifts to 63·0 cm, what is the e.m.f. of the second cell (*Text Quest*

Ans. Here, $E_1 = 1{\cdot}25$ V ; $l_1 = 35{\cdot}0$ cm ; $l_2 = 63{\cdot}0$ cm

Now, $$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

or $$E_2 = E_1 \times \frac{l_2}{l_1} = 1{\cdot}25 \times \frac{63{\cdot}0}{35{\cdot}0} = \mathbf{2{\cdot}25\ V}$$

Q. 2.14. Sometimes balance point may not be obtai on the potentiometer wire. Why ?

Ans. The balance point may not be obtained on potentiometer wire, in case the e.m.f. of the auxiliary bat (battery used to maintain constant current through potentiometer wire) is less than the e.m.f. of the cell to measured.

Q. 2.15. The e.m.f. of the cell used in the main cir of the potentiometer should be more than the poten difference to be measured. Why ?

Ans. If it is not so, the balance point will not be obtai on the potentiometer wire.

Also, refer to VSQ 2.14.

Q. 2.16. Name the device used for measuring internal resistance of a secondary cell. (*C.B.S.E. 19*

Ans. A potentiometer can be used to measure internal resistance of a secondary cell.

Q. 2.17. Write down the relation between the e.m. a cell and its internal resistance. (*I.S.C.E. 19* ns. The relation between e.m.f. and internal resistance ell is given by

$$r = \left(\frac{E - V}{V}\right) \times S,$$

where V is terminal potential difference between the two poles of the cell on connecting a resistance S across it.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. State Kirchhoff's laws.

(H.S.S.C.E. 1995 ; I.S.C.E. 1995)

Or

State Kirchhoff's rules for electrical network.

(H.S.S.C.E. 2002, 2001 ; C.B.S.E. 1996)

Ans. For statement of Kirchhoff's laws, refer to section

Q. 2.02. Explain the significance of Kirchhoff's law.

(P.S.S.C.E. 1992)

Ans. Ohm's law can be used only to analyse simple rical circuits. When the circuit contains more than one ce of e.m.f. and the distribution of current in various ches of the circuit is quite complicated, then one is unable alyse such a circuit by making use of Ohm's law. In such e, Kirchhoff's laws are used to analyse the circuit.

Q. 2.03. Find out the magnitude of resistance X in the it shown in Fig. 2.38, when no current flows through Ω resistor. *(I.S.C.E. 1998)*

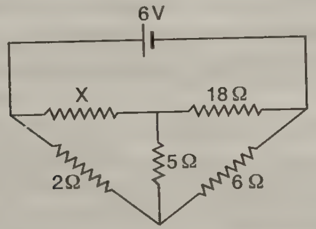


Fig. 2.38

Ans. Since no current flows through the 5 Ω resistor, the uit represents a balanced wheatstone bridge.

$$\therefore \quad \frac{X}{18} = \frac{2}{6} \quad \text{or} \quad X = \frac{2}{6} \times 18 = 6\,\Omega$$

Q. 2.04. What is the resistance between points A and n the circuit shown in Fig. 2.39 ? *(I.S.C.E. 1994)*

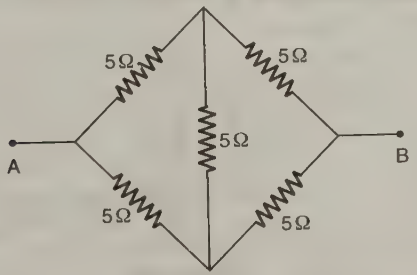


Fig. 2.39

Ans. The network represents a balanced Wheatstone idge and hence the resistance of 5 Ω connected across the idge can be considered as an open path. As such, the twork is equivalent to the circuit drawn in Fig. 2.40.

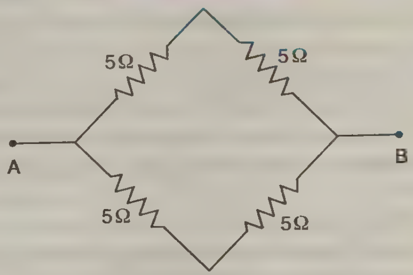


Fig. 2.40

The network is now a parallel combination of two resistive paths. The resistance of each resistive path is 5 + 5 *i.e.* 10 Ω. Hence, net resistance between points A and B,

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5} \quad \text{or} \quad R = 5\,\Omega$$

Q. 2.05. The wire AB of a slide wire bridge shown in Fig. 2.41 is 400 cm long. Where should the free end of the galvanometer be connected on AB, so that the galvanometer shows zero deflection ? *(C.B.S.E. 1997)*

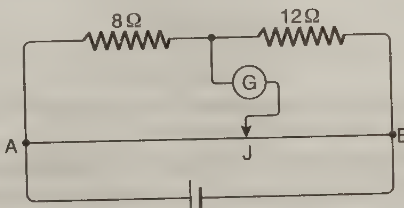


Fig. 2.41

Ans. Suppose that the galvanometer shows zero deflection, when the free end of the galvanometer is connected at point J. If length of the portion of wire AJ = l, then

$$JB = 400 - l$$

Since the bridge is balanced, we have

$$\frac{l}{400 - l} = \frac{8}{12} \quad \text{or} \quad l = 160 \text{ cm}$$

Q. 2.06. The variation of potential difference V with length *l* in case of two potentiometers X and Y is as shown in the Fig. 2.42. Which one of these two will you prefer for comparing e.m.fs. of two cells and why ? *(C.B.S.E. 1997)*

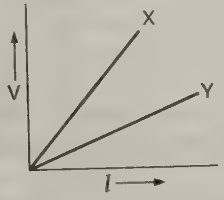


Fig. 2.42

Ans. A potentiometer is said to be more sensitive, if fall of potential per unit length along the wire *i.e.* potential gradient $\left(\frac{dV}{dl}\right)$ is small. The slope of V – l graph *i.e.* $\frac{dV}{dl}$ is smaller for wire Y than that for wire X. Therefore, potentiometer using wire Y will be preferred for comparing e.m.fs. of two cells.

Q. 2.07. How can you make a potentiometer of given wire length more sensitive using a resistance box ? *(H.S.S.C.E. 1994)*

Ans. When resistance is introduced by using a resistance box, the current passing through the potentiometer wire decreases. It results in lesser value of fall of potential per unit length of the wire *i.e.* lesser value of the least count.

Q. 2.08. Draw circuit diagram for the comparison of e..m.fs. of two cells with the help of a potentiometer. *(H.S.S.C.E. 2002)*

Sol. Refer to Fig. 2.07.

Q. 2.09. Draw the circuit diagram of a potentiometer for using it to find the internal resistance of a cell. *(H.S.S.C.E. 2001, P.S.S.C.E. 1998)*

Ans. Refer to Fig. 2.08.

Q. 2.10. In the circuit shown in Fig. 2.43, AB is a resistance wire of uniform cross-section in which a potential gradient of 0·01 V cm^{-1} exists.

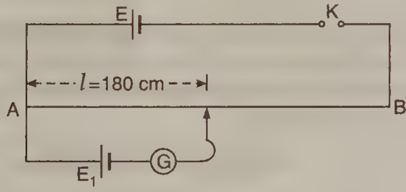


Fig. 2.43

(*i*) If the galvanometer G shows zero deflection, what is the e.m.f. E_1 of the cell used ?

(*ii*) If the internal resistance of the driver cell increases on some account, how will it change the balance point in the experiment ? *(C.B.S.E. 1998 S)*

Ans. (*i*) E_1 = potential drop across wire AJ

$= 180 \times 0{\cdot}01 = \mathbf{1{\cdot}8\ V}$

(*ii*) When the internal resistance of the driver cell increases, the current through wire AB and hence potential gradient along wire AB will decrease.

As such, the cell of e.m.f. E_1 (=1·8 V) will be bala across a length greater than 180 cm *i.e.* balance point J will towards right.

Q. 2.11. A cell of e.m.f. 4 V and of negligible int resistance is connected in series with a potentiometer of length 400 cm. The e.m.f. of Leclanche cell is four balance on 150 cm of the potentiometer wire. Find the e of the Leclanche cell. *(Karnataka, 19*

Ans. The fall of potential per unit length of pote meter wire $= \frac{4}{400} = 10^{-2}$ V cm^{-1}

Since Leclanche cell is found to be balanced ag 150 cm of potentiometer wire,

∴ the e.m.f. of Leclanche cell $= 150 \times 10^{-2} = \mathbf{1{\cdot}5\ V}$

Q. 2.12. Why do we prefer a potentiometer w longer bridge wire ? *(C.B.S.E.*

Ans. The smallest potential difference, which a p tiometer can measure gives the sensitivity of the pote meter. Therefore, for a potentiometer to be more sensitiv potential drop per unit length of its wire should be as s as possible. Obviously, if a potentiometer has a longer b wire, the potential drop per unit length will be lesser hence such a potentiometer will be able to measure very s potential differences.

Q. 2.13. In what respect is the potentiometer b than a moving coil voltmeter in comparing the e.m. two cells ? *(H.S.S.C.E. 2*

Or

Why is the use of a potentiometer preferred over of a voltmeter for measurement of e.m.f. of a cell ? *(C.B.S.E. 1998,*

Ans. When a voltmeter is connected across the two of a cell, it draws a small current from the cell. As a re the voltmeter measures terminal potential difference bet the two poles of the cell, which is always less than e.m the cell.

On the other hand, when a potentiometer is used fo measurement of e.m.f. of a cell, it does not draw any cu from the cell. Hence, it measures e.m.f. of the cell. For reasons, a potentiometer is preferred over a voltmete measuring e.m.f. of a cell.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For **a**mbitious, **b**rilliant & **c**urious Students

Q. 2.01. If each of the resistances in network shown in Fig. 2.44 is *r*, what is the resistance between terminals A and B ? *(H.S.S.C.E. 2001 ; H.P.S.S.C.E. 1995 ; I.I.T. 1978)*

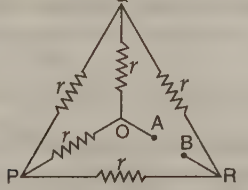


Fig. 2.44

Ans. The given network can be redrawn as shown in Fig. 2.45. It represents a balanced Wheatstone bridge *i.e.* if a source of e.m.f. is connected across terminals A and B, no curren flow along the path PQ. Therefore, the path PQ in the net is equivalent to an open path and the given network ca represented as shown in Fig. 2.46.

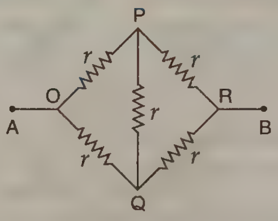


Fig. 2.45

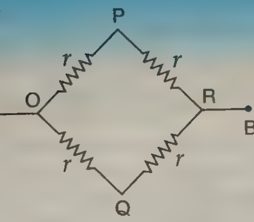


Fig. 2.46

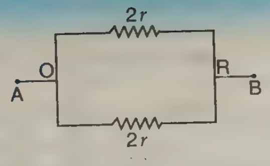


Fig. 2.47

The resistance of each of the two paths PR and QR is $2r$ own in Fig. 2.47. If R is the resistance of the network een terminals A and B, then

$$\frac{1}{R}=\frac{1}{2r}+\frac{1}{2r}=\frac{1}{r} \quad \text{or} \quad R=r$$

Q. 2.02. In the circuit shown in Fig. 2.48, the resistances P and R are unequal. If the reading of the galvano- r remains the same with switch S open or closed, then is the value of current through galvanometer ? *(I.I.T. 1999)*

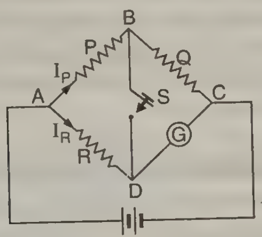


Fig. 2.48

Ans. Since the closing or opening of the switch S does affect the reading of the galvanometer, the points B and e at same electric potential. The current I_P flows as such ugh the resistance Q, while the current I_R flows through galvanometer. Thus, the current through the anometer is equal to I_R.

Q. 2.03. A battery of e.m.f. 9 V and negligible internal resistance is connected to a 3 kΩ resistor. The potential p across a part of the resistor (between points A and B) easured by (*a*) a 20 kΩ voltmeter [Fig. 2.49], (*b*) a 1 kΩ tmeter [Fig. 2.50]. In (*c*) both the voltmeters are nected across AB [Fig. 2.51].

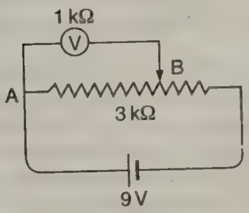


Fig. 2.49

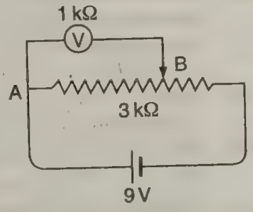


Fig. 2.50

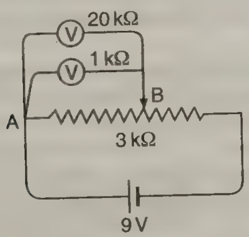


Fig. 2.51

In which case would you get the (*i*) highest, (*ii*) lowest ading ?

Ans. The voltmeter, which will draw minimum current (so that potential difference across AB will decrease by a minimum amount), will give the highest reading.

In case (*a*), $R_V = 20\ \text{k}\Omega$

In case (*b*), $R_V = 1\ \text{k}\Omega$

In case (*c*), $\frac{1}{R_V}=\frac{1}{20}+\frac{1}{1}=\frac{1+20}{20}$ or $R_V=\frac{20}{21}\ \text{k}\Omega$

(*i*) Since in case (*a*), the resistance of voltmeter is maximum (20 kΩ) ; it will draw minimum current and hence will give maximum reading.

(*ii*) Since in case (*c*), R_V is minimum $\left(\frac{20}{21}\ \text{k}\Omega\right)$, it will give lowest reading.

Q. 2.04. Fig. 2.52 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor R = 10 Ω is found to be 58·3 cm, while that with the unknown resistance X is 68·5 cm. (*a*) Determine the value of X. (*b*) What would you do, if you failed to find a balance point with the given cell E ? *(Text Question)*

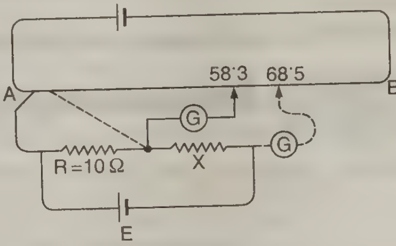


Fig. 2.52

Ans. (*a*) Let E_1 and E_2 be potential drops across R and X respectively.

Obviously, $\frac{E_2}{E_1}=\frac{X}{R}$

or $X=\left(\frac{E_2}{E_1}\right)R=\left(\frac{l_2}{l_1}\right)R$

Here, $R = 10\ \Omega$; $l_1 = 58{\cdot}3$ cm and $l_2 = 68{\cdot}5$ cm

$\therefore$ $X=\frac{68{\cdot}5}{58{\cdot}3}\times 10 = \mathbf{11{\cdot}75\ \Omega}$

(*b*) We fail to get balance point with the given cell of e.m.f. E, if potential drop across R or X is greater than the potential difference across the wire AB. In order to obtain the balance point, either a suitable **resistance should be put in series with R and X** (so as to decrease current through wire AB and hence to decrease potential drop across the wire) or **a cell of lesser e.m.f. should be used.**

Q. 2.05. Fig. 2.53. shows a potentiometer with a cell of 2·0 V and internal resistance 0·4 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant e.m.f. E_1 = 1·02 V (for very moderate currents up to a few ampere) gives a balance point at 67·3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown e.m.f. E_2 and the balance point found similarly turns out to be at 82·3 cm length of the wire.

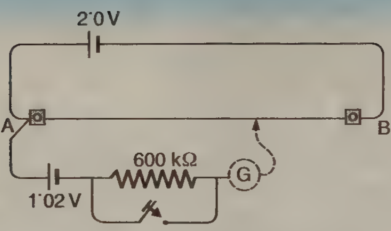


Fig. 2.53

(a) What is the value of E_2 ?

(b) What purpose does the high resistance of 600 kΩ have ?

(c) Is the balance point affected by this high resistance ?

(d) Is the balance point affected by the internal resistance of the driver cell ?

(e) Would the method work in the above situation, if the driver cell of the potentiometer had an e.m.f. of 1·0 V instead of 2·0 V ?

(f) Would the circuit work well for determining extremely small e.m.f., say of the order of a few mV as the typical e.m.f. of a thermocouple) ? (*Text Ques*

Ans. (*a*) Here, $l_1 = 67{\cdot}3$ cm ; $l_2 = 82{\cdot}3$ cm ; $E_1 = 1{\cdot}0$

Now, $$\frac{E_2}{E_1} = \frac{l_2}{l_1}$$

or $$E_2 = \frac{l_2}{l_1} \times E_1 = \frac{82{\cdot}3}{67{\cdot}3} \times 1{\cdot}02$$

$$= \mathbf{1{\cdot}247\ V}$$

(*b*) The purpose of using high resistance is to **allow a very small current** to flow through the galvanometer, the balance point has not been obtained.

(*c*) No, the balance point is **not** affected by the pre of high resistance.

(*d*) No, the balance point is **not** affected by the int resistance of the driver cell.

(*e*) No, the arrangement will **not** work. If the e.m.f. driver cell is less than that of the cell whose e.m.f. is found, the balance point will not be obtained.

(*f*) The circuit is **not** suitable for measuring extre small e.m.f. It is because in such a case, the balance poin be just close to the end A.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. State and explain Kirchhoff's laws for electrical circuits. (*H.S.S.C.E. 2002, 1996 ; P.S.S.C.E. 2001, 1999 S ; H.P.S.S.C.E. 1999 S ; P.S.S.C.E. 1999 S, 1994 ; Pre-degree Kerala, 1994 ; C.B.S.E. 1992*)
2. State Kirchhoff's laws for electrical circuits and explain them giving illustrations. (*H.S.S.C.E. 1998*)
3. Apply Kirchhoff's laws to obtain the condition of balanced Wheatstone bridge. (*P.S.S.C.E. 1998 S ; H.S.S.C.E. 1998 S ; H.P.S.S.C.E. 1998*)
4. Derive the principle of Wheatstone bridge using Kirchhoff's laws ? (*H.S.S.C.E. 2002 ;P.S.S.C.E. 1999, 1997 S ; I.S.C.E. 1994, 1993*)
5. Using Kirchhoff's laws, derive the condition for balance of a Wheatstone bridge circuit ? (*H.S.S.C.E. 2002 ; P.S.S.C.E. 2001 ; C.B.S.E. 1995*)
6. Explain with the help of a circuit diagram, how the value of an unknown resistance can be determined using a Wheatstone bridge. Give the formula used. (*C.B.S.E. 1996*)
7. Explain the principle of Wheatstone bridge for determining an unknown resistance. How is it realised in actual practice in the laboratory ? (*C.B.S.E. 2002*)
8. What is a slide wire bridge ? How can you find unknown resistance by it ? (*H.P.S.S.C.E. 2002*)
9. With a circuit diagram, briefly expalin how a metre bridge can be used to find the unknown resistance of a given wire. State the formula used. (*C.B.S.E. 2001*)
10. Draw a circuit diagram of a metre bridge arranged to compare two resistances. Explain the principle of the experiment and give the formula used. (*C.B.S.E. 1995*)
11. Draw a circuit diagram of a metre bridge arranged to determine an unknown resistance. Explain the principle of the experiment and give the formula used. (*C.B.S.E. 1995*)
12. Draw a neatly labelled diagram of a simple potentiometer and explain its principle. (*C.B.S.E. 1998 ; I.S.C.E. 1995*)
13. Explain the principle of a potentiometer. What are its uses ? (*H.S.S.C.E. 2002*)
14. Explain the principle on which working of a potentio is based. Why is the use of a potentiometer preferre that of a voltmeter for measurement of e.m.f. of a c (*H.P.S.S.C.E.*
15. What is a potentiometer ? Explain, how it is us compare the e.m.fs. of two cells. (*H.P.S.S.C.E.2002,*
16. Define e.m.f. of a cell. Explain how will you compa e.m.fs. of two cells with the help of a potentiomete (*P.S.S.C.E.*
17. What is the principle of a potentiometer ? How is i to compare e.m.fs of two cells. (*P.S.S.C.E. 1999 S ; C.B.S.E. 1999, 1998, 1994, H.P.S.S.C.E. 1997 ; H.S.S.C.E.*
18. With the help of a circuit diagram ; explain h potentiometer can be used to compare e.m.fs. of twc State the formula used. (*H.S.S.C.E. 2002 ; C.B.S.E. 2000 ; H.P.S.S.C.E.*
19. Explain, how you will compare the e.m.fs.' of two ce a potentiometer. (*C.B.S.E.*
20. Draw a circuit diagram and explain how the e.m the two primary cells can be compared by us potentiometer ? State the formula used. (*C.B.S.E. 2000,*
21. Define e.m.f. of a cell. Explain how the internal resis of a primary cell can be determined using a potentior (*P.S.S.C.E.*
22. Explain, with the help of a circuit diagram, the u potentiometer for determination of internal resistar a primary cell. Derive the necessary mathem expression. (*C.B.S.E. 1999, 1998 ; H.P.S.S.C.E.*
23. Give the principle of a potentiometer. Explain hov you use it to measure the internal resistance of a ce (*P.S.S.C.E. 1999 S ; H.S.S.C.E. 1*
24. With the help of a circuit diagram, explain how the in resistance of a cell can be determined usi potentiometer. Write the formula used. (*H.S.S.C.E. 2002 ; C.B.S.E. 2000,*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

State and explain Kirchhoff's laws. Derive the condition for obtaining balance in a Wheatstone bridge.
(H.P.S.S.C.E. 2000 ; P.S.S.C.E. 2000 ; C.B.S.E. 1990)

State Kirchhoff's laws of electric circuits and using these laws, derive Wheatstone bridge principle ?
(P.S.S.C.E. 2002, 2001)

What is Wheatstone bridge ? Derive the condition for balance of a Wheatstone bridge. How does it form the principle of a slide wire bridge ? Draw a circuit diagram to compare two resistances with the help of a slide wire bridge.

4. Describe the principle and construction of potentiometer. Explain, how a potentiometer can be used to compare the e.m.fs. of two cells. *(P.S.S.C.E. 2002)*

5. Give the principle of a potentiometer. Explain, how will you use a potentiometer to (*i*) measure the internal resistance of a cell and (*ii*) to compare the e.m.fs. of two cells.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

A. On Kirchhoff's laws

Fig. 2.54 shows a part of an electric current consisting of resistors of resistances r, $2r$ and $3r$. The potentials at the points A, B and C are 3 E, 2 E and E respectively. Find the potential at point O. Also, find the value of currents I_1, I_2 and I_3.

$$\left[\textbf{Ans. } \frac{26\,E}{11};\ I_1=\frac{7\,E}{11\,r};\ I_2=\frac{2\,E}{11\,r};\ I_3=\frac{5\,E}{11\,r}\right]$$

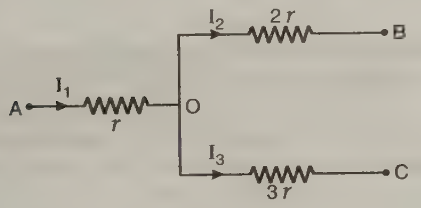


Fig. 2.54

Two cells of e.m.f. $E_1 = 1{\cdot}5$ V and $E_2 = 2{\cdot}0$ V and internal resistance of $r_1 = 1{\cdot}0\ \Omega$ and $r_2 = 1{\cdot}5\ \Omega$ respectively are connected to an external resistance of 2·5 Ω as shown in Fig. 2.55. Find the current in the circuit and the potential difference between points B and C. **[Ans. 0·1 A, 1·85 V]**

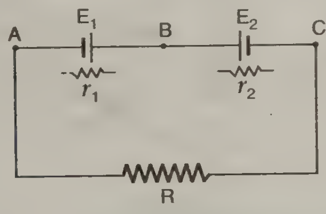


Fig. 2.55

3. In the circuit given in Fig. 2.56, find the potential difference between the points A and B. Assume that both the batteries have zero internal resistance. **[Ans. 9·6 V]**

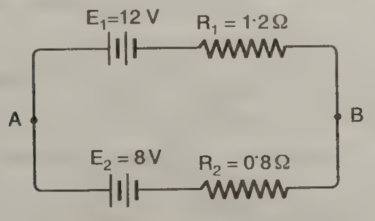


Fig. 2.56

4. In the network shown in the Fig. 2.57 :

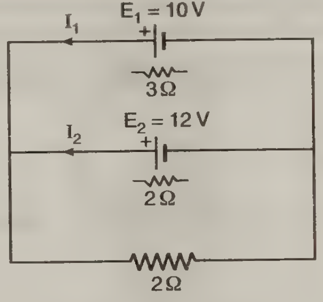


Fig. 2.57

(*i*) What is current in 2 Ω resistor ?

(*ii*) What is terminal voltage of each battery ?

[Ans. (*i*) 3·5 A ; (*ii*) across E_1 or E_2 = 7 V**]**

5. The circuit diagram shown in Fig. 2.58 has two cells E_1 and E_2 with e.m.f. 4 V and 2 V respectively, each one having an internal resistance 2 Ω. The external resistance R is of 8 Ω. Find the magnitude and direction of the currents flowing through the two cells. *(I.S.C.E. 1998)*

[Ans. $I_1 = 2/3$ A, $I_2 = -1/3$ A**]**

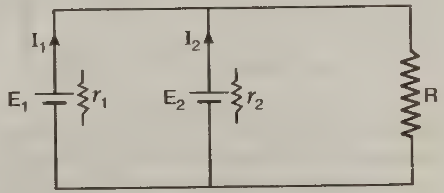


Fig. 2.58

6. Using Kirchhoff's laws, determine the currents I_1, I_2 and I_3 for the network shown in Fig. 2.59. *(C.B.S.E. 1999 S)*

[Ans. $I_1 = 3$ A, $I_2 = -1{\cdot}5$ A, $I_3 = 4{\cdot}5$ A**]**

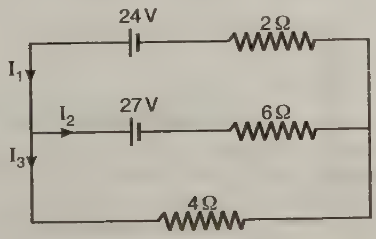


Fig. 2.59

7. A cell of e.m.f. E_1 (= 2 V) and internal resistance 1 Ω is connected in parallel with another cell of e.m.f. E_2 (= 1·5 V) and internal resistance 2 Ω. When the combination is in parallel with a resistance of 4 Ω, find the current through each branch and also the potential difference across the 4 Ω resistance. *(Karnataka, 1990)*

$$\left[\textbf{Ans. } I_1 = \frac{3}{7}\text{ A}, I_2 = -\frac{1}{28}\text{ A}; I = \frac{11}{28}\text{ A}; \frac{11}{7}\text{ V}\right]$$

8. Two cells of e.m.fs. 6 V and 12 V and internal resistances 1 Ω and 2 Ω respectively are connected in parallel so as to send current in the same direction through an external resistance of 15 Ω. Draw the circuit diagram. Using Kirchhoff's laws, calculate : (*i*) current through each branch of two circuit, (*ii*) potential difference across the 15 Ω resistance. *(H.S.S.C.E. 2002)*

[**Ans.** (*i*) I_1 due to 6 V cell) = – 1·66 A, I_2 (due to 12 V cell) = 2·17 A, I (through 15 Ω) = 0·51 A (*ii*) 7·65 V)

9. Find the currents through all the arms in the network shown in Fig. 2.60 of a cell and resistances.

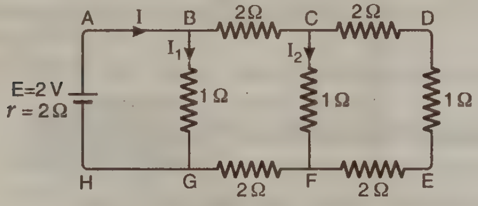


Fig. 2.60

$$\left[\textbf{Ans. } I = \frac{70}{99}\text{ A}; I_1 = \frac{58}{99}\text{ A}; I_2 = \frac{10}{99}\text{ A}; I_{BC} = I_{FG} = \frac{-12}{99}\text{ A}\right.$$

$$\left.\text{and } I_{CD} = I_{DE} = I_{EF} = \frac{2}{99}\text{ A}\right]$$

10. In the circuit shown in Fig. 2.61, find the current through 4 Ω resistor. *(I.I.T. 1998)* [**Ans.** 0·25 A]

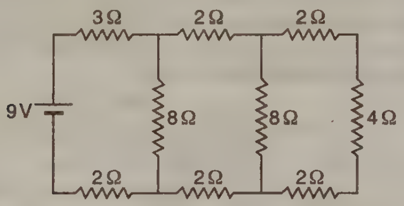


Fig. 2.61

11. Two cells of e.m.f. of 1· 6 V and 2·4 V have internal resistance 2 Ω and 4 Ω respectively. The two positive poles are connected to resistance of 6 Ω and negative poles connected to 8 Ω coil. If another wire of 10 Ω is connected between the mid-points of these wires, what is the potential difference across it. *(Karnataka, 1984)* [**Ans.** 1·31 V]

12. In the network shown in the Fig. 2.62 :

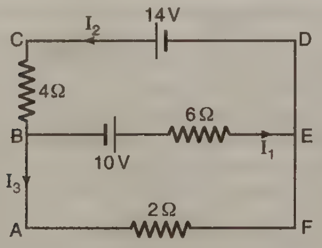


Fig. 2.62

(*i*) Find the current I_1, I_2 and I_3.

(*ii*) Find the potential difference between points B and E.

[**Ans.** (*i*) I_1 = 2 A, I_2 = 3 A, I_3 = 1 A ; (*ii*) 2 V]

13. Find the current in the three resistors shown in Fig.

[**Ans.** Zero in each resista

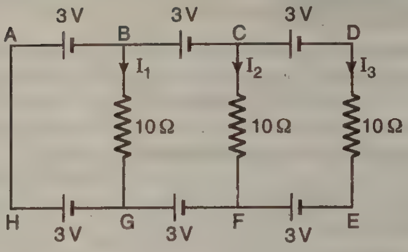


Fig. 2.63

14. In the network, given in the Fig. 2.64 :

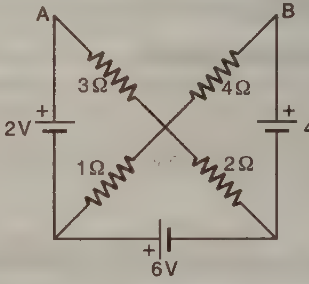


Fig. 2.64

(*i*) Calculate the current through the 6 V battery.

(*ii*) Determine the potential difference between points A B. [**Ans.** (*i*) 1·3 A ; (*ii*)

15. Determine the values of currents I_1, I_2 and I_3 in the netw shown in Fig. 2.65. Given that $R_1 = R_2 = 2\ \Omega$, $R_3 = 1\ \Omega$, $R_5 = 4\ \Omega$, $E_1 = 5$ V and $E_2 = 10$ V.

[**Ans.** $I_1 = I_3 = 1{\cdot}875$ A, $I_2 = 2$

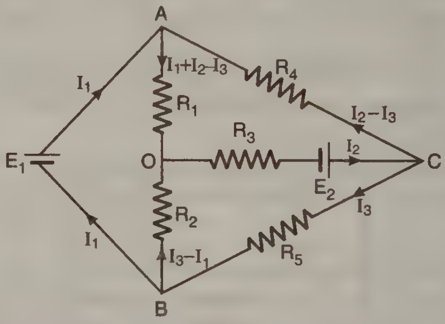


Fig. 2.65

Type B. On Wheatstone Bridge

16. Five resistances are connected as shown in Fig. 2.66. W is the effective resistance between points X and Y ? *(H.S.S.C.E. 1996)* [**Ans.** 3·33 c

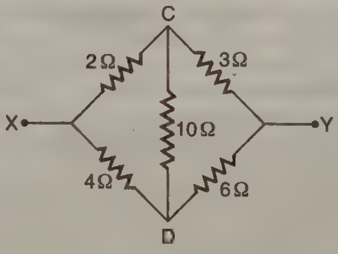


Fig. 2.66

7. Calculate the resistance between points X and Y in the circuit shown in the Fig. 2.67. **[Ans. 8 Ω]**

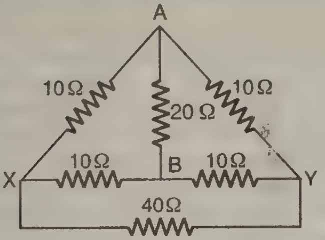


Fig. 2.67

3. Find the equivalent resistance of the network shown in Fig. 2.68 between points X and Y. **[Ans. 10 Ω]**

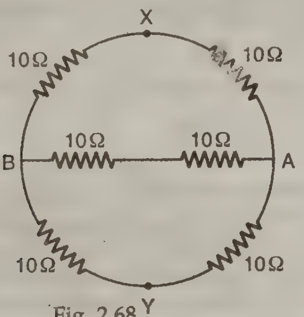


Fig. 2.68

9. Six equal resistors, each of 5 Ω are connected to a battery of e.m.f. 3 V as shown in Fig. 2.69. What is the current drawn by the circuit from the battery ? **[Ans. 1·2 A]**

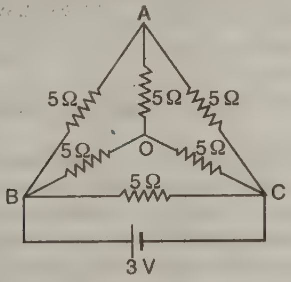


Fig. 2.69

0. For the network shown in the Fig. 2.70, determine the value of R and the current through it, if the current through the branch AO is zero. **[Ans. 6 Ω, 0·5 A]**

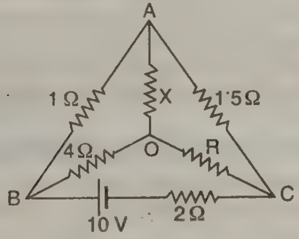


Fig. 2.70

1. Five equal resistors, each of 2 Ω, are connected in a network as shown in Fig. 2.71. Find the equivalent resistance between points X and Y. (*H.S.S.C.E. 1999 S*) **[Ans. 2 Ω]**

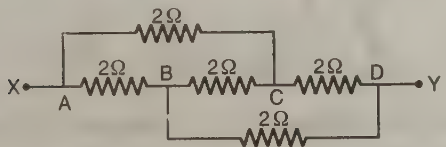


Fig. 2.71

22. Calculate the current drawn by the network of resistors from the battery as shown in Fig. 2.72. **[Ans. 2 A]**

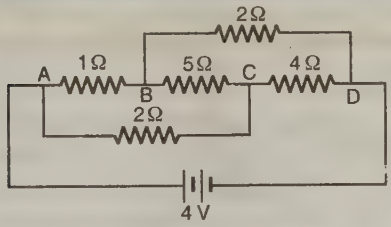


Fig. 2.72

23. In a Wheatstone's bridge network, resistances P = 3 Ω, Q = 2 Ω, R = 6 Ω, S = 4 Ω and the galvanometer resistance G = 5 Ω. Calculate the currents in different branches of the network. E.M.F. of the cell is 2 V. Ignore internal resistance of the cell (*H.S.S.C.E. 2002*)
[Ans. I = 0·6 A ; $I_P = I_Q$ = 0·4 A, $I_R = I_S$ = 0·2 A, I_G = 0]

24. In the circuit shown in Fig. 2.73, find the currents I, I_1, I_2 and I_3. Given that e.m.f. of the battery = 2 V, internal resistance of the battery = 2 Ω and resistance of the galvanometer = 4 Ω. **[Ans. I = 47/91 A, I_1 = 17/91 A, I_2 = 30/91 A, I_3 = − 1/91 A]**

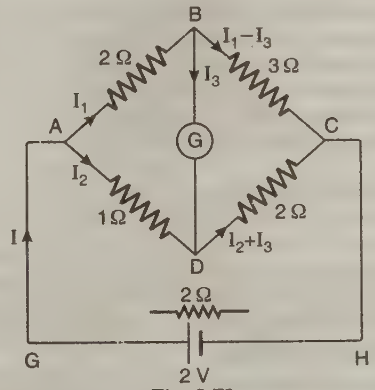


Fig. 2.73

25. Determine the current in each branch of the network as shown in the Fig. 2.74. **[Ans. I = 10/17 A, I_1 = 4/17 A, I_2 = 6/17 A, I_3 = − 2/17 A, I_4 = 6/17 A, I_5 = 4/17 A]**

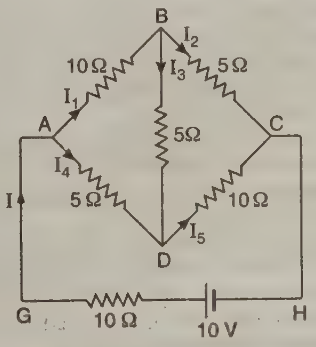


Fig. 2.74

Type C. On Slide wire Bridge

26. Two parallel resistors of 5 ohm and 20 ohm are connected in left arm of a metre bridge. If the null point is at 40 cm from left end of the wire, calculate the value of resistance connected in right arm. **[Ans. 6 Ω]**

27. In the circuit shown in Fig. 2.75, a metre bridge is shown in its balanced state. The metre bridge has a resistance of 1 Ω cm^{-1}. Calculate the value of the unknown resistance X and the current drawn from the battery of e.m.f. 6 V and

negligible internal resistance. (*C.B.S.E. 1996*)
[**Ans.** 2 Ω, 1·26 A]

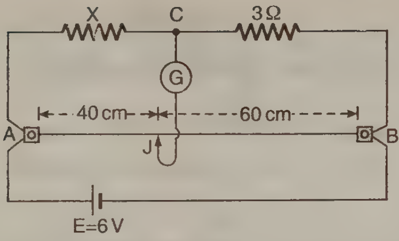


Fig. 2.75

28. In a slide wire bridge, when a resistor of resistance 2 Ω is in left gap and unknown resistance in right gap, the balance point is obtained at 40 cm from the zero end of the wire. Find the shift in the balance point, when the unknown resistor is shunted with a resistor of 2 Ω.
[**Ans.** 22·5 cm]

29. In a metre bridge experiment, resistors of resistance 5 Ω and 2 Ω are connected in the left and right gap respectively. When the resistor of 5 Ω is shunted by a wire, the balance point is found at 62·5 cm mark. What is the resistance of the wire ? [**Ans.** 10 Ω]

30. In a metre bridge experiment, a resistance R is kept in the right gap in all the observations. When resistances R_1 and R_2 are connected in trun in the left gap, the balance point is obtained at 40 cm and 50 cm from the zero end of the wire. Find the position of balance point, when the left gap contains R_1 and R_2 (*a*) in series and (*b*) in parallel.
[**Ans.** (*a*) 62·5 cm (*b*) 28·6 cm]

31. In comparing the resistance of two coils P and Q with a metre bridge, the balance point is obtained at 30 cm from the zero end. The coils P and Q are interchanged and the balance point is obained at 120 cm from the zero end. Find (*a*) the ratio of the resistance of the two coils and (*b*) the length of the bridge wire. [**Ans.** (*a*) 1 : 4 (*b*) 150 cm]

Type D. On Potentiometer

32. A cell gives a balance with 85 cm of a potentiometer wire. When the terminals of the cell are shorted through a resistance of 7·5 Ω, the balance is obtained at 75 cm. Find the internal resistance of the cell. (*I.S.C.E. 1995*)
[**Ans.** 1 Ω]

33. It is found that 125 cm length of a potentiometer wire is required to balance the e.m.f. of a Daniel cell but that only 100 cm of the wire is required for the balance, if the poles of the cell are joined to a resistance of 2 ohm. Calculate the internal resistance of the cell. [**Ans.** 0·5 ohm]

34. A cell can be balanced against 110 cm and 100 cm of potentiometer wire respectively, when open circuited and when short circuited through a resistance of 10 ohm. Find the internal resistance of the cell. [**Ans.** 1 Ω]

35. In an experiment to determine the internal resistance of a cell, the null point is obtained at 220 cm, when the cell is shunted by a resistance of 5 Ω. When the cell is shunted by a resistance of 20 Ω, the null point is obtained at 300 cm. Find the internal resistance of the cell. [**Ans.** 2·76 Ω]

36. In the circuit diagram shown in Fig. 2.76, AB is a uniform wire of resistance 10 Ω and length 1 m. It is connected series arrangement of cell E_1 of e.m.f. 2 V and negligi internal resistance and a resistor R. Terminal A is a connected to an electrochemical cell E_2 of e.m.f. 100 and a galvanometer G. In this set up, a balancing poin obtained at 40 cm mark from A. Calculate the resista of R. If E_2 were to have an e.m.f. of 300 mV, where will expect the balancing point to be ? (*C.B.S.E. 1999*
[**Ans.** 70 Ω, balance point will not be obtain

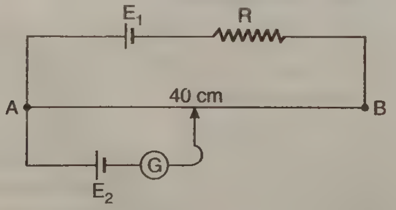


Fig. 2.76

37. Two cells of e.m.f. E_1 and E_2 ($E_1 > E_2$) are connected shown in Fig. 2.77. When a potentiometer is connec between points A and B, the balancing length of potentiomter wire is 300 cm. On connecting the sa potentiometer between points A and C, the balanci length is 100 cm. Calculate the ratio of the e.m.fs. of the t cells. (*C.B.S.E. 1994*) [**Ans.** 3

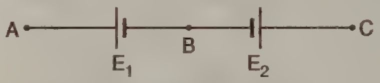


Fig. 2.77

38. A potentiometer has five segments of wire, each of len 1 m. A battery is used to pass a steady current through It is found that the e.m.f. of a cell can be balanced at cm from the zero end. Now, the battery is connected acr the four segments of wire. Find the length of potentiometer wire that will balance the e.m.f. of the c
[**Ans.** 280 c

39. A cell of e.m.f. 2·56 V and negligible internal resistanc in series with a 1 m long potentiometer wire and a resis The resistance per unit length of the wire is 0·025 ohm cm. The potential difference across a length 50·9 cm of potentiometer wire balances the e.m.f. of 1·018 V c standard cell. Find the resistance of the resistor.
[**Ans.** 0·7

40. A standard cell of e.m.f. 1·08 V is balanced by the poten difference across 91 cm of a metre long wire supplied a cell of e.m.f. 2 V through a series resistor of resista 2 Ω. The internal resistance of the 2 V cell is zero. Find resistance per unit length of the potentiometer wire.
[**Ans.** 0·0292 Ω cm

41. A potentiometer wire carries current. The potent difference across 70 cm of it balances the potent difference across a 2 ohm coil supplied by a cell of e. 2 volt. When a 1 ohm coil is placed in parallel with 2 ohm coil, a length equal to 50 cm of the potentiome wire is required to balance the potential difference acr the parallel combination. What is the internal resistance the 2 V cell ? [**Ans.** 0·5

MISCELLANEOUS PROBLEMS

42. Find the resistance between points X and Y of the netw of resistors shown in Fig. 2.78, if each resistor is resistance *r*.

[**Ans.** $\frac{8}{7}$

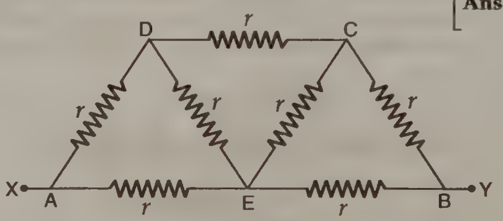


Fig. 2.78

Find the equivalent resistance of the network shown in Fig. 2.79 between the points A and B. **[Ans.** 0·8 r**]**

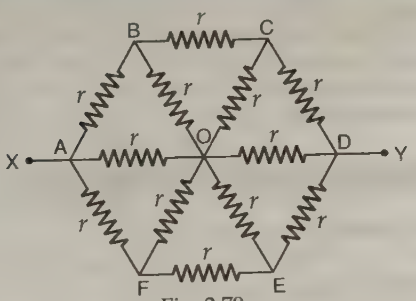


Fig. 2.79

. Find the resistance between points X and Y of the network of resistors shown in Fig. 2.80, if each resistor is of resistance r. $\left[\textbf{Ans. } \frac{10}{9} r\right]$

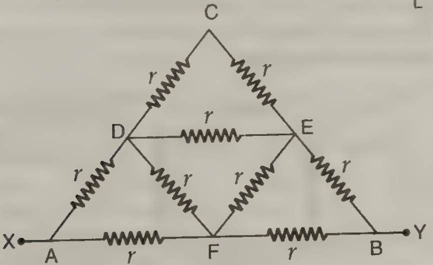


Fig. 2.80

5. Find the effective resistance between the points A and B of the network shown in Fig. 2.81. Each resistor has shown in Fig. 2.81. **[Ans.** 0·5 r**]**

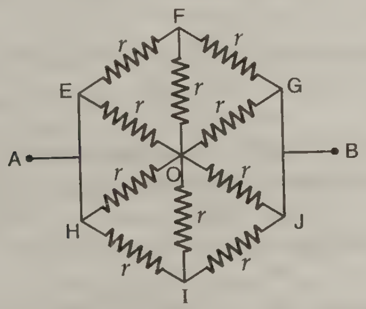


Fig. 2.81

6. Eleven resistors, each of resistance r are connected in the form of an incomplete cube as shown in Fig. 2.82. Find the resistance of the network between the points A and B of the network. $\left[\textbf{Ans. } \frac{7}{5} r\right]$

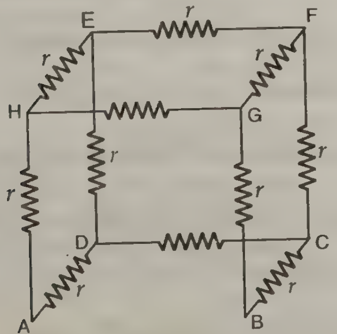


Fig. 2.82

47. ABCD is a uniform circular wire of resistance 2 Ω. AOC and BOD are two wires (each of resistance 1 Ω) forming diameters at right angles to each other. Show that the resistance of the network is 15/14 Ω, if the battery is placed in AD.

48. In the circuit shown in Fig. 2.83, calculate the potential difference across the capacitor C. **[Ans.** Zero**]**

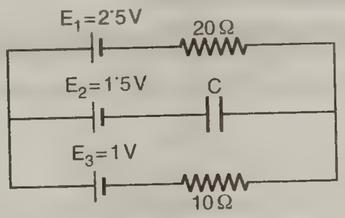


Fig. 2.83

49. In the network shown in Fig. 2.84, P = 2 Ω, Q = 2 Ω, R = 3 Ω, S = 7 Ω, C = 3 μF and E = 6 V. (*a*) Find the charge and energy stored in the capacitor. (*b*) For what value of Q, the charge on the capacitor will be zero ?

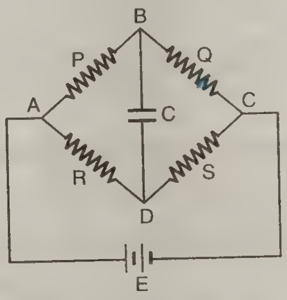


Fig. 2.84

[Ans. (*a*) 3·6 μC ; 2·16 × 10^{-6} J (*b*) = 14/3 Ω**]**

50. A certain unknown resistance is connected in the left gap and a resistance box in the right gap of a metre bridge. By introducing a resistance of 10 Ω with the help of resistance box, the balance point is determined. If the balance point shifts by 20 cm on increasing the resistance from the resistance box by 12·5 Ω, find the value of unknown resistance. **[Ans.** 15 Ω**]**

51. A short circuit occurs in a telephone cable having a resistance of 0·45 Ω m^{-1}. The circuit is tested with a Wheatstone bridge. The two resistors in the ratio arms of the Wheatstone bridge network have values of 1110 Ω and 100 Ω respectively. A balance condition is found, when the variable resistor has a value of 400 Ω. Calculate the distance down the cable, where the short has occurred. **[Ans.** 40 m**]**

52. In the circuit shown in Fig. 2.85, the uniform wire AB of the metre bridge is 1 m long and is of resistance 10 Ω. If the bridge is in balanced state, find the distance of the balance point from the end A. **[Ans.** 37·5 cm**]**

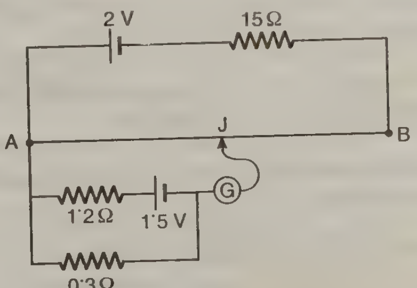


Fig. 2.85

53. A cell of e.m.f. 1·08 V is in series with a galvanometer. The series combination is placed across a resistor of resistance 108 ohm which is in series with a potentiometer wire, a battery of e.m.f. 4 V and zero internal resistance and another variable resistor. The resistance of the 1 m long potentiometer wire is 2 Ω. What should be the resistance of the variable resistor, so that the deflection in the galvanometer is to be zero? **[Ans. 290 Ω]**

54. The length of a potentiometer wire is 1200 cm and it car a current of 80 mA. For a cell of e.m.f. 4 V and inter resistance 20 Ω, the null point is found to be at 1000 cm a voltmeter is connected across the cell, the balanc length decreases by 20 cm. Find (*a*) the resistance of potentiometer wire, (*b*) reading of the voltmeter and (*c*) resistance of the voltmeter. (*C.B.S.E.* 2000 **[Ans.** (*a*) 60 Ω, (*b*) 3·92 V, (*c*) 980

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. At the junction O, according to Kirchhoff's first law, we have

$$I_1 - I_2 - I_3 = 0 \qquad ...(i)$$

Here, $V_A = 3\,E$, $V_B = 2\,E$ and $V_C = E$

Let V be potential at point O. Then,

$$V_A - V = I_1 \times r \quad \text{or} \quad 3\,E - V = I_1\, r$$

$$\text{or} \quad I_1 = \frac{3\,E - V}{r} \qquad ...(ii)$$

Again, $V - V_B = I_2 \times 2\,r$ or $V - 2\,E = 2\,I_2\, r$

$$\text{or} \quad I_2 = \frac{V - 2\,E}{2\,r} \qquad ...(iii)$$

Also, $V - V_C = I_3 \times 3\,r$ or $V - E = 3\,I_3\, r$

$$\text{or} \quad I_3 = \frac{V - E}{3\,r} \qquad ...(iv)$$

Substituting for I_1, I_2 and I_3 in equation (*i*), we have

$$\frac{3\,E - V}{r} - \frac{V - 2\,E}{2\,r} - \frac{V - 3\,E}{3\,r} = 0$$

$$\text{or} \quad 6\,(3\,E - V) - 3\,(V - 2\,E) - 2\,(V - 3\,E) = 0$$

$$\text{or} \quad 11\,V = 26\,E \quad \text{or} \quad V = \frac{26\,E}{11}$$

$$\text{Now,} \quad I_1 = \frac{3\,E - 26\,E/11}{r} = \frac{7\,E}{11\,r},$$

$$I_2 = \frac{26\,E/11 - 2\,E}{2\,r} = \frac{2\,E}{11\,r}$$

$$\text{and} \quad I_3 = \frac{26\,E/11 - E}{3\,r} = \frac{5\,E}{11\,r}$$

2. Suppose that current I flows in the circuit in the direction CBAX YC.

According to Kirchhoff's second law,

$$E_2 - E_1 = I\,(r_1 + r_2 + R)$$

$$\text{or} \quad 2{\cdot}0 - 1{\cdot}5 = I\,(1{\cdot}0 + 1{\cdot}5 + 2{\cdot}5) \quad \text{or} \quad \mathbf{I = 0{\cdot}1\ A}$$

Since current flows out from the cell E_2, potential drop across r_2 opposes its e.m.f. Therefore,

$$V_B - V_C = E_2 - I\,r_2 = 2{\cdot}0 - 0{\cdot}1 \times 1{\cdot}5 = \mathbf{1{\cdot}85\ V}$$

3. Let I be current in the circuit.

According to Kirchhoff's second law,

$$E_1 - E_2 = I\,(R_1 + R_2) \quad \text{or} \quad 12 - 8 = I\,(1{\cdot}2 + 0{\cdot}8)$$

$$\text{or} \quad I = 2\ A$$

Since current I flows out from the cell E_1, potential drop across R_1 opposes its e.m.f. Therefore,

$$V_A - V_B = E_1 - I\,R_1 = 12 - 2 \times 1{\cdot}2 = \mathbf{9{\cdot}6\ V}$$

Aliter. Since current I flows into the cell E_2, potential drop across R_2 aids its e.m.f. Therefore,

$$V_A - V_B = E_2 + I\,R_2 = 8 + 2 \times 0{\cdot}8 = \mathbf{9{\cdot}6\ V}$$

4. (*i*) By proceeding as in solved problem no. 2.02, it can be obtained that

$$I_1 = 1\ A \quad \text{and} \quad I_2 = 2{\cdot}5\ A$$

Therefore, current throgh 2 Ω resistor,

$$I = I_1 + I_2 = 1 + 2{\cdot}5 = \mathbf{3{\cdot}5\ A}$$

(*ii*) Terminal voltage across E_1,

$$V_1 = R_1 - I_1\, r_1 = 10 - 1 \times 3 = \mathbf{7\ V}$$

Terminal voltage across E_2,

$$V_2 = E_2 - I_2\, r_2 = 12 - 2{\cdot}5 \times 2 = \mathbf{7\ V}$$

5. Proceed as in problem no. 4.
6. Proceed as in problem no. 4.
7. Proceed as in solved problem no. 2.02.
8. Proceed as in solved problem no. 2.02.
9. The current through various branches of the circuit will as shown in Fig. 2.86 :

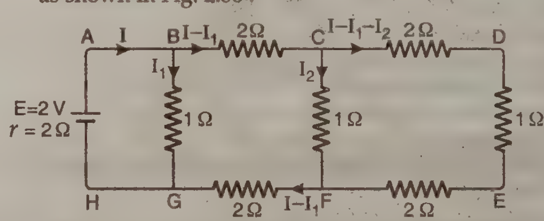


Fig. 2.86

In the closed part ABGHA of the circuit :

$$I \times 2 + I_1 \times 1 = 2$$

$$\text{or} \quad 2\,I + I_1 = 2$$

In the closed part BCFGB of the circuit :

$$(I - I_1) \times 2 + I_2 \times 1 + (I - I_1)\,2 - I_1 \times 1 = 0$$

$$\text{or} \quad 4\,I - 5\,I_1 + I_2 = 0 \qquad ...$$

In an closed part CDEFC of the circuit :

$$(I - I_1 - I_2) \times (2 + 1 + 2) - I_2 \times 1 = 0$$

$$\text{or} \quad 5\,I - 5\,I_1 - 6\,I_2 = 0 \qquad ...($$

Solving equations (*i*), (*ii*) and (*iii*), we get

$$I = \frac{70}{99}\,A\,;\ I_1 = \frac{58}{99}\,A\,;\ I_2 = \frac{10}{99}\,A$$

$$\therefore \quad I_{BC} = I_{FG} = I - I_1 = \frac{12}{99}\,A$$

$$\text{and} \quad I_{CD} = I_{DE} = I_{EF} = I - I_1 - I_2 = \frac{2}{99}\,A$$

10. Proceed as in problem no. 9.
11. Proceed as in solved problem no. 2.04.
12. (*i*) From Kirchhoff's first law, at the junction B, we ha

$$I_2 = I_1 + I_3$$

In the closed part ABEFA of the circuit :

$$6\,I_1 - 2\,I_3 = 10 \qquad ...$$

In the closed part ACDFA of the circuit :

$$4\,I_2 + 2\,I_3 = 14 \qquad ...($$

Solving equation (*i*), (*ii*) and (*iii*), we get

$$\mathbf{I_1 = 2\ A,\ I_2 = 3\ A\ \text{and}\ I_3 = 1\ A}$$

(*ii*) Potential difference between points B and E

$$= \text{potential drop across } 2\ \Omega = 2 \times I_3 = 2 \times 1 =$$

13. *In the closed part ABGHA of the circuit :*

$$3 + (-3) = I_1 \times 10 \quad \text{or} \quad I_1 = 0$$

In the closed part ACFHA of the circuit :

$$3 + 3 + (-3) + (-3) = I_2 \times 10 \quad \text{or} \quad I_2 = 0$$

In the closed part ADEHA of the circuit :

$3 + 3 + 3 + (-3) + (-3) + (-3) = I_3 \times 10$ or $I_3 = \mathbf{0}$

4. (*i*) Let the distribution of current in various branches of the circuit be as shown in Fig. 2.87 :

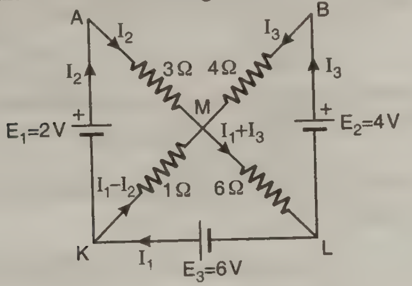


Fig. 2.87

In the closed part AKMA of the circuit :

$I_2 \times 3 - (I_1 - I_2) \times 1 = 2$

or $4\,I_2 - I_1 = 2$ ...(*i*)

In the closed part BLMB of the circuit :

$I_3 \times 4 + (I_1 + I_3) \times 6 = 4$ or $6\,I_1 + 10\,I_3 = 4$

or $3\,I_1 + 5\,I_3 = 2$ ...(*ii*)

In the closed part KLMK of the circuit :

$(I_1 - I_2) \times 1 + (I_1 + I_3) \times 6 = 6$

or $7\,I_1 - I_2 + 6\,I_3 = 6$ ...(*iii*)

Solving equations (*i*), (*ii*) and (*iii*), it can be obtained that

$$I_1 = \mathbf{1{\cdot}3\ A}$$

(*ii*) $V_A - V_B = E_1 + E_3 + (-E_2) = 2 + 6 - 4 = \mathbf{4\ V}$

15. *In the closed part E_1ABE_1 of the circuit :*

$(I_1 + I_2 - I_3) \times R_1 - (I_3 - I_1) \times R_2 = E_1$

or $(I_1 + I_2 + I_3) \times 2 - (I_3 - I_1) \times 2 = 5$

or $4\,I_1 + 2\,I_2 - 4\,I_3 = 5$ ...(*i*)

In the closed part E_1ACBE_1 of the circuit :

$-(I_2 - I_3) \times R_4 + I_3 \times R_5 = E_1$

or $-(I_2 - I_3) \times 4 + I_3 \times 4 = 5$

or $-4\,I_2 + 8\,I_3 = 5$ ...(*ii*)

In the closed part BCOB of the circuit :

$I_2 \times R_3 + I_3 \times R_5 + (I_3 - I_1) \times R_2 = E_2$

or $I_2 \times 1 + I_3 \times 4 + (I_3 - I_1) \times 2 = 10$

or $-2\,I_1 + I_2 + 6\,I_3 = 10$ ...(*iii*)

Solving equations (*i*), (*ii*) and (*iii*), it can be obtained that

$I_1 = I_3 = \mathbf{1{\cdot}875\ A}$ and $I_2 = \mathbf{2{\cdot}5\ A}$

16. Since the bridge is balanced $\left(\frac{2}{3} = \frac{4}{6}\right)$, no current flows along the path CD. Thus, 10 Ω resistance is ineffective and may be ignored.

Resistance of branch XCY, $R_1 = 2 + 3 = 5\ \Omega$

and resistance of branch XDY, $R_2 = 4 + 6 = 10\ \Omega$

Therefore, effective resistance between points X and Y,

$$R = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \mathbf{3{\cdot}33\ \Omega}$$

17. The given circuit is an arrangement of balanced Wheatstone bridge and therefore AB may be treated as an open path.

Resistance of branch XAY, $R_1 = 10 + 10 = 20\ \Omega$

Resistance of branch XBY, $R_2 = 10 + 10 = 20\ \Omega$

Therefore, effective resistance between points X and Y is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{40} = \frac{1}{20} + \frac{1}{20} + \frac{1}{40} = \frac{5}{40}$$

or $R = \mathbf{8\ \Omega}$

18. The given circuit may be arranged shown in Fig. 2.88 :

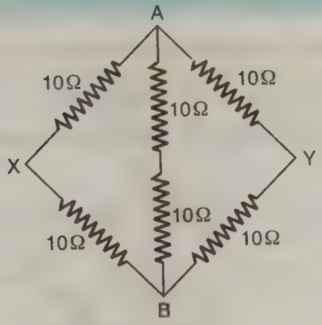


Fig. 2.88

Now, proceed as in problem no. 16.

19. The given network can be arranged as shown in Fig. 2.89 :

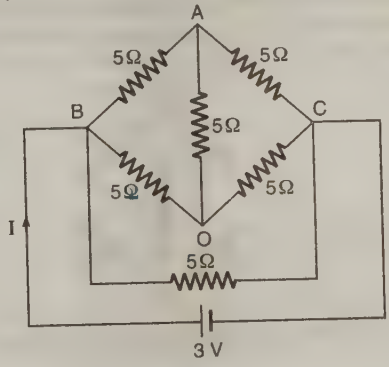


Fig. 2.89

It is a balanced Wheatstone bridge. Proceeding as in problem no. 17, it can be obtained that the effective resistance of the network,

$$R = 2{\cdot}5\ \Omega$$

Therefore, current drawn by the circuit,

$$I = \frac{3}{2{\cdot}5} = \mathbf{1{\cdot}2\ A}$$

20. The given network can be rearranged as shown in Fig. 2.90 :

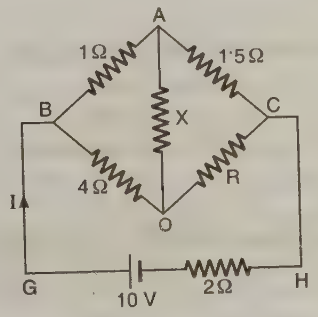


Fig. 2.90

As the current through branch AO is zero, the bridge is balanced *i.e.*

$$\frac{1}{1{\cdot}5} = \frac{4}{R} \quad \text{or} \quad R = 4 \times 1{\cdot}5 = 6\ \Omega$$

Resistance of branch BAC, $R_1 = 1 + 1{\cdot}5 = 2{\cdot}5\ \Omega$

Resistance of branch BOC, $R_2 = 4 + 6 = 10\ \Omega$

Hence, equivalent resistance of the circuit,

$$R_{equi} = \frac{R_1 R_2}{R_1 + R_2} + 2 = \frac{2{\cdot}5 \times 10}{2{\cdot}5 + 10} + 2 = 2 + 2 = 4\,\Omega$$

Now, current in the main circuit, $I = \frac{10}{R_{equi}} = \frac{10}{4} = 2\cdot5\ \text{A}$

Hence, current through R (or branch BOC),

$$I' = \frac{I \times R_1}{R_1 + R_2} = \frac{2\cdot5 \times 2\cdot5}{2\cdot5 + 10} = \mathbf{0\cdot5\ A}$$

21. The given network can be rearranged as shown in Fig. 2.91 :

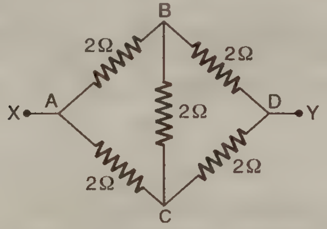


Fig. 2.91

It is a balanced Wheatstrone bridge. Its effective resistance can be found by proceeding as in problem no. 16.

22. The given network can be rearranged as shown in Fig. 2.92 :

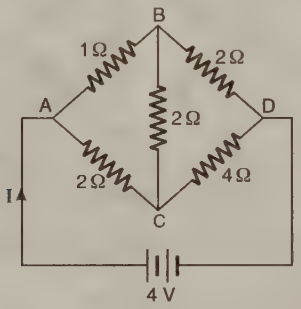


Fig. 2.92

It is a balanced Wheatstone bridge.

The effective resistance of network,

$$R = \frac{3 \times 6}{3+6} = 2\,\Omega$$

Therefore, current drawn by the circuit, $I = 4/2 = \mathbf{2\ A}$

23. Proceed as in solved problem no. 2.09.

24. At the junction A, according to Kirchhoff's first law, we have

$$I = I_1 + I_2 \qquad \text{...}(i)$$

In closed part ABDA of the circuit :

$$I_1 \times 2 + I_3 \times 4 - I_2 \times 1 = 0$$

or $$2\,I_1 - I_2 + 4\,I_3 = 0 \qquad \text{...}(ii)$$

In closed part BCDB of the circuit :

$$(I_1 - I_3) \times 3 - (I_2 + I_3) \times 2 - I_3 \times 4 = 0$$

or $$3\,I_1 - 2\,I_2 - 9\,I_3 = 0 \qquad \text{...}(iii)$$

In closed part ABCHGA of the circuit :

$$I_1 \times 2 + (I_1 - I_3) \times 3 + I \times 2 = 2$$

or $$I_1 \times 2 + (I_1 - I_3) \times 3 + (I_1 + I_2) \times 2 = 2$$

or $$7\,I_1 + 2\,I_2 - 3\,I_3 = 2 \qquad \text{...}(iv)$$

Solving equations (*ii*), (*iii*) and (*iv*), it can be obtained that

$$I_1 = \frac{\mathbf{17}}{\mathbf{91}}\ \text{A};\ I_2 = \frac{\mathbf{30}}{\mathbf{91}}\ \text{A} \quad \text{and} \quad I_3 = -\frac{\mathbf{1}}{\mathbf{91}}\ \text{A}$$

From equation (*i*), we have

$$I = I_1 + I_2 = \frac{17}{91} + \frac{30}{91} = \frac{\mathbf{47}}{\mathbf{91}}\ \text{A}$$

25. Proceed as in problem no. 24.

26. Resistance in left arm, $R_p = \frac{5 \times 20}{5+20} = 4\,\Omega$

Let X be resistance in the right arm. Then,

$$\frac{4}{X} = \frac{40}{(100-40)} \quad \text{or} \quad X = \frac{4 \times 60}{40} = \mathbf{6\ \Omega}$$

27. Proceed as in solved problem no. 2.12.

28. Let X be unknown resistance. Then,

$$\frac{2}{X} = \frac{40}{(100-40)} \quad \text{or} \quad X = \frac{2 \times 60}{40} = 3\,\Omega$$

When unknown resistance is shunted with 2 Ω resistor :

$$X' = \frac{X \times 2}{X+2} = \frac{3 \times 2}{3+2} = 1\cdot2\ \Omega$$

If balanced point is obtained at a distance l' from the left end, then,

$$\frac{2}{X'} = \frac{l'}{(100-l')} \quad \text{or} \quad \frac{2}{1\cdot2} = \frac{l'}{(100-l')}$$

$$l' = 62\cdot5\ \text{cm}$$

Therefore, shift in balance point,

$$l' - l = 62\cdot5 - 40 = \mathbf{22\cdot5\ cm}$$

29. Resistance in right gap, $X = 2\ \Omega$

Let S be the resistance of shunt wire. Then,

resistance in left gap, $R = \frac{5 \times S}{5+S} = \frac{5S}{5+S}$

Now, $\frac{R}{X} = \frac{l}{100-l}$ or $\frac{5S}{(5+S)\times 2} = \frac{62\cdot5}{100-62\cdot5}$

or $\quad S = \mathbf{10\ \Omega}$

30. Resistance in right gap, $X = R$

When resistance R_1 is in left gap :

$$\frac{R_1}{R} = \frac{40}{(100-40)} \quad \text{or} \quad R_1 = \frac{2R}{3}$$

When resistance R_2 is in left gap :

$$\frac{R_2}{R} = \frac{50}{100-50} \quad \text{or} \quad R_2 = R$$

(*a*) $R_s = R_1 + R_2 = \frac{2R}{3} + R = \frac{5R}{3}$

Now, $\frac{R_s}{R} = \frac{l}{100-l}$ or $\frac{5R/3}{R} = \frac{l}{100-l}$

or $\quad l = \mathbf{62\cdot5\ cm}$

(*b*) $R_p = \frac{R_1\,R_2}{R_1+R_2} = \frac{(2R/3)\times R}{(2R/3)+R} = 0\cdot4\,R$

Now, $\frac{R_p}{R} = \frac{l}{100-l}$ or $\frac{0\cdot4\,R}{R} = \frac{l}{100-l}$

or $\quad l = \mathbf{28\cdot6\ \ cm}$

31. Let l be total length of the wire of metre bridge.

Now, $$\frac{P}{Q} = \frac{30}{l-30} \qquad \text{...}(i)$$

When coils P and Q are interchanged :

$$\frac{Q}{P} = \frac{120}{l-120} \qquad \text{...}(ii)$$

Multiplying equations (*i*) and (*ii*), we have

$$\frac{P}{Q} \times \frac{Q}{P} = \frac{30}{l-30} \times \frac{120}{l-120} \quad \text{or} \quad (l-30)\,(l-120) = 3600$$

or $\quad l^2 - 150\,l + 3600 = 3600 \quad$ or $\quad l = \mathbf{150\ cm}$

Substituting for l in equation (*i*), we have

$$\frac{P}{Q} = \frac{30}{150-30} = \frac{\mathbf{1}}{\mathbf{4}}$$

32. $r = \left(\frac{E}{V} - 1\right)S = \left(\frac{l_1}{l_2} - 1\right)S = \left(\frac{85}{75} - 1\right) \times 7\cdot5 = \mathbf{1\ \Omega}$

33. Proceed as in problem no. 32.

34. Proceed as in problem no. 32.

35. ***When shunted by 5 Ω resistor :***

$$r=\left(\frac{l_1}{220}-1\right)\times 5 \qquad ...(i)$$

When shunted by 20 Ω resistor :

$$r=\left(\frac{l_1}{300}-1\right)\times 20 \qquad ...(ii)$$

$$\therefore\ \left(\frac{l_1}{220}-1\right)\times 5=\left(\frac{l_1}{300}-1\right)\times 20 \quad \text{or} \quad l_1 = 341{\cdot}4 \text{ cm}$$

Substituting for l_1 in equation (*i*), we have

$$r=\left(\frac{341\cdot 4}{220}-1\right)\times 5=\mathbf{2{\cdot}76\ \Omega}$$

36. Current through potentiometer wire,

$$I=\frac{E_1}{R+R_{AB}}=\frac{2}{R+10}$$

Resistance of balancing length of the wire,

$$R'=\frac{10}{100}\times 40=4\,\Omega$$

$\therefore$ E_2 = potential drop across R′

$$\text{or } 100\times 10^{-3}=4\times\frac{2}{R+10}$$

$$\text{or} \qquad R=\frac{8}{100\times 10^{-3}}-10=\mathbf{70\ \Omega}$$

Potential drop across whole length of potentiometer wire,

$$V=R_{AB}\times I=10\times\frac{2}{10+10}=0\cdot 1\text{ V}=100\text{ mV}$$

Since V is less than 300 mV, balance point **will not be obtained,** if E_2 were to have an e.m.f. of 300 mV.

37. ***When potentiometer is connected between the points A and B :***

$$E_1 \propto 300 \qquad ...(i)$$

When potentiometer is connected between points A and C :

The net e.m.f. of the two cells = $E_1 - E_2$

$$\therefore \qquad E_1 - E_2 \propto 100 \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{E_1-E_2}{E_1}=\frac{100}{300} \quad \text{or} \quad 1-\frac{E_2}{E_1}=\frac{1}{3}$$

$$\text{or} \qquad \frac{E_1}{E_2}=\frac{3}{2}$$

38. Let *r* be resistance of 1 cm length of the potentiometer wire. Then, total resistance of potentiometer wire = 500 *r*

In first case : If E′ is e.m.f. of the battery used to pass current through the potentiometer wire, then

$$I_1=\frac{E'}{500\,r}$$

If E is e.m.f. of the cell, then

E = potential drop across 350 cm long wire

$$=350\,r\times I_1=\frac{350\,r\times E'}{500\,r}$$

$$\text{or} \qquad E'=\frac{10\,E}{7}$$

In second case : Current through potentiometer wire,

$$I_2=\frac{E'}{400\,r}=\frac{10\,E}{7\times 400\,r}=\frac{E}{280\,r}$$

Suppose that the e.m.f. of the cell is balanced against length *l* of the potentiometer wire.

Then, E = potential drop across length *l* of the wire

$$\text{or} \qquad E=\frac{E}{280\,r}\times l\,r \qquad \text{or} \qquad l=\mathbf{280\ cm}$$

39. Let R be resistance of the resistor.

Resistance of the potentiometer wire

$$=100\times 0{\cdot}025=2{\cdot}5\ \Omega$$

Therefore, current through the potentiometer wire,

$$I=\frac{2\cdot 56}{R+2\cdot 5}$$

Now, 1·018 = potential drop across 50·9 cm of potentiometer wire

$$=50\cdot 9\times 0\cdot 025\times\frac{2\cdot 56}{R+2\cdot 5}$$

$$\text{or} \qquad R=\frac{50\cdot 9\times 0\cdot 025\times 2\cdot 56}{1\cdot 018}-2\cdot 5=3\cdot 2-2\cdot 5$$

$$=\mathbf{0{\cdot}7\ \Omega}$$

40. Proceed as in problem no. 39.

41. Let *r* be internal resistance of the cell.

In first case : Current sent by the cell through 2 Ω coil,

$$I_1=\frac{\text{e.m.f.}}{\text{total resistance}}=\frac{2}{2+r}$$

Potential drop acorss 2 Ω coil,

$$V_1=2\times I_1=2\times\frac{2}{2+r}=\frac{4}{2+r}$$

$$\text{Now,} \qquad V_1 \propto 70\text{ cm}$$

$$\text{or} \qquad \frac{4}{2+r}\propto 70 \qquad ...(i)$$

In second case : Let R_p be resistance of the parallel combination of 2 Ω and 1 Ω coils. Then,

$$R_p=\frac{2\times 1}{2+1}=\frac{2}{3}\ \Omega$$

Current sent by the cell through R_p,

$$I_2=\frac{\text{e.m.f.}}{\text{total resistance}}=\frac{2}{2/3+r}$$

Potential drop across R_p,

$$V_2=R_p\times I_2=\frac{2}{3}\times\frac{2}{2/3+r}=\frac{4}{2+3r}$$

$$\text{Now,} \qquad V_2 \propto 50\text{ cm}$$

$$\text{or} \qquad \frac{4}{2+3r}\propto 50 \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*), we have

$$\frac{4}{2+r}\times\frac{2+3r}{4}=\frac{70}{50} \qquad \text{or} \qquad r=\mathbf{0{\cdot}5\ \Omega}$$

42. Consider that a cell is connected across the points X and Y, so that current I enters at point A and leaves at point B. Due to symmetry, the distribution of current in various branches will be as shown in Fig. 2.93 :

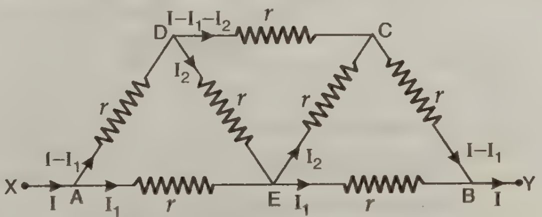


Fig. 2.93

Since current in branch DE is same as that in branch EC, it will remain so even when contract between branch DEC and AEB is broken at point E as shown in Fig. 2.94 :

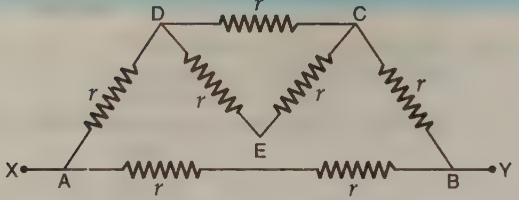


Fig. 2.94

Therefore, resistance of the given network is same as that of the circuit as shown above.

Now, resistance of branch AB, $R_1 = r + r = 2r$

and resistance of branch ADCB,

$$R_2 = r + \frac{r \times 2r}{r + 2r} + r = \frac{8r}{3}$$

Since R_1 and R_2 are in parallel, resistance of the network between points X and Y,

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{2r \times 8r/3}{2r + 8r/3} = \frac{8r}{7}$$

Aliter. *For closed part ADEA of the circuit :*

$$(I - I_1)\, r + I_2\, r - I_1\, r = 0$$

or $$I - 2I_1 + I_2 = 0 \qquad ...(i)$$

For closed part CDEC of the circuit :

$$I_2\, r + I_2\, r - (I - I_1 - I_2)\, r = 0$$

or $$I - I_1 - 3I_2 = 0 \qquad ...(ii)$$

From equations (*i*) and (*ii*), by eliminating I_2, we have

$$I_1 = \frac{4}{7} I \qquad ...(iii)$$

If R is resistance of the network, then

$$V_A - V_B = I R \qquad ...(iv)$$

Also, $V_A - V_B = I_1 r + I_1 r$

or $$V_A - V_B = 2 I_1 r \qquad ...(v)$$

From equations (*iv*) and (*v*), we have

$$I R = 2 I_1 r = 2 \times \frac{4}{7} I \times r \qquad \left(\because I_1 = \frac{4}{7} I\right)$$

or $$R = \frac{8}{7} r$$

43. By proceeding as in problem no. 42, it can be shown that the given network is equivalent to the circuit shown in Fig. 2.95 :

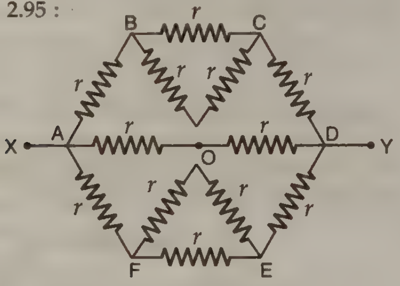


Fig. 2.95

As obtained earlier,

resistance of branch ABCD, $R_1 = \frac{8r}{3}$

resistance of branch AD, $R_2 = 2r$

and resistance of branch AFED, $R_3 = \frac{8r}{3}$

Since R_1, R_2 and R_3 are in parallel, resistance of the network between points X and Y is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{8r} + \frac{1}{2r} + \frac{3}{8r} = \frac{10}{8r}$$

or $$R = 0{\cdot}8\, r$$

Note. The problem can also be solved by applying Kirchh laws as in problem no. 42.

44. As discussed in problem no. 42, the resistance of g network will remain the same, if the contact betweer branches AB and ADCEB is broken at point F.

Resistance of the branch AB, $R_1 = r + r = 2r$

Resistance of the part CDFE of the branch ADCEB (w contact is broken at point F) is given by

$$\frac{1}{R_p} = \frac{1}{2r} + \frac{1}{r} + \frac{1}{2r} \qquad \text{or} \qquad R = \frac{r}{2}$$

Therefore, resistance of the branch ADCEB,

$$R_2 = r + R_p + r = r + \frac{r}{2} + r = \frac{5r}{2}$$

Hence, resistance of the given network between the p X and Y,

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{2r \times 5r/2}{2r + 5r/2} = \frac{10r}{9}$$

44. Since $\frac{R_{EF}}{R_{FG}} = \frac{R_{EO}}{R_{OG}}$,

no current will flow along path FO and hence FO ac an open path. For the same reason, OI also acts as an path. As such, the given network can be rearrange shown in Fig. 2.96 :

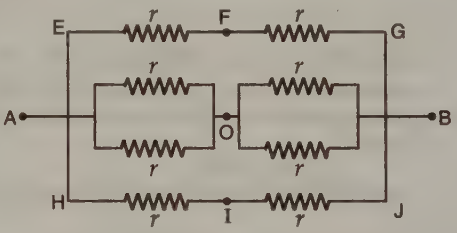


Fig. 2.96

Now, resistance of the branch EFG, $R_1 = r + r = 2r$,

resistance of the branch HIJ, $R_2 = r + r = 2r$

and resistance of the branch AOB, $R_3 = r/2 + r/2 =$

The resistances R_1, R_2 and R_3 of these branches a parallel. Therefore, effective resistance of the netwo

$$\frac{1}{R} = \frac{1}{2r} + \frac{1}{2r} + \frac{1}{r} = \frac{2}{r} \qquad \text{or} \qquad R = 0{\cdot}5\ \Omega$$

46. Suppose that a battery of e.m.f. E is connected a the points A and B of the incomplete cube as show Fig. 2.97 :

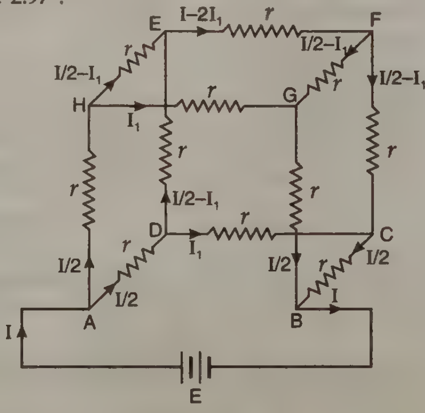


Fig. 2.97

Further, suppose that a current I enters the cube at point A. As the paths ADCB and AHGB are of equal resistances, current $\frac{I}{2}$ will flow along each of the two paths. Suppose that at point D, a part I_1 flows along the path DC. Then, the remaining part $\frac{I}{2} - I_1$ will along the path DE. In a similar manner, current equal to I_1 will flow along the path HG and $\frac{I}{2} - I_1$ along the path HE. The distribution of current in the other branches of the circuit will be as shown in the figure. If R is the resistance of the cube between the points A and B, then

$$R = \frac{E}{I} \qquad \ldots(i)$$

In the closed part ADCBA of the circuit :

$$E = \frac{I}{2} \times r + I_1 \times r + \frac{I}{2} \times r$$

or $$E = I\,r + I_1\,r \qquad \ldots(ii)$$

In the closed part EFGHE of the circuit :

$$(I - 2I_1) \times r + \left(\frac{I}{2} - I_1\right) r - I_1 \times r + \left(\frac{I}{2} - I_1\right) r = 0$$

or $$(I - 2I_1) + \left(\frac{I}{2} - I_1\right) - I_1 + \left(\frac{I}{2} - I_1\right) = 0$$

or $$I_1 = \frac{2}{5} I$$

Substituting for I_1 in equation (*ii*), we have

$$E = I\,r + \frac{2}{5} I\,r = \frac{7}{5} I\,r$$

or $$\frac{E}{I} = \frac{7}{5} r \qquad \ldots(iii)$$

From equations (*i*) and (*iii*), we have

$$R = \frac{7}{5} r$$

47. Suppose that a battery of e.m.f. E is connected in AD. Let the distribution of currents in various branches of the circuit be as shown in Fig. 2.98 :

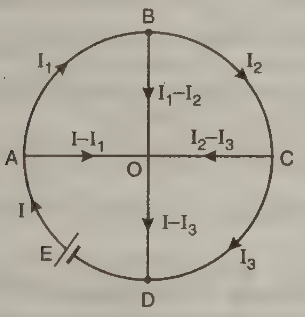


Fig. 2.98

It follows that $R_{AB} = R_{BC} = R_{CD} = R_{DA} = \frac{2}{4} = 0{\cdot}5\ \Omega$

Also, $R_{AO} = R_{OC} = R_{BO} = R_{OD} = \frac{1}{2} = 0{\cdot}5\ \Omega$

If R is resistance of the network, then

$$R = \frac{E}{I} \qquad \ldots(i)$$

In the closed part AODA of the circuit :

$$I \times 0{\cdot}5 + (I - I_1) \times 0{\cdot}5 + (I - I_3) \times 0{\cdot}5 = E$$

or $$1{\cdot}5\,I - 0{\cdot}5\,I_1 - 0{\cdot}5\,I_3 = E \qquad \ldots(ii)$$

In the closed part ABOA of the circuit :

$$I_1 \times 0{\cdot}5 + (I_1 - I_2) \times 0{\cdot}5 - (I - I_1) \times 0{\cdot}5 = 0$$

or $$3\,I_1 - I_2 = I \qquad \ldots(iii)$$

In the closed part BCOB of the circuit :

$$I_2 \times 0{\cdot}5 + (I_2 - I_3) \times 0{\cdot}5 - (I_1 - I_2) \times 0{\cdot}5 = 0$$

or $$I_1 - 3\,I_2 + I_3 = 0 \qquad \ldots(iv)$$

In the closed part CODC of the circuit :

$$I_3 \times 0{\cdot}5 - (I_2 - I_3) \times 0{\cdot}5 - (I - I_3) \times 0{\cdot}5 = 0$$

or $$I_2 = 3\,I_3 - I \qquad \ldots(v)$$

Substituting for I_2 in equation (*iii*), we have

$$3\,I_1 - (3\,I_3 - I) = I \quad \text{or} \quad I_3 = I_1$$

Substituting for I_3 in equation (*v*), we have

$$I_2 = 3\,I_1 - I$$

Setting $I_2 = 3\,I_1 - I$ and $I_3 = I_1$ in equation (*iv*), we have

$$I_1 - 3\,(3\,I_1 - I) + I_1 = 0 \quad \text{or} \quad 7\,I_1 = 3\,I$$

or $$I_1 = \frac{3}{7} I$$

Setting $I_1 = I_3 = \frac{3}{7} I$ in equation (*ii*), we have

$$1{\cdot}5\,I - 0{\cdot}5 \times \frac{3}{7} I - 0{\cdot}5 \times \frac{3}{7} I = E$$

or $$\frac{E}{I} = 1{\cdot}5\left(1 - \frac{1}{7} - \frac{1}{7}\right)$$

or $$\frac{E}{I} = \frac{15}{14} \qquad \ldots(vi)$$

From equations (*i*) and (*vi*), we have

$$R = \frac{15}{14}\ \Omega$$

48. In the steady state, current can not flow through the capacitor and hence the branch of the circuit containing capacitor can be considered as an open path. Likewise, the given network is equivalent to the circuit as shown in Fig. 2.99 :

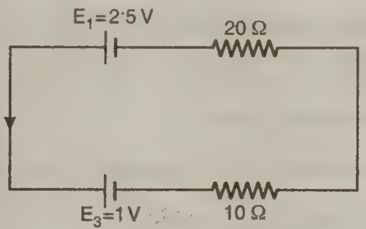


Fig. 2.99

Let I be current in the circuit.

According to Kirchhoff's second law,

$$20 \times I + 10\,I = E_1 - E_3 \quad \text{or} \quad 30\,I = 2{\cdot}5 - 1$$

or $$I = 0{\cdot}05 \text{ A}$$

Potential difference across C

= terminal potential difference across E_1 (or E_3) − E_2

Now, terminal potential difference across E_1

$$= E_1 - I \times 20$$
$$= 2{\cdot}5 - 0{\cdot}05 \times 20 = 1{\cdot}5 \text{ V}$$

Therefore, potential difference across C = 1·5 − 1·5 = 0

49. In steady state, the capacitor behaves as an open path.

Now, $V_A = 6$ V and $V_C = 0$

Since battery is connected across series combination of P and Q,

$$\frac{V_A - V_B}{V_B - V_C} = \frac{P}{Q} = \frac{2}{2} = 1$$

or $$V_A - V_B = V_B - V_C$$

or $$6 - V_B = V_B - 0 \quad \text{or} \quad V_B = 3 \text{ V}$$

Since battery is connected across series combination of R and S,

$$\frac{V_A - V_D}{V_D - V_C} = \frac{R}{S} = \frac{3}{7}$$

or $7(V_A - V_D) = 3\,(V_D - V_C)$

or $7\,(6 - V_D) = 3\,(V_D - 0)$ or $V_D = 4{\cdot}2$ V

$\therefore$ $V_D - V_B = 4{\cdot}2 - 3 = 1{\cdot}2$ V

(*a*) Charge stored in the capacitor,

$$q = C\,(V_D - V_B) = 3 \times 1{\cdot}2 = \mathbf{3{\cdot}6\ \mu C}$$

Energy stored in the capacitor,

$$U = \frac{1}{2} C\,(V_D - V_B)^2 = \frac{1}{2} \times 3 \times 10^{-6} \times (1{\cdot}2)^2$$

$$= \mathbf{2{\cdot}16 \times 10^{-6}\ J}$$

(*b*) When $V_B = V_D$ *i.e.* when bridge is balanced, no charge will be stored in the capacitor. The value of Q for balanced bridge is given by

$$\frac{2}{Q} = \frac{3}{7} \quad \text{or} \quad Q = \frac{14}{3}\,\Omega$$

50. Suppose that the balance point is obtained at a distance l from the zero mark. Then,

$$\frac{X}{R} = \frac{l}{100 - l} \quad \text{or} \quad \frac{X}{10} = \frac{l}{100 - l}$$

or $$X = \frac{10\,l}{100 - l} \qquad ...(i)$$

On increasing resistance by 12·5 Ω :

Since R has been increased, balancing length will decrease. Suppose that it becomes equal to l'.

Here, $R' = 10 + 12{\cdot}5 = 22{\cdot}5\ \Omega$ and $l' = l - 20$

Now, $$\frac{X}{R'} = \frac{l'}{100 - l'} \quad \text{or} \quad \frac{X}{22{\cdot}5} = \frac{l - 20}{100 - (l - 20)}$$

or $$X = \frac{22{\cdot}5\,(l - 20)}{120 - l} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{10\,l}{100 - l} = \frac{22{\cdot}5\,(l - 20)}{(120 - l)} \quad \text{or} \quad \frac{l}{100 - l} = \frac{2{\cdot}25\,(l - 20)}{120 - l}$$

On solving, we get

$l = 60$ cm

Substituting for l in equation (*i*), we have

$$X = \frac{10 \times 60}{100 - 60} = \mathbf{15\ \Omega}$$

51. Suppose that the short circuit occurs in the telephone cable at a distance d. Then,

length of cable, $l = 2\,d$

resistance of telephone cable, $X = 2\,d \times 0{\cdot}45\ \Omega$

Now, $$X = R\frac{Q}{P}$$

Here, $P = 1110\ \Omega$, $Q = 100\ \Omega$ and $R = 400\ \Omega$

$$\therefore \quad 2\,d \times 0{\cdot}45 = \frac{400 \times 100}{1110}$$

or $d = \mathbf{40\ m}$

52. Current through the potentiometer wire,

$$I = \frac{2}{15 + R_{AB}} = \frac{2}{15 + 10} = 0{\cdot}08\ A$$

Potential drop across wire AB = 0·08 × 10 = 0·8 V

Potential drop per unit length of wire AB

$$= \frac{0{\cdot}8}{1} = 0{\cdot}8\ V\ m^{-1}$$

Let AJ be equal to l.

Then, potential drop across portion AJ of the wire,

$V = l \times 0{\cdot}8$

Let I′ be the current sent by 1·5 V cell through the series combination of 1·2 Ω and 0·3 Ω resistors. Then,

$$I' = \frac{1{\cdot}5}{1{\cdot}2 + 0{\cdot}3} = 1\ A$$

In the balanced state,

V = potential drop across 0·3 Ω resistor

$= I' \times 0{\cdot}3 = 1 \times 0{\cdot}3$

or $V = 0{\cdot}3$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$l \times 0{\cdot}8 = 0{\cdot}3$

or $$l = \frac{0{\cdot}3}{0{\cdot}8} = 0{\cdot}375\ m = \mathbf{37{\cdot}5\ cm}$$

53. The various components are connected as shown in Fig. 2.100 :

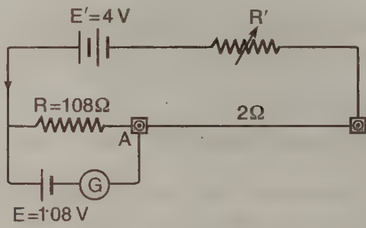


Fig. 2.100

Let R′ be the resistance of the variable resistor, which makes deflection on the galvanometer equal to zero.

Total resistance of the circuit = $R' + R + R_{AB}$

$= R' + 108 + 2 + R' + 110$

Therefore, current through the circuit,

$$I = \frac{E'}{R' + 110} = \frac{4}{R' + 110}$$

and potential difference across resistor R,

$$V = I\,R = \frac{4 \times 108}{R' + 110}$$

Since galvanometer does not show deflection,

$$V = E \quad \text{or} \quad \frac{4 \times 108}{R' + 110} = 1{\cdot}08$$

or $$R' = \frac{4 \times 108}{1{\cdot}08} - 110 = 400 - 110 = \mathbf{290\ \Omega}$$

54. (*a*) Current through the potentiometer wire,

$I = 80\ mA = 80 \times 10^{-3}$ A

The e.m.f. 4 V of the cell is balanced against 1000 cm of the potentiometer wire. Therefore,

$$\text{resistance of 1000 cm of wire, } R' = \frac{4}{80 \times 10^{-3}} = 50\ \Omega$$

Hence, resistance of the whole length of potentiometer wire,

$$R = \frac{R'}{1000} \times 1200 = \frac{50}{1000} \times 1200 = \mathbf{60\ \Omega}$$

(*b*) When voltmeter is connected across the cell, it will draw some current. The terminal potential difference across the two poles of the cell is balanced against 980 cm length of the potentiometer wire. Therefore,

$$\text{reading of voltmeter} = \frac{4}{1000} \times 980 = \mathbf{3{\cdot}92\ V}$$

(*c*) Let R_V be resistance of the voltmeter. Then,

$$r = \left(\frac{E}{V} - 1\right) R_V = \left(\frac{l_1}{l_2} - 1\right) R_V$$

or $$20 = \left(\frac{1000}{980} - 1\right) R_V \quad \text{or} \quad R_V = \mathbf{980\ \Omega}$$

CHAPTER

3

Thermal Effects of Current

3.01. THERMAL EFFECT OF CURRENTS

In 1841, Joule experimentally showed the heating effect of the electric current. When electric current is passed through the element of a heater or an electric bulb, it becomes hot due to conversion of electrical energy into heat energy.

When potential difference is applied across the two ends of a conductor, an electric field is set up across its ends. Under the effect of the electric field, the electrons get accelerated and move in a direction opposite to the direction of the electric field. As they move, they collide against the ions or atoms in the conductor and in this process, the electrons transfer their kinetic energy to the atoms. As a result, the average kinetic energy of the atoms increases and consequently the temperature of the conductor rises. It is termed as heating effect of the electric currents.

Key point

On passing current, heat is produced in a conductor due to transfer of energy from electrons (constituting the current) to the ions and atoms in the conductor.

3.02. ELECTRIC POWER

The rate at which work is done by the source of e.m.f. in maintaining the electric current in a circuit is called electric power of the circuit.

If an amount of work W is done in maintaining electric current for a time t in the circuit, then electric power of the circuit,

$$P = \frac{W}{t} \qquad ...(3.01)$$

Consider a resistor of resistance R, across which a potential difference V is applied [Fig. 3.01]. According to Ohm's law, the current I flowing through the resistor is given by

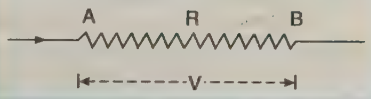


Fig. 3.01

$$V = I R$$

Suppose that the steady current I flows through the resistor for a time t. Then, total charge that crosses through the resistor is given by

$$q = I t$$

If current* flows from end A to B, then potential of end A must be higher than that of B. Since potential difference across the conductor is V, the potential of end A is higher than that of B by an amount V. From the definition of potential difference, it follows that if a unit charge is made to cross the resistor from the end B to A (from the end at lower potential to that at higher potential), work equal to V has to be done. Conversely, if a unit charge flows from end A to B, then energy equal to V will be consumed from the source of e.m.f. and it will appear as heat energy across the resistor. If current I flows for time t *i.e.* charge equal to I t crosses the resistor from the end A to B, then electrical energy dissipated is given by

$$W = V I t \qquad ...(3.02)$$

If V is in **volt**, I in **ampere** and t in **seconds**, then energy dissipated will be in **joule**.

From equations (3.01) and (3.02), we have

$$P = V I \qquad ...(3.03)$$

The SI unit of electric power is **watt**.

1 watt = 1 volt × 1 ampere or 1 W = 1 V × 1 A

The electric power of a circuit or a device is said to be one watt, if one ampere of current flows through it, when a constant potential difference of one volt is applied across it.

The bigger units of electric power are **kilowatt (kW)** and **megawatt (MW)**.

$$\mathbf{1\ kW = 10^3\ W \text{ and } 1\ MW = 10^6\ W}$$

The power of electric circuit in kilowatt is given by

$$P\,(\text{in kW}) = \frac{V\,(\text{in volt}) \times I\,(\text{in ampere})}{1000}$$

---

*Conventional current *i.e.* the current which flows from positive to negative pole of a battery.

Expression for electric power in terms of I and R. Since V = I R, equation (3.03) becomes

$$P = I^2 R \qquad \text{...(3.04)}$$

Expression for electric power in terms of V and R. Since $I = \frac{V}{R}$, equation (3.03) becomes

$$P = \frac{V^2}{R} \qquad \text{...(3.05)}$$

3.03. ELECTRIC ENERGY

The total work done (or energy supplied) by the source of e.m.f. in maintaining the electric current in the circuit for a given time is called electric energy consumed in the circuit.

The equation (3.02) gives electric energy consumed in the circuit in time t. Since V I = P (electric power of the circuit), equation (3.02) may be rewritten as

$$W = P\,t \qquad \text{...(3.06)}$$

i.e. *electric energy = electric power × time*

Thus, the total amount of electric energy consumed by an electric circuit depends upon its electric power and the time for which the power is used.

Self-test Questions

What do kilowatt and kilowatt hour represent ?

The SI unit of electric energy is **joule** but another unit called **watt hour** is oftenly used.

1 watt hour = 1 watt × 1 hour

The energy dissipated or consumed in an electric circuit is called one watt hour, if a device of electric power of one watt is used for one hour.

The bigger unit of electric energy is **kilowatt hour (kWh).** It is also known as **Board of Trade Unit (B.O.T.)**.

The energy dissipated or consumed is called one kilowatt hour, if a device of power of one kilowatt is used for one hour.

1 kilowatt hour = 10^3 watt hour

Since $1\text{ kW} = 10^3\text{ J s}^{-1}$ and $1\text{ hour} = 60 \times 60\text{ s}$,

$$1\text{ kWh} = (10^3\text{ J s}^{-1}) \times (60 \times 60\text{ s})$$

or $\mathbf{1\ kWh = 3{\cdot}6 \times 10^6\ J}$

The electric energy consumed in kWh is given by

$$W\text{ (in kWh)} = \frac{V\text{ (in volt)} \times I\text{ (in ampere)} \times t\text{ (in hour)}}{1000}$$

3.04. JOULE'S LAWS OF HEATING EFFECT OF CURRENTS

When a conductor having some resistance is connected to a source of e.m.f., a current flows through it. In fact, free electrons inside the conductor immediately start moving from the end at lower potential to the end at higher potential. This flow of electrons constitutes the electric current. In the course of their motion, the electrons collide against one another and also against the ions and atoms of the conductor. Due to such collisions, the conductor appears to offer some opposition to the motion of electrons *i.e.* to the flow of current. This opposition to the current is termed as resistance of the conductor. Apart from this, at each collision, a part of the kinetic energy of the electron gets converted into heat and the temperature of the conductor increases. It is known as heating effect of the current. The working of electric iron, electric bulb, electric heater, etc are a few examples of heating effects of current.

In 1841, Joule experimentally studied the heating effect of electric current. He found that amount of heat produced in a conductor due to the flow of current is

(i) directly proportional to the square of the current i.e.

$$Q \propto I^2 \qquad \text{...(3.07)}$$

(ii) directly proportional to the resistance of the conductor i.e.

$$Q \propto R \qquad \text{...(3.08)}$$

(iii) directly proportional to the time for which the current is passed i.e.

$$Q \propto t \qquad \text{...(3.09)}$$

Combining factors (3.07), (3.08) and (3.09), we have

$$Q \propto I^2 R\, t$$

$$Q = \frac{I^2 R\, t}{J}, \qquad \text{...(3.10)}$$

where J is a conversion factor, called ***Joule's mechanical equivalent of heat*** and its value is **4·18 J cal^{-1}**. The equation (3.10) is called mathematical form of Joule's laws of heating.

In SI, the heat produced (in joule) due to the flow of current through a resistance is given by

$$Q = I^2 R t \qquad ...(3.11)$$

3.05. EFFICIENCY OF A SOURCE OF E.M.F.

The efficiency of an electric source of e.m.f. is defined as the ratio of the output power to the input power. It is defined by η.

Thus,
$$\eta = \frac{\text{output power}}{\text{input power}} \qquad ...(3.12)$$

Consider that an electric source of e.m.f. E and having internal resistance r is connected to an external resistance R. Then, current flowing through the circuit,

$$I = \frac{E}{R + r} \qquad ...(3.13)$$

Now, input power = E I

The output power of the source is always less than the input power. It is because, the power is wasted in the form of heat developed across the internal resistance of the source. Thus,

output power = $E I - I^2 r$

Therefore, efficiency of the source of e.m.f.,

$$\eta = \frac{E I - I^2 r}{E I} = 1 - \frac{I r}{E}$$

From equation (3.13), substituting for I, we have

$$\eta = 1 - \frac{E}{(R + r)} \cdot \frac{r}{E} = 1 - \frac{r}{R + r}$$

or
$$\eta = \frac{R}{R + r} \qquad ...(3.14)$$

3.06. MAXIMUM POWERTHEOREM

It states that the output power of a source of e.m.f. is maximum, when the external resistance in the circuit is equal to the internal resistance of the source.

Consider that a source of e.m.f. E and having internal resistance r is connected to an external resistance R as shown in Fig. 3.02. The current flowing through the circuit is given by

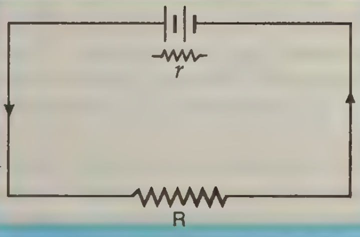


Fig. 3.02

$$I = \frac{E}{R + r}$$

Therefore, output power (electric power dissipated across external resistance) of the source of e.m.f.,

$$P = I^2 R = \left(\frac{E}{R + r}\right)^2 R = \frac{E^2 R}{(R + r)^2} \qquad ...(3.15)$$

For the output power of the source to be maximum,

$$\frac{dP}{dR} = 0 \quad \text{or} \quad \frac{d}{dR}\left[\frac{E^2 R}{(R + r)^2}\right] = 0 \quad \text{or} \quad \frac{d}{dR}\left[\frac{R}{(R + r)^2}\right] = 0$$

or
$$\frac{d}{dR}[R (R + r)^{-2}] = 0 \quad \text{or} \quad [1 \times (R + r)^{-2} + R (-2) (R + r)^{-3}] = 0$$

or
$$(R + r)^{-2} - 2 R (R + r)^{-3} = 0 \quad \text{or} \quad 1 - \frac{2 R}{R + r} = 0$$

or
$$R = r \qquad ...(3.16)$$

Hence, the output power of a source of e.m.f. is maximum, when the external resistance in the circuit is equal to the internal resistance of the source.

Key point

Maximum power is consumed in a circuit, when external resistance in the circuit is equal to the internal resistance of the sourceb of e.m.f.

Maximum output power. The expression for of maximum output power can be obtained by setting r = R in equation (3.15). Therefore, maximum output power,

$$P_{max} = \frac{E^2 R}{(R + R)^2}$$

or
$$P_{max} = \frac{E^2}{4 R} \qquad ...(3.17)$$

3.07. APPLICATIONS OF HEATING EFFECT OF CURRENT

The heating effect of electric current has a wide range of applications in everyday life. Different electric appliances have been designed by making use of heating effect of electric current. A few important applications of this phenomenon are as given below :

1. Incandescent electric bulb. It consists of a filament of fine metallic wire enclosed in a glass blub filled with some inert gas at a suitable pressure. The material of the filament wire has a large value of resistivity and high melting point. Therefore, usually tungsten is used to make the filament of the bulb. When current is passed through the filament, it gets heated to the stage of incandescence. The electric power and voltage specification of the electric bulb are always marked over it. The following points should be noted about the electric bulbs (or electrical appliances in general) of different electric powers :

(i) Since P = V I,

it follows that *current drawn by an electric bulb is directly proportional to its power.*

(ii) Since $P = \frac{V^2}{R}$,

it follows that *resistance of the filament of an electric bulb is inversely proportional to its power.*

The jargon

The emission of light by a substance, when heated to a high temperature, is called incandescence.

2. Electric heating appliances. Electric heater, electric iron, electric immersion rod, electric geyser, electric stove, etc are some of the useful electric appliances, whose working is based on heating effect of electric current. In all such appliances, the heating element used is made of nichrome (an alloy of nickel and chromium). The heating element is made of nichrome for the following reasons :

(i) Its resistivity is large.
(ii) Its melting point is high.
(iii) It does not get oxidised easily, when heated in air.
(iv) It can be easily drawn into wires.

Key point

The filament of an electric bulb or the heating element of an electric appliance is made from a metal (or alloy) possessing large resistivity and high melting point.

3. Arc-lamp. In an arc lamp, light is produced by an electric arc bewteen two electrodes. The most common arc-lamp consists of two carbon electrodes separated by a small gap between them. When voltage is applied across the two electrodes, a spark jumps across the gap between the electrodes and a very intense light is emitted.

4. Electric fuse. It is a safety device used to protect electric installations against overloading. It is also used in electric appliances to save them from getting damaged due to flow of heavy current through them or to prevent electric shock due to live wire coming in contact with the metal casing of the appliance. In the household electric installation and the appliances, the safety fuse is put in series with the live wire from the mains supply.

Watch out !

The load in an electric circuit implies the current drawn circuit from the supply line. When a current flows through the circuit beyond the safe value for it, the current is said to be overloaded !

A safety fuse is basically a piece of wire having a suitable high resistivity and low melting point. The fuse wire is generally prepared from tin-lead alloy (63% tin and 37% lead). When current in the circuit grows to a large value, the fuse wire gets heated, melts and breaks the circuit. The fuse wire of suitable current rating is used in an electric circuit depending on the load in the circuit.

Key point

The safety fuse wire is made from a metal (or alloy) possessing large resistivity and low melting point.

SOLVED NUMERICAL PROBLEMS

Type A. On Electric Power

Problem 3.01. A lamp of 100 W works at 220 volt. What is its resistance and current capacity ?

Sol. Here, power of lamp, P = 100 W ;

operating voltage, V = 220 volt

Now, $P = VI$

Therefore, current capacity of the lamp,

$$I = \frac{P}{V} = \frac{100}{220} = \mathbf{0 \cdot 455\ A}$$

Now, resistance of the lamp, $R = \frac{V^2}{P} = \frac{(220)^2}{100} = \mathbf{484\ \Omega}$

Problem 3.02. Calculate the number of electrons moving per second through the filament of a lamp of 100 watt, operating at 200 volt. Given, charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C.

Sol. Here, charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C

Power of the lamp, P = 100 W ;

operating voltage, V = 200 volt

Now, $P = VI$

$$\therefore \quad I = \frac{P}{V} = \frac{100}{200} = 0 \cdot 5\ A$$

Charge passing through the lamp in 1 s,

$$q = I \times t = 0{\cdot}5 \times 1 = 0{\cdot}5\ C$$

Therefore, number of electrons moving through the filament per second,

$$N = \frac{q}{e} = \frac{0 \cdot 5}{1 \cdot 6 \times 10^{-19}} = \mathbf{3 \cdot 125 \times 10^{18}}$$

Problem 3.03. A 60 W – 220 V bulb and 100 W – 220 V bulb are connected in parallel to mains supply. Which bulb will draw more current ?

Ans. Here, $P_1 = 60$ W and $P_2 = 100$ W

When the two bulbs are connected in parallel to the mains supply, the voltage across both of them is same. If I_1 and I_2 are the currents through the two bulbs, then

$$P_1 = V I_1 \quad \text{and} \quad P_2 = V I_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{P_1}{P_2} = \frac{60}{100} = 0{\cdot}6 \quad \textit{i.e.} \quad I_2 > I_1$$

Hence, **100 W bulb** draws more current.

Problem 3.04. A 60 W – 220 V bulb and 100 W – 220 V bulb are connected in series to mains supply. (*a*) Which bulb will dissipate more power ? (*b*) Which bulb will glow brighter ?

Sol. Here, $P_1 = 60$ W and $P_2 = 100$ W

(*a*) When the two bulbs are connected in series to the mains supply, the current through both of them will be same. If R_1 and R_2 are the resistances of the two bulbs, then

$$P_1 = I^2 R_1 \quad \text{and} \quad P_2 = I^2 R_2$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{R_1}{R_2} \qquad \ldots(i)$$

$$\text{Now,} \quad R_1 = \frac{V^2}{P_1} = \frac{V^2}{60} \quad \text{and} \quad R_2 = \frac{V^2}{P_2} = \frac{V^2}{100}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{V^2}{60} \times \frac{100}{V^2} = \frac{5}{3} \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{P_1}{P_2} = \frac{5}{3} \quad \textit{i.e.} \quad P_1 > P_2$$

Hence, when 60 W and 100 W bulbs are connected in series to mains supply, **60 W bulb** will dissipate more power.

(*b*) Since 60 W bulb is dissipating more power, it glows brighter.

Problem 3.05. An electric heater and an electric bulb are rated 500 W-220 V and 100 W-220 V respectively. Both are connected in series to a 220 a.c. mains. Calculate the power consumed by (*i*) the heater and (*ii*) electric bulb.

(*C.B.S.E. 1997*)

Ans. Power of electric heater, $P_1 = 500$ W ;

operating voltage, $V_1 = 220$ V

power of electric bulb, $P_2 = 100$ W ;

operating voltage, $V_2 = 220$ V

Let R_1 and R_2 be resistances of electric heater and bulb respectively. Since $P = \dfrac{V^2}{R}$, we have

$$R_1 = \frac{V_1^2}{P_1} = \frac{(220)^2}{500} = 96{\cdot}8\ \Omega$$

$$\text{and} \quad R_2 = \frac{V_2^2}{P_2} = \frac{(220)^2}{100} = 484\ \Omega$$

When series combination of electric heater and bulb is connected to 220 V main, current in the circuit is given by

$$I = \frac{V}{R_1 + R_2} = \frac{220}{96{\cdot}8 + 484} = 0{\cdot}38 \text{ A}$$

Now, power consumed by heater,

$$P_1' = I^2 R_1 = (0{\cdot}38)^2 \times 96{\cdot}8 = \mathbf{13{\cdot}98\ W}$$

and power consumed by bulb,

$$P_2' = I^2 R_2 = (0{\cdot}38)^2 \times 484 = \mathbf{69{\cdot}89\ W}$$

Problem 3.06. A 100 W - 220 V bulb is connected to 110 V source. Calculate the power consumed by the bulb.

(*P.S.S.C.E. 1999 ; Roorkee 1986*)

Sol. Here, P = 100 W ;

marked operating voltage, V = 200 V

Therefore, resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

Actual operating voltage, $V' = 110$ V

Therefore, power consumed by the bulb,

$$P' = \frac{V'^2}{R} = \frac{(110)^2}{484} = \mathbf{25\ W}$$

Problem 3.07. A voltage stabiliser controls the voltage output to 220 V ± 1%. If a refrigerator rated at 550 W – 220 V is connected to it, what will be the minimum and maximum power consumed by it ?

Sol. Here, power of refrigerator, P = 550 W ;

marked operating voltage, V = 220 V

Therefore, resistance of the refrigerator,

$$R = \frac{V^2}{P} = \frac{(220)^2}{550} = 88\ \Omega$$

The refrigerator will consume maximum power, when supply voltage becomes

$$V_1 = 220 \text{ V} + 1\% = 220 + 2{\cdot}2 = 222{\cdot}2 \text{ V}$$

$$\therefore \quad P_{max} = \frac{V_1^2}{R} = \frac{(222{\cdot}2)^2}{88} = \mathbf{561\ W}$$

The refrigerator will consume minimum power, when supply voltage becomes

$$V_2 = 220 \text{ V} - 1\% = 220 - 2{\cdot}2 = 217{\cdot}8 \text{ V}$$

$$\therefore \quad P_{min} = \frac{V_2^2}{R} = \frac{(217{\cdot}8)^2}{88} = \mathbf{539\ W}$$

Problem 3.08. An electric bulb marked 60 W - 220 V will get fused, if the electric power fed to it becomes 85 W or more. What voltage fluctuation can the bulb withstand.

Sol. Here, P = 60 W ; V = 220 V

Therefore, resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{(220)^2}{60} = 806{\cdot}67\ \Omega$$

Let V′ be the supply voltage, corresponding to which the power fed to the bulb becomes 85 W *i.e.* bulb gets fused. Then,

$$\frac{V'^2}{R} = 85 \quad \text{or} \quad V' = \sqrt{85\,R}$$

$$\text{or} \quad V' = \sqrt{85 \times 806{\cdot}67} = \mathbf{261{\cdot}8\ V}$$

Problem 3.09. A generator is supplying power to a factory by cables of resistance 20 Ω. If the generator is generating 50 kW power at 5000 volt, what is the power received by the factory ? (*P.S.S.C.E. 1996 S*)

Sol. Here, $P = 50 \text{ kW} = 50 \times 10^3$ W ; V = 5000 volt

Therefore, current supplied by the generator,

$$I = \frac{P}{V} = \frac{50 \times 10^3}{5000} = 10 \text{ A}$$

As this current flows through the cables of resistance 20 Ω, a part of power will be wasted in the form of heat energy developed across the cable.

Power wasted across cable,

$$P = I^2 R = (10)^2 \times 20 = 2000 \text{ W} = 2 \text{ kW}$$

Therefore, power received by the factory

$$= P - P' = 50 - 2 = \mathbf{48\ kW}$$

Problem. 3.10. Three resistors R_1, R_2 and R_3 are connected as shown in Fig. 3.03. Each resistor can withstand a maximum power of 18 W (otherwise it will melt). What maximum power, the whole circuit can possess ?

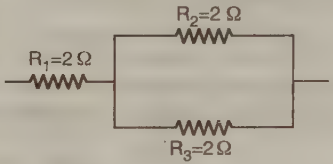


Fig. 3.03

Sol. Here, $R_1 = R_2 = R_3 = 2\ \Omega$

Each of the three resistances can withstand a maximum power of 18 W. If I_{max} is the value of maximum current, which can flow through the resistance without causing it to melt, then

$$I_{max}^2 \times 2 = 18 \quad \text{or} \quad I_{max} = \sqrt{\frac{18}{2}} = 3\text{ A}$$

Electric power of resistance R_1,

$$P_1 = I_{max}^2 \times R_1 = (3)^2 \times 2 = 18\text{ W}$$

The resistance of the parallel combination of R_2 and R_3,

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{2 \times 2}{2+2} = 1\ \Omega$$

Therefore, electric power of the parallel combination of R_2 and R_3,

$$P_{23} = I_{max}^2 \times R_{23} = (3)^2 \times 1 = 9\text{ W}$$

Hence, the maximum power, the circuit can possess

$$= P_1 + P_{23} = 18 + 9 = 27\text{ W}$$

Type B. On Joule's heating effect of currents

Problem 3.11. A power line of resistance 0·5 Ω carries a constant current of 100 A. How much energy is being lost per day in the form of heat in the line ? *(H.S.S.C.E. 2002)*

Sol. Here, $R = 0{\cdot}5\ \Omega$; $I = 100$ A ;

$$t = 1\text{ day} = 24 \times 60 \times 60\text{ s}$$

The amount of heat lost per day in the power line

$$Q = I^2 R t = (100)^2 \times 0{\cdot}5 \times 24 \times 60 \times 60$$
$$= \mathbf{4{\cdot}32 \times 10^8\ J}$$

Problem 3.12. A 10 volt storage battery of negligible internal resistance is connected across a 50 Ω resistor made of alloy manganin. How much heat energy is produced in the resistor in 1 h ? What is the source of this energy ? *(Text Problem)*

Sol. Here, $R = 50\ \Omega$; $V = 10$ volt ; $t = 1\text{ h} = 3600$ s ;

Current in the circuit, $I = \frac{V}{R} = \frac{10}{50} = 0{\cdot}2$ A

Heat produced, $W = I^2 R t = (0{\cdot}2)^2 \times 50 \times 3600 = \mathbf{7200\ J}$

The source of the energy is the **chemical energy** stored in the battery.

Problem 3.13. An electric bulb is marked 100 W – 200 V. If the supply voltage drops to 100 V, what is the heat and light energy produced by the bulb in 10 minutes ? *(P.S.S.C.E. 2002)*

Sol. Here, $P = 100$ W ; $V = 200$ volt

Let R be resistance of the filament of the bulb.

Now, electric power, $P = VI = \frac{V^2}{R}$

Therefore, $R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400\ \Omega$

When the voltage drops to 100 volt, heat and light ene produced by the bulb in 10 minutes is given by

$$W = VIt = \frac{V^2}{R}t = \frac{(100)^2}{400} \times 10 \times 60 = \mathbf{15{,}00}$$

Problem 3.14. Three resistors R_1, R_2 and R_3 connected to a battery of 12 V as shown in Fig. 3. Calculate the heat produced in each of the three resist in 5 minutes.

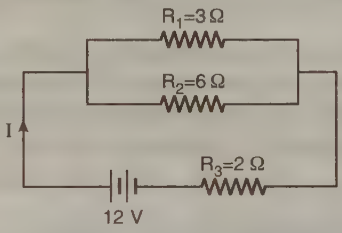


Fig. 3.04

Sol. Here, $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$; $R_3 = 2\ \Omega$;

$$V = 12\text{ volt and } t = 5\text{ mintues} = 5 \times 60 = 300$$

The resistance of parallel combination of R_1 and R_2

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 6}{3+6} = 2\ \Omega$$

Total resistance of the circuit,

$$R = R_{12} + R_3 = 2 + 2 = 4\ \Omega$$

Therefore, current through the circuit,

$$I = \frac{V}{R} = \frac{12}{4} = 3\text{ A}$$

Let I_1 and I_2 be currents through resistances R_1 anc respectively.

Then, $$I_1 = \frac{I \times R_2}{R_1 + R_2} = \frac{3 \times 6}{3+6} = 2\text{ A}$$

and $$I_2 = \frac{I \times R_1}{R_1 + R_2} = \frac{3 \times 3}{3+6} = 1\text{ A}$$

Therefore, heat produced in resistance R_1,

$$Q_1 = I_1^2 R_1 t = (2)^2 \times 3 \times 300 = \mathbf{3600\ J}$$

Heat produced in resistance R_2,

$$Q_2 = I_2^2 R_2 t = (1)^2 \times 6 \times 300 = \mathbf{1800\ J}$$

Heat produced in resistance R_3,

$$Q_3 = I^2 R_3 t = (3)^2 \times 2 \times 300 = \mathbf{5400\ J}$$

Problem 3.15. An electric kettle has two coils. Wl one coil is switched on, it takes 5 mintues to boil water when second coil is switched on, it takes 10 minutes. H long will it take to boil water, when both the coils are u in series ? *(P.S.S.C.E. 20*

Sol. Let R_1 and R_2 be the resistances of the two coils Q be the amount of heat required to boil the water in kettle.

In first case : $t_1 = 5\text{ minutes} = 5 \times 60 = 300$ s

$$\therefore \quad Q = \frac{V^2}{R_1}t_1 = \frac{V^2}{R_1} \times 300$$

or $$R_1 = \frac{300\ V^2}{Q}$$

In second case : $t_2 = 10$ minutes $= 10 \times 60 = 600$ s

$$\therefore \quad Q = \frac{V^2}{R_2} t_2 = \frac{V^2}{R_2} \times 600$$

$$\text{or} \quad R_2 = \frac{600\, V^2}{Q} \qquad \ldots(ii)$$

Let t be the time taken to boil the water, when both the coils are used in series. Then,

$$Q = \frac{V^2}{(R_1 + R_2)} t \quad \text{or} \quad t = \frac{Q}{V^2}(R_1 + R_2)$$

From equation (*i*) and (*ii*), substituting for R_1 and R_2, we have

$$t = \frac{Q}{V^2}\left(\frac{300\, V^2}{Q} + \frac{600\, V^2}{Q}\right) = 900 \text{ s} = \mathbf{15\ minutes}$$

Problem 3.16. A thin metallic wire of resistance 100 ohm is immersed in a calorimeter containing 250 g of water at 10°C and a current of 0· 5 ampere is passed through it for half an hour. If the water equivalent of calorimeter is 10 g, find the rise of the temperature. (*H.S.S.C.E. 1995*)

Sol. Here, $R = 100\ \Omega$; $I = 0{\cdot}5$ A ;

$$t = \frac{1}{2} \text{ h} = \frac{1}{2} \times 60 \times 60 = 1800 \text{ s}$$

Heat produced, $Q = I^2 R t = (0{\cdot}5)^2 \times 100 \times 1800$
$= 4{\cdot}5 \times 10^4$ J

Water equivalent of calorimeter, $w = 10$ g $= 10 \times 10^{-3}$ kg

Mass of water, $m = 250$ g $= 250 \times 10^{-3}$ kg

Let θ be rise in temperature. Since specific heat of water is $4{\cdot}2 \times 10^3$ J kg^{-1} °C^{-1}, we have

$$Q = (m + w)\, c\, \theta$$

$$\text{or} \quad \theta = \frac{Q}{(m + w) \times c}$$

$$= \frac{4{\cdot}5 \times 10^4}{(10 \times 10^{-3} + 250 \times 10^{-3})\, 4{\cdot}2 \times 10^3} = \mathbf{41{\cdot}2°\,C}$$

Problem 3.17. A house is fitted with 10 lamps rated 100 W each, four fans each consuming 0·5 A, an electric kettle of resistance 100 Ω and an electric iron of resistance 121 Ω. If the energy is supplied at 200 V and costs Rs. 2·50 per kWh, calculate the bill for running the appliances for four hours in a day for the month of February, 2000.

Sol. Power of 10 lamps (of 100 W each),

$$10 \times P = 10 \times 100 = 1000 \text{ W}$$

Power of 4 fans (consuming 0·5 A),

$$4 \times V I = 4 \times 200 \times 0{\cdot}5 = 400 \text{ W}$$

Power of the100 Ω electric kettle,

$$\frac{V^2}{R} = \frac{(200)^2}{100} = 400 \text{ W}$$

Power of the 121 Ω electric iron,

$$\frac{V^2}{R} = \frac{(200)^2}{121} = 330{\cdot}58 \text{ W}$$

Total power of the appliances $= 1000 + 400 + 400 + 330{\cdot}58$
$= 2130{\cdot}58$ W $= 2{\cdot}13058$ kW

Number of days in February, 2000 = 29

Therefore, time for which electric energy is used in the month of February, 2000, $= 29 \times 4 = 116$ h

Total electrical energy consumed
$=$ power × time $= 2{\cdot}13058 \times 116 = 247{\cdot}15$ kWh

Cost of electrical energy = Rs. 2·50 per kWh

Therefore, bill for February, 2000 $= 247{\cdot}15 \times 2{\cdot}50$
$=$ **Rs. 617·90**

Problem 3.18. Following electric items are used in a house :

Item	**Number**	**Power**	**Time**
T.V	1	60 W	2 hours a day
Fridge	1	60 W	24 hours a day
Tubes	3	40 W each	4 hours a day (each)
Bulbs	2	60 W each	2 hours a day (each)
Press	1	600 W	$\frac{1}{2}$ hour a day

What will be the electric bill of the house for two months (each month of 30 days), if the rate of power is Rs. 1·35 per unit ? (*H.S.S.C.E. 1994*)

Sol. Electric energy consumed by T.V. per day
$= 60 \times 10^{-3} \times 2 = 0{\cdot}12$ kWh

Electric energy consumed by fridge per day
$= 60 \times 10^{-3} \times 24 = 1{\cdot}44$ kWh

Electric energy consumed by 3 tubes per day
$= 3 \times 40 \times 10^{-3} \times 4 = 0{\cdot}48$ kWh

Electric energy consumed by 2 bulbs per day
$= 2 \times 60 \times 10^{-3} \times 2 = 0{\cdot}24$ kWh

Electric energy consumed by press per day

$$= 600 \times 10^{-3} \times \frac{1}{2} = 0{\cdot}30 \text{ kWh}$$

Total electric energy consumed by all electricals per day
$= 0{\cdot}12 + 1{\cdot}44 + 0{\cdot}48 + 0{\cdot}24 + 0{\cdot}30 = 2{\cdot}58$ kWh

Electric energy consumed in two months (each month of 30 days) $= 2 \times 30 \times 2{\cdot}58 = 154{\cdot}8$ kWh

Cost of electrical energy = Rs. 2·70 per kWh

Therefore, electric bill for 2 months
$= 1{\cdot}35 \times 154{\cdot}8 =$ **Rs. 417·96**

Problem 3.19. An electric power station (100 MW) transmits power to a distant load through long and thin cables. Which of the two modes of transmission would result in lesser power wastage, power transmission at (*i*) 20,000 V or (*ii*) 200 V ? (*Text Problem*)

Sol. Let R be the resistance of the cables.

Here, $P = 100$ MW $= 100 \times 10^6$ W

When transmission is done at $V_1 = 20{,}000$ V : If I_1 is the current through the cables, then

$$I_1 = \frac{P}{V_1} = \frac{100 \times 10^6}{20{,}000} = 5000 \text{ A}$$

Therefore, rate of heat dissipation along the cable at 20,000 V,

$$Q_1 = I_1^2 R = (5000)^2 R = 25 \times 10^6 R \text{ watt}$$

When transmission is done at $V_2 = 200$ V : If I_2 is the current through the cables, then

$$I_2 = \frac{100 \times 10^6}{200} = 5 \times 10^5 \text{ A}$$

Therefore, rate of heat dissipation along the cable at 200 V,

$$Q_2 = I_2^2 R = (5\times10^5)^2 R = 25\times10^{10} R \text{ watt}$$

Hence, there would be lesser power wastage, when transmission is done at **20,000 V.**

Problem 3.20. Two ribbons are given with the following particulars :

Parameter	Constantan	Nichrome
Length (m)	8·456	4·235
Width (mm)	1·0	2·0
Thickness (mm)	0·03	0·06
Resistivity (Ω m)	4.9×10^{-7}	1.1×10^{-6}

For a fixed voltage supply, which of the two ribbons corresponds to a greater rate of heat production ?

(*Text Problem*)

Sol. *For constantan ribbon* :

Resistivity, $\rho_1 = 4.9\times10^{-7}$ Ω m ; length, $l_1 = 8.456$ m ;
width, $b_1 = 1.0$ mm ; thickness, $t_1 = 0.03$ mm

Therefore, area, $A_1 = b_1 \times t_1$

$$= 1.0\times0.03 = 0.03 \text{ mm}^2 = 0.03\times10^{-6} \text{ m}^2$$

Now, $$R_1 = \rho_1 \frac{l_1}{A_1} = \frac{4.9\times10^{-7}\times8.456}{0.03\times10^{-6}}$$

$$= 138.115\ \Omega$$

If V is fixed supply voltage, then rate of heat production,

$$\frac{V^2}{R_1} = \frac{V^2}{138.115} \text{ W} \qquad ...(i)$$

For nichrome ribbon :

Resistivity, $\rho_2 = 1.1\times10^{-6}$ Ω m ; length, $l_1 = 4.235$
width, $b_1 = 2.0$ mm; thickness, $t_1 = 0.06$ mm

$$\therefore \quad A_2 = b_2 \times t_2 = 2.0\times0.06$$

$$= 0.12 \text{ mm}^2 = 0.12\times10^{-6} \text{ m}^2$$

$$\therefore \quad R_2 = \rho_2 \frac{l_2}{A_2} = \frac{1.1\times10^{-6}\times4.235}{0.12\times10^{-6}}$$

$$= 38.821\ \Omega$$

Therefore, rate of heat production,

$$\frac{V^2}{R_2} = \frac{V^2}{38.821} \text{ W}$$

From equations (*i*) and (*ii*), it follows that for a f supply voltage, **nichrome ribbon** corresponds to greater of heat production.

Problem 3.21. An electric motor operating on a 50 V supply draws a current of 12 A. If the efficiency of the m is 30%, estimate the resistance of the windings of the mo

(*P.S.S.C.E. 1992 ; Text Prob*

Sol. Here, V = 50 volt ; I = 12 A

Therefore, power of the electric motor

$$= V\times I = 50\times12 = 600 \text{ W}$$

Since efficiency of the motor is 30%, it dissipates 70 the power across the resistance of the windings of the m

Therefore, power dissipated $= 600\times\frac{70}{100} = 420$ W

If R is resistance of the windings of the motor, ther

$$I^2 R = 420 \quad \text{or} \quad R = \frac{420}{I^2} = \frac{420}{(12)^2} = 2.92\ \Omega$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 3.22. A series battery of 6 lead accumulators each of e.m.f. 2·0 volt and internal resistance 0·50 Ω is charged by a 100 V d.c. supply. What series resistance should be used in the charging circuit in order to limit the current to 8·0 A ? Using the required resistor, obtain (*a*) the power supplied by the d.c. source, (*b*) the power dissipated as heat, (*c*) the power supplied by the d.c. energy stored in the battery in 15 minutes. (*Text Problem*)

Sol. For charging the lead accumulators, their positive terminal have to be connected to positive terminal of d.c. supply. Therefore, e.m.f. of the six lead accumulators will oppose the 100 d.c. supply used for charging them.

Hence, net e.m.f. in the circuit = 100 – 6 × 2·0 = 88 volt

Let R be the series resistance that should be used to limit the current to 8·0 A. Therefore,

total resistance of the charging circuit

= R + internal resistance of the six lead accumulators

= R + 6 × 0·50 = R + 3

Now, $$I = \frac{\text{net e.m.f.}}{\text{total resistance}} \quad \text{or} \quad 8 = \frac{88}{R+3}$$

or $$R = 8\ \Omega$$

(*a*) Power supplied by d.c. source

= e.m.f. of source × current supplied

= 100 × 8·0 = **800 W**

(*b*) Power dissipated as heat

$= I^2$ (R + internal resistance of the accumulators)

$= (8.0)^2 \times (8 + 6\times0.50) = 64\times11 =$ **704 W**

(*c*) Power stored in the battery (lead accumulators

= 800 – 704 = **96 W**

Therefore, energy stored in the battery in 15 minut

= 96 × 15 × 60 = **86400 J**

Problem 3.23. Power from a 64 V d.c. supply goe charge a battery of 8 lead accumulators each of e. 2·0 V and internal resistance 1/8 Ω. The charging cur also runs an electric motor placed in series with the bat If the resistance of the windings of the motors is 7·0 Ω the steady supply current is 3·5 A, obtain (*a*) the me nical energy yielded by the motor and (*b*) the chem energy stored in the battery during charging in 1 h.

(*Text Prob*

Sol. Here, e.m.f. of d.c. supply = 64 V ;

e.m.f. of each of the 8 lead accumulators = 2·0 V

Total e.m.f. of 8 accumulators = 2·0 × 8 = 16·0 V

Internal resistance of each lead accumulator $= \frac{1}{8}$ Ω

Total internal resistance, $r = \frac{1}{8}\times8 = 1.0\ \Omega$

Resistance of the windings of the motor, R = 7·0 Ω

Steady current in the circuit, I = 3·5 A

Rate of consumption of energy in the circuit

= 64 × 3·5 = 224 W

Rate of storing chemical energy in the battery

= 16 × 3·5 = 56 W

Rate of energy dissipation as heat in the circuit

$$= I^2 (R + r) = (3{\cdot}5)^2 \times (7{\cdot}0 + 1{\cdot}0) = 98 \text{ W}$$

Rate of mechanical energy yielded by motor

$$= 224 - (56 + 98) = 70 \text{ W}$$

(*a*) Mechanical energy yielded by motor in 1 h

$$= 70 \times 3600 = \mathbf{252000 \text{ J}}$$

(*b*) Chemical energy stored in the battery during charging in 1 h

$$= 56 \times 3600 = \mathbf{201600 \text{ J}}$$

Problem 3.24. A battery of internal resistance r (= 4 Ω) is connected to the network of resistanes as shown in Fig. 3.05. What must be the value of R, so that maximum power is delivered to the network ? What is the maximum power ? *(I.I.T. 1995)*

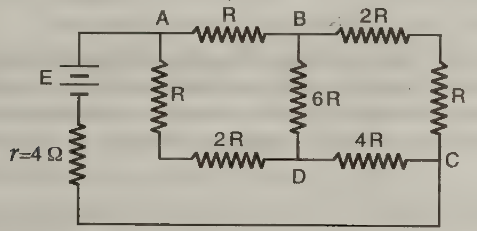


Fig. 3.05

Sol. The given network is equivalent to the circuit shown in Fig. 3.06.

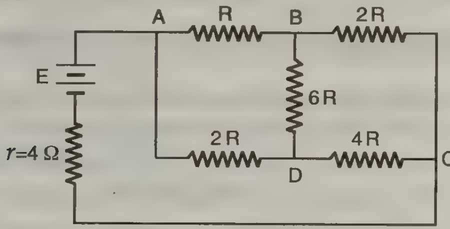


Fig. 3.06

It follows that the circuit represents a Wheatstone bridge, where

Further, resistance in arm AB (P arm) = R

resistance in arm BC (Q arm) = 2 R,

resistance in arm AD (R arm) = 2 R

and resistance in arm DC (S arm) = 4 R

Further, as $\frac{P}{Q} = \frac{R}{S}$, it is a balanced Wheatstone bridge. Since the resistance 6 R connected across the points B and D is ineffective, the circuit shown in Fig. 3.06 is equivalent to the circuit shown in Fig. 3.07.

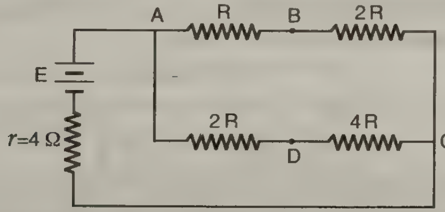


Fig. 3.07

Therefore, effective resistance of path ABC,

$$R_1 = R + 2\,R = 3\,R$$

and effective resistance of path ADC,

$$R_2 = 2\,R + 4\,R = 6\,R$$

If R_{12} is equivalent resistance of R_1 and R_2, then

$$R_{12} = \frac{R_1\,R_2}{R_1 + R_2} = \frac{3\,R \times 6\,R}{3\,R + 6\,R} = 2\,R$$

If E is e.m.f. of the battery, then current in the main circuit,

$$I = \frac{E}{R_{12} + r} = \frac{E}{2\,R + 4}$$

Therefore, electric power of the network of resistances,

$$P = I^2\,R_{12} = \left(\frac{E}{2\,R + 4}\right)^2 \times 2\,R \qquad \text{...}(i)$$

According to maximum power theorem, maximum power will be delivered to the network, when

resistance of network = internal resistance of battery

i.e. when $R_{12} = r$ or $2\,R = 4$

or $R = 2\,\Omega$

Setting R = 2 Ω in equation (*i*), we have maximum power of the network,

$$P_{max} = \left(\frac{E}{2 \times 2 + 4}\right)^2 \times 2 \times 2 = \frac{\mathbf{E^2}}{\mathbf{16}}$$

Problem 3.25. A 24 V battery of internal resistance 4 Ω is connected to a variable resistor. At what value of the current drawn from the battery is the rate of heat produced in the resistor maximum ? *(Text Problem)*

Sol. Here, e.m.f. of battery = 24 V ;

internal resistance, $r = 4\,\Omega$

Let R be the resistance of the variable resistor, at which rate of heat produced (power) is maximum.

Then, total resistance in the circuit $= R + r = R + 4$

Therefore, current drawn from the battery

$$I = \frac{\text{e.m.f.}}{\text{total resistance}} = \frac{24}{R + 4}$$

Therefore, rate at which heat is produced,

$$P = I^2\,R = \left(\frac{24}{R + 4}\right)^2 R$$

For rate of heat produced to be maximum, $\frac{dP}{dR} = 0$

$$\text{or} \qquad \frac{d}{dR}\left[\left(\frac{24}{R + 4}\right)^2 R\right] = 0$$

$$\text{or} \qquad \frac{d}{dR}\left[576\,R\,(R + 4)^{-2}\right] = 0$$

$$\text{or} \qquad \frac{d}{dR}\left[R\,(R + 4)^{-2}\right] = 0$$

$$\text{or} \quad R\,(-2)\,(R + 4)^{-3} + (R + 4)^{-2} = 0$$

$$\text{or} \qquad -2\,R\,(R + 4)^{-1} + 1 = 0 \quad \text{or} \quad \frac{2\,R}{R + 4} = 1$$

$$R = 4\;\Omega$$

Therefore, the required value of the current drawn from the battery,

$$I = \frac{24}{R + 4} = \frac{24}{4 + 4} = \mathbf{3\text{ A}}$$

It may be noted that the rate of heat produced is maximum, when external resistance is equal to internal resistance of the battery.

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. How is energy related to electric power ?

Ans. Electric energy = electric power × time

Q. 3.02. What is practical unit of electric energy ?

Ans. The practical unit of electric energy is kilowatt hour (kWh).

Q. 3.03. What is B.O.T ? Convert it into joule. *(H.S.S.C.E. 1996)*

Ans. B.O.T. (Board Of Trade unit) is unit of electric power. It is also called kilowatt hour (kWh).

Now, $1 \text{ B.O.T.} = 1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ hour}$
$= (10^3 \text{ J s}^{-1}) \times 3600 \text{ s} = \mathbf{3{\cdot}6 \times 10^6 \text{ J}}$

Q. 3.04. Calculate the number of joule in 1 kWh. *(H.P.S.S.C.E. 1999 ; H.S.S.C.E. 1996)*

Ans. $1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ hour} = (10^3 \text{ J s}^{-1}) \times 3600 \text{ s}$
$= 3{\cdot}6 \times 10^6 \text{ J}$

Q. 3.05. How many kilowatt hours (kWh) are there in one joule ? *(C.B.S.E. 1999 S)*

Ans. From VSQ 3.04, it follows that

$$1 \text{ J} = \frac{1}{3{\cdot}6 \times 10^6} \text{ kWh}$$

Q. 3.06. Distinguish between kilowatt and kilowatt hour.

Ans. Kilowatt is unit of electric power, while kilowatt hour is unit of electric energy.

Q. 3.07. Which consumes more electric energy in a given time – a 1000 watt heater or a 1000 watt bulb ?

Ans. Both consume same amount of electric energy in a given time.

Q. 3.08. Calculate the current through a lamp of 60 W, operating at 220 V. *(H.P.S.S.C.E. 1998)*

Ans. Here, P = 60 W ; V = 220 volt

Now, $$I = \frac{P}{V} = \frac{60}{220} = 0{\cdot}273 \text{ A}$$

Q. 3.09. What is the resistance of 1 kW heater marked for 220 V ? *(H.P.S.S.C.E. 1999 S ; H.S.S.C.E. 1998)*

Ans. Here, P = 1 kW = 1000 W ; V = 220 volt

Now $$R = \frac{V^2}{P} = \frac{(220)^2}{1000} = 48{\cdot}4 \, \Omega$$

Q. 3.10 Which has a greater resistance–1 kW electric heater or a 100 W filament bulb both marked for 220 V ? *(C.B.S.E. 2001 S)*

Ans. Now, $P = \frac{V^2}{R}$ or $R = \frac{V^2}{P}$

Here, power of electric heater, $P_1 = 1 \text{ kW} = 1000 \text{ W}$;
power of bulb = 100 W

Therefore, resistance of heater, $$R_1 = \frac{(220)^2}{1000} = 48{\cdot}4 \, \Omega$$

and resistance of bulb, $$R_2 = \frac{220^2}{100} = 484 \, \Omega$$

Hence, the **100 W bulb** has more resistance than the electric heater.

Q. 3.11. Which lamp has greater resistance, a 100 W and 60 W lamp when connected to the same source of supply ? *(C.B.S.E. 1999 S, 1998 ; H.S.S.C.E. 1999 S ; H.S.S.C.E. 1992)*

Ans. Now, $R = \frac{V^2}{P}$

Therefore, resistance of 100 W bulb,

$$R_1 = \frac{(220)^2}{100} = 484 \, \Omega$$

and resistance of 60 W bulb, $$R_2 = \frac{(220)^2}{60} = 806{\cdot}67 \, \Omega$$

Thus, the **60 W bulb** has greater resistance.

Q. 3.12. Two heating coils, one of fine wire and other of thick wire, made of same material and of the same length, are connected one by one to a source of electricity. Which one of the two coils will produce heat at a greater rate ? *(C.B.S.E. 1999 S)*

Ans. Since $R \propto \frac{1}{A}$, the heating coil made of thick wire possesses lesser resistance.

As $P = \frac{V^2}{R}$ *i.e.* $P \propto \frac{1}{R}$, heat will be produced at a greater rate in the coil of lesser resistance *i.e.* in the coil made of **thick wire**.

Q. 3.13. The maximum power dissipated in a 10,000 Ω resistor is 1 W. What is the maximum current ? *(I.S.C.E. 1993)*

Ans. Here, $P_{max} = 1 \text{ W}$; $R = 10{,}000 \, \Omega$

Now, $P_{max} = I_{max}^2 \, R$

$$\therefore \quad I_{max} = \sqrt{\frac{P_{max}}{R}} = \sqrt{\frac{1}{10000}} = \mathbf{0{\cdot}01 \text{ A}}$$

Q. 3.14. How much charge flows through a 250 V –1000 W heater in one minute ? *(I.S.C.E. 1996)*

Ans. Here, P = 1000 W ; V = 250 volt ;
$t = 1 \text{ minute} = 60 \text{ s}$

Now, $$I = \frac{P}{V} = \frac{1000}{250} = 4 \text{ A}$$

Therefore, charge flowing in 1 minute,
$q = I\,t = 4 \times 60 = \mathbf{240 \text{ C}}$

Q. 3.15. What is the largest voltage you can safely put across a 98 Ω – 0·5 W resistor ? *(I.S.C.E. 1997)*

Ans. Here, P = 0·5 W ; R = 98 Ω

Now, $$P = \frac{V^2}{R}$$

$$\therefore \quad V = \sqrt{PR} = \sqrt{0{\cdot}5 \times 9{\cdot}8} = \mathbf{7 \text{ V}}$$

Q. 3.16. A current of 5·0 A flows through an electric press of resistance 11 Ω. Calculate the energy consumed by the press in 5 minutes. *(C.B.S.E. 1994)*

Ans. Here, I = 5·0 ; R = 11 Ω ;
$t = 5 \text{ minutes} = 5 \times 60 = 300 \text{ s}$

Electric energy consumed in 5 minutes
$= I^2 R t = (5{\cdot}0)^2 \times 11 \times 300 = \mathbf{8{\cdot}25 \times 10^4 \text{ J}}$

Q. 3.17. Nichrome and copper wires of the same length and same radius are connected in series. Current I is passed through them. Why does the nichrome wire get heated first? *(C.B.S.E. 2001 S)*

Ans. The nichrome and copper wires are of same length and same radius but the resistivity of nichrome is large as compared to that of copper. Due to this, the resistance of nichrome wire is very large in comparison to that of copper wire. When two wires are connected in series, the same amount of current flows through each of them. Since heat produced in a wire is directly proportional to its resistance, **nichrome** wire gets heated first.

Q. 3.18. Three bulbs 40 W, 60 W and 100 W are connected to 220 V mains. Which bulb will glow brightly, if they are joined in series ? *(H.P.S.S.C.E. 2001)*

Sol. From the relation, $R = \dfrac{V^2}{P}$, it follows that resistance of 40 W bulb will be maximum. When the bulbs are connected in series, the same amount current flows through all the bulbs. Since heat produced and hence brightness of a bulb is directly proportional to the resistance of its filament, **40 W** bulb will glow brightly.

Q. 3.19. A heater joined in series with 60 W bulb is connected to the mains. If 60 W bulb is replaced by a 100 W bulb, will the rate of heat produced by the heater be more, less or remain the same ? *(C.B.S.E. 2001 (S))*

Sol. The resistance of 100 W bulb is less than that of 60 W bulb. Therefore, when 60 W bulb is replaced by 100 W bulb, the current through the circuit will increase. As a result, heater will produce **more** heat, when 60 W bulb is replaced by 100 W bulb.

Q. 3.20. What are the characteristics of a fuse wire ?

Ans. A fuse wire is made of a material having high resistivity and low melting point. However, the value of its melting point is slightly above the temperature that the wire attains, when the current equal to maximum safe value flows through the fuse wire.

Q. 3.21. What is a safety fuse ? Explain its function. *(P.S.S.C.E. 1999)*

Ans. Refer to section 3.07.

Q. 3.22. What are the characteristics of the filament of a heater ?

Ans. The material of the filament of a heater has high resistivity and high melting point.

Q. 3.23. Name a few practical applications of heating effect of current.

Ans. The appliances, such as (*i*) electric lamp, (*ii*) electric heater, (*iii*) electric iron, (*iv*) electric geyser, (*v*) safety fuse, etc are based on heating effect of current.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. Define the terms electric energy and electric power. Give their units. *(H.S.S.C.E. 1992)*

Ans. For definition of electric energy and electric power, refer to sections 3.03 and 3.02.

The unit of electric energy is kilowatt hour and that of electric power is kilowatt.

Q. 3.02. Explain, why heat is produced in a conductor, when an electric current is passed through it.

Ans. The flow of electric current through a conductor means the flow of electrons through it. In the course of their motion, the electrons collide against one another and also against the atoms of the conductor. At each collision, a part of kinetic energy of the electron gets converted into heat, thereby causing a rise in the temperature of the conductor.

Q. 3.03. State Joule's heating law. *(H.P.S.S.C.E. 2001)*

Ans. Refer to section 3.04.

Q. 304. State and explain Joule's law for the rate of production of heat in a conductor carrying current. *(H.S.S.C.E. 2002)*

Ans. Refer to section 3.04.

Q. 3.05. A given potential difference is applied to a good conductor and a bad conductor of same dimension. Which one shall produce more heat and why ?

Ans. If a good conductor and a bad conductor have same dimensions, then resistance of the good conductor will be less than that of the bad conductor. When a given potential difference is applied to each of them, current in case of conductor will be more. Since heat produced $Q \propto I^2$, it follows that heat produced in case of **good conductor** will be more.

Q. 3.06. Two resistors of 2 Ω and 4 Ω are connected in parallel to a constant d.c. voltage. In which resistor is more heat produced ? *(C.B.S.E. 1998 S)*

Ans. Here, $R_1 = 2\ \Omega\ ;\ R_2 = 4\ \Omega$

Let V be the applied d.c. voltage. When the two resistors are connected in parallel to the d.c. supply of voltage V, the potential difference across both the resistances will be same *i.e.* V. If H_1 and H_2 are heat produced in the two resistors in 1 s, then

$$H_1 = \frac{V^2}{R_1} \text{ and } H_2 = \frac{V^2}{R_2}$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{V^2}{R_1} \times \frac{R_2}{V^2} = \frac{R_2}{R_1} = \frac{4}{2} = 2$$

Therefore, heat produced in **2 Ω** resistance will be more.

Q. 3.07. A 6 V storage battery of negligible internal resistance is connected across a 60 Ω resistor. How much heat is produced in the resistor in 2 hours ? What is the source of energy ? *(H.S.S.C.E. 2002, 1996)*

Ans. Here, $V = 6$ volt ; $R = 60\ \Omega$; $t = 2\text{ h} = 7200$ s

Heat energy produced in 1 h

$$= V\,I\,t = \frac{V^2}{R}\,t = \frac{(6)^2}{60} \times 7200 = \mathbf{4320\ J}$$

The source of heat energy is the **chemical energy** stored in the storage battery.

Q. 3.08. A heater coil is cut into two parts and only one of them is used in the heater. What is the ratio of the heat produced by this half coil to that by the original coil ?

Ans. Suppose that the resistance of the heater coil is R and it is used on a supply line of V volt.

Heat energy produced by the heater coil in time t,

$$Q = \frac{V^2}{R} t$$

The resistance of the one half of the coil $= \frac{R}{2}$

Heat energy produced by the half coil,

$$Q_2 = \frac{V^2}{R/2} t = \frac{2V^2}{R} t$$

$$\therefore \quad \frac{Q_2}{Q_1} = \frac{2V^2}{R} t \Big/ \frac{V^2}{R} t = 2$$

Q. 3.09. Three equal resistors connected in series across a source of e.m.f. together dissipate 10 W of power. What would be the power dissipated, if the same resistors are connected in parallel across the same source of e.m.f. ?

Ans. Let the e.m.f. of the source be V and the resistance of each resistor be R.

Resistance of the three resistors in series = 3 R

As power consumed by the three resistors in series is 10 W, we have

$$\frac{V^2}{3R} = 10$$

$$\text{or} \quad \frac{V^2}{R} = 30 \qquad \ldots(i)$$

Resistance of the three resistors in parallel $= \frac{R}{3}$

Therefore, power consumed by parallel combination of the resistors, $P = \frac{V^2}{R/3} = \frac{3V^2}{R}$

Using equation (*i*), we have

$$P = 3 \times 30 = 90 \text{ W}$$

Q. 3.10. In a house having 220 V-line, the following appliances are operating : a 60 W bulb, a 1000 W heater and a 40 W radio. Calculate (*a*) the current drawn by heater and (*b*) the current passing through fuse for this line. (*M.N.R. 1989*)

Ans. Here, V = 220 volt

Power rating of bulb, $P_1 = 60$ W;

power rating of heater, $P_2 = 1000$ W ;

power rating of radio, $P_3 = 40$ W

(*a*) Current drawn by heater $= \frac{P_2}{V} = \frac{1000}{220} = \frac{50}{11}$ A

Current drawn by bulb $= \frac{P_1}{V} = \frac{60}{220} = \frac{3}{11}$ A

Current drawn by radio $= \frac{P_3}{V} = \frac{40}{220} = \frac{2}{11}$ A

(*b*) Current passing through fuse for the line

$$= \frac{50}{11} + \frac{3}{11} + \frac{2}{11} = 5 \text{ A}$$

Q. 3.11. A toaster produces more heat than a light bulb, when connected in parallel. Which has greater resistance ? (*C.B.S.E. 1999 S*)

Ans. Let R_1 and R_2 be resistances of the toaster and the light bulb respectively. If V is the voltage applied across their parallel combination, then

heat produced per second in toaster, $P_1 = \frac{V^2}{R_1}$

and heat produced per second in light bulb, $P_2 = \frac{V^2}{R_2}$

Since toaster produces more heat than the light bulb,

$$P_1 > P_2$$

$$i.e. \quad \frac{V^2}{R_1} > \frac{V^2}{R_2} \quad \text{or} \quad R_1 < R_2$$

i.e. the resistance of the **light bulb** is more than that of the toaster.

Q. 3.12. Find the cost of electricity for running an electric motor of 1 h.p. for 5 hours a day @ Rs. 1·50 per unit for the month of November. (*H.S.S.C.E. 1995*)

Ans. Here, power of electric motor,

$$P = 1 \text{ h.p.} = 746 \text{ W} = 0{\cdot}746 \text{ kW}$$

Number of hours for which motor is run

$$= 5 \times 30 = 150 \text{ h}$$

Electrical energy consumed by the motor

$$= 0{\cdot}746 \times 150 = 111{\cdot}9 \text{ kWh}$$

Cost of running the motor $= 111{\cdot}9 \times 1{\cdot}50 =$ **Rs. 167·85**

Q. 3.13. The element of a heater is very hot, while the connecting wires carrying the current are cold. Why ?

Ans. The same current passes through the element of the heater and the connecting wires. The element of the heater becomes very hot as its resistance is very high as compared to that of the connecting wires.

Q. 3.14. Explain why an electric bulb becomes dim, when an electric heater in parallel circuit is switched on. Why dimness decreases after some time ?

Ans. The electric power of a heater is more than that of a bulb. Since power of an appliance is inversely proportional to its resistance $\left(P \propto \frac{1}{R}\right)$, the resistance of heater is less than that of the electric bulb.

When the heater connected in parallel to the bulb is switched on, it draws more current in the circuit due to its lesser resistance. As a result, current through the electric bulb decreases and it becomes dim.

When the heater coil becomes sufficently hot, its resistance becomes more and then it draws a little lesser current. As a result of it, the current through the electric bulb recovers. It is for this reason that after some time, the dimness of the bulb decreases.

Q. 3.15. Water boils in an electric kettle in time *t*, after it has been switched on. Using the same main supply, it is desired to make water boil in a lesser time. Should the length of the heating element be increased or decreased ?

Ans. Let l, A and ρ be the length, area of cross-section and resistivity of the material of the heating element. Then, its resistance is given by

$$R = \rho \frac{l}{A}$$

If V is the supply voltage, then heat required to boil the water in kettle,

$$Q = \frac{V^2 t}{R} = \frac{V^2 A t}{\rho l}$$

$$\text{or} \quad l = \frac{V^2 A t}{Q \rho} \quad \text{or} \quad l \propto t$$

Therefore, if water is to be boiled in lesser time, the ength of the heating element should be **decreased**.

Q. 3.16. What is the difference between heating wire nd a fuse wire ? *(H.S.S.C.E. 2001)*

Ans. A heating wire is made of a material having high alues of both resisitivity and melting point. The high value f resistivity ensures the production of a large amount of heat, whereas the high value of melting point saves it from melting.

On the other hand, a fuse wire is made of a material having a suitable high resisitivity but low melting point. When the current in electric circuit exceeds a safe limit, the high value of resistivity results in the production of heat and the low value of melting point causes it to melt. It, thus, saves the electrical appliances from gettting damaged.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 3.01. Prove that total power consumed by a number of electrical appliances (when connected in parallel) s equal to the sum of the powers consumed by individual ppliances.

Ans. Consider three electrical appliances of powers $_1$, P_2 and P_3. Let R_1, R_2 and R_3 be their respective resistances. ince appliances are connected in parallel, their total esistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{...}(i)$$

Suppose that the appliances are operated at the voltage . Multiplying both sides of equation (*i*) by V^2, we have

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$

or $\quad P = P_1 + P_2 + P_3$

Hence, total power consumed by the three appliances is qual to the sum of the powers of the individual appliances.

Q. 3.02. A frill of 50 bulbs joined in series is connected to mains. On getting one bulb fused, the remaining 9 bulbs are again put in series and connected to the same ain supply. Will the light emitted by the frill increase or ecrease ?

Ans. Let R be the resistance of the 50 bulbs, when joined n series.

Therefore, power of the frill of 50 bulbs, $P = \frac{V^2}{R}$,

here V is the voltage of mains supply.

When one bulb gets fused, the resistance of remaining 9 bulbs of the frill will decrease. Suppose that it becomes R'.

Therefore, power of the frill of 49 bulbs, $P' = \frac{V^2}{R'}$,

Since $R' < R$, it follows that $\frac{V^2}{R'} > \frac{V^2}{R}$. Hence, the light mitted by the frill of 49 bulbs will be **more**.

Q. 3.03. The maximum power rating of a 30 Ω resistor is 2·0 kW. (That is, this is the maximum power the esistor can dissipate as heat without melting or changing n some other undesirable way). Would you connect this esistor directly across a 300 V d.c. source of negligible nternal resistance ? Explain your answer.

(H.S.S.C.E. 1997 S ; Text Question)

Ans. Here, R = 30 Ω ;

Power rating of resistor P = 2·0 kW

If the resistor is connected to 300 V d.c., then rate of dissipation of heat,

$$P = \frac{V^2}{R} = \frac{300^2}{30} = 3000 \text{ W} = 3\cdot0 \text{ kW}$$

Since power rating of the resistor is 2·0 kW, it will melt out. Hence, it **should not be connected** to the 300 V d.c. directly.

Q. 3.04. A current is passed through a steel wire, heating it to red hot. The half of the wire is immersed in cold water. Which half of the wire will heat up more and why ?

Ans. The part of the steel wire outside the cold water will heat up more. The reason is that the temperature of the part of the wire outside the water will be more than that of the part inside the water. Since resistance of the wire increases with temperature, the resistance of the part of the wire outside the water will become more than that of the part inside the water. Since same current is flowing through the two parts, more heat will be produced in the part outside the water.

Q. 3.05. A 60 W electric bulb connected (*a*) in series (*b*) in parallel with a room heater is further connected across the mains. If 60 W bulb is now replaced by 100 W bulb in each case, will the heat produced by heater be smaller, remain the same or be larger and why ?

(C.B.S.E. Sample Q. Paper)

Ans. Let V be mains supply voltage and R be the resistance of room heater.

Resistance of 60 W bulb, $R_1 = \frac{V^2}{P} = \frac{V^2}{60}$

Resistance of 100 W bulb, $R_2 = \frac{V^2}{P} = \frac{V^2}{100}$

Obviously, R_2 is less than R_1.

(a) When bulb is connected in series with the room heater :

Current flowing through heater and 60 W bulb,

$$I_1 = \frac{V}{R + R_1}$$

Current flowing through heater and 100 W bulb,

$$I_2 = \frac{V}{R + R_2}$$

Now heat produced is directly proportional to square of the current flowing through the heater. Since R_2 is less than R_1, I_2 will be greater than I_1 and consequently heat produced by the heater will be **more**, when 60 W bulb connected in series is replaced by 100 W bulb.

(b) When bulb is connected in parallel with the room heater : When bulb and heater are connected in parallel to the mains, the potential difference across each is equal to the mains supply voltage. Therefore, current and consequently heat produced by the heater will remain **unchanged** on replacing 60 W bulb by 100 W bulb.

Q. 3.06. A storage battery of e.m.f. E is shorted by an external circuit first with a resistance R_1 and then with

a resistance R_2. Find the value of the internal resistance of the battery, if the quantities of heat liberated by the battery in the two cases are same.

Ans. Let r be internal resistance of the battery.

Heat produced in time t, when shorted with the resistance R_1,

$$Q_1 = \left(\frac{E}{R_1 + r}\right)^2 R_1 t \qquad \text{...}(i)$$

Also, heat produced in time t, when shorted with the resistance R_2,

$$Q_2 = \left(\frac{E}{R_2 + r}\right)^2 R_2 t \qquad \text{...}(ii)$$

Since same quantity of heat is produced in two cases,

$$\left(\frac{E}{R_1 + r}\right)^2 R_1 t = \left(\frac{E}{R_2 + r}\right)^2 R_2 t$$

$$\text{or} \quad R_1 (R_2 + r)^2 = R_2 (R_1 + r)^2 \quad \text{or} \quad r = \sqrt{R_1 R_2}$$

Q. 3.08. A steel wire has a resistance twice as great as a copper wire. Which wire will liberate more heat : (*a*) in parallel ; (*b*) in series, when the combination is connected to a battery ?

Ans. Let 2 R and R be the resistances of the steel and copper wires respectively.

(a) When the two wires are connected in parallel : Let I_1 and I_2 be currents through the steel and copper wires respectively [Fig. 3.08].

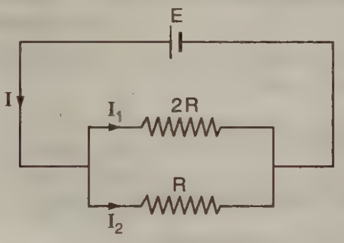


Fig. 3.08

$$\text{Then,} \qquad I_1 = \frac{E}{2R} \text{ and } I_2 = \frac{E}{R}$$

Heat produced in steel wire in time t,

$$Q_1 = I_1^2 (2R) t = \left(\frac{E}{2R}\right)^2 (2R) t = \frac{E^2}{2R} t$$

Heat produced in copper wire in time t,

$$Q_2 = I_2^2 R t = \left(\frac{E}{R}\right)^2 R t = \frac{E^2}{R} t$$

As $Q_2 > Q_1$, **copper wire** will liberate more heat.

(b) When two wires are connected in series : Let I be the current flowing through the series combination of the two wires [Fig. 3.09].

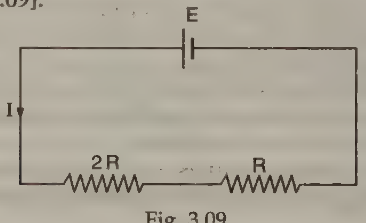


Fig. 3.09

$$\text{Then,} \qquad I = \frac{E}{2R + R} = \frac{E}{3R}$$

Heat produced in steel wire in time t,

$$Q_1 = I^2 (2R) t = \left(\frac{E}{3R}\right)^2 (2R) t = \frac{2}{9}\frac{E^2}{R} t$$

Heat produced in copper wire in time t,

$$Q_2 = I^2 R t = \left(\frac{E}{3R}\right)^2 R t = \frac{1}{9}\frac{E^2}{R} t$$

As $Q_1 > Q_2$, **sheet wire** will liberate more heat.

Q. 3.08. A dry cell of e.m.f. 1·5 volt and interr resistance 0·10 Ω is connected across a resistance series with a very low resistance ammeter. When the circ is switched on, the ammeter reading settles to a steady val of 2·0 A. What is the steady

(*a*) rate of chemical energy consumption of the cel

(*b*) rate of energy dissipation inside the cell,

(*c*) rate of energy dissipation inside the resistor an

(*d*) power output of the source ? *(Text Questic*

Sol. Here, V = 1·5 volt ; r = 0·10 Ω ; I = 2·0 A

Let R be the external resistance. Then,

$$I = \frac{V}{R + r} \quad \text{or} \quad R + r = \frac{V}{I} = \frac{1{\cdot}5}{2{\cdot}0} = 0{\cdot}75\,\Omega$$

$$\text{or} \quad R = 0{\cdot}75 - 0{\cdot}10 = 0{\cdot}65\,\Omega$$

(*a*) Rate of consumption of chemical energy of the c

$V \times I = 1{\cdot}5 \times 2{\cdot}0 = \mathbf{3{\cdot}0\ W}$

(*b*) Rate of energy dissipation inside the cell,

$I^2\, r = (2{\cdot}0)^2 \times 0{\cdot}10 = \mathbf{0{\cdot}4\ W}$

(*c*) Rate of energy dissipation inside the resistor,

$I^2\, R = (2{\cdot}0)^2 \times 0{\cdot}65 = \mathbf{2{\cdot}6\ W}$

(*d*) Power output of the source

= total power

– rate of energy dissipiation inside the c

$= 3{\cdot}0 - 0{\cdot}4 = \mathbf{2{\cdot}6\ W}$

Q. 3.09. Two wires made of tinned copper havi identical cross-section (= 10^{-6} m^2) and lengths 10 and 15 cm are to be used as fuses. Show that the fuses w melt at the same value of current in each case.

(Text Questi

Ans. Let there be a wire of length l, radius r and mater of resistivity ρ.

Now, resistance of the wire, $R = \rho \dfrac{l}{A} = \rho \dfrac{l}{\pi r^2}$

Suppose that a current I is passed through the wire.

Heat produced per second $= I^2 R = I^2 \rho \dfrac{l}{\pi r^2}$

The hot wire will heat radiate from its surface to t surroundings. Let the rate of loss of heat due to radiation fr the surface of the wire per unit its surface area be equal to

Neglecting the area of the end faces,

$$\text{surface area of wire} = 2 \pi r l$$

Therefore, heat radiated per second by the wire

$$= 2 \pi r l H$$

As current flows through the wire, heat is produced a its temperature keeps on rising. At the same time, heat is l to the surroundings due to radiation at a rate, which increa with the increase in temperature of the wire. The wire w be saved from melting, if its temperature rises say up to θ (below the melting point of the material of the wire), at which the rate of production of heat in the wire just becomes equal to the rate of loss of heat due to radiation from it. Therefore, in equilibrium,

heat lost per second = heat produced per second

$$2\pi r l H = I^2 \rho \frac{l}{\pi r^2}$$

or
$$H = I^2 \cdot \frac{\rho}{2\pi^2 r^3}$$

It follows that rate of loss of heat (which depends upon the temperature of the wire) does not depend upon length of the wire. Hence, fuses of different length will melt at the same value of current in each case.

Q. 3.10. A fuse with a circular cross-sectional radius of 0·15 mm blows at 15 A. What should be the radius of cross-section of a fuse made of the same material, which will blow at 30 A ? *(Text Question)*

Ans. From conceptual SAQ 3.09, it follows that

$$\frac{I^2}{r^3} = \frac{2\pi^2 H}{\rho} = \text{constant}$$

If I_1 and I_2 are current ratings of the fuse wires (currents at which, the fuse wires will blow) of radii r_1 and r_2, then

$$\frac{I_1^2}{r_1^3} = \frac{I_2^2}{r_2^3} \quad \text{or} \quad r_2 = r_1\left(\frac{I_2}{I_1}\right)^{2/3}$$

Here, $I_1 = 15$ A ; $r_1 = 0{\cdot}15$ mm and $I_2 = 30$ A

$$\therefore \quad r_2 = 0{\cdot}15 \times \left(\frac{30}{15}\right)^{2/3} = 0{\cdot}15 \times 2^{2/3} = 0{\cdot}15 \times 1{\cdot}5874$$

$$= 0{\cdot}238 \text{ mm}$$

Q. 3.11. A nichrome heating element across 230 V supply consumes 1·5 kW of power and heats up to a temperature of 750°C. A tungsten bulb across the same supply operates at a much higher temperature of 1600°C in order to be able to emit light. Does it mean that the tungsten bulb necessarily consume greater power ? *(Text Question)*

Ans. As worked out in conceptual SAQ 3.09, the steady temperature of a current carrying wire depends on rate of loss of heat (which depends on surface area, emissivity, etc) from the wire in addition to the power consumed. Therefore, it does not necessarily mean that tungsten bulb working at higher temperature consumes greater power.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What do you understand by heating effect of current ? Explain its cause. *(H.P.S.S.C.E. 2002)*
2. Prove that heat produced in good conductor is given by
$$H = \frac{I^2 R t}{4{\cdot}2} \text{ (in cal)},$$
where the symbols used have usual meanings.
3. Prove that electric energy (W) consumed in a conductor is given by the formula :
$$W = I^2 R t$$
4. A potential difference V is applied across a resistance R. Find a relation for the power consumed. *(P.S.S.C.E. 1992)*
5. Obtain an expression for the heat produced in a given conductor on passing current. *(H.S.S.C.E. 1999)*
6. Define electric power and electric energy. Give their SI units. *(P.S.S.C.E. 1993)*
7. Derive Joule's law for heating effects of currents. What are the units of electric power and electric energy ?
8. State and explain Joule's law of heating effects of current. *(H S S C E 1997)*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. (*a*) Explain, why heat is produced, when a current passes through a resistance. Obtain an expression for the heat produced, when current is passed through a conductor.
(*b*) Define electric power and electric energy. Define their SI units.
2. (*a*) State and explain Joule's laws for heating effect of current.
(*b*) Explain in brief three important applications of heating effect of current.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Electric power

1. A lamp of 40 W works at 200 volt. What is its resistance and current capacity ? **[Ans.** 1000 Ω, 0·2 A]
2. How many 60 watt bulbs may be safely run on 220 V using a 5 A fuse ? *(C.B.S.E. 1999)* **[Ans.** 18]
3. An electric heater consists of 10 m long nichrome wire of $2{\cdot}5 \times 10^{-6}$ m^2 cross-sectional area. Calculate the wattage of the heater, when the potential difference across the heater is 200 V. Given that resistivity of nichrome = $1{\cdot}1 \times 10^{-6}$ Ω m. **[Ans.** 9·091 kW]
4. Three equal resistances connected in series across a source of e.m.f. consumes 20 watt. If the same resistances are connected in parallel across the same source of e.m.f., what would be power dissipated ? *(P.S.S.C.E. 1999 ; C.B.S.E. 1999)* **[Ans.** 180 W]

Type B. On Joule's heating effect of current

5. A heating element of resistance 40 ohm is connected to 200 V supply ? How much heat will be produced in one minute ? *(P.S.S.C.E. 1999)* **[Ans.** 6×10^4 J]
6. An electric bulb is marked 100 W-200 V. If the supply voltage drops to 110 V, how much heat and light energy is produced by the bulb in half hour ? *(P.S.S.C.E. 1998)* **[Ans.** 54450 J]
7. An electric bulb is marked 100 W-230 V. If the supply voltage drops to 115 V, what is the heat and light energy produced by the bulb in 20 minutes ? *(P.S.S.C.E. 1997 S ; Text Problem)* **[Ans.** 30,000 J]
8. Calculate the amount of heat produced per second (in calories), when a bulb of 100 W, 220 V glows assuming that only 20% of electric energy is converted into light ($J = 4{\cdot}2$ J cal^{-1}). *(H.S.S.C.E. 2001)* **[Ans.** 19·05 cal]

9. In the circuit shown in Fig. 3.10, if the heat developed in 4 Ω resistance due to the current flowing through it is 40 cal s^{-1}, find the rate at which heat is produced in 2 Ω resistance. [Ans. 80 cal s^{-1}]

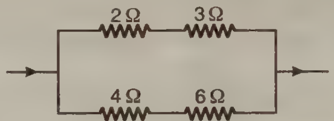


Fig. 3.10

10. Three resistors R_1, R_2 and R_3 each of 240 Ω are connected across a 120 V suppy as shown in Fig. 3.11. Find (*a*) the potential difference across each resistor and (*b*) the total heat developed across the three resistors in 1 minute. [Ans. $V_1 = 80$ V, $V_2 = V_3 = 40$ V ; 2400 J]

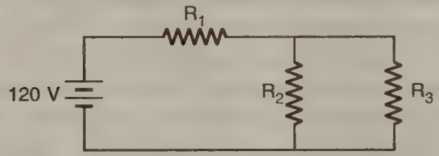


Fig. 3.11

11. A 2·5 kW heater is used for 4 hours daily. If electric energy is being sold at the rate of Rs. 2·5 per kWh, what will be the cost of the bill for a month ? (*P.S.S.C.E. 1999*) [Ans. Rs. 750]

12. What will be the cost of running 500 W heater for 20 hours at the rate of Re. 2·40 per unit ? [Ans. Rs. 24]

13. By mistake, a 100 watt electric lamp remains switched on for 30 days. What is the loss, if electricity costs Re. 3·00 per kilowatt hour ? [Ans. Rs. 216]

14. What is the cost of electrically heating 50 litres of water from 40°C to 100°C at Re. 2·50 per kWh ? Given, J = 4·2 J cal^{-1}. [Ans. Rs. 8·75]

15. A room heater is marked 1000 watt and 220 volt. Calculate (*a*) the current it draws (*b*) its resistance (*c*) cost of running it for 12 hours at the rate of Rs. 2·50 per unit. [Ans. 4·545A, 48·4 Ω, Rs. 30]

16. A house is fitted with two electric lamps, each of 100 W ; one heater of resistance 110 Ω and two fans, each consuming 0·25 A. If electric energy is supplied at 200 V and each appliance works for 5 hours a day, find the monthly bill at the rate of Rs. 3·0 per kWh. (*P.S.S.C.E. 1998 S*) [Ans. Rs. 308·58]

17. A house is fitted with eight lamps (each of 40 watt) and two fans, each taking a current of 0·25 ampere. The energy is supplied at 220 volt. If the lamps are lighted for 3 hours a day and the fans work for 6 hours a day, find the bill for 30 days. The cost of energy is at the rate of Rs. 2·40 per kWh. [Ans. Rs. 116·64]

18. A house is fitted with 10 lamps of 60 W each, 10 fans consuming 0·5 A each and an electric kettle of resistance 110 Ω. If the energy is supplied at 220 V and costs Rs 2·50 per kWh ; calculate bill for 10 days, if all the appliances are used for 6 hours daily. (*H.P.S.S.C.E. 1998 S*) [Ans. Rs. 321]

19. A house is fitted with 20 lamps rated 100 W each, 10 fans consuming 0·5 A each and electric kettle of resistance 100 ohm. Electricity is supplied at 220 V and electrical energy is sold at Re. 2·50 per kWh. Calculate the bill for 10 days, if all the appliances are used for 6 hours daily. [Ans. Rs. 537·60]

20. Calculate the amount of water at 26°C that will converted into steam by an electric heater of power 2 kW one hour. Given, specific heat of water = 4·2 × 10^3 J kg^{-1}°C and latent heat of steam = 2·26 × 10^6 J kg^{-1}. [Ans. 2·8 k

21. A copper electric kettle of 1 kg contains 900 g of water 20°C. It takes 12 minutes to raise the temperature to 100° If electric energy is supplied at 210 volt, calculate t strength of the electric current, assuming 10% of the he is wasted. Take specific heat of copper = 0·1. [Ans. 2·47

22. A heater coil is rated 100 W, 200 V. It is cut into two identi parts. Both parts are connected together in parallel to t same source of 200 V. Calculate the energy liberated p second in the new combination. (*C.B.S.E. 200* [Ans. 400

23. Two electric lamps are rated at 220 V- 40 W and 220 V- W. Find the heat generated in each per second, when th are connected in series across 220 V line. J = 4·2 J cal^{-1}. [Ans. 3·429 cal ; 2·286 c

24. For what value of load resistance, the power transfer maximum, when two identical cells, each of e.m.f. E a internal resistance *r* are connected in (*a*) series and parallel. [Ans. (*a*) 2 *r* (*b*) *r*/

25. How will you connect 36 cells, each of internal resistar 1·5 Ω so as to get maximum power out across a load 6 Ω ? [Ans. Three rows, each row containing 12 cel

26. An electric motor operates on a 50 V supply and draw current of 15 A. If the motor yields a mechanical power 150 W, estimate the power dissipated across its windin Also, find the efficiency of the motor ? [Ans. 600 W ; 20

MISCELLANEOUS PROBLEMS

27. A cell sends a current through a resistance R_1 for time *t* is, then, used to send current through another resistar for the same time *t*. If the heat developed in the two cas is same, then calculate the internal resistance of the ce [Ans. $\sqrt{R_1 \, R}$

28. Water boils in an electric kettle in 15 minutes, after it h been switched on. If the length of the heating element decreased to two thirds of its original length calculate t time in which the same amount of water will boil, wh kettle is connected to the same supply. [Ans. 10 mi

29. An electric kettle heater has two coils. When one coil switched on, the water in the kettle begins to boil after minutes and when the other is switched on the wa begins to boil after 30 minutes. How soon will the wa in the kettle begin to boil, if both the coils are connect (*a*) in series ; (*b*) in parallel ? [Ans. (*a*) 45 min ; (*b*) 10 m

30. Twenty one electric bulbs are connected in series to 2 V supply. After one bulb is fused, the remaining 20 bul are again connected in series to the same supply. By wh precentage will the illumination of (*a*) one bulb change a (*b*) all the bulbs change ? [Ans. (*a*) 10·25% (*b*) 5

31. A room is lit with a 60 W lamp. The voltage in the mai is 220 V. A 1000 W heater is, then, switched on. If t resistance of the wires connecting the electrical devices the room with the mains is 10 Ω, by how much will t voltage supplied to the lamp be changed, when the hea is switched on ? [Ans. 36·83

32. A line having a total resistance of 0·2 Ω delivers 10 kW 220 V to a small factory. What is the efficiency transmission ? [Ans. 96

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Proceed as in solved problem no. 3.01.

2. Current drawn by 60 W bulb, $I = \frac{P}{V} = \frac{60}{220}$ A

Therefore, number of bulbs that can be run on 5 A fuse,

$$n = \frac{\text{current rating of fuse}}{I} = \frac{5}{60/220} = 18 \cdot 33$$

i.e. $n = \mathbf{18}$

3. Now, $R = \rho \frac{l}{A} = 1 \cdot 1 \times 10^{-6} \times \frac{10}{2 \cdot 5 \times 10^{-6}} = 4 \cdot 4\,\Omega$

Also, $P = \frac{V^2}{R} = \frac{(200)^2}{4 \cdot 4} = 9091\text{ W} = \mathbf{9{\cdot}091\ kW}$

4. Let V be e.m.f. of the source and R, the resistance each of the three resistors.

When resistors are connected in series :

$$R_s = R + R + R = 3R$$

Now, $P = \frac{V^2}{R_s}$ or $20 = \frac{V^2}{3R}$

or $\frac{V^2}{R} = 60$ ...(*i*)

When resistors are connected in parallel :

$$R_p = \frac{R}{3}$$

Now, $P' = \frac{V^2}{R_p}$ or $P' = \frac{V^2}{R/3} = 3\frac{V^2}{R}$

or $P' = 3 \times 60 = \mathbf{180\ W}$

5. $Q = \frac{V^2}{R} t = \frac{(200)^2}{40} \times 60 = \mathbf{6 \times 10^4\ J}$

6. Proceed as in solved problem no. 3.13.

7. Proceed as in solved problem no. 3.13.

8. Power of bulb, P = 100 W

Therefore, electric energy consumed per second = 100 J

Electric energy converted into heat,

$$Q = \frac{100 \times 80}{100} = 80\text{ J} = \frac{80}{4 \cdot 2} = \mathbf{19{\cdot}05\ cal}$$

9. Resistance of series combination of 2 Ω and 3 Ω resistors,

$$R_1 = 2 + 3 = 5\,\Omega$$

Also, resistance of series combination of 4 Ω and 6 Ω resistors,

$$R_2 = 4 + 6 = 10\,\Omega$$

Let I be the total current in the circuit. Then, part of current through R_1,

$$I_1 = \frac{I \times R_2}{R_1 + R_2} = \frac{I \times 10}{5 + 10} = \frac{2I}{3}$$

and part of current through R_2,

$$I_2 = \frac{I \times R_1}{R_1 + R_2} = \frac{I \times 5}{5 + 10} = \frac{I}{3}$$

Now, heat produced per second in 2 Ω resistor,

$Q_1 \propto I_1^2 \times 2$ ...(*i*)

and heat produced per second in 4 Ω resistor.

$Q_2 \propto I_2^2 \times 4$ ...(*ii*)

$\therefore \quad \frac{Q_1}{Q_2} = \frac{I_1^2 \times 2}{I_2^2 \times 4}$ or $\frac{Q_1}{40} = \frac{(2I/3)^2 \times 2}{(I/3)^2 \times 4}$

or $\mathbf{Q_1 = 80\ cal\ s^{-1}}$

10. (*a*) Total resistance of the circuit,

$$R = R_1 + \frac{R_2 \times R_3}{R_2 + R_3} = 240 + \frac{240 \times 240}{240 + 240}$$

$$= 240 + 120 = 360\Omega$$

Therefore, current in the main circuit,

$$I = \frac{V}{R} = \frac{120}{360} = \frac{1}{3}\text{ A}$$

and potential difference across R_1,

$$V_1 = IR_1 = \frac{1}{3} \times 240 = \mathbf{80\ V}$$

Since $R_2 = R_3$, current equal to $\frac{I}{2}$ *i.e.* $\frac{1}{6}$ A will flow through each of the two resistances.

Therefore, potential difference across R_2 or R_3 *i.e.*

$$V_2 = V_3 = \frac{1}{6} \times 240 = \mathbf{40\ V}$$

(*b*) Heat developed across the three resistors in 1 minute,

$$Q = I^2 R t = \left(\frac{1}{3}\right)^2 \times 360 \times 60 = \mathbf{2400\ J}$$

11. Energy consumed in 1 day = 2·5 × 4 = 10 kWh

Energy consumed in 1 month = 10 × 30 = 300 kWh

Cost of monthly bill = 300 × 2·5 = **Rs. 750**

12. Proceed as in problem no. 11.

13. Proceed as in problem no. 11.

14. Volume of water, V = 50 litre = 50×10^3 cm^3

Mass of water, $m = V\rho = 50 \times 10^3 \times 1 = 50 \times 10^3$ g

$= 50$ kg

Heat required to raise the temperature of water from 40°C to 100°C

$$Q = mc(\theta_2 - \theta_1)$$

Specific heat of water = 1 cal g^{-1} °C^{-1}

$= 4{\cdot}2 \times 10^3$ J kg^{-1} °C^{-1}

$\therefore \quad Q = 50 \times 4{\cdot}2 \times 10^3 \times (100 - 40) = 1{\cdot}26 \times 10^7$ J

Now, 1 kWh = $3{\cdot}6 \times 10^6$ J

$$\therefore \quad Q = \frac{1 \cdot 26 \times 10^{-7}}{3 \cdot 6 \times 10^6} = 3 \cdot 5\text{ kWh}$$

Cost of electricity = 3·5 × 2·50 = **Rs. 8·75**

15. Here, P = 1000 W, V = 220 V

(*a*) $I = \frac{P}{V} = \frac{1000}{220} = \mathbf{4{\cdot}545\ A}$

(*b*) $R = \frac{V}{I} = \frac{220 \times 220}{1000} = \mathbf{48{\cdot}4\ \Omega}$

(*c*) Energy consumed in 12 hours,

$$Q = P \times t = \frac{1000}{1000} \times 12 = 12\text{ kWh}$$

Cost of electricity = 12 × 2·50 = **Rs. 30**

16. Proceed as in solved problem no. 3.17.

17. Proceed as in solved problem no. 3.17.

18. Proceed as in solved problem no. 3.17.

19. Proceed as in solved problem no. 3.17.

20. Heat produced by heater in 1 hour,

$$Q = P \times t = 2 \times 10^3 \times 60 \times 60$$

or $Q = 7{\cdot}2 \times 10^6$ ...(*i*)

Suppose that this amount of heat Q can convert mass *m* (in kg) of water at 26°C into steam.

Then, $Q = mc(\theta_2 - \theta_1) + mL = m[c(\theta_2 - \theta_1) + L]$

$= m[4{\cdot}2 \times 10^3 (100 - 26) + 2{\cdot}26 \times 10^6]$

or $Q = 2{\cdot}57 \times 10^6\, m$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$2{\cdot}57 \times 10^6\, m = 7{\cdot}2 \times 10^6$ or $\mathbf{m = 2{\cdot}8\ kg}$

21. Mass of kettle, M = 1 kg = 1000 g ;

mass of water, *m* = 900 g

specific heat of copper, *c* = 0·1,

specific heat of water, *c'* = 1,

$\theta_1 = 20$°C and $\theta_2 = 100$°C

Therefore, amount of heat required to heat water in kettle from 20°C to 100°C,

$$Q = M\,c\,(\theta_2 - \theta_1) + m\,c'\,(\theta_2 - \theta_1)$$
$$= (M\,c + m\,c')\,(\theta_2 - \theta_1)$$

or $Q = (1000 \times 0{\cdot}1 + 900 \times 1) \times (100 - 20)$
$= 8 \times 10^4 \text{ cal} = 8 \times 4{\cdot}2 \times 10^4 \text{ J}$

Let I be the current passed through the kettle. Therefore, heat produced by kettle in 12 minutes,

$$Q' = V\,I\,t = 210 \times I \times 12 \times 60 \text{ J}$$

As 10% of heat produced is wasted, useful heat produced,

$$Q = \frac{Q' \times 90}{100}$$

or $$Q = \frac{210 \times I \times 12 \times 60 \times 90}{100} \text{ J} \quad ...(ii)$$

From equations (*i*) and (*ii*) we have

$$\frac{210 \times I \times 12 \times 60 \times 90}{100} = 8 \times 4{\cdot}2 \times 10^4$$

or $I = 2{\cdot}47$ **A**

22. Resistance of heater coil, $R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400\ \Omega$

When coil is cut into two parts, resistance of each of the two parts,

$$R_1 = R_2 = \frac{400}{2} = 200\ \Omega$$

Resistance of the parallel combination of two parts of the coil,

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{200 \times 200}{200 + 200} = 100\ \Omega$$

Therefore, energy liberated per second by the parallel combination,

$$P = \frac{V^2}{R_p} = \frac{(200)^2}{100} = \mathbf{400\ W}$$

23. Proceed as in solved problem no. 3.05.

24. (*a*) When cells are connected in series,
total internal resistance = $2\,r$
For maximum power transfer,
$R = 2\,r$
(*b*) When cells are connected in parallel,
total internal resistance = $r/2$
For maximum power transfer,
$$R = \frac{r}{2}$$

25. Suppose that the 36 cells are arranged in *m* rows, each row containing *n* cells. Then,

$$m\,n = 36 \quad ...(i)$$

Let *r* be internal resistance each cell.

Total internal resistance of the cells = $\frac{n\,r}{m}$

The grouping cells will deliver maximum current and hence maximum power, when
total internal resistance = external resistance

or $\frac{n\,r}{m} = R$ or $\frac{n \times 1{\cdot}5}{m} = 6$ or $n = 4\,m$

Substituting for *n* in equation (*i*), we have

$m \times 4\,m = 36$ or $m = 3$

Also, $n = 4\,m = 4 \times 3 = \mathbf{12}$

26. Total power of the motor, $P = V\,I = 50 \times 15 = 750$ W
Mechanical power of the motor = 150 W
Therefore, power dissipated across windings of motor
$P' = 750 - 150 =$ **600 W**

Efficiency of motor = $\frac{P'}{P} \times 100 = \frac{150}{750} \times 100 =$ **20%**

27. Let E be e.m.f. and *r*, the internal resistance of the cell. Heat produced in resistance R_1 in time *t*,

$$Q_1 = \left(\frac{E}{R_1 + r}\right)^2 R_1\,t$$

Also, heat produced in resistance R_2 in time *t*,

$$Q_2 = \left(\frac{E}{R_2 + r}\right)^2 R_2\,t$$

Since $Q_1 = Q_2$, we have

$$\left(\frac{E}{R_1 + r}\right)^2 R_1\,t = \left(\frac{E}{R_2 + r}\right)^2 R_2\,t$$

or $$\frac{R_1}{(R_1 + r)^2} = \frac{R_2}{(R_2 + r)^2}$$

or $R_1\,(R_2 + r)^2 = R_2\,(R_1 + r)^2$

On simplifying, we get

$$r = \sqrt{\mathbf{R_1\,R_2}}$$

28. Let R be resistance of the heating element of the kettle a
Q be the amount of heat required to boil the water in t
kettle. Then,

$$Q = \frac{V^2}{R}\,t$$

In first case :

$$Q = \frac{V^2}{R} \times 15 \times 60 \quad ...$$

In second case : When length of the heating element
decreased to two thirds of its original length, resistance
heating element becomes,

$$R' = \frac{2\,R}{3}$$

Suppose that water now boils in time *t*. Then,

$$Q = \frac{V^2}{R'} \times t = \frac{V^2}{2\,R/3} \times t$$

or $$Q = \frac{1{\cdot}5\,V^2}{R}\,t \quad ...($$

From equations (*i*) and (*ii*), we have

$$\frac{1{\cdot}5\,V^2}{R}\,t = \frac{V^2}{R} \times 15 \times 60$$

or $$t = \frac{15 \times 60}{1{\cdot}5} = 600\ \text{s} = \frac{600}{60} = \mathbf{10\ minutes}$$

29. (*a*) Proceed as in solved problem no. 3.15.

(*b*) For first coil, $Q = \frac{V^2}{R_1} \times 15 \times 60$

or $$R_1 = \frac{V^2 \times 900}{Q}$$

and for second coil, $Q = \frac{V^2}{R_2} \times 30 \times 60$

or $$R_2 = \frac{V^2 \times 1800}{Q}$$

The resistance of the parallel combination of two coils,

$$R = \frac{R_1\,R_2}{R_1 + R_2}$$

If water, now, boils in time *t*, then

$$Q = \frac{V^2}{R}\,t = \frac{V^2(R_1 + R_2)}{R_1\,R_2}\,t = V^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)t$$

Substituting for R_1 and R_2, we get

$$Q = V^2\left(\frac{Q}{V^2 \times 900} + \frac{Q}{V^2 \times 1800}\right)t$$

or $$\frac{1}{t} = \frac{1}{900} + \frac{1}{1800}$$

or $$t = \frac{1800 \times 900}{1800 + 900} = 600 \text{ s} = \textbf{10 minutes}$$

30. Let R be resistance of each bulb. Then,
resistance of series combination of all the 21 bulbs = 21 R

Current through the bulbs, $I = \frac{220}{21\,R}$

Illuminating power of one bulb,

$$P_1 = I^2\,R = \left(\frac{220}{21\,R}\right)^2 R = \frac{109 \cdot 75}{R}$$

Illuminating power of all the bulbs,

$$P = 21 \times P_1 = \frac{21 \times 109 \cdot 75}{R} = \frac{2304 \cdot 75}{R}$$

When one bulb gets fused :
Resistance of series combination of all the 20 bulbs = 20 R

Current through the bulbs, $I' = \frac{220}{20\,R}$

Illuminating power of one bulb,

$$P_1' = I_1'\,R = \left(\frac{220}{20\,R}\right)^2 R = \frac{121}{R}$$

Illuminating power of all the bulbs,

$$P' = 20 \times P_1' = \frac{20 \times 121}{R} = \frac{2420}{R}$$

(*i*) % increase in illuminating power of one bulb,

$$\frac{P_1' - P_1}{P_1} \times 100 = \frac{121/R - 109 \cdot 75/R}{109 \cdot 75/R} \times 100$$

$$= \mathbf{10 \cdot 25\ \%}$$

(*ii*) % increase in illuminating power of all the bulbs,

$$\frac{P' - P}{P} \times 100 = \frac{2420/R - 2304 \cdot 75/R}{2304 \cdot 75/R} \times 100$$

$$= \mathbf{5\ \%}$$

31. Resistance of bulb, $R_1 = \frac{V^2}{P_1} = \frac{(220)^2}{60} = 806 \cdot 67\ W$

Resistance of heater, $R_2 = \frac{V^2}{P_2} = \frac{(220)^2}{1000} = 48 \cdot 4\ \Omega$

Resistance of the wires connecting the devices to the mains, $R_0 = 10\ \Omega$

Before the heater is switched on :
Current in the circuit,

$$I = \frac{V}{R_1 + R_0} = \frac{220}{806 \cdot 67 + 10} = 0 \cdot 2694\ A$$

Voltage drop across the connecting wires,

$$V_0 = I\,R_0 = 0 \cdot 2694 \times 10 = 2 \cdot 694\ V$$

After the heater is switched on :
Current in the circuit,

$$I' = \frac{V}{\dfrac{R_1 \times R_2}{R_1 + R_2} + R_0} = \frac{220}{\dfrac{806 \cdot 67 \times 48 \cdot 4}{806 \cdot 67 + 48 \cdot 4} + 10} = 3 \cdot 9525\ A$$

Voltage drop across the connecting wires

$$V_0' = I'\,R_0 = 3 \cdot 9525 \times 10 = 39 \cdot 525\ V$$

Therefore, change in voltage supplied to the lamp,

$$V_0' - V_0 = 39 \cdot 525 - 2 \cdot 694 = \mathbf{36 \cdot 83\ V}$$

32. Actual power delivered to factory, P = 10 kW
Current along the transmission line,

$$I = \frac{P}{V} = \frac{10 \times 10^3}{220} = 45 \cdot 455\ A$$

Resistance of power line, $R = 0 \cdot 2\ \Omega$
Therefore, loss of power along the transmission line

$$= I^2\,R = (45 \cdot 455)^2 \times 0 \cdot 2 = 413 \cdot 2\ W = 0 \cdot 4132\ kW$$

Hence, power supplied to transmission line,

$$P' = 10 + 0 \cdot 4132 = 10 \cdot 4132\ kW$$

Therefore, efficiency of transmission line,

$$\eta = \frac{P}{P'} \times 100 = \frac{10}{10 \cdot 4132} \times 100 = \mathbf{96\%}$$

CHAPTER
4

Chemical Effects of Curren

4.01. CHEMICAL EFFECT OF ELECTRIC CURRENTS

The chemical effects of electric current were studied by Faraday in 1833. When electric current is passed through a conductor, it gets heated. The magnetic field is also produced around a current carrying conductor. However, no chemical effect is produced, when current is passed through a conductor. The chemical effects of current may be observed in liquids. If electric current is passed through a liquid, it may or may not allow the passage of current through it. In fact, on the basis of their electric behaviour, the liquids may be divided into three classes :

(i) The liquids which do not allow current to pass through them. For example, distilled water, vegetable oil, etc.

(ii) *The liquids which allow current to pass through them but do not dissociate into ions.* For example, mercury.

(iii) *The liquids which allow current to pass through them by dissociating into ions.* For example, salt solutions, acids and bases.

The liquids, whcih dissociate into ions on passing current through them are called ***electrolytes*** *and the process of dissociation of a liquid into ions on passing current through it, is called* ***electrolysis****.*

Therefore, passage of electric current through the electrolytes produces chemical effect.

Key point

**When current passes through a metal
conductor, heating and magnetic effe
occur. No chemical effect is produced
a metallic conductor on passing curre
through it. Only a liquid may sho
chemical effects of electric curre
through it.**

4.02. ELECTROLYTE

The substances, which dissociate into ions and when in solution form, allow electric current to pass through them, are called ***electrolytes.*** For example, acids, bases, acidulated water, salt solutions, such as copper sulphate, silver nitrate, etc.

On the basis of their behaviour towards the passage of electric current, the electrolytes are of the following two types :

1. Strong electrolytes. *Those electrolytes, which are more or less completely ionised in their solutions, are called strong electrolytes.* For example, HCl, NaOH and NaCl.

2. Weak electrolytes. *Those electrolytes, which are ionised to a small extent in their solutions, are called weak electrolytes.* For example, NH_4Cl, CH_3COOH and $H_2 O_3$.

Key point

**The electrolytes are also known
conductors of second kind.**

4.03. ELECTROLYSIS

The process of liberating free elements from an electrolytic solution, when electric current is passed through it, is called ***electrolysis.***

When an electrolyte, such as copper sulphate ($CuSO_4$) is dissolved in water, it dissociates into ions as below :

$$CuSO_4 \longrightarrow Cu^{++} + SO_4^{--}$$

Consider that the copper sulphate solution is taken in a vessel and current is passed through it by using two electrodes and a source of e.m.f. The electrode, through which the current enters the electrolytic solution, is called ***anode,*** while the other electrode, through which current leaves, is called ***cathode.***

On passing current, Cu^{++} ions drift to the cathode, while SO_4^{--} ions towards the anode. At the cathode, Cu^{++} ions discharge themselves and copper atoms are liberated at the cathode.

The vessel, in which electrolysis is carried out is called ***voltameter.*** It is also called an ***electrolytic cell.***

4.04. THEORY OF ELECTROLYTES

An atom, a group of atoms or radical having charge is called an ion.

Substances like sodium chloride (NaCl), copper sulphate ($CuSO_4$), silver nitrate ($AgNO_3$), etc, when dissolved in water dissociate into ions. The dissociation of these substances into ions may be represented as below :

$$\mathrm{NaCl} \longrightarrow \mathrm{Na}^+ + \mathrm{Cl}^-$$
$$\mathrm{CuSO_4} \longrightarrow \mathrm{Cu}^{++} + \mathrm{SO_4^{--}}$$
$$\mathrm{AgNO_3} \longrightarrow \mathrm{Ag}^+ + \mathrm{NO_3^-}$$

If an atom, a molecule or a radical has electrons less than what it would have in its normal state, it is called a ***positive ion***. The positive ions are called ***cations*** ; as during electrolysis, they would collect at the cathode. On the other hand, if an atom, a molecule or a radical has electrons more than what it would have in its normal state, it is called a ***negative ion***. The negative ions collect at the anode during electrolysis and accordingly, they are called ***anions***. Let us first understand the mechanism of dissociation of a substance into ions.

In crystalline NaCl, Na^+ and Cl^- ions have large force of attraction between them. To remove Na^+ and Cl^- ions from each other, an energy of 7·9 eV per molecule is required. The average thermal agitation energy is of the order of 0·03 eV and it is thus insufficient to separate Na^+ and Cl^- ions, when it is in crystalline state.

Water has dielectric constant equal to 80. When sodium chloride is dissolved in water, Na^+ and Cl^- ions find themselves in water as continuous medium. Due to this, the potential energy of the sodium chloride molecule reduces by a factor of 80 and the average thermal agitation energy becomes sufficient to take Na^+ and Cl^- ions away from electric field. Electrolytes conduct electricity but their conductivity is very low. At room temperature, the conductivity of an electrolyte is about 10^{-5} to 10^{-6} times that of a good metallic conductor. It is because of the following three reasons :

Key point

An electrolyte dissociates into ions on being dissolved in water due to the reason that high value of the dielectric constant of water reduces the force of attraction between its positive and negative ions.

1. The number of free ions per unit volume in an electrolyte is quite smaller than the number of free electrons per unit volume in a metallic conductor.
2. In comparison to electrons, ions have very large mass. Therefore, when electric field is applied, they drift much more slowly than electrons.
3. Since an electrolytic solution is quite dense, the ions experience more difficulty to drift through it.

Key point

The conductivity of electrolytes is very small as compared to that of a metallic conductor due to the fact that number density of free ions in them is very small as compared to number density of free electrons in a metallic conductor.

Water also dissociates into positive and negative ions as represented below :

$$\mathrm{H_2O} \longrightarrow \mathrm{H}^+ + \mathrm{OH}^-$$

In pure water, its dissociation into ions is of a very low degree (about one part in 10^7). As such its conductivity is very low. However, when an acid or an alkali is added to water, its conductivity increases by a very large amount. Thus, water containing say a few drops of an acid also behaves as an electrolyte.

4.05. ELECTROLYSIS OF COPPER SULPHATE

A copper voltameter is used to carry out the electrolysis of copper sulphate solution. It consists of a glass vessel, which contains two copper electrodes A and C and the copper sulphate solution as the electrolyte. A battery, a rheostat, an ammeter and a one-way key are connected across the two copper electrodes A and C as shown in Fig. 4.01. The rheostat is used to adjust the current in the circuit, while he ammeter is used to measure the current. The copper electrode A to which positive ole of battery is connected, is called *anode*, while the electrode C connected to the egative pole is called *cathode*.

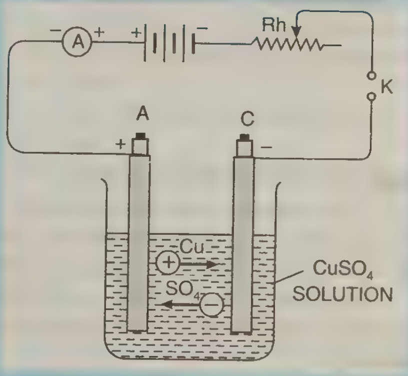


Fig. 4.01

When the circuit is switched on, current inside the voltameter is carried by the ons of the electrolyte and outside the voltameter, the current is carried by electrons. f the electrolyte is made acidic, the conductivity of the electrolyte improves without ffecting the reactions that take place inside the voltameter.

Mechanism of electrolysis. When the acidic copper sulphate solution issociates, the ions are produced as below :

$$\mathrm{CuSO_4} \longrightarrow \mathrm{Cu}^{++} + \mathrm{SO_4^{--}}$$
$$\mathrm{H_2O} \longrightarrow \mathrm{H}^+ + \mathrm{OH}^-$$

The Cu^{++} ions drift towards the cathode C. As soon as a Cu^{++} ion reaches the athode, two electrons from the negative pole of the battery flow into the cathode nd combine with the Cu^{++} ion to produce a neutral Cu atom. The Cu atom so roduced due to the discharge of Cu^{++} ion gets deposited at the cathode. The ollowing reaction takes place at the cathode :

$$\mathrm{Cu}^{++} + 2\,e^- \longrightarrow \mathrm{Cu}$$

As soon as a Cu^{++} ion discharges at the cathode, a Cu atom at the anode A eleases two electrons and the Cu^{++} ion so produced goes into the $\mathrm{CuSO_4}$ solution.

The two electrons so released flow from the anode to the positive pole of the battery. The following reaction takes place at the anode :

$$Cu \longrightarrow Cu^{++} + 2e^-$$

Therefore, the circuit completes due to the flow of ions inside the electrolyte and flow of electrons in the metallic connecting wires outside the voltameter. There is no accumulation of charge anywhere in the voltameter. The result of current flow in the circuit is simply the ***plating out of copper*** from the anode and its ***deposition*** at the cathode of the copper voltameter. However, the concentration of $CuSO_4$ solution remains the same during the process of electrolysis.

It may be noted that one Cu atom is deposited at the cathode for each pair of electrons flowing through the connecting wires. Therefore, the amount of copper deposited at the cathode during electrolysis is always directly proportional to the charge flowing through the circuit.

Key point

In an electric circuit containing a voltameter, current inside the voltameter i carried by ions and outside the voltamete current is carried by electrons.

4.06. ELECTROLYSIS OF WATER

A water voltameter consists of a glass vessel, at the base of which two platinum electrodes are provided. The vessel contains acidic water. Two long tubes filled with acidic water are held in inverted position over the two electrodes [Fig. 4.02]. A battery, a rheostat, an ammeter and a one-way key are connected to the two electrodes.

When current is passed through the acidic water, hydrogen gas is collected in the inverted tube put over cathode and oxygen gas is collected in the tube placed over anode.

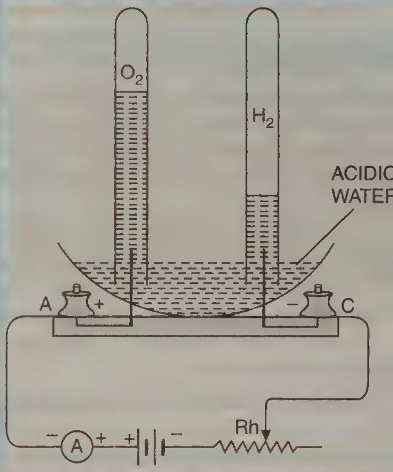


Fig. 4.02

Mechanism of electrolysis. The electrons from the negative terminal of the battery enter the cathode through the connecting wire and cause reduction of the water. The reaction at the cathode takes place as below :

$$4H_2O + 4e^- \longrightarrow 4OH^- + 2H_2$$

Due to this, hydrogen gas is liberated at the cathode. Also, the concentration of OH^- ions increases.

At the anode, concentration of OH^- ions is low. Due to dissociation of sulphuric acid (used to make water acidic), SO_4^{--} ions are also present at the anode but they are difficult to oxidize. As such, the main reaction at the anode is the direct oxidation of the water. The reaction takes place as below :

$$2H_2O \longrightarrow +4H^+ + 4e^- + O_2$$

The oxygen gas is liberated at the anode. Also, the released electrons reach the positive terminal of the battery after entering the anode and passing through the connecting wire. Further, due to this reaction, the concentration of H^+ ions increases. In the electrolyte, H^+ and OH^- move towards the cathode and anode respectively and neutralize according to following reaction :

$$4H^+ + 4OH^- \longrightarrow 4H_2O$$

If we add the respective sides of the above three reactions, we obtain

$$2H_2O \longrightarrow 2H_2 + O_2$$

4.07. FARADAY'S LAWS OF ELECTROLYSIS

The detailed experimental study of the electrolysis was carried out by Michael Faraday. The conclusions drawn by him are commonly known as ***Faraday's laws of electrolysis***. They are stated and discussed as below :

First law. *It states that the mass of substance deposited at the cathode during electrolysis is directly proportional to the quantity of electricity (total charge) passed through the electrolyte.*

Let m be the mass of the substance liberated, when a charge q is passed through the electrolyte. Then, according to Faraday's first law of electrolysis

$$m \propto q$$

or
$$m = z\,q, \qquad (4.01)$$

where the constant of proportionality z is called ***electrochemical equivalent* (E.C.E.) *of the substance***.

If a constant current I is passed through the electrolyte for time t, then the total charge passing through the electrolyte is given by

$$q = I\,t$$

Therefore, from equation (4.01), we have

$$m = z\,I\,t \qquad ...(4.02)$$

If $q = 1$ coulomb, then from equation (4.01), we have

$$m = z \times 1$$

or
$$z = m$$

Hence, *the electrochemical equivalent of substance may be defined as the mass of its substance deposited at the cathode, when one coulomb of charge passes through the electrolyte.*

The SI unit of electrochemical equivalent of a substance is **kilogram coulomb^{-1} (kg C^{-1})**. In practice, E.C.E. of a substance is expressed as **gram coulomb^{-1}** (g C^{-1}). The values of E.C.E. of a few common substances are given in the following table :

Element	Valency	Atomic weight	E.C.E. (g C^{-1})
Hydrogen	1	1 · 008	$104 \cdot 4 \times 10^{-7}$
Oxygen	2	16 · 000	829×10^{-7}
Chromium	3	52 · 010	1796×10^{-7}
Nickel	2	58 · 680	3040×10^{-7}
Copper	2	63 · 670	3294×10^{-7}
Zinc	2	65 · 380	3387×10^{-7}
Gold	3	197 · 200	6812×10^{-7}
Silver	1	107 · 880	11180×10^{-7}

Second law. *If same quantity of electricity is passed through different electrolytes, masses of the substances deposited at the respective cathodes are directly proportional to their chemical equivalents.*

The jargon .....

Chemical equivalent = $\dfrac{\text{atomic weight}}{\text{valency}}$

Let m be the mass of the ions of a substance liberated, whose chemical equivalent is E. Then, according to Faraday's second law of electrolysis,

$$m \propto \mathrm{E}$$

or
$$m = \text{constant} \times \mathrm{E}$$

or
$$\frac{m}{\mathrm{E}} = \text{constant} \qquad \text{...(4.03)}$$

Relation between chemical equivalent and electrochemical equivalent. Suppose that on passing same amount of electricity q through two different electrolytes, masses of the two substances liberated are m_1 and m_2. If E_1 and E_2 are their chemical equivalents, then from Faraday's second law, we have

$$\frac{m_1}{m_2} = \frac{\mathrm{E}_1}{\mathrm{E}_2} \qquad \text{...(4.04)}$$

Further, if z_1 and z_2 are the respective electrochemical equivalents of the two substances, then from Faraday's first law, we have

$$m_1 = z_1 q \qquad \text{and} \qquad m_2 = z_2 q$$

$$\therefore \qquad \frac{m_1}{m_2} = \frac{z_1}{z_2} \qquad \text{...(4.05)}$$

From equations (4.04) and (4.05), we have

$$\frac{z_1}{z_2} = \frac{\mathrm{E}_1}{\mathrm{E}_2}$$

or
$$z_2 = z_1 \times \frac{\mathrm{E}_2}{\mathrm{E}_1} \qquad \text{...(4.06)}$$

It gives the relation between chemical equivalents and electrochemical equivalents of two substances. Knowing z_1, E_1 and E_2, the above relation can be used to find electrochemical equivalent z_2 of an unknown substance. Further, from equation (4.06), it follows that

$$\frac{\mathrm{E}}{z} = \text{constant} \qquad \text{...(4.07)}$$

Note. The ratio E/z for a substance is known as *Faraday constant* and is equal to 96500 C mol^{-1}.

4.08. FARADAY CONSTANT

Faraday constant is the quantity of charge required to liberate one gram equivalent of the substance on an electrode during the process of electrolysis. It is denoted by F. It has a fixed value of 96500 C mol^{-1}.

Relation between Faraday constant and E.C.E. of a substance. Suppose that on passing electric charge q through an electrolyte, mass m of a substance is liberated.

If z is electrochemical equivalent of the substance, then from Faraday's first law, we have

$$m = z\,q$$

or
$$q = \frac{m}{z} \qquad \text{...(4.08)}$$

From definition of Faraday constant, it follows that when $m = \text{E}$ (chemical equivalent of the substance), $q = \text{F}$

Setting the above condition in equation (4.08), we have

$$\text{F} = \frac{\text{E}}{z} \qquad \text{...(4.09)}$$

Thus, *Faraday constant is equal to the ratio of chemical equivalent of a substance to its electrochemical equivalent.*

Key point

The ratio of chemical equivalent and electrochemical of a substance is known as ***Faraday constant***. Its value is same (96500 C mol^{-1}) irrespective of the nature of substance.

Further, equation (4.09) gives

$$z = \frac{\text{E}}{\text{F}} \qquad \text{...(4.10)}$$

Hence, *electrochemical equivalent of a substance is equal to the ratio of its chemical equivalent to Faraday constant.*

To express Faraday constant in terms of Avogadro number and electronic charge. Again, from Faraday's first law of electrolysis, we have

$$m = z\,q \qquad \text{...(4.11)}$$

Let us first calculate the quantity of charge required to liberate one mole of the substance. Suppose that the valency of the atoms of the substance is p. From the concept of the valency, it follows that in order to deposit one atom of the substance, p electrons have to pass through the electrolyte. In other words, when electric charge equal to pe is passed through the electrolyte, one atom of the substance will be deposited at the electrode. If N is Avogadro number, then

charge required to deposit one mole of the substance = N ($p\,e$)

Let M be mass of one mole of the substance. Then it follows that

when $q = \text{N}\,(p\,e), \quad m = \text{M}$

Setting the above condition in equation (4.11), we have

$$\text{M} = z\,\text{N}\,(p\,e)$$

or
$$\frac{\text{M}}{p} = z\,\text{N}\,e$$

But $\frac{\text{M}}{p} = \text{E}$, the chemical equivalent of the substance. Therefore, the above equation becomes

$$\text{E} = z\,\text{N}\,e$$

Substituting for E in equation (4.09), we have

$$\text{F} = \frac{z\,\text{N}\,e}{z}$$

or
$$\text{F} = \text{N}\,e \qquad \text{...(4.12)}$$

i.e. Faraday constant is equal to the product of Avogadro number and electronic charge.

Key point

Faraday constant is given by

$$\text{F} = \frac{\text{E}}{z} = \text{N}\,e,$$

where the letters have their usual meanings.

Since $\text{N} = 6{\cdot}0229 \times 10^{23}$ and $e = 1{\cdot}602 \times 10^{-19}$ C, we have

$$\text{F} = \text{N}\,e = 6{\cdot}0229 \times 10^{23} \times 1{\cdot}602 \times 10^{-19} = 96487 \text{ C} \approx 96500 \text{ C}$$

4.09. USES OF ELECTROLYSIS

The phenomenon of electrolysis has been put to many technical and commercial uses as detailed below :

1. Electroplating. The process of depositing a thin layer of a metal on any surface by electrolysis is called ***electroplating***. The article to be electroplated is made cathode and the metal to be deposited is made anode. A soluble salt of the metal (the metal to be deposited) is used as electrolyte. When the current is passed, a thin layer of the metal deposits on the material.

2. Extraction of metals from the ores. Certain metals are extracted from their ores by electrolysis. For example, aluminium is extracted by passing current through the fused ores bauxite (Al_2O_3) and cryolite ($Na_3\,AlF_6$).

3. Purification of metals. The impure metal is made anode and the pure metal is made cathode. The soluble salt of the pure metal is used as electrolyte. On passing current, pure metal gets deposited on the cathode. In this manner, copper has been purified upto 99·99%.

4. Electrotyping. Exact copies of the metallic type used in printing works and of the engraved blocks on metals can be prepared by the process of electrolysis. The impression of the type of block is first obtained on a soft material, such as wax. The mould (impression on wax) is made conducting by coating with graphite powder. Then, a thin film of copper is deposited by the process of electrolysis. This sheet gives a copy of the type or the engraved block.

5. Manufacture of caustic soda. By carrying out electrolysis of sodium chloride solution, caustic soda is prepared.

6. Electrolytic capacitors. An electrolytic capacitor consists of two aluminium electrodes dipping in a mixture of boric acid, glycerine and ammonia water. When current is passed, a thin film of aluminium oxide is formed on anode due to electrolysis, which acts as dielectric. As the thickness of the oxide layer is very small, the capacitor has a very large capacitance.

7. Noden valve. It is used to rectify an alternating e.m.f. up to 40 V and works on the phenomenon of electrolysis. It consists of two electrodes of lead and aluminium dipping in aqueous solution of an alkali borate. When an alternating e.m.f. is applied, the current flows only when aluminium electrode is negative. It, thus, acts as a rectifier.

8. Medical applications. Electrolysis is used for local ***anaesthesia*** (when current is passed through a nerve, it is rendered insensitive to pain) ; for ***nerve stimulation*** especially for polio ; for removing unwanted hair on any part of body, etc.

9. Chemical analysis. The process of electrolysis has been used for determining the composition of many chemical salts.

4.10. SOURCE OF E.M.F.

We know that water always flows from a place at higher level to the place at lower level and this movement of water due to difference in levels of the two places may be termed as water flow. Similarly, when two ends of a conductor are at different levels of electricity, electric charge flows from one end of the conductor to the other. This charge flow is called flow of electric current.

A conductor contains free electrons, which are always in random motion. The flow of free electrons in a definite direction constitutes the electric current through the conductor. It can be achieved by maintaining electric potential difference across the two ends of the conductor. The in electric potential difference across the two ends of the conductor performs the work in moving free electrons through the conductor in a definite direction. Such a device or an arrangement, which can maintain potential difference across a conductor is called ***source of e.m.f.***

Therefore, *a source of e.m.f. is an arrangement, which supplies energy in some form so as to bring about the movement of free electrons in a conductor in a particular direction.*

There are following types of sources of e.m.f. :

1. Electrochemical cell. *An electrochemical cell or simply a cell is an arrangement, in which chemical reaction takes place at a steady rate, so as to convert chemical energy into electrical energy.*

The total amount of electrical energy that can be provided by a cell is limited by the amount of reactants present in the cell.

The electrochemical cells are of two types namely ***primary cell*** and ***secondary cells***. The secondary cells are also known as ***storage cells.***

2. Photo cell. *A device in which light energy is converted into electrical energy is called a photo cell.*

Such a cell is based on the phenomenon ***of photoelectric effect***.

3. Thermocouple. A *thermocouple is a source of e.m.f., in which heat energy is converted into electrical energy.*

It is an arrangement of two wires of different materials joined at their ends, so as to form two junctions. When one junction is kept hot and the other cold, electric current flows through the arrangement. It is called ***Seebeck effect*** or ***thermoelectric effect.***

4. Electrostatic machines. By making use of some electrostatic phenomena, machines have been developed which may be used to provide very high voltages. For example, electrophorus, Wimshurst machine, Van de Graaff generator, etc.

5. Electric generator. *It is a device in which mechanical energy required to rotate a conducting loop in a magnetic field is converted into electrical energy.*

An electric generator is based on the phenomenon of ***electromagnetic induction.***

6. Other cells. (*i*) Certain radioactive substances (β-emitters) emit electrons over long periods. Such radioactive substances are used in constructing ***atomic cells.***

(*ii*) When a mechanical stress is applied in a suitable direction on certain crystals, such as tourmaline, quartz and Rochelle salt, the opposite faces become electrically charged. In other words, in such crystals, mechanical energy gets transformed into electrical energy. Such crystals are called ***piezo crystals.*** The transformation of mechanical energy into electrical energy in such crystals is known as ***piezo electric effect*** and such a mechanism constitutes a ***piezo electric cell*** or ***piezo electric generator.***

4.11. PRIMARY AND SECONDARY CELLS

Primary cell. *It is an electrochemical cell, which once discharged, cannot be put to use again by passing electric current from an external source.*

Therefore, when a primary cell gets discharged, the chemicals inside the cell have to be replaced completely. Daniel cell, Leclanche cell and Bunsen cell are examples of primary cells.

Secondary cell. *It is an electrochemical cell, which has to be charged initially by passing electric current from an external source, so as to convert the stored chemical energy into electrical energy.*

On getting discharged, a secondary cell can again be put into use by recharging it. In other words, the chemical reactions that take place inside a secondary cell are reversible in nature. Since a secondary cell stores energy when it is charged, it is also called a ***storage cell*** or an ***accumulator.*** Lead acid cell and alkali cells are two commonly used secondary cells.

Advantages of a secondary cell over a primary cell. 1. A secondary cell can be put to use again and again by recharging it ; while a primary cell once discharged, cannot be recharged.

2. The internal resistance of a secondary cell is less than that of a primary cell. As such, a secondary cell enables to draw much larger current.

Disadvantages of a secondary cell. 1. A secondary cell has to be charged first, so as to obtain electrical energy from it. In other words, a secondary cell requires a lot of time before it can be put to use, while a primary cell starts functioning immediately after its construction.

2. The initial cost of a secondary cell is quite large as compared to that of a primary cell.

4.12. PRIMARY CELLS

Luigi Galvani, an Italian professor of anatomy accidentally made a discovery, which led to the development of first primary cell. He observed that when the two ends of the exposed nerves in the leg of a freshly dissected frog were connected through a copper and iron wire, the leg jerked violently. He concluded that the effect was due to animal electricity in the frog's leg.

Alessandro Volta investigated the accidental discovery of animal electricity made by Galvani. He concluded that two dissimilar metals (copper and iron wires) immersed in a conducting fluid (fluid in frog's leg) led to a chemical reaction capable of producing an electric current, which caused the frog's leg to jerk violently. Ultimately, Volta invented the first primary cell, now called the ***simple voltaic cell.***

This cell suffered from a few defects. In order to remove them, other cells were developed. In addition to simple voltaic cell ; Daniel cell, Bunsen cell and Leclanche cell are the other primary cells.

4.13. SIMPLE VOLTAIC CELL

It was invented by Allexandro de Volta in the year 1800.

Construction. It consists of a glass vessel containing dilute sulphuric acid as electrolyte. Two rods, one of copper and the other made of zinc, are placed in dilute sulphuric acid as shown in Fig. 4.03. Due to the chemical reactions inside the cell, the zinc rod acquires negative charge and the copper rod positive charge. As a result, a potential difference is established between the copper and the zinc rods. The copper rod is called *positive pole,* while the zinc rod is called *negative pole* or *terminal* of the cell.

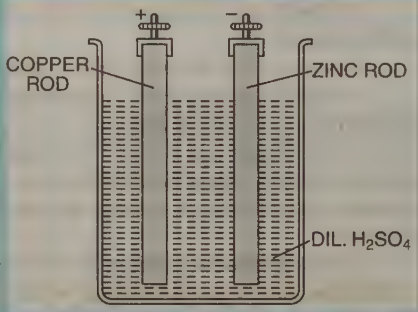


Fig. 4.03

Action. The dilute sulphuric acid dissociates into positive hydrogen ions (H^{++}) and negative sulphate ions (SO_4^{--}) as represented below :

$$H_2SO_4 \longrightarrow 2H^+ + SO_4^{--}$$

When the copper and zinc rods are placed in dilute sulphuric acid, some of the neutral zinc atoms go into the electrolyte as Zn^{++} ions as represented below :

$$Zn \longrightarrow Zn^{++} + 2e^-$$

For each Zn^{++} ion so produced, two electrons ($2e^-$) are left on the zinc rod. As more and more Zn^{++} ions enter the electrolyte, the zinc rod becomes more and more negative. Also, the concentration of Zn^{++}ions in the electrolyte goes on increasing. The H^+ ions drift to the copper rod. On reaching the copper rod, the H^+ ions extract electrons from the rod and form neutral hydrogen atoms. As a result of it, copper rod acquires positive charge. As more and more H^+ ions discharge at the copper rod, it becomes more and more positive. Due to positive charges building up on copper rod and negative charge on the zinc rod, the potential difference between the two rods goes on increasing. It continues, till the potential gradient along the electrolyte between copper and zinc rod just restricts the drift of H^+ ions to the copper rod. The maximum e.m.f. developed in voltaic cell is found to be 1·08 V.

Defects in a voltaic cell. A voltaic cell suffers from the following two defects :

1. ***Local action.*** The commercial zinc usually contains traces of iron and lead. When zinc rod containing such impurity atoms of iron and lead is dipped in dilute sulphuric acid, tiny voltaic cells are formed locally on the zinc rod and in these cells, local electric currents are set up. Due to these local currents, zinc is converted into $ZnSO_4$ and hence wasted even when the cell is not in use. This defect in voltaic cell is called ***local action***.

Key point ✓

A simple voltaic cell suffers from the defects of local action and polarisation.

To remove this defect, the zinc rod is rubbed with mercury. The impurity atoms of iron and lead are insoluble in mercury and hence they get covered with a layer of mercury. On the other hand, zinc forms a mercury amalgam. Due to this, whereas the impurity atoms of iron and lead are no longer exposed to dilute sulphuric acid, the zinc atoms remain exposed. Thus, on amalgamating the zinc rod, the chemical action inside the cell is sustained without any local action.

Key point ✓

The defect of local action arises in a simple voltaic cell due to formation of tiny voltaic cells on the zinc rod.Such cells are formed due to the reason that commercial zinc contains traces of iron and lead.

2. ***Polarisation.*** This defect arises due to the formation of a neutral layer of hydrogen gas on copper rod, when the H^+ ions on reaching copper rod extract electrons from it and form hydrogen atoms. The layer of hydrogen gas on the copper rod weakens the action of the cell in the following two ways :

(i) The layer of hydrogen is a bad conductor of electricity. Due to this, internal resistance of the cell increases.

(ii) Due to the formation of a layer of hydrogen gas, H^+ ions moving towards the copper rod are unable to transfer their positive charge on it. As such, H^+ ions set up an electric field from the layer of hydrogen to zinc rod and oppose the motion of H^+ ions towards the copper rod. It results in ***back electromotive force***, which sends the current in opposite direction and weakens the action of the cell. This defect in the cell is called ***polarisation***. This defect can be removed by brushing off the hydrogen from time to time or by using a depolariser (an oxidising agent), which will convert hydrogen gas into water.

Key point ✓

The defect of polarisation arises in a voltaic cell due to deposition of a layer of hydrogen on copper rod, which is bad conductor of electricity.

4.14. DANIEL CELL

It was developed by Daniel in the year 1836.

Construction. It consists of a copper vessel containing copper sulphate ($CuSO_4$) solution. The copper vessel itself acts as the positive pole of the cell. Inside the $CuSO_4$ solution, a porus pot containing an amalgamated zinc rod and dilute H_2SO_4 acid is placed as shown in Fig. 4.04. Whereas the porus pot prevents the dilute H_2SO_4 and the $CuSO_4$ solution from mixing with each other, it allows the H^+ ions produced in the porus pot to diffuse through its walls into the $CuSO_4$ solution. The amalgamated zinc rod is used in order to avoid the defect of local action from occurring in the cell. The amalgamated zinc rod acts as the negative pole of the cell and both the $CuSO_4$ solution and dil. H_2SO_4 serve as the electrolyte. However, the $CuSO_4$ solution serves as depolariser also.

When the cell functions, the concentration of $CuSO_4$ solution falls. In order to keep the concentration of $CuSO_4$ solution constant, the crystals of $CuSO_4$ are placed on the perforated shelf provided along the walls of the copper vessel.

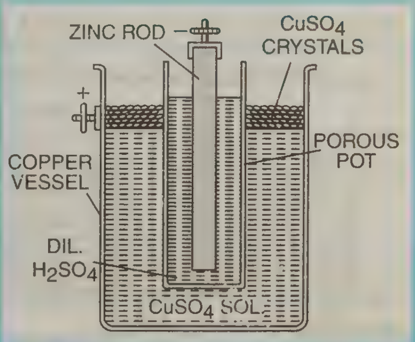


Fig. 4.04.

Action. Inside the porus pot, the dilute H_2SO_4 acid dissociates into H^+ and SO_4^{--} ions. As the zinc rod is dipping inside the dilute H_2SO_4 acid, some of the zinc atoms go into the solution as Zn^{++} ions. For each Zn^{++} ion going into the solution, two electrons are left on the zinc rod. As the concentration of Zn^{++} ions in the solution increases, zinc rod becomes more and more negative and the H^+ ions diffuse through the walls of the porus pot into the $CuSO_4$ solution. The reactions taking place inside the porus pot are as below :

$$H_2SO_4 \longrightarrow 2\,H^+ + SO_4^{--}$$
$$Zn \longrightarrow Zn^{++} + 2\,e^-$$
$$Zn^{++} + SO_4^{--} \longrightarrow ZnSO_4$$
$$\overline{\underline{Zn + H_2SO_4 \longrightarrow ZnSO_4 + 2\,H^+ + 2\,e^-}}$$

The formation of $ZnSO_4$ on the porus pot does not affect the working of the cell, until crystals of $ZnSO_4$ are deposited along its walls.

On diffusing out of porus pot, H^+ ions react with copper sulphate solution as below :

$$2\,H^+ + CuSO_4 \longrightarrow H_2SO_4 + Cu^{++}$$

As the Cu^{++} ions deposit on the copper vessel, it acquires positive charge. Due to building up of the positive charge on the copper vessel and negative charge on the zinc rod, the potential difference between the two poles of the cell goes on increasing. When equilibrium is attained, the cell developes an e.m.f. of 1·1 V.

It may be pointed out that the use of $CuSO_4$ solution makes Cu^{++} ions to discharge at the copper vessel in place of H^+ ions. In case, H^+ ions discharged at the copper vessel, the defect of polarisation would occur due to the production of H_2 gas. Therefore, due to the use of $CuSO_4$ solution, the defect of polarisation is avoided in Daniel cell. The $CuSO_4$ solution is said to be acting as ***depolariser***. However, as the cell works, the concentration of $CuSO_4$ solution falls and this in turn decreases the e.m.f. of the cell. But before this may happen, some $CuSO_4$ crystal from the perforated shelf get dissolved into $CuSO_4$ solution and the concentration of $CuSO_4$ solution recovers to the original value. As the concentration of $CuSO_4$ solution is maintained, the cell provides the steady e.m.f., while in use.

Key point

In a Daniel cell, the defect of polarisation is avoided by using $CuSo_4$ solution as depolariser.

4.15. LECLANCHE CELL

It was invented by George Leclanche in the year 1865.

Construction. It consists of a glass vessel containing a strong solution of ammonium chloride (NH_4Cl) as electrolyte. An amalgamated zinc rod dipping in the NH_4Cl solution acts as the negative pole of the cell. The use of amalgamated zinc rod avoids the defect of local action from occurring in the cell. A porus pot containing the carbon rod is placed inside the NH_4Cl solution. The carbon rod acts as the positive pole of the cell. The empty space in the porus pot is filled with manganese dioxide (MnO_2) and charcoal powder [Fig. 4.05]. Manganese dioxide is used as *depolariser*. The use of charcoal powder makes MnO_2 conducting and thus, decreases the internal resistance of the cell.

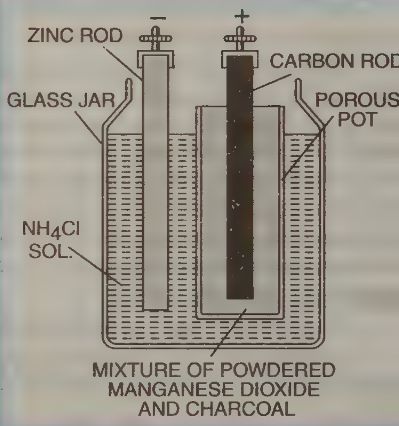


Fig. 4.05

Action. The electrolyte NH_4Cl solution dissociates into NH_4^+ and Cl^- ions. As zinc rod is dipping inside the NH_4Cl solution, some of the atoms go into the solution as Zn^{++} ions. For each Zn^{++} ion so produced, two electrons are left on the zinc rod and hence it becomes negative pole of the cell. The NH_4^+ ions are repelled by Zn^{++} ions and they diffuse into the MnO_2 and charcoal mixture through the walls of the porus pot. Inside the glass vessel, Zn^{++} ions combine with Cl^- ions to form $ZnCl_2$. The reactions taking place inside the glass vessel are as below :

$$2\,NH_4Cl \longrightarrow 2\,NH_4^+ + 2\,Cl^-$$
$$Zn \longrightarrow Zn^{++} + 2\,e^-$$
$$Zn^{++} + 2\,Cl^- \longrightarrow ZnCl_2$$
$$\overline{\underline{Zn + 2\,HN_4Cl \longrightarrow ZnCl_2 + 2\,NH_4^+ + 2\,e^-}}$$

On diffusing into porus pot, NH_4^+ ions extract electrons from the carbon rod making carbon rod as positively charged and producing ammonia and hydrogen gas as represented below :

$$2\,NH_4^+ + 2\,e^- \longrightarrow 2\,NH_3 + H_2$$

Whereas the ammonia gas escapes from the cell, hydrogen gas is neutralised by MnO_2 producing manganese trioxide (Mn_2O_3) and water as below :

$$2\,MnO_2 + H_2 \longrightarrow Mn_2O_3 + H_2O$$

It may be pointed out that by converting hydrogen gas into water, MnO_2 prevents hydrogen gas from causing the defect of polarisation in the cell. Therefore, MnO_2 acts as depolariser in Leclanche cell. The e.m.f. of Leclanche cell is nearly 1·5 V.

However, the defect of polarisation is found to occur, if the cell is used continuously. Being a solid depolariser, MnO_2 reacts with the hydrogen gas at a much slower rate, than at which it is liberated. Therefore, after some time, there is partial polarisation due to accumulation of hydrogen gas and the e.m.f. of the cell falls.

It is found that if the cell is allowed to rest for some time by switching off the circuit, the e.m.f. of the cell recovers. It is because, during this period, Mn_2O_3 changes back into MnO_2 by taking oxygen from the atmosphere as represented below :

$$2\,Mn_2O_3 + O_2 \longrightarrow 4\,MnO_2$$

Because of this reason, Leclanche cell is used in experiments, where only intermittent supply of electric current is required.

Key point ✓

In a Leclanche cell, the defect of polarisation is avoided by using MnO_2 as depolariser. Being a solid, MnO_2 reacts with hydrogen gas at a much slower rate. Therefore, if the cell is used continuosly, there is partial polarisation due to accumulation of hydrogen gas. The cell is found to recover, if rested for a small time. For this reason, Leclanche cell is used for obtaining current for intermittent intervals.

4.16. DRY CELL

It is a portable form of a Leclanche cell.

Construction. In a dry cell, a moist paste of ammonium chloride containing zinc chloride is used as an electrolyte. Zinc chloride being highly hydroscopic, it is added to ammonium chloride in order to keep it moistened. The paste of NH_4Cl and $ZnCl_2$ is contained in a small cylindrical zinc vessel, which acts as the negative pole of the cell. A carbon rod fitted with a brass cap is placed in the middle of the zinc vessel. It acts as the positive pole of the cell. The carbon rod is surrounded by a closely packed mixture of MnO_2 and charcoal powder in a muslin bag. While the MnO_2 acts as depolariser, the charcoal powder reduces the internal resistance of the cell by making MnO_2 electrically conducting. The zinc container and its contents are sealed at the top with pitch or shellac as shown in Fig. 4.06. A small hole is provided at the top, so as to allow ammonia gas formed during chemical reactions to escape the cell.

Action. A dry cell is only a modification of a wet Leclanche cell. Therefore, the action of a dry cell as regards the chemical reactions that take place are same as in case of Leclanche cell.

The dry cells are manufactured in different sizes and shapes to suit particular needs. Irrespective of the size of the cell, the e.m.f. of the cell is nearly 1·5 V. Its internal resistance may vary from $0{\cdot}1\ \Omega$ to$10\ \Omega$. Further, an electric current of about 0·25 A can be continuously drawn from a dry cell.

Key point ✓

A dry cell is portable form of Lenclanche cell.

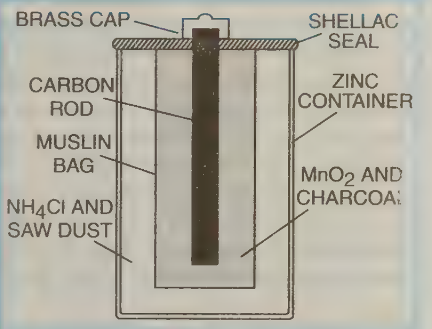


Fig. 4.06

4.17. SECONDARY CELLS

A secondary cell is one in which chemical energy is converted into electrical energy but they do so only when they are charged by passing current through them by some source.

These are also called accumulators or storage cells. These are of two types :

1. Lead acid cell or lead accumulator **2.** Edison alkali cell

4.18. LEAD ACCUMULATOR

It is a secondary cell and is the most common type of storage battery used in automobiles. It is also known as lead acid cell. It was invented by French Physicist, Gaston Plante in the year 1859.

Construction. It consists of a hard rubber, glass or celluloid container containing dilute sulphuric acid as the electrolyte. Each of the two electrodes consists of a set of alternate parallel mesh type perforated plates made of lead. The set of plates to be made the positive pole is filled with a paste of lead dioxide (PbO_2), while the set of plates to be made the negative pole is filled with a paste of spongy lead (Pb). The positive and negative plates are kept separated and insulated from each other by porus separators made of rubber, plastic or glass fibre. This arrangement of positive and negative plates is placed inside the dilute sulphuric acid and connected to the lead terminals provided on the hard and rigid cover of the container [Fig. 4.07].

The e.m.f. of each cell of a fully charged lead accumulator is 2·05 V and a lead accumulator of such six cells produces an e.m.f. of nearly 12 V. The specific gravity of the electrolyte (dilute sulphuric acid) in a fully charged lead accumulator is 1·28.

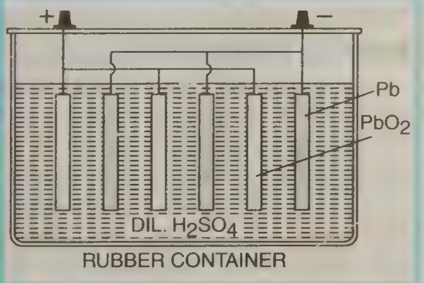


Fig. 4.07

However, when discharged, both the e.m.f. of the cell and the specific gravity of the electroyte fall. Whereas, e.m.f. of a cell of the accumulator should not be allowed to fall below 1·8V, the specific gravity of the electrolyte should not drop below 1·12.

Discharging. When the cell is connected to an external load circuit, it sends current in the circuit and starts discharging. The dilute sulphuric acid dissociates into H^+ and SO_4^{--} ions as below :

$$H_2SO_4 \longrightarrow 2H^+ + SO_4^{--}$$

The H^+ ions drift towards the positive plate, while SO_4^{--} ions move towards the negative plate of the accumulator. The chemical reactions take place at the two plates of the accumulators as given below :

At the negative plate :

$$Pb + SO_4^{--} - 2e^- \longrightarrow PbSO_4$$

At the positive plate :

$$\begin{array}{r}PbO_2 + 2H^+ + 2e^- \longrightarrow PbO + H_2O\\ PbO + H_2SO_4 \longrightarrow PbSO_4 + H_2O\\ \hline PbO_2 + H_2SO_4 + 2H^+ + 2e^- \longrightarrow PbSO_4 + 2H_2O\\ \hline\end{array}$$

Therefore, during discharging, the materials of each plate is converted into $PbSO_4$. The lead sulphate produced in these reactions is a soft form and is chemically more active than the hard lead sulphate. Further, H_2SO_4 molecules do not exist in the solution as such. As said above, it dissociates into H^+ and SO_4^{--}. Therefore, the reaction at positive plate may be written as below :

$$PbO_2 + 4H^+ + SO_4^{--} + 2e^- \longrightarrow PbSO_4 + 2H_2O$$

Further, in the discharging reactions, sulphuric acid is consumed and water is formed. As a result, the specific gravity of the sulphuric acid falls.

Charging. In order to charge a lead accumulator, it is connected to a battery charger capable of supplying e.m.f. greater than that of the lead accumulator. While charging an accumulator, a high series resistance is included in the circuit. In the absence of such a resistance, large current flows through the accumulator during charging. Due to this, a non-porus layer of $PbSO_4$ is formed on both the plates, which does not decompose during subsequent charging. Further, the positive plate of the accumulator has to be connected to the positive terminal of the battery charger *i.e.* the battery charger has to force the charging current through the accumulator in the direction opposite to that of the discharging current. It will, then, make H^+ions to move towards negative plate and SO_4^{--} ions towards the positive plate. The following chemical reactions take place at the two plates :

Key point

During charging of a storage cell, curren from the battery charger flows into th cell. As a result, the terminal potentia difference between the two terminals of storage cell during charging is more tha its e.m.f. It is because, potential dro across internal resistance of the cell aid its e.m.f.

At the negative plate :

$$PbSO_4 + 2H^+ + 2e^- \longrightarrow Pb + H_2SO_4$$

At the positive plate :

$$\begin{array}{r}PbSO_4 + SO_4^{--} - 2e^- \longrightarrow PbO_2 + 2SO_3\\ 2SO_3 + 2H_2O \longrightarrow 2H_2SO_4\\ \hline PbSO_4 + 2H_2O + SO_4^{--} - 2e^- \longrightarrow PbO_2 + 2H_2SO_4\end{array}$$

Therefore, during the charging process, $PbSO_4$ at both electrodes is converted back into spongy lead (Pb) at negative plate and PbO_2 at positive plate. Further, in the charging reactions, water is consumed and sulphuric acid is produced. As a result, the specific gravity of the sulphuric acid rises.

Key point

While charging a storage cell, a larg resistance is put in series with the storag cell. In case, it is not done so, a larg current flows through the storage cell. I is rendered useless due to the formation o a non-porus layer of $PbSO_4$ on the plate of the accumulator.

4.19. EDISON ALKALI CELL

It is also called nickel-iron (Ni-Fe) cell. Its positive plate consists of a number of tubes of perforated steel ribbon, wound spirally and held together by steel rings. The tubes are nickel plated. The active material consisting of $Ni(OH)_4$ and flakes of metallic nickel are packed into the nickel plated steel tubes in alternate layers.

The negative plate is made from finely perforated nickelled steel strip, stamped into pockets. The pockets are filled with powdered iron oxide.

The plates are separated from one another by hard rubber strips. The chemical changes during charging and discharging can be represented by the following reversible reaction :

$$Ni(OH)_4 + KOH + Fe \rightleftharpoons Ni(OH)_2 + KOH + Fe(OH)_2$$

The e.m.f. of the fully charged cell is 1·45 volt and it falls to 1·2 volt on discharge.

This cell is light portable and does not need special care. It is more durable and robust. Its internal resistance is greater than the lead accumulator and its efficiency is low.

4.20. CAPACITY OF A SOURCE OF E.M.F.

The capacity of source of e.m.f., such as an accumulator, is measured in *ampere hour.*

The capacity of a source of e.m.f. is called one ampere hour, if it can discharge one ampere of current for one hour.

For example, a lead accumulator having capacity of 80 ampere hour can supply 8 ampere of current for 10 hours or 4 ampere current for 20 hours.

4.21. BUTTON CELL

The button cells are usually *pallet type flat* in construction and look like a button in shape. Owing to their small sizes, they are used in small electronic devices so as to minimise space and weight. Different types of button cells have been developed which make use of positive and negative electrodes of different nature. However, all types of button cells are basically primary cells.

Fig. 4.08 shows the various parts of a ***zinc-air button cell***. In this cell, zinc (in powder form) acts as the negative electrode, while air as the positive electrode. It contains potassium hydroxide as the electrolyte. The whole arrangement is made inside a small metal can having a small hole at its bottom. A layer of polytetrafluroethane (abbreviated as PTFE) allows air into the cell through the hole but prevents the electrolyte from leaking out. PTFE is a chemically inert solid plastic and is known by the trade names teflon and fluon. It is heat resistant and is stable upto 300°C. Under the catalytic action of carbon, the oxygen from the air reacts with potassium hydroxide to form water.

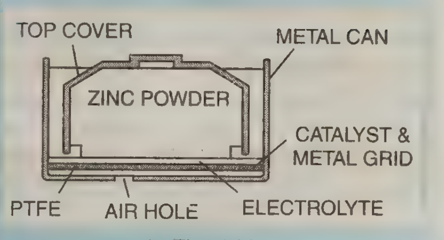


Fig. 4.08

A zinc-air botton cell provides an e.m.f. of 1·4 V. It is used in electronic devices, such as watches, pocket calculators, hearing aid, etc. A few other types of button cells are as listed below :

1. Nickel-cadmium button cell
2. Lithium-manganese button cell
3. Silver-zinc button cell
4. Alkaline manganese button cell
5. Silver oxide button cell
6. Mercury oxide button cell

SOLVED NUMERICAL PROBLEMS

Type A. On Faraday's first law of electrolysis

Problem 4.01. For what time must a constant current of 2 A pass through a copper voltameter to deposit 1 g of copper on the cathode ? Given that the E.C.E. of copper = 3·3 ×10⁻⁷ kg C⁻¹. *(H.S.S.C.E. 2002)*

Sol. Here, $m = 1\text{ g} = 10^{-3}\text{ kg}$; $I = 2\text{ A}$;

$z = 3{\cdot}3 \times 10^{-7}\text{ kg C}^{-1}$

From Faraday's first law of electrolysis,

$$m = zIt$$

$$\therefore \quad t = \frac{m}{zI} = \frac{10^{-3}}{3{\cdot}3 \times 10^{-7} \times 2} = 1515{\cdot}2\text{ s}$$

Problem 4.02. A steady current deposits 0·15 g of copper in 15 minutes from $CuSO_4$ solution. An ammeter in series with the voltameter reads 0·6 A. What correction is to be applied to the ammeter reading ? Given that E.C.E. of copper = 3·3 × 10⁻⁷ kg C⁻¹.

Sol. Here, $m = 0{\cdot}15\text{ g} = 0{\cdot}15 \times 10^{-3}\text{ kg}$;

$z = 3{\cdot}3 \times 10^{-7}\text{ kg C}^{-1}$

$t = 15\text{ minutes} = 15 \times 60 = 900\text{ s}$

Let I be the current passed through $CuSO_4$ solution. Then

$$I = \frac{m}{zt} = \frac{0{\cdot}15 \times 10^{-3}}{3{\cdot}3 \times 10^{-7} \times 900} = 0{\cdot}5\text{ A}$$

The value of current noted from ammeter, $I' = 0{\cdot}6$ A

Therefore, correction to be applied to the ammeter reading,

$$I - I' = 0{\cdot}5 - 0{\cdot}6 = -\,0{\cdot}1\text{ A}$$

Problem 4.03. A brass rectangular plate 12 cm × 3 cm is to be electroplated with copper. If we wish to coat it with a layer 0·02 mm thick on both sides, how much time will it take with a constant current of 5 A ? Given E.C.E. for copper is 33 × 10⁻⁵ g C⁻¹ and density of copper is 8·9 g cm⁻³.

Sol. Total area of the plate to be coated

$= 2 \times (12 \times 3)\text{ cm}^2 = 72\text{ cm}^2$

Thickness of copper layer to be coated

$= 0{\cdot}02\text{ mm} = 0{\cdot}002\text{ cm}$

Therefore, volume of the copper to be deposited

$= 72 \times 0{\cdot}002 = 0{\cdot}144\text{ cm}^3$

Mass of the copper to be deposited,

$m = \text{volume} \times \text{density}$

$= 0{\cdot}144\text{ cm}^3 \times 8{\cdot}9\text{ g cm}^{-3} = 1{\cdot}2816\text{ g}$

Let t be the time taken to deposit the copper. Then,

$$m = zIt$$

Here, $z = 33 \times 10^{-5}\text{ g C}^{-1}$; $I = 5$ A

$$\therefore \quad t = \frac{m}{zI} = \frac{1{\cdot}2816}{33 \times 10^{-5} \times 5} = 777\text{ s}$$

Problem 4.04. A silver and a copper voltameter are connected in parallel across a 6 V battery of negligible internal resistance. In half hour, 1 g of copper and 2 g of silver are deposited. Calculate the rate at which the energy is supplied by the battery. Given, the E.C.E. of copper is 3294×10^{-7} g C^{-1} and that of silver is 1118×10^{-6} g C^{-1}.

Sol. Let z_1 and z_2 be the E.C.E. of silver and copper respectively and m_1 and m_2 be the masses of the silver and copper deposited. If I_1 and I_2 be the currents through the silver and copper voltameters, then

$$m_1 = z_1 I_1 t \qquad ...(i)$$

and $$m_2 = z_2 I_2 t, \qquad ...(ii)$$

where t is the time for which current flows through each electrolyte.

Here, $m_1 = 2$ g ; $m_2 = 1$ g ; $t = \frac{1}{2}$ hour $= 1800$ s

$z_1 = 1118 \times 10^{-6}$ g C^{-1} ; $z_2 = 3294 \times 10^{-7}$ g C^{-1}

Therefore, from equations (*i*) and (*ii*), we have

$$I_1 = \frac{m_1}{z_1 t} = \frac{2}{1118 \times 10^{-6} \times 1800} = 0{\cdot}994 \text{ A}$$

Also, from equation (*ii*), we have

$$I_2 = \frac{m_2}{z_2 t} = \frac{1}{3294 \times 10^{-7} \times 1800} = 1{\cdot}687 \text{ A}$$

Total current drawn from the battery,

$$I = I_1 + I_2 = 0{\cdot}994 + 1{\cdot}687 = 2{\cdot}681 \text{ A}$$

Power of the circuit,

P = current × e.m.f. = 2·681 × 6 = **16·086 watt**

Problem 4.05. A silver and copper voltameter are connected in series with a 12 V battery of negligible resistance. It is found that 0·403 g of silver is deposited in 15 minutes. Find (*a*) the mass of the copper deposited and (*b*) the energy supplied by battery. Given that

E.C.E. of silver = $1{\cdot}12 \times 10^{-6}$ kg C^{-1} and

that of copper = $3{\cdot}3 \times 10^{-7}$ kg C^{-1}.

Ans. Here, mass of silver deposited,

$$m_1 = 0{\cdot}403 \text{ g} = 0{\cdot}403 \times 10^{-3} \text{ kg}$$

E.C.E. of silver, $z_1 = 1{\cdot}12 \times 10^{-6}$ kg C^{-1};

E.C.E. of copper, $z_2 = 3{\cdot}3 \times 10^{-7}$ kg C^{-1}

Time for which current is passed,

t = 15 minutes = 15 × 60 = 900 s

E.M.F. of battery, E = 12 V

(*a*) Since the two voltameters are in series, the same current passes through each of them. If I is current through the two voltameters, then

$$m_1 = z_1 I t \text{ and } m_2 = z_2 I t$$

$$\therefore \quad \frac{m_1}{m_2} = \frac{z_1}{z_2}$$

or $$m_2 = m_1 \times \frac{z_2}{z_1} = \frac{0{\cdot}403 \times 10^{-3} \times 3{\cdot}3 \times 10^{-7}}{1{\cdot}12 \times 10^{-6}}$$

$$= \mathbf{0{\cdot}119 \times 10^{-3} \text{ kg}}$$

(*b*) The energy supplied by the battery = E I t

$$= E\left(\frac{m_1}{z_1 t}\right)t \qquad (\because m_1 = z_1 I t)$$

$$= \frac{12 \times 0{\cdot}403 \times 10^{-3}}{1{\cdot}12 \times 10^{-6}} = \mathbf{4{\cdot}32 \times 10^3 \text{ J}}$$

Problem 4.06. A copper voltameter is connected in series with a coil of resistance 100 ohm. A steady current is passed through the circuit for 10 minutes and deposits 0·1 g of copper. How many joule of heat generated in the coil ? Take electrochemical equivalent copper $3{\cdot}3 \times 10^{-7}$ kg C^{-1}. (*C.B.S.E. 19*

Sol. Here, R = 100 ohm ;

t = 10 minutes = 60 × 10 = 600 s ;

$z = 3{\cdot}3 \times 10^{-7}$ kg C^{-1} ; $m = 0{\cdot}1$ g $= 10^{-4}$ kg

Now, $$m = z I t$$

or $$I = \frac{m}{zt} = \frac{10^{-4}}{3{\cdot}3 \times 10^{-7} \times 600} = 0{\cdot}505 \text{ A}$$

Heat produced in the resistance coil,

$$Q = I^2 R t = (0{\cdot}505)^2 \times 100 \times 600 = \mathbf{15301{\cdot}5}$$

Type B. On Faraday's second law of electrolysis

Problem 4.07. The mass of hydrogen (H_2) evolved p unit coulomb of electricity passed through a wa voltameter is $1{\cdot}05 \times 10^{-8}$ kg C^{-1}. The corresponding figu for copper deposited in a copper voltameter and sil deposited in a silver voltameter are $3{\cdot}29 \times 10^{-7}$ kg C^{-1} a $1{\cdot}12 \times 10^{-6}$ kg C^{-1} respectively. What conclusion is impl by these measure-ments ? Predict the amount of ir deposited per coulomb in a voltameter with Fe electro in $FeCl_3$. Relative atomic masses of hydrogen, copper, sil and iron are1·008, 63·54, 107·9 and 55·85 respectively.

(*Text Probl*

Sol. *For water voltameter* :

Atomic mass of hydrogen = 1·008 ; valency = 1

Therefore, chemical equivalent,

$$E_1 = \frac{1{\cdot}008}{1} = 1{\cdot}008 \text{ g mole}^{-1}$$

Also, mass of hydrogen liberated,

$$m_1 = 1{\cdot}05 \times 10^{-8} \text{ kg } C^{-1}$$

$$\therefore \quad \frac{m_1}{E_1} = \frac{1{\cdot}05 \times 10^{-8}}{1{\cdot}008} = 1{\cdot}0417 \times 10^{-8}$$

For copper voltameter :

Atomic mass of copper = 63·54 ; valency = 2

Therefore, chemical equivalent,

$$E_2 = \frac{63{\cdot}54}{2} = 31{\cdot}77 \text{ g mole}^{-1}$$

Also, mass of copper deposited,

$$m_2 = 3{\cdot}29 \times 10^{-7} \text{ kg } C^{-1}$$

$$\therefore \quad \frac{m_2}{E_2} = \frac{3{\cdot}29 \times 10^{-7}}{31{\cdot}77} = 1{\cdot}0356 \times 10^{-8}$$

For silver voltameter :

Atomic mass of silver =107·9 ; valency = 1

Therefore, chemical equivalent,

$$E_3 = \frac{107{\cdot}9}{9} = 107{\cdot}9 \text{ g mole}^{-1}$$

Also, mass of silver deposited, $m_3 = 1{\cdot}12 \times 10^{-6}$ kg

$$\therefore \quad \frac{m_3}{E_3} = \frac{1{\cdot}12 \times 10^{-6}}{107{\cdot}9} = 1{\cdot}0380 \times 10^{-8}$$

It follows that $\dfrac{m_1}{E_1} \approx \dfrac{m_2}{E_2} \approx \dfrac{m_3}{E_3}$

or $$\frac{m}{E} = \text{constant}$$

Therefore, these measurements lead to the verificat of Faraday's second law of electrolysis.

For iron voltameter :

Atomic mass of iron = 55·85 ; valency = 3

Therefore,chemical equivalent,

$$E_4 = \frac{55{\cdot}85}{3} = 18{\cdot}617 \text{ g mole}^{-1}$$

If m_4 is mass of iron deposited, then

$$\frac{m_1}{E_1} = \frac{m_4}{E_4}$$

or $$m_4 = \frac{m_1}{E_1} \times E_4 = 1{\cdot}0417 \times 10^{-8} \times 18{\cdot}617$$

$$= \mathbf{1{\cdot}94 \times 10^{-7} \text{ kg C}^{-1}}$$

Type C. On Faraday constant

Problem 4.08. A steady current of 10·0 A is maintained in a copper voltameter. Calculate the time required to deposit 2·5 g of copper. Relative atomic mass of copper = 63·5. (Take F = 96500 C mol^{-1}.

(C.B.S.E. 1991 ; Pre-degree Kerala, 1991)

Sol. Here, m = 2·5 g ; I = 10·0 A ; F = 96500 C mol^{-1} ; relative atomic mass of copper = 63·5 ; valency = 2

Therefore, chemical equivalent of copper,

$$E = \frac{63{\cdot}5}{2} = 31{\cdot}75 \text{ g mole}^{-1}$$

Now, $$m = z\,I\,t = \frac{E}{F} I\,t$$

or $$t = \frac{m\,F}{E\,I} = \frac{2{\cdot}5 \times 96500}{31{\cdot}75 \times 10{\cdot}0} = 759{\cdot}8 \text{ s}$$

Problem 4.09. In a copper plating experiment, the difference between final and initial mass of the cathode is measured carefully and is found to be 16·43 g. The time duration of electrolysis is noted to be 4000 s. The steady reading of current in an ammeter placed in series with the voltameter is 12·6 A. What is the error in the ammeter reading ? (F = 96485 C mol^{-1}, relative atomic mass of Cu = 63·54).

Sol. Here, m = 16·43 g ; t = 4000 s ; F = 96485 C mol^{-1} ; atomic mass of Cu = 63·54 ; valency of copper = 2

Therefore, chemical equivalent of copper,

$$E = \frac{63{\cdot}54}{2} = 31{\cdot}77 \text{ g mole}^{-1}$$

Now, $$m = z\,I\,t = \frac{E}{F} I\,t$$

or $$I = \frac{m\,F}{E\,t} = \frac{16{\cdot}43 \times 96485}{31{\cdot}77 \times 4000} = 12{\cdot}474 \text{ A}$$

The value of current as noted from the ammeter = 12·6 A

Therefore, error in the ammeter reading

= 12·6 – 12·474 = **0·126 A (reads more)**

Problem 4.10. A steady potential difference of 1·62 V is maintained across two platinum electrodes placed in a solution of CuCl$_2$. At the end of 600 s, the mass of copper deposited on the cathode is measured to be 5·92 g. The back e.m.f. of the voltameter is given to be 1·34 V. Estimate the resistance of voltameter. Faraday constant = 96500 C mol^{-1}, relative atomic mass of copper = 63·5.

Sol. Here, t = 600 s ; m = 5·92 g ; F = 96500 C mol^{-1}

Steady potential difference applied = 1·62 V;

back e.m.f. produced = 1·34 V

Therefore, net e.m.f. of the voltameter,

$$V = 1{\cdot}62 - 1{\cdot}34 = 0{\cdot}28 \text{ volt}$$

Atomic mass of copper = 63·5 ; valency of copper = 2

Therefore, chemical equivalent of copper,

$$E = \frac{63{\cdot}5}{2} = 31{\cdot}75 \text{ g mole}^{-1}$$

Now, $$m = z\,I\,t = \frac{E}{F} I\,t$$

or $$I = \frac{m\,F}{E\,t} = \frac{5{\cdot}92 \times 96500}{31{\cdot}75 \times 600} = 29{\cdot}99 \text{ A}$$

Therefore, resistance of the voltameter,

$$R = \frac{\text{net e.m.f.}}{I} = \frac{0{\cdot}28}{29{\cdot}99} = 9{\cdot}34 \times 10^{-3}\ \Omega = \mathbf{9{\cdot}34\ m\Omega}$$

Type D. On Charging of storage battery

Problem 4.11. A storage battery of e.m.f. 8 V, internal resistance 1 Ω is being charged by a 120 V d.c. source using a 15 Ω resistor in series in the circuit. Calculate (*a*) the current in the circuit, (*b*) terminal voltage across the battery during charging and (*c*) chemical energy stored in the battery in 5 minutes. *(C.B.S.E. 2001)*

Sol. Fig. 4.09 shows the circuit used to charge a storage battery of e.m.f. E and internal resistance r with the help of source of e.m.f. E′ and external resistance R.

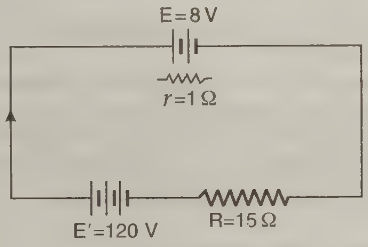


Fig. 4.09

Here, E = 8 V ; r = 1 Ω ; R = 15 Ω,
E′ = 120 V and t = 5 minutes = 5 × 6 = 300 s

(*a*) Let I be current in the circuit. Then,

$$I = \frac{\text{net e.m.f.}}{\text{total resistance}} = \frac{120 - 8}{15 + 1} = \mathbf{7\ A}$$

(*b*) Since current due to d.c. source flows into the storage cell, the potential difference across the internal resistance of the storage cell aids its e.m.f. Therefore, terminal voltage across the battery during charging,

$$V = E + I\,r = 8 + 7 \times 1 = \mathbf{15\ V}$$

(*c*) The total energy supplied by the d.c. source,

$$E'\,I\,t = 120 \times 7 \times 300 = 252000 \text{ J}$$

The energy dissipated as heat,

$$I^2 (R + r) \cdot t = (7)^2 \times (15 + 1) \times 300 = 235200 \text{ J}$$

Therefore, energy stored in the battery

= energy supplied by the d.c. source
– energy dissipated as heat
= 25200 0– 235200 = **16800 J**

Alternative method

The energy stored in the battery,

$$E\,I\,t = 8 \times 7 \times 300 = \mathbf{16800\ J}$$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 4.12. A steady current of 10·0 A is passed through a water voltameter for 300 s. Estimate the volume of H_2 evolved at standard temperature and pressure. Use the value of Faraday constant as 96500 C mol^{-1}. Relative molecular mass of H_2 is 2·016 and molar volume = 22·4 litres (volume of 1 mole of an ideal gas at S.T.P.).

(Text Problem)

Sol. Here, I = 10·0 A ; t = 300 s

Therefore, charge passed, $q = It = 10{\cdot}0 \times 300 = 3000$ C

Molar mass of hydrogen = 2·016

Hydrogen molecule is diatomic.

$$\text{Therefore, atomic mass of hydrogen} = \frac{2\cdot016}{2} = 1\cdot008$$

Valency of hydrogen = 1

Therefore, chemical equivalent of hydrogen,

$$E = \frac{1\cdot008}{1} = 1\cdot008 \text{ g mole}^{-1}$$

Therefore, charge required to liberate one molar mass of

$$\text{hydrogen} = \frac{96500}{1\cdot008} \times 2\cdot016 = 96500 \times 2 \text{ C}$$

Now, one molar mass of hydrogen at S.T.P. occupies volume = 22·4 litres

Therefore, volume of hydrogen evolved at S.T.P. on passing 3000 C of charge,

$$V = \frac{22\cdot4}{96500 \times 2} \times 3000 = \mathbf{0\cdot3482\ litre}$$

Problem 4.13. The potential difference across the terminals of a battery of e.m.f. 12 V and internal resistance 2 Ω drops to 10 V, when it is connected to a silver voltameter. Calculate the silver deposited at the cathode in half an hour. Atomic weight of silver is107·9 g mol^{-1}.

(C.B.S.E. 1998)

Ans. Here, atomic weight of silver = 107·9 g mol^{-1}

E.M.F. of battery, E = 12 V ;

Internal resistance of the battery, r = 2 Ω

Terminal potential difference of the battery on connecting the voltameter, V = 10 volt

If R is resistance offered by silver voltameter, then

$$r = \frac{E - V}{V} \times R$$

or

$$R = r \times \frac{V}{E - V} = \frac{12 \times 10}{12 - 10} = 10\ \Omega$$

Therefore, current passing through the silver voltameter,

$$I = \frac{E}{R + r} = \frac{12}{10 + 2} = 1 \text{ A}$$

Since valency of silver is 1, its chemical equivalent is given by

$$E = \frac{\text{atomic weight}}{\text{valency}} = \frac{107\cdot9}{1} = 107\cdot9 \text{ g mol}^{-1}$$

We know that Faraday constant, F = 96500 C mol^{-1}

Therefore, electrochemical equivalent of silver,

$$z = \frac{E}{F} = \frac{107\cdot9}{96500} \text{ g C}^{-1}$$

If m is mass of silver deposited in half an hour *i.e.* 180 s, then

$$m = zIt = \frac{107\cdot9}{96500} \times 1 \times 1800 = \mathbf{2\cdot01\ g}$$

Problem 4.14. In a silver plating system, an electrolysi current of 5·0 A is used for a certain time and 0·5 mo of silver is deposited. How many moles of copper and iro will be deposited in their respective plating system, if a electrolysis current of 10·0 A is passed for twice the time fo silver plating ? (Relative atomic mass of silver = 107· of copper = 63·54, of iron = 55·85). Take Faraday constar as 96500 C mol^{-1}.

(Text Problem

Sol. *For silver* : Here, I = 5· 0A ;

silver deposited = 0·5 mole

Silver is monoatomic and its valency is 1. Therefore, it molar mass is same as its chemical equivalent.

Now, charge required to liberate 0·5 mole (0·5 chemica equivalent) of silver,

$$q = 96500 \times 0\cdot5 \text{ C}$$

Now, $\quad q = It$

Therefore, time for which current is passed,

$$t = \frac{q}{I} = \frac{96500 \times 0\cdot5}{5\cdot0} = 9650 \text{ s}$$

For copper : Here, I = 10 A ; t = 9650 × 2 s ;

atomic mass of copper = 63·54 ; valency = 2

Therefore, chemical equivalent,

$$E = \frac{63\cdot54}{2} \text{ g mole}^{-1}$$

$$\text{Now, } m = zIt = \frac{E}{F}It = \frac{63\cdot54 \times 10 \times 9650 \times 2}{2 \times 96500} = 63\cdot54 \text{ g}$$

Since copper is monoatomic, therefore, its molar mass i equal to its atomic mass *i.e.*63·54.

Hence, **one mole** of copper will be liberated.

For iron : Here, I = 10 A ; t = 9650 × 2 s ;

atomic mass of iron = 55·85, valency = 3

Therefore, chemical equivalent,

$$E = \frac{55\cdot85}{3} \text{ g mole}^{-1}$$

$$\text{Now,} \quad m = \frac{E}{F}It = \frac{55\cdot85 \times 10 \times 9650 \times 2}{3 \times 96500} = \frac{2}{3} \times 55\cdot85 \text{ g}$$

Since iron is monoatomic, its molar mass is also 55·85.

Hence, $\frac{2}{3}$ **mole** of iron will be deposited.

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. Are all pure liquids bad conductors of electricity ?

Ans. All pure liquids are not bad conductors of electricity. For example, mercury in pure form is a liquid and is a goo conductor of electricity.

Q. 4.02. What is electrolysis ? *(P.S.S.C.E. 1999*

Ans. Refer to section 4.03.

Q. 4.03. Name the liquid, which is good conductor of electricity but does not undergo electrolysis on passage of the electric current.

Ans. Mercury is a liquid which is good conductor of electricity and it does not undergo electrolysis on passing current through it.

Q. 4.04. What is difference between a voltameter and a voltmeter ? *(P.S.S.C.E. 1999, 1998 S)*

Ans. Whereas voltmeter is an instrument used to measure potential difference, voltameter is a vessel used to carry out electrolysis of an electrolyte.

Q. 4.05. What are the carriers of current in a copper voltameter normally ? *(P.S.S.C.E. 1997 S)*

Ans. Normally, Cu^{++} and SO_4^{--} ions are considered as carriers of current in a copper voltameter.

Note. In a copper voltameter, the electrolyte is copper sulphate dissolved in water. Copper sulphate dissociates into Cu^{++} and SO_4^{--} ions, while water dissociates into H^+ and OH^- ions. Therefore, in a copper voltameter, the carriers of current are Cu^{++}, H^+, SO_4^{--} and OH^- ions.

Q. 4.06. What are cations ? *(H.P.S.S.C.E. 2000)*

Ans. *The positive ions, which collect at cathode during the process of electrolysis, are called cations.*

Q. 4.07. What are anions ? *(H.P.S.S.C.E. 2000)*

Ans. *The negative ions, which collect at anode during the process of electrolysis, are called anions.*

Q. 4.08. If pure water is used, no electrolysis takes place. Why ?

Ans. It is because, pure water does not dissociate into ions.

Q. 4.09. Are the drift speeds of positive and negative ions equal to each other in the electrolysis of an electrolyte ? *(H.S.S.C.E. 1992)*

Ans. The drift speeds of positive and negative ions are not equal in the electrolysis of an electrolyte.

Q. 4.10. Explain V – I graph for a copper voltameter. *(P.S.S.C.E. 2001 ; C.B.S.E. 2001 S)*

Ans. The V – I graph for the copper voltameter is a straight line [Fig. 4.10]. It indicates that the electrolyte used in copper voltameter obeys Ohm's law. It may be pointed that V – I graph remains a straight line, till the current is made not too high.

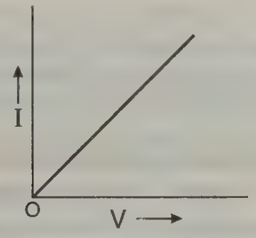


Fig. 4.10

Note. In a copper voltameter, no back e.m.f. is produced in general. The electrolytes obey Ohm's law, if the applied voltage is greater than the back e.m.f.

Q. 4.11. How is the electrical conductivity of an electrolyte affected by increase of temperature ? *(P.S.S.C.E. 2001, C.B.S.E. 1995)*

Ans. Due to increase of temperature, the electrical conductivity of an electrolyte increases (resistance of electrolyte decreases).

Q. 4.12. Draw the graph showing variation of mass of ions deposited with time during electrolysis, when a steady current is passed during the process. *(C.B.S.E. 2002)*

Ans. From Faraday's first law of electrolysis

$$m = z\,I\,t$$

When a steady current is passed during electrolysis,

$$m = \text{constant} \times t$$

The above equation represents a straight line. Hence, graph between mass of ions deposited with time will be a straight line, when steady current is passed during electrolysis. [Fig. 4.11]

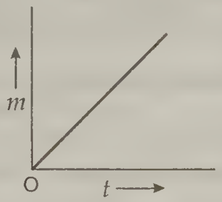


Fig. 4.11

Q. 4.13. Fig. 4.12 shows mass deposited (m) versus time (t) graph of a voltameter for two currents I_1 and I_2. Is I_1 more than, less than or equal to I_2 ?

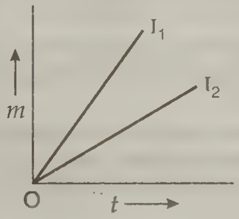


Fig. 4.12

Ans. From Faraday's first law of electrolysis,

$$m \propto I$$

From Fig. 4.12, it follows that in a given time, mass deposited is more for current I_1 than I_2. Hence, current I_1 is greater than current I_2.

Q. 4.14. Are electrochemical equivalent and chemical equivalent of a substance same ? *(H.S.S.C.E. 1992)*

Ans. No, electrochemical equivalent and chemical equivalent of a substance are not the same quantities.

Q. 4.15. Define Faraday constant. *(P.S.S.C.E. 2002 ; H.S.S.C.E. 2000)*

Or

Define one faraday of charge. *(H.S.S.C.E. 1995)*

Ans. *One faraday is the quantity of charge required to liberate one gram equivalent of the substance on an electrode during the process of electrolysis.* It has a value of 96500 coulomb mole^{-1}.

Q. 4.16. Can we make use of a.c. in electrolysis ? Explain briefly. *(H.P.S.S.C.E. 2000, 1998 S)*

Or

Why can't we use alternating current for producing electrolysis ? *(P.S.S.C.E. 1998)*

Ans. No, we cannot use a.c. for carrying out electrolysis. If a.c. is used, the electrolysis will take place alternately in opposite directions and hence no deposit of ions will take place on any electrode.

Q. 4.17. Name a few practical applications of the phenomenon of electrolysis.

Ans. A few practical applications of electrolysis are electroplating, purification of metals, chemical analysis, electrolytic capacitors, etc.

Q. 4.18. Does electromotive force have units of force ?

Ans. No, electromotive force does not have the unit of force. It has the unit of work done per unit charge.

Q. 4.19. What is the most important property of a standard cell ? *(I.S.C.E. 1997)*

Ans. The standard cell is one, whose e.m.f. does not vary practically with time and temperature.

Q. 4.20. Name a few primary cells.

Ans. Simple voltaic cell, Daniel cell, Leclanche cell, Bunsen cell.

Q. 4.21. Name the electrolyte in a simple voltaic cell.

Ans. In a simple voltaic cell, the dilute sulphuric acid is used as electrolyte.

Q. 4.22. Mention two defects of a voltaic cell. *(C.B.S.E. 2001; H.S.S.C.E. 1999)*

Ans. Local action and polarisation.

Q. 4.23. What is ploarisation in a simple voltaic cell ? *(P.S.S.C.E. 2002)*

Ans. When the H^+ ions reach the copper rod in a simple voltaic cell, they extract electrons from it so as to form hydrogen atoms. It results in the formation of a neutral layer of hydrogen gas on the copper rod, which weakens the action of the cell.

Q. 4.24. Name the metals, which are used to make a positive and negative poles in a (*i*) Daniel cell (*ii*) Leclanche cell.

Ans. In both the cells, zinc is used to make negative pole of the cell. Whereas in Daniel cell, copper is used to make positive pole, carbon is used to make the positive pole in Leclanche cell.

Q. 4.25. Name the electrodes, which act as anode and cathode in a Daniel cell.

Ans. In a Daniel cell, the copper vessel acts as cathode, while zinc rod acts as anode.

Q. 4.26. Name the depolarisers in Daniel cell and Leclanche cell.

Ans. Copper sulphate solution and manganese dioxide respectively act as depolarisers in Daniel and Leclanche cell.

Q. 4.27. Give the role of MnO_2 in a dry cell. *(C.B.S.E. 2002 ; P.S.S.C.E. 2001)*

Or

What is the function of MnO_2 in Leclanche cell ? *(P.S.S.C.E. 2002 ; H.S.S.C.E. 1998 S)*

Ans. By converting hydrogen gas into water, MnO_2 prevents hydrogen gas from causing the defect of polarisation. As such, MnO_2 is said to act as depolariser.

Q. 4.28. What is a dry cell ?

Ans. A dry cell is portable form of a Leclanche cell.

Q. 4.29. What is the function of saw dust in a dry cell ? *(P.S.S.C.E. 1999*

Ans. To keep ammonium chloride moistened.

Q. 4.30. You are given a primary and a secondary cel of the same e.m.f. From which cell, will you be able to drav larger current and why ? *(C.B.S.E. 1994*

Ans. As the internal resistance of a secondary cell is les than that of a primary cell, the secondary cell will provide higher value of current.

Q. 4.31. Name the carriers of electric current in a voltai cell and a lead accumulator. *(Text Question*

Ans. Both in voltaic cell and a lead accumulator (bein charged by an external supply), the carriers of electric curren are H^+ and SO_4^{--} ions.

Q. 4.32. Which type of cell would you want to use, i your device required (*i*) a current of 100 A for 20 s (*ii*) current of 10 mA occasionally ? *(Text Question*

Ans. (*i*) A lead accumulator having very low interna resistance. (*ii*) A dry cell.

Q. 4.33. What do you mean by capacity of a storag cell ?

Ans. The capacity of a storage cell implies its ability t supply electrical energy.

Q. 4.34. Define the unit for measuring the capacity o a storage cell ?

Ans. The capacity of a storage cell is measured i ampere-hour.

Storage capacity of a cell is called one ampere hour, if it ca supply one ampere of current for one hour continuously.

Q. 4.35. A storage battery is marked a capacity of 3·5 h at 1 h discharge rate. What does this signify ? Would th cell provide 14 A for 15 minutes ?

Ans. It indicates that the storage battery can supply current of 3·5 A for 1 hour. If a large current is drawn, th battery exhausts at a faster rate than expected. Therefore, i will not provide 14 A current for 15 minutes.

Q. 4.36. Under what condition, can we draw maximun current from a secondary cell ? *(C.B.S.E. 2001*

Ans. The maximum current can be drawn from secondary cell, when its internal resistance is minimum.

Note. The power consumed in the circuit will b maximum, when external load (resistance) is equal to interna resistance of the secondary cell.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. What is the difference between the electric conduction in the solutions and metals ?

Ans. The conduction of electricity in metals is due to the drift of free electrons, while the conduction of electricity in electrolyte is due to the movement of positive and negative ions. Further, during electric conduction, whereas no chemical change takes place in metals ; the chemical decomposition takes place in electrolytes. Further, whereas no matter is transported during electric conduction in metals, matter is transported in the form of positive and negative ions in case of electrolytes.

Q. 4.02. Conductivity of electrolytic solutions is much lower as compared to that of the metals. Give reasons. *(C.B.S.E. 1999 S, 1996*

Ans. 1. Number of ions in electrolytes is very small a compared to number of free electrons in metallic conductors

2. Due to large mass, the mobility of ions in electrolyte is very low as compared to that of electrons in metalli conductors.

Q. 4.03. Why is the conductivity of molten electrolyte smaller than common metallic conductors ? *(C.B.S.E. 1998 S*

Ans. A solid electrolyte, such as NaCl dissociates into ions but it cannot conduct electricity. It is because, its ions are not mobile as such. When NaCl is in molten state, the ions acquire a small mobility. Hence, it can conduct electricity, which is very small as compared to that of metallic conductor. It may be pointed out that the conductivity of NaCl further increases on dissolving it in water.

Q. 4.04. What are conductors of the second kind ?

Ans. The electrolytes, in which the current is conducted through the ions, are called the conductors of the second kind. The metals are called the conductors of the first kind. The current is conducted in metallic conductors by the free electrons.

Q. 4.05. Briefly explain the dissociation theory of electrolysis. *(H.S.S.C.E. 2001)*

Ans. Refer to section 4.04.

Q. 4.06. Why are electrolytes dissociated, when dissolved in liquids ? *(H.S.S.C.E. 2001)*

Ans. The polar molecules of a liquid, such as water have a large value of dielectric constant (≈ 80). When an electrolyte is dissolved in a liquid, the ionic bonds between the ions of the electrolyte become very weak. Then, the thermal energy of the ions takes the ions away from each other leading to the dissociation of the electrolyte.

Q. 4.07. Explain the theory of electrolysis. *(H.S.S.C.E. 2001)*

Ans. Refer to section 4.04.

Q. 4.08. Explain, what do you understand by back e.m.f. in an electrolytic cell.

Ans. The e.m.f. that exists due to contact difference of potential between two different materials is called back e.m.f. It sends a current through the electrolyte in a direction opposite to that sent by the externally applied e.m.f.

Q. 4.09. The anode of the silver voltameter is made of silver. Why ?

Ans. When current is passed through silver nitrate in silver voltameter, silver ions (Ag^+) move towards cathode and nitrate ions (NO_3^-) towards anode. The silver ions deliver charge on the silver cathode and become silver atoms. On the other hand, nitrate ions on reaching the anode form silver nitrate, which goes into the silver nitrate solution. In this manner, whereas the concentration of electrolyte is maintained, the mass of cathode increases and that of anode decreases. So that this process goes on, it is essential that in a silver voltameter, anode should be made of silver.

Q. 4.10. State Faraday's laws of electrolysis. *(H.S.S.C.E. 2002 ; H.P.S.S.C.E. 2001, 1999 S ; P.S.S.C.E. 1999 S ; C.B.S.E. 1994, 1991, 1990)*

Ans. For Faraday's laws of electrolysis, refer to section 4.07.

Q. 4.11. Draw a neat labelled diagram of a copper voltameter. Define electrochemical equivalent of a substance. *(H.S.S.C.E. 2002)*

Ans. For diagram of copper voltameter, refer to Fig. 4.01.

Electrochemical equivalent of a substance. *It may be defined as the mass of the substance deposited at the cathode, when one coulomb of charge passes through the electrolyte.*

Q. 4.12. Define electrochemical equivalent. Give its units. *(C.B.S.E. 1992)*

Ans. Refer to SAQ 4.11.

Q. 4.13. In two voltameters with different electrolytes in solution, same mass is deposited when the currents in the two are in the ratio of 1 : 2 and times of deposit are in the ratio of 2 : 3. What is the ratio of electrochemical equivalents of the electrolytes in the two cases ? *(C.B.S.E. 2001 S)*

Ans. From Faraday's first law of electrolysis,

$$m = z\,I\,t$$

Let I_1 and I_2 be the currents passed for times t_1 and t_2 respectively through the two electrolytes. Further, let z_1 and z_2 be their electrochemical equivalents. Since same mass of the ions is deposited in the two cases,

$$z_1 I_1 t_1 = z_2 I_2 t_2 \quad \text{or} \quad \frac{z_1}{z_2} = \frac{I_2}{I_1} \times \frac{t_2}{t_1}$$

$$\text{Here,} \quad \frac{I_1}{I_2} = \frac{1}{2} \quad \text{and} \quad \frac{t_1}{t_2} = \frac{2}{3}$$

$$\textit{i.e.} \quad \frac{I_2}{I_1} = 2 \quad \text{and} \quad \frac{t_2}{t_1} = \frac{3}{2}$$

$$\therefore \quad \frac{z_1}{z_2} = 2 \times \frac{3}{2} = 3$$

Hence, the ratio of electrochemical equivalents of the two electrolytes is **3 : 1.**

Q. 4.14. Explain, how you can use a voltameter for measuring current. *(P.S.S.C.E. 1992)*

Ans. A voltameter can be used to measure current indirectly. Suppose that on passing current through an electrolyte for a known time t, the increase in mass of the cathode is m. Then,

$$m = z\,I\,t,$$

where z is electrochemical equivalent of the substance. It follows that

$$I = \frac{m}{z\,t}$$

Knowing m, t and z, the value of current passed through the electrolyte can be calculated.

Q. 4.15. A voltameter measures current more accurately than an ammeter. Why ?

Ans. A voltameter measures current indirectly in terms of mass of ions deposited and E.C.E. of the substance

$$\left(I = \frac{m}{z\,t} \right)$$

Since values of m and z are measured to 3rd decimal place (upto one milligram) and 5th decimal place (refer to the values of z from tables) respectively, the relative error in the measurement of current by voltameter will be very small as compared to that when measured by ammeter directly.

Q. 4.16. How can the accuracy of a given ammeter be determined with the help of copper voltameter. *(P.S.S.C.E. 1999 S ; C.B.S.E. 1998 S)*

Ans. By passing current for a known time t, mass of copper deposited on the cathode is measured. Since $m = z\,I\,t$, we have

$$I = \frac{m}{z\,t}$$

Knowing m, t and z (electrochemical equivalent of copper), the current I flowing through the voltameter can be calculated. The comparison of the calculated value of current

I with the value noted from the ammeter gives the accuracy of the ammeter.

Q. 4.17. How the process of electrolysis can be used to find zero error in an ammeter ?

Ans. As explained above, the current passed through an electrolyte can be calculated by noting m, t and z in an experiment. If the ammeter is also put in the circuit, the difference in the calculated value of I and the reading of ammeter gives the zero zero for the ammeter.

Q. 4.18. State and explain Faraday's second law of electrolysis. *(H.S.S.C.E. 2002)*

Ans. Refer to section 4.07.

Q. 4.19. Define chemical equivalent and electrochemical equivalent of a substance. Establish the relationship between the two.

(C.B.S.E. 1997 ; H.P.S.S.C.E. 1997)

Ans. Chemical equivalent. *The chemical equivalent of an element is defined as the ratio of its atomic weight to the valency of its atom.*

$$\text{Thus, chemical equivalent, } E = \frac{\text{atomic weight}}{\text{valency}}$$

Electrochemical equivalent. *Electrochemical equivalent of a substance may be defined as the mass of ions liberated, when one coulomb of charge passes through the electrolyte.*

For relation between chemical equivalent and electrochemical equivalent, refer to section 4.07.

Q. 4.20. What is electrolysis ? What is the relation between electrochemical equivalent and chemical equivalent of a substance ? *(H.S.S.C.E. 2002)*

Ans. Electrolysis. *The process of liberating free elements from an electrolytic solution, when electric current is passed through it, is called electrolysis.*

Relation between electrochemical equivalent and chemical equivalent. It is given by

$$\frac{E}{z} = \text{constant}$$

Q. 4.21. Define Faraday constant. How is it related to E.C.E. of a substance ?

Ans. For definition of Faraday constant, refer to section 4.08.

Faraday constant and E.C.E. of a substance are related to each other by the relation

$$z = \frac{E}{F},$$

where E is chemical equivalent of the substance.

Q. 4.22. Explain the process of electroplating.

(P.S.S.C.E. 2002)

Ans. Refer to section 4.09.

Q. 4.23. What is the difference between primary and secondary cells ? Give one example of each.

(P.S.S.C.E. 2002, 1994 ; H.S.S.C.E. 2002 ; Pre-degree Kerala, 1991)

Ans. An electrochemical cell, which cannot be recharged and the chemicals have to be replaced, is called a primary cell.

On the other hand, an electrochemical cell in which the chemicals of the cell can be recharged again and again is called a secondary cell. A Leclanche cell is an example of a primary cell, while a lead-acid cell is a secondary cell.

Q. 4.24. Name the two defects of a simple voltaic cell. How these defects can be eliminated ? *(P.S.S.C.E. 1996)*

Ans. A voltaic cell suffers from the two defects name local action and polarisation.

For removal of these defects, refer to section 1.05.

Q. 4.25. What do you mean by local action in a volt cell ? How is it elliminated ? *(H.S.S.C.E. 20*

Ans. The zinc rod used in the voltaic cell contains tra of iron and lead. When such a zinc rod is dipped in dil sulphuric acid, tiny voltaic cells are formed locally on the z rod. Due to the electric currents set up in these local cells, z is converted into $ZnSO_4$, even when the cell is not in use. T defect in a voltaic cell is called local action.

Q. 4.26. Draw a labelled diagram of a Daniel cell.

(H.S.S.C.E. 2002, 2001 ; P.S.S.C.E. 1999 S ; C.B.S.E. 19

Ans. Refer to Fig. 4.04.

Q. 4.27. Draw a labelled diagram of Daniel cell. W is the function of copper sulphate and sulphuric a solutions used in it ? *(C.B.S.E. 19*

Ans. For labelled diagram, refer to Fig. 4.04.

In a voltaic cell, a neutral layer of hydrogen gas is forn on the copper rod (anode). It weakens the action of the cell a this defect in voltaic cell is called polarisation.

However, it does not happen so in a Daniel cell, wl dilute sulphuric acid and copper sulphate solutions are us In this cell, the hydrogen ions formed due to the reactior zinc with dilute sulphuric acid, react with copper sulph solution so as to form sulphuric acid again. As such, polarisation due to hydrogen can take place due to the use copper sulphate solution. The copper sulphate solution is s to act as depolariser.

Q. 4.28. Draw a labelled diagram of Leclanche cell

(H.S.S.C.E. 2002, 2001 ; P.S.S.C.E. 1999

Ans. Refer to Fig. 4.05.

Q. 4.29. Draw labelled diagram of a Leclanche c Write the function of charcoal and manganese dioxide u in its porus pot. *(P.S.S.C.E. 1999 S ; C.B.S.E. 19*

Ans. For labelled diagram, refer to Fig. 4·05.

In a Leclanche cell, manganese dioxide is used depolariser. The hydrogen ions formed due to the reactior zinc with ammonium chloride solution enter the porus through its pores. Inside the porus pot, the hydrogen ions re with MnO_2 to form Mn_2O_3. However, by taking oxygen fr the atmosphere, Mn_2O_3 gets oxidised to MnO_2 again. T charcoal powder is used in order to make manganese diox electrically conducting and its use helps in decreasing internal resistance of the cell.

Q. 4.30. A Leclanche cell is used, when constant sup of electric current is not required. Why ?

Ans. The e.m.f. of a Leclanche cell falls, if it is u continuously. It is because of the partial polarisation due accumulation of hydrogen gas. In case, Leclanche cell is u in experiments, where current is drawn after short brea then during each break, hydrogen gas escapes and Mn converts into MnO_2 by taking oxygen from the atmosph As a result, the cell regains its original e.m.f.

Q. 4.31. Draw a neat, labelled diagram of a dry cell

(H.S.S.C.E. 20

Ans. Refer to Fig. 4.06.

Q. 4.32. Why a series resistance is put while charg a storage cell ?

Ans. In case, a large current flows through the storage cell during charging, then a non-porous layer of $PbSO_4$ is formed on the plates, which does not decompose during subsequent recharging. As a result, the storage cell is rendered useless.

Q. 4.33. A storage cell should not be short circuited. Why ?

Ans. During short circuiting, a large current flows through the cell and as said in SAQ 4.32, a non-porous layer of $PbSO_4$ is formed. This layer does not decompose during charging and the cell is rendered useless.

Q. 4.34. Why is the terminal voltage across a battery more than its e.m.f. during recharging ? (*C.B.S.E. 1998 S*)

Ans. During charging, the positive plate of the accumulator is connected to positive terminal and its negative plate to the negative terminal of the battery charger. As a result, the terminal voltage across the battery becomes more than its e.m.f.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For **a**mbitious, **b**rilliant & **c**urious Students

Q. 4.01. Molten sodium chloride conducts electricity, while sodium chloride below its boiling point is non-conducting liquid. Why ? (*Text Question*)

Ans. Sodium chloride is ionic in solid state. But in the solid state, ions are bound. When sodium chloride is in molten state, the ions become free to move. Due to this reason, molten sodium chloride conducts electricity.

Q. 4.02. The solution of HCl in water is strongly conducting. Why ? (*Text Question*)

Ans. HCl is not ionic and therefore in molten state, it is not conducting. When HCl is dissolved in water, the hydrogen molecules having great affinity for H_2O molecules, react with water and Cl^- ions are set free. As a result, the solution of HCl in water becomes strongly conducting.

Q. 4.03. Why do some covalent salts (e.g. Al_2Cl_6), which are not ionic in solid state, become conducting, when dissolved in water ? (*Text Question*)

Ans. Some covalent salts, which are not ionic and hence non- conducting become conducting when dissolved in water. The reason is that the metal ions in covalent salts have strong affinity for water and react with water. The ions other than metal ions in the salts are set free. These ions act as current carriers. It explains how the covalent salts when dissolved in water become conducting.

Q. 4.04. What is a strong electrolyte and a weak electrolyte ? Give examples. (*Text Question*)

Ans. Strong electrolytes. *Those electrolytes, which are more or less completely ionised in their solutions are called strong electrolytes.* For example, HCl, NaOH and NaCl.

Weak electrolytes. *Those electrolytes, which are ionised to a small extent in their solutions are called weak electrolytes.* For example, NH_4Cl, CH_3COOH and H_2CO_3.

Q. 4.05. What causes the dissociation of the solute molecules in the solution ? (*Text Question*)

Ans. The ionic bonds between the ions of the solute are weakened by the polar molecules of the solvent like water. Due to this, the ions of the solute get dissociated. The chaotic motion of the solvent and solute molecules helps in the process of dissociation.

Q. 4.06. Define international ampere.

Ans. In 1910, the International Conference of Weights and Measures defined the unit of current (ampere) on the basis of the electrolysis of the silver salt and ampere defined in this manner is called international ampere.

From Faraday's first law of electrolysis, we have

$$m = z\,I\,t$$

Suppose that $I = 1$ ampere, $t = 1$ s

Then, $m = z \times 1 \times 1 = z$

Now, value of z for silver is 0·001118 g C^{-1}.

Hence, *international ampere is defined as the steady current, which when passed through a silver voltameter, deposits 0·001118 g of silver in one second on the cathode.*

Q. 4.07. A simple voltaic cell has an e.m.f. equal to 0·1 V. When the circuit is open, is there a net field, which would give rise to a force on a test charge (*i*) inside the electrolyte of the cell (*ii*) outside the cell ? (*Text Question*)

Ans. (*i*) Inside the cell, there are two fields. One field is the electrostatic field between the plates due to the e.m.f. of the cell and another field is of non-electrostatic origin. However, the two fields just balance each other and consequently net field inside the cell is zero.

(*ii*) Outside the cell, the net field is the electrostatic field between the two plates of the cell.

Q. 4.08. Consider a voltaic cell in open circuit with copper plate at higher potential w.r.t. zinc plate. Since the electrolyte separating them is a conducting medium, why are the positive and negative charges on the electrodes not neutralized immediately ? (*Text Question*)

Ans. The positive and negative charges on the two electrodes are prevented from being neutralized by the non-electrostatic field existing inside the cell. The electrostatic field is opposed by the non-electrostatic field.

Q. 4.09. During the action of voltaic cell, copper rod is positively charged and zinc rod is negatively charged. However, copper rod is called cathode and zinc rod is called anode. Why ?

Ans. The electrode of the cell, at which oxidation reaction (release of electrons) takes place is called anode ; while the electrode, at which reduction reaction (addition of electrons) takes place is called cathode.

Since in voltaic cell, oxidation and reduction reactions take place at zinc and copper rods respectively, zinc rod is called anode and the copper rod is said to act as cathode.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Give the ionic theory of electrolysis.
2. Explain the terms : electrode, anode, cathode, anion, cation, electrolyte, chemical equivalent, gram equivalent, electrochemical equivalent and faraday.
3. What are (*i*) copper voltameter (*ii*) silver voltameter (*iii*) water voltameter ? Discuss.

4. What is electrolysis ? State its laws. (*P.S.S.C.E. 2002, 2000*)
5. State and explain Faraday's laws of electrolysis. (*H.P.S.S.C.E. 2000 ; P.S.S.C.E. 1999 ; H.S.S.C.E. 1997 S, 1997*)
6. State Faraday's laws of electrolysis. Draw a labelled circuit diagram to illustrate deposition of copper on a given metallic object. (*C.B.S.E. 1995*)
7. What is electrolysis ? State and explain Faraday's laws of electrolysis. (*P.S.S.C.E. 2001*)
8. State Faraday's law of electrolysis. Derive the relation connecting chemical equivalent and electrochemical equivalent of an element. (*C.B.S.E. 1999, 1996*)
9. State Faraday's laws of electrolysis. How is Faraday constant related to E.C.E. of a substance ? (*H.P.S.S.C.E. 1996*)
10. Prove that Faraday constant $F = \frac{qE}{m}$ and hence define Faraday constant. (*H.P.S.S.C.E. 2002*)
11. Define Faraday constant. How is Faraday constant related to E.C.E. of a substance ? (*P.S.S.C.E. 1999 ; H.P.S.S.C.E. 1996*)
12. Write notes on : (*a*) Electroplating (*b*) Refining of metals by electrolysis.
13. What is an electrochemical cell ? What is the differen
between primary and secondary cells ? Give one examp
of each type. (*H.S.S.C.E. 1998 ; P.S.S.C.E. 1998, 1997, 199*
14. Write two advantages and two disadvantages of a seco
dary cell over a primary cell. (*C.B.S.E. 199*
15. Explain the construction and action of a simple voltaic ce
16. What is a source of e.m.f. ? Describe the construction a
working of a Daniel cell.
17. Explain with the help of a labelled diagram, the con
truction and working of a Daniel cell. (*C.B.S.E. 199*
18. What is a primary cell ? Give the construction a
necessary theory of Leclanche cell.
19. Name the defects of voltaic cell. How are they removed
Leclanche cell ? Draw its labelled diagram.
(*P.S.S.C.E. 199*
20. Draw a labelled diagram of a Daniel cell. Write down t
chemical reactions taking place at the anode and t
cathode. (*C.B.S.E. 199*
21. Draw a labelled diagram of a dry cell. Explain its acti
with the help of chemical reactions taking place at catho
and anode.
22. With the help of a suitable diagram, explain the constru
tion and working of a dry cell. (*C.B.S.E. 199*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. (*a*) State Faraday's laws of electrolysis and the definition of strong electrolyte. (*H.S.S.C.E. 1995*)
   (*b*) Discuss a method to determine the electrochemical equivalent of a substance.
2. (*a*) State and explain Faraday's laws of electrolysis. Define electrochemical equivalent. (*C.B.S.E. 1992*)
   (*b*) Show that the Faraday's constant is equal to the produ
   of Avogadro number and the electronic charge.
3. What is the difference between a primary and a seconda
cell ? Describe the action of a lead-acid cell with the he
of a diagram and the reactions taking place inside. Expla
in brief the charging and discharging process in a lead ac
cell.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Faraday's first law of electrolysis

1. How long will it take to deposit 1·5 g of silver by a constant current of 0·75 A in a silver voltameter ? Given E.C.E. of silver 0·001118 $g\ C^{-1}$. [**Ans.** 29 minutes 49 seconds]
2. It is desired to deposit 0·315 kg of copper on the cathode of a copper voltameter. How long will it take to deposit this amount, if a steady current of 10 A is maintained ? Given that E.C.E. of copper $= 3{\cdot}15 \times 10^{-7}\ kg\ C^{-1}$. (*H.P.S.S.C.E. 1998 S*) [**Ans.** 1,000 s]
3. Calculate the value of current required to deposit 0·972 g of chromium in 3 hours. E.C.E. of chromium is 0·00018 $g\ C^{-1}$. [**Ans.** 0·5 A]
4. A copper voltameter and an ammeter are connected in series with a battery through a resistance. In 150 minutes, 2·97 g copper is deposited on the cathode plate. The ammeter reads 0·95 A. Calculate the error in the reading of ammeter. Given that E.C.E. of copper $= 3{\cdot}3 \times 10^{-4}\ g\ C^{-1}$. (*C.B.S.E. 2000 S*) [**Ans.** Reads 0·05 A less]
5. An ammeter is suspected to be giving incorrect reading of current. To check the error, it is connected in series with a copper voltameter. When a steady current is passed through the voltameter for one hour, 2·02 g of copper is deposited. If the ammeter reads 2 A, what is the error in it ? Given that E.C.E. of copper is 0·00033 $g\ C^{-1}$. [**Ans.** Reads 0·3 A more]
6. A steady current deposits 0·15 g of copper in 15 minutes from copper sulphate solution. An ammeter in series read 0·5 A. What is the correction to be applied to the ammeter reading ? Given E.C.E. of copper $= 3{\cdot}3 \times 10^{-4}\ g\ C^{-1}$.
(*Pre-degree Kerala, 1992*) [**Ans.** Reads 0·005 A mo
7. A silver and a copper voltameter are connected in paral
to a 12 volt battery having negligible internal resistance.
half hour, 1 g of silver and 1·8 g of copper are deposite
Calculate the rate of dissipation of energy from the batte
Given that E.C.E. of copper is $3294 \times 10^{-7}\ g\ C^{-1}$and that
silver is $11180 \times 10^{-7}\ g\ C^{-1}$. [**Ans.** 42·4 wa
8. A silver and a copper voltameter are connected in series wi
a 12·0 V battery of negligible internal resistance. 0·806 g
silver is deposited in half an hour in the silver voltamet
Calculate (*i*) magnitude of current flowing in the circuit a
(*ii*) mass of copper deposited in the copper voltamet
during the same period. Given that E.C.E. of silver 1·12
$10^{-6}\ kg\ C^{-1}$and E.C.E. of copper $= 3{\cdot}3 \times 10^{-7}\ kg\ C^{-1}$.
(*C.B.S.E. 1998*) [**Ans.** (*i*) 0·4 A (*ii*) 0·238
9. A copper voltameter is in series with a heater coil
resistance 0·3 Ω. A steady current flows in the circuit for
minutes. A mass of 0·99 g of copper is deposited at t
cathode. If the electrochemical equivalent of copper
0·00033 $g\ C^{-1}$, calculate the heat generated in the coil ?
(*C.B.S.E. 2000*) [**Ans.** 1500
10. A copper voltameter is connected in series with a coil
resistance 20 ohm. A steady current is passed through t
circuit for 10 minutes and it deposits 0·1 g of copp
Calculate the amount of heat generated in the coil. T
E.C.E. of copper is 0·000329 $g\ C^{-1}$. [**Ans.** 3079·55
11. A plate of area 10 cm^2 is to be electroplated with copp
to a thickness of 0·001 cm on both sides using a batte

of 10 volt. Calculate the energy spent in the process. Density of copper is 8·9 g cm^{-3} and E.C.E. of copper is 0·00033 g C^{-1}. **[Ans. 5393·94 J]**

12. A spoon having 10 cm^{2} area is to be electroplated with silver. A current of 0·12 A is passed for 48 hours through the electrolyte. Calculate the thickness of silver deposited. Given that density of silver = 10·5 × 10^{3} kg m^{-3} and E.C.E. of silver = 1·118 × 10^{-6} kg C^{-1} **[Ans. 2·2 mm]**

Type B. On Faraday's second law of electrolysis

13. A copper voltameter is connected in series with a silver voltameter. When same current is passed through both the voltameters for same time, 1·5 g of silver is deposited on the cathode of the silver voltameter. Calculate the copper deposited on the cathode of the copper voltameter. Take the chemical equivalents of silver and copper as 108 and 31·5 respectively. **[Ans. 0·4375 g]**

14. Two voltameters are connected in series and are arranged to deposit copper and silver respectively. How much copper will be deposited in copper voltameter, if 1 g of silver is deposited in silver voltameter. Given that chemical equivalents of copper and silver are 31·5 and 108 respectively. **[Ans. 0·292 g]**

15. Two voltameters are connected in series. One of them is water voltameter and the other is copper voltameter. How much oxygen will be liberated in the water voltameter, when 0·5 g of copper is deposited in the copper voltameter? Chemical equivalents of copper and oxygen are 32 and 8 respectively. **[Ans. 0·125 g]**

16. Find the strength of current that deposits 0·777 g of metallic copper in half an hour in a copper voltameter. The atomic weight of copper is 63, E.C.E. of hydrogen is 1·036 × 10^{-5} g C^{-1} and chemical equivalent of hydrogen is 1·008. **[Ans. 1·333 A]**

17. Find the value of Faraday constant by considering the electrolysis of NaCl. Given that atomic weight of Na = 23 and E.C.E. of Na = 2383·4 × 10^{-7} g C^{-1}. *(P.S.S.C.E. 1999)* **[Ans. 96500 C]**

Type C. On Faraday constant

18. Given that 96500 C of charge liberates 1 gram-equivalent of a substance. How long will it take for a current of 0·15 A to deposit 500 mg of copper from a copper sulphate solution ? Take chemical equivalent of copper as 32. **[Ans. 167·54 minutes]**

19. It is desired to deposit 0·54 kg of silver per hour on the cathode of silver voltameter, whose resistance is 0·72 mΩ. How much potential difference must be maintained between the plates of the voltameter during electrolysis. Use the known vlaue of Faraday constant. Relative atomic mass of silver is 108. **[Ans. 96·4 mV]**

20. A current of 1 A is passed through a dilute solution of sulphuric acid for some time to liberate 1 g of oxygen. How much hydrogen is liberated during this period ? How long was the current passed ? Given that Faraday constant = 96500 C mole^{-1}. **[Ans. 0·126 g ; 201 minutes]**

MISCELLANEOUS PROBLEMS

21. One ampere current flows for one minute through a silver voltameter. It deposits 0·067 g of silver on the cathode. Calculate (*a*) the electrochemical equivalent of silver. (*b*) How much charge will be required to deposit 108 g of silver ? (*c*) How much charge is carried by silver ion ? Given that Avogadro number = 6·024 × 10^{26} per kg mole. *(Roorkee 1981)*
[Ans. (*a*) 1·1167 × 10^{-6} kg C^{-1} (*b*) 9·67 × 10^{4} C (*c*) 1·6 × 10^{-19} C]

22. In producing chlorine through electrolysis, 100 kW power of 125 V is being consumed. How much chlorine per minute is liberated ? Given that E.C.E. of chlorine = 0·367 × 10^{-6} kg C^{-1}. *(Roorkee 1982)* **[Ans. 1·762 × 10^{-2} kg]**

23. A charged capacitor of capacitance 5 × 10^{-2} F is discharged through a resistor of 20 Ω and a copper voltameter of internal resistance 30 Ω is connected in series to it. If 4·62 × 10^{-6} kg of copper is deposited, calculate the heat generated in the resistor. Given that E.C.E. of copper = 3·3 × 10^{-7} kg C^{-1}. *(M.N.R. 1994)* **[Ans. 784 J]**

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Proceed as in solved problem no. 4.01.
2. Proceed as in solved problem no. 4.01.
3. Now, $m = z\,I\,t$ or $I = \dfrac{m}{z\,t}$

   or $$I = \frac{0\cdot972}{0\cdot00018 \times 3 \times 60 \times 60} = 0\cdot5\ \text{A}$$
4. Proceed as in solved problem no. 4.02.
5. Proceed as in solved problem no. 4.02.
6. Proceed as in solved problem no. 4.02.
7. Proceed as in solved problem no. 4.04.
8. (*i*) Proceed as in problem no. 3.
   (*ii*) Proceed as in solved problem no. 4.05.
9. Proceed as in solved problem no. 4.06.
10. Proceed as in solved problem no. 4.06.
11. Mass of copper deposited,
    $$m = (2\ \text{A}) \times d \times \rho = 2 \times 10 \times 0\cdot001 \times 8\cdot9\ \text{g}$$
    Energy spent in the process, $W = V\,I\,t$

    But $m = z\,I\,t$ or $I\,t = \dfrac{m}{z}$
    $$\therefore \quad W = \frac{V \times m}{z} = \frac{2 \times 10 \times 0\cdot001 \times 8\cdot9}{0\cdot00033} = 5393\cdot94\ \text{J}$$
12. Let d be thickness of silver deposited.
    Therefore, mass of silver deposited,
    $$m = A\,d\,\rho = 10 \times 10^{-4} \times d \times 10\cdot5 \times 10^{3} = 10\cdot5\,d\ \text{kg}$$
    Now, $m = z\,I\,t$

    or $10\cdot5\,d = 1\cdot118 \times 10^{-6} \times 0\cdot12 \times 48 \times 60 \times 60$

    or $d = 2\cdot2 \times 10^{-3}\ \text{m} = 2\cdot2\ \text{mm}$
13. Now, $\dfrac{m_1}{E_1} = \dfrac{m_2}{E_2}$

    or $$m_2 = m_1 \times \frac{E_2}{E_1} = \frac{1\cdot5 \times 31\cdot5}{1\cdot8} = 0\cdot4375\ \text{g}$$
14. Proceed as in problem no. 13.
15. Proceed as in problem no. 13.
16. Chemical equivalent of copper,
    $$E_1 = \frac{\text{atomic weight}}{\text{valency}} = \frac{63}{2} = 31\cdot5$$
    Chemical equivalent of hydrogen, $E_2 = 1\cdot008$

    E.C.E. of hydrogen, $z_2 = 1\cdot036 \times 10^{-5}$ g C^{-1}

    Therefore, E.C.E. of copper,
    $$z_1 = z_2 \times \frac{E_1}{E_2} = \frac{1\cdot036 \times 10^{-5} \times 31\cdot5}{1\cdot008}$$
    $$= 3\cdot2375 \times 10^{-4}\ \text{g C}^{-1}$$
    Now, $$I = \frac{m_1}{z_1 \times t} = \frac{0\cdot777}{3\cdot2375 \times 10^{-4} \times 30 \times 60} = 1\cdot333\text{A}$$

17. Chemical equivalent of Na,

$$E = \frac{\text{atomic weight}}{\text{valency}} = \frac{23}{1} = 23$$

Now, $F = \frac{E}{z} = \frac{23}{2383 \cdot 4 \times 10^{-7}} = \mathbf{96500\ C}$

18. Now, $z = \frac{E}{F} = \frac{32}{96500} = 3 \cdot 316 \times 10^{-4} \text{g C}^{-1}$

Also, $t = \frac{m}{zI} = \frac{500 \times 10^{-3}}{3 \cdot 316 \times 10^{-4} \times 0 \cdot 15} = 10052 \cdot 3 \text{ s}$

$= \mathbf{167{\cdot}54\ minutes}$

19. $E = \frac{\text{atomic weight}}{\text{valency}} = \frac{108}{1} = 108$

Now, $z = \frac{E}{F} = \frac{108}{96500} = 1 \cdot 12 \times 10^{-3} \text{ g C}^{-1}$

$= 1{\cdot}12 \times 10^{-6} \text{ kg C}^{-1}$

Also, $I = \frac{m}{zt} = \frac{0 \cdot 54}{1 \cdot 12 \times 10^{-6} \times 60 \times 60} = 133 \cdot 93 \text{ A}$

Therefore potential difference to be applied,

$V = IR = 133{\cdot}93 \times 0{\cdot}72 = \mathbf{96{\cdot}4\ mV}$

20. Here, chemical equivalent of oxygen, $E_1 = 8$, chemical equivalent of hydrogen, $E_2 = 1{\cdot}008$ and mass of oxygen deposited, $m_1 = 1$ g

Therefore, mass of hydrogen liberated,

$$m_2 = m_1 \times \frac{E_2}{E_1} = \frac{1 \times 1 \cdot 008}{8} = \mathbf{0{\cdot}126\ g}$$

Now, $z_1 = \frac{E_1}{F} = \frac{8}{96500} = 8 \cdot 29 \times 10^{-5} \text{g C}^{-1}$

$\therefore \quad t = \frac{m_1}{z_1 \times I} = \frac{1}{8 \cdot 29 \times 10^{-5} \times 1} = 12063 \text{ s}$

$= \mathbf{201\ minutes}$

21. (*a*) Now, $z = \frac{m}{It} = \frac{0 \cdot 067}{1 \times 60} = 1 \cdot 1167 \times 10^{-3} \text{g C}^{-1}$

$= \mathbf{1{\cdot}1167 \times 10^{-6}\ kg\ C^{-1}}$

(*b*) Also, $q = \frac{m}{z} = \frac{108 \times 10^{-3}}{1 \cdot 1167 \times 10^{-6}} = \mathbf{9{\cdot}67 \times 10^4\ C}$

(*c*) Since 108 is chemical equivalent of silver, the quantit of charge of $9{\cdot}67 \times 10^4$ C is the value of Faraday constant.

Now, $F = Ne$

Here, $N = 6{\cdot}024 \times 10^{26}$ per kg molecule

$= 6{\cdot}024 \times 10^{23}$ per gram molecule

$\therefore \quad e = \frac{F}{N} = \frac{9 \cdot 67 \times 10^4}{6 \cdot 024 \times 10^{23}} = \mathbf{1{\cdot}6 \times 10^{-19}\ C}$

22. Current passed through voltameter,

$$I = \frac{P}{V} = \frac{100 \times 10^3}{125} = 800 \text{ A}$$

Now, $m = zIt = 0{\cdot}367 \times 10^{-6} \times 800 \times 60 = \mathbf{1{\cdot}762 \times 10^{-2}\ kg}$

23. Charge passing through copper voltameter due t discharge of capacitor,

$$q = \frac{m}{z} = \frac{4 \cdot 62 \times 10^{-6}}{3 \cdot 3 \times 10^{-7}} = 14 \text{ C}$$

Therefore, energy stored in the capacitor before it discharge,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{14 \times 14}{2 \times 5 \times 10^{-2}} = 1960 \text{ J}$$

When the capacitor discharges, energy stored in th capacitor will generate heat both in series resistance (R) an in the copper voltameter due to its internal resistance (R_{in}

Therefore, heat generated in series resistance R,

$$Q = \frac{U \times R}{R + R_{in}} = \frac{1960 \times 20}{20 + 30} = 784 \text{ J}$$

CHAPTER

5

Thermoelectric Effect

5.01. THERMOELECTRICITY

We know that when electric current is passed through a resistance, electrical energy gets converted into heat energy. This phenomenon is known as ***Joule heating***. If we reverse the direction of flow of current, heat is still produced in the resistance. Thus, Joule heating is an irreversible phenomenon and it cannot be used to convert heat into electrical energy.

Key point

Joule heating is not a reversible effect.

From thermodynamics, we know that heat can be converted into mechanical energy and it led to the development of heat energies. However, heat can be converted into electrical energy by making use of a ***thermocouple***.

A thermocouple is an arrangement of two suitable dissimiliar metallic wires (wires of different materials) joined at their ends to form junctions.

Fig. 5.01. shows a Cu-Fe thermocouple obtained by joining the copper and iron wires at their ends. When one junction of the thermocouple is kept cold and the other hot, a small e.m.f. (≈ a few μV) is developed, which sends current through it. The e.m.f. so produced is called ***thermo-e.m.f.*** and the current so produced is called ***thermo-electric current***. The flow of current can be detected by including a sensitive galvanometer in the thermocouple. The electrical energy is produced at the expense of a net amount of heat absorbed in the thermocouple.

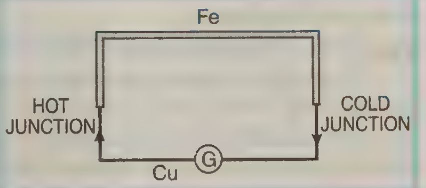


Fig. 5.01

The conversion of heat into electrical energy by using a thermocouple is called thermoelectricity.

Key point

A thermocouple produces thermo-e.m.f. by converting heat into electric energy.

In a Cu-Fe thermocouple, current flows from copper to iron through hot junction as shown in the figure.

5.02. ORIGIN OF THERMO E.M.F.

A conductor always contains free electrons. The number of free electrons per unit volume in a conductor depends upon the nature of its material. Further, the number of free electrons per unit volume, called electron density, increases with rise in temperature.

When two metallic wires of different materials are joined at their ends to form a thermocouple, electrons from a metal having greater electron density diffuse into the other with lower value of electron density. Due to this, a small potential difference is established across the junction of the two metals. The potential difference so established is called contact potential and its value depends upon the temperature of the junction. Obviously, if the two junctions are at the same temperature, the contact potentials at the two junctions will be equal. The equal contact potentials at the two junctions tend to send equal currents but in opposite directions ; and as such no net current flows through the thermocouple.

Self-test Questions

No current flows through a thermocouple, when its two junctions are at the same temperature. Why ?

However, if one of the two junctions is heated, then diffusion of electrons at the hot junction becomes more and likewise the contact potential at the hot junction also becomes more than that at the cold junction. Hence, when the two junctions of a thermocouple are at different temperatures, a net e.m.f., called thermo-e.m.f. is produced.

5.03. SEEBECK EFFECT

The phenomenon of producing thermoelectric e.m.f. (or thermoelectric current) in a thermocouple by keeping its two junctions at different temperatures is called thermoelectric effect.

This effect was discovered by Thomas Johann Seebeck in 1821, which after his name is also known as *Seebeck effect*.

Seebeck effect can be demonstrated with any two dissimilar metals. As said earlier, in a Cu-Fe thermocouple, the current flows from copper to iron through hot junction and from iron to copper through cold junction.

Cause of Seebeck effect. Suppose that a current I flows through the thermocouple, when its two junctions are kept at different temperatures. The two wires used to make the thermocouple and the galvanometer have got some resistance, say R. It implies that an amount of electrical energy $I^2 R$ is being converted into heat every second. It is natural to ask, where is this energy coming from ? Obviously, there must be a source of e.m.f. somewhere in the thermocouple, which is causing the flow of current. Experimental studies showed that a thermocouple constantly absorbs some heat (Q) from the hot junction and rejects a part of it (Q′) to the cold junction. The net amount of heat absorbed (Q – Q′) is converted into electrical energy. Thus, a thermocouple acts like a heat engine and the net heat absorbed (Q – Q′) is the source of e.m.f. in the circuit.

Key point

Seebeck effect is reversible in nature.

It may be pointed out that Seebeck effect, in fact, is the resultant of two effects*, namely ***Peltier effect*** and ***Thomson effect***.

Seebeck effect is reversible. Consider that a cell is put in the circuit so that it sends current in a direction from copper to iron through cold junction *i.e.* in a direction opposite to the direction of flow of current in a Cu-Fe thermocouple as shown in Fig 5.02. It is found that heat is now evolved (liberated) at the hot junction and absorbed at the cold junction. In other words, the hot junction is found to become hotter and the cold junction colder than before. It shows that Seebeck effect is reversible in nature.

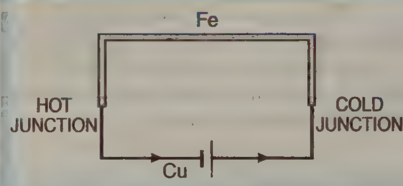


Fig. 5.02

5.04. THERMOELECTRIC SERIES

The magnitude and direction of the thermo-e.m.f. produced in a thermocouple depends upon the nature of the metals forming the thermocouple and to the temperature difference between the two junctions. Keeping this fact in view, Seebeck studied the behaviour of a many pair of metals. On the basis of his experimental results, he arranged a number of metals in the form of series, called ***thermoelectric series.*** Some of the metals forming the series are as below :

Sb, As, Fe, Zn, Ag, Au, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Pt, Co, Ni and *Bi*.

It is also called ***Seebeck series.***

The following are two main uses of the thermoelectric series :

1. *To know direction of current.* If a thermocouple is formed with wires of any two metals from the thermoelectric series, *the direction of current will be from a metal occurring earlier in this series to the metal occurring later in the series through the cold junction.* Therefore, in copper-iron (Cu-Fe) thermocouple, the current will flow from iron to copper through cold junction or from copper to iron through the hot junction. The words **Hot Coffee** serve as an aid to remember the direction of flow of current in copper-iron thermocouple. In antimony-bismuth (Sb-Bi) thermocouple, the current flows from antimony to bismuth through the cold junction. The direction of flow of current can be remembered with the help of letters **ABC,** where A stands for antimony, B for bismuth and C for cold junction.

Key point

The thermoelectric series can be used to know the direction of flow of thermo-electric current in a thermocouple formed of two metals from the series. Further, it helps to choose the metals, so as to obtain large thermo-e.m.f.

2. *To know magnitude of thermo-e.m.f.* If two metals chosen to form the thermocouple are more widely separated in the series, then the thermo-e.m.f. produced (for the same temperature difference between the two junctions of the thermocouple) is also greater. As a rule, *more the metals are separated in the thermoelectric series, greater is the thermo-e.m.f. produced.*

The thermo-e.m.f. produced for a difference of temperature equal to 100°C is about 0·0013 V for Cu-Fe thermocouple and about 0·008 V for Sb-Bi thermocouple.

5.05. VARIATION OF THERMO-E.M.F. WITH TEMPERATURE

The magnitude of thermo-e.m.f. increases, when the temperature of hot junction is increased and the temperature of other junction (cold junction) is kept constant.

To study the effect of difference of temperature between the two junctions, consider a Cu-Fe thermocouple. Its one junction is kept hot by immersing in oil bath, heated with a burner. The other junction is kept cold by immersing it in pounded ice and as shown in Fig. 5.03. The temperature of the hot junction can be measured by the thermometer T placed in the hot oil bath.

As the temperature of the hot junction is increased by keeping the temperature of the cold junction constant at 0°C, the deflection in the galvanometer goes on increasing. The deflection in the galvanometer is directly proportional to the thermoelectric current and hence the thermo-e.m.f. The graph between thermo e.m.f.

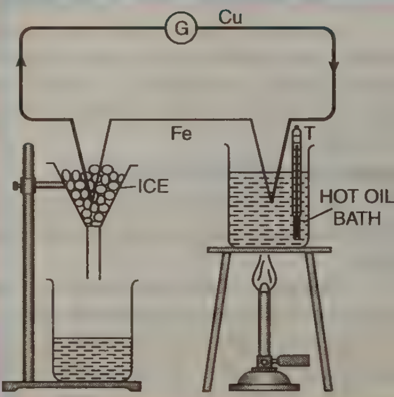


Fig. 5.03

---

*For the discussion of Peltier effect and Thomson effect, refer to sections 5.09 and 5.11.

and the temperature of hot junction is found to be parabolic in shape and is as shown in Fig. 5.04. As the temperature of hot junction is further increased, a stage comes, when the thermo-e.m.f. becomes maximum.

The temperature of hot junction, at which the thermo-e.m.f. produced in the thermocouple becomes maximum, is called **neutral temperature.** *It is denoted by* θ_n.

The value of neutral temperature :

(*i*) *is independent of the temperature of the cold junction of the thermocouple and is always constant for a given thermocouple.*

(*ii*) *depends on the nature of the materials of the two metals used to form the thermocouple.*

For example, whatever may be the temperature of the cold junction, for Cu-Fe thermocouple, neutral temperature is always 270°C.

As the temperature of the hot junction is further increased, the thermo-e.m.f. decreases and ultimately becomes zero. If the temperature of the junction is still increased, direction of the thermo-e.m.f. reverses.

The temperature of the hot junction, at which the direction of the thermo e.m.f. reverses, is called the ***temperature of inversion.*** It is denoted by θ_i.

The value of temperature of inversion :

(*i*) *depends on the temperature of the cold junction of the thermocouple and therefore is not constant for a given thermocouple.*

It is found that *the temperature of inversion is as much above the neutral temperature as the neutral temperature is above the temperature of the cold junction.* Thus, if θ_c is temperature of the cold junction, then

$$\theta_n - \theta_c = \theta_i - \theta_n$$

or

$$\theta_n = \frac{\theta_i + \theta_c}{2} \qquad \text{...(5.01)}$$

Therefore, the neutral temperature is the mean of the temperature of inversion θ_i and temperature of the cold junction θ_c. For Cu-Fe thermocouple $\theta_n = 270°$. If cold junction is at 0°C, then it follows that $\theta_i = 540°C$.

(*ii*) *depends on the nature of the materials of the two metals forming the thermocouple.*

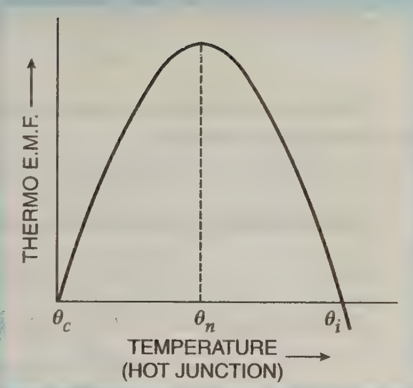


Fig. 5.04

5.6. RELATION BETWEEN THERMO E.M.F. AND TEMPERATURE OF HOT JUNCTION

When cold junction of a thermocouple is kept at 0°C and temperature of hot junction is increased, the graph between the thermo-e.m.f. and the temperature of hot junction is found to be parabolic in nature. Further, for a number of metals, the parabolic graph between thermo-e.m.f. (E) and temperature of hot junction was found to satisfy the simple relation

$$E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2, \qquad \text{...(5.02)}$$

where α and β are constants, called ***thermoelectric constants.*** The value of these constants depend upon the nature of the two metals forming the thermocouple. However, the above relation between E and θ holds over a limited range of temperature.

The following table gives the value of α and β for the thermocouples formed of the various metals with lead. These values of α and β have been found in the temperature range 200 K to 400 K.

Substance	α (μV °C^{-1})	β (μV °C^{-2})
Aluminium	$-0 \cdot 76$	$+0 \cdot 0039$
Bismuth	$-43 \cdot 7$	$-0 \cdot 465$
Copper	$+1 \cdot 34$	$+0 \cdot 0094$
Constantan	$-38 \cdot 1$	$-0 \cdot 089$
Iron	$+17 \cdot 2$	$-0 \cdot 048$
Platinum	$-3 \cdot 04$	$-0 \cdot 033$

Self-test Question

Using equation (5.02), find the units of α and β.

5.07. THERMOELECTRIC POWER

The rate of change of thermo-e.m.f. with the change in the temperature of the hot junction is called thermoelectric power.

Thermoelectric power is also called **Seebeck coefficient** and is denoted by S.

Key point

Thermoelectric power is also called Seebeck coefficient.

Thus, thermoelectric power (or Seebeck coefficient),

$$S = \frac{dE}{d\theta} \qquad ...(5.03)$$

We know that thermo-e.m.f. varies with the temperature of the hot junction according to the relation

$$E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2 \qquad ...(5.04)$$

Differentiating both sides of equation (5.04) w.r.t. θ, we have

$$\frac{dE}{d\theta} = \frac{d}{d\theta}(\alpha\,\theta + \frac{1}{2}\beta\,\theta^2) = \alpha + \beta\,\theta$$

Therefore, equation (5.03) becomes

$$S = \frac{dE}{d\theta} = \alpha + \beta\,\theta \qquad ...(5.05)$$

The equation (5.05) tells that if a graph is plotted between θ (along X-axis) and $dE/d\theta$ (along Y-axis), the graph will be a straight line having slope β and making intercept α on Y-axis. Thus, the graph between $\frac{dE}{d\theta}$ and θ will be as shown in Fig. 5.05.

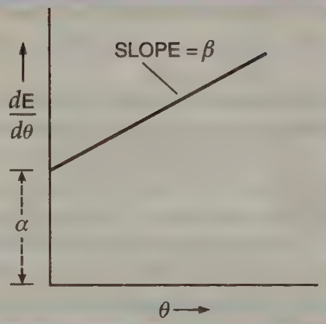


Fig. 5.05

We know that at the neutral temperature of the thermocouple, the thermo-e.m.f. is maximum *i.e.* rate of change of thermo-e.m.f. with temperature becomes zero. Therefore,

when $\qquad \theta = \theta_n, \; \frac{dE}{d\theta} = 0$

Setting the above condition in equation (5.05), we have

$$\alpha + \beta\,\theta_n = 0$$

or $\qquad \theta_n = -\frac{\alpha}{\beta} \qquad ...(5.06)$

It gives the value of neutral temperature of the thermocouple in terms of its thermoelectric constants α and β.

Key point ✓

At neutral temperature of the thermocouple, its seeback coefficient becomes zero.

5.08. SEEBECK EFFECT–A RESULTANT OF TWO EFFECTS

As said earlier, Seebeck effect is a resultant of two effects, called the Peltier effect and the Thomson effect. When the two junctions of a thermocouple are kept at different temperatures, the different parts of the two wires constituting the thermocouple are also at different temperatures. The e.m.f. developed in the thermocouple is partly due to different temperatures of the two junctions and partly due to different temperatures along the length of the two wires of the thermocouple. As explained below, Peltier effect accounts for the e.m.f. developed due to different temperatures of the two junctions, while Thomson effect is responsible for the e.m.f. developed due to different temperatures along the length of the two wires.

Key point ✓

The thermo-e.m.f. due to Seeback effect is resultant of e.m.fs. produced due to Peltier and Thomson effects.

5.09. PELTIER EFFECT

This effect was discovered by the French physicist Jean Charles Peltier in 1834.

Peltier found that when two metals, say bismuth and copper are connected to form a thermocouple, an e.m.f. exists across each of the two junctions of the thermocouple. In case of Bi-Cu thermocouple, copper is at higher potential as compared to bismuth. The e.m.f. across the junction of two metals increases with increase of temperature of the junction Let π_A abd π_B be the e.m.fs. developed due to Peltier effect at the hot and cold junction respectively [Fig. 5.06]. The thermocouple is, thus, equivalent to a circuit containing two sources of e.m.f. π_A and π_B, such that π_A is greater than π_B. Obviously, the resultant e.m.f. of the circuit is $\pi_A - \pi_B$. This resultant e.m.f. causes a current to flow from bismuth to copper at hot junction *i.e.* from a metal at lower potential to the one at higher potential. Since energy is required for current to flow from lower potential to higher potential, heat is absorbed at the hot junction. On the other hand, current flows from copper to bismuth at the cold junction *i.e.* from a metal at higher potential to the metal at lower potential. Hence, energy is evolved at the cold junction.

This absorption or evolution of heat at a junction of two dissimilar metals, when a current is passed, is known as Peltier effect.

Key point ✓

Peltier effect accounts for the e.m.f. developed in the thermocouple due to difference in temperatures of the two junctions of the thermocouple.

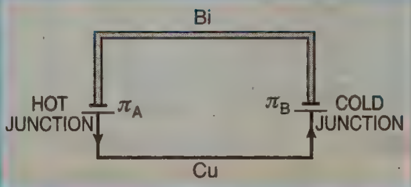


Fig. 5.06

Peltier effect is reversible. If the direction of flow of current is reversed, heat will be evolved at the hot junction and absorbed at the cold junction. Thus, Peltier effect is reversible in nature.

Peltier coefficient. *The amount of heat energy absorbed or evolved per second at a junction, when a unit current is passed through it, is known as Peltier coefficient.* It is denoted by π.

Suppose the cold junction of the thermocouple is at temperature T and hot junction at T+ dT. If dE is the thermo-e.m.f. produced, then it is found that

$$\frac{\pi}{T}=\frac{dE}{dT} \qquad ...(5.07)$$

Here, $\frac{dE}{dT}$ is rate of charge of thermo-e.m.f. As said earlier, it is known as *thermoelectric power* or *Seebeck coefficient* (S). Therefore, equation (5.07) may be written as

$$\frac{\pi}{T}=\frac{dE}{dT}=S \qquad ...(5.08)$$

5.10. DIFFERENCE BETWEEN JOULE HEATING AND PELTIER EFFECT

The following points of difference can be drawn between Peltier effect and Joule heating effect of current :

Peltier effect	Joule heating effect
1. Heat is evolved or absorbed at the junctions of the thermocouple.	1. Heat is evolved throughout the conductor.
2. Heat evolved or absorbed at a junction varies directly as the current passed.	2. Heat evolved in the conductor varies directly as the square of the current passed.
3. If the direction of current is reversed, heat is absorbed at the junction, where earlier heat was evolved and vice-versa.	3. There is no change in Joule effect on reversing the direction of current *i.e.* whatever may be the direction of current, always heat is evolved.
4. Both heating or cooling effect can take place at the same junction of the thermocouple. The effect produced depends upon the direction of flow of current through the junction.	4. It is always a heating effect.
5. The amount of heat evolved or absorbed at a junction depends upon the nature of the two metals forming the thermocouple.	5. It depends upon the resistance of the conductor.

5.11. THOMSON EFFECT

Thomson discovered that when two ends of a conductor are kept at different temperatures, an e.m.f. develops between the two ends. For example, in case of copper wire, the hotter end is at higher potential [Fig. 5.07]. Therefore, when current flows through a copper wire from the hotter to the colder end *i.e.* from end at higher potential to the end at lower potential, heat is evolved along the wire. As a result, the copper wire will get heated. On the other hand, when current flows from the colder to hotter end *i.e.* from lower to higher potential, heat is absorbed along the wire. It will result in cooling of the wire. A few other such metals are antimony, silver, zinc, etc. All other substances, are said to possess ***positive Thomson effect.***

In the case of a bismuth wire, the hotter end is at lower potential as shown in Fig. 5.08. Therefore, when current flows through a bismuth wire from the hotter to the colder end, heat is absorbed along the wire and when current flows from the colder to the hotter end, heat is evolved along the wire. A few other such metals are iron, cobalt, platinum, etc. All such metals, are said to possess ***negative Thomson effect.***

This absorption or evolution of heat along the length of a conductor on passing current through it, when its two ends are kept at different temperatures is known as Thomson effect.

For lead, Thomson effect is zero. Further, Thomson effect is also reversible in nature.

Thomson coefficient. *The amount of heat energy absorbed or evolved per second between two points of a conductor having a unit temperature difference, when a unit current is passed, is known as Thomson coefficient for the material of a conductor.* It is denoted by σ.

Thomson coefficient of the material of a conductor is found by forming its

Key point

Thomson effect accounts for the e.m.f. developed in a thermocouple due different temperatures along the length of each of the two wires forming the thermocouple.

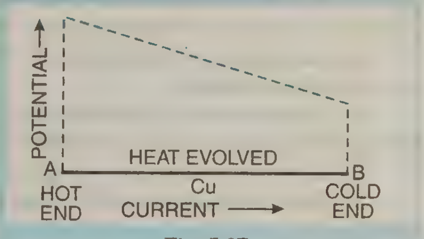


Fig. 5.07

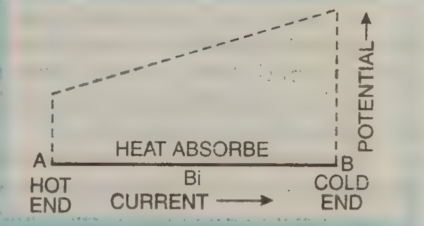


Fig. 5.08

Key point

Thomson effect may be positive, negati
or zero depending on the nature of met

thermocouple with a lead wire (Thomson coefficient of lead is zero). It can be proved that Thomson coefficient of the material of conductor is given by

$$\sigma = -T\frac{d^2E}{dT^2} \qquad ...(5.09)$$

Now, Seebeck coefficient is given by

$$S = \frac{dE}{dT}$$

$$\therefore \qquad \frac{dS}{dT} = \frac{d^2E}{dT^2}$$

Thus,

$$\sigma = -T\frac{d^2E}{dT^2} = -T\left(\frac{dS}{dT}\right) \qquad ...(5.10)$$

5.12. THERMOELECTRIC THERMOMETER (To measure temperature)

It is a device used to measure both low and high temperatures.

Principle. *It is based on Seebeck effect.*

Construction. A thermoelectric thermometer is basically a thermocouple. Two metals used to construct the thermoelectric thermometer are chosen depending upon the range of measurement of temperature as given below :

Thermocouple	Range of temperature
Copper-gold and iron alloy	1 K to 50 K
Copper-constantan	50 K to 400 K
Platinum-platinum and iridium alloy	400 K to 1300 K
Platinum-platinum rhodium alloy	1500 K to 2000 K

The wires to be used for forming the thermocouple are welded together at one end and this end forms the hot junction. The two wires near the hot junction are kept separate from each other by enclosing them in hard glass capilliary tubes (C.T.). The wires, then, pass through mica discs (D), which are arranged one above the other in a porcelain tube T. The ends of the two wires are connected to the terminals T_1 and T_2 fitted on the lid of the porcelain tube. To shift the cold junction to a convenient place, two wires L_1 and L_2 made of same material as those used for making the thermocouple, are connected to the terminals T_1 and T_2 [Fig. 5.09]. The wires L_1 and L_2 are called compensating leads.

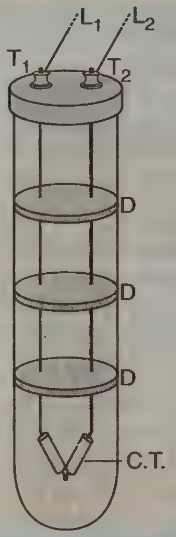


Fig. 5.09

Theory. Before measuring the temperature of an unknown hot body, a graph between thermo-e.m.f. and the temperature of hot junction is plotted, keeping cold junction at 0°C. The graph so obtained is called calibration curve of the thermocouple and it is as shown in Fig. 5.04. Then, to measure the temperature of the hot body, one junction is kept at 0°C and the other junction is placed in contact with the hot body. The thermo-e.m.f. produced in the thermocouple is measured. The temperature of the hot body is read from the calibration curve against the measured thermo-e.m.f. It may be pointed out that instead of plotting calibration curve, the galvanometer or the micro voltmeter included in the thermocouple may be calibrated, so as to directly read the temperature of the hot body.

Advantages. A thermoelectric thermometer has following advantages over mercury thermometer :

1. Thermoelectric thermometers have much wider range of measurement of temperature (from – 200°C to 1600°C).

2. These thermometers are quite cheap and can be easily constructed.

3. They are quite sensitive and can measure temperature accurately up to 0·05C.

4. They can be used to measure the temperature of the specific part of a hot body.

5. Due to the low thermal capacity of the junction of two metals, they attain the temperature of the hot body very quickly. As such, they are found to be useful for measuring rapidly varying temperatures.

6. They form the basis of the radiation pyrometer for measuring temperature of a hot body from a distance.

Disadvantages. 1. These thermometers are not direct reading thermometers and hence they cannot be used in experiments on calorimetery.

2. Each thermoelectric thermometer has to be calibrated separately before using it to measure the temperature.

3. For measurment of temperature over different ranges, different thermoelectric thermometers (thermocouples made of different materials) have to be used.

5.13. THERMOPILE (To detect heat radiation)

It is a device used to detect and measure the intensity of heat radiations. It is based on Seebeck effect.

Key point

When n identical thermocouples are connected in series, thermo-e.m.f. produced by the arrangement is n times the thermo-e.m.f. produced by a single thermo-couple for a given difference between the temperatures of the two junctions.

Principle. For a given small difference in temperature of two junctions of a thermocouple, Bi-Sb thermocouple produces a comparatively large e.m.f. and it can be used to detect the heat radiation. When, a number of such thermocouples are connected in series, the arrangement becomes very much sensitive to detect heat radiation as the thermo-e.m.fs. of the thermocouples get added.

Construction. A series combination of a large number of Bi-Sb thermocouples is enclosed in a funnel or a horn-shaped enclosure.

The junction A of each thermocouple is coated with lamp black, while the junction B of each thermocouple is well polished and covered with insulating material. The extreme ends of the arrangement are connected to the terminals T_1 and T_2, across which a sensitive galvanometer is connected. Such an arrangement forms a thermopile and is as shown in Fig. 5.10.

Working. When heat radiations fall on the funnel shaped end of the thermopile, the set of junctions A coated with lamp black absorb the heat radiation. As a result, the temperature of set of junctions A relative to junctions B rises and thermo-e.m.f. is developed in each thermocouple. The thermoelectric current flows in the same direction (from Sb to Bi through cold junction) in all the thermocouples. Therefore, a large current flowing through the circuit produces deflection in the galvanometer, which in turn detects the heat radiation falling on it.

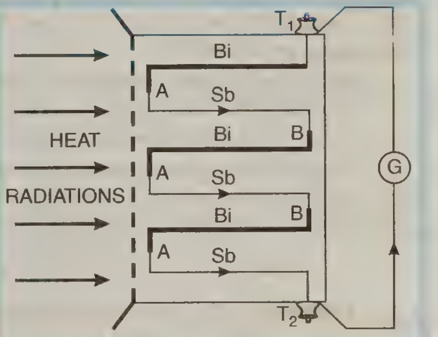


Fig. 5.10

5.14. APPLICATIONS OF THERMOELECTRIC EFFECT

The thermoelectric effect has the following important applications :

1. To measure temperature. A thermocouple is preferred and used to measure temperatures in industries and laboratories for the following reasons :

(*i*) Since the junction is very small, it absorbs only a very small amount of heat and therefore it does not change the temperature of the object.

(*ii*) It quickly attains the temperature of the hot object.

(*iii*) The accuracy in the measurement of temperature is quite high.

2. To detect heat radiation. A thermopile is a combination of a large number of thermocouples in series. It can be used to detect the heat radiation and to note the small difference or the variation in temperatures, if any.

3. Thermoelectric refrigerator. As discussed earlier, if a current is passed through a thermocouple, then due to Peltier effect, heat is removed at one junction and is absorbed at the other junction. In case, if on the whole, heat is removed, then the thermocouple acts as a thermoelectric refrigerator. The advantage is that it has no compressor. No doubt, the cooling effect produced is much low as compared to that in case of a conventional refrigerator. A thermoelectric refrigerator is used, when the region to be cooled is very small and the noise is not acceptable. The thermocouple used as a thermoelectric refrigerator should have following three characteristics :

(*i*) It should have low resistivity. Otherwise, loss of energy in the form of heat will be large.

(*ii*) It should have low thermal conductivity. It will help in maintaining large temperature difference between the two junctions.

(*iii*) It should produce high thermopower.

Researches have shown that these requisites are met best by the semiconductor $Bi_2 Te_3$.

4. Thermoelectric generator. Thermocouples can be used to generate thermoelectric power in remote areas. It may be done by heating one junction in a flame and exposing the other junction to air. The thermo-e.m.f. developed has been used to power radio receivers, etc.

SOLVED NUMERICAL PROBLEMS

Type A. On Neutral temperature and Temperature of inversion

Problem 5.01. In a given thermocouple, the temperature of cold junction is 15 °C, while the neutral temperature is 270 °C. What is the value of the temperature of inversion ?

Sol. Here, temperature of cold junction, $\theta_c = 15\ °C$;

neutral temperature, $\theta_n = 270°C$

If θ_i is temperature of inversion, then

$$\theta_i = 2\,\theta_n - \theta_c = 2 \times 270 - 15 = 540 - 15 = \mathbf{525\ °C}$$

Problem 5.02. The cold junction of a thermocouple is kept at 10 °C. Calculate the temperature at which thermo-e.m.f. would be maximum. Given that the thermo-e.m.f. changes sign at 800 K.

Sol. Here, temperature of cold junction, $\theta_c = 10°C$

Thermo-e.m.f. changes sign at temperature of inversion.

$$\therefore \qquad \theta_i = 800\ K = 800 - 273 = 527\ °C$$

When the temperature of the hot junction is equal to neutral temperature for thermocouple, thermo-e.m.f. produced is maximum. The neutral temperature is given by

$$\theta_n = \frac{\theta_i + \theta_c}{2} = \frac{527 + 10}{2} = \mathbf{268 \cdot 5\ °C}$$

Problem 5.03. Near room temperature, the thermo-e.m.f. of copper-constantan thermocouple is 40 μV °C^{-1} at room temperature. What is the smallest temperature difference that can be detected with single such thermocouple and a galvanometer of 100 ohm resistance capable of detecting currents as low as 10^{-6} ampere ?

Sol. Resistance of galvanometer $= 100\ \Omega$

Least current that can be detected $= 10^{-6}$ A

Therefore, least e.m.f. that can be detected

$$= 100 \times 10^{-6} = 10^{-4}\ V$$

e.m.f. developed per °C in the thermocouple

$$= 40\ \mu V = 40 \times 10^{-6}\ V$$

Therefore, least temperature that can be detected

$$= \frac{10^{-4}}{40 \times 10^{-6}} = \mathbf{2 \cdot 5\ °C}$$

Type B. On Thermo-e.m.f.

Problem 5.04. Calculate the thermo-e.m.f. of a silve iron thermocouple with junction at 0 °C and 80 °C. Give that $\alpha = +\ 13 \cdot 31\ \mu V\ °C^{-1}$ and $\beta = 0 \cdot 038\ \mu V\ °C^{-2}$. Als calculate the neutral temperature. *(Karnataka 198*

Sol. If θ is temperature difference between the cold an hot junction of a thermocouple, then thermo-e.m.f. is give by

$$E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2$$

Here, $\theta = 80 - 0 = 80°C$; $\alpha = +\ 13 \cdot 31\ \mu V\ °C^{-1}$;

$\beta = -\ 0 \cdot 038\ \mu V\ °C^{-2}$;

$$\therefore \qquad E = 13 \cdot 31 \times 80 + \frac{1}{2}(-\ 0 \cdot 038) \times 80^2 = \mathbf{943 \cdot 2\ \mu V}$$

Also, neutral temperature,

$$\theta_n = -\frac{\alpha}{\beta} = -\frac{13 \cdot 31}{(-\ 0 \cdot 038)} = \mathbf{350 \cdot 26\ °C}$$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 5.05. The e.m.f. E of a Cu-Fe thermocouple varies with the temperature θ of the hot junction (cold junction at 0 °C) as

$$\mathbf{E\ (\mu V) = 14\ \theta\ -\ 0 \cdot 02\ \theta^2}$$

Determine the neutral temperature.

(H.S.S.C.E. 2001 ; C.B.S.E. 1990)

Sol. Here, the thermo-e.m.f. of thermocouple at temperature θ of hot junction,

$$E\ (\mu V) = 14\ \theta\ -\ 0 \cdot 02\ \theta^2 \qquad ...(i)$$

The expression for thermo-e.m.f. is given by

$$E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2 \qquad ...(ii)$$

Comparing the equations (*i*) and (*ii*), we have

$$\alpha = 14\ \mu V\ °C^{-1}\ ;\ \beta = -\ 0 \cdot 04\ \mu V\ °C^{-2}$$

Now, neutral temperature,

$$\theta_n = -\frac{\alpha}{\beta} = -\frac{14}{-\ 0 \cdot 04} = \mathbf{350°\ C}$$

Aliter. Here, $E = 14\ \theta - 0 \cdot 02\ \theta^2$ ...(*i*)

The thermo-e.m.f. is maximum at neutral temperature. In other words,

$$\text{when} \qquad \theta = \theta_n, \quad \frac{dE}{d\theta} = 0 \qquad ...(ii)$$

Differentiating the equation (*i*) *w.r.t.* θ, we have

$$\frac{dE}{d\theta} = 14 - 0 \cdot 02 \times 2\,\theta = 14 - 0 \cdot 04\,\theta$$

Applying the condition given by equation (*ii*), we have

$$0 = 14 - 0 \cdot 04\ \theta_n \quad \text{or} \quad \theta_n = 14 / 0 \cdot 04 = \mathbf{350°\ C}$$

Problem 5.06. The thermoelectric e.m.f. E of a copp constantan thermocouple and the temperature θ of th hot junction (with cold junction at 0°C) are found to satisf approximately the following relation :

$$\mathbf{E = \alpha\,\theta + \beta\,\theta^2}$$

where E is in μV, θ in °C. Also, $\alpha = 41\ \mu V\ °C^{-1}$ and $\beta = 0 \cdot 04$ $\mu V\ °C^{-2}$. What is the temperature of the hot junction, whe the thermo-electric e.m.f. is measured to be 5·5 mV ?

(Text Problen

Sol. The thermo-e.m.f. of the thermocouple at tempe rature θ of the hot junction is given by

$$E = \alpha\,\theta + \beta\,\theta^2 \qquad ...($$

Here, $E = 5 \cdot 5\ mV = 500\ \mu V$;

$\alpha = 41\ \mu V\ °C^{-1}$ and $\beta = 0 \cdot 041\ \mu V\ °C^{-2}$

Therefore, equation (*i*) becomes

$$5500 = 41\ \theta + 0 \cdot 041\ \theta^2$$

$$\text{or} \qquad 0 \cdot 041\ \theta^2 + 41\ \theta - 5500 = 0$$

$$\text{or} \qquad \theta = \frac{-\ 41 \pm \sqrt{(41)^2 - 4 \times 0 \cdot 041 \times (-5500)}}{2 \times 0 \cdot 041}$$

$$= \frac{-\ 41 \pm \sqrt{1681 + 902}}{0 \cdot 082} = \frac{-\ 41 \pm 50 \cdot 82}{0 \cdot 082}$$

Since the temperature of hot junction has to be greate than 0°C, it follows that

$$\theta = \frac{-\ 41 + 50 \cdot 82}{0 \cdot 082} = \mathbf{119 \cdot 76\ °C}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. Give the direction in which thermoelectric current flows at the hot junction of a Cu-Fe thermocouple. (*C.B.S.E. 2001*)

Ans. From Cu to Fe.

Q. 5.02. Give the direction of thermoelectric current (*i*) at the cold junction of Cu-Bi (*ii*) at the hot junction of Fe-Cu (*iii*) at the cold junction of platinum-lead thermocouple.

Ans. (*i*) From Cu to Bi. (*ii*) From Cu to Fe. (*iii*) From lead to platinum.

Q. 5.03. Give any two factors on which thermoelectric e.m.f. produced in a thermocouple depends. (*C.B.S.E. 1997*)

Ans. Thermo- e.m.f. produced in a thermocouple depends upon (*i*) the temperature of hot junction and (*ii*) the nature of the two metals forming the thermocouple.

Q. 5.04. How does the thermo-e.m.f. vary with the temperature of the hot junction ? (*H.S.S.C.E. 2001*)

Ans. The thermo-e.m.f. increases with increase in temperature of hot junction, till the temperature becomes equal to the neutral temperature for the thermocouple. As the temperature is increased beyond the neutral temperature, thermo-e.m.f. starts decreasing.

Q. 5.05. Draw a graph showing the variation of thermo-e.m.f. with temperature, indicating the position of neutral temperature and temperature of inversion. (*I.S.C.E. 1994*)

Ans. For the graph, refer to Fig. 5.04.

The temperature corresponding to the peak of the graph is neutral temperature (θ_n) and the temperature, corresponding to which, the thermo e.m.f. reverses sign, is temperature of inversion (θ_i).

Q. 5.06. What is the nature of the plot between thermo-e.m.f. and the temperature of hot junction ?

Ans. It is parabolic in nature.

Q. 5.07. Define neutral temperature of a thermocouple. (*P.S.S.C.E. 2002*)

Ans. *The temperature of the hot junction, at which the thermo-e.m.f. produced in the thermo-couple becomes maximum, is called neutral temperature.*

Q. 5.08 How does the neutral temperature of a thermocouple vary with the temperature of the cold junction ? (*C.B.S.E. 2001*)

Ans. The neutral temperature of a thermocouple does not vary with the temperature of the cold junction.

Q. 5.09. Is the neutral temperature for a thermocouple constant ?.

Ans. Yes, the neutral temperature for a thermocouple is a constant.

Q. 5.10. Is the temperature of inversion for a thermocouple constant ?

Ans. No, the temperature of inversion for a thermocouple is not a constant.

Q. 5.11. Write the expression connecting the thermoelectric e.m.f. of a thermocouple with the temperature difference of its cold and hot junctions. (*H.S.S.C.E. 1999 ; C.B.S.E. 1995*)

Ans. The relation between thermo-e.m.f (E) and temperature difference (θ) between the cold and the hot junction is given as

$$E = \alpha\theta + \frac{1}{2}\beta\theta^2$$

Q. 5.12. What are the units of α and β in relation to thermoelectricity ?

Ans. The unit of α is μV °C^{-1} and that of β is μV °C^{-2}.

Q. 5.13. Define Seebeck coefficient. (*P.S.S.C.E. 2001*)

Ans. *The rate of change of thermo-e.m.f. with the change in temperature of the hot junction of a thermocouple is called Seebeck coefficient.*

It is also known as thermoelectric power.

Q 5.14. What is the value of Seebeck coefficient at the neutral temperature of a thermocouple ? (*C.B.S.E. 2002*)

Ans. It is zero.

Q. 5.15. What is Peltier effect ? (*P.S.S.C.E. 2001*)

Ans. Refer to section 5.09.

Q. 5.16. Define Peltier coefficient. (*P.S.S.C.E. 2001*)

Ans. *The amount of heat energy absorbed or evolved per second at a junction, when a unit current is passed through it, is known as Peltier coefficient of the junction*

Q. 5.17. What is Thomson effect ? (*P.S.S.C.E. 2001*)

Ans. Refer to section 5.11.

Q. 5.18. Define Themsom coefficient.(*P.S.S.C.E. 2001*)

Ans. *The amount of heat energy absorbed or evolved per second between two points of a conductor having a unit temperature difference, when a unit current is passed, is known as Thomson coefficient for the material of the conductor.*

Q. 5.19. Give one practical application of thermoelectricity. (*C.B.S.E. 1991*)

Ans. Thermoelectric effect is used to measure temperature and to detect heat radiation.

Q. 5.20. What is a thermopile ?

Ans. It is a combination of a large number of thermocouples in series. As such, for a small temperature difference between its hot and cold junctions, comparatively a large e.m.f. is produced. It can be used to detect the heat radiation and to note the small variation or difference in temperature.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. What is thermo-e.m.f. ? Explain its origin. (*P.S.S.C.E. 2002, 1999 S*)

Ans. Thermo e.m.f. *The e.m.f. produced in a thermocouple, when its two junctions are kept at different temperatures, is called thermo-e.m.f.*

Origin of thermo e.m.f. Refer to section 5.02.

Q. 5.02. Briefly explain the origin of thermo-e.m.f. (*H.S.S.C.E. 2001*)

Ans. Refer to section 5.02.

Q. 5.03. What is Seebeck effect ? Give one example. (*P.S.S.C.E. 2000 ; C.B.S.E. 1990*)

Ans. Seebeck effect. *The phenomenon of production of an electric current in a thermocouple, when its two junctions are kept at different temperatures, is known as Seebeck effect.*

For example, when a thermocouple is formed of the two wires of copper and iron, the current flows from copper to iron through hot junction.

Q. 5.04. What is Seebeck effect ? On what factors, magnitude of thermo-e.m.f. depends ? (*H.S.S.C.E. 2001*)

Ans. Seebeck effect. Refer to section 5.02.

The magnitude of thermo e.m.f. depends upon the difference in the temperature between the two junctions and the nature of metals used to form a thermocouple. The thermo-e.m.f. produed increases, when the two metals chosen to form the thermocouple are widely separated in the thermoelectric series.

Q. 5.05. How does thermoelectric series help to predict the direction of flow of current in a thermocouple ?

Ans. It helps to know the direction in which current will flow, when a thermocouple is formed with the wires of any two metals in the series. The direction of current will be from a metal occurring earlier in this series to the metal occurring later in the series through the cold junction.

Q. 5.06. Why do we generally prefer Sb-Bi thermocouple ? (*H.P.S.S.C.E. 1999 S, 1999*)

Ans. The metals Sb and Bi are at the two extreme ends of the thermoelectric series and hence for the given temperatures of cold and hot junctions, the thermo-e.m.f. produced is maximum. It is because, more the metals are separated in the series, the greater is the thermo-e.m.f. produced.

Q. 5.07. What is thermo-e.m.f. ? Show its variation with temperature by a graph. (*P.S.S.C.E. 1997*)

Ans. *The e.m.f. developed in a thermocouple when its tw junctions are maintained at different temperature, is called therm e.m.f.*

For the graph, refer to Fig. 5.04.

Q. 5.08. Plot a graph showing variation of thermo electric e.m.f. with temperature of hot junction. Defin neutral temperature. (*H.S.S.C.E. 200*

Ans. For graph, refer to Fig. 5.04.

Neutral temperature. Refer to VSQ 5.07.

Q. 5.09. Define temperature of inversion. State tw factors on which the temperature of inversion of a thermo couple depends. (*C.B.S.E. 2000, 1998 S*

Ans. Temperature of inversion. Refer to section 5.05.

The temperature of inversion of a thermocoupl depends upon the temperature of cold junction and the natur of the metals forming the thermocouple.

Q. 5.10. Define thermoelectric power. What is it magni-tude at neutral temperature ?

(*C.B.S.E. 2000 ; H.S.S.C.E. 1998 S*

Ans. For definition, refer to section 5.07.

At neutral temperature, thermoelectric power is **zero**

Q. 5.11. State and explain Seebeck effect. Mention it one application. (*C.B.S.E. 199*

Ans. For seebeck effect, refer to section 5.03.

A thermopile is based on Seebeck effect. It is used t detect heat radiation or to detect any small variation i temperature.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 5.01 A sensitive microphone cannot withstand currents greater than 0·05 A. When connected across a thermocouple of e.m.f. 8·5 mV, the current in a very low resistance ammeter placed in series in the circuit reads 34 mA. What is the resistance of the microphone ?

(*Text Question*)

Ans. Here, $E = 8{\cdot}5 \text{ mV} = 8{\cdot}5 \times 10^{-3}$ V

Current passing through the microphone,

$$I = 34 \text{ mA} = 34 \times 10^{-3} \text{ A}$$

Therefore, resistance of the microphone,

$$R = \frac{E}{I} = \frac{8{\cdot}5 \times 10^{-3}}{34 \times 10^{-3}} = 0{\cdot}25\,\Omega$$

Q. 5.02. What is the importance of the relation

$$E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2 \text{ ?}$$

Ans. From the relation ; $E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2$, we have

$$\frac{dE}{d\theta} = \alpha + \beta\,\theta \qquad ...(i)$$

When the temperature of the hot junction is equal to neutral temperature, thermo-e.m.f. is maximum *i.e.* the rate of change of thermo-e.m.f. becomes zero.

Therefore, when $\theta - \theta_n$, $\quad \frac{dE}{d\theta} = 0$

Applying the above condition, the equation (*i*) become

$$\alpha + \beta\,\theta_n = 0 \qquad \text{or} \qquad \theta_n = -\frac{\alpha}{\beta}$$

Therefore, knowing the values of the constants α and for a thermocouple, neutral temperature of the thermo-coupl can be found.

Q. 5.03. Heat is produced at a junction of two metals when a current passes through. When the direction o current is reversed, heat is absorbed at the junction (*i.e.* th junction gets cooler). Is the usual formula ($I^2 R$ =powe dissipated as heat) applicable for this situation ? If no why not ?

Ans. This phenomenon of absorption of heat at a junctio on reversing the direction of current is known as Peltier effec The formula describing Joule's heating effect of current is no applicable to Peltier effect. The reason is that heat absorbe or evolved at a junction is directly proportional to the curren passing through the thermocouple. Further, Peltier effect i reversible, whereas Joule's heating effect is not reversible i nature.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is Seebeck effect ? What is the cause of thermo-e.m.f. ?
(*H.P.S.S.C.E. 2002, 1995 ; P.S.S.C.E. 1998 S ; 1996 S*)
2. What is seebeck effect ? Discuss the thermoelectric serie
(*C.B.S.E. 199*
3. What is Seebeck effect ? What do you understand b neutral temperature and temperature of inversion of

thermocouple ? Are they constant for a given thermocouple ? *(C.B.S.E. 2001 S)*

4. Draw a graph showing the variation of thermo-electric e.m.f. with temperature of hot junction. How does its neutral temperature vary with the temperature of the cold junction ? *(C.B.S.E. 2000 ; H.S.S.C.E. 1998, 1992)*
5. What is thermocouple ? Give two applications of thermocouple. If the temperature of cold junction of a thermocouple is lowered, what will be the effect on its neutral temperature and temperature of inversion ? *(C.B.S.E. 2001 S)*
6. Discuss the effect of temperature on thermo-e.m.f. of a thermocouple. *(P.S.S.C.E. 1998 S)*
7. Explain neutral temperature and temperature of inversion. Establish a relation between them. *(H.P.S.S.C.E. 1998, P.S.S.C.E. 1997 S)*
8. Explain the terms thermo-emf, neutral temperature and temperature of inversion. *(P.S.S.C.E. 1994)*
9. What is Seebeck effect ? Give a mathematical relationship between e.m.f. E and the temperature of the hot junction θ, while the other junction is kept at 0°C. *(I.S.C.E. 1997, 1993)*
10. The thermo-e.m.f. of a thermocouple is given by

$$E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2,$$

where θ is the temperature of the hot junction ; α and β are the constants of the thermocouple. Deduce an expression for the neutral temperature. *(C.B.S.E. 2000, 1998 S)*

11. What is Seebeck effect ? Explain its cause. How thermoelectric power changes with temperature and hence find neutral temperature ? *(H.P.S.S.C.E., 1995)*
12. (*a*) Explain neutral temperature and temperature of inversion. Establish a relation between them. *(H.S.S.C.E. 2002)*

    (*b*) How is thermocouple used for measuring temperature ?
13. What is Peltier effect and Peltier coefficient ? How is Peltier coefficient related to thermoelectric power ?
14. What is the difference between Joule's heating effect and Peltier effect ? *(H.P.S.S.C.E. 1997)*
15. What is Thomson effect and Thomson coefficient ? How is Thomson coefficient related to thermo-e.m.f. and Seebeck coefficient ?
16. How is a thermocouple used for measuring unknown temperature ? Give the advantages and disadvantages of using thermocouple as a temperature measuring device.
17. Explain a few important practical applications of thermoelectric effect.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. (*a*) What is Seebeck effect ? How does the e.m.f. of a thermocouple vary with temperature of the hot junction ? (*b*) Explain the terms neutral temperature and temperature of inversion in the case of thermocouple. *(Pre-degree Kerala, 1994)*
2. What is a thermocouple ? Which of the two thermocouples Sb-Bi or Cu-Fe produces greater e.m.f. for the same temperature difference between hot and cold junctions ? Graphically sketch the variation of thermo e.m.f. generated as a function of temperature of the hot junction keeping the cold junction at 0°C. Give two applications of thermocouples. *(C.B.S.E. 1999 S)*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Neutral temperature and Temperature of inversion

1. The neutral temperature of Cu-Fe thermocouple is 270 °C. If the temperature of the cold junction is 20 °C, what will be the temperature of inversion ? [**Ans.** 520 °C]
2. The e.m.f. of a thermocouple changes sign at 700 K. If the neutral temperature is 210 °C, what is the temperature of cold junction ? [**Ans.** 7 °C]

Type B. On Thermo-e.m.f.

3. Calculate the thermo-e.m.f. of silver-iron thermocouple with junctions at 20°C and 80°C.

   Given that $\alpha = +\,13{\cdot}3\ \mu\text{V}\ {}^\circ\text{C}^{-1}$ and $\beta = -\,0{\cdot}019\ \mu\text{V}\ {}^\circ\text{C}^{-2}$. *(Karnataka, 1985)* [**Ans.** 763·8 μV]
4. For a Cu-Fe thermocouple, neutral temperature is 270 °C. If $\alpha = 15{\cdot}86\ \mu\text{V}\ {}^\circ\text{C}^{-1}$, then find the value of β. [**Ans.** – 0·0587 $\mu\text{V}\ {}^\circ\text{C}^{-2}$]
5. The e.m.f. of a copper-iron thermocouple varies with the temperature of hot junction as

$$E\ (\text{in } \mu\text{V}) = 11\,\theta - 0{\cdot}02\,\theta^2$$

   Determine the neutral temperature, if the cold junction is maintained at 0°C. *(H.S.S.C.E. 2002)* [**Ans.** 275 °C]
6. The thermo-e.m.f. of a thermocouple is given by the expression $E = \alpha\,\theta - \beta\,\theta^2$, where θ is the temperature of the hot junction. If $\alpha = 42\ \mu\text{V}\ {}^\circ\text{C}^{-1}$, $\beta = 0{\cdot}035\ \mu\text{V}\ {}^\circ\text{C}^{-2}$, calculate the (*i*) neutral temperature and (*ii*) temperature of inversion of the thermocouple, if the cold junction is at 20°C. *(C.B.S.E. 2001)* [**Ans.** (*i*) 600 °C (*ii*) 1180 °C]
7. The thermo-e.m.f. of a thermocouple varies with the temperature of a hot junction according to the relation $E = 14\,\theta - 0{\cdot}02\,\theta^2$, where θ is the temperature af the hot junction. Calculate : (*i*) neutral temperature of the thermocouple, (*ii*) temperature of inversion, assuming that cold junction is at 20°C and (*iii*) value of Seebeck coefficient at neutral temperature. *(C.B.S.E. 2002)* [**Ans.** (*i*) 350°C (*ii*) 680 °C (*iii*) 0]
8. For a copper-iron and a chromel-alumel thermocouple, the plots between thermo-e.m.f. and the temperature of the hot junction (when the cold junction is at 0°C) are found to satisfy approximately the equation $E\ (\text{in } \mu\text{V}) = \alpha\,\theta + \beta\,\theta^2$, where for copper-iron : $\alpha = 11\ \mu\text{V}\ {}^\circ\text{C}^{-1}, \beta = -\,0{\cdot}02\ \mu\text{V}\ {}^\circ\text{C}^{-2}$ and for chromel-alumel : $\alpha = 41\ \mu\text{V}\ {}^\circ\text{C}^{-1}, \beta = -\,0{\cdot}03\ \mu\text{V}\ {}^\circ\text{C}^{-2}$ Which of the two thermocouples, would you use to measure temperature in the range of about 500° to 600°C. [**Ans.** Chromel-alumel]
9. The thermo-e.m.f. of copper-constantan thermocouple is $50\ \mu\text{V}\ {}^\circ\text{C}^{-1}$ at room temperature. A galvanometer of resistance 50 Ω and capable of detecting currents of the order of 2 μA is used. Calculate the smallest temperature difference that can be detected with this galvanometer. [**Ans.** 2°C]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Now, $\theta_i - \theta_n = \theta_n - \theta_c$

   or $\theta_i = 2\,\theta_n - \theta_c = 2 \times 270 - 20 = \mathbf{520\ °C}$

2. Proceed as in solved problem no. 5.01.

3. Here, $\alpha = 13{\cdot}3\ \mu\text{V °C}^{-1}$; $\beta = -\,0{\cdot}019\ \mu\text{V °C}^{-2}$

   and $\theta = 80 - 20 = 60$°C

   Now, $E = \alpha\,\theta + \frac{1}{2}\,\beta\,\theta^2 = 13{\cdot}3 \times 60 + \frac{1}{2} \times (-\,0{\cdot}019) \times (60)^2$

   $= 798 - 34{\cdot}2 = \mathbf{763{\cdot}8\ \mu V}$

4. Now, $\theta_n = -\dfrac{\alpha}{\beta}$

   or $\beta = -\dfrac{\alpha}{\theta_n} = -\dfrac{15{\cdot}86}{270} = \mathbf{-\,0{\cdot}0587\ \mu V\ °C^{-2}}$

5. Here, $E = 11\,\theta - 0{\cdot}02\,\theta^2$ ...(*i*)

   But $E = \alpha\,\theta + \frac{1}{2}\,\beta\,\theta^2$ ...(*ii*)

   Comparing equations (*i*) and (*ii*), we have

   $\alpha = 11\ \mu\text{V °C}^{-1}$ and $\beta = -\,0{\cdot}02 \times 2 = -\,0{\cdot}04\ \mu\text{V °C}^{-2}$

   Now, $\theta_n = -\dfrac{\alpha}{\beta} = \dfrac{11}{-\,0{\cdot}04} = \mathbf{275\ °C}$

6. Here, $E = \alpha\,\theta - \beta\,\theta^2$

   (*i*) Now, $\dfrac{dE}{d\theta} = \dfrac{d}{d\theta}(\alpha\,\theta - \beta\,\theta)^2 = \alpha - 2\,\beta\,\theta$

   When $\theta = \theta_n$, $\dfrac{dE}{d\theta} = 0$

   $\therefore$ $0 = \alpha - 2\,\beta\,\theta_n$ or $\theta_n = \dfrac{\alpha}{2\,\beta}$

   or $\theta_n = \dfrac{42}{2 \times 0{\cdot}035} = \mathbf{600\ °C}$

   (*ii*) $\theta_i = 2\,\theta_n - \theta_c = 2 \times 600 - 20 = \mathbf{1180\ °C}$

7. Proceed as in problem no. 6.

   (*iii*) Seebeck coefficient, $S = \dfrac{dE}{d\theta}$

   At neutral temperature, $\dfrac{dE}{d\theta} = 0$

   Therefore, at neutral temperature,

   $$S = 0$$

8. A thermocouple can be used to measure temperature in the range 500°C to 600°C, if neutral temperature for the thermocouple is fairly above the range of temperature measurement.

   Here, $E = \alpha\,\theta + \beta\,\theta^2$

   $\therefore$ $\dfrac{dE}{d\theta} = \dfrac{d}{d\theta}(\alpha\,\theta + \beta\,\theta^2) = \alpha + 2\,\beta\,\theta$

   When $\theta = \theta_n$, $\dfrac{dE}{d\theta} = 0$

   $\therefore$ $0 = \alpha + 2\,\beta\,\theta_n$ or $\theta_n = -\dfrac{\alpha}{2\,\beta}$

   For copper-iron thermocouple :

   $$\theta_n = -\frac{\alpha}{2\,\beta} = -\frac{11}{2 \times (-0{\cdot}02)} = 275\,°\text{C}$$

   For chromel-alumel thermocouple :

   $$\theta_n = -\frac{\alpha}{2\,\beta} = -\frac{41}{2 \times (-0{\cdot}03)} = 683{\cdot}33\,°\text{C}$$

   Therefore, **chromel-alumel thermocouple** can be used to measure temperature in the range 500°C to 600°C.

9. Resistance of galvanometer, $G = 50\ \Omega$

   The minimum current that can be detected by the galvanometer,

   $$I_{min} = 2\ \mu A$$

   Therefore, minimum e.m.f. that can be detected by the galvanometer,

   $$E_{min} = I_{min} \times G = 2 \times 50 = 100\ \mu V$$

   For a difference of temperature of 1°C, copper-constantan thermocouple produces thermo-e.m.f. of 50 μV.

   Therefore, smallest temperature that can be detected by the galvanometer,

   $$\theta = \frac{100}{50} = 2\ °\text{C}$$

Competitive Examination File (Unit II)

REVISION AT A GLANCE

Chapter 1. Electric current, Resistance and E.M.F.

☐ **Electric current.** *It is the rate of flow of electric charge through a conductor.*

Mathematically : $I = \frac{dq}{dt}$

Unit. In SI, the unit of electric current is **ampere (A).**

1 ampere (A) = 1 coulomb second^{-1} (C s^{-1})

☐ **Ohm's law.** It states that *physical conditions remaining unchanged, the current flowing through a conductor is always directly roportional to the potential difference across its two ends.*

Mathematically : $V \propto I$ or $V = RI$

Here, R is called resistance of the conductor.

Unit. The unit of resistance is **ohm (Ω)**

1 ohm (Ω) = 1 volt ampere^{-1} (V A^{-1})

☐ **Resistance of a conductor.** The resistance of a conductor of length l and area of cross-section A is given by

$$R = \rho \frac{l}{A}$$

Here, ρ is ***resistivity*** of the material of the conductor.

☐ **Resistivity.** *The resistivity of the material of a conductor is the resistance offered by a wire of this material of unit length and nit area of cross-section.* It is also known as ***specific resistance*** of the material of the wire.

Unit. The SI unit of resistivity is **ohm metre (Ω m)**

☐ **Conductance.** *The reciprocal of the resistance of a conductor is called its conductance* (G). Thus,

$$G = \frac{1}{R}$$

Unit. The SI unit of conductance is **ohm^{-1} (Ω^{-1})** or **siemen** (S). ohm^{-1} is also written as **mho.**

☐ **Conductivity.** *The reciprocal of the resistivity of the material of a conductor is called its conductivity* (σ). Thus,

$$\sigma = \frac{1}{\rho}$$

Unit. The SI unit of conductivity is **ohm^{-1} metre^{-1} (Ω^{-1} m^{-1})** or **siemen metre^{-1} (S m^{-1}).** ohm^{-1} metre^{-1} is also written s **mho metre^{-1}.**

☐ **Drift velocity.** *It is the velocity with which a free electron in the conductor gets drifted under the influence of the applied external lectric field.*

Mathematically : $v_d = \frac{eE}{m}\tau = \frac{I}{nAe}$

Here, τ is average relaxation time and n is number of free electrons per unit volume in the conductor. The other symbols ave their usual meanings.

☐ **Temperature coefficient of resistance.** *It is defined as the change in resistance per unit resistance per degree rise in temperature.*

If resistance increases linearly up to temperature θ, then temperature coefficient,

$$\alpha = \frac{R_\theta - R_0}{R_0 \times \theta}$$

Unit. The unit of temperature coefficient is **°C^{-1}.**

☐ **Thermistor.** *A thermally sensitive resistance is called a thermistor.*

☐ **Resistors in series.** The total resistance of resistors R_1, R_2 and R_3 in series is given by

$$R_s = R_1 + R_2 + R_3$$

☐ **Resistors in parallel.** The total resistance of resistors R_1, R_2 and R_3 in parallel is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

☐ **E.M.F.** *The work done per unit charge by the source in taking the charge from its one terminal to the other is called the electromotive orce or e.m.f. of the source.*

It is equal to potential difference between the two terminals of the source, when no current is drawn from it.

☐ **Internal resistance.** *The resistance offered by the electrolyte of the cell, when the electric current flows through it is known as nternal resistance of the cell.*

If V is potential difference across the two terminals of a cell, when a current I is drawn from it, then

$$V = E - Ir = E\left(1 - \frac{r}{R+r}\right) \qquad \left(\because I = \frac{E}{R+r}\right.$$

Here, E is e.m.f. of the cell and R is the external resistance in the circuit.

Mathematically : Internal resistance, $r = \left(\frac{E}{V} - 1\right)R$

Some Useful Facts :

1. $6{\cdot}25 \times 10^{18}$ electrons passing through a conductor per second constitute a current of one ampere.

2. In most of the substances, the carriers of electric current are electrons. In electrolytes and ionic crystals, the curren carriers are positive and negative ions.

3. The resistivity of a material is inversely proportional to the number of electrons per unit volume. It is also inversel proportional to the average relaxation time.

4. The resistivity of metals is in the range $10^{-8}\ \Omega$ m to $10^{-6}\ \Omega$ m. The resistivity of the insulators, such as glass and rubbe is in the range of $10^{10}\ \Omega$ m to $10^{14}\ \Omega$ m. The resistivity of semiconductors lies between that of metals and insulators.

5. The materials used for making standard resistances should have very small temperature coefficient and a high valu of resistivity.

6. Resistors are connected in series, when the resistance in the circuit is to be increased. When the resistors are connecte in series, same current passes through all the resistors.

7. Resistors are connected in parallel, when the resistance in the circuit is to be decreased. When the resistors are connecte in parallel, potential difference across all the resistors is same.

8. Electromotive force is not a force.

9. The cells are connected in series, when the external resistance of the circuit is very large as compared to the total intern resistance of the cells.

10. The cells are connected in parallel, when the external resistance of the circuit is very small as compared to the tot internal resistance of the cells.

Chapter 2. Electrical measurements

☐ **Kirchhoff's Laws.** These laws are used to analyse electric circuits.

First law. *It states that the algebraic sum of the currents meeting at a point in an electrical circuit is always zero.*

Second law. *It states that in any closed part of an electrical circuit, the algebraic sum of the e.m.fs. is equal to the algebraic sum the products of the resistances and the currents flowing through them.*

☐ **Potentiometer.** It is a device used to compare e.m.fs. of two sources or to measure internal resistance of the sourc

Principle. *When a constant current is passed through a wire of uniform area of cross-section, the potential drop across any portio of the wire is directly proportional to the length of that portion.*

☐ **Wheatstone bridge.** It is an arrangement of four resistances used to determine an unknown resistance.

In a balanced Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{X}.$$

where P, Q, R and X are resistances in four arms of the wheatstone bridge.

☐ **Slide wire bridge.** It is a device used to measure unknown resistance and it is based on the principle of Wheatston bridge. It is also called ***metre bridge.***

Some Useful Facts

1. Kirchhoff's first law and second law are also known as ***junction rule*** and ***loop rule*** respectively.

2. In steady state, the branch of the circuit containing a capacitor acts as an open circuit.

3. In steady state, the inductor acts as a conductor having no resistance.

4. When current I flows through a cell of e.m.f. E and internal resistance r, the potential difference across its two termina is given by

(*a*) $V = E - Ir$, if the current flows *out of* the cell.

(*b*) $V = E + Ir$, if the current flows *into* the cell.

In former case, the potential drop across internal resistance ***opposes*** the e.m.f., whereas in the latter case, it ***aids*** the e.m.

Chapter 3. Thermal effects of current

☐ **Cause of heating effect of current.** The potential difference applied across the two ends of conductor sets up electr field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in th conductor. The energy of electrons transferred to the atoms and ions in the conductor appears as heat.

☐ **Electric power.** *The rate of dissipation of electrical energy in an electric circuit is called electric power.*

Mathematically : $P = VI = \frac{V^2}{R} = I^2 R$

Unit. In SI, the unit of electric power is **watt (W).**

The electric power of a circuit is said to be one watt, if one ampere of current flows through it, when a constant potential difference f one volt is applied across it.

The bigger unit is **kilowatt (kW)** and **megawatt (MW).**

$$\mathbf{1\ kW = 10^3\ W} \quad \text{and} \quad \mathbf{1\ MW = 10^6\ W}$$

☐ **Electric energy.** The total amount of electric energy consumed by an electric circuit depends upon its electric power nd the time for which the circuit is kept on.

Mathematically : $W = P \times t = VIt = \frac{V^2}{R}t = I^2 Rt$

In SI, the unit of electric energy is **joule (J)** but the practical unit of electric energy is **kilowatt hour (kWh).** It is also nown as **Board Of Trade unit (B.O.T.).**

$$W \text{ (in kWh)} = \frac{V \text{ (in volt)} \times I \text{ (in ampere)} \times t \text{ (in hour)}}{1000}$$

Also, $\mathbf{1\ kWh = 3{\cdot}6 \times 10^6\ J}$

Some Useful Facts

1. The heat produced in a conductor on passage of current does not depend upon the direction of flow of current.

2. The fuse wire for an electric circuit is chosen keeping in view the value of safe current through the circuit. The material of fuse wire should have high resistivity and low melting point. However, the melting point of the material of fuse wire should e above the temperature that will be reached on the passage of the safe current through the circuit.

3. The material of the filament of a heater should have high resistivity and high melting point.

Chapter 4. Chemical effects of current

☐ **Electrolysis.** *The process of dissociation of a liquid into ions on passing current through it is called electrolysis.*

☐ **Electrolyte.** *The liquids (or solutions) which allow current through them by dissociating into ions are called electrolytes.*

☐ **Ion.** *An atom, a group of atom or radical having charge is called ion.*

The ions are of two types namely ***cations*** and ***anions***.

Cations are the positive ions and collect at cathode during electrolysis.

Anions are the negative ions and collect at anode during electrolysis.

☐ **Voltameter.** *A vessel containing two electrodes and the electrolyte for carrying out electrolysis is called voltameter.*

☐ **Faraday's first law of electrolysis.** *The mass of substance deposited at the cathode during electrolysis is directly proportional to the total charge passing through the electrolyte.*

Mathematically : $m = zq = zIt$

Here, z is called ***electrochemical equivalent*** of the substance.

☐ **Faraday's second law of electrolysis.** *If same quantity of electricity is passed through different electrolytes, mass of the different substances deposited at the respective cathodes are proportional to their chemical equivalents.*

Mathematically : $\frac{m}{E} = \text{constant}$

Here, E is chemical equivalent (ratio of atomic mass to the valency) of the substance.

For given two substances of chemical equivalents E_1 and E_2, we have

$$\frac{m_1}{m_2} = \frac{E_1}{E_2} = \frac{z_1}{z_2},$$

where letters have their usual meanings.

☐ **Electrochemical equivalent of a substance.** *It is defined as the mass of ions deposited at the cathode, when one coulomb of charge passes through the electrolyte.*

The practical unit of electrochemical equivalent of a substance is **gram coulomb^{-1} (g C^{-1}).**

☐ **Faraday constant.** *It is the quantity of charge required to liberate one gram equivalent of the substance.*

1 faraday (F) = 96500 C mol^{-1}

It has a fixed value.

☐ **Electrochemical cell.** *An arrangement in which chemical reaction can be carried out at a steady rate so as to convert chemical energy into electrical energy is called an electrochemical cell.*

☐ **Primary cell.** *An electrochemical cell which cannot be recharged but the chemicals have to be replaced is called a primary cell.*

☐ **Secondary cell.** *An electrochemical cell in which the chemicals of the cell can be recharged again and again is called a secon cell.*

☐ **Storage capacity of a cell.** *It is the ability of a cell to supply electrical energy.*

Unit. It is measured in **ampere hour (A h).**

Storage capacity of a cell is called one ampere hour, if it can supply one ampere of current for one hour.

Some Useful Facts

1. Usually a liquid is not a conductor of electricity. However, mercury is conductor of electricity and it does not dissoc into ions on passage of electricity through it.

2. Water molecule is polar in nature *i.e.* it possesses permanent electric dipole moment. Further, water molecules h tendency to dissociate. Because of these two facts, water is highly polarizable medium and it has a large value of dielec constant ($K \approx 80$).

3. The crystalline NaCl consists of NA^+ and Cl^- ions. The two ions are held together due to electrostatic force of attract When NaCl is dissolved in water, the electrostatic force of attraction between Na^+ and Cl^- ions is reduced by a factor o (equal to dielectric constant of water). Due to thermal agitation at room temperature, the NaCl molecules in the solutio NaCl, dissociate into free Na^+ and Cl^- ions.

4. (a) *Electrolysis of $CuSO_4$* :

At the cathode : $Cu^{++} + 2e^- \longrightarrow Cu$

At the anode : $Cu \longrightarrow Cu^{++} + 2e^-$

(b) *Electrolysis of $AgNO_3$* :

At the cathode : $Ag^+ + e^- \longrightarrow Ag$

At the anode : $Ag \longrightarrow Ag^+ + e^-$

(c) *Electrolysis of water* :

At the cathode : $4\,H_2O + 4\,e^- \longrightarrow 4\,OH^- + 2\,H_2$

At the anode : $2\,H_2O \longrightarrow 4\,H^+ + 4\,e^- + O_2$

$4\,H^+ + 4\,OH^- \longrightarrow 4\,H_2O$

The net reaction : $2H_2O \longrightarrow 2\,H_2 + O_2$

5. Faraday's second law suggests a relation between electric charge and valency. It leads to the conclusion that the an elementary unit of charge e, which is common to all substances so that all the charges are integral multiples of e.

6. Chemical equivalent, $E = \dfrac{M}{p}$, where M is atomic mass and p is valency of the substance.

7. The total charge required to deposit mass of a substance equal to its atomic mass is equal to Npe, where N is Avoga number and p is valency. Thus,

$$M = z\,(N\,p\,e) \qquad \text{or} \qquad z = \frac{M}{N\,p\,e}$$

8. Faraday constant, $F = N\,e$

9. Electrochemical equivalent of a substance, $z = \dfrac{E}{F}$

10. Faraday's first law may be mathematically expressed as

$$m = \frac{M}{N\,p\,e}\,I\,t = \frac{E}{F}\,I\,t$$

11. Leclanche cell is used in experiments, where constant supply of electric current is not needed. It is because, as cell is used, hydrogen gas causes polarisation of the carbon rod and its e.m.f. falls. But when it is used in experiments requi intermittent current, hydrogen gas escapes (when the cell is not in use) and the cell regains its original e.m.f.

12. A storage cell should not be short circuited. It is because ; during short circuiting, a large current flows through cell and a non-porus layer of $PbSO_4$ is formed. This layer does not decompose during subsequent recharging and the becomes useless.

13. A series resistance is put, while charging a storage cell. It is done so that the cell may not get damaged due to he charging current.

Chapter 5. Thermoelectric effect

☐ **Thermocouple.** *The arrangement of two wires of different materials joined at their ends so as to form junctions is call thermocouple.*

☐ **Seebeck effect.** *The phenomenon of production of e.m.f. causing a current to flow in a thermocouple, when its two junct are kept at different temperatures, is known as Seebeck effect.*

☐ **Thermoelectric series.** The metals forming the thermoelectric series are :

Sb, As, Fe, Zn, Ag, Au, Mo, Cr, Sn, Pb, Hg, Mn, Cu, Pt, Co, Ni and Bi.

1. If a thermocouple is formed of wires of any two metals from the above series, the direction of current is from a metal occurring earlier in the series to the metal occurring latter in the series.

2. More the two metals (used for forming a thermocouple) are separated in the series, the greater is the thermo- e.m.f. produced in the thermocouple.

☐ **Neutral temperature.** *The temperature of the hot junction, at which the thermo-e.m.f. produced in the thermocouple becomes maximum, is called neutral temperature* θ_n.

For a given thermocouple, it is a fixed temperature.

☐ **Temperature of inversion.** *The temperature of the hot junction, at which the thermo e.m.f. reverses its direction, is called temperature of inversion* θ_i.

For a given thermocouple, it changes with temperature of the cold junction (θ_c).

However, *the temperature of inversion is as much above the neutral temperature, as the neutral temperature is above the temperature of the cold junction.*

Mathematically : $\theta_n = \dfrac{\theta_i + \theta_c}{2}$

☐ **Thermoelectric power.** *The rate of change at thermo-e.m.f. with the change in temperature of the hot junction is called thermoelectric power.*

Mathematically : Thermoelectric power $= \dfrac{dE}{d\theta}$

☐ **Laws of Thermoelectricity.** The following two laws have been established experimentally :

1. Law of intermediate metals. *It states that if a number of metals are put between two metals so as to form a chain, then the effective thermo-e.m.f. between the two extreme metals is equal to the sum of the e.m.fs. between intermediate metals, provided all the junctions are at the same temperature.*

Mathematically : If a metal B is put between two metals A and C, then

$$E_A^C = E_A^B + E_B^C$$

2. Law of intermediate temperatures. *It states that the thermo-e.m.f. for a green thermocouple having its junctions at temperatures* θ_1 *and* θ_n *is equal to the sum of the e.m.fs. of the thermocouples of the same metals with junctions at temperatures* θ_1 *and* θ_2 ; θ_2 *and* θ_3 ; θ_3 *and* θ_4 ; .... θ_{n-1} *and* θ_n.

Mathematically : $E_{\theta_1}^{\theta_n} = E_{\theta_1}^{\theta_2} + E_{\theta_2}^{\theta_3} + E_{\theta_3}^{\theta_4} + \ldots\ldots\ldots + E_{\theta_{n-1}}^{\theta_n}$

☐ **Peltier effect.** *The absorption or evolution of heat at a junction of two dissimilar metals, when current is passed, is known as Peltier effect.*

☐ **Peltier coefficient.** *It is defined as the amount of heat energy absorbed or evolved per second at a junction, when a unit current is passed through it.*

☐ **Thomson effect.** *The absorption or evolution of heat along the length of a conductor, when current is passed through the conductor, whose one end is hot and other is kept cold, is known as Thomson effect.*

☐ **Thomson coefficient.** *It is defined as the amount of heat energy absorbed or evolved per second between two points of a conductor having a unit temperature difference, when a unit current is passed, through it.*

☐ **Thermopile.** *An arrangement of a large number of thermocouples connected in series is called a thermopile.*

Some Useful Facts

1. Seebeck effect is reversible in nature. If current is passed through a thermocouple in a direction opposite to the direction of flow of current in the thermocouple, then heat is evolved at the hot junction and absorbed at the cold junction.

2. For given temperatures of cold and hot junctions, Sb-Bi thermocouple produces maximum thermo-e.m.f. It is because, Sb and Bi are the most separated metals in the thermoelectric series.

3. The graph between thermo-e.m.f. (E) and the temperature (θ) of the hot junction is parabolic in nature and is found to satisfy the equation

$$E = \alpha\,\theta + \frac{1}{2}\beta\,\theta^2,$$

4. At neutral temperature, theromo-e.m.f. is maximum *i.e.* $\dfrac{dE}{d\theta} = 0$

It leads to the relation,

$$\theta_n = -\frac{\alpha}{\beta}$$

5. Thermoelectric power is also called ***Seeback coefficient.***

7. Peltier effect and Thomson effect are also reversible in nature.

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Electric current, Resistance and E.M.F.

Problem 1.01. A hollow copper tube of 1 metre length has got external diameter equal to 10 cm and its walls are 5 mm thick. Find its resistance if the specific resistance of copper is $1{\cdot}7 \times 10^{-8}\ \Omega$ m. *(I.S.M. Dhanbad, 1982)*

Sol. Here, $l = 1$ m ; $\rho = 1{\cdot}7 \times 10^{-8}\ \Omega$ m

External radius of hollow tube,

$$r_2 = \frac{10}{2} = 5 \text{ cm} = 0{\cdot}5 = 0{\cdot}05 \text{ m}$$

Internal radius of hollow tube, $r_1 = r_2 -$ thickness of tube

Since thickness of tube is 5 mm *i.e.* 0·005 m ;

$$r_1 = 0{\cdot}005 - 0{\cdot}005 = 0{\cdot}045 \text{ m}$$

Area of cross-section of the tube

$$= \pi\left(r_2^2 - r_1^2\right) = \pi\left(0{\cdot}05^2 - 0{\cdot}045^2\right)$$

The resistance of copper tube,

$$R = \rho = \frac{l}{A} = 1{\cdot}7 \times 10^{-8} \times \frac{1}{\pi\,(0{\cdot}05^2 - 0{\cdot}045^2)}$$

$$= \mathbf{1{\cdot}031 \times 10^{-5}\ \Omega}$$

Problem 1.02. If a copper wire is stretched to make it 0·1% longer, what is the percentage change in its resistance ? *(M.N.R. 1990 ; I.I.T. 1978)*

Sol. Let the initial length of the wire be l_1 and its area of cross-section be A_1. If ρ is resistivity of the material of the wire, then

initial resistance of the wire, $R_1 = \rho \dfrac{l_1}{A_1}$

On stretching the wire, the increased length of the wire,

$$l_2 = l_1 \times \frac{100{\cdot}1}{100}$$

If A_2 is the new area of cross-section, then final resistance of the wire, $R_2 = \rho \dfrac{l_2}{A_2}$

Since volume of the wire remains same, $l_1 A_1 = l_2 A_2$

$$\therefore\ R_2 = \rho \frac{l_2}{l_1 A_1 / l_2} = \rho \frac{l_2^2}{l_1 A_1}$$

Substituting for l_2, we have

$$R_2 = \rho \frac{(l_1 \times 100{\cdot}1)^2}{(100)^2 \times l_1 A_1} = 1{\cdot}002 \times \rho \frac{l_1}{A_1} = 1{\cdot}002\, R_1$$

Therefore, % increase in resistance, $\dfrac{R_2 - R_1}{R_1} \times 100$

$$= \frac{1{\cdot}002\, R_1 - R_1}{R_1} \times 100$$

$$= (1{\cdot}002 - 1) \times 100 = 0{\cdot}2\%$$

Problem 1.03. A resistance is made by joining two wires of the same material. The radii of the two wires are 1 mm and 3 mm respectively, while their lengths are 3 cm and 5 cm respectively. A battery of e.m.f. 16 V and negligible internal resistance is connected across the resistance. What is the potential drop along the shorter wire ? *(I.I.T. 1970)*

Sol. In Fig. 1.01, AB and BC are two wires of same material connected to a battery of e.m.f. E = 16 V.

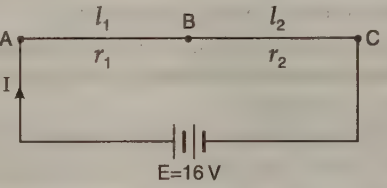


Fig. 1.01

Let ρ be resistivity of the materials of the two wires. If R_1 and R_2 are the resistances of two wires, then

$$R_1 = \rho \frac{l_1}{A_1} = \rho \frac{l_1}{\pi r_1^2}\ ; \text{ and } R_2 = \rho \frac{l_2}{\pi r_2^2}$$

Here, $l_1 = 3$ cm $= 3 \times 10^{-2}$ m, $l_2 = 5$ cm $= 5 \times 10^{-2}$ m, $r_1 = 1$ mm $= 10^{-3}$ m ; $r_2 = 3$ mm $= 3 \times 10^{-3}$ m

$$\therefore\quad R_2 = \rho \frac{3 \times 10^{-2}}{\pi \times (10^{-3})^2} = \frac{3\rho}{\pi} \times 10^{-4}\ ;$$

$$R_2 = \rho \frac{5 \times 10^{-2}}{\pi \times (3 \times 10^{-3})^2} = \frac{5\rho}{9\pi} \times 10^{-4}$$

Total resistance of wire AC, $R = R_1 + R_2$

$$= \frac{3\rho}{\pi} \times 10^4 + \frac{5\rho}{9\pi} \times 10^4 = \frac{32\rho}{9\pi} \times 10^4$$

If I is current through the circuit, then

$$I = \frac{E}{R} = \frac{16 \times 9\pi}{32\rho \times 10^4} = \frac{9\pi}{2\rho} \times 10^{-4}$$

Therefore, potential difference across shorter wire,

$$I R_1 = \frac{9\pi}{2\rho} \times 10^{-4} \times \frac{3\rho}{\pi} \times 10^{-4} = \mathbf{13{\cdot}5\ V}$$

Problem 1.04. An electric current of 5 A is divided into 3 parallel branches, in which the lengths of wire are in the ratio 2 : 3 : 4 and the diameters 3 : 4 : 5. Find the currents in each branch, if the wires are of the same material. *(I.I.T. 1975)*

Sol. Let the lengths of the wires in three branches be $2l$, $3l$ and $4l$ respectively and their diameter be 3 D, 4 D and 5 D respectively [Fig. 1.02].

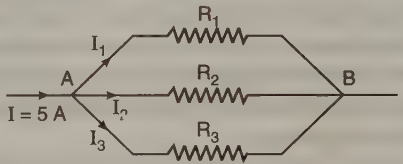


Fig. 1.02.

If ρ is resistivity of the material of the three wires, then

$$R_1 = \rho \frac{2l}{\pi\,(3D)^2/4} = \frac{8\pi l}{9\pi D^2}$$

$$R_2 = \rho \frac{2l}{\pi\,(4D)^2/4} = \frac{3\pi l}{4\pi D^2}$$

and $$R_3 = \rho \frac{4l}{\pi\,(5D)^2/4} = \frac{16\pi l}{25\pi D^2}$$

Let I_1, I_2 and I_3 be the currents through the three ranches as shown in Fig. 2.02. Then,

$$I_1 + I_2 + I_3 = 5 \qquad \ldots(i)$$

As the three wires are connected between the same two oints A and B, we have

$$I_1 R_1 = I_2 R_2 = I_3 R_3 = V \text{ (say)}$$

Then, $I_1 = \dfrac{V}{R_1} = \dfrac{9\pi D^2 V}{16\rho l}$; $I_2 = \dfrac{V}{R_2} = \dfrac{4\pi D^2 V}{3\rho l}$

and $I_3 = \dfrac{V}{R_3} = \dfrac{25\pi D^2 V}{16\rho l}$

Substituting for I_1, I_2 and I_3 in equation (*i*), we have

$$\frac{9\pi D^2 V}{8\rho l} + \frac{4\pi D^2 V}{3\rho l} + \frac{25\pi D^2 V}{16\rho l} = 5$$

or $$\frac{\pi D^2 V}{\rho l}\left(\frac{9}{8} + \frac{4}{3} + \frac{25}{16}\right) = 5$$

or $$\frac{\pi D^2 V}{\rho l} \times \frac{193}{48} = 5$$

or $$\frac{\pi D^2 V}{\rho l} = \frac{240}{193}$$

$\therefore$ $$I_1 = \frac{9\pi D^2 V}{8\rho l} = \frac{9}{8} \times \frac{240}{193} = \mathbf{1{\cdot}40\ A},$$

$$I_2 = \frac{4\pi D^2 V}{3\rho l} = \frac{4}{3} \times \frac{240}{193} = \mathbf{1658\ A}$$

and $$I_3 = \frac{25\pi D^2 V}{16\rho l} = \frac{25}{16} \times \frac{240}{193} = \mathbf{1{\cdot}943\ A}$$

Problem 1.05. The resistances of the four sides of a uare ABCD are 10 Ω, 5 Ω, 7 Ω and 2 Ω in that order. A sistance of 10 Ω is connected in between A and C. alculate the equivalent resistance between A and B. *(I.S.M. Dhanbad, 1984)*

Sol. The given resistances are connected as shown in Fig. 03.

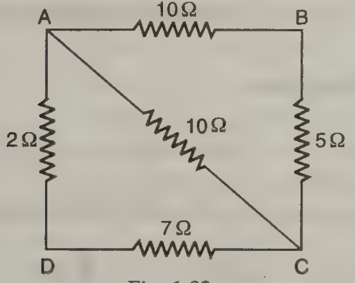


Fig. 1.03

Consider the part ADC of the network of the resistances. follows that the resistances AD and DC are connected in ries. Therefore, their effective resistance,

$$R_1 = 2 + 7 = 9\ \Omega$$

The resistance of path ADC and resistance AC are in rallel. Therefore, their effective resistance,

$$R_2 = \frac{R_1 \times 100}{R_1 + 10} = \frac{9 \times 10}{9 + 10} = \frac{90}{19}\ \Omega$$

The effective resistance R_2 of part ADC and resistance BC are in series. Therefore, their effective resistance,

$$R_3 = R_2 + 5 = \frac{90}{19} + 5 = \frac{185}{19}\ \Omega$$

The effective resistance R_3 and resistance AB are in parallel. Therefore, equivalent resistance of the network,

$$R = \frac{R_3 \times 10}{R_3 + 10} = \frac{\frac{185}{19} \times 10}{\frac{185}{19} + 10} = \mathbf{4{\cdot}933\ \Omega}$$

Problem 1.06. Calculate the steady state current in the 2 Ω resistor shown in the circuit in Fig. 1.04. The internal resistance of the battery is negligible and the capacitance of the capacitor C is 0·2 microfarad. *(I.I.T. 1982)*

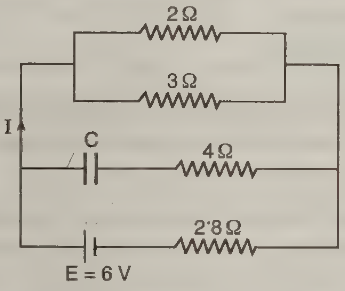


Fig. 1.04

Thought Process

In steady state, a capacitor offers infinite resistance to d.c. and it acts as open path.

Sol. The effective resistance of the parallel combination of 2 Ω and 3 Ω resistors is given by

$$R' = \frac{2 \times 3}{2 + 3} = 1{\cdot}2\ \Omega$$

Since capacitor offers infinite resistance to d.c., it acts as open path. Therefore, current cannot flow through the branch containing the capacitor and the 4 Ω resistance and hence the given network may be redrawn as shown in Fig. 1.05.

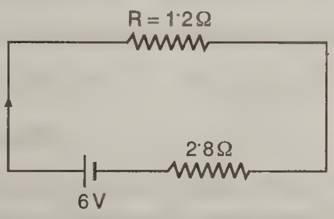


Fig. 1.05

Therefore, total resistance of the circuit,

$$R = R' + 2{\cdot}8 = 1{\cdot}2 + 2{\cdot}8 = 4\ \Omega$$

Current drawn from the cell, $I = \dfrac{E}{R} = \dfrac{6}{4} 1{\cdot}5\ A$

Potential difference across R′(the parallel combination of 2 Ω and 3 Ω resistance),

$$V = I R' = 1{\cdot}5 \times 1{\cdot}2 = 1{\cdot}8\ V$$

Therefore, current through 2 Ω resistor

$$= \frac{V}{2} = \frac{1{\cdot}8}{2} = 0{\cdot}9\ A$$

Problem 1.07. An infinite ladder network of resistances is constricted with 1 Ω and 2 Ω resistances as

shown in Fig. 2.06. The 6 V battery between A and B has negligible internal resistance.

(*a*) Show that the effective resistance between A and B is 2 Ω.

(*b*) What is the current that passes through the 2 Ω resistance nearest to the battery ? (*Roorkee 1989 ; I.I.T. 1987*)

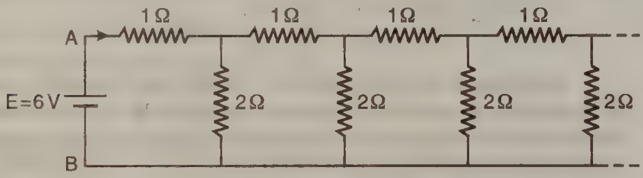


Fig. 1.06

Thought Process

As the network of resistances is infinite, adding one more set of the resistances of 1 Ω and 2 Ω will not affect the total resistance of the network.

Sol. (*a*) Let the equivalent resistance of the infinite ladder network of the resistances be X. Since the network is infinite, adding one more set of resistances of 1 Ω and 2 Ω across the terminals A and B will not affect the total resistance X of the network. The network on adding one set of two resistances of 1 Ω and 2 Ω will appear as shown in Fig. 1.07.

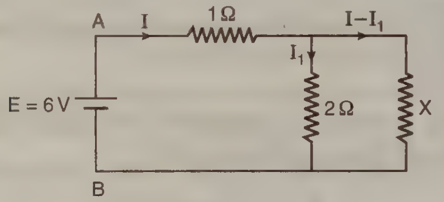


Fig. 1.07

Now, total resistance across A and B

= 1 + resistance equivalent to the parallel combination of 2 Ω and X

$$= 1 + \frac{2\,X}{2+X}$$

Since total resistance of the network remains unaffected,

$$1 + \frac{2\,X}{2+X} = X \qquad \text{or} \qquad X_2 - X - 2 = 0$$

or $\qquad X = -1\,\Omega \qquad$ or $\qquad 2\,\Omega$

Since X cannot be equal to – 1 Ω, we have

$$X = 2\,\Omega$$

(*b*) If I is current from the battery, then

$$I = \frac{E}{X} = \frac{6}{2} = 3\text{ A}$$

Suppose a part I_1 of the current flows throug resistance and the remaining part $(I - I_1)$ passes thro resistance X. Then,

$$2 \times I_1 = (I - I_1)\,X \text{ or } 2 \times I_1 = (3 - I_1) \times 2$$

or $\qquad I_1 = 1{\cdot}5$ A

Problem 1.08. 12 cells each having the same e.m. connected in series and are kept in a closed box. Some c cells are wrongly connected. This battery is connect series with an ammeter and two cells identical wit others. The current is 3 A , when the cells and batter each other and 2 A, when the cells and battery oppose other. How many cells in the battery are wrongly connec

(*Roorkee, 1993 ; I.I.T.*

Sol. Let e.m.f. of each cell be E. If *n* cells out of 1 wrongly connected, then (12–*n*) cells will send current i direction, while *n* cells in opposite direction. Therefore

net e.m.f. of the battery=$(12-n)\,E - n\,E = (12 - 2$

When the two cells aid the battery :

Then, total e.m.f. of the circuit = $(12 - 2\,n)\,E + 2$

Let R be the resistance of the circuit. Then,

$$\text{current in the circuit, } \frac{(12-2\,n)\,E + 2\,E}{R} = 3$$

When the two cells oppose the battery :

Total e.m.f. of the circuit = $(12 - 2\,n)\,E - 2\,E$

$$\text{Then, current in the circuit, } \frac{(12-2\,n)\,E - 2\,E}{R} = 2$$

Dividing equation (*i*) by (*ii*), we have

$$\frac{(12-2\,n)\,E+2\,E}{(12-2\,n)\,E-2\,E} = \frac{3}{2} \text{ or } \frac{(12-2\,n)+2}{(12-2\,n)-2} = \frac{3}{2}$$

Solving for *n*, we get

$$n = 1$$

Hence, **one cell has been wrongly connected.**

Chapter 2. Electrical Measurements

Problem 2.01. Three resistors are joined to form a triangle ABC, such that AB = 1 Ω, BC = 2 Ω and CA = 3 Ω. A cell of e.m.f. 3 V and internal resistance 1 Ω is connected to points A and C. Determine the current in the three arms and also the potential difference at the cell terminals.

(*I.S.M. Dhanbad, 1986*)

Sol. The three resistors of 1Ω, 2 Ω and 3 Ω are connected to form a triangle ABC as shown in Fig. 2.01.

The cell of e.m.f. E = 2 V and internal resistance *r* = 1 Ω are connected across points A and C. Since resistance of path ABC is 1 + 2 *i.e.* 3 Ω, it easily follows that the resistance of the triangular net work ABC,

$$R = \frac{3 \times 3}{3+3} = 1{\cdot}5\,\Omega$$

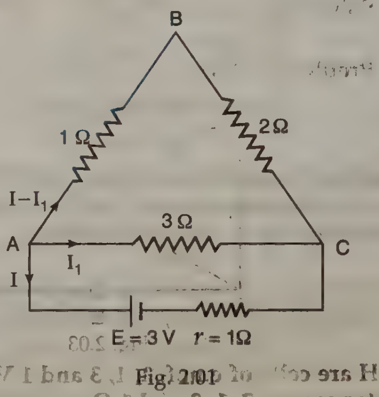


Fig. 2.01

If I is current from the cell, then

$$I = \frac{E}{R + r} = \frac{3}{1{\cdot}5 + 1} = 1{\cdot}2\ A$$

Suppose that part I_1 of the current flows through arm AC, so that part $(I - I_1)$ flows along the path ABC. Obviously,

$I_1 \times$ resistance of path AC

$= (I - I_1) \times$ resistance of path ABC

or $\quad I_1 \times 3 = (1{\cdot}2 - I_1) \times 3$

or $\quad I_1 = 0{\cdot}6\ A$

Therefore, current through arm AC, $I = 0{\cdot}6\ A$

Also, the current through the arms AB and BC,

$$I - I_1 = 1{\cdot}2 - 0{\cdot}6 = 0{\cdot}6\ A$$

Potential difference at the cell terminals,

$$V = E - I r = 3 - 1{\cdot}2 \times 1 = 1{\cdot}8\ V$$

Problem 2.02. A 10 V battery having an internal resistance of 1 Ω is joined in parallel with another of 20 V and internal resistance 2 Ω. The combination is placed across an external resistance of 30 Ω. Apply Kirchhoff's laws, find the current flowing through each battery.

(I.S.M. Dhanbad, 1993)

Sol. The given two batteries of e.m.fs. $E_1 = 10$ V and $E_2 = 20$ V are connected in parallel to an external resistance R = 30 Ω as shown in Fig. 2.02. Their internal resistances are $r_1 = 1 Ω$ and $r_2 = 2\ Ω$ respectively. Let I_1 and I_2 be currents due to the batteries E_1 and E_2 respectively.

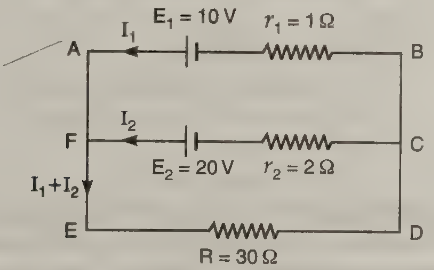


Fig. 2.02.

For the closed part ABDEA :

$$E_1 = I_1 r_1 + (I_1 + I_2) R$$

or $\quad 10 = I_1 \times 1 + (I_1 + I_2) \times 30$

or $\quad 31\ I_1 + 30\ I_2 = 10 \qquad ...(i)$

For the closed part CDEFC :

$$E_2 = I_2 r_2 + (I_1 + I_2) R$$

or $\quad 10 = I_2 \times 2 + (I_1 + I_2) \times 30$

or $\quad 15\ I_1 + 16\ I_2 = 10 \qquad ...(ii)$

Solving equations (*i*) and (*ii*), we have

$$I_1 = -\,3{\cdot}043\ A,\ I_2 = 3{\cdot}478\ A$$

Problem 2.03. In the circuit shown in Fig. 2.03, E, F, G and H are cells of e.m.f. 2, 1, 3 and 1 V and their internal resistances are 2, 1, 3 and 1 Ω respectively. Calculate (*a*) the potential difference between B and D and (*b*) the potential difference across the terminals of each of the cells G and H.

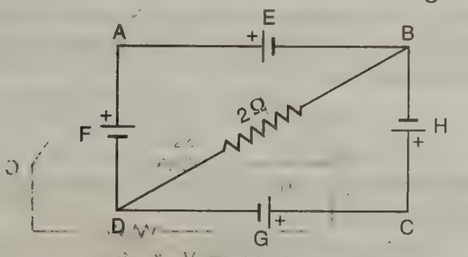


Fig. 2.03

(I.I.T. 1984)

Thought Process

In a circuit, when the current flows through a cell from its positive to negative terminal, the potential difference across its internal resistance aids the e.m.f. of the cell. On the other hand, when the current flows from its negative to positive terminal, the potential difference across its internal resistance opposes the e.m.f. of the cell.

Sol. Let E_1, E_2, E_3 and E_4 be e.m.fs. and r_1, r_2, r_3 and r_4 be internal resistances of the four cells E, F, G and H respectively.

The given circuit can be redrawn after incorporating the internal resistances of the cells as shown in Fig. 2.04. The distribution of currents in various branches of the circuit has also been shown in the figure.

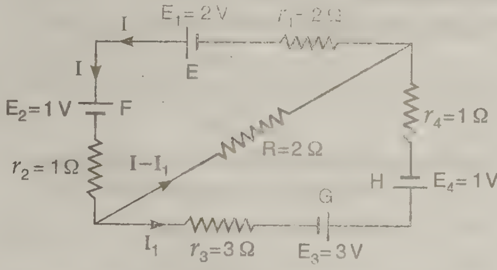


Fig. 2.04

For the closed part ABDA :

$$E_1 - E_2 = I r_1 + I r_2 + (I - I_1) R$$

or $\quad 2 - 1 = I \times 2 + I \times 1 + (I - I_1) \times 2$

or $\quad 5\,I - 2\,I_1 = 1 \qquad ...(i)$

For the closed part BCDB :

$$E_1 - E_2 = I r_1 + I_1 r_1 + (I - I_1) R$$

or $\quad 3 - 1 = I_1 \times 3 + I_1 \times 1 - (I - I_1) \times 2$

or $\quad -I + 3\,I_1 = 1 \qquad ...(ii)$

Solving the equations (*i*) and (*ii*), we obtain

$$I_1 = \frac{6}{13}\ A \text{ and } I = \frac{5}{13}\ I$$

(*a*) Potential difference between points D and B

$$= (I - I_1) R = \left(\frac{5}{13} - \frac{6}{13}\right) \times 2 = -\frac{2}{13}\ V$$

Therefore, potential difference between points B and D

= – (potential difference between points D and B)

$$= -\left(-\frac{2}{13}\right) = \frac{2}{13}\ V$$

(*b*) The current I_1, flows through cell G from its negative to positive terminal potential. Therefore, potential difference across the terminals of cell G,

$$V_G = E_3 - I_1 r_3 = 3 = \frac{6}{13} \times 3 = \frac{21}{13}\ V$$

The current I_1 flows through cell H from its positive to negative terminal. Therefore, potential difference across the terminals of cell H,

$$V_H = E_4 + I_1 r_4 = 1 + \frac{6}{13} \times 1 = \frac{19}{13}\ V$$

Problem 2.04. In the circuit shown in Fig. 2.05, $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V and the resistances $R = r_1 = r_2 = r_3 = 1\,\Omega$

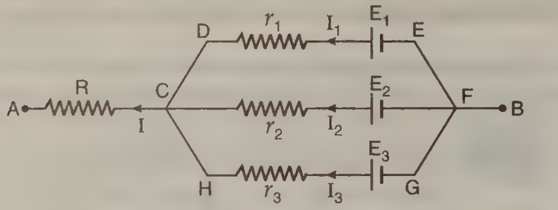


Fig. 2.05

(*a*) Find the potential difference between the points A and B and the current through each branch.

(*b*) If r_2 is short circuited and the point A is connected to the point B, find the currents through E_1, E_2 and E_3 and the resistor R. (*I.I.T. 1981*)

Sol. (*a*) Let I_1, I_2 and I_3 be currents from the cells of e.m.f. $I_1 = 3$ V, $E_2 = 2$ V and $E_3 = 1$ V respectively.

Further, $R = r_1 = r_2 = r_3 = 1\,\Omega$

In the closed part CDEFC of the circuit :

$$I_1 r_1 - I_2 r_2 = E_1 - E_2$$

or $$I_1 \times 1 - I_2 \times 1 = 3 - 2$$

or $$I_1 - I_2 = 1 \qquad \text{...}(i)$$

In the closed part CDEFGHC of the circuit :

$$I_1 r_1 - I_3 r_3 = E_1 - E_3$$

or $$I_1 \times 1 - I_3 \times 1 = 3 - 1$$

or $$I_1 - I_3 = 2 \qquad \text{...}(ii)$$

Since the other end A of resistance R is not connected to point B, no current flows through R *i.e.* I = 0. Therefore, at point C, we have

$$I_1 + I_2 + I_3 = 0 \qquad \text{...}(iii)$$

Solving equations (*i*), (*ii*) and (*iii*), we have

$$I_1 = 1\text{ A};\ I_2 = 0 \text{ and } I_3 = -1\text{ A}$$

The negative value of I_3 indicates that current I_3 does not flow along the indicated path FGHC, but flows along the path CHGF.

The potential difference between points C and F can be calculated across any one of the three paths between C and F. Since $I_2 = 0$,

potential difference between points C and F

$$= E_2 - I_2 r_2 = 2 - 0 \times 1 = 2\text{ V}$$

As there is no current in branches AC and FB,

potential difference between A and B

= potential difference between C and F = **2 V**

(*b*) When resistance r_2 is short circuited and point A is joined to B, the given circuit becomes of the form as shown in Fig. 2.06. The distribution of current is shown in the figure.

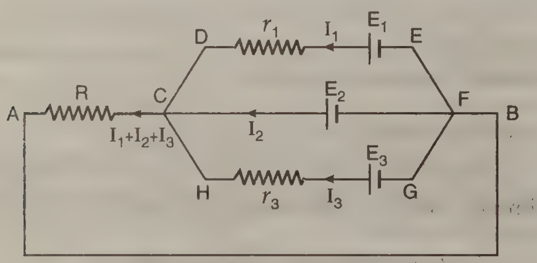


Fig. 2.06

For the closed part CDEFC of the circuit :

$$I_1 r_1 = E_1 - E_2$$

or $$I_1 \times 1 = 3 - 2$$

or $$I_1 = 1\text{ A}$$

For the closed part CFGHC of the circuit :

$$-I_3 r_2 = E_2 - E_3$$

or $$-I_3 \times 1 = 2 - 1$$

or $$I_3 = -1\text{ A}$$

For the closed part ACDEFBKJA of the circuit :

$$I_1 r_1 + (I_1 + I_2 + I_3) R = E_1$$

or $$1 \times 1 + (1 + I_2 - 1) \times 1 = 3$$

or $$I_2 = 2\text{ A}$$

The current through resistor R

$$= I_1 + I_2 + I_3 = 1 + 2 + (-1) = 2\text{ A}$$

Problem 2.05. An electrical circuit is shown in Fig. 2.07. Calculate the potential difference across the resistor of 400 Ω as will be measured by the voltmeter V of resistance 400 Ω, either by applying Kirchhoff's rules or otherwise. (*I.I.T. 1996*)

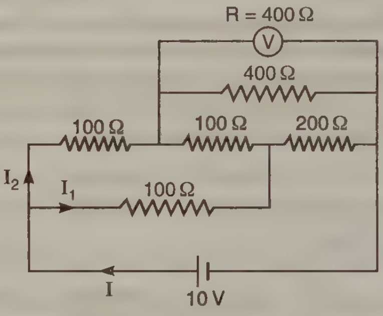


Fig. 2.07

Sol. The parallel combination of the resistance of 400 Ω and the voltmeter of resistance 400 Ω connected across it is equivalent to a resistance of 200 Ω. The given circuit is equivalent to the circuit redrawn as in Fig. 2.08.

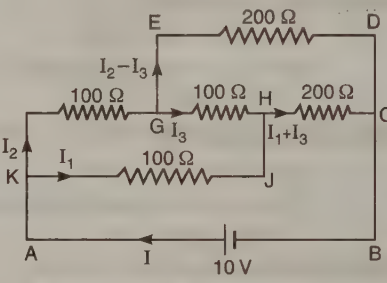


Fig. 2.08

The currents in the various branches of the circuit will be distributed in accordance with Kirchhoff's first law.

In the closed part FGHJKF of the circuit : According to Kirchhoff's second law,

$$I_2 = 100 + I_3 \times 100 - I_1 \times 100 = 0$$

or $$I_1 - I_2 - I_3 = 0 \qquad \text{...}(i)$$

In the closed part ABCHJKA of the circuit : According to Kirchhoff's second law,

$$10 = I_1 \times 100 + (I_1 + I_3) \times 200$$

or $$3 I_1 + 2 I_3 - 0{\cdot}1 = 0 \qquad \text{...}(ii)$$

In the closed part CDEGHC of the circuit : According to Kirchhoff's second law,

$$(I_2 - I_3) \times 200 - (I_1 + I_3) \times 200 - I_3 \times 100 = 0$$

or $I_1 - I_2 + 2{\cdot}5\, I_3 = 0$ ...(*iii*)

Adding equations (*i*) and (*iii*), we have

$$(I - I_2 - I_3) + (I_1 - I_2 + 2{\cdot}5\, I_3) = 0$$

or $1{\cdot}5\, I_3 = 0$ or $I_3 = 0$

Setting $I_3 = 0$ in equation (*ii*), we have

$$2\, I_1 + 2 \times 0 - 0{\cdot}1 = 0 \text{ or } I_1 = \frac{0\cdot1}{3} = \frac{1}{30}\text{ A}$$

Setting $I_1 = \frac{1}{30}$ A and $I_3 = 0$ in equation (*i*), we have

$$\frac{1}{30} - I_2 - 0 = 0 \text{ or } I_2 = \frac{1}{30}\text{ A}$$

The potential difference (measured by voltmeter across 400 Ω resistance of the circuit shown in Fig. 2.07) is equal to potential difference across 200 Ω resistance (in branch DE of the circuit shown in Fig. 2.08). It is given by

$$V = (I_2 - I_3) \times 200 = \left(\frac{1}{30} - 0\right) \times 200 = \mathbf{6{\cdot}67\ V}$$

Problem 2.06. A part of the circuit in a steady state along with the currents flowing in the branches, the values of resistances, etc are shown in the Fig. 2.09. Calculate the energy stored in the capacitor. (*I.I.T. 1986*)

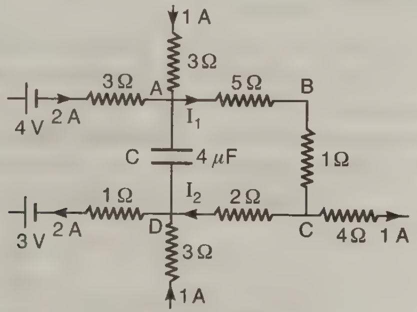


Fig. 2.09

Thought Process

The capacitor offers infinite resistance to d.c. and therefore the branch containing capacitor acts as open path. However, there is potential difference across the two plates of the capacitor and charge is stored inside the capacitor.

Sol. Let I_1 and I_2 be currents in the branches AB and CD of the circuit in the directions as shown in Fig. 2.09. According to Kirchhoff's first law,

at point A : $2 + 1 - I_1 = 0$ or $I_3 = 3$ A

at point D : $1 + 1 + I_2 - 2 = 0$ or $I_2 = 1$ A

For closed part ABCDA of the circuit : Let V be potential difference across the two plates of the capacitor. Then, according to Kirchhoff's second law,

$$I_1 \times 5 + I_1 \times 1 + I_2 \times 2 = V$$

or $V = 3 \times 5 + 3 \times 1 + 1 \times 2 = 20$ V

Now, capacitance of the capacitor,

$$C = 4\,\mu F = 4 \times 10^{-6}\text{ F}$$

Therefore, energy stored in capacitor,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (20)^2 = \mathbf{8 \times 10^{-4}\ J}$$

Problem 2.07. In the circuit shown in Fig. 2.10, $E_1 = 3\ E_2 = 2\ E_3 = 6$ V '; C = 5 μF ; $R_1 = 2\ R_2 = 6\ \Omega$; $R_3 = 2\ R_4 = 4\ \Omega$. Find the current in R_3 and the energy stored in the capacitor. (*I.I.T. 1988*)

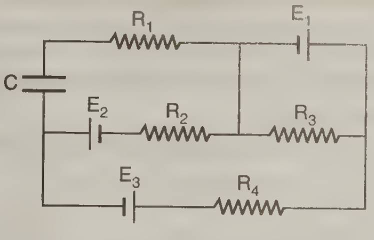


Fig. 2.10

Thought Process

In steady state, a capacitor offers infinite resistance to d.c. and the branch containing the capacitor acts as open path. Therefore, no current flows through the branch of the circuit containing C and R_1 and there is no potential drop across resistance R_1. Hence, the potential difference across the capacitor is equal to potential difference between points A and C.

Sol. The distribution of currents in the various branches of the circuit is as shown in Fig. 2.11.

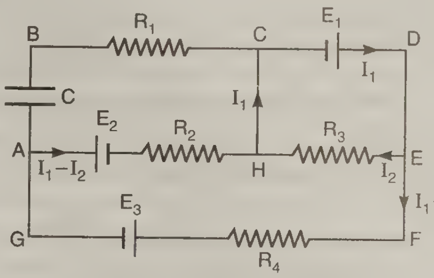


Fig. 2.11

Here, $E_1 = 6$ V ; $E_2 = 2$ V ; $E_3 = 3$ V ;

$C = 5\,\mu F = 5 \times 10^{-6}$ F ;

$R_1 = 6\ \Omega$; $R_2 = 3\ \Omega$, $R_3 = 4\ \Omega$; $R_4 = 2\ \Omega$

In the closed part CDEHC of the circuit : $I_2 R_3 = E_1$

or $I_2 = 4 = 6$ or $I_2 = 1{\cdot}5$ A

Therefore, current through resistance $R_3 = 1{\cdot}5$ A

In the closed part AHEFGA of the circuit :

$$(I_1 - I_2) \times R_2 - I_2 R_3 + (I_1 - I_2) R_4 = -E_2 - E_3$$

or $(I_1 - 1{\cdot}5) \times 3 - 1{\cdot}5 \times 4 + (I_1 - 1{\cdot}5) \times 2 = -2 - 3$

or $I_1 = 1{\cdot}7$ A

Let V_A an V_C be potentials at points A and C respectively. As the current is flowing along the path AHC,

$V_A = V_C$ + potential drop across $R_2 + E_2$

or $V_A - V_C = (I_1 - I_2) R_2 + E_2$

$= (1{\cdot}7 - 1{\cdot}5) \times 2 + 2 = 2{\cdot}4$ V

As the branch ABC contains capacitor, no current flows through R_1 and hence there is no potential difference across R_1. Therefore,

potential difference across capacitor

$= V_A - V_C = 2{\cdot}4$ V

Energy stored in capacitor, $U = \frac{1}{2} C V^2$

$$= \frac{1}{2} \times 5 \times 10^{-6} \times (2{\cdot}4)^2 = \mathbf{1{\cdot}44 \times 10^{-5}\ J}$$

Problem 2.08. A circuit containing a two position switch S is shown in Fig. 2.12. $E_1 = 12$ V, $E_2 = 3$ V, $R_1 = R_2 = R_3 = 2\ \Omega$, $R_4 = 3\ \Omega$, $R_5 = 1\ \Omega$, $C = 2\ \mu F$, $L = 10$ mH.

(a) The switch S is in position 1. Find the potential difference $(V_A - V_B)$ and the rate of production of Joule heat in R_1.

(b) If now the switch S is put in position 2, find the steady state current in R_4. *(I.I.T. 1991)*

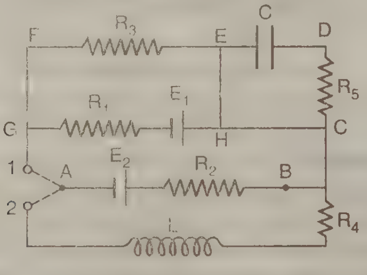


Fig. 2.12

Thought Process

In the steady state, a capacitor offers infinite resistance to d.c. but an inductor offers zero resistance to d.c. Therefore, in the steady state, the capacitor acts as an open path and the inductor as a conducting path.

Sol. (a) When the switch S is in position 1 : In this position of the switch, only the part of the circuit shown in Fig. 2.13 is effective. The distribution of currents is also shown in the figure.

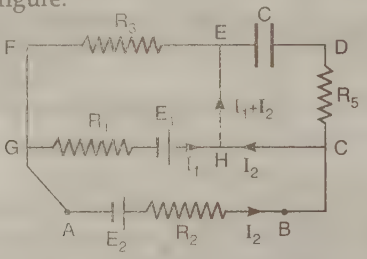


Fig. 2.13

For the closed part ABCHGA of the circuit : According to Kirchhoff's second law,

$$I_2 R_2 - I_1 R_1 = E_2 - E_1$$

or $\quad I_2 \times 2 - I_1 \times 2 = 3 - 12$

or $\quad 2 I_1 - 2 I_2 = 9 \quad \quad ...(i)$

For the closed part EFGHE of the circuit : Applying Kirchhoff's second law,

$$I_1 R_1 - I_1 + (I_1 + I_2) R_3 = E_1$$

or $\quad I_1 \times 2 + (I_1 + I_2) \times 2 = 12$

or $\quad 2 I_1 + I_2 = 6 \quad \quad ...(ii)$

Solving equations (*i*) and (*ii*), we have

$$I_1 = 3{\cdot}5\text{ A} ; I_2 = -1\text{ A}$$

Let V_A and V_B be potentials at points A and B respectively. As the negative terminal of battery E_1 is connected to point A, we have

$$V_A - V_B = -3 + I_2 R_2 = -3 + (-1) \times 2 = \mathbf{-5\ V}$$

Rate of production of heat in $R_1 = I_1^2 R_1$

$$= (3{\cdot}5)^2 \times 2 = \mathbf{24{\cdot}5\ W}$$

(b) When the switch S is in position 2 : In this position of the switch, only the part of the circuit shown in Fig. 2.14 is effective. In the steady state, the ohmic resistance of inductance is zero.

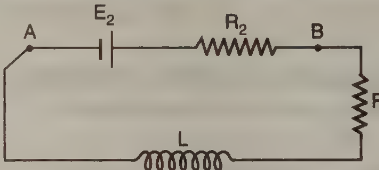


Fig. 2.14

Therefore, total resistance of the circuit in steady state,

$$= R = R_2 + R_4 = 2 + 3 = 5\ \Omega$$

The current in the resistance R_4 will be the same as the current in the circuit in steady state. Therefore,

$$\text{current in resistance } R_4 = \frac{E_2}{R} = \frac{3}{5} = \mathbf{0{\cdot}6\ A}$$

Problem 2.09. In a Wheatstone bridge, the four resistance arms of the bridge are AB = 2 Ω, BC = 3 Ω, CD = 4 Ω and DA = 1Ω. A cell of e.m.f. 2 V and negligible internal resistance is connected across AC and a galvanometer of resistance 10 Ω, is connected between B and D. Find the current through the galvanometer. *(Karnataka, 1992)*

Sol. Let $P = 2\ \Omega$; $Q = 3\ \Omega$; $S = 4\ \Omega$;

$$R = 1\ \Omega, E = 2\text{ V and } G = 10\ \Omega$$

The distribution of current in various branches is as shown in Fig. 2.15.

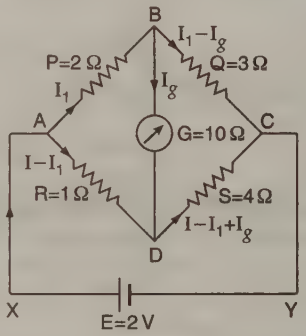


Fig. 2.15

For the closed part ABDA of the circuit :

$$I_1 \times P + I_g \times G - (I - I_1) R = 0$$

or $\quad I_1 \times 2 + I_g \times 10 - (I - I_1) 1 = 0$

or $\quad 3 I_1 + 10 I_g - I = 0 \quad \quad ...(i)$

For the closed part BCDB of the circuit :

$$(I_1 - I_g) Q - (I - I_1 + I_g) S - I_g \times G = 0$$

or $\quad (I_1 - I_g) \times 3 - (I - I_1 + I_g) \times 4 - I_g \times 10 = 0$

or $\quad 7 I_1 - 17 I_g - 4 I = 0 \quad \quad ...(ii)$

For the closed part ABC YXA of the circuit :

$$I_1 \times P + (I_1 = I_g) Q = E$$

or $\quad I_1 \times 2 + (I_1 - I_g) \times 3 = 2$

or $\quad 5 I_1 - 3 I_g = 2 \quad \quad ...(iii)$

From equation (*i*), we have

$$I = 3 I_1 + 10 I_g$$

Substituting for I in equation (*ii*), we have

or $\quad 7 I_1 - 17 I_g - 4 (3 I_1 + 10 I_g) = 0$

or $\quad 5 I_1 + 57 I_g = 0 \quad \quad ...(iv)$

Subtracting equation (*iii*) from (*iv*), we have

$$60\, I_g = -2 \text{ or } I_g = -\frac{1}{30} \text{ A}$$

The negative sign shows that current flows from point B.

Problem 2.10. A thin uniform wire AB of length 1 m, nknown resistance X and a resistance of 12 Ω are ected by thick conducting strips as shown in Fig. 2.16.

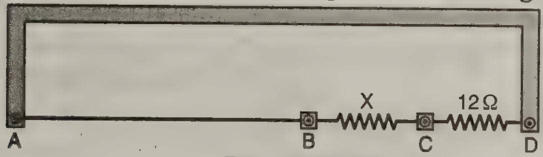


Fig. 2.16

attery and a galvanometer (with a sliding jockey ecting to it) are also available. Connections are to be e to measure the unknown resistance X using the ciple of Wheatstone bridge. Answer the following tions :

(*a*) Are there positive and negative terminals on the anometer ?

(*b*) Copy the figure in your answer book and show the ery and the galvanometer (with jockey) connected at opriate points.

(*c*) After appropriate connections are made, it is found no deflection takes place in the galvanometer, when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X. (*I.I.T. 2002*)

Sol. (*a*) A galvanometer is used to detect current in the circuit. There are **no positive or negative terminals** on the galvanometer.

(*b*) To measure the unknown resistance, the galvanometer (G) and battery should be connected in the circuit as shown in Fig. 2.17.

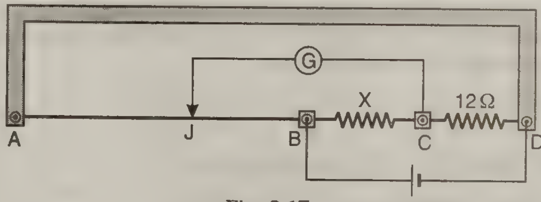


Fig. 2.17

(*c*) Here, AJ = 60 cm

∴ BJ = 100 – 60 = 40 cm

Since the Wheatstone bridge is balanced,

$$\frac{X}{12} = \frac{R_{BJ}}{R_{AJ}} \quad \text{or} \quad \frac{X}{12} = \frac{40}{60}$$

$$\text{or} \quad X = \frac{12 \times 40}{60} = 8\ \Omega$$

Chapter 3. Thermal effects of current

Problem 3.01. The walls of a closed cubical box of 50 cm are made of a material of thickness 1 mm and nal conductivity 4×10^{-4} s^{-1} cm^{-1}°C^{-1}. The interior of ox maintained at 100°C above the outside temperature heater placed inside the box and connected across a 400 :. source. Calculate the resistance of the heater. (*I.I.T. 1971*)

Sol. Here, $K = 4 \times 10^{-4}$ cal s^{-1} cm^{-1} °C^{-1} ;

$$\theta_2 - \theta_1 = 100°\text{C} \,;\, d = 1 \text{ mm} = 0{\cdot}1 \text{ cm} \,;$$

Surface area of the six faces of the cubical box,

$$A = 6 \times (50 \times 50) = 15000 \text{ cm}^2$$

Let Q_1 be the heat escaping the box from its six faces per nd. Then,

$$Q_1 = \frac{K A (\theta_2 - \theta_1)}{d} = \frac{4 \times 10^{-4} \times 15000 \times 100}{0{\cdot}1}$$

$$= 6000 \text{ cal } = 6000 \times 4{\cdot}2 \text{ J}$$

If R is the resistance of the heater, then heat produced econd,

$$Q_2 = I^2 R\, t = \frac{V^2}{R} = \frac{(400)^2}{R}$$

Since the heater maintains a constant temperature inside ox, it follows that $Q_1 = Q_2$

$$\text{or} \quad \frac{(400)^2}{R} = 6000 \times 4{\cdot}2 \text{ or } R = 6{\cdot}35 \text{ W}$$

Problem 3.02. A fuse made of lead wire has an area of -section 0·2 mm². On short circuiting, the current in use wire reaches 30 A. How long after the short ting will the fuse begin to melt ? For lead, specific heat cal g^{-1} °C^{-1}, melting point = 327 °C, density = 11·34 $^{-3}$ and the resistivity = 22× 10^{-6} Ω cm. Initial tempera- of the wire is 20°C. Neglect heat losses. (*I.I.T. 1976*)

Sol. Let *l* (cm) be length of the fuse wire.

Here, area of cross-section of wire,

$$A = 0{\cdot}2 \text{ mm}^2 = 0{\cdot}2 \times 10^{-2} \text{ cm}^2$$

volume of wire, $A\, l = 0{\cdot}2 \times 10^{-2} \times l$ cm^3

and density of the material of wire, = 11·34 g cm^{-3}

Therefore, mass of wire, $m = A\ l \times$ density

$$= 0{\cdot}2 \times 10^{-2} \times l \times 11{\cdot}34 = 2{\cdot}268 \times 10^{-2}\, l$$

Initial temperature, $\theta_1 = 20°$C ; melting point, $\theta_2 = 327°$C

Specific heat of the fuse wire, $c = 0{\cdot}032$ cal g^{-1} °C^{-1}

Therefore, heat required by the wire before it starts melting,

$$H = m\,c\,(\theta_1 - \theta_2) = 2{\cdot}268 \times 10^{-2}\, l \times 0{\cdot}032 \times (327 - 20)$$

$$= 0{\cdot}223\, l \text{ cal} = 0{\cdot}223 \times 4{\cdot}21\, l \text{ joule} \qquad \ldots(i)$$

Now, resistivity of wire, $\rho = 22 \times 10^{-6}\ \Omega$

Area of cross-section of the wire,

$$A = 0{\cdot}22 \text{ mm}^2 = 0{\cdot}2 \times 10^{-2} \text{ cm}^2$$

$$\text{Resistance of wire, } R = \rho \frac{l}{A} = 22 \times 10^{-6} \times \frac{l}{0{\cdot}2 \times 10^{-2}}$$

$$= 1{\cdot}1 \times 10^{-2}\, l$$

Current through the wire on short circuiting, I = 30 A

Suppose that the wire begins to melt after time *t*.

Then, $H = I^2 Rt = 30^2 \times 1{\cdot}1 \times 10^{-2}\, l\, 5\, t = 9{\cdot}9\, l\, t \qquad \ldots(ii)$

From equations (*i*) and, (*ii*), we have

$9{\cdot}9\, l\, t = 0{\cdot}223 \times 4{\cdot}2\, l$ or $t = 0{\cdot}095$ s

Problem 3.03. An electric kettle has two thermal coils. When a current is passed in one of them, water in kettle boils in 6 minutes and when current is passed in other one, the water boils in 8 minutes. If both the coils are joined in (*a*) series (*b*) parallel, how long will it take in boiling water ? (*I.I.T. 1975*)

Sol. Let R_1 and R_2 be the resistances of the first and second thermal coil respectively. Suppose that V is the voltage of main line. Further, suppose that the heat required to boil the water in the kettle is H.

If t_1 is the time in which water boils, when first thermal coil is used, then,

$$H = \frac{V^2}{R_1} t_1 \qquad \text{or} \qquad R_1 = \frac{V^2}{H} t_1$$

Similarly, if t_2 is the time in which water boils, when second thermal coil is used, then

$$H = \frac{V^2}{R_2} t_1 \qquad \text{or} \qquad R_2 = \frac{V^2}{H} t_2$$

(a) When coils are joined in series :

The net resistance of the two coils, $R = R_1 + R_2$

If water boils is time t, then

$$H = \frac{V^2}{R} t$$

$$\text{or} \qquad t = \frac{H}{V^2} R = \frac{H}{V^2}(R_1 + R_2)$$

$$= \frac{H}{V^2}\left(\frac{V^2}{H} t_1 + \frac{V^2}{H} t_2\right)$$

$$= t_1 + t_2 = 6 + 8 = \mathbf{14\ min}$$

(b) When coils are joined in parallel :

The net resistance of the two coils, $R' = \dfrac{R_1 R_2}{R_1 + R_2}$

If water boils in time t', then

$$H = \frac{V^2}{R'} t'$$

$$\text{or} \qquad t' = \frac{H}{V^2} R' = \frac{H}{V^2}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$= \frac{H}{V^2}\left[\frac{\frac{V^2}{H} t_1 \times \frac{V^2}{H} t_2}{\frac{V^2}{H} t_1 + \frac{V^2}{H} t_2}\right]$$

$$= \frac{t_1 t_2}{t_1 + t_2} = \frac{6 \times 8}{6 + 8} = \frac{\mathbf{24}}{\mathbf{7}}\ \mathbf{min}$$

Problem 3.04. A heater is designed to operate with a power of 1000 W in 100 V line. It is connected to two resistances of 10 Ω and R as shown in Fig. 3.01. If the hea is now giving a power of 62·5 W, calculate the value of resistance R. *(I.I.T. 19*

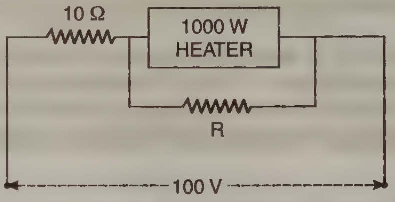


Fig. 3.01

Sol. Here, power of the heater in 100 V line,

$$P_1 = 1000\ \text{W}$$

Therefore, resistance of the heater,

$$R_1 = \frac{V_1^2}{P_1} = \frac{100^2}{1000} = 10\ \Omega$$

In the circuit shown in Fig. 3.01, the heater hav resistance R_1 (= 10 Ω) is giving a power 62·5 W.

If V_2 is potential difference across the parallel combi tion of heater and the resistance R, then,

$$R_1 = \frac{V_2^2}{P_2}$$

$$\text{or} \qquad V_2 = \sqrt{P_2 R_1} = \sqrt{62 \cdot 5 \times 10} = 25\ \text{V}$$

Therefore, potential difference across 10 Ω resistanc 100 – 25 = 75 V

If I is current through 10 Ω resistance, then

$$I = \frac{75}{10} = 7 \cdot 5\ \text{A}$$

It follows that current through parallel combinatior heater and resistance R is also 7·5 A. If X is resistance of parallel combination of heater and resistance R, then

$$\frac{25}{X} = 7 \cdot 5$$

$$\text{or} \qquad X = \frac{25}{7 \cdot 5} = \frac{10}{3}\ \Omega$$

$$\text{But} \qquad \frac{1}{X} = \frac{1}{R} + \frac{1}{R_1}$$

$$\text{or} \qquad \frac{1}{R} = \frac{1}{X} - \frac{1}{R_1} = \frac{3}{10} - \frac{1}{10} = \frac{1}{5}$$

$$\therefore \qquad R = 5\ \Omega$$

Chapter 4. Chemical effects of current

Problem 4.01. A piece of metal weighing 200 g is to be electroplated with 5% of its weight in gold. If the strength of the available current is 2 A, how long would it take to deposit the required amount of gold ? E.C.E. of hydrogen = 1·044 × 10^{-5} g C^{-1}, atomic weight of gold = 197·1, atomic weight of hydrogen = 1·008. *(Roorkee, 1988)*

Sol. Here, mass of the piece of metal = 200 g

Mass of gold to be deposited, $m = \dfrac{200 \times 5}{100} = 10\ \text{g}$

Current passed, $I = 2\ \text{A}$

Electrochemical equivalent of hydrogen,

$$z_1 = 1 \cdot 044 \times 10^{-5}\ \text{g C}^{-1}$$

Chemical equivalent of hydrogen,

$$E_1 = \frac{\text{atomic weight}}{\text{valency}} = \frac{1 \cdot 008}{1} = 1 \cdot 008\ \text{g mole}$$

Chemical equivalent of gold,

$$E_2 = \frac{\text{atomic weight}}{\text{valency}} = \frac{197 \cdot 1}{3} = 65 \cdot 7\ \text{g mole}$$

Let z_2 be electrochemical equivalent of gold. Then, fr Faraday's second law of electrolysis,

$$z_2 = z_1 \times \frac{E_2}{E_1} = 1 \cdot 044 \times 10^{-5} \times \frac{65 \cdot 7}{1 \cdot 008}$$

$$= 68 \cdot 05 \times 10^{-5}\ \text{g C}^{-1}$$

If t is the time taken to deposit the required amount of gold, then

$$m = z_2 I t$$

or

$$t = \frac{m}{z_2 I} = \frac{10}{68 \cdot 05 \times 10^{-5} \times 2} = \mathbf{7347 \cdot 5 \ s}$$

Chapter 5. Thermoelectric effect

Problem 5.01. A thermocouple is made of iron and nickel and the difference of temperature between their junctions is 100°C. Find the thermo-e.m.f. developed across such a thermocouple, when the thermo-e.m.f. for iron-lead thermocouple is 12·5 μV °C^{-1} and for nickel-lead thermocouple is 15 μV °C^{-1}.

Sol. According to law of intermediate metals,

$$E_{Fe}^{Ni} = E_{Fe}^{Pb} + E_{Pb}^{Ni}$$

Here, $E_{Fe}^{Pb} = 12{\cdot}5 \ \mu V \ °C^{-1}$

and $E_{Pb}^{Ni} = 15 \ \mu V \ °C^{-1}$

$\therefore$ $E_{Fe}^{Ni} = 12{\cdot}5 + 15 = 27{\cdot}5 \ \mu V \ °C^{-1}$

Hence, when the difference of temperature between the junctions is 100°C,

thermo e.m.f. developed = 27·5 × 100 = **2750 μV**

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Electric current, Resistance and E.M.F.

PART I. TEXT-BASED QUESTIONS

1. One ampere is equivalent to
(A) 1 coulomb second (B) 1 coulomb second^{-1}
(C) 1 joule second (D) 1 joule second^{-1}

2. Assuming that the charge of an electron is $1{\cdot}6 \times 10^{-19}$ C, the number of electrons passing through a section of wire per second, when the wire carries a current of 1 A is
(A) $0{\cdot}625 \times 10^{19}$ (B) $1{\cdot}6 \times 10^{-19}$
(C) $1{\cdot}6 \times 10^{19}$ (D) $0{\cdot}625 \times 10^{17}$
(N.C.E.R.T. 1974)

3. In the Bohr's model of hydrogen atom, the electron moves around the nucleus in a circular orbit of radius 5×10^{-11} m. Its time period is $1{\cdot}5 \times 10^{-16}$ s. The current associated with the electron motion is
(A) zero (B) $1{\cdot}6 \times 10^{-19}$ A
(C) 0·17 A (D) $1{\cdot}07 \times 10^{-3}$ A
(M.N.R. 1992)

4. Unit of resistance is
(A) volt × ampere (B) volt^2 × ampere
(C) volt/ampere (D) ampere/volt
(A.F.M.C. 1997)

5. A solenoid is at potential difference of 60 V. If the current flowing through it is 15 A, then resistance of the solenoid will be
(A) 4 Ω (B) 8 Ω
(C) 0·25 Ω (D) 2 Ω *(A.F.M.C. 1995)*

6. The resistance of a wire is R. If the length of the wire is doubled, then its resistance will be
(A) 2 R (B) 4 R
(C) R (D) 0·25 R
(A.F.M.C. 1995 ; Karnataka, 1992, 1989)

7. Three copper wires of lengths and cross-sectional areas are (l, A) ; (2 l, A/2) and (l/2, 2 A). Resistance is minimum in
(A) wire of cross-sectional area A/2.
(B) wire of cross-sectional area A.
(C) wire of cross-sectional area 2 A.
(D) same in all the three cases. *(C.B.S.E. 1997)*

8. The specific resistance of a wire 1·1 m long, 0·4 mm in diameter and having a total resistance of 4·2 ohm will be
(A) $4{\cdot}97 \times 10^{5}$ Ω m (B) 48×10^{-8} Ω m
(C) 48×10^{4} Ω m (D) none of these.

9. Two copper wires, one of length 1 m and the other of length 9 m, are found to have the same resistance. Their diameters are in the ratio
(A) 3 : 1 (B) 1 : 9
(C) 9 : 1 (D) 1 : 3
(Karnataka Entrance, 1991)

10. A metallic wire of resistance 40 Ω is stretched to twice its length. Its new resistance would be approximately :
(A) 20 Ω (B) 80 Ω
(C) 120 Ω (D) 160 Ω *(N.C.E.R.T. 1974)*

11. A wire of resistance 10 Ω is elongated by 10%. The resistance of the elongated wire is
(A) 10·1 Ω (B) 11·1 Ω
(C) 12·1 Ω (D) 13·1 Ω
(C.B.S.E. 1999, 1991 ; A.F.M.C. 1998)

12. When a piece of aluminium wire of finite length is drawn through a series of dies to reduce its diameter to half its original value, its resistance will become
(A) two times (B) four times
(C) eight times (D) sixteen times.
(N.C.E.R.T. 1974)

13. A certain piece of copper is to be shaped into a wire of minimum resistance. Its length and diameter should be
(A) L, D (B) 2 L, D/ $\sqrt{2}$
(C) L/4, 2 D (D) L/2, $\sqrt{2}$ D
(Karnataka Entrance 1987, 1985)

14. A uniform wire of resistance R is uniformly compressed along its length, until its radius becomes n times the original radius. Now, the resistance of the wire becomes
(A) R/n (B) n R
(C) R/n^2 (D) R/n^4 *(Karnataka, 2000)*

15. Two wires of the same metal have same length, but their cross-sections are in the ratio 3 : 1. They are joined in series. The resistance of thicker wire is 10 Ω. The total resistance of the combination will be
(A) 5/2 Ω (B) 40/3 Ω
(C) 40 Ω (D) 100 Ω *(C.B.S.E. 1995)*

16. The appropriate material to be used in the construction of resistance boxes out of the following is :
(A) copper (B) iron
(C) manganin (D) aluminium
(N.C.E.R.T. 1990)

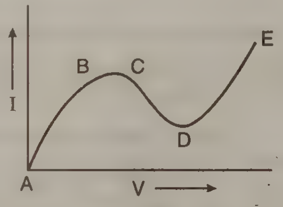
17. Identify the material commonly used for making coils of a resistance box :
(A) Molybdenum (B) Manganese
(C) Manganin (D) Magnesium
(Karnataka Entrance, 1987)

18. The reciprocal of resistance is
(A) conductance (B) specific resistance
(C) voltage (D) current *(A.F.M.C. 1995)*

19. If the resistivity of an alloy is ρ' and that of constituent metals is ρ, then
(A) $\rho' = \rho$ (B) $\rho' < \rho$
(C) $\rho' > \rho$
(D) there is no simple relation between ρ' and ρ.
(Karnataka Entrance, 1994)

20. The resistance of a discharge tube is
(A) ohmic (B) non-ohmic
(C) zero (D) both (A) and (B)
(A.F.M.C. 1996)

21. From the graph between current (I) and voltage (V) as shown in the figure, identify the portion corresponding to negative resistance :



(A) AB (B) BC
(C) CD (D) DE *(C.B.S.E. 1997)*

22. Carbon resistors used in electronic circuits are marked for their resistance value and tolerance by a colour scheme. A given resistor has a colour scheme brown, black, green and gold. Its value (in ohm) is

(A) $3{\cdot}2 \times 10^5 \pm 5\,\%$ (B) $1{\cdot}0 \times 10^6 \pm 10\%$
(C) $1{\cdot}0 \times 10^6 \pm 5\%$ (D) $1{\cdot}0 \times 10^3 \pm 5\%$
(Karnataka Entrance, 1994)

23. When a current flows in a conductor, the order of magnitude of drift velocity of electrons through it is
(A) 10^{-7} cm s^{-1} (B) 10^{-2} cm s^{-1}
(C) 10^4 mm s^{-1} (D) 0·5 mm s^{-1}
(C.P.M.T. 1986)

24. The ratio of drift velocity to that of the thermal velocity of an electron in a conductor is :
(A) 10^{-6} (B) 10^{-8}
(C) 10^{-5} (D) 10^{-10}

25. Assume that each atom of copper contributes one free electron. The density of copper is 9 g cm^{-3} and atomic weight of copper is 63. If the current flowing through a copper wire of 1 mm diameter is 1·1 ampere, the drift velocity of electrons will be
(A) 0·1 mm s^{-1} (B) 0·2 mm s^{-1}
(C) 0·3 mm s^{-1} (D) 0·5 mm s^{-1} *(C.P.M.T. 1989)*

26. Resistance of a conductor increases with the rise of temperature, because
(A) relaxation time decreases.
(B) relaxation time increases.
(C) electron density decreases.
(D) electron density increases.

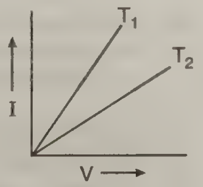
27. Si and Cu are cooled to a temperative of 300 K. Then resistivity
(A) for Si increases and for Cu decreases
(B) for Cu increases and for Si decreases
(C) decreases for both Si and Cu
(D) increases for both Si and Cu. *(C.B.S.E. 2001)*

28. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
(A) each of them increases.
(B) each of them decreases.
(C) copper increases, germanium decreases.
(D) copper decreases and germanium increases.
(I.I.T. 1980)

29. A piece of germanium and another of copper are stated to experience a rise in temperature. The resistance of
(A) each of them increases.
(B) each of them decreases.
(C) copper increases and germanium decreases.
(D) copper decreases and germanium increases.
(C.E.T. 1998)

30. The specific resistance of a conductor increases with
(A) increase in temperature
(B) increase in cross-sectional area
(c) decrease in length.
(d) decrease in cross-scetional area. *(C.B.S.E. 2002)*

31. The current I and voltage V curves for a given metallic wire at two different temperatures T_1 and T_2 are shown in the figure. Then,
(A) $T_1 > T_2$
(B) $T_1 < T_2$
(C) $T_1 = T_2$
(D) $T_1 = 2\,T_2$



(I.I.T. 1989)

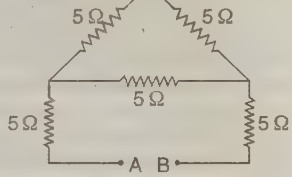
32. The resistance of a conductor is 5 ohm at 50°C, and 6 ohm at 100°C. What is its resistance at 0°C ?
(A) 1 ohm (B) 2 ohm
(C) 3 ohm (D) 4 ohm
(Karnataka Entrance, 2000, 1989, 1988)

33. The temperature coefficient of resistance of wire is 0·00125 °C^{-1}. At 300 K, its resistance is 1 ohm. The resistance of the wire will be 2 ohm at
(A) 1154 K (B) 1100 K
(C) 1400 K (D) 1127 K *(I.I.T. 1980)*

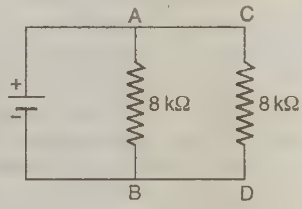
34. Which of the following has a negative temperature coefficient ?
(A) C (B) Fe
(C) Mn (D) Ag *(A.F.M.C. 1995)*

35. Three resistances each of 4 Ω are connected to form a triangle. The resistance between any two terminals is
(A) 12 Ω (B) 2 Ω
(C) 6 Ω (D) 8/3 Ω
(C.B.S.E. 1993 ; Karnataka 1992, 1990)

36. What is the resistance between points A and B ?
(A) 15 Ω
(B) 13·3 Ω
(C) 6 Ω
(D) 25 Ω

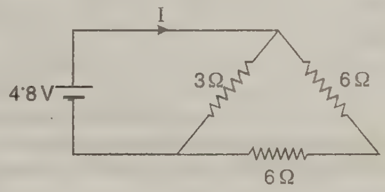


37. Find the equivalent resistance of a two branched circuit as shown in figure :
(A) 2 kΩ
(B) 3 kΩ
(C) 4 kΩ
(D) 5 kΩ



(A.F.M.C. 1997)

38. The current in the given circuit is



(A) 8·31 A (B) 6·82 A
(C) 4·92 A (D) 2 A
(C.B.S.E. 1999, 1997 ; similar C.E.T. 1999)

39. Two resistances r_1 and r_2 ($r_1 < r_2$) are joined in parallel. The equivalent resistance R is such that
(A) $r_2 < R < r_1 + r_2$ (B) $R > r_1 + r_2$
(C) $R < r_1$ (D) $r_1 < R < r_2$
(Karnataka, 2000)

40. A metal wire of resistance R is cut into three equal pieces that are then connected side by side to form a new wire, the length of which is equal to one third of the original length. The resistance of this new wire is
(A) R (B) 3 R
(C) R/9 (D) R/3 *(C.E.T. 1998)*

41. n equal resistors are first connected in series and then connected in parallel. What is the ratio of the maximum to the minimum resistance ?
(A) n (B) $1/n^2$
(C) n^2 (D) $1/n$
(Karnataka Entrance, 1994)

42. ***n*** **conductors, each of resistance** ***r*****, when connected in parallel, give an effective resistance R. What will be resistance, if they are connected in series ?**

(A) $n^2 R$ (B) $\frac{R}{n^2}$

(C) $\frac{R^2}{n}$ (D) $\frac{n^2}{R}$

43. A student has 10 resistors each of resistance ***r*****. The minimum resistors made by him from the given resistors is**

(A) $10\,r$ (B) $\frac{r}{10}$

(C) $\frac{r}{100}$ (D) $\frac{r}{5}$ *(A.F.M.C. 1995)*

44. An unknown resistance is connected in parallel with a resistance of 4 Ω. Similarly, two other resistances of 2 and 3 Ω are connected in parallel. The effective resistance, when these two combinations are connected in series, is 3·6 Ω. The value of the unknown resistance is (in Ω)

(A) 1 (B) 4

(C) 5 (D) 6 *(Karnataka Entrance, 1987)*

45. Four resistances R_1, R_2, R_3 and R_4 are connected in parallel. The resultant resistance R is

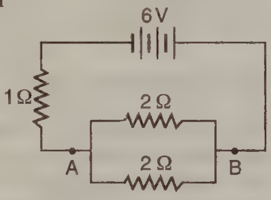
(A) equal to sum of the four resistances.

(B) greater than the sum of the four resistances.

(C) less than the sum of the four resistances.

(D) less than any of the four resistances.

46. Refer to the figure. Assuming the internal resistance of the battery to be negligible, a voltmeter connected between the points A and B should read



(A) 0 V

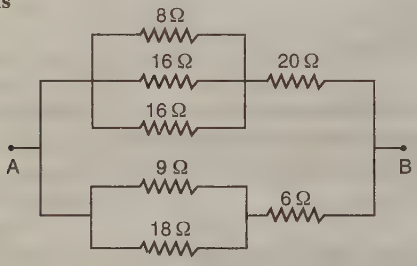
(B) 1·2 V

(C) 3·0 V

(D) 4·8 V

(Karnataka Entrance, 1992, 1988)

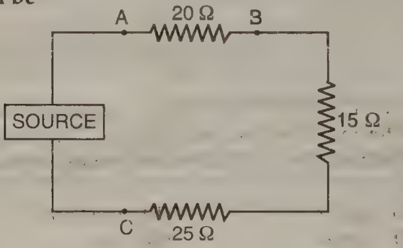
47. The equivalent resistance of the arrangement of resistances shown between the points A and B in the figure given below is



(A) 6 Ω (B) 8 Ω

(C) 16 Ω (D) 24 Ω *(C.P.M.T. 1990)*

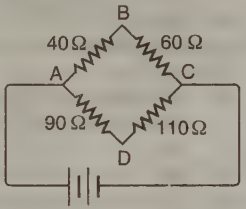
48. In the electric circuit given below, the voltage between the points A and B is 10 V. The voltage between the points B and C will be



(A) 5 V (B) 10 V

(C) 15 V (D) 20 V

49. Four resistances of 40 Ω, 60 Ω, 90 Ω and 110 Ω are connected as shown to a battery of e.m.f. 4 V and negligible internal resistance. Then, the potential difference between the points B and D is



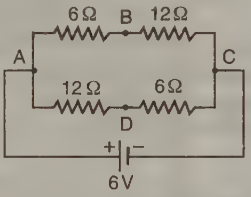
(A) −0·2 V

(B) 0·2 V

(C) 1 V

(D) −1 V

50. In the circuit as shown in the figure, the potential difference between the points B and D is



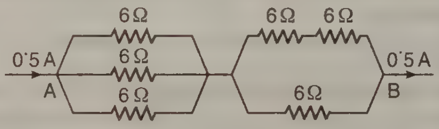
(A) 1 V

(B) 2 V

(C) 3 V

(D) 4 V

51. Resistance of 6 ohm each are connected in the manner shown in given figure. With the current 0·5 ampere as shown in figure, the potential difference $V_A - V_B$ is



(A) 3·0 volt (B) 3·6 volt

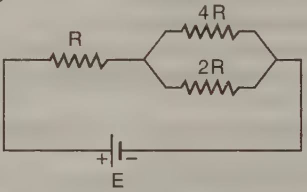
(C) 6·0 volt (D) 7·2 volt *(C.P.M.T. 1989)*

52. Three resistors of 4 Ω, 6 Ω and 12 Ω are connected in parallel and the combination is connected in series with 4 V battery with internal resistance of 2 Ω. The battery current is

(A) 0·5 A (B) 1 A

(C) 2 A (D) 10 A *(C.E.T. 1991)*

53. As shown in figure, the current flowing in the resistor 2 R (in ohm) is



(A) $\frac{2\,E}{R}$ (B) $\frac{2\,E}{7\,R}$

(C) $\frac{E}{7\,R}$ (D) $\frac{E}{R}$ *(C.P.M.T. 1980)*

54. For a cell, the terminal potential difference is 2·2 V, when circuit is open and reduces to 1·8 V, when cell is connected to a resistance R = 5 Ω. The internal registance of cell (***r*****) is**

(A) 10/9 Ω (B) 9/10 Ω

(C) 11/9 Ω (D) 5/9 Ω *(C.B.S.E. 2002)*

55. A primary cell has an e.m.f. 1·5 volt. When short circuited, it gives a current of 3 ampere. The internal resistance of the cell is

(A) 4·5 Ω (B) 2 Ω

(C) 0·5 Ω (D) $\frac{1}{4 \cdot 5}$ Ω *(C.P.M.T. 1983)*

56. **A cell supplies a current of 0·3 A through a resistance of 7 Ω and a current of 0·9 A through a resistance of 2 Ω. The internal resistance of the cell is**
(A) 2·0 Ω (B) 1·2 Ω
(C) 1·0 Ω (D) 0·5 Ω

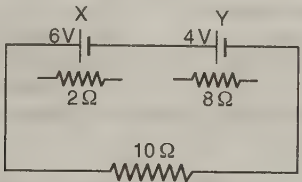
57. **A cell of e.m.f. E is connected across a resistance *r*. The potential difference between the terminals of the cell is found to be V. The internal resistance of the cell must be**
(A) $\frac{2(E-V)V}{r}$ (B) $\frac{2(E-V)r}{E}$
(C) $\frac{(E-V)r}{V}$ (D) $(E-V)r$ *(M.N.R. 1987)*

58. **If E is the e.m.f. of a cell of internal resistance *r* and external resistance R, then potential difference across R is given as**
(A) $V=\frac{E}{(1+r)}$ (B) $V=\frac{ER}{(R+r)}$
(C) $V=\frac{E}{(1+r/R)}$ (D) $V=\frac{E}{1+R/r}$
(Karnataka Entrance, 1986, 1984)

59. **The internal resistance of a cell of e.m.f. 2 V is 0·1 Ω. It is connected to a resistance of 3·9 Ω. The voltage across the cell will be :**
(A) 0·5 V (B) 1·5 V
(C) 1·95 V (D) 2 V *(C.B.S.E. 1999)*

60. **The terminal voltage of a cell of e.m.f. E on short circuting will be**
(A) E (B) E/2
(C) 2 E (D) zero *(A.F.M.C. 1998)*

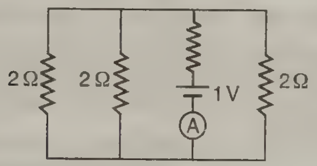
61. **Two cells X and Y are connected to a resistance of 10 Ω as shown in the figure :**



The terminal voltage of cell Y is
(A) zero (B) 2 V
(C) 4 V (D) 10 V

62. **In the above figure, if the polarity of the cell Y is reserved, then the terminal voltage of the cell Y will be :**
(A) 2·8 V (B) 3·8 V
(C) 4·8 V (D) 5·8 V

63. **The reading of the ammeter in the circuit is :**

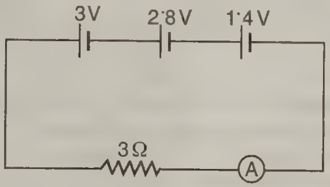


(A) $\frac{1}{8}$ A (B) $\frac{3}{8}$ A
(C) $\frac{1}{2}$ A (D) 2 A

64. **A current of 2·0 A passes through a cell of e.m.f. 1·5 V having internal resistance of 0·15 Ω. The potential difference (measured in volt) across both the terminals of the cell is**
(A) 1·35 (B) 1·50
(C) 1·00 (D) 1·20 *(M.N.R. 1991*

65. ***n* cells each of e.m.f. E volt and internal resistance *r* ohm are connected in series with an external resistance R. The current will increase *n* times that of a single cell, if**
(A) R is very large as compared to the internal resistance.
(B) R is very small as compared to the internal resistance.
(C) R is equal to total internal resistance
(D) regardless of relative magnitudes of R and *r*.
(C.E.T. 1998)

66. **Three cells of e.m.f. 3 V, 2·8 V and 1·4 V are connected in series. Their internal resistances are 0·02 Ω, 0·6 Ω and 1 Ω respectively. If the battery is connected to an external resistance of 3 Ω via a very low resistance ammeter, what would be the reading of the ammeter ?**



(A) 1·0 A (B) 1·5 A
(C) 2·6 A (D) 5 A *(A.F.M.C. 1997)*

67. **If *n* batches of *m* cells (each of emf E and internal resistance *r*) in series are connected in parallel with an external resistance R, then the current I through the external resistance is given by**
(A) $I=\frac{mnE}{(mr+nR)}$ (B) $I=\frac{mnE}{(mR+nr)}$
(C) $I=\frac{mE/n}{(mR+nr)}$ (D) $I=\frac{nE/m}{(mr+nR)}$
(Karnataka Entrance, 1986, 1984)

68. **To get maximum current in a resistance of 3 ohm, one can use *n* rows of *m* cells (connected in series) connected in parallel. If the total number of cells is 24 and the internal resistance of a cell is 0·5 ohm, then**
(A) $m = 12, n = 2$ (B) $m = 8, n = 3$
(C) $m = 6, n = 4$ (D) $m = 2, n = 12$
(N.C.E.R.T. 1980)

69. **Suppose 4 branches of 5 identical cells (in series) are connected in parallel, the internal resistance of each cell being 10 ohm. If the group of cells sends a current of 0·2 ampere through an external resistance of 25 ohm, then the e.m.f. of each cell is (in volt)**
(A) 1·5 (B) 1·25
(C) 2·5 (D) 1·75
(Karnataka Entrance, 1987)

PART II. THOUGHT-BASED QUESTIONS

70. **The potential difference applied to an X-ray tube is 5 kV and the current through it is 3·2 mA. Then the number of electrons striking the target per second is**
(A) 2×10^{16} (B) 5×10^{6}
(C) 1×10^{17} (D) 4×10^{15} *(I.I.T. 2002)*

71. **In a hydrogen discharge tube, it is observed that through a given cross-section $3{\cdot}13 \times 10^{15}$ electrons are moving from right to left and $3{\cdot}12 \times 10^{15}$ protons are moving from left to right. What is the electric current in the discharge tube and what is its direction ?**
(A) 1 mA towards right (B) 1 mA towards left.
(C) 2 mA towards right. (D) 2 mA towards left.
(A.F.M.C. 1996)

72. **A small sphere that carries a charge *q* is whirled in a circle at the end of an insulating string. The angular frequency of rotation is 2 ω. This rotating charge represents a current of**
(A) $q\omega/\pi$ (B) $q\omega/2\pi$
(C) $q\pi/\omega$ (D) $q\omega$ *(C.E.T. 1998)*

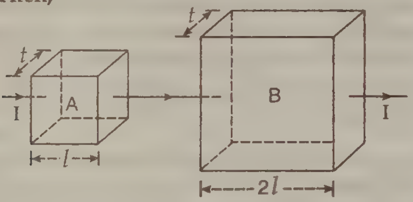
73. A quantity X is given by $\varepsilon_0 L \frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time internal. The dimensional formula for X is the same as that of :
(A) resistance (B) charge
(C) voltage (D) current *(I.I.I. 2001)*

74. Masses of three wires of same metal are in the ratio 1 : 2 : 3 and their lengths in the ratio 3 : 2 : 1. Electrical resistance of these wires will be in the ratio of :
(A) 1 : 1 : 1 (B) 1 : 2 : 3
(C) 9 : 4 : 1 (D) 27 : 6 : 1

75. As the temperature of a metallic resistor is increased, the product of resistivity and conductivity
(A) increases. (B) decreases.
(C) may increase or decrease.
(D) remains constant.

76. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length 2 L. The temperature of the wire is raised by the same amount ΔT in the same time. The value of N is
(A) 4 (B) 6
(C) 8 (D) 9 *(I.I.T. 2001)*

77. The two metal plates A and B of same material are square in shape and have the same thickness. The side of B is twice that of A. They are connected in the circuit as shown in figure. Then,



(A) the resistance of B is twice that of A.
(B) the resistance of A is twice that of B.
(C) both have the same resistance.
(D) the resistance of A is four times that of B.

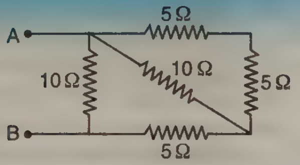
78. A conductor of resistance 8 Ω is bent in the form of circle. What will be resistance between two points on any diameter of the circle ?
(A) 1 Ω (B) 2 Ω
(C) 8 Ω (D) 16 Ω

79. When current flows in a conductor, then the ratio of the intensity of electric field E at any point within the conductor and the current density at a point is called
(A) resistance (B) conductance
(C) specific resistance (D) inductance *(A.F.M.C. 1996)*

80. Which of the following quantities do not change, when a resistor is heated by passing current through it ?
(A) Resistance (B) Resistivity
(C) Drift velocity (D) Number of free electrons.

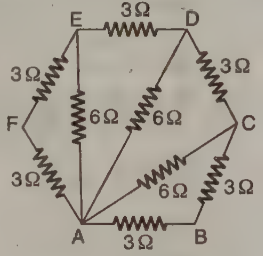
81. You are given several identical resistors each of resistance R = 10 Ω and each capable of carrying a maximum current 1 A. It is required to make suitable combination of these resistors to produce a resistance of 5 Ω, which can carry a current of 4 A . The minimum number of such resistors required is
(A) 4 (B) 10
(C) 8 (D) 20 *(C.B.S.E. 1990)*

82. The effective resistance between the terminals A and B in the circuit is :



(A) 25 Ω
(B) 35 Ω
(C) 10 Ω
(D) 5 Ω

83. Six resistors of 3 Ω are connected along the sides of a hexagon and three resistors of 6 Ω each are connected along AC, AD and AE as shown in the figure. The equivalent resistance between the points A and B is equal to



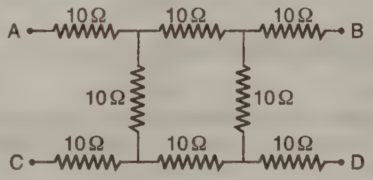
(A) 3 Ω
(B) 9 Ω
(C) 2 Ω
(D) 6 Ω *(A.F.M.C. 1995)*

84. Given three resistors of different values. How many different combinations of these three resistances can be made ?
(A) Six (B) Five
(C) Four (D) Eight

85. Given three equal resistors. How many different combinations of these three resistances can be made ?
(A) Three (B) Four
(C) Five (D) Six *(N.C.E.R.T. 1980)*

86. A technician has only two resistors. By using them singly in series or in parallel, he is able to obtain the resistances of values 3, 4, 12 and 16 Ω. What are the resistances of the two resistors ?
(A) 6 and 10 Ω (B) 4 and 12 Ω
(C) 7 and 9 Ω (D) 5 and 12 Ω
(Karnataka Entrance, 1993)

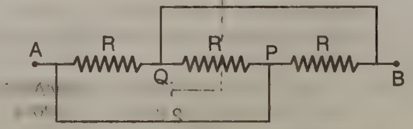
87. What will be the equivalent resistance between the points A and D ?



(A) 10 Ω (B) 20 Ω
(C) 30 Ω (D) 40 Ω *(C.B.S.E. 1996)*

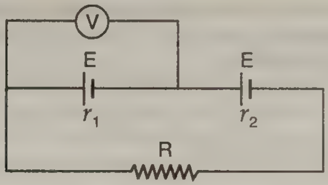
88. An electrical cable of copper has just one wire of radius 9 mm. Its resistance is 5 ohm. The single copper wire of the cable is replaced by 6 different well insulated copper wires each of radius 3 mm. The total resistance of the cable will now be equal to
(A) 7·5 Ω (B) 45 Ω
(C) 90 Ω (D) 270 Ω *(C.P.M.T. 1988)*

89. Find the resistance of the portion of the circuit between the points A and B in the circuit shown in the figure. Points A and P are connected by a copper wire of negligible resistance. Similarly, points B and Q are also connected by a copper wire of negligible resistance.



(A) 3 R
(B) $\frac{R}{3}$
(C) $\frac{3R}{2}$
(D) $\frac{2R}{3}$ *(C.P.M.T. 1990)*

90. **In the diagram shown, the two cells have equal e.m.f. E, but internal resistances are r_1 and r_2. If the reading of the voltmeter is zero, then**



(A) $R = r_1 - r_2$ (B) $R = r_1 + r_2$
(C) $R = 2r_1 - r_2$ (D) $R = 2r_1 + r_2$

(Roorkee, 1983)

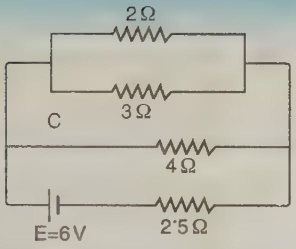
91. **The steady state current in a 2 Ω resistor (see figure), when the internal resistance of the battery is negligible and the capacitance of the capacitor is 0·1 μF is**

(A) 0·3 A

(B) 0·6 A

(C) 0·9 A

(D) 1·3 A



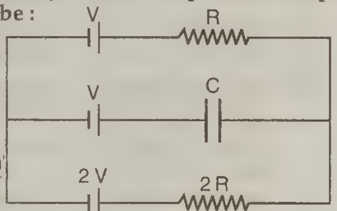
92. In the given circuit with steady current, the potential drop across he capacitor must be :

(A) V

(B) V/2

(C) V/3

(D) 2 V/3 (I.I.T. 2001)



Chapter 2. Electrical measurements

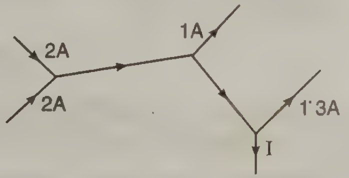
PART I. TEXT-BASED QUESTIONS

1. **Kirchhoff's first law, *i.e.* Σ I = 0 at a junction, deals with the conservation of**

(A) charge (B) energy
(C) momentum (D) angular momentum

(C.B.S.E. 1997)

2. **Figure shows current in a part of electrical circuit. Then, current I is**



(A) 1·7 A (B) 3·7 A
(C) 1·3 A (D) 1 A

(Karnataka, 2000 ; C.P.M.T. 1981)

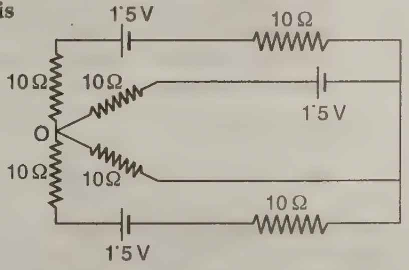
3. **The algebraic sum of the currents at the point O in the following circuit is**

(A) 0 A

(B) 0·75 A

(C) 0·375 A

(D) 1·5 A

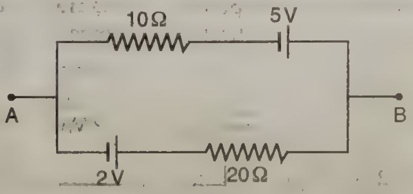


(Karnataka Entrance, 1989)

4. **Two cells of 1·25 V and 0·75 are connected in parallel. The effective voltage will be**

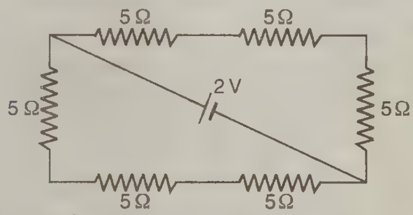
(A) 0·75 V (B) 1·25 V
(C) 2·0 V (D) 0·50 V (C.P.M.T. 1992)

5. **The current in the given circuit is :**



(A) 0·1 A (B) 0·2 A
(C) 0·3 A (D) 0·4 A *(C.B.S.E. 1999)*

6. The potential difference between the points A and B in the figure will be



(A) $\frac{2}{3}$ V (B) $\frac{8}{9}$ V

(C) $\frac{4}{3}$ V (D) 2 V (C.P.M.T. 1991)

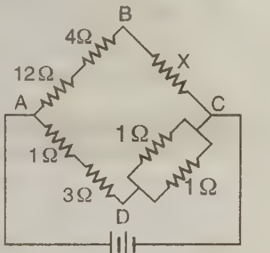
7. In the arrangement of resistances shown in the circuit, the potential difference between points B and D will be zero, when the unknown resistance X is

(A) 4 Ω

(B) 3 Ω

(C) 2 Ω

(D) 1 Ω (C.P.M.T. 1988)



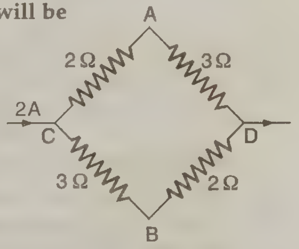
8. A current of 2 A flows in conductors as shown. The potential difference $V_A - V_B$ will be

(A) +2 V

(B) +1 V

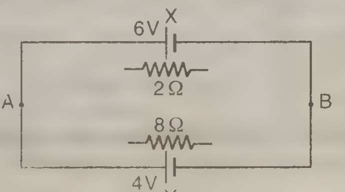
(C) –1 V

(D) +4 V



(C.P.M.T. 1976)

9. The cells X and Y are connected as shown : The potential difference between terminals A and B is



(A) 5·4 V (B) 5·6 V
(C) 5·8 V (D) 6·0 V

10. The instrument for the accurate measurement of the e.m.f. of a cell is
(A) a voltmeter (B) an ammeter
(C) a potentiometer (D) a slide wire bridge.
(Karnataka Entrance, 1993)

11. Potentiometer measures the potential difference more accurately than a voltmeter, because
(A) it has a wire of high resistance
(B) it has a wire of low resistance.
(C) It does not draw current from external circuit
(D) it draws a heavy current from external circuit.
(C.B.S.E. 2002)

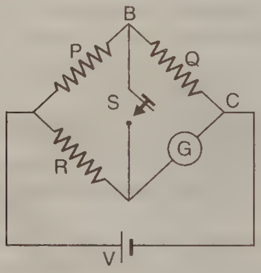
12. The resistivity of potentiometer wire is 10^{-7} Ω m and its area of cross section is 10^{-6} m^2. When a current I = 0·1 A flows through the wire, its potential gradient is
(A) 10^{-2} V m^{-1} (B) 10^{-4} V m^{-1}
(C) 0·1 V m^{-1} (D) 10 V m^{-1} *(C.B.S.E. 2001)*

13. In a potentiometer experiment, for measuring internal resistance of a cell, the balance point has been obtained on fourth wire. The balance point can be shifted to fifth wire by
(A) decreasing the current due to auxiliary battery.
(B) increasing the current due to auxiliary battery.
(C) putting a suitable resistance in series to the cell.
(D) putting a shunt resistance in parallel to the cell.

14. It is observed in a potentiometer experiment that no current passes through the galvanometer, when the terminals of the cell are connected across a certain length of the potentiometer wire. On shunting the cell by a 2 Ω resistance, the balancing length is reduced to half. The internal resistance of the cell is
(A) 4 Ω (B) 2 Ω
(C) 9 Ω (D) 18 Ω

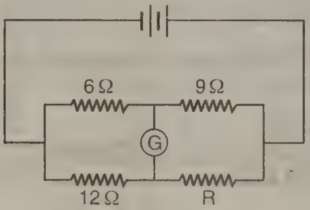
15. The sensitivity of the potentiometer can be increased by :
(A) increasing the e.m.f. of primary cell.
(B) increasing the potential gradient.
(C) increasing the length of potentiometer wire.
(D) decreasing the length of potentiometer wire.
(A.F.M.C. 1998)

16. In the circuit shown in the figure, P≠R. The reading of the galvanometer is same with switch S open or closed. Then,



(A) $I_R = I_G$ (B) $I_P = I_G$
(C) $I_Q = I_G$ (D) $I_Q = I_R$ *(I.I.T. 1999)*

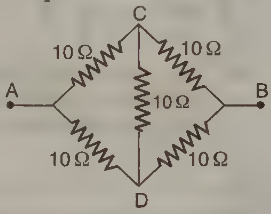
17. When the four resistors are connected as shown, it is found that the current flowing through the galvanometer G is zero. The resistance of R is



(A) 3 ohm (B) 4·5 ohm
(C) 18 ohm
(D) impossible to determine without knowing the e.m.f of the battery. *(Karnataka Entrance, 1988)*

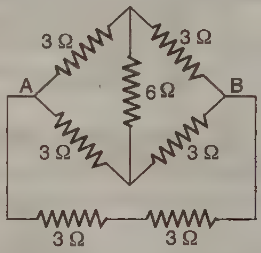
18. In a typical Wheatstone's network, the resistances in cyclic order are P = 10 Ω, Q = 5 Ω, S = 4 Ω and R = 4 Ω. For the bridge to balance,
(A) 10 Ω should be connected in series with P.
(B) 10 Ω should be connected in parallel with P.
(C) 5 Ω should be connected in series with Q.
(D) 5 Ω should be connected in parallel with Q.
(Karnataka, 2000)

19. The effective resistance between points A and B in the given circuit A is



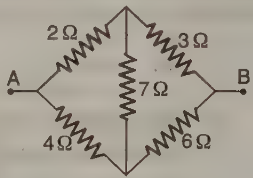
(A) 10 Ω
(B) 20 Ω
(C) 40 Ω
(D) 50 Ω
(C.B.S.E. 2001, N.C.E.R.T. 1974)

20. The resistance of the circuit between points A and B is :



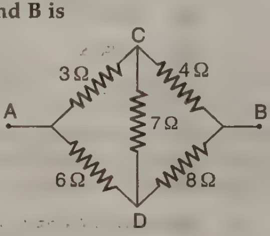
(A) 2 Ω
(B) 18 Ω
(C) 6 Ω
(D) 2·4 Ω

21. Five resistances are connected as shown in the figure. The effective resistance between the points A and B is



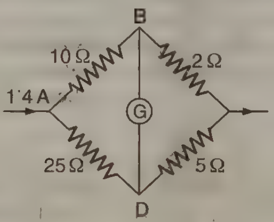
(A) $\frac{10}{3}$ Ω
(B) $\frac{20}{3}$ Ω
(C) 15 Ω
(D) 6 Ω *(I.I.T. 1976)*

22. A bridge circuit is shown in the figure. The equivalent resistance between points A and B is



(A) 21 Ω
(B) 7 Ω
(C) 252/85 Ω
(D) 14/3 Ω *(C.B.S.E. 2000)*

23. In the given circuit, when galvanometer G shows no deflection, the current in the 2 ohm resistor is



(A) 1·4 A
(B) 1·2 A
(C) 1·0 A
(D) 0·4 A
(C.P.M.T. 1989)

24. In a metre bridge, the balancing length from the left end (standard resistance of 1 Ω is in the right gap) is found to be 20 cm. The value of the unknown resistance is

(A) 0·8 Ω (B) 0·5 Ω
(C) 1·4 Ω (D) 0·25 Ω (*C.B.S.E. 1999*)

25. Two resistances are connected in the two gaps of a metre bridge. The balance points is 20 cm from the zero end. When a resistance of 15 ohm is connected in series with the smaller of the two resistances, the null point shifts to 40 cm. The smaller of the two resistances has the value (in ohm)

(A) 8 (B) 9
(C) 10 (D) 12

(*Karnataka Entrance, 1985*)

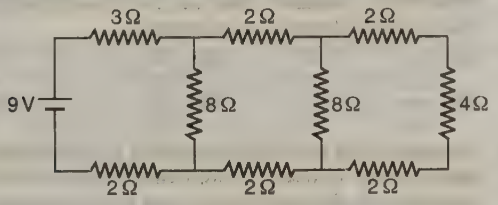
26. An unknown resistance R_1 is connected in series with a resistance of 10 ohm. This combination is connected to one gap of a metre bridge, while other gap is connected to another resistance R_2. The balance point is at 50 cm. Now, when the 10 ohm resistance is removed, the balance point shifts to 40 cm. Then, the value of R_1 (in ohm) is

(A) 60 (B) 40
(C) 20 (D) 10

(*Karnataka Entrance, 1987*)

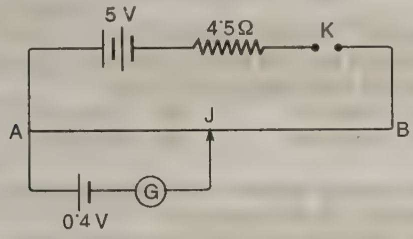
PART II. THOUGHT-BASED QUESTIONS

27. In the circuit shown in the figure, the current through the



(A) 3 Ω resistor is 0·50 A.
(B) 3 Ω resistor is 0·25 A.
(C) 4 Ω resistor is 0·50 A.
(D) 4 Ω resistor is 0·25 A. (*I.I.T. 1998*)

28. In figure, the potentiometer wire AB has a resistance of 5 Ω and length 10 m. The balancing length AJ for the e.m.f. of 0·4 V is



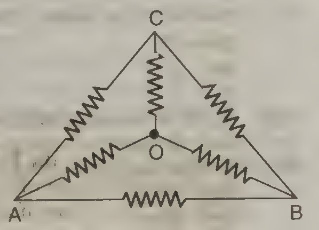
(A) 4 m (B) 1·5 m
(C) 0·8 m (D) 0·4 m (*A.F.M.C. 1995*)

29. When balance point is obtained in a potentiometer for finding the internal resistance of a cell ; the current through the potentiometer wire is due to

(A) the cell, whose internal resistance is to be found.
(B) the auxiliary battery.
(C) both cell and the auxiliary battery.
(D) neither the cell nor the battery.

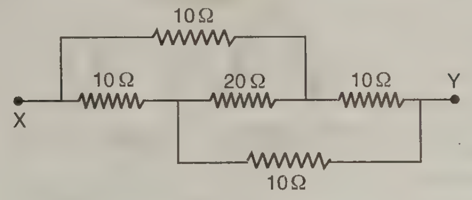
30. Six equal resistances each of 4 Ω are connected to form the following network. The resistance between any two terminals of the network is

(A) 4 Ω
(B) 1 Ω
(C) 2 Ω
(D) 0·5 Ω



31. The equivalent resistance between the points X and Y of the following circuit is

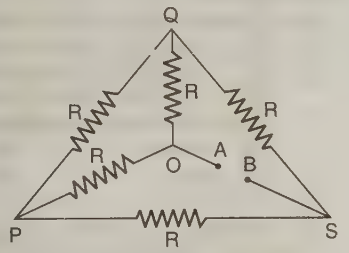
(A) 10 Ω (B) 30 Ω
(C) 20 Ω (D) 50 Ω



(*C.P.M.T. 1988*)

32. Each of resistance in the network shown in the figure is equal to R. The resistance between the terminals A and B is

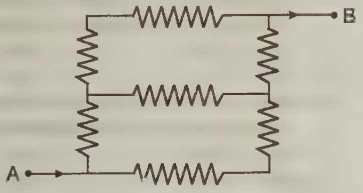
(A) R
(B) 5 R
(C) 3 R
(D) 6 R



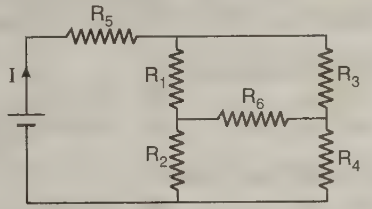
(*C.B.S.E. 1995 ; I.I.T. 1978*)

33. Seven resistances each of 5 Ω are connected as shown in figure. The equivalent resistance between the points A and B is

(A) 5 Ω
(B) 7 Ω
(C) 14 Ω
(D) 35 Ω



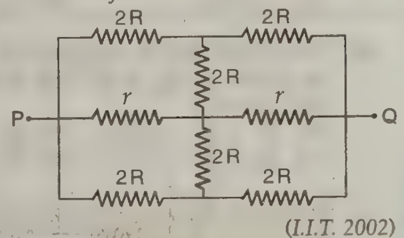
34. In the given ciruit, it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy :



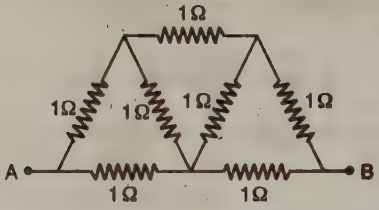
(A) $R_1 R_4 = R_2 R_3$
(B) $R_1 R_3 = R_2 R_4$
(C) $R_1 R_2 R_5 = R_3 R_4 R_6$
(D) $\frac{1}{R_5}+\frac{1}{R_6}=\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4}$ (*I.I.T. 2001*)

35. In the given ciruit, it is observed that the current I is independent of the value of the resistance R_6. Then the registance values must satisfy :

(A) $\frac{2Rr}{R+r}$
(B) $\frac{8R(R+r)}{3R+r}$
(C) $2r+4R$
(D) $2r+5R/2$ (*I.I.T. 2002*)



36. **In the network shown in figure, each resistance is 1 Ω. The effective resistance between the points A and B is**



(A) $\frac{4}{3}\Omega$ (B) $\frac{3}{2}\Omega$
(C) 7 Ω (D) $\frac{8}{7}\Omega$ (C.B.S.E. 1990

Chapter 3. Thermal effects of Current

PART I. TEXT-BASED QUESTIONS

1. **You have the the following electrical appliances :**
(*i*) 1 kW-250 V electric heater
(*ii*) 1 kW-250 V electric kettle
(*iii*) 1 kW-250 V electric bulb
Which of these have highest resistance ?
(A) Heater (B) Kettle
(C) Bulb (D) All have equal resistances.
(*N.C.E.R.T. 1991*)

2. **If a power of 100 W is being supplied across a potential difference of 200 V, current flowing is**
(A) 0·5 A (B) 1 A
(C) 2 A (D) 20 A (*A.F.M.C. 1993*)

3. **A current of 2 A passing through conductor produces 80 J of heat in 10 seconds. The resistance of the conductor is**
(A) 0·5 Ω (B) 2 Ω
(C) 4 Ω (D) 20 Ω (*C.B.S.E. 1993*)

4. **How many calories of heat will approximately be developed in a 210 W electric bulb in 5 minutes.**
(A) 15000 (B) 1050
(C) 63000 (D) 80000 (*M.N.R. 1985*)

5. **Two bulbs 25 W – 220 V and 100 W – 220 V are given. Which has higher resistance ?**
(A) 25 W bulb (B) 100 W bulb
(C) Both bulbs will have equal resistances
(D) Resistances of the bulbs cannot be compared.
(*C.B.S.E. 2000*)

6. **A kilowatt-hour is a unit of**
(A) energy (B) power
(C) electric charge (D) electric current.
(*N.C.E.R.T. 1985*)

7. **One kilowatt hour equals**
(A) 36×10^5 J (B) 36×10^3 J
(C) 36×10^{-5} J (D) 36×10^{-3} J
(*C.B.S.E. 1997 ; N.C.E.R.T. 1974*)

8. **Watt-hour meter measures :**
(A) electrical energy (B) current
(C) voltage (D) power.
(*Karnataka Entrance, 1994*)

9. **A 2 kW boiler used for 1 hour everyday consumes the following electrical energy in thirty days :**
(A) 60 units (B) 120 units
(C) 15 units (D) None of above.
(*Karnataka Entrance, 1993*)

10. **An electric lamp is marked 60 W–230 V. The cost of 1 kWh of electric energy is Rs. 1·25. The cost of using this lamp for 8 hours is :**
(A) Rs. 1·20 (B) Rs. 4·00
(C) Re. 0·25 (D) Re. 0·60.
(*Karnataka Entrance, 1994*)

11. **If current flowing in a conductor changes by 1%, then power will change by**
(A) 1% (B) 2%
(C) 10% (D) 100% (*A.F.M.C. 1996*

12. **A 5 °C rise in temperature is observed in a conductor b passing the current. When the current is doubled, the ris in temperature will be approximately**
(A) 5 °C (B) 10 °C
(C) 20 °C (D) 40 °C (*C.B.S.E. 1998*

13. **If according to Joule's law of heating, the potentia difference across a conductor having a material o resistivity ρ remains constant, then heat produced in th conductor is directly proportional to**
(A) ρ (B) ρ^2
(C) $\frac{1}{\sqrt{\rho}}$ (D) $\frac{1}{\rho}$ (*P.E.T. 1986*

14. **A heater coil is cut into two parts of equal lengths and onl one of them is used in the heater. The ratio of the hea produced by this half coil to that by the original coil is**
(A) 2 : 1 (B) 1 : 2
(C) 1 : 4 (D) 4 : 1

15. **In an ordinary electric heater, if the length of the coil i reduced to half, a given quantity of water will boil in**
(A) more time (B) less time
(C) same time
(D) a time, which depends upon the resistivity of the wir
(*Karnataka Entrance, 199*

16. **On 220 V supply, a heater heats a volume of water i 5 minutes time. On the supply of 110 V, the same heater wi heat the same volume of water in**
(A) 5 minutes (B) 8 minutes
(C) 10 minutes (D) 20 minutes
(*A.F.M.C. 199*

17. **The heating element of a heater should be made with material, which should have**
(A) high resistivity and high melting point.
(B) high resistivity and low melting point.
(C) low resistivity and low melting point.
(D) low resistivity and high melting point.
(*M.N.R. 198*

18. **Two heater wires of equal lengths are first connected i series and then in parallel. The ratio of heat produced i two cases is**
(A) 2 : 1 (B) 1 : 2
(C) 4 : 1 (D) 1 : 4.
(*C.E.T. 1999 ; M.N.R. 198*

19. **If two bulbs, whose resistances are in the ratio of 1 : 2, a connected in series. The power dissipated in them has th ratio of**
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 1 : 4 (*C.B.S.E. 199*

20. **A 500 W heating unit is designed to operate from a 115 line. If the line voltage drops to 110, the percentage dro in heat output will be**
(A) 10·2% (B) 8·1%
(C) 8·6% (D) 7·6% (*Dhanbad, 199*

21. A heating coil is labelled 100 W-220 V. The coil is cut in two halves and the two pieces are joined in parallel to the same source. The energy now liberated per second is
(A) 25 J (B) 50 J
(C) 200 J (D) 400 J (C.B.S.E. 1995)

22. Three equal resistors connected across a source of e.m.f. together dissipate 10 W of power. What will be the power dissipated, if the same resistors are connected in parallel across the same source of e.m.f. ?
(A) 5 W (B) 10 W
(C) 30 W (D) 90 W (C.B.S.E. 1998)

23. A 25 W-220 V bulb and a 100 W-220 V bulb are joined in series and connected to the mains. Which bulb will glow brighter ?
(A) 25 W bulb (B) 100 W bulb
(C) First 25 W bulb and then 100 W bulb.
(D) Both will glow with same brightness. *(I.I.T. 1972)*

24. If R_1 and R_2 are respectively the filament resistance of a 200 W bulb and a 100 W bulb designed to operate on the same voltage, then
(A) $R_1 = 2R_2$ (B) $R_2 = 2R_1$
(C) $R_2 = 4R_1$ (D) $R_1 = 4R_2$
(Karnataka Entrance , 1990 ; C.P.M.T. 1990)

25. Of the two bulbs in a house, one glows brighter than the other. Which of the two has a larger resistance ?
(A) The brighter bulb. (B) The dim bulb.
(C) Both have the same resistance.
(D) The brightness does not depend upon the resistance. *(C.P.M.T. 1985)*

26. A fuse is used in an electrical circuit to
(A) indicate the consumption of power.
(B) break the circuit, when the power is off.
(C) maintain constant current.
(D) break the circuit in case of overloading.
(Karnataka Entrance , 1988)

PART II. THOUGHT-BASED QUESTIONS

7. *(i)* The product of volt and coulomb is joule,
(ii) The product of volt and ampere is joule per second.
(iii) The product of volt and watt is horse power.
(iv) Watt-hour can be measured in terms of electron-volt.
(A) All the four are correct.
(B) *(i)*, *(ii)* and *(iv)* are correct.
(C) *(i)* and *(iii)* are correct.
(D) *(iii)* and *(iv)* are correct. *(N.C.E.R.T. 1978)*

28. Two heating coils, one of fine wire and the other of thick wire made of the same material and of the same length, are connected in series and in parallel. Which of the following statement is correct ?
(A) In series, fine wire ; and in parallel, thick wire will liberate more energy.
(B) In series, fine wire will liberate less energy ; while in parallel more energy.
(C) In series, thick wire will liberate more energy ; while in parallel, less energy.
(D) Both will liberate equal energy. *(N.C.E.R.T. 1972)*

29. If two bulbs of wattage 25 and 100 respectively each rated at 220 volt are connected in series with the supply of 440 volt, which bulb will get fused ?
(A) 100-watt bulb (B) 25-watt blub
(C) none of them (D) both of them
(Karnataka Entrance , 1988 ; M.N.R. 1988)

30. If a wire of resistance 20 Ω is covered with ice and a voltage of 210 V is applied across the wire, then the rate of melting of ice is
(A) 6·6 g s^{-1} (B) 4·2 g s^{-1}
(C) 2·1 g s^{-1} (D) 1·1 g s^{-1} *(A.F.M.C. 1997)*

31. Household electrical appliances are not usually connected in series, because
(A) switching off an appliance would switch off the rest.
(B) a fuse would blow, as soon as one appliance is used.
(C) power consumption would be very much greater.
(D) the appliances would get damaged due to high current.
(Karnataka Entrance, 1991)

32. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled, if
(A) both the length and radius of wire are halved.
(B) both length and radius of wire are doubled.
(C) the radius of wire is doubled.
(D) the length of the wire is doubled. *(I.I.T. 1980)*

33. A resistor R_1 dissipates the power P, when connected to certain voltage supply. If a resistor R_2 is put in series with R_1, the power dissipated by R_1
(A) decreases. (B) increases.
(C) remains same.
(D) Any of the above, depending upon the relative values of R_1 and R_2. *(C.P.M.T. 1985)*

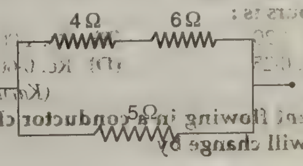
34. Two identical batteries each of e.m.f. E = 2 V and internal resistance $r = 1\ \Omega$ are available to produce heat in an external resistance R = 0·5 Ω by passing a current through it. The maximum Joulean power that can be developed across R using these batteries is
(A) 1·28 W (B) 2·0 W
(C) 8/9 W (D) 3·2 W (C.B.S.E. 1990)

35. A lamp of 6 V and 30 W is used in a laboratory, but the supply is 220 V. For this, following may be done :
(i) A resistance may be used in parallel with the lamp.
(ii) A resistance may be used in series with the lamp.
(iii) The resistance should be of 42·8 Ω.
(A) *(i)*, *(ii)* and *(iii)* (B) *(i)* and *(ii)*
(C) *(i)* and *(iii)* (D) *(ii)* or *(iii)* (N.D.A. 1994)

36. It is well known that the resistance of the filament of an electric bulb changes with temperature. If an electric bulb rated 220 volt-100 watt is connected to a source of voltage 220 × 0·8 volt, then actual power will be
(A) 100 × 0·8 watt. (B) $100 \times (0{\cdot}8)^2$ watt.
(C) more than 100 × 0·8 watt, but less than 100 watt.
(D) more than $100 \times (0{\cdot}8)^2$ watt, but less than 100 × 0·8 watt.
(C.P.M.T. 1989)

37. You are given resistance wire of length 50 cm and a battery of negligible resistance. In which case, the maximum amount of heat is generated ?
(A) When the wire is connected across the battery directly.
(B) When the wire is divided into two parts and both the parts are connected across the battery in parallel.
(C) When the wire is divided into four parts and all the four parts are connected across the battery in parallel.
(D) When only half the wire is connected across the battery.
(N.C.E.R.T. 1974)

38. In the circuit as shown in figure, the heat produced in the 5 Ω resistor due to the current flowing in it is 10 cal s^{-1}. The heat generated in 4 Ω is



(A) 1 cal s^{-1} (B) 2 cal s^{-1}
(C) 3 cal s^{-1} (D) 4 cal s^{-1} *(I.I.T. 1985)*

39. A battery of emf 10 V and internal resistance 0·5 ohm is connected across a variable resistance R. The value of R, for which the power delivered in it is maximum, is given by *(C.B.S.E. 1992)*
(A) 2·0 ohm (B) 0·25 ohm
(C) 1·0 ohm (D) 0·5 ohm

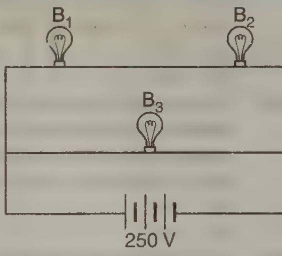
40. A factory is served by a 220 V power supply line. In a circuit protected by a fuse marked 10 A, the maximum number of 100 W lamps in parallel that can be turned on is
(A) 11 (B) 22
(C) 31 (D) 45 *(C.E.T. 1998)*

41. What is immaterial for an electric fuse ?
(A) Its specific resistance
(B) Its radius
(C) Its length
(D) Current flowing through it
(E) None of above. *(M.N.R. 1985)*

42. A fuse wire with a radius of 1 mm blows at 1·5 A. If the fuse wire of the same material should blow at 3·0 A, the radius of the wire must be
(A) $4^{1/3}$ mm (B) 2 mm
(C) 0·5 mm (D) 8·0 mm
(Karnataka Entrance , 1991)

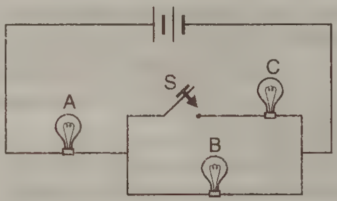
43. Forty electric bulbs are connected in series across a 220 V supply. After one bulb gets fused, the remaining thirty nine bulbs are again connected in series across the same supply. The illumination will be
(A) equal in both the cases.
(B) more with 40 bulbs than with 39.
(C) more with 39 bulbs than with 40.
(D) in the ratio $40^2 : 39^2$.

44. A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source as shown in the figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, E and B_3 respectively. Then



(A) $W_1 > W_2 = W_3$
(B) $W_1 > W_2 > W_3$
(C) $W_1 < W_2 = W_3$
(D) $W_1 < W_2 < W_3$

45. Three identical electric bulbs A, B and C are connected a shown :



What changes occur in brightness of the bulbs, when th switch is closed ?
(A) Brightness of A increases, but that of B decreases.
(B) Brightness of A remains the same, but that of decreases.
(C) Brightness of both A and B decreases.
(D) Brightness of A increases, but that of B remains th same.

46. If 2·2 kW power is transmitted through a 10 Ω line a 22,000 V, the power loss in the form of heat will be
(A) 0·1 W (B) 1 W
(C) 10 W (D) 100 W *(M.P. 199*

Chapter 4. Chemical effects of Current

PART I. TEXT-BASED QUESTIONS

1. A voltameter is an instrument
(A) to measure the potential difference.
(B) to determine the electrochemical equivalent.
(C) to find the electrical power.
(D) made up of a pile of voltmeters.
(Karnataka Entrance, 1993)

2. In an electrolyte, electric conduction is due to the flow of :
(A) electrons (B) negative ions only
(C) positive ions only
(D) both the negative and positive ions.

3. When current is passed through water voltameter, hydrogen is liberated at :
(A) anode (B) cathode
(C) escapes to atmosphere
(D) remains in water.

4. When a current is passed through water acidified with a dilute sulphuric acid, the gases formed at the platinum electrodes are :
(A) 1 volume hydrogen (cathode) and 2 volume oxygen (anode)
(B) 2 volume hydrogen (cathode) and 1 volume oxygen (anode)
(C) 1 volume hydrogen (cathode) and 1 volume oxygen (anode)
(D) 1 volume oxygen (cathode) and 2 volume hydrogen (anode) *(Karnataka Entrance, 1994)*

5. Copper sulphate solution has a direct current passe through it using platinum electrodes. The substance liberated at the electrode are :
(A) oxygen at anode and hydrogen at cathode.
(B) hydrogen at anode and oxygen at cathode.
(C) hydrogen at anode and copper at cathode.
(D) oxygen at anode and copper at cathode.

6. The current inside a copper voltameter
(A) is half the outside value.
(B) is the same as the outside value.
(C) is twice the outside value.
(D) depends on the concentration of $CuSO_4$.
(Roorkee, 199

7. The electrochemical equivalent of a metal is $3{\cdot}3 \times 10$ kg C^{-1}. The mass of the metal liberated at the cathod when a 3 A current is passed for 2 s will be :
(A) $1{\cdot}1 \times 10^{-7}$ kg (B) $6{\cdot}6 \times 10^{-7}$ kg
(C) $9{\cdot}9 \times 10^{-7}$ kg (D) $19{\cdot}8 \times 10^{-7}$ kg
(S.C.R.A. 199

8. If nearly 10^5 C of charge liberates 1 gram equivalent o aluminium, then the amount of aluminium (equivaler weight = 9) is deposited through electrolysis in 20 min b a current of 50 A will be
(A) 0·6 g (B) 1·8 g
(C) 5·4 g (D) 10·8 g *(C.B.S.E. 199*

9. In an electroplating experiment, *m* gram of silver deposited, when 4 ampere of current flows for 2 minute

The amount (in gram) of silver deposited by 6 ampere of current flowing for 40 seconds will be

(A) 4 m (B) $m / 2$
(C) $m / 4$ (D) 2 m *(M.N.R. 1991)*

10. **In producing chlorine through the process of electrolysis, 100 kW power at 125 V is being consumed. E.C.E. for chlorine = $0{\cdot}367 \times 10^{-6}$ kg C^{-1}. How much chlorine per minute is being liberated ?**
(A) 367 g (B) 3·67 g
(C) 17·6 g (D) 18·6

11. **A current passing through a copper voltameter deposits 0·002 kg of copper on the cathode plate in 100 minutes. If there are 10^{25} atoms in 1 kg of copper, the electric charge delivered to the cathode by Cu^{++} ions per second will be**
(A) 0·53 C (B) 0·71 C
(C) 1·06 C (D) 10·06 C *(A.M.U. 1999)*

12. **In view of chemical effect of electricity, Faraday constant is equal to the ratio of**
(A) electrochemical equivalent to the chemical equivalent.
(B) electrochemical equivalent to the gram equivalent.
(C) gram equivalent to the electrochemical equivalent.
(D) chemical equivalent to the electrochemical equivalent. *(S.C.R.A. 1994)*

3. **In electrolysis, the mass deposited on an electrode is directly proportional to**
(A) current (B) square of current
(C) concentration of solution
(D) inverse of current. *(C.B.S.E. 2000)*

14. **The masses of different substances liberated in electrolysis by the same quantity of electricity are proportional to their :**
(A) atomic masses.
(B) ratios of the atomic mass and valency.
(C) valencies.
(D) products of atomic mass and valency.

5. **Silver and zinc voltameters are connected in series and a current is passed through them for time *t*. If the mass of zinc liberated is *m*, the mass of silver deposited would be nearly**
(A) m (B) 1·7 m
(C) 2·4 m (D) 3·3 m *(N.C.E.R.T. 1976)*

6. **A copper voltameter and silver voltameter are connected in series in a circuit. The rate of the increase in the weights of the cathodes in the two voltameters will be :**
(A) in the ratio of the densities of Cu and Ag.
(B) in the ratio of the atomic weights of Cu and Ag.
(C) in the ratio of half the atomic weight of Ag to the atomic weight of Cu.
(D) in the ratio of half the atomic weight of Cu to the atomic weight of Ag.

7. **Faraday constant F, Avogadro number N and electronic charge *e* are related with each other by :**
(A) $F = \frac{N}{e}$ (B) $F = \frac{e}{N}$
(C) $F = N e$ (D) $F = N e^2$

18. **On passing a charge of 2 faraday through a copper voltameter, the mass of copper ions liberated will be :**
(A) 128 g (B) 16 g
(C) 32 g (D) 64 g

19. **The commercial aluminium (trivalent, atomic weight=27) is generally obtained by electrolysis. The total charge required (1 faraday = 96500 C) to deposit 9 g of aluminium is :**
(A) 3266·6 C (B) 96500 C
(C) 65900 C (D) 289·5 C *(M.N.R. 1982)*

20. **Electroplating does not help metals to**
(A) give fine finish to the surface.
(B) give shining appearance.
(C) become hard.
(D) to protect against corrision. *(A.I.I.M.S. 1998)*

21. **Which of the following statements is not correct ?**
(A) In a voltaic cell, the anode and cathode are copper and zinc rod respectively.
(B) Leclanche cell suffers from polarisation.
(C) Dry cell is basically a lechanche cell.
(D) Edison alkali cell is a primary cell.

22. **The equation**
$$Zn + 2\,NH_4Cl \longrightarrow 2\,NH_3 + ZnCl_2 + 2\,H + 2\,e$$
represents the :
(A) dry cell (B) Daniel cell
(C) Leclanche cell (D) none of these. *(A.I.I.M.S. 1988)*

23. **Which of the following statements is not correct ?**
(A) A primary cell is an electrochemical cell.
(B) After charging, a primary cell can be again put to use.
(C) Dry cell is a primary cell.
(D) Leclanche cell is used in experiments, where constant supply of current is not needed.

24. **The efficiency of a modern accumulator is of the order of :**
(A) 100% (B) 70%
(C) 50% (D) 33%

PART II. THOUGHT-BASED QUESTIONS

25. **The E.C.E. of a substance is equal to the product of its chemical equivalent and the E.C.E. of :**
(A) oxygen (B) hydrogen
(C) copper (D) platinum

26. **How much electricity must pass through acidulated water to release 22,400 cm^3 of hydrogen at N.T.P. ?**
(A) 96500 C (B) 193000 C
(C) 22·4 C (D) 95·5 C

27. **The electric charge for electrode deposition of one gram equivalent of substance is**
(A) one ampere per second.
(B) 96500 coulomb per second.
(C) one ampere per hour.
(D) charge on one mole electron. *(I.I.T. 1984)*

28. **In a lead accumulator, the plates are placed close to each other ; as**
(A) it makes the lead accumulator sturdy.
(B) it increases the e.m.f. of the accumulator.
(C) it decreases the internal resistance of the accumulator.
(D) it increases the life span of the accumulator.

29. **Which of the following statements is not correct ? A lead accumulator of 20 ampere-hour can supply :**
(A) 20 A current for 20 hours.
(B) 1 A current for 20 hours.
(C) 20 A current for 1 hour.
(D) 0·1 A current for 200 hours.

Chapter 5. Thermoelectric effect

RT I. TEXT-BASED QUESTIONS

1. **When current flows through a conductor, heat is produced. This is**
(A) Seebeck effect (B) Thomson effect
(C) Peltier effect (D) Joule effect

2. **Which of the following statements is correct ? The thermo e.m.f. of a thermocouple :**
(A) is independent of the nature of the pair of metals used.
(B) depends upon the temperature of cold junction alone.
(C) depends upon the temperature of hot junction alone.
(D) depends upon the temperature of cold junction and the

3. For a thermocouple, the thermo e.m.f. may be:
(A) zero (B) positive
(C) negative (D) any of the above.

4. As the temperature of hot junction is increased, the thermo-e.m.f. produced :
(A) is always positive.
(B) may be positive or negative.
(C) always increases with temperature.
(D) first increases with temperature and then decreases, but never becomes negative.

5. The temperature of a hot junction, at which the thermo-e.m.f. is maximum, is
(A) neutral temperature.
(B) transitional temperature.
(C) temperature of inversion.
(D) null temperature. *(Karnataka Entrance, 1984)*

6. Above neutral temperature, thermo-e.m.f. :
(A) changes sign. (B) is constant.
(C) increases with rise in temperature.
(D) decreases with rise in temperature.

7. The thermoelectric power is the rate of change of thermo-e.m.f. with
(A) temperature (B) time
(C) resistance (D) current.
(Karnataka Entrance, 1990)

8. The thermo e.m.f. E (in volt) of a certain thermocouple is found to vary with temperature θ (in °C) in accordance with the relation,

$$E = 40\,\theta = \frac{\theta^2}{20},$$

where θ is the temperature of the hot junction, the cold junction being kept at 0°C. The neutral temperature of thermocouple is
(A) 100°C (B) 200°C
(C) 300°C (D) 400°C

9. A thermopile is a number of thermocouples connected in
(A) series (B) parallel
(C) either series or parallel
(D) both series and parallel. *(Karnataka Entrance , 1989)*

10. Thermopile is used for :
(A) measuring the e.m.f.
(B) measuring the current.
(C) converting heat energy into electrical energy.
(D) measuring and detecting the heat radiation.

11. When different parts of a metal are kept at different temperatures and current is passed through it, the heat is either evolved or absorbed. This effect is called
(A) Peltier effect (B) Seebeck effect
(C) Thomson effect (D) none of the above.

PART II. THOUGHT-BASED QUESTIONS

12. The respective temperature of cold and hot junctions of Sb-Bi and a Cu-Fe thermocouples are same. If E_1 and E_2 a thermo-e.m.f. developed in the two cases, then :
(A) $E_1 = E_2$ (B) $E_1 < E_2$
(C) $E_1 > E_2$ (D) cannot be predicted.

13. A copper wire AB and an iron wire AC are joined at A. T junction A is kept at 0°C and the free ends of t arrangement of the two wires are maintained at 100° Then, a potential difference exists between
(A) the two ends of the copper wire.
(B) the two ends of the iron wire.
(C) the free ends of the arrangement of two wires.
(D) the ends of the two wires forming the junction.

14. At the temperature of inversion, thermo-electric current a Cu-Fe thermocouple
(A) flows from Cu to Fe through hot junction.
(B) flows from Fe to Cu through hot junction.
(C) is maximum.
(D) does not depend upon the temperature of cold junctic

15. The plot showing the variation of thermo-e.m.f. wi temperature of the hot junction of a thermocouple is
(A) parabolic (B) circular
(C) hyperbolic (D) elliptical.

16. The cold junction of a silver-lead thermocouple is kept 0°C. If the values of constants α and β for the thermocoup are 2·50 μV °C^{-1} and 0·012 μV °C^{-2} respectively, then increasing the temperature of the other junction,
(A) no thermo-e.m.f. will be produced.
(B) no thermoelectric current will be produced.
(C) no neutral temperature will be obtained.
(D) no inversion temperature will be obtained.

17. In a certain range of temperature, a copper-constant thermocouple produces a thermo-e.m.f. of 40 μV °C^{-1}. galvanometer of resistance 10 Ω and capable of detecti current of the order of 1 μA is used to measure t temperature. What is the smallest temperature that c be measured by the galvanometer ?
(A) $\frac{1}{400}$°C (B) $\frac{1}{4}$°C
(C) 4 °C (D) 400 °C

ANSWERS TO MULTIPLE CHOICE QUESTIONS

Chapter 1. Electric current, Resistance and E.M.F.

1. (B)	2. (A)	3. (D)	4. (C)	5. (A)	6. (B)	7. (C)	8. (B)	9. (D)	10. (D)
11. (C)	12. (D)	13. (C)	14. (D)	15. (C)	16. (C)	17. (C)	18. (A)	19. (D)	20. (B)
21. (C)	22. (C)	23. (B)	24. (D)	25. (A)	26. (A)	27. (A)	28. (D)	29. (C)	30. (A)
31. (B)	32. (D)	33. (D)	34. (A)	35. (D)	36. (B)	37. (C)	38. (D)	39. (C)	40. (C)
41. (C)	42. (A)	43. (B)	44. (D)	45. (D)	46. (C)	47. (B)	48. (D)	49. (B)	50. (B)
51. (A)	52. (B)	53. (B)	54. (A)	55. (C)	56. (D)	57. (C)	58. (B)	59. (C)	60. (D)
61. (A)	62. (C)	63. (B)	64. (D)	65. (A)	66. (B)	67. (A)	68. (A)	69. (A)	70. (A)
71. (A)	72. (A)	73. (D)	74. (D)	75. (D)	76. (B)	77. (C)	78. (B)	79. (C)	80. (D)
81. (C)	82. (D)	83. (C)	84. (D)	85. (B)	86. (B)	87. (C)	88. (A)	89. (B)	90. (A)
91. (C)	92. (C)								

Chapter 2. Electrical measurements

1. (A)	2. (A)	3. (A)	4. (D)	5. (A)	6. (A)	7. (C)	8. (B)	9. (B)	10. (C)
11. (C)	12. (A)	13. (A)	14. (B)	15. (C)	16. (A)	17. (C)	18. (B or C)	19. (A)	20. (D)
21. (A)									
22. (D)	23. (C)	24. (D)	25. (B)	26. (C)	27. (D)	28. (B)	29. (B)	30. (C)	31. (A)
32. (A)	33. (B)	34. (A)	35. (A)	36. (D)					

hapter 3. Thermal effects Of Currents

1. (D) **2.** (A) **3.** (B) **4.** (A) **5.** (A) **6.** (A) **7.** (A) **8.** (A) **9.** (A) **10.** (D)
11. (B) **12.** (C) **13.** (D) **14.** (A) **15.** (B) **16.** (D) **17.** (A) **18.** (D) **19.** (B) **20.** (C)
21. (D) **22.** (D) **23.** (A) **24.** (B) **25.** (B) **26.** (D) **27.** (B) **28.** (A) **29.** (B) **30.** (A)
31. (A) **32.** (B) **33.** (A) **34.** (B) **35.** (D) **36.** (D) **37.** (C) **38.** (B) **39.** (D) **40.** (B)
41. (C) **42.** (B) **43.** (C) **44.** (D) **45.** (A) **46** (A)

hapter 4. Chemical effects of Currents

1. (B) **2.** (D) **3.** (B) **4.** (B) **5.** (D) **6.** (B) **7.** (D) **8.** (C) **9.** (B) **10.** (C)
11. (C) **12.** (C) **13.** (A) **14.** (B) **15.** (D) **16.** (D) **17.** (C) **18.** (D) **19.** (B) **20.** (C)
21. (D) **22.** (C) **23.** (B) **24.** (B) **25.** (B) **26.** (B) **27.** (D) **28.** (C) **29.** (A)

hapter 5. Thermoelectric effects

1. (D) **2.** (D) **3.** (D) **4.** (B) **5.** (A) **6.** (D) **7.** (A) **8.** (D) **9.** (A) **10.** (D)
11. (C) **12.** (C) **13.** (A, B, C & D) **14.** (B) **15.** (A) **16.** (C & D) **17.** (B)

UNIT
III

MAGNETIC EFFECTS OF CURRENTS & MAGNETISM

1. **Magnetic Effect of Current**
2. **Motion of Charged Particle**
3. **Magnetic Dipole**
4. **Earth's Magnetism**
5. **Classification of Magnetic Materials**

> "Microscopic currents within magnets account for their magnetism."
>
> – **Andre-Marie Ampere**

CHAPTER 1

Magnetic Effect of Current

1.01. CONCEPT OF MAGNETIC FIELD

The space aound a current carrying conductor, in which its magnetic effect can be xperienced, is called magnetic field.

When current is passed through a conductor, it modifies the space around the onductor. If a magnetic pole[*] is placed at a point near the conductor, the magnetic ole experiences force due to the current carrying conductor. The force between the nagnetic pole and the current carrying conductor may be visualised as a two-step rocess. The current through the conductor produces something, called ***magnetic eld***, in the space around the conductor and this magnetic field exerts a force on ny magnetic pole placed in the field. The magnetic field has its own existence and s present even if there is no magnetic pole to experience the force. The magnetic eld due to a current carrying conductor falls off inversely as the distance from the onductor.

The electric and magnetic fields differ from each other on the following ccounts :

1. Whereas the source of electric field is electric charge (a scalar quantity), the ource of magnetic field is current element (a vector quantity). To elaborate, the ource of magnetic field is not a magnetic charge, the analogue of electric charge. is because, an isolated magnetic pole does not exist.

2. Further, there is a qualitative difference between the lines of electric field due a charge distribution with net charge and the lines of magnetic field due to a urrent distribution. Whereas the lines of electric field due to a charge distribution tart from one point and end at some other point, the lines of magnetic field due a current element start and end at the same point *i.e.* they form closed loops.

Quotable Quote

"I have little to say on M. Oersted's discovery, for I must confess I do not quite understand it."

– Michael Faraday

Watch out !

The source of electric field is electric charge. Since an isolated magnetic pole does not exist, source of magnetic field is not a magnetic charge (analogue of electric charge) !

1.02. OERSTED'S EXPERIMENT

In 1820, the Danish scientist Oersted discovered the magnetic effect of electric urrent.

Oersted found that when a wire AB carrying current was held over a magnetic eedle NS, such that electric current passed from the end A of the wire to the end [Fig. 1.01], the north pole of the needle got deflected towards west (into the paper). n holding the wire below the magnetic needle, the north pole of the needle got eflected in the opposite direction. It indicated that magnetic lines of force below nd above the needle are directed in the opposite directions. Thus, Oersted's xperiment led to the discovery that when electric current passes through a onductor, magnetic field is produced around it. If a strong current is passed through conductor, then magnetic field produced around the conductor is also very strong nd the earth's magnetic field may be neglected in its comparison. In such a case, nagnetic lines of force near the conductor are found to be circular. The direction of nagnetic field lines due to a straight current carrying conductor may be found by pplying ***right hand thumb rule*** or ***Maxwell's cork screw rule***[†].

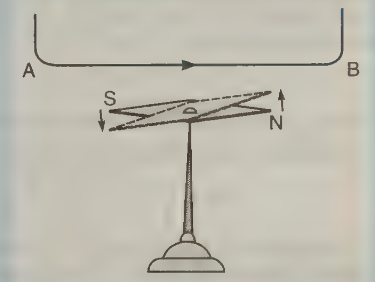


Fig. 1.01

When electric current is passed through a circular conductor, the magnetic field nes near the centre of the conductor are almost straight. The direction of magnetic eld lines may be found by applying ***right hand fist rule.***

1.03. BIOT SAVART'S LAW

Biot Savart's law is a basic law of electricity and magnetism. It is used to find ne magnetic field at a point due to a current carrying conductor.

---

[*] Till today, an isolated pole (or magnetic monopole) has not been detected.

[†] For discussion of the rules, refer to section 1.04.

It states that the magnetic field produced due to a small conductor of length *dl* carrying a current I at a point at distance *r* is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{r^2},$$

where θ is the angle between the direction of flow of current and the line joining the small conductor to the observation point.

Consider a conductor XY, through which current I is flowing [Fig. 1.02]. Due to flow of current, a magnetic field is produced around the conductor XY. Suppose that we have to find magnetic field at point P due to the current flowing through the conductor XY. Consider a small elementary portion AB = *dl* of the conductor. Let *r* be the distance of the observation point P (the point, at which strength of magnetic field is to be found) from the centre C of the elementary portion AB. Biot and Savart found that the small magnetic field produced at point P due to current flowing through the elementary portion AB is

(*i*) *directly proportional to the current i.e.*

$$dB \propto I$$

(*ii*) *directly proportional to the length of the elementary portion i.e.*

$$dB \propto dl$$

(*iii*) *directly proportional to sine of the angle θ between the direction of the flow of current and the line joining the elementary portion to the observation point i.e.*

$$dB \propto \sin\theta$$

It may be noted that for all the points along the length of the current element, θ = 0 and hence at such points, *d*B = 0. Further, for all points lying in a plane to which the current element is perpendicular, θ = 90° and hence at points at a given distance in this plane, the magnetic field *d*B is maximum.

(*iv*) *inversely proportional to the square of the distance r of the point P from the elementary portion i.e.*

$$dB \propto \frac{1}{r^2}$$

It is known as ***inverse square law.***

Combining all the above factors, we have

$$dB \propto \frac{I\, dl \sin\theta}{r^2}$$

or

$$dB = \frac{k\, I\, dl \sin\theta}{r^2}, \qquad \text{...(1.01)}$$

where *k* is the constant of proportionality. Its value depends upon the nature of medium between the observation point and the current carrying conductor and the system of units used to measure the quantities involved. If the conductor and the observation point lie in vacuum, then in SI,

$$k = \frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{T\,A^{-1}\,m} = 10^{-7}\ \mathrm{Wb\,A^{-1}\,m^{-1}} \quad (\because 1\,\mathrm{T} = 1\,\mathrm{Wb\,m^{-2}})$$

Here, μ_0 is called ***absolute permeability*** of vacuum or free space.

Therefore, in SI, Biot Savart's law may be expressed as

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl \sin\theta}{r^2} \qquad \text{...(1.02)}$$

The equation (1.02) gives the magnitude of the magnetic field produced due to a small current element. When the current flows through the elementary portion AB in the direction as shown in Fig. 1.01, the magnetic field at point P is directed perpendicular to the plane of paper and in inward direction.

The direction of magnetic field produced can be known easily by expressing Biot Savart's law in vector form. If $\hat{r}$ is a unit vector along CP, then the small magnetic field (both in magnitude and direction) produced at point P is given by

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \hat{r}}{r^2} \qquad \text{...(1.03)}$$

Here, $I\,\vec{dl}$ is taken as a vector in the direction of flow of current.

If $\overrightarrow{CP} = \vec{r}$, then the above result may be expressed as

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \vec{r}}{r^3} \qquad \text{...(1.04)}$$

The jargon .....

Biot Savart's law is also known as Laplace's law.

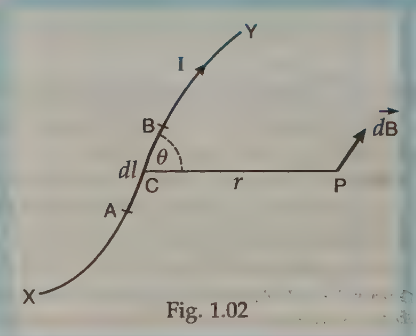


Fig. 1.02

Self-test Question ?

When the observation point lies along the length of the current element $I\vec{dl}$, magnetic field is zero. On the other hand, for a given distance from the current element, field is maximum, when the observation point is located along a direction perpendicular to it. Why ?

Self-test Question ?

Show that in terms of current density $\vec{j}$, equation (1.04) may be expressed as

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{\vec{j} \times \vec{r}}{r^3}\, dV$$

The magnetic field due to the whole conductor XY is obtained by integrating $\vec{dB}$ over the entire length of the conductor *i.e.* magnetic field due to the whole conductor at the point P is given by

$$\vec{B} = \int \vec{dB} = \frac{\mu_0}{4\pi} \int \frac{I\,\vec{dl} \times \hat{r}}{r^2} \qquad ...(1.05)$$

The integral in the above equation can be evaluated by knowing the proper limits of the problem.

Direction of $\vec{dB}$. The direction of $\vec{dB}$ is same as that of the vector $I\,\vec{dl} \times \hat{r}$. It follows that at point P, the vector $\vec{dB}$ *is perpendicular to the plane of the paper and is directed inwards.*

1.04. RULES TO FIND DIRECTION OF MAGNETIC FIELD

The direction of magnetic field due to a current carrying conductor can be found by applying the following two rules :

Right hand thumb rule. *If we grasp the conductor in the palm of the right hand so that the thumb points in the direction of the flow of current, then the direction in which the fingers curl, gives the direction of magnetic field lines.* This rule is illustrated in Fig. 1.03.

It follows that magnetic field is in the form of concentric circles, whose centres lie on the straight conductor.

Maxwell's cork screw rule. *If a right handed cork screw is rotated so that it moves in the direction of flow of current through the conductor, then the direction of the rotation of the screw gives the direction of magnetic field lines.* This rule is illustrated in Fig. 1.04.

For the current flowing through the conductor in the direction shown in Figs. 1.03 or 1.04, both the rules predict that magnetic field lines will be in anticlockwise direction, when seen from above.

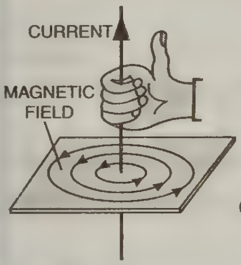


Fig. 1.03

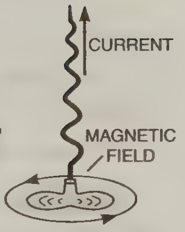


Fig. 1.04

1.05. MAGNETIC FIELD DUE TO CURRENT FLOWING IN A STRAIGHT CONDUCTOR

Consider a long straight conductor XY, through which current I is flowing from the ends X to Y as shown in Fig. 1.05. Then, magnetic field will be set up around the conductor. Suppose that we want to find magnetic field at point P, which is located at a perpendicular distance PM = a from the conductor. To do so, consider an infinitesimally small portion AB of length dl. Suppose that PA = r and $\angle$ PAM = θ. Then, according to Biot-Savart's law, strength of magnetic field at point P due to small portion AB is given by

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \hat{r}}{r^2},$$

where $I\,\vec{dl}$ is a vector in the direction of flow of current and $\hat{r}$ is a unit vector along AP. The direction of magnetic field $\vec{dB}$ at point P due to the small current element AB points in the direction $I\,\vec{dl} \times \hat{r}$ *i.e.* perpendicular to the plane of the paper and in inward direction. The magnitude of the small magnetic field at point P due to current element AB is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2} \qquad ...(1.06)$$

Since the element AB is infinitesimally small, practically $\angle$ PBM = $\angle$ PAM = θ. Drop AN perpendicular to BP. Then, from right angled triangle ABN, we have

$$AN = dl \sin\theta \qquad ...(1.07)$$

Let $\angle$ MPA = ϕ and $\angle$ APB = $d\phi$. Then,

$$AN = r\,d\phi \qquad ...(1.08)$$

From equations (1.07) and (1.08), we have

$$dl \sin\theta = r\,d\phi$$

Substituting for $dl \sin\theta$ in equation (1.05), we have

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,(r\,d\phi)}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{I\,d\phi}{r} \qquad ...(1.09)$$

From right angled triangle AMP, we have

$$\cos\phi = \frac{a}{r} \quad \text{or} \quad r = \frac{a}{\cos\phi}$$

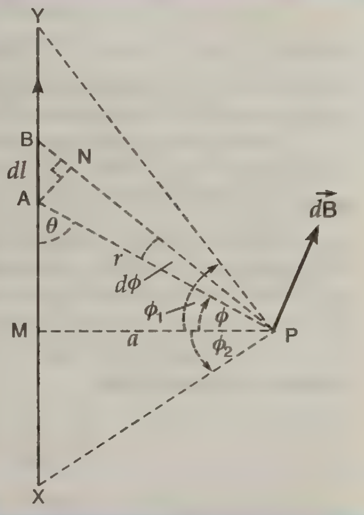


Fig. 1.05

Substituting for r in equation (1.09), we have

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \cos\phi \, d\phi$$

Join PX and PY. Let $\angle$ MPY $= \phi_1$ (in clockwise direction) and $\angle$ MPX $= -\phi_2$ (in anticlockwise direction). Then, magnetic field due to the whole length of the conductor can be found by integrating dB given by the above equation within limits $-\phi_2$ to ϕ_1. Thus, the magnetic field due to current flowing through the whole conductor XY,

$$B = \int_{-\phi_2}^{\phi_1} dB = \int_{-\phi_2}^{\phi_1} \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \cos\phi \, d\phi = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \int_{-\phi_2}^{\phi_1} \cos\phi \, d\phi$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \left| \sin\phi \right|_{-\phi_2}^{\phi_1} = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \left| \sin\phi_1 - \sin(-\phi_2) \right|$$

or $$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin\phi_1 + \sin\phi_2) \qquad \text{...(1.10)}$$

It gives the magnetic field due to a current carrying straight conductor of finite length.

Self-test Question

Show that magnetic field due to a
infinitely long current carrying conducto
at a point near its one end is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}$$

Special case. If the conductor is infinitely long, then $\phi_1 = \phi_2 = \frac{\pi}{2}$. Hence, the magnetic field at point P due to current flowing through an infinitely long conductor is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2} \right)$$

or $$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} \qquad \text{...(1.11)}$$

The direction of magnetic field is determined by the cross product of the vector $I\vec{dl}$ and $\hat{r}$ *i.e.* by $I\vec{dl} \times \hat{r}$, where $\hat{r}$ is a unit vector along AP. Therefore, at point P, the direction of magnetic field due to the small current element or due to the whole conductor will be along perpendicular to the plane of paper and in inward direction.

The direction of the magnetic field can also be found by making use of the right hand thumb rule or Maxwell's cork screw rule described in section 1.04.

1.06. MAGNETIC FIELD AT THE CENTRE OF A CIRCULAR LOOP

Consider a circular loop of radius a, centre O and carrying current I as shown in Fig. 1.06. In order to find magnetic field at point O, the centre of the loop, consider an infinitesimally small length AB = dl of the loop. Join the centre O of the loop to the middle point C of the current element AB. Obviously, $\angle$ACO is equal to 90°.

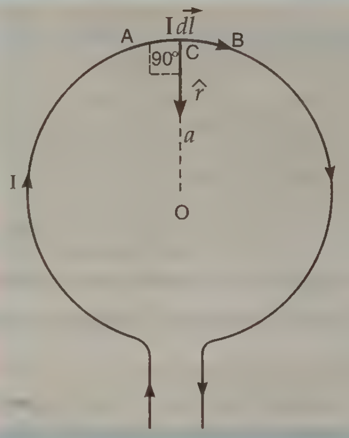


Fig. 1.06

According to Biot-Savart's law, the magnetic field at point O due to the current element AB is given by

$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\vec{dl} \times \hat{r}}{a^2},$$

where $I\vec{dl} = I(\vec{AB})$ is a vector in the direction of flow of current through AB and $\hat{r}$ is a unit vector along CO. The magnetic field $\vec{dB}$ at point O due to the small current element AB points in the direction of $I\vec{dl} \times \hat{r}$ *i.e.* perpendicular to the plane of the loop and in inward direction. Since angle between $I\vec{dl}$ and $\hat{r}$ is 90°, the magnitude of magnetic field at point O due to the current element AB is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{a^2}$$

Hence, the magnetic field at point O due to the whole circular loop is given by

$$B = \oint dB = \oint \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{a^2} = \frac{\mu_0}{4\pi} \cdot \frac{I}{a^2} \oint dl$$

Now, $\oint dl =$ circumference of the loop $= 2\pi a$

Key point

When current I flows through tw
concentric circular conductors of radii a
and a_2, net magnetic field at their centr
is given by

$$B = \frac{\mu_0}{4\pi} \cdot I \left(\frac{1}{a_1} + \frac{1}{a_2} \right),$$

when current flows in same direction an
by the relation

$$B = \frac{\mu_0}{4\pi} \cdot I \left(\frac{1}{a_1} - \frac{1}{a_2} \right),$$

when current flows in opposite directio
through the two circular conductors.

Therefore,
$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a^2} (2\pi a)$$

or
$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a} \qquad ...(1.12)$$

In case the circular loop has n turns, then
$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} \qquad ...(1.13)$$

Direction of magnetic field. The whole circular loop may be supposed to be divided into a large number of current elements, such as AB. Since the direction of magnetic field produced at point O due to each current element is same, the magnetic field due to whole of the circular loop will be same as that due to the current element AB *i.e.* magnetic field B will be perpendicular to the plane of the loop and in inward direction. In general, if current through a loop flows in clockwise direction, then direction of magnetic field at its centre is perpendicular to the plane of the loop and in inward direction. On the other hand, if the current flows through the loop in anticlockwise direction, then direction of magnetic field at its centre is perpendicular to the plane of the loop and in outward direction.

Self-test Question ?

Show that magnetic field due to a part of current carrying circular conductor is
$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}\,\theta,$$
where θ is angle, which the circular part subtends at its centre.

Key point ✓

Magnetic field due to a current carrying circular conductor is non-uniform. However, it may be considered as uniform (for practical purposes) at its centre.

Fig. 1.08 explains a useful method to know the direction of the magnetic field, when the current flows through a circular loop. The face of the loop, in which the current appears to flow anticlockwise [Fig. 1.07 (*a*)], acts as magnetic north pole. On the other hand, if the current appears to flow through the face of the loop in clockwise direction [Fig. 1.07 (*b*)], then the face acts as magnetic south pole. In both the cases, the magnetic field at the centre of the loop is directed along normal to the plane of the loop.

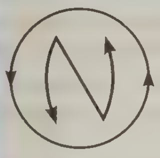


Fig. 1.07 (*a*)

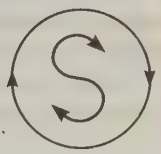


Fig. 1.07 (*b*)

1.07. MAGNETIC FIELD AT A POINT ONTHE AXIS OF A LOOP

Consider a circular loop of radius a, centre O and carrying a current I as shown in Fig. 1.08. The plane of the loop (YZ-plane) is perpendicular to the plane of the paper, while the axis of the loop (*i.e.* OX or X-axis) lies in the plane of the paper.

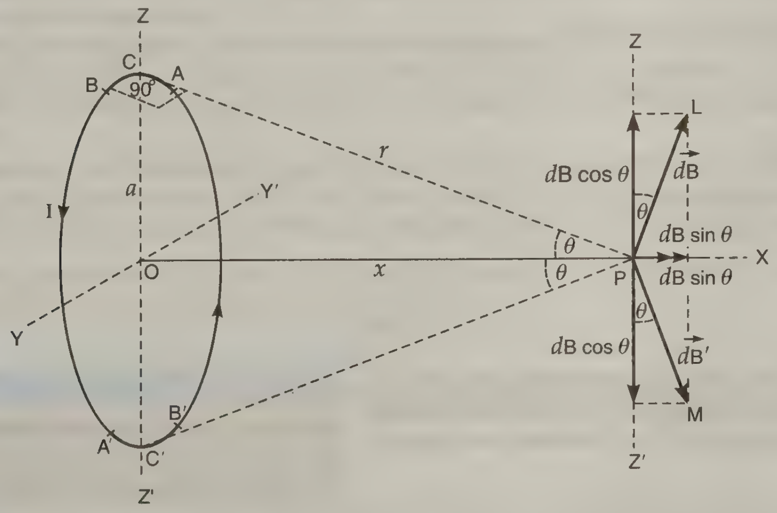


Fig. 1.08

Let P be the point on the axis of the loop at a distance OP = x from its centre O, where the magnetic field due to the circular loop has to be determined. In order to do so, consider an infinitesimally small length AB = dl of the loop. Join the point P to the middle point C of the current element AB. Suppose that CP = r. Obviously, ∠ BCP (or ∠ ACP) is equal to 90°.

According to Biot Savart's law, the magnetic field due to the current element AB at point P is given by
$$\vec{dB} = \frac{\mu_0}{4\pi} \cdot \frac{I\,\vec{dl} \times \hat{r}}{r^2},$$
where $I\,\vec{dl} = I\,(\overrightarrow{AB})$ is a vector in the direction of flow of current through AB and $\hat{r}$ is a unit vector along CP. The magnetic field $\vec{dB}$ at point P due to the small current

element AB points in the direction of vector $\mathrm{I}\,\overrightarrow{dl} \times \hat{r}$ and it has been represented by the vector $\overrightarrow{\mathrm{PL}}$. As the angle between $\mathrm{I}\,\overrightarrow{dl}$ and $\hat{r}$ is 90°, the magnitude of $\overrightarrow{d\mathrm{B}}$ is given by

$$d\mathrm{B} = \frac{\mu_0}{4\pi} \cdot \frac{\mathrm{I}\,dl}{r^2} \qquad \text{...(1.14)}$$

Let us consider the small element A′B′ = dl located just opposite to the element AB. The magnetic field $\overrightarrow{d\mathrm{B}'}$ due to the current element A′B′ has the magnitude same as that due to the current element AB but its direction is that of vector $\overrightarrow{\mathrm{PM}}$. Thus,

$$|\overrightarrow{d\mathrm{B}}| = |\overrightarrow{d\mathrm{B}'}|$$

Both $\overrightarrow{d\mathrm{B}}$ and $\overrightarrow{d\mathrm{B}'}$ can be resolved into two mutually perpendicular components along PX and ZZ′. The components of the magnetic fields due to current elements AB and A′B′ along ZZ′ cancel each other, as they are equal in magnitude and opposite in direction. The same will hold for such other pairs of current elements over the entire circumference of the loop. Therefore, due to the various current elements, the components of magnetic field only along PX will contribute to the magnetic field due to the whole loop at point P.

If $\angle \mathrm{OPC} = \angle \mathrm{OPC}' = \theta$, then $\angle \mathrm{ZPL} = \angle \mathrm{Z'PM} = \theta$. Therefore,

effective component of magnetic field due to a current element = $d\mathrm{B}\sin\theta$ (along PX)

The magnetic field at point P due to the whole loop is obtained by integrating $d\mathrm{B}\sin\theta$ over the circumference of the loop. Therefore, the magnetic field at point P due to the whole circular loop,

$$\mathrm{B} = \oint d\mathrm{B}\sin\theta = \oint \frac{\mu_0}{4\pi} \cdot \frac{\mathrm{I}\,dl}{r^2}\sin\theta = \frac{\mu_0}{4\pi} \cdot \frac{\mathrm{I}}{r^2}\sin\theta \oint dl$$

Now, $\oint dl = 2\pi a$, circumference of the loop of radius a

$$\therefore \quad \mathrm{B} = \frac{\mu_0}{4\pi} \cdot \frac{\mathrm{I}}{r^2}\sin\theta\,(2\pi a) = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \mathrm{I} a}{r^2}\sin\theta$$

From right angled triangle OPC, $\sin\theta = \frac{a}{r}$

$$\therefore \quad \mathrm{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \mathrm{I} a}{r^2} \times \frac{a}{r} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \mathrm{I} a^2}{r^3}$$

But $\quad r = (a^2 + x^2)^{1/2}$

$$\therefore \quad \mathrm{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \mathrm{I} a^2}{(a^2 + x^2)^{3/2}} \qquad \text{...(1.15)}$$

If the circular loop consists of n turns, then strength of the magnetic field at point P due to the loop will be n times the magnetic field due to a loop of single turn *i.e.* n times the magnetic field given by the equation (1.15). Therefore, for a loop of n turns,

$$\mathrm{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n \mathrm{I} a^2}{(a^2 + x^2)^{3/2}} \qquad \text{...(1.16)}$$

Key point ✓

For a point on the axis of a circular coil carrying current, magnetic field is maximum at the centre of the coil.

The equations (1.15) and (1.16) give the magnitude of the magnetic field produced due to the flow of current through a circular loop at a point on its axis. For the current flowing in the direction as shown in Fig. 1.10, the direction of the magnetic field produced due to the loop is along PX.

Special cases. Let us consider the following two cases :

1. ***Magnetic field at the centre of the loop.*** When the point P lies at the centre of the loop, $x = 0$. Therefore, setting $x = 0$ in equation (1.15), we have

$$\mathrm{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \mathrm{I}}{a} \qquad \text{...(1.17)}$$

It gives the magnetic field due to a current carrying circular loop of radius a at its centre and is the same as obtained in the form of equation (1.12).

For a circular loop of n turns, the expression will be

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} \qquad ...(1.18)$$

2. ***When the observation point lies far away from the centre of the loop.*** In case e observation point P lies at a distance very large as compared to the radius of e loop *i.e.* if $x >> a$, then in equation (1.15), a^2 can be neglected as compared to . Therefore, in such a case,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{x^3} \qquad ...(1.19)$$

But $\pi a^2 = A$, the area of the circular loop . Therefore, equation (1.19) becomes

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2 I A}{x^3} \qquad ...(1.20)$$

The equations (1.19) and (1.20) give the magnetic field due to a current carrying rcular loop at a far away point on the axis of the loop.

Similarly, for a loop of n turns,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I a^2}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{2 n I A}{x^3} \qquad ...(1.21)$$

1.08. AMPERE'S CIRCUITAL LAW

It states that *the line integral of magnetic field ($\vec{B}$) around any closed path or circuit equal to μ_0 (absolute permeability of free space) times the total current (I) threading the osed circuit.*

Mathematically, $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$ ...(1.22)

Ampere's circuital law can be proved, when a current threads a closed path any shape. However, before proving Ampere's circuital law for any arbitrary osed path, we first prove it for a special case *i.e.* for a circular path around a long irrent carrying conductor.

(*a*) To prove Ampere's circuital law for a circular path. Consider a long straight onductor XY, such that a current I flows through it from the end X to Y [Fig. 1.09]. ue to flow of current through the conductor, the magnetic field will be produced round it.

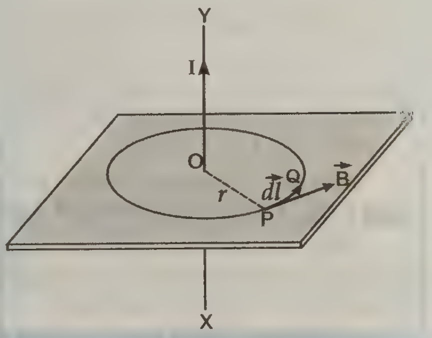


Fig. 1.09

Consider a small element of length $\overrightarrow{PQ}$ $(= \vec{dl})$ of the circular path. Let $\vec{B}$ be the rength of magnetic field at point P. According to the right hand thumb rule, the irection of magnetic field at point P is along tangent to the circular path at that point.

bviously, the vectors $\vec{B}$ and $\vec{dl}$ are along the same direction. Hence, the line integral f magnetic field over the closed circular path is given by

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl \cos 0° = \oint B\, dl \qquad ...(1.23)$$

The magnitude of magnetic field due to the current carrying straight conductor point P is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$$

In fact, the magnitude of magnetic field is same for all the points on the circular ath around the conductor. Substituting for B in equation (1.23), we have

$$\oint \vec{B} \cdot \vec{dl} = \oint \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}\, dl = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} \oint dl$$

Now, $\oint dl$ = circumference of the circle of radius $r = 2\pi r$

$$\therefore \quad \oint \vec{B} \cdot \vec{dl} = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} \cdot 2\pi r \quad \text{or} \quad \oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

It proves Ampere's circuital law.

It may be noted that the magnitude of magnetic field at a point on the circular ath changes with the change in radius of the circular path but the line integral f $\vec{B}$ over any closed circular path will be independent of its radius *i.e.* equal to μ_0 mes the current threading the circle.

(b) To prove Ampere's circuital law for any arbitrary path. Consider any arbitrary closed path around a long straight conductor XY carrying current I as shown in Fig. 1.10. Divide the circular path into a large number of segments of infinitesimally small lengths $\overrightarrow{AB}\ (=\vec{dl_1})$, $\overrightarrow{BC}\ (=\vec{dl_2})$, $\overrightarrow{CD}\ (=\vec{dl_3})$, ... Suppose that these segments of small lengths subtend angles $d\theta_1, d\theta_2, d\theta_3$ ..... at the point O, through which the conductor passes. Obviously,

$$d\theta_1 + d\theta_2 + d\theta_3 ..... = 2\pi \qquad ...(1.24)$$

Suppose that all the segments AB, BC, CD, ... are small circular arcs of radii r_1, r_2, r_3, .. respectively. Then,

$$d\theta_1 = \frac{dl_1}{r_1}, d\theta_2 = \frac{dl_2}{r_2}, d\theta_3 = \frac{dl_3}{r_3}, ... \qquad ...(1.25)$$

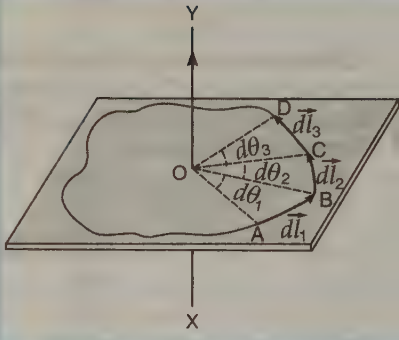


Fig. 1.10

If $\vec{B_1}, \vec{B_2}, \vec{B_3}$..... are strengths of $\vec{B}$ along $\vec{dl_1}, \vec{dl_2}, \vec{dl_3}$ ...., then the line integral of $\vec{B}$ along the arbitrary closed path can be expressed as

$$\oint \vec{B} \cdot \vec{dl} = \vec{B_1} \cdot \vec{dl_1} + \vec{B_2} \cdot \vec{dl_2} + \vec{B_3} \cdot \vec{dl_3} + ... = B_1(dl_1) + B_2(dl_2) + B_3(dl_3) + ...$$

Now, $$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_1}; B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_2}; B_3 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_3}; .....$$

$$\therefore \quad \oint \vec{B} \cdot \vec{dl} = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_1}(dl_1) + \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_2}(dl_2) + \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_3}(dl_3) + ...$$

$$= \frac{\mu_0}{4\pi} \cdot 2I\left(\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \frac{dl_3}{r_3} + ...\right)$$

Using equation (1.25), we obtain

$$\oint \vec{B} \cdot \vec{dl} = \frac{\mu_0}{4\pi} \cdot 2I(d\theta_1 + d\theta_2 + d\theta_3 + ...)$$

Again, using equation (1.24), we have

$$\oint \vec{B} \cdot \vec{dl} = \frac{\mu_0}{4\pi} \cdot 2I(2\pi)$$

or $$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

It proves Ampere's circuital law in a general case.

1.09. APPLICATIONS OF AMPERE'S CIRCUITAL LAW

We now apply Ampere's circuital law so as to obtain expressions for magnetic field due to current flowing through a long straight conductor, a long straight solenoid and a toroidal solenoid.

1. Magnetic field due to a long current carrying wire. Consider an infinitely long wire XY carrying current I as shown in Fig. 1.09. Suppose that P is the point at a distance r from the conductor, where the magnetic field due to the current carrying wire is to be determined. Draw a circle of radius r passing through point P, such that the wire XY is normal to the plane of the circle and passes through its centre. If $\vec{B}$ is the magnetic field at point P due to the current carrying wire and $\overrightarrow{PQ} = \vec{dl}$ is small element of the circle drawn around the wire, then

line integral of magnetic field along the circular path $= \oint \vec{B} . \vec{dl}$

Since the current threading the circular path is I ; according to Ampere's circuital law, we have

$$\oint \vec{B} . \vec{dl} = \mu_0 I \qquad ...(1.26)$$

Now, the angle between the vectors $\vec{B}$ and $\vec{dl}$ is 0°. Therefore,

$$\oint \vec{B} . \vec{dl} = \oint B\, dl \cos 0° = \oint B\, dl = B \oint dl$$

But $\oint dl = 2\pi r =$ circumference of all the circular path of radius r

$$\therefore \quad \oint \vec{B} . \vec{dl} = B \times 2\pi r \qquad ...(1.27)$$

From equations (1.26) and (1.27), we have

$$B \times 2\pi r = \mu_0 I$$

or
$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$$

It gives the strength of magnetic field due to an infinitely long current carrying wire at a point at distance r from it.

2. Magnetic field due to a long straight solenoid. *A solenoid is a tightly wound helical loop from an insulated wire, such that its length is very large as compared to its diameter.*

Consider a long straight solenoid having n turns per unit length and wound on a cylindrical former. When current I is passed through it, magnetic field is produced due to the solenoid as shown in Fig. 1.11.

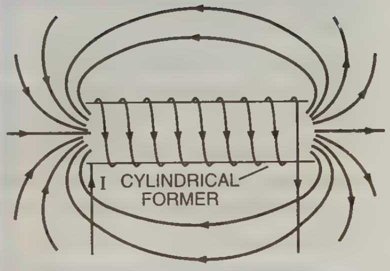


Fig. 1.11

It may be pointed out that the magnetic field produced by a straight solenoid is similar to that produced by a magnetic dipole. At points well inside the solenoid, the magnetic field is uniform and parallel to the length of the solenoid. Let $\vec{B}$ be the strength of the magnetic field at a point well inside the solenoid.

In order to determine $\vec{B}$, consider a rectangular closed path ABCD as shown in Fig. 1.12. Let the length of the rectangle be AB = L. Obviously, the number of turns of the solenoid enclosed by the rectangle is equal to n L. Since the current threads the rectangle nL times, according to Ampere's circuital law, line integral of magnetic field $\vec{B}$ along the path ABCD is equal to μ_0 (n L I), *i.e.*

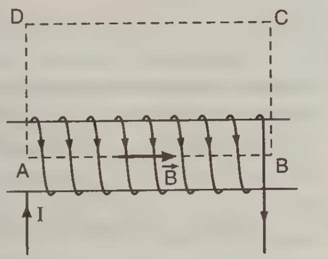


Fig. 1.12

$$\oint_{ABCD} \vec{B} \cdot \vec{dl} = \mu_0 (n L I) \qquad \text{...(1.28)}$$

Now,
$$\oint_{ABCD} \vec{B} \cdot \vec{dl} = \int_A^B \vec{B} \cdot \vec{dl} + \int_B^C \vec{B} \cdot \vec{dl} + \int_C^D \vec{B} \cdot \vec{dl} + \int_D^A \vec{B} \cdot \vec{dl} \qquad \text{...(1.29)}$$

Since direction of $\vec{B}$ is perpendicular to BC and AD,

$$\int_B^C \vec{B} \cdot \vec{dl} = \int_D^A \vec{B} \cdot \vec{dl} = 0$$

For a solenoid, whose length is much larger than its diameter, the magnetic field at points outside the solenoid is practically zero. Therefore,

$$\int_C^D \vec{B} \cdot \vec{dl} = 0$$

Using the above results, equation (1.29) becomes

$$\oint_{ABCD} \vec{B} \cdot \vec{dl} = \int_A^B \vec{B} \cdot \vec{dl} + 0 + 0 + 0 = \int_A^B \vec{B} \cdot \vec{dl}$$

As along the path AB, $\vec{B}$ and $\vec{dl}$ are along the same direction, the angle between $\vec{B}$ and $\vec{dl}$ is zero. Therefore,

$$\oint_{ABCD} \vec{B} \cdot \vec{dl} = \int_A^B B\, dl \cos 0° = B \int_A^B dl$$

Now, $\int_A^B dl$ = length of the side AB of the rectangle ABCD = L

$$\therefore \qquad \oint_{ABCD} \vec{B} \cdot \vec{dl} = B L \qquad \text{...(1.30)}$$

From equations (1.28) and (1.30), we have

$$B L = \mu_0 (n L I)$$

or $$B = \mu_0 n I \qquad ...(1.31)$$

It may be pointed out that in case of a straight long solenoid, magnetic field is given by the above formula only at a point well inside the solenoid. At points near its end, the magnetic field is given by

$$B_{end} = \frac{1}{2} \mu_0 \, n I \qquad ...(1.32)$$

Note. If the solenoid is wound on a core of a material having relative permeability μ_r, then

$$B = \mu_0 \mu_r n I \qquad ...(1.33)$$

Since $\mu_0 \mu_r = \mu$, the absolute permeability of the material of the core,

$$\therefore \quad B = \mu n I \qquad ...(1.34)$$

3. Magnetic field due to a toroidal solenoid. Consider a toroidal solenoid consisting of an anchor ring of radius r, over which a large number of turns of a metallic wire are wound. Let n be the number of turns per unit length and the point O be its centre [Fig. 1.13].

The jargon

A toroidal solenoid is an anchor ri
(torus), around which a large number
turns of a metallic wire are wound.

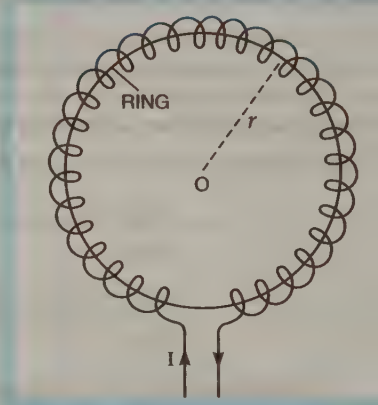


Fig. 1.13

Suppose that the current I is passed through the solenoid. The magnetic field produced will be the same at all points on the circumference of the ring of the solenoid and at any point, the direction of the magnetic field is along tangent to the ring.

According to Ampere's circuital law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \times \text{(the current threading the ring)} \qquad ...(1.35)$$

The current threading the ring is not simply equal to I. In fact, the current threads the ring as many times as there are turns in the solenoid wound on the ring of radius r. Since number of turns in the solenoid of radius r is equal to $2\pi r n$,

the total current threading the ring of radius r

$$= I\,(2 \pi r n) \qquad ...(1.36)$$

Further, $\vec{B}$ and $\vec{dl}$ are along same direction.

$$\therefore \quad \oint \vec{B} \cdot \vec{dl} = \oint B\, dl = B \oint dl = B\,(2 \pi r) \qquad ...(1.37)$$

Using equations (1.36) and (1.37), equation (1.35) becomes

$$B\,(2 \pi r) = \mu_0 \times I\,(2 \pi r n)$$

or $$B = \mu_0 n I \qquad ...(1.38)$$

It may be noted that magnetic field due to a toroidal solenoid depends upon its number of turns per unit length and the magnitude of the current passing through it. It does not depend upon the radius of the ring, over which the solenoid is wound.

SOLVED NUMERICAL PROBLEMS

Type A. On Magnetic field due to a straight conductor

Problem 1.01. A horizontal overhead power line carries a current of 90 A in the east to west direction. What is the magnitude and direction of magnetic field due to the current at a point 1·5 m below the line ? *(Text Problem)*

Sol. Here, $I = 90$ A ; $a = 1{\cdot}5$ m

Now, $$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} = 10^{-7} \times \frac{2 \times 90}{1{\cdot}5} = \mathbf{1{\cdot}2 \times 10^{-5}\ T}$$

Right hand thumb rule tells that for a current in east to west direction, the direction of magnetic field at a point below the line will be **towards south.**

Problem 1.02. A straight wire carries a current of 3 A. Calculate the magnitude of the magnetic field at a point 15 cm away from the wire. Draw a diagram to show the direction of the magnetic field. *(C.B.S.E. 1996)*

Sol. Here, $I = 3$ A ; $a = 15$ cm $= 0{\cdot}15$ m

Now, $$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} = \frac{10^{-7} \times 2 \times 3}{0{\cdot}15} = \mathbf{4 \times 10^{-6}\ T}$$

The direction of magnetic field will be perpendicula
the plane of paper and in inward direction as shown in F
1.14.

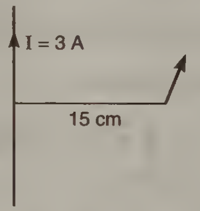


Fig. 1.14

Problem 1.03. Fig. 1.15 shows a right angled isosce triangle PQR having its base equal to a. A current c ampere is passing downwards along a thin straight w cutting the plane of the paper normally as shown at Likewise a similar wire carries an equal current passi normally upwards at R. Find the magnitude and directi

the magnetic induction B at P. Assume the wires to be initely long. (*I.C.S.E. 1997*)

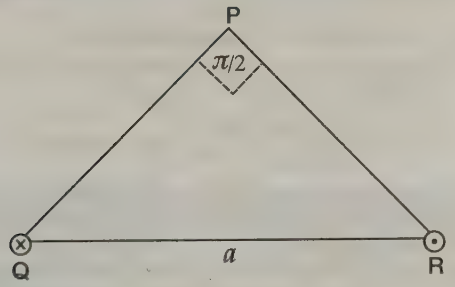


Fig. 1.15

Ans. It follows that ∠ PQR = ∠ PRQ = 45°.

Therefore, $PQ = PR = QR \sin 45° = a \times \frac{1}{\sqrt{2}} = \frac{a}{\sqrt{2}}$

Magnetic induction at point P due to current through nductor at point Q,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{PQ} = \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}\,I}{a} \qquad \text{(along PR)}$$

Also, magnetic induction at point P due to current rough conductor at point R,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{PR} = \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}\,I}{a} \qquad \text{(along PQ)}$$

The resultant magnetic induction at point P is given by

$$B = \sqrt{B_1^2 + B_2^2 + 2\,B_1 B_2 \cos \pi/2} = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \frac{\mu_0}{4\pi} \cdot \frac{4I}{a} = \frac{\mu_0 I}{\pi a}$$

The direction of the resultant magnetic induction will be ong bisector of ∠ QPR and towards the line joining the ints Q and R.

Problem 1.04. Calculate the magnetic field induction the centre of a coil bent in the form of a square of side cm carrying current of 10 A. (*P.S.S.C.E. 2001*)

Sol. Let B′ be the magnetic field induction at the centre of the square due to current passing through one side of e square. Since the magnetic field induction at point O due all the four sides is same and perpendicular to the plane the paper in outward direction, the total magnetic field duction due to the square,

$$B = 4\,B'$$

Now, $B' = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin \phi_1 + \sin \phi_2)$,

here a is the perpendicular distance of point O from any side the square [Fig. 1.16].

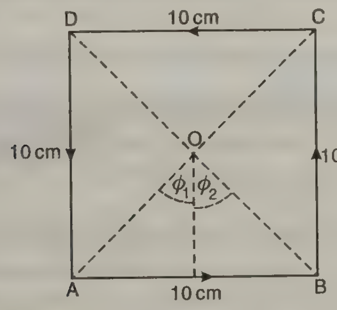


Fig. 1.16

Here, $I = 10\text{ A}, a = \frac{AD}{2} = \frac{10}{2} = 5\text{ cm} = 0{\cdot}05\text{ m}$

and $\phi_1 = \phi_2 = 45°$

$$\therefore \quad B' = 10^{-7} \times \frac{10}{0{\cdot}05} (\sin 45° + \sin 45°) = 2{\cdot}83 \times 10^{-5}\text{ T}$$

Hence, magnetic field induction due to the whole square,

$$B = 4 \times 2{\cdot}83 \times 10^{-5} = 1{\cdot}13 \times 10^{-4}\text{ T}$$

(perpendicular to paper in outward direction)

Type B. On Magnetic field due to a circular coil at its centre

Problem. 1.05. The plane of a circular coil is horizontal. It has 10 turns, each of 8 cm radius. A current of 2 A flows through it, which appears clockwise from a point vertically above it. Find the magnitude and direction of the magnetic field at the centre of the coil due to the current.

Sol. Here, $n = 10\,;\ a = 8\text{ cm} = 0{\cdot}08\text{ m}\,;\ I = 2\text{ A}$

We know, $\frac{\mu_0}{4\pi} = 10^{-7}\text{ Wb A}^{-1}\text{ m}^{-1}$

Now, magnetic field at the centre of the coil,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} = \frac{10^{-7} \times 2\pi \times 10 \times 2}{0{\cdot}08}$$

$$= 1{\cdot}57 \times 10^{-4}\text{ T}$$

Since the current is clockwise as seen from a point vertically above the coil, the direction of the field is **vertically downwards.**

Problem 1.06. The radius of the first electron orbit of a hydrogen atom is 0·5 Å. The electron moves in this orbit with a uniform speed of 2·2 ×10^{-6} m s^{-1}. What is the magnetic field produced at the centre of the nucleus due to the motion of this electron ? Use $\frac{\mu_0}{4\pi} = 10^{-7}$ N A^{-2} and electronic charge = 1·6 × 10^{-19} C. (*I.S.C.E. 1998*)

Sol. Here, uniform speed of electron,

$$v = 2{\cdot}2 \times 10^{-6}\text{ m s}^{-1}\,;$$

Radius of the electron orbit in hydrogen atom,

$$a = 0{\cdot}5\text{ Å} = 0{\cdot}5 \times 10^{-10}\text{ m}\,;$$

and charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C

and $\frac{\mu_0}{4\pi} = 10^{-7}\text{ N A}^{-2}$

Time taken by the electron to complete one revolution in the orbit of hydrogen atom,

$$T = \frac{\text{circumference of orbit}}{\text{speed of electron}} = \frac{2\pi a}{v}$$

Therefore, at any point on the electron orbit, a charge equal to e (charge on the electron) crosses after every T seconds. Therefore, revolution of the electron in the orbit is equivalent to the current I, given by

$$I = \frac{q}{t} = \frac{e}{T} = \frac{e\,v}{2\pi a}$$

$$= \frac{1{\cdot}6 \times 10^{-19} \times 2{\cdot}2 \times 10^{6}}{2\pi \times 0{\cdot}5 \times 10^{-10}} = 1{\cdot}12 \times 10^{-3}\text{ A}$$

If B is the magnetic field produced at the centre of the nucleus due to the motion of the electron, then

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a} = \frac{10^{-7} \times 2\pi \times 1{\cdot}12 \times 10^{-3}}{0{\cdot}5 \times 10^{-10}}$$

$$= 14{\cdot}07\text{ T}$$

Problem 1.07. A straight wire carrying a current of 12 A is bent into a semicircular arc of radius 2·0 cm as shown in Fig. 1.17.

(i) What is the direction and magnitude of magnetic field at the centre of the arc ? (*P.S.S.C.E. 1991*)

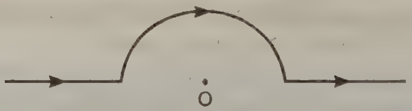


Fig. 1.17

(ii) Would your answer change, if the wire were bent into a semicircular arc of the same radius but in the opposite way as shown in Fig. 1.18 ? (*Text Problem*)

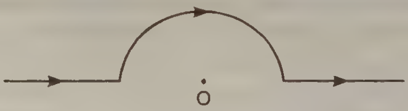


Fig. 1.18

Sol. (*i*) ***For Fig. 1.17*** **:** The magnetic field at the centre of a current carrying circular coil is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{a}$$

For magnetic field at the centre O of the semicircular conductor, the above equation will modify to

$$B = \frac{1}{2}\times\frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{a} = \frac{\mu_0}{4\pi}\cdot\frac{\pi I}{a}$$

Here, $I = 12$ A ; $a = 2{\cdot}0$ cm $= 2{\cdot}0\times10^{-2}$ m

$$\therefore \quad B = 10^{-7}\times\frac{\pi\times12}{2{\cdot}0\times10^{-2}} = \mathbf{1{\cdot}885\times10^{-4}\ T}$$

The direction of the field will be **normal to plane of the paper** and in **inward direction.**

(*ii*) ***For Fig. 1.18*** **:** The field will be of the same magnitude *i.e.* **$1{\cdot}885\times10^{-4}$ T**, but its direction will be **normal to the plane of paper** and in **outward direction.**

Problem 1.08. The wire shown in the Fig. 1.19. carries a current of 10 A. Determine the magnitude of magnetic field induction at the centre O. Given that radius of the bent coil is 3 cm. (*P.S.S.C.E. 2001*)

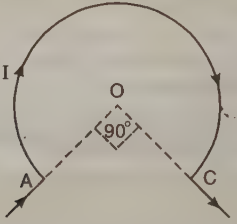


Fig. 1.19

Sol. The magnetic field induction at the centre of a current carrying circular wire of radius *a* is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{a}$$

A circular wire subtends an angle of 2 π radians at its centre. Suppose that the given circular part ABC of the wire subtends an angle θ at its centre. Then, magnetic field at point O due to circular part ABC is given by

$$B' = B\times\frac{\theta}{2\pi} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{a}\times\frac{\theta}{2\pi}$$

or $$B' = \frac{\mu_0}{4\pi}\cdot\frac{I}{a}\theta$$

Here, $I = 10$ A ; $a = 3$ cm $= 0{\cdot}03$ m

and $\theta = 360° - 90° = 270° = \dfrac{3\pi}{2}$ rad

$$\therefore \quad B' = \frac{10^{-7}\times10\times3\pi}{0{\cdot}03\times2} = \mathbf{1{\cdot}57\times10^{-4}\ T}$$

Note. The two end straight parts of the coil do no produce any magnetic field at point O. It is because, point lies on these straight parts.

Problem 1.09. As shown in Fig. 1.20, a cell is connecte across points X and Y of a uniform circular conductor. Prov that the magnetic field at its centre will be zero. (*P.S.S.C.E. 1999*

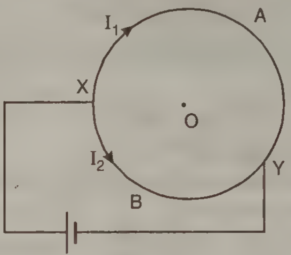


Fig. 1.20

Sol. Let I_1 and I_2 be the currents flowing through th parts XAY and XBY of the circular conductor. Suppose tha the parts XAY and XBY subtend angles θ_1 and θ_2 at the centr O. If *a* is radius of the circular conductor, then magnetic fiel due to part XAY at point O,

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{I_1}{a}\theta_1 \quad \text{...(}$$

(perpendicular to paper and in inward directio

Also, magnetic field due to part XBY at point O,

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{I_2}{a}\theta_2 \quad \text{...(}i$$

(perpendicular to paper and in outward directio

If *r* is resistance per unit length of the circular conducto then resistances of the parts XAY and XBY are given by

$$R_1 = (a\,\theta_1)\,r \quad \text{and} \quad R_2 = (a\,\theta_2)\,r$$

Since the two parts are connected between the same tw points, potential difference across their ends will be same *i.*

$$I_1 R_1 = I_2 R_2 \quad \text{or} \quad I_1\,(a\,\theta_1)\,r = I_2\,(a\,\theta_2)\,r$$

or $$I_1\,\theta_1 = I_2\,\theta_2$$

From equations (*i*) and (*ii*), it follows that magnetic fiel B_1 and B_2 due to the two parts of the circular conductor a equal and opposite. Hence, the magnetic field at the centr of the circular conductor is **zero.**

Type C. On Magnetic field due to a circular coil at point on axis

Problem 1.10. Calculate the magnetic field due to circular coil of 500 turns and of diameter 0·1 m, carrying current of 7 A (*a*) at a point on the axis of the coil dista 0·12 m (*b*) at the centre of the coil.

Sol. (*a*) Here $n = 500$; $I = 7$ A ; $a = \dfrac{0{\cdot}1}{2} = 0{\cdot}05$ m ;

$x = 0{\cdot}12$ m

Now, magnetic field at a point on axis of the coil,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi n I a^2}{(a^2+x^2)^{3/2}} = \frac{10^{-7}\times2\pi\times500\times7\times(0{\cdot}05)^2}{\{(0{\cdot}05)^2+(0{\cdot}12)^2\}^{3/2}}$$

$$= \frac{2\pi\times500\times7\times25\times10^{-4}\times10^{-7}}{(0\cdot0169)^{3/2}}$$

$= 2\cdot5\times10^{-3}$ **tesla**

(*b*) Now, magnetic field at the centre of the coil,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi n I}{a} = \frac{10^{-7}\times2\pi\times500\times7}{0\cdot05}$$

$= 4\cdot4\times10^{-2}$ **tesla**

Problem 1.11. An electric current is flowing in a cular wire of radius *a*. At what distance from the centre the axis of the circular wire will the magnetic field be th of its value at the centre ?

Sol. Let B be the magnetic field at the centre of the cular wire, when a current I passes through it. Then,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{a} \qquad ...(i)$$

Let x be the distance on the axis of the circular wire from centre, where the magnetic field is $\frac{B}{8}$. Then,

$$\frac{B}{8} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I a^2}{(a^2+x^2)^{3/2}} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{\mu_0}{4\pi}\cdot\frac{2\pi I a^2}{(a^2+x^2)^{3/2}} = \frac{1}{8}\times\frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{a}$$

or $$\frac{a^2}{(a^2+x^2)^{3/2}} = \frac{1}{8a}$$

or $(a^2+x^2)^{3/2} = 8a^3$ or $a^2+x^2 = 4a^2$

or $x = \sqrt{3}\,a$

pe D. On Ampere's Circuital law

Problem 1.12. A long straight solid conductor of radius m carries a current of 2 A, which is uniformly distributed er its circular cross-section. Find the magnetic field duction at a distance of 3 cm from the axis of the nductor.

Sol. To find the magnetic field at a point P at distance = 3 cm) due to current carrying conductor, imagine a cular path of radius *r* around the conductor, such that int P lies on it. If R is the radius of the solid conductor, en current enclosed by the circular path,

$$I' = \frac{I}{\pi R^2}\times\pi r^2 = \frac{I r^2}{R^2}$$

. Let B be magnetic field at point P due to the current carr- ng conductor. The magnetic field B acts tangential to the cular path and its magnitude is same at every point on it. Therefore, according to Ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I'$$

or $$B\times2\pi r = \mu_0\frac{I r^2}{R^2}$$

or $$B = \frac{\mu_0}{2\pi}\cdot\frac{I r}{R^2} = \frac{\mu_0}{4\pi}\cdot\frac{2 I r}{R^2}$$

Here, $I = 2$ A, $r = 3$ cm $= 0\cdot03$ m and R = 5 cm = 0·05 m

$$\therefore \quad B = \frac{10^{-7}\times2\times2\times0\cdot03}{(0\cdot05)^2} = 4\cdot8\times10^{-6}\ \mathrm{T}$$

Note. As the point lies inside the conductor, the magnetic field produced will be μ_r times the value obtained above, where μ_r is permeability of the material of the conductor.

Type E. On Magnetic field due to a Solenoid

Problem 1.13. A 0·5 m long solenoid has 500 turns and has a flux density of $2\cdot52\times10^{-3}$ T at its centre. Find the current in the solenoid. Given, $\mu_0 = 4\pi\times10^{-7}$ H m^{-1}.
(*I.S.C.E. 1995*)

Ans. Here, $B = 2\cdot52\times10^{-3}$ T ; $\mu_0 = 4\pi\times10^{-7}$ H m^{-1}

Length of solenoid, $l = 0\cdot5$ m ; total number of turns in the solenoid, N = 500

Therefore, number of turns per unit length of the solenoid,

$$n = \frac{N}{l} = \frac{500}{0\cdot5} = 1000\ \mathrm{m^{-1}}$$

If I is the current through the solenoid, then

$$B = \mu_0 n I$$

or $$I = \frac{B}{\mu_0 n} = \frac{2\cdot52\times10^{-3}}{4\pi\times10^{-7}\times1000} = 2\cdot0\ \mathrm{A}$$

Problem 1.14. A solenoid is 2 · 0 m long and 3 · 0 cm in diameter. It has 5 layers of winding of 1000 turns each and carries a current of 5 · 0 A. What is the magnetic field at its centre ? Use the standard value of μ_0. (*P.S.S.C.E. 1997 S*)

Sol . Here, current in the solenoid, I = 5 · 0 A ;
length of the solenoid, $l = 2\cdot0$ m
number of turns in each layer of winding = 1000 ;
number of layers = 5

Therefore, total number of turns in the solenoid,

N = 1000 × 5 = 5000

and the number of turns per unit length of the solenoid,

$$n = \frac{N}{l} = \frac{5000}{2\cdot0} = 2500\ \mathrm{m^{-1}}$$

Now, $B = \mu_0 n I$

Since $\mu_0 = 4\pi\times10^{-7}$ N A^{-2}, we have

$B = 4\pi\times10^{-7}\times2500\times5\cdot0 = 1\cdot57\times10^{-2}$ **T**

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 1.15. Two concentric circular coils X and Y of radii 16 cm and 10 cm respectively lie in the same rtical plane containing the north-south direction. Coil X s 20 turns and carries a current of 16 A ; coil Y has 25 rns and carries a current of 18 A. The sense of the rrent in X is anticlockwise and in Y clockwise for an server looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centres. (*Text Problem*)

Sol. *For coil X* : $a = 16$ cm $= 16\times10^{-2}$ m ; $n = 20$; I = 16 A

Magnetic field at the centre of the coil X,

$$B_x = \frac{\mu_0}{4\pi}\cdot\frac{2\pi n I}{a} = 10^{-7}\times\frac{2\pi\times20\times16}{16\times10^{-2}}$$

$= 4\pi\times10^{-4}$ T (towards east)

For coil Y : $a = 10 \text{ cm} = 10 \times 10^{-2}$ m ; $n = 25$; I = 18 A
Magnetic field at the centre of the coil Y,

$$B_y = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} = 10^{-7} \times \frac{2\pi \times 25 \times 18}{10 \times 10^{-2}}$$

$$= 9\pi \times 10^{-4} \text{ T (towards west)}$$

Therefore, net magnetic field at the centre of the two coils,

$$B = B_y - B_x = 9\pi \times 10^{-4} - 4\pi \times 10^{-4} = 5\pi \times 10^{-4} \text{ T}$$

(towards west)

Problem 1.16. Two infinitely long straight current carrying conductors are held at right angles to each other so that their common end lies at the origin [Fig. 1.21]. If both the conductors carry the same current I as shown in the figure, find the value of magnetic field at the point P (*a*, *b*).

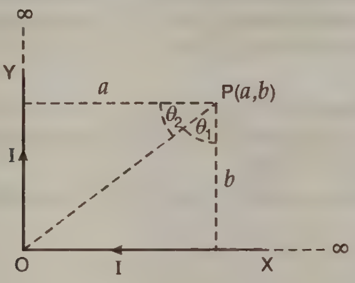


Fig. 1.21

Sol. The magnetic field due to a straight conductor is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin \phi_1 + \sin \phi_2) \quad ...(i)$$

Let B_1 and B_2 be the magnetic fields at point P due to the conductors lying along X-axis and Y-axis respectively.

For the conductor lying along X-axis,

$$\phi_1 = \theta_1 ; \phi_2 = \pi/2 \text{ and } a = b$$

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} (\sin \theta_1 + \sin \pi/2)$$

$$\text{or} \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \left(\frac{a}{\sqrt{a^2 + b^2}} + 1 \right)$$

For the conductor lying along Y-axis,

$$\phi_1 = \theta_2 ; \phi_2 = \pi/2 \text{ and } a = a$$

$$\therefore \quad B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin \theta_2 + \sin \pi/2)$$

$$\text{or} \quad B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \left(\frac{b}{\sqrt{a^2 + b^2}} + 1 \right)$$

According to the right hand thumb rule, both B_1 and B_2 act perpendicular to the plane of paper and in inward direction. Therefore, resultant magnetic field at point P,

$$B = B_1 + B_2$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{b} \left(\frac{a}{\sqrt{a^2 + b^2}} + 1 \right) + \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \left(\frac{b}{\sqrt{a^2 + b^2}} + 1 \right)$$

$$= \frac{\mu_0 I}{4\pi} \cdot I \left[\frac{1}{\sqrt{a^2 + b^2}} \left(\frac{a}{b} + \frac{b}{a} \right) + \left(\frac{1}{b} + \frac{1}{a} \right) \right]$$

$$= \frac{\mu_0}{4\pi} I \left[\frac{1}{\sqrt{a^2 + b^2}} \left(\frac{a^2 + b^2}{a b} \right) + \frac{a + b}{a b} \right]$$

$$\text{or} \quad B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a b} \left[\sqrt{a^2 + b^2} + a + b \right]$$

Problem 1.17. An infinitely long conductor is bent ir the shape as shown in Fig. 1.22. The part semicircul in shape lies in YZ-plane, while of the two straight par one is along X-axis and the other parallel to Y-axis a distance *r* from it. If the conductor carries a current I shown in the figure, find the magnetic field at the cent C of the semi-circular loop.

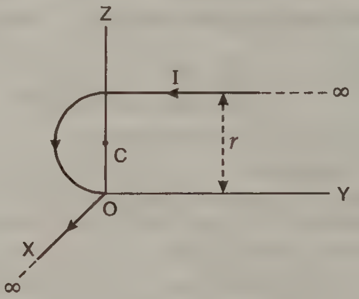


Fig. 1.22

Sol. Let $\vec{B}_1$, $\vec{B}_2$ and $\vec{B}_3$ be respectively the magne fields at point C due to straight part lying along X-axis, sen circular part and straight part lying parallel to Y-axis.

Due to a current carrying straight conductor,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin \phi_1 + \sin \phi_2)$$

For straight part lying along X-axis

$$\phi_1 = \pi/2, \quad \phi_2 = 0 \text{ and } \quad a = r/2$$

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r/2} (\sin \pi/2 + \sin 0)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} \quad \text{(along negative Y-axis)}$$

$$\therefore \quad \vec{B}_1 = -\frac{\mu_0}{4\pi} \cdot \frac{2I}{r} \hat{j}$$

The semicircular part subtends angle π at its centre. Al radius of semi-circular part is $r/2$. Therefore, for semi-circu part lying in YZ-plane.

$$\therefore \quad B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{(r/2)} \pi$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \quad \text{(along positive X-axis)}$$

$$\text{or} \quad \vec{B}_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \hat{i}$$

For straight part lying parallel to Y-axis,

$$\phi_1 = \pi/2, \quad \phi_2 = 0 \text{ and } \quad a = r/2$$

$$\therefore \quad B_3 = \frac{\mu_0}{4\pi}\cdot\frac{I}{r/2}(\sin \pi/2 + \sin 0)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2I}{r} \text{ (along positive X-axis)}$$

or $$\vec{B}_3 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}\hat{i}$$

Therefore, total magnetic field at point C,

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$= -\frac{\mu_0}{4\pi}\cdot\frac{2I}{r}\hat{j} + \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{r}\hat{i} + \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}\hat{i}$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}\left[(\pi+1)\hat{i} - \hat{j}\right]$$

Problem 1.18. A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm around vhich 3500 turns of a wire are wound. If the current in the vire is 11 A, what is the magnetic field (*a*) outside the toroid b) inside the core of the toroid (*c*) in the empty space urrounded by the toroid ? (*Text Problem*)

Sol. Here, I = 11 A ; total number of turns = 3500

Mean radius of toroid,

$$r = \frac{25+26}{2} = 25\cdot5 \text{ cm} = 25\cdot5\times10^{-2} \text{ m}$$

Total length (circumference) of the toroid

$$= 2\pi r = 2\pi\times 25\cdot5\times10^{-2} = 51\cdot0\times10^{-2}\,\pi \text{ m}$$

Therefore, number of turns per unit length,

$$n = \frac{3500}{51\cdot0\times10^{-2}\,\pi}$$

(*a*) The field is non-zero only inside the core surrounded by the windings of the toroid. Therefore, the field outside the toroid is **zero.**

(*b*) The field inside the core of the toroid,

$$B = \mu_0 n I = 4\pi\times10^{-7}\times\frac{3500}{51\cdot0\times10^{-2}\,\pi}\times 11$$

$$= \mathbf{3\cdot02\times10^{-2}\ T}$$

(*c*) For the reason given in (*a*), the field in the empty space surrounded by the toroid is also zero.

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. Is the source of magnetic field analogue to the ource of electric field ? (*H.S.S.C.E. 1992*)

Ans. No. It is because, the source of magnetic field is not magnetic charge. In case of electric field, the source of lectric field is electric charge.

Q. 1.02. State Biot Savart's law. (*I.S.C.E. 1994*)

Ans. It states that magnetic field due to small conductor f length *dl*, carrying a current I at a point at distance *r*, is given y

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\sin\theta}{r^2},$$

vhere θ is the angle between the direction of flow of urrent and the line joining the small conductor to the bservation point.

Q. 1.03. A conductor carrying current is placed somevhere. What is the magnitude and direction of the force due o a small part of the conductor at a point near it ? (*P.S.S.C.E. 1996*)

Or

Write Biot Savart expression for the magnetic field due o a current element in vector form. (*C.B.S.E. 1991 S, 1991*)

Ans. $$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I\,d\vec{l}\times\vec{r}}{r^3},$$

vhere the letters have their usual meanings.

Q. 1.04. What is the SI unit of permeability ? (*I.S.C.E. 1996*)

Ans. The SI unit of permeability is **weber ampere^{-1} netre^{-1}(Wb A^{-1}m^{-1})** or **tesla ampere^{-1} metre (T A^{-1} m).**

Q. 1.05. What is the value of constant $\frac{\mu_0}{4\pi}$?

Sol. $\frac{\mu_0}{4\pi} = 10^{-7}$ T A^{-1} m

Q. 1.06. Write the dimensional formula of μ_0.

Sol. The dimensional formula of μ_0 is [M L T^{-2} A^{-2}].

Q. 1.07. Write an expression for the magnetic field produced by an infinitely long straight wire carrying a current I, at a short perpendicular distance *a* from itself. (*I.S.C.E. 1998*)

Ans. It is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{a}$$

Q. 1.08. What kind of magnetic field is produced due to a current carrying straight conductor ?

Sol. Magnetic field lines are concentric circles lying in a plane perpendicular to the straight conductor. The centres of the circular magnetic lines of force lie on the conductor.

Q. 1.09. Show the magnetic lines of force around a straight conductor carrying current. (*P.S.S.C.E. 1997 S*)

Ans. Refer to Fig. 1.04.

Q. 1.10. State the rule that is used to find the direction of field acting at a point near a current carrying straight conductor. (*P.S.S.C.E. 1998*)

Ans. Right hand thumb rule can be used to find the direction of magnetic field at a point near a current carrying conductor.

For statement, refer to section 1.04.

Q. 1.11. Does a current carrying circular coil produce uniform magnetic field ?

Sol. No, magnetic field produced due to a current carrying circular coil is not uniform. However, it may be considered as uniform at the centre of the circular coil.

Q. 1.12. What is the effect of increasing the number of turns on magnetic field produced due to a circular coil ?

Ans. The magnetic field produced by a coil of *n* turns is *n* times the magnetic field produced by a coil of single turn.

Q. 1.13. How will the magnetic field intensity at the centre of a circular coil carrying current change, if the current through the coil is doubled and the radius of the coil is halved ? (*C.B.S.E. 2001*)

Sol. The magnetic field at the centre of a circular coil is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a}$$

$$\therefore \quad \frac{B'}{B} = \frac{I'}{a'} \times \frac{a}{I} \qquad \text{(number of turns unchanged)}$$

If $I' = 2I$ and $a' = a/2$, then

$$\frac{B'}{B} = \frac{2I}{a/2} \times \frac{a}{I} = 4 \quad \text{or} \quad B' = 4B$$

Q. 1.14. Looking at a circular coil, the current is found to be flowing in anticlockwise direction. Predict the direction of magnetic field produced at a point on the axis of the coil on the same side as the observer.

Ans. The direction of magnetic field is perpendicular to the plane of the coil and directed towards the observer.

Q. 1.15. Write down an expression for the flux density B inside a long straight solenoid, explaining the symbol used. (*I.S.C.E. 1995*

Or

How much is the flux density B at the centre of a lon solenoid ? (*I.S.C.E. 1997*

Ans. Magnetic field due to a long straight solenoid at point well inside is given by

$$B = \mu_0 n I$$

Here, μ_0 is the permeability of free space, n is th number of turns per unit length and I is the current flowin through it.

Q. 1.16. What kind of magnetic field is produced du to straight solenoid ?

Ans. The magnetic field produced by a straight solenoi is similar to that produced by a bar magnet.

Q. 1.17. What is a toroid ?

Ans. An anchor ring, around which a large number o turns of a metallic wire are wound, is called a toroid.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What is the basic difference between magnetic and electric field ? (*P.S.S.C.E. 2002*)

Or

In what way is an electric field different from a magnetic field ?

Ans. A charged particle always experiences force due to an electric field. It does not matter ; whether the charged particle is at rest or is in motion. However, due to magnetic field, force on a charged particle acts only, when it moves inside the magnetic field and its motion is not along the direction of the magnetic field. Further, when electric field acts, the kinetic energy of the charged particle is affected. However, there is no effect on the kinetic energy of a charged particle due to its motion inside the magnetic field.

Q. 1.02. In what respect does a wire carrying a current differ from a wire, which carries no current ? (*P.S.S.C.E. 1996*)

Ans. A current carrying wire produces magnetic field. It is because, when current flows through a wire, electrons move inside it along a definite direction. On the other hand, in a wire which carries no current, electrons are in motion in random direction. Such a wire does not produce any magnetic field.

Q. 1.03. Explain the rule, with illustration, related to the direction of current through a conductor and magnetic field. (*P.S.S.C.E. 1997*)

Sol. When current passes through a straight conductor, magnetic field is in the form of concentric circles, whose centres lie on the conductor. The sense of the circular field lines can be known by using ***right hand thumb rule.***

For statement of right hand thumb rule, refer to section 1.04.

Q. 1.04. A metallic wire is bent into a square of each side *a*. If a current I is passed through the wire, what is the magnetic field at the centre of the square ?

Sol. Refer to Fig. 1.17. Then, magnetic field at the centre of the square is given by

$$B = 4 \times \frac{\mu_0}{4\pi} \cdot \frac{I}{a/2} (\sin 45° + \sin 45°)$$

$$= 4 \times \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0}{4\pi} \cdot \frac{8\sqrt{2}\, I}{a}$$

Q. 1.05. A long wire is first bent into a circular coil o one turn and then into a circular coil of smaller radiu having *n* turns. If the same current passes in both the cases find the ratio of the magnetic fields produced at the centre in the two cases.

Sol. Let *r* be the radius of the coil formed, when the give wire is bent into a smaller circular coil of one turn. Then magnetic field produced at the centre of the coil,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r}$$

Let *r′* be the radius of the coil formed, when the give wire is bent into a smaller circular coil of *n* turns. Then magnetic field produced at the centre of the coil,

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{r'}$$

Now, $n(2\pi r') = 2\pi r$ or $r' = r/n$

$$\therefore \quad B' = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{r/n} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n^2 I}{r} = n^2 B$$

Q. 1.06. An electron revolves in a circular orbit o radius *r* with uniform angular speed ω. From the expression for magnetic field due to a current carrying circula conductor, deduce the expression for magnetic field at th centre of the electron orbit.

Sol. Magnetic field due to current carrying circular conductor of radius *r* is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \qquad \text{...(i}$$

The period of revolution of the electron, $T = \frac{2\pi}{\omega}$

The motion of electron in circular orbit is equivalent t current given by

$$I = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{e\omega}{2\pi}$$

Substituting for I in equation (*i*), we have

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi}{r} \cdot \frac{e\omega}{2\pi} = \frac{\mu_0 e\omega}{4\pi r}$$

Q. 1.07. Consider the circuit as shown in Fig. 1.23, where APB and AQB are semi-circles. What will be the magnetic field at the centre C of the circular loop ? (*C.B.S.E. 1995*)

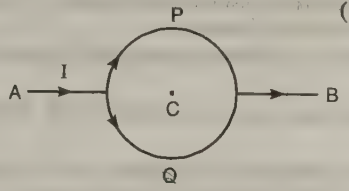


Fig. 1.23

Sol. The magnitude of magnetic field due to current flowing through semi-circular part APB or AQB is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{a}$$

The direction of magnetic field due to the part APB is erpendicular to the paper in inward direction, while that due to part AQB is in outward direction.

Further, the parts APB and AQB produce equal and pposite magnetic fields at the centre C. Hence, the magnetic ield at the centre C is zero.

Q. 1.08. A wire loop is formed by joining two semi-ircular wires of radii r_1 and r_2 as shown in Fig. 1.24. If the oop carries a current I, find the magnetic field at the entre O.

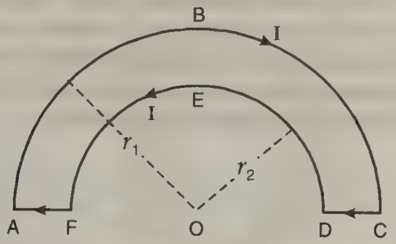


Fig. 1.24

Sol. Magnetic field at O due to semicircular part ABC,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_1} \text{ (inwards)}$$

Magnetic field at O due to semicircular part DEF,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_2} \text{ (outwards)}$$

The point O lies on the straight parts AF and CD of the vire. Hence, magnetic field due to these straight parts at point) is zero.

Therefore, net magnetic field at point O due to wire loop,

$$B = B_2 - B_1 = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_2} - \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_1}$$

or $$B = \frac{\mu_0}{4\pi} \cdot \pi I \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \textbf{ (in outward direction)}$$

Q. 1.09. A wire loop is formed by joining two semi-ircular wires of radii r_1 and r_2 as shown in Fig. 1.25. If the oop carries a current I, find the magnetic field at the entre O.

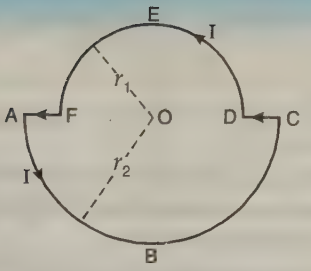


Fig. 1.25

Sol. Magnetic field at O due to semicircular part ABC,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_1} \qquad \text{(outwards)}$$

Magnetic field at O due to semicircular part DEF,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_2} \qquad \text{(outwards)}$$

Since point O lies on the straight parts AF and CD of the loop, the magnetic field due to these parts at point O is zero.

Therefore, net magnetic field at point O due to wire loop,

$$B = B_1 + B_2 = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_1} + \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r_2}$$

or $$B = \frac{\mu_0}{4\pi} \cdot \pi I \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \textbf{ (in outward direction)}$$

Q. 1.10. Two circular segments of a metallic wire having radii r_1 and r_2 are joined to form a wire loop [Fig. 1.26]. The two segments subtend an angle θ at the centre O. If a current I passes through the wire loop in the direction as shown in figure, find the magnetic field at the centre O.

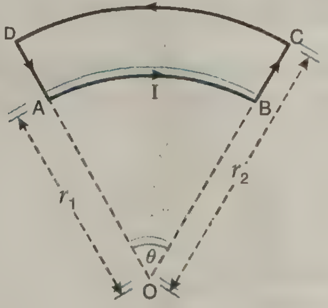


Fig. 1.26

Sol. For a current carrying circular segment of radius *a* subtending an angle θ at its centre, magnetic field* at the centre is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} \theta$$

Therefore, magnetic field at O due to circular segment AB,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r_1} \theta \qquad \text{(inwards)}$$

and magnetic field at O due to circular segment CD,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r_2} \theta \qquad \text{(outwards)}$$

Since point O lies on the straight parts AD and BC of the loop, the magnetic field due to these parts at point O is zero.

Therefore, net magnetic field at point O due to wire loop,

---

*For details, refer to solved numerical problem no. 1.08.

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\cdot\frac{I}{r_1}\theta - \frac{\mu_0}{4\pi}\cdot\frac{I}{r_2}\theta$$

or $$B = \frac{\mu_0}{4\pi}\cdot I\theta\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$ **(in inward direction)**

Q. 1.11. Give expressions for magnetic field due to circular coil (at its centre) and due to long straight wire, when these are carrying currents.

Ans. Magnetic field due to a circular coil at its centre,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi n I a^2}{(a^2 + x^2)^{3/2}}$$

and magnetic field due to a long straight wire,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{a}$$

Here, the letters have their usual meanings.

Q. 1.12. What is a solenoid ? Give the magnitude of magnetic field developed at a point well inside a solenoid carrying current. *(H.S.S.C.E. 1998)*

Ans. A solenoid is a circular coil of a large number turns, such that the turns of the solenoid run parallel to it length.

Magnetic field at a point well inside the solenoid,

$$B = \mu_0 n I$$

Here, n is number of turns per unit length of the solenoi

Q. 1.13. An air-core solenoid having N turns of lengt l is carrying a current I ampere.

(*i*) Sketch the pattern of magnetic lines of force in th solenoid.

(*ii*) If an iron core is inserted in it, how is its fiel modified ?

(*iii*) Write down the expression for the magnetic fiel at the centre of the solenoid along its axis.*(C.B.S.E. 1998 S*

Ans. (*i*) For the pattern of magnetic field, refer to Fi 1.12.

(*ii*) The magnetic field lines come closer to each othe

(*iii*) The magnetic field at the centre of the solenoid,

$$B = \mu_0 \frac{N}{l} I$$

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. Assuming that the earth's magnetic field is due to a large circular loop of current in the interior of the earth, what is the plane of the loop and what is the direction of the current around it ?

Ans. The direction of magnetic field at the centre of the earth is from north to south (It is on the surface of the earth that the magnetic field is from south to north). The magnetic field will be produced along the NS direction [Fig. 1.27], if the circular loop is held in equitorial plane and the current flows in clockwise direction, when seen from north of the earth.

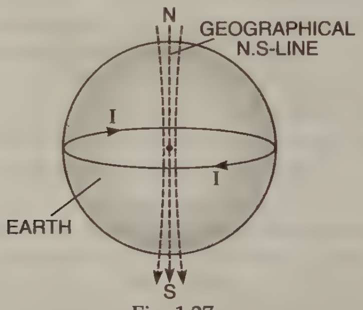


Fig. 1.27

Q. 1.02. Obtain the dimensional formula of (*a*) ε_0 (permittivity of the vacuum) and (*b*) μ_0 (permeability of the vacuum) in terms of M (mass), L (length), T (time) and I (electric current). *(I.I.T. 1998)*

Ans. (*a*) From Coulomb's law in electrostatics, we have

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$$

$$\therefore \quad [\varepsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[I\,T][I\,T]}{[M L T^{-2}][L^2]}$$

$$= [M^{-1} L^{-3} T^4 I^2]$$

(*b*) Force per unit length between two infinitely long current carrying conductors,

$$F = \frac{\mu_0}{4\pi}\cdot\frac{2 I_1 I_2}{r}$$

Since F stands for force per unit length, its dimensiona formula is $[M L^0 T^{-2}]$

$$\therefore \quad [\mu_0] = \frac{[F][r]}{[I_1][I_2]} = \frac{[M L^0 T^{-2}][L]}{[I][I]} = [M L T^{-2} I^{-2}]$$

Q. 1.03. A circular branching made from a unifor conductor is connected to a battery as shown i Fig. 1.28. What force will the magnetic field of the curren in the branching exert on the magnetic pole placed at i centre O ? Deduce the result qualitatively.

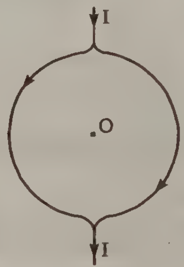


Fig. 1.28

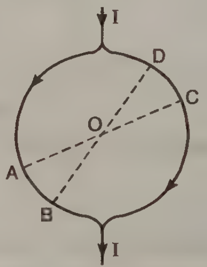


Fig. 1.29

Ans. Suppose that the circular branching is divided int a large number of elementary portions. Consider tw elementary portions AB and CD of equal lengths situate diametrically opposite to each other [Fig. 1.29]. The intensitie of magnetic field produced by the portions AB and CD a point O will be equal in magnitude but opposite in directio The net magnetic field due to the portions AB and CD at poi O will be zero. Similarly, the magnetic fields due to the othe elementary portions will cancel out at point O in pairs. Thu magnetic field due to the complete circular branching at poi O will be zero. Hence, the force on magnetic pole placed a point O will be **zero**.

Q. 1.04. A circular loop of radius R carrying current lies in XY-plane with its centre at origin. What is th magnetic flux through the XY-plane ? *(I.I.T. 199*

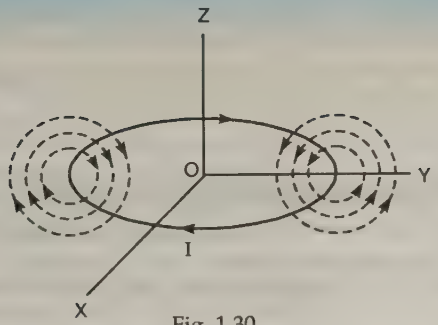


Fig. 1.30

Ans. The magnetic flux in the region of XY-plane lying inside the circular loop will be directed along negative Z-axis, while in the region outside the circular loop, equal magnetic flux will be directed along positive Z-axis as shown in Fig. 1.30. Hence, due to the circular loop, the magnetic flux through the XY-plane will be **zero.**

Q. 1.05. A current I flows along an infinite straight thin-walled pipe [Fig. 1.31]. What is the intensity of the magnetic field inside the pipe ? Assume that current is distributed uniformly along the entire section of the pipe.

Ans. The current flowing along the pipe may be regarded as the sum of many identical currents uniformly distributed over the surface of the pipe. The intensity of magnetic field at any point in the space inside the pipe may be considered as the sum of the intensities of the magnetic fields produced by such linear currents.

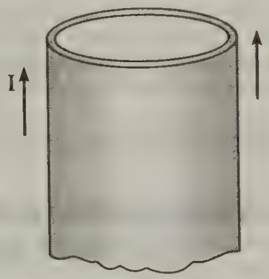


Fig. 1.31

Fig. 1.32 shows the flow of current through the cross-section of the pipe. To find intensity of magnetic field at point P inside the pipe, let us first find the intensities of the magnetic field produced at this point by the linear currents passing through the elementary portions AB and CD.

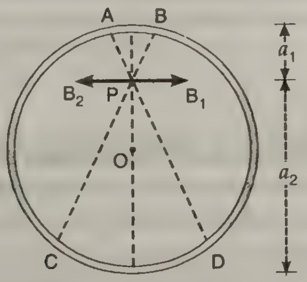


Fig. 1.32

Let I_1 and I_2 be the currents flowing through the portions AB and CD respectively. Obviously,

$$\frac{I_1}{I_2} = \frac{AB}{CD}$$

Let a_1 and a_2 be the distances of the portions AB and CD from the point P. If dB_1 and dB_2 are the intensities of the magnetic fields produced by currents I_1 and I_2 flowing through the elementary portions AB and CD, then

$$dB_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{a_1} \text{ and } dB_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{a_2}$$

From the right hand thumb rule, it follows that if dB_1 is towards right, then dB_2 will be towards left. Thus, the resultant intensity of the magnetic field due to portions AB and CD is given by

$$dB = dB_1 - dB_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{a_1} - \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{a_2}$$

$$= \frac{\mu_0}{2\pi}\left(\frac{I_1}{a_1} - \frac{I_2}{a_2}\right) \qquad ...(ii)$$

Since triangles PAB and PCD are similar,

$$\frac{AB}{CD} = \frac{a_1}{a_2} \qquad ...(iii)$$

From equations (*i*) and (*iii*), we have

$$\frac{I_1}{I_2} = \frac{a_1}{a_2} \text{ or } \frac{I_1}{a_1} = \frac{I_2}{a_2}$$

Setting $\frac{I_1}{a_1} = \frac{I_2}{a_2}$ in equation (*ii*), we have

$$dB = 0$$

Similarly, the contribution to the intensity of the magnetic field at point P by other such pairs of elementary portions of the cross-section of the pipe will be zero. Hence, the intensity of the magnetic field inside an infinite straight pipe carrying current is zero.

Q. 1.06. A current I flows upwards along the inner conductor of a co-axial cable [Fig. 1.33] and returns down along the external shell of the cable. What is the intensity of the magnitude field at points inside the cable ?

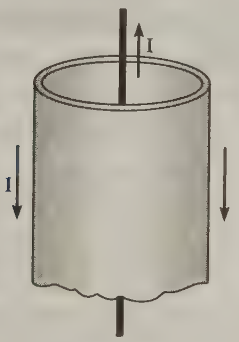


Fig. 1.33

Ans. At a point inside the cable, the magnetic field will be produced only due to the inner conductor. The magnetic field produced due to the external shell of the cable will be zero as proved in conceptual SAQ 1.06.

At a point at distance *a* from the inner conductor, the magnetic field due to the infinitely long inner conductor will be

$$B = \frac{\mu_0}{4\pi} \frac{2I}{a}$$

Q. 1.07. The magnetic field at a point near the centre but outside a current carrying solenoid is zero. Explain, why.

Ans. Consider a vertical plane through the centre of the solenoid and perpendicular to its length. The plane divides the solenoid into two equal parts. At a point outside th solenoid on this plane, corresponding turns of the two halve of the solenoid produce equal and opposite magnetic fields Hence, magnetic field at such a point is zero.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. State and explain Biot-Savart's theorem for a current carrying finite conductor. (*P.S.S.C.E., 2002, 2000*)
2. State Biot-Savart law for magnetic field produced at a point due a small current element. How will you find the direction of magnetic field ? (*P.S.S.C.E. 1996 ; H.S.S.C.E. 1992 ; C.B.S.E. 1991*)
3. State Biot Savart law for the magnetic field produced at a point due to a current element. How will you find the direction of magnetic field ? Define SI unit of magnetic field and give its dimensional formula. (*P.S.S.C.E. 1995*)
4. State Biot Savart law. Use it to obtain an expression for magnetic field at the centre of a current carrying circular loop. (*H.S.S.C.E. 2002 ; C.B.S.E. 1999, 1995*)
5. State Biot-Savart law. Using Biot-Savart law, derive an expression for the magnetic field at the centre of a circular coil of number of turns *n*, radius *r* carrying a current I. (*C.B.S.E. 2002*)
6. Write the expression for Biot-Savart law for the magnetic field due to a small current carrying element. Using the expression, calculate the magnetic field at the centre of current carrying circular coil of radius *r* having *n* number of turns. (*C.B.S.E. 2001 S*)
7. Using Biot Savart law, calculate the magnetic field at the centre of a circular coil. (*C.B.S.E. 2000 ; P.S.S.C.E. 2000, 1999, 1991*)
8. Derive a relation for the magnetic field due to a current carrying circular coil at a point on its axis. (*H.S.S.C.E. 2002, 2001 ; P.S.S.C.E. 1999 S, 1992 H.P.S.S.C.E. 1997*
9. Derive an expression for the magnetic field at the centr of a circular coil of *n* turns. (*I.S.C.E. 1996*
10. State Biot-Savart's law. Use this law to find the magneti field at point on the axis of a circular coil carrying curren (*H.P.S.S.C.E. 2001*
11. Write the expression for Biot-Savart's law for the magneti field due to a small current element. Using this expressior calculate the magnetic field intensity at a point on the axi of a circular current carrying conductor.(*C.B.S.E. 2001 S*
12. State Biot Savart law. Obtain an expression for the magneti field at a point situated at a distance of *x* metre from th centre of a circular coil of *n* turns and of *r* metre radiu carrying a current of I A. (*H.S.S.C.E. 1992*
13. State Ampere's circuital law and prove the Ampere' circuital law for a circular path around a long curren carrying conductor. (*H.P.S.S.C.E. 1998*
14. State and prove Ampere's circuital law. (*H.S.S.C.E. 2001, 1998 S ; P.S.S.C.E. 1995 S*
15. Calculate, using Ampere circuital law, the magnetic fiel at a point well inside a straight solenoid carrying curren
16. Calculate using Ampere's circuital theorem, the magneti field due to an infinitely long wire carrying current I. (*C.B.S.E. 1990*
17. Find the magnetic field intensity for a toroidal solenoid (*H.S.S.C.F. 2000*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. State the Biot-Savart law. Derive an expression for the magnetic field at a point near a long straight wire carrying a current. (*H.S.S.C.E. 1997 ; Pre degree, Kerala 1994 ; I.S.C.E. 1993*)
2. Using Biot-Savart law, derive an expression for magnetic field intensity produced at a point due to the current flowing through an infinite long straight conductor. (*P.S.S.C.E. 2002, 2000, 1999, 1998 S*)
3. Find the magnetic field intensity at a distance *a* from a current carrying infinitely long, straight, thin metallic conductor. Also draw the sketch of magnetic field. (*P.S.S.C.E. 2000, 1999 S ; H.P.S.S.C.E. 1995*)
4. State Biot Savart's law. Using this law, obtain an expressio for the magnetic field induction at a point situated on th axis of a current carrying circular coil. (*P.S.S.C.E. 2001*
5. State and explain Ampere's circuital law. Derive a expression for magnetic field due to current in a toroid. (*P.S.S.C.E. 2001 ; H.P.S.S.C.E. 1998*
6. State and explain Ampere's circuital law. By applying thi law, obtain an expression for the magnetic field at a poin well inside the solenoid carrying current. (*P.S.S.C.E. 2000, 1999 S, 1999, 1995*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Magnetic field due to a straight conductor

1. A current of 5 A is flowing south to north in a wire kept along north-south direction. Find the magnetic field due to a 1 cm piece of the wire at a point 2 m north-east from the piece of wire. **[Ans.** $8{\cdot}34 \times 10^{-10}$ T (vertically downwards)]
2. Calculate the magnetic field at a distance of 5 m from an infinite straight conductor carrying a current of 100 A. **[Ans.** 4×10^{-6} T]
3. A current of 10 A is flowing east to west in an infinitel long wire kept along east-west direction. Find magneti field in a horizontal plane at a distance (*a*) 20 cm north an (*b*) 10 cm south from the wire. **[Ans.** (*a*) 10^{-5} T (vertically downwards (*b*) 2×10^{-5} T (vertically upwards)
4. A current of 5 A is flowing east to west in an infinitel long wire kept along east-west direction. Find magneti field in a vertical plane at a distance (*i*) 50 cm below th

wire and (*ii*) 40 cm above the wire.

[**Ans.** (*i*) 2×10^{-6} T (horizontally southwards)
(*ii*) $2{\cdot}5 \times 10^{-6}$ T (horizontally northwards)]

5. Earth's magnetic field is 5×10^{-5} T. A cable is 10 m above the ground. Calculate the current in the cable, so that it may produce a neutral point with earth's magnetic field on the surface of the earth just below it. [**Ans.** 2500 A]

6. Two straight long conductors AOB and COD are perpendicular to each other and carry currents I_1 and I_2 respectively. Find the magnitude of magnetic field at a point P at a distance *a* from the point O in a direction perpendicular to the plane ABCD.

$$\left[\textbf{Ans.}\ \frac{\mu_0}{4\pi}\cdot\frac{2\,(I_1^{\,2}+I_2^{\,2})^{1/2}}{a}\right]$$

7. Calculate the magnetic field induction at the centre of a coil bent in the form of a square of perimeter 40 cm carrying a current of 8 A. (*P.S.S.C.E. 2001*)
[**Ans.** $9{\cdot}04 \times 10^{-5}$ in outward direction]

8. A rectangular loop of metallic wire is of length *a* and breadth *b* and carries a current I. Find the magnitude of magnetic field at the centre O of the loop.

$$\left[\textbf{Ans.}\ \frac{\mu_0}{4\pi}\cdot\frac{8\sqrt{a^2+b^2}\,I}{a\,b}\right]$$

9. A hexagonal loop of a metallic wire is of each side *a* and carries a current I. Find the magnitude of magnetic field at the centre of the loop.

$$\left[\textbf{Ans.}\ \frac{\mu_0}{4\pi}\cdot\frac{4\sqrt{3}\,I}{a}\right]$$

Type B. On Magnetic field due to a circular coil at its centre

10. Calculate the current in a circular coil of radius 5 cm and 100 turns to produce a field of 2×10^{-5} T at its centre.
[**Ans.** $1{\cdot}59 \times 10^{-2}$ A]

11. The electron in a hydrogen atom circles around the proton with a speed of $2{\cdot}18 \times 10^{6}$ m s^{-1} in an orbit of radius $5{\cdot}3 \times 10^{-11}$ m. Calculate (*i*) the equivalent current, (*ii*) magnetic field produced at the proton. Given that $e = 1{\cdot}6 \times 10^{-19}$ C and $\mu_0 = 4\pi \times 10^{-7}$ SI units.
[**Ans.** (*i*)$1{\cdot}047 \times 10^{-3}$ A ; (*ii*) 12·41 T]

12. Calculate the strength of magnetic field due to an electron revolving in a circle of radius 2×10^{-10} m with a speed of 5×10^{6} m s^{-1} at its centre. [**Ans.** 2 T]

13. In Bohr model of hydrogen atom, the electron circulates around the nucleus in a path of radius $5{\cdot}1 \times 10^{-11}$m at a frequency of $6{\cdot}8 \times 10^{15}$ Hz. What is the value of magnetic field set up at the centre of the orbit ?
(*H.S.S.C.E. 1997 S*) [**Ans.** 13·4 T]

14. In an atom, the radius of the electron orbit is 4×10^{-11} m. The electron revolves with frequency of 10^{15} revolutions per second. Charge on the electron is $1{\cdot}6 \times 10^{-19}$ C. Calculate the magnetic field at the centre of the orbit.
[**Ans.** 2·513 T]

15. Two identical coils each of radius *r* and having number of turns *n* are lying in perpendicular planes, such that they have common centre. Find the magnetic field at the centre of the coils, if they carry currents equal to I and $\sqrt{3}$ I respectively.

$$\left[\textbf{Ans.}\ \frac{\mu_0\, n I}{r}\right]$$

16. A semicircular arc of radius 20 cm carries a current of 10 A. Calculate the magnitude of magnetic field at the centre of the arc. (*C.B.S.E. 2002*) [**Ans.** $1{\cdot}57 \times 10^{-5}$ T]

17. An electric current I is flowing through an infinitely long conductor bent into a loop of the shape as shown in Fig. 1.34. Find the magnitude and direction of magnetic field at the centre O of the circular part of the loop.

$$\left[\textbf{Ans.}\ \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}(\pi+1) \text{ in outward direction}\right]$$

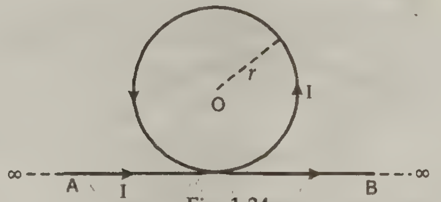


Fig. 1.34

18. An electric current I is flowing through an infinitely long conductor bent into a loop of the shape as shown in Fig. 1.35. Find the magnitude and direction of magnetic field at the centre O of the circular part of the loop.

$$\left[\textbf{Ans.}\ \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}(\pi-1) \text{ in inward direction}\right]$$

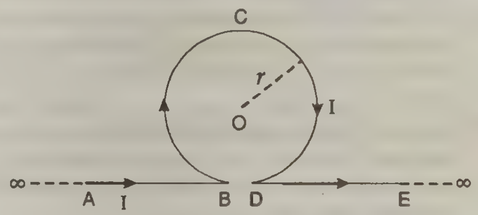


Fig. 1.35

19. An electric current I is flowing through an infinitely long conductor bent into the form as shown in Fig. 1.36. Find the magnitude and direction of magnetic field at the centre of the semi-circular part of the conductor.

$$\left[\textbf{Ans.}\ \frac{\mu_0}{4\pi}\cdot\frac{I}{r}(\pi+2) \text{ in outward direction}\right]$$

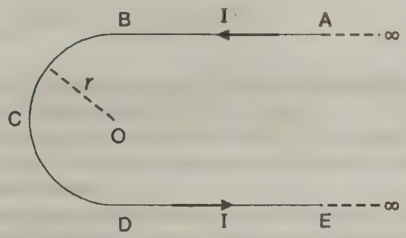


Fig. 1.36

20. A circular segment of radius 0·1 m subtends an angle of 60° at its centre as shown in Fig. 1.37. A current of 6 A is flowing through it. Find the magnitude and direction of the magnetic field produced at the centre O.
[**Ans.** $6{\cdot}28 \times 10^{-6}$ T]

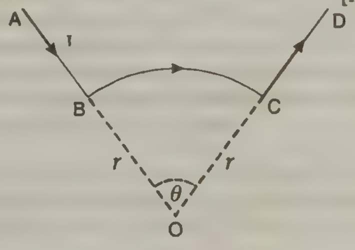


Fig. 1.37

21. A metallic wire is bent into the form as shown in Fig. 1.38 and carries a current I. If point O is the common centre of all the three circular segments of the wires of radii r, $2r$ and $3r$, find the magnetic field at point O.

$$\left[\textbf{Ans. } \frac{\mu_0}{4\pi}\cdot\frac{5\,I}{6\,r}\,\theta \text{ in inward direction}\right]$$

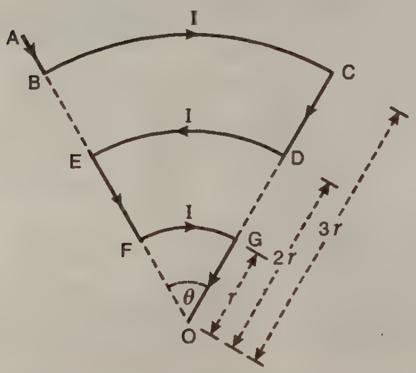


Fig. 1.38

Type C. On Magnetic field due to a circular coil at a point on axis

22. A current of 10 A flows through a circular coil of 50 turns. Calculate the magnetic field at a point situated at a distance of 3 m on the axis from the centre of the coil. The radius of the coil is 10 cm. **[Ans.** $1{\cdot}162 \times 10^{-7}$ T**]**

23. Show that the magnetic field along the axis of a current carrying circular coil of radius a at a distance x ($x << a$) from the centre of the coil is smaller by a factor $1{\cdot}5\, x^2/a^2$ than the field at the centre of the coil.

Type D. On Ampere's circuital law

24. A long straight metallic conductor of radius R carries a current I, which is uniformly distributed over its circular cross-section. Find the magnetic field induction at a point at distance r from the axis of the wire, when the point lies (*a*) outside and (*b*) inside the conductor.

$$\left[\textbf{Ans. } (a)\ \frac{\mu_0}{4\pi}\cdot\frac{2\,I}{r},\quad (b)\ \frac{\mu_0}{4\pi}\cdot\frac{2\,\mu_r\,I\,r}{R^2}\right]$$

Type E. On Magnetic field due to a solenoid

25. Length of a solenoid is 0·2 m and it has 120 turns. Find the magnetic field in its interior, if a current of 2·5 A is flowing through it. Given, $\mu_0 = 4\,\pi \times 10^{-7}$ SI units. **[Ans.** $1{\cdot}885 \times 10^{-3}$ T**]**

26. A solenoid 50 cm long has 4 layers of windings of 350 turns each. If the current carried is 6·0 A, estimate the magnitude of magnetic field (*a*) near the centre of the solenoid on its axis (*b*) near its ends on its axis. (*c*) outside the solenoid near its centre. **[Ans.** (*a*) $2{\cdot}1 \times 10^{-2}$ T ; (*b*) $1{\cdot}05 \times 10^{-2}$ T ; (*c*) zero**]**

27. A solenoid of length 1 m and 3 cm in diameter has five layers of windings of 850 turns each and carries a current of 5 A. What is the magnetic field through a cross-sectional end of the solenoid ? Also calculate the magnetic flux through the cross-section of the solenoid at its centre. **[Ans.** $1{\cdot}335 \times 10^{-2}$ T, $9{\cdot}44 \times 10^{-6}$ Wb**]**

MISCELLANEOUS PROBLEMS

28. Two insulating infinitely long wires are lying mutually perpendicular to each other as shown in Fig. 1.39. If the two wires carry currents I_1 and I_2 as shown in the figure, find the magnetic field at point P (a, b).

$$\left[\textbf{Ans. } \frac{\mu_0}{2\pi}\cdot\left(\frac{I_2}{b}-\frac{I_1}{a}\right) \text{ in inward direction}\right]$$

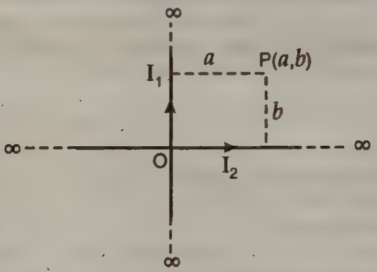


Fig. 1.39

29. Two insulating infinitely long wires are lying mutually perpendicular to each other as shown in Fig. 1.40. If the two wires carry currents I_1 and I_2 as shown in figure, find the locus of the point, where the magnetic field due to two wires is zero.

$$\left[\textbf{Ans. } y = \frac{I_2}{I_1}\,x\right]$$

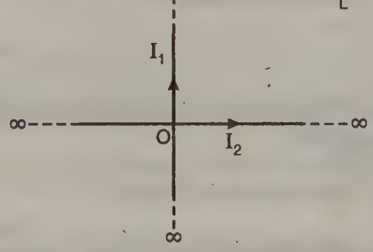


Fig. 1.40

30. A metallic wire carrying a current I is bent into the form as shown in Fig. 1.41. The circular portion ABC of the wire is of radius r and the straight portion AC subtends an angle $2\,\theta$ at the centre O. Find the magnetic field due to the conductor at point O.

$$\left[\textbf{Ans. } \frac{\mu_0}{2\pi}\cdot\frac{I}{r}(\pi-\theta+\tan\theta) \text{ in inward direction}\right]$$

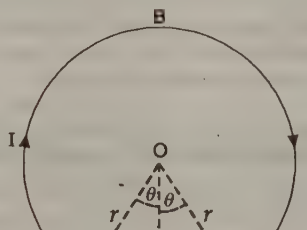


Fig. 1.41

31. A current I is flowing in an infinitely long metallic conductor bent into the form as shown in Fig. 1.42. If the radius of the curved path is r, find the magnetic field at the centre O.

$$\left[\textbf{Ans. } \frac{\mu_0}{4\pi}\cdot\frac{I}{r}\left(\frac{3\pi}{2}+1\right) \text{ in outward direction}\right]$$

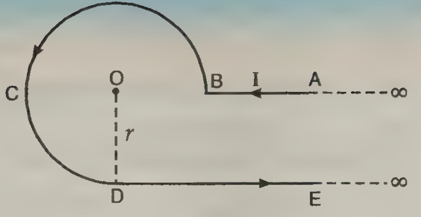


Fig. 1.42

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\,\sin\theta}{r^2}$

Since the wire is kept along north-south direction and observation point is north-east from the piece of wire, $\theta = 45°$.

$$\therefore \quad dB = \frac{10^{-7}\times 5\times 10^{-2}\sin 45°}{(2)^2}$$

$$= 8{\cdot}34 \times 10^{-10}\ \text{T (Vertically downwards)}$$

2. $B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{a} = \frac{10^{-7}\times 2\times 100}{5} = 4\times 10^{-6}\ \text{T}$

3. (*a*) $B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{a} = \frac{10^{-7}\times 2\times 10}{20\times 10^{-2}} = 10^{-5}\ \text{T (downwards)}$

(*b*) $B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{a} = \frac{10^{-7}\times 2\times 10}{10\times 10^{-2}} = 2\times 10^{-5}\ \text{T (upwards)}$

4. Proceed as in problem no. 3.

5. So as to produce neutral point, magnetic field due to the current carrying cable at that point should be equal and opposite to the earth's magnetic field at that point. Thus,

$$\frac{\mu_0}{4\pi}\cdot\frac{2I}{a} = 5\times 10^{-5} \quad \text{or} \quad \frac{10^{-7}\times 2\times I}{10} = 5\times 10^{-5}$$

or $\quad I = 2500\ \text{A}$

6. Let B_1 and B_2 be magnetic fields produced at point O by conductors AOB and COD respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I_1}{a} \quad \text{and} \quad B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{a}$$

Since the two conductors are perpendicular to each other, magnetic fields B_1 and B_2 will also be perpendicular to each other. The resultant magnetic field is given by

$$B = \sqrt{B_1^2 + B_2^2} = \left[\left(\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{a}\right)^2 + \left(\frac{\mu_0}{4\pi}\cdot\frac{2I_2}{a}\right)^2\right]^{1/2}$$

or $\quad B = \frac{\mu_0}{4\pi}\cdot\frac{2\,(I_1^2 + I_2^2)^{1/2}}{a}$

7. Proceed as in solved problem no. 1.04.

8. Suppose that current flows through the rectangle loop ABCD as shown in Fig. 1.43. The magnetic fields produced due to all the four sides at the centre O will be perpendicular to the plane of the loop and in outward direction.

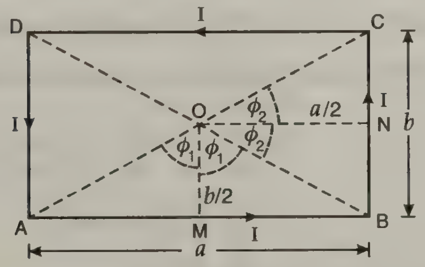


Fig. 1.43

Therefore, total magnetic field at point O,

$$B = B_1 + B_2 + B_3 + B_4,$$

where B_1, B_2, B_3 and B_4 are magnetic fields produced due to sides AB, BC, CD and DA respectively.

$$\text{Now,} \quad B_1 = B_3 = \frac{\mu_0}{4\pi}\cdot\frac{I}{(b/2)}(\sin\phi_1 + \sin\phi_1)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{4I}{b}\sin\phi_1$$

From right angled Δ OMA, we have

$$\sin\phi_1 = \frac{AM}{OA} = \frac{a/2}{\sqrt{a^2+b^2}/2} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\therefore \quad B_1 = B_3 = \frac{\mu_0}{4\pi}\cdot\frac{4I}{b}\times\frac{a}{\sqrt{a^2+b^2}}$$

Similarly, it can be obtained that

$$B_2 = B_4 = \frac{\mu_0}{4\pi}\cdot\frac{4I}{a}\times\frac{b}{\sqrt{a^2+b^2}}$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\cdot\frac{4I}{\sqrt{a^2+b^2}}\left[\frac{a}{b}+\frac{b}{a}+\frac{a}{b}+\frac{b}{a}\right]$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{4I}{\sqrt{a^2+b^2}}\times\frac{a^2+b^2+a^2+b^2}{ab}$$

$$\text{or} \quad B = \frac{\mu_0}{4\pi}\cdot\frac{8\sqrt{a^2+b^2}\,I}{ab}$$

9. Suppose that the current flows through the hexagonal loop ABCDEF as shown in the Fig. 1.44.

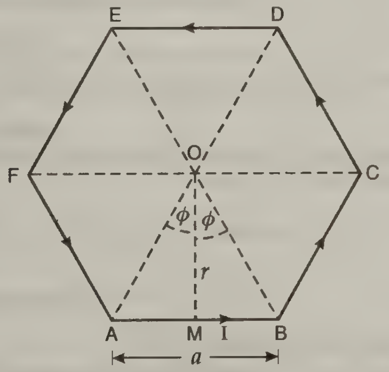


Fig. 1.44

The magnetic field produced at point O due to each of the six sides of the hexagonal loop is same and perpendicular to its plane and in outward direction. If perpendicular distance of point O from the side AB *i.e.* OM = r, then magnetic field at point O due to side AB of the loop,

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{I}{r} (\sin\phi + \sin\phi) = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r} \sin\phi$$

Here $\phi = 30°$

Also, from right angled Δ AMO, we have

$$\tan\phi = \frac{AM}{OM} \quad \text{or} \quad \tan 30° = \frac{a/2}{r}$$

or $$r = \frac{a}{2\tan 30°} = \frac{\sqrt{3}\,a}{2}$$

∴ $$B' = \frac{\mu_0}{4\pi} \cdot \frac{2I}{\sqrt{3}\,a/2} \sin 30° = \frac{\mu_0}{4\pi} \cdot \frac{2I}{\sqrt{3}\,a}$$

Hence, magnetic field due to the whole hexagonal loop,

$$B = 6\,B' = \frac{\mu_0}{4\pi} \cdot \frac{4\sqrt{3}\,I}{a}$$

10. Proceed as in solved problem no. 1.05.

11. Proceed as in solved problem no. 1.06.

12. Proceed as in solved problem no. 1.06.

13. If ν is frequency of revolution of electron, then

$$I = e\nu$$

Now, proceed as in solved problem no. 1.06.

14. Proceed as in problem no. 13.

15. Let B_1 and B_2 be magnetic fields produced by the two coils. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I_1}{r} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{r} = \frac{\mu_0 n I}{2r}$$

and $$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I_2}{r} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n \times \sqrt{3}\,I}{r} = \frac{\sqrt{3}\,\mu_0 n I}{2r}$$

Since the planes of the two coils are perpendicular to each other, magnetic fields B_1 and B_2 produced by them will also be perpendicular to each other. The resultant magnetic field is given by

$$B = \sqrt{B_1^2 + B_2^2} = \left[\left(\frac{\mu_0 n I}{2r}\right)^2 + \left(\frac{\sqrt{3}\,\mu_0 n I}{2r}\right)^2\right]^{1/2}$$

$$= \frac{\mu_0 n I}{2r}(1+3)^{1/2} = \frac{\mu_0 n I}{r}$$

16. Proceed as in solved problem no. 1.07.

17. Let B_1 and B_2 be magnetic fields produced due to circular and straight parts of the loop. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r}$$ (perpendicular to the plane of loop and in outward direction)

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r}$$ (perpendicular to the plane of loop and in outward direction)

Since B_1 and B_2 are in same direction, total magnetic field produced by the loop,

$$B = B_1 + B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} + \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}(\pi + 1)$$ **in outward direction**

18. Proceed as in problem no. 17.

Here, B_1 will be perpendicular to plane of loop and in inward direction, while B_2 will be in outward direction. Therefore,

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}(\pi - 1)$$ **in inward direction**

19. Let B_1, B_2 and B_3 be the magnetic fields produced at p
O by straight part AB, semicircular part BCD and strai
part DE of the current carrying conductor ABCDE. Th

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(\sin\phi_1 + \sin\phi_2)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(\sin 90° + \sin 0°)$$

or $$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r},$$ (perpendicular to the pl
and in outward directi

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r},$$ (perpendicular to the pl
and in outward directi

and $$B_3 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(\sin\phi_1 + \sin\phi_2)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(\sin 90° + \sin 0°)$$

or $$B_3 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}$$ (perpendicular to the pl
and in outward durecti

Therefore, total magnetic field produced by the conduc

$$B = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r} + \frac{\mu_0}{4\pi} \cdot \frac{\pi I}{r} + \frac{\mu_0}{4\pi} \cdot \frac{I}{r}$$

or $$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(\pi + 2)$$ **in outward direction**

20. Let B_1, B_2 and B_3 be the magnetic fields produced at p
O by the straight part AB, circular part BC and straight
CD of the current carrying conductor ABCD. Since p
O lies on straight parts AB and CD,

$$B_1 = B_3 = 0$$

Further, as circular segment BC subtends angle θ at p
O,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{r} \times \frac{\theta}{2\pi} = \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{r}$$

$$= \frac{10^{-7} \times 6 \times \pi/3}{0 \cdot 1} = 6 \cdot 28 \times 10^{-6}\ \text{T}$$

Hence, total magnetic field at point O,

$$B = B_1 + B_2 + B_3 = 0 + 6{\cdot}28 \times 10^{-6} + 0$$

$$= \mathbf{6{\cdot}28 \times 10^{-6}\ T}$$

21. For the reason explained in problem no. 20, magnetic fi
produced at point O by the straight parts AB, CD, EF
GO of the conductor will be zero. Let B_1, B_2 and B
magnetic fields produced at point O due to circular p
BC, DE and FG respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{3r},$$ (in inward direction)

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{2r}$$ (in outward direction)

and $$B_3 = \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{r}$$ (in inward direction)

Therefore, total magnetic field produced by the conduc

$$B = B_1 - B_2 + B_3$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{3r} - \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{2r} + \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{r}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I\theta}{r}\left(\frac{1}{3} - \frac{1}{2} + 1\right)$$

or $$B = \frac{\mu_0}{4\pi} \cdot \frac{5I\theta}{6r}$$ **(in inward direction)**

22. Proceed as in solved problem no. 1.10.

23. Let B be magnetic field at a point at distance x from the centre of the coil and B_0, at the centre of the coil.

Then, $$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2+x^2)^{3/2}} \quad \text{...(i)}$$

and $$B_0 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a} \quad \text{...(ii)}$$

It follows that the magnetic field at a point on the axis of the coil is smaller than that at its centre by an amount, $B_0 - B$. Hence, the factor by which B is smaller than B_0,

$$n = \frac{B_0 - B}{B_0} = 1 - \frac{B}{B_0}$$

Using equations (*i*) and (*ii*), we have

$$n = 1 - \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2+x^2)^{3/2}} \times \frac{4\pi}{\mu_0} \cdot \frac{a}{2\pi I}$$

$$= 1 - \frac{a^3}{(a^2+x^2)^{3/2}} = 1 - \frac{a^3}{a^3(1+x^2/a^2)^{3/2}}$$

$$= 1 - \left(1 + \frac{x^2}{a^2}\right)^{-3/2} = 1 - \left(1 - \frac{3}{2}\frac{x^2}{a^2} + \ldots\right)$$

$$= 1 - \left(1 - \frac{3}{2}\frac{x^2}{a^2}\right) = \frac{3}{2}\frac{x^2}{a^2} = 1.5\frac{x^2}{a^2}$$

24. Proceed as in solved problem no. 1.12.

25. $$B = \mu_0 n I = \mu_0 \frac{N}{l} I = 4\pi \times 10^{-7} \times \frac{120}{0.2} \times 2.5$$
$$= 1.885 \times 10^{-3}\ \text{T}$$

26. **(a)** Proceed as in solved problem no. 1.14

(b) Magnetic field near the end on axis is equal to one half of that at the centre of the solenoid.

(c) Outside the solenoid, magnetic field is zero.

For explanation, refer to conceptual SAQ 1.07.

27. **(a)** Proceed as in problem no. 26 (*b*)

(b) Magnetic flux, $\phi = B_{end} \times \frac{\pi D^2}{4}$

28. $$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1}{a} \text{ (in inward direction)}$$

and $$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{a} \text{ (in outward direction)}$$

As $b < a$, $B_2 > B_1$

Therefore, net magnetic field at point P,

$$B = B_2 - B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{b} - \frac{\mu_0}{4\pi} \cdot \frac{2 I_1}{a}$$

$$= \frac{\mu_0}{2\pi}\left(\frac{I_2}{b} - \frac{I_1}{a}\right)$$

29. Let magnetic field be zero at a point, whose coordinates are (x, y).

Then, $$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1}{x} \text{ (in inward direction)}$$

and $$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{y} \text{ (in outward direction)}$$

As magnetic field at the observation point is zero,

$B_1 = B_2$ (in magnitude)

or $$\frac{\mu_0}{4\pi} \cdot \frac{2 I_1}{x} = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{y} \quad \text{or} \quad y = \frac{I_2}{I_1} x$$

30. Let B_1 and B_2 be magnetic fields produced at point O due to circular part ABC and straight part AC of the current carrying metallic wire.

The circular part ABC of the metallic wire subtends an angle $2\pi - 2\theta$ at the centre O.

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I(2\pi - 2\theta)}{r}$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{I}{r}(\pi - \theta) \quad \text{(in inward direction)}$$

Let ON = a. Then,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}(\sin\theta + \sin\theta)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}\sin\theta \quad \text{(in inward direction)}$$

From right angled Δ ANO, we have

ON = OA cos θ or $a = r\cos\theta$

$$\therefore \quad B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r\cos\theta}\sin\theta = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}\tan\theta$$

Hence, total magnetic field at point O,

$$B = B_1 + B_2 = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}(\pi - \theta) + \frac{\mu_0}{2\pi} \cdot \frac{I}{r}\tan\theta$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{I}{r}(\pi - \theta + \tan\theta) \text{ in inward direction}$$

31. Let B_1, B_2 and B_3 be magnetic fields produced at point O due to straight part AB, circular part BCD and straight part DE of the current carrying conductor.

As point O lies on the straight part AB,

$B_1 = 0$

The circular part BCD subtends an angle of 270° *i.e.* $\frac{3\pi}{2}$ at the centre O. Therefore,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r} \times \frac{3\pi}{2} \text{ (in outward direction)}$$

Also, $$B_3 = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(\sin 90° + \sin 0°) = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(1+0)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{r} \text{ (in outward direction)}$$

Therefore, resultant magnetic field at point O,

$$B = B_1 + B_2 + B_3 = 0 + \frac{\mu_0}{4\pi} \cdot \frac{I}{r} \times \frac{3\pi}{2} + \frac{\mu_0}{4\pi} \cdot \frac{I}{r}$$

or $$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}\left(\frac{3\pi}{2} + 1\right) \text{ in outward direction}$$

CHAPTER

2

Motion of Charged Particle

2.01. MOTION OF A CHARGED PARTICLE INSIDE ELECTRIC FIELD

Suppose that a uniform electric field is set up in the plane of paper along OY by applying potential difference between two plates parallel to horizontal [Fig. 2.01]. If V is the potential difference between the parallel plates held at a distance d apart, then strength of the electric field,

$$E = \frac{V}{d} \qquad ...(2.01)$$

Suppose that a positively charged particle having charge q and mass m enters the electric field along OX (normal to the direction of electric field) with velocity v. Due to the action of the electric field, the path of the charged particle no longer remains straight. Suppose that the charged particle moves along the curved path OA. On emerging out of the electric field, it will move along the straight path AB, tangent to the curved path OA at point A. As the particle moves along path OA, it covers a horizontal distance OQ = x and a vertical distance QA = y.

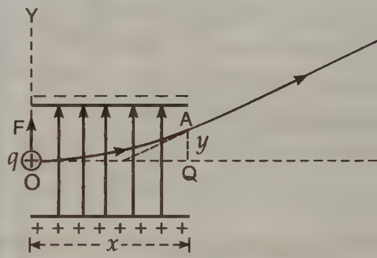


Fig. 2.01

Due to the electric field, force acts on the charged particle only along OY and it does not experience any force along OX. Therefore, the horizontal distance x is covered by the charged particle with uniform velocity v. Therefore, time taken by the charged particle to cover the horizontal distance OQ = x is given by

$$t = \frac{x}{v}$$

Along vertical direction OY, the charged particle has zero initial velocity. Due to electric field, it experiences a force,

$$F = qE \qquad \text{(along OY)}$$

and the acceleration in the path of charged particle along OY is given by

$$a = \frac{qE}{m}$$

The distance y covered along OY is given by

$$y = (0)\,t + \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}\cdot\left(\frac{x}{v}\right)^2$$

or

$$y = \frac{qE}{2mv^2}x^2 \qquad ...(2.02)$$

This is the equation of a parabola and hence inside the electric field, a charged particle moves along the curved path OA, which is ***parabolic*** in nature.

Key point

A charge always experiences a force
electric field, whether it is stationary o
in motion.

2.02. FORCE ON A CHARGE MOVING IN A MAGNETIC FIELD

Consider that a charge $+q$ is moving inside the magnetic field of strength $\vec{B}$. Suppose that the magnetic field acts along Z-axis, while the charge $+q$ moves with velocity $\vec{v}$ in YZ-plane making angle θ with the direction of $\vec{B}$ and as shown in Fig. 2.02.

As such, the charge $+q$ experiences force $\vec{F}$ along X-axis. It is found that the magnitude of the force is :

(*i*) *directly proportional to the magnitude of the strength of magnetic field i.e.*

$$F \propto B$$

(*ii*) *directly proportional to the magnitude of the charge i.e.*

$$F \propto q$$

(*iii*) *directly proportional to the component of velocity along a direction perpendicular to the direction of magnetic field i.e.*

$$F \propto v \sin\theta$$

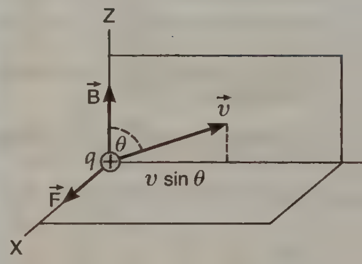


Fig. 2.02

Combining the three factors, we have

$$F \propto B\,q\,v \sin\theta$$

or $$F = k\,B\,q\,v \sin\theta,$$

here k is the constant of proportionality. In SI, when B is measured in tesla, q in ulomb and v in metre second^{-1}, the value of the constant k is found to be 1. herefore,

$$F = B\,q\,v \sin\theta, \qquad ...(2.03)$$

The force on a charged particle moving inside a magnetic field is called *agnetic Lorentz force*. The equation (2.03) gives the magnitude of the magnetic orentz force on a charged particle. In vector notation,

$$\vec{F} = q(\vec{v} \times \vec{B}) \qquad ...(2.04)$$

The equation (2.04) gives both the magnitude and the direction of the magnetic orentz force on the charged particle moving inside the magnetic field. Thus, if the harged particle moves along Y-axis with velocity $\vec{v}$ and the magnetic field $\vec{B}$ is irected along Z-axis [Fig. 2.03], then force on the charge $+q$ acts in the direction of $\vec{v} \times \vec{B})$ *i.e.* along X-axis.

Key point

The force on a charge moving inside a magnetic field results due to interaction between the magnetic field produced by the moving charge (which is equivalent to current) and the applied field.

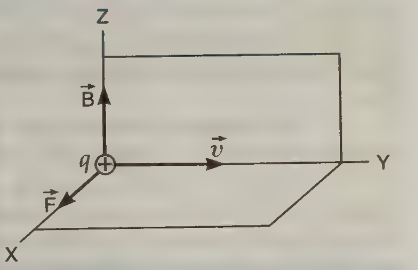


Fig. 2.03

Special cases. Let us consider the following cases :

1. ***When the charged particle is at rest.*** If the charged particle is at rest, hen $v = 0$. From equation (2.03), the force on charged particle,

$$F = B\,q\,(0) \sin\theta = 0$$

Thus, a charged particle at rest inside the magnetic field does not experience ny force due to a magnetic field.

2. ***When the charged particle moves parallel to the magnetic field.*** If $\vec{v}$ is arallel to $\vec{B}$, then $\theta = 0°$ or 180°. Then, from equation (2.03), the force on the charged article,

$$F = B\,q\,v\,(0) = 0 \qquad (\because \sin 0° = \sin 180° = 0)$$

In other words, a charged particle experiences force inside a magnetic field nly if it does not move parallel to the direction of the magnetic field.

Key point

1. A stationary charge does not experience any force in magnetic field.
2. A moving charge experiences force in magnetic field, provided it is not moving parallel to the direction of the field.

3. ***When the charged particle moves perpendicular to the magnetic field.*** If $\vec{v}$ is erpendicular to $\vec{B}$, then $\theta = 90°$. In that case, from equation (2.03), the force xperienced by the charged particle,

$$F = B\,q\,v\,(1) = B\,q\,v \textit{ (maximum)} \qquad (\because \sin 90° = 1)$$

Unit of strength of magnetic field. In SI, the unit of strength of magnetic field s **tesla (T).** The equation (2.03) may be used to define the unit of magnetic field as xplained below :

From equation (2.03), we have

$$B = \frac{F}{q\,v \sin\theta}$$

Therefore, $$\mathbf{1\ tesla\ (T)} = \frac{\mathbf{1\,N}}{\mathbf{1\,C \times 1\,m\,s^{-1} \times 1}} = \mathbf{1\,N\,A^{-1}\,m^{-1}} \qquad (\because C\,s^{-1} = A)$$

The strength of magnetic field at a point is called one tesla, if a charge of one coulomb, when moving with a velocity of 1 m s^{-1} along a direction perpendicular to the direction of the magnetic field, experiences a force of one newton.

Note. 1. The strength of magnetic field at a point is also measured as the magnetic flux crossing per unit area normally at that point.

The strength of magnetic field at a point is called one tesla, if a magnetic flux of one weber crosses an area of 1 m^2 along normal to the surface.

Thus $$\mathbf{1\ tesla\ (T) = 1\ weber\ metre^{-2}\ (Wb\ m^{-2})}$$

2. The strength of magnetic field is commonly expressed as **gauss (G).**

$$\mathbf{1\ gauss\ (G) = 10^{-4}\ tesla\ (T)}$$

2.03. LORENTZ FORCE

When a charge q moves with velocity $\vec{v}$ inside magnetic field of strength $\vec{B}$, then force on the charge,

$$\vec{F} = q(\vec{v} \times \vec{B})$$

It is called ***magnetic Lorentz force*** on the charge due to its motion inside the magnetic field.

If in addition to magnetic field of strength $\vec{B}$, an electric field of strength $\vec{E}$ also acts, then total force on the charge q is given by

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \qquad ...(2.05)$$

The force $\vec{F}$ given by equation (2.05) is called the ***total Lorentz force*** on the charge q.

Key point

If a charge is moving along the directio of electric field, only magnitude of it velocity is affectd. The direction o motion is affected, when the charge enter the field obliquely.

The following points may be noted about Lorentz force :

1. The magnetic Lorentz force is in the direction of vector $\vec{v} \times \vec{B}$ *i.e.* perpendicular to both the direction of motion of charge and the direction of magnetic field.

2. The magnetic Lorentz force does no work on the charge, as it always acts perpendicular to the motion of the charge.

3. Magnetic Lorentz force on a charge, in addition to the strength of the magnetic field, depends upon its velocity also, whereas force on the charge due to electric field depends only on the strength of the electric field. These facts can be used to tell whether the charge is moving inside an electric or magnetic field as discussed below :

(a) *If at a point, a charge moving in a field experiences force, which varies directly as the strength of the field (but does not depend upon the velocity of the charge), then the charge is moving inside electric field only.*

(b) *If at a point, a charge moving in a field experiences force, which varies directly as the strength of the field as well as its velocity (becomes zero, when charge is at rest), then the charge is moving inside magnetic field only.*

Key point

In a field, if the force experienced by charge depends only upon the field an does not depend upon the velocity of th particle, the field must be electric i nature.

2.04 MOTION OF A CHARGED PARTICLE INSIDE A UNIFORM MAGNETIC FIELD

When a charged particle having charge q moves inside a magnetic field $\vec{B}$ with velocity $\vec{v}$, it experiences a force given by

$$\vec{F} = q(\vec{v} \times \vec{B}) \qquad ...(2.06)$$

The direction of force on the charged particle due to magnetic field is always in a direction perpendicular to both its velocity $\vec{v}$ and the magnetic field $\vec{B}$. Let us study the motion of the charged particle inside magnetic field in the following two cases :

***(a)* When $\vec{v}$ is perpendicular to $\vec{B}$**. Consider that the uniform magnetic field of strength $\vec{B}$ is directed along OZ and the charged particle is moving in XY-plane. Suppose that at any instant, the charged particle is at point A, so that the vectors representing its velocity $\vec{v}$ and the magnetic field $\vec{B}$ are perpendicular to each other as shown in Fig. 2.04. For the orientations of $\vec{v}$ and $\vec{B}$ at point A, the equation (2.06) tells that the force $\vec{F}$ on the charged particle acts along AO *i.e.* along perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force $\vec{F}$ acts on the charged particle perpendicular to its velocity, *it does not do any work.* Hence, magnitude of the velocity of the charged particle remains constant and the force on the charged particle brings about a change only in the direction of the velocity of the charged particle. In other words, the force $\vec{F}$ on the charged particle acts as the centripetal force and makes it to move along a circular path. At points B and C in the XY-plane, the charged particle experiences force along BO and CO respectively. If m is the mass of the charged particle and r, the radius of the circular path followed by it, then

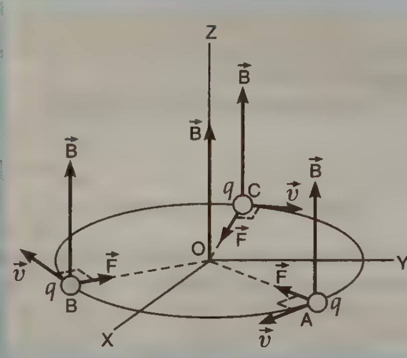


Fig. 2.04

$$|q(\vec{v} \times \vec{B})| = \frac{mv^2}{r} \qquad ...(2.07)$$

Since $\vec{v}$ and $\vec{B}$ are at right angles to each other, we have

$$|q(\vec{v} \times \vec{B})| = Bqv \qquad ...(2.08)$$

Therefore, from equations (2.07) and (2.08), we have

$$\mathrm{B}\,q\,v = \frac{m\,v^2}{r}$$

or
$$r = \frac{m\,v}{\mathrm{B}\,q} \qquad \text{...(2.09)}$$

It follows that

(*i*) *faster the particle moves (v large), the larger is the radius of circular path and*
(*ii*) *heavier the charged particle (m large), the larger is the radius of the circular path.*

The period of circular motion of the charged particle is given by

$$\mathrm{T} = \frac{2\pi r}{v} = \frac{2\pi}{v} \times \frac{m\,v}{\mathrm{B}\,q}$$

or
$$\mathrm{T} = \frac{2\pi m}{\mathrm{B}\,q} \qquad \text{...(2.10)}$$

The angular frequency of the charged particle moving inside the uniform agnetic field is given by

$$\omega = \frac{2\pi}{\mathrm{T}} = 2\pi \Big/ \frac{2\pi m}{\mathrm{B}\,q}$$

or
$$\omega = \frac{\mathrm{B}\,q}{m} \qquad \text{....(2.11)}$$

From equations (2.10) and (2.11), it follows that the period of the circular motion nd the angular frequency of a charged particle moving in a uniform magnetic field *either depends upon the magnitude of its velocity nor upon the radius of the circular path.* his fact is made use of to accelerate positive ions and such an accelerator is called *yclotron*. The angular frequency ω is oftenly called ***cyclotron frequency***.

(*b*) **When $\vec{v}$ and $\vec{\mathrm{B}}$ are inclined to each other.** Suppose that at any instant, the harged particle is moving with velocity $\vec{v}$ inside the uniform magnetic field $\vec{\mathrm{B}}$ naking an angle θ with the direction of the magnetic field as shown in Fig. 2.05.

The velocity $\vec{v}$ of the charged particle can be resolved into the following two omponents :

(*i*) The component v_B along the direction of the magnetic field *i.e.* along OZ. It is given by

$$v_\mathrm{B} = v\cos\theta$$

(*ii*) The component v_n along normal to the direction of magnetic field *i.e.* in XY-plane. It is given by

$$v_n = v\sin\theta$$

As the magnetic field acts perpendicular to the velocity component v_n, the harged particle moves along a circular path. Therefore, as obtained in equation 2.09), the radius of the circular path followed by the charged particle is given by

$$r = \frac{m\,v_n}{\mathrm{B}\,q} = \frac{m\,v\sin\theta}{\mathrm{B}\,q} \qquad \text{...(2.12)}$$

The period of the circular path is given by

$$\mathrm{T} = \frac{2\pi r}{v_n} = \frac{2\pi}{v\sin\theta} \times \frac{m\,v\sin\theta}{\mathrm{B}\,q}$$

or
$$\mathrm{T} = \frac{2\pi m}{\mathrm{B}\,q} \qquad \text{...(2.13)}$$

It is the same expression as given by equation (2.10). It implies that the period of revolution of the charged particle remains same, whether $\vec{v}$ is parallel to or nclined to $\vec{\mathrm{B}}$.

As the charged particle moves along circular path in XY-plane due to the velocity component v_n, it also advances linearly along OZ due to the velocity component v_B. As a result, the charged particle will move along the path as shown n Fig. 2.06. Such a path is called ***helical path***.

Key point

When a charge moves inside the magnetic field, magnitude of its velocity is not affected by the field. However, direction of motion of the charge changes continuously and it moves along a circular path..

Key point

A charge follows circular path, when it enters a magnetic field in a direction perpendicular to the field. In case charge enters the field obliquely, it follows helical path due to component of its velocity along the field.

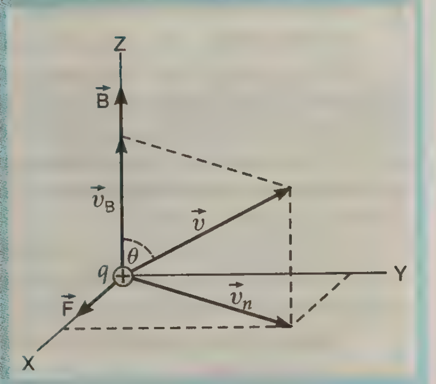


Fig. 2.05

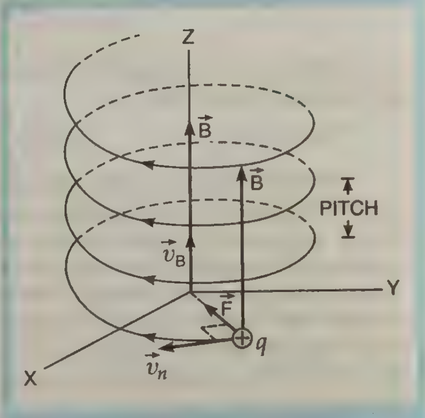


Fig. 2.06

The distance travelled by the charged particle, along the direction of magnetic field in a time it completes one revolution, is called ***pitch of the helical path.*** Therefore,

$$\text{pitch of the helical path} = v_B \times T = v \cos\theta \times \frac{2\pi m}{Bq}$$

or

$$\text{pitch} = \frac{2\pi m v \cos\theta}{Bq} \qquad \text{...(2.14)}$$

Note. Pitch of the helical path is equal to the spacing between its two consecutive circular paths.

2.05. CYCLOTRON

A cyclotron is used for accelerating positive ions, so that they acquire energy large enough to carry out nuclear reactions. Another accelerator, called ***linear accelerator***, was also developed for this purpose. However, it has a main drawback that its length would very large, if the positive ions are to be accelerated to a very high energy. It is because, the cylindrical tubes of increasing lengths have to be used, as the positive ions gain more and more energy.

Cyclotron was designed by Lawrence and Livingstone in 1931 in order to overcome the drawbacks of the linear accelerator. In a cyclotron, the positive ions cross again and again the same alternating (radio frequency) electric field and thereby gain the energy. It is achieved by making them to move along spiral path under the action of a strong magnetic field. It is also known as ***magnetic resonance accelerator.***

Principle. It is based on the principle that *a positive ion can acquire sufficiently large energy with a comparatively smaller alternating potential difference by making them to cross the same electric field time and again by making use of a strong magnetic field.*

Construction. It consists of two D-shaped hollow semicircular metal chambers D_1 and D_2, called *dees*. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} mm mercury. The whole apparatus is placed between the two poles of a strong electromagnet NS as shown in Fig. 2.07. The magnetic field acts perpendicular to the plane of the dees.

The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used ; while for producing alpha-particles, helium gas is used.

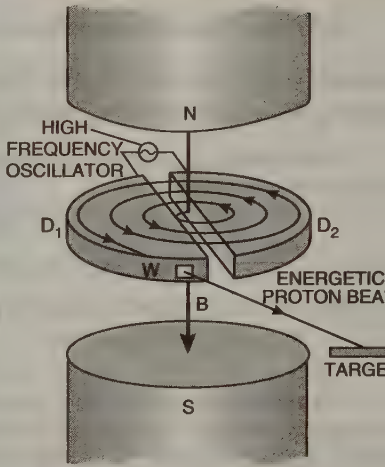


Fig. 2.07

Theory. Consider that a positive ion is produced at the centre of the gap at the time, when the dee D_1 is at positive potential and the dee D_2 is at a negative potential. The positive ion will move from dee D_1 to dee D_2. As the magnetic field acts normally to the motion of the positive ion, the positive ion experiences force. The force on the positive ion due to magnetic field provides the centripetal force to the positive ion and it is deflected along a circular path. If B is strength of the magnetic field and m, v and q, are respectively the mass, velocity and charge of the positive ion, then

$$B q v = \frac{m v^2}{r} \qquad \text{...(2.15)}$$

Here, r is the radius of the semi-circular path along which the positive ion will move inside the dee D_2.

Thus,

$$r = \frac{m v}{B q} \qquad \text{...(2.16)}$$

After moving along the semi-circular path inside the dee D_2, the positive ion reaches the gap between the dees. At this stage, the polarity of the dees just reverses due to alternating* electric field *i.e.* dee D_1 becomes negative and dee D_2 becomes positive. The positive ion again gains the energy, as it is attracted by the dee D_1. After moving along the semi-circular path inside the dee D_1, the positive ion again reaches the gap and it again gains the energy. This process repeats itself time and again. It is because, *the positive ion spends the same time inside a dee irrespective of its velocity or the radius of the circular path.* It can be proved as below :

---

*The alternating electric field used is of radio-frequency.

The time spent inside a dee to cover semi-circular path,

$$t = \frac{\text{length of the semi circular path}}{\text{velocity}} = \frac{\pi r}{v}$$

Using equation (2.16), we have

$$t = \frac{\pi m}{Bq} \qquad ...(2.17)$$

As required, the time which a positive ion of mass m and charge q spends inside dee is indeed independent of its velocity and radius of the semi-circular path. It ay again be pointed out that decrease in time spent inside a dee due to increasing elocity of positive ions is exactly compensated by the increase in length of the semi-rcular path. Due to this condition, the positive ion always crosses the alternating ectric field across the gap in correct phase.

Key point ✓

Time period of revolution of a charged particle inside the dees (magnetic field) is independent of the speed and radius of the circular path described. It depends upon the value of its charge, mass and strength of the field.

Cyclotron frequency. Under the action of the given magnetic field (B fixed), the ven positive ion (e/m fixed) will cover the semi-circular path in a fixed time only, t in equation (2.17) is equal to $\frac{T}{2}$ where T is the time period of the electric field.

$$\therefore \qquad \frac{T}{2} = \frac{\pi m}{Bq}$$

$$\text{or} \qquad T = \frac{2\pi m}{Bq}$$

The cyclotron angular frequency, $\omega = \frac{2\pi}{T} = \frac{Bq}{m}$ ...(2.18)

Also, cyclotron frequency, $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$ ...(2.19)

It is also known as ***magnetic resonance frequency*** (non-relativistic).

Maximum energy of the positive ions. Let v_{max} be the velocity acquired by the ositive ions, when it moves along the largest circular path *i.e.* path of radius equal the radius R of the dees. Then,

Key point ✓

A charged particle gets accelerated only due to the electric field. There is no effect of magnetic field on velocity (or kinetic energy) of the charged particle. The magnetic field simply makes the charged particle to cross the electric field again and again by making it to move along circular path.

$$B q v_{max} = \frac{m v_{max}^2}{R}$$

$$\text{or} \qquad v_{max} = \frac{BqR}{m} \qquad ...(2.20)$$

Therefore, maximum kinetic energy gained by the positive ion,

$$E_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} m \left(\frac{BqR}{m}\right)^2$$

$$\text{or} \qquad E_{max} = \frac{1}{2}\frac{B^2 q^2 R^2}{m} \qquad ...(2.21)$$

The maximum energy acquired by the positive ion can be expressed in nother form as given below :

If V is the potential difference applied between the dees and N is the number f times, the positive ion crosses the gap between the dees before leaving the dees, en

$$E_{max} = N(Vq) \qquad ...(2.22)$$

Obviously, $N = 2n$, where n is the number of rotations completed by the positive n before the radius of its circular path becomes equal to that of the dees.

Limitations of cyclotron. 1. When a charged particle is accelerated, its mass so starts increasing with increase in its speed. When its speed becomes comparable that of light, the mass of the charged particle becomes quite large as compared its rest mass. If m_0 is rest mass, then mass of charged particle when moving with eed v is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Substituting for m in equation (2.17), we have

time spent inside a dee, $t = \dfrac{\pi m_0}{qB\sqrt{1 - v^2/c^2}}$

Therefore, as v increases, t also increases *i.e.* the charged particle starts taking more and more time to complete the semi-circular path inside the dee. Since electric field changes the polarity of the dees after a fixed interval, the charged particle starts lagging behind the electric field and it is ultimately lost by colliding against the walls of the dees. However, this problem is overcome in the following two ways :

(*a*) As v increases, $\sqrt{1-v^2/c^2}$ decreases. Therefore, B is increased in such a manner that the factor $B\sqrt{1-v^2/c^2}$ and hence t always remains constant. Such a cyclotron, in which the strength of magnetic field is adjusted to overcome the problem due to relativistic variation in mass of the positive ion, is called ***synchrotron.***

(*b*) The frequency of revolution of charged particle inside the dees may be expressed as

$$\nu = Bq\frac{\sqrt{1-v^2/c^2}}{2\pi m_0}$$

It follows that as v increases, $\sqrt{1-v^2/c^2}$ decreases and hence ν decreases. If frequency of the electric field is adjusted to be always equal to the frequency of revolution of the charged particle, then such a cyclotron is called ***synchro-cyclotron*** or ***frequency modulated cyclotron.***

2. Cyclotron is used to accelerate heavy charged particles, such as protons. It is not suitable for accelerating electrons.

The reason is that due to small mass, the electrons gain in speed quickly and likewise the relativistic variation in mass quickly makes them out of step with the oscillating electric field.

Note. The charge on a proton is represented by the symbol e. In such a case, the symbol q may be replaced by e in the expressions obtained above.

2.06. FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN A MAGNETIC FIELD (FORCE ON A CURRENT)

Consider a straight conductor PQ of length l placed along Y-axis. Suppose that a uniform magnetic field of strength $\vec{B}$ acts along Z-axis *i.e.* perpendicular to the length of the conductor [Fig. 2.08]. Further suppose that current I flows through the conductor from its end P to Q, so that the vector $\vec{I\,l}$ (current element vector) is along OY.

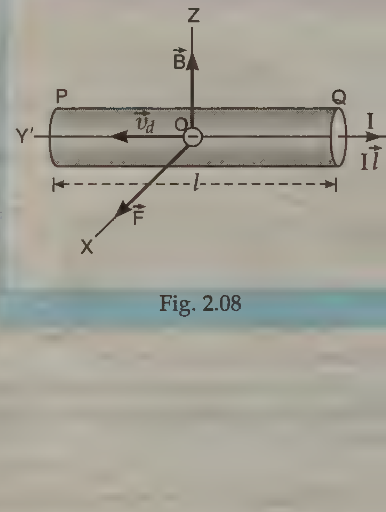


Fig. 2.08

H.A. Lorentz was the first to suggest that the electric current in metals is due to the flow of electrons. In a conductor, current flows due to drift velocity (v_d) of the electrons. If the conductor has area of cross-section A and the number of free electrons per unit volume equal to n, then the current flowing through the conductor is given by

$$I = nAev_d$$

Multiplying both sides by l, we have

$$Il = nAlev_d$$

The direction of flow of the conventional current I is opposite to that of the drift velocity of the electrons. In other words, $\vec{I\,l}$ and $\vec{v}_d$ have opposite directions and likewise the above equation may be written as

$$\vec{I\,l} = -nAle\,\vec{v}_d \qquad ...(2.23)$$

The magnetic Lorentz force on an electron having charge $-e$ and moving with velocity $\vec{v}_d$ inside the magnetic field of strength $\vec{B}$ is given by

$$\vec{f} = -e(\vec{v}_d \times \vec{B}) \qquad ...(2.24)$$

Each of the free electron inside the conductor experiences force $\vec{f}$ under the effect of magnetic field. If N is the total number of electrons in the conductor, then force on the current carrying conductor placed inside the magnetic field is given by

$$\vec{F} = N \times \vec{f} \qquad ...(2.25)$$

Now, the total number of free electrons in the conductor,

$$N = n \times \text{volume of conductor} = n\,A\,l$$

Substituting for N and $\vec{f}$ in equation (2.25), the force on the conductor,

$$\vec{F} = n\,A\,l\left(-e\,(\vec{v}_d \times \vec{B})\right) = (-n\,A\,l\,e\,\vec{v}_d) \times \vec{B}$$

Using equation (2.23), we have

$$\vec{F} = \mathrm{I}\,\vec{l} \times \vec{B} \qquad \text{...(2.26)}$$

Magnitude of force. From equation (2.26), the magnitude of the force on the urrent carrying conductor is given by

$$F = B\,I\,l \sin\theta, \qquad \text{...(2.27)}$$

here θ is the angle between the direction of magnetic field and the direction of flow f current.

(*i*) If $\theta = 0°$ or $180°$ *i.e.* $\sin\theta = 0$, then

$$F = B\,I\,l\,(0) = 0$$

Thus, if the current carrying conductor is placed ***parallel*** to the direction of agnetic field, it ***does not*** experience any force.

(*ii*) If $\theta = 90°$, then

$$F = B\,I\,l \sin 90° = B\,I\,l \qquad \textit{(maximum)}$$

In other words, a current carrying conductor experiences ***maximum*** force in a agnetic field, when it is placed ***perpendicular*** to the direction of magnetic field.

Direction of force. If $\mathrm{I}\,\vec{l}$ and $\vec{B}$ are oriented along OY and OZ respectively as hown in Fig. 2.08, then equation (2.26) tells that the force $\vec{F}\;(= \mathrm{I}\,\vec{l} \times \vec{B})$ on the onductor will act along OX.

In practice, to find the direction of force on a current carrying conductor, ***leming's left hand rule*** is used.

2.07. FLEMING'S LEFT HAND RULE

It states that *if the fore-finger, central finger and the thumb of left hand are stretched utually perpendicular to each other, such that the fore-finger points in the direction of agnetic field* ($\vec{B}$) *and the central finger in the direction of current (I), then the thumb points the direction of the force* ($\vec{F}$) *experienced by the conductor.*

The rule has been illustrated in Fig. 2.09.

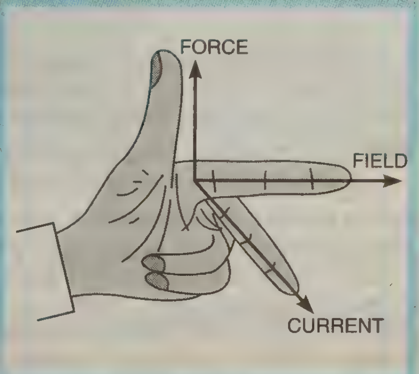


Fig. 2.09

Note. The equation (2.26) holds only *if the magnetic field is uniform over the whole ngth of the conductor.* If magnetic field is uniform over a small element of length dl nly, then force on the small element of the conductor is given by

$$d\vec{F} = \mathrm{I}\,d\vec{l} \times \vec{B} \qquad \text{...(2.28)}$$

2.08. FORCE BETWEEN TWO INFINITELY LONG PARALLEL CURRENT CARRYING CONDUCTORS

Consider two infinitely long conductors X_1Y_1 and X_2Y_2 placed parallel to each ther at a distance r apart. The currents I_1 and I_2 are flowing through the conductors $_1Y_1$ and X_2Y_2 in the same direction as shown in Fig. 2.10.

Let us find the *force experienced per unit length* by the current carrying conductor $_1Y_1$ due to the magnetic field produced by the current carrying conductor X_2Y_2.

Magnetic field at point P (on the conductor X_1Y_1) due to curernt I_2 flowing rough the infinitely long conductor X_2Y_2 is given by

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{r}$$

According to the right hand thumb rule, the direction of the magnetic field B_2 point P is perpendicular to the plane of paper and in inward direction. Now, e conductor X_1Y_1 carrying current I_1 lies in the magnetic field B_2 produced by the onductor X_2Y_2. Since $F = B\,I\,l$, the force experienced by the *unit length* of the onductor X_1Y_1 due to magnetic field B_2 is given by

$$F = B_2 \times (I_1 \times 1) = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{r} \times I_1$$

or

$$F = \frac{\mu_0}{4\pi}\cdot\frac{2I_1 I_2}{r} \qquad \text{...(2.29)}$$

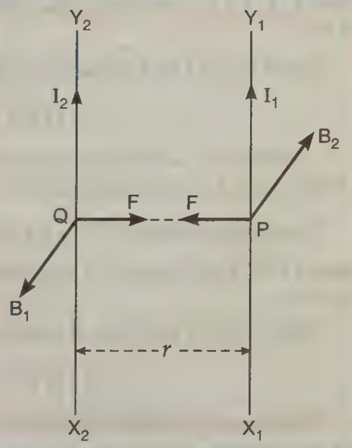


Fig. 2.10

Applying Fleming's left hand rule, it follows that the force F on the conductor X_1Y_1 acts in the plane of the paper and towards left. If we proceed in a similar manner, then it can be proved that the conductor X_2Y_2 experiences an equal force in the plane of the paper but towards right.

Therefore, the two parallel conductors carrying currents in the same direction attract each other and the magnitude of force, that each conductor exerts on the unit length of the other conductor, is given by equation (2.29).

On the other hand, it can be easily shown *that if the two parallel conductors carry currents in opposite directions, they repel each other* and the magnitude of the force experienced per unit length by one conductor due to the other is same as given by equation (2.29).

Definition of ampere. Let $I_1 = I_2 = 1$ A ; $r = 1$ m

Then, from equation (2.29), we have

$$F = \frac{10^{-7} \times 2 \times 1 \times 1}{1} = 2 \times 10^{-7} \text{ N m}^{-1}$$

Thus, *one ampere is that current, which when flowing through each of the two parallel conductors of infinite length and placed in free space at a distance of one metre from each other, produces between them a force of* 2×10^{-7} *newton per metre of their lengths.*

Note. When two infinitely long current carrying conductors are placed parallel to each other, then equation (2.29) for force experienced per unit length by one conductor due to the other has been obtained in a qualitative manner. A general expression for force can be obtained between two short parallel current carrying conductors by making use of equation (2.28) for force on a short current carrying conductor placed inside magnetic field. It can be proved that when two short conductors of lengths dl_1 and dl_2 and carrying currents I_1 and I_2 respectively are placed parallel to each other at a distance r apart, then force on one short conductor due to the other is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{I_1 I_2 \, dl_1 \, dl_2}{r^2} \qquad \text{...(2.30)}$$

When the current flows in same direction through the two elements, they attract each other and when the two parallel conductors carry currents in opposite directions, then the current elements repel each other with equal forces.

2.09. TORQUE ON A CURRENT LOOP PLACED IN A MAGNETIC FIELD

Consider that a rectangular coil ABCD is suspended in a uniform magnetic field $\vec{B}$ acting in the plane of the paper from left to right [Fig. 2.11]. Let l be length (AB or CD) and b be breadth (AD or BC) of the coil. Suppose that the coil carries current I in the direction ABCD and the normal to the plane of the coil makes an angle θ with the direction of the magnetic field. Let $\vec{F_1}, \vec{F_2}, \vec{F_3}$ and $\vec{F_4}$ be the forces acting on the four current carrying arms DA, BC, AB and CD of the coil respectively due to the magnetic field. It follows that the force on arm AD,

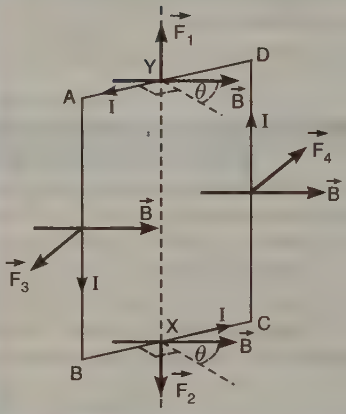


Fig.2.11

$$\vec{F_1} = I(\overrightarrow{DA}) \times \vec{B},$$

where $I(\overrightarrow{DA})$ is a vector in the direction of flow of current through the arm DA of the coil.

Since $I(\overrightarrow{DA})$ and $\vec{B}$ make angle $(90° + \theta)$, we have

$$F_1 = |I(\overrightarrow{DA})| \; |\vec{B}| \sin(90° + \theta) = (I\,b)\,B \cos\,\theta = B\,I\,b \cos\,\theta$$

The force $\vec{F_1}$ acts on arm DA in the direction of vector $I(\overrightarrow{DA}) \times \vec{B}$ *i.e.* in the plane of the coil and in upward direction.

The force on arm BC, $\vec{F_2} = I(\overrightarrow{BC}) \times \vec{B}$,

where $I(\overrightarrow{BC})$ is a vector in the direction of flow of current through the arm BC of the coil.

Since $I(\overrightarrow{BC})$ and $\vec{B}$ and make angle $(90° - \theta)$, we have

$$F_2 = |I(\overrightarrow{BC})| \; |\vec{B}| \sin(90° - \theta) = (I\,b)\,B \cos\,\theta = B\,I\,b \cos\,\theta$$

The force $\vec{F_2}$ acts in the direction of vector $I(\overrightarrow{BC}) \times \vec{B}$ *i.e.* in the plane of the coil and in downward direction.

Thus, forces $\vec{F_1}$ and $\vec{F_2}$ acting on the coil are equal in magnitude and opposite in direction. As the forces $\vec{F_1}$ and $\vec{F_2}$ have the same line of action, their resultant effect on the coil is zero.

Now, the force on arm AB, $\vec{F_3} = I(\overrightarrow{AB}) \times \vec{B}$,

where $I(\overrightarrow{AB})$ is the vector in the direction of flow of current through the arm AB of the coil.

The angle between the direction of magnetic field $\vec{B}$ and the side AB is 90°.

Therefore, $F_3 = |I(\overrightarrow{AB})|\,|\vec{B}|\sin 90° = (I\,l)\,B\,(1) = B\,I\,l$

The force $\vec{F_3}$ acts in the direction of vector $I(\overrightarrow{AB}) \times \vec{B}$ *i.e.* perpendicular to the paper and towards the reader.

Finally, the force on arm CD, $\vec{F_4} = I(\overrightarrow{CD}) \times \vec{B}$,

where $I(\overrightarrow{CD})$ is the vector in the direction of flow of current through the arm CD of the coil.

As the angle between the direction of magnetic field $\vec{B}$ and the side CD is 90°, we have

$$F_4 = |I(\overrightarrow{CD})|\,|\vec{B}|\sin 90° = (I\,l)\,B\,(1) = B\,I\,l$$

The force $\vec{F_4}$ acts in the direction of vector $I(\overrightarrow{CD}) \times \vec{B}$ *i.e.* perpendicular to the plane of the paper and away from the reader.

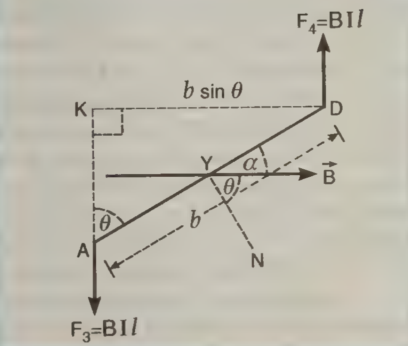


Fig. 2.12

Thus, the two forces $\vec{F_3}$ and $\vec{F_4}$ are equal in magnitude and opposite in direction. Fig. 2.12 shows the orientations of the forces $\vec{F_3}$ and $\vec{F_4}$ on the arms AB and CD, when seen from the top of the coil. As the two forces have different lines of action, they constitute a torque, whose magnitude is given by

$$\tau = \text{either force} \times KD,$$

where KD is the perpendicular distance between the forces $\vec{F_3}$ and $\vec{F_4}$.

As already said, the direction of the magnetic field $\vec{B}$ makes an angle θ with the normal (YN or Y′N′) to plane of the coil. Therefore, ∠ KAD = θ*. From right angled triangle AKD, we have

$$KD = AD \sin\theta = b \sin\theta$$

$$\therefore \quad \tau = BI\,l \times b \sin\theta$$

Since $l \times b = A$, the area of the coil ABCD, it follows that

$$\tau = B\,I\,A \sin\theta \qquad \text{...(2.31)}$$

As seen from top of the coil, the torque so produced will rotate the coil in anticlockwise direction. In vector notation,

$$\vec{\tau} = (I\,\vec{A}) \times \vec{B}, \qquad \text{...(2.32)}$$

where $\vec{A}$ is the vector representing the area of the coil and is directed along YN (or Y′N′), normal to the plane of the coil.

Key point

Torque on a planar current loop depends upon current, strength of magnetic field and area of the loop. It is independent of the shape of the loop.

Now, $I\,\vec{A} = \vec{M}$, the magnetic dipole moment of the current loop. Therefore,

$$\vec{\tau} = \vec{M} \times \vec{B} \qquad \text{...(2.33)}$$

Note. 1. If the coil has n turns, then

$$\tau = n\,B\,I\,A \sin\theta \qquad \text{...(2.34)}$$

2. If the direction of the magnetic field makes an angle α with the plane of the coil as shown in Fig. 2.12, then

$$\theta + \alpha = 90° \quad \text{or} \quad \theta = 90° - \alpha \quad \text{or} \quad \sin\theta = \sin(90° - \alpha) = \cos\alpha$$

Then, equation (2.34) becomes

$$\tau = n\,B\,I\,A \cos\alpha \qquad \text{...(2.35)}$$

3. The above expressions for the torque on the coil has been obtained for a rectangular loop. In fact, these expressions hold for a planar current loop of any shape.†

---

*Since the angle between two lines is same as the angle between their perpendiculars, it can be easily shown that ∠ KAD = θ.

†For detailed explanation, refer to S.A.Q. 2.27.

2.10. MOVING COIL GALVANOMETER

The moving coil galvanometer was first devised by Kelvin and later on modified by D' Arsonaval. It is also known as ***American type galvanometer.***

It is a device used to detect small current flowing in an electric circuit. With suitable modifications, a moving coil galvanometer can be used to measure current and potential difference.

Principle. *When a current carrying coil is placed in a magnetic field, it experiences a torque.*

Construction. It consists of a coil ABCD suspended between the poles of a permanent magnet NS as shown in Fig. 2.13. The coil is wound on a non-metallic frame and has a central soft iron core. The coil is suspended from the torsion head with a phosphor bronze wire. The other end of the coil is attached to a hair spring. A small concave mirror M is attached to the suspension wire. The whole arrangement is enclosed in a non-magnetic case to avoid disturbance due to air. The galvanometer can be levelled with the help of levelling screws, so that the coil can rotate freely without touching the iron core or the poles of the magnet.

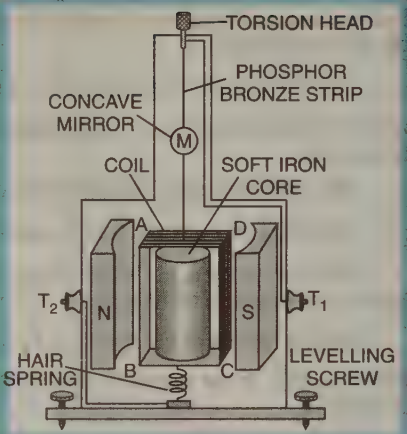


Fig. 2.13

To connect the galvanometer in the circuit, the binding screws T_1 and T_2 are provided at the back of the case of the galvanometer.

Theory. Suppose that the rectangular coil ABCD having n turns is of length l (= AB or CD) and breadth b (= AD or BC). Let B be the strength of the magnetic field due to the magnet NS. Initially *i.e.* (before passing the current through the galvanometer), the plane of the coil is parallel to the magnetic field. When current is passed through the coil, forces act on all the arms of the coil. The forces on arms DA and BC being equal and opposite*, cancel the effect of each other. If current is passed in the direction of ABCD, then

force on arm AB, $F = n\,B\,I\,l$ (normally outwards)

and force on arm CD, $F = n\,B\,I\,l$ (normally inwards)

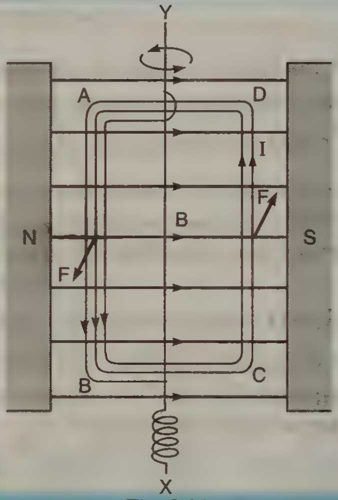


Fig. 2.14

The two forces are equal, opposite, parallel and act at different points as shown in Fig. 2.14. Hence, the two forces constitute a torque. As the coil rotates under the effect of the torque, the suspension wire gets twisted and a restoring torque is developed in the suspension wire. The coil will rotate, till *the deflecting torque acting on the coil due to flow of current is balanced by the restoring torque developed in the suspension wire due to twisting.* Therefore, in equilibrium,

deflecting torque = restoring torque (in magnitude)

Suppose the coil comes to rest after rotating through an angle α. In this position, the plane of the coil makes angle α with the direction of the field as shown in Fig. 2.15. The perpendicular distance between the forces acting on arms AB and CD is given by

$$DK = AD\cos\alpha = b\cos\alpha$$

Therefore, deflecting torque acting on coil = either force × DK = $n\,B\,I\,l \times b\cos\alpha$

$$= n\,B\,I\,A\cos\alpha, \qquad ...(2.36)$$

where $A = l \times b$ is the area of the coil. If k is the restoring torque per unit twist for the material of the suspension wire, then

$$\text{restoring torque} = k\,\alpha$$

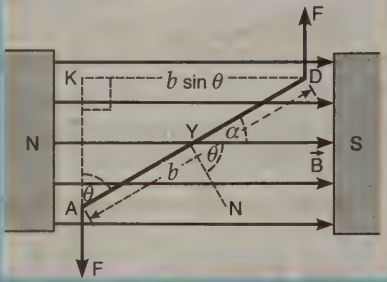


Fig. 2.15

Since in equilibrium, the restoring torque is just equal and opposite to the deflecting torque,

$$n\,B\,I\,A\cos\alpha = k\,\alpha$$

or
$$I = \frac{k}{n\,B\,A}\cdot\frac{\alpha}{\cos\alpha}$$

Since the factor $\dfrac{k}{n\,B\,A}$ is constant for the given galvanometer,

$$I \propto \frac{\alpha}{\cos\alpha}$$

As the deflection (α) produced is not directly proportional to the current (I) passing through the galvanometer, a linear scale cannot be used for measuring the current. In order that deflection produced in the coil of the galvanometer may be directly proportional to the current passed, the poles of the magnet are made concave so as to produce ***radial magnetic field.*** In such a field, the plane of the coil is parallel to the magnetic lines of force in all positions of the coil [Fig. 2.16] and hence

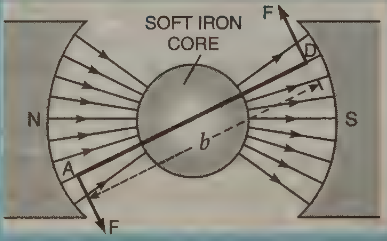


Fig. 2.16

---

* For details refer to section 2.09.

rpendicular distance between the forces on the two arms of the coil is always equal b, the breadth of the coil. Therefore, in case of radial magnetic field,

deflecting torque $= n\,B\,I\,l \times b = n\,B\,I\,A$

In equilibrium, $n\,B\,I\,A = k\,\alpha$

or $$I = \frac{k}{n\,B\,A}\,\alpha \qquad ...(2.37)$$

or $$I = G\,\alpha, \qquad ...(2.38)$$

here $G = \dfrac{k}{n\,B\,A}$ is constant for a galvanometer and is called ***galvanometer nstant.***

Hence, $I \propto \alpha$

deflection produced is directly proportional to the current passed through the lvanometer. Such a galvanometer will have a linear scale.

Sensitivity. *A galvanometer is said to be sensitive, if it gives a large deflection, even nen a small current is passed through it or a small voltage is applied across its coil.*

Current sensitivity. *It is defined as the deflection produced in the galvanometer on ssing unit current through its coil.* Therefore, if α is the deflection produced on passing rrent I, then

$$\text{current sensitivity } \frac{\alpha}{I} = \frac{n\,B\,A}{k} \qquad ...(2.39)$$

Voltage sensitivity. *It is defined as the deflection produced in the galvanometer, hen a unit voltage is applied across its coil.* Therefore, if α is the deflection produced n applying voltage V, then

$$\text{voltage sensitivity} = \frac{\alpha}{V}$$

If R is the resistance of coil and I is the current that passes through coil on pplying voltage V, then

$$V = I\,R$$

Therefore, voltage sensitivity, $$\frac{\alpha}{V} = \frac{\alpha}{I\,R} = \frac{n\,B\,A}{k\,R} \qquad ...(2.40)$$

It follows that a galvanometer will be highly sensitive, if

(*i*) *n is large ;* (*ii*) *B is large ;* (*iii*) *A is large ;* (*iv*) *k is small* and (*v*) *R is small.*

However, n and A cannot be increased beyond a certain limit, as otherwise the ze of the galvanometer and its resistance will become large. Therefore, B is made s large as possible and k is made as small as possible. To increase B, a very strong ermanent magnet is used. To decrease k, the suspension wire is made of phosphor ronze ; as for this material, k is very small. The value of k further decreases, if the vire is hammered in to a flat strip. In very sensitive galvanometers, quartz fibre is sed for suspension of the coil ; as for quartz, k is still smaller.

2.11. AMMETER

An ammeter is an instrument used for measuring current in the electrical circuits.

A galvanometer is a low resistance instrument and as such it cannot be used o measure current in a circuit. It is because, when even a small current is passed hrough the galvanometer, it produces full scale deflection. If a large current is assed, the galvanometer may be damaged because of the following two reasons :

(*i*) The large current will cause the coil of the galvanometer to deflect through large angle and the pointer of the galvanometer, in an attempt to go out of scale, nay break.

(*ii*) The large current on its passage through the coil of the galvanometer will roduce a large amount of heat. The excessive heating may also damage the alvanometer.

However, when a galvanometer is converted into ammeter, it can measure urrents without causing any damage to it.

To convert a galvanometer into an ammeter. Suppose that the galvanometer s to be converted into an ammeter of range 0 to I ampere. Further, suppose that he galvanometer gives full scale deflection, when current I_g (usually very small as compared to I) is passed through it. In order to convert the galvanometer into an ammeter of the required range, a small resistance S is connected in parallel to he coil of the galvanometer. Its value is so adjusted that when current I is passed, only the part I_g passes through the coil of the galvanometer and the remaining part $(I - I_g)$ flows through the resistance S [Fig. 2.17].

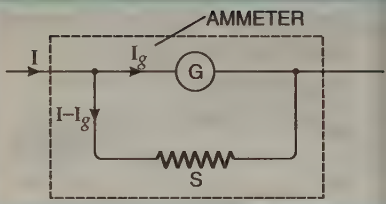


Fig. 2.17

Let G be the resistance of the coil of the galvanometer. Since the coil of the galvanometer and the shunt are connected across the same two points,

potential difference across shunt = potential difference across the coil

i.e. $$(I - I_g) \times S = I_g \times G$$

or $$S = \frac{I_g}{I - I_g} G \qquad ...(2.41)$$

Knowing the values of I_g, I and G, the value of S can be calculated. By connecting shunt of this value, the given galvanometer will work as an ammeter of the given range (0 to I ampere).

Note. 1. The parallel combination of galvanometer and the shunt constitutes the ammeter.

2. The ammeter so obtained has the resistance R_A, which is given by

$$\frac{1}{R_A} = \frac{1}{G} + \frac{1}{S} = \frac{S + G}{GS}$$

or $$R_A = \frac{GS}{S + G} \qquad ...(2.42)$$

In practice, G is large as compared to S. Therefore, practically $R_A \approx S$. Thus, resistance of the ammeter formed will be very low as compared to that of galvanometer. However, an ideal ammeter has ***zero resistance.***

3. As ammeter is an instrument of low resistance, it is always connected in series with the circuit. Because of the low resistance of the ammeter, on its introduction in the circuit, the resistance of the circuit practically remains unchanged and consequently the current in the circuit also remains unaffected.

Key point

1. The resistance of an ammeter is always much smaller than the resistance of the galvanomter, from which it is obtained.

2. The range of an ammeter can be increased but it cannot be decreased.

2.12. VOLTMETER

A voltmeter is an instrument used for measuring potential difference across the two ends of a conductor carrying current.

A galvanometer cannot be used to measure potential difference across a conductor in the circuit. It is because, when a galvanometer is connected in parallel to the conductor in order to measure potential difference across it ; due to low resistance of the galvanometer, a large part of the current will flow through it. As a result, current flowing through the conductor and hence potential difference across it will decrease on connecting galvanometer across it. Therefore, a galvanometer is converted into a voltmeter, so that it can measure potential difference across a conductor without causing any change (decrease) in the current flowing through it.

To convert a galvanometer into a voltmeter. Suppose the galvanometer is to be converted into a voltmeter of range 0 to V volt. Further, suppose that the galvanometer gives full scale deflection, when current I_g is passed through it. The resistance R is connected in series to the galvanometer so as to convert it into the required voltmeter [Fig. 2.18]. For this, the value of R is adjusted so that when a battery of V volt (maximum value of potential difference to be measured) is connected across the series combination of galvanometer and resistance R, the galvanometer gives full scale deflection *i.e.* current equal to I_g flows through it. If G is the resistance of the galvanometer, then

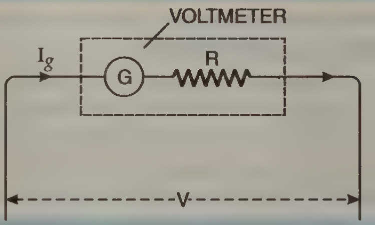


Fig. 2.18

$$I_g = \frac{V}{G + R}$$

or $$R = \frac{V}{I_g} - G \qquad ...(2.43)$$

Knowing V, I_g and G, the value of the R can be found. When the resistance of this value is connected in series to the galvanometer, it will work as a voltmeter of range 0 to V volt.

Note. 1. The series combination of galvanometer and the resistance R constitutes the voltmeter.

2. The voltmeter so obtained has resistance,

$$R_V = G + R \qquad ...(2.44)$$

In practice, it is found that R_V is very large as compared to the value of G. Hence, voltmeter is a high resistance instrument. An ideal voltmeter should possess *ıfinite resistance.*

3. Because of the high resistance of the voltmeter, it is connected in parallel to ıe conductor across which potential difference is to be measured. It is because, as ıe voltmeter will not practically draw any current due to its high resistance, the ırrent through the conductor will not change.

Key point

1. The resistance of a voltmeter is very large as compared to the resistance of the galvanometer, from which it is obtained.
2. The range of a voltmeter can be both increased and decreased.

ype A. On Motion of Charged Particle inside Electric field

Problem 2.01. A proton is initially travelling with orizontal velocity $v_x = 5{\cdot}0 \times 10^8$ cm s^{-1}. Calculate the ansverse deflection in travelling horizontal distance, = 10 cm in electric field of $E_y = 200$ volt cm^{-1}. Mass of proon, $m = 1{\cdot}6 \times 10^{-24}$ g ; charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C.

Sol. Here, $m = 1{\cdot}6 \times 10^{-24}\text{ g} = 1{\cdot}6 \times 10^{-27}\text{ kg}$;

$$e = 1{\cdot}6 \times 10^{-19}\text{ C}$$

Strength of electric field along Y-axis,

$$E_y = 200\text{ V cm}^{-1} = 200\text{ V} \times \left(\frac{1}{100}\text{ m}\right)^{-1} = 2 \times 10^4\text{ V m}^{-1}$$

Distance travelled along X-axis, $x = 10\text{ cm} = 0{\cdot}1\text{ m}$

Velocity along X-axis,

$$v_x = 5{\cdot}0 \times 10^8\text{ cm s}^{-1} = 5{\cdot}0 \times 10^6\text{ m s}^{-1}$$

If t is the time for which electron remains inside the ectric field, then

$$t = \frac{x}{v_x} = \frac{0{\cdot}1}{5{\cdot}0 \times 10^6} = 2 \times 10^{-8}\text{ s}$$

Force on proton along Y-axis,

$$F_y = e\,E_y = 1{\cdot}6 \times 10^{-19} \times 2 \times 10^4\text{ N}$$

Therefore, acceleration of proton along Y-axis,

$$a = \frac{F_y}{m} = \frac{1{\cdot}6 \times 10^{-19} \times 2 \times 10^4}{1{\cdot}6 \times 10^{-27}} = 2 \times 10^{12}\text{ m s}^{-2}$$

Initial velocity along Y-axis, $u = 0$;

time for which electric field acts, $t = 2 \times 10^{-8}$ s

Therefore, displacement along Y-axis,

$$y = u\,t + \frac{1}{2}a\,t^2$$

$$= (0) \times 2 \times 10^{-8} + \frac{1}{2} \times 2 \times 10^{12} \times (2 \times 10^{-8})^2$$

$$= 4 \times 10^{-4}\text{ m} = \mathbf{0{\cdot}4\text{ mm}}$$

ype B. On Force on a charge moving in magnetic field

Problem 2.02. Copper has $8{\cdot}0 \times 10^{28}$ conduction ectrons per metre^3. A copper wire of length 1 m and crossectional area $8{\cdot}0 \times 10^{-6}$ m^2 carrying a current and lying at ght angle to a magnetic field of strength 5×10^{-3} T periences a force of $8{\cdot}0 \times 10^{-2}$ N. Calculate the drift velocity f free electrons in the wire. (*C.B.S.E. Sample Q. Paper*)

Sol. In a copper wire of length l, area of cross-section A nd having number of free electrons per unit volume equal n, the total charge in the wire in the form of free electrons,

q = volume of wire × number of electrons per unit volume × charge on electron

$$= (A\,l) \times n \times e = n\,A\,l\,e$$

If v_d is drift velocity of the electron, then force on the opper wire,

$$F = B\,q\,v_d \qquad (\text{angle between } \vec{B} \text{ and } \vec{v}_d \text{ is } 90°)$$

or $$v_d = \frac{F}{B\,q} = \frac{F}{B\,n\,A\,l\,e}$$

Here, $n = 8{\cdot}0 \times 10^{28}\text{ m}^{-2}$; $l = 1$ m ; $A = 8{\cdot}0 \times 10^{-6}\text{ m}^2$;

$B = 5 \times 10^{-3}$ T ; $F = 8{\cdot}0 \times 10^{-2}$ N ;

Also, $e = 1{\cdot}6 \times 10^{-9}$ C

$$\therefore\ v_d = \frac{8{\cdot}0 \times 10^{-2}}{5 \times 10^{-3} \times 8{\cdot}0 \times 10^{28} \times 8{\cdot}0 \times 10^{-6} \times 1 \times 1{\cdot}6 \times 10^{-19}}$$

$$= \mathbf{1{\cdot}5625 \times 10^{-4}\ m\ s^{-1}}$$

Problem 2.03. An electron is moving northwards with a velocity 10^7 m s^{-1} in a magnetic field of 3 T directed eastwards. Calculate the instantaneous force on the electron. Given that charge on electron = $1{\cdot}6 \times 10^{-19}$ C.

Sol. Here, $q = 1{\cdot}6 \times 0^{-19}$ C ; $v = 10^7$ m s^{-1} ; B = 3 T

As electron is moving northwards and the magnetic field is east-wards [Fig. 2.19], the angle between $\vec{v}$ and $\vec{B}$ is 90°.

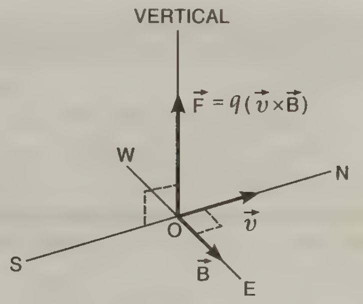


Fig. 2.19

Now, $$F = q\,v\,B \sin 90°$$

$$= 1{\cdot}6 \times 10^{-19} \times 10^7 \times 3 \times 1$$

$$= \mathbf{4{\cdot}8 \times 10^{-12}\ N}$$

Since the charge on electron is negative, force on electron,

$$\vec{F} = -q(\vec{v} \times \vec{B})$$

The direction of force on electron will be that of vector $-q(\vec{v} \times \vec{B})$ *i.e.* along vertically upwards [Fig. 2.19].

Problem 2.04. A long straight wire carries a current of 2 A. An electron travels with a velocity of 4×10^4 m s^{-1} parallel to the wire at a distance of 0·1 m from it and in a direction opposite to the current. What force does the magnetic field of current exert on the moving electron ? Given, $\mu_0 = 4\,\pi \times 10^{-7}$ T A^{-1}m and charge on electron, $q = 1{\cdot}6 \times 10^{-19}$ C.

Sol. Here, I = 2 A ; $\mu_0 = 4\,\pi \times 10^{-7}$ T A^{-1} m

Distance of the observation point from the conductor,

$$a = 0{\cdot}1\text{m}$$

Magnetic field at the observation point,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} = \frac{10^{-7} \times 2 \times 2}{0 \cdot 1} = 4 \times 10^{-6} \text{ tesla}$$

The magnetic field acts along normal to the plane, in which the wire and the observation point lies. As the electron moves in a direction parallel to wire and opposite to the current, the angle between $\vec{v}$ and $\vec{B}$ is 90°. Therefore,

force of element, $F = B\,q\,v$

Here $q = 1{\cdot}6 \times 10^{-19}$ C and $v = 4 \times 10^4$ m s^{-1}

$\therefore \quad F = 4 \times 10^{-6} \times 1{\cdot}6 \times 10^{-19} \times 4 \times 10^4$

$= \mathbf{2{\cdot}56 \times 10^{-20}}$ **N**

Problem 2.05. A long straight wire AB carries a current of 4 A. A proton P travels at 4 × 10^6 m s^{-1} parallel to the wire, 0·2 m from it and in a direction opposite to the current as shown in Fig. 2.20. Calculate the force which the magnetic field of current exerts on the proton. Also specify the direction of the force. *(C.B.S.E. 2002)*

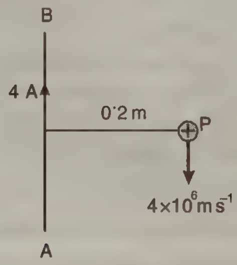


Fig. 2.20

Sol. Here, $I = 4$ A, $a = 0{\cdot}2$ m and $v = 4 \times 10^6$ m s^{-1}

Magnetic field due to a current carrying straight conductor is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} = \frac{10^{-7} \times 2 \times 4}{0 \cdot 2}$$

$= 4 \times 10^{-6}$ T

According to right hand thumb rule, the magnetic field will be directed perpendicular to the plane of paper and in inward direction [Fig. 2.21].

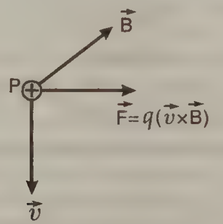


Fig. 2.21

Since proton is moving parallel to the conductor, the magnetic field produced by the current carrying straight wire is perpendicular to the direction of motion of the proton. If q is charge on proton and v is its velocity, then force on the proton due to magnetic field,

$$F = B\,q\,v = 4 \times 10^{-6} \times 1{\cdot}6 \times 10^{-19} \times 4 \times 10^6$$

$= \mathbf{2{\cdot}56 \times 10^{-18}}$ **N**

The direction of force on the proton is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

i.e. force on the proton acts **parallel to horizontal towards right.**

Type C. On Motion of a charge inside Magnetic fiel

Problem 2.06. An electron is projected with a velocity of 10^5 m s^{-1} at right angles to a magnetic field of 0·019 G. Calculate the radius of the circular path described by the electron, if $e = 1{\cdot}6 \times 10^{-19}$ C, $m = 9{\cdot}1 \times 10^{-31}$ kg.

(C.B.S.E. Sample Q. Paper

Sol. Here, $v = 10^5$ m s^{-1} ; $e = 1{\cdot}6 \times 10^{-19}$ C ;

$m = 9{\cdot}1 \times 10^{-31}$ kg ;

$B = 0 \cdot 019$ G $= 0 \cdot 019 \times 10^{-4}$ T

Now, $\quad B\,e\,v = \dfrac{m\,v^2}{r}$

or $\quad r = \dfrac{m\,v}{B\,e} = \dfrac{9 \cdot 1 \times 10^{-31} \times 10^5}{0 \cdot 019 \times 10^{-4} \times 1 \cdot 6 \times 10^{-19}} = \mathbf{0 \cdot 299\ m}$

Problem 2.07. Electron (with a charge of 1·6 × 11^{-19} C on each) moving at right angles to the uniform magnetic fiel completes a circular orbit in 10^{-6} s. Calculate the value o magnetic field. Given, mass of electron = 9 × 10^{-31} kg.

Sol. Here, $e = 1{\cdot}6 \times 10^{-19}$ C ; $m = 9 \times 10^{-31}$ kg ; $T = 10^{-6}$

Now, $\quad \dfrac{m\,v^2}{r} = B\,e\,v \quad$ or $\quad v = \dfrac{B\,e\,r}{m}$

Therefore, period of revolution,

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{B\,e\,r/m} = \frac{2\pi m}{B\,e}$$

$$\therefore B = \frac{2\pi m}{e\,T} = \frac{2\pi \times 9 \times 10^{-31}}{1 \cdot 6 \times 10^{-19} \times 10^{-6}} = \mathbf{3 \cdot 534 \times 10^{-5}\ T}$$

Problem 2.08. A proton and an alpha particle having the same kinetic energy are allowed to pass through uniform magnetic field perpendicular to their direction o motion. Compare the radii of the paths of proton and alpha particle. *(C.B.S.E. 1996*

Sol. Let q, m and v be the charge, mass and velocity o proton and Q, M and V be the respective values for alpha particle. Obviously,

$$Q = 2\,q \text{ and } M = 4\,m$$

Let r and R be the radii of circular paths followed by proton and alpha particle, while passing through the magnetic field of strength B.

Then, for proton : $\dfrac{m\,v^2}{r} = B\,q\,v$

or $\quad r = \dfrac{m\,v}{B\,q} \quad$ ...(*i*)

Similarly, for alpha particle : $R = \dfrac{M\,V}{B\,Q} = \dfrac{(4\,m)\,V}{B\,(2\,q)}$

or $\quad R = \dfrac{2\,m\,V}{B\,q} \quad$ ...(*ii*)

Dividing equation (*i*) by (*ii*), we have

$$\frac{r}{R} = \frac{m\,v}{B\,q} \times \frac{B\,q}{2\,m\,V} = \frac{1}{2}\left(\frac{v}{V}\right) \quad \text{...}(iii)$$

Since the proton and alpha particle have the same kinetic energy,

$$\frac{1}{2}\,m\,v^2 = \frac{1}{2}\,M\,V^2$$

or $\quad \dfrac{v}{V} = \sqrt{\dfrac{M}{m}} = \sqrt{\dfrac{4\,m}{m}} = 2 \quad$ ...(*iv*)

From equations (*iii*) and (*iv*), we have

$$\frac{r}{R} = \frac{1}{2} \times 2 = 1$$

ype D. On Cyclotron

Problem 2.09. A cyclotron's oscillator frequency is MHz. What should be the operating magnetic field for celerating protons? If the radius of its 'dees' is 60 cm, what the kinetic energy of the proton beam produced by the celerator ? Given, $e = 1{\cdot}60 \times 10^{-19}$ C, $m = 1{\cdot}67 \times 10^{-27}$ kg. press your answer in units of MeV.

Given, 1 MeV = $1{\cdot}6 \times 10^{-13}$ J. *(Text Problem)*

Sol. Here, $m = 1{\cdot}67 \times 10^{-27}$ kg ; $e = 1{\cdot}60 \times 10^{-19}$ C ;

$$\nu = 10 \text{ MHz} = 10^7 \text{ Hz} ; R = 60 \text{ cm} = 0{\cdot}6 \text{ m}$$

Now, $$\nu = \frac{1}{T} = \frac{Be}{2\pi m}$$

or $$B = \frac{2\pi m \nu}{e} = \frac{2\pi \times 1{\cdot}67 \times 10^{-27} \times 10^7}{1{\cdot}60 \times 10^{-19}}$$

$$= 0{\cdot}656 \text{ T}$$

Now, $$E_{max} = \frac{B^2 e^2 R^2}{2m}$$

$$= \frac{(0{\cdot}656)^2 \times (1{\cdot}60 \times 10^{-19})^2 (0{\cdot}6)^2}{2 \times 1{\cdot}67 \times 10^{-27}}$$

$$= 11{\cdot}874 \times 10^{-13} \text{ J}$$

$$= \frac{11{\cdot}874 \times 10^{-13}}{1{\cdot}6 \times 10^{-13}} = 7{\cdot}421 \text{ MeV}$$

pe E. On force on a current carrying conductor

Problem 2.10. A horizontal wire 0·1 m long carries a rrent of 5 A . Find the magnitude and direction of the gnetic field, which can support the weight of the wire. sume wire to be of mass 3×10^{-3} kg m^{-1}.

Sol. Here, $I = 5$ A ; $l = 0{\cdot}1$ m ;

mass per unit length of the wire $= 3 \times 10^{-3}$ kg m^{-1}

Therefore, weight of the wire,

$$mg = 3 \times 10^{-3} \times 0{\cdot}1 \times 9{\cdot}8 = 2{\cdot}94 \times 10^{-3} \text{ N}$$

Let B be the strength of the magnetic field applied. Then,

$$F = BIl = B \times 5 \times 0{\cdot}1 = 0{\cdot}5 \text{ B}$$

In equilibrium, $F = mg$

$\therefore$ $0{\cdot}5 \text{ B} = 2{\cdot}94 \times 10^{-3}$

or $$B = \frac{2{\cdot}94 \times 10^{-3}}{0{\cdot}5} = 5{\cdot}88 \times 10^{-3} \text{ tesla}$$

pe F. On Force between two long straight current carrying conductors

Problem 2.11. Calculate the force per unit length on ong straight wire carrying current of 4 A due to a allel wire carrying 6 A current. Distance between the es = 3 cm. *(H.S.S.C.E. 1998 ; C.B.S.E. 1991)*

Sol. Here, $I_1 = 4$ A ; $I_2 = 6$ A ; $r = 3$ cm $= 0{\cdot}03$ m

Now, $$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} = \frac{10^{-7} \times 2 \times 4 \times 6}{0{\cdot}03}$$

$$= 1{\cdot}6 \times 10^{-4} \text{ N m}^{-1}$$

Problem 2.12. Two long parallel wires are 0·4 m apart air and carry currents of 5 A and 12 A respectively. culate the force on each metre of the wire, if the rents are in (*i*) the same direction and (*ii*) in the osite direction. Given, $\mu_0 = 4\pi \times 10^{-7}$ T A^{-1} m.

Sol. Here, $I_1 = 5$ A ; $I_2 = 12$ A ; $r = 0{\cdot}4$ m ;

$$\mu_0 = 4\pi \times 10^{-7} \text{ T A}^{-1} \text{ m}$$

(i) When currents flow in the same direction. The conductors will *attract* each other. The force on one metre of a wire due to the whole length of the other wire,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} = \frac{10^{-7} \times 2 \times 2 \times 5 \times 12}{0{\cdot}4}$$

$$= 3 \times 10^{-5} \text{ N m}^{-1}$$

(ii) When currents flow in the opposite directions. The two conductors will *repel* each other, In this case,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} = 3 \times 10^{-5} \text{ N m}^{-1}$$

Problem 2.13. The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires, if they are 70 cm long and 1·5 cm apart ? Is the force attractive or repulsive ?

(H.S.S.C.E. 2001 ; P.S.S.C.E. 1995 ; Text Problem)

Sol. Force on a unit length of a current carrying conductor due to another *infinitely long* parallel current carrying conductor is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r}$$

Here $I_1 = I_2 = 300$ A ; $r = 1{\cdot}5$ cm $= 1{\cdot}5 \times 10^{-2}$ m

$\therefore$ $$F = \frac{10^{-7} \times 2 \times 300 \times 300}{1{\cdot}5 \times 10^{-2}} = 1{\cdot}2 \text{ N m}^{-1}$$

$$\left(\because \frac{\mu_0}{4\pi} = 10^{-7} \text{ T A m}^{-1} \right)$$

In the two wires connecting the battery to the starting motor, the current will flow in opposite directions. Hence, the force is **repulsive.**

Type G. On Torque on a current loop

Problem 2.14. Calculate the torque on a 100 turn rectangular coil of length 40 cm and breadth 20 cm, carrying a current of 10 A, when placed making an angle of 60° with a magnetic field of 5 T.

Sol. Here, $I = 10$ A ; $n = 100$;

$A = 40 \text{ cm} \times 20 \text{ cm} = 800 \text{ cm}^2 = 8 \times 10^{-2} \text{ m}^2$; $B = 5$ T ;

$\alpha = 60°$ (angle between field and plane of coil)

Now, $\tau = nBIA\cos\alpha = 100 \times 5 \times 10 \times 8 \times 10^{-2} \times \cos 60°$

$= \mathbf{200 \text{ N m}}$

Problem 2.15. (*a*) A circular coil of 30 turns and radius 8·0 cm carrying a current of 6·0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1·0 T. The field lines make an angle of 60° with the normal to the coil. Calculate the magnitude of the counter torque, that must be applied to prevent the coil from turning.

(*b*) Would your answer change, if the circular coil in (*a*) were replaced by a planar coil of some irregular shape that encloses the same area ? (All other particulars are also unaltered). *(Text Problem)*

Sol. Here, $n = 30$; $I = 6{\cdot}0$ A ; $B = 1{\cdot}0$ T ;

$\theta = 60°$ (angle between field and normal to coil) ;

radius of coil, $r = 8{\cdot}0 \text{ cm} = 8{\cdot}0 \times 10^{-2}$ m

Therefore, area of coil,

$$A = \pi r^2 = \pi \times (8{\cdot}0 \times 10^{-2})^2 = 2{\cdot}01 \times 10^{-2} \text{ m}^2$$

(*a*) Now, torque acting on the current carrying coil due to magnetic field,

$$\tau = n\,\mathrm{B\,I\,A}\sin\theta = 30 \times 1{\cdot}0 \times 6{\cdot}0 \times 2{\cdot}01 \times 10^{-2} \sin 60°$$
$$= 30 \times 1{\cdot}0 \times 6{\cdot}0 \times 2{\cdot}01 \times 10^{-2} \times 0{\cdot}866 = \mathbf{3{\cdot}133\ N\ m}$$

To prevent the coil from rotating, a torque equal and opposite to $\tau = (= 3{\cdot}133\ \mathrm{N\ m})$ has to be applied.

(*b*) Since torque on a planar loop is independent of its shape (provided area is same), the torque will remain **unchanged.**

Problem 2.16. Given a uniform magnetic field of 100 G in an east to west direction and a 44 cm long wire with a current carrying capacity of at most 10 A. What is the shape and orientation of the loop made of this wire, which yields maximum turning effect on the loop ? What is the magnitude of the maximum torque ? (*Text Problem*)

Sol. Here, $\mathrm{B} = 100\ \mathrm{G} = 100 \times 10^{-4}\ \mathrm{T}$; $\mathrm{I} = 10\ \mathrm{A}$

The torque on a planar loop will be maximum, if it has maximum area. For a given perimeter, a circle encloses maximum area. If 44 cm long wire is bent into a circular loop of radius *r*, then

$$2\pi r = 44$$

or $$r = \frac{44}{2\pi} = \frac{44 \times 7}{2 \times 22} = 7\ \mathrm{cm}$$

Area of loop,

$$\mathrm{A} = \pi r^2 = \pi \times 7^2 = \frac{22}{7} \times 49 = 154\ \mathrm{cm}^2$$
$$= 154 \times 10^{-4}\ \mathrm{m}^2$$

Now, $\tau = \mathrm{B\,I\,A}\sin\theta$

For maximum turning effect, $\theta = 90°$ *i.e.* $\sin\theta = 1$.

$$\therefore \quad \tau_{max} = \mathrm{B\,I\,A} = 100 \times 10^{-4} \times 10 \times 154 \times 10^{-4}$$
$$= \mathbf{1{\cdot}54 \times 10^{-3}\ N\ m}$$

Problem 2.17. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0·10 T normal to the plane of the coil. If the current in the coil is 5·0 A, what is the (*a*) total torque on the coil, (*b*) total force on the coil, (*c*) average force on each electron in the coil due to the magnetic field. The coil is made of copper wire of cross-sectional area $10^{-5}\ \mathrm{m}^2$ and the free electron density in copper is given to be about $10^{29}\ \mathrm{m}^{-3}$.

Sol. Here, $n = 20$; $r = 10\ \mathrm{cm} = 10 \times 10^{-2}\ \mathrm{m}$;
$\mathrm{B} = 0{\cdot}10\ \mathrm{T}$; $\mathrm{I} = 5{\cdot}0\ \mathrm{A}$;
angle between field and normal to the coil, $\theta = 0°$
Area of the coil, $\mathrm{A} = \pi r^2 = \pi \times (10 \times 10^{-2})^2 = \pi \times 10^{-2}\ \mathrm{m}^2$

(*a*) $\tau = n\,\mathrm{B\,I\,A}\sin\theta = 20 \times 0{\cdot}10 \times 5{\cdot}0 \times \pi \times 10^{-2} \sin 0°$
$= 20 \times 0{\cdot}10 \times 5{\cdot}0 \times \pi \times 10^{-2} \times 0 = 0$

(*b*) Net force on a planar current loop in a magnetic field is always zero.

(*c*) Force on one electron $= \mathrm{B}\,e\,v_d = \mathrm{B}\,e\left(\dfrac{\mathrm{I}}{n\,e\,\mathrm{A}}\right) = \dfrac{\mathrm{BI}}{n\,\mathrm{A}}$

Here, $n = 10^{29}\ \mathrm{m}^{-3}$; $\mathrm{A} = 10^{-5}\ \mathrm{m}^2$

Therefore, force on one electron

$$= \frac{0{\cdot}10 \times 5{\cdot}0}{10^{29} \times 10^{-5}} = 5 \times 10^{-25}\ \mathrm{N}$$

Type H. On Moving coil galvanometer

Problem 2.18. A rectangular coil of area $5{\cdot}0 \times 10^{-4}\ \mathrm{m}^2$ and 60 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 90 G ('radial' here means the field lines are in the plane of the coil for ar orientation). What is the torsional constant of the ha springs connected to the coil, if a current of 0·20 m produces an angular deflection of 18° ? (*Text Probler*

Sol. Here, $\mathrm{A} = 5{\cdot}0 \times 10^{-4}\ \mathrm{m}^2$; $n = 60$; $\alpha = 18°$;
$\mathrm{B} = 90\ \mathrm{G} = 90 \times 10^{-4}\ \mathrm{T}$;
$\mathrm{I} = 0{\cdot}20\ \mathrm{mA} = 0{\cdot}20 \times 10^{-3}\ \mathrm{A}$

Now, $$\mathrm{I} = \frac{k}{n\,\mathrm{B\,A}}\,\alpha$$

or $$k = \frac{n\,\mathrm{B\,I\,A}}{\alpha}$$
$$= \frac{60 \times 90 \times 10^{-4} \times 0{\cdot}20 \times 10^{-3} \times 5{\cdot}0 \times 10^{-4}}{18}$$
$$= \mathbf{3{\cdot}0 \times 10^{-9}\ N\ m\ per\ degree}$$

Problem 2.19. Compare the current sensitivity and vo tage sensitivity of the following moving coil galvanometer

Meter A : $n = 30$; $\mathrm{A} = 1{\cdot}5 \times 10^{-3}\ \mathrm{m}^2$; $\mathrm{B} = 0{\cdot}25\ \mathrm{T}$; R = 20

Meter B : $n = 35$; $\mathrm{A} = 2{\cdot}0 \times 10^{-3}\ \mathrm{m}^2$; $\mathrm{B} = 0{\cdot}25\ \mathrm{T}$; R = 30

You are given that the springs in the two meters ha the same torsional constants.

Sol. *For meter A :* *For meter B :*

$n_1 = 30$; $\mathrm{A}_1 = 1{\cdot}5 \times 10^{-3}\ \mathrm{m}^2$ $n_2 = 35$; $\mathrm{A}_2 = 2{\cdot}0 \times 10^{-3}$

$\mathrm{B}_1 = 0{\cdot}25\mathrm{T}$; $\mathrm{R}_1 = 20\Omega$ $\mathrm{B}_2 = 0{\cdot}25\ \mathrm{T}$; $\mathrm{R}_2 = 30\ \Omega$

Current sensitivity of a meter, $\dfrac{\alpha}{\mathrm{I}} = \dfrac{n\,\mathrm{B\,A}}{k}$

$$\therefore \quad \frac{\text{current sensitivity of meter A}}{\text{current sensitivity of meter B}} = \frac{n_1\,\mathrm{B}_1\,\mathrm{A}_1}{k_1} \times \frac{k_2}{n_2\,\mathrm{B}_2\,\mathrm{A}_2}$$

Now, springs in the two meters have same value of *k* i

$$k_1 = k_2$$

$$\therefore \quad \frac{\text{current sensitivity of meter A}}{\text{current sensitivity of meter B}} = \frac{n_1\,\mathrm{B}_1\,\mathrm{A}_1}{n_2\,\mathrm{B}_2\,\mathrm{A}_2}$$
$$= \frac{30 \times 0{\cdot}25 \times 1{\cdot}5 \times 10^{-3}}{35 \times 0{\cdot}25 \times 2{\cdot}0 \times 10^{-3}} = \frac{45}{70} = \frac{9}{14}$$

Voltage sensitivity of a meter, $\dfrac{\alpha}{\mathrm{IR}} = \dfrac{n\,\mathrm{B\,A}}{k\mathrm{R}}$

$$\therefore \quad \frac{\text{voltage sensitivity of meter A}}{\text{voltage sensitivity of meter B}} = \frac{n_1\,\mathrm{B}_1\,\mathrm{A}_1}{k_1\,\mathrm{R}_1} \times \frac{k_2\,\mathrm{R}_2}{n_2\,\mathrm{B}_2\,\mathrm{A}_2}$$

Since $k_1 = k_2$, we have

$$\frac{\text{voltage sensitivity of meter A}}{\text{voltage sensitivity of meter B}} = \frac{n_1\,\mathrm{B}_1\,\mathrm{A}_1\,\mathrm{R}_2}{n_2\,\mathrm{B}_2\,\mathrm{A}_2\,\mathrm{R}_1}$$
$$= \frac{9}{14} \times \frac{30}{20} = \frac{27}{28}$$

Problem 2.20. The current sensitivity of a moving co galvanometer increases by 20%, when its resistance increased by a factor of two. Calculate by what factor, tl voltage sensitivity changes. (*C.B.S.E. 1998*

Sol. Consider that the moving coil galvanometer consis of a coil of *n* turns and possesses area A and resistance When a potential difference V is applied, suppose that current I flows through the galvanometer and the coil deflec through angle α. Then, current sensitivity of the galvanometer, $\frac{\alpha}{I} = \frac{nAB}{k}$ ...(*i*)

and voltage sensitivity of the galvanometer,

$$\frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{nAB}{kR} \quad ...(ii)$$

The resistance of the galvanometer can be increased by increasing n or A or both n and A. Suppose that the resistance of the galvanometer increases by a factor 2 *i.e.* becomes R′= 2 R, when the values of n and A are changed to $n'$and A′respectively. When the same potential difference V is applied, suppose that current I′ flows through the galvanometer and the coil deflects through angle α'.

Then, new current sensitivity of the galvanometer,

$$\frac{\alpha'}{I'} = \frac{n'A'B}{k} \quad ...(iii)$$

and new voltage sensitivity of the galvanometer,

$$\frac{\alpha'}{V} = \frac{\alpha'}{I'R'} = \frac{n'A'B}{2kR} \quad ...(iv)$$

Since on increasing resistance from R to R′ = 2 R, the current sensitivity increases by 20%, it follows that

$$\frac{\alpha'}{I'} = \frac{120}{100} \cdot \frac{\alpha}{I}$$

Using equations (*i*) and (*iii*), we have

$$\frac{n'A'B}{k} = \frac{120}{100} \cdot \frac{nAB}{k}$$

$$\text{or} \qquad n'A' = \frac{6nA}{5} \quad ...(v)$$

From equations (*iv*) and (*v*), we have

$$\frac{\alpha'}{V} = \frac{6nA}{5} \times \frac{B}{2kR} = \frac{3nAB}{5kR} \quad ...(vi)$$

From equations (*ii*) and (*vi*), we have

$$\frac{\alpha'}{V} = \frac{3}{5} \cdot \frac{\alpha}{V}$$

i.e. on increasing resistance of the glavanometer, the voltage sensitivity decreases by a factor of 3/5 *i.e.* it becomes 60% or decreases by 40%.

Type I. On conversion of galvanometer into ammeter and voltmeter

Problem 2.21. A galvanometer of resistance 15 Ω gives full scale deflection for a current of 2 mA. Calculate the shunt resistance needed to convert it to an ammeter of range 0 – 5 A. (*C.B.S.E. 1991*)

Sol. Here, G = 15 Ω ; I_g = 2 mA = 2× 10^{-3} A; I = 5 A

$$\text{Now,} \qquad S = \frac{I_g G}{I - I_g} = \frac{2 \times 10^{-3} \times 15}{5 - 2 \times 10^{-3}} = 0 \cdot 006\,\Omega$$

Problem 2.22. A galvanometer has a resistance of 100 ohm. A difference of potential of 50 millivolt between its terminals gives a full scale deflection. Calculate shunt resistance, which will enable the instrument to read upto 5 ampere.

Sol. Here, V = 50 mV = 50 × 10^{-3} V ; G = 100 Ω

$$\therefore \qquad I_g = \frac{V}{G} = \frac{50 \times 10^{-3}\,V}{100\,\Omega} = 5 \times 10^{-4}\,A$$

$$\text{Now,} \qquad S = \frac{I_g G}{I - I_g} = \frac{5 \times 10^{-4} \times 100}{5 - 5 \times 10^{-4}} = 0 \cdot 01\,\Omega$$

Problem 2.23. An ammeter of resistance 100 Ω can measure a maximum current of 5 mA . What will you do to measure a maximum current of 5 A with it ? (*P.S.S.C.E. 1994*)

Sol. The given ammeter may be considered as a galvanometer of resistance 100 Ω, which can measure a maximum current of 5 mA. Thus,

G = 100 Ω ; I_g = 5 mA = 5 × 10^{-3} A and I = 5 A

For this, a suitable small resistance S has to be connected in parallel to the coil of the given galvanometer. It is given by

$$S = \frac{I_g \times G}{I - I_g} = \frac{5 \times 10^{-3} \times 100}{5 - 5 \times 10^{-3}} = 0 \cdot 1\,\Omega$$

Problem 2.24. A galvanometer has a resistance of 5 Ω and a full scale deflection is produced by 15 mA. Calculate what resistance must be used to enable the galvanometer to read (*a*) 1·5 A ; (*b*) 1·5 volt ? (*Pre-degree Kerala, 1992*)

Sol. Here, G = 5 Ω; I_g = 15 mA = 15 × 10^{-3} A

(*a*) ***To enable galvanometer to read 1·5 A*** :

Here, = 1·5 A

The required resistance,

$$S = \frac{I_g \times G}{I - I_g} = \frac{15 \times 10^{-3} \times 5}{1 \cdot 5 - 15 \times 10^{-3}} = 0 \cdot 05\,\Omega \quad \textbf{(in parallel)}$$

(*b*) ***To enable galvanometer to read 1·5 V*** :

Here, V = 1·5 volt

The required resistance,

$$R = \frac{V}{I_g} - G = \frac{1 \cdot 5}{15 \times 10^{-3}} - 5 = 95\,\Omega \ \textbf{(in series)}$$

Problem 2.25. A moving coil galvanometer of resistance 10 ohm produces full scale deflection, when a current of 25 mA is passed through it. Describe showing full calculations, how will you convert the galvanometer into (*a*) a voltmeter reading upto 120 V (*b*) an ammeter reading upto 20 A. (*I.S.C.E. 1994*)

Sol. Here, G = 10 Ω ; I_g = 24 mA = 25 × 10^{-3} A

(*a*) ***To convert the galvanometer into voltmeter reading upto 120 V.*** To convert a galvanometer into voltmeter of range V, a large resistance R has to be connected in series to it. The value of R is given by

$$R = \frac{V}{I_g} - G$$

Here, V = 120 volt

$$\therefore \qquad R = \frac{120}{25 \times 10^{-3}} - 10 = 4790\,\Omega$$

(*b*) ***To convert galvanometer into ammeter reading upto 20 A.*** To convert a galvanometer into ammeter of range I, a small resistance S has to be connected in parallel to it. The value of S is given by

$$S = \frac{I_g G}{I - I_g}$$

Here, I = 20 A

$$\therefore \qquad S = \frac{25 \times 10^{-3} \times 10}{20 - 25 \times 10^{-3}} = 0 \cdot 0125\,\Omega$$

Type J. On Ammer and Voltmeter

Problem 2.26. A battery of e.m.f. 3 volt and internal resistance *r* is connected in series with a resistor of 55 Ω

through an ammeter of resistance 1 Ω. The ammeter reads 50 mA. Draw the circuit diagram and calculate the value of *r*. *(H.S.S.C.E. 2002 ; Pre-degree Kerala, 1994)*

Sol. The given battery, ammeter and the resistor are connected as shown in Fig. 2.22.

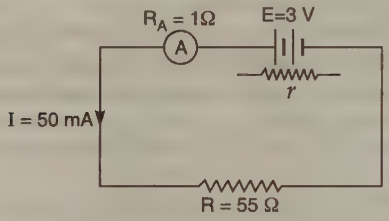


Fig. 2.22

Let R_A be the resistance of the ammeter. Then, the current in the circuit is given by

$$I = \frac{E}{R + R_A + r}$$

Here, $E = 3\text{ V}, R = 55\ \Omega, R_A = 1\ \Omega,$

$I = 50\text{ mA} = 50 \times 10^{-3}\text{ A}$

$$\therefore\ 50 \times 10^{-3} = \frac{3}{55 + 1 + r} = \frac{3}{56 + r}$$

or $$r = \frac{3}{50 \times 10^{-3}} - 56 = 60 - 56 = \mathbf{4\ \Omega}$$

Problem 2.27. In a circuit shown in Fig. 2.23 ; a voltmeter reads 30 V, when it is connected across 400 ohm resistance. Calculate what the same voltmeter will read, when connected across the 300 ohm resistance ? *(P.S.S.C.E. 1995)*

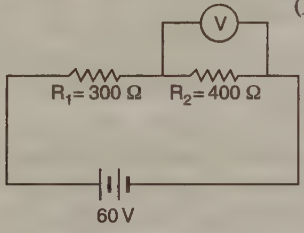


Fig. 2.23

Sol. Here, $E = 60\text{ V}\ ;\ R_1 = 400\ \Omega\ ;\ R_2 = 300\ \Omega$,

Reading of voltmeter, when connected across R_1,

$V_1 = 30\text{ V}$

Let R_V be the internal resistance of the voltmeter. If R is the resistance of the parallel combination of R_V and R_1, then

$$R = \frac{R_V \times R_1}{R_V + R_1} = \frac{R_V \times 400}{R_V + 400}$$

The current in the circuit,

$$I = \frac{E}{R_2 + R} = \frac{60}{300 + \dfrac{R_V \times 400}{R_V + 400}} = \frac{60\,(R_V + 400)}{700\,R_V + 12000}$$

Since potential difference across R is measured to be 30 V,

$IR = 30$

or $$\frac{60\,(R_V + 400)}{700\,R_V + 12000} \times \frac{400\,R_V}{(R_V + 400)} = 30 \text{ or } R_V = 1200\ \Omega$$

Suppose that when the voltmeter is connected across R_2, the resistance of their parallel combination becomes R′. Then,

$$R' = \frac{R_V \times R_2}{R_V + R_2} = \frac{1200 \times 300}{1200 + 300} = 240\ \Omega$$

If I′ is current in the circuit in this case, then

$$I' = \frac{E}{R' + R_1} = \frac{60}{240 + 400} = \frac{3}{32}\text{ A}$$

Therefore, reading of the voltmeter, when connecte across R_2,

$$V = I'R' = \frac{3}{32} \times 240 = \mathbf{22{\cdot}5\ V}$$

Problem 2.28. A voltmeter V of resistance 400 Ω i used to measure the potential difference across a 100 resistor in the circuit as shown in Fig. 2.24.

(*a*) What will be the reading on the voltmeter ? (b Calculate the potential difference across 100 Ω resisto before the voltmeter is connected. *(C.B.S.E. 1998*

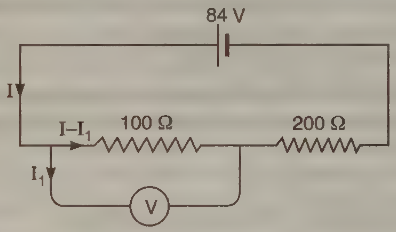


Fig. 2.24

Sol. Here, resistance of voltmeter, $R_V = 400\ \Omega$

E.M.F. of the battery, E = 84 V

Suppose that a current I flows through the main circui and a part I_1 of this current flows through the voltmeter a shown in the figure. Then, the remaining part $(I - I_1)$ will flov through the resistance of 100 Ω .

(*a*) Since voltmeter and 100 Ω resistance are connecte in parallel to each other, the potential drop across them i same *i.e.*

$I_1 \times R_V = (I - I_1) \times 100$

or $I_1 \times 400 = (I - I_1) \times 100$

or $I_1 = I/5$ ...(*i*

Also, if R is the equivalent resistance of the paralle combination of R_V (resistance of voltmeter) and 100 resistance, then

$$\frac{1}{R} = \frac{1}{R_V} + \frac{1}{100} = \frac{1}{400} + \frac{1}{100} = \frac{5}{400}$$

or $R = 80\ \Omega$

If R_1 is the total resistance of the circuit, then

$R_1 = R + 200 = 80 + 200 = 280\ \Omega$

Hence, current in the main circuit, $I = \dfrac{E}{R_1} = \dfrac{84}{280} = 0{\cdot}3\text{ A}$

Substituting for I in equation (*i*), we have

$I_1 = 0{\cdot}3/5 = 0{\cdot}06\text{ A}$

Therefore, reading of the voltmeter = potential dro across voltmeter $= I_1 \times R_V = 0{\cdot}06 \times 400 =$ **24 V**

(*b*) When the voltmeter is not connected, the tota resistance of the circuit will become,

$R_2 = 100 + 200 = 300\ \Omega$

If I_2 is current in the circuit, then

$$I_2 = \frac{E}{R_2} = \frac{84}{300} = 0{\cdot}28\text{ A}$$

Hence, potential difference across 100 Ω resistance

$$= I_2 \times 100 = 0{\cdot}28 \times 100 = \mathbf{28\ V}$$

Problem 2.29. (*a*) A d.c. supply of 120 V is connected to a large resistance X. A voltmeter of resistance 10 k Ω placed in series in the circuit reads 4 V. What is the value of X ? *(P.S.S.C.E. 1992)*

(*b*) What do you think is the purpose in using a voltmeter, instead of an ammeter, to determine the large resistance X ? *(Text Problem)*

Sol. (*a*) Here, $E = 120\ V$; $R_V = 10\ k\Omega = 10^4\ \Omega$

When the voltmeter is put in series with unknown resistance X,

the total resistance in the circuit $= R_V + X = 10^4 + X$

The current in the circuit, $I = \dfrac{E}{R_V + X} = \dfrac{120}{10^4 + X}$

Therefore, potential difference across the voltmeter itself

$$= I \times R_V = \frac{120}{10^4 + X} \times 10^4$$

Since reading of the voltmeter is 4 V, we have

$$\frac{120}{10^4 + X} \times 10^4 = 4$$

or $\qquad 10^4 + X = 30 \times 10^4$

or $\qquad X = 29 \times 10^4 = \mathbf{290\ k\Omega}$

(*b*) If an ammeter is used, then its reading will be too small to be measured.

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 2.30. A magnetic field set up using Helmholtz coils is uniform in a small region and has a magnitude of 0·75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single-species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected, when the electrostatic field is $9{\cdot}0 \times 10^5\ V\ m^{-1}$; make a simple guess as to what the beam contains. Why is the answer not unique ? *(Text Problem)*

Sol. Here, $B = 0{\cdot}75\ T$; $E = 9{\cdot}0 \times 10^5\ V\ m^{-1}$

Let *e* be charge, *m* be mass and *v* be velocity of the charged particles.

Since beam of charged particles remains undeflected, the force on a charged particle due to magnetic and electric fields must be equal and opposite. Therefore,

$$B\,e\,v = E\,e$$

or $\qquad v = \dfrac{E}{B} = \dfrac{9{\cdot}0 \times 10^5}{0{\cdot}75} = 1{\cdot}12 \times 10^6\ m\,s^{-1} \qquad ...(i)$

Now, kinetic energy of proton, $\frac{1}{2} m v^2 = V e$

Here, $\qquad V = 15\ kV = 15 \times 10^3\ V$

$$\therefore \quad \frac{1}{2} m v^2 = 15 \times 10^3 \times e$$

or $\qquad \dfrac{e}{m} = \dfrac{v^2}{30 \times 10^3} \qquad ...(ii)$

From equations (*i*) and (*ii*), we have

$$\frac{e}{m} = \frac{(1{\cdot}2 \times 10^6)^2}{30 \times 10^3} = 4{\cdot}8 \times 10^7\ C\,kg^{-1}$$

For proton, the value of e/m is $9{\cdot}6 \times 10^7\ C\,kg^{-1}$. It follows that the charged particle under reference has the value of e/m half of that for proton and therefore beam may be of **deutrons** (assuming that charge on particle is $1{\cdot}6 \times 10^{-19}$ C).

The answer is not unique because this ratio of charge to mass *i.e.* $4{\cdot}8 \times 10^7\ C\,kg^{-1}$ can be satisfied by many other charged particles such as He^{++}, Li^{+++}, etc.

Problem 2.31. A monoenergetic (18 keV) electron beam initially in the horizontal direction is subject to a horizontal magnetic field of 0·40 G normal to the initial direction. Estimate the up or down deflection of the beam over a distance of 30 cm. Given, $m = 9{\cdot}11 \times 10^{-31}$ kg and $e = 1{\cdot}60 \times 10^{-19}$ C.

Sol. Here, $m = 9{\cdot}11 \times 10^{-31}$ kg ; $e = 1{\cdot}60 \times 10^{-19}$ C ;

$$B = 0{\cdot}40\ G = 0{\cdot}40 \times 10^{-4}\ T\ ;$$

$$\frac{1}{2} m v^2 = 18\ keV = 18 \times 1{\cdot}6 \times 10^{-16}\ J$$

Suppose electron moves along a circular path of radius *r* under the effect of magnetic field.

Then, $\qquad \dfrac{m v^2}{r} = B\,e\,v$

or $\qquad r = \dfrac{m v}{B e} = \dfrac{\sqrt{2 m \times \frac{1}{2} m v^2}}{B e}$

$$= \frac{\sqrt{2 \times 9{\cdot}11 \times 10^{-31} \times 18 \times 1{\cdot}6 \times 10^{-16}}}{0{\cdot}40 \times 10^{-4} \times 1{\cdot}60 \times 10^{-19}}$$

$$= 11{\cdot}32\ m$$

Fig 2.25 shows an electron covering a circular path PQ (= 30 cm = 0·3 m) and of radius OP (= OQ =11·32 m).

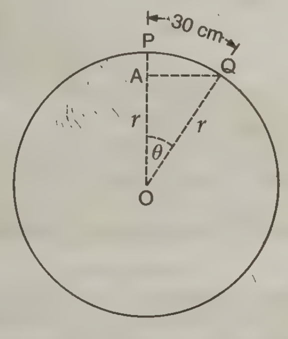


Fig. 2.25

It follows that the electron will undergo a deflection PA in downward direction, while it covers a distance PQ. If *θ* is angle subtended by PQ at the point O, then

$$PA = OP - OA = r - r\cos\theta = r\,(1 - \cos\theta)$$

Now, $\qquad \theta = \dfrac{\overset{\frown}{PQ}}{OP} = \dfrac{0{\cdot}30}{11{\cdot}32} = 0{\cdot}0265\ rad = 1{\cdot}52°$

$$\therefore \quad PA = 11{\cdot}32\,(1 - \cos 1{\cdot}52°) = 11{\cdot}32\,(1 - 0{\cdot}99965)$$

$$= \mathbf{3{\cdot}962 \times 10^{-4}\ m = 4\ mm}$$

Problem 2.32. A straight horizontal conducting rod of length 0·45 m and mass 60 g is suspended by the vertical wires at its ends. A current of 5·0 A is set up in the rod through the wires.

(*a*) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero ?

(*b*) What will be the total tension in the wires, if the direction of current is reversed, keeping the magnetic field same as before ? (Ignore the mass of the wire) $g = 9{\cdot}8\ \text{m s}^{-2}$. *(Text Problem)*

Sol. Here, mass of rod, $m = 60\text{ g} = 60 \times 10^{-3}\text{ kg}$;

$l = 0{\cdot}45\text{ m}; \text{I} = 5{\cdot}0\text{ A}$

(*a*) There will be no tension in the supporting wires, if the force on the rod due to the magnetic field is equal to the weight of the rod and is in vertically upward direction. Fleming's left hand rule tells that the magnetic field should be applied along horizontal. Therefore, for no tension in supporting wires,

$$\text{B I } l = mg$$

or $$\text{B} = \frac{mg}{\text{I } l} = \frac{60 \times 10^{-3} \times 9{\cdot}8}{5{\cdot}0 \times 0{\cdot}45} = \mathbf{0{\cdot}261\ T}$$

(*b*) If the direction of the current is reversed, force on the rod will act in the direction of its weight and hence tension in the supporting wires will become

$$\text{T} = 2 \times \text{weight of the rod} = 2 \times 60 \times 10^{-3} \times 9{\cdot}8$$
$$= \mathbf{1{\cdot}176\ N}$$

Problem 2.33. On a smooth plane inclined at 30° with the horizontal, a thin current-carrying metallic rod is placed parallel to the horizontal ground. The plane is located in a uniform magnetic field of 0·15 T in the vertical direction. For what value of current can the rod remain stationary ? The mass per unit length of the rod is $0{\cdot}30\ \text{kg m}^{-1}$. *(Text Problem)*

Sol. A rod XY of length say l is placed on an inclined plane as shown in Fig. 2.26.

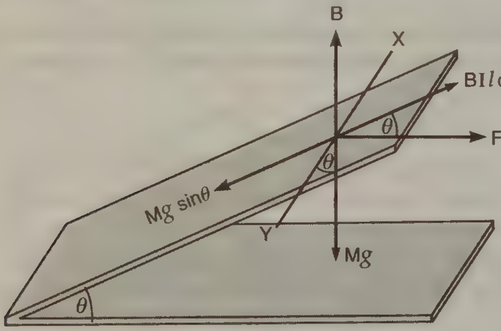


Fig. 2.26

Let θ be inclination of the inclined plane with the horizontal. If current I flows through the rod XY from end Y to X, then following forces act on the wire XY :

(*i*) Weight Mg acts vertically downward. It follows that weight Mg makes an angle θ with normal to inclined plane.

(*ii*) Due to magnetic field, force B I l acts parallel to horizontal and towards right. The force B I l makes an angle θ with the inclined plane.

The rod will remain stationary, if component of the weight Mg of the rod along the inclined plane in downward direction is just balanced by the component of force B I l along the inclined plane in upward direction *i.e.*

$$\text{M}g \sin\theta = \text{B } l \text{ I} \cos\theta$$

If m is mass per unit length of the rod, then

$$\text{M} = m\,l$$

$$\therefore \quad m\,l\,g \sin\theta = \text{B I } l \cos\theta$$

or $$\text{I} = \frac{m\,g \tan\theta}{\text{B}}$$

Here, $\text{B} = 0{\cdot}15\text{ T}\ ;\ m = 0{\cdot}30\text{ kg m}^{-1}$;

$\theta = 30°\ ;\ g = 9{\cdot}8\text{ m s}^{-2}$;

$$\therefore \quad \text{I} = \frac{0{\cdot}30 \times 9{\cdot}8 \times \tan 30°}{0{\cdot}15} = \frac{0{\cdot}30 \times 9{\cdot}8 \times 0{\cdot}5774}{0{\cdot}15}$$
$$= \mathbf{11{\cdot}32\ A}$$

Problem 2.34. A short conductor of length 5·0 cm is placed parallel to a long conductor of length 1·5 m nea its centre. The conductors are carrying currents 4·0 A and 3·0 A respectively in the same direction. What is the total force experienced by the long conductor, when they are 3· cm apart ? *(Text Problem)*

Sol. Here, $\text{I}_1 = 4{\cdot}0\text{ A}$ (through short conductor);

$\text{I}_2 = 3{\cdot}0\text{ A}$ (through long conductor)

$r = 3{\cdot}0\text{ cm} = 3{\cdot}0 \times 10^{-2}\text{ m}$

Force acting per unit length on the short conductor due to a long conductor is given by

$$\text{F} = \frac{\mu_0}{4\pi} \times \frac{2\text{I}_1\text{I}_2}{r}$$

$$\therefore \quad \text{F} = \frac{10^{-7} \times 2 \times 4{\cdot}0 \times 3{\cdot}0}{3{\cdot}0 \times 10^{-2}} = 8{\cdot}0 \times 10^{-5}\text{ N}$$

Therefore, total force on short conductor,

$$\text{F}' = \text{F} \times \text{length of the short conductor}$$
$$= 8{\cdot}0 \times 10^{-5} \times (5{\cdot}0 \times 10^{-2})$$
$$= \mathbf{4{\cdot}0 \times 10^{-6}\ N\ (attractive\ force)}$$

According to Newton's third law, the long conducto will also experience an equal attractive force **($4{\cdot}0 \times 10^{-6}$ N)** due to the short conductor.

Problem 2.35. A rectangular loop of sides 25 cm and 10 cm carrying a current of 15 A is placed with its longer side parallel to a long straight conductor 2·0 cm apart carrying a current of 25 A. What is the net force on the loop ? *(Pre-degree Kerala, 1991 ; Text Problem)*

Sol. Consider the rectangular loop ABCD placed nea a long straight conductor XY as shown in Fig. 2.27.

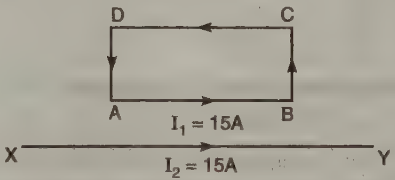


Fig. 2.27

The arm AB will get attracted, while CD will get repelled. Forces on arms BC and AD will cancel each other.

Here, $\text{AB} = 25\text{ cm} = 25 \times 10^{-2}\text{ m}$;

$\text{BC} = 10\text{ cm} = 10 \times 10^{-2}\text{ m}$;

Distance of AB from XY,

$r_1 = 2{\cdot}0\text{ cm} = 2{\cdot}0 \times 10^{-2}\text{ m}$;

Distance of CD from XY,

$r_2 = 10{\cdot}0 + 2{\cdot}0 = 12{\cdot}0\text{ cm} = 12{\cdot}0 \times 10^{-2}\text{ m}$;

Current through the rectangular loop, $\text{I}_1 = 15\text{ A}$;

Current through the long wire XY, $\text{I}_2 = 25\text{ A}$

Force on side AB, $F_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r_1}$

$\times$ length of the conductor AB

$$= \frac{10^{-7} \times 2 \times 15 \times 25}{2\cdot 0 \times 10^{-2}} \times 25 \times 10^{-2}$$

$= 9\cdot 375 \times 10^{-4}$ N (attractive)

Force on side CD, $F_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r_2}$

$\times$ length of the conductor CD

$$= \frac{10^{-7} \times 2 \times 15 \times 25}{12\cdot 0 \times 10^{-2}} \times 25 \times 10^{-2}$$

$= 1\cdot 5625 \times 10^{-4}$ N (repulsive)

Therefore, net force on the loop,

$$F = F_1 - F_2 = 9\cdot 375 \times 10^{-4} - 1\cdot 5625 \times 10^{-4}$$

$$= \mathbf{7\cdot 8125 \times 10^{-4}\ N\ (towards\ XY)}$$

Problem 2.36. A current balance (or ampere balance) is a device for measuring currents. The current to be measured is arranged to go through two long parallel wires of equal lengths in opposite directions, one of which is linked to the pivot of the balance. The resulting repulsive force on the wire is balanced by putting a suitable mass in the scale pan hanging on the other side of the pivot. In one measurement, the mass in the scale pan is 30·0 g, the length of the wires is 50·0 cm each and the separation between them is10·0 mm. What is the value of the current being measured ? Take $g = 9\cdot 8$ m s^{-2} and assume that the arms of the balance are equal. *(Text Problem)*

Sol. Here, length of each of the two wires,

$$l = 50 \text{ cm} = 0\cdot 5 \text{ m};$$

distance between the two wires, $r = 10$ mm $= 10^{-2}$ m

Since length of each wire is quite large as compared to distance between them, the two wires may be considered as straight long conductors. Therefore, force experienced by unit length of one wire due to the other is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r}$$

and total force on the wire attached to the scale pan,

$$F' = F \times l = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} l$$

When the pan is balanced, $F' = M g$

or $$\frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} l = M g$$

Here, $M = 30\cdot 0 \text{ g} = 30\cdot 0 \times 10^{-3}$ kg ; $g = 9\cdot 8$ m s^{-2} ;

$l = 0\cdot 5$ m ; $r = 10^{-2}$ m

Since the same current flows through the two wires,

$$I_1 = I_2 = I \text{ (say)}$$

$$\therefore\ 10^{-7} \times \frac{2 I \times I}{10^{-2}} \times 0\cdot 5 = 30\cdot 0 \times 10^{-3} \times 9\cdot 8$$

or $$I = \sqrt{\frac{30\cdot 0 \times 10^{-3} \times 9\cdot 8 \times 10^{-2}}{10^{-7} \times 2 \times 0\cdot 5}} = \mathbf{171\cdot 46\ A}$$

Problem 2.37. A solenoid 60 cm long and of radius 4·0 cm has 3 layers of windings of 300 turns each. A 2·0 cm long wire of mass 2·5 g lies inside the solenoid near its centre and normal to its axis ; both the wires and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6·0 A in the wire. What value of current in the windings of the solenoid can support the weight of the wire ? $g = 9\cdot 8$ m s^{-2}. *(Text Problem)*

Sol. Let I be the current passed through the solenoid.

Then, magnetic field produced along axis of the solenoid,

$$B = \mu_0 n I$$

Here, total number of turns $= 300 \times 3 = 900$;

length $= 60$ cm $= 0\cdot 6$ m

Therefore, number of turns per unit length,

$$n = \frac{\text{total number of turns}}{\text{length of solenoid}} = \frac{900}{0\cdot 6}$$

$$= 1500 \text{ turns m}^{-1}$$

Now, $$B = \mu_0 n I = 4\pi \times 10^{-7} \times 1500 \times I$$

$$= 6\pi \times 10^{-4} I \text{ tesla}$$

Length of the wire placed inside the solenoid,

$$l = 2\cdot 0 \text{ cm} = 0\cdot 02 \text{ m}$$

Current through the wire, $I' = 6\cdot 0$ A

Force on the wire,

$$F = B I' l = (6\pi \times 10^{-4} I) \times 6\cdot 0 \times 0\cdot 02$$

$$= 2\cdot 262 \times 10^{-4} I \text{ newton}$$

The weight of the wire will be supported, if force F due to magnetic field on the current carrying wire is just equal and opposite to the weight of wire *i.e.*

$$2\cdot 262 \times 10^{-4} I = m g$$

Here, $m = 2\cdot 5 \text{ g} = 2\cdot 5 \times 10^{-3}$ kg

Therefore, $2\cdot 262 \times 10^{-4} I = 2\cdot 5 \times 10^{-3} \times 9\cdot 8$

or $$I = \frac{2\cdot 5 \times 10^{-3} \times 9\cdot 8}{2\cdot 262 \times 10^{-4}} = \mathbf{108\cdot 3\ A}$$

Problem 2.38. The coil of a galvanometer is $0\cdot 02 \times 0\cdot 08$ m^2. It consists of 200 turns of fine wire and is in a magnetic field of 0·2 tesla. The restoring torque constant of the suspension fibre is 10^{-6} N m per degree. Assuming the magnetic field to be radial,

(*i*) what is the maximum current that can be measured by this galvanometer, if the scale can accommodate 30° deflection ? (*ii*) what is the smallest current that can be detected, if the minimum observable deflec-tion is 0·1° ? *(Text Problem)*

Sol. Here $A = 0\cdot 02 \times 0\cdot 08 = 1\cdot 6 \times 10^{-3}$ m^2 ;

$n = 200$; $B = 0\cdot 2$ T

Restoring torque constant of the suspension fibre,

$$k = 10^{-6} \text{ N m per degree}$$

(*i*) Maximum deflection, $\theta_{max} = 30°$

Let I_{max} be the maximum current that can be measured by the galvanometer. Then,

$$n B A I_{max} = k \theta_{max}$$

or $$I_{max} = \frac{k \theta_{max}}{n B A} = \frac{10^{-6} \times 30}{200 \times 0\cdot 2 \times 1\cdot 6 \times 10^{-3}}$$

$$= \mathbf{4\cdot 69 \times 10^{-4}\ A}$$

(ii) Minimum deflection, $\theta_{min} = 0\cdot1°$

Let I_{min} be the minimum current that can be measured by the galvanometer. Then,

$$I_{min} = \frac{k\,\theta_{min}}{n\,B\,A} = \frac{10^{-6}\times0\cdot1}{200\times0\cdot2\times1\cdot6\times10^{-3}} = \mathbf{1{\cdot}56\times10^{-6}\ A}$$

Problem 2.39. A voltmeter reads 5·0 V at full scale deflection and is graded according to its resistance per volt at full scale deflection as 5000 Ω V^{-1}. How will you convert it into a voltmeter that reads 20 V at full scale deflection ? *(C.B.S.E. 2001 S)*

Will it still be graded as 5000 Ω s^{-1}? Will you prefer this voltmeter to one that is graded as 2000 Ω V^{-1}?

(Text Problem)

Sol. Resistance per volt at full scale deflection

$$= 5000\ \Omega\ V^{-1}$$

Therefore, current for maximum deflection*,

$$I_g = \frac{1}{5000} = 2\times10^{-4}\ A$$

Reading of the voltmeter at full scale deflection = 5 V

Therefore, resistance of the instrument,

$$G = \frac{5}{2\times10^{-4}} = 2\cdot5\times10^{4}\ \Omega$$

New range of voltmeter, V = 20 volt

Let R be the resistance that should be connected in series, so that voltmeter may read upto 20 V.

Then, $$R = \frac{V}{I_g} - G = \frac{20}{2\times10^{-4}} - 2\cdot5\times10^{4}$$

$$= 10\times10^{4} - 2\cdot5\times10^{4} = \mathbf{7{\cdot}5\times10^{4}\ \Omega}$$

Resistance of the new voltmeter,

$$R + G = 7\cdot5\times10^{4} + 2\cdot5\times10^{4} = 10^{5}\ \Omega$$

Resistance per volt of the new voltmeter

$$= \frac{10^{5}}{20} = \mathbf{5000\ \Omega\ V^{-1}}$$

Therefore, new voltmeter will be still graded as **5000 Ω V^{-1}.**

A voltmeter graded as 5000 Ω V^{-1} is better than one graded as 2000 Ω V^{-1}. It is because, higher the resistance per volt, lesser is the current it draws from the circuit and hence the better it is.

Problem 2.40. A voltmeter of resistance 400 Ω connected for measurement to a section of a circuit having a resistance of 20 Ω shows a reading of 100 V. What is the error in the reading of the voltmeter, if the current in the main circuit is assumed to remain constant before and after connecting the voltmeter ?

Ans. Here, potential difference across R as measured by the voltmeter, V′ = 100 V

When the voltmeter is connected across the resistance of 20 Ω, it draws a part of the total current I flowing through the circuit. Let I_1 be the current drawn by the voltmeter as shown in Fig. 2.28.

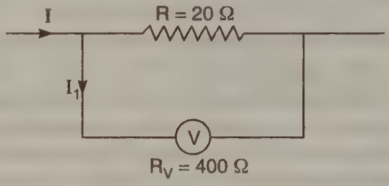


Fig. 2.28

Then, $$I_1 = \frac{V'}{R_V} = \frac{100}{400} = 0\cdot25\ A$$

Therefore, on connecting the voltmeter, decrease in the potential difference across the resistance,

$$\Delta V = I_1\times20 = 0\cdot25\times20 = 5\ V$$

Hence, potential difference across the resistance before the voltmeter was connected,

$$V = V' + \Delta V = 100 + 5 = 105\ V$$

Since the voltmeter measures potential difference across R as 100 V, error in the reading of the voltmeter is **5 V.**

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. An electric charge enters in electric field at right angles to the direction of electric field. What is the nature of the path followed ?

Ans. The electric charge will move along a parabolic path.

Q. 2.02. What is the magnitude of transverse acceleration produced in the motion of the electric charge, when it passes through the electric field ?

Ans. If a charge q having mass m passes transversely through an electric field E, then

$$\text{acceleration, } a = \frac{qE}{m}$$

Q. 2.03. What is magnetic Lorentz force ? *(P.S.S. C.E. 2001)*

Ans. The force experienced by a charged particle, when moving inside magnetic field is called magnetic Lorentz force. It is given by

$$\vec{F} = q(\vec{v}\times\vec{B})$$

The magnitude of force is given by

$$F_m = B\,q\,v\sin\theta$$

Q. 2.04. In a chamber of uniform magnetic field $\vec{B}$, an electron beam enters with velocity $\vec{v}$. Write the expression for the force experienced by the electrons. *(H.S.S. C.E. 2002)*

Ans. If $-e$ is charge on the electron, then force on the electron is given by

$$\vec{F} = -e(\vec{v}\times\vec{B})$$

Q. 2.05. The force $\vec{F}$ experienced by a particle of charge e moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = e(\vec{v}\times\vec{B})$. Of these, name the pairs of vectors, which are always at right angles to each other. *(C.B.S.E. 1998)*

Ans. The pair of vectors $\vec{v}$ and $\vec{F}$; and the pair $\vec{B}$ and $\vec{F}$ are always at right angles to each other.

---

*I_g = 1/resistance per volt at full scale deflection.

Q. 2.06. What is the magnitude of force on a charge moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$? (*C.B.S.E. 1991*)

Ans. Refer to VSQ 2.03. The magnitude of force on the moving charge due to magnetic field is given by

$$F_m = B\,q\,v \sin\theta$$

If the charged particle moves at right angle to the direction of the field, then

$$F_m = B\,q\,v \qquad (\because \theta = 90°)$$

Q. 2.07. A particle of charge q moves with a velocity v at an angle θ to a magnetic field B. What is the force experienced by the particle ? (*I.S.C.E. 1993*)

Ans. Force on the charged particle, $F = B\,q\,v \sin\theta$

Q. 2.08. Under which condition, an electron moving through a magnetic field experiences maximum force ? (*H.S.S.C.E. 2002 ; 1994*)

Ans. An electron moving through a magnetic field experiences maximum force, when it moves perpendicular to the direction of magnetic field.

Q. 2.09. Under what condition is the force acting on a charge moving through a uniform magnetic field maximum ? (*H.P.S.S.C.E. 2002*)

Or

A certain proton moving through a magnetic field region experiences maximum force. When does this occur ? (*C.B.S.E. 1997*)

Ans. Again, $\vec{F} = q(\vec{v} \times \vec{B})$

Obviously, magnitude of $\vec{F}$ will be maximum, if the direction of $\vec{v}$ is perpendicular to that of $\vec{B}$.

Q. 2.10. Under what condition is the force acting on a charge moving through a uniform magnetic field minimum ? (*H.S.S.C.E. 2002 ; C.B.S.E. 1995*)

Ans. A charge moving through a magnetic field, experiences no force (minimum), when it moves along the direction of magnetic field.

Q. 2.11. What is the force experienced by a stationary charge in magnetic field ? (*H.P.S.S.C.E. 2002*)

Or

A stationary charge experiences no magnetic Lorentz force. Why ? (*P.S.S.C.E. 2001*)

Ans. Refer to VSQ 2.03. For a stationary charge, $v = 0$

$$\therefore \quad F_m = B\,q\,(0) \sin\theta = 0$$

Q. 2.12. Does an electric charge moving along the direction of magnetic field experience a force ?

Ans. Refer to VSQ 2.03. For a charge moving along the direction of magnetic field, $\theta = 0°$ *i.e.* $\sin\theta = 0$.

$$\therefore \quad F_m = B\,q\,v\,(0) = 0$$

Q. 2.13. In a certain arrangement, a proton does not get deflected, while passing through a magnetic field region. Under what condition is it possible ? (*C.B.S.E. 1997*)

Ans. Now, $\vec{F} = e(\vec{v} \times \vec{B})$

Obviously, $\vec{F}$ will be zero, if $\vec{v} \times \vec{B} = 0$ *i.e.* $\vec{v} \parallel \vec{B}$

Therefore, if a proton does not get deflected while passing through a magnetic field, it must be moving along the direction of magnetic field.

Q. 2.14. An electron is not deflected, while moving through a certain region of space. Can we be sure that there is no magnetic field in the region ? (*H.P.S.S.C.E. 2002, 1996*)

Ans. No. The electron may be moving parallel to the direction of magnetic field. In such a case, the force

$$\vec{F} = -e(\vec{v} \times \vec{B}) = 0 \qquad (\because \vec{v} \text{ is parallel to } \vec{B})$$

Q. 2.15. An electron is projected in the direction of magnetic field. How will its motion be affected by the action of magnetic field ?

Ans. Refer to VSQ 2.12. No force acts on the electron due to the magnetic field, when it is projected in the direction of magnetic field. Hence, its motion will not be affected.

Q. 2.16. What will be the path of a charged particle moving along the direction of a uniform magnetic field ? (*C.B.S.E. 1995*)

Ans. When the charged particle moves along the direction of a uniform magnetic field, it experiences no force and therefore it will move along its original straight path.

Q. 2.17. A test charge of $1{\cdot}6 \times 10^{-19}$ C is moving with velocity $\vec{v} = (2\hat{i} + 3\hat{j})\ \text{m s}^{-1}$ in a magnetic field $\vec{B} = (2\hat{i} + 3\hat{j})\ \text{Wb m}^{-2}$. Find the force acting on the test charge. (*H.S.S.C.E. 1998 S*)

Ans. Here, $e = 1{\cdot}6 \times 10^{-19}$ C ; $\vec{v} = (2\hat{i} + 3\hat{j})\ \text{m s}^{-1}$;

$$\vec{B} = (2\hat{i} + 3\hat{j})\ \text{Wb m}^{-2}.$$

Now, $\vec{F} = e(\vec{v} \times \vec{B})$

$$= 1{\cdot}6 \times 10^{-19}\,[(2\hat{i} + 3\hat{j}) \times (2\hat{i} + 3\hat{j})] = 0$$

Q. 2.18. A charged particle in a plasma trapped in a magnetic bottle leaks out after a millisecond. What is the total work done by the magnetic field during the time, the particle is trapped ?

Ans. As the force due to magnetic field on a moving charged particle always acts perpendicular to its direction of motion (velocity), no work is done by the magnetic field.

Q. 2.19. Two protons P and Q, moving with the same speed enter magnetic fields B_1 and B_2 respectively at right angles to the field directions. If B_2 is greater than B_1, for which of the protons, P and Q, the circular path in the magnetic field will have a smaller radius ? (*C.B.S.E. 1998*)

Ans. From the expression : $B\,e\,v = \dfrac{m\,v^2}{r}$,

it follows that

$$B \propto \frac{1}{r}.$$

If $B_2 > B_1$, then $r_2 < r_1$ *i.e.* in magnetic field B_2, circular path will have a smaller radius.

Q. 2.20. An electron and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. For which of the two particles will the radius of circular path be smaller. (*C.B.S.E. 1998*)

Ans. Again from the expression,

$$B\,e\,v = \frac{m\,v^2}{r},$$

it follows that $r \propto m$. As mass of electron is less than that of proton, radius of the circular path of **electron** will be smaller.

Q. 2.21. Write down the epxression for the Lorentz force on a charged particle.

(H.P.S.S.C.E. 1999 ; P.S.S.C.E. 1999, 1998 S)

Ans. The force experienced by a charge, when moving inside the electric and magnetic fields is called Lorentz force. It is given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Q. 2.22. Does a stationary charge experience a force in an electric field ?

Ans. A stationary charge experiences force in an electric field. It is because, the force due to electric field does not depend, whether the charge is at rest or is in motion.

The force due to electric field, $|\vec{F}_e| = |q\vec{E}| = qE$

Q. 2.23. In a field, the force experienced by a charge depends only upon the magnitude of the field and does not depend upon the velocity. Is field electric or magnetic in nature ?

Ans. Refer to VSQ 2.22. The field is electric in nature.

Q. 2.24. In a field, the force experienced by a charge depends upon its velocity and becomes zero, when it is at rest. What is the nature of the field ?

Ans. The field is magnetic in nature. It is because, force due to electric field is not affected, whether the charge is at rest or is in motion.

Q. 2.25 State the principle of cyclotron.

(P.S.S.C.E. 2001)

Ans. *It states that positive ions can acquire a large amount of energy with a comparatively smaller alternating potential difference by making them to cross the same electric field time and again by making use of a strong magnetic field.*

Q. 2.26. Does time spent by a proton inside the dee of the cyclotron depend upon (*i*) the radius of the circular path (*ii*) the velocity of the proton ?

Ans. The time spent by a proton inside the dee of a cyclotron is independent of both the radius of circular path and the velocity of the proton.

Q. 2.27. What are the primary functions of electric field and the magnetic field in a cyclotron ?

Ans. The magnetic field makes the charged particle to cross the gap between the dees again and again by making it to move along circular path, while the oscillating electric field applied across the dees accelerates the charged particle again and again.

Q. 2.28. What is meant by cyclotron frequency ?

Ans. Refer to section 2.05.

Q. 2.29. The frequency of a charge circulating inside the dees of a cyclotron does not depend upon the speed of the charge. Why ?

Ans. The radius of the circular path inside the dee increases in direct proportion to the velocity of the charge. As a result, time period of the charge and hence its frequency remains independent of the speed of the charge.

Q. 2.30. Can neutrons be accelerated in a cyclotron ? Why ? *(H.P.S.S.C.E. 1998 S)*

Ans. No, neutrons cannot be accelerated by using a cyclotron. This is because, a cyclotron can accelerate charged particles and is particularly suitable for protons and positive ions.

Q. 2.31. State Fleming's left hand rule.

(P.S.S.C.E. 2001)

Ans. Refer to section 2.07.

Q. 2.32. What is the nature of force, when the two parallel conductors carry currents in the (*i*) same direction ; (*ii*) opposite direction ?

Ans. (*i*) Force is attractive. (*ii*) Force is repulsive.

Q. 2.33. Define an ampere in terms of the force between current carrying conductors. *(I.S.C.E. 1996)*

Ans. *One ampere is that current, which when flowing through each of the two parallel conductors of infinite length and placed in free space at a distance of one metre from each other, produces between them a force of 2×10^{-7} newton per metre of their length.*

Q. 2.34. Does the torque on a planar current loop in magnetic field change, when its shape is changed without changing its geometrical area ?

Ans. The torque on a planar current loop in a magnetic field does not change, when its shape is changed without changing the area of the loop.

Q. 2.35. Estimate the torque on a closed current loop placed in the magnetic field $\vec{B}$. What is the main function of soft iron core used in a moving coil galvanometer ?

Ans. When a closed current loop is suspended in magnetic field, torque on the coil is given by

$$|\vec{\tau}| = |\vec{M} \times \vec{B}| = nBIA \sin\theta,$$

where the letters have their usual meanings.

Q. 2.36. A current carrying loop free to turn is placed in a uniform magnetic field $\vec{B}$. What will be its orientation relative to $\vec{B}$ in the equilibrium state ? *(C.B.S.E. 1990)*

Ans. In equilibrium state, the current carrying loop will orient itself, such that $\vec{B}$ is perpendicular to the plane of the coil. It is because of the fact that in this orientation, the torque on the current loop becomes zero.

Q. 2.37. Under what circumstances, will a current carrying loop not rotate in the magnetic field ?

Ans. If the current carrying loop is placed in a magnetic field, with its plane perpendicular to the field, then it will not rotate.

Q. 2.38. State the principle of moving coil galvanometer. *(P.S.S.C.E. 2001 ; H.S.S.C.E. 2001 ; 1997 S)*

Ans. It is based on the principle that when a current carrying conductor is placed in magnetic field, it experiences a torque.

Q. 2.39. What is the nature of magnetic field in a moving coil galvanometer ? *(C.B.S.E. 1996)*

Ans. It is radial in nature.

Q. 2.40 State two properties of the material of the wire used for suspension of the coil in a moving coil galvanometer. *(C.B.S.E. 2001)*

Ans. 1. Its restoring torque per unit twist should be low.

2. Its tensile strength should be high so that it does not break under the weight of the coil.

Q. 2.41. Define current sensitivity of a galvanometer.

Ans. Refer to section 2.10.

Q. 2.42. Define current sensitivity of a moving coil alvanometer and state its SI unit. (*P.S.S.C.E. 1999 S*)

Ans. For current sensitivity of a moving coil galvanoneter, refer to section 2.10.

The SI unit of current sensitivity is **radian ampere^{-1}.**

Q. 2.43. Give two factors by which the current ensitivity of a moving coil galvanometer can be increased. (*C.B.S.E. 2001*)

Ans. Current sensitivity $= \dfrac{nBA}{k}$,

vhere the letters have their usual meanings. Since n and A annot be increased beyond a limit, current sensitivity may e increased by (*i*) increasing B and (*ii*) decreasing k.

Q. 2.44. Explain voltage sensitivity of a galvanometer. (*P.S.S.C.E. 1991*)

Ans. Refer to section 2.10.

Q. 2.45. Give two factors by which voltage sensitivity of a moving coil galvanometer can be increased. (*C.B.S.E. 2001*)

Ans. Voltage sensitivity $= \dfrac{nBA}{kR}$,

where the letters have their usual meanings. Since n and A can not be increased beyond a limit, voltage sensitivity may be increased by **(*i*)** increasing B, **(*ii*)** decreasing k and **(*iii*)** decreasing R.

Q. 2.46. What is shunt ? State its SI units. (*P.S.S.C.E. 1999 ; H.S.S.C.E. 1996*)

Ans. A small resistance connected in parallel to the coil of the galvanometer is called shunt. Its SI unit is ohm.

Q. 2.47. How will you convert a moving coil galvanometer into an ammeter ? (*H.S.S.C.E. 2002*)

Or

How can a galvanometer be converted into an ammeter ? How will you convert a moving coil galvanometer into an ammeter ? (*C.B.S.E. 2001; P.S.S.C.E. 2000 ; H.P.S.S.C.E. 1998 ; I.S.C.E. 1997*)

Ans. A galvanometer can be converted into an ammeter by connecting a suitable small resistance in parallel to it.

Q. 2.48. Is the resistance of an ammeter greater than or less than that of the galvanometer of which it is formed ?

Ans. The resistance of an ammeter is always less than that of the galvanometer of which it is formed.

Q. 2.49. An ammeter and a milliammeter are converted from the same galvanometer. Out of the two, which current measuring instrument has higher resistance ? (*C.B.S.E. 2002*)

Ans. From the relation,

$$S = \frac{I_g \times G}{I - I_g},$$

it follows that value of S needed to convert the galvanometer into milliammeter is larger than that required to convert into ammeter.

Since the resistance of current measuing instrument,

$$R = \frac{GS}{G+S},$$

it follows that **milliammeter** will possess higher resistance.

Q. 2.50. Why should an ammeter have a low resistance ? (*H.S.S.C.E. 2002*)

Ans. For measuring current in a circuit, an ammeter is connected in series. For the current in the circuit to remain practically unchanged, the resistance of the ammeter should be low.

Q. 2.51. How is an ammeter connected in an electric circuit ?

Ans. An ammeter is connected in series in an electric circuit.

Q. 2.52. How can a galvanometer be converted into voltmeter ? (*H.P.S.S.C.E. 2001 ; C.B.S.E. 2001*)

Ans. A galvanometer can be converted into a voltmeter by connecting a suitable high resistance in series to its coil.

Q. 2.53. Is the resistance of a voltmeter greater than or less than that of the galvanometer of which it is formed ?

Ans. The resistance of a voltmeter is always greater than that of the galvanometer, of which it is formed.

Q. 2.54. What is the resistance of an ideal voltmeter and an ammeter ?

Ans. The resistance of an ideal voltmeter is infinite and that of an ammeter is zero.

Q. 2.55. How is a voltmeter connected in an electric circuit ?

Ans. A voltmeter is connected in parallel in an electric circuit.

Q. 2.56. How does (*i*) an ammeter (*ii*) a voltmeter differ from a galvanometer ? (*P.S.S.C.E. 1998*)

Ans. (*i*) An ammeter is a galvanometer, in which a very small suitable resistance is connected in parallel to it.

(*ii*) A voltmeter is a galvanometer, in which a very high suitable resistance is connected in series to it.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Describe qualitatively, the path of a charged particle moving in (*a*) a uniform electrostatic field with initial velocity (*i*) parallel to the field, (*ii*) perpendicular to the field, (*iii*) at an arbitrary angle with the field direction.

Ans. (*i*) The force due to uniform electrostatic field will act along the direction of the field and hence path will be a straight line.

(*ii*) It will follow parabolic path, as the force acts along a direction perpendicular to initial velocity.

(*iii*) It will still follow parabolic path. The velocity of electron may be resolved into two components. The velocity component along the field will not affect its path, while the velocity component perpendicular to the field will make it to follow parabolic path.

Q. 2.02. Why does not a charged particle moving at right angle to the direction of an electric field follow a circular path ?

Ans. A charged particle moving at right angle to the direction of electric field experiences force in the plane of electric field. The force acts at right angle to its path only at the instant the charged particle enters the electric field and with the passage of time, it no longer remains at right angle to the path. Since the force on the charged particle *does not remain always perpendicular to the path,* it does not move along circular path.

Q. 2.03. An electron and a proton are freely situated in an electric field. Will the electric force on them be equal ? Will their acceleration be equal ? Justify your answer. (*P.S.S.C.E. 1999 S*)

Ans. Force on a charged particle having charge e and situated inside an electric field E,

$$F = e\,E$$

Since the magnitude of charge on an electron is same as that on a proton, the electric force on them will be **equal.**

Now, acceleration of electron, $a_1 = \dfrac{e\,E}{m_1}$

and acceleration of proton, $a_2 = \dfrac{e\,E}{m_2}$,

where m_1 and m_2 are respective masses of electron and proton. Since $m_1 < m_2$, the acceleration in the path of **electron** will be greater than that of proton.

Q. 2.04. An electron travelling west to east enters a chamber having a uniform electrostatic field in a north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron deflecting from its straight line path. (*Text Question*)

Ans. Under the action of electrostatic field, the electron will be deflected towards north (towards positive plate). It will remain undeflected, if force due to magnetic field is towards south. As the velocity $\vec{v}$ of electron is from west to east, the expression for magnetic Lorentz force *i.e.*

$$\vec{F}_m = -e\,(\vec{v} \times \vec{B})$$

or Fleming's left hand rule tells that magnetic field $\vec{B}$ should be applied along vertical and in downward direction.

Q. 2.05. When is the force on a moving charge due to a magnetic field maximum and when is it minimum ?

Ans. We know that $F_m = B\,q\,v \sin\theta$

For force to be maximum, $\sin\theta = 1$ *i.e.* $\theta = 90°$ *i.e.* when the charged particle moves perpendicular to the direction of magnetic field.

For force to be minimum, $\sin\theta = 0$ *i.e.* $\theta = 0°$ *i.e.* when the charged particle moves along the direction of magnetic field.

Q. 2.06. A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle ? (*Text Question*)

Ans. The force on a charged particle moving inside magnetic field is given by

$$\vec{F}_m = q\,(\vec{v} \times \vec{B})$$

The force on the charged particle will be zero (will remain undeflected), if $\vec{v} \times \vec{B}$ is equal to zero *i.e.* if initial velocity $\vec{v}$ is either parallel or antiparallel to the magnetic field $\vec{B}$.

Q. 2.07. Is any work done by a magnetic field on a moving charge ?

Ans. Force on a charge q moving inside a magnetic field of strength $\vec{B}$ with velocity is $\vec{v}$ given by

$$\vec{F}_m = q\,(\vec{v} \times \vec{B})$$

Thus, $\vec{F}_m$ is perpendicular to $\vec{v}$ *i.e.* perpendicular to the direction of motion of the charge.

Therefore, work done by magnetic field,

$$W = \vec{F}\,.\,\vec{S} = F\,S \cos 90° = 0$$

i.e. no work is done by magnetic field on a moving charge.

Q. 2.08. Describe qualitatively, the path of a charged particle in a uniform magnetic field with initial velocity (*i*) parallel to the field, (*ii*) perpendicular to the field, (*iii*) at an arbitrary angle with the field direction.

Ans. **(*i*)** No force will act due to uniform magnetic field and hence charged particle will move along straight line.

(*ii*) It will move along a circle in the plane perpendicular to the magnetic field.

(*iii*) It will move along a helical path with its axis parallel to magnetic field.

Q. 2.09. Why does a charged particle moving at right angle to the direction of a magnetic field follow a circular path ?

Ans. When a charged particle moves at right angle to the direction of a magnetic field, it experiences force which *always acts perpendicular to the velocity.* Hence, magnitude of its velocity remains constant and only the direction of the velocity of the charged particle changes. In other words, the force on the charged particle acts as centripetal force and it follows circular path.

Q. 2.10. An electric charge moving in a direction making an angle ϕ ($\neq 90°$) with the direction of magnetic field follows a helical path. Why ?

Ans. When a charge moves with velocity v making an angle ϕ with the direction of magnetic field, the component of velocity $v \sin\phi$ (perpendicular to the magnetic field) makes the charge move along circular path, while the component of velocity $v \cos\phi$ (along the magnetic field) drifts the charge along the direction of the field. As a result, the path of the charge becomes helical.

Q. 2.11. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields, which are parallel to each other. What will be the nature of the path followed by the charged particle ? (*I.I.T. 1999*)

Ans. Under the effect of electric field, the charged particle will move in a straight line along the direction of electric field. The magnetic field will not affect the motion of the charged particle, as it is parallel to the direction of electric field *i.e.* parallel to the direction of motion of the charged particle.

Q. 2.12. The energy of a charged particle moving in a uniform magnetic field does not change. Why ? Explain. (*C.B.S.E. 2002 ; P.S.S.C.E. 2001*)

Ans. The force on a charged particle moving in a uniform nagnetic field always acts in a direction perpendicular to the lirection of motion of the charge. As work done by magnetic ield on the charge is zero, the energy of the charged particle loes not change.

Q. 2.13. A beam of α-particles and of protons, of the ame velocity v, enters a uniform magnetic field at right ngles to the field lines. The particles describe circular aths. Calculate the ratio of the two paths.

(C.B.S.E. 2001 S)

Ans. Let m_1 and q_1 be mass and charge on α-particle. Let $_1$ be radius of the circular path described by the α-particle, vhen it moves in a uniform magnetic field B with a veloity v. Then,

$$B q_1 v = \frac{m_1 v^2}{r_1} \quad \text{or} \quad r_1 = \frac{m_1 v}{B q_1}$$

Let m_2 and q_2 be mass and charge on proton. Then, adius of the circular path described by the proton, when moving with same velocity v in magnetic field B, is given by

$$r_2 = \frac{m_2 v}{B q_2}$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{m_1 v}{B q_1} \times \frac{B q_2}{m_2 v} = \frac{m_1 / m_2}{q_1 / q_2} = \frac{4}{2} = 2$$

Q. 2.14. An electron beam passes through a region of crossed electric and magnetic fields of intensities E and B respectively. For what value of electron speed will the beam remain undeflected ? *(C.B.S.E. 2001 S)*

Ans. Let v be the electron speed, for which beam remains undeflected while crossing through crossed electric and magnetic fields. For the beam to remain undeflected, the force on an electron due to two fields should be equal and opposite

i.e. $\quad B e v = e E \quad$ or $\quad v = \dfrac{E}{B}$

Q. 2.15. A point charge + q is moving with speed v perpendicular to a magnetic field B as shown in Fig. 2.30. Explain, what should be the magnitude and direction of the applied electric field, so that net force acting on the charge is zero. *(C.B.S.E. Sample Q. Paper)*

Ans. The magnetic field as shown in Fig. 2.29 is perpendicular to the plane of paper and in inward direction.

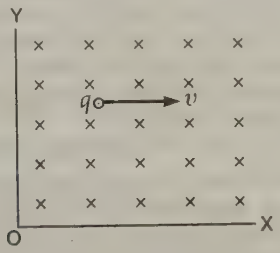


Fig. 2.29

The force on the charge due to magnetic field is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

i.e. the magnitude of the force will be equal to B q v and the force will be along OY. If E is the strength of electric field, then force on charge q due to electric field will be q E. So that the net force on charge is zero, q E should be equal and opposite to B q v *i.e.*

$$q E = B q v$$

or $\quad E = B v$

The direction of electric field will be along **YO.**

Q. 2.16. `Cyclotron is not suitable for accelerating electrons.' Explain the reason.

(P.S.S.C.E. 2002 ; H.S.S.C.E. 1997)

Or

Why is a cyclotron not suitable for accelerating electrons ? *(P.S.S.C.E. 1996 S)*

Ans. Electrons have very small mass. Even a small increase in the energy of the electron imparts a very high speed to it. Due to the high speed, the electrons go quickly out of step with the oscillating electric field. Also, the fast relativistic variation in mass of electrons poses a serious problem, when cyclotron is used to accelerated electrons.

Q. 2.17. What is the force that a conductor $\vec{dl}$ carrying a current I experiences, when placed in a magnetic field $\vec{B}$? What is the direction of the force ?

(H.P.S.S.C.E. 1996 ; C.B.S.E. 1990)

Ans. The force is given by

$$\vec{dF} = I\, \vec{dl} \times \vec{B}$$

The direction of force is given by the cross-product $I\, \vec{dl} \times \vec{B}$. It can also be found by using Fleming's left hand rule. For Fleming's left hand rule, refer to section 2.07.

Q. 2.18. In the Fig. 2.30, the straight wire AB is fixed, while the loop is free to move under the influence of the electric currents flowing in them. In which direction does the loop begin to move ? Give reason for your answer.

(C.B.S.E. 1999, 1997)

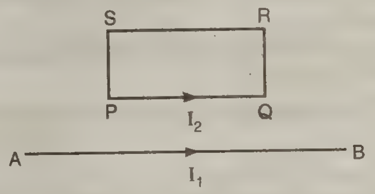


Fig. 2.30

Ans. The currents in arm PQ and wire AB are in same direction and hence wire will attract the arm PQ with a force, say F_1. On the other hand, the arm RS of the loop will experience a force of repulsion, say F_2. Since the arm PQ is closer to wire AB, $F_1 > F_2$ *i.e.* the loop will move towards the wire.

Q. 2.19. Write the relation for force between two parallel wires carrying current and hence define unit of current.

(P.S.S.C.E. 1996 S)

Ans. Force between two parallel wires carrying current is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r},$$

where the letters have their usual meanings.

For definition of unit of current *i.e.* ampere, refer to VSQ 2.33.

Q. 2.20. Why do the two parallel conductors carrying current exert force on each other ?

Ans. The current flows in a conductor due to the motion of electrons through it. When a current carrying conductor is placed inside a magnetic field, the electrons moving inside the conductor experience force due to the magnetic field in a direction perpendicular to the length of the conductor. As the electrons are confined to the conductor, the force on the electrons shows its effect as the force on the conductor.

Q. 2.21. Two parallel conducting wires carrying electric current in the same direction attract each other. Explain, why. *(P.S.S.C.E. 2001)*

Ans. The current through one conducting wire produces magnetic field and the other parallel current carrying wire experiences force due to this magnetic field. The application of Biot Savart's law tells that when the current in the two wires is in same direction, the two wires attract each other.

Q. 2.22. Why do two straight parallel metallic wires carrying current in the opposite directions repel each other. *(P.S.S.C.E. 2001 ; H.S.S.C.E. 1999)*

Ans. The current through one metallic wire produces magnetic field and the other parallel current carrying metallic wire experiences force due to this magnetic field. The application of Biot Savart's law tells that when currents flow through the two conductors in opposite directions, one parallel wires repels the other.

Q. 2.23. Why should a solenoid tend to contract, when a current passes through it ?

Ans. When current flows through a solenoid, the currents in the various turns of the solenoid are parallel and in the same direction. Since the currents flowing through parallel wires in the same direction lead to force of attraction between them, the turns of the solenoid will also attract each other and as a result, the solenoid tends to contract.

Q. 2.24. A rectangular loop of area A, having N turns and carrying a current of I ampere is held in a uniform magnetic field B.

(*i*) Write down the expression for the maximum torque experienced by the loop.

(*ii*) In which orientation, will the loop be in stable equilibrium ? *(C.B.S.E. 1998 S)*

Ans. (*i*) $\tau_{max} = \text{N B I A}$

(*ii*) When the plane of the coil becomes parallel to the direction of magnetic field, it will experience no torque and hence it will be in stable equilibrium.

Q. 2.25. A current carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium ? Show that in this orientation, the flux of the total field (external field and the field produced by the loop) is maximum. *(Text Question)*

Ans. The stable equilibrium position will be that in which vector $\vec{A}$ representing the area of the circular loop and $\vec{B}$ are parallel. As both the fields (applied field and the field produced by current loop) are in same direction, the flux of the total field is maximum in this orientation of the loop.

Q. 2.26. A small coil carrying a current is placed in a uniform magnetic field. How does the coil tend to orient itself relative to the magnetic field ?

Ans. The torque acting on a coil is given by

$$\tau = n\,\text{B I A} \cos\alpha,$$

where α is the angle between the plane of the coil the and the direction of magnetic field. When $\alpha = 90°$,then

$$\tau = 0$$

The coil tries to orient itself in this position. Thus, in equilibrium, the coil will acquire a position, such that its plane makes an angle of 90°with the direction of magnetic field.

Q. 2.27. The torque on a planar current loop depends upon its area and is independent of the shape of the loop of the planar loop. Explain.

Ans. A planar loop of any shape can be divided into a large number of rectangular strips as shown in Fig. 2.31.

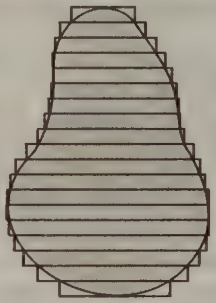


Fig. 2.31

Each rectangular strip is acted upon by a small torque. In the horizontal arms of any two adjacent rectangular strips, the current flows in opposite directions and hence net force on the horizontal arms of the rectangular strips is zero. The vertical arms of the strips will coincide more and more with the boundary of the planar loop, as the number of strips into which the planar loop is divided, is increased. Therefore, in the limiting case, magnetic moment $(\text{I}\vec{\text{A}})$ of the planar loop is equal to current times the total area of the rectangular strips *i.e.* the area of the planar loop and hence torque $\vec{\tau}\,(= \text{I}\,\vec{\text{A}} \times \vec{\text{B}})$ depends upon its area and is independent of the shape of the loop.

Q. 2.28. Explain, why earth's magnetic field does not affect the working of a moving coil galvanometer.

Ans. The field magnet used in a moving coil galvanometer is very strong. The earth's magnetic field is quite weak as compared to the magnetic field produced by the field magnet. Practically, the coil rotates under the effect of the strong magnetic field due to the field magnet and the weak magnetic field due to the earth does not affect the working of the moving coil galvanometer.

Q. 2.29. What is the importance of radial magnetic field in a moving coil galvanometer ?

(C.B.S.E. 2001 ; P.S.S.C.E. 2001, 2000 ; H.P.S.S.C.E. 1995)

Ans. The coil of the galvanometer experiences torque due to the action of magnetic field produced by the field magnet. In case the field magnet has flat poles, the torque on the coil decreases as it rotates from its equilibrium position (when plane of the coil is parallel to the magnetic field). It is because, in the new position, the plane of the coil is not parallel to the magnetic field. Moreover, as such, the deflection in the coil is not directly proportional to the current passed through the coil.

When the magnetic field is made radial by using a eld magnet with concave poles, the plane of the coil always mains parallel to the direction of the magnetic field. Due this, the galvanometer scale becomes linear.

Q. 2.30. What is the main function of soft iron core used a moving coil galvanometer ?

Ans. When a soft iron core is used, the magnetic field es tend to crowd through the core. It is because, soft iron re is ferromagnetic in nature. As a result, the strength of the agnetic field due to the field magnet increases, which in turn creases the sensitivity of the galvanometer.

Q. 2.31. Explain the action of a shunt.

(Pre-degree Kerala, 1992)

Ans. A shunt is a small resistance connected in parallel a galvanometer. The resistance of a galvanometer is small nd if the whole of the current in an electric circuit passes rough it, it may get damaged. The shunt allows only a small art of the current to flow through the galvanometer by acting a by pass to the major part of the current.

Q. 2.32. It is desired to pass only 5% of the current rough a galvanometer of resistance 95 Ω. What shunt esistance should be connected across it ?

Ans. Here, $G = 95\ \text{W}\ ; I_g = \frac{5\,I}{100}$ or $I = 20\,I_g$

Now, $S = \frac{I_g\,G}{I - I_g} = \frac{I_g \times 95}{20\,I_g - I_g} = 5\,\Omega$

Q. 2.33. A galvanometer has a resistance of 50 ohm. A esistance of 5 ohm is connected across its terminals. What art of total current will flow through the galvanometer ?

(H.S.C.C.E. 2001, 1992)

Ans. We know, $S = \frac{I_g\,G}{I - I_g}$ or $\frac{I_g}{I} = \frac{S}{G + S}$

Here $G = 50\,\Omega$ and $S = 5\,\Omega$

$\therefore\quad \frac{I_g}{I} = \frac{5}{50 + 5} = \frac{1}{11}$

Q. 2.34. A galvanometer coil has a resistance of 15 Ω nd the meter shows full scale deflection for a current of 4 nA. How will you convert the meter into an ammeter of ange 0 to 6 A ? *(Text Question)*

Ans. Here, $G = 15\,\Omega;\ I_g = 4\text{ mA} = 4 \times 10^{-3}\text{ A}\ ; I = 6\text{ A}$

To convert a galvanometer into an ammeter, a small esistance S has to be connected in parallel to the coil of the alvanometer. Then,

$$S = \frac{I_g\,G}{I - I_g} = \frac{4 \times 10^{-3} \times 15}{6 - 4 \times 10^{-3}} = 0{\cdot}01\,\Omega$$

Q. 2.35. A galvanometer has a resistance of 100 ohm. resistance of one ohm is connected across its terminals. What part of the total current flows through the galvano-neter ? Draw the necessary circuit diagram.

(P.S.S.C.E. 1996 S)

Ans. Here, $G = 100\,\Omega\ ; S = 1\,\Omega$

Let I be the total current in the circuit and the part I_g flow hrough the galvanometer as shown in Fig.2.32.

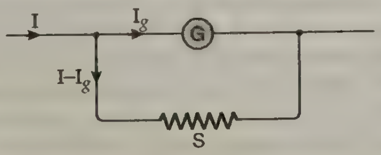


Fig. 2.32

Then $I_g \times G = (I - I_g) \times S$

or $I_g\,(G + S) = I\,S$

or $\frac{I_g}{I} = \frac{S}{G + S} = \frac{1}{100 + 1} = \frac{1}{101}$

Q. 2.36. Why an ammeter should have as low a resistance as possible ? Explain.

Ans. An ammeter is connected in series in an electric circuit to measure the current. In case the resistance of ammeter is not low, the current in the circuit will decrease on connecting it in the circuit. For this reason, the ammeter should have as low a resistance as possible.

Q. 2.37. When an ammeter is put in a circuit, does it read slightly less or more than the actual current in the original circuit ? Explain.

Ans. Yes, when an ammeter is connected in a circuit, it reads slightly less than the actual current in the circuit. It is because, an ammeter has a small resistance and when it is connected in the circuit, it decreases the current by a small amount.

Q. 2.38. A galvanometer gives full scale deflection with the current I_g. Can it be converted into an ammeter of range $I < I_g$?

Ans. The required value of shunt is given by

$$S = \frac{I_g \times G}{I - I_g}$$

For $I < I_g$, the above formula gives negative value for S. Hence, it cannot be converted into an ammeter of range $I < I_g$.

Q. 2.39. Can we increase/decrease the range of a given ammeter ?

Ans. As discussed above, the range of given ammeter cannot be decreased. It is because, S can not be negative.

However, the range of an ammeter can be increased by selecting a proper shunt. In the formula for S (in last question), I_g will represent the present range of the ammeter.

Q. 2.40. How does a voltmeter differ from ammeter ?

(P.S.S.C.E. 1999)

Or

Compare a voltmeter and an ammeter.

(P.S.S.C.E. 1998 S)

Ans. 1. An ammeter is a galvanometer to which a small resistance is connected in parallel, while a voltmeter is a galvanometer to which a high resistance is connected in series.

2. An ammeter is a low resistance instrument, while a voltmeter is a high resistance instrument.

3. An ammeter is always connected in series, while a voltmeter is connected in parallel to the part of the circuit across which potential difference is to measured.

4. An ammeter is used to measure current, while a voltmeter is used to measure potential difference in a circuit.

Q. 2.41. How can a moving coil galvanometer be converted into a voltmeter ? *(C.B.S.E. 1991)*

Ans. A moving coil galvanometer can be converted into a voltmeter by connecting a suitable high resistance R in series with its coil. The value of R is given by

$$R = \frac{V}{I_g} - G,$$

where I_g, G and V have their usual meanings.

Q. 2.42. A voltmeter of resistance R is to be used to measure a voltage 100 times as large as it can measure without an additional resistance in series. How large a resistance is to be used in series with the meter ?

Ans. Suppose that a voltmeter of resistance R can measure potential difference upto V. Obviously, when potential difference V is applied, the deflection in the voltmeter is maximum *i.e.*

$$I_g = \frac{V}{R}$$

So that the voltmeter can measure potential difference as large as 100 times V *i.e.* 100 V, suppose that the value of the resistance R′ to be connected in series with the meter is R′. Then,

$$R' = \frac{(100\ V)}{I_g} - R = \frac{100\ V}{V/R} - R = 99\ R$$

Q. 2.43. A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA. How will you convert the meter into a voltmeter of range 0 to 18 V ? *(Text Question)*

Ans. Here, $G = 12\ \Omega$; $I_g = 3\ mA = 3 \times 10^{-3}\ A$; $V = 18$ volt

To convert galvanometer into voltmeter, a large resistance R has to be connected in series to the coil of the galvanometer. Then,

$$R = \frac{V}{I_g} - G = \frac{18}{3 \times 10^{-3}} - 12 = 5988\ \Omega$$

Q. 2.44. A good quality voltmeter should have a large istance. Explain.

Ans. A voltmeter is connected in parallel to the conductor, across which the potential difference is to be measured. Obviously, the potential difference across the conductor should not change on connecting the voltmeter across the conductor. It will be so, only if the voltmeter draws a negligible current *i.e.* if its resistance is very large.

Q. 2.46. Draw a circuit, showing how an ammeter and a voltmeter can be connected to a resistor to measure the current and voltage at a given instant. *(I.S.C.E. 1996)*

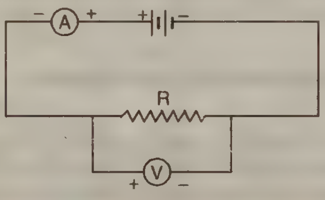


Fig. 2.33

Ans. In order to measure current through the resistor and to measure voltage across it, ammeter (A) and voltmete are connected as shown in Fig. 2.34.

Q. 2.45. When a voltmeter is put across a part of th circuit, does it read slightly less or more than the origina voltage drop across that part ? Explain.

Ans. When a voltmeter is put across a part of the circui it reads slightly less than the original potential drop acros that part. It is because, the voltmeter draws some curren form the part of the circuit across which it is connected an as a result, the potential drop across that part decreases.

Q. 2.47. Why is an ammeter connected in series, whil the voltmeter is connected in parallel with the rest of th circuit ? *(H.S.S.C.E. 2000*

Ans. An ammeter is a low resistance instrument, whil a voltmeter is a high resistance instrument.

When an ammeter is connected in series, the resistanc of the circuit remains practically the same. In case it i connected in parallel, most of the current will flow throug it and the current in the main circuit will decrease.

When a voltmeter is connected in parallel with the res of the circuit, only a very small amount of current will flov through the voltmeter and hence current in the circuit remain the same. In case, the voltmeter is connected in series, th current in the circuit will decrease appreciably due to hig resistance of the voltmeter.

Q. 2.48. Can we increase/decrease the range of a give voltmeter ?

Ans. The range of a given voltmeter can both b increased and decreased.

We know that for converting a galvanometer o resistance G into a voltmeter of range V, a high resistance given by

$$R = \frac{V}{I_g} - G$$

has to be connected in series to the coil of the galvanomete

In order to increase the range of the voltmeter, R has t be increased. It can be done by putting a suitable resistanc in series with the resistance R.

On the other hand, to decrease the range of th voltmeter, R has to be decreased. It can be done by putting suitable resistance in parallel with the resistance R.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 2.01. A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction and comes out of it following a complicated trajectory. Would its final speed equal to initial speed, if it suffered no collisions with the environment ? *(Text Question)*

Ans. The magnetic field exerts force on the charged particle always perpendicular to its motion and hence does no work. Therefore, charged particle will have its final speed equal to its initial speed, provided it suffered no collision with the environment.

Q. 2.02. A cloud chamber photograph shows a pair of circular tracks emerging from a common point. The tracks have similar density of droplets, but curve i opposite directions in a plane normal to the magnetic fiel maintained in the chamber. If one of the ionizing particle is established to be an electron, guess the high energy even that took place at the common point of the track. *(Text Question*

Ans. The similarity of the two paths shows that the othe charged particle has same mass and same magnitude o charge as that of electron. As the tracks are curved in opposit directions, the nature of charge on the other particle i opposite to that of electron. This other particle is calle positron. Thus, at the common point of the track, the hig energy event that took place is conversion of a γ-ray photo into electron-positron pair.

Q. 2.03. A similar event as described in conceptual SAQ 2.02 photographed in a liquid hydrogen bubble mber shows spiral tracks instead of circular tracks. lain, why ? *(Text Question)*

Ans. A charged particle produces much more ionization liquid than in a gas. Therefore, in liquid hydrogen bubble mber, the charged particles produce ionization and gressively lose energy due to collisions encountered with environment. Since, the radius of circular path followed charged particle inside magnetic field is proportional to peed, the radius of circular path also goes on decreasing gressively. Hence, the charged particle follows spiral path.

Q. 2.04. A soft spiral spring hangs freely with its upper end fixed to a rigid support. The lower end of spring is immersed in a cup of mercury. The spring the cup are connected to a d.c. source as shown in 2.34. What will happen to the spring after the switch closed ?

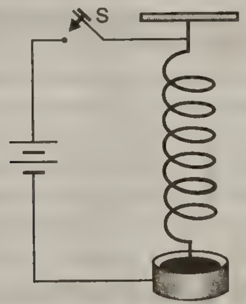


Fig. 2.34

Ans. When the switch S is closed, the current flows ough each coil of the spring in the same direction. As such, coils of the spring will attract each other and the spring l be compressed. On being compressed, the lower end of spring will get out of the mercury cup and the current will interrupted. As it happens so, the coils of the spring will se to attract each other and the spring will then straighten lf out. As soon as the lower end of the spring enters the rcury cup, the current will begin to flow through the spring l the entire process will be repeated.

Q. 2.05. A beam of electrons is moving with a velocity of 3×10^6 m s^{-1} and carries a current of 1 μA.

(a) How many electrons per second pass a given int ?

(b) How many electrons are in 1 m of the beam ?

(c) What is the total force on all the electrons in 1 m of beam, if it passes through a field of 0·1 N A^{-1} m^{-1}?

(d) What is the force on a single electron, if the ctrons experience the same force ?

Ans. Here, $v = 3 \times 10^6 \text{ m s}^{-1}$; $I = 1\ \mu A = 10^{-6}$ A

(a) The number of electrons passing a given point per cond,

$$n = \frac{I}{e} = \frac{10^{-6}}{1 \cdot 6 \times 10^{-19}} = \mathbf{6 \cdot 25 \times 10^{12}}$$

(b) Since electrons traverse a distance of 3×10^6 m in 1 s, number of electrons in 1 m of the beam

$$= \frac{6 \cdot 25 \times 10^{12}}{3 \times 10^6} = \mathbf{2 \cdot 08 \times 10^6\ m^{-1}}$$

(c) Force on 1 m beam of electrons,

$$F = B\,I\,l = 0{\cdot}1 \times 10^{-6} \times 1 = \mathbf{10^{-7}\ N}$$

(d) Since 1 m contains $2{\cdot}08 \times 10^6$ electrons, force on one electron,

$$f = \frac{F}{2 \cdot 08 \times 10^6} = \frac{10^{-7}}{2 \cdot 08 \times 10^6} = \mathbf{4 \cdot 81 \times 10^{-14}\ N}$$

Q. 2.06. A straight conductor carrying current I_1 is passed along the axis of a circular conductor carrying current I_2 [Fig. 2.35]. With what force are the currents interacting ?

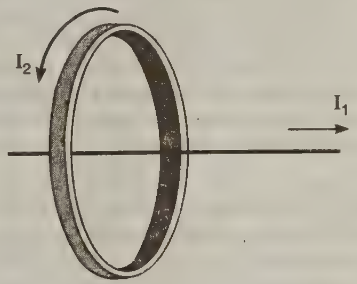


Fig. 2.35

Ans. The lines of force of the magnetic field B_2 produced by the current I_2 flowing through the circular conductor will be along the length of the straight conductor carrying current I_1. As the angle between magnetic field B_2 and the current I_1 is 0°, the straight conductor will not experience any force due to the current through circular conductor.

The lines of force of the magnetic field B_1 produced by the current I_1 flowing through the straight conductor are circular. The angle between the magnetic field B_1 and all the elements of circular conductor with current I_2 is 0°. Therefore, the current carrying circular conductor will also not experience any force due to the magnetic field produced by the straight conductor.

Thus, currents I_1 and I_2 are interacting with **no force.**

Q. 2.07. A rubber band is covered with a conducting coating and current is passed through it in the direction as shown in Fig. 2.36.

(a) What happens, when it is placed in the magnetic field $\vec{B}$ shown in the figure ?

(b) What happens, if the current is reversed in the rubber band ?

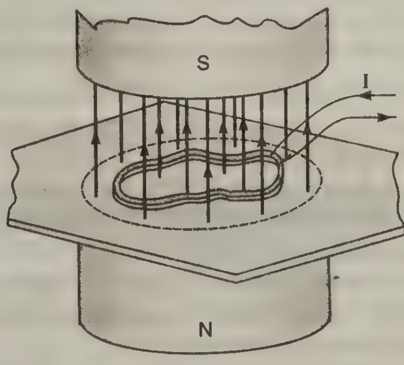


Fig. 2.36

Ans. (*a*) From Fleming's left hand rule, it follows that the force on each portion of the rubber band acts in the plane of the rubber band but is directed outwards. If we consider that the rubber band is made of a large number of small straight portions and since the current is the same in every portion of the rubber band, the force on each small portion is equal in magnitude and directed outwards. By symmetry, outward forces move the rubber band until it assumes a circular shape. The stable shape of the rubber band occurs, when the magnetic force on each small portion is balanced by the elastic forces at each end of that portion.

(*b*) When the direction of the current is reversed, the force on each small portion is now inward instead of outward. The rubber band collapses inward toward the centre and then continues outward away from the centre, until it again acquires a circular shape, each portion of which has come from a position diametrically opposite.

Q. 2.08. A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape ?

(*P.S.S.C.E. 1996 ; Text Question*)

Ans. It will assume circular shape with its plane normal to the field in order to maximise the magnetic flux through it. The reason is that for a given perimeter, of all the geometrical shapes, a circle has maximum area.

For details, refer to conceptual SAQ. 2.07.

Q. 2.09. The magnetic field $\vec{B}$ inside a long solenoid is uniform and parallel to the axis of the solenoid and is equal to 10^{-2} N A m^{-1}. A wire ABCD carrying a current of 1 A and supported on a light rectangular ebonite strip is placed inside the solenoid and parallel to its axis [Fig. 2.37].

(*a*) What is the force on portion AB (or CD) of the wire, if its length is 0 · 1m ?

(*b*) What is the force on portion BC of the wire, if its length is 0·02 m ?

(*c*) How much mass M should be attached to the left end of the ebonite strip to keep it form tilting ? Assume that strip forms an equal-arm balance.

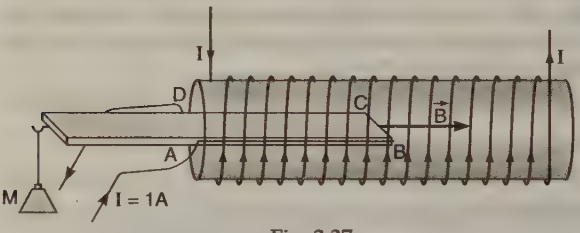


Fig. 2.37

Ans. (*a*) The current in the portion AB (or CD) is parallel to the magnetic field produced by the solenoid along its axis. Since there is no component of the magnetic field $\vec{B}$ perpendicular to the current in AB (or CD), the force on such a portion of the wire is **zero.**

(*b*) The current in portion CD is perpendicular to the direction of the magnetic field $\vec{B}$. The magnitude of force on portion CD is given by

$$F = B\,I\,l = 10^{-2} \times 1 \times 0{\cdot}02 = \mathbf{2 \times 10^{-4}\ N}$$

(*c*) The strip will not tilt, if a mass M is attached to its left end. Therefore, in equilibrium,

$$F = Mg \qquad \text{or} \qquad Mg = 2 \times 10^{-4}$$

$$\text{or} \qquad M = \frac{2\times10^{-4}}{g} = \frac{2\times10^{-4}}{9{\cdot}8} = \mathbf{2{\cdot}04\times10^{-5}\ kg}$$

Q. 2.10. The two parallel circular loops of wire show in Fig. 2.38 each have a radius R = 20 cm and ar separated by a distance *a* = 0·5 cm. The bottom loop is hel fixed and the top loop hangs from one end of the arm of a equal-arm balance. Since *a* is much smaller than R, th magnetic field produced by one loop at the other i practically the same as if both were long straight wires.

(*a*) If the current in each loop is 20 A, how big is th force of attraction of one loop for the other ?

(*b*) How much additional mass *m* must be added to th right-hand pan of the balance to keep the loops separate by 0·5 cm, when the currents are turned on ?

Ans. Here, radius of each loop, R = 20 cm = 0·2 m ; distance between the loops, a = 0·5 cm = 5 × 10^{-3} m

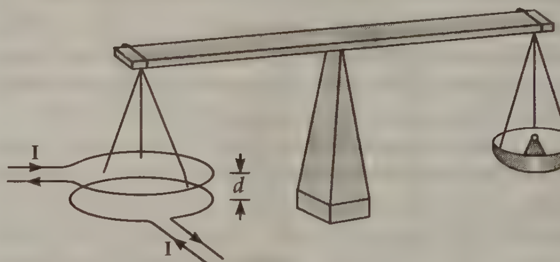


Fig. 2.38

(*a*) Assuming the magnetic field produced by the lowe loop to be practically the same as due to a straight conducto magnetic field produced by lower loop at the top loop,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{a} = \frac{10^{-7}\times2\times20}{5\times10^{-3}} = 8\times10^{-4}\ \text{T}$$

Again considering the top loop as a straight conducto force on the top loop due to the lower loop,

$$F = B\,I\ (\text{circumference of the top loop})$$
$$= B\,I \times 2\pi R = 8\times10^{-4}\times20\times2\pi\times0{\cdot}2$$
$$= \mathbf{2\times10^{-2}\ N}$$

(*b*) The mass *m* added to the right-pan of the balance i given by

$$F = m\,g$$

$$\text{or} \qquad m = \frac{F}{g} = \frac{2\times10^{-2}}{9{\cdot}8} = \mathbf{2{\cdot}04\times10^{-3}\ kg}$$

Q. 2.11. Currents are passed through two free straigh conductors AB and CD arranged at right angles a shown in Fig. 2.39. How will the interaction of the magneti fields of the currents change the position of the conductor relative to each other ?

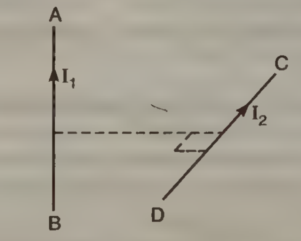


Fig. 2.39

Ans. Fig. 2.40 shows the part of magnetic field produced

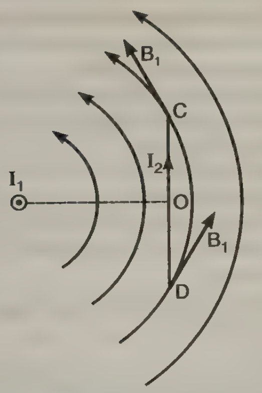


Fig. 2.40

conductor AB carrying current I_1. Suppose that the rength of magnetic field due to the current carrying nductor AB at the ends C and D is B_1. Force on the end C the conductor CD acts perpendicular to the plane of paper lane containing conductor CD and the magnetic field B_1) d in outward direction, while an equal force acts on end perpendicular to the plane of paper and in inward rection. Under the action of these forces, the conductor CD ill turn and the two conductors will become parallel to each her. As they do so, currents in the two conductors flow in me direction and the two will get mutually attracted wards each other.

Q. 2.12. An infinitely long current caryimg straight conductor AB has near it a uniform straight conductor D of finite length and capable of moving freely. Initially, e whole of the conductor CD lies on one side of AB and a plane passing through AB [Fig. 2.41]. What will happen the conductor CD, if current is passed through it in the irection from C to D ?

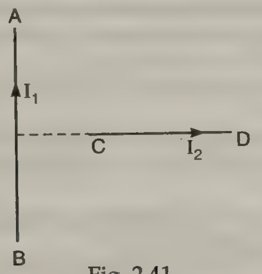


Fig. 2.41

Ans. The strength of the magnetic field produced due o current carrying conductor AB is not uniform over the ength of the conductor CD. The strength of the magnetic field t the end C will be greater than that at the end D. Therefore, ifferent elements of the current carrying conductor CD will experience different amounts of force due to the magnetic field produced by the current carrying conductor AB. Larger forces will act on the elements adjoining the end C than on the identical elements adjoining the end D. The resultant F of the forces on all the elements of the conductor CD will lie to the left of its centre of gravity G [Fig. 2.42].

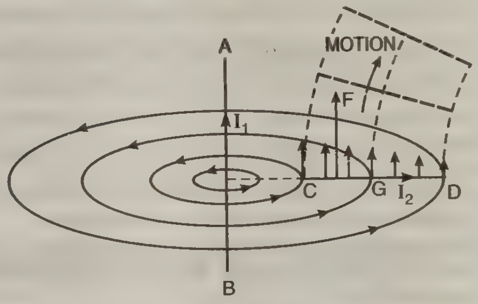


Fig. 2.42

As the resultant force F does not act at the centre of gravity of the conductor CD, it will move up and turning at the same time, till it also becomes parallel to the conductor AB. As the current through the two conductors flow in opposite directions, the conductor CD will get repelled away by the conductor AB.

Q. 2.13. Two parallel wires carrying currents in same direction attract each other, while two beams of electrons travelling in the same direction repel each other. Explain, why. *(P.S.S.C.E. 1995)*

Ans. We know that when current flows through two parallel wires in the same direction, the force F_m experienced by one wire is due to the magnetic field produced by the other wire and is attractive in nature.

In case of two beams of electrons moving in the same direction, in addition to the force F_m, another force F_e (repulsive in nature) also acts on them. Since $\frac{F_m}{F_e} \approx 10^{-27}$, electrons appear to experience a strong repulsive force only.

Q. 2.14. A current carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (*i.e.* turns about the vertical axis) ? *(Text Question)*

Ans. The magnetic field cannot be set up to turn the loop around itself. We know that

$$\vec{\tau} = \vec{M} \times \vec{B} = I\,\vec{A} \times \vec{B},$$

where $\vec{A}$ is vector representing area of the circular loop.

For turning the loop around itself, $\vec{\tau}$ should be along vertical. Since area vector $\vec{A}$ acts perpendicular to the plane of loop *i.e.* along vertical, $\vec{\tau}$ cannot act along vertical.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Show that the path of a charged particle moving inside the electric field $\vec{E}$ is parabolic in nature. *(H.P.S.S.C.E. 1999 S)*
2. Show that the trajectory of an electron moving in a direction perpendicular to the electric field is parabolic. *(H.P.S.S.C.E. 1997)*

3. A stream of electrons travelling with speed v m s^{-1} at right angles to a uniform magnetic field E, is deflected in a circular path of radius r. Prove that $\frac{e}{m} = \frac{v^2}{r\,E}$. *(C.B.S.E. 2002)*
4. Write an expression for the force $\vec{F}$ acting on a charge q moving with a velocity $\vec{v}$ in region, where magnetic induction $\vec{B}$ is uniform. How does the speed change, as the charge moves ? Under what circumstances, the force $\vec{F}$ shall be zero ? *(I.S.C.E. 1996)*
5. Write the expression for the force on a charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$. Discuss the cases, when the force is maximum and minimum.
6. Discuss the circular motion of a charged particle under the action of a magnetic field. Hence, find expression for its time period.
7. A charge q moving in a straight line is accelerated by a potential difference V. It enters a uniform magnetic field B perpendicular to its path. Deduce in terms of V an expression for the radius of the circular path in which it travels. *(C.B.S.E. 2000)*
8. An electron beam of velocity v moving in the plane of paper enters a region of uniform magnetic field, which is perpendicular to the plane of the paper and travels in a circular path. (*a*) Deduce an expression for the time period of the electron. (*b*) Does the kinetic energy of electron change during this time ? Give reason. *(C.B.S.E. 1998 S)*
9. A particle with charge q moving with a velocity v in the plane of the paper enters a uniform magnetic field B, acting perpendicular to the plane of the paper. Deduce an expression for the time period of the charge, as it moves in a circular path in the field. Why does the kinetic energy of the charge not change, while moving in the magnetic field ? *(C.B.S.E. 2001)*
10. A charged particle of mass m, charge q, moving at uniform velocity v, enters a uniform magnetic field B acting normal to the plane of the paper. Deduce expression for the (*i*) radius of the circular path in which it travels, (*ii*) kinetic energy of the particle (assuming $v << c$). Why does the kinetic energy of the charge not change, when moving through the magnetic field ? *(C.B.S.E. 2001)*
11. Discuss the motion of a charged particle moving in a direction making angle θ with the direction of the magnetic field.
12. Explain the construction and theory of a cyclotron. *(P.S.S.C.E. 2000)*
13. Find the frequency of electric field to be applied between dees of a cyclotron to accelerate charged particles by it. *(H.S.S.C.E. 1994 ; C.B.S.E. 1994)*
14. Show that the frequency of a charge circulating at right angle to a uniform magnetic field does not depend upon the speed of the charge.
15. Explain, by drawing a labelled diagram, how a positively charged particle is accelerated in a cyclotron. *(C.B.S.E. 1994)*
16. Derive an expression for the force experienced by a curre
    carrying conductor placed in a magnetic field.
    (P.S.S.C.E. 1999 S, 1996 ; H.S.S.C.E. 199
17. Define the SI unit of magnetic field. Deduce an expressio
    for the force F on a conductor of length l carrying a curre
    I in a uniform magnetic field B. *(C.B.S.E. 200*
18. Derive the relation for the force per unit length betwe
    two infinitely long parallel straight wires carrying curren
    Hence, define one ampere. *(H.S.S.C.E. 200*
19. Derive a mathematical expression for the force per ur
    length acting on each of the two straight parallel metal
    conductors carrying current in the same direction and ke
    near each other. Why do such current carrying conducto
    attract each other ? *(C.B.S.E. 199*
20. Find the magnitude and direction of the force between tv
    parallel conducting wires carrying current and hen
    define ampere. *(H.S.S.C.E. 1999 S ; P.S.S.C.E. 1999, 199*
21. Find the force between the two long parallel wires carryi
    current in the same direction.
    (H.S.S.C.E. 2000 ; H.P.S.S.C.E. 200
22. Two long thin straight parallel wires, each carrying
    current I in the same direction, are separated by a distan
    d. Indicate the direction of the force between them. Wri
    down the expression for the magnitude of force per ur
    length of the wire and hence define an ampere.
    (C.B.S.E. 1999
23. Derive a formula for the force between two parall
    straight conductors carrying current in opposi
    directions and write the nature of the force. Hence, defi
    an ampere. *(C.B.S.E. 199*
24. Derive an expression for the torque acting on a curre
    carrying loop suspended in a uniform magnetic field.
    (P.S.S.C.E. 200
25. Obtain an expression for torque acting on a rectangul
    current loop, when placed inclined at an angle θ with t
    direction of magnetic field B. *(H.S.S.C.E. 199*
26. Derive an expression for the torque on a rectangular c
    of area A, carrying a current I and placed in a unifor
    magnetic field B. The angle between the direction of B a
    vector perpendicular to the plane of the coil is θ.
    (H.S.S.C.E. 2002 ; C.B.S.E. 199
27. State the principle of a moving coil galvanometer. Show th
    the current passing through the coil is directly proportion
    to the deflection of the coil. *(H.S.S.C.E. 200*
28. What is a galvanometer ? How can it be converted into
    ammeter ? *(H.P.S.S.C.E. 200*
29. What is a galvanometer ? With the help of a schematic di
    gram, explain how it can be converted into an ammete
    (H.P.S.S.C.E. 200
30. Explain, how a galvanometer can be converted into an a
    meter of a given range.
    (H.S.S.C.E. 2001 ; P.S.S.C.E. 1999 S ; 1998
31. Which one has more resistance : ammeter or galvanomet
    and why ? Also explain, why ammeter is connected
    series in the circuit. *(P.S.S.C.E. 1999*
32. Explain, how you will convert a galvanometer into
    voltmeter of a given range ?
    (P.S.S.C.E. 1999 S ; C.B.S.E. 199

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Prove that two parallel conductors of infinite lenghts, carrying currents in the same direction, attract each other. Deduce the expression for the force per unit length exerperienced by each conductor. *(P.S.S.C.E. 2002)*
2. State the principle, construction, theory and limitations
   a cyclotron. *(P.S.S.C.E. 2002 ; C.B.S.E. 200*
3. Discuss the principle, construction, theory and working
   a cyclotron. Why is a cyclotron not suitable for accelerati
   electrons ? *(P.S.S.C.E. 1999 S, 1998 S, 1998, 1997, 1996*

4. Discuss the principle, construction, theory and working of a cyclotron. *(P.S.S.C.E. 2001)*
5. Explain with the help of a labelled diagram, the underlying principle, construction and working of a cyclotron. *(C.B.S.E. 2001, 1997)*
6. Derive an expression for the force acting on a current carrying conductor placed in a uniform magnetic field and using this result derive the expression for torque acting on a current carrying loop suspended in a uniform magnetic field. *(P.S.S.C.E. 2002)*
7. (*a*) Derive an expression for the force experienced by a current carrying straight conductor placed in a magnetic field.
   (*b*) Under what conditions, is this force (*i*) zero and (*ii*) maximum ? State the rule used to determine the direction of this force. *(C.B.S.E. 1998)*
8. Explain with the help of a labelled diagram, the underlying principle, construction and working of a moving coil galvanometer. *(C.B.S.E. 1997)*
9. Describe the principle, construction, theory and two merits of a moving coil galvanometer. *(P.S.S.C.E. 2002)*
10. Discuss the principle, construction and working of a moving coil galvanometer. *(P.S.S.C.E. 2001, 1999, 1998 S ; 1997 S ; Pre-degree Kerela, 1992)*
11. Give the principle, construction and working of a moving coil galvanometer. How can its sensitivity be increased ?
12. Discuss the principle, construction and theory of a dead-beat galvanometer. State its drawback. *(P.S.S.C.E. 2002, 2000)*
13. Explain how a galvanometer can be converted into (*i*) ammeter (*ii*) voltmeter of the required range. *(P.S.S.C.E. 2002)*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

ype A. On motion of Charged particle inside Electric field

1. An electron beam is deflected by 1·76 mm in travelling a distance of 10 cm through an electric field of 1800 V m^{-1} acting perpendicular to its path. Find the electron charge to mass ratio, if average velocity of the electrons is 3×10^7 m s^{-1}. **[Ans.** $1{\cdot}76 \times 10^{11}$ C kg^{-1}**]**

ype B. On force on a charge moving in magnetic field

2. A proton enters a magnetic field of flux density 2·5 tesla with a velocity of $1{\cdot}5 \times 10^7$ m s^{-1} at an angle of 30° with the field. Find the force on the proton. Charge of the proton = $1{\cdot}602 \times 10^{-19}$ coulomb. *(Pre-degree Kerala, 1994)* **[Ans.** $3{\cdot}0 \times 10^{-12}$ N**]**
3. An electron enters a magnetic field of flux density 3 T with a velocity of 10^6 m s^{-1} at an angle of 30° with the field. Calculate the magnitude of force on the electron. Charge on electron = $1{\cdot}6 \times 10^{-19}$ C. **[Ans.** $2{\cdot}4 \times 10^{-13}$ N**]**
4. A proton of velocity 10^8 cm s^{-1} is moving perpendicular to a uniform magnetic field of 10 kilogauss. What is the sideway force acting on the proton of charge $e = 4{\cdot}8 \times 10^{-10}$ e.s.u. ? **[Ans.** $1{\cdot}6 \times 10^{-13}$ N**]**
5. An electron is moving at 10^6 m s^{-1} in a direction parallel to a current of 5 A, flowing through an infinitely long straight wire, separated by a perpendicular distance of 10 cm in air. Calculate the magnitude of the force experienced by the electron. *(C.B.S.E. 1999)* **[Ans.** $1{\cdot}6 \times 10^{-18}$ N**]**
6. A 5 MeV proton moves vertically downward through a magnetic field of 1·5 weber m^{-2} pointing horizontally from south to north. Calculate the direction of force acting on the proton. Mass of proton = $1{\cdot}7 \times 10^{-27}$ kg and charge = $1{\cdot}6 \times 10^{-19}$ C. **[Ans.** $7{\cdot}37 \times 10^{-12}$ N, from west to east**]**

ype C. On motion of charge inside Magnetic field

7. Electrons move at right angles to a magnetic field of 100 gauss and enter it with a velocity of 2×10^{10} cm s^{-1}. Find the radius of their circular path. Given, $e/m = 1{\cdot}7 \times 10^7$ e.m.u. g^{-1}. **[Ans.** 11·765 cm**]**
8. Find the flux density of the magnetic field to cause 62·5 eV electron to move in a circular path of radius 5 cm. Given, $m = 9{\cdot}1 \times 10^{-31}$ kg , $e = 1 \cdot 6 \times 10^{-19}$ C. **[Ans.** $5{\cdot}335 \times 10^{-4}$ T**]**
9. An electron after being accelerated through a potential difference of 100 V enters a uniform magnetic field of 0·004 T perpendicular to its direction of motion. Calculate the radius of the path described by the electron. Given, $m = 9{\cdot}1 \times 10^{-31}$ kg and $e = 1{\cdot}6 \times 10^{-19}$ C. *(C.B.S.E. 1992)* **[Ans.** 8·42 mm**]**

Type D. On Cyclotron

10. A cyclotron has an oscillator frequency of 12 megahertz and dees of radius 0·5 metre. Calculate the magnetic induction B needed to accelerate deutrons of mass $3{\cdot}34 \times 10^{-27}$ kg and a charge $1{\cdot}6 \times 10^{-19}$ coulomb. Also, calculate the energy acquired by deutrons. **[Ans.** 1·5744 T, 14·843 MeV**]**
11. In a cyclotron, magnetic induction of 1·55 Wb m^{-2} is used. (*i*) What must be the high voltage oscillating frequency of the cyclotron to accelerate deutrons of mass $3{\cdot}34 \times 10^{-27}$ kg and charge numerically equal to that of an electron ? (*ii*) If the radius of the dee is 0·50 m, what is the maximum attainable energy of the deutrons in MeV ? **[Ans.** (*i*) 11·82 MHz ; (*ii*) 14·39 MeV**]**
12. In a cyclotron to accelerate proton, the magnetic field is 15000 gauss and radius of the dees is 15 cm. Calculate the energy of the proton beam and the frequency of the oscillator. Take e/m for proton to be 9580·84 e.m.u. g^{-1} and mass of proton = $1{\cdot}67 \times 10^{-24}$ g. **[Ans.** 2·425 MeV, 22·87 MHz**]**

Type E. On Force on a current carrying conductor

13. A straight conductor carries a current of 1·5 A and is 2 m long. Find the force on it, when it is (*a*) inclined at an angle at 30°, (*b*) perpendicular and (*c*) parallel to a uniform field of 2×10^{-2} T. **[Ans.** (*a*) 0·03 N (*b*) 0·06 N (*c*) zero**]**
14. What is the force on a wire of length 4·0 cm placed inside a solenoid near its centre, making an angle of 60° with its axis ? The wire carries a current of 12 A and the magnetic field due to the solenoid has a magnitude of 0·25 T. **[Ans.** 0·104 N**]**

Type F. On Force between two long straight current carrying conductors

15. A current of 10 A flows through each of two parallel long wires. The wires are 5 cm apart. Calculate the force acting per unit length of each wire. Use the standard value of constant required. *(P.S.S.C.E. 1998)* **[Ans.** 4×10^{-4} N m^{-1}**]**

16. Calculate the force per unit length between two parallel long straight wires 2 cm apart in air, each carrying a current of 4 ampere. (*H.S.S.C.E. 1995*) [**Ans.** $1{\cdot}6 \times 10^{-4}$ N m^{-1}]

17. A current of 5·0 A flows through each of two parallel long wires. The wires are 2·5 cm apart. Calculate the force acting per unit length of each wire. Use standard value of constant required. What will be the direction of force, if both currents flow in the same direction ? (*P.S.S.C.E. 1999*) [**Ans.** 2×10^{-4} N m^{-1}, **repulsive**]

18. Two long straight conductors are placed parallel at a distance of 2 cm from each other. When one of them carries a current of 60 A, the force on 1 m length of the other is 1200 dyne. Calculate current in the other. [**Ans.** 20 A]

19. Two straight wires A and B of lengths 10 m and 12 m carrying currents of 4·0 A and 6·0 A respectively in opposite directions lie parallel to each other at a distance of 4·0 cm. Estimate the force on a 10 cm section of the wire B near its centre. (*H.S.S.C.E. 2001*) [**Ans.** $1{\cdot}2 \times 10^{-5}$ N]

Type G. On Torque on a current loop

20. What torque acts on a 40 turn coil of 100 cm^2 area carrying a current of 10 ampere held with its axis at right angle to magnetic field of flux density 0·2 tesla and plane of the coil parallel to the field ? [**Ans.** 0·8 N m]

21. Calculate the torque on a 50 turn circular coil of radius 10 cm, when placed with its plane at 60° with a magnetic field of $3 \cdot 1 \times 10^{-5}$ T. The current through the coil is 2 A. [**Ans.** $4 \cdot 87 \times 10^{-5}$ N m]

22. A circular coil of 25 turns and radius 6·0 cm, carrying current of 10 A, is suspended vertically in a uniform magnetic field of magnitude 1·2 T. The field lines run horizontally in the plane of the coil. Calculate the force and torque on the coil due to the magnetic field. In which direction, should a balancing torque be applied to prevent the coil from turning ?
[**Ans.** Zero, 3·393 N m (vertically upwards), 3·393 N m (vertically downwards)]

23. A rectangular coil of sides 8 cm and 6 cm having 2000 turns and carrying a current of 200 mA is placed in a uniform magnetic field of 0·2 T directed along positive X-axis. (*a*) What is the maximum torque that the coil can experience ? Also, tell the orientation of the coil (*b*) For which orientation of the coil is the torque zero ?
[**Ans.** (*a*) 0·384 N m (whenever X-axis lies in plane of the coil) ; (*b*) whenever the coil lies in YZ plane]

Type H. On Moving coil galvanometer

24. A moving coil meter has the following particulars. Number of turns, $n = 24$; area of coil ; $A = 2{\cdot}0 \times 10^{-3}$ m^2 ; magnetic field strength, $B = 0{\cdot}20$ T and resistance of the coil, $R = 14\ \Omega$.
(*a*) Indicate a simple way to increase in current sensitivity of the meter by 25%. (It is not easy to change A or B).
(*b*) If in doing so, the resistance of the coil increases to 21 Ω, is the voltage sensitivity of the modified meter greater or less than the original meter ?
[**Ans.** (*a*) increase n from 24 to 30 ; (*b*) lesser]

25. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that its new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change ? (*C.B.S.E. 2001*) [**Ans.** 25% (decrease)]

26. In an exercise to increase current sensitivity by 25%, its resistance is also increased by 50%. How will the voltage sensitivity of the galvanometer be affected ? (*P.S.S.C.E. 1999 S*) [**Ans.** 16·67% (decreases)]

Type I. On Conversion of galvanometer into ammete and voltmeter

27. A resistance of 900 Ω is connected in series with
galvanometer of resistance 100 Ω. A potential difference
1 V produces a deflection of 100 divisions in th
galvanometer. Find the figure of merit of galvanometer
[**Ans.** 10^{-5} A division

28. How will you convert a 1·0 mA full scale deflection mete
of resistance 100 Ω into an ammeter to read upto 1·0 A
(*P.S.S.C.E. 1997*) [**Ans.** 0·1 Ω in paralle

29. Resistance of a galvanometer is 25 Ω. It can carry a curre
of 0·5 mA. How can it be used to measure a current up
one ampere ? (*Pre-degree Kerala, 199*
[**Ans.** $1{\cdot}25 \times 10^{-2}$ Ω (in parallel

30. It is intended to measure a maximum current of 25 A wi
an ammeter of range 2·5 A and resistance of 0·9 Ω. Ho
can it be done ? [**Ans.** 0·1 Ω (in parallel

31. A galvanometer has a resistance of 10 ohm. It give
maximum deflection for a current of 50 mA. Find the shu
resistance required to convert it into ammeter of rang
2·5 A. What is the resistance of ammeter so constructed
[**Ans.** 0·204 Ω (in parallel), 0·2

32. In a galvanometer, there is a deflection of 10 divisions p
mA. The internal resistance of the galvanometer is 60
If a shunt of 2·5 Ω is connected to the galvanometer an
there are 50 divisions in all on the scale of the galvanomete
calculate the maximum current which the galvanomet
can read. (*C.B.S.E. 2001 S*) [**Ans.** 0·125

33. The coil of a galvanometer has a resistance 100 Ω. It give
full scale deflection for a current of 5×10^{-4} A. How w
you convert it into a voltmeter of 5 V range ?
(*H.S.S.C.E. 1999 S*) [**Ans.** 9900

34. How will you convert a 1 mA full scale deflection met
of resistance 100 Ω into an ammeter to read 1 A (full sca
deflection) and into a voltmeter to read 1 volt (full sca
deflection). (*C.B.S.E. Sample Q. Pape*
[**Ans.** 0·1 Ω (in parallel) ; 900 Ω (in series

35. A galvanometer with a coil of resistance 12·0 Ω shows fu
scale deflection for a current of 2·5 mA. How will yo
convert the meter into (*a*) an ammeter of range 0 to 7·5
and (*b*) a voltmeter of range 0 to 10·0 V ? Determine th
net resistance of the meter in each case. (*Text Proble*
[**Ans.** (*a*) 4×10^{-3} Ω ; 4×10^{-3} Ω, (*b*) 3988 Ω ; 4000

36. A voltmeter reads up to 3 V. Its resistance is 200 oh
It is to be used to measure a potential difference, whic
may be as large as 60 V. What measures you would ta
to protect the voltmeter ?
[**Ans.** connect a series resistance of 3800

Type J. On Ammeter and Voltmeter

37. A battery of e.m.f. 3 volt and internal resistance r
connected in series with a resistor of 55 Ω through a
ammeter of resistance 1 Ω. The ammeter reads 50 mA
Calculate the value of r. (*Pre-degree Kerala, 199*
[**Ans.** 4

38. A cell of e.m.f. 2 volt and internal resistance 0·1 oh
supplies a current through a coil of resistance 11·9 ohn
The current is being measured by an ammeter, whos
resistance is 6 ohm. What reading does it give ? Wh
is the percentage difference from the actual current, whe
the meter is not used ? [**Ans.** 1/9 A, 33·3%

39. Two resistance coils of 100 Ω and 200 Ω respectively a
connected in series across 100 V. A moving coil voltmet
of 200 Ω is connected in turn across each coil. What will
read in each case ? (*Karnataka, 1986*) [**Ans.** 25 V, 50

40. A battery of e.m.f. 12 V supplies a current through a coil of resistance 48 ohm. The internal resistance of the battery is 1·2 ohm. A voltmeter of resistance 72 ohm is used to measure the potential difference across the coil. What value would it show ? [**Ans.** 11·52 V]

41. A current is being measured in a circuit, consisting of a battery of 12 V and a resistor of resistance 22 ohm. The potential difference measured across the resistor by an accurate voltmeter is 11 V. If the current is to be measured with an ammeter with an error not greater than 10%, what is the maximum permissible resistance of the ammeter ? [**Ans.** 2·67 Ω]

MISCELLANEOUS PROBLEMS

42. A uniform magnetic field of 3000 G is established along the positive Z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the four cases shown in the Fig. 2.43 ?

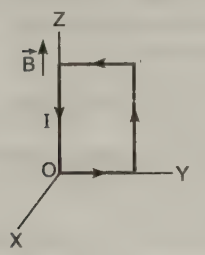


Fig. 2.43 (*a*)

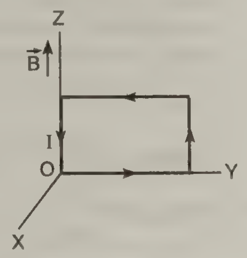


Fig. 2.43 (*b*)

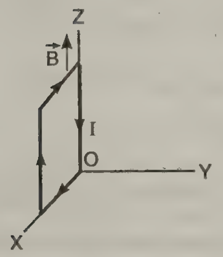


Fig. 2.43 (*c*)

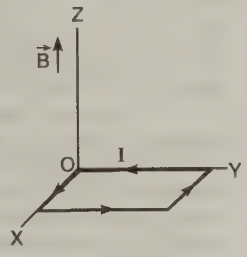


Fig. 2.43 (*d*)

[**Ans.** (*a*) $1\cdot8\times10^{-2}$ N m (along Y′-axis), (*b*) same as in (*a*), (*c*) $1\cdot8\times10^{-2}$ N m (along X′-axis), (*d*) zero]

43. A galvanometer coil is 2 cm square and contains 100 turns. Its period is 20 s and its moment of inertia is 2 g cm^2. The field of the magnet is 0·2 T. Determine the current sensitivity of the galvanometer. [**Ans.** 23·2° μA^{-1}]

44. A galvanometer has a sensitivity of 60 divisions A^{-1}. When a shunt is used, its sensitivity becomes 10 divisions A^{-1}. What is the value of shunt used, if the resistance of galvanometer is 20 Ω. [**Ans.** 4 Ω]

45. A galvanometer has a resistance of 8 Ω and gives full scale deflection, when a current of 10 mA is passed through it. The only shunt resistance available is 0·02 Ω, which is not appropriate to convert the galvanometer into an ammeter of range 5 A. To achieve this, a resistance R is connected in series with the coil of the galvanometer as shown in Fig. 2.44. Find the value of R. [**Ans.** 1·98 Ω]

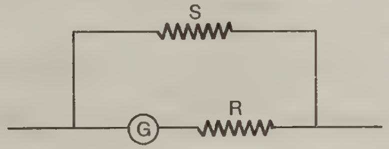


Fig. 2.44.

46. A voltmeter reads 5·0 V at full scale deflection and is graded according to the resistance per volt at full scale deflection as 10,000 Ω V^{-1}. How will you convert it into a voltmeter that reads 25 V at full scale deflection ?

(*C.B.S.E. 2001 S*) [**Ans.** 200 kΩ (in series)]

47. It is required to convert a galvanometer of current range 15 mA and voltage range 750 mV into (*a*) an ammeter of range 25 A and (*b*) a voltmeter of range 150 V. Calculate the value of necessary resistance in each case.

(*Text Problem*) [**Ans.** (*a*) 0·03 Ω, (*b*) 9950 Ω]

48. The scale of a galvanometer is divided into 150 equal divisions. The galvanometer has the current sensitivity of 10 divisions mA^{-1} and the voltage sensitivity of 2 divisions mV^{-1}. How the galvanometer be designed to read (*a*) 6 A division^{-1} and (*b*) 1 V division^{-1}. (*Roorkee 1999*)

[**Ans.** (*a*) $8\cdot33\times10^{-5}$ Ω (in parallel), (*b*) 9995 Ω (in series)]

49. In the circuit shown in Fig. 2.45, the voltmeter reads 117 V and the ammeter reads 0·13 A. The resistance of voltmeter and ammeter are 9000 Ω and 0·015 Ω respectively. Find the value of R. [**Ans.** 1000 Ω]

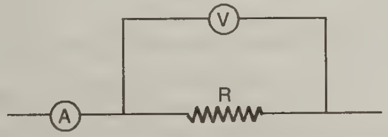


Fig. 2.45

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Time taken by electron to traverse the electric field,

$$t=\frac{10\times10^{-2}}{3\times10^{7}}=3\cdot33\times10^{-9}\text{ s}$$

Acceleration of electron along the electric field,

$$a=\frac{e\text{E}}{m}=\frac{e}{m}\times1800\text{ m s}^{-2}$$

Now, $\text{S}=ut+\frac{1}{2}at^2$

Here, S = 1·76 mm = $1\cdot76\times10^{-3}$ m and $u=0$,

$$\therefore\ 1\cdot76\times10^{-3}=0\times3\cdot33\times10^{-9}+\frac{1}{2}\frac{e}{m}\times1800\times(3\cdot33\times10^{-9})^2$$

or $\frac{e}{m}=\mathbf{1{\cdot}76\times10^{11}\ C\ kg^{-1}}$

2. $\text{F}=qv\text{B}\sin\theta=1\cdot602\times10^{-19}\times1\cdot5\times10^{7}\times2\cdot5\times\sin30°$

$=\mathbf{3{\cdot}0\times10^{-12}\ N}$

3. Proceed as in problem no. 2.

4. $\text{F}=\text{B}qv\sin\theta$

Here, B = 10 kilogauss $=10\times10^{3}\text{ G}=\frac{10\times10^{3}}{10^{4}}=1\text{ T}$,

$v=10^{8}\text{ cm s}^{-1}=10^{6}\text{ m s}^{-1}$,

$q=4\cdot8\times10^{-10}$ e.s.u. of change

$=\frac{4\cdot8\times10^{-10}}{3\times10^{9}}=1\cdot6\times10^{-19}\text{ C}$

$\therefore$ $\text{F}=1\times1\cdot6\times10^{-19}\times10^{6}\sin90°=\mathbf{1{\cdot}6\times10^{-13}\ N}$

Aliter. B = 10 kilograms = 10^4 G,

$q=\frac{4\cdot8\times10^{-10}}{3\times10^{10}}=1\cdot6\times10^{-20}$ e.m.u. of charge

Now, $\text{F}=\text{B}qv\sin\theta=10^{4}\times1\cdot6\times10^{-20}\times10^{8}\times\sin90°$

$=1\cdot6\times10^{-8}\text{ dyne}=\mathbf{1{\cdot}6\times10^{-13}\ N}$

5. Magnetic field produced due to straight conductor,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} = \frac{10^{-7} \times 2 \times 5}{10 \times 10^{-2}} = 10^{-5}\ \text{T}$$

Now, $F = B\,q\,v \sin\theta = 10^{-5} \times 1{\cdot}6 \times 10^{-19} \times 10^{6} \sin 90°$

$= \mathbf{1{\cdot}6 \times 10^{-18}\ N}$

6. Here, $\frac{1}{2} m v^2 = 5\ \text{MeV} = 5 \times 1{\cdot}6 \times 10^{-13} = 8 \times 10^{-13}\ \text{J}$

$$\therefore \quad v = \sqrt{\frac{2 \times 8 \times 10^{-13}}{m}} = \sqrt{\frac{2 \times 8 \times 10^{-13}}{1{\cdot}7 \times 10^{-27}}}$$

$= 3{\cdot}07 \times 10^{7}\ \text{m s}^{-1}$

Now, $F = B\,q\,v \sin\theta$

$= 1{\cdot}5 \times 1{\cdot}6 \times 10^{-19} \times 3{\cdot}07 \times 10^{7} \times \sin 90°$

$= \mathbf{7{\cdot}37 \times 10^{-12}\ N}$

To find direction of force : Force on the proton acts in the direction of vector $q(\vec{v} \times \vec{B})$ *i.e.* from **west to east** as depicted in Fig. 2.46

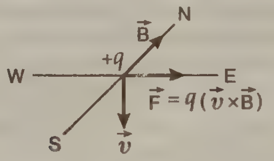


Fig. 2.46

7. Now, $\frac{m v^2}{r} = B\,e\,v$ or $r = \frac{m v}{B e}$

or $r = \frac{v}{B(e/m)} = \frac{2 \times 10^{10}}{100 \times 1{\cdot}7 \times 10^{7}} = \mathbf{11{\cdot}765\ cm}$

8. Here, $\frac{1}{2} m v^2 = 62{\cdot}5\ \text{eV} = 62{\cdot}5 \times 1{\cdot}6 \times 10^{-19} = 10^{-17}\ \text{J}$

or $v = \sqrt{\frac{2 \times 10^{-17}}{m}} = \sqrt{\frac{2 \times 10^{-17}}{9{\cdot}1 \times 10^{-31}}}$

$= 4{\cdot}69 \times 10^{6}\ \text{m s}^{-1}$

Now, $\frac{m v^2}{r} = B\,e\,v$ or $B = \frac{m v}{e r}$

$$\therefore \quad B = \frac{9{\cdot}1 \times 10^{-31} \times 4{\cdot}69 \times 10^{6}}{1{\cdot}6 \times 10^{-19} \times 5 \times 10^{-2}} = \mathbf{5{\cdot}335 \times 10^{-4}\ T}$$

9. Here, energy of electron, $\frac{1}{2} m v^2 = 100\ \text{eV}$

Now, proceed as in problem no. 8.

10. Proceed as in solved problem no. 2.09.

11. *(i)* $\nu = \frac{B e}{2\pi m} = \frac{1{\cdot}55 \times 1{\cdot}6 \times 10^{-19}}{2\pi \times 3{\cdot}34 \times 10^{-27}}$

$= 11{\cdot}82 \times 10^{6}\ \text{Hz} = \mathbf{11{\cdot}82\ MHz}$

(ii) Proceed as in solved problem no. 2.09.

12. $E_{max} = \frac{B^2 e^2 R^2}{2m} = \frac{B^2 (e/m)^2 R^2 m}{2}$

$$= \frac{(15000)^2 \times (9580{\cdot}84)^2 \times (15)^2 \times 1{\cdot}67 \times 10^{-24}}{2}$$

$= 3{\cdot}88 \times 10^{-6}\ \text{erg} = 3{\cdot}88 \times 10^{-13}\ \text{J}$

$= \frac{3{\cdot}88 \times 10^{-13}}{1{\cdot}6 \times 10^{-13}} = \mathbf{2{\cdot}425\ MeV}$

Also, $\nu = \frac{B e}{2\pi m} = \frac{B(e/m)}{2\pi} = \frac{15000 \times 9580{\cdot}84}{2\pi}$

$= 22{\cdot}87 \times 10^{6}\ \text{Hz} = \mathbf{22{\cdot}87\ MHz}$

13. $F = B\,I\,l \sin\theta$

(a) $F = 2 \times 10^{-2} \times 1{\cdot}5 \times 2 \times \sin 30° = \mathbf{0{\cdot}03\ N}$

(b) $F = 2 \times 10^{-2} \times 1{\cdot}5 \times 2 \times \sin 90° = \mathbf{0{\cdot}06\ N}$

(c) $F = 2 \times 10^{-2} \times 1{\cdot}5 \times 2 \times \sin 0° = \mathbf{0}$

14. Proceed as in problem no. 13.

15. Proceed as in solved problem no. 2.11.

16. Proceed as in solved problem no. 2.11.

17. Proceed as in solved problem no. 2.11.

18. Here, $F = 1200$ dyne $= 1200 \times 10^{-5}$ N,

$I_1 = 60$ A, $r = 2$ cm $= 0{\cdot}02$ m

Now, $F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r}$

or $1200 \times 10^{-5} = 10^{-7} \times \frac{2 \times 60 \times I_2}{0{\cdot}02}$

or $\mathbf{I_2 = 20\ A}$

19. The lengths of two conductors is quite large as compared to the separation between them. Therefore, the two conductors may be considered as infinitely long and hence force on unit length of wire B due to A,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} = \frac{10^{-7} \times 2 \times 4 \times 6}{4{\cdot}0 \times 10^{-2}} = 1{\cdot}2 \times 10^{-4}\ \text{N m}^{-1}$$

Therefore, force on 10 cm (or 0·1 m) section of wire B,

$F' = F \times 0{\cdot}1 = 1{\cdot}2 \times 10^{-4} \times 0{\cdot}1 = \mathbf{1{\cdot}2 \times 10^{-5}\ N}$

20. Here, $\alpha = 0°$

Now, $\tau = n\,B\,I\,A \cos\alpha = 40 \times 0{\cdot}2 \times 10 \times 100 \times 10^{-4} \cos 0°$

$= \mathbf{0{\cdot}8\ N\ m}$

21. Here, $A = \pi r^2 = \pi \times (10)^2\ \text{cm}^2 = \pi \times 10^{-2}\ \text{m}^2$ and $\alpha = 60°$

Now, proceed as in problem no. 20.

22. Proceed as in problem no. 21.

23. Here $A = 8 \times 6 = 48\ \text{cm}^2 = 48 \times 10^{-4}\ \text{m}^2$,

$I = 200\ \text{mA} = 200 \times 10^{-3}\ \text{A}$

(a) Now, $\tau = n\,B\,I\,A \cos\alpha$,

where α is angle between plane of the coil and direction of magnetic field. Therefore, coil will experience maximum torque, when $\alpha = 0°$.

Hence, $\tau_{max} = 2000 \times 0{\cdot}2 \times 200 \times 10^{-3} \times 48 \times 10^{-4} \times \cos 0°$

$= \mathbf{0{\cdot}384\ N\ m}$

The angle α will be zero, whenever magnetic field is parallel to the plane of the coil *i.e.* X-axis is in the plane of coil.

(b) The torque on coil will be zero, when $\alpha = 90°$ *i.e.* magnetic field is perpendicular to the plane of the coil. In other words, when coil lies in YZ-plane, torque on the coil is zero.

24. *(a)* Current sensitivity, $\frac{\alpha}{I} = \frac{n A B}{k}$

Since it is not easy to change A or B, current sensitivity may be increased by changing n (number of turns in the coil). Suppose that number of turns in the coil are changed from n to n' so as to increase current sensitivity by 25%.

Then, modified current sensitivity,

$$\frac{\alpha'}{I} = \frac{n' A B}{k}$$

Now, $\frac{\alpha'}{I} = \frac{125}{100} \times \frac{\alpha}{I}$ or $\frac{n' A B}{k} = \frac{125}{100} \times \frac{n A B}{k}$

or $n' = \frac{125}{100} \times n = \frac{125 \times 24}{100} = \mathbf{30}$

(b) Initial voltage sensitivity, $\frac{\alpha}{I R} = \frac{n A B}{k R}$

$$= \frac{24 \times 2{\cdot}0 \times 10^{-3} \times 0{\cdot}20}{k \times 14} = \frac{6{\cdot}857 \times 10^{-4}}{k}$$

Voltage sensitivity of modified meter, $\frac{\alpha}{IR'} = \frac{n'AB}{kR'}$

$$= \frac{30\times 2\cdot0\times10^{-3}\times0\cdot20}{k\times21} = \frac{5\cdot714\times10^{-4}}{k}$$

i.e. voltage sensitivity becomes **lesser**.

25. Proceed as in solved problem no. 2.20.
26. Proceed as in solved problem no. 2.20.
27. Here, G = 100 Ω, R = 900 Ω, V = 1 V

Therefore, current through the galvanometer,

$$I_g = \frac{V}{R+G} = \frac{1}{900+100} = 10^{-3}\text{ A}$$

Now, figure of merit

= current that produces a deflection of 1 division

$$= \frac{10^{-3}}{100} = \mathbf{10^{-5}\text{ A division}^{-1}}$$

28. Proceed as in solved problem no. 2.21.
29. Proceed as in solved problem no. 2.21.
30. Proceed as in solved problem no. 2.21.
31. Proceeding as in solved problem no. 2.21, it can be obtained that

S = **0·204 Ω (in parallel)**

Therefore, resistance of ammeter so constructed,

$$R_A = \frac{GS}{G+S} = \frac{10\times0\cdot204}{10+0\cdot204} = \mathbf{0\cdot2\ \Omega}$$

32. Here, $I_g = \frac{1}{10}\times50 = 5\text{ mA} = 5\times10^{-3}\text{ A}$

G = 60 Ω and S = 2·5 Ω

Now, $S = \frac{I_g G}{I-I_g}$ or $I = \frac{I_g G}{S}+I_g$

or $I = \frac{5\times10^{-3}\times60}{2\cdot5}+5\times10^{-3} = 125\times10^{-3}\text{ A}$

= **0·125 A**

33. Proceed as in solved problem no. 2.24 (*b*).
34. Proceed in as solved problem no. 2.24.
35. Proceed in as solved problem no. 2.24.
36. Here, $R_V = 200\ \Omega$

Current for full scale deflection, $I_g = \frac{3}{200} = 1\cdot5\times10^{-2}\text{ A}$

To increase the range of voltmeter from 3 V to 30 V, the resistance to be connected in series to the voltmeter is given by

$$R = \frac{V}{I_g} - R_V = \frac{60}{1\cdot5\times10^{-2}} - 200 = \mathbf{3800\ \Omega}$$

37. Here E = 3 V ; R = 55 Ω and $R_A = 1\ \Omega$

Now, $I = \frac{E}{R+r+R_A}$ or $50\times10^{-3} = \frac{3}{55+r+1}$

or $r = \frac{3}{50\times10^{-3}} - 56 = 60-56 = \mathbf{4\ \Omega}$

38. Here, E = 2 V, r = 0·1 Ω, R = 11·9 Ω, $R_A = 6\ \Omega$

Therefore, reading of the ammeter,

$$I = \frac{E}{R+r+R_A} = \frac{2}{11\cdot9+0\cdot1+6} = \mathbf{\frac{1}{9}\text{ A}}$$

Actual current in the circuit (in the absence of ammeter),

$$I' = \frac{E}{R+r} = \frac{2}{11\cdot9+0\cdot1} = \frac{1}{6}\text{ A}$$

$$\%\text{ difference} = \frac{I'-I}{I'}\times100 = \frac{1/6-1/9}{1/6}\times100 = \mathbf{33\cdot3\%}$$

39. Here, $R_1 = 100\ \Omega$; $R_2 = 200\ \Omega$, $R_V = 200\ \Omega$ and E = 100 V

When voltmeter is connected across R_1:

Let R_p be resistance of parallel combination of R_1 and R_v. Then,

$$R_p = \frac{R_1\times R_V}{R_1+R_V} = \frac{100\times200}{100+200} = \frac{200}{3}\ \Omega$$

Therefore, current in circuit,

$$I = \frac{E}{R_2+R_p} = \frac{100}{200+200/3} = \frac{3}{8}\text{ A}$$

Hence, reading of voltmeter

$$= I\times R_p = \frac{3}{8}\times\frac{200}{3} = \mathbf{25\text{ V}}$$

When voltmeter is connected across R_2:

$$R_p' = \frac{R_2\times R_V}{R_2+R_V} = \frac{200\times200}{200+200} = 100\ \Omega$$

$$I' = \frac{E}{R_1+R_p'} = \frac{100}{100+100} = 0\cdot5\text{ A}$$

Hence, reading of voltmeter $= I'\times R_p' = 0{\cdot}5\times200 = \mathbf{50\text{ V}}$

40. Here, E = 12 V ; r = 1·2 Ω ; R = 48 Ω and $R_V = 72\ \Omega$

Let R_p be resistance of series combination of R and R_V. Then,

$$R_p = \frac{R\times R_V}{R+R_V} = \frac{48\times72}{48+72} = 28\cdot8\ \Omega$$

Therefore, current in the circuit,

$$I = \frac{E}{R_p+r} = \frac{12}{28\cdot8+1\cdot2} = 0\cdot4\text{ A}$$

Hence, reading of the voltmeter

$= I\times R_p = 0{\cdot}4\times28{\cdot}8 = \mathbf{11{\cdot}52\text{ V}}$

41. Here E = 12 V ; R = 22 Ω

Let *r* be the internal resistance of the battery. Then,

current in the circuit, $I = \frac{E}{R+r}$

or $$I = \frac{12}{22+r} \qquad \text{...}(i)$$

Since potential difference across R is 11 V,

$$I = \frac{V}{R} = \frac{11}{22} = 0\cdot5\text{ A}$$

Substituting for I in equation (*i*), we have

$$0\cdot5 = \frac{12}{22+r} \quad \text{or} \quad r = 2\ \Omega$$

Let R_A be resistance of the ammeter. When ammeter is connected in circuit, current in the circuit will decrease due to resistance of the ammeter. Let I′ be the current measured by the ammeter. Then,

$$I' = \frac{E}{R+r+R_A} = \frac{12}{22+2+R_A}$$

or $$I' = \frac{12}{24+R_A}$$

Now, $I' = I\times\frac{90}{100}$ or $\frac{12}{24+R_A} = 0\cdot5\times\frac{90}{100} = 0\cdot45$

or $R_A = \frac{12}{0\cdot45} - 24 = 26\cdot67 - 24 = \mathbf{2{\cdot}67\ \Omega}$

42. Here, $B = 3000\text{ G} = 3000\times10^{-4} = 0{\cdot}3\text{ T}$,

$A = 10\times5 = 50\text{ cm}^2 = 50\times10^{-4}\text{ m}^2$, I = 12 A,

$\therefore IA = 50\times10^{-4}\times12 = 0{\cdot}06\text{ A m}^2$

(*a*) Here, $I\vec{A} = 0{\cdot}06\,\hat{i}\ \text{A m}^2$ and $\vec{B} = 0{\cdot}03\,\hat{k}\ \text{T}$

Now, $\vec{\tau} = (I\vec{A}) \times \vec{B} = 0{\cdot}06\,\hat{i} \times 0{\cdot}03\,\hat{k} = -1{\cdot}8 \times 10^{-2}\,\hat{j}\ \text{N m}$
i.e. torque equal to **$1{\cdot}8 \times 10^{-2}$ N m acts along the negative Y-axis.**

(*b*) Here, $I\vec{A} = 0{\cdot}06\,\hat{i}\ \text{A m}^2$ and $I\vec{B} = 0{\cdot}03\,\hat{k}\ \text{T}$

Since $I\vec{A}$ and $\vec{B}$ are same as in case (*a*), torque in this case also is equal to **$1{\cdot}8 \times 10^{-2}$ N m and acts along the negative Y-axis.**

(*c*) Here, $I\vec{A} = -0{\cdot}06\,\hat{j}\ \text{A m}^2$ and $\vec{B} = 0{\cdot}03\,\hat{k}\ \text{T}$

Now, $\vec{\tau} = (I\vec{A}) \times \vec{B} = -0{\cdot}06\,\hat{j} \times 0{\cdot}03\,\hat{k} = -1{\cdot}8 \times 10^{-2}\,\hat{i}\ \text{N m}$
i.e. torque equal to **$1{\cdot}8 \times 10^{-2}$ N m acts along the negative X-axis.**

(*d*) Here, $I\vec{A} = 0{\cdot}06\,\hat{k}\ \text{A m}^2$ and $\vec{B} = 0{\cdot}03\,\hat{k}\ \text{T}$

Now, $\vec{\tau} = (I\vec{A}) \times \vec{B} = 0{\cdot}06\,\hat{k} \times 0{\cdot}03\,\hat{k} = 0$

43. Here, $A = 2 \times 2 = 4\ \text{cm}^2 = 4 \times 10^{-4}\ \text{m}^2$; $n = 100$;
$T = 20$ s, $B = 0{\cdot}2$ T
and moment of inertia of the coil, $I = 2\ \text{g cm}^2$
$= 2 \times (10^{-3}\ \text{kg}) \times (10^{-2}\ \text{m})^2 = 2 \times 10^{-7}\ \text{kg m}^2$
When a coil of moment of inertia I is suspended from a wire having restoring torque per unit twist *k*, its period of oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{k}}$$

$$\text{or}\quad k = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2 \times 2 \times 10^{-7}}{(20)^2} = 1{\cdot}974 \times 10^{-8}\ \text{N m rad}^{-1}$$

$$\text{Now, current sensitivity} = \frac{nAB}{k} = \frac{100 \times 4 \times 10^{-4} \times 0{\cdot}2}{1{\cdot}974 \times 10^{-8}}$$
$$= 4{\cdot}053 \times 10^5\ \text{rad A}^{-1}$$
$$= 4{\cdot}053 \times 10^5 \times \left(\frac{180°}{\pi}\right) \times \left(10^6\ \mu\text{A}\right)^{-1}$$
$$= \mathbf{23{\cdot}2°\ \mu A^{-1}}$$

44. Here, $G = 20\ \Omega$
Let *n* be total number of divisions on the galvanometer scale.

Then, $I_g = \dfrac{n}{60}$ A and $I = \dfrac{n}{10}$ A

$$\text{Now,}\quad S = \frac{I_g G}{I - I_g} = \frac{(n/60) \times 20}{n/10 - n/60} = \frac{20}{6 - 1} = \mathbf{4\ \Omega}$$

45. Here, $G = 8\ \Omega$, $I_g = 10\ \text{mA} = 0{\cdot}01$ A, $I = 5$ A and $S = 0{\cdot}02\ \Omega$
When the resistance R is connected in series to the galvanometer, it behaves as a galvanometer of effective resistance,
$G' = G + R = 8 + R$

$$\text{Now,}\quad S = \frac{I_g G'}{I - I_g} \quad \text{or} \quad 0{\cdot}02 = \frac{0{\cdot}01 \times (8 + R)}{5 - 0{\cdot}01}$$

$$\text{or}\quad R = \frac{0{\cdot}02 \times (5 - 0{\cdot}01)}{0{\cdot}01} - 8 = 9{\cdot}98 - 8 = \mathbf{1{\cdot}98\ \Omega}$$

46. Proceed as solved problem no. 2.39.

47. Current range of galvanometer and voltage range of the galvanometer imply the values of current and voltage, which produce full scale deflection in the galvanometer. Thus,
$I_g = 15\ \text{mA} = 15 \times 10^{-3}$ A and $V_g = 750\ \text{mV} = 750 \times 10^{-3}$ V

$$\therefore\quad G = \frac{V_g}{I_g} = \frac{750 \times 10^{-3}}{15 \times 10^{-3}} = 50\ \Omega$$

(*a*) Here, I = 25 A

$$\therefore\ S = \frac{I_g G}{I - I_g} = \frac{15 \times 10^{-3} \times 50}{25 - 15 \times 10^{-3}} = \mathbf{0{\cdot}03\ \Omega}$$

(*b*) Here, V = 150 V

$$\therefore R = \frac{V}{I_g} - G = \frac{150}{15 \times 10^{-3}} - 50 = \mathbf{9950\ \Omega}$$

48. Total number of divisions on galvanometer scale = 150
Current sensitivity of galvanometer = 10 divisions mA⁻¹
Therefore, current required for full scale deflection,

$$I_g = \frac{150}{10} = 15\ \text{mA} = 15 \times 10^{-3}\ \text{A}$$

Voltage that produces full scale deflection,

$$V_g = \frac{150}{2} = 75\ \text{mV}$$

$$\text{Therefore, resistance of galvanometer, } G = \frac{V_g}{I_g} = \frac{75}{15} = 5\ \Omega$$

(*a*) Range of required ammeter, I = 6 × 150 = 900 A

$$\therefore\ S = \frac{I_g G}{I - I_g} = \frac{15 \times 10^{-3} \times 5}{900 - 15 \times 10^{-3}} = \mathbf{8{\cdot}33 \times 10^{-5}\ \Omega\ (in\ parallel)}$$

(*b*) Range of required voltmeter, V = 1 × 150 = 150 V

$$\therefore\ R = \frac{V}{I_g} - G = \frac{150}{15 \times 10^{-3}} = \mathbf{9995\ \Omega\ (in\ series)}$$

49. Here, $R_V = 9000\ \Omega$, $R_A = 0{\cdot}015\ \Omega$
Reading of voltmeter = 117 V
and reading of ammeter = 0·13 A
Suppose that the e.m.f. of the battery in the circuit is E.
Resistance of parallel combination of R_V and R,

$$R_p = \frac{R_V \times R}{R_V + R} = \frac{9000\,R}{9000 + R} \qquad \text{...(}i\text{)}$$

$$\text{Therefore, reading of ammeter, } \frac{E}{R_p + R_A} = 0{\cdot}13$$

$$\text{Also, reading of voltmeter, } \frac{E}{R_p + R_A} \times R_p = 117$$

or $0{\cdot}13 \times R_p = 117$ or $R_p = 900\ \Omega$
Substituting for R_p in equation (*i*), we have

$$\frac{9000\,R}{9000 + R} = 900$$

or $\mathbf{R = 1000\ \Omega}$

CHAPTER

3

Magnetic dipole

3.01. INTRODUCTION

Historically, Thales of Miletus knew that pieces of lodestone or magnetite (black ron oxide *i.e.* Fe_2O_3 – a naturally occurring iron ore) could attract small pieces of ron. Further, these small pieces of iron could themselves acquire this property, if hey came in contact with this ore (lodestone). Later, it was discovered by the Chinese hat a linear piece of lodestone, when suspended freely would come to rest approximately along geographical N – S line. This directional property forms the basis of the principle of the magnetic compass used as an aid to navigation. The Chinese used it as a primitive compass and because of this use, the iron ore 'magnetite' was given the name *lodestone* or *leading stone*. William Gilbert suggested hat the earth itself was a huge magnet. It is due to the earth's magnetism that a magnetic compass needle aligns itself along N– S direction.

The magnetic force and magnetism are basically the effects produced by electrical charges in motion. Oersted discovered the magnetic effect of electric currents. However, the phenomenon of magnetism and some of the basic properties of magnetism were known long before.

Quotable Quote

"The disgrace of the nineteenth century is its ignorance of the subject of magnetism, will the twentieth bring knowledge ?"

– Hermann Von Helmholtz

3.02. ARTIFICIAL BAR MAGNET AND ITS PROPERTIES

Usually, the natural magnets are not strong enough magnetically. Further, because of their odd shapes, the natural magnets are not very convenient for practical use. However, pieces of iron of suitable shapes can be made magnets by rubbing them with a natural magnet. It is found that a steel rod does not acquire magnetism that easily, with which a soft iron rod acquires. But the steel rod retains the magnetism, it has acquired and does not lose it, unless heated to a very high temperature. On the other hand, the soft iron rod which acquires magnetism, even when merely placed near a permanent magnet, loses this property when removed away from it.

When a bar magnet is brought near a heap of iron filings, they cling most near the two ends of the magnet called the poles of the magnet.

Magnetic pole. *The preferred regions of attraction near the two ends of a magnet, where the magnetic force due to bar magnet is maximum, are called the poles of the magnet.*

A bar magnet has the following properties :

1. ***Attractive property.*** A magnet can attract small pieces of magnetic substances like iron, steel, cobalt, nickel, etc. The attraction is maximum at poles. It may be pointed out that the poles are situated near the ends of the magnet and not exactly at the ends.

2. ***Directive property.*** A magnet, when suspended freely, aligns itself approximately along geographical N–S line, such that north pole of the magnet is towards the north of the earth and the south pole is towards the south of the earth as shown in Fig. 3.01.

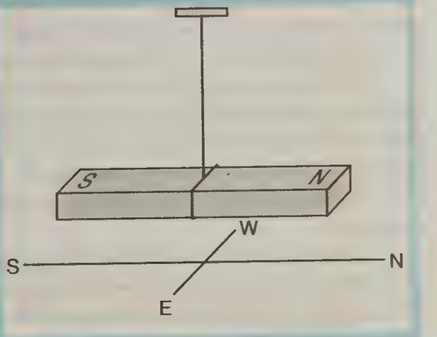


Fig. 3.01

3. ***Unlike poles attract and like poles repel.*** If the S-pole of a magnet is brought near the N-pole of another magnet, they are found to attract each other [Fig. 3.02 (*a*)].

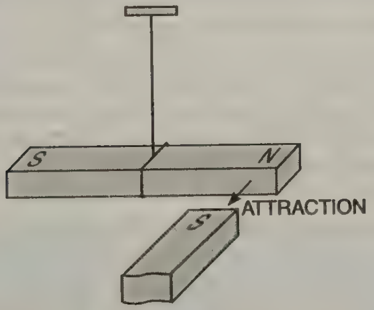


Fig. 3.02 (*a*)

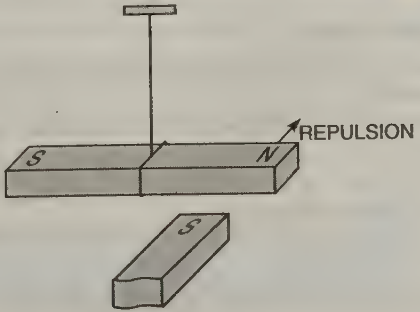


Fig. 3.02 (*b*)

On the other hand, if the N-pole of a magnet is brought near the N-pole of another magnet (or S-pole of a magnet is brought near the S-pole of another magnet), they are found to repel each other [Fig. 3.02 (*b*)]. Therefore, unlike poles attract each other and like poles repel each other.

Key point

An isolated magntic pole does not exist.

4. ***Magnetic poles exist in pairs.*** If a magnet is cut* into two equal parts transverse to its length as shown in Fig. 3.03, N and S-poles of the magnet donot get separated. Each of the two parts is found to be a magnet having north and south poles at the two ends as shown in figure. Further, each part of the magnet possesses the pole strength same as that of the original magnet.

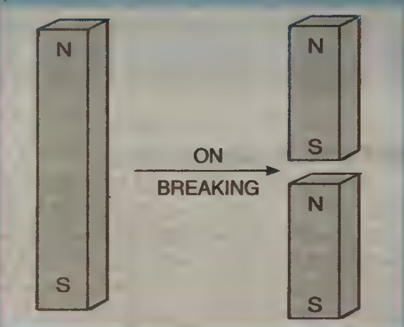


Fig. 3.03

On the other hand, if the magnet is cut into two equal parts along the length of the magnet, each part is found to be a magnet having pole strength exactly half of the pole strength of the original magnet.

Key point

When a magnet having pole strength *m* is cut in two equal parts

(*i*) longitudinally, the pole strength of each new magnet obtained is *m*/2.

(*ii*) transversely, the pole strength of each new magnet still remains *m*.

The important difference in electricity and magnetism is that in electricity, an isolated charge exists, whereas in magnetism, an isolated magnetic pole *i.e.* monopole does not exist.

5. ***Inductive property.*** When a piece of a substance, such as soft iron, steel, cobalt, nickel, etc is placed near a bar magnet, it acquires magnetism. The magnetism so acquired is called *induced magnetism*.

6. ***Repulsion is the surest test for distinguishing between a magnet and a piece of iron.*** The pole of a magnet attracts a piece of iron as well as the unlike pole of an other magnet. Therefore, when the magnet used for distinguishing between a magnet and a piece of iron is brought near the two ends of a specimen, it will show attraction if it is an iron rod ; but will show repulsion at one end, if it is a magnet. Thus, repulsion is the surest test to distinguish between a magnet and a piece of iron.

3.03. ATOMIC VIEW OF MAGNETISM

Ewing modified the molecular theory of magnetism given by Weber. Since a current loop behaves as a magnetic dipole, each atom of a magnetic substance can be considered as a magnetic dipole. It is because, electrons revolving around the nucleus in an atom are equivalent to small current loops. According to Ewing, the molecules of a magnetic substance, whether magnetised or unmagnetised, are all complete magnets. As shown in Fig. 3.04, in an unmagnetised magnetic substance, these molecular magnets (represented by arrows) are randomly oriented and form closed chains. The north pole of one molecular magnet attracts the south pole of its neighbouring molecular magnet, while its own south-pole is attracted by the north pole of another and so on. In this manner, the molecular magnets neutralize the effect of each other and the unmagnetised specimen exhibits no magnetism.

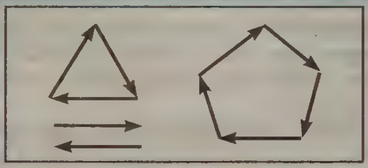


Fig. 3.04

In a bar magnet, the molecular magnets are aligned in the same direction *i.e.* the north-poles of all the molecular magnets point in one direction, while the south-poles in the opposite direction [Fig. 3.05]. Hence, one end behaves as a strong north-pole and other as a strong south-pole. Thus, the difference between a magnetised and unmagnetised substance lies only in the arrangement of these molecular magnets.

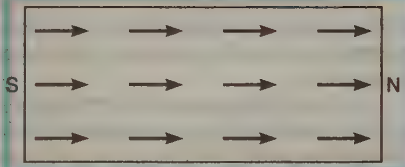


Fig. 3.05

It may be pointed out that Ampere had suggested that the elementary atomic magnets in the magnetic substances were tiny circulating current loops. In principle, this insight of Ampere has turned out to be substantially correct. Now, it is agreed that the atomic magnetism is of electrical origin. However, most part of the atomic magnetism is contributed by the spinning motion of electrons in the atoms, rather than their orbital motion.

3.04. COULOMB'S LAW IN MAGNETISM

In 1785, by making use of his torsion balance, Coulomb discovered the law governing force between two magnetic poles†.

Suppose that two magnetic poles of strengths m_1 and m_2 are placed a distance r apart. Then, force of attraction or repulsion between the two poles is found to be directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them *i.e.*

$$F \propto \frac{m_1 m_2}{r^2}$$

---

*For details, refer to VSQ 3.03.

†As isolated magnetic poles do not exist, two long magnets were used by Coulomb. It was assumed that the pole of one magnet does not experience any appreciable force due to the distant pole of the other magnet.

or $$F = k\,\frac{m_1\,m_2}{r^2}$$

In SI, the constant of proportionality, $k = \frac{\mu_0}{4\pi} = 10^{-7}$ Wb A^{-1} m^{-1} .

$$\therefore \qquad F = \frac{\mu_0}{4\pi}\cdot\frac{m_1\,m_2}{r^2} = 10^{-7}\times\frac{m_1\,m_2}{r^2} \qquad ...(3.01)$$

Here, μ_0 is called the ***absolute permeability*** of free space.

Unit of magnetic pole strength. Suppose that $m_1 = m_2 = m$ (say)

$r = 1$ m and $F = 10^{-7}$ N

From equation (3.01), we have

$$10^{-7} = 10^{-7}\times\frac{m\times m}{1^2} \qquad \text{or} \qquad m^2 = 1$$

or $m = \pm 1$ ampere metre (A m)

Therefore, *strength of a magnetic pole is said to be one ampere metre, if it repels an qual and similar pole with a force of* 10^{-7} *N, when placed in vacuum* (or air) *at a distance f one metre from it.*

3.05. MAGNETIC DIPOLE AND MAGNETIC DIPOLE MOMENT

A bar magnet has two magnetic poles. It is sometimes also called as magnetic ipole.

Magnetic dipole. *An arrangement of two magnetic poles of equal and opposite trengths separated by a finite distance is called a magnetic dipole.*

The two poles of a magnetic dipole (or a magnet), called north and south poles, re always of equal strength and of opposite nature. Further, such two magnetic oles always exist in pair. In other words, an isolated north or a south pole does ot exist.

Magnetic length. *The distance between the two poles of a bar magnet is called the agnetic length of the magnet.*

Magnetic length is a vector from S-pole of the magnet to its N-pole and is lenoted by $2\vec{l}$ [Fig. 3.06].

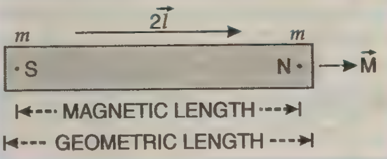


Fig. 3.06

Since the magnetic poles are situated within the magnet, the distance between ts two magnetic poles called its ***magnetic length*** is always less than its ***geometric ength.***

Magnetic dipole moment. *The product of the strength of either pole and the magnetic ength of the magnet is called magnetic dipole moment.*

It is a vector quantity and is represented by $\vec{M}$. Its direction is same as that of $2\vec{l}$ *.e.* from S-pole of the magnet to its N-pole. Thus,

$$\vec{M} = m\,(2\vec{l}) \qquad ...(3.02)$$

In SI, the unit of magnetic dipole moment is **ampere metre^2 (A m^2).**

Note. In place of bar magnet, the term 'magnetic dipole' is also often used for an arrangement of two equal and unlike magnetic poles separated by certain distance.

3.06. MAGNETIC FIELD

The space around a magnet (or a conductor carrying current), *in which its magnetic ffect can be experienced, is called the magnetic field.*

The magnetic field in a region is said to be uniform, if the magnitude of its strength and direction is same at all the points in that region.

A uniform magnetic field is represented by equidistant parallel arrows.

The SI unit of strength of magnetic field (also known as magnetic induction or magnetic induction field or magnetic flux density) is **tesla (T).** It can be measured in terms of force on a charge moving inside the magnetic field or by using the concept of magnetic flux linked with a surface.

1 tesla (T) = 1 newton ampere $^{-1}$ metre^{-1} (N A^{-1} m^{-1})

= 1 weber metre^{-2} (Wb m^{-2})

Note. 1. The strength of magnetic field is also measured in **gauss (G).**

1 gauss (G) = 10^{-4} tesla (T)

2. Usually, the strength of magnetic field B is simply called magnetic field.

3.07. MAGNETIC FIELD LINES

The magnetic field line is the path along which an isolated north pole will tend to move, if it is free to do so.

Since an isolated magnetic pole does not exist, a small magnetic needle is used to plot the field lines of a magnetic field. The magnetic field lines do not exist in reality. The concept of magnetic field lines was developed to visualize the strength of the magnetic field in different regions of the field. The regions, where the magnetic field lines are close enough (crowded), the magnetic field is considered as quite strong there.

Key point

The relative closeness of magnetic field lines in different regions of a magnetic field can be used to adjudge the field strength.

Properties of magnetic field lines. The following are a few important properties of magnetic field lines :

1. The magnetic field lines are closed continuous loops extending through the body of the magnet.
2. Outside the body of the magnet, the magnetic field lines run from north pole to south pole of the magnet.
3. The tangent to the magnetic field line at any point gives the direction of magnetic field at that point.
4. No two magnetic field lines can intersect each other.
5. The magnetic field lines dilate laterally and contract longitudinally.

It may be noted that the electric field of an electric dipole and magnetic field of a bar magnet are also similar. However, there is a fundamental difference between the two fields. In case of the electric field of an electric dipole, the electric lines of force originate from positive charge and end at the negative charge. Since isolated magnetic poles do not exist ; in case of a bar magnet, the magnetic field lines are closed loops. In other words, magnetic field lines do not start or end anywhere. One may imagine them to be extending through the body of the magnet. It is only at far away points that the two fields (electric field of an electric dipole and magnetic field of a bar magnet) look very similar.

Key point

In contrast to electric lines of force, magnetic field lines are always closed loops.

3.08. MAGNETIC FIELD ON AXIAL LINE OF A BAR MAGNET

Consider a bar magnet NS, whose each pole is of strength m. Let $2l$ be magnetic length of the magnet and O be its centre. Let P be a point on the axial line of the magnet at a distance r from the centre of the magnet [Fig. 3.07]

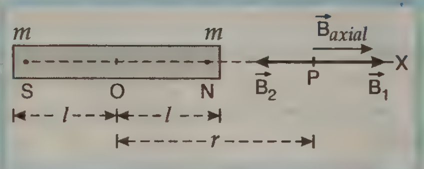


Fig. 3.07

The magnetic field $\vec{B}_{axial}$ at point P due to the bar magnet will be the resultant of the magnetic fields $\vec{B}_1$ (due to N-pole of the magnet) and $\vec{B}_2$ (due to S-pole of the magnet) *i.e.*

$$\vec{B}_{axial} = \vec{B}_1 + \vec{B}_2$$

Now, $$|\vec{B}_1| = \frac{\mu_0}{4\pi} \cdot \frac{m}{NP^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r-l)^2} \quad \text{(along PX)}$$

and $$|\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{m}{SP^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r+l)^2} \quad \text{(along PS)}$$

It follows that $|\vec{B}_1|$ is greater than $|\vec{B}_2|$.

Since $\vec{B}_1$ and $\vec{B}_2$ act along the same line but in opposite directions,

$$|\vec{B}_{axial}| = |\vec{B}_1| - |\vec{B}_2| \quad \text{(along PX)}$$

or $$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r-l)^2} - \frac{\mu_0}{4\pi} \cdot \frac{m}{(r+l)^2}$$

$$= \frac{\mu_0}{4\pi} \cdot m \left[\frac{(r+l)^2 - (r-l)^2}{(r^2 - l^2)^2}\right] = \frac{\mu_0}{4\pi} \cdot \frac{m(4rl)}{(r^2 - l^2)^2}$$

Now, $m(2l) = M$, magnitude of the magnetic dipole moment of the magnet.

$$\therefore \quad B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l^2)^2} \quad \text{(along PX)} \qquad \text{...(3.03)}$$

It may be noted that the direction of magnetic field due to a bar magnet at a point on its axial line is from S-pole to N-pole *i.e.* same as that of magnetic dipole moment of the bar magnet. Therefore, equation (3.03) may be expressed as

$$\vec{B}_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{M}r}{(r^2 - l^2)^2} \qquad \text{...(3.04)}$$

Key point

Magnetic field due to a bar magnet at a point on its axial line has the same direction as that of its magnetic dipole moment vector.

When bar magnet is of very small length. For a bar magnet of very small ngth,

$$l << r$$

d hence in equation (3.03), l^2 can be neglected in comparison to r^2. Therefore, for very short bar magnet,

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} \qquad \text{(along PX)} \qquad ...(3.05)$$

3.09. MAGNETIC FIELD ON EQUITORIAL LINE OF A BAR MAGNET

Consider a bar magnet NS, whose each pole is of strength m. Let $2\,l$ be the agnetic length of the bar magnet and O be its centre. Let P be a point on the quitorial line of the magnet at a distance r from the centre of the magnet [Fig. 3.08].

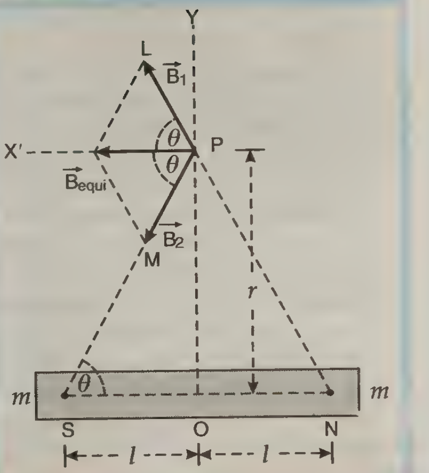


Fig. 3.08

Let $\vec{B}_1$ and $\vec{B}_2$ be the magnetic fields at point P due to N-pole and S-pole of the r magnet. Then, resultant magnetic field at point P due to the bar magnet is given

$$\vec{B}_{equi} = \vec{B}_1 + \vec{B}_2$$

Now, $$|\vec{B}_1| = \frac{\mu_0}{4\pi} \cdot \frac{m}{NP^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2+l^2)^2} \qquad \text{(along NP)}$$

and $$|\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{m}{SP^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2+l^2)^2} \qquad \text{(along PS)}$$

It follows that $|\vec{B}_1|$ is equal to $|\vec{B}_2|$. To find the resultant magnetic field at point due to the bar magnet, resolve $\vec{B}_1$ and $\vec{B}_2$ into rectangular components along PX' nd OY. If $\angle PSO = \theta$, then $\angle LPX' = \angle MPX' = \theta$. The components of $\vec{B}_1$ and $\vec{B}_2$ along Y are $|\vec{B}_1| \sin\theta$ and $|\vec{B}_2| \sin\theta$ respectively. As the two components are equal and pposite, they cancel each other. Therefore, resultant magnetic field at point P due the bar magnet,

$$|\vec{B}_{equi}| = |\vec{B}_1| \cos\theta + |\vec{B}_2| \cos\theta \qquad \text{(along PX')}$$

or $$B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2+l^2)^2} \cos\theta + \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2+l^2)^2} \cos\theta$$

or $$B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{(r^2+l^2)^2} \cos\theta$$

From right angled Δ SOP,

$$\cos\theta = \frac{SO}{SP} = \frac{l}{(r^2+l^2)^{1/2}}$$

Hence, $$B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{(r^2+l^2)} \cdot \frac{l}{(r^2+l^2)^{1/2}}$$

or $$B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2+l^2)^{3/2}} \qquad \text{(along PX')} \qquad ...(3.06)$$

Here, $M = 2\,m\,l$ is magnetic dipole moment of the bar magnet.

It may be noted that direction of magnetic field due to a bar magnet on its equitorial line is from N-pole to S-pole of the magnet *i.e.* in a direction opposite to hat of the magnetic dipole moment of the magnet. Therefore, equation (3.06) may e expressed as

$$\vec{B}_{equi} = -\frac{\mu_0}{4\pi} \cdot \frac{\vec{M}}{(r^2+l^2)^{3/2}} \qquad ...(3.07)$$

Key point

The direction of magnetic field at a point on its equitorial line is opposite to that of its magnetic dipole moment vector.

When bar magnet is of very small length. For a bar magnet of very small length,

$$l << r$$

and hence in equation (3.06), l^2 can be neglected in comparison to r^2. Therefore, for a very short bar magnet,

$$B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \qquad \text{(along PX')} \qquad ...(3.08)$$

Note. From equations (3.05) and (3.08), it follows that magnetic field at a point at a certain distance on the equitorial line of a short bar magnet is just one half of that at the same distance on its axial line.

3.10. TORQUE ON A BAR MAGNET IN A MAGNETIC FIELD

Let a bar magnet N S having pole strength m and of length $2\vec{l}$ be placed in a uniform magnetic field of strength B making an angle θ with the direction of the magnetic field [Fig. 3.09].

Force on N-pole of the magnet = m B (along the direction of magnetic field B)

Force on S-pole of the magnet = m B (opposite to the direction of magnetic field B)

Thus, the bar magnet is acted upon by two equal, parallel and opposite forces. The two forces constitute a torque and it tends to rotate the magnet in the clockwise direction. The magnitude of the torque is given by

$$\tau = \text{either force} \times \text{perpendicular distance between the two forces}$$
$$= mB \times KN$$

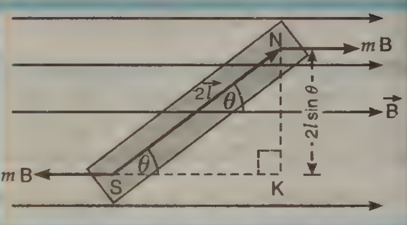


Fig. 3.09

From the right angled triangle N K S, we have

$$KN = NS\sin\theta = 2l\sin\theta$$

$$\therefore \quad \tau = mB \times 2l\sin\theta = m(2l)B\sin\theta$$

Since $m(2l) = M$, the magnetic dipole moment of the bar magnet, we have

$$\tau = MB\sin\theta \qquad ...(3.09)$$

If B = 1 and $\theta = 90°$ *i.e.* $\sin\theta = 1$, then

$$\tau = M \times 1 \times 1 \text{ or } M = \tau$$

Hence, *the magnetic dipole moment of a magnetic dipole is numerically equal to the torque acting on the dipole, when placed perpendicular to the direction of a uniform magnetic field of unit strength.*

In vector notation,

$$\vec{\tau} = \vec{M} \times \vec{B} \qquad ...(3.10)$$

If the vectors $\vec{M}$ and $\vec{B}$ are in the plane of paper [Fig. 3.10], then torque vector $\vec{\tau}$, which acts in the direction of $\vec{M} \times \vec{B}$ will be perpendicular to the plane of the paper and in inward direction.

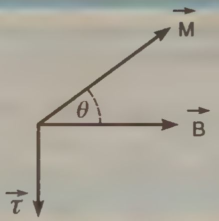


Fig. 3.10

Unit of magnetic dipole moment. From equation (3.09), we have

$$M = \frac{\tau}{B\sin\theta}$$

Therefore, unit of $M = \dfrac{\text{newton metre}}{\text{tesla}} = \text{N m T}^{-1} = \text{J T}^{-1}$

Therefore, magnetic dipole moment may be measured as **newton metre tesla^{-1} (N m T^{-1})** or as **joule tesla^{-1}** (J T^{-1})or as explained in section 1.05, it may be measured as **ampere meter^2** (Am^2). Since tesla is weber metre^{-2}, the magnetic dipole moment is dimensionally equivalent to **joule metre^2 weber^{-1} (J m^2 Wb^{-1}).**

3.11. POTENTIAL ENERGY OF A BAR MAGNET PLACED IN A MAGNETIC FIELD

Let a bar magnet of dipole moment M be placed in a uniform magnetic field of strength B, such that the magnet makes an angle θ with the direction of the field. Then, magnitude of the torque acting on the dipole is given by

$$\tau = MB\sin\theta$$

This torque tends to align the magnet along the direction of the field. If the magnet is rotated against the action of this torque, work has to be done. Suppose that the magnet is rotated through an infinitesimally small angle $d\theta$ under the action of the constant torque τ.

Then, small amount of work done in rotating the magnet through an infinitesimally small angle $d\theta$,

$$dW = \tau\, d\theta = MB\sin\theta\, d\theta$$

If the magnet is rotated from initial position $\theta = \theta_1$ to the final position $\theta = \theta_2$, then the total work done is given by

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = MB\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = MB\left|-\cos\theta\right|_{\theta_1}^{\theta_2}$$
$$= -MB(\cos\theta_2 - \cos\theta_1) = MB(\cos\theta_1 - \cos\theta_2)$$

The work done in rotating the magnet is stored inside the magnet as its potential rgy (U). Thus, potential energy of the magnet inside the magnetic field,

$$U = M B (\cos \theta_1 - \cos \theta_2) \qquad ...(3.11)$$

Suppose that the magnet is initially perpendicular to the direction of the gnetic field *i.e.* $\theta_1 = 90°$. Then, potential energy of the magnet in any position king angle θ with the direction of the field can be obtained by setting $\theta_1 = 90°$ $\theta_2 = \theta$ in equation (3.11) *i.e.*

$$U = M B (\cos 90° - \cos \theta)$$

or

$$U = -M B \cos \theta \qquad ...(3.12)$$

In vector notation, $U = -\vec{M} \cdot \vec{B}$ ...(3.13)

Special cases : Let us find the potential energy of the dipole in following cases :

(i) When $\theta = 0°$, $U = -M B \cos 0° = -M B$ (minimum)

(ii) When $\theta = 90°$, $U = -M B \cos 90° = 0$

iii) When $\theta = 180°$, $U = -M B \cos 180° = M B$ (maximum)

Thus, a magnetic dipole possesses minimum potential energy, when $\vec{M}$ and $\vec{B}$ like parallel and maximum potential energy, when $\vec{M}$ and $\vec{B}$ are antiparallel.

Key point

The energy possessed by a magnetic dipole placed in a magnetic field is minimum, when it is placed along the direction of the field and maximum, when placed anti-parallel to the field.

3.12. CURRENT LOOP AS A MAGNETIC DIPOLE

Consider a circular coil carrying current I. Suppose that the current flows in i-clockwise direction, when seen from above [Fig. 3.11]. The magnetic field lines e to each elementary portion of the circular coil will be of the shape of circular ps near that portion and almost straight near the centre of the circular coil. From t-Savart's law (or right hand thumb rule), it follows that the magnetic field lines m to enter at the lower face of the coil and leave at its upper face. Thus, the lower e of the coil, through which magnetic field lines enter, behaves as the south pole ; ile the upper face, through which the field lines leave, behaves as the north pole. nce, the current carrying circular coil behaves as a magnetic dipole.

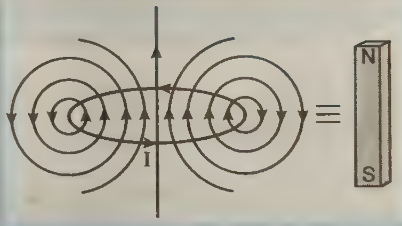


Fig. 3.11

It is found that magnetic dipole moment of the current loop

(i) *is directly proportional to strength of current through the loop i.e.*

$$M \propto I \qquad ...(3.14)$$

(ii) *is directly proportional to area of the current loop i.e.*

$$M \propto A \qquad ...(3.15)$$

Combining equations (3.14) and (3.15), we have

$$M \propto I A$$

or

$$M = k I A,$$

ere k is the constant of proportionality. The constant of proportionality k is found e equal to 1, when current is measured in ampere, area in metre^2 and magnetic ole moment in ampere metre^2. Thus,

$$M = I A \qquad ...(3.16)$$

Since the upper face of the current loop behaves as north pole and the lower e as south pole, the magnetic dipole moment vector $\vec{M}$ acts perpendicular to the ne of the current loop and in upward direction as shown in Fig. 3.12. If $\vec{A}$ is area ctor representing the area of the current loop, then equation (3.16) may be pressed as

$$\vec{M} = I \vec{A} \qquad ...(3.17)$$

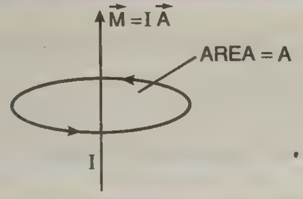


Fig. 3.12

Key point

A planar current loop (of any shape) behaves as a magnetic dipole.

In case the current carrying circular coil has n turns, then

$$M = n I A \qquad ...(3.18)$$

The factor n I is called ***ampere turns*** of the circular current loop. Thus, *magnetic dipole moment of current loop = ampere turns × area of the coil*

The equation (3.18) has been obtained for a circular current loop. In fact, it is e for planar current loop of any shape.

3.13. ATOM AS A MAGNETIC DIPOLE (Magnetic dipole moment of a revolving electron)

In an atom, electrons revolve around the nucleus and as such the circular orbits electrons may be considered as the small current loops. Due to this, an atom ssesses magnetic dipole moment and hence behaves as a magnetic dipole.

Expression for magnetic moment. Let us calculate the magnetic dipole moment an atom due to orbital motion of the electron. Suppose that an electron of mass

m_e and charge e revolves in a circular orbit of radius r around the positive nucleus in anticlockwise direction, as seen from above [Fig. 3.13]. The angular momentum of the electron due to its orbital motion is given by

$$L = m_e\, v\, r \qquad \text{...(3.19)}$$

For the sense of orbital motion of electron shown in the figure, the angular momentum vector $\vec{L}$ acts along normal to the plane of the electron orbit and in upward direction.

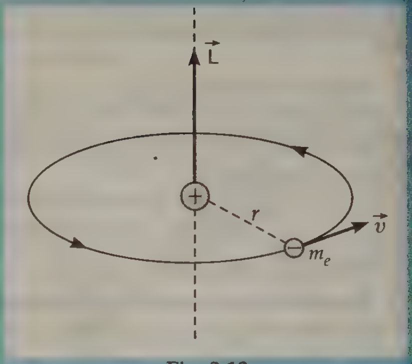


Fig. 3.13

Suppose that the period of orbital motion of the electron is T. Then, the electron crosses any point on its orbit after every T seconds or $\frac{1}{T}$ times in one second. Therefore, orbital motion of electron is equivalent to a current,

$$I = e\left(\frac{1}{T}\right)$$

Now, the period of revolution of the electron is given by

$$T = \frac{2\pi r}{v}$$

Therefore, $$I = e\left(\frac{1}{2\pi r/v}\right) = \frac{e\,v}{2\pi r}$$

The area of the electron orbit (area of current loop), $A = \pi r^2$

Therefore, the magnetic dipole moment of the atom,

$$M = \frac{e\,v}{2\pi r} \times \pi r^2 = \frac{e\,v\,r}{2}$$

Using equation (3.19), we have

$$M = \left(\frac{e}{2\,m_e}\right) L \qquad \text{...(3.20)}$$

In vector notation,

$$\vec{M} = -\left(\frac{e}{2\,m_e}\right)\vec{L} \qquad \text{...(3.21)}$$

The negative sign has been included for the reason that electron has negative charge. The equation (3.21) tells that the magnetic dipole moment vector is directed in a direction opposite to that of angular momentum vector.

According to Bohr's theory, angular momentum of electron in a stationary orbit can have only those values, which are integral multiples of $\frac{h}{2\pi}$ *i.e.*

$$L = n\frac{h}{2\pi},\ \text{where } n = 1, 2, 3... \qquad \text{...(3.22)}$$

From equations (3.20) and (3.22), we have

$$M = \left(\frac{e}{2\,m_e}\right)\frac{n\,h}{2\pi}$$

or $$M = n\left(\frac{e\,h}{4\pi\,m_e}\right) \qquad \text{...(3.23)}$$

The equation (3.23) gives the magnetic moment of an electron due to its orbital motion.

Bohr magneton. *It is defined as the magnetic dipole moment associated with an atom due to orbital motion of an electron in the first orbit of hydrogen atom.*

From equation (3.23), it follows that $\frac{e\,h}{4\pi\,m_e}$ is the least value of the magnetic moment. This natural unit of magnetic moment is called ***Bohr magneton***. It is denoted by μ_B. Its value is

$$\mu_B = \frac{(1{\cdot}6\times10^{-19}\ \text{C})\times(6{\cdot}62\times10^{-34}\ \text{J s})}{4\pi\times(9{\cdot}1\times10^{-31}\ \text{kg})} = 9{\cdot}27\times10^{-24}\ \text{A m}^2$$

We know that in an atom, in addition to orbital motion, an electron has got spin motion also. The electron possesses magnetic moment due to its spin motion also.

e total magnetic moment of the electron is the vector sum of its magnetic moments e to orbital and spin motion.

3.14. BAR MAGNET AS AN EQUIVALENT SOLENOID

Consider a straight solenoid of length L, area of cross-section A and carrying rrent I as shown in Fig. 3.14 (*a*). Each turn of the solenoid behaves as a small agnetic dipole having magnetic dipole moment I A. Therefore, a solenoid can be eated as the arrangement of small magnetic dipoles placed in line with each other shown in Fig. 3.14 (*b*). The number of such small magnetic dipoles is equal to the umber of turns in the solenoid. The south and north poles of these small magnetic poles neutralise each other except at the ends. Thus, a current carrying solenoid n be replaced by just a single south pole and a single north pole separated by a stance equal to the length of the solenoid [Fig. 3.14 (*c*)]. Since each turn of the lenoid behaves a s a small magnetic dipole having dipole moment I A, the uivalent magnetic dipole moment of the solenoid having n turns is n I A.

The magnetic field of a bar magnet or a current carrying solenoid can be plotted using a compass needle. Fig. 3.15 shows the magnetic field produced by a bar agnet, while Fig. 3.16 shows the magnetic field produced by a current carrying lenoid. It follows that the magnetic field produced by a bar magnet is identical that produced by a current carrying solenoid.

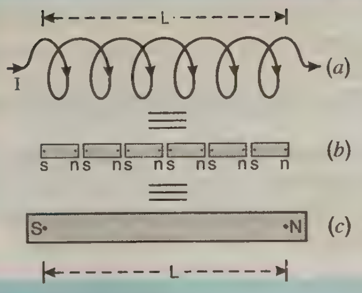


Fig. 3.14

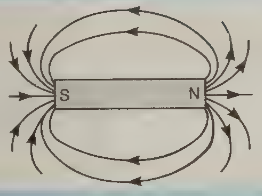


Fig. 3.15

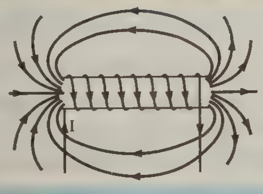


Fig. 3.16

3.15. GAUSS'S LAW IN MAGNETISM*

According to Gauss's law in electrostatic, the surface integral of electric eld $\vec{E}$ over a closed surface S is equal to $\frac{1}{\varepsilon_0}$ times the total charge q enclosed by the rface *i.e.*

$$\oint_S \vec{E}\,.\,d\vec{S} = \frac{q}{\varepsilon_0}$$

An isolated magnetic pole does not exist. In other words, a surface may enclose magnetic dipole *i.e.* a pair of equal and opposite magnetic poles so that net pole rength enclosed by the surface is zero. Therefore, the magnetic analogue of Gauss's w in electrostatic may be stated as

$$\oint_S \vec{B}\,.\,d\vec{S} = 0$$

. surface integral of magnetic field $\vec{B}$ over a closed surface S is always zero. It is called ***auss's law in magnetism.***

Following are a few direct consequences of Gauss's law in magnetism :

1. If a number of magnetic field lines are leaving a closed surface, an equal umber of field lines must also be entering the surface.

2. Isolated magnetic poles do not exist *i.e.* magnetic poles exist in unlike pairs f equal strengths.

Key point

The fact that a magnetic monopole does not exist is a direct consequence of Gauss's law in magnetism.

Key point

Due to the fact that magnetic poles exist in pairs of equal and opposite strengths or that a magnetic monopole does not exist, magnetic flux through a closed surface must always be zero.

SOLVED NUMERICAL PROBLEMS

ype A. On Force between Magnetic poles

Problem 3.01. Two equal magnetic poles placed 5 cm air attract each other with a force of 14·4 × 10^{-4} N. How r from each other should they be placed so that the force f attraction will be 1·6 × 10^{-4} N ?

Sol. Let m be the pole strength of each pole.

In first case : F = 14·4 × 10^{-4} N ; r = 5 cm = 0·05 m

Now, $F = \frac{\mu_0}{4\pi}\cdot\frac{m \times m}{r^2}$ or $14 \cdot 4 \times 10^{-4} = 10^{-7} \times \frac{m^2}{(0 \cdot 05)^2}$

or $m = \pm 6$ A m

In second case : Let r' be the distance between the poles, hen force of attraction becomes F′ = 1·6 × 10^{-4} N.

Now, $F' = \frac{\mu_0}{4\pi}\cdot\frac{m \times m}{r'^2}$ or $1 \cdot 6 \times 10^{-4} = 10^{-7} \times \frac{6 \times 6}{r'^2}$

or r' = **0·15 m**

Problem. 3.02. Two poles, one of which is 5 times as strong as the other, exert on each other a force equal to 0·8 × 10^{-3} kgf, when placed 10 cm apart in air. Find the strength of each pole.

Sol. Let m and 5 m be pole-strengths of the two poles.

Here, F = 0·8 × 10^{-3} kgf = 0·8 × 10^{-3} × 9·8 N,

r = 10 cm = 0·1 m

Now, $F = \frac{\mu_0}{4\pi}\cdot\frac{m_1\, m_2}{r^2}$

or $0 \cdot 8 \times 10^{-3} \times 9 \cdot 8 = \frac{10^{-7} \times m \times 5\,m}{(0 \cdot 1)^2}$

or m = **12·52 A m**

and 5 m = 5 × 12·52 = **62·6 A m**

---

*For additional knowledge only.

Type B. On Magnetic moment

Problem 3.03. Two identical thin bar magnets, each of length L and pole-strength *m* are placed at right angles to each other, with the N-pole of one touching the south pole of the other. Find the magnetic moment of the system.

(I.I.T. 1967)

Sol. When the magnets $N_1 S_1$ and $N_2 S_2$, each of pole strength *m* and length L, are placed at right angles to each other [Fig. 3.17], the system behaves as a magnet, whose pole strength is *m* and length is $N_1 S_2$.

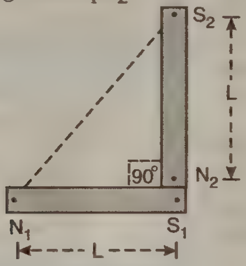


Fig. 3.17

Now, $N_1 S_2 = \sqrt{N_1 S_1^2 + N_2 S_2^2} = \sqrt{L^2 + L^2} = \sqrt{2}\,L$

Therefore, magnetic moment of the system,

$$M = \text{pole strength} \times N_1 S_2 = m \times \sqrt{2}\,L = \sqrt{2}\,m\,L$$

Type C. On Magnetic field due to a bar magnet

Problem 3.04. A bar magnet of length 0 · 1 m has a pole strength of 50 A m. Calculate the magnetic field at distance of 0 · 2 m from its centre on (*i*) its axial line and (*ii*) its equitorial line.

Sol. Here, $m = 50$ A m ; $r = 0{\cdot}2$ m ; $2l = 0{\cdot}1$ m or $l = 0{\cdot}05$ m

Therefore, $M = m\,(2\,l) = 50 \times 0{\cdot}1 = 5$ A m^2

$$(i)\ B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2\,M\,r}{(r^2 - l^2)^2}$$

$$= \frac{10^{-7} \times 2 \times 5 \times 0{\cdot}2}{(0{\cdot}2^2 - 0{\cdot}05^2)^2} = \frac{10^{-7} \times 2 \times 5 \times 0{\cdot}2}{(0{\cdot}0375)^2}$$

$$= 1 \cdot 42 \times 10^{-4}\ T$$

$$(ii)\ B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}}$$

$$= \frac{10^{-7} \times 5}{(0{\cdot}2^2 + 0{\cdot}05^2)^{3/2}} = \frac{10^{-7} \times 5}{(0{\cdot}0425)^{3/2}} = \frac{5 \times 10^{-7}}{0{\cdot}00876}$$

$$= 5 \cdot 708 \times 10^{-5}\ T$$

Problem 3.05. Two identical short bar magnets each of magnetic moment 12·5 A m^2 are placed at a separation of 10 cm between their centres, such that their axes are perpendicular to each other. Find the magnetic field at a point midway between the two magnets.

Sol. Suppose that the two identical magnets $N_1 S_1$ and $N_2 S_2$ are placed as shown in Fig. 3.18. Let P be the point midway between the two magnets.

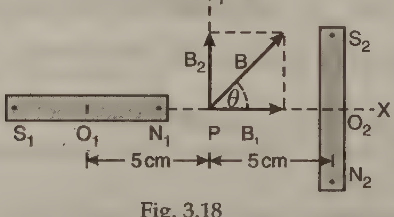


Fig. 3.18

Here, $M_1 = M_2 = 12{\cdot}5$ A m^2

and $O_1P = O_2P = 5$ cm $= 0{\cdot}05$ m

The point P lies on axial line of magnet $N_1 S_1$. If B_1 magnetic field due to magnet $N_1 S_1$ at point P, then

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\,M_1}{(O_1\,P)^3} = \frac{10^{-7} \times 2 \times 12{\cdot}5}{(0{\cdot}05)^3}$$

$= 0{\cdot}02$ T (along P

The point P lies on equitorial line of magnet $N_2 S_2$. If is magnetic field due to magnet $N_2 S_2$ at point P, then

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M_2}{(O_2\,P)^3} = \frac{10^{-7} \times 12{\cdot}5}{(0{\cdot}05)^3}$$

$= 0{\cdot}01$ T (along P

The resultant magnetic field at point P,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(0{\cdot}02)^2 + (0{\cdot}01)^2}$$

$$= \mathbf{2{\cdot}237 \times 10^{-2}\ T}$$

If θ is the angle which the resultant field makes wi X-axis, then

$$\tan\theta = \frac{B_2}{B_1} = \frac{0{\cdot}01}{0{\cdot}02} = 0{\cdot}5 \text{ or } \theta = \mathbf{26{\cdot}57°}$$

Type D. On Torque on a Bar magnet

Problem 3.06. A bar magnet placed in a unifor magnetic field of strength 0·3 T, with its axis at 30° to th field, experiences a torque of 0·06 N m. What is the magnet mo-ment of the bar magnet ? *(I.S.C.E. 199*

Sol. Here, $B = 0{\cdot}3$ T; $\theta = 30°$; $\tau = 0{\cdot}06$ N m

Now, $\tau = M\,B\sin\theta$

$$\therefore \quad M = \frac{\tau}{B\sin\theta} = \frac{0{\cdot}06}{0{\cdot}3 \times \sin 30°} = \mathbf{0{\cdot}4\ A\ m^2}$$

Problem 3.07. A bar magnet has poles of strength 48 A which are 25 cm apart. (*i*) What is the magnetic moment the magnet ? (*ii*) What torque is required to hold th magnet at an angle 30° with a uniform field of flux densi 0·15 T ?

Sol. Here, $m = 48$ A m ; $2\,l = 25$ cm $= 0{\cdot}25$ m

(*i*) Magnetic moment, $M = m \times 2\,l = 48 \times 0{\cdot}25 =$ **12 A n**

(*ii*) Here, $\theta = 30°$; $B = 0{\cdot}15$ T

Now, $\tau = M\,B\sin\theta = 12 \times 0{\cdot}15 \sin 30° =$ **0·9 N m**

Problem 3.08. A short bar magnet placed with its ax at 30° to a uniform magnetic field of 0·2 T experiences torque of 0·06 N m. (*a*) Calculate the magnetic moment the magnet and (*b*) find out what orientation of the magn corresponds to the stable equilibrium in the magnetic fiel

(C.B.S.E. 200

Sol. Here, $B = 0{\cdot}2$ T ; $\theta = 30°$ and $\tau = 0{\cdot}06$ N m

(*a*) Now, $\tau = M\,B\sin\theta$

$$\therefore \quad M = \frac{\tau}{B\sin\theta} = \frac{0{\cdot}06}{0{\cdot}2 \times \sin 30°} = \mathbf{0{\cdot}6\ A\ m^2}$$

(*b*) The magnet will be in stable equilibrium in th magnetic field, if it experiences no torque due to magnet field *i.e.*

$M\,B\sin\theta = 0$ or $\sin\theta = 0$

or $\theta = 0°$

i.e. when the magnet aligns itself parallel to the magnet field.

Type E. On Potential energy of a Bar magnet

Problem 3.09. A bar magnet of moment 1·5 J T^{-1} li aligned with the direction of a uniform magnetic field

22 T. What is the amount of work required to turn the agnet, so as to align its magnetic moment normal to the eld direction ? What is the torque on the magnet now ? *(Pre-degree Kerala, 1991 ; Text Problem)*

Sol. Here, $M = 1{\cdot}5\ J\ T^{-1}$; $B = 0{\cdot}22\ T$; $\theta_1 = 0°$; $\theta_2 = 90°$

Now, work done to rotate magnet from position θ_1 to , is given by

$$W = M\,B\,(\cos\theta_1 - \cos\theta_2)$$

$$\therefore \quad W = 1{\cdot}5 \times 0{\cdot}22\,(\cos 0° - \cos 90°)$$

$$= 1{\cdot}5 \times 0{\cdot}22\,(1 - 0) = 1{\cdot}5 \times 0{\cdot}22 = \mathbf{0{\cdot}33\ J}$$

Now, $\tau = M\,B \sin\theta$

Therefore, torque on the magnet, when it is normal to e field,

$$\tau = 1{\cdot}5 \times 0{\cdot}22 \sin 90° = 1{\cdot}5 \times 0{\cdot}22 \times 1$$

$$= \mathbf{0{\cdot}33\ N\ m}$$

ype F. On Magnetic moment of a current loop

Problem 3.10. A circular coil of 50 turns and radius 2 m carries a current of 12 A. Find (*a*) magnetic field at e centre of the coil and (*b*) magnetic moment associated ith it.

Sol. Here, $n = 50$; $a = 0{\cdot}2$ m and $I = 12$ A

(*a*) Now,
$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} = \frac{10^{-7} \times 2\pi \times 50 \times 12}{0{\cdot}2}$$

$$= 1{\cdot}885 \times 10^{-3}\ T$$

(*b*) Magnetic moment associated with the current carrying coil,

$$M = n\,I\,A = n\,I \times \pi a^2$$

$$= 50 \times 12 \times \pi \times (0{\cdot}2)^2 = \mathbf{75{\cdot}4\ A\ m^2}$$

Type G. On Magnetic moment of revolving electron

Problem 3.11. An electron in an atom revolves around the nucleus in an orbit of radius 0·53 Å. Calculate the equivalent magnetic moment, if the frequency of revolution of electron is $6{\cdot}8 \times 10^9$ MHz. *(C.B.S.E. 1998)*

Sol. Here, radius of the orbit of electron,

$$r = 0{\cdot}53\ Å = 0{\cdot}53 \times 10^{-10}\ m$$

Frequency of revolution of electron,

$$\nu = 6{\cdot}8 \times 10^9\ MHz = 6{\cdot}8 \times 10^{15}\ Hz$$

If *e* is the charge on electron and T is the time taken to complete one revolution, then the motion of electron equivalent to the current I given by

$$I = \frac{e}{T} = e\,\nu \qquad \left(\because \text{frequency} = \frac{1}{\text{time period}}\right)$$

The electron revolving in circular orbit behaves as a current loop and hence possesses magnetic moment given by

$$M = I\,A = e\,\nu \times \pi r^2 = \pi\,e\,\nu\,r^2$$

Since $e = 1{\cdot}6 \times 10^{-19}$ C,

$$M = \pi \times 1{\cdot}6 \times 10^{-19} \times 6{\cdot}8 \times 10^{15} \times (0{\cdot}53 \times 10^{-10})^2$$

$$= \mathbf{9{\cdot}6 \times 10^{-24}\ A\ m^2}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & Curious Students

Problem 3.12. Each atom of an iron bar (having dimensions 5 cm × 1 cm × 1 cm) has a magnetic dipole oment of $1{\cdot}8 \times 10^{-23}$ Am². (*a*) What will be the magnetic oment of bar in the state of magnetic saturation ? (*b*) What ill be torque required to place this magnetised bar erpendicular to magnetic field of 15,000 G ? Given, density f iron = $7{\cdot}8 \times 10^3$ kg m^{-3}, atomic weight of iron = 56 and vogadro number, N = $6{\cdot}02 \times 10^{26}$ kg mol^{-1}.

Sol. Here, magnetic moment of one atom of iron,

$$M' = 1{\cdot}8 \times 10^{-23}\ A\ m^2$$

Volume of iron bar, $V = 5 \times 1 \times 1 = 5\ cm^3 = 5 \times 10^{-6}\ m^3$

Mass of iron bar, $m = V\rho = 5 \times 10^{-6} \times 7{\cdot}8 \times 10^3$

$$= 3{\cdot}9 \times 10^{-2}\ kg$$

Therefore, number of atoms in the iron bar,

$$n = \frac{N}{A} \times m = \frac{6{\cdot}02 \times 10^{26}}{56} \times 3{\cdot}9 \times 10^{-2}$$

$$= 4{\cdot}19 \times 10^{23}$$

(*a*) In the state of magnetic saturation, all the atomic agnets will align themselves in the same direction and then, e magnetic moment of iron bar will become

$$M = M' \times n = 1{\cdot}8 \times 10^{-23} \times 4{\cdot}19 \times 10^{23}$$

$$= 7{\cdot}54\ A\ m^2$$

(*b*) Here, $B = 15{,}000\ G = 1{\cdot}5\ T$ and $\theta = 90°$

The required torque,

$$\tau = M\,B \sin\theta = 7{\cdot}54 \times 1{\cdot}5 \sin 90°$$

$$= 11{\cdot}31\ N\ m$$

Problem 3.13. A magnetic dipole is under the influence of two magnetic fields B_1 and B_2 [Fig. 3.19]. The angle etween the field directions is 60° and one of the fields has magnitude of $1{\cdot}2 \times 10^{-2}$ T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field ? *(Text Problem)*

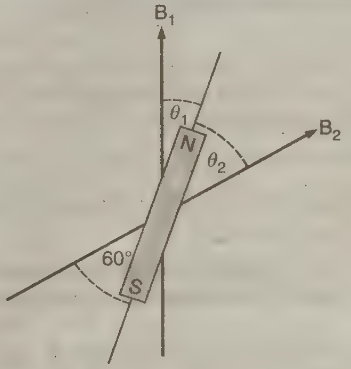


Fig. 3.19

Sol. Here, $B_1 = 1{\cdot}2 \times 10^{-2}$ T ;

Inclination of dipole with B_1, $\theta_1 = 15°$

Therefore, inclination of dipole with B_2,

$$\theta_2 = 60° - 15° = 45°$$

As the dipole is in equilibrium ; the torque on the dipole due to the field B_1 is equal and opposite to that due to B_2. If M is magnetic dipole moment of the dipole, then

$$M\,B_1 \sin\theta_1 = M\,B_2 \sin\theta_2$$

or
$$B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2} = \frac{1{\cdot}2 \times 10^{-2} \sin 15°}{\sin 45°}$$

$$= \frac{1{\cdot}2 \times 10^{-2} \times 0{\cdot}2588}{0{\cdot}707} = \mathbf{4{\cdot}39 \times 10^{-3}\ T}$$

Problem 3.14. A closely wound solenoid of 1000 turns and area of cross-section $2{\cdot}0 \times 10^{-4}$ m² carries a current

of 2·0 A. It is placed with its horizontal axis at 30°with the direction of a uniform horizontal magnetic field of 0·16 T as shown in Fig. 3.20.

(*a*) What is the torque experienced by the solenoid due to the field ?

(*b*) If the solenoid is free to turn about the vertical direction, specify its orientations of stable and unstable equilibrium. What is the amount of work needed to displace the solenoid from its stable orientation to its unstable orientation ? (*Text Problem*)

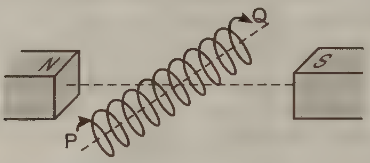


Fig. 3.20

Sol. Here, number of turns = 1000 ;

$$I = 2{\cdot}0\ \text{A} ; A = 2{\cdot}0 \times 10^{-4}\ \text{m}^2 ;$$

The magnetic moment of the solenoid,

$$M = (I\,A) \times \text{number of turns}$$
$$= 2{\cdot}0 \times 2{\cdot}0 \times 10^{-4} \times 1000$$
$$= 0{\cdot}4\ \text{A m}^2$$

(*a*) Now, $\tau = M\,B \sin\theta$

Here, $B = 0{\cdot}16$ T, $\theta = 30°$

$\therefore$ $\tau = 0{\cdot}4 \times 0{\cdot}16 \times \sin 30° = \mathbf{0{\cdot}032\ N\ m}$

(*b*) The solenoid will be in ***stable equilibrium***, when $\vec{M}$ *i* *parallel* $\vec{B}$ *(θ = 0°) and it will be in* ***unstable equilibrium***, *whe* $\vec{M}$ *is antiparallel* $\vec{B}$ (θ = 180°).

Now, $W = M\,B\,(\cos\theta_1 - \cos\theta_2)$

Therefore, work done to displace the solenoid fro stable orientation to unstable orientation,

$$W = M\,B\,(\cos 0° - \cos 180°) = 2\,M\,B = 2 \times 0{\cdot}4 \times 0{\cdot}1$$
$$= \mathbf{0{\cdot}128\ J}$$

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. What is a magnet ? (*P.S.S.C.E. 2002*)

Ans. A magnet is an arrangement of two equal and opposite magnetic poles separated by certain distance. The attractive and directive are two main properties of a magnet.

Q. 3.02. What do you mean by directive property of a magnetic dipole ?

Ans. A freely suspended magnet always aligns itself along the N-S line.

Q. 3.03 What happens to dipole moment, if a bar magnet is cut into two equal pieces parallel to its length ? (*H.P.S.S.C.E. 2002*)

Ans. Let m and $2\,l$ be pole strength and length of the given bar magnet. When the magnet is cut into two equal pieces parallel to its length, each piece will be a magnet having length $2\,l$ (unchanged) and pole strength $\frac{m}{2}$. Therefore, magnetic moment of each piece will be $\frac{m}{2}(2\,l) = m\,l$ *i.e.* one half of that of original magnet.

Q. 3.04. A bar magnet is cut into two equal pieces transverse to its length. What happens to its dipole moment ? (*H.S.S.C.E. 2002*)

Ans. Let m and $2\,l$ be pole strength and length of the given bar magnet. When the magnet is cut into two equal pieces transverse to its length, each piece will be a magnet having pole strength m (unchanged) and length l. Therefore, magnetic moment of each piece will be $m\,l$ *i.e.* one half of that of original magnet.

Q. 3.05. Why ordinarily a piece of iron does not behave as a magnet ?

Ans. In an ordinary piece of iron, the molecular magnets are randomly oriented and form closed chains. Since the molecular magnets cancel the effect of each other, the ordinary iron piece does not behave as a magnet.

Q. 3.06. What is the source of magnetic field (magnetism) ? (*P.S.S.C.E. 1999 S*)

Ans. Magnetism is of electrical origin. The electrons revolving in an atom behave as tiny current loops and these current loops give rise to magnetism.

Q. 3.07. Does an isolate magnetic pole exist like an isolate electric charge ?

Ans. No, an isolate magnetic pole does not exist.

Q. 3.08. What is the unit of magnetic pole strength ?

Ans. Unit of magnetic pole strength : ampere metre (Am

Q. 3.09. Define the term magnetic dipole moment. (*H.S.S.C.E. 2002 ; C.B.S.E. 1995*

Ans. Refer to section 3.05.

Q. 3.10. What is the unit of magnetic dipole moment

Ans. Unit of magnetic dipole moment : ampere metre (A m^2).

Q. 3.11. What is the direction of magnetic dipol moment ? (*H.S.S.C.E. 2002*

Ans. It is from S-pole to N-pole of the magnetic dipole

Q. 3.12. What is the basic difference between magneti lines of force and electric lines of force ? (*H.S.S.C.E. 1994*

Ans. The electric lines of force originate from positiv charge and end at negative charge and are thus discontinuou curves. But as isolated magnetic poles do not exist, th magnetic field lines are closed loops.

Q.3.13. Why two magnetic lines of force donot cros each other ? (*P.S.S.C.E. 1999*

Ans. Two magnetic field lines can not intersect eac other. It is because, if they do so, then at the point o intersection, the magnetic field will have two directions alon the tangents to the two field lines.

Q. 3.14. What is the torque experienced by a magneti dipole of magnetic moment M placed with its axis at a angle ϑ with a uniform external magnetic field B ? (*I.S.C.E. 1993*

Ans. Torque on the dipole, $\tau = M\,B \sin\theta$

Q. 3.15. A short bar magnet placed with its axis makin an angle θ with a uniform external field B experiences torque. What is the magnetic moment of the magnet ? (*C.B.S.E. 1990*

Ans. Torque on a dipole, $\tau = M\,B \sin\theta$

$$\therefore \qquad M = \frac{\tau}{B \sin\theta}$$

Q. 3.16. Does a bar magnet exert a torque on itself due o its own field ? Does one element of a current-carrying vire exert a force on another element of the same wire ? *(Text Question)*

Ans. No, a bar magnet does not exert a force or torque n itself due to its own field.

But an element of a current carrying conductor xperiences force due to another element of the conductor.

Q. 3.17. A bar magnet is stationary in magnetic meriian. Another similar magnet is kept parallel to it, such that he centres lie on their perpendicular bisectors. If the econd magnet is free to move, then what type of motion t will have : translatory, rotatory or both ? *(I.I.T. 1977)*

Ans. There will be only translatory motion. It can be hown by finding the net forces on the two poles of the second nagnet due to the poles of the first by drawing a vector liagram.

Q. 3.18. When does a magnetic dipole possess maxinum potential energy inside a magnetic field ?

Ans. A magnetic dipole possesses maximum potential nergy, when its magnetic moment $\vec{M}$ and magnetic ield $\vec{B}$ are antiparallel.

Q. 3.19. Can a current loop be treated as magnetic lipole ?

Ans. A current loop can be treated as a magnetic dipole. f the current loop has area A and carries current I, then its magnetic dipole moment is given by

$$M = I A$$

Q. 3.20. Compare the magnetic fields due to a straight solenoid and a bar magnet. *(P.S.S.C.E. 1996 S, 1994)*

Ans. The magnetic field of a bar magnet and a straight solenoid are identical. The two ends of the straight solenoid behave as the north and south poles as in case of a bar magnet.

Q. 3.21. Define Bohr magneton and write its value. *(P.S.S.C.E. 1999 S)*

Ans. *Bohr magneton is defined as the magnetic dipole moment associated with an atom due to orbital motion of an electron in the first orbit of hydrogen atom.*

Bohr magneton, $\mu_B = 9{\cdot}27 \times 10^{-24}$ A m^2

Q. 3.22. What is the magnetic moment of an electron due to its orbital motion ?

Ans. Magnetic moment of electron due to its orbital motion,

$$M = n\left(\frac{e\,h}{4\,\pi\,m_e}\right),$$

where the letters have their usual meanings.

Q. 3.23. Find out the value of 1 Bohr magneton. Given $h = 6{\cdot}62 \times 10^{-34}$ J s, $e = 1{\cdot}6 \times 10^{-19}$ C and $m_e = 9{\cdot}1 \times 10^{-31}$ kg. *(H.S.S.C.E. 2001)*

Ans. 1 Bohr magneton

$$= \frac{e\,h}{4\,\pi\,m_e} = \frac{1{\cdot}6\times10^{-19}\times6{\cdot}62\times10^{-34}}{4\,\pi\times9{\cdot}1\times10^{-31}}$$

$$= 9{\cdot}27 \times 10^{-24} \text{ A m}^2$$

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. What happens, if a bar magnet is cut into wo equal pieces *(i)* transverse to its length *(ii)* along its ength ? *(Text Question)*

Ans. In both the cases, one will get two magnets. If the nagnet has a pole strength m, length $2\,l$ and magnetic noment M, then pole strength, length and magnetic moment n case *(i)* will be m, l and $\frac{M}{2}$; while in case *(ii)*, the orresponding values will be $\frac{m}{2}$, $2\,l$ and $\frac{M}{2}$.

Also refer to VSQ 3.03 and 3.04.

Q. 3.02. State four properties of bar magnet. *(P.S.S.C.E. 2002)*

Ans. Refer to section 3.04.

Q. 3.03. You are given two identical looking bars A and 3. One of these is a bar magnet and the other an ordinary piece of iron. Give an experiment to identify, which one of he two is a bar magnet. You are not to use any additional naterials for the experiment. *(H.P.S.S.C.E. 2001 ; C.B.S.E. 1998)*

Or

Two identical looking iron bars, A and B are given, one of which is definitely known to be magnetised. How would one ascertain, whether or not both are magnetised ? If only one is magnetised, how does one ascertain which one ? Using nothing else, but the two bars A and B). *(I.I.T. 1982 ; Text Question)*

Ans. If on bringing different ends of the two bars close o each other, they produce repulsion in some case, then both he iron bars are magnetised. Otherwise, one bar is nagnetised and the other is simply iron bar. To find out which one is magnetised, call one bar as A and the other as B. Place bar B on table. Bring one end of bar A close to the one end of bar B. The bar A will experience attraction. Now, move the bar A to the middle of bar B. If A experiences no force, then A is iron bar and B is magnetised. On the other hand, if bar A experiences force, then A is magnetised and B is iron bar.

Q. 3.04. A steel wire of length l has a magnetic moment M. It is then bent into a semicircular arc. What is the new magnetic moment ? *(N.C.E.R.T. 1975)*

Ans. If m is pole strength, then

$$m = \frac{M}{l}$$

When the wire is bent into a semicircular arc, the separation between the two poles changes from l to $2\,r$, where r is radius of the semicircular arc. Since $l = \pi\,r$ or $r = \frac{l}{\pi}$, the new magnetic moment of the steel wire,

$$M' = m \times 2\,r = \frac{M}{l} \times \frac{2\,l}{\pi} = \frac{2\,M}{\pi}$$

Q. 3.05. Magnetic lines of force should better be called as field lines. Explain.

Ans. It is misleading to call magnetic field lines as magnetic lines of force. To elaborate, consider the expression for magnetic Lorentz force :

$$\vec{F}_m = q\,(\vec{v} \times \vec{B})$$

The magnetic force $\vec{F}_m$ is perpendicular to magnetic field $\vec{B}$ *i.e.* magnetic force is not along the direction of magnetic field. Hence, magnetic lines of force should be called only as magnetic field lines.

Q. 3.06. Define magnetic dipole moment of a magnet and write its unit by taking into consideration the torque acting on it, when placed in magnetic field. *(H.P.S.S.C.E. 1998)*

Ans. We know, $\tau = M\,B \sin\theta$

If $B = 1$ and $\theta = 90°$, then $\tau = M \times 1 \sin 90° = M \times 1 \times 1$

or $M = \tau$

Hence, the magnetic moment of a magnet is numerically equal to the torque acting on the magnet, when placed perpendicular to the direction of a uniform magnetic field of unit strength.

Again, $M = \dfrac{\tau}{B \sin\theta}$

Therefore, unit of magnetic dipole moment

$$= \frac{\text{newton metre}}{\text{tesla}} = \mathbf{N\,m\,T^{-1}}$$

Q. 3.07. Define magnetic dipole moment of a magnet by taking into consideration the torque acting on it, when placed in magnetic field. Is it a vector or a scalar ? *(H.S.S.C.E. 1992)*

Ans. For definition of magnetic dipole moment, refer to section 3.10.

Magnetic dipole moment is a vector.

Q. 3.08. A magnetised needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why ? *(C.B.S.E. 2001 S ; Text Question)*

Ans. The force and torque act on the nail due to the induced magnetic moment acquired by it. The iron needle also will not experience any force, if magnetic field is uniform. The magnetic field due to a bar magnet is not uniform. Therefore, an iron nail experiences both a force and torque, when placed near a bar magnet.

It may be pointed that the nail experiences a net attractive force. It is because the attractive force on the near induced pole of the nail is greater than the repulsive force on the far induced pole.

Q. 3.09. Why does a magnetic dipole possess potential energy, when placed at some inclination with the direction of the field ?

Ans. In equilibrium, a magnetic dipole always align itself along the direction of magnetic field. When the magneti dipole is displaced from the equilibrium position, a restorin torque acts on the dipole to bring it back. Therefore, to plac the dipole at some inclination with the field, work has to b done against the restoring torque. This work done is store in the dipole as its potential energy.

Q. 3.10. A magnetic dipole is free to rotate in a uniforn magnetic field. For what orientation of the magnet w.r.t. th magnetic field, (*a*) torque is maximum, (*b*) potential energ is maximum (*c*) rate of change of torque with angle o orientation is maximum ?

Ans. (*a*) We know, $\tau = M\,B \sin\theta$

Therefore, torque on the dipole will be maximum, whe $\sin\theta$ is maximum *i.e.* equal to 1 or θ = **90°.**

(*b*) Also, $U = -M\,B \cos\theta$

The maximum value of potential energy of the dipole i equal to M B. Therefore, potential energy of the dipole wil be maximum, when $\cos\theta$ is equal to – 1. *i.e.* θ = **180°.**

Now, rate of change of torque with angle of orientation

$$\frac{d\tau}{d\theta} = \frac{d}{d\theta}(M\,B \sin\theta) = M\,B \cos\theta$$

Therefore, $\dfrac{d\tau}{d\theta}$ will be maximum, when $\cos\theta$ i maximum *i.e.* equal to 1 or $\theta = 0°$

Q. 3.11. A bar magnet of magnetic moment M i aligned parallel to the direction of a uniform magnetic fiel B. What is the work done to turn the magnet, so as to alig its magnetic moment (*i*) opposite to the field direction an (*ii*) normal to the field direction ? *(C.B.S.E. 2001*

Ans. The work done to rotate a bar magnet from it initial position $\theta = \theta_1$ to the final position $\theta = \theta_2$ is given by

$W = M\,B\,(\cos\theta_1 - \cos\theta_2)$,

where θ is angle, which magnet (or M) makes with th direction of magnetic field (B).

(*i*) Here, $\theta_1 = 0°$ and $\theta_2 = 180°$

$\therefore\ W = M\,B\,(\cos 0° - \cos 180°) = M\,B = [1 - (-1)] = \mathbf{2\,M}$

(*ii*) Here, $\theta_1 = 0°$ and $\theta_2 = 90°$

$\therefore\ W = M\,B\,(\cos 0° - \cos 90°) = M\,B\,[1 - 0] = \mathbf{M\,B}$

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 3.01. An unmagnetised piece of iron is attracted to a bar magnet. Explain the origin of this attractive force. *(Roorkee, 1980)*

Ans. An atom is equivalent to a current loop and hence behaves as a tiny magnet. In an unmagnetised piece of iron (or in the absence of an external magnetising field), these current loops are so oriented that the iron piece does not have any net magnetism.

When unmagnetised iron piece is brought near the bar magnet, the current loops located at the ends of the bar magnet and the iron piece facing each other align themselves. When they do so, the currents in the current loops happen to be in the same direction and as a consequence, the attractive force occurs between the two.

Q. 3.02. Must every magnetic field configuration have a north pole and a south pole ? What about the field due to a toroid ? *(Text Question)*

Ans. It is not necessary that every magnetic fiel configuration must have a north pole and a south pole. Th poles exist only, if the source has a net non-zero magneti moment.

A toroid and infinite straight conductor do not hav north and south poles.

Q. 3.03. Can you think of a magnetic field configu ration with three poles ? *(Text Question*

Ans. Magnetic poles always occur in pairs. However, tw bar magnets can be arranged to produce a magnetic fiel configuration of three poles. It can be done by putting nort poles or south poles of the two magnets together.

Q. 3.04. Magnetic field arises due to charges in motion Can a system have magnetic moment, even though it net charge is zero ? *(Text Question*

Ans. A system can have a magnetic moment eve though its net charge is zero. It is because, the average charg of a system may be zero, but it is not necessary that magnetic moments due to various current loops will also be zero. For example, a neutron has zero charge, but possesses non-zero magnetic moment.

Q. 3.05. An iron bar is magnetised with the help of another magnet or by subjecting it to a magnetising field. The magnetism acquired by the magnet is assumed due to the alignment of molecular magnets. Does the length of the iron bar undergo a change during the magnetisation process ?

Ans. Yes, the length of the iron bar increases in the direction of magnetisation. This effect is called ***magnetostriction*** and is used for producing ultrasonic waves.

Q. 3.06. Magnetic field lines show the direction (at every point), which a small magnetised needle takes up (at the point). Do the magnetic field lines also represent `the lines of force' on a moving charged particle at every point ? *(Text Question)*

Ans. Force on a charged particle moving in a magnetic field is given by

$$\vec{F}_m = e(\vec{v} \times \vec{B})$$

Therefore, force $\vec{F}$ is always normal to $\vec{B}$ *i.e.* lines of force on a moving charged particle are normal to the magnetic field lines. Hence, the lines of force do not represent magnetic field lines.

Q. 3.07. If a magnetic needle is secured to a cork and the cork is floated on water, the terrestrial magnetic field will cause the needle to turn and set itself along the magnetic meridian, but the needle will not move northwards or southwards. If a bar magnet is placed near the needle, the needle will set itself in the direction of the lines of force and will begin to move toward the magnet. What are the reasons for the different behaviour of the needle in the magnetic fields of the earth and the magnet ?

Ans. In the region of the dimensions of the magnetic needle, the magnetic field of earth is practically uniform *i.e.* its intensity remains constant both in magnitude and direction. For this reason, the magnetic field of earth produces only a torque and no net force on the magnetic needle. As a result, the magnetic needle sets itself along the magnetic meridian and does not move northwards or southwards.

In the region of the dimensions of the magnetic needle, the magnetic field of the magnet is heterogeneous. The intensity of the magnetic field of the magnet at one end of the magnetic needle is stronger than that at the other. As a result, the magnetic field of the magnet exerts a torque and a net resultant force on the magnetic needle. The torque on the needle makes it to set in magnetic meridian and the resultant force produces translational motion.

Q. 3.08. The poles of a magnet cannot be separated. How does this statement derive support from the magnetic dipole bahaviour of a current loop ?

Ans. A current loop behaves as a magnetic dipole. Its one face behaves as *n*-pole, while the other as *s*-pole. As the two faces of the current loop cannot be separated from each other, it follows that the magnetic poles developed on the two faces also cannot be separated from each other.

Q. 3.09. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why ? *(Text Question)*

Ans. If the field lines were entirely confined between the two ends of a straight solenoid, the flux through the cross-section of the solenoid at each end would be non-zero. But the flux of a field through any closed surface must always be zero. Since toroid has no ends, it can entirely confine the field in its core.

Q. 3.10. An electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit. The atom is placed in uniform magnetic induction $\vec{B}$, such that the plane normal to the electron orbit makes an angle of 30° in the magnetic induction. Find the torque experienced by the orbiting electron.

(I.I.T. 1996)

Ans. The magnetic moment of the electron in an hydrogen atom is given by

$$M = \frac{e\,h}{4\pi m_e}$$

The magnetic moment vector of the electron is directed along normal to the plane of the electron orbit.

When the atom is placed in magnetic field $\vec{B}$, such that plane normal to the electron orbit *i.e.* magnetic moment vector of the electron makes an angle of 30° with the magnetic induction $\vec{B}$, the magnitude of the torque experienced by the orbiting electron is given by

$$\tau = |M\,B \sin 30°| = \frac{e\,h}{4\pi m_e} B \times \frac{1}{2} = \frac{e\,h}{8\pi m_e} B$$

Q. 3.11. Two particles, each of mass *m* and charge *q* are attached to the two ends of a light rigid rod of length 2 R. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. Find the ratio of the magnetic moment of the system and its angular momentum about the axis through the centre of the rod.

(I.I.T. 1998)

Ans. Suppose that the rod is rotated at constant angular speed ω about the perpendicular axis ZZ′ through its centre [Fig. 3.21].

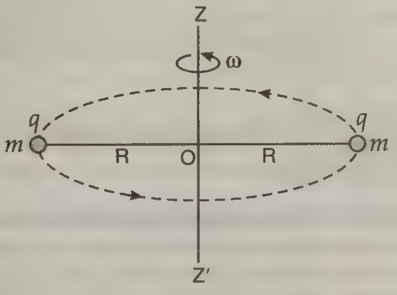


Fig. 3.21

Angular momentum of the system of two particles about ZZ′,

$$L = m R^2 \omega + m R^2 \omega = 2\,m R^2 \omega$$

When the rod is rotated, the two charged particles move along a circular path of radius R with linear speed R ω. If T is period of revolution of the rod, then a charge equal to q crosses any point on the circular path after a time,

$$t = \frac{T}{2} = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{\omega}$$

Thus, rotational motion of the charged particles is equivalent to current,

$$I = \frac{q}{t} = \frac{q}{\pi / \omega} = \frac{q\,\omega}{\pi}$$

Therefore, magnetic moment of the system,

$$M = I\,A = \frac{q\,\omega}{\pi} \times \pi\,R^2 = q\,R^2\,\omega$$

Hence $$\frac{M}{L} = \frac{q\,R^2\,\omega}{2\,m\,R^2\,\omega} = \frac{q}{2\,m}$$

Q. 3.12. If magnetic monopoles existed, how would Gauss's law of magnetism be modified ?

(Text Question)

Ans. Gauss's law of magnetism states that the flux through any closed surface is always zero. If monopole existed, then $\oint_S \vec{B} \cdot d\vec{S}$ would not be zero but it would be equal to the magnetic flux due to monopole enclosed by the closed surface.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. List the main properties of a magnet.
2. Explain the magnetism on the basis of atomic theory.
3. What is magnetic pole ? Express mathematically the force between two magnetic poles placed a finite distance apart. Hence, define unit magnetic pole strength.
4. What is a magnetic dipole ? Define magnetic dipole moment. Give its S.I. unit.
5. What are magnetic field lines ? Give their important properties. *(H.S.S.C.E. 1994)*
6. Derive an expression for the magnetic field due to a magnetic dipole in broad-side on position at a distance *r* from its centre. The length of the dipole is 2 *l* and its magnetic moment is M. *(I.S.C.E. 1998)*
7. Obtain an expression for the torque acting on a magnetic dipole placed in a magnetic field $\vec{B}$, making an angle θ with the field. *(H.P.S.S.C.E. 2000)*
8. Derive an expression for the torque experienced by a magnetic dipole in a uniform magnetic field and hence define magnetic dipole moment. *(H.P.S.S.C.E. 1996)*
9. Derive an expression for torque acting on a bar magnet held at an angle with the direction of the uniform magnetic field. *(H.S.S.C.E. 1992)*
10. Derive expression for potential energy of a bar magnet, when placed in a uniform magnetic field. *(P.S.S.C.E. 2001 ; H.P.S.S.C.E. 1998 S)*
11. A current loop behaves as a magnetic dipole. Obtain expression for magnetic dipole moment of a current loop.
12. In a hydrogen atom, an electron of charge *e* revolves in an orbit of radius *r* with a speed *v*. Prove that the magnetic moment associated with the electron is given by $\frac{e\,v\,r}{2}$. *(C.B.S.E. 2002)*
13. Derive an expression for the magnetic dipole moment of an atom. *(H.P.S.S.C.E. 1999 S)*
14. Explain how does an atom behave as magnetic dipole. Derive an expression for its magnetic dipole moment.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What is a magnetic dipole ? Obtain expression for the strength of magnetic field at a point at distance *r* from its centre on (*i*) axial line and (*ii*) equitorial line. Find the strength of the field in special case, when the length of the magnet is very small as compared to the distance *r*.
2. (*a*) Derive an expression for the torque acting on a bar magnet, when placed at an angle θ with the direction of the uniform magnetic field. Hence define magnetic dipole moment of the magnet.
   (*b*) Find the amount of work done in rotating a magnet through angle θ against the torque acting due to uniform magnetic field. What happens to the work done ?
3. (*a*) Derive an expression for the torque acting on a bar magnet placed in the uniform magnetic field.
   (*b*) Compare the magnetic fields of a bar magnet and current carrying solenoid. *(P.S.S.C.E. 2001)*
4. Explain how does an atom behave as magnetic dipole. Derive an expression for the magnetic dipole moment of the atom. Also define Bohr magneton. *(P.S.S.C.E. 2001)*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Force between Magnetic poles

1. Two magnetic poles, one of which is four times stronger than the other exert a force of 5 gf on each other, when placed at distance of 10 cm in air. Find the strength of each pole. **[Ans. 35 A m, 140 A m]**
2. Two like magnetic poles of strength 25 A m and 64 A m are situated 1·0 m apart in air. At what point on the line, joining the two poles, the magnetic field will be zero ?
   [Ans. Either at 0·385 m from pole of 25 A m (towards 64 A m) or at 1·667 m from pole of 25 A m (on the other side of 64 A m)]
3. Two identical magnets with a length 10 cm and weight 50 gf each are arranged freely wih their like poles facing in a vertical glass tube [Fig. 3.22]. The upper magnet hangs in the air above the lower one so that the distance between the nearest poles of the magnets is 3 mm. Determine the pole strength of the poles of these magnets. **[Ans. 6·64 A m]**

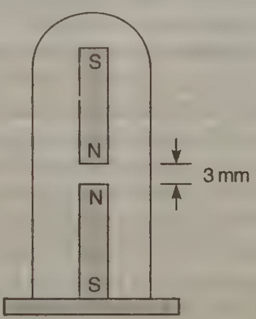


Fig. 3.22

Type B. On Magnetic moment of a Bar Magnet

4. A magnetised steel wire 31·4 cm long has a pole strength of 0·2 A m. It is then bent in the form of a semicircle. Calculate the magnetic moment of this needle.
[**Ans.** 0·04 A m^2]

5. Two thin bar magnets of pole strengths 4 A m and 7 A m respectively and lengths 0·21 m and 0·12 m respectively are placed at right angles to each other with the N-pole of the first touching the S-pole of the second. Find the magnetic moment of the system. [**Ans.** 1·19 A m^2]

Type C. On Magnetic field due to a Bar magnet

6. A bar magnet is 0·1 m long and has a pole strength of 10 A m. Calculate the magnetic field at a distance of 0·2 m from its centre at a point (*i*) on its axis and (*ii*) on its right bisector through the centre.
[**Ans.** (*a*) $2{\cdot}84 \times 10^{-5}$ T ; (*b*)$1{\cdot}14 \times 10^{-5}$ T]

7. A short bar magnet has a magnetic moment of 0·48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (*i*) the axis (*ii*) the equitorial line of the magnet. (*Text Problem*)
[**Ans.** $9{\cdot}6 \times 10^{-5}$ T (parallel to $\vec{M}$),
$4{\cdot}8 \times 10^{-5}$ T (opposite to $\vec{M}$)]

8. Two short bar magnets of magnetic moments 96 A m^2 and 81 A m^2 are placed along mutually perpendicular straight lines meeting at a point P. Find the magnetic field at point P, if it lies at distances of 0·4 m and 0·3 m respectively from the centres of the two magnets. [**Ans.** $6{\cdot}7 \times 10^{-4}$ T]

9. Two short bar magnets $N_1 S_1$ and $N_2 S_2$ of magnetic moments 32 A m^2 and 27 A m^2 are placed on the table as shown in Fig. 3.23. Find the magnitude and direction of the magnetic fields produced by these magnets at point P located on equitorial lines of both the magnets at distances of 0·4 m and 0·3 m respectively from their centres.
[**Ans.** $1{\cdot}12 \times 10^{-4}$ T (making an angle of 63·4° with N_1S_1)

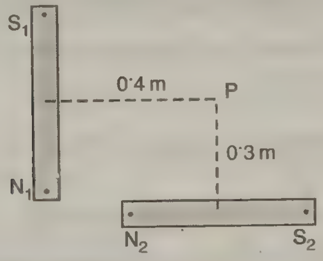


Fig. 3.23

Type D. On Torque on a Bar magnet

10. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0·25 T experiences a torque of magnitude equal to $4{\cdot}5 \times 10^{-2}$ J. What is the magnitude of magnetic moment of the magnet?
(*Text Problem*) [**Ans.** 0·36A m^2]

11. A magnetised needle of magnetic moment $4{\cdot}8 \times 10^{-2}$ J T^{-1} is placed at 30° with the direction of uniform magnetic field of magnitude 3×10^{-2} T. Calculate the torque acting on the needle. (*C.B.S.E. 2001 S*) [**Ans.** $7{\cdot}2 \times 10^{-4}$ N m]

12. A short bar magnet of magnetic moment 0·9 J T^{-1} is placed with its axis at 60° to a uniform magnetic field. It experiences a torque of 0·063 N m. (*i*) Calculate the strength of the magnetic field and (*ii*) what orientation of the bar magnet corresponds to its equilibrium position in the magnetic field? (*C.B.S.E. 2000*)
[**Ans.** (*i*) 0·081 T (*ii*) when $\vec{M} \parallel \vec{B}$]

13. A closely wound solenoid of 2000 turns and area of cross-section $1{\cdot}6 \times 10^{-4}$ m^2, carrying a current of 4·0 A is suspended through its centre allowing it to turn in a horizontal plane.
(*a*) What is the magnetic moment associated with the solenoid?
(*b*) What is the force and torque on the solenoid, if a uniform horizontal magnetic field of $7{\cdot}5 \times 10^{-2}$ T is set up at an angle of 30° with the axis of the solenoid?
(*Text Problem*) [**Ans.** (*i*) 1·28 A m^2 (*ii*) 0, 0·048 N m]

14. A closely wound solenoid of 800 turns and area of cross-section $2{\cdot}5 \times 10^{-4}$ m^2 carries a current of 3·0 A. What is its associated magnetic moment? If the solenoid is free to turn about the vertical direction and a uniform horizontal magnetic field of 0·25 T is applied, what is the magnitude of torque on the solenoid, when its axis makes an angle of 30° with the direction of the applied field? (*Text Problem*)
[**Ans.** 0·6 A m^2 ; $7{\cdot}5 \times 10^{-2}$ N m]

Type E. On Potential energy of a Bar magnet

15. Calculate the work done in rotating a bar magnet of magnetic moment 3 J T^{-1} through an angle of 60° from its position along a magnetic field of strength $0{\cdot}34 \times 10^{-4}$ T.
[**Ans.** $5{\cdot}1 \times 10^{-5}$ J]

16. A circular coil of 100 turns and having an effective radius of 5 cm carries a current of 0·1 A. How much work is required to turn it in an external magnetic field of 1·5 Wb m^{-2} through 180° about an axis perpendicular to magnetic field. The plane of the coil is initially perpendicular to the magnetic field. (*Roorkee, 1986*)
[**Ans.** 0·2356 J]

17. A short bar magnet of magnetic moment 0·32 J T^{-1} is placed in a uniform external magnetic field of 0·15 T. If the bar is free to rotate in the plane of the field, which orientations would correspond to its (*i*) stable and (*ii*) unstable equilibrium? What is the potential energy of the magnet in each case? (*Text Problem*)
[**Ans.** (*i*) $\vec{M}$ parallel $\vec{B}$, $-4{\cdot}8 \times 10^{-2}$ J ;
(*ii*) $\vec{M}$ antiparallel $\vec{B}$, $4{\cdot}8 \times 10^{-2}$ J]

18. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0·16T experiences a torque of magnitude 0·032 N m. (*a*) Estimate the magnetic moment of the magnet. (*b*) If the bar were free to rotate, which orientations would correspond to its (*i*) stable and (*ii*) unstable equilibrium? What is its potential energy in the field for cases (*i*) and (*ii*)?
[**Ans.** (*a*) 0·4 A m^2 ; (*b*) (*i*) $\vec{m}$ parallel $\vec{B}$, $-0{\cdot}064$ J
(*ii*) $\vec{m}$ antiparallel $\vec{B}$, $+0{\cdot}064$ J]

Type F. On Magnetic moment of a current loop

19. A planar loop of irregular shape enclosed an area of $7{\cdot}5 \times 10^{-4}$ m^2 and carries a current of 12 A. The sense of flow of current appears to be clockwise to an observer. What is the magnitude and direction of the magnetic moment vector associated with the current loop?
(*Text Problem*)
[**Ans.** 9×10^{-3} A m^2, normal to plane of loop and away from observer]

20. A circular coil of 300 turns and diameter 14 cm carries a current of 15 A. What is the magnitude of magnetic moment associated with the coil? (*Text Problem*)
[**Ans.** 69·3 A m^2]

Type C. On Magnetic moment of revolving electron

21. An electron in an atom revolves around the nucleus in an orbit of radius 0·5 Å. Calculate the equivalent magnetic moment, if the frequency of revolution of electron is 10^{10} MHz. *(C.B.S.E. 1998)* [**Ans.** $1\cdot26 \times 10^{-23}$ A m^2]

22. In a hydrogen atom, the electron moves in an orbit of radius 0·5 Å making 10^{16} r.p.s. Calculate the magnetic moment associated with the orbital motion of electron. *(I.I.T. 1988)* [**Ans.** $1\cdot26 \times 10^{-23}$ A m^2]

23. In hydrogen atom, an electron is making $6\cdot6 \times 10^{15}$ r.p.s. around the nucleus in an orbit of radius 0·523 Å. Calculate the equivalent magnetic dipole moment. *(C.B.S.E. 1992)* [**Ans.** $9\cdot07 \times 10^{-24}$ A m^2]

24. The electron in the hydrogen atom is moving with a speed of $2\cdot3 \times 10^6$ m s^{-1} in an orbit of radius 0·53 Å. Calculate the magnetic moment of the revolving electron. *(Roorkee, 1990)* [**Ans.** $9\cdot75 \times 10^{-24}$ A m^2]

MISCELLANEOUS PROBLEMS

25. Earth's magnetic field may be supposed to be due to a small bar magnet located at the centre of the earth. If the magnetic field at a point on the magnetic equator is 0·3 G, what is the magnetic moment of such a bar magnet? What is the magnetic field at the north pole? Given, radius of earth = $6\cdot4 \times 10^6$ m. *(I.I.T. 1973)* [**Ans.** $7\cdot864 \times 10^{22}$ A m^2, 0·6 G]

26. A short magnet of magnetic moment 2 A m^2 is lying in a horizontal plane with its north pole pointing 60° east of north. Find the net horizontal magnetic field at a point north of the magnet 0·2 m away from it. Horizontal component of earth's magnetic field = $0\cdot3 \times 10^{-4}$ T. *(Roorkee, 1991)* [**Ans.** $0\cdot59 \times 10^{-4}$ T, 21·48° west of north]

27. A bar magnet with poles 25 cm apart and pole strength 14·4 A m rests with its centre on a frictionless pivot. It is held in equilibrium at 60° to a uniform magnetic field of induction 0·25 T by applying a force F at right angle to its axis, 10 cm from its pivot. Calculate F? What will happen, if the force is removed? *(I.I.T. 1978)* [**Ans.** 6·24 N, the bar magnet will tend to align along the field]

28. Two small bar magnets each of magnetic moment 12·5 A m^2 are placed at the opposite corners A and C of the square ABCD as shown in Fig. 3.24. The length of the side of the square is 10 cm. Find the magnitude and direction of the magnetic field due to these magnets at the corners B and D.

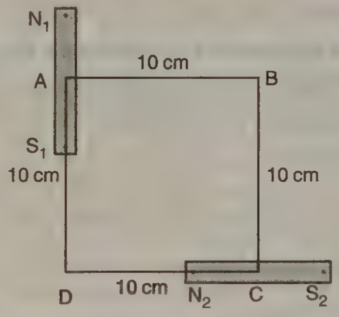


Fig. 3.24

[**Ans.** $1\cdot77 \times 10^{-3}$ T (making an angle of 45° with BC, $3\cdot54 \times 10^{-3}$ T making an angle of 45° with DA)

29. A small coil of radius 0·002 m is placed on the axis of a magnet of magnetic moment 10^5 J T^{-1} and length 0·1 m at a distance of 0·15 m from the centre of the magnet. The plane of the coil is perpendicular to the axis of the magnet. Find the force on the coil, when the current of 2·0 A is passed through it. *(Roorkee, 1989)* [**Ans.** $4\cdot4 \times 10^{-3}$ N]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Here, F = 5 gf = 5×10^{-3} kgf = $5 \times 10^{-3} \times 9\cdot8$ N ;

r = 10 cm = 0·1 m

Let m and 4 m be strengths of two poles.

Now, $F = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m \times 4m}{r^2}$

or $5 \times 10^{-3} \times 9\cdot8 = \dfrac{10^{-7} \times m \times 4m}{(0\cdot1)^2}$

or m = **35 A m** and $4m = 4 \times 35 =$ **140 A m**

2. Here, m_1 = 25 A m ; m_2 = 64 Am and r = 1·0 m

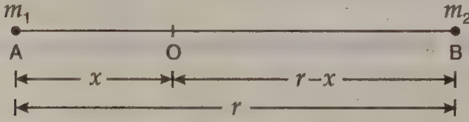


Fig. 3.25

Let magnetic field be zero at point O, such that AO = x.

Then, $\dfrac{\mu_0}{4\pi} \cdot \dfrac{m_1}{x^2} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{m_2}{(r-x)^2}$ or $\dfrac{25}{x^2} = \dfrac{64}{(1\cdot0 - x)^2}$

or $64x^2 = 25(1\cdot0 - x)^2$ or $8x = \pm 5(1\cdot0 - x)$

or $8x = 5(1\cdot0 - x)$ or $-5(1\cdot0 - x)$

or x = **0·385 m** or **– 1·667 m**

3. Since length of the magnets is quite large as compared to separation between them, the interaction is effective only between their N-poles.

Let m be strength of the poles of the two magnets. Then,

$\dfrac{\mu_0}{4\pi} \cdot \dfrac{m \times m}{r^2}$ = weight of the magnet

Here, weight of the magnet = 50 gf = 50×10^{-3} kgf

$= 50 \times 10^{-3} \times 9\cdot8$ N

and r = 3 mm = 3×10^3 m

$\therefore \dfrac{10^{-7} \times m^2}{(3 \times 10^{-3})^2} = 50 \times 10^{-3} \times 9\cdot8$ or m = **6·64 A m**

4. Proceed as in SAQ 3.04.

5. $M_1 = 4 \times 0\cdot21 = 0\cdot84$ A m^2 and $M_2 = 7 \times 0\cdot12 = 0\cdot84$ A m^2

Let M be the magnetic moment of the combination of two magnets placed at right angles to each other. Then,

$M = \sqrt{M_1^2 + M_2^2} = \sqrt{(0\cdot84)^2 + (0\cdot84)^2} =$ **1·19 A m^2**

6. Proceed as in solved problem no. 3.04.

7. *(i)* $B_{axial} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2M}{r^3} = \dfrac{10^{-7} \times 2 \times 0\cdot48}{(10 \times 10^{-2})^3}$

$=$ **$9\cdot6 \times 10^{-5}$ T** **(parallel to $\vec{M}$)**

(ii) $B_{equi} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{M}{r^3} = \dfrac{10^{-7} \times 0\cdot48}{(10 \times 10^{-2})^3}$

$=$ **$4\cdot8 \times 10^{-5}$ T** **(opposite to $\vec{M}$)**

8. The two magnets are placed as shown in Fig. 3.26 :

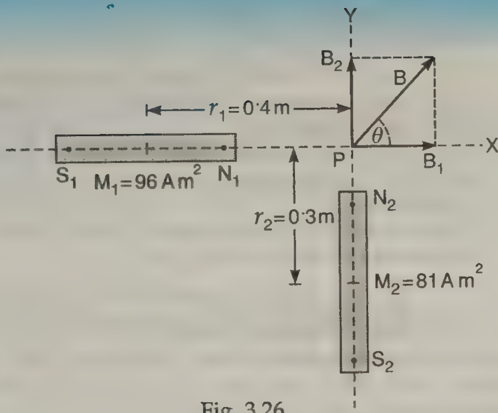


Fig. 3.26

The point P lies on axial line of both the magnets.

Let B_1 and B_2 be magnetic fields at point P due to the magnets $N_1 S_1$ and $N_2 S_2$ respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2 M_1}{r_1^3} = \frac{10^{-7} \times 2 \times 96}{(0 \cdot 4)^3}$$

$$= 3 \times 10^{-4} \text{ T} \qquad \text{(along PX)}$$

and $$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 M_2}{r_2^3} = \frac{10^{-7} \times 2 \times 81}{(0 \cdot 3)^3}$$

$$= 6 \times 10^{-4} \text{ T} \qquad \text{(along PY)}$$

Therefore, resultant magnetic field at point P,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(3 \times 10^{-4})^2 + (6 \times 10^{-4})^2} = \mathbf{6{\cdot}7 \times 10^{-4}\,T}$$

9. As shown in Fig. 3.27, the point P lies on the equitorial line of both the magnets.

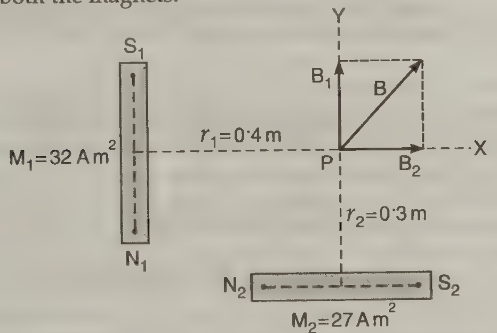


Fig. 3.27

Let B_1 and B_2 be magnetic fields at point P due to the magnets $N_1 S_1$ and $N_2 S_2$ respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{M_1}{r_1^3} = \frac{10^{-7} \times 32}{(0 \cdot 4)^3} = 0 \cdot 5 \times 10^{-4} \text{ T (along PY)}$$

and $$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M_2}{r_2^3} = \frac{10^{-7} \times 27}{(0 \cdot 3)^3} = 10^{-4} \text{ T (along PX)}$$

Therefore, resultant magnetic field at point P,

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(0{\cdot}5 \times 10^{-4})^2 + (10^{-4})^2} = \mathbf{1{\cdot}12 \times 10^{-4}\,T}$$

If the direction of B makes an angle θ with the X-axis (or $N_1 S_1$), then

$$\tan\theta = \frac{B_2}{B_1} = \frac{10^{-4}}{0 \cdot 5 \times 10^{-4}} = 2 \qquad \text{or} \qquad \boldsymbol{\theta = 63{\cdot}4^\circ}$$

10. Proceed as in solved problem no. 3.06.
11. Proceed as in solved problem no. 3.07.
12. Proceed as in solved problem no. 3.08.
13. Proceed as in solved problem no. 3.14.
14. Proceed as in solved problem no. 3.14.
15. Proceed as in solved problem no. 3.09.
16. $M = n I A = 100 \times 0{\cdot}1 \times \pi \times (0{\cdot}05)^2 = 7{\cdot}854 \times 10^{-2}$ A m²

$W = M B (\cos\theta_1 - \cos\theta_2)$

$= 7{\cdot}854 \times 10^{-2} \times 1{\cdot}5 (\cos 0^\circ - \cos 180^\circ)$

$= 7{\cdot}854 \times 10^{-2} \times 1{\cdot}5 [1 - (-1)] = \mathbf{0{\cdot}2356\ J}$

17. (*i*) When $\vec{M}$ is parallel to $\vec{B}$, the magnet is in stable equilibrium.

$U = - M B = - 0{\cdot}32 \times 0{\cdot}15 = \mathbf{- 4{\cdot}8 \times 10^{-2}\ J}$

(*ii*) When $\vec{M}$ is antiparallel to $\vec{B}$, the magnet is in unstable equilibrium.

$U = M B = 0{\cdot}32 \times 0{\cdot}15 = \mathbf{4{\cdot}8 \times 10^{-2}\ J}$

18. (*a*) Proceed as in solved problem no. 3.06.
(*b*) Proceed as in problem no. 16.
19. $M = I A = 12 \times 7{\cdot}5 \times 10^{-4} = \mathbf{9 \times 10^{-3} A\ m^2}$

The direction of magnetic moment is perpendicular to the plane of the loop and away from the observer.

20. Proceed as in solved problem 3.10 (*b*).
21. Proceed as in solved problem 3.11.
22. Proceed as in solved problem 3.11.
23. Proceed as in solved problem 3.11.
24. Period of revolution of electron,

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0 \cdot 53 \times 10^{-10}}{2 \cdot 3 \times 10^6} = 1 \cdot 448 \times 10^{-16} \text{ s}$$

The orbital motion of electron is equivalent to current,

$$I = \frac{e}{T} = \frac{1 \cdot 6 \times 10^{-19}}{1 \cdot 448 \times 10^{-16}} = 1 \cdot 105 \times 10^{-3} \text{ A}$$

Therefore, magnetic moment of the revolving electron,

$M = I A = I \times \pi r^2 = 1{\cdot}105 \times 10^{-3} \times \pi \times (0{\cdot}53 \times 10^{-10})^2$

$= \mathbf{9{\cdot}75 \times 10^{-24}\ Am^2}$

25. A point on equator lies on the equitorial line of magnet and at a distance equal to radius of the earth. Therefore,

$B_{equi} = 0{\cdot}3\ \text{G} = 0{\cdot}3 \times 10^{-4}\ \text{T}\ ; r = 6{\cdot}4 \times 10^6$ m

For a short bar magnet,

$$B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \qquad \text{or} \qquad 0 \cdot 3 \times 10^{-4} = \frac{10^{-7} \times M}{(6 \cdot 4 \times 10^6)^3}$$

or $\quad \mathbf{M = 7{\cdot}864 \times 10^{22}\ A\ m^2}$

The point at the north pole of the earth lies on its axial line at a distance equal to radius of the earth.

For a short bar magnet,

$B_{axial} = 2\ B_{equi}$ (at the same distance)

$= \mathbf{2 \times 0{\cdot}3 = 0{\cdot}6\ G}$

26. The magnet NS of magnetic moment M is placed with its north pole pointing 60° east of north as shown in Fig. 3.28.

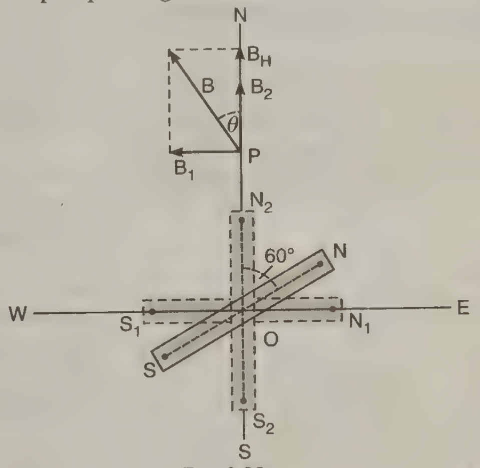


Fig. 3.28

The point P lies at a distance OP = 0·2 m north of the magnet.

Here, $M = 2\ A\ m^2$, $B_H = 0{\cdot}3 \times 10^{-4}\ T$

The magnetic moment M of the magnet NS can be resolved into two components :

(*i*) M sin 60° along EW-line and

(*ii*) M cos 60° along NS-line.

In other words, magnet NS of magnetic moment M placed 60° east of north can be replaced by two magnets N_1S_1 (of magnetic moment M sin 60° placed along EW-line) and N_2S_2 (of magnetic moment M cos 60° placed along NS-line). Let B_1 and B_2 be magnetic fields at point P due to magnets N_1S_1 and N_2S_2 respectively. Then,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{M \sin 60°}{(OP)^3} = \frac{10^{-7} \times 2 \times \sin 60°}{(0{\cdot}2)^3}$$

$$= 0{\cdot}2165 \times 10^{-4}\ T$$

$$\text{and } B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 M \cos 60°}{(OP)^3} = \frac{10^{-7} \times 2 \times 2 \times \cos 60°}{(0{\cdot}2)^3}$$

$$= 0{\cdot}25 \times 10^{-4}\ T$$

Resultant magnetic field at point P,

$$B = \sqrt{(B_1)^2 + (B_2 + B_H)^2}$$

$$= \sqrt{(0{\cdot}2165 \times 10^{-4})^2 + (0{\cdot}25 \times 10^{-4} + 0{\cdot}3 \times 10^{-4})^2}$$

$$= \mathbf{0{\cdot}59 \times 10^{-4}\ T}$$

Suppose that resultant magnetic field B is θ° west of north as shown in the figure. Then,

$$\tan\theta = \frac{B_1}{B_2 + B_H} = \frac{0{\cdot}2165 \times 10^{-4}}{0{\cdot}25 \times 10^{-4} + 0{\cdot}3 \times 10^{-4}} = 0{\cdot}3936$$

or $\theta = \mathbf{21{\cdot}48°}$ **(west of north)**

27. Here, $m = 14{\cdot}4$ A m ; $2\,l = 25$ cm $= 0{\cdot}25$ m, $B = 0{\cdot}25$ T

The magnet is held in equilibrium at 60° to the direction of magnetic field B by applying force F as shown in Fig. 3.29.

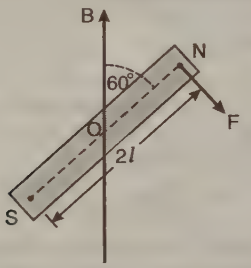


Fig. 3.29

For the equilibrium of the magnet,

$F \times ON = M\,B \sin\theta$ or $F \times l = (m \times 2\,l)\,B \sin 60°$

or $F = 2\,m\,B \sin 60° = 2 \times 14{\cdot}4 \times 0{\cdot}25 \times \sin 60° = \mathbf{6{\cdot}24\ N}$

If the force is removed, magnet will tend to align itsel along the magnetic field.

28. To find magnetic field at point B, proceed as in problem no. 9 and to find magnetic field at point B, proceed as in problem no. 8.

29. The circular coil carrying current is placed on the axis of the magnet as shown in Fig. 3.30.

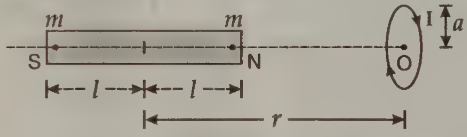


Fig. 3.30

Here, $a = 0{\cdot}002$ m, $I = 2$ A ; $2\,l = 0{\cdot}1$ m ; $r = 0{\cdot}15$ m and $M = 10^5\ J\ T^{-1}$

Therefore, pole strength of the magnet,

$$m = \frac{M}{2l} = \frac{10^5}{0{\cdot}1} = 10^6\ A\ m$$

Magnetic field at a distance *x* from the centre of a curren carrying coil of radius *a*,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{x^3} \qquad (a << x)$$

If B_1 and B_2 are magnetic fields due to the coil at north and south poles of the magnet, then force on the magnet due to coil,

$$F = m\,B_2 - m\,B_1$$

According to Newton's third law, magnet will exert an equal and opposite force on the coil. Therefore, force on the coil due to the magnet,

$$F = m\,B_1 - m\,B_2 = m\,(B_1 - B_2)$$

$$= m\left[\frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(r-l)^3} - \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(r+l)^3}\right]$$

$$= \frac{\mu_0}{4\pi} \cdot 2\pi I a^2 m \left[\frac{1}{(r-l)^3} - \frac{1}{(r+l)^3}\right]$$

$$= 10^{-7} \times 2\pi \times 2 \times (0{\cdot}002)^2 \times 10^6$$

$$\times \left[\frac{1}{(0{\cdot}15 - 0{\cdot}05)^3} - \frac{1}{(0{\cdot}15 + 0{\cdot}05)^3}\right]$$

$$= 5{\cdot}026 \times 10^{-7}\,(1000 - 125) = \mathbf{4{\cdot}4 \times 10^{-3}\ N}$$

CHAPTER
4

Earth's magnetism

4.01. EARTH'S MAGNETISM

Whenever a bar magnet is suspended freely (near the equator), its N-pole points wards the geographical north and S-pole towards the geographical south of the rth. William Gilbert suggested that earth itself behaves as a huge magnet. It was ssumed that a huge magnet is lying deep inside the earth with its S-pole towards e geographical north and N- pole towards the south of the earth. It, then, easily xplains as to why the N-pole of a freely suspended bar magnet always points wards the geographical north of the earth. It may be pointed out that the line ining the geographic north and south of the earth, called geographic axis (or eographical N-S line) makes a small angle (≈ 20°)with the magnetic axis (or agnetic N-S line). Fig. 4.01 represents the magnetic field lines around the earth.

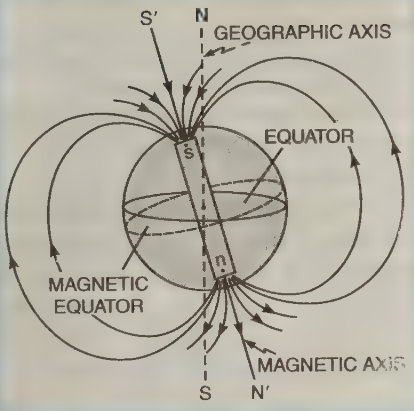


Fig. 4.01

The axis of rotation of earth is called ***geographic axis*** and the points, where it uts the surface of earth are called ***geographic poles***. The points N and S represent e geographic north and south poles. The great circle on the earth's surface erpendicular to the geographic axis is called ***equator***.

*A vertical plane passing through the geographic axis is called the **geographic meridian**.*

The axis of the huge magnet assumed to be lying inside the earth is called ***agnetic axis*** of the earth and the points, where the magnetic axis cuts the earth's urface are called ***earth's magnetic poles***. The points N′ and S′ indicate the magnetic orth and south of the earth. The great circle on the earth's surface perpendicular the magnetic axis is called ***magnetic equator***.

*A vertical plane passing through the magnetic axis of the earth is called **magnetic eridian**.*

The geographic and magnetic axis of the earth do not coincide with each other. he magnetic axis of earth makes an angle of about 20° with the geographic axis. It ollows that when a magnet is freely suspended, its north pole will point towards agnetic south (S′) and south pole towards magnetic north (N′). In other words, he poles of a freely suspended magnet only approximately point towards eographic north and south. The magnetic north and south poles are in fact at a quite arge distance from the geographic poles. At present, the magnetic south pole is ocated in Northern Canada and is about 1600 km from the geographic north at 73° orth lattitude and 96° west longitude. The north magnetic pole is at the iametrically opposite point *i.e.* at 75° south lattitude and 84° east longitude.

Cause of earth's magnetism. It may be pointed out that the source of earth's agnetism is not a large solid mass of magnetic material located inside the earth. t is guessed that earth's magnetism is due to molten charged metallic fluid. It ives rise to a current flowing inside the core of the earth. This core has a radius of bout 3500 km. Upto a distance of about 3×10^4 km (≈ five earth's radii), the agnetic field is entirely due to earth's magnetism. At larger distances, the motion f the ions influence the earth's magnetic field. The motion of the ions is affected by thin hot gas expelled by the sun, called ***solar wind***.

Prof. Blackett studied earth's magnetism in detail and suggested that since very substance is made up of charged particles (protons and electrons), hence a ubstance rotating about an axis will become magnetised. The earth's magnetism s also due to the rotation of the earth about its own axis.

There is another theory about the earth's magnetism. The gases in the tmosphere are in the ionised states. The high energy radiation coming from the un ionises the atoms in the upper part of the atmosphere. The radioactivity of the tmosphere and cosmic rays also ionise the gases in the atmosphere. Hence, due o rotation of the earth, strong electric currents flow. Due to action of these currents, he earth is magnetised.

Key point

The earth's magnetism cannot be due to the magnetisation of iron deposits inside the earth. It is because iron (if magnetised) can not retain magnetism at the high temperature inside the earth.

The third view regarding earth's magnetism is that the core of the earth is made of iron and **nickel**. Due to rotation, the earth behaves as a dynamo and becomes magnetised.

4.02. MAGNETIC ELEMENTS

The physical quantities, which determine the intensity of earth's total magnetic field completely (both in magnitude and direction), are called ***magnetic elements.***

Following are three magnetic elements of earth :

1. Magnetic declination. *Declination at a place is the angle between the geographic meridian and the magnetic meridian.* It is denoted by θ.

If ABCD and AB′C′D represent magnetic and geographical meridian, then $\angle$ BAB' = θ represents the magnetic declination [Fig. 4.02].

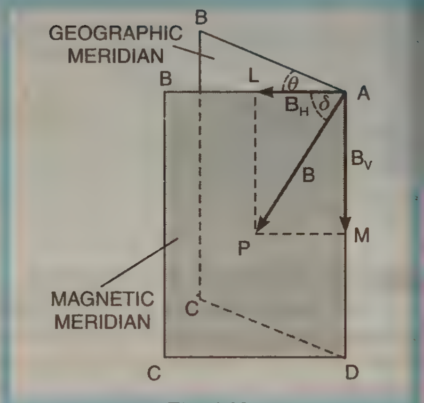


Fig. 4.02

2. Magnetic inclination or dip. *Dip at a place is defined as the angle made by the direction of the earth's total magnetic field with the horizontal direction.* It is denoted by δ.

Suppose that a small bar magnet is suspended freely at point A and it points in direction AP. Then, AP is the direction of the intensity of earth's total magnetic field (B). The line AP lies in magnetic meridian.

Consider that AB is a horizontal line in the magnetic meridian. Then, $\angle$ BAP = δ represents the dip at point A.

3. Horizontal component of earth's magnetic field. *It is the component of earth's magnetic field along the horizontal direction.*

It is denoted by B_H.

Drop PL perpendicular to AB and PM perpendicular to AD.

Then, AL and AM represent horizontal component (B_H) and vertical component (B_V) of earth's total magnetic field. In right angled triangle ALP,

$$\cos\delta = \frac{AL}{AP} = \frac{B_H}{B}$$

or

$$B_H = B\cos\delta \qquad ...(4.01)$$

Key point

The direction of horizontal component of earth's magnetic field is from geographic south to geographic north above the surface of earth.

Also,

$$\sin\delta = \frac{LP}{AP} = \frac{AM}{AP} = \frac{B_V}{B}$$

or

$$B_V = B\sin\delta \qquad ...(4.02)$$

Squaring and adding the equations (4.01) and (4.02), we have

$$B_H{}^2 + B_V{}^2 = B^2\cos^2\delta + B^2\sin^2\delta = B^2$$

or

$$B = \sqrt{B_H{}^2 + B_V{}^2} \qquad ...(4.03)$$

Dividing equation (4.02) by (4.01), we have

$$\frac{B\sin\delta}{B\cos\delta} = \frac{B_V}{B_H}$$

or

$$\tan\delta = \frac{B_V}{B_H} \qquad ...(4.04)$$

Watch out !

A magnet suspended freely at equator will orient itself parallel to the surface of earth, while one suspended at pole will stand vertically !

4.03. NEUTRAL POINT ANDTO FIND MAGNETIC MOMENT OF A BAR MAGNET

Neutral point. *A neutral point in the magnetic field of a bar magnet is that point, where the field due to the magnet is completely neutralised by the horizontal component of earth's magnetic field.*

Thus, at the neutral point, the magnetic field due to the bar magnet (B) is just equal and opposite to the horizontal component of earth's magnetic field (B_H) *i.e.*

$$B = B_H \qquad ...(4.05)$$

To find magnetic moment of a bar magnet. The magnetic moment of a bar magnet can be found by plotting magnetic field of the magnet, locating the neutral points and then using the equation (4.05).

(a) When magnet is placed with its north pole towards north of the earth. If we plot the magnetic field due to a small bar magnet by placing its north pole towards north of the earth, then plot of the magnetic field of the magnet (which is superposed by earth's magnetic field) will be as shown in Fig. 4.03.

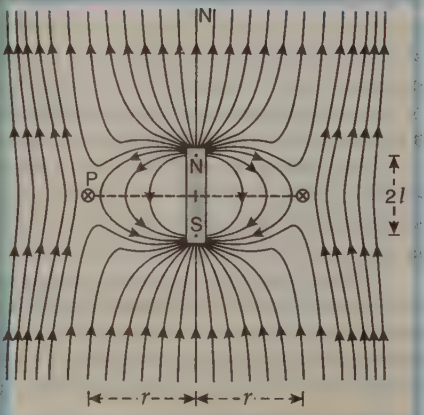


Fig. 4.03

The horizontal component of earth's magnetic field and the magnetic field due to the bar magnet cancel out at the two cross-marks on the equitorial line of the magnet (the normal bisector of the magnet) and these points are called *neutral points.* Let *r* be distance of a neutral point from the centre of the magnet. As the neutral nt lies on the equitorial line of the magnet, the magnetic field due to the magnet ne neutral point is given by

$$B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}},$$

where M is magnetic dipole moment of the magnet.

Since at the neutral point, magnetic field due to the magnet is equal to B_H, we e

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}} = B_H \qquad ...(4.06)$$

In case, if the magnet is so small that l^2 can be neglected as compared to r^2, then

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H \qquad ....(4.07)$$

Knowing r and B_H, the magnetic dipole moment of the bar magnet can be ulated from the equations (4.06) and (4.07).

Key point

When a magnet is placed with its S-pole towards north of the earth, neutral points are obtained on its axial line. On the other hand, when magnet is placed with its N-pole towards north of the earth, neutral points lie on its equitorial line. Thus, neutral points are displaced through 90° on rotating the magnet through 180°. In general, when a magnet is rotated through an angle θ, neutral points turn through an angle $\theta/2$.

(b) When magnet is placed with its south pole towards north of the earth. The of the magnetic field of a bar magnet in this case will be as shown in Fig. 4.04.

In this case, the neutral points are found to be on axial line of the magnet. If tral point is at a distance r from the centre of the magnet, then magnetic field to the magnet at the neutral point (point on the axial line of the magnet) is given

$$B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l^2)^2}$$

Since at the neutral point, the magnetic field due to the magnet is equal to B_H, have

$$\frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l^2)^2} = B_H \qquad ...(4.08)$$

In case, if the magnet is so small that l^2 can be neglected as compared to r^2, then

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H \qquad ...(4.09)$$

Knowing r and B_H, the equations (4.08) and (4.09) can be used to find the gnetic dipole moment of the bar magnet.

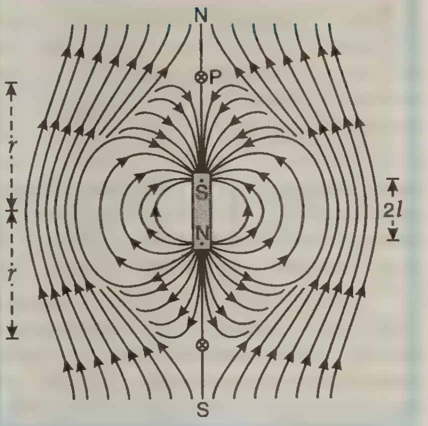


Fig. 4.04

Note. From the plot of magnetic field in the two cases *i.e.* when magnet is placed h its N-pole towards north of earth and when magnet is placed with its S-pole ards north of the earth, it follows that when magnet is turned through 180°, the tral points turn through 90°.

4.04. TANGENT LAW

Tangent law. *If a small bar magnet is suspended in two mutually perpendicular form magnetic fields $\vec{B}$ and $\vec{B}_H$, such that it comes to rest making an angle θ with the ction of field $\vec{B}_H$, then*

$$B = B_H \tan\theta \qquad ...(4.10)$$

Proof. Consider that a small bar magnet NS is placed in two uniform magnetic ds B and B_H acting at right angles to each other. Let m be the pole strength of h pole and $2\,l$ be the length of the magnet. The force on the north pole of the gnet due to the magnetic field will act along the direction of the field, while the ce on south-pole will be along a direction opposite to the direction of the field . 4.05]. Each magnetic field produces a torque on the magnet. The torque due he magnetic field B_H tends to rotate the magnet in anticlockwise direction, while t due to magnetic field B in clockwise direction. Suppose that the magnet comes he equilibrium position, where it makes an angle θ with the direction of magnetic d B_H.

Torque due to the magnetic field B_H,

$$\tau_1 = m\,B_H \times KN$$

Torque due to the magnetic field B,

$$\tau_2 = m\,B \times KS$$

In equilibrium, $\tau_1 = \tau_2$

i.e. $m\,B \times KS = m\,B_H \times KN$

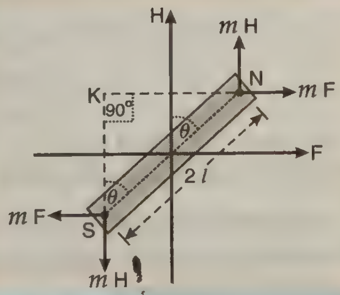


Fig. 4.05

or $$B = B_H \times \frac{KN}{KS}$$

But from the right angled triangle S K N,

$$\frac{KN}{KS} = \tan\theta$$

From the above two equations, we have

$$B = B_H \tan\theta$$

It proves the tangent law. The above result holds only, when the two magnetic fields are uniform over the entire length of the magnet.

4.05. TANGENT GALVANOMETER

A tangent galvanometer is an instrument used to measure very small currents. Further, it is a galvanometer of ***moving magnet and fixed coil type.***

Principle. It is based on tangent law in magnetism.

Construction. It consists of a non-magnetic circular frame mounted on a horizontal turn table provided with levelling screws [Fig. 4.06]. Three coils having 2 ; 50 and 500 turns are wound on the circular frame and their ends are connected to binary terminals provided over the base of the instrument. The coil can be rotated about a vertical axis passing through the centre of the instrument. A circular compass box made of non-magnetic material is mounted at the centre of the coil. The compass box has a small magnetic needle *n s* pivoted at its centre, which is free to move over the horizontal circular scale of the compass box. The circular scale is divided into four quadrants of 0° – 90° each. To note the position of the magnetic needle accurately, a long aluminium pointer is attached at right angle to the magnetic needle. Also, a plane mirror is fixed below the pointer to remove error due to parallax, while reading the position of the pointer. The reading of the pointer is taken in the position, where the image of the pointer coincides with the pointer itself.

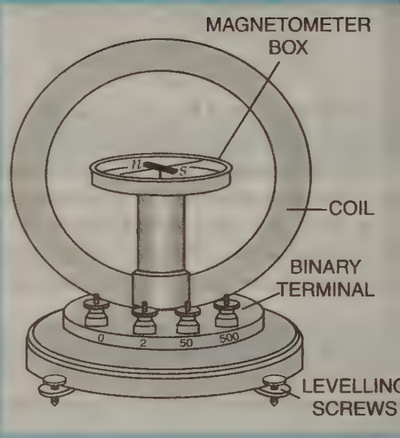


Fig. 4.06

Setting. The tangent galvanometer is levelled with the help of levelling screws and the coil is rotated, till its plane becomes parallel to the length of the magnetic needle. In this position, the plane of the coil lies along the magnetic meridian *i.e.* the horizontal component of earth's magnetic field is in the plane of the coil. Now, the compass box is rotated, so that the pointer comes along 0°- 0° line.

Theory. When current is passed through the coil, the magnetic field (B) is produced at the centre of the coil in a direction perpendicular to the plane of the coil. Since, galvanometer has been set in a position such that the horizontal component of earth's magnetic field (B_H) is in the plane of the coil ; the magnetic needle will be under the action of two mutually perpendicular magnetic fields. If θ is deflection of the needle, then according to tangent law,

$$B = B_H \tan\theta \qquad \text{...(4.11)}$$

If the coil has *n* turns and radius *a*, then magnetic field produced at the centre of the coil on passing current I is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} \qquad \text{...(4.12)}$$

From equations (4.11) and (4.12), we have

$$\frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} = B_H \tan\theta$$

or $$I = \frac{2a}{n\mu_0} B_H \tan\theta \qquad \text{...(4.13)}$$

Knowing *a*, *n*, θ and B_H ($0{\cdot}32 \times 10^{-4}$ tesla approximately), the value of current I through the coil can be calculated.

In the above equation, the factor $\frac{n\mu_0}{2a}$ is a constant for the given galvanometer and likewise it is called ***galvanometer constant***. It is denoted by G. Therefore, equation (4.13) reduces to

$$I = \frac{B_H}{G} \tan\theta \qquad \text{...(4.14)}$$

Further, the factor $\frac{B_H}{G}$ is called ***reduction factor*** of the tangent galvanometer and is denoted by K. Therefore, equation (4.14) becomes

$$I = K \tan\theta \qquad \text{...(4.15)}$$

Key point

Reduction factor of a tangent galvano
meter depends upon the geometry of it
coil. It increases with increase of radiu
and decrease in number of turns of the coi
of the galvanometer.

If $\theta = 45°$, then $I = K \tan 45°$ or $K = I$

Thus, *reduction factor of a tangent galvanometer is numerically equal to the current* *mpere) through the galvanometer, which produces a deflection of 45°, when the plane* *s coil lies in the magnetic meridian.*

4.06. VIBRATION MAGNETOMETER

It is an instrument used to compare magnetic moments of two bar magnets or etermine the horizontal component of earth's magnetic field.

Principle. *It is based on the principle that when a bar magnet suspended freely in a* *orm magnetic field is displaced from its equilibrium position, it starts executing simple* *nonic motion about the equilibrium position.*

Construction. A vibration magnetometer consists of a wooden box with its es made of glass. A long brass or glass tube is fixed vertically at the top of the . At the upper end of tube, a screw is provided to suspend *an unspun silken thread* *air of a horse.* The lower end of the silken thread carries an arrangement, called rup (a hook of peculiar shape and made of non-magnetic substance), in which magnet can be placed. A plane mirror is fitted over the base of the instrument, he centre of which a line is drawn in a direction parallel to its length [Fig. 4.07]. s line is called the *index line.*

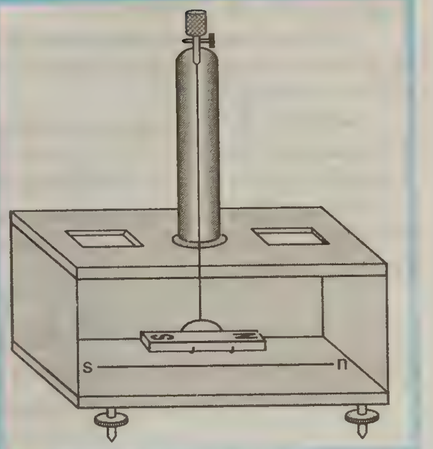


Fig. 4.07

The magnet placed in the stirrup suspends horizontally above the index line. en the magnet vibrates, its vibrations can be counted by observing its vibratory tion through the glass walls.

Setting. To set the vibration magnetometer, place the magnetic compass needle the index line. The magnetometer is rotated, till the compass needle becomes allel to the index line. The line now represents the magnetic N-S line. In order make the suspension fibre torsionless, a brass bar nearly of the size of the magnet laced in the stirrup. The screw is then adjusted to bring the brass bar in the gnetic meridian. The brass bar is then replaced by the magnet, with its north pole inting towards north of the earth.

Theory. Consider that a magnet having the magnetic dipole moment M is ely suspended in earth's magnetic field B_H. In equilibrium position, it will point the N-S direction. If the magnet is displaced through an angle θ, a torque equal $M B_H \sin\theta$ acts on it, which tends to bring it back to its equilibrium position. As magnet reaches the mean position, it acquires kinetic energy and overshoots the uilibrium position and thus executes oscillations.

If I is moment of inertia of the magnet about the suspension axis and α, the gular acceleration produced, then

$$I\alpha = -M B_H \sin\theta$$

If θ is small, then $\sin\theta \approx \theta$ and hence the angular acceleration (in magnitude) given by

$$\alpha = \frac{M B_H}{I}\theta \qquad \text{...(4.16)}$$

For a given magnet, the factor $\frac{M B_H}{I}$ is a constant and therefore, angular celeration (α) is directly proportional to angular displacement (θ). Hence, the riodic motion of the vibrating magnet is S.H.M. Therefore, time period of vibration the magnet is given by

$$T = 2\pi\sqrt{\frac{\text{angular displacement}}{\text{angular acceleration}}} = 2\pi\sqrt{\frac{\theta}{\alpha}}$$

Using equation (4.16), we have

$$T = 2\pi\sqrt{\frac{I}{M B_H}} \qquad \text{...(4.17)}$$

For a rectangular bar magnet, the moment of inertia of the magnet about the is of oscillation through its C.G. is given by

$$I = \text{mass} \times \frac{a^2 + b^2}{12},$$

here a and b are the geometrical length and breadth of the bar magnet.

The relation (4.17) may be used to find the magnetic dipole moment of a bar magnet by noting the period of vibration. In case B_H is not known, the relation can be used to compare magnetic dipole moments of two bar magnets.

4.57. USES OF A VIBRATION MAGNETOMETER

A vibration magnetometer can be used to compare the magnetic moments of two bar magnets, to compare horizontal components of earth's magnetic field at two different places ; and together with a deflection magnetometer, it can be used to find the magnetic moment of a magnet and the horizontal component of earth's magnetic field at a given place.

1. To compare magnetic moments of two bar magnets. Let us study the use of vibration magnetometer, when the two given bar magnets are of (*a*) equal sizes and (*b*) of unequal sizes.

(a) When the bar magnets are of equal sizes. Consider two bar magnets A and B of same size and mass. Let M_1 and M_2 be the magnetic moments of the magnets A and B respectively and I be the moment of inertia of each magnet.

Set the vibration magnetometer, so that the suspension fibre is torsionless and the stirrup rests along N-S line. Place the magnet A in the stirrup and find its time period T_1. Then,

$$T_1 = 2\pi\sqrt{\frac{I}{M_1 B_H}}, \qquad \text{...(4.18)}$$

where B_H is horizontal component of the earth's magnetic field.

By proceeding in the same manner, find the time period T_2 of magnet B. Then,

$$T_2 = 2\pi\sqrt{\frac{I}{M_2 B_H}} \qquad \text{...(4.19)}$$

Dividing equation (4.19) by (4.18), we have

$$\frac{T_2}{T_1} = \sqrt{\frac{I}{M_2 B_H}} \times \sqrt{\frac{M_1 B_H}{I}} = \sqrt{\frac{M_1}{M_2}}$$

or $$\frac{M_1}{M_2} = \frac{T_2^2}{T_1^2} \qquad \text{...(4.20)}$$

Knowing T_1 and T_2, the ratio M_1/M_2 can be found.

(b) When the bar magnets are of unequal sizes. Consider two unequal magnets A and B having moment of inertia I_1 and I_2 and magnetic moments M_1 and M_2 respectively.

Set the vibration magnetometer, so that the suspension fibre is torsionless and the stirrup rests along the N-S line. By placing the two magnets A and B together in the stirrup, so that their like poles are in the same direction [Fig. 4.08], the time period T_1 of the combination is found. Then,

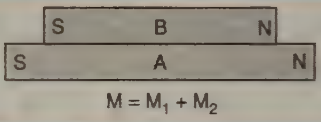


Fig. 4.08

$$T_1 = 2\pi\sqrt{\frac{I}{M B_H}},$$

where I and M are the vlaues of moment of inertia and magnetic moment of the combination of the two magnets. Since the like poles of the two magnets are in same direction,

$$M = M_1 + M_2$$

$$\therefore \qquad T_1 = 2\pi\sqrt{\frac{I}{(M_1 + M_2) B_H}} \qquad \text{...(4.21)}$$

Now, the two magnets are placed in the stirrup, so that their opposite poles are in same direction [Fig. 4.09]. If T_2 is time period of the combination in this case, then

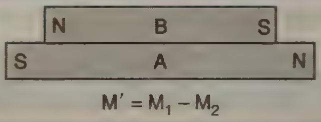


Fig. 4.09

$$T_2 = 2\pi\sqrt{\frac{I}{M' B_H}}$$

Here, M′ is magnetic moment of the combination of the two magnets in this case. Since, the like poles of the two magnets are in opposite direction,

$$M' = M_1 - M_2$$

$$\therefore \qquad T_2 = 2\pi\sqrt{\frac{I}{(M_1 - M_2)\,B_H}} \qquad \text{...(4.22)}$$

Dividing equation (4.21) by (4.22), we have

$$\frac{T_2}{T_1} = \sqrt{\frac{I}{(M_1 - M_2)\,B_H}} \times \sqrt{\frac{(M_1 + M_2)\,B_H}{I}} = \sqrt{\frac{M_1 + M_2}{M_1 - M_2}}$$

$$\text{or} \qquad \frac{M_1 + M_2}{M_1 - M_2} = \frac{T_2^2}{T_1^2}$$

Applying componendo and dividendo, we have

$$\text{or} \qquad \frac{(M_1 + M_2) + (M_1 - M_2)}{(M_1 + M_2) - (M_1 - M_2)} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

$$\text{or} \qquad \frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} \qquad \text{...(4.23)}$$

Knowing T_1 and T_2, the ratio M_1/M_2 can be found.

2. To compare horizontal components of earth's magnetic field at two places. Consider a magnet having moment of inertia I and magnetic moment M. Let B_H and B_H' be the values of horizontal components of earth's magnetic field at two places A and B respectively.

If T and T′ are the values of time period of the magnet at the place A and B respectively, then

$$T = 2\pi\sqrt{\frac{I}{M\,B_H}} \qquad \text{...(4.24)}$$

$$\text{and} \qquad T' = 2\pi\sqrt{\frac{I}{M\,B_H'}} \qquad \text{...(4.25)}$$

Dividing equation (4.25) by (4.24), we have

$$\frac{T'}{T} = \sqrt{\frac{I}{M\,B_H'}} \times \sqrt{\frac{M\,B_H}{I}} = \sqrt{\frac{B_H}{B_H'}}$$

$$\text{or} \qquad \frac{B_H}{B_H'} = \frac{T'^2}{T^2} \qquad \text{...(4.26)}$$

Knowing T and T′, the ratio B_H/B_H' can be found.

SOLVED NUMERICAL PROBLEMS

Type A. On Magnetic elements

Problem 4.01. The declination at a place is 12°west of north. Find in what direction, a ship should stear so that it reaches a place due east ?

Sol. The dotted N-S line represents geographical N S-line and the compass needle *n s* points 12° west of geographical north [Fig. 4.10]. Therefore, the ship would reach a place due east, if it stears 12° + 90° *i.e.* **102°** ***with the direction of the compass needle.***

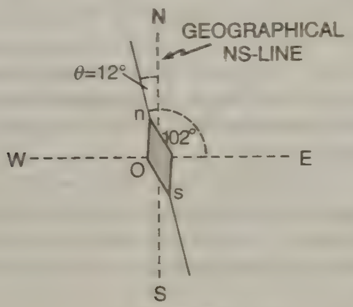


Fig. 4.10

Problem 4.02. A ship is sailing due east according to Mariner's compass. If the declination of that place is 18° east of north, what is the true direction of the ship ?

Sol. The compass needle *n s* points along magnetic N S-line and the ship is sailing along OP *i.e.* towards east (as indicated by compass needle). As the declination of that place is 18° east of north, the geographical N-S line is as shown by dotted N-S line [Fig. 4.11]. Thus, true direction of ship is 18° ***south of east.***

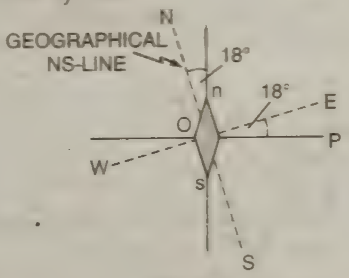


Fig. 4.11

Problem 4.03. At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the

magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0·16 G. Specify the direction and magnitude of the earth's field at the location. *(Text Problem)*

Sol. Here, angle of dip, $\delta = 60°$

horizontal component of earth field, $B_H = 0{\cdot}16$ G

Let earth's magnetic field be equal to B.

Then $\qquad B_H = B \cos \delta$

$$\text{or} \qquad B = \frac{B_H}{\cos \delta} = \frac{0 \cdot 16}{\cos 60°} = \frac{0 \cdot 16}{0 \cdot 5} = 0 \cdot 32 \text{ G}$$

Thus, earth's magnetic field has a magnitude of 0·32 G and lies in a vertical plane 12° *west of geographic meridian making an angle of* 60° *(upwards) with the horizontal.*

Problem 4.04. The true value of dip at a place is 45°. If the plane of the dip circle is turned through 60° from the magnetic meridian, what will be the apparent value of the dip ?

Sol. On turning the dip circle through 60°, the apparent value of the horizontal component of earth's magnetic field becomes

$$BH' = B_H \cos 60°$$

However, vertical component of earth's field remains unchanged. If δ' is apparent value of dip, then

$$\tan \delta' = \frac{B_V'}{B_H'} = \frac{B_V}{B_H \cos 60°} \qquad (\because B_V' = B_V)$$

$$\text{Now} \qquad \frac{B_V}{B_H} = \tan \delta,$$

where δ is true dip at that place.

$$\text{Therefore, } \tan \delta' = \frac{\tan \delta}{\cos 60°} = \frac{1}{0 \cdot 5} = 2 \cdot 0$$

$$\text{or} \qquad \delta' = 63{\cdot}4°$$

Problem 4.05. If δ_1 and δ_2 be the angles of dip observed in two planes at right angles to each other and δ is the true angle of dip, prove that *(P.S.S.C.E. 1996)*

$$\cot^2 \delta_1 + \cot^2 \delta_2 = \cot^2 \delta$$

Sol. Let B_H and B_V be the horizontal and vertical components of the earth's magnetic field. As δ is true angle of dip,

$$\tan \delta = \frac{B_V}{B_H} \qquad \text{...}(i)$$

Planes 1 and 2 are at right angles to each other. If plane 1 makes an angle θ with the magnetic meridian, then plane 2 will make angle $(90° - \theta)$ with the magnetic meridian as shown in Fig. 4.12.

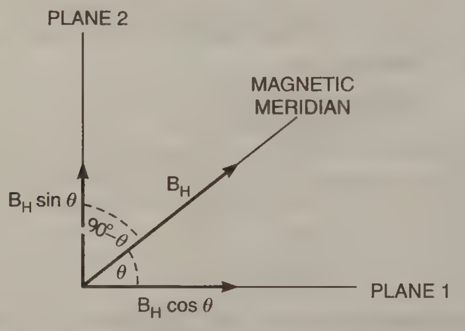


Fig. 4.12

Whereas the vertical component of earth's magnetic field remains unaffected in the two planes, the effective horizontal components of earth's magnetic field in plane 1 and 2 a $B_H \cos \theta$ and $B_H \sin \theta$ respectively. Therefore, angle of d δ_1 in plane 1 is given by

$$\tan \delta_1 = \frac{B_V}{B_H \cos \theta} \qquad \text{or} \qquad \cos \theta = \frac{B_V}{B_H \tan}$$

Using equation (*i*), we have

$$\cos \theta = \frac{\tan \delta}{\tan \delta_1} \qquad \text{...(}$$

Further, angle of dip δ_2 in plane 2 is given by

$$\tan \delta_2 = \frac{B_V}{B_H \sin \theta}$$

Again using equation (*i*), we obtain

$$\sin \theta = \frac{\tan \delta}{\tan \delta_2} \qquad \text{...(}$$

Squaring and adding equations (*ii*) and (*iii*), we hav

$$\frac{\tan^2 \delta}{\tan^2 \delta_1} + \frac{\tan^2 \delta}{\tan^2 \delta_2} = \cos^2 \theta + \sin^2 \theta = 1$$

$$\text{or} \qquad \frac{1}{\tan^2 \delta_1} + \frac{1}{\tan^2 \delta_2} = \frac{1}{\tan^2 \delta}$$

$$\text{or} \qquad \cot^2 \delta_1 + \cot^2 \delta_2 = \cot^2 \delta$$

Type B. On Neutral points

Problem 4.06. A bar magnet 30 cm long is placed in t magnetic meridian with its north pole pointing south. T neutral point is observed at a distance of 30 cm from its o end. Calculate the pole strength of the magnet. Give horizontal component of earth's field = 0·34 G.

Sol. Here, $2\,l = 30$ cm or $l = 15$ cm $= 0{\cdot}15$ m ;

$r = 30$ cm $= 0{\cdot}30$ m,

$B_H = 0{\cdot}34 \text{ G} = 0{\cdot}34 \times 10^{-4}$ T

When magnet is placed with its north pole pointi south, neutral point is obtained on its axial line. Therefo at the neutral point,

$$B_{axial} = B_H$$

$$\text{or} \qquad \frac{\mu_0}{4\pi} \times \frac{2\,M\,r}{(r^2 - l^2)^2} = B_H$$

$$\text{or} \qquad M = \frac{4\pi}{\mu_0} \times \frac{B_H (r^2 - l^2)^2}{2r}$$

$$= \frac{1}{10^{-7}} \times \frac{0 \cdot 34 \times 10^{-4} \times (0 \cdot 30^2 - 0 \cdot 15^2)^2}{2 \times 0 \cdot 30}$$

$$= \frac{0 \cdot 34 \times 10^{-4} \times (0 \cdot 0675)^2}{10^{-7} \times 2 \times 0 \cdot 30} = 2 \cdot 582 \text{ A m}^2$$

The pole strength of the magnet,

$$m = \frac{M}{2\,l} = \frac{2 \cdot 582}{0 \cdot 30} = 8 \cdot 606 \text{ A m}$$

Problem 4.07. A bar magnet is 10 cm long. Its magne moment is 0·4 A m^2. It is placed in magnetic meridian wi its north pole towards north. If the neutral point is obtain at a distance of 10 cm from the centre of the magnet, fi the horizontal component of earth's magnetic field.

Sol. Here, $M = 0{\cdot}4 \text{ A m}^2$; $r = 10$ cm $= 0{\cdot}1$ m ;

$2\,l = 10$ cm or $l = 5$ cm $= 0{\cdot}05$ m

When magnet is placed with its north pole towards no of earth, the neutral point is obtained on the equitorial li If B_H is horizontal component of earth's field, then at t neutral point

$$B_{equi} = B_H$$

$$\text{or } B_H = \frac{\mu_0}{4\pi} \times \frac{M}{(r^2 + l^2)^{3/2}}$$

$$= \frac{10^{-7} \times 0 \cdot 4}{(0 \cdot 1^2 + 0 \cdot 05^2)^{3/2}} = \frac{10^{-7} \times 0 \cdot 4}{(0 \cdot 0125)^{3/2}} = \frac{10^{-7} \times 0 \cdot 4}{0 \cdot 0014}$$

$$= \mathbf{0 \cdot 286 \times 10^{-4}\ T}$$

Problem 4.08. A long straight horizontal cable carries current of 2·5 A in the direction 10° north of east. The agnetic meridian of the place happens to be 10° west of e geographic meridian. The earth's magnetic field at the cation is 0·33 G and the angle of dip is zero. Locate the ne of neutral point. (Ignore the thickness of the ble). *(Text Problem)*

Sol. The magnetic $n - s$ line is 10° west of geographical –S line as shown in Fig. 4.13. The straight horizontal cable Y is 10° north of east.

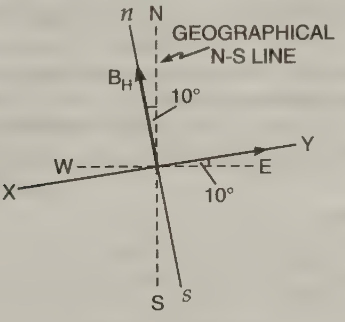


Fig. 4.13

Here, $\delta = 0°$ and $B = 0 \cdot 33$ G

The horizontal component of earth's field,

$$B_H = \frac{B}{\cos \delta} = \frac{0 \cdot 33}{\cos 0°} = 0 \cdot 33\ G = 0 \cdot 33 \times 10^{-4}\ T$$

At a point below the cable, the direction of the magnetic eld produced is same as that of B_H, while at a point above ne cable, the direction of the field produced is opposite to that f B_H. Therefore, neutral point will lie above the cable. If the istance of the neutral point from the wire is a, then,

$$\frac{\mu_0}{4\pi} \cdot \frac{2I}{a} = B_H \qquad \text{or} \qquad a = \frac{\mu_0}{4\pi} \times \frac{2I}{B_H}$$

Here $\quad I = 2 \cdot 5$ A

$$\therefore \quad a = 10^{-7} \times \frac{2 \times 2 \cdot 5}{0 \cdot 33 \times 10^{-4}} = 1 \cdot 5 \times 10^{-2}\ m = \mathbf{1 \cdot 5\ cm}$$

ype C. On Tangent law and Tangent Galvanometer

Problem 4.09. A current carrying planar loop suspen-ed in a vertical plane normal to the magnetic meridian is n stable equilibrium. The horizontal component of earth's agnetic field is 0·32 G. Another horizontal magnetic field of magnitude 0·48 G parallel to the plane of the loop is set up along magnetic west to east. Specify the new stable equilibrium. *(Text Problem)*

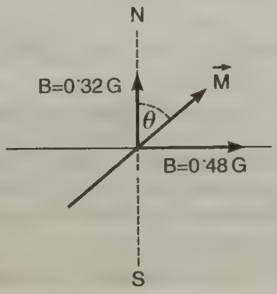


Fig. 4.14

Sol. Suppose that the current loop behaves as a magnetic dipole of magnetic moment $\vec{M}$. It will come to equilibrium in a position making angle θ with B_H [Fig. 4.14], such that

$$\frac{B}{B_H} = \tan \theta \qquad \text{(tangent law)}$$

Here, $\quad B_H = 0 \cdot 32$ G ; $B = 0 \cdot 48$ G

$$\therefore \quad \tan \theta = \frac{0 \cdot 48}{0 \cdot 32} = 1 \cdot 5$$

$$\text{or} \quad \theta = \mathbf{56 \cdot 3°} \quad \textbf{(east of north)}$$

Problem 4.10. A tangent galvanometer has a coil of 22 turns and mean radius 0·1 m. Find the reduction factor at a place, where $B_H = 0 \cdot 3 \times 10^{-4}$ T. *(Karnataka, 1986 S)*

Sol. Here, $\quad n = 22$; $a = 0 \cdot 1$ m ; $B_H = 0 \cdot 3 \times 10^{-4}$ T

We know that $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

Reduction factor,

$$K = \frac{2a}{n\mu_0} B_H = \frac{2 \times 0 \cdot 1}{22 \times 4\pi \times 10^{-7}} \times 0 \cdot 3 \times 10^{-4}$$

$$= \mathbf{0 \cdot 217\ A}$$

Problem 4.11. A tangent galvanometer has a coil of 40 turns and of radius 12 cm. A steady current of 0·15 A, when passed through the coil, produces a deflection of 45° in the galvanometer. Calculate the strength of the horizontal component of the earth's magnetic field at the centre of the coil of the galvanometer. *(H.S.S.C.E. 1998)*

Sol. Here, $n = 40$; $a = 12$ cm $= 0 \cdot 12$ m ; $I = 0 \cdot 15$ A ; $\theta = 45°$

Also, $\quad \mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

$$\text{Now,} \quad I = \frac{2a}{n\mu_0} B_H \tan \theta$$

$$\text{or} \quad B_H = \frac{n\mu_0}{2a} \cdot \frac{I}{\tan \theta}$$

$$= \frac{40 \times 4\pi \times 10^{-7}}{2 \times 0 \cdot 12} \times \frac{0 \cdot 15}{\tan 45°} = \mathbf{0 \cdot 314 \times 10^{-4}\ T}$$

Problem 4.12. Two tangent galvanometers having equal radius of coils are connected in parallel to an electric cell. The ratio of the currents in the galvanometers are 1 : 2 and respective mean deflections are 30° and 45°. Compare the ratio of the number of turns of the tangent galvano-meters. *(C.B.S.E. 2002)*

Sol. The current passing through a tangent galvano-meter is given by

$$I = \frac{2a}{n\mu_0} B_H \tan \theta$$

Let n_1 and n_2 be number of turns of the two tangent galvanometers. Since the two galvanometers have coils of equal radii,

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} \cdot \frac{\tan \theta_1}{\tan \theta_2} \qquad \text{or} \qquad \frac{n_1}{n_2} = \frac{I_2}{I_1} \cdot \frac{\tan \theta_1}{\tan \theta_2}$$

$$\text{or} \quad \frac{n_1}{n_2} = \frac{1}{I_1 / I_2} \cdot \frac{\tan \theta_1}{\tan \theta_2}$$

$$\text{Here,} \quad \frac{I_1}{I_2} = \frac{1}{2} ; \theta_1 = 30° \qquad \text{and} \qquad \theta_2 = 45°$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{1}{1/2} \cdot \frac{\tan 30°}{\tan 45°} = \frac{2 \times 0 \cdot 5774}{1} = \mathbf{1 \cdot 155}$$

Type D. On Vibration Magnetometer

Problem 4.13. A circular coil of 16 turns and radius 10 cm carrying a current of 0·75 A rests with its plane normal to an external field of magnitude $5{\cdot}0 \times 10^{-2}$ T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2{\cdot}0\ \text{s}^{-1}$. What is the moment of inertia of the coil about its axis of rotation ?

Sol. Here, $n = 16$; $a = 10$ cm = 0·1 m ;

current* through the coil, $i = 0{\cdot}75$ A

The magnetic dipole moment of the coil,

$$M = n\,i\,A = n\,i \times \pi a^2$$
$$= 16 \times 0{\cdot}75 \times \pi \times (0{\cdot}1)^2 = 0{\cdot}377\ \text{A m}^2$$

The time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{M B_H}}$$

$$\therefore \quad I = \frac{T^2 M B_H}{4\pi^2} = \frac{M B_H}{4\pi^2 \nu^2},$$

where $\nu = \frac{I}{T}$ is frequency of oscillation.

Here, $B_H = 5{\cdot}0 \times 10^{-2}$ T ; $\nu = 2{\cdot}0\ \text{s}^{-1}$

$$\therefore \quad I = \frac{0{\cdot}377 \times 5{\cdot}0 \times 10^{-2}}{4\pi^2 \times (2{\cdot}0)^2}$$
$$= \mathbf{1{\cdot}194 \times 10^{-4}\ kg\ m^2}$$

Problem 4.14. Two bar magnets having their momer of inertia in the ratio 1 : 1·5 oscillate in a horizontal plar with time periods of 2·5 s and 4·5 s respectively. Compar their magnetic moments. (*C.B.S.E. 200*

Sol. The time period of a bar magnet is given by

$$T = 2\pi\sqrt{\frac{I}{M B_H}}$$

Let M_1 and M_2 be the magnetic moments of the tw magnets ; and I_1 and I_2 be their moment of inertia. If T_1 ar T_2 are their respective time periods, then

$$\frac{T_1}{T_2} = \sqrt{\frac{I_1 M_2}{M_1 I_2}} \quad \text{or} \quad \frac{M_1}{M_2} = \frac{I_1}{I_2} \times \frac{T_2^2}{T_1^2}$$

Here, $\frac{I_1}{I_2} = \frac{1}{1{\cdot}5}$, $T_1 = 2{\cdot}5$ s and $T_2 = 4{\cdot}5$

$$\therefore \quad \frac{M_1}{M_2} = \frac{1}{1{\cdot}5} \times \frac{(4{\cdot}5)^2}{(2{\cdot}5)^2} = \mathbf{2{\cdot}16}$$

Problem 4.15. Two bar magnets placed together in vibration magnetometer take 3 s for 1 vibration. If or magnet is reversed, the combination takes 4 s for vibration. Find the ratio of their magnetic moments. (*H.S.S.C.E. 1998*

Sol. Here, $T_1 = 3$ s ; $T_2 = 4$ s

We know, $$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{4^2 + 3^2}{4^2 - 3^2} = \frac{16+9}{16-9} = \frac{25}{7}$$
$$= 3{\cdot}57$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 4.16. A telephone cable at a place has four long straight horizontal wires carrying a current of 1·0 A from east to west (in the same direction in all the wires). The earth's magnetic field at the place is 0·39 G and angle of dip is 35º. The magnetic declination is nearly zero. What are the resultant magnetic fields at points 4·0 cm below and above the cable ? (*Text Problem*)

Sol. Here, earth's magnetic field, $B = 0{\cdot}39$ G ; $\delta = 35°$

Therefore, horizontal component of earth's magnetic field,

$$B_H = B\cos\delta$$
or $$B_H = 0{\cdot}39\cos 35° = 0{\cdot}3195\ \text{G}$$

Vertical component of earth's magnetic field,

$$B_V = B\sin\delta$$
or $$B_V = 0{\cdot}39\sin 35° = 0{\cdot}2237\ \text{G}$$

Magnetic field produced by telephone cable having four wires,

$$B' = \left(\frac{\mu_0}{4\pi}\cdot\frac{2I}{a}\right) \times 4$$

Here, $I = 1{\cdot}0$ A, $a = 4{\cdot}0$ cm $= 4 \times 10^{-2}$ m

$$\therefore \quad B' = 10^{-7} \times \frac{2 \times 1{\cdot}0 \times 4}{4 \times 10^{-2}} = 0{\cdot}2 \times 10^{-4}\ \text{T} = 0{\cdot}2\ \text{G}$$

Resultant field below the cable : According to right har thumb rule, the direction of B′ at a point below the cable w be opposite to B_H.

Therefore, at a point 4 cm below the cable, resultar horizontal component of earth's field,

$$R_H = B_H - B' = 0{\cdot}3195 - 0{\cdot}2 = 0{\cdot}1195\ \text{G}$$

Resultant vertical component of earth's field,

$$R_V = B_V = 0{\cdot}2237\ \text{G (unchanged)}$$

Therefore, resultant earth's field,

$$R = \sqrt{R_H^2 + R_V^2} = \sqrt{0{\cdot}1195^2 + 0{\cdot}2237^2}$$
$$= \sqrt{0{\cdot}0143 + 0{\cdot}0500} = \sqrt{0{\cdot}0643} = \mathbf{0{\cdot}254\ G}$$

Resultant field above the cable : According to right har thumb rule, at a point above the cable, B′ will be in the sam direction as B_H. Therefore, at a point 4 cm above the cable

$$R_H = B_H + B' = 0{\cdot}3195 + 0{\cdot}2 = 0{\cdot}5195\ \text{G},$$
$$R_V = B_V = 0{\cdot}2237\ \text{G}$$

$$\therefore \quad R = \sqrt{R_H^2 + R_V^2} = \sqrt{0{\cdot}5195^2 + 0{\cdot}2237^2}$$
$$= \mathbf{0{\cdot}566\ G}$$

Problem 4.17. A short magnet of magnetic momer $5{\cdot}25 \times 10^{-2}\ \text{J T}^{-1}$ is placed with its axis perpendicul to the earth's field direction. At what distance from th centre of the magnet on (*i*) its normal bisector, (*ii*) its axi is the resultant field inclined at 45° with the earth's fiel Magnitude of the earth's field at the place is given to be 0· G. Ignore length of the magnet in comparison to th distances involved. (*Text Problen*

---

*In order to distinguish between the symbols for current and moment of inertia, current has been denoted by *i* (in place of I) in this problem.

Sol. Here, $M = 5{\cdot}25 \times 10^{-2}\ \text{J T}^{-1}$; $B_H = 0{\cdot}42\ \text{G} = 0{\cdot}42 \times 10^{-4}\ \text{T}$

(i) On its normal bisector : Fig. 4.15 shows the point P n the normal bisector of the magnet, where the resultant field is inclined at 45° with the earth's field.

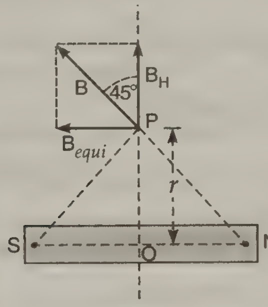


Fig. 4.15

It can happen only, if B_{equi} is equal to the horizontal omponent of the earth's field (B_H) *i.e.*

$$B_{equi} = B_H$$

For a short magnet, $B_{equi} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{M}{r^3}$

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H$$

or $$r = \left(\frac{\mu_0}{4\pi} \cdot \frac{M}{B_H}\right)^{1/3} = \left(\frac{10^{-7} \times 5{\cdot}25 \times 10^{-2}}{0{\cdot}42 \times 10^{-4}}\right)^{1/3}$$

$$= (125 \times 10^{-6})^{1/3} = 5 \times 10^{-2}\ \text{m} = \mathbf{5{\cdot}0\ cm}$$

(ii) On its axis : Fig. 4.16 shows the point P on the axis f the magnet, where the resultant field is inclined at 45° with he earth's magnetic field.

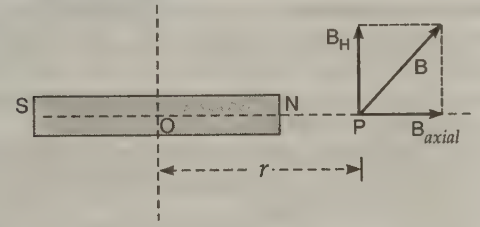


Fig. 4.16

It will be so, if

$$B_{axial} = B_H$$

For a short magnet, $B_{axial} = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2M}{r^3}$

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H$$

or $$r = \left(\frac{\mu_0}{4\pi} \cdot \frac{2M}{B_H}\right)^{1/3} = \left(\frac{10^{-7} \times 5{\cdot}25 \times 10^{-2}}{0{\cdot}42 \times 10^{-4}}\right)^{1/3}$$

$$= (250 \times 10^{-6})^{1/3} = 6{\cdot}3 \times 10^{-2}\ \text{m} = \mathbf{6{\cdot}3\ cm}$$

Problem 4.18. A compass needle free to turn in a horizontal plane is placed at the centre of a circular coil f 30 turns and radius 12 cm. The coil is in a vertical plane aking an angle of 45° with the magnetic meridian. When he current in the coil is 0·35 A, the needle points west to ast.

***(a)* Determine the horizontal component of the earth's agnetic field at the location.**

***(b)* The current in the coil is reversed and the coil is rotated about its vertical axis by an angle of 90° in the anticlockwise sense looking from the above. Predict the direction of the needle. Take the magnetic declination at the place to be zero.** *(Text Problem)*

Sol. *(a)* Horizontal component of earth's magnetic field (B_H) acts along magnetic N-S line. As the coil is placed in a vertical plane making angle 45° with magnetic meridian, the magnetic field B produced by the coil on passing current through it, will be along normal to the plane of the coil *i.e.* in a direction making angle of 45° with W-E line [Fig. 4.17].

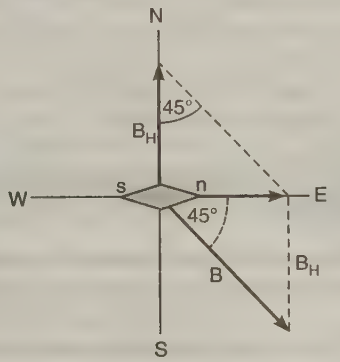


Fig. 4.17

The compass needle placed at the centre of the coil points west to east. Therefore, from the figure, it follows that

$$B_H = B \sin 45°$$

But magnetic field produced by the coil,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a}$$

$$\therefore \quad B_H = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} \sin 45°$$

Here, $I = 0{\cdot}35\ \text{A}$; $n = 30$; $a = 12\ \text{cm} = 12 \times 10^{-2}\ \text{m}$

$$\therefore \quad B_H = 10^{-7} \times \frac{2\pi \times 30 \times 0{\cdot}35 \times 0{\cdot}707}{12 \times 10^{-2}}$$

$$= 3{\cdot}9 \times 10^{-5}\ \text{T} = 0{\cdot}39\ \text{G}$$

(b) The needle will reverse its original direction *i.e.* it will point *east to west.*

Problem 4.19. A small magnet A makes 10 vibrations in 90 seconds in earth's field. When another magnet B of short length is placed 0·1 m due south of the direction of earth's field, the magnet A makes 10 vibrations in 45 seconds. Calculate the magnetic moment of magnet B. Given that $B_H = 0{\cdot}34 \times 10^{-4}$ T.

Sol. *In earth's magnetic field* : Suppose I_1 and M_1 are values of moment of inertia and magnetic moment of the magnet A. Its time period of vibration in earth's magnetic field is given by

$$t_1 = 2\pi \sqrt{\frac{I_1}{M_1 B_H}} \qquad \text{...(i)}$$

In combined magnetic field of magnet B and B_H : Suppose time period of magnet A becomes t_2, when magnet B is placed due south. Since, magnet A lies on axial line of the magnet B, the magnet A vibrates in magnetic field

$$B_H' = B_H + B_{axial}$$

Here, B_{axial} is magnetic field on the axial line of the magnet B at the point, where magnet A vibrates.

$$\therefore \quad t_2 = 2\pi\sqrt{\frac{I_1}{M_1 B_H'}} \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*), we have

$$\frac{t_1}{t_2} = \sqrt{\frac{B_H'}{B_H}} \quad \text{or} \quad B_H' = B_H\frac{t_1^2}{t_2^2}$$

$$\text{or} \quad B_H + B_{axial} = B_H\frac{t_1^2}{t_2^2}$$

$$\text{or} \quad B_{axial} = B_H\left(\frac{t_1^2}{t_2^2} - 1\right)$$

Here, $t_1 = \frac{90}{10} = 9\text{ s}; t_2 = \frac{45}{10} = 4.5\text{ s}$

and $B_H = 0.34 \times 10^{-4}$ T

$$\therefore \quad B_{axial} = 0.34\times10^{-4}\left(\frac{9^2}{4.5^2} - 1\right) = 1.02\times10^{-4}\text{ T}$$

If M_2 is magnetic moment of the magnet B (a short ba magnet), then

$$B_{axial} = \frac{\mu_0}{4\pi}\cdot\frac{2M_2}{r^3}$$

$$\therefore \quad \frac{\mu_0}{4\pi}\cdot\frac{2M_2}{r^3} = 1.02\times10^{-4}$$

Here, $\frac{\mu_0}{4\pi} = 10^{-7}\text{ T A}^{-1}\text{ m}$ and $r = 0.1$ m

$$\therefore \quad 10^{-7}\times\frac{2M_2}{(0.1)^3} = 1.02\times10^{-4}$$

or $\mathbf{M_2 = 0.51\ A\ m^2}$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. The earth's core is known to contain iron, but geologists do not regard this as a source of magnetic field. Why ? *(H.P.S.S.C.E. 2002)*

Ans. The temperature inside the earth is so high that it is impossible for the iron core to remain as a magnet and act as a source of magnetic field. The magnetic field due to earth is considered to be due to circulating electric currents induced in the iron in molten state and other conducting materials inside the earth.

Q. 4.02. Why does a bar magnet always stand in NS-direction, when suspended freely ? *(P.S.S.C.E. 2002)*

Ans. It is assumed that a huge magnet is lying deep inside the earth with its S-pole towards the geographical north and N-pole towards the geographical south.

Q. 4.03. What is geographic meridian ?

Ans. A vertical plane passing through the geographical N-S line is called geographical meridian.

Q. 4.04. What is magnetic meridian ? What is geomagnetic equator ? *(C.B.S.E. 1992)*

Ans. A vertical plane passing through the magnetic axis of the earth is called magnetic meridian.

The great circle on the earth's surface, whose plane is perpendicular to the magnetic axis is called magnetic equator.

Q. 4.05. Name the elements of earth's magnetic field. *(H.S.S.C.E. 2002, P.S.S.C.E. 1994)*

Or

Write the names of parameters of earth's magnetic field. *(P.S.S.C.E. 1999 S)*

Ans. Magnetic elements : declination, dip and horizontal component of earth's magnetic field.

Q. 4.06. Name the parameters needed to completely specify the earth's magnetic field at a point on the earth's surface. *(C.B.S.E. 1991)*

Ans. The parameters needed to completely specify the earth's magnetic field are declination, dip and horizontal component of earth's magnetic field.

Q. 4.07. A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field. *(Text Question)*

Ans. The three quantities are : (*i*) magnetic declinatio (*ii*) angle of dip and (*iii*) horizontal component of earth magnetic field.

Q. 4.08. Define declination at a place. *(H.S.S.C.E. 200*

Ans. Declination at a place is the angle between th geographic and magnetic meridian.

Q. 4.09. At a certain location in Africa, a compas needle points 15° west of the geographic north. What is th angle of declination at that point ? *(H.S.S.C.E. 199*

Ans. The declination, θ = 15°west of the geograph north.

Q. 4.10. How does the knowledge of declination at place help in navigation ? *(H.S.S.C.E. 1999*

Ans. Declination at a place is the angle between th geographic and magnetic meridians. By knowing th declination, the ship can be steered in the required directio so as to reach the destination.

Q. 4.11. A small magnet is pivoted to move freely in th magnetic meridian. At what place on the earth's surfac will the magnet be vertical ? *(C.B.S.E. 199*

Ans. At the magnetic poles of the earth.

Q. 4.12. What would be the direction of a compas needle, when placed exactly at the geo-magnetic north pol *(C.B.S.E. 1998*

Ans. It will stand vertical with its north pole downwar and south pole upwards.

Q. 4.13. Which direction would a compass point to, located right on the geomagnetic north or south pole ? *(C.B.S.E. 2000 ; Text Questio*

Ans. The magnetic compass needle can point in an direction on the geomagnetic north or south pole. It because, horizontal component of earth's field is zero at thes points. (The earth's field is vertical).

Q. 4.14. Define angle of dip at a given place ? *(H.S.S.C.E. 2002, 199*

Ans. Dip at a place is the angle made by the directio of earth's total magnetic field with the horizontal directio

Q. 4.15. What is the value of angle of dip at any plac situated on the magnetic equator of the earth ? *(H.S.S.C.E. 1999 ; C.B.S.E. 199*

Ans. It is 0°.

Q. 4.16. The angle of dip at a location in southern India is about 18°. Would you expect a greater or lesser dip angle in Britain ? *(Text Question)*

Ans. Britain is close to the magnetic north pole. Therefore, angle of dip is greater in Britain than in India. It is about 70°.

Q. 4.17. What is the expected value of dip at the magnetic poles of the earth ? *(H.P.S.S.C.E. 2000)*

Or

What is the maximum value of angle of dip ? At what places, does it occur ?

Ans. It is 90°. The magnet becomes vertical ($\delta = 90°$) at both the magnetic north and south poles.

Q. 4.18. Define neutral point in the magnetic field of a bar magnet. *(P.S.S.C.E. 2002, 1999)*

Ans. Refer to section 4.03.

Q. 4.19. It is found that the neutral points lie along the axis of a magnet placed on a table. What is the orientation of the magnet w.r.t. the earth's magnetic field ? *(I.I.T. 1976, 1974)*

Ans. The neutral points lie along the axis of the magnet ; when the magnet is placed along magnetic N-S line, with its north pole towards the south of the earth.

Q. 4.20. A magnet is placed with its north pole towards north of the earth. Predict the position of the neutral points.

Ans. On equitorial line of the magnet.

Q. 4.21. A magnet is placed with its north pole towards south of the earth. Predict the position of the neutral points.

Ans. On axial line of the magnet.

Q. 4.22. What is the principle of tangent galvanometer?

Ans. It is based on tangent law.

Q. 4.23. Write the mathematical form of tangent law in magnetism. *(H.S.S.C.E. 2002, 1994)*

Ans. $B = B_H \tan\theta$

Q. 4.24. State tangent law in magnetism. *(P.S.S.C.E. 2002 ; C.B.S.E. 2000 ; H.S.S.C.E. 1998)*

Ans. If a small bar magnet is suspended in two mutually perpendicular uniform magnetic fields $\vec{B}$ and $\vec{B}_H$, such that it comes to rest making an angle θ with the direction of field $\vec{B}_H$, then $B = B_H \tan\theta$

Q. 4.25. Define reduction factor of a tangent galvanometer. *(C.B.S.E. 2001)*

Ans. *The reduction factor of a tangent galvanometer is numerically equal to the current (in ampere) through the galvanometer, which produces a deflection of 45°, when the plane of its coil lies in the magnetic meridian.*

Q. 4.26. What is the principle of a vibration magnetometer ?

Ans. When a bar magnet freely suspended in earth's magnetic field is displaced from its equilibrium position, it executes simple harmonic motion.

Q. 4.27. Enumerate the factors on which the time of vibrations depends in a vibration magnetometer. *(I.S.C.E. 1995)*

Ans. The time period of vibration in a vibration a magnetometer depends on *(i)* the moment of inertia of the bar magnet, *(ii)* its magnetic moment and *(iii)* the horizontal component of earth's magnetic field.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. What is the probable cause of earth's magnetism ? *(P.S.S.C.E. 2002, 2000, 1999 ; H.P.S.S.C.E. 1997)*

Ans. The exact cause of earth's magnetism is not yet known. It must be basically due to magnetic effect of currents. Earth rotates about its axis of rotation. Since every substance is made of charged particle, the rotating earth is equivalent to circulating current and it gives rise to earth's magnetism.

Q. 4.02. Define three independent quantities conventionally used to specify the earth's magnetic field at a point. *(H.P.S.S.C.E. 1999 S)*

Or

Name and define the three magnetic elements at a place. *(P.S.S.C.E. 1997, 1992 ; C.B.S.E. 1991)*

Or

What are the parameters of earth's magnetic field ? Define each. *(P.S.S.C.E. 1999, 1993)*

Ans. Three magnetic elements are : declination, dip and horizontal components of earth's magnetic field.

For definitions, refer to section 4.02.

Q. 4.03. A magnetic needle is placed on a cork floating on a still lake in the northern hemisphere. Does this needle together with the cork move towards the north of the lake ? *(I.I.T. 1973)*

Ans. The magnetic needle does not experience any net force and hence the needle together with the cork will not move towards the north of the lake. However, if the needle is inclined to the magnetic meridian, a torque will act on the needle and after a few oscillations, the needle will set itself along the magnetic meridian.

Also refer to chapter 3 (Magnetic dipole)-conceptual SAQ 3.07.

Q. 4.04. Define angle of dip. Deduce the relation connecting angle of dip and horizontal component of components of earth's magnetic field. *(C.B.S.E. 2002)*

Ans. *The angle of dip at a place is defined as the angle made by the direction of the earth's total magnetic field with the horizontal direction.*

For the derivation of relation between δ and B_H, refer to section 4.02.

Q. 4.05. Define the terms magnetic inclination and horizontal component of earth's magnetic field at a place. Establish the relationship between the two with the help of a diagram. *(C.B.S.E. 1998)*

Ans. Refer to section 4.02.

Q. 4.06. Define the term angle of dip. Derive the relation between angle of dip and the resultant magnetic field of earth at a place. *(C.B.S.E. 2001)*

Ans. Refer to section 4.02.

Q. 4.07. Horizontal and vertical components of earth's magnetic field at a place are 0·22 tesla and 0·38 tesla respectively. Find the resultant intensity of earth's magnetic field. *(H.S.S.C.E. 1995)*

Ans. Here, $B_H = 0{\cdot}22$ T ; $B_V = 0{\cdot}38$ T

Now,
$$B = \sqrt{B_H^2 + B_V^2}$$
$$= \sqrt{(0\cdot 22)^2 + (0\cdot 38)^2} = \sqrt{0\cdot 1928} = \mathbf{0{\cdot}44\ T}$$

Q. 4.08. Horizontal components and vertical component of earth's magnetic field at a place are equal. What is the angle of dip at this place ?

(H.S.S.C.E. 2002 ; C.B.S.E. 2000, 1999, 1997)

Or

The ratio of vertical component to the horizontal component of earth's magnetic field at a given place is 1. What is the angle of dip at that place ?

(C.B.S.E. 1998 S)

Ans. Here, $B_V = B_H$

Now, $\tan\delta = \frac{B_V}{B_H} = 1$ or $\delta = 45°$

Q. 4.09. Horizontal component of earth's magnetic field at a place is $\sqrt{3}$ times the vertical component. What is the value of angle of dip at this place ? *(C.B.S.E. 1997)*

Ans. Here, $B_H = \sqrt{3}\, B_V$

Now, $\tan\delta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3}\, B_V} = \frac{1}{\sqrt{3}}$ or $\delta = 30°$

Q. 4.10. The ratio of the horizontal component to the resultant magnetic field of earth at a given place is $\frac{1}{\sqrt{2}}$. What is the angle of dip at that place ? *(C.B.S.E. 1998 S)*

Ans. Here, $\frac{B_H}{B} = \frac{1}{\sqrt{2}}$

Now, $\cos\delta = \frac{B_H}{B} = \frac{1}{\sqrt{2}}$ or $\delta = 45°$

Q. 4.11. The horizontal component of earth's magnetic field at a place is $0{\cdot}3 \times 10^{-4}$ T. If the angle of dip is 60°, what is the total earth's magnetic field ? *(H.S.S.C.E. 2001, 1992,)*

Ans. Here, $\delta = 60°$; $B_H = 0{\cdot}3 \times 10^{-4}$ T

Now, $B_H = B\cos\delta$

or $$B = \frac{B_H}{\cos\delta} = \frac{0{\cdot}3 \times 10^{-4}}{\cos 60°} = \frac{0{\cdot}3 \times 10^{-4}}{0{\cdot}5}$$
$$= 0{\cdot}6 \times 10^{-4}\ \text{T}$$

Q. 4.12. The horizontal component of earth's magnetic field is 0·2 gauss and total magnetic field is 0·4 gauss. Find angle of dip. *(H.S.S.C.E. 1996)*

Ans. Here, $B_H = 0{\cdot}2$ G ; $B = 0{\cdot}4$ G

Now, $B_H = B\cos\delta$

or $\cos\delta = \frac{B_H}{B} = \frac{0{\cdot}2}{0{\cdot}4} = 0{\cdot}5$ or $\delta = 60°$

Q. 4.13. If a compass needle be placed on the magnetic north pole of the earth, then how does it behave ? If a dip needle be placed at the same place, then what will be its behaviour ?

Ans. A compass needle is free to rotate in horizontal plane, while a dip needle can freely rotate in vertical plane.

The earth has only vertical component of its magnetic field at the magnetic poles. The vertical component of earth's field will exert torque on the magnetic needle so as to align it along its direction. As the compass needle can not rotate in vertical plane, it will rest horizontally, when placed on the magnetic north pole of the earth. On the other hand, the dip needle will come to rest along vertical.

Q. 4.14. Define dip and tangent law in magnetism.

(H.S.S.C.E. 1993)

Ans. For dip, refer to section 4.02 and for tangent law, refer to section 4.04.

Q. 4.15. State tangent law in magnetism. Which are t two required magnetic fields in a tangent galvanometer

(P.S.S.C.E. 1999 S, 199

Ans. For statement of tangent law, refer to section 4.0

One of the two required magnetic fields in a tange galvanometer is the field produced by the coil of t galvanometer on passing current and the other is t horizontal component of earth's magnetic field.

Q. 4.16. Why do we set the coil of tangent galvanomet in the magnetic meridian ? How is accuracy of adjustme tested ? *(C.B.S.E. 1999*

Ans. A tangent galvanometer is based on tangent la The magnet in the magnetometer box has to be placed in tw uniform mutually perpendicular magnetic fields. So that t magnetic field produced on passing current through the c of the galvanometer is perpendicular to the horizont component of earth's magnetic field, the coil is set in magne meridian.

The accuracy of the setting of the galvanometer can tested by rotating the coil through exactly 90° and th passing current through it. In case, the setting was accura the galvanometer will not show any deflection on passi current.

Q. 4.17. State the working principle of a tange galvano-meter. Write the mathematical expression f measuring currents using the tangent galvanometer.

(C.B.S.E. 199

Ans. It is based on tangent law in magnetism. F statement of principle, refer to section 2.04.

The current measured by using a tangent galvanomet is given by

$$I = K\tan\theta,$$

where $K = \frac{B_H}{G} = \frac{2a}{n\mu_0} B_H$ is called reduction factor.

Q. 4.18. State two factors by which a tangent galvan meter can be made more sensitive. Why is a tange galvanometer not considered to be practically a very usef instrument ? *(C.B.S.E. 200*

Ans. The tangent galvanometer can be made mo sensitive by (*i*) increasing the number of turns of its coil a (*ii*) decreasing the radius of the coil.

In a tangent galvanometer deflection produced is n directly proportional to the current passed through it. Sin it can not measure current directly, it is not considered to practically a very useful instrument. A tangent galvanomet is usually used to detect the current in an electric circuit.

Q. 4.19. What is reduction factor of a tangent galvan meter ? Explain. *(H.S.S.C.E. 199*

Ans. Refer to section 4.05.

Q. 4.20. Two tangent galvanometers of the same radi of coils, have their number of turns in the ratio 2 : 3. Whi galvanometer has larger value of reduction factor ?

(C.B.S.E. 200

Ans. Let K_1 and K_2 be reduction factors of the tv tangent galvanometers having number of turns 2 n and 3 respectively. Then,

$$K_1 = \frac{2a}{(2n)\mu_0} B_H \quad \text{and} \quad K_2 = \frac{2a}{(3n)\mu_0} B$$

It follows that galvanometer having lesser number turns has larger value of reduction factor.

Q. 4.21. Deduce an expression for comparing the reduction factors of two tangent galvanometers, connected in series, showing mean deflections θ_1, θ_2 for a given current. *(C.B.S.E. 2001)*

Ans. Let K_1 and K_2 be reduction factors of the two galvanometers. Since the two galvanometers are connected in series, same current I will pass through them. Therefore,

$$I = K_1 \tan\theta_1 \quad \text{and} \quad I = K_2 \tan\theta_2$$

Thus, $K_1 \tan\theta_1 = K_2 \tan\theta_2$ or $\dfrac{K_1}{K_2} = \dfrac{\tan\theta_2}{\tan\theta_1}$

Q. 4.22. Give three advantages of a moving coil galvanometer over the tangent galvanometer. *(P.S.S.C.E. 1999 S)*

Ans. 1. In a moving coil galvanometer, the deflection produced is directly proportional to the amount of current passed through it.

2. A moving coil galvanometer does not require any particular setting. The coil of a tangent galvanometer has to be set in magnetic meridian.

3. The magnetic field produced by the coil of tangent galvanometer is not uniform over the whole length of the small magnet of the magnetometer box.

Q. 4.23. A magnet makes angular oscillations in a horizontal plane with time periods T_1 and T_2 at two places, where the horizontal components of earth's magnetic field are B_H and B'_H respectively. Deduce an expression to compare the horizontal components at the two places. *(C.B.S.E. 2001)*

Ans. Let M be magnetic moment and I be moment of inertia of the magnet.

Then, $T_1 = 2\pi\sqrt{\dfrac{I}{M B_H}}$ and $T_2 = 2\pi\sqrt{\dfrac{I}{M B_H'}}$

$\therefore$ $\dfrac{T_1}{T_2} = \sqrt{\dfrac{B_H'}{B_H}}$ or $\dfrac{B_H}{B_H'} = \dfrac{T_2^2}{T_1^2}$

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For **a**mbitious, **b**rilliant & **c**urious Students

Q. 4.01. If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground ? *(Text Question)*

Ans. The magnetic field lines of earth would seem to come out of the ground. It is because, Melbourne (Australia) is in southern hemisphere.

Q. 4.02. The earth's core is known to contain iron. Yet geologists do not regard this as a source of earth's magnetism. Why ? *(Text Question)*

Ans. The molten iron is above its Curie temperature and hence it cannot retain magnetism.

Q. 4.03. The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion ? *(Text Question)*

Ans. At large distances, the earth's magnetic field gets modified by the field produced by the motion of ions in the earth's ionosphere.

Q. 4.04. The earth field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} J T^{-1} located at its centre. Check the order or magnitude of this number in some way. *(Text Question)*

Ans. The magnetic field at a point on equatorial line of the magnetic dipole (earth as magnetic dipole) is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$

Here, $M = 8 \times 10^{22}$ J T^{-1}

and $r = 6{\cdot}4 \times 10^6$ m (radius of earth)

$$\therefore \quad B = 10^{-7} \times \frac{8 \times 10^{22}}{(6{\cdot}4 \times 10^6)^3} = 0{\cdot}3 \times 10^{-4}\ \text{T} = 0{\cdot}3\ \text{G}$$

It is really of the order of the observed field of earth.

Q. 4.05. Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence ? Explain. *(Text Question)*

Ans. A weak field of the order of 10^{-12} T can cause the charged particles to move along circular paths of very large radii. Over a small distance, we may not be able to notice the deflection in the path of the charged particles but over large interstellar distance, the deflection is quite noticeable.

Q. 4.06. The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might bethe 'battery' (*i.e.* the source of energy) to sustain these currents ? *(Text Question)*

Ans. A possible explanation can be the phenomenon of radioactivity.

Q. 4.07. The earth's magnetic field varies from point to point in space. Does it also change with time ? If so, on what time scale does it change appreciably ? *(Text Question)*

Ans. Yes, it changes with time. After a few hundred years, the earth's magnetic field undergoes an appreciable change.

Q. 4.08. The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past ? *(Text Question)*

Ans. Analysis of the rock magnetism (earth's magnetic field gets recorded in certain rocks during solidification, though weekly) offers clues to geomagnetic history.

Q. 4.09. A dip needle with a circular scale is mounted on a horizontal axis [Fig. 4.18]. How can the direction of the magnetic meridian be found with the help of this needle ?

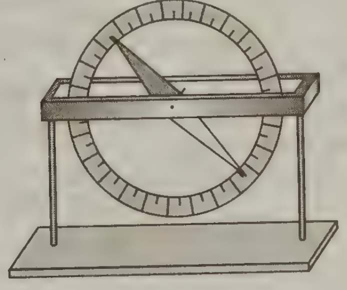


Fig. 4.18

Ans. Rotate the needle, till its take a vertical position. In this position, the two poles of the dip needle experience force due to the vertical component of the earth's magnetic field. The horizontal component of the earth's magnetic field exerts only a sideways pressure on the needle. In this position, the dip needle lies in a plane perpendicular to the magnetic meridian. If the needle is turned through 90° about a vertical axis, it will lie in magnetic meridian.

Q. 4.10. Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all ? *(Text Question)*

Ans. The earth's magnetic field is only approximately a dipole field. The local N-S poles may be oriented due to the different deposits of magnetised minerals.

Q. 4.11. In order to keep the dip needle [refer to Fig. 4.19] in a horizontal position, a load of 0·01 gf is suspended from its top end. Find the magnitudes of the horizontal and vertical components of the intensity of earth's magnetic field. Given that the angle of dip is 40° and the pole strength of the dip needle is 0·98 A m.

Ans. Let m be pole strength of each pole of the dip needle and $2l$ be its length. The dip needle experiences a torque due to the vertical component of the earth magnetic field. The torque due to the load Mg suspended from its top end opposes the torque on the needle due to the vertical component of the earth's magnetic field [Fig. 4.19]. When the dip needle is horizontal,

$$m\,B_V \times 2l = Mg \times l$$

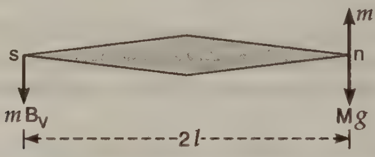


Fig. 4.19

or $$B_V = \frac{Mg}{2m}$$

Here $m = 0{\cdot}98$ A m

$Mg = 0{\cdot}01$ gf $= 0{\cdot}01 \times 980 \times 10^{-5}$ N ;

$$\therefore \quad B_V = \frac{0{\cdot}01 \times 980 \times 10^{-5}}{2 \times 0{\cdot}98} = 5 \times 10^{-5}\ \text{T}$$

Since angle of dip, $\delta = 40°$, the horizontal component earth's magnetic field is given by

$$B_H = \frac{B_V}{\tan\delta} = \frac{5\times10^{-5}}{\tan 40°} = \frac{5\times10^{-5}}{0{\cdot}8391} = 5{\cdot}96 \times 10^{-5}\ \text{T}$$

Q. 4.12. How many neutral points on a horizont board are there and why, when a magnet is hel vertically on the board ? *(I.I.T. 198*

Ans. Consider a magnet NS placed vertically on a hor zontal board as shown in Fig. 4.20. The neutral point on th horizontal board will be obtained, where the component

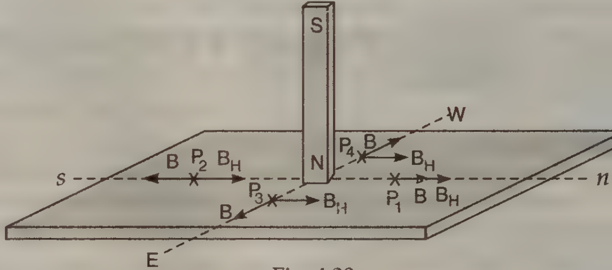


Fig. 4.20

magnetic field (B) due to magnet in the plane of board neutralised by the horizontal component of earth's magnet field (B_H). The directions of magnetic fields B and B_H hav been shown at four points P_1, P_2, P_3 and P_4. It follows th the two fields can balance each other only at point P Therefore, when a magnet is placed vertically on a horizont board, only one neutral point will be obtained.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain the possible causes of earth's magnetic field. *(P.S.S.C.E. 1998 S)*
2. Define parameters of earth's magnetic field. *(P.S.S.C.E. 2000)*
3. Explain the three magnetic elements of earth at a place. *(H.S.S.C.E. 2002)*
4. Define magnetic elements of earth's magnetic field at a place. *(P.S.S.C.E. 2000)*
5. Explain the terms : (*i*) Declination, (*ii*) Dip and (*iii*) the magnetic field due to earth at a point.
6. Explain briefly magnetic elements. *(P.S.S.C.E. 1991)*
7. What are the elements of the earth's magnetic field ? Explain them briefly. *(Pre-degree Kerala, 1994 ; P.S.S.C.E. 1993)*
8. Name the elements of earth's magnetic field at a place. Explain their meaning. *(C.B.S.E. 1995)*
9. Derive tangent law in magnetism. *(H.S.S.C.E. 1996 ; Karnataka, 1988)*
10. State and prove tangent law in magnetism. *(P.S.S.C.E. 2001, 2000, 1999 ; H.S.S.C.E. 2001, 1998 S, 199 (H.P.S.S.C.E. 199*
11. Describe the working of a tangent galvanometer. Defi its reduction factor. *(H.S.S.C.E. 200*
12. State the underlying principle of a tangent galvanomet Write briefly, how it can be used to determine th horizontal component of earth's magnetic field at a give place. *(C.B.S.E. 1998*
13. Describe the use of a vibration magnetometer for com paring magnetic moments of two bar magnets of same si and mass. *(C.B.S.E. 1998 ,199*
14. Explain, how can magnetic moments of two unequ magnets be compared with the help of vibratio magnetometer. *(C.B.S.E. 1996 ; H.S.S.C.E. 199*
15. How will you compare the magnetic moments of two b magnets of equal size using a vibration magnetometer *(H.S.S.C.E. 200*
16. Describe, how the horizontal components of earth magnetic field at two places may be compared by using vibration magnetometer. *(C.B.S.E. 199*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. (*a*) Explain the three magnetic elements of earth's magnetic field at a place.
(*b*) What are the possible causes of Earth's magnetism ? Explain. (*P.S.S.C.E. 2001*)
2. Define neutral point. How will you find the magnetic moment of a bar magnet by locating its neutral point, when magnet is placed with its north pole towards (*a*) north pole of the earth and (*b*) south pole of the earth ?
3. Explain the principle, construction and theory of tangent galvanometer. (*P.S.S.C.E. 1999, 1998 S, 1993 ; H.S.S.C.E. 1999 S ; C.B.S.E. 1993*)
4. With the help of a diagram, explain the principle and working of a tangent galvanometer. (*C.B.S.E. 1996*)
5. Explain the construction, principle and theory of tangent galvanometer. State its merits also. (*P.S.S.C.E. 2000*)
6. Describe with the necessary theory, the working of a tangent galvanometer. How is it used for the measurement of current ? (*Pre-degree Kerala, 1991*)
7. Describe the principle and construction of a deflection magnetometer. How will you compare the magnetic moments of two equal bar magnets by placing the deflection magnetometer in end-on position.
8. Describe the principle and construction of a deflection magnetometer. How will you compare the magnetic moment of two equal bar magnets by placing the deflection magnetometer in broad side-on position.
9. Describe briefly the principle, construction and theory of a vibration magnetometer. (*I.S.C.E. 1994*)
10. State and derive tangent law in magnetism. Describe giving relevant formulae, how you will determine the magnetic moment (M) of a magnet and the horizontal component (B_H) of the earth's magnetic field using deflection and vibration magnetometers. (*H.S.S.C.E. 1995*)

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Magnetic elements

1. A ship is sailing due west according to a Mariner's compass. If the declination of that place is 18° east of north, what is the true direction of the ship ? [**Ans.** 72° west of north]
2. The vertical and horizontal components of earth's magnetic field at a place are 0·2 G and 0·3464 G respectively. Calculate the angle of dip and earth's magnetic field at that place. [**Ans.** 30°, 0·4 G]
3. At 52° from the magnetic meridian, a magnetic needle in a vertical plane makes an angle of 45° with the horizontal plane. Find the actual angle of dip at that place. [**Ans.** 31·62°]
4. A dip circle is so set that it moves freely in the magnetic meridian and shows angle of dip equal to 45°. Now the dip circle is rotated through 30°. What value of dip, will the needle show now ? (*Roorkee, 1983*) [**Ans.** 49·1°]

Type B. On Neutral points

5. A small bar magnet has a magnetic moment 5 A m^2. The neutral point is obtained on axial line, when it is placed in magnetic meridian with its north pole pointing south of earth and neutral point is obtained on equitorial line, when it is placed with its north pole pointing north of earth. If horizontal component of earth's field is 0·38 G, find the position of neutral points in two cases. [**Ans.** 29·7 cm, 23·6 cm]
6. A short bar magnet is placed in a horizontal plane with its axis in the magnetic meridian. Null points are found on its equitorial line (*i.e.* its normal bisector) at 12·5 cm from the centre of the magnet. The earth's magnetic field at the place is 0·38 G and the angle of dip is zero.
(*a*) What is the total magnetic field at points on the axis of the magnet located at the same distance (12·5 cm) as the null-points from the centre ?
(*b*) Locate the null-points, when the bar is turned around by 180°.
Assume that the length of the magnet is negligible as compared to the distance of the null-points from the centre of the magnet. [**Ans.** (*a*) 1·14 G ; (*b*) 15·7 cm]
7. A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0·36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null points (*i.e.* 14 cm) from the centre of the magnet ? (*Text Problem*) [**Ans.** 0·54 G]
8. If the bar magnet in problem no. 7 is turned around by 180°, where will the new null points be located ? (*Text Problem*) [**Ans.** 11·1 cm]

Type C. On Tangent law and Tangent galvanometer

9. In a tangent galvanometer, when a current of 10 mA is passed, the deflection is 31°. By what percentage, the current has to be increased, so as to produce a deflection of 42°. [**Ans.** 50%]
10. A 2 turn coil of radius 10 cm is placed with its plane in the magnetic meridian. A small magnetic needle is suspended at the centre of the coil by a torsion free silk thread. On passing a current through the coil, the needle is deflected through 45°. Calculate the strength of the current, if the horizontal component of the earth's magnetic field at the location is 1·6 × 10^{-5} T.
Given, $\mu_0 = 4\,\pi \times 10^{-7}$ T m A^{-1}. (*I.S.C.E. 1993*) [**Ans.** 1·273 A]
11. A tangent galvanometer with a coil of 35 turns and mean radius 11 cm is used at a place, where horizontal component of earth's magnetic field, $B_H = 0{\cdot}32$ gauss. When a current is passed through the galvanometer, the deflection is 35°. Calculate the current. [**Ans.** 0·112 A]
12. A circular coil of radius 20 cm and 20 turns of wire is mounted vertically with its plane in magnetic meridian. A small magnetic needle is placed at the centre of this coil and it is deflected through 45°, when a current is passed through the coil. Calculate current in the coil, if H = 0·34 × 10^{-4} T. (*I.I.T. 1983*) [**Ans.** 0·54 A]
13. The coil in a tangent galvanometer is 16 cm in radius. Find the number of turns of the wire that should be wound on it, if a current of 40 mA is to produce a deflection of 45°. Horizontal component of earth's magnetic field = 0 · 36 × 10^{-4} T. (*C.B.S.E. 2001 S*) [**Ans.** 229]
14. The radius of a tangent galvanometer is 16 cm. Find the number of turns in it, if a current of 4·0 A is to produce

a deflection of 45°. Horizontal component of earth's magnetic field is $0{\cdot}31 \times 10^{-4}$ T. (*H.S.S.C.E. 1997*) **[Ans. 2]**

15. Two tangents galvanometers have coils of the same radii, differing only in their number of turns. They are connected in series. When a steady current is passed in the circuit, the mean deflections in the galvanometers are θ_1, θ_2. Deduce an expression for the ratio of the number of turns of the galvanometers. (*C.B.S.E. 2000*)

[Ans. $\frac{n_1}{n_2} = \frac{\tan\theta_1}{\tan\theta_2}$]

16. Two tangent galvanometers having number of turns in the ratio $\sqrt{3} : 2$ are connected in series to a d.c. source. If the mean deflections of the two tangent galvanometers are 30° and 45° respectively, calculate the ratio of radii of the coils of the tangent galvanometers. (*C.B.S.E. 2002*)

[Ans. 1·5]

17. Two tangent galvanometers A and B have their number of turns in the ratio 1 : 3 and diameters in the ratio 1 : 2. Explain with reason : (*a*) Which galvanometer has a greater reduction factor ? (*b*) Which galvanometer shows greater deflection, when both are connected in series to a d.c. source ? (*C.B.S.E. 1999*) **[Ans. (*a*) A ; (*b*) B]**

Type D. On Vibration magnetometer

18. A magnet executes 55 vibrations in one minute, where horizontal component of earth's field is 0·36 G. If the moment of inertia of the bar magnet is 10^{-5} kg m^2, find the magnetic moment of the bar magnet. **[Ans. 9·215 A m^2]**

19. A magnetic needle pivoted through its centre of mass and free to rotate in a plane containing a uniform magnetic field of 100 G is displaced slightly from its stable equilibrium. The frequency of its angular oscillations of small amplitudes is measured to be 1·5 s^{-1}. If the moment of inertia of the needle about its axis of rotation is $7{\cdot}5 \times 10^{-6}$ kg m^2, determine the magnetic moment of the needle. **[Ans. 0·067 A m^2]**

20. A magnet makes 30 vibrations per minute at a place, where horizontal component of earth's field is 0·18 G. At another place, if it takes 1·6 seconds to complete one vibration. What is the value of horizontal component of earth's field there ? **[Ans. 0·28 G]**

MISCELLANEOUS PROBLEMS

21. A magnetic needle free to rotate about the vertical direction (compass) points 3·5° west of the geographic north. Another magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 18° with the horizontal. The magnitude of th horizontal component of the earth's magnetic field at th place is known to be 0·40 G. What is the direction ar magnitude of the earth's magnetic field at the place ?

[Ans. 0·42 G making an angle of 18° with horizont in the magnetic meridian ; while the magnet meridian is 3·5° west of geographic meridiar

22. A bar magnet of length 8 cm and having a pole streng of 1 A m is placed vertically on a horizonal table with i N-pole on the table. A neutral point is found on the tab at a distance of 6 cm south of the magnet. Calculate th earth's horizontal magnetic field. **[Ans.** $2{\cdot}18 \times 10^{-5}$

23. Two tangent galvanometers have radii 0·075 m and 0·1 r number of turns 15 and 10 and resistances of 8 ohm ar 12 ohm respectively. If on connecting them in parallel a circuit, the first galvanometer gives a deflection of 6C then calculate deflection in the second galvanometer.

[Ans. 30

24. A bar magnet, held horizontally, is set into angul oscillations in earth's magnetic field. It has time perio T and T′ at two places, where the angles of dip are δ ar θ_2 respectively. Deduce an expression for the ratio of tl resultant magnetic fields at the two places.

(*C.B.S.E. 1999*) $\left[\textbf{Ans. } \frac{B}{B'} = \frac{\cos\delta'}{\cos\delta} \times \left(\frac{T'}{T}\right)^2\right.$

25. A magnet is suspended so that it may oscillate in tl horizontal plane. It performs 20 oscillations per minute a place where the angle of dip is 30° and 15 oscillations p minute, where the angle of dip is 60°. Compare the earth total magnetic field at these two places. **[Ans.** $16 : 9\sqrt{3}$

26. A magnet makes 10 oscillations per minute at a plac where the angle of dip is 45° and the total intensity earth's magnetism is 0·4 G. Calculate the number oscillations made per second by the same magnet another place, where the angle of dip is 60° and intensi of the field is equal to 0·5 G. (*M.N.R. 198*

[Ans. 0 · 157 s

27. A magnet free to oscillate under the earth's horizont field has a period of T seconds. Another magnet of leng 16 cm is kept east of it with its middle point at a distan of 20 cm and its axis in magnetic meridian. The tin period of the first magnet is reduced to half. Find the pc strength of the other magnet. Given, horizont component of earth's field = 0·2 G. **[Ans.** 37·5 A r

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Proceed as in solved problem no. 4.02.

2. $\tan\delta = \frac{B_V}{B_H} = \frac{0{\cdot}2}{0{\cdot}3464} = 0{\cdot}5774$ or $\delta = 30°$

Now, $B_H = B\cos\delta$ or $B = \frac{B_H}{\cos\delta} = \frac{0{\cdot}3464}{\cos 30°} = \mathbf{0{\cdot}4\ G}$

3. At 52° of magnetic meridian, the apparent value of horizontal component of earth's magnetic field,

$$B_H' = B_H \cos 52°$$

Apparent angle of dip δ' is given by

$$\tan\delta' = \frac{B_V}{B_H'} \quad \text{or} \quad \tan 45° = \frac{B_V}{B_H \cos 52°}$$

or $\frac{B_V}{B_H} = \cos 52° = 0{\cdot}6157$

But $\frac{B_V}{B_H} = \tan\delta$,

where δ is true angle of dip.

$\therefore \tan\delta = 0{\cdot}6157$ or $\delta = \mathbf{31{\cdot}62°}$

4. Proceed as in solved problem no. 4.04.

5. ***In first case*** **:** At the neutral point on axial line of a sho magnet,

$$\frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3} = B_H \quad \text{or} \quad \frac{10^{-7}\times 2\times 5}{r^3} = 0{\cdot}38 \times 10^{-4}$$

or $r^3 = 0{\cdot}0263$ or $r = 29{\cdot}7 \times 10^{-2}$ m = **29·7 cm**

In second case **:** At the neutral point on equitorial line o short magnet,

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = B_H \quad \text{or} \quad \frac{10^{-7}\times 5}{r^3} = 0{\cdot}38 \times 10^{-4}$$

or $r^3 = 0{\cdot}0132$ or $r = 23{\cdot}6 \times 10^{-2}$ m = **23·6 cm**

6. At the neutral point on equitorial line of a short magne

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = B_H \quad \text{or} \quad \frac{10^{-7}\times M}{(12{\cdot}5\times 10^{-2})^3} = 0{\cdot}38 \times 10^{-4}$$

(*a*) Now, $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = \frac{10^{-7} \times 2 \times 0{\cdot}74}{(12{\cdot}5 \times 10^{-2})^3}$

$$= 0{\cdot}76 \times 10^{-4}\ \text{T} = 0{\cdot}76\ \text{G}$$

Therefore, total magnetic field,

$$B = B_{axial} + B_H = 0{\cdot}76 + 0{\cdot}38 = \mathbf{1{\cdot}14\ G}$$

(*b*) When magnet is turned through 180°, neutral points will be obtained on axial line.

Now, at the neutral point on axial line of a short magnet,

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H \quad \text{or} \quad \frac{10^{-7} \times 2 \times 0{\cdot}74}{r^3} = 0{\cdot}38 \times 10^{-4}$$

or $\quad r^3 = 3{\cdot}895 \times 10^{-3} \quad$ or $\quad r = 0{\cdot}157\ \text{m} = \mathbf{15{\cdot}7\ cm}$

7. At the neutral point on axial line of a short magnet,

$$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = B_H \quad \text{or} \quad \frac{10^{-7} \times 2 \times M}{(14 \times 10^{-2})^3} = 0{\cdot}36 \times 10^{-4}$$

or $\quad M = 0{\cdot}494\ \text{A m}^2$

Now, $B_{equi} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = \frac{10^{-7} \times 0{\cdot}494}{(14 \times 10^{-2})^3}$

$$= 0{\cdot}18 \times 10^{-4}\ \text{T} = 0{\cdot}18\ \text{G}$$

Therefore, total magnetic field,

$$B = B_{equi} + B_H = 0{\cdot}18 + 0{\cdot}36 = \mathbf{0{\cdot}54\ G}$$

8. When magnet is turned through 180°, neutral points will be obtained on equitorial line.

Now, at the neutral point on equitorial line of a short magnet,

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = B_H \quad \text{or} \quad \frac{10^{-7} \times 0{\cdot}494}{r^3} = 0{\cdot}36 \times 10^{-4}$$

or $\quad r^3 = 1{\cdot}37 \times 10^{-3} \quad$ or $\quad r = 0{\cdot}111\ \text{m} = \mathbf{11{\cdot}1\ cm}$

9. Now, $\quad I = K \tan\theta$

$$\therefore \quad \frac{I_1}{I_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

or $\quad I_2 = I_1 \times \frac{\tan\theta_2}{\tan\theta_1} = 10 \times \frac{\tan 42^\circ}{\tan 31^\circ} = 15\ \text{mA}$

% increase required in the value of current,

$$\frac{I_2 - I_1}{I_1} \times 100 = \frac{15 - 10}{10} \times 100 = \mathbf{50\%}$$

10. $I = \frac{2a}{n\mu_0} B_H \tan\theta = \frac{2 \times 10 \times 10^{-2} \times 1{\cdot}6 \times 10^{-5} \times \tan 45^\circ}{2 \times 4\pi \times 10^{-7}}$

$$= \mathbf{1{\cdot}273\ A}$$

11. Proceed as in problem no. 10.
12. Proceed as in problem no. 10.
13. $I = \frac{2a}{n\mu_0} B_H \tan\theta \quad$ or $\quad n = \frac{2a}{\mu_0 I} B_H \tan\theta$

or $n = \frac{2 \times 16 \times 10^{-2} \times 0{\cdot}36 \times 10^{-4} \times \tan 45^\circ}{4\pi \times 10^{-7} \times 40 \times 10^{-3}} = \mathbf{229}$

14. Proceed as in problem no. 13.
15. Let n_1 and n_2 be number of turns in the two tangent galvanometers. Suppose that a current I passes through the two galvanometers, when they are connected in series.

For first tangent galvanometer,

$$I = \frac{2a}{n_1 \mu_0} B_H \tan\theta_1 \quad \ldots(i)$$

For second tangent galvanometer,

$$I = \frac{2a}{n_2 \mu_0} B_H \tan\theta_2 \quad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{2a}{n_1 \mu_0} B_H \tan\theta_1 = \frac{2a}{n_2 \mu_0} B_H \tan\theta_2$$

or $\quad \frac{n_1}{n_2} = \frac{\tan\theta_1}{\tan\theta_2}$

16. Let a_1 and a_2 be radii of the coils of the two tangent galvanometers. As the number of turns of the coils of the two galvanometers are in the ratio $\sqrt{3} : 2$, let $\sqrt{3}\,n$ and $2\,n$ be the their number of turns. Suppose that a current I passes though the galvanometers, when they are connected in series.

For first galvanometer, $\theta = 30^\circ$

$$\therefore \quad I = \frac{2a_1}{\sqrt{3}\,n\mu_0} B_H \tan 30^\circ \quad \ldots(i)$$

For second galvanometer, $\theta = 45^\circ$

$$\therefore \quad I = \frac{2a_2}{2n\mu_0} B_H \tan 45^\circ \quad \ldots(ii)$$

From equation (*i*) and (*ii*), we have

$$\frac{2a_1}{\sqrt{3}\,n\mu_0} B_H \tan 30^\circ = \frac{2a_2}{2n\mu_0} B_H \tan 45^\circ$$

or $\quad \frac{a_1}{a_2} = \frac{\sqrt{3} \times \tan 45^\circ}{2 \times \tan 30^\circ} = \frac{\sqrt{3} \times 1}{2 \times 1/\sqrt{3}} = \frac{3}{2} = \mathbf{1{\cdot}5}$

17. Here, $n_1 = n$, $n_2 = 3\,n$, $a_1 = a$ and $a_2 = 2\,a$

(*a*) Reduction factor, $K = \frac{2a}{n\mu_0} B_H$

$$\therefore \quad K_A = \frac{2a_1}{n_1\mu_0} B_H = \frac{2a}{n\mu_0} B_H$$

and $\quad K_B = \frac{2a_2}{n_2\mu_0} B_H = \frac{2 \times 2a}{3n \times \mu_0} B_H = \frac{2}{3} K_A$

Therefore, $K_A > K_B$ *i.e.* **galvanometer A has a greater reduction factor.**

(*b*) Let θ_1 and θ_2 be deflections produced in the two galvanometers, when they are connected in series.

For first galvanometer,

$$I = K_A \tan\theta_1 = \frac{2a}{n\mu_0} B_H \tan\theta_1 \quad \ldots(i)$$

For second galvanometer,

$$I = K_B \tan\theta_2 = \frac{4a}{3n\mu_0} B_H \tan\theta_2 \quad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{2a}{4\mu_0} B_H \tan\theta_1 = \frac{4a}{3n\mu_0} B_H \tan\theta_2$$

or $\quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{2}{3}$

Therefore, $\theta_1 < \theta_2$ *i.e.* **galvanometer B shows greater** deflection.

18. Now, $T = 2\pi\sqrt{\frac{I}{M B_H}} \quad$ or $\quad M = \frac{4\pi^2 I}{T^2 B_H}$

Here, $T = \frac{60}{55}$ s, $B_H = 0{\cdot}36\ \text{G} = 0{\cdot}36 \times 10^{-4}\ \text{T}$

and $\quad I = 10^{-5}\ \text{kg m}^2$

$$\therefore \quad M = \frac{4\pi^2 \times 10^{-5}}{(60/55)^2 \times 0{\cdot}36 \times 10^{-4}} = \mathbf{9{\cdot}215\ Am^2}$$

19. Here, $B = 100\ \text{G} = 100 \times 10^{-4} = 0{\cdot}01\ \text{T}$, $I = 7{\cdot}5 \times 10^{-6}\ \text{kg m}^2$

$$\nu = 1{\cdot}5\ \text{s}^{-1}$$

Now, $T = 2\pi\sqrt{\frac{I}{MB}}$

or $\quad M = \frac{4\pi^2 I}{T^2 B} = \frac{4\pi^2 I \nu^2}{B}$

$$= \frac{4\pi^2 \times 7{\cdot}5 \times 10^{-6} \times (1{\cdot}5)^2}{0{\cdot}01} = \mathbf{0{\cdot}067\ A\ m^2}$$

20. Let B_H and B_H' be horizontal components of earth's magnetic field at the two places. If T and T′ are periods of oscillation at the two places, then

$$T = 2\pi\sqrt{\frac{I}{M B_H}} \quad \text{and} \quad T' = 2\pi\sqrt{\frac{I}{M B_H'}}$$

$$\therefore \quad \frac{T}{T'} = \sqrt{\frac{B_H'}{B_H}} \quad \text{or} \quad B_H' = B_H \times \frac{T^2}{T'^2}$$

Here, $B_H = 0{\cdot}18$ G ; $T = 60/30 = 2$ s and $T' = 1{\cdot}6$ s

$$\therefore \quad B_H' = \frac{0{\cdot}18 \times (2)^2}{(1{\cdot}6)^2} = \mathbf{0{\cdot}28\ G}$$

21. Here, $\theta = 3{\cdot}5°$ west of geographic north,
$\delta = 18°$ and $B_H = 0{\cdot}40$ G

Now, $B_H = B \cos \delta$

$$\text{or} \quad B = \frac{B_H}{\cos \delta} = \frac{0{\cdot}40}{\cos 18°} = 0{\cdot}42\ \text{G}$$

Thus, magnetic field at the place is **0·42 G** making an angle of 18° with horizontal in magnetic meridian, where the magnetic meridian is 3·5° west of geographic meridian.

22. Refer to conceptual SAQ 4.12.
The neutral point will be obtained at point P_2 on the horizontal table as shown in Fig. 4.21.

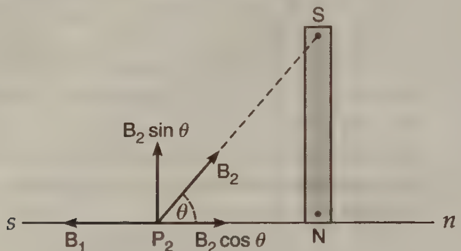


Fig. 4.21

At the neutral point, the horizontal component of earth's magnetic field (B_H) is balanced by the horizontal component of the magnetic field due to the magnet.

Magnetic field due to N-pole of the magnet at the neutral point P_2,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(NP_2)^2} = \frac{10^{-7} \times 1}{(0{\cdot}06)^2}$$

$$= 2{\cdot}78 \times 10^{-5}\ \text{T (along } P_2\, s)$$

Magnetic field due to S-pole of the magnet at the neutral point P_2,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(SP_2)^2}$$

$$\text{Now, } SP_2 = \sqrt{(NS)^2 + (NP_2)^2}$$

$$= \sqrt{(0{\cdot}08)^2 + (0{\cdot}06)^2} = 0{\cdot}1\ \text{m}$$

$$\therefore \quad B_2 = \frac{10^{-7} \times 1}{(0{\cdot}1)^2} = 10^{-5}\ \text{T (along } P_2\, S)$$

The magnetic field B_2 makes an angle θ with the horizontal table as shown in the figure.

Therefore, component of field B_2 along horizontal

$$= B_2 \cos\theta = B_2 \times \frac{NP_2}{SP_2} = 10^{-5} \times \frac{0{\cdot}06}{0{\cdot}1}$$

$$= 0{\cdot}6 \times 10^{-5}\ \text{T} \qquad \text{(along } P_2\, N)$$

Therefore, net horizontal component of the field of magn

$$= B_1 - B_2 \cos\theta = 2{\cdot}78 \times 10^{-5} - 0{\cdot}6 \times 10^{-5}$$

$$= 2{\cdot}18 \times 10^{-5}\ \text{T} \qquad \text{(along } P_2\, s)$$

At the neutral point,

$$B_H = B_1 - B_2 \cos\theta = \mathbf{2{\cdot}18 \times 10^{-5}\ T}$$

23. Here, $a_1 = 0{\cdot}075$ m ; $a_2 = 0{\cdot}1$ m ; $n_1 = 15$; $n_2 = 10$;
$R_1 = 8\ \Omega$; $R_2 = 12\ \Omega$, $\theta_1 = 60°$

Let I be total current in the circuit. As the tw galvanometers are connected in parallel, current throug the first galvanometer,

$$I_1 = \frac{I R_2}{R_1 + R_2} = \frac{I \times 12}{8 + 12} = 0{\cdot}6\ I$$

and current through the second galvanometer,

$$I_2 = \frac{I R_1}{R_1 + R_2} = \frac{I \times 8}{8 + 12} = 0{\cdot}4\ I$$

For first galvanometer,

$$I_1 = \frac{2 a_1}{n_1 \mu_0} B_H \tan\theta_1$$

$$\text{or} \quad 0{\cdot}6\ I = \frac{2 \times 0{\cdot}075}{15 \times \mu_0} B_H \tan 60°$$

$$\text{or} \quad I = \frac{2{\cdot}89 \times 10^{-2}\ B_H}{\mu_0} \qquad \text{...(}$$

For second galvanometer,

$$I_2 = \frac{2 a_2}{n_2 \mu_0} B_H \tan\theta_2$$

$$\text{or} \quad 0{\cdot}4\ I = \frac{2 \times 0{\cdot}1}{10 \times \mu_0} B_H \tan\theta_2$$

$$\text{or} \quad I = \frac{5 \times 10^{-2}\ B_H}{\mu_0} \tan\theta_2 \qquad \text{...(}$$

From equations (*i*) and (*ii*), we have

$$\frac{5 \times 10^{-2}\ B_H}{\mu_0} \tan\theta_2 = \frac{2{\cdot}89 \times 10^{-2}\ B_H}{\mu_0}$$

$$\text{or} \quad \tan\theta_2 = 0{\cdot}578$$

$$\text{or} \quad \theta_2 = \mathbf{30°}$$

24. Let B and B′ be earth's magnetic field, at the two place Then, horizontal components of earth's magnetic field the two places are given by

$$B_H = B \cos\delta \quad \text{and} \quad B_H' = B \cos\delta'$$

$$\text{Now,} \quad T = 2\pi\sqrt{\frac{I}{M B_H}}$$

$$\therefore \quad \frac{T}{T'} = \sqrt{\frac{B_H'}{B_H}} = \sqrt{\frac{B' \cos\delta'}{B \cos\delta}}$$

$$\text{or} \quad \frac{B}{B'} = \frac{\cos\delta'}{\cos\delta} \times \left(\frac{T'}{T}\right)^2$$

25. By proceeding as in problem no. 24, it can be obtained th

$$\frac{B}{B'} = \frac{\cos\delta'}{\cos\delta} \times \left(\frac{T'}{T}\right)^2$$

Here, $T = 60/20 = 3$ s, $T' = 60/15 = 4$ s ;
$\delta = 30°$ and $\delta' = 60°$

$$\therefore \quad \frac{B}{B'} = \frac{\cos 60°}{\cos 30°} \times \left(\frac{4}{3}\right)^2 = \frac{1/2}{\sqrt{3}/2} \times \frac{16}{9} = \frac{\mathbf{16}}{\mathbf{9\sqrt{3}}}$$

26. As obtained in problem no. 24,

$$\frac{B}{B'} = \frac{\cos\delta'}{\cos\delta} \times \left(\frac{T'}{T}\right)^2$$

$$\therefore \quad T' = T\sqrt{\frac{B\cos\delta}{B'\cos\delta'}}$$

Here, $T = 60/10 = 6\text{ s}$; $\delta = 45°$, $B = 0{\cdot}4\text{ G}$;
$\delta' = 60°$ and $B' = 0{\cdot}5\text{ G}$

$$\therefore \quad T' = 6\sqrt{\frac{0{\cdot}4\times\cos 45°}{0{\cdot}5\times\cos 60°}} = 6{\cdot}382\ s$$

Therefore, number of oscillations made by the magnet per second,

$$v' = \frac{1}{T'} = \frac{1}{6{\cdot}382} = 0{\cdot}157\ \text{s}^{-1}$$

27. ***When magnet oscillates in earth's magnetic field :***
Let B_H be horizontal component of earth's magnetic field. Then,

$$T = 2\pi\sqrt{\frac{I_1}{M_1\,B_H}}, \qquad \text{...}(i)$$

where M_1 and I_1 are magnetic moment and moment of inertia of the first magnet.

When magnet oscillates in the combined field of earth and the second magnet :

Let B_H' be effective magnetic field along horizontal.
Then, $\quad B_H' = B_H \pm B_2,$
where B_2 is magnetic field produced by the second magnet at the location of the first magnet.

If T' is period of oscillation of the first magnet in the combined field, then

$$T' = 2\pi\sqrt{\frac{I_1}{M_1\,B_H'}}$$

Since $T' = T/2$ *i.e.* $T' < T$, B_H' must be greater than B_H *i.e.* $B_H' = B_H + B_2$

$$\therefore \quad \frac{T}{2} = 2\pi\sqrt{\frac{I_1}{M_1\,(B_H + B_2)}} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$2\pi\sqrt{\frac{I_1}{M_1\,B_H}} = 2\times 2\pi\sqrt{\frac{I_1}{M_1\,(B_H+B_2)}}$$

$$\text{or} \quad B_H + B_2 = 4\,B_H \quad \text{or} \quad B_2 = 3\,B_H$$

$$\text{or} \quad B_2 = 3\times 0{\cdot}2\times 10^{-4} = 0{\cdot}6\times 10^{-4}\ \text{T}$$

From the statement of the problem, it follows that the first magnet oscillates at a point, which lies on the equitortial line of the second magnet.

If M_2 is magnetic moment of the second magnet, then

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{M_2}{(r^2 + l_2^2)^{3/2}},$$

where $\quad r = 20\text{ cm} = 0{\cdot}2\text{ m}$
and $\quad l_2 = 16/2 = 8\text{ cm} = 0{\cdot}08\text{ m}$

$$\therefore \quad 0{\cdot}6\times 10^{-4} = \frac{10^{-7}\times M_2}{\left[(0{\cdot}2)^2 + (0{\cdot}08)^2\right]^{3/2}}$$

$$\text{or} \quad 0{\cdot}6\times 10^{-4} = 10^{-5}\,M_2$$

$$\text{or} \quad M_2 = \frac{0{\cdot}6\times 10^{-4}}{10^{-5}} = 6\ \text{A m}^2$$

Therefore, pole strength of the second magnet,

$$m_2 = \frac{M_2}{2\,l_2} = \frac{6}{16\times 10^{-2}} = \mathbf{37{\cdot}5\ A\ m}$$

CHAPTER 5

Classification of Magnetic Materials

5.01. SOME IMPORTANT TERMS

The following terms and concepts are used to classify the magnetic substances and to understand their response to the magnetic fields :

1. Magnetic intensity. Consider that inside vacuum, a magnetic field B_0 exists. Then, magnetic intensity is defined by the relation

$$H = \frac{B_0}{\mu_0}, \qquad \text{...(5.01)}$$

where $\mu_0 = 4\pi \times 10^{-7}$ tesla metre ampere^{-1} is absolute permeability of vacuum. Since unit of magnetic field is tesla, the unit of magnetic intensity is

$$[H] = \frac{\text{tesla}}{\text{tesla metre ampere}^{-1}} = \textbf{ampere meter}^{-1}\ (\textbf{A m}^{-1})$$

Magnetic intensity is also known as **H-field** or **magnetic field strength.** The unit of magnetic intensity *i.e.* **A m^{-1}** is also equivalent to **N m^{-2} T^{-1}** or **N Wb^{-1}** or **J m^{-3} T^{-1}** or **J m^{-1} Wb^{-1}.**

The older unit of magnetic intensity is **oersted.** It is related to gauss (the older unit of magnetic field) as given below :

$$\textbf{1 oersted} = \frac{\textbf{1 gauss}}{\mu_0}$$

$$\text{Therefore, } \textbf{1 oersted} = \frac{10^{-4}\,\text{T}}{4\pi \times 10^{-7}\,\text{T m A}^{-1}} \simeq \textbf{80 A m}^{-1}$$

2. Intensity of magnetisation. *It is defined as the magnetic moment developed per unit volume, when a magnetic specimen is subjected to magnetising field.* It is denoted by I.

If a magnetic specimen of volume V acquires magnetic dipole moment M due to the magnetising field, Then,

$$I = \frac{M}{V} \qquad \text{...(5.02)}$$

Since unit of magnetic moment is ampere metre^2, the unit of intensity of magnetisation is

$$[I] = \frac{\text{ampere metre}^2}{\text{metre}^3} = \textbf{ampere metre}^{-1}\ (\textbf{A m}^{-1})$$

i.e. intensity of magnetisation and the magnetic intensity have the same units. The intensity of magnetisation is also called simply **magnetisation.**

If the magnetic specimen has area of cross-section a and length $2\,l$, then its volume,

$$V = a \times 2\,l$$

Further, if m is pole strength developed, then magnetic moment of the specimen,

$$M = m \times 2\,l$$

Therefore, equation (5.02) gives

$$I = \frac{m \times 2l}{a \times 2l} = \frac{m}{a} \qquad \text{...(5.03)}$$

Hence, *intensity of magnetisation may also be defined as the pole strength developed per unit area of cross-section of the specimen.*

3. Magnetic flux. *Magnetic flux through a surface is defined as the number of magnetic field lines passing normally through the surface.* It is denoted by ϕ.

Consider that a surface area $\Delta \vec{S}$ is held in a magnetic field of induction $\vec{B}$. hen, magnetic flux passing normally through the surface is given by

$$\phi = \vec{B} . \Delta \vec{S}$$

The SI unit of magnetic flux is **weber (Wb).**

4. Magnetic induction. Consider that a soft iron bar (ferromagnetic material) s placed in a uniform magnetic field B_0 set up in vacuum. The uniform magnetising ield B_0 has been represented by parallel lines in Fig. 5.01.

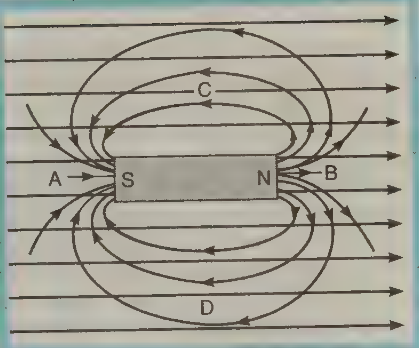


Fig. 5.01

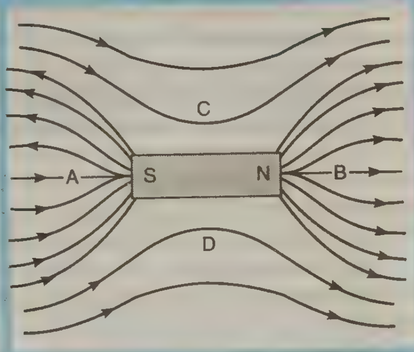


Fig. 5.02

As soon as the iron bar is placed inside the magnetic field, it gets magnetised. ts left end becomes S-pole, while its right end becomes N-pole. Because of induced nagnetism, it produces its own magnetic field. In the regions A and B, the nagnetising field is strengthened, while in the regions C and D, the magnetising ield is opposed by the field produced due to the magnetisation of soft iron bar. The esultant field is as shown in Fig. 5.02.

The magnetic induction is defined as the number of magnetic lines of induction (magnetic field lines inside the material) crossing per unit area normally through the magnetic substance. It is denoted by B.

The magnetic induction B is the sum of the magnetic field B_0 and the magnetic field $\mu_0 I$ produced due to the magnetisation of the substance. Thus,

$$B = B_0 + \mu_0 I = \mu_0 H + \mu_0 I$$

or $$B = \mu_0 (H + I) \qquad ...(5.04)$$

Magnetic induction is also known as **magnetic flux density** or simply **magnetic field.** The SI unit of magnetic induction is **tesla (T)** or **weber metre^{-2} (Wb m^{-2}).** These units of magnetic induction are equivalent to **N m^{-1} A^{-1}** or **J A^{-1} m^{-2}.**

The older unit of magnetic induction is **guass (G).**

1 tesla = 10^4 gauss (G)

5. Magnetic susceptibility. *The magnetic susceptibility of a magnetic substance is defined as the ratio of the intensity of magnetisation to the magnetic intensity.* It is denoted by χ_m.

Therefore $$\chi_m = \frac{I}{H} \qquad ...(5.05)$$

The magnetic susceptibility of a magnetic substance gives the measure of its aptness to acquire magnetism. As magnetic susceptibility is the ratio of two quantities having same units (A m^{-1}), it has ***no units.***

6. Magnetic permeability. *The magnetic permeability of a magnetic substance is defined as the ratio of the magnetic induction to the magnetic intensity.* It is denoted by μ.

Therefore, $$\mu = \frac{B}{H} \qquad ...(5.06)$$

The magnetic permeability of a magnetic substance gives the measure of its conducting power towards the passage of magnetic lines of induction. Its unit in SI is

$$[\mu] = \frac{\text{tesla}}{\text{ampere metre}^{-1}} = \textbf{tesla metre ampere}^{-1} \textbf{ (T m A}^{-1}\textbf{)}$$

i.e. both μ and μ_0 have the same units.

Dividing both sides of the equation (5.04) by H, we have

$$\frac{B}{H} = \mu_0 \left(1 + \frac{I}{H}\right)$$

Using equations (5.05) and (5.06), we have

$$\mu = \mu_0 (1 + \chi_m)$$

or $$\frac{\mu}{\mu_0} = 1 + \chi_m \qquad ...(5.07)$$

Since $\frac{\mu}{\mu_0} = \mu_r$, the ***relative permeability*** of the magnetic substance, the equation (5.07) becomes

$$\mu_r = 1 + \chi_m \qquad ...(5.08)$$

5.02. CLASSIFICATION OF MAGNETIC MATERIALS

Iron and steel are not the only substances, which are attracted by a magnet or which can be magnetised. In fact, all the substances possess magnetic properties. On the basis of their magnetic behaviour, Faraday divided the magnetic materials into three classes :

(i) **Diamagnetic.** *Those substances, which when placed in a magnetic field are feebly magnetised in a direction opposite to that of the magnetising field, are called diamagnetic substances.*

When a diamagnetic substance is placed inside an external magnetic field, the magnetic field inside the diamagnetic is found to be slightly less than the external magnetic field. It is observed that when a diamagnetic sample is placed inside a non-uniform magnetic field, it tends to move from stronger part to the weaker part of the magnetic field. It may be pointed out that the diamagnetic effects are too feeble to be detected, unless the applied magnetic field is strong. The behaviour of a diamagnetic substance is independent of temperature. Further, a diamagnetic substance has the nature similar to that of a dielectric having non-polar atoms. A few examples of diamagnetic substances are copper, zinc, bismuth, silver, gold, lead, glass, marble, water, helium, argon sodium chloride, etc.

(ii) **Paramagnetic.** *Those substances, which when placed in a magnetic field are feebly magnetised in the direction of the magnetising field, are called paramagnetic substances.*

When a paramagnetic substance is placed inside an external magnetic field, the magnetic field inside the paramagnetic is found to be a slightly greater than the external magnetic field. In contrast to the behaviour of diamagnetics, a paramagnetic substance tends to move from weaker part of the magnetic field to stronger part, when placed in a non-uniform magnetic field. Further, in contrast to diamagnetic, the behaviour of a paramagnetic is temperature dependent. Also, the paramagnetic effects are perceptible only with a strong magnetic field. The nature of a paramagnetic is similar to that of a dielectric having polar atoms. A few examples of paramagnetic substances are aluminium, sodium, antimony, platinum, copper chloride, manganese, chromium, liquid oxygen, etc.

(iii) **Ferromagnetic.** *Those substances, which when placed in a magnetic field are strongly magnetised in the direction of the magnetising field, are called ferromagnetic substances.*

When a ferromagnetic substance is placed inside a magnetic field, the field inside the ferromagnetic substance gets greatly enhanced. As a result, when a ferromagnetic is placed in a non-uniform magnetic field, it quickly moves from weaker part to stronger part of the magnetic field. In other words, the ferromagnetic effects are perceptible even in the presence of weak magnetic field. Further, the ferromagnetic behaviour of a substance becomes temperature dependent above certain temperature, which is characteristic of that substance. It is called ***Curie temperature.*** The nature of a ferromagnetic may be compared to that of a ferroelectric type dielectric. A few examples of ferromagnetic materials are iron, nickel, cobalt, alnico, etc.

The nature of diamagnetic, paramagnetic and ferromagnetic substances can be successfully explained on the basis of ***electron-theory of magnetism,*** which makes use of the fact that an atom may possess magnetic moment due to orbital and spin motion of the electron in the atom.

5.03. DIAMAGNETISM ON THE BASIS OF ELECTRON THEORY

We know that in an atom, electrons revolve in circular orbits and as such they behave as tiny current loops and possess magnetic dipole moments. The magnetic dipole moment of an electron revolving with velocity v in an orbit of radius r is given by

$$\mathrm{M} = \frac{e\,v\,r}{2}, \qquad \text{...(5.09)}$$

where e is magnitude of charge on the electron. Now, angular momentum of electron is given by

$$\mathrm{L} = m_e\,v\,r,$$

where m_e is mass of electron.

$$\therefore \qquad v = \frac{\mathrm{L}}{m_e\,r}$$

Substituting for v in equation (5.09), we have

$$\mathrm{M} = \frac{e}{2\,m_e}\mathrm{L} \qquad \text{...(5.10)}$$

In vector notation, equation (5.10) may be expressed as

$$\vec{M} = -\frac{e}{2\,m_e}\vec{L} \qquad ...(5.11)$$

The equation (5.11) tells that magetic dipole moment vector is directed in a lirection opposite to that of angular momentum vector.

In a diamagnetic material, there are no unpaired electrons. Further, the two lectrons of a pair possess orbital motion in opposite directions. Fig. 5.03 shows the rbital motion of an electron revolving in clockwise direction. The angular nomentum vector $\vec{L}$ of such an electron is along OZ′ and the magnetic dipole noment vector $\vec{M}$ is directed along OZ. For the orbital motion of an electron in anti-lockwise direction, the orientation of vectors $\vec{M}$ and $\vec{L}$ will be as shown in Fig. 5.04 .e. the angular momentum vector $\vec{L}$ is along OZ, while the magnetic dipole moment ector $\vec{M}$ is along OZ′. Thus, the two electrons of a pair possess magnetic moments qual in magnitude and opposite in direction and hence the magnetic moment of any pair of electrons due to their orbital motion is zero. For similar reasons, the diamagnetic atoms do not possess magnetic moments due to spin motion of electrons.

However, when external magnetic field $\vec{B}$ is applied, then magnetic Lorentz orce

$$\vec{F} = -e(\vec{v} \times \vec{B})$$

acts on the electron.

In case of an electron revolving in clockwise direction [Fig. 5.05], magnetic Lorentz force $\vec{F}$ acts radially outwards and it tends to decrease the centripetal force on the electron. As a result, its velocity decreases to $v - \Delta v$ and hence magnetic dipole moment to $\vec{M} - \Delta\vec{M}$. On the other hand, in case of electron revolving in anti-clockwise direction [Fig. 5.06], the force $\vec{F}$ acts radially inwards and it tends to increase the centripetal force on the electron. It results in increase of velocity of the electron to $v + \Delta v$ and hence magnetic dipole moment to $\vec{M} + \Delta\vec{M}$. Thus, on applying external magnetic field, the magnetic dipole moments of the two electrons in a pair no longer remain equal. The pair of electrons possesses a net magnetic dipole moment $2\,\Delta\vec{M}$ in a direction opposite to the direction of applied magnetic field. The magnetic dipole moments of paired electrons in all atoms of the diamagnetic add up to account for the small magnetic moment of the specimen along a direction opposite to that of the applied field. The diamagnetic material, thus, gets feebly magnetised along a direction opposite to that of the applied field, which explains the behaviour of diamagnetics, when placed in external magnetic field.

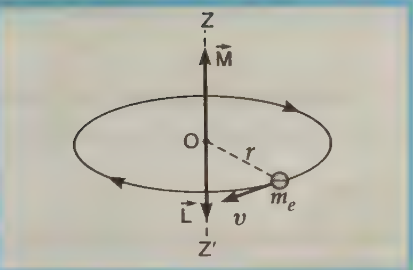


Fig. 5.03

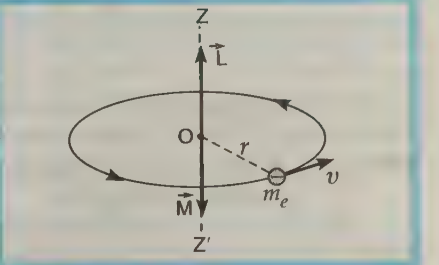


Fig. 5.04

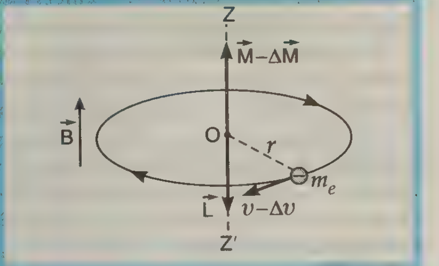


Fig. 5.05

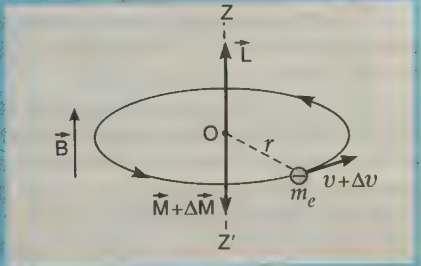


Fig. 5.06

5.04. PARAMAGNETISM ON THE BASIS OF ELECTRON THEORY

The atom of a paramagnetic substance possesses magnetic moment due to the orbital motion of the unpaired electron in the atom or due to spin motion of the electrons. The interaction between the atomic magnetic dipoles of the paramagnetic substances is very weak and therefore they may be regarded as independent of each other. Due to thermal agitation, the atomic magnetic dipoles are randomly oriented as shown in Fig. 5.07.

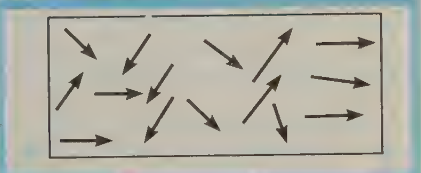


Fig. 5.07

When external magnetic field is applied, the field tries to align the magnetic dipoles along its own direction but the thermal agitation hampers the alignment and the magnetic dipoles try to go in random directions. Therefore, when the temperature is low and the applied magnetic field is increased, more and more magnetic dipoles align themselves along the direction of the applied magnetic field. At a given temperature, the maximum or saturation value of the alignment of magnetic dipoles will occur for a certain value of the applied magnetic field [Fig. 5.08].

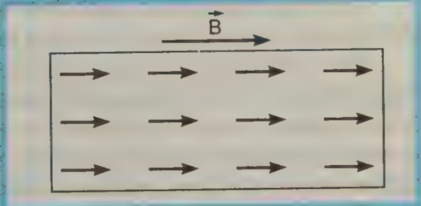


Fig. 5.08

For paramagnetic materials, it is observed that before such saturation occurs, the intensity of magnetisation of the material is

(*i*) *directly proportional to the magnetic intensity i.e.*

$$I \propto H$$

(*ii*) *inversely proportional to the absolute temperature of the material i.e.*

$$I \propto \frac{1}{T}$$

Combining the above two factors, we have

$$I \propto \frac{H}{T}$$

or $$I = C\frac{H}{T}, \qquad \text{...(5.12)}$$

where the constant of proportionality C is called ***Curie's constant***. From equation (5.12), we have

$$\frac{I}{H} = \frac{C}{T}$$

or $$\chi_m = \frac{C}{T} \qquad \text{...(5.13)}$$

i.e. the magnetic susceptibility of paramagnetic material is inversely proportional to its absolute temperature. It is called ***Curie's law in magnetism.***

5.05. FERROMAGNETISM ON THE BASIS OF ELECTRON THEORY (DOMAINTHEORY)

The atom of a ferromagnetic material also possesses non-zero magnetic moment as in case of a paramagnetic substance. However, due to a quantum mechanical effect, called ***exchange interaction,*** an unpaired electron in one atom interacts strongly with the unpaired electron in the neighbouring atom in such a way that they spontaneously align themselves in a common direction over a small volume of the material. These small volumes of uniform magnetisation are called ***domains.*** Although domains are extremely small in size ($\approx 10^{-18}$ m^3 in volume), yet each domain contains a large number of atoms ($\approx 10^{11}$ atomic magnetic dipoles).

The magnetic moments of the atoms in one domain are parallel to each other and hence a domain possesses a small value of net magnetic dipole moment. In the absence of external magnetic field, the direction of magnetic moments in different domains are oriented randomly in different directions [Fig. 5.09]. The result is that one domain cancels the effect of the other, so that the net magnetic moment of the material is zero. The ferromagnetic material, thus, appears unmagnetised in the absence of an external magnetic field.

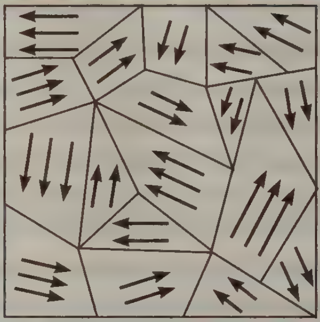


Fig. 5.09

When external magnetic field is applied, say from left to right ; the magnetic moments of domains progressively align along the direction of the external field in the following two ways :

(*i*) If the ferromagnetic material is homogeneous and pure, then the domains in which the magnetic moments are parallel to the external field expand continually at the expense of remaining domains and ultimately the magnetic moments align themselves along the direction of the magnetic field. As shown in Fig. 5.10 (*a*), the magnetic moments in domain A only are parallel to the direction of external magnetic field. Then, as illustrated through Fig. 5.10 (*b*), (*c*) and (*d*), the domain A continually expands at the expense of other three domains and ultimately all the magnetic moments become parallel to the direction of external magnetic field.

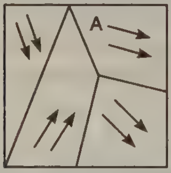


Fig. 5.10 (*a*)

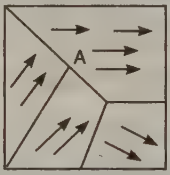


Fig. 5.10 (*b*)

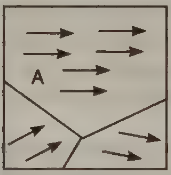


Fig. 5.10 (*c*)

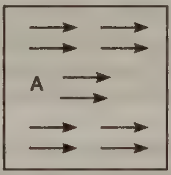


Fig. 5.10 (*d*)

(*ii*) If the external magnetic field is very strong, then magnetic moments in all the domains align themselves along the direction of applied magnetic field.

Once the magnetic moments align themselves along the direction of applied magnetic field, the ferromagnetic material appears magnetised. If the magnetised ferromagnetic material is heated, the alignment of magnetic moments within the domain gets disturbed. The order of disturbance increases with increase in temperature and at the ***Curie point,*** the material is reduced to paramagnetic material.

The susceptibility of a ferromagnetic substance above its curie temperature is inversely *proportional to the excess of temperature above the Curie temperature.* It is called *Curie-Weiss law.*

5.06. PROPERTIES OF DIA, PARA AND FERROMAGNETIC SUBSTANCES

The three types of magnetic materials possess the properties as listed below :

(*a*) Properties of diamagnetic substances. 1. *A diamagnetic substance is feebly repelled by a magnet.*

When a diamagnetic substance is placed near a magnet (*i.e* in the magnetic field due to the magnet), its atoms acquire a small magnetic dipole moment in a direction opposite to that of the external magnetic field. As a result, the diamagnetic substance moves from stronger part of the magnetic field to its weaker part *i.e.* it is feebly repelled by the magnet.

2. *When a rod of diamagnetic substance is suspended inside a magnetic field, it slowly sets itself at right angles to the direction of the field as shown in Fig. 5.11*

It happens, because a diamagnetic substance is weakly repelled by a magnet.

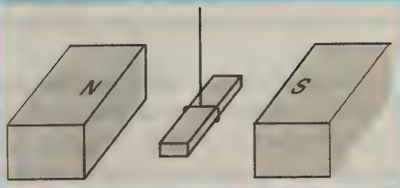


Fig. 5.11

3. *If a diamagnetic liquid contained in a watch glass is placed on two closely spaced pole-pieces of a magnet, it suffers a slight depression in the middle [Fig. 5.12]. However, the liquid shows a rise in the middle, when the pole-pieces are moved apart [Fig. 5.13].*

When the pole-pieces of the magnet are close to each other, the magnetic field in the middle is stronger than that near the poles of the magnet. Since a diamagnetic substance moves from stronger part of the magnetic field to the weaker part, the liquid depresses in the middle.

When the pole-pieces are moved apart, the magnetic field becomes weaker in the middle. As a result, the diamagnetic liquid shows a rise in the middle.

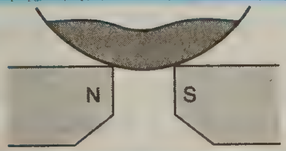


Fig. 5.12

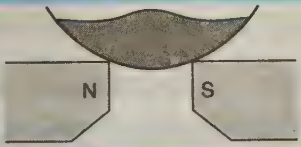


Fig. 5.13

4. *For a diamagnetic substance, the intensity of magnetisation (I) has a small negative value.*

It is because, on being placed in a magnetic field, a diamagnetic substance gets slightly magnetised in a direction opposite to that of the field.

5. *When a diamagnetic material is placed inside a magnetic field, the magnetic field lines become slightly less dense in the diamagnetic material [Fig. 5.14].*

When a diamagnetic substance is placed in a magnetic field, a small magnetization occurs opposite to the direction of the field. As a result, the magnetic field lines become less dense inside the diamagnetic material.

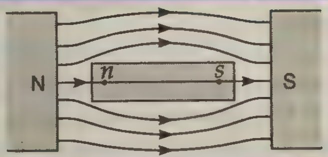


Fig. 5.14

Key point

On placing a bar of diamagnetic material between two poles of a magnet, its end facing N-pole of the magnet becomes south pole, while the other end facing S-pole becomes north pole [Fig. 5.14]. It is because, due to the magnetic field of the magnet, a small magnetization occurs opposite to the direction of the field.

6. *The magnetic susceptibility (χ_m) of a diamagnetic substance has a small negative value.*

Since for a diamagnetic substance, I has a small negative value, from the relation

$$\chi_m = \frac{I}{H},$$

it follows that χ_m will have a small negative value. It is of the order of 10^{-6} to 10^{-3} (negative).

7. *The relative permeability (μ_r) of a diamagnetic substance is slightly less than 1.*

Since for a diamagnetic substance, I has a small negative value, from the relation

$$\mu_r = 1 + \chi_m,$$

it follows that μ_r will be slightly less than 1.

8. *When a diamagnetic substance is placed inside a magnetising field (H), the magnetic induction or magnetic flux density (B) in the diamagnetic substance is lesser than that in vacuum (μ_0 H).*

Since for a diamagnetic substance, I has a negative value, from the relation

$$B = \mu_0 (H + I),$$

it follows that B will be less than μ_0 H.

9. *The diamagnetic substances do not obey Curie's law.*

It is because, the susceptibility of a diamagnetic substance does not change with temperature for practical purposes. However, bismuth (at low temperature) is an exception to this general property of diamagnetic substances.

(*b*) Properties of paramagnetic substances. 1. *A paramagnetic substance is feebly attracted by a magnet.*

When a paramagnetic substance is placed near a magnet, its atoms (possessing small magnetic dipole moments) align themselves along the direction of the magnetic field due to magnet. As such, the paramagnetic substance moves from weaker part of the magnetic field to its stronger part *i.e.* it is feebly attracted by the magnet.

2. *When a rod of paramagnetic substance is suspended inside a magnetic field, it slowly sets itself parallel to the direction of the magnetic field [Fig. 5.15].*

It happens, because a paramagnetic substance is weakly attracted by a magnet.

3. *If a paramagnetic liquid contained in a watch glass is placed on two closely spaced pole-pieces of a magnet, it shows a slight rise in the middle [Fig. 5.16]. On the other hand, when the poles are moved apart, the paramagnetic liquid gets depressed in the middle [Fig. 5.17]*

As said earlier, when the pole-pieces of the magnet are close to each other, the magnetic field in the middle is stronger than that near the poles of the magnet. Since a paramagnetic substance moves from weaker part of the magnetic field to the stronger part, the liquid rises in the middle. The reverse happens, when the pole-pieces of the magnet are moved apart.

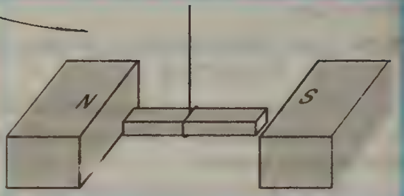


Fig. 5.15

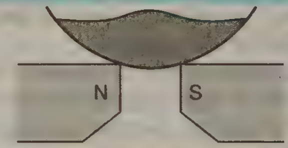


Fig. 5.16

4. *For paramagnetic substances, the intensity of magnetisation (I) has a small positive value.*

It is because, on being placed in a magnetic field, a paramagnetic substance gets slightly magnetised in the direction of the field.

5. *When a paramagnetic material is placed inside a magnetic field, the magnetic field lines become slightly more dense in the paramagnetic material [Fig. 5.18].*

When a paramagnetic substance is placed in a magnetic field, a small magnetisation occurs in the direction of the field. As a result, the magnetic field lines become more dense inside the paramagnetic substance.

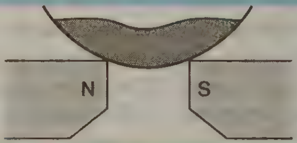


Fig. 5.17

6. *The magnetic susceptibility (χ_m) of a paramagnetic substance has a small positive value.*

Since for a paramagnetic substance, I has a small positive value, from the relation

$$\chi_m = \frac{I}{H},$$

it follows that χ_m also has a small positive value. It is of the order of 10^{-5} to 10^{-3}.

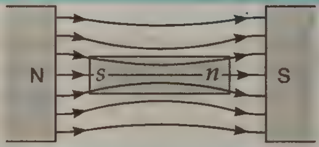


Fig. 5.18

7. *The relative permeability (μ_r) of a paramagnetic substance is slightly greater than 1.*

Since for a paramagnetic substance, I has a small positive value, from the relation

$$\mu_r = 1 + \chi_m,$$

it follows that μ_r will be slightly greater than 1.

8. *When a paramagnetic substance is placed inside a magnetising field (H), the magnetic flux density (B) in the paramagnetic substance is greater than that in vacuum (μ_0 H).*

Since for a paramagnetic substance, I has a small positive value, from the relation

$$B = \mu_0 (H + I),$$

it follows that B will be greater than μ_0H.

9. *The paramagnetic substances obey Curie's law i.e. the susceptibility of paramagnetic substances is inversely proportional to its absolute temperature.*

It follows that a paramagnetic substance tends to lose its magnetic character on being heated.

(c) Properties of ferromagnetic substances. The properties of ferromagnetic substances are similar in nature to those of paramagnetic substances but the effect is much more pronounced in case of ferromagnetics. Their properties are as listed below :

1. *A ferromagnetic substance is strongly attracted by a magnet.*

2. *When a rod of ferromagnetic substance is suspended in a magnetic field, it quickly aligns itself along the direction of the magnetic field as shown in Fig. 5.15.*

3. *The ferromagnetic materials move from weaker part of the magnetic field to the stronger part.*

Ferromagnetism is not found in liquids and gases. However, if a finely powdered ferromagnetic solid contained in a watch glass is placed on the two pole-pieces of a magnet, the material rises in the middle as in the case of paramagnetics [Fig. 5.16]. When the pole-pieces are moved apart, the material depresses in the middle [Fig. 5.17].

4. *When a ferromagnetic material is placed inside a magnetic field, the magnetic field lines becomes highly dense in the ferromagnetic substance as shown in Fig. 5.19.*

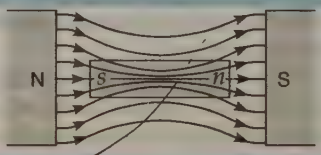


Fig. 5.19

When a ferromagnetic material is placed in a magnetic field, a large magnetisation occurs in the direction of magnetic field.

5. *For a ferromagnetic substance, the intensity of magnetisation (I) has a large positive value.*

It is because, on being placed in a magnetic field, a ferromagnetic substance gets strongly magnetised in the direction of the field.

6. *The magnetic susceptibility (χ_m) of a ferromagnetic substance has a large positive value.*

It follows from the relation : $\chi_m = \frac{I}{H}$

It is of the order of several thousands.

7. *The relative permeability (μ_r) of a ferromagnetic substance is extremely large as compared to 1.*

It follows from the relation : $\mu_r = 1 + \chi_m$

It is of the order of several thousands.

8. *When a ferromagnetic substance is placed inside a magnetising field (H), the magnetic flux density (B) in the ferromagnetic substance is much larger than that in vacuum (μ_0 H).*

It follows from the relation : $B = \mu_0 (H + I)$.

9. *The ferromagnetic substances do not obey Curie's law.*

The susceptibility of ferromagnetic substances decreases with the rise of temperature in a complicated manner. At a certain temperature, called ***Curie point*** or ***Curie temperature***, a ferromagnetic substance starts behaving as a paramagnetic substance. In case of iron, the Curie point is 1043 K. After Curie point, the susceptibility of a ferromagnetic substance varies inversely with its absolute temperature. In other words, a ferromagnetic substance obeys Curie's law only above its Curie point.

Key point

Both in paramagnetic and ferromagnetic substances, magnetism decreases, when they are heated. In is because, as temperature rises, the alignment of molecular magnets gradually decreases due to thermal agitation.

5.07. HYSTERESIS

Consider that a specimen of ferromagnetic material is placed in a magnetising field, whose strength and direction can be changed. Suppose that the specimen is unmagnetised initially. When the magnetising field (H) is increased, the intensity of magnetisation (I) of the material of the specimen also increases. It is found that when the magnetising field is made zero, the intensity of magnetisation does not become zero but still has some finite value. It becomes zero only, when magnetising field is increased in reverse direction. In other words, intensity of magnetisation does not become zero on making magnetising field zero but does so a little late and this effect is called ***hysteresis***.

The jargon . . . . .

The dictionary meaning of the word hysteresis is 'coming late'

The lag of intensity of magnetisation behind the magnetisation field during the process of magnetisation and demagnetisation of a ferromagnetic material is called hysteresis.

Fig. 5.20 shows the magnetisation curve of a ferromagnetic material, when it is taken over a complete cycle of magnetisation. Corresponding to point O, the magnetising field (H) is zero and likewise intensity of magnetisation (I) is also zero. As magnetising field is increased, intensity of magnetisation also increases along OA. Corresponding to point A, the intensity of magnetisation becomes maximum. The increase in value of the magnetising field beyond H_0 does not produce any increase in the intensity of magnetisation. In other words, corresponding to point A, the specimen of the ferromagnetic material acquires a state of magnetic saturation. If magnetising field is now decreased slowly, intensity of magnetisation decreases but not along the path AO. It decreases along the path AB. Corresponding to point B, magnetising field becomes zero but some magnetisation equal to OB is still left in the specimen. Here, OB gives the measure of ***retentivity*** of the material of the specimen.

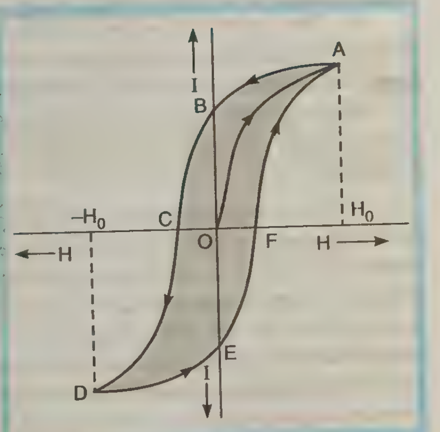


Fig. 5.20

The value of the intensity of magnetisation of a material, when the magnetising field is reduced to zero, is called retentivity of the material. It is also known as ***residual magnetism*** or ***remanent magnetism*** or ***remanence***.

To reduce intensity of magnetisation to zero, the magnetising field has to be increased in reverse direction. As it is done so, the intensity of magnetisation decreases along BC, till it becomes zero corresponding to point C. Thus, to make intensity of magnetisation zero, magnetising field equal to OC has to be applied in reverse direction. Here, OC gives the measure of ***coercivity*** of the material of the specimen.

Key point

Hysteresis is shown by only ferromagnetic substances.

The value of reverse magnetising field required so as to reduce residual magnetism to zero, is called coercivity of the material.

When the magnetising field is further increased in reverse direction, intensity of magnetisation increases along CD as the magnetising field is increased. Corresponding to point D (when the magnetising field becomes $-H_0$), it again acquires a saturation value, which is symmetrical to that corresponding to point A. If the magnetising field is decreased from $-H_0$ to zero, the intensity of magnetisation follows the path DE. Finally, when magnetising field is increased in original direction, the point A is reached via EFA. If the magnetising field is repeatedly changed between H_0 and $-H_0$, the curve ABCDEFA is retraced. The curve ABCDEFA is called the ***hysteresis loop***. It is found that the area of the ***hysteresis loop*** (I – H curve) is proportional to the net energy absorbed per unit volume by the specimen, as it taken over a complete cycle of magnetisation and demagnetisation. The energy so absorbed by the specimen appears as the heat energy.

Note. If hysteresis loop is drawn by plotting a graph between magnetic induction (B) and intensity of magnetisation, then area of the hysteresis loop is numerically equal to the work done per unit volume (or energy absorbed per unit volume) in taking the magnetic specimen over a complete cycle of magnetisation.

Self-test Question

The integral $\oint B\, d I$ represents the area of I – B curve when a magnetic substance is taken over a complete cycle of magnetisation. Show that it has dimensions of energy per unit volume.

Key point

The area of I – H curve is proportional to and area of I – B curve for a magnetic substance is numerically equal to net energy absorbed per unit volume by it, as it is taken over a complete cycle of magnetisation.

5.08. COMPARISON OF HYSTERESIS LOOPS FOR SOFT IRON AND STEEL

The shape of the hysteresis loop is a characteristics of a ferromagnetic substance. It gives the idea about many important magnetic properties of the substance.

Fig. 5.21. shows hysteresis loops for soft iron and steel. Whereas the hysteresis loop for soft iron is narrow, the hysteresis loop for steel is quite wide. The following conclusions can be drawn from the study of the hysteresis loops of soft iron and steel.

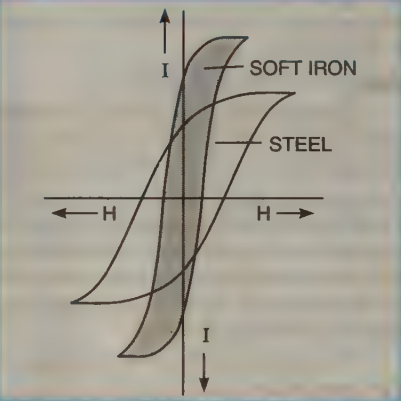


Fig. 5.21

1. The area of hysteresis loop for soft iron is much smaller than that for steel, therefore loss of energy per unit volume in case of soft iron will be very small as compared to that in case of steel, when they are taken over a complete cycle of magnetisation and demagnetisation.

2. Soft iron acquires maximum intensity of magnetisation for comparatively much lesser value of magnetising field than in case of steel. In other words, soft iron is much strongly magnetised (or more susceptible to magnetism) than steel.

3. The retentivity of soft iron is greater than that of steel. On removing magnetising field, quite a large amount of magnetisation is retained by soft iron.

4. The coercivity of steel is much larger than that of soft iron. Therefore, the residual magnetism in steel can not be destroyed that easily as in case of soft iron.

5.09. APPLICATIONS OF FERROMAGNETIC SUBSTANCES

The choice of a particular ferromagnetic material for a practical application is decided from the magnetic properties, such as retentivity, coercivity and area of the hysteresis loop.

1. Permanent magnets. The materials used for making permanent magnets should possess a high value of both retentivity and coercivity. Soft iron has high retentivity but due to small value of coercivity, it is unsuitable for making permanent magnets. Steel possesses fairly larger value of retentivity and a high value of coercivity. No doubt, large magnetising fields are needed to magnetise steel but once magnetised, the magnetisation is retained to a large extent due to its large retentivity. The magnetisation in steel is not easily destroyed, even if it gets exposed to stray reverse magnetic fields or when handled roughly. The area of hysteresis loop is large for steel but it is of no consideration as a permanent magnet has never to be taken through a cycle of magnetisation. Cobalt steel (52% iron, 36% cobalt, 7% tungsten, 3·5% chromium, 0·5% manganese and 0 · 7 % carbon) and carbon steel (98% iron, 0·86% carbon and 0·9% manganese) are used for making permanent magnets. The alloy alnico (55% iron, 10% aluminium, 17% nickel, 12% cobalt and 6% copper) is also very suitable for making permanent magnets. The only disadvantage is that alnico is brittle. The following table gives the retentivity and coercivity of cobalt steel, carbon steel and alnico :

Key point

The ferromagnetic materials used for making permanent magnets should possess high value of both retentivity and coercivity.

Material	Retentivity (A m^{-1})	Coercivity (T)
Cobalt steel	18×10^3	0·95
Carbon steel	$3{\cdot}6 \times 10^3$	0·95
Alnico	42×10^3	0·76

2. Electromagnets. These are the magnets, which can be turned on and off by switching the current on and off. Obviously, the materials used for making an electromagnet should possess high retentivity and low coercivity. Since such a material has to undergo magnetisation and demagnetisation again and again, the area of hysteresis loop of the material used for making electromagnet should also be small. All these requirements are best fulfilled by soft iron. An electromagnet is made by winding a coil of insulated wire around a soft iron core. When electric current is passed through the wire, magnetic field set up in the coil magnetises the soft iron core. When the current is switched off, the magnetic field disappears and the soft iron core loses its magnetism. The electromagnets are used in electric bells, relays, telephones, etc. In factories, electromagnets are used to lift heavy machines and bulks of iron scrap.

3. Transformer cores. For better inductive effect, the two coils of a transformer are always wound on a core of ferromagnetic material. When alternating current is passed through a transformer, the core of the transformer undergoes a large number of cycles of magnetisation and demagnetisation every second. During each cycle, energy proportional to the area of the hysteresis loop for the material of core is wasted in the form of heat. So that efficiency of the transformer is high, the material chosen for making the core of the transformer should have narrow hysteresis loop. On account of this, soft iron is used to make the core of transformers. Mumetal (76% nickel, 17% iron, 5% copper and 2% chromium) also possesses a narrow hysteresis loop and is used for making the core of transformers.

Key point

The ferromagnetic materials used for making electromagnets or core of a transformer should possess high retentivity, low coercivity and small area of I – H curve.

SOLVED NUMERICAL PROBLEMS

Type A. On Magnetisation, Permeability and Susceptibility

Problem 5.01. The permeability of mumetal is measured to be 0 · 12 T A^{-1} m. Find its relative permeability and susceptibility.

Sol. Here, $\mu = 0 \cdot 12 \text{ T A}^{-1} \text{ m}$

We know, $\mu_0 = 4\pi \times 10^{-7} \text{ T A}^{-1} \text{ m}$

$$\therefore \quad \mu_r = \frac{\mu}{\mu_0} = \frac{0{\cdot}12}{4\pi \times 10^{-7}} = \mathbf{9 \cdot 59 \times 10^4}$$

Also, $\chi_m = \mu_r - 1 = 9 \cdot 59 \times 10^4 - 1 = \mathbf{9 \cdot 59 \times 10^4}$

Problem 5.02. An iron rod of 0 · 5 cm^2 area of cross-section is subjected to a magnetising field of 1200 A m^{-1}. If susceptibility of iron is 599, calculate (*i*) μ (*ii*) B and (*iii*) ϕ (magnetic flux) produced.

Sol. Here, $A = 0 \cdot 5 \text{ cm}^2 = 0 \cdot 5 \times 10^{-4} \text{ m}^2$;

$H = 1200 \text{ A m}^{-1}$; $\chi_m = 599$

We know, $\mu_0 = 4\pi \times 10^{-7} \text{ T A}^{-1} \text{ m}$

(*i*) $\mu = \mu_0 (1 + \chi_m) = 4\pi \times 10^{-7} (1 + 599)$

$= \mathbf{7 \cdot 54 \times 10^{-4} \text{ T A}^{-1} \text{ m}}$

(*ii*) $B = \mu H = 7 \cdot 54 \times 10^{-4} \times 1200 = \mathbf{0 \cdot 905 \text{ T}}$

(*iii*) $\phi = B A = 0 \cdot 905 \times 0 \cdot 5 \times 10^{-4}$

$= \mathbf{4 \cdot 525 \times 10^{-5} \text{ Wb}}$

Problem 5.03. A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field in the core for a magnetising current of 1 · 2A ? (*Text Problem*)

Sol. Here, $I = 1 \cdot 2 \text{ A}$; $\mu_r = 800$; $a = 15 \text{ cm} = 0 \cdot 15 \text{ m}$;

Total number of turns = 3500

Length over which the wire is wound,

$l = 2\pi a = 2\pi \times 0 \cdot 15 \text{ m}$

Therefore, number of turns per unit length,

$$n = \frac{3500}{l} = \frac{3500}{2\pi \times 0{\cdot}15}$$

If u_r is relative permeability of the core, then

$$B = \mu_0 \mu_r n I = \frac{4\pi \times 10^{-7} \times 800 \times 3500 \times 1{\cdot}2}{2\pi \times 0{\cdot}15}$$

$= \mathbf{4 \cdot 48 \text{ T}}$

Type B. On Curie's Law

Problem 5.04. For a magnetising field of intensity 2×10^3 A m^{-1}, aluminium at 280 K acquires intensity of magnetisation of $4 \cdot 8 \times 10^{-2}$ A m^{-1}. Find the susceptibility of aluminium at 280 K. If the temperature of the aluminium is raised to 320 K, what will be its susceptibility and intensity of magnetisation ?

Sol. Here, $H = 2 \times 10^3 \text{ A m}^{-1}$;

$I = 4 \cdot 8 \times 10^{-2} \text{ A m}^{-1}$ and $T = 280 \text{ K}$

$$\text{Now,} \quad \chi_m = \frac{I}{H} = \frac{4{\cdot}8 \times 10^{-2}}{2 \times 10^3} = \mathbf{2 \cdot 4 \times 10^{-3}}$$

Aluminium is a paramagnetic substance and hence obeys Curies' law. If χ_m' is susceptibility of aluminium at temperature T′ (= 320 K), then

$$\frac{\chi_m'}{\chi_m} = \frac{T}{T'} \quad \text{or} \quad \chi_m' = \chi_m \times \frac{T}{T'}$$

$$\text{or} \quad \chi_m' = 2 \cdot 4 \times 10^{-3} \times \frac{280}{320} = \mathbf{2 \cdot 1 \times 10^{-5}}$$

If I′ is intensity of magnetisation of aluminium at 320 K, then

$I' = \chi_m' H = 2 \cdot 1 \times 10^{-5} \times 2 \times 10^3$

$= \mathbf{4 \cdot 2 \times 10^{-2} \text{ A m}^{-1}}$

Type C. On Hysteresis loss

Problem 5.05. An iron sample having mass 8 · 4 kg is repeatedly taken over cycles of magnetisation and demagnetisation at a frequency of 50 cycles s^{-1}. It is found that energy equal to $3 \cdot 2 \times 10^4$ J is dissipated as heat in the sample in 30 minutes. If the density of the iron is 7200 kg m^{-3}, calculate the value of energy dissipated per unit volume per cycle in the iron sample.

Sol. Let Q be the energy dissipated per unit volume per second in the given iron sample. Then, total energy lost by the sample in a given time t,

$$W = Q \times V \times \nu \times t, \qquad \ldots(i)$$

where V is volume of the sample and ν is frequency of magnetisation and demagnetisation.

Here, $W = 3 \cdot 2 \times 10^4$ J; $V = \dfrac{\text{mass}}{\text{density}} = \dfrac{8 \cdot 4}{7200}$ kg m^{-3},

$\nu = 50$ cycle s^{-1} and $t = 30$ minutes $= 30 \times 60$ s

From equation (*i*), we have

$$Q = \frac{W}{V \times \nu \times t} = \frac{3 \cdot 2 \times 10^4 \times 7200}{8 \cdot 4 \times 50 \times 30 \times 60}$$

$$= \mathbf{304 \cdot 8 \ J \ m^{-3} \ cycle^{-1}}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For **a**mbitious, **b**rilliant & **c**urious Students

Problem 5.06. A bar magnet has pole strength 4·5 A m, magnetic length 12 cm and cross-sectional area 0·9 cm^2. Find (*a*) intensity of magnetisation (I), (*b*) magnetic intensity (H) at the centre and (*c*) magnetic induction (B) at the centre of the magnet.

Sol. Here, $m = 4 \cdot 5$ A m ; $2l = 12$ cm $= 0 \cdot 12$ m and

$$A = 0 \cdot 9 \ cm^2 = 0 \cdot 9 \times 10^{-4} \ m^2$$

(*a*) Now, $I = \dfrac{m}{A} = \dfrac{4 \cdot 5}{0 \cdot 9 \times 10^{-4}} = \mathbf{5 \times 10^4 \ A \ m^{-1}}$

(from S-pole to N-pole)

(*b*) The magnetic intensity due to a magnetic pole at a distance r is given by

$$H = \frac{1}{4\pi} \cdot \frac{m}{r^2}$$

For a point at the centre of the magnet,

$$r = \frac{2l}{2} = \frac{0 \cdot 12}{2} = 0 \cdot 06 \ m$$

Therefore, magnetic intensity at the centre of magnet due to its N-pole,

$$H_1 = \frac{1}{4\pi} \cdot \frac{4 \cdot 5}{(0 \cdot 06)^2}$$

$= 99 \cdot 5$ A m^{-1} (towards S-pole)

Also, magnetic intensity at the centre of magnet due to its S-pole,

$$H_2 = \frac{1}{4\pi} \cdot \frac{4 \cdot 5}{(0 \cdot 06)^2}$$

$= 99 \cdot 5$ A m^{-1} (towards S-pole)

The resultant magnetic intensity at the centre of the magnet,

$$H = H_1 + H_2 = 99 \cdot 5 + 99 \cdot 5$$

$= \mathbf{199 \ A \ m^{-1}}$ **(towards S-pole)**

(*c*) Now, magnetic induction at the centre of the magnet,

$$B = \mu_0 (H + I)$$

Since H and I have opposite direction and I is greater than H,

$$B = 4\pi \times 10^{-7} (-199 + 5 \times 10^4)$$

$= \mathbf{6 \cdot 26 \times 10^{-2} \ T}$ **(towards N-pole)**

Problem 5.07. A sample of paramagnetic salt contains $2 \cdot 0 \times 10^{24}$ atomic dipoles, each of dipole moment $1 \cdot 5 \times 10^{-23}$ J T^{-1}. The sample is placed under homogeneous magnetic field of 0 · 84 T and cooled to a temperature of 4 · 2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0 · 98 T and a temperature of 2 · 8 K (assume Curie's law) ? (*Text Problem*)

Sol. Magnetic dipole moment of each dipole

$$= 1 \cdot 5 \times 10^{-23} \ J \ T^{-1}$$

Number of atomic dipoles $= 2 \cdot 0 \times 10^{24}$

Therefore, possible magnetic dipole moment of the sample,

$$M = 1 \cdot 5 \times 10^{-23} \times 2 \cdot 0 \times 10^{24} = 30 \ J \ T^{-1}$$

At temperature of 4 · 2 K, the magnetic saturation is 15%.

Therefore, dipole moment achieved at 4 · 2 K,

$$M_1 = 30 \times \frac{15}{100} = 4 \cdot 5 \ J \ T^{-1}$$

According to Curie's law,

$$M \propto \frac{B}{T} \quad \text{or} \quad \frac{M_1}{M_2} = \frac{B_1}{T_1} \times \frac{T_2}{B_2}$$

or $$M_2 = M_1 \times \frac{T_1}{T_2} \times \frac{B_2}{B_1}$$

Here, $M_1 = 4 \cdot 5$ J T^{-1} ; $T_1 = 4 \cdot 2$ K ; $T_2 = 2 \cdot 8$ K ;
$B_1 = 0 \cdot 84$ T and $B_2 = 0 \cdot 98$ T

$$\therefore \quad M_2 = \frac{4 \cdot 5 \times 4 \cdot 2 \times 0 \cdot 98}{2 \cdot 8 \times 0 \cdot 84} = \mathbf{7 \cdot 875 \ J \ T^{-1}}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. Define intensity of magnetisation of a magnetic material. How does it vary with temperature of a paramagnetic material ? (*C.B.S.E. 2001*)

Ans. *Intensity of magnetisation of a substance is defined as the magnetic moment developed per unit volume, when it is subjected to a magnetising field.*

The intensity of magnetisation of a paramagnetic substance is inversely proportional to its absolute temperature.

Q. 5.02. Write down the dimensional formula of magnetic flux. (*H.S.S.C.E. 2001*)

Ans. The dimensional formula of magnetic flux is [M L^2 T^{-2} A^{-1}]. For details, refer to SAQ 4.03.

Q. 5.03. Name the physical quantity which is measured in weber ampere^{-1}. (*H.S.S.C.E. 1994*)

Ans. $$\left[\frac{B}{M}\right] = \frac{\text{weber metre}^{-2}}{\text{ampere metre}^{-2}} = \text{weber ampere}^{-1}$$

Thus, ratio of magnetic induction to the magnetic noment is measured in weber ampere^{-1}.

Q. 5.04. Define magnetic susceptibility. (*H.S.S.C.E. 2001*)

Ans. *It is defined as the ratio of the intensity of magnetisation f a substance to the intensity of magnetising field.*

Q. 5.05. Define magnetic permeability. (*I.S.C.E. 1998*)

Ans. It is defined as the ratio of the magnetic induction B) to the magnetic intensity (H).

Mathematically, $\mu = \frac{B}{H}$.

Q. 5.06. An iron rod of 0 · 1 m² area of cross-section is ubjected to a magnetising field of 1000 A m^{-1}. Calculate ts magnetic permeability. Given susceptibility of iron is 599. (*H.S.S.C.E. 2001*)

Ans. Here, $A = 0 \cdot 1\ m^2$; $H = 1000\ A\ m^{-1}$ and $\chi_m = 599$

We know, $\mu_0 = 4\pi \times 10^{-7}\ T\ A^{-1}\ m$

Now, $\mu = \mu_0 (1 + \chi_m) = 4\pi \times 10^{-7} (1 + 599)$

$= \mathbf{7 \cdot 54 \times 10^{-4}\ T\ A^{-1}\ m}$

Q. 5.07. Name the source of magnetic properties of materials.

Ans. Magnetism is an effect of electric charges in motion. The magnetic properties of the materials due to the atomic magnets, which are the tiny circulating currents in the form of electrons revolving in atomic orbits.

Q. 5.08. What is meant by the non-magnetic material ?

Ans. Non-magnetic materials are those materials, which cannot be affected very much by the strong magnetic field.

Q. 5.09. What is a diamagnetic substance ? (*P.S.S.C.E. 1997*)

Ans. A substance, which when placed in a magnetic field gets feebly magnetised in a direction opposite to that of the magnetising field, is called a diamagnetic substance.

Q. 5.10. Name any two diamagnetic substances. (*P.S.S.C.E. 1999 S*)

Ans. 1. Zinc 2. Bismuth.

Q. 5.11. What happens, when a diamagnetic substance is placed in a varying magnetic field ? (*P.S.S.C.E. 2002, 1998*)

Ans. It moves from stronger part of the magnetic field to the weaker part.

Q. 5.12. Why is diamagnetism independent of temperature ? (*H.P.S.S.C.E. 1996 ; Text Question*)

Ans. The atoms of a diamagnetic do not have intrinsic magnetic dipole moment. In a diamagnetic sample, the magnetic moment of the sample on placing in a magnetic field is always opposite to the direction of the field. It is not affected by the thermal motion of the dipoles.

Q. 5.13. The magnetic susceptibility for a sample has a small negative value. To which class of magnetic substances, does the specimen belong ?

Ans. Diamagnetic.

Q. 5.14. What is paramagnetic substance ? (*P.S.S.C.E. 1999, 1998 S*)

Ans. A substance, which when placed in a magnetic field gets feebly magnetised in the direction of the magnetising field, is called a paramagnetic substance.

Q. 5.15. Name any two paramagnetic substances. (*P.S.S.C.E. 1999 S*)

Ans. 1. Aluminium 2. Manganese.

Q. 5.16. The value of intensity of magnetisation is small positive for a specimen. Is it diamagnetic or paramagnetic or ferromagnetic ?

Ans. Paramagnetic.

Q. 5.17. Why does a paramagnetic sample display greater magnetisation (for the same magnetising field), when cooled ? (*Text Question ; H.P.S.S.C.E. 1998 S*)

Ans. When cooled, the tendency of the thermal agitation to disrupt the alignment of magnetic dipoles decreases in case of paramagnetic materials. Hence, they display greater magnetisation.

Q. 5.18. State Curie law in magnetism. (*H.P.S.S.C.E. 2001*)

Ans. Refer to section 5.04.

Q. 5.19. What is Curie point ?

Ans. It is the temperature for a ferromagnetic substance above which, it behaves as paramagnetic.

Q. 5.20. How does the magnetic induction of a paramagnetic material vary with temperature ? (*C.B.S.E. 2000, 1999*)

Ans. It decreases with increase of temperature.

Q. 5.21. How does the intensity of magnetisation of a paramagnetic material vary with increasing applied magnetic field ? (*C.B.S.E. 1996*)

Ans. The intensity of magnetisation of a paramagnetic material increases directly with increase in applied magnetic field.

Q. 5.22. How can paramagnetic and diamagnetic materials be distinguished by studying their behaviour in a magnetic field ? (*I.S.C.E. 1994*)

Ans. In a non-uniform magnetic field, a paramagnetic substance will move from weaker part of the magnetic field to the stronger part, while a diamagnetic substance will move from stronger to weaker part of the field.

Q. 5.23. State two properties of ferromagnetic substances. (*P.S.S.C.E. 2002*)

Ans. Refer to section 5.06.

Q. 5.24. Why do magnetic field lines prefer to pass through ferromagnetic substances (*e.g.* iron) than through air ? (*C.B.S.S.E. 1999*)

Ans. Ferromagnetic substances have relative permeability quite greater than 1. Hence, the magnetic field lines prefer to pass through them.

Q. 5.25. What type of magnetic material is used in making permanent magnets ? (*C.B.S.E. 1999 S, 1996*)

Ans. For making permanent magnet, a material having high coercivity is used.

Q. 5.26. Why is it possible to make permanent magnets out of steel ? (*I.S.C.E. 1998*)

Or

Why do we prefer to use the alloy alnico for making permanent magnets ? (*H.P.S.S.C.E. 1998*)

Ans. It is because, steel (or alnico) has a large coercivity.

Q. 5.27. Soft iron is used to make electromagnets. Why ? (*H.S.S.C.E. 2000*)

Ans. It is because of the fact that hysteresis loop for soft iron is narrow. Due to this, loss of energy per unit volume is small, when soft iron is taken over complete cycle of magnetisation.

Q. 5.28. Which is the material used in making the core of a transformer or a moving coil galvanometer ? *(C.B.S.E. 1999 S, 1996)*

Ans. Soft iron is used in making the core of the galvanometer. It makes the magnetic field strong as magnetic field lines tend to cross through a ferromagnetic.

Q. 5.29. Why soft iron is used in making the core of a transformer ? *(P.S.S.C.E. 2002)*

Ans. The area of hysteresis loop for soft iron is very small. Since energy dissipated during a complete cycle of magnetisation and demagnetisation is proportional to area of the hysteresis loop, a small amount of energy will be wasted, when core of the transformer is made of soft iron.

Q. 5.30. The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy ? *(Text Question)*

Ans. The carbon steel piece will dissipate greater heat energy as its hysteresis loop has greater area.

Q. 5.31. State two methods to destroy the magnetism of a magnet. *(P.S.S.C.E. 2002)*

Ans. 1. By applying magnetising field in reverse direction.

2. By heating the magnet.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. What is meant by magnetic field and magnetic field intensity ? *(H.S.S.C.E. 1994)*

Ans. Refer to section 5.01.

Q. 5.02. Define magnetic flux. Write its dimensions. *(P.S.S.C.E. 2000, 1998 S)*

Ans. Magnetic flux through a surface is defined as the number of magnetic field lines passing normally through that surface. Mathematically,

$$\phi = \vec{B} \cdot \Delta\vec{S},$$

where the letters have their usual meanings.

Dimensional formula of magnetic flux $[\phi] = [ML^2T^{-2}A^{-1}]$

Q. 5.03. Define magnetic susceptibility and magnetic permeability.

Ans. Refer to section 5.01.

Q. 5.04. What is relative permeability of a magnetic material ? How is it related to the magnetic susceptibility ? *(I.S.C.E. 1997)*

Ans. The relative permeability of a material is defined as the ratio of its absolute permeability to the absolute permeability of vacuum.

Mathematically, $\mu_r = \dfrac{\mu}{\mu_0}$

Relative permeability (μ_r) and magnetic susceptibility are related to each other as

$$\mu_r = 1 + \chi_m$$

Q. 5.05. What do we infer from the large value of the susceptibility of the material ? For example, susceptibility of iron is more than that of copper. Explain.

Ans. The susceptibility of a material is defined as the ratio of the intensity of magnetisation (I) to the magnetic intensity (H) *i.e.*

$$\chi_m = \frac{I}{H}$$

In other words, greater the value of the susceptibility of a material, greater will be the value of intensity of magnetisation for it *i.e.* more easily it can be magnetised. Thus, greater value of susceptibility for iron means that iron can be magnetised more easily than copper.

Q. 5.06. Distinguish between dia, para and ferro magnetic substances. Give one example of each. *(H.S.S.C.E. 1999 ; C.B.S.E. 1994)*

Or

Give one difference each between diamagnetic, para-magnetic and ferromagnetic substances. *(C.B.S.E. 1992)*

Ans. A diamagnetic is feebly magnetised in a direction opposite to the direction of magnetising field, whereas a paramagnetic is magnetised feebly and ferromagnetic strongly in the direction as that of the field.

For examples, refer to section 5.06.

Q. 5.07. Give any four properties of a diamagnetic substance. *(P.S.S.C.E. 1999 S)*

Ans. Refer to section 5.06.

Q. 5.08. The susceptibility of a magnetic material is $-0 \cdot 085$. Identify the magnetic type of the substance. A specimen of this material is kept in a uniform magnetic field. Draw the modified field pattern. *(C.B.S.E. 2001)*

Ans. Since the susceptibility of the material has a small negative value, the given material is diamagnetic in nature.

When this material is kept in a uniform magnetic field, the magnetic field lines will become slightly less dense as shown in Fig. 5.14.

Q. 5.09. State two properties each of a para and dia-magnetic substances. *(P.S.S.C.E. 2000, 1999, 1998 S, 1997 S)*

Ans. Properties of paramagnetic substances. 1. When placed in magnetic field, a paramagnetic material gets feebly magnetised along the direction of field.

2. It obeys Curie's law in magnetism.

Properties of diamagnetic substances. 1. When placed in magnetic field a diamagnetic substance gets feebly magnetised in a direction opposite to that of the field.

2. It does not obey Curie's law.

Q. 5.10. Distinguish between a diamagnetic substance and a paramagnetic substance stating two points of difference. *(C.B.S.E. 2001 S)*

Ans. Refer to section 5.06.

Q. 5.11. Two substances A and B have their relative permeabilities slightly greater and less than unity respectively. What do you conclude about A and B ? *(I.S.C.E. 1997)*

Ans. We know, $\mu_r = 1 + \chi_m$

Since relative permeability of substance A is slightly greater than 1, it follows that its susceptibility is small positive and hence it is a **paramagnetic** substance.

On the other hand, as relative permeability of B is ightly less than 1, its susceptibility is small negative and ence it is a **diamagnetic** substance.

Q. 5.12. Give any four properties of a paramagnetic ubstance. (*P.S.S.C.E. 1999 S*)

Ans. Refer to section 5.06.

Q. 5.13. Does the magnetisation of a paramagnetic salt epend on temperature ? Give reason for your answer. (*C.B.S.E. 1990*)

Ans. Yes, the magnetisation of a paramagnetic salt epends on temperature. The atoms of a paramagnetic aterial possess small magnetic dipole moments but these tomic dipoles are randomly oriented. When external agnetic field is applied, the magnetic dipoles try to align but e thermal agitation hampers the alignment. Hence, the agnetisation of a paramagnetic salt depends on the emperature.

Q. 5.14. Distinguish between diamagnetic and paraagnetic substances in terms of their susceptibility, permebility and nature of magnetism, when placed in a uniform agnetic field. (*I.S.C.E. 1995*)

Ans. Refer to section 5.06.

Q. 5.15. Would the maximum possible magnetisation f a paramagnetic sample be of the same order of magnitude s the magnetisation of a ferromagnetic ? (*Text Question*)

Ans. Yes, a paramagnetic sample with saturated agnetisation will have the same order of magnetisation as e magnetisation of a ferromagnet. However, the saturated agnetisation will require magnetising field too high to chieve. Further, there may be a minor difference in the trengths of the atomic dipoles of paramagnetic and erromagnetic materials.

Q. 5.16. Why are all pieces of iron not magnet even hough iron is a ferromagnetic material ?

Ans. In iron, the atomic magnetic dipoles are randomly riented in the form of small domains, such that the net agnetic moment of the specimen is zero. It becomes magnet nly on applying external magnetic field. When external field s applied, the magnetic dipoles align along the direction of he field and the specimen becomes magnet as it now ossesses non-zero magnetic moment.

Q. 5.17. An iron bar magnet is heated to 1000° C and hen cooled in a magnetic field free space. Will it retain agnetism ? (*C.B.S.E. 1990*)

Ans. At 1000°C, which is quite above the Curie emperature for iron (770° C), the iron will behave as aramagnetic. Moreover, when a magnet is heated, it loses its magnetism. Hence, iron bar magnet on heating to 1000° C and then cooled, will not retain its magnetism.

Q. 5.18. Is the permeability of a ferromagnetic material independent of the magnetic field ? If not, is it more for lower or higher fields ? (*Text Question*)

Ans. No, permeability of a ferromagnetic material depends upon the applied magnetic field. It is more for lower value of applied field. Graph between B and H is of the shape as shown in Fig. 5.20. Obviously, the value of B will be more for lesser value of H.

Q. 5.19. Give two points to compare the magnetic properties of steel and soft iron. (*C.B.S.E. 1990*)

Ans. 1. The area of the hysteresis loop for steel is larger than that for soft iron. On account of this, the loss of energy per unit volume in case of steel will be much larger as compared to that in case of soft iron, when they are taken over a complete cycle of magnetisation. For this reason, soft iron is used to make core of a transformer.

2. The coercivity for steel is very large as compared to that for soft iron. For this reason, steel is used to make permanent magnet.

Q. 5.20. What are the uses of studying the hysteresis curve for a given material ?

Ans. The main use of the study of hysteresis curve of a material is that it gives an estimate of the *hysteresis loss*. Moreover, the study of the hysteresis curve helps in knowing the retentivity and coercivity of the material. Their knowledge helps in selecting proper materials for making core of transformers, permanent magnets and electromagnets.

Q. 5.21. Why do we use steel or alnico for making permanent magnets ?

Ans. The material used for making a permanent magnet should have high value of coercivity and also high value of retentivity. Moreover, the area of hysteresis curve should also be large. A permanent magnet is never taken through a complete cycle of magnetisation and hence large area of hysteresis curve will not be a demerit of such a material. Steel and alnico fulfil all these requirements.

Q. 5.22. What sort of hysteresis loop should a material have, if it is to be used for the core of a transformer ?

Ans. The core of a transformer undergoes cycles of magnetisation again and again. During each cycle of magnetisation, energy numerically equal to the area of the hysteresis loop is spent per unit volume of the core. Therefore, for high efficiency of transformer, the energy loss will be lesser, if the hysteresis loop is of lesser area *i.e.* narrow. Since, soft iron has narrow hysteresis loop, it should be used as the core of a transformer.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 5.01. A strong horse-shoe magnet is closed by a piece A made of iron [Fig. 5.22 (*a*)]. The weight of the iron iece A corresponds to the lifting force of the magnet and he magnet can easily hold the piece of iron. If the poles of he magnet are now touched on the sides with a piece B made of soft iron [Fig. 5.22 (*b*)], the iron piece A will drop t once. Explain the above observation with a proper xplanation.

Ans. When the horse-shoe magnet is closed by piece of ron A, the magnetic field lines run through the magnet and he piece of iron A as shown in Fig. 5.22 (*a*). As a result of the

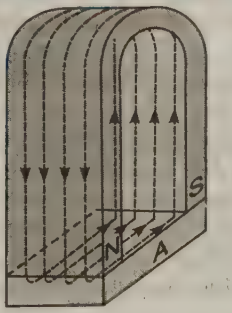


Fig. 5.22(a)

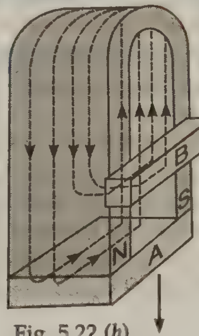


Fig. 5.22 (b)

strong attraction, the piece of iron A is held by the horse-shoe magnet.

When the piece of iron B is placed in contact on the sides of horse-shoe magnet, part of the magnetic field lines are short circuited through this piece of iron [Fig. 5.22 (*b*)]. The number of lines of force penetrating the piece A sharply decreases. As a result, the attraction between the magnet and the iron piece A also decreases and it drops.

Q. 5.02. A long rod R made of soft iron is placed in vertical position. If a strong magnet NS is brought to the top of the rod as shown in Fig. 5.23 (*a*), the rod gets magnetised so intensely that it is able to hold at its other end several small pieces of iron. If the same magnet NS is placed so as to stick to the rod at its side near the bottom, the magnetisation of the rod is too weak to hold the pieces of iron [Fig. 5.23 (*b*)]. Explain, why the magnet N S acts differently in these two cases.

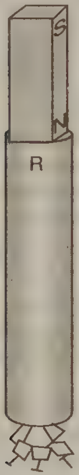


Fig. 5.23 (*a*)

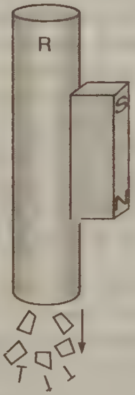


Fig. 5.23. (*b*)

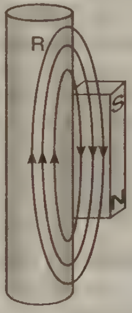


Fig. 5.23. (*c*)

Ans. The rod holds the small pieces of iron as it gets intensely magnetised due to the magnet placed at its top as shown in Fig. 5.23 (*a*).

When the magnet is placed on the side of the rod [Fig. 5.23 (*b*)], most of the magnetic lines of force are short-circuited inside the part of rod adjoining the magnet as shown in Fig. 5.23 (*c*). As such, the rod does not remain intensely magnetised as before. The result is that the pieces of iron drop.

Q. 5.03. A certain region of space is to be shielded from magnetic fields. Suggest a method. (*Text Question*)

Ans. It can be done by surrounding the region with soft iron rings. Magnetic field lines will be drawn into the rings and the enclosed space will be free of magnetic field.

Q. 5.04. If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty ? (*Text Question*)

Ans. Since bismuth is diamagnetic, the field in the core will be slightly less than that when the core is empty.

Q. 5.05. Several steel needles are freely suspended on hooks from a small brass disc as shown in Fig. 5.24. If the pole of a strong magnet is brought up to the needles from below ; the needles will first be drawn apart and then will again assume a vertical position, when the magnet is brought right up to them. As the magnet is removed, the needles will again be drawn apart forming a cone-shaped bunch. Explain the causes of such behaviour of the needles.

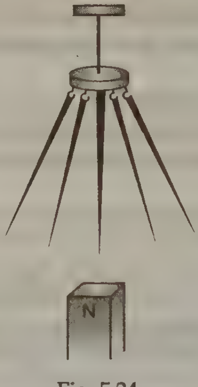


Fig. 5.24

Ans. All the steel needles develop south-poles at their lower ends due to induced magnetism and are drawn apart due to mutual force of repulsion between them.

When the magnet is sufficiently close to the needles, the force of attraction between the needles and the magnet exceeds the mutual force of repulsion between them. Under the effect of net attractive force due to the magnet, the steel needles assume a vertical position.

When the magnet is removed, the needles are again drawn apart due to the mutual repulsive force between the needles because of the residual magnetism left on the needles.

Q. 5.06. Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point). Why ?

(*Text Question*)

Ans. The exact explanation is beyond the scope of the book. However, it may be said that when a material has $\mu_r >> 1$, the field lines meet the material nearly normally.

Q. 5.07. A small thin iron nail is suspended from a light fire-proof thread [Fig. 5.25]. A strong electro-magnet is placed near the nail and a flame from a gas burner is produced between the nail and the electromagnet. The flame licks the nail, when it is attracted by the magnet. As the current through the solenoid of the electro-magnet is switched on, the nail will be at once deflected into the flame and will then get out of the flame to assume its original position. After a lapse of time, the nail will again be drawn to the magnet. Explain, what causes these periodic oscillations of the nail.

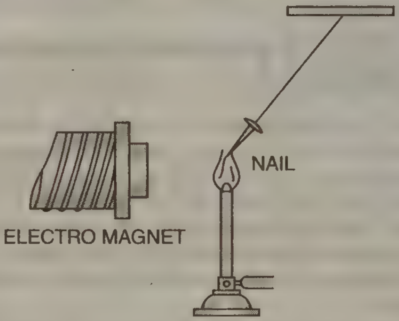


Fig. 5.25

Ans. When the iron nail is drawn into the flame on being attracted by the electromagnet, it loses its ferromagnetism on getting heated. As a result, the interaction between the nail and he electromagnet decreases abruptly and it returns to its nitial position. As the nail cools, it regains its ferromagnetism nd again gets attracted towards the magnet and is drawn nto the flame. The process repeats itself and the nail executes periodic oscillations.

Q. 5.08. Suppose a man proposes a theory that the earth's magnetic field is due to permanent magnetisation of molten iron core of earth. Will you accept this theory ? Give reasons for your answer. *(Roorkee, 1980)*

Ans. Curie temperature of iron is 770°C and the temperature of earth increases by $6°C\ km^{-1}$. Since radius of earth is 6400 km, the temperature of the iron core of earth will be so high that it can not retain magnetism. Hence, such a theory is not acceptable.

Q. 5.09. What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player or for building 'memory stores' in a modern computer ? *(Text Question)*

Ans. Ceramics are used for coating magnetic tapes in a cassette player or for building memory stores in a modern computer. Ceramics are specially treated barium iron oxides and are also called *ferrites*.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain the terms (*a*) magnetic intensity, (*b*) intensity of magnetisation, (*c*) magnetic flux, (*d*) magnetic induction, (*e*) magnetic susceptibility and (*f*) magnetic permeability. Give their SI units.
2. Distinguish between paramagnetic, diamagnetic and ferromagnetic substances. *(H.S.S.C.E. 2002 ; C.B.S.E. 1992)*
3. What is the difference between ferromagnetic, paramagnetic and diamagnetic substances ? Classify the following materials in respect of the magnetic behaviour : (*i*) Mercury (*ii*) Water (*iii*) Iron (*iv*) Aluminium.
4. What are the categories, in which magnetic materials are classified ? Explain their differences. *(C.B.S.E. 1992 S)*
5. What are dia, para and ferromagnetic substances ? Give one example of each. *(H.S.S.C.E. 2002)*
6. Classify materials on the basis of their behaviour in a magnetic field. Under which category does iron come ? How does the magnetic property of iron change with increase of temperature ? *(C.B.S.E. 1995)*
7. What are diamagnetic substances ? Explain diamagnetism on the basis of electron theory. *(H.S.S.C.E. 2001)*
8. How are materials classified according to their behaviour in a magnetic field ? Why does the magnetisation of a paramagnetic salt increase on cooling ? *(C.B.S.E. 1991)*
9. State and explain Curie's law in magnetism. *(P.S.S.C.E. 2001, 2000 ; C.B.S.E. 1997)*
10. What are ferromagnetic substances ? Explain briefly domain theory to explain ferromagnetism. *(H.P.S.S.C.E. 2002)*
11. Give any six properties of diamagnetic substances. *(P.S.S.C.E. 2001)*
12. Compare any four properties of diamagnetic and paramagnetic substances. *(H.S.S.C.E. 2001)*
13. Explain magnetic hysteresis. *(H.S.S.C.E. 2001)*
14. What is hysteresis loop ? Explain with its help the terms related to it. *(H.P.S.S.C.E. 1999)*
15. What is a hysteresis loop ? Explain the different portions of a complete hysteresis loop.
16. Explain the phenomenon of hysteresis. Draw a hysteresis loop showing the remanence and coercive force. *(I.S.C.E. 1996)*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Distinguish between ferromagnetic, paramagnetic and diamagnetic materials. Explain diamagnetism and paramagnetism on the basis of electron theory. Give main properties of diamagnetic materials.
2. Describe briefly, how ferromagnetism can be explained on the basis of the domain theory. Give the important properties of ferromagnetic and diamagnetic substances.
3. What are dia, para and ferromagnetic materials ? Discuss their important properties. *(H.P.S.S.C.E. 1994 ; C.B.S.E. 1992, 1991 ; P.S.S.C.E. 1992)*
4. Define coercivity and remanence. How are soft iron and hard steel distinguished by the values of these quantities ? Sketch the hysteresis loop for soft iron and hard steel. What do you infer about these materials from their hysteresis loops ?

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Magnetisation, Permeability and Susceptibility

1. A magnet weighs 75 g and its magnetic moment is $2 \times 10^{-4}\ A\ m^2$. If the density of the material of the magnet is $7{\cdot}5 \times 10^3\ kg\ m^{-3}$, calculate the intensity of magnetisation. **[Ans.** $20\ A\ m^{-1}$**]**
2. The magnetic induction and magnetising field in a sample of magnetic material are $1{\cdot}0\ Wb\ m^{-2}$ and $2 \times 10^3\ A\ m^{-1}$ respectively. Find (*a*) magnetic permeability (*b*) relative permeability of the material, (*c*) magnetic susceptibility and (*d*) intensity of magnetisation. Given, $\mu_0 = 4\pi \times 10^{-7}\ T\ A^{-1}\ m$. **[Ans.** (*a*) $5 \times 10^{-4}\ T\ m\ A^{-1}\ m$; (*b*) 397·9 ; (*c*) 396·9 ; (*d*) $7{\cdot}94 \times 10^5\ A\ m^{-1}$**]**
3. The magnetising field of $1600\ A\ m^{-1}$ produces a magnetic flux of $2{\cdot}4 \times 10^{-5}$ weber in a bar of iron of cross-section $0{\cdot}2\ cm^2$. Calculate relative permeability, intensity of magnetisation and susceptibility of the bar. **[Ans.** 596·8, $9{\cdot}534 \times 10^5\ A\ m^{-1}$, 595·8**]**
4. The core of a toroid having 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. The magnetic field

in the core for a current of 0·70 A is 2·5 T. What is the relative permeability of the core ? *(Text Problem)* **[Ans.** 684·5**]**

5. A Rowland ring has 10^3 turns per unit length. On passing a current of 2 A, magnetic induction is measured to be 1·5 Wb m^{-2}. Calculate (*i*) relative permeability of the core, (*ii*) magnetic susceptibility, (*iii*) magnetising field intensity and (*iv*) magnetisation. **[Ans.** (*i*) 596·8 ; (*ii*) 595·8 ; (*iii*) 2000 A m^{-1} ; (*iv*) $1·192 \times 10^6$ A m^{-1}**]**

Type B. On Curie's law

6. The susceptibility of magnesium at 300 K is $1·2 \times 10^{-5}$. At what temperature will the susceptibility equal to $1·44 \times 10^{-5}$? **[Ans.** 250 K**]**

Type C. On Hysteresis loop

7. The hysteresis loss for a specimen of iron weighing 12 kg is equivalent to 300 J m^{-3} cycle^{-1}. Find the loss of energy per hour at 50 cycles s^{-1}. Given, density of iron = 7500 kg m^{-3}. **[Ans.** $8·64 \times 10^4$ J**]**

MISCELLANEOUS PROBLEMS

8. Assume that each iron atom has a permanent magnetic moment equal to 2 Bohr magneton (1 Bohr magneton = $9·27 \times 10^{-24}$ A m^2). The number density of atoms in iron is $8·52 \times 10^{28}$ m^{-3}. Find the maximum value of (*a*) intensity of magnetisation and (*b*) magnetic induction in an iron bar. **[Ans.** (*a*) $1·58 \times 10^6$ A m^{-1} (*b*) 1·99 T**]**

9. A paramagnetic gas consists of atoms each with a dipole moment of $1·5 \times 10^{-23}$ J T^{-1}. The temperature of the gas is 27°C and its number density (number of atoms per unit volume) is 2×10^{26} m^{-3}. (*i*) What is the maximum magnetisation of the sample possible, when it is placed in an external magnetic field ? (*ii*) Do you think, the sample can achieve close to maximum magnetisation for a magnetic field of 3 T ? Given, Boltzmann's constant = $1·38 \times 10^{-23}$ J molecule^{-1} K^{-1}. *(Text Problem)* **[Ans.** 3×10^3 A m^{-1}, No**]**

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. If V is volume of the magnet, then

$$I = \frac{M}{V}$$

Now, $$V = \frac{\text{mass}}{\text{density}} = \frac{75 \times 10^{-3}}{7·5 \times 10^3} = 10^{-5}\ m^3$$

$$\therefore \quad I = \frac{2 \times 10^{-4}}{10^{-5}} = \mathbf{20\ A\ m^{-1}}$$

2. Here, B = 1·0 Wb m^{-2} ; H = 2×10^3 A m^{-1}

(*a*) Magnetic permeability,

$$\mu = \frac{B}{H} = \frac{1·0}{2 \times 10^3} = \mathbf{5 \times 10^{-4}\ T\ m\ A^{-1}}$$

(*b*) Relative permeability, $$\mu_r = \frac{\mu}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = \mathbf{397·9}$$

(*c*) Magnetic susceptibility, $\chi_m = \mu_r - 1 = 397·9 - 1 = \mathbf{396·9}$

(*d*) Intensity of magnetisation,

$$I = \chi_m H = 396·9 \times 2 \times 10^3$$
$$= \mathbf{7·94 \times 10^5\ A\ m^{-1}}$$

3. Here, H = 1600 A m^{-1} ; $\phi = 2·4 \times 10^{-5}$ Wb,

$$A = 0·2\ cm^2 = 0·2 \times 10^{-4}\ m^2$$

Now, magnetic induction, $$B = \frac{\phi}{A} = \frac{2·4 \times 10^{-5}}{0·2 \times 10^{-4}} = 1·2\ T$$

Also, $$\mu = \frac{B}{H} = \frac{1·2}{1600} = 7·5 \times 10^{-4}\ T\ m\ A^{-1}$$

Therefore, relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{7·5 \times 10^{-4}}{4\pi \times 10^{-7}} = \mathbf{596·8}$$

Again, $B = \mu_0 (H + I)$

$$\therefore \quad I = \frac{B}{\mu_0} - H = \frac{1·2}{4\pi \times 10^{-7}} - 1600$$
$$= 9·55 \times 10^5 - 1600 = \mathbf{9·534 \times 10^5\ A\ m^{-1}}$$

Finally, $\chi_m = \mu_r - 1 = 596·8 - 1 = \mathbf{595·8}$

4. Mean radius of toroid,

$$r = \frac{11 + 12}{2} = 11·5\ cm = 11·5 \times 10^{-2}\ m$$

Therefore, number of turns per unit length of the toroid,

$$n = \frac{\text{total number of turns}}{2\pi r} = \frac{3000}{2\pi \times 11·5 \times 10^{-2}}$$
$$= 4·152 \times 10^3\ m^{-1}$$

Now, magnetic field in the core of solenoid,

$$B = \mu_0 \mu_r n I \quad \text{or} \quad \mu_r = \frac{B}{\mu_0 n I}$$

or $$\mu_r = \frac{2·5}{4\pi \times 10^{-7} \times 4·152 \times 10^3 \times 0·70} = \mathbf{684·5}$$

5. Here, $n = 10^3\ m^{-1}$; I = 2 A ; B = 1·5 Wb m^{-2}

(*i*) Magnetic induction due to the solenoid,

$$B = \mu_0 \mu_r n I \quad \text{or} \quad \mu_r = \frac{B}{\mu_0 n I}$$

or $$\mu_r = \frac{1·5}{4\pi \times 10^{-7} \times 10^3 \times 2} = \mathbf{596·8}$$

(*ii*) $\chi_m = \mu_r - 1 = 596·8 - 1 = \mathbf{595·8}$

(*iii*) $$H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} = \frac{1·5}{4\pi \times 10^{-7} \times 596·8}$$
$$= \mathbf{2000\ A\ m^{-1}}$$

(*iv*) $I = \chi_m H = 595·8 \times 2000 = \mathbf{1·192 \times 10^6\ A\ m^{-1}}$

6. According to Curie's law ; $\chi_m = \frac{C}{T}$

$$\therefore \quad \frac{\chi_m}{\chi_m'} = \frac{T'}{T}$$

or $$T' = T \times \frac{\chi_m}{\chi_m'} = \frac{300 \times 1·2 \times 10^{-5}}{1·44 \times 10^{-5}} = \mathbf{250\ K}$$

7. Here, hysteresis loss per unit volume per cycle

$$= 300\ J\ m^{-3}\ cycle^{-1}$$

Time, t = 1 hour = 3600 s, frequency, ν = 50 cycles s^{-1}

Therefore, number of hysteresis cycle in 1 hour,

$$N = \nu \times t = 50 \times 3600 = 1·8 \times 10^5$$

Volume of the iron specimen,

$$V = \frac{\text{mass}}{\text{density}} = \frac{12}{7500} = 1·6 \times 10^{-3}\ m^3$$

Hence, loss of energy in one hour,

W = hysteresis loss per unit volume per cycle × V × N

$$= 300 \times 1·6 \times 10^{-3} \times 1·8 \times 10^5 = \mathbf{8·64 \times 10^4\ J}$$

8. *(a)* Here, number of atoms per unit volume in iron,

$$n = 8{\cdot}52 \times 10^{28}\ \text{m}^{-3}$$

Magnetic moment of each iron atom $= 2\,\mu_B$

$$= 2 \times 9{\cdot}27 \times 10^{-24} = 1{\cdot}854 \times 10^{-23}\ \text{A m}^2$$

Now, intensity of magnetisation is equal to magnetic moment per unit volume.

Therefore, the maximum value of magnetisation,

$$I_{max} = n \times 2\,\mu_B = 8{\cdot}52 \times 10^{28} \times 1{\cdot}854 \times 10^{-23}$$

$$= \mathbf{1{\cdot}58 \times 10^6\ A\ m^{-1}}$$

(b) Now, $B = \mu_0\,(H + I)$

Since no magnetic field is applied, $H = 0$

$$\therefore \quad B = \mu_0\, I = 4\,\pi \times 10^{-7} \times 1{\cdot}58 \times 10^6 = \mathbf{1{\cdot}99\ T}$$

9. *(i)* Here, number of atoms per unit volume in the paramagnetic gas,

$$n = 2 \times 10^{26}\ \text{m}^{-3}$$

Magnetic moment of each gas atom $= 1{\cdot}5 \times 10^{-23}\ \text{J T}^{-1}\ \text{m}$

Therefore, maximum value of magnetisation,

$$I_{max} = 2{\cdot}0 \times 10^{26} \times 1{\cdot}5 \times 10^{-23} = \mathbf{3 \times 10^3\ A\ m^{-1}}$$

(ii) The maximum magnetisation will occur, if the atomic dipole remain aligned along the direction of applied magnetic field at the temperature of the gas.

In other words, maximum magnetisation will be achieved on applying magnetic field of 3 T, if the thermal energy of an atomic dipole is quite small as compared to its potential energy.

Now, potential energy of an atomic dipole,

$$U = M\,B = 1{\cdot}5 \times 10^{-23} \times 3 = 4{\cdot}5 \times 10^{-23}\ \text{J}$$

Also, thermal energy of an atomic dipole.

$$U' = \frac{3}{2}\,k\,T = \frac{3}{2} \times 1{\cdot}38 \times 10^{-23} \times (273 + 27)$$

$$= 6{\cdot}21 \times 10^{-21}\ \text{J}$$

Since $U' >> U$, all the atomic dipoles can not remain aligned to the applied field. Hence, maximum magnetisation **cannot be achieved** in the paramagnetic gas under given conditions.

Competitive Examination File (Unit III)

REVISION AT A GLANCE

Chapter 1. Magnetic Effect of Current

☐ **Biot Savart's Law.** According to this law, *the magnetic field (in magnitude) due to a current element of length dl carrying a current I at a point at distance r from it, is given by*

$$|\vec{dB}| = \frac{\mu_0}{4\pi} \cdot \frac{|\vec{dl} \times \vec{r}|}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2},$$

where θ is the angle between the direction of the current and the line joining the current element to the point and μ_0 is absolute permeability of the free space. The direction of magnetic field $\vec{dB}$ is that of $I\,\vec{dl} \times \vec{r}$. It can also be found by using ***right hand thumb rule.***

The absolute permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\,A^{-1}\,m}$.

☐ **Magnetic field due to a current carrying conductor.** The magnetic field at a point at perpendicular distance a from a straight conductor carrying current I is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}(\sin\phi_1 + \sin\phi_2),$$

where ϕ_1 and ϕ_2 are angles, which the lines joining the two ends of the conductor to the observation point make with the perpendicular from the observation point to the conductor.

In case, the straight conductor is of infinite length $\left(\phi_1 = \phi_2 = \frac{\pi}{2}\right)$, the magnetic field is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

☐ **Magnetic field due to a current carrying circular coil.** For a coil of radius a, consisting of n turns and carrying current I,

(*i*) the magnetic field at a point on axis at distance x from its centre is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I a^2}{(a^2 + x^2)^{3/2}}$$

(*ii*) the magnetic field at its centre is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a}$$

☐ **Ampere's circuital law.** It states that *the line integral of magnetic field $\vec{B}$ around any closed path or circuit is equal to μ_0 times the total current I threading through the closed circuit.*

Mathematically : $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

☐ **Magnetic field due to a solenoid.** *A long insulated wire closely wound in the form of a helix, such that its length is very large as compared to its diameter is called a solenoid.*

If a solenoid of n turns per unit length carries a current I, then

(*i*) magnetic field at a point well inside the solenoid is given by

$$B = \mu_0 n I$$

(*ii*) magnetic field at a point on one end of the solenoid is given by

$$B = \frac{1}{2}\mu_0 n I$$

Some Useful Facts

1. Whereas the source of electric field is electric charge, the source of magnetic field is current element (not a magnetic charge).

2. The magnatic field due to a straight conductor is zero, when the observation point lies along the length of the conductor.

3. The magnetic field due to a part of circular conductor of radius a and carrying current I is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}\theta,$$

where θ is the angle, which the circular part subtends at its centre.

Chapter 2. Motion of charged particle

☐ **Force on a charge in electric field.** A charge q inside an electric field of strength $\vec{E}$ experiences force $\vec{F}$, which is given by

$$|\vec{F}| = |q\vec{E}| = qE$$

Force is in **newton**, when q is in **coulomb** and E in **newton coulomb^{-1}**.

☐ **Motion of a charge inside electric field.** If a potential difference V is applied between two parallel plates, a uniform electric field is set up between the plates. Its strength is given by

$$E = \frac{V}{d}$$

A charge q of mass m experiences force

$$F = qE,$$

which produces an acceleration,

$$a = \frac{qE}{m}$$

The charge moving inside an electric field follows a ***parabolic*** path.

☐ **Force on a charge moving inside a magnetic field.** A charge q moving with velocity $\vec{v}$ inside a magnetic field of strength $\vec{B}$ is given by

$$|\vec{F}| = q|\vec{v} \times \vec{B}| = Bqv \sin\theta$$

Force is in **newton**, when B is in **tesla**, q in **coulomb** and v in **m s^{-1}**.

The charge does not experience any force, if it is at rest or if it moves along the direction of magnetic field. The force is maximum, when charge moves perpendicular to the direction of magnetic field.

The above expression is used to define **tesla**, the unit of strength of magnetic field.

The strength of magnetic field is called one tesla, if a charge of one coulomb, when moving with a velocity of 1 m s^{-1} along a direction perpendicular to the direction of the magnetic field experiences a force of one newton.

1 tesla (T) = 1 weber metre^{-2} (Wb m^{-2}) = 1 newton ampere^{-1} metre^{-1} (N A^{-1} m^{-1})

☐ **Lorentz force.** The total force experienced by a charge moving inside the electric and magnetic fields is called Lorentz force. It is given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

☐ **Motion of charge inside magnetic field.** A charge q of mass m moving with velocity $\vec{v}$ inside a magnetic field of strength $\vec{B}$ experiences force,

$$|\vec{F}| = q|\vec{v} \times \vec{B}| = Bqv \sin\theta,$$

where θ is angle between the direction of motion of charge $(\vec{v})$ and the direction of magnetic field $(\vec{B})$. This force acts perpendicular to both $\vec{v}$ and $\vec{B}$ *i.e.* the direction of motion of the charge and the direction of applied magnetic field.

(*i*) If $\vec{v}$ and $\vec{B}$ are perpendicular ($\theta = 90°$), the force on the charged particle makes it to move along circular path, whose radius is given by

$$r = \frac{mv}{Bq}$$

(*ii*) If $\vec{v}$ and $\vec{B}$ act at an angle θ, then due to the component of velocity $v \sin\theta$ (perpendicular to $\vec{B}$, the charge moves along circular path of radius r, which is given by

$$r = \frac{mv \sin\theta}{Bq}$$

while due to the component of velocity $v \cos\theta$ (along $\vec{B}$), the charge at the same time moves along the direction of magnetic field. As a result, the charge moves along a ***helical path*** and pitch of the helical path is given by

$$\text{pitch} = \frac{2\pi mv \cos\theta}{Bq}$$

☐ **Cyclotron.** It is a particle accelerator and is used to accelerate positive ions. Under the action of magnetic field, the positive ions move along spiral path and gain energy as they cross the alternating electric field again and again.

☐ **Cyclotron frequency.** In a cyclotron, the frequency of applied alternating electric field is equal to frequency of oscillation of the positive ion and this frequency is called cyclotron frequency. It is given by

$$\nu = \frac{Bq}{2\pi m},$$

where m and q are mass and charge of the positive ion and B is strength of the magnetic field.

☐ **Maximum energy attained by positive ions.** The positive ions of charge q and mass m attain maximum energy is given by

(*i*) $E_{max} = \frac{1}{2} \cdot \frac{B^2 q^2 R^2}{m}$

where R is radius of the dees of the cyclotron.

(*ii*) $E_{max} = 2n(Vq)$

where n is number of revolutions completed by the positive ions before leaving the dees.

☐ **Force on a current carrying conductor placed inside a magnetic field.** A conductor of length l carrying current I and placed inside a magnetic field of strength B is given by

$$|\vec{F}| = |I\,\vec{l} \times \vec{B}| = BIl \sin\theta$$

The conductor experiences maximum force, when the magnetic field acts at right angle to the length of the conductor ; and the force is zero, when the length of the conductor is parallel to the direction of the magnetic field.

☐ **Force between two infinitely long parallel current carrying conductors.** When two infinitely long parallel conductors carrying currents I_1 and I_2 are placed a distance r apart, then force on the unit length of a conductor due to the other conductor is given by

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r}$$

The force is attractive, if currents in two conductors is in same direction ; and repulsive, if currents are in opposite directions.

The above expression gives definition of **ampere,** the unit of current.

One ampere is that current, which when flowing through each of the two parallel conductors of infinite length and placed in vacuum at a distance of 1 metre from each other, produces between them a force of 2×10^{-7} newton per metre of their lengths.

☐ **Torque on a current carrying coil placed inside a magnetic field.** When a coil of area A having n turns and carrying current I is suspended inside a magnetic field of strength B, then torque on the coil is given by

$$\tau = nBIA\sin\theta,$$

where θ is angle between the direction of magnetic field and normal to the plane of the coil.

If the direction of magnetic field makes an angle α with the plane of the coil, then

$$\tau = nBIA\cos\alpha$$

The torque on the coil is maximum, when the plane of the coil is parallel to the magnetic field *i.e.* $\theta = 90°$ or $\alpha = 90°$.

☐ **Sensitivity of a galvanometer.** *A galvanometer is said to be sensitive, if it gives a large deflection, even when a small current is passed through it or when a small voltage is applied across its coil.*

☐ **Current sensitivity.** *It is defined as the deflection produced in the galvanometer on passing unit current through its coil.*

Current sensitivity , $\frac{\theta}{I} = \frac{nBA}{k}$ $\left(\textbf{rad } \mu\textbf{A}^{-1}\right)$

☐ **Voltage sensitivity.** *It is defined as the deflection produced in the galvanometer, when a unit voltage is applied across its coil.*

Voltage sensitivity, $\frac{\theta}{V} = \frac{nBA}{kR}$ $\left(\textbf{rad } \mu\textbf{V}^{-1}\right)$

Here, R is resistance of the coil of the galvanometer.

Shunt. *A small resistance usually put in parallel to the coil of a galvanometer is called shunt.*

The most part of the current in the circuit passes through the shunt and thus shunt allows only a very small part of current to pass through the galvanometer.

Ammeter. *It is an instrument used to measure current in an electrical circuit.*

A galvanometer of resistance G can be converted into an ammeter of range I by putting a small resistance S in parallel to its coil, which is given by

$$S = \frac{I_g \times G}{I - I_g}$$

Here, I_g is maximum current that can pass through the galvanometer.

The resistance of the ammeter so obtained is given by

$$R_A = \frac{GS}{G+S}$$

Ammeter is a low resistance instrument and it is always connected in series to the circuit.

☐ **Voltmeter.** *It is an instrument used to measure potential difference across a conductor in an electrical circuit.*

A galvanometer of resistance G can be converted into a voltmeter to read upto V by connecting a large resistance R in series to its coil, which is given by

$$R = \frac{V}{I_g} - G$$

The resistance of the voltmeter so obtained is given by

$$R_V = G + R$$

Voltmeter is a high resistance instrument and it is always connected in parallel to the conductor, across which potential difference is to be measured.

Some Useful Facts

1. A stationary charge experiences force inside electric field. A magnetic field does not exert any force on a stationary charge.

2. The expression for torque on a current carrying coil *i.e.*

$$\tau = n\,B\,I\,A\sin\theta$$

holds for a planar current loop of any shape having the area A.

3. A magnetic field, in which the plane of the coil in all position remains parallel to the direction of magnetic field is called ***radial magnetic field.*** A field magnet having concave poles produces radial magnetic field.

Chapter 3. Magnetic dipole

☐ **Magnetic dipole.** *An arrangement of two unlike poles of equal strength and separated by a small distance is called magnetic dipole.*

The distance between the two magnetic poles is called the magnetic length of the magnetic dipole. It is denoted by $2\vec{l}$, a vector from south to north pole of the magnetic dipole.

☐ **Magnetic dipole moment.** *The product of the pole strength of either magnetic pole and the magnetic length of the magnetic dipole is called its magnetic dipole moment.* It is denoted by $\vec{M}$.

Mathematically : $\vec{M} = m(2\vec{l})$ **(ampere metre^2)**

Here, m is pole strength of the magnetic dipole.

☐ **Magnetic field due to a bar magnet.** The magnetic field due to a bar magnet of length $2l$ and having magnetic dipole moment M at a distance r from its centre

(*i*) on its axial line is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2Mr}{(r^2 - l^2)^2}$$

(*ii*) on its equitorial line is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{M}{(r^2 + l^2)^{3/2}}$$

☐ **Torque on a magnetic dipole in a magnetic field.** When a magnetic dipole of magnetic dipole moment $\vec{M}$ is placed in a uniform magnetic field of strength $\vec{B}$ making an angle θ with the direction of magnetic field, it experiences a torque, which is given by

$$|\vec{\tau}| = |\vec{M} \times \vec{B}| = M\,B\sin\theta$$

☐ **Potential energy stored in a magnetic dipole on rotating inside a magnetic field.** The work done in rotating a magnetic dipole against the torque acting on it, when placed in magnetic field is stored inside it in the form of potential energy. When magnetic dipole is rotated from initial position $\theta = \theta_1$ to final position $\theta = \theta_2$, then potential energy stored is given by

$$U = M\,B\,(\cos\theta_1 - \theta_2)$$

☐ **Current loop and magnetic dipole.** A current loop of area A carrying current I behaves as a magnetic dipole having magnetic dipole moment,

$$M = I\,A \text{ (ampere metre}^2\text{)}$$

Some Useful Facts

1. The atom or molecule of a substance behaves is a complete magnet.

2. An atom behaves as a magnetic dipole due to the fact that electrons revolving around the nucleus constitute small current loops.

3. A circular current loop produces magnetic field, in the same manner as a magnetic dipole does.

4. A free magnetic pole does not exist.

5. Magnetic lines of force should be better called as magnetic field lines.

Chapter 4. Earth's magnetism

☐ **Geographic meridian.** *A vertical plane passing through the axis of rotation of the earth is called the geographic meridian.*

☐ **Magnetic meridian.** *A vertical plane passing through the axis of a freely suspended magnet is called the magnetic* meridian.

☐ **Magnetic elements.** The quantities, ***magnetic declination, magnetic inclination (dip)*** and ***horizontal component of earth's magnetic field*** completely determine the earth's magnetic field at a given place and are called magnetic elements.

***Magnetic declination** at a place is the angle between the geographic meridian and magnetic meridian.* It is denoted by θ.

***Magnetic inclination (dip)** at a place is the angle between the direction of the intensity of the total earth's magnetic field and th horizontal.* It is dentoed by δ.

Horizontal component of earth's magnetic field is the component of earth's total magnetic field along the horizontal It is denoted by B_H.

If B is intensity of earth's total magnetic field, then the horizontal component of earth's magnetic field is given by

$$B_H = B\cos\delta$$

Also, the vertical component of earth's magnetic field,

$$B_V = B\sin\delta$$

so that $$B = \sqrt{B_H^2 + B_V^2}$$

and $$\tan\delta = \frac{B_V}{B_H}$$

☐ **Neutral point.** *It is that point, where the magnetic field due to a bar magnet is completely cancelled by the horizontal componen of earth's magnetic field.*

(*i*) When a bar magnet is placed with its north pole towards south of the earth, the neutral points are obtained on axia line of the magnet. If d is the distance of the neutral point from the magnet,

$$\frac{\mu_0}{4\pi}\cdot\frac{2Md}{(r^2-l^2)^2} = B_H$$

(*ii*) When a bar magnet is placed with its north pole towards north of the earth, the neutral points are obtained on equitoria line of the magnet. If d is the distance of the neutral point from the magnet,

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{(r^2-l^2)^{3/2}} = B_H$$

☐ **Tangent law.** *It states that when a short bar magnet is suspended freely under the combined action of two uniform magneti fields of intensities B and B_H acting at 90° to each other, the magnet comes to rest making an angle θ with the direction of magnetic fiel B_H, such that*

$$B = B_H\tan\theta$$

☐ **Tangent galvanometer.** A tangent galvanometer is a moving magnet and fixed coil type galvanometer. It is base on tangent law and is used to measure very small currents.

If a tangent galvanometer has coil of radius R and number of turns n, then deflection θ produced on passing current is given by

$$I = \frac{2a}{n\mu_0}B_H\tan\theta = \frac{B_H}{G}\tan\theta = K\tan\theta$$

Here, $G = \frac{n\mu_0}{2a}$ is called ***galvanometer constant*** and

$K = \frac{B_H}{G} = \frac{2a}{n\mu_0}B_H$ is called ***reduction factor*** of tangent galvanometer.

☐ **Vibration magnetometer.** It is based on the principle that a magnet suspended inside a magnetic field experience a torque. It is used to compare magnetic moments of two bar magnets or to determine horizontal component of earth's magneti field.

If a bar magnet of magnetic dipole moment M and moment of inertia I is free to oscillate under the action of horizonta component of earth's magnetic field B_H, then its time period is given by

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Some Useful Facts

1. The magnetic field of earth is same as that would be due to a huge bar magnet located deep inside the earth with it north pole towards south of the earth.

2. The magnetic field due to a short bar magnet at a distance r on its axial line is twice the magnetic field at the sam distance on its equitorial line.

3. The reduction factor of a tangent galvanometer increases due to increase in the radius of its coil and decrease in it number of turns.

Chapter 5. Classification of Magnetic materials

☐ **Diamagnetic substances.** *Those substances, which when placed in a magnetic field are feebly magnetised in a direction opposit to that of the magnetising field.*

☐ **Paramagnetic substances.** *Those substances, which when placed in a magnetic field are feebly magnetised in the direction o the magnetising field.*

☐ **Ferromagnetic substances.** *Those substances, which when placed in a magnetic field are strongly magnetised in the direction the magnetising field.*

☐ **Intensity of magnetisation.** *It is defined as the magnetic dipole moment developed per unit volume or the pole strength developed r unit area of cross-section of the specimen.* It is given by

$$I = \frac{M}{V} = \frac{m}{a}$$

Here, V is volume and a is area of cross-section of the specimen. Magnetic induction, intensity of magnetisation and agnetic intensity are related to each other as below :

$$B = \mu_0 (H + I)$$

☐ **Magnetic permeability.** *The magnetic permeability of a material is defined as the ratio of the magnetic induction (B) of the aterial to the strength of magnetising field (H).* It is given by

$$\mu = \frac{B}{H}$$

If μ_r is relative permeability of a medium, then

$$\mu_r = \frac{\mu}{\mu_0}$$

☐ **Magnetic susceptibility.** *The magnetic susceptibility of a material is defined as the ratio of the intensity of magnetisation (I) nd the strength of magnetising field (H).* It is given by

$$\chi_m = \frac{I}{H}$$

It can be proved that $\mu = \mu_0 (1 + \chi_m)$

o that $\mu_r = 1 + \chi_m$

☐ **Curie temperature.** *It is the temperature for a ferromagnetic substance above which, it behaves as a paramagnetic substance.*

☐ **Curie's law.** It states that *the magnetic susceptibility of a paramagnetic substance varies inversely with its absolute temperature.*

Mathematically : $\chi_m \propto \frac{1}{T}$ or $\chi_m T = \text{constant}$

☐ **Hysteresis.** *The lagging of intensity of magnetisation (or magnetic induction) behind the magnetising field, when a magnetic pecimen is taken through a cycle of magnetisation, is called hysteresis.*

☐ **Retentivity.** *The value of intensity of magnetisation of the magnetic material, when the magnetising field is reduced to zero, is alled its retentivity.*

☐ **Coercivity.** *The value of the reverse magnetising field, which has to be applied to the magnetic material so as to reduce the residual nagnetisation to zero, is called its coercivity.*

ome Useful Facts

1. The atoms of a diamagnetic substance do not possess any magnetic moment, while those of paramagnetic and erromagnetic substances possess a net non-zero magnetic moment.

2. The diamagnetism is fundamental to all the magnetic substances. However, diamagnetism is overshadowed by aramagnetism and ferromagnetism in paramagnetic and ferromagnetic materials respectively.

3. The energy is spent in taking a ferromagnetic substance over a complete cycle of magnetisation. It is numerically equal o the area of I-H curve or $\frac{1}{4\pi}$ times the area of B-H curve.

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Magnetic Effect of Current

Problem 1.01. A current of 1 A is flowing in the sides of an equilateral triangle of side 4·5 × 10⁻². Find the magnetic field at the centroid of the triangle.

(Roorkee, 1991)

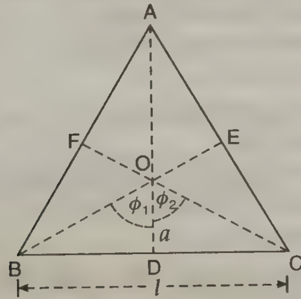


Fig. 1.01.

Thought Process

Let O be the centroid of the equilateral triangle ABC [Fig 1.01]. The current through each of the three sides of the triangle will produce magnetic field at point O having the same magnitude and direction. If B is magnetic field produced due to any one of the three sides at point O, then the net magnetic field at point O due to the whole ΔABC,

$$B' = 3B$$

If If $\Delta\, BOD = \phi$ and $OD = a$, then as obtained in section 1.03,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin \phi_1 + \sin \phi_2)$$

Sol. Here, I = 1 A, each side of the Δ ABC, $l = 4{\cdot}5 \times 10^{-2}$ m

It follows that

$$\phi_1 = \phi_2 = 60°,\ a = \frac{BD}{\tan 6°} = \frac{l/2}{\sqrt{3}} = \frac{4{\cdot}5 \times 10^{-2}}{2\sqrt{3}}\ \text{m}$$

Hence, magnetic field at the centroid O due to current flowing through Δ ABC,

$$B' = 3 \times \frac{\mu_0}{4\pi} \cdot \frac{I}{a} (\sin \phi_1 + \sin \phi_2)$$

$$= \frac{3 \times 10^{-7} \times 1 \times 2 \times \sqrt{3}}{4{\cdot}5 \times 10^{-2}} (\sin 60° + \sin 60°)$$

$$= \frac{6\sqrt{3} \times 10^{-7}}{4{\cdot}5 \times 10^{-2}} \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) = \mathbf{4 \times 10^{-5}\ tesla}$$

Problem 1.02. Two long straight parallel wires are 2 m apart (perpendicular to the plane of the paper as shown in Fig. 1.02. The wire A carries current of 9·6 A directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P at a distance $\frac{10}{11}$ m from the wire B is zero. Find

(*i*) the magnitude and direction of the current in B.

(*ii*) the magnitude of the magnetic field of induction at the point S.

(*iii*) the force per unit length on the wire B.

(I.I.T. 1987)

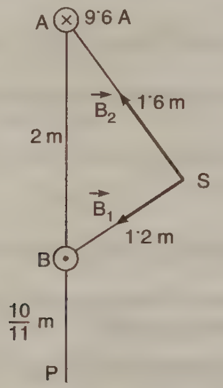


Fig. 1.02.

Sol. Here, current through the wire A,

$I_1 = 9{\cdot}6$ A (directed into the paper)

Distance of point P from wire A, $a_1 = 2 + \frac{10}{11} = \frac{32}{11}$ m

Distance of point P from wire B, $a_2 = \frac{10}{11}$ m

(*i*) Suppose that I_2 is the current through the wire, so that magnetic field induction at point P due to the two wires becomes zero. For this, current in wire B has to be directed outwards of the paper, so that the magnetic fields produced at point P due to the two wires are equal in magnitude and opposite in direction. Thus, for magnetic field induction at point P to be zero,

$$\frac{\mu_0}{4\pi} \cdot \frac{2 I_1}{a_1} = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{a_2}$$

or, $$I_2 = I_1 \times \frac{a_2}{a_1} = \frac{9{\cdot}6 \times 10/11}{32/11}$$

= 3 A (directed outwards of the paper)

(*ii*) In fact, points A, B and S form a right angled triangle. It is because, $AB^2 = AS^2 + BS^2$.

Therefore, magnetic field induction at point S due to wire A ,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{BS} = \frac{10^{-7} \times 2 \times 9{\cdot}6}{1{\cdot}6}$$

$= 12 \times 10^{-7}$ T (along perpendicular to AS *i.e.* along $\overrightarrow{SB}$)

Magnetic field induction at point S due to wire B,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{BS} = \frac{10^{-7} \times 2 \times 3}{1{\cdot}2}$$

$= 5 \times 10^{-7}$ T (along perpendicular to BS *i.e.* along $\overrightarrow{SA}$)

As the two fields B_1 and B_2 are linclined at 90°, the resultant magnetic field at point S is given by

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(12 \times 10^{-7})^2 + (5 \times 10^{-7})^2}$$

$= \mathbf{13 \times 10^{-7}\ T}$

(*iii*) The force per unit length on the wire B,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{AB} = \frac{10^{-7} \times 2 \times 9{\cdot}6 \times 3}{2}$$

$= \mathbf{2{\cdot}88 \times 10^{-6}\ N\ m^{-1}}$

Problem 1.03. Two long parallel wires carrying current 2·5 ampere and I ampere in the same direction (directed into the plane of the paper) are held at P and Q respectively, such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5 metres and 2 metres respectively from a collinear point R as shown in Fig. 1.03.

(*i*) An electron moving with a velocity of 4×10^5 m s^{-1} along the positive X-direction experiences a force of magnitude $3{\cdot}2 \times 10^{-20}$ N at the point R. Find the value of I.

(*ii*) Find all the positions at which a third long parallel wire carrying a current of magnitude 2·5 ampere may be placed, so that the magnetic induction at R is zero.

(I.I.T. 1990)

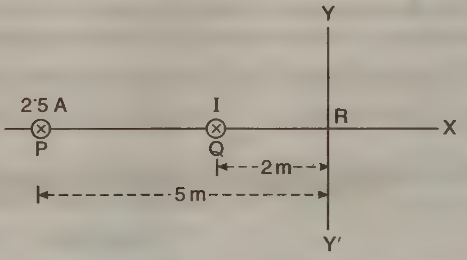


Fig. 1.03.

Sol. Here, current through the wire at point P,

$I_1 = 2{\cdot}5$ A

current through the wire at point Q, $I_2 = I = I$

distance of wire at point P from R,

$r_1 = PR = 5$ m

distance of wire at point Q from R,

$r_2 = QR = 2$ m

If B_1 and B_2 are magnetic fields produced at point R due to the wires at points P and Q respectively, then the net magnetic field at point R,

$$B = B_2 + B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1}{r_1} + \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{r_2}$$

$$= \frac{\mu_0}{4\pi} \times 2\left(\frac{I_1}{r_1}+\frac{I_2}{r_2}\right)$$

$$= 10^{-7} \times 2\left(\frac{2\cdot5}{5}+\frac{I}{2}\right) = 10^{-7}\,(1+I)$$

The direction of magnetic field B at point R will be along negative Y-axis. Therefore,

$$\vec{B} = 10^{-7}\,(1+I)\,(-\hat{j}) = -10^{-7}\,(1+I)\,\hat{j}$$

(*i*) velocity of electron, $\vec{v} = 4\times10^{5}$ m s^{-1} along X-axis

$$= 4\times10^{5}\,\hat{i}\ (\text{m s}^{-1})$$

Also, charge on electron, $q = -e = -1{\cdot}6\times10^{-19}$ C

Therefore, force on electron,

$$\vec{F} = q\,(\vec{v}\times\vec{B})$$

$$= (-1{\cdot}6\times10^{-19})\,[(4\times10^{5}\,\hat{i})\times\{-10^{-7}\,(1+I)\,\hat{j}\}]$$

$$= 6{\cdot}4\times10^{-21}\,(1+I)\,\hat{k}$$

But $F = 3{\cdot}2\times10^{-20}$ N

$$\therefore \quad 6{\cdot}4\times10^{-21}\,(1+I) = 3{\cdot}2\times10^{-20}$$

or $$1+I = \frac{3\cdot2\times10^{-20}}{6\cdot4\times10^{-21}} = 5$$

or $$I = 4\text{ A}$$

(*ii*) The magnetic induction at point R will be zero, if the third wire produces magnetic field just equal and opposite to the combined field produced by the two wires at points P and Q. Suppose that the third conductor carrying current I = 2·5 A is placed at a distance *r* from the point R, then

$$\frac{\mu_0}{4\pi}\cdot\frac{2I_3}{r} = 10^{-7}\,(1+I)$$

or $$\frac{10^{-7}\times2\times2\cdot5}{r} = 10^{-7}\,(1+4)$$

or $$r = 1\text{ m}$$

If the third wire is placed at 1 m on right side of point R, the current through it should be in the same direction as through the wires at P and Q. On the other hand, the current through the third wire should be in opposite direction, if it is placed on left of the point R. Thus, the third conductor can be placed in the two positions explained above.

Chapter 2. Motion of Charged particle

Problem 2.01. The region between $x = 0$ and $x = L$ is filled with uniform, steady magnetic field $B_0\,\hat{k}$. A particle of mass *m*, positive charge *q* and velocity $v_0\,\hat{i}$ travels along X-axis and enters the region of the magnetic field. Neglect gravity throughout region.

(*a*) Find the value of L, if the particle emerges from the region of magnetic field with its final velocity at angle 30° to its initial velocity.

(*b*) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto 2·1 L. (*I.I.T. 1999*)

Sol. (*a*) Fig. 2.01 shows magnetic field $B_0\,\hat{k}$ (represented by ⊙) acting along Z-axis in the region $x = 0$ and $x = L$. The positive charge *q* moves with velocity $v_0\,\hat{i}$ along OX.

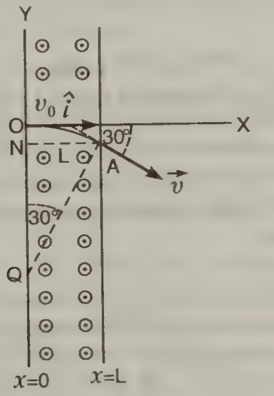


Fig. 2.01.

The force on charge *q* is given by

$$\vec{F} = q\,(\vec{v}\times\vec{B}) = q\,(v_0\,\hat{i}\times B_0\,\hat{k}) = -B_0\,v_0\,q\,\hat{j}$$

The force on positive charge *q* acts along negative Y-axis. Under the effect of this force, the charge moves along a circular path. If *r* is the radius of the circular path, then

$$\frac{m\,v_0^2}{r} = -B_0\,v_0\,q \quad\text{or}\quad r = \frac{m\,v_0}{B_0\,q} \qquad ...(i)$$

The velocity vector $\vec{v}$ of the positive charge at point A acts along tangent to the circular pathOA and it makes an angle of 30° with OX. If Q is centre of the circular path OA, then ∠OQA = 30°. From point A, drop perpendicular AN on Y-axis. Then, NA = L. From right angled ∠QNA, we have

$$\sin 30^\circ = \frac{NA}{QA} = \frac{L}{r}$$

or $$L = r\sin 30^\circ = \frac{m\,v_0}{B_0\,q}\times\frac{1}{2}$$

or $$L = \frac{m\,v_0}{2\,B_0\,q} \qquad ...(ii)$$

(*b*) From equations (*i*) and (*ii*), its follows that

$$r = 2\,L$$

If the magnetic field extends upto 2·1 L [Fig. 2.02], the

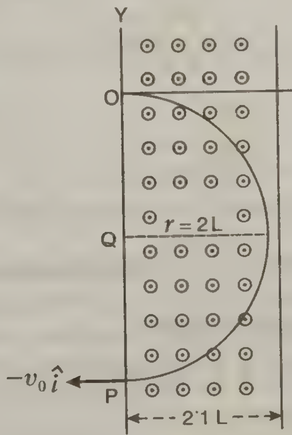


Fig. 2.02

positive charge will complete its semi-circular path inside the magnetic field and will emerge out of magnetic field at point P with the final velocity,

$$v = -v_0\,\hat{i}$$

The time spent by the positive charge inside the magnetic field,

$$t = \frac{\pi r}{v_0} = \frac{\pi}{v_0} \times \frac{m v_0}{B_0\, q}$$

or $$t = \frac{\pi m}{B_0\, q}$$

Problem 2.02. A beam of protons with a velocity 4 × 10^5 m s^{-1} enters the uniform magnetic field of 0·3 tesla at an angle of 60° to magnetic field. Find the radius of the helical path taken by the proton beam. Also, find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation). Given, mass of proton = 1·67 × 10^{-27} kg.

(I.I.T. 1986, 1981)

Hint. Refer to section 2.04. If q is charge on proton, then the radius of helical path, $r = \dfrac{m v \sin\theta}{Bq}$

Setting $B = 0{\cdot}3$ T ; $m = 1{\cdot}67 \times 10^{-27}$ kg ;

$q = 1{\cdot}6 \times 10^{-19}$ C ; $v = 4 \times 10^5$ m s^{-1} ; and $\theta = 60°$;

it can be obtained that radius of the helical path,

$r = 1{\cdot}205 \times 10^{-2}$ m = **1·205 m**

Also, pitch of the helix

$$= v \cos\theta \times \text{time period of circular path}$$

$$= v \cos 60° \times \frac{2\pi m}{Bq}$$

It can be obtained that pitch of the helix

$= 4{\cdot}372 \times 10^{-2}$ m = **4·372 cm**

Problem 2.03. An electron gun G emits electrons of energy 2 keV travelling in the positive X-direction. The electrons are required to hit the spot S, where GS = 0·1 m and the line GS makes an angle of 60° with the X-axis as shown in the Fig. 2.03. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electron hit S.

(I.I.T. 1993)

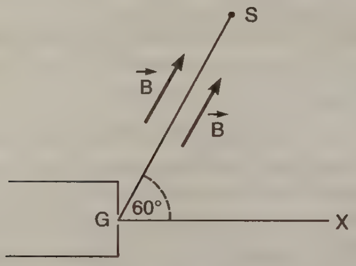


Fig. 2.03.

Sol. Suppose that v is the velocity, with which electrons are emitted from the gun.

Here, kinetic energy of electron,

$$\frac{1}{2} m v^2 = 2 \text{ keV} = 2 \times 10^3 \times 1{\cdot}6 \times 10^{-19}$$

$$= 3{\cdot}2 \times 10^{-16} \text{ J}$$

$$\therefore \quad v = \sqrt{\frac{2 \times 3{\cdot}2 \times 10^{-16}}{m}}$$

Since mass of electron, $m = 9{\cdot}1 \times 10^{-31}$ kg, we have

$$v = \sqrt{\frac{2 \times 3{\cdot}2 \times 10^{-16}}{9{\cdot}1 \times 10^{-31}}}$$

$$= 2{\cdot}65 \times 10^7 \text{ m s}^{-1}$$

The velocity v of the electron can be resolved into two components :

(*i*) $v \sin 60°$ *perpendicular to GS*. Under the effect of magnetic field, the velocity component $v \sin 60°$ will make the electron to move along circular path of radius, say r. Then,

$$\frac{m (v \sin 60°)^2}{r} = B e (v \sin 60°)$$

or $$r = \frac{m v \sin 60°}{B e}$$

The time period of circular motion,

$$T = \frac{2\pi r}{v \sin 60°} = \frac{2\pi m v \sin 60°}{B e \times v \sin 60°} = \frac{2\pi m}{B e}$$

(*ii*) $v \cos 60°$ *along GS*. The velocity component v cos 60° will drift the electron along GS and as such, it will move along helical path. The electron will hit S, if the time (t) taken by electron to cover the distance GS (= 0·1 m) is equal to the integral multiple of the period (T) of the circular path *i.e.* $t = n\,T$

or $$\frac{GS}{v \cos 60°} = n\left(\frac{2\pi m}{B e}\right)$$

or $$B = n\left(\frac{2\pi m v \cos 60°}{e \times GS}\right)$$

For B to be minimum, $n = 1$ *i.e.*

$$B_{min} = \frac{2\pi m v \cos 60°}{e \times GS}$$

$$= \frac{2\pi \times 9{\cdot}1 \times 10^{-31} \times 2{\cdot}65 \times 10^7 \times 0{\cdot}5}{1{\cdot}6 \times 10^{-19} \times 0{\cdot}1}$$

= **4·735 × 10^{-3} tesla**

Problem 2.04. A circular loop of radius R is bent along a diameter and given a shape as shown in Fig. 2.04. One of the semicircles (KNM) lies in the XZ-plane and the other one (KLM) in the YZ-plane with their centres at origin. Current I is flowing through each of the semicircles as shown in figure.

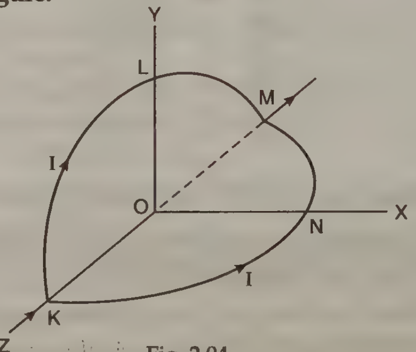


Fig. 2.04

(a) A particle of charge q is released at the origin with velocity $\vec{v} = -v_0\,\hat{i}$. Find the instantaneous force $\vec{F}$ on the article. Assume that space is gravity free.

(b) If an external uniform magnetic field $B_0\,\hat{j}$ is pplied, determine the force $\vec{F_1}$ and $\vec{F_2}$ on the semicircles LM and KNM due to the field and the net force $\vec{F}$ on the oop. *(I.I.T. 2000)*

Sol. Let $\vec{B_1}$ and $\vec{B_2}$ be respectively the magnetic fields roduced at point O due to current carrying semicircular arts KNM and KLM of the circular loop of radius R. Then,

$$\vec{B_1} = \frac{\mu_0}{4\pi}\cdot\frac{\pi I}{R}(\hat{j}) = \frac{\mu_0 I}{4R}\,\hat{j}$$

$$\text{and } \vec{B_2} = \frac{\mu_0}{4\pi}\cdot\frac{\pi I}{R}(-\hat{i}) = -\frac{\mu_0 I}{4R}\,\hat{i}$$

Therefore, total magnetic field produced by the two arts of the loop,

$$\vec{B} = \vec{B_1} + \vec{B_2} = \frac{\mu_0 I}{4R}\,\hat{j} + \left(-\frac{\mu_0 I}{4R}\,\hat{i}\right)$$

$$\text{or } \vec{B} = \frac{\mu_0 I}{4R}(-\hat{i} + \hat{j})$$

(*a*) Force on charge q due to magnetic field produced by urrent loop,

$$\vec{F} = q(\vec{v} \times \vec{B}) = q\left[(-v_0\,\hat{i}) \times \frac{\mu_0 I}{4R}(-\hat{i} + \hat{j})\right]$$

$$\text{or } \vec{F} = -\frac{\mu_0\, q\, v_0\, I}{4R}\,\hat{k}$$

(*b*) When a current carrying semicircular wire KLM or KNM (in general, of any shape) is placed in a magnetic field, then force on the wire is same as that on a current carrying straight conductor of length KM (*i.e.* length between the end points of the semicircular wire).

Therefore, force on current carrying semicircular part KNM of the loop,

$$\vec{F_1} = I(\overrightarrow{KM}) \times B_0\,\hat{j} = I[2R(-\hat{k})] \times B_0\,\hat{j} = 2B_0 I R\,\hat{i}$$

Similarly, force on current carrying semicircular part KLM of the loop,

$$\vec{F_2} = 2B_0 I R\,\hat{i}$$

Therefore, force on the whole loop,

$$\vec{F} = \vec{F_1} + \vec{F_2} = 2B_0 I R\,\hat{i} + 2B_0 I R\,\hat{i} = \mathbf{4B_0 I R}\,\hat{i}$$

Problem 2.05. A current of 10 A flows around a closed path in a circuit, which is in the horizontal plane as shown in Fig. 2.05. The circuit consists of eight alternating arcs of radii $r_1 = 0{\cdot}08$ m and $r_2 = 0{\cdot}12$ m. Each subtends the same angle at the centre.

(a) Find the magnetic field produced by this circuit at the centre.

(b) An infinitely long straight wire carrying a current of 10 A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre? *(I.I.T. 2001)*

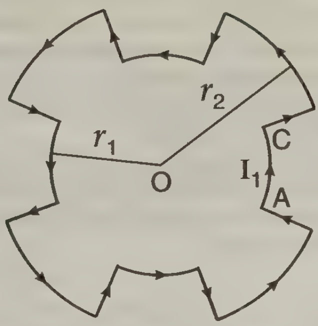


Fig. 2.05

Thought Process

1. *The four inner arcs are equivalent to a semi circular wire of radius r_1 and four outer arcs are equivalent to a semicircular wire of radius r_2. Both the sets of arcs produce magnetic fields perpendicular to plane of the paper and in outward direction.*
2. *Since direction of magnetic field produced due to the closed circuit is along the current carrying straight conductor through the centre, force on the straight conductor is zero.*
3. *Since magnetic field produced due to the straight conductor is tangential to all the arcs, no force will act on the current carrying arcs (such as AC) of the circuit.*
4. *Force will act on the straight parts (such as CD) of the circuit due to the straight conductor. It is because, magnetic field produced by the straight conductor through point O acts perpendicular to the straight part CD.*

Sol. Here, $r_1 = 0{\cdot}08$ m ; $r_2 = 0{\cdot}12$ m

Current through the circuit, $I_1 = 10$ A

Current through the straight conductor passing through centre O, $I_2 = 10$ A

(a) The four inner circular arcs constitute a semicircular wire of radius r_1. Therefore, magnetic field produced due to four inner acrs at centre O,

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{\pi I_1}{r_1} = \frac{10^{-7} \times \pi \times 10}{0{\cdot}08}$$

$$= 3{\cdot}93 \times 10^{-5}\text{ T} \qquad \text{(outwards)}$$

Similarly, magnetic field produced due to four outer arcs at centre O,

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{\pi I_1}{r_2} = \frac{10^{-7} \times \pi \times 10}{0{\cdot}12}$$

$$= 2{\cdot}62 \times 10^{-5}\text{ T} \qquad \text{(outwards)}$$

Therefore, net magnetic field produced by the circuit at centre O,

$$B = B_1 + B_2 = 33 \times 10^{-5} + 2{\cdot}62 \times 10^{-5}$$

$$= \mathbf{6{\cdot}55 \times 10^{-5}\ T\ (outwards)}$$

(b) ***Force on straight conductor through centre*:** When a current carrying conductor is placed in a magnetic field, force experienced by the conductor,

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

Since direction of current through the straight conductor passing through centre and the direction of magnetic field produced by the circuit are along the length of the straight conductor (antiparallel), force on the conductor is **zero.**

Force on arc AC : The magnetic field produced by the straight conductor at the circular arc AC is along tangent to the arc. As explained above, force on arc AC due to the straight conductor is also **zero.**

Force on straight segment CD : Let PQ = dx be an elementary portion of the straight segment CD of the circuit at a distance x from the straight conductor passing through the centre O [Fig. 2.06].

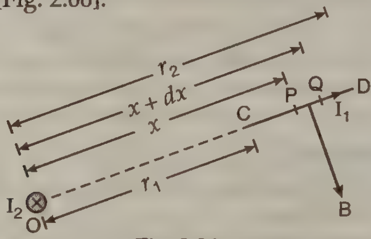


Fig. 2.06

Mganetic field B produced due to the straight conductor at point P acts normally to PQ and is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{x}$$

Since the straight segment carries current I_1, force on elementary portion PQ,

$$dF = B I_1 \, dx = \frac{\mu_0}{4\pi} \cdot \frac{2 I_2}{x} \times I_1 \times dx$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{x} dx \qquad \text{(inwards)}$$

Therefore, force on straight segment CD,

$$F = \int_{r_1}^{r_2} dF = \frac{\mu_0}{4\pi} \cdot 2 I_1 I_2 \int_{r_1}^{r_2} \frac{1}{x} dx = \frac{\mu_0}{4\pi} \cdot 2 I_1 I_2 \left| \log_e x \right|_{r_1}^{r_2}$$

$$= \frac{\mu_0}{4\pi} \cdot 2 I_1 I_2 \left| \log_e r_2 - \log_e r_1 \right| = \frac{\mu_0}{4\pi} \cdot 2 I_1 I_2 \cdot \log_e \frac{r_2}{r_1}$$

$$= 10^{-7} \times 2 \times 10 \times 10 \times \log_e \frac{0{\cdot}12}{0{\cdot}08}$$

$$= \mathbf{8{\cdot}11 \times 10^{-6} \ N} \qquad \textbf{(inwards)}$$

Problem 2.06. A straight segment OC (of length L metre) of a circuit containing a current I ampere is placed along the X-axis. Two infinitely long straight wires A and B each extending from $z = -a$ to $+a$ are fixed at $y = -a$ metre and $y = +a$ metre respectively as shown in the Fig. 2.07. If the wires A and B each carry a current I ampere into the plane of the paper, obtain the expression for the force acting on the segment OC. What will be the force on OC, if the current in the wire B is reversed ? (I.I.T. 1992)

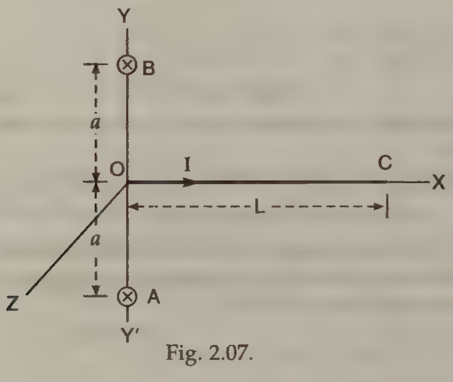


Fig. 2.07.

Thought Process

The resultant magnetic field due to the two wires is not uniform over the whole length of the straight segment OC. Therefore, force on segment OC is found by first finding force on an infinitesimally small length OC and then integrating over the whole length of the segment OC.

Sol. Consider an infinitesimally small portion PQ = dx of the segment located at distance x from the origin. The magnetic field $\vec{B}_1$ produced by the wire A will be along perpendicular to AP, while the magnetic field $\vec{B}_2$ due to the wire B will be along perpendicular to BP [Fig. 2.08].

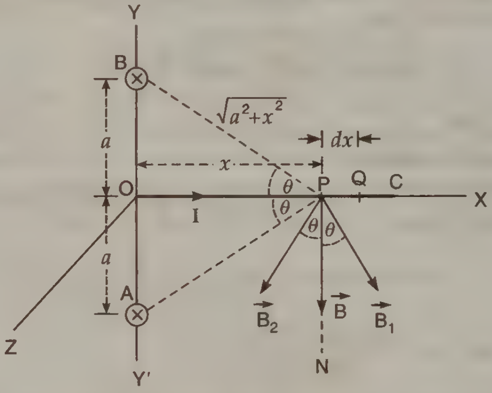


Fig. 2.08

Obviously, $AP = BP = \sqrt{a^2 + x^2}$

Now, $|\vec{B}_1| = \frac{\mu_0}{4\pi} \cdot \frac{2I}{AP} = \frac{\mu_0 I}{2\pi \sqrt{a^2 + x^2}}$

and $|\vec{B}_2| = \frac{\mu_0}{4\pi} \cdot \frac{2I}{BP} = \frac{\mu_0 I}{2\pi \sqrt{a^2 + x^2}}$

Draw PN perpendicular to OX. Suppose $\vec{B}_1$ and $\vec{B}_2$ make angle θ with PN. The components of $\vec{B}_1$ and $\vec{B}_2$ along OX are equal and opposite and hencecancel out ; while those along PN, add up. Therefore, net magnetic field at the elementary position PQ,

$$|\vec{B}| = \frac{\mu_0 I}{2\pi \sqrt{a^2 + x^2}} \cos\theta + \frac{\mu_0 I}{2\pi \sqrt{a^2 + x^2}} \cos\theta$$

$$= \frac{\mu_0 I}{\pi \sqrt{a^2 + x^2}} \cos\theta$$

Now, $\angle APO = \angle BPO = \theta$ and $\cos\theta = \frac{x}{\sqrt{a^2 + x^2}}$

Hence, $|\vec{B}| = \frac{\mu_0 I}{\pi (a^2 + x^2)} \cdot \frac{x}{\sqrt{a^2 + x^2}}$

$$= \frac{\mu_0 I x}{\pi (a^2 + x^2)} \quad \text{along PN}$$

or $$\vec{B} = \frac{\mu_0 I x}{\pi(a^2+x^2)}(-\hat{j}) = -\frac{\mu_0 I x}{\pi(a^2+x^2)}\hat{j}$$

Therefore, force on elementary portion PQ of the egment OC,

$$\vec{dF} = I\,\vec{dx} \times \vec{B}$$

Since $I\,\vec{dx} = I\,dx\,\hat{i}$, we have

$$\vec{dF} = I dx\,\hat{i} \times \left(-\frac{\mu_0 I x}{\pi(a^2+x^2)}\hat{j}\right) = -\frac{\mu_0 I^2}{\pi(a^2+x^2)}x\,dx\,\hat{k}$$

Force on the whole segment OC can be found by ntegrating the above between limits $x = 0$ to $x = L$ *i.e.*

$$\vec{F} = \int_0^L \vec{dF} = (-\hat{k})\frac{\mu_0 I^2}{\pi}\int_0^L \frac{x}{a^2+x^2}dx$$

Let $a^2 + x^2 = t$

$\therefore$ $0 + 2x\,dx = dt$ or $x\,dx = \frac{1}{2}dt$

The new limits of integration will be as obtained below :

When $x = 0,\ t = a^2 + 0^2 = a^2$;

and when $x = L,\ t = a^2 + L^2$

Hence, $$\vec{F} = -(\hat{k})\frac{\mu_0 I^2}{2\pi}\int_{a^2}^{(a^2+L^2)}\frac{1}{t}dt$$

$$= -(\hat{k})\frac{\mu_0 I^2}{2\pi}\Big|\log_e t\Big|_{a^2}^{(a^2+L^2)}$$

$$= -(\hat{k})\frac{\mu_0 I^2}{2\pi}\left[\log_e(a^2+L^2) - \log_e a^2\right]$$

$$= -\frac{\mu_0 I^2}{2\pi}\log_e\left(\frac{a^2+L^2}{a^2}\right)(\hat{k})$$

$$= \frac{\mu_0 I^2}{2\pi}\log_e\left(1+\frac{L^2}{a^2}\right)(-\hat{k})$$

i.e. $$\mathbf{F} = \frac{\mu_0 I^2}{2\pi}\log_e\left(1+\frac{L^2}{a^2}\right)$$ **(along negative Z-axis)**

Problem 2.07. A long horizontal wire AB which is free to move in a vertical plane and carries a steady current of 20 A is in equilibrium at a height of 0·01 m over another parallel long wire CD, which is fixed in a horizontal plane and carries a steady current of 30 A as shown in Fig. 2.09. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations.

(I.I.T. 1994)

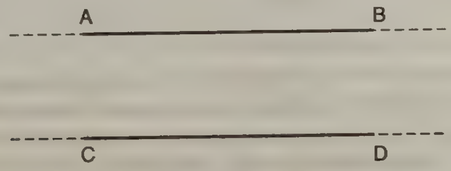


FIg. 2.09.

Sol. Force per unit length on wire AB due to the magnetic field produced by the wire CD

$$= \frac{\mu_0}{4\pi}\cdot\frac{2 I_1 I_2}{r}$$

where I_1 (= 20 A) and I_2 (= 30 A) are currents through the wires AB and CD respectively and r (= 0·1 m) is the height of wire AB above the wire CD.

If L is length of the wire AB, then force on the total length of the wire AB due to the wire CD,

$$F = \frac{\mu_0}{4\pi}\cdot\frac{2 I_1 I_2}{r}\times L = \frac{\mu_0 I_1 I_2}{2\pi r}\times L$$

The weight M g of the wire AB can balance the force F, only if F is in vertically upward direction. In other words, the wire AB can be in equilibrium, if the wire CD repels it *i.e.* currents I_1 and I_2 are in opposite direction. Therefore, in equilibrium

$$\frac{\mu_0 I_1 I_2}{2\pi r}\times L = M g \qquad ...(i)$$

Suppose that the wire AB is depressed through a small distance y. Since the distance between the two wires become $(r - y)$, the force on the wire AB increases to

$$F' = \frac{\mu_0 I_1 I_2}{2\pi(r-y)}\times L = \frac{\mu_0 I_1 I_2}{2\pi r}\left(1-\frac{y}{r}\right)^{-1}\times L$$

$$\approx \frac{\mu_0 I_1 I_2}{2\pi(r-y)}\left(1+\frac{y}{r}\right)^{-1}\times L$$

On releasing the wire AB, the wire moves under the effect of restoring force f given by

$$f = F' - F = \frac{\mu_0 I_1 I_2}{2\pi(r-y)}\left(1+\frac{y}{r}\right)^{-1}\times L - \frac{\mu_0 I_1 I_2}{2\pi(r-y)}\times L$$

$$= \frac{\mu_0 I_1 I_2}{2\pi(r-y)}\times L$$

The acceleration produced in the motion of wire AB is given by

$$a = \frac{f}{M} = \frac{\mu_0 I_1 I_2}{2\pi r^2 M}y\times L$$

or $a \propto y$

Hence, motion of the wire AB is simple harmonic in nature and its time period is given by

$$T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{2\pi r^2 M}{\mu_0 I_1 + I_2 L}}$$

From the equation (i), substituting the value of M, we have

$$T = 2\pi\sqrt{\frac{2\pi r^2}{\mu_0 I_1 I_2 L}\times\frac{\mu_0 I_1 + I_2 L}{2\pi r g}}$$

$$= 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{0{\cdot}01}{9{\cdot}8}} = 0{\cdot}2\text{ s}$$

Problem 2.08. A wire loop carrying a current I is placed in the XY-plane as shown in the Fig. 2.10. The extremities M and N of the current loop subtend an angle of 120° at the centre (P) of the circular part of the loop.

(a) If a particle with charge + Q and mass m is placed at the centre P and given a velocity $\vec{v}$ along NP, find its instantaneous acceleration.

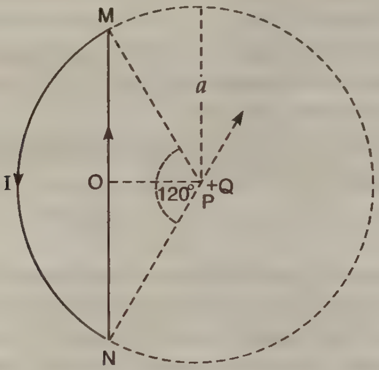


Fig. 2.10.

(b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due this field. *(I.I.T. 1991)*

Thought Process

From point P, drop PO perpendicular to MN. Join PN and PM.

Obviously, $\angle NOP = \angle MPO = 60°$,

$OP = a \cos 60° = \dfrac{a}{2}$ *and* MN = 2

$OM = 2\,a \sin 60° = \sqrt{3}\,a$

Now, the magnetic field due to the wire loop may be considered as the combined effect of the magnetic field ($\vec{B}_1$) *produced by the circular arc (which is one third of a complete turn) and the magnetic field* ($\vec{B}_2$) *produced by the straight conductor MN, lying at distance OP and its two ends subtending angles* $\phi_1 = \phi_2 = \dfrac{1}{2}$ ($\angle$ MPN) = 60° *at the observation point P.*

Sol. (a) Here, the radius of the circular arc is a. For a circular loop of n turns,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} = \frac{\mu_0 n I}{2a}$$

Therefore, magnetic field due to the portion of the circular loop of wire (one third of a complete turn),

$$B_1 = \frac{\mu_0 \left(\frac{1}{3}\right) I}{2a} = \frac{\mu_0 I}{6a}$$

or
$$\vec{B} = \frac{\mu_0 I}{6a}\hat{k}$$

Magnetic field due to the straight portion MN of the wire loop,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{OP}(\sin\phi_1 + \sin\phi_2)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{a/2}(\sin 60° + \sin 60°$$

$$= \frac{\sqrt{3}\,\mu_0 I}{2\pi a} \qquad \text{(along Z-axis)}$$

or
$$\vec{B}_2 = \frac{\sqrt{3}\,\mu_0 I}{2\pi a}(-\hat{k}) = -\frac{\sqrt{3}\,\mu_0 I}{2\pi a}\hat{k}$$

Therefore, net magnetic field due to the wire loop at point P,

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{6a}\hat{k} + \left(-\frac{\sqrt{3}\,\mu_0 I}{2\pi a}\hat{k}\right)$$

$$= -\frac{0{\cdot}109\,\mu_0 I}{a}\hat{k}$$

Since the charged particle moves with velocity $\vec{v}$ along NP *i.e.* making an angle of 60° with X-axis, we have

$$\vec{v} = (v\cos 60°)\,\hat{i} + (v\cos 60°)\,\hat{j} = v\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

Force on the charged particle due to the magnetic field produced by the wire loop,

$$\vec{F} = Q(\vec{v} \times \vec{B})$$

$$= Q\left\{v\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)\right\} \times \left\{\frac{-0{\cdot}109\,\mu_0 I}{a}\hat{k}\right\}$$

$$= \frac{0{\cdot}109\,Q\,v\,\mu_0 I}{a}\left[-\frac{1}{2}(-\hat{j}) - \frac{\sqrt{3}}{2}\hat{i}\right]$$

$$= \frac{0{\cdot}109\,Q\,v\,\mu_0 I}{2a}(-\sqrt{3}\,\hat{i} + \hat{j})$$

If m is mass of the charged particle, then acceleration is given by

$$\vec{a} = \frac{\vec{F}}{m} = \frac{0{\cdot}109\,Q\,v\,\mu_0 I}{2ma}(-\sqrt{3}\,\hat{i} + \hat{j})$$

$$\therefore\ |\vec{a}| = \frac{0{\cdot}109\,Q\,v\,\mu_0 I}{2ma}\sqrt{(-\sqrt{3})^2 + (1)^2}$$

$$= \frac{0{\cdot}109\,Q\,v\,\mu_0 I}{2ma}$$

(b) Net force on the wire loop due to applied magnetic field is zero for obvious reasons.

Now, torque on the wire loop is given by

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Here, $\vec{B} = \hat{i}\,B$ and $\vec{A}$ = area vector representing the area of wire loop

Now, $|\vec{A}| = \dfrac{1}{3}$ (area of circle)

– area of Δ PMN $= \dfrac{1}{3}\pi a^2 - \dfrac{1}{2}\sqrt{3}\,a \cdot \dfrac{a}{2} = 0{\cdot}6136\,a^2$

As the wire loop lies in XY-plane, the area vector representing the area of the wire loop is given by

$$\vec{A} = 0{\cdot}6136\,a^2\,\hat{k}$$

Hence, $\vec{\tau} = I(0{\cdot}6136\,a^2\,\hat{k}) \times (B\,\hat{i}) = 0{\cdot}6136\,I\,B\,a^2\,\hat{j}$

$\therefore\ \tau = |\vec{\tau}| = 0{\cdot}6136\,I\,B\,a^2$

Problem 2.09. A galvanometer having 30 divisions has a current sensitivity of 20μA per division. It has a resistance of 25 Ω. How will you convert it into an ammeter measuring upto 1 A ? How will you now convert this ammeter into a voltmeter reading upto 1 V ? *(Roorkee ,1987)*

Sol. Here, current sensitivity of galvanometer

$$= 20\ \mu\text{A division}^{-1} = 20 \times 10^{-6}\ \text{A division}^{-1}$$

Since galvanometer has 30 divisions, current for full scale deflection,

$$I_g = 30 \times 20 \times 10^{-6} = 6 \times 10^{-4}\ \text{A}$$

Conversion of galvanometer into ammeter. If S is the resistance of required shunt that should be connected in parallel to the galvanometer, then

$$S = \frac{I_g}{I - I_g} G$$

Here, I = 1 A, G = 25 Ω

$$\therefore \quad S = \frac{6 \times 10^{-4}}{1 - 6 \times 10^{-4}} \times 25$$

$$= \mathbf{0.015\ \Omega\ (in\ parallel)}$$

Conversion of ammeter into voltmeter. If R is the resistance that should be connected in series to the ammeter, then

$$R = \frac{V}{I} - R_A$$

Here, I is current that can pass through the ammeter ; and R_A is the resistance of the ammeter and is given by

$$R_A = \frac{GS}{G+S} = \frac{25 \times 0.015}{25 + 0.015} = 0.015\ \Omega$$

Since V = 1 V and I = 1 A,

$$R = \frac{1}{1} - 0.015 = \mathbf{0.985\ \Omega} \qquad \textbf{(in series)}$$

Problem 2.10. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. What is the value of the resistance, which can convert this galvanometer into a meter giving a full scale deflection for a current of 10 A ?

A resistance of the required value is available but it will get burnt, if the energy dissipated in it is greater than 1 W. Can it be used for the above described conversion of the galvanometer ? When this modified galvanometer is connected across the terminals of battery, it shows a current of 4 A . The current drops to 1 A , when a resistance of 1·5 Ω is connected in series with the modified galvanometer. Find the e.m.f. and the initial resistance of the battery.

(I.I.T. 1972)

Sol. Here, G = 100 Ω, $I_g = 1\ \text{mA} = 10^{-3}$ A and I = 10 A

If S is the required shunt resistance, then

$$S = \frac{I_g}{I - I_g} G = \frac{10^{-3}}{10 - 10^{-3}} \times 100 = \frac{\mathbf{100}}{\mathbf{9999}}\ \mathbf{\Omega}\ \textbf{(in parallel)}$$

The current that will pass through shunt is given by

$$I_s = I - I_g = 1 - 10^{-3} = 9.999\ \text{A}$$

Therefore, power dissipated in the shunt

$$= I_s^2 \times S = (9.999)^2 \times \frac{100}{9999} = 0.9999\ \text{W}$$

Since in the available shunt resistance, the power dissipated can be 1 W, it **can be safely used** for the conversion of galvanometer into the ammeter.

If R_A is resistance of the ammeter obtained, then

$$R_A = \frac{GS}{G+S} = \frac{100 \times \frac{100}{9999}}{100 + \frac{100}{9999}} = 0.01\ \Omega$$

Let E be e.m.f. of the battery and r be its internal resistance. When modified galvanometer is connected to the battery, then

$$I = \frac{E}{R_A + r}$$

Here, I = 4 A

$$\therefore \quad 4 = \frac{E}{0.01 + r}$$

or $\quad E = 0.04 + 4r \qquad$ ...(*i*)

When a resistance of 1·5 Ω is connected in series, then

$$I' = \frac{E}{R_A + r + 1.5}$$

Here, I′ = 1 A

$$\therefore \quad 1 = \frac{E}{0.01 + r + 1.5}$$

or $\quad E = 0.01 + r + 1.5 \qquad$ ...(*ii*)

Solving the equations (*i*) and (*ii*), we obtain

E = 2 V and **r = 0·49 Ω**

Problem 2.11. Two resistors 400 Ω and 800 Ω are connected in series with 6 V battery. It is desired to measure the current in the circuit. An ammeter of 10 Ω resistance is used for this purpose. What will be the reading in the ammeter ? Similarly, if a voltmeter of 10,000 Ω resistance is used to measure the potential difference across the 400 Ω resistor, what will be the reading in voltmeter ?

(M.N.R. 1990 ; I.I.T. 1982)

Sol. When ammeter is connected in circuit, its resistance R_A also comes in series with the resistances R_1 and R_2 [Fig. 2.11].

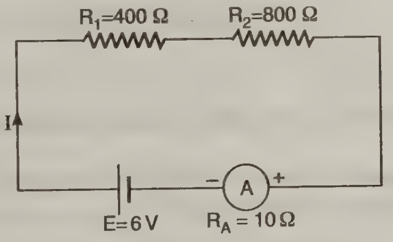


Fig. 2.11.

If R is total resistance of the circuit, then

$$R = R_1 + R_2 + R_A = 400 + 800 + 10 = 1210\ \Omega$$

If I is current in the circuit, then

$$I = \frac{E}{R} = \frac{6}{1210} = \mathbf{4.96 \times 10^{-3}\ A}$$

Therefore, reading of the ammeter = 4.96×10^{-3} A

When voltmeter is connected across the resistance R_1 [Fig. 2.12], then the resistances R_V and R_1 are in parallel and the combination is in series with R_2.

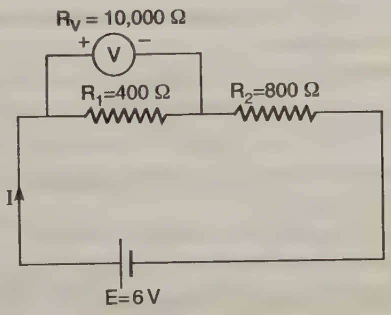


Fig. 2.12.

If R′ is effective resistance of R_1 and R_V, then

$$R' = \frac{R' \times R_V}{R_1 + R_V} = \frac{400 \times 10000}{400 + 10000} = 384 \cdot 6\ \Omega$$

The equivalent resistance of the circuit is given by

$$R = R' + R_2 = 384{\cdot}6 + 800 = 1184{\cdot}6\ \Omega$$

Therefore, current in the circuit,

$$I = \frac{E}{R} = \frac{6}{1184 \cdot 6} = 5 \cdot 065 \times 10^{-3}\ A$$

Hence, reading of the voltmeter,

$$I R' = 5{\cdot}06 \times 10^{-3} \times 384{\cdot}6 = \mathbf{1{\cdot}948\ V}$$

Chapter 3. Magnetic dipole

Problem 3.01. A small magnet of magnetic moment M is placed at broad-side on position of a magnet of magnetic moment M′, length 2 l′ in such a way that the axis of former coincides with the perpendicular bisector of the latter. The separation between their centres is *d*. Calculate the nature of interactions (force or torque) among them. What is its limiting value, when *d* becomes very large ?

(Roorkee, 1986)

Sol. The small magnet NS of magnetic moment M and pole strength *m* is placed at broad-side on position of magnet N′ S′ of magnetic moment M′ and length 2 l′ as shown in Fig. 3.01.

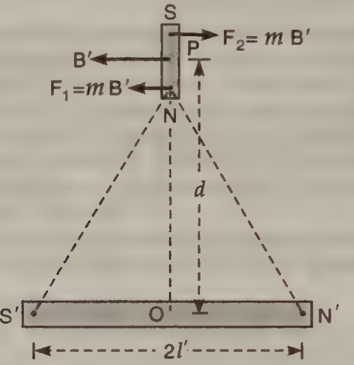


Fig. 3.01.

The distance between the centres of two magnets *i.e.* OP = *d*.

The magnetic field B′ due to the magnet N′ S′ at point P (a point on equitorial line of N′ S′),

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{M'}{(d^2 + l^2)^{3/2}}$$

Let F_1 and F_2 be the forces experienced by north and south-pole of the small magnet NS due to the magnetic field B′. As the magnet NS is small, force on the two poles will be same in magnitude, say F. Therefore,

$$F = F_1 = F_2 = m\,B'$$

The forces on the two poles of the magnet NS are equal in magnitude but opposite in direction. Therefore, magnet NS is acted upon by a torque given by

$$\tau = F \times NS = m\,B' \times NS$$

Since $m \times NS = M$, the magnetic moment of magnet NS ;

$$\tau = M\,B'$$

Substituting for B′, we have

$$\tau = \frac{\mu_0}{4\pi} \cdot \frac{M M'}{(d^2 + l^2)^{3/2}}$$

Therefore, between the two magnets, the nature of interaction is **torque.**

When *d* becomes very large *i.e.* if $d >> l'$, then $d^2 + l^2 \approx d^2$.

The limiting value of torque, $\tau = \mathbf{\dfrac{\mu_0}{4\pi} \cdot \dfrac{M M'}{d^3}}$

Problem 3.02. A coil in the shape of an equilateral triangle of side 0·02 m is suspended from a vertex, such that it is hanging in a vertical plane between the pole pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T [Fig. 3.02]. Find the couple acting on the coil, when a current of 0·1 A is passed through it and the magnetic field is parallel to its plane. *(Roorkee, 1991)*

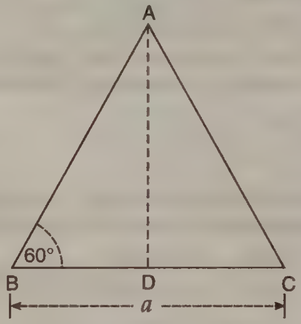


Fig. 3.02.

Sol. Here I = 0·1 A, $B = 5 \times 10^{-2}$ T

Angle between normal to the plane of the coil and the applied magnetic field, $\theta = 90°$

When a current I is passed through a coil having area A, then magnetic dipole moment of the coil is given by

$$M = I\,A$$

Since the coil of the shape of equilateral triangle has side,

$$a = 0{\cdot}02\ \text{m},$$

$$\text{area of coil, } A = \frac{1}{2}\text{base} \times \text{height} = \frac{1}{2}\, a \times a \sin 60°$$

$$= \frac{1}{2} \times 0 \cdot 02 \times 0 \cdot 02 \times \frac{\sqrt{3}}{2} = \sqrt{3} \times 10^{-4}\ \text{m}^2$$

Therefore, magnetic dipole moment of the coil,

$$M = 0 \cdot 1 \times \sqrt{3} \times 10^{-4} = \sqrt{3} \times 10^{-5}\ \text{A m}^2$$

The torque acting on the coil,

$$\tau = M\,B \sin\theta = \sqrt{3} \times 10^{-5} \times 5 \times 10^{-2} \times \sin 90°$$

$$= \mathbf{5\sqrt{3} \times 10^{-7}\ N\,m}$$

Problem 3.03. A rectangular loop PQRS made from a uniform wire has length *a*, width *b* and mass *m*. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the Y-axis [Fig. 3.03]. Take the vertically upward direction as the Z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})\,B_0$ exists in the region. The loop is held in the XY-plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium.

(*a*) What is the direction of the current I and PQ ?

(*b*) Find the magnetic force on the arm RS.

(*c*) Find the expression for I in terms of B_0, *a*, *b* and *m*.

(I.I.T. 2002)

Thought Process

The loop will stay horizontal, when the torque on loop due to magnetic field is equal and opposite to the torque of weight of the loop about PQ.

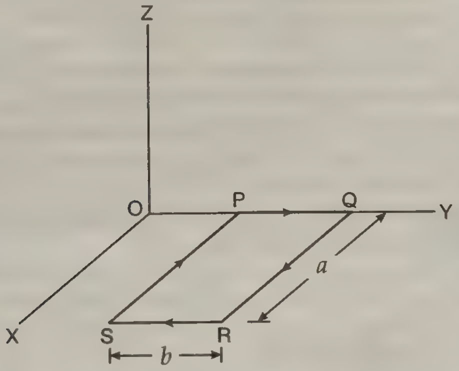


Fig. 3.03

Sol. (*a*) Here $\vec{B} = (3\,\hat{i} + 4\,\hat{k})\,B_0$;

$PQ = RS = b$, $PS = QR = a$ and mass of loop $= m$

Weight of the loop, $m\vec{g} = m\,g(-\hat{k}) = -m\,g\,\hat{k}$

The centre of gravity of loop lies at a distance $a/2$ along OX.

$$\therefore \qquad \vec{r} = \frac{a}{2}\,\hat{i}$$

Hence, torque of weight of the loop about PQ,

$$\vec{\tau}_1 = \vec{r} \times m\,\vec{g} = \left(\frac{a}{2}\,\hat{i}\right) \times (-m\,g\,\hat{k}) = \frac{1}{2}\,m\,g\,a\,\hat{j}$$

The direction of current through the loop should be such that torque $\vec{\tau}_2$ on the loop due to magnetic field acts opposite to $\vec{\tau}_1$.

Suppose that current I is passed through the loop in the direction PQRS. Then, area vector $\vec{A}$ of the loop is given by

$$\vec{A} = a\,b\,(-\hat{k}) = -a\,b\,\hat{k}$$

and magnetic moment of the current loop,

$$\vec{M} = I\,\vec{A} = -a\,b\,I\,\hat{k}$$

Hence, torque on the current loop due to magnetic field,

$$\vec{\tau}_2 = \vec{M} \times \vec{B} = (-a\,b\,I\,\hat{k}) \times (3\,\hat{i} + 4\,\hat{k})\,B_0 = -3\,a\,b\,I\,B_0\,\hat{j}$$

Thus, $\vec{\tau}_1$ and $\vec{\tau}_2$ act in opposite direction, if current is passed through the loop in the direction PQRS *i.e.* from **point P to Q in arm PQ.**

(*b*) Magnetic force on arm RS,

$$\vec{F} = I\,(\overrightarrow{RS}) \times \vec{B} = I\,(-b\,\hat{j}) \times (3\,\hat{i} + 4\,\hat{k})\,B_0$$

$$= \mathbf{b\,I\,B_0\,(3\,\hat{i} - 4\,\hat{k})}$$

(*c*) Since loop is in equilibrium,

$$|\vec{\tau}_1| = |\vec{\tau}_2|$$

$$\text{or} \qquad \frac{1}{2}\,m\,g\,a = 3\,a\,b\,I\,B_0$$

$$\text{or} \qquad I = \frac{m\,g}{6\,b\,B_0}$$

Problem 3.04. A magnet is suspended in the magnetic meridian with a untwisted wire. The upper end of wire is rotated through 180° to deflect the magnet by 30° from magnetic meridian. Now this magnet is replaced by another magnet and the upper end of the wire is rotated through 270° to deflect the magnet 30° from magnetic meridian. Compare the magnetic moments of magnets.

(*M.N.R. 1985*)

Thought Process

When wire is twisted through angle ϕ, the magnet gets deflected through angle θ. Therefore, the net twist produced in the wire will be $(\phi - \theta)$. If k is restoring torque per unit twist of the wire, then in equilibrium.

$$k\ (\phi - \theta) = M\,B \sin\theta$$

Sol. ***For First magnet :***

$$\phi - \theta = 180° - 30° = 150° = \frac{150 \times \pi}{180}\ \text{rad}$$

If M_1 is magnetic moment of the first magnet, then

$$k \times \frac{150\,\pi}{180} = M_1\,B \sin 30° \qquad \ldots(i)$$

For Second magnet :

$$\phi - \theta = 270° - 30° = 240° = \frac{240 \times \pi}{180}\ \text{rad}$$

If M_2 is magnetic moment of the second magnet, then

$$k \times \frac{240\,\pi}{180} = M_2\,B \sin 30° \qquad \ldots(ii)$$

Dividing equation (*i*) by (*ii*), we have

$$\frac{M_1\ B \sin 30°}{M_2\ B \sin 30°} = \left(k = \frac{150\,\pi}{180}\right) \bigg/ \left(\frac{k \times 240\,\pi}{180}\right)$$

$$\text{or} \qquad \frac{M_1}{M_2} = \frac{5}{8}$$

Problem 3.05. A magnet of magnetic moment M is freely suspended in a constant uniform magnetic field B. Calculate the work done in deflecting the magnet through an angle θ from the direction of B. (*Roorkee, 1990*)

Hint. Work done in rotating a magnet inside the magnetic field,

$$W = M\,B\,(\cos\theta_1 - \cos\theta_2) \qquad \ldots(i)$$

As the magnet is to be rotated from the direction of B,

$$\theta_1 = 0° \text{ and } \theta_2 = \theta$$

Therefore, equation (*i*) becomes

$$W = M\,B\,(\cos 0° - \cos\theta)$$

$$= \mathbf{M\,B\,(1 - \cos\theta)}$$

Chapter 4. Earth's Magnetism

Problem 4.01. A circular coil of radius 0·157 m has 50 turns. It is placed, such that its axis is in magnetic meridian. A dip needle is supported at the centre of the coil with its axis of rotation horizontal and in the plane of the coil. The angle of dip is 30°, when a current flows through the coil. The angle of dip becomes 60° on reversing the current. Find

the current in the coil assuming that the magnetic field due to the coil is smaller than the horizontal component of earth's magnetic field (Horizontal component of earth's field = 3 × 10^{-5} T). *(Roorkee, 1994)*

Thought Process

The horizontal component of earth's magnetic field B_H *is aided by the magnetic field B′ produced by the coil, when current is passed in one direction and is opposed by the magnetic field B′ on reversing the direction of the current. It follows that*

$$\textit{when } \delta = 30°, \tan\delta = \frac{B_V}{B_H - B'}$$

$$\textit{and when } d = 60°, \tan\delta = \frac{B_V}{B_H - B'} \qquad (B' < B_H)$$

Sol. Here, $BH = 3 \times 10^{-5}$ T

In first case, $\delta = 30°$

$$\therefore \quad \tan 30° = \frac{B_V}{B_H + B'}$$

$$\text{or} \quad \frac{B_V}{B_H + B'} = \frac{1}{\sqrt{3}} \qquad ...(i)$$

On reversing the current, $\delta = 60°$

$$\therefore \quad \tan 60° = \frac{B_V}{B_H - B'}$$

$$\text{or} \quad \frac{B_V}{B_H - B'} = \sqrt{3} \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{B_V}{B_H - B'} \times \frac{B_H + B'}{B_V} = \sqrt{3} \times \frac{\sqrt{3}}{1}$$

$$\text{or} \quad \frac{B_H + B'}{B_H - B'} = 3$$

$$\text{or} \quad B' = \frac{1}{2} B_H = \frac{1}{2} \times 3 \times 10^{-5} = 1{\cdot}5 \times 10^{-5} \text{ T}$$

But magnetic field at the centre of a circular coil,

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a}$$

$$\therefore \quad \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} = 1{\cdot}5 \times 10^{-5}$$

Here, $n = 50$, $a = 0{\cdot}157$ m

$$10^{-7} \times \frac{2\pi \times 50 \times I}{0{\cdot}157} = 1{\cdot}5 \times 10^{-5}$$

$$\text{or} \quad I = \frac{1{\cdot}5 \times 10^{-5} \times 0{\cdot}157}{10^{-7} \times 2 \times 3{\cdot}14 \times 50} = \mathbf{0{\cdot}075\ A}$$

Problem 4.02. A small magnet of magnetic moment $\pi = 10^{-10}$ is placed on the Y- axis at a distance of 0·1 m from the origin with its axis parallel to the X-axis. A coil having 169 turns and radius 0·05 m is placed on the X-axis at a distance of 0·12 m from the origin with the axis of the coil coinciding with X-axis. Find the magnitude and direction of the current in the coil for a compass needle placed at the origin to point in the north-south direction. *(Roorkee, 1993)*

Thought Process

A compass needle placed at the origin will point in the north-south direction, if the magnetic field produced at the origin by the magnet and the coil are equal in magnitude and opposite in direction. Accordingly, as seen from origin, the direction of flow of current through the coil is in anticlockwise direction.

Sol. Let B be the magnetic field produced by the small magnet NS at the origin and B′ be the magnetic field produced by the coil on passing current through it. Since magnetic field B due to the magnet is along OX, the magnetic field B′ due to the coil would be along OX′. It will be so, if on looking from the origin O, the direction of flow of current through the coil is in anticlockwise direction [Fig. 4.01].

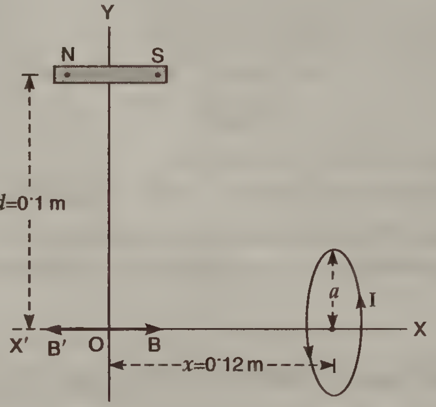


Fig. 4.01.

The magnetic field due to small magnet at point O,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$

Here, $M = \pi \times 10^{-10}$ A m^2 ; $r = 0{\cdot}1$ m

$$\therefore \quad B = 10^{-7} \times \frac{\pi \times 10^{-10}}{(0{\cdot}1)^3} = \pi \times 10^{-14} \text{ T}$$

The magnetic field due to circular coil at point O,

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + x^2)^{3/2}}$$

Here, $n = 169$; $a = 0{\cdot}65$ m ; $x = 0{\cdot}12$ m

$$\therefore \quad B' = 10^{-7} \times \frac{2\pi \times 169 \times I \times (0{\cdot}05)^2}{[(0{\cdot}05)^2 + (0{\cdot}12)^2]^{3/2}}$$

$$= 10^{-7} \times \frac{2\pi \times 169 \times I \times (0{\cdot}05)^2}{(0{\cdot}13)^2} = 3{\cdot}846\,\pi \times 10^{-5}\, I$$

For compass needle to point in the north-south direction,

$3{\cdot}846\,\pi \times 10^{-5}\, I = \pi \times 10^{-14}$

or **$I = 2{\cdot}6 \times 10^{-10}$ A**

(in anticlockwise direction, when seen from the origin)

Problem 4.03. A vibration magnetometer consists of two identical bar magnets placed one over the other, such that they are mutually perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is 4 s. If one of the magnets is taken away, find the period of the other in the same field. *(Roorkee, 1992)*

Thought Process

When two magnets, each of magnetic moment M and moment of inertia I are placed perpendicular to each other, the moment of inertia of the combination becomes

$$I' = 2I$$

and the magnetic moment of the combination is given by

$$M' = \sqrt{M^2 + M^2 + 2\,M\,M\cos 90^\circ} = \sqrt{2}\,M$$

Sol. Let T′ be the time period of the combination and T, the time period of a magnet when the other is taken away.

For combination : $T' = 2\pi\sqrt{\dfrac{I'}{M'B}}$

$$\therefore \qquad 4 = 2\pi\sqrt{\frac{2I}{\sqrt{2}\,MB}} \qquad \text{...(i)}$$

When one magnet is taken away : $T = 2\pi\sqrt{\dfrac{I}{MB}}$ ...(ii)

Dividing equation (ii) by (i), we have

$$T = 2\pi\sqrt{\frac{I}{MB}} \Big/ 2\pi\sqrt{\frac{2I}{\sqrt{2}\,MB}} = (2)^{1/4}$$

or $\qquad T = 4 \times (2)^{1/4} = 4 \times 1{\cdot}189 =$ **4·76 s**

Chapter 5. Classification of Magnetic materials

Problem 5.01. An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns cm^{-1}. If a current of 0·5 A is passed through the solenoid, find the magnetic moment of the rod.

Sol. Here, $\mu_r = 1000$; $n = 5$ turns cm^{-1} = 500 turns m^{-1}

volume of the rod, $V = 10^{-4}$ m^3 ;

current through the solenoid, $i = 0{\cdot}5$ A

The magnetic field produced by solenoid,

$$B = \mu_0\mu_r\,n\,i = 4\pi \times 10^{-7} \times 1000 \times 500 \times 0{\cdot}5 = \frac{\pi}{10}\text{ tesla}$$

Now, $\qquad B = \mu_0\,(H + I)$

$$\text{or} \qquad I = \frac{B}{\mu_0} - H = \frac{B}{\mu_0} - \frac{B}{\mu} \qquad \left(\because H = \frac{B}{\mu}\right)$$

$$\text{or} \qquad \frac{B}{\mu_0}\left(1 - \frac{\mu_0}{\mu}\right) = \frac{B}{\mu_0}\left(1 - \frac{1}{\mu_r}\right)$$

$$\text{or} \qquad I = \frac{B}{\mu_0\,\mu_r}(\mu_r - 1) = \frac{\pi/10}{4\pi \times 10^{-7} \times 1000}(1000 - 1)$$

$$= 2{\cdot}5 \times 10^5 \text{ A m}^{-1}$$

Now, $I = \dfrac{M}{V}$,

where M is magnetic moment and V is volume of the rod.

$$\therefore \qquad M = I \times V = 2{\cdot}5 \times 10^5 \times 10^{-4} = \mathbf{25\ A\ m^2}$$

Problem 5.02. A bar magnet has coercivity 4×10^3 A m^{-1}. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. What current should be sent through the solenoid ?

Sol. The coercivity of 4×10^3 A m^{-1} of the bar magnet implies that magnetic intensity $H = 4 \times 10^3$ A m^{-1} (in opposite direction) is required to demagnetise it.

Here, number of turns per unit length of the solenoid,

$$n = \frac{60}{12} = 5 \text{ turns cm}^{-1} = 500 \text{ turns m}^{-1}$$

Now, $\qquad B = \mu_0\,n\,I$

$$\text{or} \qquad I = \frac{B}{\mu_0\,n} = \frac{H}{n} = \frac{4 \times 10^3}{500} = 8\text{ A}$$

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Magnetic Effect of Current

PART I. TEXT-BASED QUESTIONS

1. A current flows in a conductor from east to west. The direction of the magnetic field at a point above the conductor is
(A) towards north. (B) towards south.
(C) towards east. (D) towards west.
(Karnataka Entrance, 1992, 198 7)

2. The magnetic field at a point due to a current carrying conductor is directly proportional to
(A) resistance of the conductor.
(B) thickness of the conductor.
(C) current flowing through the conductor.
(D) distance from the conductor. *(C.E.T. 1999)*

3. In SI system, permeability has the units :
(A) weber metre^{-1} ampere^{-1}
(B) weber metre ampere^{-1}
(C) weber metre^{-1}ampere^{-2}
(D) weber metre ampere *(Pre-Medical/Dental ,1993)*

4. The magnetic field $\overrightarrow{dB}$ due to a small current element dl at a distance $\vec{r}$ and element carrying current I is
(A) $\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{I\,\vec{dl} \times \vec{r}}{r}$ (B) $\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{I\,\vec{dl} \times \vec{r}}{r^2}$
(C) $\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{I\,\vec{dl} \times \vec{r}}{r^3}$ (D) $\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{I\,\vec{dl} \times \vec{r}}{r^4}$
(C.B.S.E. 1996)

5. The magnetic field at a distance *r* from a long wire carrying current I is 0·4 tesla. The magnetic field at a distance 2 *r* is
(A) 0·1 tesla (B) 0·2 tesla
(C) 0·8 tesla (D) 1·6 tesla *(C.B.S.E. 1992)*

6. A straight wire of diameter 0·5 mm carrying a current of 1 A is replaced by another wire of 1 mm diameter carrying the same current. The strength of the magnetic field far away is
(A) twice the earlier value.
(B) one half of the earlier value.
(C) one quarter of the earlier value.
(D) same as the earlier value. *(C.B.S.E. 1999, 1997)*

7. A circular coil A has a radius *a* and the current flowing in it is I. Another circular coil B has a radius 2 *a* and if 2 I is the current flowing through it, then the magnetic fields at the centre of the circular coils are in the ratio of
(A) 1 : 1 (B) 2 : 1
(C) 3 : 1 (D) 4 : 1 *(C.B.S.E. 1993)*

8. Magnetic field intensity at the centre of coil of 50 turns, radius 0·5 m and carrying a current of 2 A is
(A) $0{\cdot}5 \times 10^{-5}$ T (B) $1{\cdot}26 \times 10^{-4}$ T
(C) 3×10^{-5} T (D) 4×10^{-5} T *(C.B.S.E. 1999)*

9. A helium nucleus makes a full rotation in a circle of radius 0·8 m in 2 s. The vlaue of the magnetic field B at the centre of circle will be
(A) $\frac{10^{-19}}{\mu_0}$ (B) $10^{-19}\,\mu_0$
(C) $2 \times 10^{-19}\,\mu_0$ (D) $\frac{2 \times 10^{-19}}{\mu_0}$ *(C.P.M.T. 1988)*

10. A particle carrying a charge equal to 100 times the charge on an electron is revolving per second in a circular path of radius 0·8 m. The magnetic field produced at the centre wil be :
(A) $10^{-7}/\mu_0$ (B) $10^{-17}\,\mu_0$
(C) $10^{-6}\,\mu_0$ (D) $10^{-7}\,\mu_0$

11. A positively charged particle moving due east enters region of uniform magnetic field directed verticall upwards. The particle will
(A) get deflected in vertically upward direction.
(B) move in circular path with an increased speed.
(C) move in circular path with a decreased speed.
(D) move in a circular path with a uniform speed.
(C.B.S.E. 1997

12. A charged particle moves with velocity $\vec{v}$ in a unifor magnetic field $\vec{B}$. The magnetic force experienced by th particle is
(A) always zero.
(B) never zero.
(C) zero, if $\vec{B}$ and $\vec{v}$ are perpendicular.
(D) zero, if $\vec{B}$ and $\vec{v}$ are parallel.
(Pre-Medical/Dental ,1990

13. A 2 MeV proton is moving perpendicular to a unifor magnetic field of 2·5 tesla. The force on the proton is
(A) $2{\cdot}5 \times 10^{-10}$ N (B) $7{\cdot}84 \times 10^{-11}$ N
(C) $2{\cdot}5 \times 10^{-11}$ N (D) $7{\cdot}84 \times 10^{-12}$ N
(C.P.M.T. 1989

14. Tesla is a unit of magnetic
(A) field (B) flux
(C) induction (D) moment. *(C.P.M.T. 1992*

15. Weber m^{-2} is equal to
(A) tesla (B) henry
(C) watt (D) dyne. *(A.F.M.C. 1997*

16. The unit of magnetic induction is
(A) A m^{-1} (B) A m
(C) weber (D) tesla *(Karnataka, 1993*

17. A power line lies along east-west direction and carries current of 10 ampere. The force per metre due to earth' magnetic field of 10^{-4} tesla is
(A) 10^{-2} N (B) 10^{-3} N
(C) 10^{-4} N (D) 10^{-5} N *(Roorkee, 1992*

18. The magnetic field of a U-shaped magnet is parallel to th surface of the paper, when the magnet is placed in the plan of paper with its north pole towards left. A conductor i placed in the field, so that it is perpendicular to the pape If the direction of flow of current through the conducto is outwards, it will tend to move :
(A) downwards. (B) upwards.
(C) to the right. (D) to the left.

19. Two parallel wires carrying currents in the same directio attract each other, because of
(A) potential difference between them.
(B) mutual inductance between them.
(C) electric forces between them.
(D) magnetic forces between them. *(N.C.E.R.T. 1983*

20. Two long straight wires carrying the same current I an separated by a distance *r* exert force F per unit length o each other. If the current is increased to 2 I and separatio is reduced to *r* / 2, then force will become :

(A) 8 F (B) 2 F
(C) F/2 (D) F/8

21. **Two thin long parallel wires are separated by a distance r and carry a current I each. The magnitude of the force per unit length experienced by one wire due to the other is :**

(A) $\frac{\mu_0 I^2}{r^2}$ (B) $\frac{\mu_0 I^2}{2\pi r}$
(C) $\frac{\mu_0 I}{2\pi r}$ (D) $\frac{\mu_0 I}{2\pi r^2}$ *(I.I.T. 1986)*

22. **Two long parallel wires are carrying currents of 5 A and 12 A respectively. If the force per unit length between these wires is 3×10^{-5} N m^{-1}, the distance between them should be**

(A) 400 m (B) 4 m
(C) 0·4 m (D) 0·04 m *(C.E.T. 1999)*

23. **Two long parallel wires are at a distance of 1 m. If both of them carry 1 A of current, then the force of attraction per unit length between the two wires is**

(A) 2×10^{-7} N m^{-1} (B) 2×10^{-8} N m^{-1}
(C) 5×10^{-8} N m^{-1} (D) 5×10^{-7} N m^{-1}
(C.B.S.E. 1998)

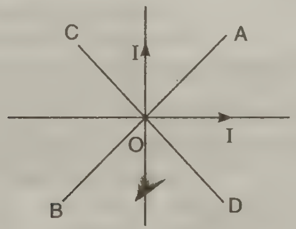
24. **Two parallel wires in free space are 10 cm apart and each carries a current of 10 A in the same direction. The force exerted by one wire on the other (per metre length) is**

(A) 2×10^{-4} N attractive.
(B) 2×10^{-7} N attractive.
(C) 2×10^{-4} N repulsive.
(D) 2×10^{-7} N repulsive. *(C.B.S.E. 1997)*

25. **A 15 cm radius ring has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field in the core for a magnetising current 1·2 A ?**

(A) 0·39 oersted (B) 5·76 oersted
(C) 5·76 tesla (D) 4·48 tesla
(Karnataka Entrance, 1991)

26. **Two equal electric currents are flowing perpendicular to each other as shown in the figure. AB and CD are perpendicular to each other and symmetrically placed w.r.t. the currents. Where do we expect the resultant magnetic field to be zero ?**



(A) on AB. (B) on CD.
(C) on both AB and CD. (D) on both OD and BO.
(C.B.S.E. 1996)

27. **A circular loop of radius 10 cm carrying a current of 1 A is placed in a uniform magnetic field of 0·1 T. If the magnetic field is perpendicular to the plane of the loop, the force experienced by the loop is**

(A) 0·001 N (B) 0·314 N
(C) 0·52 N (D) zero *(C.E.T. 1998)*

28. **A circular loop of radius R carrying a current I is placed in a uniform magnetic field with its plane perpendicular to B. The force on the loop is**

(A) $2\pi B I R$ (B) $2\pi B I^2 R$
(C) $4\pi B I R$ (D) zero *(C.P.M.T. 1993)*

29. **The torque on the loop in Q. 28. is**

(A) $\pi B R^2$ (B) $\pi B I^2 R^2$
(C) $\pi B^2 I R^2$ (D) zero *(C.P.M.T. 1993)*

30. **A current loop of area 0·01 m^2 and carrying a current of 10 ampere is held perpendicular to a magnetic field of intensity of 0·1 tesla. The torque (in N m) acting on the loop is**

(A) 0 (B) 0·001
(C) 0·01 (D) 1·1
(Pre-Medical/Dental, 1994)

31. **A current carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend upon**

(A) shape of the loop.
(B) size of the loop.
(C) value of the current.
(D) magnetic field. *(C.E.T. 1998)*

32. **The basic instrument employed to detect current is**

(A) galvanometer (B) ammeter
(C) wattmeter (D) voltmeter.
(Karnataka, 1992 S)

33. **A galvanometer is said to be sensitive, if it gives a**

(A) small deflection for a small current.
(B) small deflection for a large current.
(C) large deflection for a large current.
(D) large deflection for a small current. *(C.E.T. 1999)*

34. **The sensitivity of a moving coil galvanometer can be increased by**

(A) increasing the number of turns.
(B) decreasing the number of turns.
(C) decreasing the area of the coil.
(D) none of the above. *(Karnataka Entrance, 1992, 1989)*

35. **To convert a galvanometer into an ammeter, we connect**

(A) low resistance in series.
(B) low resistance in parallel.
(C) high resistance in series.
(D) high resistance in parallel.
(C.B.S.E. 2002 ; A.F.M.C. 1995, 1993)

36. **A moving coil galvanometer has a resistance of 900 Ω. In order to send only 10% of the main current through this galvanometer, the resistance of the required shunt is**

(A) 0·9 Ω (B) 90 Ω
(C) 100 Ω (D) 900 Ω *(Roorkee ,1992)*

37. **A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance 2 Ω. If the total current is 1 A, the part of it passing through the shunt will be**

(A) 1·2 A (B) 0·8 A
(C) 0·5 A (D) 0·3 A *(C.B.S.E. 1998)*

38. **The deflection in a moving coil galvanometer falls from 50 to 10 divisions, when a shunt of 12 Ω is connected across it. The resistance of the coil of the galvanometer is**

(A) 2·4 Ω (B) 12 Ω
(C) 48 Ω (D) 60 Ω.

39. **To increase the range of an ammeter,**

(A) a small resistance is connected in series.
(B) a small resistance is connected as shunt.
(C) a large resistance is connected in series.
(D) a large resistance is connected as shunt.
(Karnataka Entrance, 1988)

40. **To convert a galvanometer into a voltmeter,**

(A) a high resistance is connected in parallel.
(B) a low resistance is connected in series.
(C) a low resistance is connected in parallel.
(D) a high resistance is connected in series.
(C.E.T. 1999 ; Karnataka Entrance, 1993)

41. **A galvanometer can measure a current up to 20 milliampere. When it is connected in series with a**

resistance of 2970 ohm, it can read up to 60 volt. The galvanometer has a resistance (in ohm)

(A) 130
(B) 30
(C) 230
(D) 60 *(Karnataka Entrance, 1987)*

42. **The e.m.f. of generator is 6 V and its internal resistance is 0·5 kΩ. The reading of a voltmeter having an internal resistance of 2·5 kΩ is**

(A) 10^{-3} V
(B) 1 V
(C) 5 V
(D) 10 V *(C.E.T. 1991)*

43. **A voltmeter has a resistance of 20 kΩ. When it is connected in series with a resistance R across a 230 V supply, it reads 160 V. The value of R is**

(A) 3·75 kΩ
(B) 4·75 kΩ
(C) 6·75 kΩ
(D) 8·75 kΩ *(C.E.T. 1999)*

PART II. THOUGHT-BASED QUESTIONS

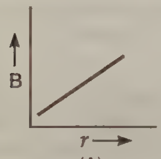
44. **If a long hollow copper pipe carries a current, the magnetic field produced will be**

(A) inside the pipe only.
(B) outside the pipe only.
(C) neither inside nor outside the pipe.
(D) both inside and outside the pipe. *(C.B.S.E. 1999)*

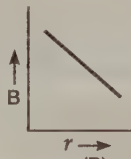
45. **A current I flows along the length of an infinitely long, straight and thin walled pipe. Then, the magnetic field**

(A) at all points inside the pipe is the same, but not zero.
(B) at any point inside the pipe is zero.
(C) is zero only on the axis of the pipe.
(D) is different at different points inside the pipe. *(I.I.T. 1993)*

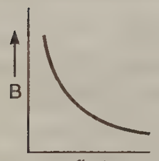
46. **A constant current is flowing through a straight conductor. The variation of magnetic induction B with distance r of the observation point will be as shown :**



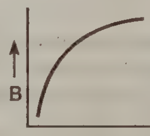
(A)



(B)



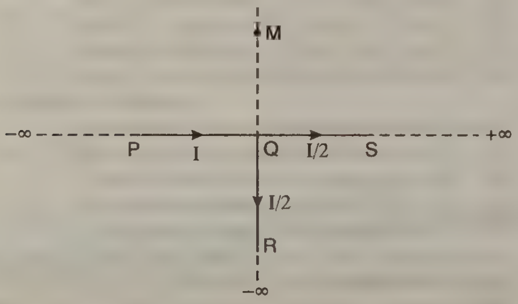
(C)



(D)

(A.F.M.C. 1997)

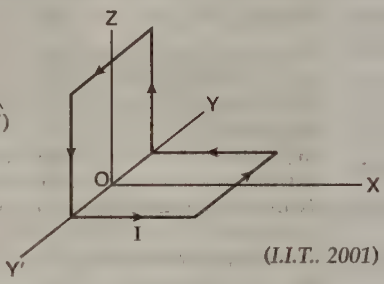
47. **An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is B_1. Now, another infinitely long straight conductor QS is connected at Q, so that the current is I/2 in QR as well as QS, the current in PQ remaining unchanged. The magnetic field at M is now B_2. The ratio B_1/B_2 is given by**



(A) 1/2
(B) 1
(C) 2/3
(D) 2 *(I.I.T. 2000)*

48. **A non-polar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length 2 *a*. The magnetic field due to this loop at the point P (*a*, 0, *a*) points in the direction**

(A) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$
(B) $\frac{1}{\sqrt{3}}(-\hat{j}+\hat{k}+\hat{i})$
(C) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$
(D) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$



(I.I.T.. 2001)

49. **A long straight along the Z-axis carries a current I in the negative Z-direction. The magnetic field vector $\vec{B}$ at a point having coordinate (*x*, *y*) on the *z* = 0 plane is**

(A) $\frac{\mu_0}{2\pi}\cdot\frac{I(y\hat{i}-x\hat{j})}{(x^2+y^2)}$
(B) $\frac{\mu_0}{2\pi}\cdot\frac{I(y\hat{i}+x\hat{j})}{(x^2+y^2)}$
(C) $\frac{\mu_0}{2\pi}\cdot\frac{I(x\hat{j}-y\hat{i})}{(x^2+y^2)}$
(D) $\frac{\mu_0}{2\pi}\cdot\frac{I(x\hat{i}-y\hat{j})}{(x^2+y^2)}$ *(I.I.T. 2002)*

50. **Two wires are held perpendicular to the plane of paper at 5 m apart. They carry currents of 2·5 A and 5 A in same direction. Then the magnetic field strength (B) at a point midway between the wires will be :**

(A) $\frac{\mu_0}{4\pi}$ T
(B) $\frac{\mu_0}{2\pi}$ T
(C) $\frac{3\mu_0}{2\pi}$ T
(D) $\frac{3\mu_0}{4\pi}$ T *(C.B.S.E. 2001)*

51. **The magnetic field of a given length of wire carrying a current for a single turn circular coil at centre is B. Then its value for the same wire, when same current passing through it, is**

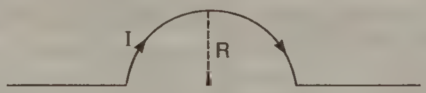
(A) B/4
(B) B/2
(C) 2 B
(D) 4 B *(C.B.S.E. 2002)*

52. **A wire of certain length is bent to form a circular coil of a single turn. If the same wire is bent into a coil of smaller radius so as to have two turns, then magnetic field produced at the centre by the same value of current is :**

(A) one quarter of its value in first case.
(B) one half of its value in first case.
(C) two times its value in first case.
(D) four times its value in first case.

(C.B.S.E. 1998 ; Karnataka Entrance, 1991, 1988 ; A.I.I.M.S. 1990)

53. **A portion of a conducting wire is bent in the form of a semicircle of radius R as shown in figure. At the centre of the semicircle, the magnetic induction will be :**



(A) $\frac{\mu_0 I}{2R}$
(B) $\frac{\mu_0 I}{4R}$
(C) $\frac{\mu_0 I}{2\pi R}$
(D) $\frac{\mu_0 I}{4\pi R}$

54. The current that flows through a conductor of length 1 cm bent into the form of an arc of a circle of radius 2 cm to produce a magnetic field of 0·1 oersted at its centre is

(A) 1 ampere (B) 4 ampere
(C) 0·5 ampere (D) 0·1 ampere

55. A coil having n turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is

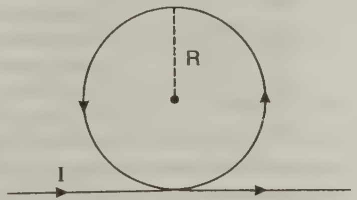
(A) $\frac{\mu_0 n I}{b}$ (B) $\frac{2 \mu_0 n I}{a}$

(C) $\frac{\mu_0 n I}{2(a-b)} \log_e \frac{b}{a}$ (D) $\frac{\mu_0 I^n}{2(b-a)} \log_e \frac{b}{a}$ (*I.I.T. 2001*)

56. Two coils are placed, such that their planes are at right angles to each other and have common centre. If each coil carries the same current and produces same magnetic field B, then the resultant field will be :

(A) 2 B (B) $\sqrt{2}$ B
(C) B / $\sqrt{2}$ (D) zero

57. An infinitely long straight conductor is bent into the shape as shown. If it carries a current I and the radius of the circular loop is R, then magnetic induction at the centre of the circular part is :



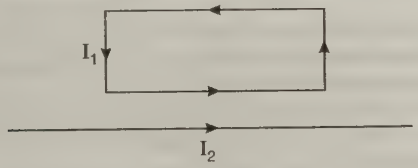
(A) $\frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{R}$

(B) $\frac{\mu_0}{4\pi} \cdot \frac{2I}{R}(\pi + 1)$

(C) $\frac{\mu_0}{4\pi} \cdot \frac{2I}{R}(\pi - 1)$

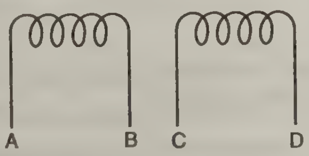
(D) cannot be found.

58. A rectangular loop carries a current I_1 and is placed near a long straight wire. If the wire carries a current I_2 and it is parallel to one of the sides of the loop, then the loop will



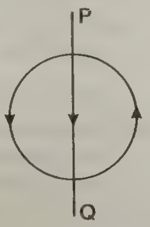
(A) move towards the wire.
(B) move away from the wire.
(C) rotate about some axis parallel to the wire.
(D) remain stationary.

59. Two current carrying coils are held as shown in the figure. If the two coils attract each other, then direction of flow of electrons must be same at the ends



(A) A and B.
(B) A and C.
(C) A and D.
(D) C and D.

60. In the diagram, PQ is a long current carrying wire, which is placed near a current carrying circular coil. The direction of the force acting on PQ is



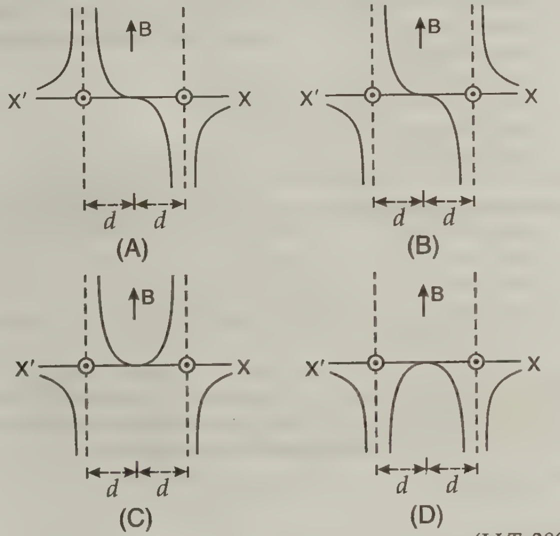
(A) parallel to PQ towards Q.
(B) parallel to PQ towards P.
(C) perpendicular to PQ towards right.
(D) perpendicular to PQ towards left.

61. Two very long, straight and parallel wires carry steady currents I and – I respectively. The distance between the wires is r. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. Its instantaneous velocity v is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(A) $\frac{\mu_0 I q v}{2\pi r}$ (B) $\frac{\mu_0 I q v}{\pi r}$

(C) $\frac{2 \mu_0 I q v}{\pi r}$ (D) 0 (*I.I.T. 1998*)

62. Two long parallel wires are at a distance 2 d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by



(A) (B) (C) (D)

(*I.I.T. 2000*)

63. Current I is flowing in a coil of area A and number of turns n. The magnetic moment of the coil is

(A) n I A (B) n^2 I A

(C) $\frac{nI}{A}$ (D) $\frac{nI}{\sqrt{A}}$ (*C.B.S.E. 2001*)

64. The resistance of an ideal ammeter is

(A) very low. (B) very high.
(C) infinity. (D) can have any value.

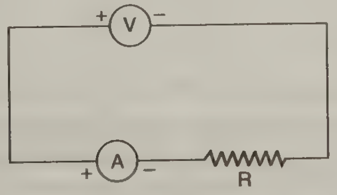
(*C.E.T. 1999 ; Karnataka Entrance, 1993*)

65. The resistance of an ideal voltmeter is

(A) very high. (B) nearly zero.
(C) very low. (D) none of the above.

(*Karnataka Entrance, 1984*)

66. A candidate connects a moving coil voltmeter V and a moving coil ammeter A and resistance R as shown in figure. If the voltmeter reads 20 V and the ammeter reads 4 A, then R is



(A) equal to 5 Ω. (B) greater than 5 Ω.
(C) less than 5 Ω.
(D) greater or less than 5 Ω depending upon its material.

(*A.F.M.C. 1998*)

Chapter 2. Motion of Charged Particle

PART I. TEXT-BASED QUESTIONS

1. **The mass of a proton is about 2,000 times the mass of an electron. An electron and a proton are injected into a uniform electric field at right angles to the direction of the field with the same initial kinetic energy. Then,**
   (A) the electron trajectory will be less curved than that of the proton.
   (B) the proton trajectory will be less curved than that of the electron.
   (C) both the trajectories will be equally curved.
   (D) both the trajectories will be straight.
   (Karnataka Entrance, 1988)

2. **A deuterium nucleus consists of one proton and one neutron. If a deuterium nucleus and a helium nucleus are both placed in the same electric field, the acceleration of deuterium is**
   (A) equal to that of the helium.
   (B) greater than that of helium.
   (C) less than that of helium.
   (D) not related in a fixed way to that of helium.
   (Karnataka Entrance, 1986, 1985)

3. **A charged particle is moving through magnetic field. Then, magnetic field :**
   (A) always exerts a force on the particle.
   (B) never exerts a force on the particle.
   (C) exerts a force, if the particle is moving at right angles to the field.
   (D) exerts a force, if the particle is moving along the field.

4. **An electric charge e moves with a constant velocity v parallel to the lines of force of a uniform magnetic field B, the force experienced by the charge is :**
   (A) Bev (B) $\frac{ev}{B}$
   (C) zero (D) $\frac{e}{Bv}$

5. **A uniform magnetic field acts at right angles to the direction of motion of electrons. As a result, the electron moves in a circular path of radius 2 cm. If the speed of the electrons is doubled, then the radius of the circular path will be**
   (A) 2·0 cm (B) 0·5 cm
   (C) 4·0 cm (D) 1·0 cm *(C.B.S.E., 1991)*

6. **An electron moves with a velocity of $10^3\ \text{m s}^{-1}$ in a magnetic field of induction 0·3 T at an angle 30°. If e/m of electron is $1{\cdot}76 \times 10^{11}\ \text{C kg}^{-1}$, the radius of the path is nearly**
   (A) 10^{-8} m (B) 2×10^{-8} m
   (C) 10^{-6} m (D) 10^{-10} m *(C.B.S.E. 2000)*

7. **A charged particle of charge q and mass m enters perpendicularly in a magnetic field $\vec{B}$. Kinetic energy of the particle is E. Then frequency of rotation is**
   (A) $\frac{qB}{\pi m}$ (B) $\frac{qB}{2\pi m}$
   (C) $\frac{qBE}{2\pi m}$ (D) $\frac{qB}{2\pi E}$ *(C.B.S.E. 2001)*

8. **A proton having charge e enters a magnetic field of strength B with a velocity v in a direction making angle θ with the direction of the field. The force on proton is :**
   (A) Bev (B) zero
   (C) $Bev\cos\theta$ (D) $Bev\sin\theta$

9. **Two particles X and Y having equal charges, after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular paths of radii r_1 and r_2 respectively. The ratio of the mass of X to that of Y is**
   (A) $\left(\frac{r_1}{r_2}\right)^{1/2}$ (B) $\left(\frac{r_2}{r_1}\right)$
   (C) $\left(\frac{r_1}{r_2}\right)^2$ (D) $\left(\frac{r_1}{r_2}\right)$
   (C.P.M.T. 1991 ; I.I.T. 1988)

10. **A proton is projected in a direction perpendicular to a uniform magnetic field and an electron is projected along the direction of the field. Then, the path followed by**
    (A) electron will be circular, while that of proton will be straight.
    (B) proton will be circular, while that of the electron will be straight.
    (C) both of them will be parabolic.
    (D) both of them will be straight.

11. **The radius of curvature of the paths of a charged particle in uniform magnetic field is directly proportional to :**
    (A) the charge on the particle.
    (B) the energy of the particle.
    (C) the momentum of the particle.
    (D) the intensity of the field.

12. **A deutron of kinetic energy 50 keV is describing a circular orbit of radius 0·5 m in a plane perpendicular to magnetic field $\vec{B}$. The kinetic energy of the proton that describes a circular orbit of radius 0·5 m in the same plane with the same $\vec{B}$ is**
    (A) 25 keV (B) 50 keV
    (C) 200 keV (D) 100 keV
    (Pre-Medical/Dental, 1991)

13. **A proton and an alpha particle enter in a uniform magnetic field with the same velocity. The time period of rotation of the alpha particle will be**
    (A) four times that of the proton.
    (B) two times that of the proton.
    (C) three times that of the proton.
    (D) same as that of the proton. *(Pre-Medical/Dental 1990)*

14. **A charge q moves in a regin, where electric field $\vec{E}$ and magneic field $\vec{B}$ both exist. Then the force on it is :**
    (A) $q(\vec{v} \times \vec{B})$ (B) $q\vec{E} + q(\vec{v} \times \vec{B})$
    (C) $q\vec{E} + q(\vec{B} \times \vec{v})$ (D) $q\vec{B} + q(\vec{E} \times \vec{v})$
    (C.B.S.E. 2002)

15. **In a certain region of space, electric field $\vec{E}$ and magnetic field $\vec{B}$ are perpendicular to each other and an electron enters in region perpendicular to the direction of $\vec{B}$ and $\vec{E}$ both and moves undeflected. Then velocity of electron is**
    (A) $\frac{|\vec{E}|}{|\vec{B}|}$ (B) $\frac{|\vec{B}|}{|\vec{E}|}$
    (C) $\vec{E} \times \vec{B}$ (D) $\vec{E} \cdot \vec{B}$ *(C.B.S.E. 2001)*

16. **Cyclotron is a device to**
    (A) measure charge. (B) measure voltage.
    (C) accelerate protons. (D) accelerate electrons.
    (A.F.M.C. 1997)

17. **The phenomenon of production of e.m.f. in a current carrying metallic strip on applying a magnetic field along a direction perpendicular to the direction of flow of current is known as**
(A) Half effect (B) Joule effect
(C) Seebeck effect (D) Peltier effect
(A.F.M.C. 1996)

PART II. THOUGHT-BASED QUESTIONS

18. **An electron is projected into a magnetic field with components of velocity parallel and normal to the field. The path followed by electron will be :**
(A) straight. (B) parabolic.
(C) helical. (D) elliptical.

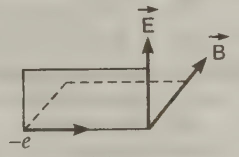
19. **An electron describes half a revolution in a circle of radius r inside a magnetic field of strength B. The energy acquired by the electron is :**
(A) $\frac{1}{2} m v^2$ (B) $m v^2$
(C) $\pi r \times B e v$ (D) zero.

20. **A proton and alpha particles are projected with the same kinetic energy at right angles to a uniform magnetic field. Then**
(A) the alpha-particle will move along a circular path of smaller radius than the proton.
(B) the alpha-particle will move along a circular path of greater radius than the proton.
(C) both will move along circular paths of same radius.
(D) both will follow a straight path.

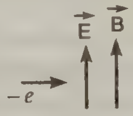
21. **After being accelerated through a potential difference of V (volt), an electron passes through a transverse magnetic field and experiences a force F. If the electron is accelerated through a potential difference of 2 V (in volt), then force on electron will be :**
(A) $\frac{F}{\sqrt{2}}$ (B) $\frac{F}{2}$
(C) $\sqrt{2}\,F$ (D) 2 F

22. **A charged particle passes through a region of space without change in its velocity. If E and B represent the electric and magnetic fields respectively, then for this region of space, which alternative may not be possible ?**
(A) $E = 0, B = 0$ (B) $E \neq 0, B \neq 0$
(C) $E = 0, B \neq 0$ (D) $E \neq 0, B = 0$

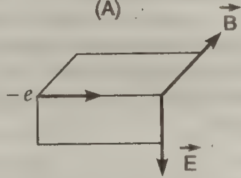
23. **An electron passes a region of space in which electric ($\vec{E}$) and magnetic field ($\vec{B}$) act over the same region. The electron may emerge undeflected, if the fields are oriented as shown :**



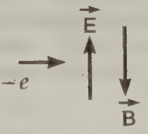
(A)



(B)



(C)



(D)

24. **An electron and a proton both having same linear momentum are projected at right angles to a uniform magnetic field. Then trajectory of**
(A) both will be equally curved.
(B) both will be straight.
(C) electron will be less curved than that of proton.
(D) proton will be less curved than that of electron.

25. **A particle of mass m and charge q moves with a consant velocity v along the positive X-direction. It enters a region containing a uniform magnetic field B directed along the negative Z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is**
(A) $\frac{q b B}{m}$ (B) $\frac{q (b - a) B}{m}$
(C) $\frac{q a B}{m}$ (D) $\frac{q (b + a) B}{2 m}$ *(I.I.T. 2002)*

Chapter 3. Magnetic dipole

PART I. TEXT-BASED QUESTIONS

1. **Which one of the following statement is correct ? The field strength of a bar magnet :**
(A) is same throughout the entire length of the bar magnet.
(B) is greater at the centre of the bar magnet.
(C) is the greatest at the poles.
(D) increases as the magnet ages.

2. **When a bar magent is immersed in a heap of iron filings and is then taken out, it is observed that iron filings stick to the surface of the bar magnet. A close exmination would reveal that iron filings**
(A) stick uniformly throughout the surface of bar magent.
(B) stick randomly on the surface of bar magnet.
(C) have maximum concentration at the mid portion of bar magnet.
(D) have maximum concentration at the two ends of the bar magent. *(C.E.T. 1998)*

3. **The magnetic field lines :**
(A) intersect at the neutral point.
(B) intersect near north and south poles.
(C) cannot intersect at all.
(D) depend upon the position of the magnet.
(M.N.R. 1987)

4. **A uniform magnetic field is obtained in :**
(A) a bar magnet.
(B) a horseshoe magnet.
(C) a circular coil carrying a current.
(D) a cylindrical coil carrying a current.
(Karnataka Entrance, 1992)

5. **The strength of magnetic field at a distance r (in metre) from an isolated magnetic pole of strength m (in SI units) in air is :**
(A) $\frac{\mu_0}{4\pi} \cdot \frac{m}{r}$ (B) $\frac{\mu_0}{4\pi} \cdot \frac{m}{r^2}$
(C) $\frac{\mu_0}{4\pi} m r$ (D) $\frac{\mu_0}{4\pi} m r^2$ *(M.N.R. 1987)*

6. **The magnetic field due to a small magnetic dipole of magnetic moment M at a point on the axis of the dipole and at a distance r from it is given by**
(A) $\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$ (B) $\frac{\mu_0}{4\pi} \cdot \frac{M}{r^2}$
(C) $\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$ (D) $\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^2}$ *(A.F.M.C. 1998)*

7. The magnetic field at a distance d from a short bar magnet in longitudinal and transverse positions are in the ratio :
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 3 : 1 *(C.E.T. 1999)*

8. A bar magnet of magnetic moment $\vec{M}$ is placed in a magnetic field of induction $\vec{B}$. The torque exerted on it is
(A) $\vec{M} . \vec{B}$ (B) $-\vec{M} . \vec{B}$
(C) $\vec{M} \times \vec{B}$ (D) $-\vec{M} \times \vec{B}$ *(C.B.S.E. 1999)*

9. The work done in turning a magnet of magnetic moment M through an angle θ from the meridian is :
(A) $M B (1 - \cos \theta)$ (B) $M B \cos \theta$
(C) $m B$ (D) $M B \sin \theta$
(A.F.M.C. 1997 ; M.N.R. 1992)

10. The work done in turning a magnet of magnetic moment M by an angle of 90° from the meridian is n times the corresponding work done to turn it through an angle of 60°. The value of n is given by
(A) 2 (B) 1
(C) 0·5 (D) 0·25 *(C.B.S.E. 1995)*

PART II. THOUGHT-BASED QUESTIONS

11. A bar magnet of magnetic moment M is cut into two parts of equal lengths. The magnetic moment and pole strength of either part is :
(A) $\frac{M}{2}, \frac{m}{2}$ (B) $M, \frac{m}{2}$
(C) $\frac{M}{2}, m$ (D) M, m
(C.B.S.E. 1997 ; N.C.E.R.T. 1983)

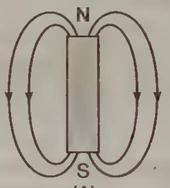
12. A bar magnet has been cut equally lengthwise and breadthwise to give four equal pieces. The pole strength of each piece as compared to the pole strength of original bar magnet is :
(A) same (B) 1/8
(C) 1/4 (D) 1/2

13. A large magnet is broken into two pieces, so that their lengths are in the ratio 2 : 1. The pole strengths of the two pieces will be :
(A) in the ratio 4 : 1 (B) in the ratio 2 : 1
(C) in the ratio 1 : 2 (D) equal.

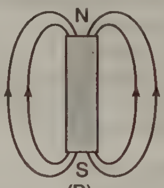
14. A steel wire of length L has a magnetic moment M. It is then bent into a semicircular arc. The new magnetic moment is :
(A) M (B) M/L
(C) $2 M/\pi$ (D) M L *(C.P.M.T. 1984)*

15. The direction of magnetic field lines of a bar magnet is
(A) from south pole to north pole.
(B) from north pole to south pole.
(C) across the bar magnet.
(D) from south pole to north pole inside the magnet and from north pole to south pole outside the magnet.
(A.F.M.C. 1995)

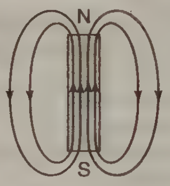
16. The magnetic field lines due to a bar magnet are correctly shown in *(I.I.T. 2002)*



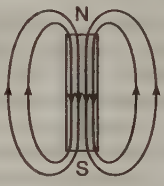
(A)



(B)



(C)



(D)

17. Two small magnets each of magnetic moment 10 A m^2 are placed in end-on position 0·1 m apart from their centres. The force acting between them is :
(A) $0{\cdot}6 \times 10^7$ N (B) $0{\cdot}06 \times 10^7$ N
(C) 0·6 N (D) 0·06 N *(M.N.R. 1994)*

18. A bar magnet is placed inside a non-uniform magnetic field. It experiences :
(A) a force and a torque.
(B) a force but not a torque.
(C) a torque but not a force.
(D) neither a force nor a torque. *(I.I.T. 1982)*

19. The absence of magnetic monopole is explained by :
(A) Coulomb's law. (B) Lorentz's law.
(C) Gauss's law. (D) Fleming's law.
(Karnataka Entrance, 1991)

Chapter 4. Earth's magnetism

PART I. TEXT-BASED QUESTIONS

1. The line joining the points of horizontal component of earth's magnetic field is called :
(A) magnetic axis. (B) magnetic line.
(C) magnetic meridian. (D) magnetic equator.
(M.N.R. 1987)

2. The angle between the earth's magnetic axis and earth's geographic axis is :
(A) 0° (B) 17°
(C) 23° (D) none of the above.
(M.N.R. 1977)

3. Earth's magnetic field always has a horizontal component except at :
(A) equator (B) poles
(C) both at equator and poles
(D) none of the above.

4. Which of the following relation is correct in magnetism ?
(A) $B = \sqrt{B_V^2 + B_H^2}$ (B) $B = B_V + B_H$
(C) $B_V = \sqrt{B^2 + B_H^2}$ (D) $B_H = \sqrt{B^2 + B_V^2}$
(A.F.M.C. 1996)

5. At magnetic poles, the angle of dip is :
(A) 45° (B) 30°
(C) 0° (D) 90° *(C.P.M.T. 1991)*

6. At a certain place, the angle of dip is 30° and the horizontal component of earth's magnetic field is 0·50 oersted. The earth's total magnetic field is :
(A) $\sqrt{3}$ (B) 1
(C) $1/\sqrt{3}$ (D) 1/2 *(C.P.M.T. 1990)*

7. The angle of dip at the magnetic equator is :
(A) 0° (B) 45°
(C) 90° (D) 30°

8. The angle of dip is θ at a place, where the horizontal and vertical components of earth's magnetic fields are equal. The value of θ is :
(A) 0° (B) 30°
(C) 45° (D) 90°

9. At a place, where $B_H = 0{\cdot}32$ oersted ; a tangent galvanometer with a coil of 20 turns carrying a current of 0·64 ampere produces a deflection of 45°, provided the radius of the coil in cm is ($\pi = 22/7$) :
(A) 25 (B) 37·5
(C) 2·25 (D) 1·75
(Karnataka Entrance, 1987)

10. The sensitivity of a tangent galvanometer is increased, if
(A) number of turns decreases.
(B) field increases.
(C) number of turns increases.
(D) field decreases. *(A.F.M.C. 1995)*

11. The period of oscillation of a magnet in a vibration magnetometer is 2 s. The period of oscillation of a magnet of same size but having magnetic moment four times will be :
(A) 1 s (B) 4 s
(C) 8 s (D) 0·05 s *(C.P.M.T. 1990)*

12. A magnet suspended in earth's field can vibrate in horizontal plane. A second magnet is so placed that the field becomes less than the horizontal component of earth's field. In this situation, the time period will :
(A) increase (B) decrease
(C) remain unchanged (D) cannot be predicted.

13. The time period of oscillation of a freely suspended magnet does not depend upon :
(A) the length of the magnet.
(B) the pole strength of the magnet.
(C) the horizontal component of the earth's magnetic field.
(D) the length and nature of the material of the suspension.
(C.P.M.T. 1980)

14. Two bar magnets having same geometry with magnetic moments M and 2 M are firstly placed in such a way that their similar poles are on the same side ; then its period of oscillation is T_1. Now the polarity of one of the magnets is reversed. The time period of oscillations will be :
(A) $T_1 < T_2$ (B) $T_1 > T_2$
(C) $T_1 = T_2$ (D) $T_2 = \infty$ (C.B.S.E. 2002)

PART II. THOUGHT-BASED QUESTIONS

15. Isogonic lines on magnetic map represent lines of :
(A) zero declination. (B) equal declination.
(C) equal dip. (D) equal horizontal field.
(C.P.M.T. 1992)

16. Due to earth's magnetic field, the charged cosmic rays particles
(A) require greater kinetic energy to reach the equator than pole.
(B) require less kinetic energy to reach the equator than pole.
(C) can never reach the pole.
(D) can never reach the equator. *(C.B.S.E. 1997)*

17. A circular loop is carrying current and is said to be equivalent to a magnetic dipole. Then, a point on the axis of the loop lies in its
(A) end-on position.
(B) broad side-on position.
(C) both end-on and broad side-on positions.
(D) none of the above.

18. A circular loop is carrying current and is said to be equivalent to a magnetic dipole. Then, a point on the loop lies in its
(A) end-on position.
(B) broad side-on position.
(C) both end-on and broad-side on positions.
(D) none of the above

19. A dip circle is at right angle to the magnetic meridian. What will be its apparent dip ?
(A) 0° (B) 30°
(C) 60° (D) 90° *(A.F.M.C. 1995)*

20. If a magnet is suspended at an angle 30° to the magnetic meridian, the dip needle makes an angle of 45° with the horizontal. The actual angle of dip is :
(A) $\tan^{-1}\frac{3}{2}$ (B) $\tan^{-1}\sqrt{3}$
(C) $\tan^{-1}\frac{2}{\sqrt{3}}$ (D) $\tan^{-1}\frac{\sqrt{3}}{2}$

21. A magnet is placed vertically on a paper. Then, the neutral point obtained on the paper is :
(A) one only (B) two only
(C) three only (D) zero. *(C.P.M.T. 1985)*

22. A magnet was found to vibrate at a place with a period of T. A piece of brass of same length, breadth and mass was placed over the magnet. The new time period will be :
(A) 2 T (B) $T/\sqrt{2}$
(C) T/2 (D) $\sqrt{2}$ T

23. A bar magnet is oscillating in earth's magnetic field with a period T. What happens to its period and motion, if its mass is quadrupled ?
(A) T/2 (B) 2 T
(C) 4 T (D) unchanged.
(Pre-Medical/Dental ,1994)

24. A thin rectangular magnet suspended freely has a period of oscillation of 4 s. If it is broken into two halves (each having half the original length) and one of the pieces is suspended similarly, the period of its oscillation will be :
(A) 4 s (B) 2 s
(C) 0·5 s (D) 0·25 s *(N.C.E.R.T. 1984)*

Chapter 5. Classification of Magnetic materials

PART I. TEXT-BASED QUESTIONS

1. Tesla is the unit of
(A) electric flux (B) magnetic flux
(C) electric field (D) magnetic induction
(C.B.S.E. 1997)

2. Chose the correct answer :
(A) All electrons possess magnetic moment.
(B) All protons possess magnetic moment.
(C) All nuclei possess magnetic moment.
(D) All atoms possess magnetic moment.

3. Which of the following is ferromagnetic ?
(A) Aluminium (B) Quartz
(C) Nickel (D) Bismuth
(Karnataka Entrance, 1993)

4. An example of a ferromagnetic substance is :
(A) aluminium (B) nickel
(C) gold (D) copper *(Karnataka, 1993)*

5. Which of the following is paramagnetic ?
(A) Antimony (B) Silver
(C) Marble (D) Alnico

6. If the permanent magnetic moment of the atoms of a substance is zero, then substance is called :

(A) diamagnetic (B) paramagnetic
(C) ferromagnetic (D) antiferromagnetic
(C.P.M.T. 1993)

7. **In which type of material the mgnetic susceptibility does nto depend on temperature ?**
(A) Diamagnetic (B) Paramagnetic
(C) Ferromagnetic (D) Ferrite *(C.B.S.E. 2001)*

Here, μ_B is Bohr magneton.

8. **The susceptibility of a diamagnetic substance :**
(A) decreases with temperature.
(B) does not vary with temperature.
(C) first decreases and then increases with increase of temperature.
(D) increases with temperature.
(Karnataka Entrance, 1994)

9. **The magnetic permeability is maximum for :**
(A) paramagnetic (B) ferromagnetic
(C) diamagnetic (D) non-magnetic.

10. **Susceptibility is positive and small for a :**
(A) paramagnetic (B) ferromagnetic
(C) non-magnetic (D) diamagnetic.
(C.P.M.T. 1990)

11. **Susceptibility is positive and large for a**
(A) paramagnetic (B) ferromagnetic
(C) non-magnetic (D) diamagnetic.
(C.P.M.T. 1990, 1982)

12. **If a diamagnetic substance is brought near north or south pole of a bar magnet, it is**
(A) attracted by the poles.
(B) repelled by the poles.
(C) repelled by the north pole and attracted by the south pole.
(D) attracted by the north pole and repelled by the south pole. *(C.B.S.E 1999)*

13. **The relative permeability of a diamagnetic material is**
(A) zero. (B) equal to unity.
(C) less than unity. (D) more than unity.
(A.F.M.C. 1998)

14. **A diamagnetic solution is poured into a U- tube and one arm of this U-tube is placed between the poles of a strong magnet with the meniscus in a line with the field. Then the level of the solution will :**
(A) rise (B) fall
(C) oscillate slowly (D) remain as such.
(M.N.R. 1993)

15. **The meniscus of a liquid contained in one of the limbs of a narrow U-tube is held in an electromagnet with the meniscus in line with the field. The liquid is seen to rise. This indicates that the liquid is :**
(A) ferromagnetic (B) paramagnetic
(C) diamagnetic (D) non-magnetic.
(M.N.R. 1992)

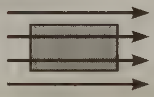
16. **Choose the correct answer :**
(A) Diamagnetism occurs in all materials.
(B) Diamagnetism is the result of partial alignment of permanent magnetic moment.
(C) The magnetic field due to induced magneitc moment is opposite to the applied field.
(D) The magnetising field intensity is always zero in free space.

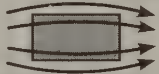
17. **Which of following is not a unit of intensity of magnetisation ?**
(A) $A\ m^{-1}$ (B) $J\ T^{-1}\ m^{-3}$
(C) $N\ T^{-1}\ m^{-2}$ (D) $A\ T^{-1}\ m^{-2}$

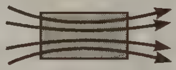
18. **Which of the following is not unit of magnetic induction ?**
(A) T (B) $Wb\ m^{-2}$
(C) $J\ A^{-1}\ m^{-2}$ (D) $N\ A^{-1}\ m^{-1}$

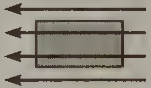
19. **Which of the following relation is not correct ?**
(A) $B = \mu_0 (I + H)$ (B) $B = \mu_0 (1 + \chi_m)H$
(C) $I = \dfrac{B - \mu_0 H}{\mu_0}$ (D) $\chi_m = \mu - 1$

20. **A uniform magnetic field (parallel to the plane of the paper) existed in space initially directed from left to right. When a bar of soft iron is placed in the field parallel to it, the lines of force passing through it, will be represented by :**


(A)


(B)


(C)


(D)

(C.P.M.T. 1988)

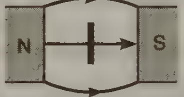
21. **At currie point, a ferromagnetic material becomes :**
(A) non-magnetic (B) diamagnetic
(C) paramagnetic (D) strongly ferromagnetic.
(N.C.E.R.T. 1990)

22. **Above curie temperature, ferromagnetic specimen :**
(A) remains ferromagnetic.
(B) becomes ferromagnetic.
(C) becomes paramagnetic.
(D) becomes diamagnetic. *(Karnataka, 1995, 1992)*

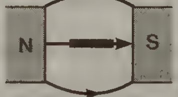
23. **Which of the following relation is not correct ?**
(A) $\chi_m = \dfrac{I}{H}$ (B) $\mu_0 = \mu_0 (1 + \chi_m)\ H$
(C) $\mu_0 = \mu\ (1 + \chi_m)$ (D) $\mu_r = 1 + \chi_m$

24. **For a paramagnetic material, the dependence of the magnetic susceptibility χ_m on the absolute temperature T is given by :**
(A) $\chi_m \propto T$ (B) $\chi_m \propto \dfrac{1}{T}$
(C) $\chi_m = e^{kT}$ (D) χ_m = constant

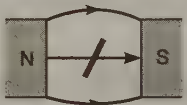
25. **A rod of a paramagnetic substance, such as platinum is placed in a non-uniform strong magnetic field. How will it align itself in the field ?**



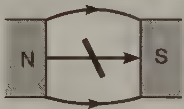
(A)



(B)



(C)



(D)

26. **A rod of a diamagnetic substance, such as bismuth is placed in a non-uniform magnetic field. How will it align itself in the field ? Refer the four orientations as shown in last question.**

27. **The hysteresis curve of a magnetic material is generally studied for :**
(A) diamagnetic materials.
(B) ferromagnetic materials.
(C) paramagnetic materials.
(D) dielectric materials. *(Karnataka Entrance, 1991)*

28. Which of the following is most suitable for the core of the electromagnets ?
(A) Air (B) Soft iron
(C) Steel (D) Cu – Ni alloy
(A.I.I.M.S. 1980)

29. Electromagnets are made of soft iron, because soft iron has :
(A) low susceptibility and low retentivity.
(B) high susceptibility and low retentivity.
(C) high susceptibility and high retentivity.
(D) low permeability and high retentivity. *(M.N.R. 1988)*

30. An electromagnet uses :
(A) soft iron core (B) steel core
(C) nickel core (D) copper core *(A.F.M.C. 1988)*

31. Soft iron is used to manufacture electromagnets, because their :
(A) magnetic saturation limit is high and retentivity and coercivity are small.
(B) retentivity is high. (C) coercivity is high.
(D) area of hysteresis loop is large. *(N.C.E.R.T. 1980)*

32. Soft iron is used in many parts of electrical machines for :
(A) low hysteresis loss and low permeability.
(B) low hysteresis loss and high permeability.
(C) high hysteresis loss and low permeability.
(D) high hysteresis loss and high permeability.

33. A magnetic needle suspended by a silk thread is vibrating in the earth's magnetic field. If the temperature of the needle is increased by 500 °C, then :
(A) the time period decreases.
(B) the time period remains unchanged.
(C) the time period increases.
(D) the needle stops vibrating. *(M.N.R. 1994)*

PART II. THOUGHT-BASED QUESTIONS

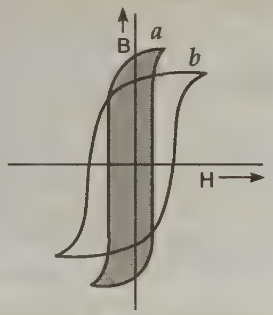
34. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length 2 R. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is
(A) $\frac{q}{2m}$ (B) $\frac{q}{m}$
(C) $\frac{2q}{m}$ (D) $\frac{q}{\pi m}$ *(I.I.T. 1998)*

35. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on
(A) ω and q (B) ω, q and m
(C) q and m (D) ω and m *(I.I.T. 2000)*

36. The magnetic moment of atomic neon is equal to :
(A) zero (B) μ_B
(C) $\mu_B/2$ (D) $3\,\mu_B/2$ *(N.C.E.R.T. 1984)*
Here, μ_B is Bohr magneton.

37. The hysteresis cycle for the material of permanent magnet is :
(A) short and wide. (B) tall and narrow.
(C) tall and wide. (D) short and narrow.

38. The B-H curves (*a*) and (*b*) in the figure are associated with
(A) a diamagnetic and paramagnetic substance respectively.
(B) a paramagnetic and a ferromagnetic substance respectively.
(C) soft iron and steel respectively.
(D) steel and soft iron respectively.
(C.P.M.T. 1989)



39. The area of the B-H hysteresis loop is an indication of :
(A) the permeability of the medium.
(B) the susceptibility of the substance.
(C) the retentivity of the material.
(D) the energy dissipated per unit volume of the substance per cycle. *(Karnataka Entrance, 1994, 1993)*

40. Area contained within the hysteresis loop represents :
(A) magnetic energy of the specimen.
(B) loss of energy of the specimen.
(C) loss of energy per unit volume per cycle.
(D) none of the above. *(Karnataka, 1995)*

41. The material of a permanent magnet has :
(A) high retentivity, low coercivity.
(B) low retentivity, high coercivity.
(C) low retentivity, low coercivity.
(D) high retentivity, high coercivity.
(Karnataka Entrance, 1994, 1988)

42. Ferromagnetic substance used in a transformer must have :
(A) high susceptibility and high hysteresis loss.
(B) high permeability and low hysteresis loss.
(C) low permeability and high hysteresis loss.
(D) low permeability and low hysteresis loss.
(Karnataka, 1992 S)

43. For protecting a sensitive equipment from external magnetic field, it should be :
(A) placed inside an iron can.
(B) wrapped with insulated wire, through which current is passed.
(C) surrounded with fine copper gauge.
(D) placed inside an aluminium can.
(C.B.S.E. 1998 ; Karnataka Entrance, 1992)

44. In cassette tapes, the sound is recorded in the form of :
(A) variable electrical resistance on the tape.
(B) sound wave on the tape.
(C) electrical energy.
(D) magnetic field on the tape.
(Karnataka Entrance, 1989)

ANSWERS TO MULTIPLE CHOICE QUESTIONS

Chapter 1. Magnetic Effect of Current

1. (A)	2. (C & D)	3. (A)	4. (C)	5. (B)	6. (D)	7. (A)	8. (B)	9. (B)	10. (B)
11. (D)	12. (D)	13. (D)	14. (C)	15. (A)	16. (D)	17. (B)	18. (B)	19. (D)	20. (A)
21. (B)	22. (C)	23. (A)	24. (A)	25. (D)	26. (A)	27. (D)	28. (D)	29. (D)	30. (A)
31. (A)	32. (A)	33. (D)	34. (A)	35. (B)	36. (C)	37. (B)	38. (C)	39. (B)	40. (D)
41. (B)	42. (C)	43. (D)	44. (B)	45. (B)	46. (C)	47. (C)	48. (D)	49. (A)	50. (B)

51. (D) **52.** (D) **53.** (B) **54.** (B) **55.** (C) **56.** (B) **57.** (B) **58.** (A) **59.** (B) **60.** (D)
61. (D) **62.** (B) **63.** (A) **64.** (A) **65.** (A) **66.** (C)

Chapter 2. Motion of Charged Particle

1. (C) **2.** (A) **3.** (C) **4.** (C) **5.** (C) **6.** (A) **7.** (B) **8.** (D) **9.** (C) **10.** (B)
11. (C) **12.** (D) **13.** (B) **14.** (B) **15.** (A) **16.** (C) **17.** (A) **18.** (C) **19.** (D) **20.** (C)
21. (C). **22.** (A & C) **23.** (C) **24.** (A) **25.** (B)

Chapter 3. Magnetic dipole

1. (C) **2.** (D) **3.** (C) **4.** (B & D) **5.** (B) **6.** (C) **7.** (C) **8.** (C) **9.** (A) **10.** (A)
11. (C) **12.** (D) **13.** (D) **14.** (C) **15.** (D) **16.** (C) **17.** (C) **18.** (A) **19.** (C)

Chapter 4. Earth's Magnetism

1. (D) **2.** (C) **3.** (B) **4.** (A) **5.** (D) **6.** (C) **7.** (A) **8.** (C) **9.** (A) **10.** (C)
11. (A) **12.** (A) **13.** (D) **14.** (A) **15.** (B) **16.** (A) **17.** (A) **18.** (B) **19.** (D) **20.** (D)
21. (A) **22.** (D) **23.** (B) **24.** (B)

Chapter 5. Classification of Magnetic materials

1. (D) **2.** (A & B) **3.** (C) **4.** (B) **5.** (A) **6.** (A) **7.** (A) **8.** (B) **9.** (B) **10.** (A)
11. (B) **12.** (B) **13.** (C) **14.** (B) **15.** (B) **16.** (A & C) **17.** (D) **18.** (D) **19.** (D) **20.** (C)
21. (C) **22.** (C) **23.** (C) **24.** (B) **25.** (B) **26.** (A) **27.** (B) **28.** (B) **29.** (C) **30.** (A)
31. (B) **32.** (B) **33.** (C) **34.** (A) **35.** (C) **36.** (A) **37.** (C) **38.** (C) **39.** (D) **40.** (C)
41. (D) **42.** (B) **43.** (A) **44.** (D)

UNIT
IV

ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

1. **Electromagnetic Induction**
2. **Transient Current**
3. **Alternating Current**
4. **Electrical Machines & Devices**

> "Perhaps all magnetic phenomena, including the magnetism of magnets is only a manifestation of charge in motion."
>
> **– Andre-Marier Ampere**

unit

VI

[illegible]

CHAPTER
1

Electromagnetic Induction

1.01. ELECTROMAGNETIC INDUCTION

As said earlier ; in 1820, Oersted discovered the magnetic effect of electric current *i.e.* when a steady current flows through a conductor, a magnetic field is produced around it. In 1831, Michael Faraday discovered the effect, called ***electromagnetic induction,*** just converse to the magnetic effect of electric current.

When a coil made of copper wire is placed inside a magnetic field, magnetic flux is linked with the coil. Faraday found that when the magnetic flux linked with the coil is changed*, an electric current starts flowing in the coil, provided the coil is a closed one. In case the coil is open, an e.m.f. is set up across the two ends of the coil. The current and the e.m.f. so produced are called ***induced current*** and ***induced e.m.f.*** respectively. The induced current and the e.m.f. in the coil last only so long as the magnetic flux linked with the coil keeps on changing.

Thus, *electromagnetic induction is the phenomenon of production of electric current (or e.m.f.) in a coil, when the magnetic flux linked with the coil is changed.*

1.02. MAGNETIC FLUX

The magnetic flux linked with a surface held in a magnetic field is defined as the number of magnetic field lines crossing the surface normally and is measured as the product of component of the magnetic field normal to the surface and the surface area.

It is a scalar quantity and is denoted by ϕ.

Mathematically, magnetic flux linked with a surface area ΔS held inside a magnetic field B is given by

$$\phi = B_n \Delta S, \quad ...(1.01)$$

where B_n is component of magnetic field B along normal to the surface area ΔS.

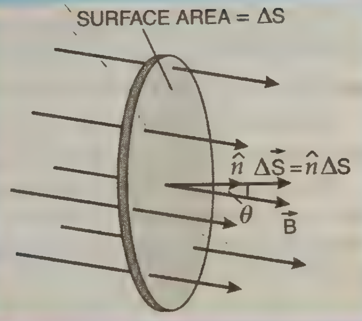


Fig. 1.01

Consider that a surface area ΔS is held inside the magnetic field $\vec{B}$, such that direction of magnetic field makes an angle θ with normal to the surface area ΔS [Fig. 1.01]. Thus, component of magnetic field along normal to the surface area,

$$B_n = B\cos\theta$$

Hence, magnetic flux linked with the surface area is given by

$$\phi = B_n \Delta S = (B\cos\theta)\,\Delta S$$

or

$$\phi = B\,\Delta S\cos\theta \quad ...(1.02)$$

If $\Delta\vec{S}$ is the area vector representing the surface area ΔS, then equation (1.01) or (1.02) can be expressed as

$$\phi = \vec{B}\cdot\Delta\vec{S} \quad ...(1.03)$$

The direction of area vector $\Delta\vec{S}$ is always along the surface area it represents. If $\hat{n}$ is unit vector along normal to the surface area, then equation (1.03) may be written as

$$\phi = \vec{B}\cdot(\hat{n}\,\Delta S) \quad ...(1.04)$$

Here, $\vec{B}\cdot\hat{n} = B_n$ gives the component of magnetic field along normal to the surface area ΔS.

Let us find the magnetic flux linked with a surface area, when held inside the magnetic field in the following cases :

1. When magnetic field is perpendicular to the surface area. Fig. 1.02 shows a surface area ΔS held inside the magnetic field B, such that the direction of magnetic field is perpendicular to the surface area. Since $\vec{B}$ and $\Delta\vec{S}$ are parallel, $\theta = 0°$.

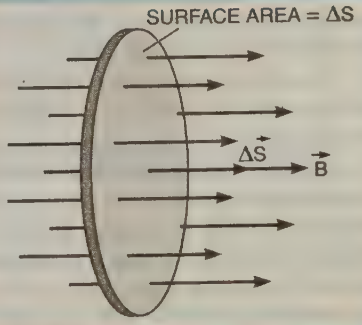


Fig. 1.02

---

* The change in magnetic flux linked with a coil may be caused by varying the strength of the magnetic field or by the relative motion between the source of magnetic flux and the coil.

From equation (1.02), we have

$$\phi = B\,\Delta S \cos 0° = B\,\Delta S$$

It follows that magnetic flux linked with a surface is maximum, when the direction of magnetic field is perpendicular to the surface area.

2. When magnetic field is parallel to the surface area. Fig. 1.03 shows a surface area ΔS held inside the magnetic field B, such that the direction of magnetic field is parallel to the surface area. Since $\vec{B}$ and $\vec{\Delta S}$ are perpendicular, $\theta = 90°$.

From equation (1.02), we have

$$\phi = B\,\Delta S \cos 90° = 0$$

Unit of magnetic flux. In SI, the unit of magnetic flux is **weber (Wb).**

$$\mathbf{1\ weber\ (Wb) = 1\ tesla\ (T) \times 1\ m^2}$$

One weber is the magnetic flux linked with a surface area of 1 metre^2, when held normally inside a uniform magnetic field of 1 tesla.

In cgs system, the unit of magnetic flux is **maxwell (Mx).**

$$\mathbf{1\ maxwell\ (Mx) = 1\ gauss\ (G) \times 1\ cm^2}$$

One maxwell is the magnetic flux linked with a surface area of 1 cm^2, when held normally inside a uniform magnetic field of 1 gauss.

$$1\ \text{Wb} = 1\ \text{T} \times 1\ \text{m}^2 = (10^4\ \text{G}) \times 10^4\ \text{cm}^2$$

or

$$\mathbf{1\ Wb = 10^8\ maxwell}$$

Dimensional formula of magnetic flux. From equation (1.02), we have

$$\phi = B\,\Delta S \cos\theta = \frac{F}{q\,v} \times \Delta S \cos\theta \qquad (\because F = B\,q\,v)$$

Therefore, dimensional formula of ϕ,

$$[\phi] = \left[\frac{F}{q\,v}\,\Delta S\cos\theta\right] = \frac{[M\,L\,T^{-2}]}{[A\,T][L\,T^{-1}]} \times [L^2]$$

or

$$[\phi] = [M\,L^2\,T^{-2}\,A^{-1}]$$

Note. Here, A stands for ampere and [A T] is dimensional formula of electric charge (coulomb).

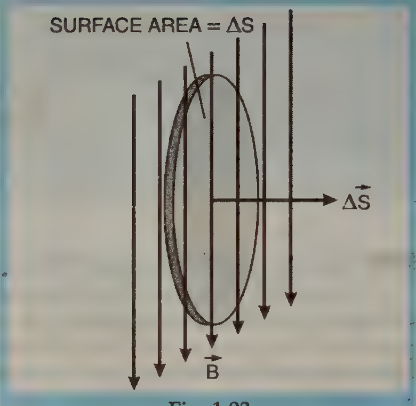


Fig. 1.03

Key point

When a surface is placed in magnetic field, the magnetic flux linked with it is maximum, when it is oriented perpendicular to the direction of magnetic field and becomes zero, when it is parallel to the direction of magnetic field.

1.03. FARADAY'S EXPERIMENT

The following experiments performed by Faraday led to the discovery of the phenomenon of electromagnetic induction :

Experiment 1. When the strength of magnetic field is varied. Consider two coils P and S wound on an iron rod. A battery and a tapping key K are connected to the coil P, while a sensitive galvanometer is connected to the coil S [Fig. 1.04]. Faraday observed that deflection is produced in the galvanometer, when the tapping key K is pressed. When the tapping key is released, deflection is again produced but the direction of deflection is opposite. The galvanometer does not show any deflection, when the tapping key is kept pressed.

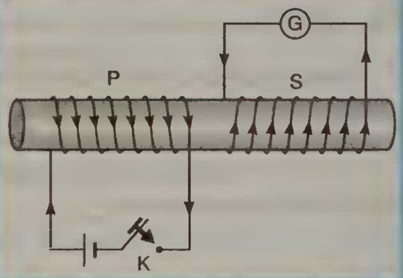


Fig. 1.04

Explanation. When current flows through the coil P, magnetic field is produced along the axis of the coil. Since the coil S is wound on the same rod, magnetic flux gets linked with this coil also.

When the tapping key is just pressed, the current through coil P starts growing and it leads to a growing magnetic field through the coil S. As the magnetic flux linked with the coil S is changing, induced current is produced. On the other hand, when the key is released, the current through the coil P starts decaying. It results in decaying magnetic field through the coil S *i.e.* the magnetic flux linked with the coil S again starts changing. Due to this, induced current is produced in the coil. Since the induced current is produced due to decaying magnetic field (earlier, the magnetic field was growing), the flow of current and hence the deflection in the galvanometer is in the opposite direction.

When the tapping key is kept pressed, the current in coil P becomes steady and hence magnetic flux linked with the coil S also becomes steady. Since induced current is produced only when the magnetic flux linked with the coil changes, the galvanometer does not show any deflection when the tapping key is kept pressed.

Experiment 2. When the source of magnetic field is moved towards or away rom a stationary coil. Consider a coil P wound on an iron rod and connected to battery. A steady current flowing through the coil will produce a steady magnetic eld along its axis. Another coil S has a galvanometer in its circuit [Fig. 1.05]. araday observed that deflection is produced in the galvanometer, when the coil is moved towards the coil S. If the coil P is moved away, the galvanometer shows deflection again, but this time, the deflection is in opposite direction. The eflection is produced in the galvanometer, only when the coil P is moved towards r away from the coil S.

Explanation. When a steady current is passed through the coil P wound on an on rod, the iron rod becomes an electromagnet. Since magnetic field lines crowd ear the poles of the magnet, the magnetic flux linked with the coil S grows when e coil P is moved towards the coil S ; and decays when it is moved away. Since ue to motion of the coil P towards or away from the coil S, the magnetic flux linked ith coil S changes, the galvanometer shows deflection due to the flow of induced urrent in the coil S. In case the coil P is kept stationary, the magnetic flux linked ith the coil S remains same and hence galvanometer does not show any deflection.

The same effects will be observed, if the electromagnet (the current carrying il P wound on the iron rod) is replaced by a bar magnet NS [Fig. 1.06].

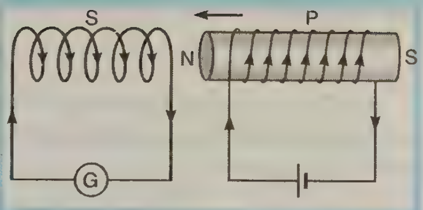


Fig. 1.05

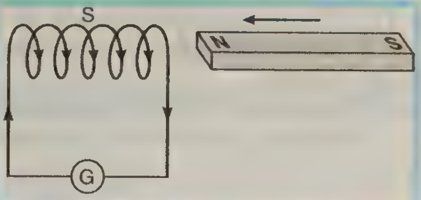


Fig. 1.06

Key point

1. Induced e.m.f. is produced in a coil, when there is relative motion between magnet and the coil.

2. No induced e.m.f. is produced, when magnet is rotated about its own axis. However, induced e.m.f. is produced, if magnet is rotated about an axis perpendicular to its length.

Experiment 3. When a coil is moved in a stationary magnetic field. Consider at a coil ABCD having a galvanometer G in its circuit is placed inside a uniform agnetic field, which is confined to the region PQRS only. The cross-marks dicate that the magnetic field is directed perpendicularly *into* the plane of the aper. When the coil is moved so that it remains entirely inside the magnetic field ig. 1.07], the galvanometer does not show any deflection. On the other hand, hen the coil is pulled out of the magnetic field (moved so that a part of the coil inside the magnetic field and a part outside) as shown in Fig. 1.08, the galvano-eter shows deflection.

Explanation. When the coil is moved entirely inside the magnetic field [Fig. 07], the magnetic flux linked with its area ABCD does not change. Due to this, no duced current is produced in the coil and the galvanometer does not show any eflection.

However, when the coil is moved as shown in Fig. 1.08, the area of the coil inside e magnetic field goes on decreasing *i.e.* the magnetic flux linked with the coil goes decreasing. As a result, the induced current flows through the coil and the lvanometer shows deflection. The e.m.f. so produced in the coil is called *motional n.f.* It is a consequence of Lorentz magnetic force on the electrons inside the wire, which the coil is made of. However, the expression for motional e.m.f. can be tained both by finding Lorentz magnetic force on the electrons inside the wire rming the coil and by applying Faraday's law of electromagnetic induction.

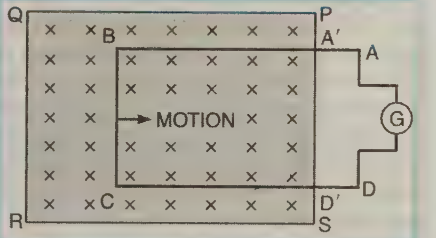


Fig. 1.07

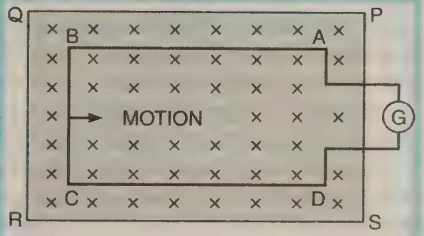


Fig. 1.08

1.04. FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

The results of Faraday's experiments on electromagnetic induction are known *Faraday's laws of electromagnetic induction.* These laws are stated as below :

1. *Whenever magnetic flux linked with a circuit (a loop of wire or a coil or an electric circuit in general) changes, induced e.m.f. is produced.*
2. *The induced e.m.f. lasts as long as the change in the magnetic flux continues.*
3. *The magnitude of the induced e.m.f. is directly proportional to the rate of change of the magnetic flux linked with the circuit.*

Let ϕ_1 and ϕ_2 be the values of magnetic flux linked with the coil initially $t = 0$) and at time t respectively. Then,

$$\text{rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t}$$

If e is the induced e.m.f. produced, then

$$e \propto \frac{\phi_2 - \phi_1}{t}$$

or

$$e = -k\,\frac{\phi_2 - \phi_1}{t}$$

Here, k is the constant of proportionality and the negative sign indicates that e induced e.m.f. has got opposing nature *i.e.* direction of flow of current due to the induced e.m.f. is such that it opposes the change in magnetic flux. In SI, the constant of proportionality* is 1. Hence,

$$e = -\frac{\phi_2 - \phi_1}{t} \qquad \text{...(1.05)}$$

In SI, e is measured in **volt,** ϕ in **weber** and t in **second.**

If $d\phi$ is small change in magnetic flux in a small time dt, then the equation (1.05) may be expressed as

$$e = -\frac{d\phi}{dt} \qquad \text{...(1.06)}$$

The equation (1.05) or (1.06) can be used to find the induced e.m.f. produced in a coil due to change in magnetic flux linked with the coil.

Key point

The negative sign in equation (1.06) i accounted for by Lenz's law.

If the coil consists of N turns, then magnetic flux ϕ is linked with each turn of the coil. Hence,

the total magnetic flux linked with the coil = N ϕ

Therefore, induced e.m.f. produced in the coil,

$$e = -\frac{d}{dt}(\mathrm{N}\,\phi)$$

or

$$e = -\mathrm{N}\frac{d\phi}{dt} \qquad \text{...(1.07)}$$

The equation (1.06) or (1.07) is commonly known as **Faraday-Lenz's law.**

1.05. LENZ'S RULE

Lenz's rule is a convenient method to determine the direction of induced current produced in a circuit. It is related to the principle of conservation of energy and it accounts for the negative sign appearing in equation (1.06) or (1.07).

Lenz's law states that *the induced current produced in a circuit always flows in such a direction that it opposes the change or the cause that produces it.*

Let us now apply Lenz's rule to find the direction of flow of induced current in the experiments described in section 1.03.

Experiment 1. Refer to Fig. 1.04. On pressing the key, the current in the coil P flows in clockwise direction, when seen from left and the magnetic field lines so produced will be directed from left to right. As the current grows through the coil P, the magnetic flux will also grow through the coil S. The direction of flow of induced current in coil S should be such that it opposes the growth of magnetic flux through it. The growing magnetic flux will be opposed, if the induced current in the coil S produces magnetic field lines from right to left. Consequently, the induced current in the coil S should flow in anticlockwise direction, when seen from left.

Experiment 2. Refer to Fig. 1.05 or 1.06. When the current carrying coil P wound on an iron rod, with its left end behaving as N-pole (or the bar magnet with its N-pole), is moved towards the coil S, the induced current produced in coil S should oppose the motion of coil P (or of the bar magnet). It will happen so; if the induced current in the coil S produces magnetic field lines from left to right *i.e.* if the induced current flows through the coil S in clockwise direction, when seen from left.

Similarly, when the coil P (or the bar magnet) is moved away from the coil S, the induced current will flow in anticlockwise direction, when seen from left.

Experiment 3. Refer to Fig. 1.08. When the coil ABCD is pulled towards right, the magnetic flux linked with the coil goes on decreasing. It is because of the reason that the area of the coil inside the magnetic field goes on decreasing. According to Lenz's law, the induced current should flow in such a direction, so that the motion of the coil is opposed *i.e.* on arm BC, force acts towards left. From Fleming's left hand rule, it follows that force on arm BC will act towards left, if the induced current in the arm BC flows from point C to B or in the direction DCBA in the coil.

1.06. LENZ'S LAW AND PRINCIPLE OF CONSERVATION OF ENERGY

Lenz's law is in accordance with the principle of conservation of energy. In electromagnetic induction, the electrical energy (in the form of induced current or induced e.m.f.) is produced at the expense of mechanical energy.

---

*The constant of proportionality depends upon the units in which e, ϕ and t are measured. In c.g.s. electromagnetic system, the constant of proportionality is $1/c$, where c is the speed of light in vacuum. In this system, e is measured in e.m.u. of e.m.f. ϕ in maxwell and t in second.

1 e.m.u. of e.m.f. = 10^8 V and 1 maxwell = 10^{-8} weber

Explanation. Consider a solenoid connected to an electric lamp (or a sensitive alvanometer) as shown in Fig. 1.09. Suppose that a magnet with its north pole facing he solenoid is moved towards the solenoid. The induced current is produced in he solenoid in clockwise direction, when seen from left. Due to the flow of current, he lamp glows and it radiates light energy. According to energy conservation rinciple, work equivalent to the electric energy (which makes the lamp glow) must e done by the agent in pushing the magnet towards the solenoid. In fact, it is exactly o. The induced current produced in the solenoid in clockwise direction (when seen rom left) produces magnetic field from left towards right. When the magnet (with s north pole towards the solenoid) is moved towards the solenoid, it experiences n opposing force due to the magnetic field and hence mechanical work has to be lone in moving the magnet. The work done in moving the magnet against this force esults in the form of electric energy (or induced e.m.f.) in the circuit. Hence, Lenz's aw is in accordance with the law of conservation of energy.

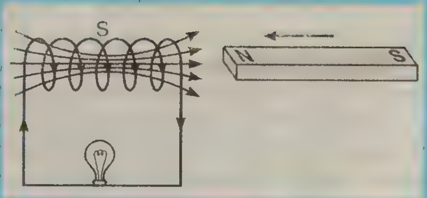


Fig. 1.09

It may be pointed out that when the magnet with its north pole facing the olenoid is moved towards the solenoid, the current cannot flow in the clockwise irection (when seen from right) *i.e.* in a direction opposite to that warranted by enz's law. It is because, if it was so, the magnetic field lines would be produced from ight to left and then the magnet would be attracted towards the solenoid and it vould gain kinetic energy. In other words, if the current were to flow in the direction pposite to that determined by Lenz's law, then the lamp would radiate energy and t the same time, the magnet would gain kinetic energy at the expense of nothing. uch a situation is against the principle of conservation of energy. Hence, the induced urrent cannot flow in a direction other than that determined by Lenz's law *i.e.* it ows in a direction so as to oppose the cause or the change that produced it.

1.07. EXPRESSION FOR MOTIONAL E.M.F.

Consider that a uniform magnetic field $\vec{B}$ is confined to the region PQRS and coil ABCD is placed inside the magnetic field. The direction of the magnetic field s perpendicular to the plane of the coil and in inward direction [Fig. 1.10]. When ne coil is moved with velocity $\vec{v}$ towards right, each electron inside the wire naking the coil) experiences magnetic Lorentz force

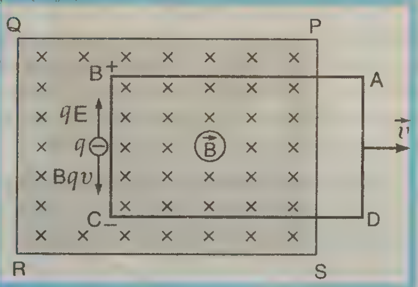


Fig. 1.10

$$\vec{f}_m = -q\,(\vec{v} \times \vec{B}), \qquad \text{...(1.08)}$$

here – q is the charge on an electron*. Since $\vec{v}$ and $\vec{B}$ are perpendicular to each other, ne magnitude of force on an electron is equal to

$$f_m = B\,q\,v$$

nd it acts from top to bottom in the plane of paper. The magnetic Lorentz force auses the electrons in the arm BC of the coil to move from its end B to C. As the lectrons accumulate at the end C, it will become negatively charged and the end will become positively charged. Due to this, an electric field is set up between the vo ends of wire BC in a direction from the end B to C. The electric field so produced xerts force on electrons in a direction from the end C to B and it opposes the agnetic Lorentz force. The force on an electron due to the electric field increases, s more and more electrons accumulate at the end C. A stage comes, when the lectrons which accumulate at the end C produce electric field just strong enough o that the force on an electron due to electric field becomes equal to magnetic orentz force on it. As the two forces are in opposite directions, they just cancel out nd there is no more motion of electrons. If E is strength of the electric field at that age, then force on electron due to electric field

$$f_e = q\,E$$

In equilibrium, $f_e = f_m$ or $q\,E = B\,q\,v$

or

$$E = B\,v \qquad \text{...(1.09)}$$

If e is the potential difference between the ends B and C and l is the length of ne arm BC, then

$$E = \frac{e}{l} \qquad \text{...(1.10)}$$

---

*Since induced e.m.f. has been denoted by e, for the sake of distinction, the charge on ectron has been denoted by $-q$ in this chapter.

From the equations (1.09) and (1.10), we have

$$\frac{e}{l} = \text{B}\, v$$

or $$e = \text{B}\, l\, v \qquad \text{...(1.11)}$$

It is the expression for ***motional e.m.f.*** produced in the coil ABCD due to its motion inside the magnetic field. The current in the coil flows from the positive end B to the negative end C along the path BADC.

Aliter. The expression for motional e.m.f. can also be obtained from Faraday's law of electromagnetic induction as explained below :

Consider that at any time t, the part BA′ = CD′ = x (say) of the coil is inside the magnetic field [Fig. 1.11]. If l is the length of the arm BC of the coil, then area of the coil inside the magnetic field at any time t,

$$\Delta\text{S} = \text{BC} \times \text{CD}' = l\, x$$

Therefore, magnetic flux linked with the coil at time t,

$$\phi = \text{B}\, \Delta\text{S} = \text{B}\, l\, x$$

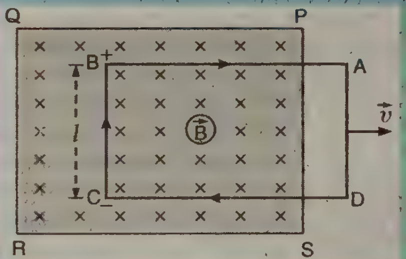


Fig. 1.11

Suppose that the coil is pulled out of the magnetic field with velocity v. As the coil is pulled out, the area of the coil inside the magnetic field and hence magnetic flux linked with the coil changes. The time rate of change of magnetic flux linked with the coil is given by

$$\frac{d\phi}{dt} = \frac{d}{dt}(\text{B}\, l\, x) = \text{B}\, l\, \frac{dx}{dt} = \text{B}\, l\, v$$

If e is the induced e.m.f. produced, then

$$e = -\frac{d\phi}{dt}$$

or $$e = -\text{B}\, l\, v$$

The negative sign shows that the induced e.m.f. opposes the coil from being pulled out of the magnetic field. It is the same expression as obtained from the concept of magnetic Lorentz force experienced by the electrons. The application of Lenz's law tells that as the coil is pulled out of the magnetic field, the induced current will flow in the coil, so as to exert force on the arm BC towards left. Fleming's left hand rule tells that the current due to induced e.m.f. will flow from the end C to B *i.e.* along the path DCBA in the coil.

Key point

No induced e.m.f is produced in a coil, when it moves by remaining wholly inside the magnetic field.

Note. Refer to Fig. 1.07. Let us investigate, why no current flows through the coil, when it remains wholly inside the magnetic field during its motion. When the coil ABCD is entirely inside the magnetic field the ends B and A of the coil become positive, while the ends C and D become negative. Thus, the potential difference between the ends B and C will be the same as that between the ends A and D. The situation will look like as if two cells of equal e.m.f. have been connected across the arms BC and AD as shown in Fig. 1.12. Obviously, no net e.m.f. exists in the coil and hence no current flows through the coil ABCD.

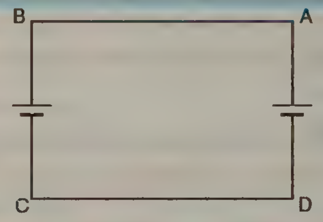


Fig. 1.12

1.08. EDDY CURRENTS

When magnetic flux linked with a coil changes, induced e.m.f. is produced in it and the induced current flows through the wire forming the coil.

Foçault found that when magnetic flux linked with a metallic conductor changes, induced currents are set up in the conductor in the form of closed loops. These currents look like *'eddies'* or *'whirl pools'* and likewise are known as *eddy currents*. **They are also known as *Focault's currents.***

Thus, *eddy currents are the currents induced in a conductor, when placed in a changing magnetic field.*

Following experiments explain the origin of eddy currents :

Experiment 1. Introduce a soft iron core inside a solenoid and connect it to a source of alternating e.m.f. Place a metallic disc over the cross-sectional face of the soft iron core as shown in Fig. 1.13. As the circuit is switched on, the metallic disc is thrown up into the air.

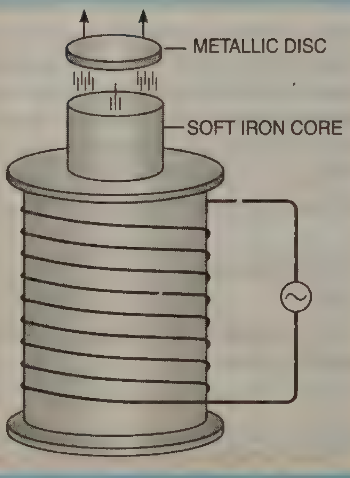


Fig. 1.13

Explanation. When the circuit is switched on, the current starts growing through the solenoid. As the current grows, the magnetic field lines along the axis of the solenoid and hence the magnetic flux through the disc also increases from zero to some finite value. Due to this, induced currents are produced in the disc and the disc is converted into a small magnet. If the upper face of the soft iron core acquires N-polarity initially, then the lower face of the disc also acquires N-polarity in accordance with Lenz's law. Due to repulsive force between the upper face of the soft iron core and the lower face of the disc, the latter is thrown up.

Experiment 2. Suspend a flat metallic plate between the two poles of an electromagnet [Fig. 1.14]. Initially, when the magnetic field is off, it is found that the metallic plate oscillates freely and its oscillatory motion can continue for a reasonably long time. However, when the magnetic field is switched on, the metallic plate comes to rest quite soon.

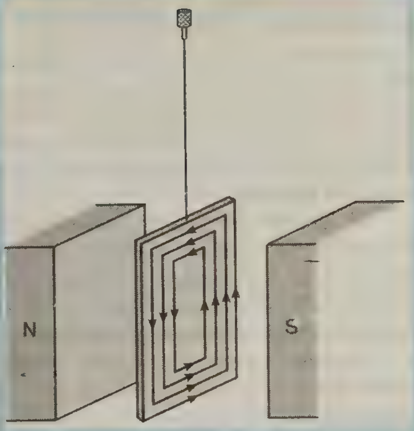


Fig. 1.14

Explanation. The damping effect on the oscillatory motion of the metallic plate can be explained by applying Lenz's law. When metallic plate is in its equilibrium position, the magnetic flux linked with the plate is maximum. As the plate starts moving towards its extreme position, its area between the two poles goes on decreasing and hence the magnetic flux linked with the plate also goes on decreasing. It results in the production of induced currents in the plate in the form of closed loops. According to Lenz's law, the induced currents oppose the plate from moving towards its extreme position. Similarly, when the metallic plate returns from its extreme position and starts entering the magnetic field, the induced currents are set up, which again oppose the motion of the metallic plate. In fact, the eddy currents produced are very large and the electric energy so produced is dissipated in the form of heat energy. The large value of eddy currents is due to the fact that the metallic plate has low resistance and even a small induced e.m.f. causes large eddy currents in it.

It may be pointed out that the production of eddy currents in the metallic plate oscillating inside the magnetic field produces damping effect. It is called *electromagnetic damping* and is made use of in a number of practical situations.

Experiment 3. Fig. 1.15 shows a metallic plate having narrow slots and suspended between the poles of a magnet. When such a metallic plate is made to oscillate, it is found that the metallic plate swings much more freely than in the case, when it did not have slots in it.

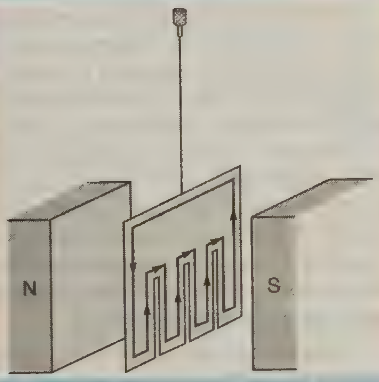


Fig. 1.15

Explanation. It is because, the magnitude of eddy currents produced in this case is much less. The reason is that for a closed loop of same area, the electrons have to cover a much larger path in this case. In traversing the path of larger length, the electrons experience more resistance ; with the result that the eddy currents produced are smaller. Therefore, it can be used as one of the methods to reduce eddy currents.

1.09. APPLICATIONS OF EDDY CURRENTS

Some of the important applications of eddy currents are as given below :

1. ***Dead beat galvanometers.*** The oscillations of a moving coil galvanometer generally take a long time to die out. But by winding its coil on a metallic frame made of copper or aluminium, the galvanometer can be made dead beat. It is because, due to the production of eddy currents in the metallic frame, the coil of the galvanometer comes to rest very soon.

It may be pointed out that the working of ballistic galvanometer requires that the damping should be minimum. It is achieved by winding the coil on a frame made of non-conducting material, such as bamboo or paper.

2. ***Electromagnetic damping.*** The oscillations of the coil of a galvanometer can also be stopped by short circuiting the two ends of the coil through a tapping key. When the key is pressed, the coil circuit is closed and the eddy currents are produced due to the motion of the coil in the magnetic field of the magnet of the galvanometer. Due to Lenz's law, the direction of eddy currents is such that it opposes the oscillations of the coil.

3. ***Induction motors.*** A rotating magnetic field is produced by means of two single phase currents. A metallic rotor placed inside the rotating magnetic field starts rotating due to large eddy currents produced in it. These motors are commonly used in fans.

4. ***Induction furnace.*** In an induction furnace, very high temperature can be produced by producing large eddy currents. The high frequency eddy currents are preferred because of the reason that the heating effect of current is proportional to the square of the frequency of the current.

5. ***Diathermy.*** Eddy currents have been used for deep heat treatment called *diathermy*.

6. *Speedometer.* In the speedometer of an automobile, a small magnet is geared to the main shaft of the vehicle and it rotates according to the speed of the vehicle. The magnet is mounted in an aluminium cylinder with the help of hair springs. When the magnet rotates, it produces eddy currents in the drum. Due to the effect of eddy currents, the drum tries to oppose the motion of the rotating magnet. In turn, the drum experiences a torque and gets deflected through certain angle. A pointer attached to the drum moves over a calibrated scale, which directly indicates the speed of the vehicle.

7. ***Energy meters.*** In energy meters, the armature coil carries a metallic aluminium disc which rotates between the poles of a pair of permanent horse shoe magnets. As the armature rotates, the current induced in the disc tends to oppose the motion of the armature coil. Due to this braking effect, the deflection is proportional to the energy consumed.

8. *Electric brake.* A metallic drum is coupled to the wheels of the train ; so that when the train runs, the drum also rotates. In order to stop the train, a strong magnetic field is applied to the rotating drum. The large eddy currents produced oppose the motion of the drum. Since the drum is connected to the wheels of the train, the latter comes to a halt.

These days, electric brakes are provided in the trains in addition to usual vacuum brakes.

Undesirable effects of eddy currents. Since the resistance of a metallic conductor is quite low, the magnitude of eddy currents produced is quite large. As such, considerable amount of heat is produced in the conductor due to Joule's heating effect. If the large eddy currents are allowed to produce in the core of a choke coil, transformer, dynamo, etc, it may produce undesirable effects.

The eddy currents produced in the core of a transformer are reduced by making use of a laminated core. The thick core of a transformer is formed by joining identical iron strips together, after insulating them from each other by coating them with varnish. Since the resistance of an iron strip will be much larger than the resistance of the thick core, only very feeble eddy currents are produced in the iron strips and the transformer is saved from getting heated.

1.10. SELF INDUCTION

Consider a coil connected to a battery and a tapping key [Fig. 1.16]. When the key K is pressed, the current starts growing through the coil and as a result, the magnetic field lines also start growing through it. This, in turn, produces induced e.m.f. in the coil. The direction of the induced e.m.f. is such that it opposes the growth of current in the circuit.

On the other hand, when the key is released, the current in the circuit starts decaying. As a result of this, the magnetic field lines linked with the coil also start decreasing. The e.m.f. is produced in the coil so that it opposes the decay of current in the circuit. Thus, during both growth and decay of current in the coil, an opposing induced e.m.f. is produced in the coil.

The phenomenon, according to which an opposing induced e.m.f. is produced in a coil as a result of change in current or magnetic flux linked with the coil, is called self-induction.

Since self-induction opposes both the growth (when the circuit is switched on) and decay (when the circuit is switched off) of the current in a coil, it is also called *back e.m.f.*

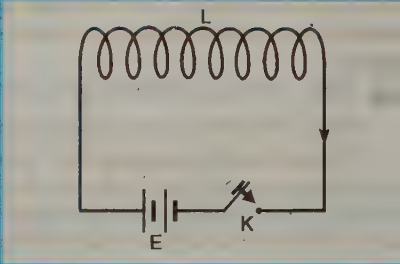


Fig. 1.16

Key point

The self-inductance of a straight conductor is zero.

1.11. COEFFICIENT OF SELF-INDUCTION

Suppose that after the key is pressed [Fig. 1.16], a current I flows through the coil at any instant. It is found that magnetic flux ϕ linked with the coil at any instant is directly proportional to the current passing through it at that instant *i.e.*

$$\phi \propto \mathrm{I}$$

or $$\phi = \mathrm{L\,I}, \qquad \text{...(1.12)}$$

where L is called *coefficient of self-induction* or simply *self-inductance* of the coil. It depends upon the number of turns, cross sectional area and permeability of the material of the core (if any), on which the coil is wound.

If $\mathrm{I} = 1$, then $\phi = \mathrm{L} \times 1$ or $\mathrm{L} = \phi$

Thus, *self-inductance of a coil is numerically equal to the magnetic flux linked with the coil, when a unit current flows through it.*

If e is the induced e.m.f. produced in the coil due to its self-inductance at the instant, when the rate of change of magnetic flux in the coil is $\frac{d\phi}{dt}$, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(\mathrm{L\,I})$$

or $$e = -\mathrm{L}\frac{d\mathrm{I}}{dt} \qquad \text{...(1.13)}$$

Here, $\frac{d\mathrm{I}}{dt}$ is the rate of change of current in the coil at that instant. The negative gn shows the opposing nature of the induced e.m.f. Considering magnitude only, om equation (1.13), we have

$$e = \mathrm{L}\frac{d\mathrm{I}}{dt}$$

If $\frac{d\mathrm{I}}{dt} = 1$, then $e = \mathrm{L} \times 1$ or $\mathrm{L} = e$

Hence, *self-inductance of a coil is numerically equal to the induced e.m.f. produced the coil, when the rate of change of current in the coil is unity.*

In SI, the unit of self-inductance is **henry (H)**.

Again, $$\mathrm{L} = \frac{e}{d\mathrm{I}/dt} \qquad \text{...(1.14)}$$

If $e = 1$ volt and $\frac{d\mathrm{I}}{dt} = 1$ ampere second^{-1}, then

$$\mathrm{L} = \frac{1 \text{ volt}}{1 \text{ ampere second}^{-1}} = 1 \text{ henry}$$

Thus, *self-inductance of a coil is said to be one henry, if a rate of change of current of ampere per second induces an e.m.f. of 1 volt in it.*

1.12. SELF INDUCTION – INERTIA OF ELECTRICITY

When a source of e.m.f. tends to send current in an electric circuit having a coil, e self inductance of the inductor (coil) behaves the same way as the mass of a body oes, when a force tends to a cause a change in its state. We know that more the ass of the body, more is the opposition it offers to the change in the state of the ody. Likewise, mass of a body gives the measure of its inertia. In the same way, ore the self-inductance of a coil, more is the opposition it offers to the change in urrent through the coil. Hence, self induction is called ***inertia of electricity***.

1.13. SELF-INDUCTANCE OF A LONG SOLENOID

Consider a long solenoid of length l, area of cross-section A and number of turns er unit length n. Suppose that a current I is passed through the solenoid. Then, agnetic field inside the solenoid is given by

$$\mathrm{B} = \mu_0\, n\, \mathrm{I}$$

In case of a long solenoid (length very large as compared to its radius), the agnetic field produced well within the solenoid is practically constant. Therefore, the magnetic flux passing through each turn of the solenoid

$$= \mathrm{B} \times \text{area of each turn} = \mu_0\, n\, \mathrm{I\,A}$$

total magnetic flux linked with the solenoid,

$$\phi = \text{magnetic flux linked with one turn} \times \text{total number of turns}$$

Now, total number of turns in the solenoid = $n\,l$

$$\therefore \quad \phi = \mu_0\, n\, \mathrm{I\,A} \times n\,l$$

or $$\phi = \mu_0\, n^2\, l\, \mathrm{A\,I} \qquad \text{...(1.15)}$$

If L is the self-inductance of the solenoid, then

$$\phi = \mathrm{L\,I} \qquad \text{...(1.16)}$$

From equations (1.15) and (1.16), we have

$$\mathrm{L} = \mu_0\, n^2\, l\, \mathrm{A} \qquad \text{...(1.17)}$$

It gives the self-inductance of a long solenoid of length l, area of cross-section A and having number of turns per unit length equal to n.

1.14. MUTUAL INDUCTION

Let us consider two coils P and S placed close* to each other. A battery and a apping key K are connected to the P-coil, while a galvanometer is connected across he S-coil [Fig. 1.17]. The coils P and S are respectively known as *primary* and *econdary coils.*

---

*If the coils are placed close to each other, then the magnetic flux linked with one coil gets linked with the other coil.

As soon as the key K is pressed, the current in the P-coil starts growing. As the current grows, the magnetic flux starts building up with the coil P and due to self-induction, an induced e.m.f. is produced in it. The induced e.m.f. opposes the growth of current in the P-coil. Since the S-coil is placed close to the P-coil, the magnetic flux also starts building up with the S-coil and hence an induced e.m.f. is produced in the S-coil also. This induced e.m.f. in the S-coil also opposes the growth of current in the P-coil.

Similarly, when the key is released, the induced e.m.f. is produced in the primary as well as the secondary coil. The sign of the induced e.m.f. produced in this case will be opposite to that produced when the key is pressed.

Thus, during both growth and decay of the current in the primary coil, an opposing induced e.m.f. is produced in the secondary coil.

The phenomenon according to which an opposing e.m.f. is produced in a coil as a result of change in current or magnetic flux linked with a neighbouring coil is called mutual induction.

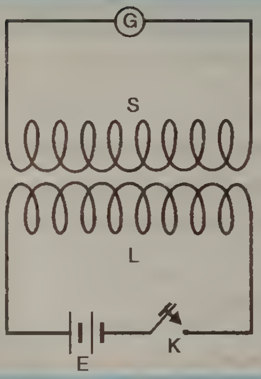


Fig. 1.17

Key point

When two coils are inductively couplec in addition to induced e.m.f. produced du to mutual induction, induced e.m.f. i produced in each of the two coils due t self-induction also.

1.15. COEFFICIENT OF MUTUAL INDUCTION

Consider two coils P and S as shown in Fig. 1.17. Suppose that a current I is flowing through the P-coil at any instant. It is found that the magnetic flux ϕ linked with the S-coil at any instant is directly proportional to the current passing through P-coil at that instant *i.e.*

$$\phi \propto \mathrm{I}$$

or
$$\phi = \mathrm{M\,I}, \qquad \text{...(1.19)}$$

where M is called ***coefficient of mutual induction*** or simply ***mutual inductance*** of the two coils.

If $\mathrm{I} = 1$, then $\phi = \mathrm{M} \times 1$ or $\mathrm{M} = \phi$

Thus, *mutual inductance of the two coils is numerically equal to the magnetic flux linked with one coil, when a unit current flows through the neighbouring coil.*

If e is the induced e.m.f. produced in the S-coil due to mutual inductance at the instant when the rate of change of magnetic flux in the coil is $\frac{d\phi}{dt}$, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(\mathrm{M\,I})$$

or
$$e = -\mathrm{M}\frac{d\mathrm{I}}{dt} \qquad \text{...(1.20)}$$

Here, $\frac{d\mathrm{I}}{dt}$ is the rate of change of current in the P-coil at that instant. The negative sign shows the opposing nature of the induced e.m.f. considering magnitude only ; from equation (1.20), we have

$$e = \mathrm{M}\frac{d\mathrm{I}}{dt}$$

If $\frac{d\mathrm{I}}{dt} = 1$, then $e = \mathrm{M} \times 1$ or $\mathrm{M} = e$

Hence, *mutual inductance of two coils is numerically equal to the induced e.m.f. produced in one coil, when the rate of change of current is unity in the other coil.*

In SI, the unit of mutual inductance is **henry (H).**

Again,
$$\mathrm{M} = \frac{e}{d\mathrm{I}/dt} \qquad \text{...(1.21)}$$

If $e = 1$ volt and $\frac{d\mathrm{I}}{dt} = 1$ ampere second^{-1}, then

$$\mathrm{M} = \frac{1 \text{ volt}}{1 \text{ ampere second}^{-1}} = 1 \text{ henry}$$

Thus, *the mutual inductance of two coils is said to be one henry, if a rate of change of current of 1 ampere per second in one coil induces an e.m.f. of 1 volt in the neighbouring coil.*

1.16. FACTORS AFFECTING MUTUAL INDUCTION BETWEEN TWO COILS

The mutual induction between two coils depends upon the shape and size of the two coils, separation between them and the permeability of the material of the core (if any), on which the two coils are wound. Apart from these factors, the mutual induction between the two coils depends upon the manner in which the two coils are oriented relative to each other. The manner in which the two coils are oriented determines the ***coefficient of coupling*** between them, which is given by

$$K = \sqrt{\frac{M}{L_1 L_2}} \qquad ...(1.18)$$

Here, L_1 and L_2 are self-inductances of the two coils and M is mutual inductance between them.

When the two coils P and S are wound on each other [Fig. 1.18], the coefficient of coupling is maximum and hence mutual induction between the coils is maximum. It, further increases, when the two coils are wound on a soft iron core. It is because, the magnetic permeability of soft iron is very large. For this reason, a transformer is wound on a core of soft iron. Mutual induction will be still greater, if the coil S is wound over the middle part of the coil P.

When the two coils are placed so as to have a common axis [Fig. 1.19], the coefficient of coupling between the two coils is still very large but less than that for the arrangement shown in Fig. 1.18. Hence, mutual induction between the two coils is lesser, when they are placed along a common axis than when they are wound on each other.

Fig. 1.20 shows the two coils placed such that their axes are perpendicular to each other. The coefficient of coupling is minimum in this case. Hence, mutual induction between two coils is minimum, when they are placed with their axes perpendicular to each other.

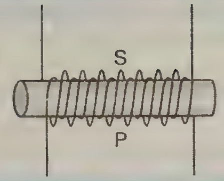


Fig. 1.18

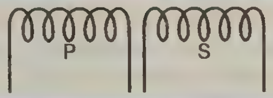


Fig. 1.19

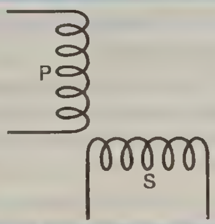


Fig. 1.20

1.17. MUTUAL INDUCTANCE OFTWO LONG SOLENOIDS

Consider two long solenoids S_1 and S_2 of same length l, such that the solenoid S_2 surrounds the solenoid S_1 completely as shown in Fig. 1.21. Let n_1 and n_2 be the number of turns per unit length of the solenoids S_1 and S_2 respectively. Suppose that current I_1 is passed through the solenoid S_1. Then, due to flow of current through the solenoid S_1, magnetic field will be produced inside it and magnetic flux will be linked with coil S_2. The magnetic flux (ϕ_{21}) linked with the solenoid S_2 is directly proportional to the current I_1 passing through the solenoid S_1 *i.e.*

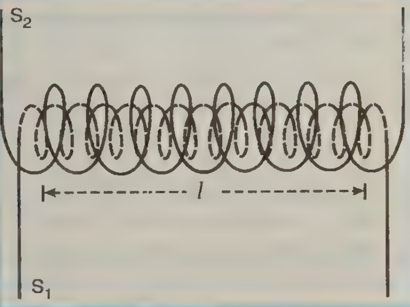


Fig. 1.21

$$\phi_{21} \propto I_1$$

or $$\phi_{21} = M_{21} I_1, \qquad ...(1.22)$$

where M_{21} is the coefficient of mutual induction of the two solenoids, when current is passed through solenoid S_1 and induced e.m.f. is produced in solenoid S_2.

Now, magnetic field produced inside the solenoid S_1 on passing current through it,

$$B_1 = \mu_0 n_1 I_1$$

When current is passed through a solenoid, the magnetic field lines are confined to the space within the solenoid only. Therefore, when the current is passed through the solenoid S_1, then in the annular region between the cross-sections of the two solenoids, the field is zero. Therefore, magnetic flux linked with each turn of the solenoid S_2 will be equal to B_1 times the area of cross-section of the solenoid S_1. If A is the area of cross-section of the solenoid S_1, then

magnetic flux linked with each turn of the solenoid $S_2 = B_1 A$

Therefore, total magnetic flux linked with the solenoid S_2,

$$\phi_{21} = B_1 A \times n_2 l = \mu_0 n_1 I_1 \times A \times n_2 l$$

or $$\phi_{21} = \mu_0 n_1 n_2 A l I_1 \qquad ...(1.23)$$

From equations (1.22) and (1.23), we have

$$M_{21} = \mu_0 n_1 n_2 A l \qquad ...(1.24)$$

Now, let us find the mutual inductance between the two solenoids, when current is passed through solenoid S_2 and induced e.m.f. is produced in solenoid S_1. If a current L_2 is passed through the solenoid S_2 and magnetic flux ϕ_{12} is linked with solenoid S_1, then

$$\phi_{12} \propto I_2$$

or $$\phi_{12} = M_{12} I_2, \qquad ...(1.25)$$

where M_{12} is coefficient of mutual inductance of the two solenoids in this case.

Now, magnetic field produced inside the solenoid S_2 on passing current through it,

$$B_2 = \mu_0 n_2 I_2$$

The magnetic flux linked with each turn of the solenoid $S_1 = B_2 A$

Key point

The coefficient of mutual inductance between two coils wound over each other does not depend upon the area of cross-section of the outer coil. It does not matter, whether current is passed through the inner or outer coil. It is because, when current is passed through the inner coil, magnetic field in the annular region between the cross-sections of the two coils is zero. On the other hand, when current is passed through the outer coil, magnetic flux is intercepted by the cross-section of inner coil only.

Therefore, total magnetic flux linked with the solenoid S_1,

$$\phi_{12} = B_2 A \times n_1 l = \mu_0 n_2 I_2 \times A \times n_1 l$$

or

$$\phi_{12} = \mu_0 n_1 n_2 A l I_2 \qquad ...(1.26)$$

From equations (1.25) and (1.26), we have

$$M_{12} = \mu_0 n_1 n_2 A l \qquad ...(1.27)$$

From equations (1.24) and (1.27), it follows that in case of two long solenoids, the coefficient of mutual induction remains the same, irrespective of whether the current is passed through the inner solenoid or the outer solenoid. Therefore,

$$M_{21} = M_{12} = M \text{ (say)}$$

Hence, coefficient of mutual induction between two long solenoids,

$$M = \mu_0 n_1 n_2 A l \qquad ...(1.28)$$

It may be pointed out that in the above expression, A represents the area of cross-section of the inner solenoid.

Key point

1. When two coils having self inductances L_1 and L_2 are connected in series keeping them far apart (so that mutual inductance between them is negligible), the equivalent inductance of the combination is given by

$$L = L_1 + L_2$$

2. When two coils are connected in parallel, the equivalent inductance is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Also refer to conceptual Problem 1.24

SOLVED NUMERICAL PROBLEMS

Type A. On Faraday's law of Electromagnetic Induction

Problem 1.01. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = (5t^3 + 4t^2 + 2t - 5)$ weber. Calculate the induced current through the coil at $t = 2$ s, if the resistance of the coil is 5 ohm. *(P.S.S.C.E. 1998 S)*

Sol. Here, resistance of the coil, $R = 5\ \Omega$

Instantaneous magnetic flux linked with the coil,

$$\phi = (5t^3 + 4t^2 + 2t - 5) \text{ wb}$$

If e is instantaneous value of induced e.m.f. produced in coil, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^3 + 4t^2 + 2t - 5)$$

$$= -[15(3t^2) + 4(2t) + 2(1) - 0]$$

or

$$e = (-15t^2 - 8t - 2) \text{ volt}$$

Therefore, induced e.m.f. produced in the coil at $t = 2$ s,

$$e = -15(2)^2 - 8(2) - 2 = -60 - 16 - 2 = -78 \text{ V}$$

Hence, magnitude of induced current in the coil at $t = 2$ s,

$$I = \frac{e}{R} = \frac{78}{5} = 15{\cdot}6 \text{ A}$$

Problem 1.02. A wire 40 cm long, bent into rectangular loop 15 cm × 5 cm, is placed perpendicular to the magnetic field, whose flux density is 0·08 Wb m^{-2}. Within 0·5 second, the loop is changed into 10 cm square and flux density increases to 1·4 Wb m^{-2}. Calculate the value of induced e.m.f.

Sol. Initial area of the loop,

$$S_1 = 15 \times 5 = 75 \text{ cm}^2 = 75 \times 10^{-4} \text{ m}^2$$

Initial flux density, $B_1 = 0{\cdot}8 \text{ Wb m}^{-2}$

Therefore, initial magnetic flux linked with the loop,

$$\phi = B_1 S_1 = 0{\cdot}8 \times 75 \times 10^{-4} = 60 \times 10^{-4} \text{ Wb}$$

Final area of the loop,

$$S_2 = 10 \times 10 = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$$

Final flux density, $B_2 = 1{\cdot}4 \text{ Wb m}^{-2}$

Therefore, final magnetic flux linked with the loop,

$$\phi_2 = B_2 S_2 = 1{\cdot}4 \times 100 \times 10^{-4} = 140 \times 10^{-4} \text{ Wb}$$

Change in magnetic flux,

$$d\phi = \phi_2 - \phi_1 = 140 \times 10^{-4} - 60 \times 10^{-4}$$

$$= 80 \times 10^{-4} \text{ Wb}$$

Time during which the change in magnetic flux takes place,

$$dt = 0{\cdot}05 \text{ s}$$

Therefore, induced e.m.f.,

$$e = -\frac{d\phi}{dt} = -\frac{80 \times 10^{-4}}{0{\cdot}5} = -0{\cdot}016 \text{ volt}$$

Problem 1.03. A 10 ohm resistance coil has 1000 turns and at a certain time, $5{\cdot}5 \times 10^{-4}$ Wb of flux passes through it. If the flux falls to $0{\cdot}5 \times 10^{-4}$ Wb in 0·1 second ; find the e.m.f. generated in volt and charge flowing through the coil in coulomb.

Sol. Initial magnetic flux linked with the coil,

$$\phi_1 = 5{\cdot}5 \times 10^{-4} \text{ Wb}$$

Final magnetic flux linked with the coil,

$$\phi_2 = 0{\cdot}5 \times 10^{-4} \text{ Wb}$$

Change in magnetic flux,

$$\phi_2 - \phi_1 = 0{\cdot}5 \times 10^{-4} - 5{\cdot}5 \times 10^{-4} = -5 \times 10^{-4} \text{ Wb}$$

Number of turns in the coil, N = 1000

Total effective change in magnetic flux,

$$d\phi = N(\phi_2 - \phi_1) = 1000 \times (-5 \times 10^{-4}) = -0{\cdot}5 \text{ Wb}$$

Time in which the change in magnetic flux takes place,

$$dt = 0{\cdot}1 \text{ s}$$

Therefore, induced e.m.f. produced,

$$e = -\frac{d\phi}{dt} = -\frac{(0{\cdot}05)}{0{\cdot}1} = \frac{5}{10} = 5 \text{ V}$$

Current through 10 ohm resistance

$$= \frac{e}{\text{resistance}} = \frac{5}{10} = 0{\cdot}5 \text{ A}$$

Charge flowing through the coil in 0·1 s

$$= 0{\cdot}5 \times 0{\cdot}1 = 0{\cdot}05 \text{ C}$$

Problem 1.04. A copper rod of length l rotates with an angular velocity ω in a uniform magnetic field B. Find the e.m.f. developed between the two ends of rod. The field is normal to the plane of rotation. *(H.S.S.C.E. 2001)*

Sol. When the copper rod rotates inside the uniform magnetic field, it intercepts magnetic field over a circular area, whose radius is equal to length (l) of the rod.

Therefore, area intercepted by the copper rod in one rotation,

$$\Delta S = \pi l^2$$

Therefore, change in magnetic flux through the copper rod in one rotation,

$$d\phi = B\,\Delta S = B \times \pi l^2$$

Since the rod rotates with angular velocity ω, time taken by it to complete one rotation,

$$dt = \frac{2\pi}{\omega}$$

If e is the induced e.m.f. produced between the two ends of the rod, then

$$e = -\frac{d\phi}{dt} = -\frac{B \times \pi l^2}{2\pi/\omega}$$

or $e = -\frac{1}{2} B l^2 \omega$

Note. If ν is frequency of rotation of the copper rod, then

$\omega = 2\pi\nu$

∴ $e = -\frac{1}{2} B l^2 \times 2\pi\nu$

or $e = -B \times \pi l^2 \times \nu$

Problem 1.05. A metal rod of length 1 m is rotated about ne of its ends in a plane at right angles to a uniform magetic field of $2{\cdot}5 \times 10^{-3}$ Wb m^{-2}. If it makes 30 r.p.s., calculate he induced e.m.f. between its ends. (*H.S.S.C.E. 2002*)

Sol. Here, $B = 2{\cdot}5 \times 10^{-3}$ Wb^{-2}, $l = 1$ m and $\nu = 30$ r.p.s.

When the metal rod rotates inside the magnetic field, it ntercepts the magnetic field over a circular area of radius qual to its length. Then, as obtained in solved problem 1.04, he induced e.m.f. produced between the two ends of the netal rod is given by

$$e = -B \times \pi l^2 \times \nu$$
$$= -2{\cdot}5 \times 10^{-3} \times \pi \times (1)^2 \times 30 = \mathbf{-0{\cdot}023\ V}$$

Problem 1.06. When a wheel with metal spokes 1·2 m ong rotates in a magnetic field of flux density 5×10^{-5} tesla ormal to the plane of the wheel, an e.m.f. of 10^{-2} volt is nduced between the rim and the axle. Find the rate of otation of the wheel.

Sol. Here, length of spoke, $l = 1{\cdot}2$ m ;

e.m.f. induced ; $e = 10^{-2}$ volt ;

magnetic flux density, $B = 5 \times 10^{-5}$ tesla

Suppose that the wheel makes ν rotations per second. Due to rotation of the wheel inside the magnetic field, the netal spoke sweeps a circular area of radius equal to its ength. As a result of change in magnetic flux, the induced .m.f. is produced across the two ends of the metal spoke. As btained in solved problem 1.04, the induced e.m.f. produced cross the two ends of the spoke is given by

$$e = B \times \pi l^2 \times \nu \qquad \text{(in magnitude)}$$

∴ $$\nu = \frac{e}{B \times \pi l^2} = \frac{10^{-2}}{5 \times 10^{-5} \times \pi\,(1{\cdot}2)^2}$$
$$= \mathbf{44{\cdot}2\ rotations\ s^{-1}}$$

Problem 1.07. A wheel with 10 metallic spokes each ·50 m long is rotated with the speed of 120 r.p.m. in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0·40 G, what is the induced e.m.f. between the axle and the rim of the wheel ? (*Text Problem*)

Sol. Here, number of spokes = 10 ;

length of spoke, $l = 0{\cdot}50$ m ; $B = 0{\cdot}40$ G $= 0{\cdot}40 \times 10^{-4}$ T

frequency of rotation, $\nu = 120$ r.p.m. $= 2$ r.p.s.

Due to rotation of the wheel inside the magnetic field, nduced e.m.f. will be produced across the two ends of each spoke. As all the ten spokes are connected with their one end at the axle and the other end at the rim, the net e.m.f. produced will be the same as that produced across a single spoke. As obtained in solved problem 1.04, the induced e.m.f. produced across the two ends of a spoke is given by

$$e = -B \times \pi l^2 \times \nu$$
$$= -0{\cdot}40 \times 10^{-4} \times \pi \times (0{\cdot}50)^2 \times 2 = \mathbf{-6{\cdot}283 \times 10^{-5}\ V}$$

Problem 1.08. A circular copper disc 10 cm in radius rotates at 20 π rad s^{-1} about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0·2 T acts perpendicular to the disc. (*a*) Calculate the potential difference developed between the axis of the disc and the rim. (*b*) What is the induced current, if the resistance of the disc is 2 Ω ? (*C.B.S.E. 2001*)

Sol. Here, $B = 0{\cdot}2$ T ;

radius of the circular disc, $r = 10$ cm $= 0{\cdot}1$ m

resistance of the disc, $R = 2\ \Omega$

angular speed of rotation of the disc, $\omega = 20\,\pi$ rad s^{-1}

Therefore, frequency of rotation of the disc,

$$\nu = \frac{\omega}{2\pi} = \frac{20\,\pi}{2\pi} = 10 \text{ r.p.s.}$$

(*a*) If e is the induced e.m.f. produced between the axis of the disc and its rim, then

$$e = -B \times \pi r^2 \times \nu$$
$$= -0{\cdot}2 \times \pi \times (0{\cdot}1)^2 \times 10 = \mathbf{-0{\cdot}0628\ V}$$

(*b*) Induced current, $I = \frac{e}{R} = \frac{0{\cdot}0628}{2} = \mathbf{0{\cdot}0314\ A}$

Note. The circular disc can be considered as a wheel having infinite number of spokes between its centre and the rim. As discussed in solved problem 1.07, the e.m.f. developed will be the same as that produced across a single spoke.

Type B. On Motional e.m.f.

Problem 1.09. A train is moving in the north-south direction with a speed of 108 km h^{-1}. Find the amount of e.m.f. generated between two wheels, if the length of axle is 2 m. Assume that the vertical component of earth's field is $8{\cdot}0 \times 10^{-5}$ Wb m^{-2}.

Sol. Here, $B_V = 8{\cdot}0 \times 10^{-5}$ Wb m^{-2} ; $l = 2$ m

$$v = 108 \text{ km h}^{-1} = 108 \times (1000 \text{ m}) \times (60 \times 60 \text{ s})^{-1}$$
$$= 30 \text{ m s}^{-1}$$

Now, $e = B_V\, l\, v = 8{\cdot}0 \times 10^{-5} \times 2 \times 30 = \mathbf{4{\cdot}8 \times 10^{-3}\ V}$

Problem 1.10. A horizontal straight wire 10 m long is extending along east and west and is falling with a speed of 5·0 m s^{-1} at right angles to the horizontal component of the earth's magnetic field of strength $0{\cdot}30 \times 10^{-4}$ wb m^{-2}. What is the instantaneous value of the e.m.f. induced in the wire ?

Sol. Here, length of the wire, $l = 10$ m ;

velocity of the wire, $v = 5{\cdot}0$ m s^{-1}

Horizontal component of earth's magnetic field,

$$B_H = 0{\cdot}30 \times 10^{-4} \text{ wb m}^{-2}$$

Now, $e = B_H\, l\, v = 0{\cdot}30 \times 10^{-4} \times 10 \times 5{\cdot}0 = \mathbf{1{\cdot}5 \times 10^{-3}\ V}$

Problem 1.11. A rectangular loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0·3 T directed normal to the loop. What is the voltage developed across the cut, if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (*i*) longer side, (*ii*) shorter side of the loop ? For how long does the induced voltage last in each case ? (*Text Problem*)

Sol. Here, $B = 0{\cdot}3$ T ; $v = 1$ cm s$^{-1} = 10^{-2}$ m s^{-1}

Now, $e = B\, l\, v$

In the above expression, l is the length of the arm, which is perpendicular to the direction of motion.

(*i*) *Direction of motion normal to longer side* : Length of longer side, $l = 8$ cm $= 8 \times 10^{-2}$ m

∴ $$e = B\, l\, v = 0{\cdot}3 \times 10^{-2} \times 8 \times 10^{-2} = 0{\cdot}24 \times 10^{-3} \text{ V}$$
$$= \mathbf{0{\cdot}24\ mV}$$

The e.m.f. will last in the loop, till it does not get out of the magnetic field *i.e.* for the time the loop takes to travel a distance equal to the length of the shorter arm. If t is the time for which the e.m.f. lasts in the loop, then

$$t = \frac{\text{length of the shorter arm}}{v} = \frac{2}{1} = \mathbf{2\ s}$$

(ii) Direction of motion normal to the shorter side : Length of the shorter side, $l = 2\ \text{cm} = 2 \times 10^{-2}$ m

$$\therefore \quad e = B\,l\,v = 0{\cdot}3 \times 10^{-2} \times 2 \times 10^{-2} = \mathbf{0{\cdot}06\ mV}$$

The time for which the e.m.f. lasts in the loop,

$$t = \frac{\text{length of the longer arm}}{v} = \frac{8}{1} = \mathbf{8\ s}$$

Problem 1.12. Fig. 1.22 shows a conducting rod PQ in contact with metal rails RP and SQ, which are 25 cm apart in a uniform field of flux density 0·4 T acting perpendicular to the plane of the paper. Ends R and S are connected through a 5 Ω resistance. What is the e.m.f., when the rod moves to the right with a velocity of 5 m s^{-1} ? What is the magnitude and the direction of the current through the 5 Ω resistor ? If the rod PQ moves towards the left with the same speed, what will be the new current and its direction ? *(I.S.C.E. 1995)*

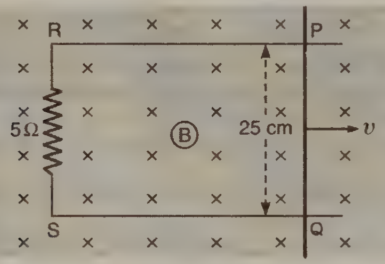


Fig. 1.22

Sol. Here $B = 0{\cdot}4$ T ; $v = 5\ \text{m s}^{-1}$;

$l = 25\ \text{cm} = 0{\cdot}25$ m

Induced e.m.f. produced,

$$e = B\,l\,v = 0{\cdot}4 \times 0{\cdot}25 \times 5 = \mathbf{0{\cdot}5\ V}$$

Current through the 5 Ω resistance,

$$I = \frac{e}{R} = \frac{0{\cdot}5}{5} = \mathbf{0{\cdot}1\ A}$$

According to Lenz's law, when the rod PQ moves towards right, the induced current should flow in a direction so that the rod PQ experiences force towards left. According to Fleming's left hand rule, then the current through the rod PQ will flow from the end Q to P *i.e.* **from the end R to S** through the resistance of 5 Ω.

If the rod PQ moves towards left with the same speed, the current of **0·1 A** will flow through 5 Ω resistance **from the end S to R.**

Problem 1.13. A jet plane is travelling west at the speed of 1800 km h^{-1}. What is the voltage difference developed between the ends of the wing 25 m long, if the earth's magnetic field at the location has a magnitude of 5·0 × 10^{-4} T and the dip angle is 30° ? *(Text Problem)*

Sol. Here, $v = 1800\ \text{km h}^{-1}$

$= 1800 \times (1000\ \text{m}) \times (60 \times 60\ \text{s})^{-1} = 500\ \text{m s}^{-1}$

Earth's magnetic field, $B = 5{\cdot}0 \times 10^{-4}\ t$;

angle of dip, $\delta = 30°$; length of the wing = 25 m

Now, $\quad B_V = B \sin \delta = 5{\cdot}0 \times 10^{-4} \sin 30° = 2{\cdot}5 \times 10^{-4}$ T

The vertical component (B_V) of the earth's field is norma to both the wings and the direction of motion.

Therefore, induced e.m.f. produced,

$$e = B_V\,l\,v = 2{\cdot}5 \times 10^{-4} \times 25 \times 500 = \mathbf{3{\cdot}125\ V}$$

Type C. On Self Induction

Problem 1.14. The self-inductance of a coil having 20 turns is 10 millihenry. Compute the total flux linked wit the coil. Also determine the magnetic flux through th cross-section of the coil corresponding to current of 4 milliampere.

Sol. Here, self-inductance of coil, L = 10 millihenry

$= 10 \times 10^{-3}$ henry ;

current through the coil, I = 4 milliampere

$= 4 \times 10^{-3}$ ampere

Total magnetic flux linked with the coil,

$$\phi = L\,I = 10 \times 10^{-3} \times 4 \times 10^{-3} = \mathbf{4 \times 10^{-5}}\ \textbf{webe}$$

Number of turns in the coil, N = 200

Therefore, magnetic flux through the cross-section of th coil (flux linked with each turn),

$$\frac{\phi}{N} = \frac{4 \times 10^{-5}}{200} = \mathbf{2 \times 10^{-7}\ weber}$$

Problem 1.15. Current in a 10 millihenry coil increase uniformly from zero to one ampere in 0·01 second. Find th direction and value of self-induced e.m.f.

Sol. Here, $L = 10$ millihenry $= 10 \times 10^{-3} = 10^{-2}$ henry

Initial current in the coil = 0 ;

final current in the coil = 1 ampere

Therefore, change in current, $dI = 1 - 0 = 1$ ampere

Time in which the current changes, $dt = 0{\cdot}01$ second

Now, induced e.m.f.,

$$e = -L\frac{dI}{dt} = -10^{-2} \times \frac{1}{0{\cdot}01} = \mathbf{-1\ V}$$

The self-induced e.m.f. will act so as to ***oppose th growth of current.***

Problem 1.16. A long solenoid with 15 turns per cm ha a small loop of area 2·0 cm^2 placed inside, normal to the axi of the solenoid. If the current carried by the solenoi changes steadily from 2 A to 4 A in 0·1 s, find the induce e.m.f. in the loop while the current is changing. *(Text Problem*

Sol. The magnetic field produced inside the solenoid,

$$B = \mu_0\,n\,I$$

If A is the area of the loop placed inside the solenoid, the magnetic flux linked with the loop,

$$\phi = B\,A = \mu_0\,n\,I\,A$$

If e is the induced e.m.f. produced due to change i current through the solenoid, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(\mu_0\,n\,I\,A)$$

or $\quad e = -\mu_0\,n\,A\,\dfrac{dI}{dt}$

Here, number of turns per unit length of the solenoid,

$$n = 15\ \text{turns cm}^{-1} = 1500\ \text{turns m}^{-1}$$

$$A = 2{\cdot}0\ \text{cm}^2 = 2{\cdot}0 \times 10^{-4}\ \text{m}^2 ; \frac{dI}{dt} = \frac{4{\cdot}0}{0{\cdot}1} = 20\ \text{A s}^{-1}$$

$$\therefore \quad e = -4\,\pi \times 10^{-7} \times 1500 \times 2{\cdot}0 \times 10^{-4} \times 20$$

$$= \mathbf{-7{\cdot}54 \times 10^{-6}\ V}$$

ype D. On Mutual Induction

Problem 1.17. Calculate the mutual inductance bet-veen two coils, when a current of 4·0 A changes to 8·0 A in ·5 s and induces an e.m.f. of 50 mV in the secondary coil. *(P.S.S.C.E. 1997 S)*

Sol. Here, induced e.m.f. produced,

$$e = 50 \text{ mV} = 50 \times 10^{-3} \text{ V}$$

Change in current in the primary coil,

$$dI = 8{\cdot}0 - 4{\cdot}0 = 4{\cdot}0 \text{ A}$$

Time in which change in current takes place, $dt = 0{\cdot}5$ s

If M is the mutual inductance between the two coils, then

$$e = -M\frac{dI}{dt}$$

or $$M = \frac{e}{dI/dt} \quad \text{(in magnitude)}$$

$$= \frac{50 \times 10^{-3}}{4{\cdot}0 - 0{\cdot}5} = \frac{0{\cdot}5 \times 50 \times 10^{-3}}{4{\cdot}0} = \mathbf{6{\cdot}25 \times 10^{-3}\ H}$$

Problem 1.18. A solenoidal coil has 50 turns per cm along its length and a cross-sectional area of 4 cm^2. 200 turns of another wire is wound round the first solenoid coaxially. The two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils. Given, $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}. *(I.S.C.E. 1998)*

Sol. Refer to Fig 1.21 ;

Here, number of turns per unit length of solenoid S_1,

$$n_1 = 50 \text{ turns cm}^{-1} = 5000 \text{ turns m}^{-1}$$

area of cross-section of solenoid S_1,

$$A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$$

total number of turns of solenoid S_2,

$$n_2 l = 200$$

Also, $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}

Therefore, mutual inductance of two coils,

$$M = \mu_0 n_1 n_2 A l = \mu_0 n_1 (n_2 l) A$$

$$= 4\pi \times 10^{-7} \times 5000 \times 200 \times 4 \times 10^{-4}$$

$$= \mathbf{5{\cdot}027 \times 10^{-4}\ H}$$

Problem 1.19. (*a*) A toroidal solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. Obtain the self-inductance of the toroid. Ignore field variation across the cross-section of the toroid.

(*b*) A second coil of 300 turns is wound closely on the toroid above. If the current in the primary coil is increased from zero to 2·0 A in 0·05 s, obtain the induced e.m.f. in the second coil. *(Text Problem)*

Sol. (*a*) Total number of turns, N = 1200 ;

area of cross-section of the solenoid,

$$A = 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$$

and average radius of toroidal solenoid,

$$r = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$$

Therefore, the number of turns per unit length,

$$n = \frac{N}{2\pi r} = \frac{1200}{2\pi \times 15 \times 10^{-2}} = \frac{4000}{\pi} \text{ m}^{-1}$$

Suppose that a current I is passed through the solenoid. Magnetic field produced along the axis of the solenoid,

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times \frac{4000}{\pi} \times I = I \times 1{\cdot}6 \times 10^{-3} \text{ T}$$

Magnetic flux linked with the solenoid itself,

$$\phi = B \times \text{area of cross-section} \times \text{total number of turns}$$

$$= I \times 1{\cdot}6 \times 10^{-3} \times 12 \times 10^{-4} \times 1200$$

$$= I \times 2{\cdot}304 \times 10^{-3} \text{ Wb}$$

Now, $$L = \frac{\phi}{I} = \frac{I \times 2{\cdot}304 \times 10^{-3}}{I} = \mathbf{2{\cdot}304 \times 10^{-3}\ H}$$

(*b*) Magnetic flux linked with the second coil,

$$\phi_2 = B \times A \times N_2,$$

where N_2 is total number of turns of the second coil.

$$\therefore \quad \phi = I \times 1{\cdot}6 \times 10^{-3} \times 12 \times 10^{-4} \times 300$$

$$= 5{\cdot}76 \times 10^{-4} \times I \text{ Wb}$$

Now, initial magnetic flux linked with the second coil,

$$\phi_i = 5{\cdot}76 \times 10^{-4} \times \text{initial current}$$

$$= 5{\cdot}76 \times 10^{-4} \times 0 = 0$$

Final magnetic flux linked with the second coil,

$$\phi_f = 5{\cdot}76 \times 10^{-4} \times \text{final current}$$

$$= 5{\cdot}76 \times 10^{-4} \times 2{\cdot}0 = 1{\cdot}152 \times 10^{-3} \text{ Wb}$$

Therefore, total change in flux, $d\phi = \phi_f - \phi_i$

$$= 1{\cdot}152 \times 10^{-3} - 0 = 1{\cdot}152 \times 10^{-3} \text{ Wb}$$

Time taken, $dt = 0{\cdot}05$ s

Now, $$e = -\frac{d\phi}{dt} = -\frac{1{\cdot}152 \times 10^{-3}}{0{\cdot}05} = \mathbf{-\,0{\cdot}023\ V}$$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 1.20. An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500 carries a current of 2·5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back e.m.f. induced across the ends of the open switch in the circuit ? Ignore the variation in magnetic field near the ends of the solenoid. *(Text Problem)*

Sol. Here, area of cross-section of the solenoid,

$$A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$$

total number of turns, N = 500 ; $l = 30$ cm $\times 10^{-2}$ m ;

Therefore, number of turns per unit length,

$$n = \frac{N}{l} = \frac{500}{30 \times 10^{-2}}$$

Current passed through the solenoid, I = 2·5 A

Magnetic field inside the solenoid,

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times \frac{500}{30 \times 10^{-2}} \times 2{\cdot}5 = 52{\cdot}36 \times 10^{-4} \text{ T}$$

Magnetic flux linked with the solenoid initially,

$$\phi_i = B \times A \times N = 52{\cdot}36 \times 10^{-4} \times 25 \times 10^{-4} \times 500$$

$$= 65{\cdot}45 \times 10^{-4} \text{ Wb}$$

Magnetic flux linked with the solenoid finally, $\phi_f = 0$

$$\therefore\ d\phi = \phi_f - \phi_i = 0 - 65{\cdot}45 \times 10^{-4} = -\,65{\cdot}45 \times 10^{-4} \text{ Wb}$$

Now, time taken, $dt = 10^{-3}$ s

Therefore, average induced e.m.f. produced,

$$e = -\frac{d\phi}{dt} = -\frac{-\,65{\cdot}45 \times 10^{-4}}{10^{-3}} = \mathbf{6{\cdot}545\ V}$$

Problem 1.21. It is desired to measure the magnitude of field between the poles of a powerful loudspeaker magnet. A small flat search coil of area 2·0 cm^2 with 25 closely wound turns is positioned normal to the field direction and then quickly snatched out of the field region. The total charge flown in the coil (measured by a ballistic galvanometer connected to the coil) is 7·5 mC. The resistance

of the coil of the galvanometer is 0·50 Ω. Estimate the field strength of the magnet. *(Text Problem)*

Sol. Here, number of turns, N = 25 ;
area of coil, $A = 2{\cdot}0 \text{ cm}^2 = 2{\cdot}0 \times 10^{-4} \text{ m}^2$
charge flown in the coil, $q = 7{\cdot}5 \text{ mC} = 7{\cdot}5 \times 10^{-3}$ C ;
resistance of coil, R = 0·50 Ω

Let B be the strength of the magnetic field. Then, magnetic flux linked with the search coil initially,

$$\phi_i = BA$$

When the search coil is brought out of the magnetic field, the magnetic flux linked with the coil finally, $\phi_f = 0$

Now, $e = -N\dfrac{d\phi}{dt}$,

where N is the number of turns in the coil.

or $$IR = -N\frac{d\phi}{dt}$$

or $$R\frac{dq}{dt} = -N\frac{d\phi}{dt}$$

or $$R\,dq = -N\,d\phi$$

Integrating both sides, we have

$$R\int_0^q dq = -N\int_{\phi_i}^{\phi_f} d\phi$$

or $$R(q-0) = -N(\phi_f - \phi i)$$

or $$Rq = -N(0 - BA) = NBA$$

or $$B = \frac{Rq}{NA} = \frac{0\cdot 50 \times 7\cdot 5 \times 10^{-3}}{25 \times 2\cdot 0 \times 10^{-4}} = 0\cdot 75 \text{ T}$$

Problem 1.22. Fig 1.23. shows a metal rod PQ resting on the rails AB and positioned between the poles of a permanent magnet. The rails, the rod and the magnetic field are in three mutually perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0·50 T, resistance of the closed loop containing the rod = 9·0 m Ω.

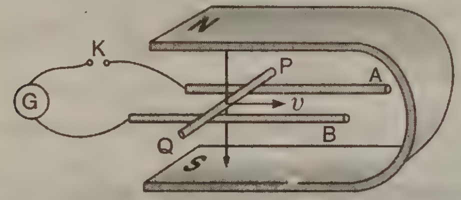


Fig. 1.23

(*a*) Suppose K is open and the rod moves with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced e.m.f.

(*b*) Is there an excess charge built up at the ends of the rods, when K is open ? What, if K is closed ?

(*c*) With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain, why.

(*d*) What is the retarding force on the rod, when K is closed ?

(*e*) How much power is required (by an external agent) to keep the rod moving at the same speed (= 12 cm s^{-1}), when K is closed ? How much power is required, when K is open ?

(*f*) How much power is dissipated as heat in the closed circuit ? What is the source of this power ?

(*g*) What is the induced e.m.f. in the moving rod, when the permanent magnet is rotated to a vertical position so that the field is parallel to the rails ? *(Text Problem)*

Sol. Here, B = 0·50 T ; $l = 15 \text{ cm} = 15 \times 10^{-2}$ m ;

$$R = 9{\cdot}0 \text{ m}\Omega = 9{\cdot}0 \times 10^{-3}\ \Omega$$

(*a*) Now, $e = B\,v\,l$

Here, $v = 12 \text{ cm s}^{-1} = 12 \times 10^{-2} \text{ m s}^{-1}$

∴ $$e = 0{\cdot}50 \times 12 \times 10^{-2} \times 15 \times 10^{-2} = \mathbf{9 \times 10^{-3}\ V}$$

If $-q$ is charge on an electron, then the electrons in the rod will experience magnetic Lorentz force $-q(\vec{v} \times \vec{B})$ along PQ. Hence, the end P of the rod will become positive and the end Q will become negative.

(*b*) On closing the switch K, electrons collect at the end Q. Therefore, excess charge is built up at the end Q. However, it does not happen, when the switch K is open.

(*c*) The magnetic Lorentz force on electron is cancelled by the electric force acting on it due to the electric field set up across the two ends due to accumulation of positive and negative charges at the ends P and Q respectively.

(*d*) Retarding force, $F = BIl = 0\cdot 50 \times \dfrac{e}{R} \times 15 \times 10^{-2}$

$$= 0\cdot 50 \times \frac{9 \times 10^{-3}}{9 \times 10^{-3}} \times 15 \times 10^{-2} = \mathbf{7{\cdot}5 \times 10^{-2}\ N}$$

(*e*) When K is closed, power required by external agent against the retarding force, $P = F\,v$

Here, speed with which the rod is moved,

$$v = 12 \text{ cm s}^{-1} = 12 \times 10^{-2} \text{ m s}^{-1}$$

∴ $$P = 7{\cdot}5 \times 10^{-2} \times 12 \times 10^{-2} = \mathbf{9 \times 10^{-3}\ W}$$

(*f*) Power dissipated as heat,

$$\frac{e^2}{R} = \frac{(9 \times 10^{-3})^2}{9 \times 10^{-3}} = \mathbf{9 \times 10^{-3}\ W}$$

The source of this power is the power of external agent.

(*g*) The motion of rod does not cut field lines, hence **no induced e.m.f.** is produced.

Problem 1.23. A circular loop of radius 0·3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm [Fig. 1.24].

(*a*) What is the flux linking the bigger loop, if a current of 2·0 A flows through the smaller loop ?

(*b*) Obtain the mutual inductance of the two loops. *(Text Problem)*

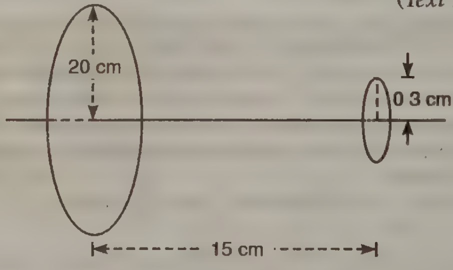


Fig. 1.24

Sol. Due to symmetry, flux linking the bigger loop due to current of 2·0 A flowing through smaller loop will be same as the flux linking the smaller loop due to current of 2·0 A flowing through bigger loop.

Bigger loop	***Smaller loop***
$a_1 = 20 \text{ cm} = 20 \times 10^{-2}$ m	$a_2 = 0{\cdot}3 \text{ cm} = 0{\cdot}3 \times 10^{-2}$ m

Distance of smaller loop from the bigger loop,

$x = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$

and current through the either loop, $I = 2{\cdot}0$ A

Magnetic field at the centre of smaller loop due to current flowing through the bigger loop is given by

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a_1^2}{(a_1^2 + x^2)^{3/2}}$$

$$= \frac{10^{-7} \times 2\pi \times 2{\cdot}0 \times (20 \times 10^{-2})^2}{\left\{(20\ 10^{-2})^2 + (15 \times 10^{-2})^2\right\}^{3/2}}$$

$$= \frac{10^{-7} \times 2\pi \times 2{\cdot}0 \times (20 \times 10^{-2})^2}{(625 \times 10^{-4})^{3/2}}$$

$$= \frac{10^{-7} \times 2\pi \times 2{\cdot}0 \times (20 \times 10^{-2})^2}{(25 \times 10^{-2})^3} = 3{\cdot}127 \times 10^{-6} \text{ T}$$

(*a*) Flux linked with smaller coil,

$$\phi = B_1 \times A_2 = B_1 \pi a_2^2$$

$$= 3{\cdot}127 \times 10^{-6} \times \pi \times (0{\cdot}3 \times 10^{-2})^2$$

$$= \mathbf{9{\cdot}096 \times 10^{-11}\ Wb}$$

(*b*) Now, $M = \frac{\phi}{I} = \frac{9{\cdot}096 \times 10^{-11}}{2{\cdot}0} = \mathbf{4{\cdot}548 \times 10^{-11}\ H}$

Problem 1.24. (*a*) A circuit contains two inductors in series with self-inductances L_1 and L_2 and mutual inductance M. Obtain a formula for the equivalent inductance in the circuit.

(*b*) Two inductors of self-inductances L_1 and L_2 are connected in parallel. The inductors are so far apart that their mutual inductance is negligible. What is the equivalent inductance of the combination ? (*Text Problem*)

Sol. (*a*) Let L_1 and L_2 be the self-inductances of two inductors and M, the mutual inductance between them. Further, let e_1 and e_2 be induced e.m.fs. produced in two inductors and *e* be the total e.m.f. produced. Then,

$$e = e_1 + e_2 \qquad ...(i)$$

If $\frac{dI}{dt}$ is the rate of change of current through the two inductors in series, then

$$e_1 = -L_1 \frac{dI}{dt} - M\frac{dI}{dt} \quad \text{and} \quad e_2 = -L_2 \frac{dI}{dt} - M\frac{dI}{dt}$$

If L is the self-inductance of the two inductors in series, then

$$e = -L\frac{dI}{dt}$$

Therefore, equation (*i*) becomes

$$-L\frac{dI}{dt} = \left(-L_1\frac{dI}{dt} - M\frac{dI}{dt}\right) + \left(-L_2\frac{dI}{dt} - M\frac{dI}{dt}\right)$$

or $\quad \mathbf{L = L_1 + L_2 + 2\,M}$

(*b*) As the two inductors are connected in parallel, induced e.m.f. produced across either of the inductors will be same. If $\frac{dI_1}{dt}$ and $\frac{dI_2}{dt}$ are the rates of change of current through the inductors L_1 and L_2 respectively, then

$$e = -L_1\frac{dI_1}{dt} \quad \text{and} \quad e = -L_2\frac{dI_2}{dt}$$

If L is the effective inductance of the two inductors in parallel, then

$$e = -L\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right)$$

Substituting for $\frac{dI_1}{dt}$ and $\frac{dI_2}{dt}$, we get

$$e = -L\left(-\frac{e}{L_1} - \frac{e}{L_2}\right)$$

or $\quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \quad$ or $\quad L = \frac{L_1 L_2}{L_1 + L_2}$

Problem 1.25. Fig 1.25 shows a short solenoid of length 4 cm, radius 2·0 cm and number of turns 100, lying inside on the axis of a long solenoid, 80 cm long and number of turns 1500.

What is the flux through the long solenoid, if a current of 5·0 A flows through the short solenoid ? Also obtain the mutual inductance of the two solenoids. (*Text Problem*)

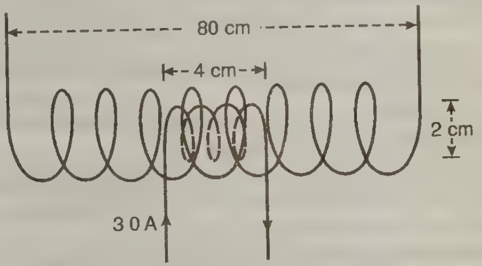


Fig. 1.25

Sol. *Long solenoid* :

Total number of turns, $N_1 = 1500$;

length, $l_1 = 80 \text{ cm} = 80 \times 10^{-2}$ m

Short solenoid :

Total number of turns, $N_2 = 100$;

length $l_2 = 4 \text{ cm} = 4 \times 10^{-2}$ m ;

radius, $r_2 = 2 \text{ cm} = 2 \times 10^{-2}$ m ;

Therefore, area of cross-section of the short solenoid,

$$A_2 = \pi r_2^2 = \pi \times (2 \times 10^{-2})^2 = 4\pi \times 10^{-4} \text{ m}^2$$

Current through the short solenoid, $I = 3{\cdot}0$ A

It is difficult to calculate magnetic flux through the long solenoid and hence the mutual inductance of the two solenoids as such. It is because, the short solenoid produces a complicated field. In section 1.17, it was proved that

$$M_{12} = M_{21}$$

Also, when same current is passed through the inner (short) or the outer (long) solenoid,

$$\phi_{21} = \phi_{12}$$

Therefore, we may assume that the current of 3 A, which actually flows through the short solenoid, is flowing through the long solenoid. Now, the magnetic flux through the long solenoid and mutual induction between the two solenoids can be easily calculated.

Magnetic flux linked with the short solenoid,

$$\phi = B_1 \times A_2 \times N_2,$$

where B_1 is field due to the long solenoid, A_2 and N_2 are area of cross-section and total number of turns of the short solenoid respectively.

$$\therefore \quad \phi = \mu_0 n_1 I \times A_2 \times N_2$$

Now, $\quad n_1 = \frac{N_1}{l_1} = \frac{1500}{80 \times 10^{-2}}$

$$\therefore \quad \phi = 4\pi \times 10^{-7} \times \frac{1500}{80 \times 10^{-2}} \times 3{\cdot}0 \times 4\pi \times 10^{-4} \times 100$$
$$= \mathbf{8{\cdot}883 \times 10^{-4}\ Wb}$$

Also, $$M = \frac{\phi}{I} = \frac{8{\cdot}883 \times 10^{-4}}{3}$$
$$= 2{\cdot}96 \times 10^{-4}\ \text{H}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What is meant by magnetic flux ? State its SI unit. *(P.S.S.C.E. 2000, 1999 S)*

Ans. The magnetic flux linked with a surface is defined as the number of magnetic field lines passing normally through that surface. Its SI unit is weber.

Q. 1.02. Weber is the unit of which physical quantity ? *(C.B.S.E. 1986)*

Ans. It is the unit of magnetic flux.

Q. 1.03. What are the SI units of (*i*) magnetic flux and (*ii*) magnetic field strength ? *(C.B.S.E. 1990)*

Ans. (*i*) Weber (*ii*) Tesla.

Q. 1.04. What is the basic cause of induced e.m.f. ? *(P.S.S.C.E. 2001 ; H.P.S.S.C.E. 1997)*

Ans. Whenever magnetic flux linked with a coil changes, induced e.m.f. is produced.

Q. 1.05. What factors govern the magnitude of the e.m.f. induced in an electric circuit ?

Ans. The magnitude of induced e.m.f. in an electric circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

Q. 1.06. What is the magnitude of the induced current in the circular loop KLMN of radius *r* [Fig. 1.26], if the straight wire PQ carries a steady current of magntude I ampere ? *(C.B.S.E. 2002)*

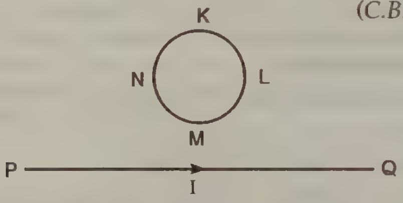


Fig. 1.26

Ans. No induced e.m.f. will be produced in the circular loop. It is because, the magnetic flux linked with the loop remains unchanged due to steady current through the wire PQ.

Q. 1.07. A conducting loop is held stationary normal to the field between the NS poles of a fixed permanent magnet. By choosing a magnet sufficiently strong, can we hope to generate current in the loop ? *(Text Question)*

Ans. No current will be induced in the loop, how so strong a magnet be chosen. It is because, there is no relative motion between the loop and the magnet. The induced current is produced only, if the magnetic flux linked with the loop changes.

Q. 1.08. Explain, whether an induced current will be developed in a conductor, if it is moved in a direction parallel to magnetic field. *(C.B.S.E. Sample Q. Paper)*

Ans. Induced current will not be developed in a conductor, if it is moved in a direction parallel to the magnetic field. It is because, in such a case, Lorentz force on free electrons in the conductor is zero and consequently no potential difference is created across the two ends of the conductor.

Q. 1.09. A closed conducting loop moves normal to the electric field between the plates of a large capacitor. Is a current induced in the loop, when it is (*i*) wholly inside the capacitor, (*ii*) partially outside the plates of the capacitor ? The electric field is normal to the plane of the loop. *(Text Question)*

Ans. The current is induced, if magnetic flux linked with the loop changes. No current is induced in either case, when the loop is wholly inside or partially inside the electric field.

Q. 1.10. Two identical loops, one of copper and another of aluminium are rotated with the same speed in the same magnetic field. In which case, the induced (*a*) e.m.f. (*b*) current will be more and why ? *(Text Question)*

Ans. The induced e.m.f. will be same in both but the induced current will be more in the copper loop as its resistance will be lesser as compared to that of the aluminium loop.

Q. 1.11. A straight conductor 1 m long moves at right angles to both its length and a uniform magnetic field. If the speed of the conductor is 2·0 m s^{-1} and the strength of the magnetic field is 10^4 gauss, find the value of induced e.m.f. in volt. *(P.S.S.C.E. 1996)*

Ans. Here, $l = 1$ m ; $v = 2{\cdot}0$ m s^{-1} ; $B = 10^4$ gauss $= 1$ tesla

Now, $e = B\ v l = 1 \times 2{\cdot}0 \times 1 = \mathbf{2{\cdot}0\ V}$

Q. 1.12. A metallic wire 1 m in length is moving normally across a field of 0·1 T with a speed of 5 m s^{-1}. Find the e.m.f. between the ends of the wire. *(I.S.C.E. 1997 ; similar H.P.S.S.C.E. 1995)*

Ans. Induced e.m.f., $e = B\,v\,l = 0{\cdot}1 \times 5 \times 1 = \mathbf{0{\cdot}5\ V}$

Q. 1.13. A wire cuts across a flux of 0·2 × 10^{-2} weber in 0·12 second. What is the e.m.f. induced in the wire ? *(I.S.C.E. 1995)*

Ans. Induced e.m.f., $$e = -\frac{d\phi}{dt} = -\frac{0{\cdot}2 \times 10^{-2}}{0{\cdot}12}$$
$$= \mathbf{-\,0{\cdot}0167\ V}$$

Q. 1.14. State Lenz's law of electromagnetic induction. *(C.B.S.E. 2002 ; P.S.S.C.E. 2002, 2001, 2000, 1999, 1998 S ; H.S.S.C.E. 2001, 1999 S, 1995 ; I.S.C.E. 1995)*

Ans. It states that the induced e.m.f. (or current) produced in a circuit always acts so that it opposes the change or the cause that produces it.

Q. 1.15. What factors govern the direction of e.m.f. ?

Ans. The direction of induced e.m.f. is always so that it opposes the change which produces it.

Q. 1.16. The induced e.m.f. is also called back e.m.f. Why ?

Ans. It is because, the induced e.m.f. opposes the applied voltage.

Q. 1.17. When current flowing in an inductive circuit is switched off, will the induced current be in the direction of main current or in the opposite direction ?

Ans. When the current is switched off, the current is in he direction of main current.

Q. 1.18. What are eddy currents ? How are they pro-luced ? *(H.S.S.C.E. 1997)*

Or

What are eddy currents ? *(H.S.S.C.E. 2001)*

Ans. The eddy currents are caused in a metallic onductor, when the magnetic flux linked with the conductor hanges.

Q. 1.19. Why the oscillations of a copper disc in a mag-etic field are lightly damped ? *(H.P.S.S.C.E. 1995)*

Ans. The eddy currents produced in the copper disc due to its motion in magnetic field) always oppose its scillatory motion and as such, the motion is damped.

Q. 1.20. Define the SI unit of self inductance.

(P.S.S.C.E. 2001, 2000, 1998 S ; H.S.S.C.E. 1998 S)

Ans. Refer to section 1.11.

Q. 1.21. If a rate of change of current of 4 A s^{-1} induces n e.m.f. of 20 mV in a solenoid, what is the self-inductance f the solenoid ? *(C.B.S.E. 1996)*

Ans. Here, $\frac{dI}{dt} = 4\text{ A s}^{-1}$; $e = 20\text{ m V} = 20 \times 10^{-3}\text{ V}$

Now, $e = L\frac{dI}{dt}$ (magnitude)

$$\therefore \quad L = \frac{e}{dI/dt} = \frac{20 \times 10^{-3}}{4} = \mathbf{5 \times 10^{-3}\ H}$$

$(\because \phi = LI)$

Q. 1.22. Magnetic flux of 5 microweber is linked with a oil, when a current of 1 mA flows through it. What is the self-nductance of the coil ? *(H.S.S.C.E. 1999 ; C.B.S.E. 1993)*

Ans. Here, $\phi = 5\,\mu\text{ Wb} = 5 \times 10^{-6}\text{ Wb}$; $I = 1\text{ mA} = 10^{-3}\text{ A}$

Now, $L = \frac{\phi}{I} = \frac{5 \times 10^{-6}}{10^{-3}} = \mathbf{5 \times 10^{-3}\ H}$

Q. 1.23. The magnetic flux threading a coil changes rom 12×10^{-3} Wb to 6×10^{-3} Wb in 0·01 s. Calculate the nduced e.m.f. *(C.B.S.E. 1990)*

Ans.
$$e = -\frac{d\phi}{dt} = -\frac{6 \times 10^{-3} - 12 \times 10^{-3}}{0{\cdot}01}$$
$$= \frac{6 \times 10^{-3}}{0{\cdot}01} = \mathbf{0{\cdot}6}$$

Q. 1.24. Explain, why the inductance coils are made of opper.

Ans. The inductance coils made of copper will have ery small ohmic resistance. Due to change in magnetic flux, large induced current will be produced in such an nductance, which will offer appreciable opposition to the low of current due to the applied e.m.f.

Q. 1.25. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely wound wires in series carrying current in the opposite senses. What do you expect about its self-inductance ? Will it be large or small ? *(Text Question)*

Ans. The self-inductance will be small due to cancellation of inductive effects. It is because, currents in two sets of wires flow in opposite directions and produce flux in opposite directions.

Q. 1.26. Coils in the resistance boxes are made from doubled up insulated wire. Why ?

Ans. It is done so as to cancel the effect of self-induced e.m.f. in the coil. In the coil made of doubled up wire, the inductive effect in the two wires will be opposite to each other.

Q. 1.27. A lamp connected in parallel with a large inductor glows brilliantly before going off, when the switch is put off. Why ?

Ans. As the switch is put off, a large e.m.f. is set up in the inductor due to break in the current through the inductor. The large e.m.f. makes the bulb glow brilliantly.

Q. 1.28. Why is spark produced in the switch of a fan, when it is switched off ?

(H.S.S.C.E. 1998 S ; H.P.S.S.C.E. 1996)

Ans. Due to the sudden break, a large induced e.m.f. is set up across the gap in the switch. Due to this, sparking takes place in the switch.

Q. 1.29. If the self-inductance of an air core inductor increases from 0·01 mH to 10 mH on introducing an iron core into it, what is the relative permeability of the core used ? *(C.B.S.E. 2002)*

Ans. Relative permeability, $\mu_r = \frac{10}{0{\cdot}01} = \mathbf{1000}$

Q. 1.30. What is meant by mutual induction ?

(H.P.S.S.C.E. 1998)

Ans. The phenomenon according to which induced e.m.f. is produced in a coil due to change in magnetic flux linked with a neighbouring coil is called mutual induction.

Q. 1.31. Define coefficient of mutual inductance for a pair of coils. *(I.S.C.E. 1998)*

Ans. Refer to section 1.15.

Q. 1.32. What is the unit of mutual inductance ?

(I.S.C.E. 1996)

Ans. Henry. For definition, refer to section 1.15.

Q. 1.33. Refer to Figs. 1.18, 1.19 and 1.20. In which case will the mutual inductance of two coils be maximum, if the number of turns remains the same in each case ?

(C.B.S.E. 1999 S)

Ans. In the arrangement shown in Fig. 1.18, the mutual inductance of the two coils will be maximum.

For details, refer to section 1.14.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What are the dimensions of magnetic flux ?

(P.S.S.C.E. 1999 S)

Ans. The magnetic flux linked with a surface,

$$\phi = B\,A\cos\theta$$

Since $F = q\,v$, we have

$$\phi = \frac{F}{q\,v}A\cos\theta$$

$$\therefore \quad [\phi] = \frac{[M\,L\,T^{-2}] \times [L^2]}{[A\,T][L\,T^{-1}]} = \mathbf{[M\,L^2\,T^{-2}\,A^{-1}]}$$

$(\because$ charge = current × time$)$

Q. 1.02. State Faraday's laws of electromagnetic induc-tion. *(P.S.S.C.E. 2002, 1999 S ; H.S.S.C.E. 2001, 1998 S ; I.S.C.E. 1995)*

Ans. Refer to section 1.04.

Q. 1.03. State Lenz's law. Show that it is in accordance with the law of conservation of energy. *(H.P.S.S.C.E. 2002 ; C.B.S.E. 1997)*

Or

Briefly explain, how Lenz's law supports the law of conservation of energy. *(C.B.S.E. 1999)*

Or

State Lenz's law. Does it violate law of conservation of energy ? *(P.S.S.C.E. 1993)*

Ans. Refer to sections 1.05 and 1.06.

Q. 1.04. Show that Lenz's law is a direct consequence of the law of conservation of energy. *(H.P.S.S.C.E. 1996)*

Or

Show that Lenz's law obeys the law of conservation of energy. *(P.S.S.C.E. 1998 S ; C.B.S.E. 1996)*

Or

Briefly explain, how Lenz's law supports the law of conservation of energy. *(C.B.S.E. 1999 S)*

Ans. Refer to section 1.06.

Q. 1.05. The electric current in a wire in the direction from B to A is decreasing. What is the direction of induced current in the metallic loop kept above the wire as shown in Fig. 1.27 ? *(C.B.S.E. 1997 ; I.I.T. 1979)*

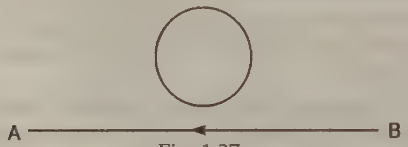


Fig. 1.27

Ans. When the increasing current flows through the wire in the direction from point B to A, the increasing magnetic field is produced ; which is directed perpendicular to the plane of the loop (or the plane of paper) and in inward direction. Due to this, induced e.m.f. is produced in the loop which opposes the magnetic field produced due to the current flowing through the wire *i.e.* induced current in the loop should flow in a direction so that it produces magnetic field perpendicular to the plane of the loop and in outward direction. Maxwell's cork screw rule tells that induced current in the loop will flow in anticlockwise direction.

Q. 1.06. A magnet is moved in the direction indicated by an arrow between two coils AB and CD as shown in Fig. 1.28. Suggest the direction of current in each coil. *(C.B.S.E. 2001 S)*

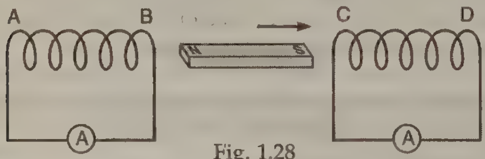


Fig. 1.28

Ans. For coil AB. N-pole of the magnet is moving away from the coil AB, the end A of the coil will behave as S-pole so as to oppose from motion of the magnet. Therefore, looking from the end A, the current in the coil AB will be in anticlockwise direction.

For coil CD. In this case, the end C of the coil CD should behave as S-pole so as to repel the approaching magnet. Looking from the end D, the direction of current in coil CD will be anticlockwise.

Q. 1.07. A cylindrical bar magnet is kept along the axis of a circular coil and near it as shown in Fig. 1.29. Will there be any induced e.m.f. at the terminals of the coil, when the magnet is rotated (*a*) about its own axis and (*b*) about an axis perpendicular to the length of the magnet ? *(C.B.S.E. 2001 S)*

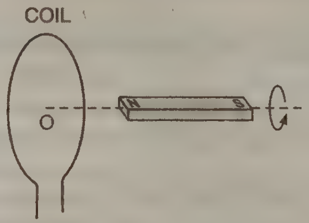


Fig. 1.29 (*a*)

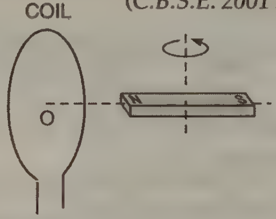


Fig. 1.29 (*b*)

Ans. Fig. 1.29. (*a*). When the magnet is rotated about its own axis, there is no change in the magnetic flux liked with the coil. Hence, no induced e.m.f. is produced in the coil.

Fig. 1.29. (*b*). When the magnet is rotated about an axis perpendicular to its length, the orientation of the magnetic field due to the magnet will change continuously. Due to this, the magnetic flux linked with the coil will also change continuously and it will result in the production of induced e.m.f. in the coil.

Q. 1.08. Figs. 1.30 (*a*) and (*b*) below show planar loops of different shapes moving out or into a region of magnetic field, which is directed normal to the plane of the loop and away from the readeı. Determine the direction of induced current. *(Text Question)*

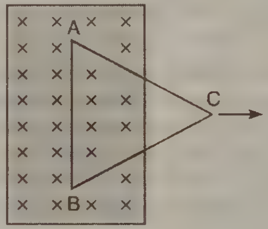


Fig. 1.30 (*a*)

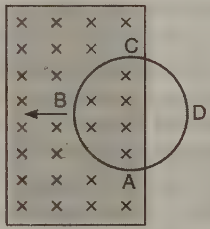


Fig. 1.30 (*b*)

Ans. ***In Fig. 1.30* (*a*) :** As the planar loop moves towards right, the magnetic flux linked with the loop decreases. The induced current will flow in a direction, so that the magnetic flux tends to increase through the loop *i.e.* the loop tends to move back into the magnetic field. For this, current in the arm AB (inside the magnetic field) should flow in such a direction so that the force on it acts towards left. It will happen so, if the induced current flows through the loop in the direction CBAC.

***In Fig. 1.30* (*b*) :** The application of Lenz's law tells that current in the part ABC (inside the magnetic field) should flow in such a direction that force on this part acts downwards. It will happen so, if the current through the part ABC flows along CBA i.e. the current through the loop flows in the direction CBADC.

Q. 1.09. Figs. 1.31 (*a*) and (*b*) below show planar loops of different shapes moving out of or into a region of magnetic

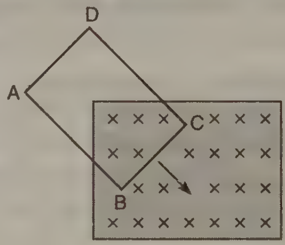


Fig. 1.31 (*a*)

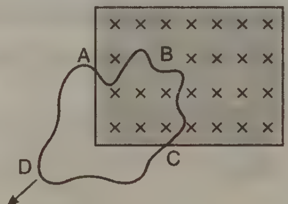


Fig. 1.31 (*b*)

ld, which is directed normal to the plane of the loops and ay from the reader. Determine the direction of induced rrent in each loop using Lenz's law. (*Text Question*)

Ans. *In Fig. 1.31 (a)* : Due to motion of loop ABCD in the ection shown, the flux linked with the loop increases. cording to Lenz's law, force on the arm BC (inside the gnetic field) should act opposite to its direction of motion. r this, current should flow through the loop in the direction DAB.

In Fig. 1.31 (b) : According to Lenz's law, the direction current in the part ABC (inside the magnetic field) should along ABC, so that it experiences force in a direction posite to the direction, in which it is moved. Hence, the ection of the current through the loop should be along CDA.

Q. 1.10. An irregular shaped wire PQRS (as shown in 1.32) placed in a uniform magnetic field perpendicular the plane of the paper changes into a circular shape. ow with reason the direction of the induced current in loop. (*C.B.S.E. 1998 S*)

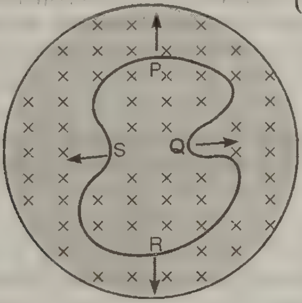


Fig. 1.32

Ans. When an irregular shaped wire PQRS changes to cular loop, the magnetic flux linked with the wire increases e to increase in area of the loop. The induced e.m.f. will use current to flow in the direction, so that the wire is pulled ward from all sides. According to Fleming's left hand rule, rce on wire PQRS will act inward from all sides, if the rrent flows in the direction PSRQ.

Note. When the wire loop changes to a circular loop, its ea increases. It is because, for a given perimeter, of all the ometrical shapes, a circle has the maximum area.

Q. 1.11. Use Lenz's law to determine the direction induced current in the situations described by the gs. 1.33 (*a*) and (*b*) given below :

In Fig. 1.33 (*a*) a wire of irregular shape turning into a rcular shape and in Fig. 1.33 (*b*), a circular loop being formed into a narrow straight wire. The cross (×) indi- tes magnetic field into the paper, and the dot (.) indicates agnetic field out of the paper. (*Text Question*)

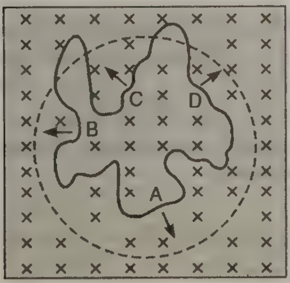


Fig. 1.33 (*a*)

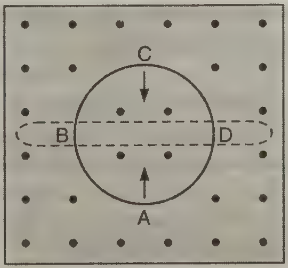


Fig. 1.33 (*b*)

Ans. *In Fig. 1.33 (a)* : When a wire of irregular shape turns into a circular loop, the magnetic flux linked with the loop increases due to increase in area (The circular loop has greater area than the loop of irregular shape). The induced e.m.f. will cause current to flow in such a direction, so that the wire forming the loop is pulled inward from all sides. It requires that current should flow in the direction ADCBA.

In Fig. 1.33 (b) : By appling Lenz's law, it follows that the current will flow in the direction ADCBA.

Q. 1.12. The closed loop PQRS is moving into a uniform magnetic field acting at right angles to the plane of the paper as shown in Fig. 1.34. State the direction in which the induced current flows in the loop. (*C.B.S.E. 2002*)

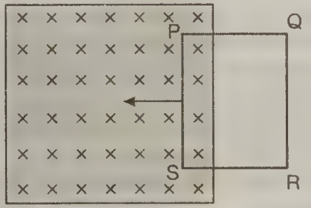


Fig. 1.34

Ans. As the closed loop PQRS moves into the uniform magnetic field, magnetic flux linked with the loop will increase. According to Lenz's law, induced e.m.f. produced in the loop should act so that the coil is opposed from moving into the magnetic field. For this to occur, force on the arm PS of the coil should act towards left. It requires that curernt through the arm PS should be from the end P to S *i.e.* current in the coil should be in the direction of SRQP.

Q. 1.13. Twelve wires of equal lengths are connected in the form of a skeleton-cube which is moving with a velocity $\vec{v}$ in the direction of a magnetic field $\vec{B}$ [Fig. 1.35]. Find the e.m.f. in each arm of the cube.

(*C.B.S.E. 2001 S*)

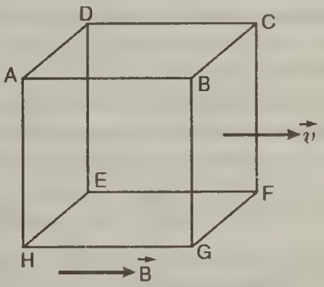


Fig. 1.35

Ans. Force on a charged particle moving inside magnetic field is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Since $\vec{v}$ and $\vec{B}$ are parallel, the force on electrons in any arm of the skeleton cube will be zero. As such, there cannot be drift of electrons in any arm from its one end to the other. Hence, no induced e.m.f. will be produced in any arm of the skeleton-cube.

Q. 1.14. A copper ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring [Fig. 1.36]. Will the acceleration of the falling magnet be equal to, greater than or less than that due to gravity ? (*P.S.S.C.E. 1995*)

Or

Why does acceleration of a magnet falling through a long solenoid decrease ? (*C.B.S.E. 1999*)

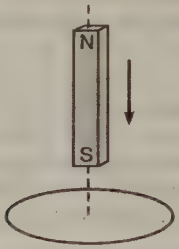


Fig. 1.36

Ans. When the magnet falls, the magnetic flux through the copper ring increases and it results in induced e.m.f. in the ring. The induced e.m.f. so produced, opposes the falling magnet. Therefore, the acceleration of the falling magnet will be less than that due to gravity.

Q. 1.15. An induced current has no direction of its own. Explain, why. (*P.S.S.C.E. 2001 ; H.S.S.C.E. 1995*)

Ans. The induced current always flows in a direction so that it opposes the cause or the change which produces it. For example, as shown in Fig. 1.36, when the magnet is dropped through the copper ring, the induced current flows in the ring in clockwise direction (when seen from above), till the magnet does not come out of the ring from the other side. But as soon as the magnet leaves the ring, the direction of the current will become anticlockwise in accordance with Lenz's law. It shows that an induced current has no direction of its own.

Q. 1.16. A magnetic field of flux density 10 T acts normal to a coil of 50 turns having 100 cm^2 area. Find e.m.f. induced, if the coil is removed from the magnetic field of 0·1 s. (*H.S.S.C.E. 1997*)

Ans. Here, $n = 50$; $A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$;

$B = 10\text{ T}$ and $dt = 0{\cdot}1\text{ s}$

Magnetic flux linked with the coil initially,

$$\phi_1 = n\,B\,A = 50 \times 10 \times 10^{-2} = 5\text{ Wb}$$

Magnetic flux linked with the coil finally *i.e.* on being removed from the magnetic field, $\phi_2 = 0$

Change in magnetic flux through the coil,

$$d\phi = \phi_2 - \phi_1 = 0 - 5 = -5\text{ Wb}$$

$$\text{Induced e.m.f., } e = -\frac{d\phi}{dt} = -\left(\frac{-5}{0{\cdot}1}\right) = \mathbf{50\ V}$$

Q. 1.17. How are eddy currents produced ? Give two applications of eddy currents.

(*P.S.S.C.E. 2000, 1999 S ; C.B.S.E. 1993, 1992*)

Or

What are eddy currents ? Name any two of its applications. (*H.P.S.S.C.E. 1997, 1995*)

Ans. Eddy currents are produced in a metallic conductor, when magnetic flux linked with the conductor changes.

Applications : 1. In energy meters **2.** In dead beat galvanometers.

Q. 1.18. How can eddy currents be reduced ?

(*H.S.S.C.E. 1997 S*)

Ans. Refer to section 1.09.

Q. 1.19. Why is the coil of a dead beat galvanometer wound on a metal frame ? (*C.B.S.E. 1991*)

Ans. On switching off the current in a galvanometer, the coil of the galvanometer does not come to rest immediately. It oscillates about its equilibrium position. But the coil of a dead beat galvanometer comes to rest immediately. It is du to the reason that the eddy currents are setup in the metalli frame, over which the coil is wound and the eddy current oppose the oscillatory motion of the coil.

Q. 1.20. Why are the to and fro oscillations completel absent in better designed galvanometer ?

Ans. Refer to section 1.09.

Q. 1.21. An iron bar falling vertically through th hollow region of a thick cylindrical shell made of coppe experiences a retarding force. What can you conclud about the iron bar ? (*Text Question*

Ans. The iron bar must be a magnet. When the ba magnet falls through the copper shell, it produces a chang in magnetic flux through it. Due to this, eddy currents ar produced in the shell. The eddy currents so produced oppos the fall of the magnet (Lenz's law) and hence bar magne experiences retarding force.

Q. 1.22. A coil A is connected to a volmeter V and th other coil B to an alternating current source [Fig. 1.37]. If large copper sheet C is placed between the two coils, hov does the induced e.m.f. in the coil A change due to curren in coil B ? (*C.B.S.E. 2001 S*

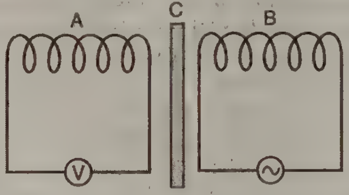


Fig. 1.37

Ans. In the absence of copper sheet, induced e.m.f. wi be produced in the coil A due to mutual induction betwee coils A and B. As a result, voltmeter will show deflectio depending on the magnitude of induced e.m.f.

When the copper sheet is placed between the two coil eddy currents will be set up in the coil. Since the eddy current have opposing effect, the magnetic flux linked with the co A due to eddy currents will always be opposite to that du to alternating current through the coil B. Hence, induce e.m.f. will get reduced.

Q. 1.23. Define the term `self-induction'. Write its S unit. Write two factors on which the self-inductance of a coi depends. (*C.B.S.E. 1999, 1995, 1993 ; H.S.S.C.E. 1995*

Ans. For definition of self-induction, refer to section 1.1 Its SI unit is henry.

The self-inductance of a coil depends upon its area o cross-section, the number of turns and the permeability of th material of the core.

Q. 1.24. How does the self inductance of a coil change when : (*i*) the number of turns in the coil is decreased (*ii*) a iron rod is introduced into it ? Justify you answer in each case

(*C.B.S.E. 1998*

Ans. Self inductance of a coil, $L = \mu_0\, n^2\, l\, A$

(*i*) When the number of turns is decreased, the sel inductance will decrease. It is because,

$$L \propto n^2$$

(*ii*) On introducing the iron rod into the coil, the sel inductance will become

$$L' = \mu_0\, \mu_r\, n^2\, l\, A,$$

where μ_r is the relative premeability of iron. Since, $\mu_r > 1$, sel induction will increase.

Q. 1.25. Define self-inductance. Write down the xpression for the self-inductance of a long solenoid of ngth *l* having N turns.

Ans. For definition of self-inductance, refer to section 11.

Self-inductance of a long solenoid, $L = \mu_0 n^2 l A$, here the letters have their usual meanings.

Q. 1.26. Self-induction is called the inertia of electricity. xplain, why. (*P.S.S.C.E. 2001*)

Ans. Self-induction of a coil is its property by virtue of hich the coil opposes any change in the current flowing rough it. It is because, the induced e.m.f. produced opposes e change in current. For this reason, self-induction is called nertia of electricity.

Q. 1.27. Define the term mutual inductance. Write its I units. Give two factors on which the coefficient of mutual nductance between a pair of coil depends. (*C.B.S.E. 1999, 1995*)

Ans. For definition of mutual inductance, refer to section .15. The SI unit of mutual inductance is henry.

The mutual inductance between two coils depends upon he number of turns of the two coils, their relative orientations nd the permeability of the material of the core.

Q. 1.28. How does the mutual inductance of a pair of oils change, when (*i*) the distance between the coils is inreased ? (*ii*) the number of turns in each coil is decreased ? Justify your answer in each case. (*C.B.S.E. 1998*)

Ans. (*i*) On increasing distance between the coil, the nagnetic flux linked with secondary coil due to current lowing through primary coil will decrease. Hence, mutual nductance of the two coils will decrease.

(*ii*) Mutual inductance of two coils, $M = \mu_0 n_1 n_2 A l$

Obviously, the mutual inductance of the two coils will decrease on decreasing the number of turns in each coil.

Q. 1.29. Making or breaking of current in a coil produces a momentary current in the neighbouring coil of another circuit. Why ?

Ans. Before making current in a coil, the current is zero and before breaking, the current is maximum. In other words, it is constant in both the cases. Obviously, on making or breaking the current in a circuit, the current starts changing. The changing current produces changing magnetic field, which in turn produces induced current in the neighbouring circuit.

Q. 1.30. Define self-inductance and mutual inductance. State their units in SI.

(*H.S.S.C.E. 1994, 1992 ; H.P.S.S.C.E. 1993 ; C.B.S.E. 1991*)

Ans. For definition of self-inductance and mutual inductance, refer to sections 1.11 and 1.15.

The SI unit of both self-inductance and mutual inductance is henry.

Q. 1.31. Distinguish between self induction and mutual induction. (*C.B.S.E. 1997*)

Ans. Refer to sections 1.10 and 1.14.

Q. 1.32. What is non-inductive wiring of coils ?

Ans. To make non-inductive coils, they are made of doubled up insulated wires. In such a coil, the direction of the current in the two wires will be opposite to each other. Due to this, the two wires will produce equal and opposite magnetic fields. Therefore, the net magnetic field linked with the coil is always zero and hence no inductive effect takes place.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. Two circular conductors are perpendicular to each other as shown in Fig. 1.38. Will a current be induced in the conductor A, if the current is changed in the conductor B ?

Ans. When the current passes through the circular conductor B, the magnetic field produced is parallel to the plane of the circular conductor A. As such, the magnetic flux linked with circular conductor A due to the magnetic field produced due to the flow of current through conductor B is zero.

When the current through conductor B is changed, the magnetic flux linked with conductor will remain zero. Hence, no current will be induced in conductor A, when current through conductor B is changed.

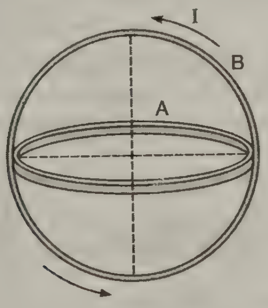


Fig. 1.38

Q. 1.02. A rectangular loop and a circular loop are moving out of a uniform magnetic field region to a field free region with a constant velocity. In which loop do you except the induced e.m.f. to be constant during the passage out of the field region ? The field is normal to the loops. (*Text Question*)

Ans. In case of a rectangular loop, induced e.m.f. will be constant. It is because, when such a loop is moved, the rate of change of its area is uniform. But when a circular loop is moved out of the magnetic field, its rate of change of area is not constant and hence the induced e.m.f. produced will not be constant.

Q. 1.03. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ [Fig. 1.39]. At the position MNQ, the speed of the ring is v. What is the potential difference developed across the ring at the position MNQ ?

(*I.I.T. 1996*)

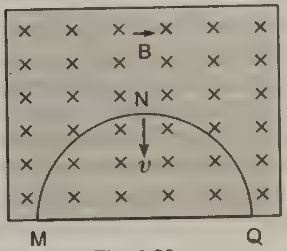


Fig. 1.39

Ans. Suppose that the semiconductor ring falls through an infinitesimally small distance dx from its initial position MNQ to M′Q′N′ in time dt [Fig. 1.40]. Then,

decrease in area of the ring inside the magnetic field,

$$dA = -\text{MQQ'M'} = -\text{M'Q'} \times \text{QQ'} = -2\,\text{R}\,dx$$

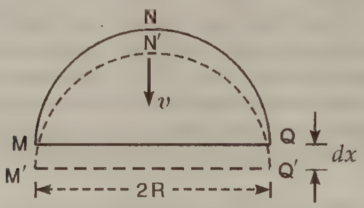


Fig. 1.40

Therefore, change in magnetic flux linked with the ring,

$$d\phi = \text{B} \times dA = \text{B} \times (-2\,\text{R}\,dx) = -2\,\text{B}\,\text{R}\,dx$$

The potential difference developed across the ring,

$$e = -\frac{d\phi}{dt} = -\left(-2\,\text{B}\,\text{R}\,\frac{dx}{dt}\right) = \mathbf{2\,B\,R}\,v$$

Here, $v = \frac{dx}{dt}$ is the speed with which the ring is falling.

Q. 1.04. An aircraft flies along the meridian. Will the potentials of the ends of its wings be the same ? Will the potential difference change, if the aircraft flies in any other direction with the same velocity ?

Ans. As the aircraft flies, magnetic flux changes through its wings due to the vertical component of the earth's magnetic field. Due to this, induced e.m.f. is produced across the wings of the aircraft. Therefore, the wings of the aircraft will not be at the same potential.

If the aircraft flies in any other direction, the value of the potential difference (induced e.m.f.) will not change. It is because, the value of potential difference depends only on the vertical component of the earth's magnetic field and the velocity of the aircraft.

Q. 1.05. (*a*) Will the earth's magnetic field induce current in an artificial satellite with a metal surface that is in orbit around the equator ? Around the poles ?

(*a*) If so how would these currents affect the motion of the satellite ?

Ans. (*a*) There will be no induced currents in the metal of the satellite, which is orbiting in equitorial plane. It is because, the magnetic flux does not change through the metal of the satellite in such a orbit. In other orbits (including orbit around the poles), the value of the magnetic field will change both in magnitude and direction. Due to this, the magnetic flux through the satellite will change and hence induced currents will be produced in the metal of the satellite.

(*b*) The induced currents in the metal of the satellite will produce magnetic field and therefore the satellite will itself act like a magnet. It will experience a small magnetic interaction with the earth so that the force on the satellite will not be purely gravitational. Therefore, there will be deflection from the gravitational path, the satellite would take if it moved in the gravitatonal field alone. Part of the effect of interaction would cause the satellite to lose energy, dissipated in the form of heat. Paradoxially, if it loses energy, it will speed up.

Q. 1.06. A copper coil L wound on a soft iron core and a 15 W-110 V lamp are connected to a 30 V battery through a tapping key as shown in Fig. 1.41. When the ke is closed, the lamp glows dimly. But when the key i suddenly opened, the lamp flashes for an instant to muc greater brightness. Explain, why.

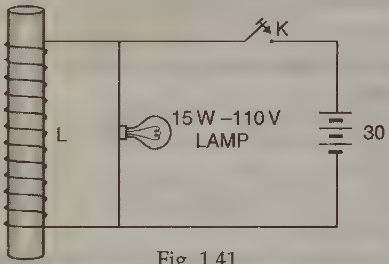


Fig. 1.41

Ans. On closing the key, the lamp glows dimly, becaus across a 110 V-lamp, the voltage applied is only 30 V. Whil the key is closed, some current also flows through the coil As the resistance of copper coil is very low, the current in th coil is much greater than the current through the lamp.

When the key is opened, the current through the lam stops and the current circulating through the coil L tends t decrease. But this tendency is opposed by the induced e.m. produced in the coil due to self induction. As the induce e.m.f. opposes the change, it tends to maintain the curren and makes it to flow through the lamp. This current can b significantly larger than would be produced in the lamp b the 30 V battery ; in fact it might be much larger tha would be produced by a 110 V battery. The large curren (driven by the induced e.m.f.) causes the lamp to flash brigh and may even burn it out.

Q. 1.07. A small square loop of wire of side l is place inside a large square loop of wire of side L (L > > l The loops are coplanar and their centres coincide [Fig. 1.42 Find the mutual inductance of the system. (*I.I.T. 1998*)

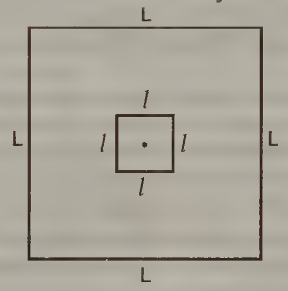


Fig. 1.42

Sol. Suppose that a current I passes through the squar loop of side L [Fig. 1.43].

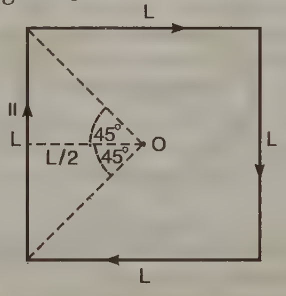


Fig. 1.43

Then, magnetic field at the centre O of the loop,

$$B = 4 \times B',$$

where B′ is magnetic field at the centre of loop due to one side of the square loop.

Now, $B' = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}(\sin\phi_1 + \sin\phi_2)$

Here, $a = \frac{L}{2}; \phi_1 = \phi_2 = 45°$

$$\therefore \quad B' = \frac{\mu_0}{4\pi} \cdot \frac{I}{L/2}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0}{2\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0}{\sqrt{2}\,\pi L}$$

Hence, $B = 4 \times \frac{\mu_0 I}{\sqrt{2}\,\pi L} = \frac{2\sqrt{2}\,\mu_0 I}{\pi L}$

Magnetic flux linked with the small square loop,

$$\phi = B \times l^2 = \frac{2\sqrt{2}\,\mu_0 I l^2}{\pi L} \quad ...(i)$$

If M is coefficient of mutual induction between the two loops, then

$$\phi = MI \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$M = \frac{2\sqrt{2}\,\mu_0 l^2}{\pi L}$$

Q. 1.08. A small resistor (say, a lamp) is usually put in parallel to the current carrying coil of an electromagnet [Fig. 1.44]. What purpose does it serve ?

(Text Question ; Pre-degree Kerala, 1991)

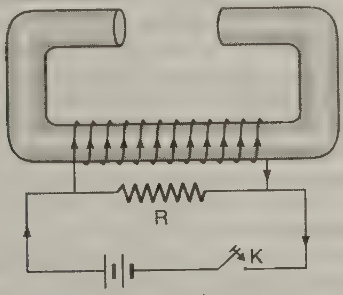


Fig. 1.44

Ans. When the current in the coil of an electromagnet is switched off, the induced e.m.f. produced at break is very large (particularly in large electromagnets). Due to this, spark is produced at the open switch. However, if a lamp (or a resistor) is connected across the coil of the magnet, the induced e.m.f. sends current through it and sparks are avoided.

Q. 1.09. A rectangular wire frame ABCD rotates with a constant velocity around one of its sides parallel to a nearby current carrying straight conductor [Fig. 1.45]. Indicate the positions, in which the minimum and maximum e.m.f.s will be induced in the frame.

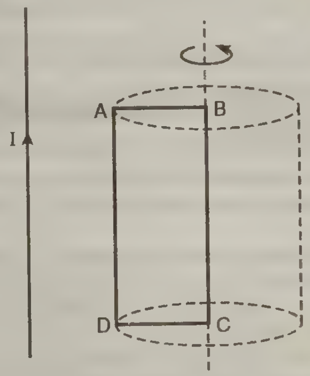


Fig. 1.45

Ans. The lines of force of the magnetic field produced by the current carrying straight conductor are circular in shape and have their centres on the straight conductor.

When the frame lies in the plane passing through the straight conductor, the magnetic field lines pass normally through it. When the magnetic field lines pass normally through the plane of the rotating rectangular frame ABCD, the magnetic flux linked with the frame is maximum but the rate of change of magnetic flux is minimum and likewise induced e.m.f. produced is also minimum. Hence, the induced e.m.f. produced is minimum, when the frame lies in the plane passing through the straight conductor.

On the other hand, when the frame is perpendicular to the plane passing through the straight conductor, the magnetic flux linked with the frame is minimum ; the rate of change of magnetic flux is maximum and likewise induced e.m.f. produced is also maximum.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain the concept of magnetic flux linked with a surface. Give its units and obtain its dimensional formula.
2. State and explain Faraday's laws of electromagnetic induction. *(H.P.S.S.C.E. 2000, 1999 ; H.S.S.C.E. 1996)*
3. State Faraday's laws of electromagnetic induction and Lenz's law. *(H.S.S.C.E. 2002)*
4. State Lenz's law of electromagnetic induction and show that it is in accordance with law of conservation of energy. *(H.S.S.C.E. 1998)*
5. State Lenz's law. Prove that the charge induced is independent of time. *(P.S.S.C.E. 1995)*
6. A straight conducting rod of length l is moved with a velocity v in a transverse magnetic field. Deduce the magnitude and direction of induced e.m.f. developed. If the ends of the rod are joined with a wire, indicate the direction of flow of induced current.
7. By calculating magnetic Lorentz force on a conductor moved in a transverse magnetic field, deduce Faraday flux law

$$e = -\frac{d\phi}{dt}$$

8. Derive an expression for the induced e.m.f. produced by changing the area of a rectangular coil placed perpendicular to a uniform magnetic field. *(C.B.S.E. 1992)*
9. A rectangular coil of N turns and area of cross-section A is held in a time-varying field given by $B = B_V \sin \omega t$, with the plane of the coil normal to the magnetic field. Deduce an expression for the e.m.f. induced in the coil. *(C.B.S.E. 2000)*

10. What are eddy currents ? Give experiments to explain their origin. *(H.P.S.S.C.E. 1997, 1996 ; P.S.S.C.E. 1996)*
11. What are eddy currents ? Discuss the effects of eddy currents in electrical appliances, where iron is used.
12. What are eddy currents and electromagnetic damping ? How are these used in a speedometer, an electric brake and a dead beat galvanometer ? *(P.S.S.C.E. 1999, 1996)*
13. What are eddy currents ? How they can be minimised ? *(H.S.S.C.E. 2002, 1997 ; H.P.S.S.C.E. 2001;)*
14. What are eddy currents ? Give their merits and demerits. *(H.S.S.C.E. 1999 S)*
15. Explain, what do you mean by 'self-induction' and 'self-inductance' of a coil. Define S.I. unit of self-inductance.
16. Explain self induction of a coil and give its unit. *(H.S.S.C.E. 2002)*
17. Explain the phenomenon of self induction. Define coefficient of self-induction. What are its units ? *(H.S.S.C.E. 1994 ; C.B.S.E. 1990)*
18. Define an expression for the self-inductance of a long solenoid. *(C.B.S.E. 2001 ; H.S.S.C.E. 2001)*
19. Calculate the coefficient of self-induction for a long air cored solenoid. *(P.S.S.C.E. 1999 S)*
20. Explain the phenomenon of mutual induction. Define coefficient of mutual induction. Give its S.I. units. *(H.S.S.C.E. 1993 ; C.B.S.E. 1990)*
21. Explain mutual inductance and define its SI unit. *(H.S.S.C.E. 2002)*
22. Define coefficient of mutual inductance of two coils. A secondary coil of n_2 turns is wound on a long solenoid of area of cross-section A, having a primary coil of n_1 turns per unit length. What is the mutual inductance of the two coils ? *(C.B.S.E. 1990)*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Explain Faraday's and Lenz's law of electromagnetic induction by describing suitable experiments. *(P.S.S.C.E. 1998, 1997 S)*
2. State Faraday's laws of electromagnetic induction and explain three methods of producing induced e.m.f. *(P.S.S.C.E. 2002, 2000, 1997 ; H.S.S.C.E. 1996)*
3. What is meant by self-induction and coefficient of self-induction ? Explain. Derive an expression for the coefficient of self-induction for a long air-cored solenoid. *(P.S.S.C.E. 2001)*
4. Explain the phenomenon of mutual induction. Define coefficient of mutual induction and state its unit. Also derive an expression for coefficient of mutual induction between two long solenoids. *(P.S.S.C.E. 2002, 2000)*
5. Explain what is meant by mutual induction and mutual inductance. Give the practical unit of mutual inductance. How it can be defined in terms of induced e.m.f. and rate of growth of current ? Derive expression for mutual inductance between two solenoids.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Faraday law of Electromagnetic Induction

1. The magnetic flux through a coil perpendicular to its plane is varying according to the relation
$$\phi = (4\,t^3 + 5\,t^2 + 8\,t + 5) \text{ weber}$$
Calculate the induced current through the coil at $t = 2$ s, if the resistance of the coil is 3·1 ohm. *(P.S.S.C.E. 1999)* [**Ans.** 24·5 A]
2. A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0·5 s. The magnetic flux between the pole pieces is known to be 8×10^{-4} Wb. Estimate the e.m.f. induced in the wire. *(Text Problem)* [**Ans.** $1{\cdot}6 \times 10^{-3}$ V]
3. A magnetic field of flux density 1·0 Wb m^{-2} acts normal to a 80 turn coil of 0·01 m^2 area. Find the e.m.f. induced in it, if this coil is removed from the field in 0·1 second. *(H.S.S.C.E. 2002)* [**Ans.** 8 V]
4. A magnetic field of flux density 10 T acts normal to a coil of 50 turns having 100 cm^2 area. Find e.m.f. induced, if the coil is removed form magnetic field in 0·1 s. *(H.S.S.C.E. 1998)* [**Ans.** 50 V]
5. Find the magnitude of e.m.f. induced in a 200 turn coil with cross-sectional area of 0·16 m^2, if the magnetic field through the coil changes from 0·10 Wb m^{-2} to 0·30 Wb m^{-2} at a uniform rate over a period of 0·05 s. *(H.S.S.C.E. 2002)* [**Ans.** 128 V]
6. Find the e.m.f. induced in a coil of 200 turns and cross-sectional area 0·2 m^2, when a magnetic field perpendicular to the plane of the coil changes from 0·1 Wb m^{-2} to 0·5 Wb m^{-2} at a uniform rate over a period of 0·05 s. *(P.S.S.C.E. 1992)* [**Ans.** 320 V]
7. A wire 88 cm long bent into a circular loop is placed perpendicular to the magnetic field of flux density 2·5 Wb m^{-2}. Within 0·5 s, the loop is changed into a square of each side 22 cm and flux density is increased to 3·0 Wb m^{-2}. Calculate the value of e.m.f. induced. *(H.P.S.S.C.E. 1990)* [**Ans.** 0·018 V]
8. A square copper coil of each side 8 cm consists of 100 turns. The coil is initially in vertical plane, such that the plane of coil is normal to uniform magnetic field of induction 0·4 weber m^{-2}. The coil is turned through 180° about a horizontal axis in 0·2 s. Find the induced e.m.f. *(P.S.S.C.E. 1999 S)* [**Ans.** 2·56 volt]
9. A conducting rod 1 m in length moves with a frequency of 50 r.p.s. with one end at the centre and the other end at the circumference of a circular metallic ring of radius 1 m about an axis passing through the centre of the coil and perpendicular to the plane of the coil. A constant magnetic field parallel to the axis is present everywhere. What is the e.m.f. developed between the centre and the metallic ring ?($B = 1{\cdot}0$ Wb m^{-2}). [**Ans.** 157·08 V]
10. A 1 m long conducting rod rotates with an angular speed 400 rad s^{-1} about an axis normal to the rod and passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0·5 T parallel to the axis exists everywhere. Calculate the e.m.f. developed between the centre and the ring. *(Text Problem)* [**Ans.** 100 V]
11. A copper disc of radius 10 cm placed with its plane normal to a uniform magnetic field completes 1200 rotations per minute. If induced e.m.f. between the centre and edge of the disc in 6·284 mV, find the intensity of the magnetic field. Take $\pi = 3{\cdot}142$. [**Ans.** 10^{-2} T]
12. A metal disc of radius 200 cm is rotated at a constant angular speed of 60 rad s^{-1} in a plane at right angles to an external field of magnetic induction 0·05 Wb m^{-1}. Find the

e.m.f. induced between the centre and a point on the rim. *(P.S.S.C.E. 1991)* **[Ans.** 6 V]

ype B. On Motional e.m.f.

13. If a 10 m long metallic bar moves in a direction at right angle to the magnetic field with a speed of $5{\cdot}0 \text{ m s}^{-1}$, 25 V e.m.f. is induced in it. Find the value of magnetic field intensity. *(P.S.S.C.E. 1999)* **[Ans.** 0·5 T]

14. A conductor 2 m long moves in a magnetic field of flux density $0{\cdot}5 \times 10^{-4}$ tesla with a speed of 36 km per hour. Calculate the e.m.f. induced in it, if it is perpendicular to both its length and field. **[Ans.** 10^{-3} volt]

15. A straight wire of length 2 m moves at a speed of 500 cm s^{-1} in a direction at right angles to a uniform field of induction 1000 gauss. Find the e.m.f. generated in volt. **[Ans.** 1 volt]

16. A metre gauge train is running due north with a constant speed of 90 km h^{-1} on a horizontal track. If the vertical component of earth's magnetic field is $3 \times 10^{-5} \text{ W b m}^{-2}$, calculate the e.m.f. induced across the axle of the train of length 1·25 m. *(H.S.S.C.E. 2002)* **[Ans.** $9{\cdot}375 \times 10^{-4}$ V]

17. Calculate the e.m.f. induced between the ends of an axle 1·8 m long of a railway carriage travelling at the rate of 50 km h^{-1}. The vertical component of the earth's magnetic field is $0{\cdot}5 \times 10^{-4}$ weber m^{-2}. **[Ans.** $1{\cdot}25 \times 10^{-3}$ V]

18. An aircraft with a wing span of 40 m flies with a speed of 1080 km h^{-1} in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetic field is $1{\cdot}75 \times 10^{-5}$ T. Find the e.m.f. that develops between the tips of the wings. *(Text Problem)* **[Ans.** 0·21 V]

19. A satellite with a 40 cm long copper wire on its bottom is revolving around the earth at 7·8 km per second, such that the wire is perpendicular to the vertical component of earth's magnetic field. Determine the e.m.f. induced across this wire, if the magnitude of the vertical component of magnetic field is 0·2 gauss. **[Ans.** 0·0624 V]

20. A railway track running north-south has two parallel rails 1·0 m apart. Calculate the value of induced e.m.f. between the rails, when a train passes at a speed of 90 km h^{-1}. Horizontal component of earth's field at that place is $0{\cdot}3 \times 10^{-4} \text{ Wb m}^{-2}$ and angle of dip is 60°. *(H.S.S.C.E. 2001)* **[Ans.** $1{\cdot}3 \times 10^{-3}$ V]

ype C. On Self Induction

21. Magnetic flux of 5μWb is linked with a coil, when a current of 1 mA flows through it. What is the self inductance of the coil ? *(H.S.S.C.E. 1999)* **[Ans.** 5 mH]

22. The self inductance of an inductor having 100 turns is 20 mH. Calculate the total magnetic flux linked with the coil and the magnetic flux through the cross-section of the inductor corresponding to a current of 4 mA. **[Ans.** 8×10^{-3} Wb, 8×10^{-5} Wb]

23. A coil has an inductance of $1{\cdot}5 \times 10^{-2}$ H. Calculate the e.m.f. induced, when current in the coil changes at the rate 200 ampere per second. **[Ans.** 3 V]

24. The current passing through a 20 H inductor changes from 9 A to 8 A in 20×10^{-3} s. What will be the value of self induced e.m.f. ? **[Ans.** 10^3 V]

25. A 5 henry inductor carries a steady current of 2 ampere. How can a 50 volt self induced e.m.f. be made to appear in the inductor ? *(P.S.S.C.E. 2001)* **[Ans.** By decreasing current from 2 A to zero in 0·2 s]

26. A coil has a self inductance of 10 mH. What is the maximum magnitude of the induced e.m.f. in the inductor, when a curent $I = 0{\cdot}1 \sin 200\, t$ ampere is sent through it ? **[Ans.** 0·2 V]

27. A long solenoid of 10 turns cm^{-1} has a small loop of area 1 cm^2 placed inside with the normal of the loop parallel to the axis. Calculate the voltage across the small loop, if the current in the solenoid is changed at a steady rate from 1 A to 2 A in 0·1 s during the duration of the change. **[Ans.** $1{\cdot}257 \mu$V]

Type D. On Mutual Induction

28. What is the mutual inductance of a pair of coils, if a current of 3 ampere in one coil causes the flux in the second coil of 1000 turns to change by 10^{-4} Wb in each turn ? **[Ans.** $3{\cdot}33 \times 10^{-2}$ H]

29. If the coefficient of mutual induction of primary and secondary of an induction coil is 6 henry and a current of 5 ampere is cut off in 2×10^{-4} s, find the induced e.m.f. in the secondary coil. **[Ans.** $1{\cdot}5 \times 10^5$ V]

30. An e.m.f. of 50 millivolt is induced in a coil, when the current in the neighbouring coil changes from 10 ampere to 5 ampere in 0·1 s. What is the mutual inductance of the coils ? **[Ans.** 10^{-3} H]

31. An e.m.f. 0·5 V is developed in the secondary coil, when current in the primary coil changes from 5·0 A to 2·0 A in 300 millisecond. Calculate the mutual inductace of the two coils. *(I.S.C.E. 1993)* **[Ans.** 0·05 H]

32. A conducting wire of 100 turns is wound over 1 cm near the centre of a solenoid of 100 cm length and 2 cm radius having 1000 turns. Calculate coefficient of mutual inductance of the two solenoids. *(H.S.S.C.E. 1994)* **[Ans.** $1{\cdot}58 \times 10^{-4}$ H]

33. An air-cored solenoid is of length 0·3 m, area of cross-section $1{\cdot}2 \times 10^{-3} \text{ m}^2$ and has 2500 turns. Around its central section, a coil of 350 turns is wound. The solenoid and the coil are electrically insulated from each other. Calculate the e.m.f. induced in the coil, if the initial current of 3 A in the solenoid is reversed in 0·25 s. *(P.S.S.C.E. 2001)* **[Ans.** 0·1056 V]

MISCELLANEOUS PROBLEMS

34. A metallic wire bent in the form of a semicircle of radius 0·1 m is moved into magnetic field of 20 mT in a direction parallel to its plane, but perpendicular to the magnetic field with a velocity of 10 m s^{-1}. Find the e.m.f. induced in the wire. **[Ans.** 0·04 V]

35. The network shown in Fig. 1.46 is a part of complete circuit. What is the potential difference $V_B - V_A$, when the current I is 5 A and is decreasing at a rate of 10^3 A s^{-1} ? **[Ans.** – 15 V]

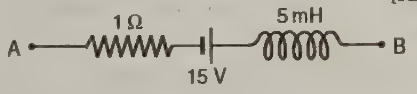


Fig. 1.46

36. A coil of wire of certain radius has 600 turns and a self inductance of 108 mH. What will be self inductance of a similar coil, which has 500 turns ? **[Ans.** 75 mH]

37. Three inductances are connected as shown in Fig. 1.47. Find the resultant inductance. *(P.S.S.C.E. 1993)* **[Ans.** 1 H]

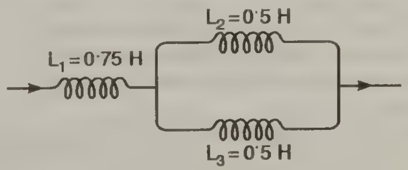


Fig. 1.47

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Proceed as in solved problem no. 1.01.

2. $e = \frac{d\phi}{dt} = \frac{8\times10^{-4}}{0\cdot5} = \mathbf{1\cdot6 \times 10^{-3}\ V}$

3. $d\phi = n\,A\,(B_2 - B_1) = 80 \times 0\cdot01\ (0 - 1\cdot0)$

   $= 0\cdot8$ Wb (in magnitude)

   Now, $e = \frac{d\phi}{dt} = \frac{0\cdot8}{0\cdot1} = \mathbf{8\ V}$

4. Proceed as in problem no. 3.

5. $d\phi = \phi_2 - \phi_1 = n\,B_2\,A - n\,B_1\,A = n\,A\,(B_2 - B_1)$

   $= 200 \times 0\cdot16\ (0\cdot30 - 0\cdot10) = 6\cdot4$ Wb

   Now, $e = \frac{d\phi}{dt} = \frac{6\cdot4}{0\cdot05} = \mathbf{128\ V}$

6. Proceed as in problem no. 5.

7. Proceed as in solved problem no. 1.02.

8. If θ is angle between the direction of magnetic field and normal to the plane of coil, then

   $$\phi = n\,B\,A\cos\theta$$

   Initial magnetic flux, $\phi_1 = n\,B\,A\cos 0° = n\,B\,A$

   and final magnetic flux, $\phi_2 = n\,B\,A\cos 180° = -\,n\,B\,A$

   Therefore, change in magnetic flux,

   $$d\phi = \phi_2 - \phi_1 = (-\,n\,BA) - n\,BA = -\,2\,n\,BA$$

   or $d\phi = 2\,n\,BA$ (in magnitude)

   $$= 2 \times 100 \times 0\cdot4 \times (8 \times 8 \times 10^{-4}) = 0\cdot512 \text{ Wb}$$

   $\therefore\quad e = \frac{d\phi}{dt} = \frac{0\cdot512}{0\cdot2} = \mathbf{2\cdot56\ V}$

9. As obtained in solved problem no. 1.04,

   $$e = B \times \pi\,l^2 \times v = 1\cdot0 \times \pi \times (1)^2 \times 50 = \mathbf{157\cdot08\ V}$$

10. As obtained in solved problem no. 1.04,

    $$e = \frac{1}{2}B\,l^2\,\omega = \frac{1}{2} \times 0\cdot5 \times (1)^2 \times 400 = \mathbf{100\ V}$$

11. As explained in solved problem no. 1.08, induced e.m.f. between the centre and edge of the disc is given by

    $$e = B \times \pi\,r^2 \times v \quad \text{(in magnitude)}$$

    Here, $v = 120$ r.p.m. $= \frac{120}{60} = 20$ r.p.s. ;

    $$e = 6\cdot284 \text{ mV} = 6\cdot284 \times 10^{-3} \text{ V}$$

    $$\therefore\quad B = \frac{e}{\pi\,r^2 \times v} = \frac{6\cdot284 \times 10^{-3}}{3\cdot142 \times (0\cdot1)^2 \times 20} = \mathbf{10^{-2}\ T}$$

12. As obtained in solved problem no. 1.04,

    $$e = \frac{1}{2}B\,r^2\,\omega$$

    Here, $r = 200$ cm $= 2$ m ; $B = 0\cdot05$ Wb m^{-2}

    and $\omega = 60$ rad s^{-1}

    $$\therefore\quad e = \frac{1}{2} \times 0\cdot05 \times (2)^2 \times 60 = \mathbf{6\ V}$$

13. Now, $e = B\,l\,v$

    or $B = \frac{e}{l\,v} = \frac{25}{10 \times 5\cdot0} = \mathbf{0\cdot5\ T}$

14. Proceed as in problem no. 13.

15. Proceed as in problem no. 13.

16. Proceed as in solved problem no. 1.09.

17. Proceed as in solved problem no. 1.09.

18. Proceed as in solved problem no. 1.09.

19. Proceed as in solved problem no. 1.09.

20. $e = B_V\,l\,v$

    Now, $B_V = B_H \tan\delta = 0\cdot3 \times 10^{-4} \times \tan 60° = 0\cdot52 \times 10^{-4}$ T

    $$\therefore\quad e = 0\cdot52 \times 10^{-4} \times 1\cdot0 \times \left(\frac{90 \times 1000}{60 \times 60}\right)$$

    $$= \mathbf{1\cdot3 \times 10^{-3}\ V}$$

21. Here, $\phi = 5\ \mu$Wb $= 5 \times 10^{-6}$ Wb and I $= 1$ mA $= 10^{-3}$ A

    Now, $\phi = L\,I$

    or $L = \frac{\phi}{I} = \frac{5 \times 10^{-6}}{10^{-3}} = 5 \times 10^{-3}$ H $= \mathbf{5\ mH}$

22. Proceed as in solved problem no. 1.14.

23. $e = L\frac{dI}{dt} = 1\cdot5 \times 10^{-2} \times 200 = \mathbf{3\ V}$

24. Proceed as in solved problem no. 1.15.

25. Suppose that current is decreased from 2 A to zero in tim *dt*. Then,

    $$dI = 2 - 0 = 2 \text{ A}$$

    Now, $e = L\frac{dI}{dt}$ or $50 = 5 \times \frac{2}{dt}$ or $dt = \mathbf{0\cdot2\ s}$

26. Here, L $= 10$ m H $= 10 \times 10^{-3}$ H and I $= 0\cdot1 \sin 200\,t$

    $$\therefore\quad \frac{dI}{dt} = \frac{d}{dt}(0\cdot1 \sin 200\,t) = (0\cdot1 \cos 200\,t) \times 200$$

    $$= 20 \cos 200\,t$$

    Now, $\left(\frac{dI}{dt}\right)_{max} = 20 \times 1 = 20$ A s^{-1}

    $$\therefore\ e_{max} = L\left(\frac{dI}{dt}\right)_{max} = 10 \times 10^{-3} \times 20 = \mathbf{0\cdot2\ V}$$

27. Proceed as in solved problem no. 1.16.

28. Total magnetic flux linked with the second coil,

    $$\phi = 1000 \times 10^{-4} = 0\cdot1 \text{ Wb}$$

    Now, $\phi = M\,I$ or $M = \frac{\phi}{I} = \frac{0\cdot1}{3} = \mathbf{3\cdot33 \times 10^{-2}\ H}$

29. Here, $dI = 5$ A ; $dt = 2 \times 10^{-4}$ s, M $= 6$ H

    Now, $e = M\frac{dI}{dt} = 6 \times \frac{5}{2 \times 10^{-4}} = \mathbf{1\cdot5 \times 10^5\ V}$

30. Proceed as in problem no. 29.

31. Proceed as in problem no. 29.

32. Here, length of the primary solenoid $= 100$ cm $= 1$ m

    Total number of turns in the primary solenoid, $N_1 = 100$

    Therefore, number of turns per unit length of the primar solenoid,

    $$n_1 = \frac{1000}{1} = 1000 \text{ m}^{-1}$$

    Radius of primary solenoid, $r = 2$ cm $= 0\cdot02$ m

    Therefore, area of cross-section of primary solenoid,

    $$A = \pi\,r^2 = \pi \times (0\cdot02)^2 = 4\,\pi \times 10^{-4} \text{ m}^2$$

    Length of the secondary solenoid, $l = 1$ cm $= 0\cdot01$ m

    Total number of turns in the secondary solenoid, $N_2 = 10$

    Therefore, number of turns per unit length of th secondary solenoid,

    $$n_2 = \frac{100}{0\cdot01} = 10^4 \text{ m}^{-1}$$

    Now, $M = \mu_0\,n_1\,n_2\,A\,l$

    $$= 4\,\pi \times 10^{-7} \times 1000 \times 10^4 \times 4\,\pi \times 10^{-4} \times 0\cdot01$$

    $$= \mathbf{1\cdot58 \times 10^{-4}\ V}$$

    Aliter. $M = \mu_0\,n_1\,n_2\,A\,l = \mu_0\,n_1\,N_2\,A$,

    where $N_2 = n_2\,l$ is total number of turns in the secondar solenoid.

    $$\therefore\quad M = 4\,\pi \times 10^{-7} \times 1000 \times 100 \times 4\,\pi \times 10^{-4}$$

    $$= \mathbf{1\cdot58 \times 10^{-4}\ V}$$

33. Here, $n_1 = \frac{2500}{0\cdot 3}\ \text{m}^{-1}$; $A = 1\cdot 2 \times 10^{-3}\ \text{m}^2$, $N_2 = 350$

$$\therefore \quad M = \mu_0\, n_1\, N_2\, A$$

$$= 4\pi \times 10^{-7} \times \frac{2500}{0\cdot 3} \times 350 \times 1\cdot 2 \times 10^{-3}$$

$$= 4\cdot 4 \times 10^{-3}\ \text{H}$$

Now, $dI = 3 - (-3) = 6$ A and $dt = 0\cdot 25$ s

$$\therefore \quad e = M\frac{dI}{dt} = 4\cdot 4 \times 10^{-3} \times \frac{6}{0\cdot 25} = \mathbf{0\cdot 1056\ V}$$

34. Refer to conceptual SAQ 1.03. It can be obtained that when a semicircular metallic wire of radius R enters into a magnetic field B with velocity v, induced e.m.f. produced

$$e = \frac{1}{2}\, B\, R\, v$$

$$\therefore \quad e = \frac{1}{2} \times (20 \times 10^{-3}) \times 0\cdot 1 \times 10 = \mathbf{0\cdot 04\ V}$$

35. It follows that

$$V_A = I\,R + (-E) + \left(-L\frac{dI}{dt}\right) = V_B$$

$$\text{or } V_A - V_B = I\,R - E - L\frac{dI}{dt}$$

$$= 5 \times 1 - 15 - 5 \times 10^{-3} \times 10^3$$

$$= 5 - 15 - 5 = \mathbf{-15\ V}$$

36. The self inductance of a coil is given by

$$L = \mu_0\, n^2\, l\, A = \frac{\mu_0\, N^2\, A}{l}$$

$$\therefore \quad \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

$$\text{or} \quad L_2 = L_1 \times \frac{N_2^2}{N_1^2} = \frac{108 \times (500)^2}{(600)^2}$$

$$= \mathbf{75\ mH}$$

37. Refer to keynote on page 508 :
The resultant inductance is given by

$$L = L_1 + \frac{L_2 \times L_3}{L_2 + L_3} = 0\cdot 75 + \frac{0\cdot 5 \times 0\cdot 5}{0\cdot 5 + 0\cdot 5}$$

$$= 0\cdot 75 + 0\cdot 25 = \mathbf{1\ H}$$

CHAPTER

2

Transient Current

2.01. TRANSIENT CURRENT

When a battery is connected to a resistor, the electric current reaches its maximum value in practically zero time. However, when the circuit contains an inductor (or a capacitor) also, the current takes some finite time to attain its maximum value. It is because, the growth of current is opposed by the induced e.m.f. produced due to the varying current. Similarly, when such an electric circuit is switched off, the current takes some finite time to decay from maximum to zero value.

The electric currents, whose magnitude vary for a small time, while growing to maximum or decaying to zero, are called transient currents.

It may be pointed out that the transient currents do not vary in direction* with time.

2.02. GROWTH AND DECAY OF CURRENT IN LR-CIRCUIT

Consider a battery of e.m.f. E connected to a series combination of an inductance L and resistance R through a two-way key as shown in Fig. 2.01.

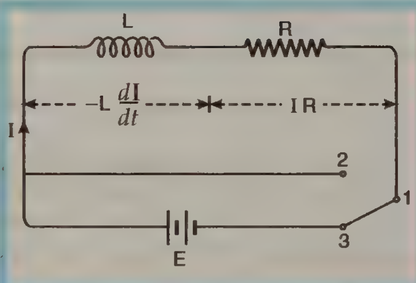


Fig. 2.01

(a) Growth of current. Initially ($t = 0$), the current in the circuit is zero. When the plug is inserted in the gap between 1 and 3 of the two-way key, the battery comes in circuit with the series combination of L and R. Due to the phenomenon of self-induction, an induced e.m.f. is set up in the circuit, which opposes the growth of current in it. If I is the current and $\frac{dI}{dt}$ is the rate of change of current in the inductance at any time t, then

potential difference across the resistance = I R

induced e.m.f. produced in the inductance $= -L\frac{dI}{dt}$

During growth of current in LR-circuit, the potential difference across the resistance will be equal to the sum of E (e.m.f. of the source) and $-L\frac{dI}{dt}$ (induced e.m.f. produced in the inductance). Thus,

$$E + \left(-L\frac{dI}{dt}\right) = IR$$

or

$$E = IR + L\frac{dI}{dt} \qquad ...(2.01)$$

Key point

When the LR-circuit is just switched on, the rate of change of current in the circuit,

$$\frac{dI}{dt} = \frac{E}{L}$$

It follows that when the current in the circuit will attain maximum value (I_0), the rate of growth of current $\left(\frac{dI}{dt}\right)$ will become zero *i.e.*

when $I = I_0$, $\quad \frac{dI}{dt} = 0$

Setting the above condition in equation (2.01), we obtain

$$E = I_0 R + L\,(0) = I_0 R$$

Setting $E = I_0 R$ in equation (2.01), we have

$$I_0 R = IR + L\frac{dI}{dt} \quad \text{or} \quad L\frac{dI}{dt} = R\,(I_0 - I)$$

or

$$\frac{dI}{I_0 - I} = \frac{R}{L}\,dt$$

Integrating both sides, we have

$$\int \frac{dI}{I_0 - I} = \frac{R}{L}\int dt$$

---

*The currents, which vary both in magnitude and direction with time, are called alternating currents.

or $$-\log_e (I_0 - I) = \frac{R}{L}t + k, \qquad ...(2.02)$$

vhere k is the constant of integration. In order to find k, we apply the boundary ondition *i.e.*

when $t = 0,\ I = 0$

Setting the above condition in equation (2.02), we have

$$-\log_e (I_0 - 0) = \frac{R}{L}(0) + k$$

or $$k = -\log_e I_0$$

Substituting for k in equation (2.02), we have

$$-\log_e (I_0 - I) = \frac{R}{L}t - \log_e I_0 \quad \text{or} \quad \log_e (I_0 - I) - \log_e I_0 = -\frac{R}{L}t$$

or $$\log_e = \frac{(I_0 - I)}{I_0} = -\frac{R}{L}t \quad \text{or} \quad \frac{(I_0 - I)}{I_0} = e^{-\frac{R}{L}t}$$

or $$I = I_0\left(1 - e^{-\frac{R}{L}\cdot t}\right) \qquad ...(2.03)$$

Key point

When the current in LR-circuit becomes steady, inductive effect of the inductor ceases.

The equation (2.03) gives the value of current at any time t during growth of current in LR-circuit.

Time constant. From equation (2.03), it follows that $\frac{L}{R}$ has got dimensions of time and is called *inductive time constant* of the LR-circuit.

When L is in *henry* and R in *ohm,* $\frac{L}{R}$ is in *seconds.*

Setting $t = \frac{L}{R}$ in equation (2.03), we obtain

$$I = I_0\left(1 - e^{-\frac{R}{L}\cdot\frac{L}{R}}\right) = I_0\,(I - e^{-1}) = I_0\,\{1 - (2{\cdot}718)^{-1}\} = I_0\,(1 - 0{\cdot}368) = 0{\cdot}632\,I_0$$

Thus, *the inductive time constant of an LR-circuit may be defined as the time in which the current grows from zero to $0{\cdot}632$ (or $63{\cdot}2\%$) of its maximum value.*

It can be calculated that when $t = \frac{2L}{R}, \frac{3L}{R}, \frac{4L}{R}$ and $\frac{5L}{R}$; the current will attain values* $0{\cdot}865\,I_0$, $0{\cdot}950\,I_0$, $0{\cdot}982\,I_0$ and $0{\cdot}993\,I_0$ respectively. Finally, when $t = \infty$,

$$I = I_0\left(1 - e^{-\frac{R}{L}\cdot\infty}\right) = I_0\,(I - e^{-\infty}) = I_0\,(1 - 0)$$

or $$I = I_0$$

i.e. in LR-circuit, current will take infinite time to grow to the maximum value. However, practically the current attains maximum value after a time equal to five time constants $\left(5\frac{L}{R}\right)$ of the LR-circuit. Fig. 2.02 shows the growth of current with time in LR-circuit.

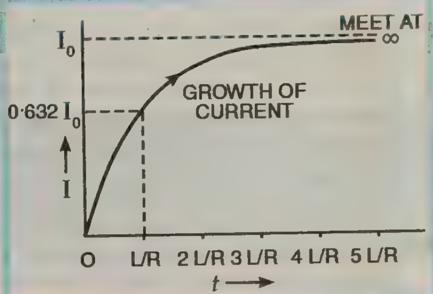


Fig. 2.02

***(b)* Decay of current.** Initially ($t = 0$), the current in the circuit is maximum (I_0). Now, as we take out plug from the gap between 1 and 3 of the two-way key and insert it in the gap between 1 and 2, the battery gets out of the circuit and the current in the circuit starts decaying [Fig. 2.03]. The decaying current produces induced e.m.f. in the circuit and it opposes the decay of the current.

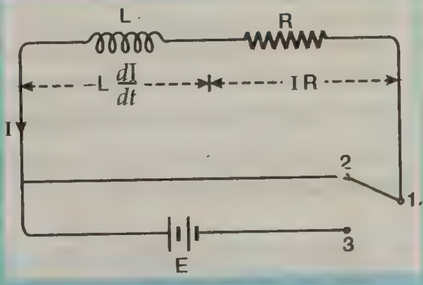


Fig. 2.03

Let I be the value of current in the circuit and $\frac{dI}{dt}$, the rate of decay of current at any time t. As battery is not present in the circuit,

$$0 + \left(-L\frac{dI}{dt}\right) = IR$$

---

*$e^{-2} = (2{\cdot}718)^{-2} = 0{\cdot}135$, $e^{-3} = (2{\cdot}718)^{-3} = 0{\cdot}050$, $e^{-4} = (2{\cdot}718)^{-4} = 0{\cdot}018$ and $e^{-5} = (2{\cdot}718)^{-5} = 0{\cdot}007$

or $$L\frac{dI}{dt} = -IR \quad \text{or} \quad \frac{dI}{I} = -\frac{R}{L}dt$$

Integrating both sides, we have

$$\int \frac{dI}{I} = -\frac{R}{L}\int dt$$

or $$\log_e I = -\frac{R}{L}t + k, \qquad ...(2.04)$$

where k is the constant of integration. In order to find k, we apply the boundary condition *i.e.*.

when $t = 0$, $I = I_0$

Setting this condition in equation (2.04), we have

$$\log_e I_0 = -\frac{R}{L}(0) + k$$

or $$k = \log_e I_0$$

Substituting for k in equation (2.04), we have

$$\log_e I = -\frac{R}{L}t + \log_e I_0 \quad \text{or} \quad \log_e I - \log_e I_0 = -\frac{R}{L}t$$

or $$\log_e \frac{I}{I_0} = -\frac{R}{L}t \quad \text{or} \quad \frac{I}{I_0} = e^{-\frac{R}{L}\cdot t}$$

or $$I = I_0\, e^{-\frac{R}{L}\cdot t} \qquad ...(2.05)$$

The equation (2.05) gives the value of current at any time t during decay of current in LR-circuit.

Again, from equation (2.05), $\frac{L}{R}$ has got dimensions of time and the ***inductive time constant*** of the LR-circuit can also be defined by using equation (2.05).

Setting $t = \frac{L}{R}$ in equation (2.05), we get

$$I = I_0\, e^{-\frac{R}{L}\cdot\frac{L}{R}} = I_0\, e^{-1} = I_0\,(0\cdot368) = 0\cdot368\, I_0$$

Therefore, *the inductive time constant of an LR-circuit may also be defined as the time in which the current decays from maximum to 0·368 (or 36·8%) of its maximum value.*

It can be calculated that at $t = \frac{2L}{R}, \frac{3L}{R}, \frac{4L}{R}$ and $\frac{5L}{R}$, the current will fall to the values $0\cdot135\, I_0$, $0\cdot050\, I_0$, $0\cdot018\, I_0$ and $0\cdot007\, I_0$ respectively. Using the equation (2.05), it can be proved that the current will become zero after infinite time. Graphically, the current will decay with time in LR-circuit as shown in Fig. 2.04.

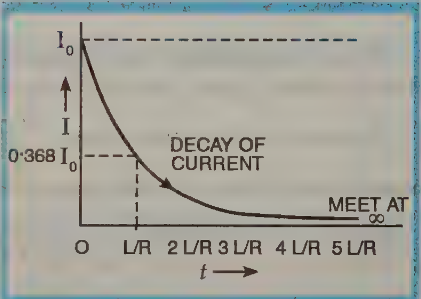


Fig. 2.04

Note. In section 2.04, we shall study that when the current in LR-circuit attains maximum value, the energy stored in the inductor is given by

$$U = \frac{1}{2}LI_0^2$$

2.03. GROWTH AND DECAY OF CHARGE IN A CR-CIRCUIT

Consider a battery of e.m.f. E connected to a series combination of a capacitor C and resistance R through a two-way key as shown in Fig. 2.05.

(a) Growth of charge. Initially ($t = 0$), the charge on the capacitor is zero. When the plug is inserted in the gap between 1 and 3 of the two-way key, the battery in the circuit starts charging the capacitor. Let q be the charge on the capacitor and I, the value of current in the circuit at any time t. Then,

potential difference across the resistance $= IR$

potential difference across the plates of capacitor $= \frac{q}{C}$

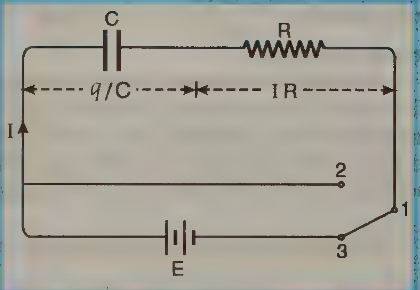


Fig. 2.05

At any time, the e.m.f. of battery must be equal to the sum of the potential differences across R and C *i.e.*

$$E = IR + \frac{q}{C} \qquad ...(2.06)$$

When the capacitor gets fully charged (q_0), the current in the circuit becomes zero *i.e.*

when $q = q_0$, $I = 0$

Setting the above condition in equation (2.06), we have

$$E = (0)\,R + \frac{q_0}{C}$$

or $$E = \frac{q_0}{C}$$

Key point

When a CR-circuit is just switched on,
$q = 0$ and $I =$ maximum
and when steady state is reached,
$q =$ maximum and $I = 0$

Substituting for E in equation (2.06), we have

$$\frac{q_0}{C} = I\,R + \frac{q}{c} \qquad \text{or} \qquad \frac{q_0 - q}{C} = I\,R$$

By definition, $$I = \frac{dq}{dt}$$

Therefore, $$\frac{q_0 - q}{C} = \left(\frac{dq}{dt}\right) R \qquad \text{or} \qquad \frac{dq}{q_0 - q} = \frac{1}{CR}\, dt$$

Integrating both sides, we have

$$\int \frac{dq}{q_0 - q} = \frac{1}{CR} \int dt$$

or $$-\log_e q_0 - q = \frac{1}{CR} t + k, \qquad \text{...(2.07)}$$

ere k is the constant of integration. In order to kind k, we apply the boundary ndition *i.e.*

when $t = 0, q = 0$

Substituting for k in equation (2.07), we have

$$-\log_e (q_0 - q) = \frac{1}{CR} t - \log_e q_0$$

$$\text{or } \log_e (q_0 - q) - \log_e q_0 = \frac{1}{CR} t \qquad \text{or} \qquad \log_e \frac{q_0 - q}{q_0} = -\frac{1}{CR} t$$

$$\text{or} \qquad \frac{q_0 - q}{q_0} = e^{-\frac{1}{CR} \cdot t} \qquad \text{or} \qquad 1 - \frac{q}{q_0} = e^{-\frac{1}{CR} \cdot t}$$

or $$q = q_0 \left(1 - e^{-\frac{1}{CR} \cdot t}\right) \qquad \text{...(2.08)}$$

The equation (2.08) gives the value of charge on the capacitor at any time t uring charging through the resistance in CR-circuit.

From equation (2.08), it follows that CR has got dimensions of time and is called *pacitive time constant* of the circuit.

When C is in *farad* and R is in *ohm*, CR is in *seconds*.

Setting $t = CR$ in equation (2.08), we have

$$q = q_0 \left(1 - e^{-\frac{1}{CR} \cdot CR}\right) = q_0 = (1 - e^{-1})$$

$$= q_0 \{1 - (2{\cdot}718)^{-1}\} = q_0 \{1 - 0{\cdot}368\} = 0{\cdot}632\, q_0$$

Thus, *the capacitive time constant of a CR-circuit may be defined as the time in which e charge on the capacitor grows from zero to 0·632 (or 63·2%) of its maximum value.*

It can be proved that when $t = 2$ CR, 3 CR, 4 CR and 5 CR, the charge on the apacitor will be $0{\cdot}865\, q_0$, $0{\cdot}950\, q_0$, $0{\cdot}982\, q_0$ and $0{\cdot}993\, q_0$ respectively.

Finally, when $t = \infty$,

$$q = q_0 \left(1 - e^{-\frac{1}{CR} \cdot \infty}\right)$$

$$= q_0 (1 - e^{-\infty}) = q_0 (1 - 0) = q_0$$

e. in a CR-circuit, the capacitor will take infinite time to charge to its maximum alue. Fig. 2.06 shows the growth of charge with time in CR-circuit. The graph is xponential in nature as suggested by equation (2.08).

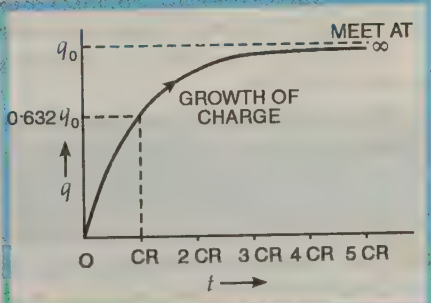


Fig. 2.06

***(b)* Decay of charge.** Consider a battery of e.m.f. E connected to a series ombination of a capacitor C and a resistance R through a two way. When key is kept pressed for some time ($t > 5\,\mathrm{C\,R}$), the charge on the capacitor attains its value q_0. Thus, initially ($t = 0$), charge on the capacitor is maximum.

As we take out plug from the gap between 1 and 3 of the two-way key and insert it in the gap between 1 and 2 [Fig. 2.07], the battery gets out of the circuit and the charged capacitor starts discharging through the resistance. Let I be the value of current in the circuit and q be the charge on the capacitor at any time t. As battery is not present in the circuit,

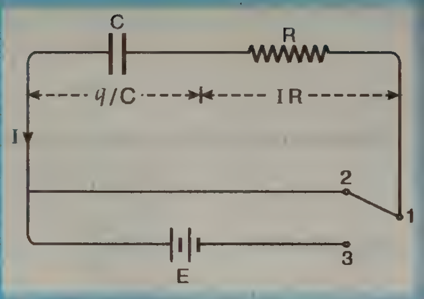


Fig. 2.07

$$0 = \mathrm{I\,R} + \frac{q}{\mathrm{C}} \qquad \text{...(2.09)}$$

By definition, $\mathrm{I} = \dfrac{dq}{dt}$

Substituting for I in equation (2.09), we have

$$\left(\frac{dq}{dt}\right)\mathrm{R} + \frac{q}{\mathrm{C}} = 0 \quad \text{or} \quad \frac{dq}{q} = -\frac{1}{\mathrm{CR}}\,dt$$

Integrating both sides, we have

$$\int \frac{dq}{q} = -\frac{1}{\mathrm{CR}} \int dt$$

or

$$\log_e q = -\frac{1}{\mathrm{CR}}\,t + k, \qquad \text{...(2.10)}$$

where k is the constant of integration. In order to find k, we apply the boundary condition *i.e.*

when $t = 0,\ q = q_0$

Setting this condition in equation (2.10), we have

$$\log_e q = -\frac{1}{\mathrm{CR}}(0) + k$$

or

$$k = \log_e q_0$$

Substituting for k in equation (2.10), we have

$$\log_e q = -\frac{1}{\mathrm{CR}}\,t + \log_e q_0$$

or

$$\log_e \frac{q}{q_0} = -\frac{1}{\mathrm{CR}}\cdot t \quad \text{or} \quad \frac{q}{q_0} = e^{-\frac{1}{\mathrm{CR}}\cdot t}$$

or

$$q = q_0\, e^{-\frac{1}{\mathrm{CR}}\cdot t} \qquad \text{...(2.11)}$$

The equation (2.11) gives the value of charge on the capacitor at any time t during discharge of the capacitor through the resistance in a CR-circuit.

Setting $t = \mathrm{CR}$ in equation (2.11), we get

$$q = q_0\, e^{-\frac{1}{\mathrm{CR}}\cdot \mathrm{CR}} = q_0\, e^{-1} = 0\cdot 368\, q_0$$

Therefore, *capacitive time constant of a CR-circuit may also be defined as the time during which the charge on the capacitor decays from maximum to 0·368 (or 36·8%) of its maximum value.*

It can be easily shown that :

(*i*) at $t = 2$ CR, 3 CR, 4 CR and 5 CR, the charge on capacitor in a CR-circuit falls to the values 0·135 q_0, 0·050 q_0, 0·018 q_0 and 0·007 q_0 respectively and

(*ii*) the charge will become zero after infinite time.

Graphically, the charge on capacitor will decay with time in a CR-circuit as shown in Fig. 2.08.

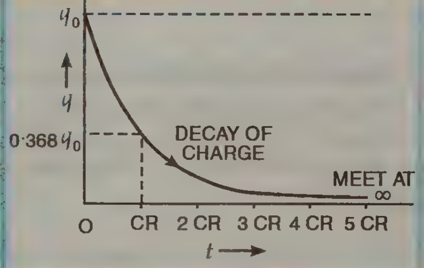


Fig. 2.08

Note. In section 2.05, we shall study that when the capacitor is charged to the maximum value q_0, the energy stored in the capacitor is given by

$$\mathrm{U} = \frac{1}{2}\frac{q_0^2}{\mathrm{C}}$$

2.04. ENERGY STORED IN AN INDUCTOR

Consider that a source of e.m.f. is connected to an inductor L. As the current starts growing, an induced e.m.f. is set up in the inductor and the induced e.m.f., then, opposes the growth of current through it. The source of e.m.f. has to spend energy in sending current through the circuit against the induced e.m.f. The energy spent by the source of e.m.f. is stored in the inductor in the form of magnetic field.

Initially ($t = 0$), the current in the inductor is zero. Consider that at any instant, e current in the inductor is I and the rate of growth of current at that time is equal $\frac{dI}{dt}$. Then,

induced e.m.f. produced in the inductor, $e = L\frac{dI}{dt}$ (in magnitude)

If the source of e.m.f. sends a constant current I through the inductor for a small ne dt, then small amount of work done by the source is given by

$$dW = e\,I\,dt = \left(L\frac{dI}{dt}\right)I\,dt = L\,I\,dI$$

The total amount of work done by the source of e.m.f., till the current increases m its initial value $I = 0$ to its final value I_0 is given as

$$W = \int_0^{I_0} L\,I\,dI = L\int_0^{I_0} I\,dI = L\left|\frac{I^2}{2}\right|_0^{I_0} = \frac{1}{2}L\,I_0^2$$

Key point

Energy stored in an inductor resides inside it in the form of magnetic field.

This work done by the source of e.m.f. in building up current from zero to I_0 stored inside the inductor in the form of energy. Therefore, energy stored in the ductor,

$$U = \frac{1}{2}L\,I_0^2 \qquad ...(2.12)$$

The energy is stored in the inductor at the expense of the energy of the source e.m.f. and it resides in the inductance in the form of magnetic field.

It may be pointed out that :

(*i*) The energy is stored in the inductor at the expense of the energy of the source of e.m.f.

(*ii*) The energy resides in the inductor in the form of magnetic field.

(*iii*) In case of an alternating source of e.m.f., during one half cycle energy is stored in the inductor from the source and during the next half cycle, the same amount of energy is returned to the source. It is for this reason that average electric power of an inductor is zero.

2.05. ENERGY STORED IN A CAPACITOR

Consider that a source of e.m.f. E is connected to a capacitor of capacitance C. s the charging of the capacitor takes place, the potential difference across the plates the capacitor increases. The source of e.m.f. has to spend energy in charging the pacitor against the potential difference across the plates of the capacitor. The work ne by the source of e.m.f. gets stored in the capacitor in the form of its energy.

Initially ($t = 0$), the charge on the capacitor is zero. Consider that at any instant, e charge on the capacitor is q and the potential difference across the plates of the pacitor at that time is V.

Then, $V = \frac{q}{C}$

Suppose that the source of e.m.f. delivers infinitesimally small amount of charge to the plates of the capacitor without producing any change in the potential fference across its plates. Then, small amount of work done by the source of e.m.f. given by

$$dW = V\,dq = \frac{q}{C}\,dq$$

Key point

Energy stored in a capacitor resides inside it in the form of electric field.

The total amount of work done by the source of e.m.f. so as to increase charge n the plates from the initial value $q = 0$ to the final value q_0 is given by

$$W = \int_0^{q_0}\frac{q}{C}\,dq = \frac{1}{C}\int_0^{q_0} q\,dq = \frac{1}{C}\left|\frac{q^2}{2}\right|_0^{q_0} \quad \frac{1}{2}\frac{q_0^{\,2}}{C}$$

This work done by the source of e.m.f. in charging the capacitor from zero to is stored inside the capacitor in the form of its energy. Therefore, energy stored side the capacitor,

$$U = \frac{1}{2}\frac{q_0^{\,2}}{C} \qquad ...(2.13)$$

When the capacitor acquires charge q_0, the potential difference across the plates of the capacitor will become equal to E, the e.m.f. of the source of e.m.f.

$$\therefore \quad q_0 = \text{C E}$$

Substituting for q_0 (= C E) in equation (2.13), we have

$$U = \frac{1}{2}\text{C E}^2 \qquad \text{...(2.14)}$$

Substituting for $C\left(= \frac{q_0}{E}\right)$ in equation (2.13), we have

$$U = \frac{1}{2}q_0 \, E \qquad \text{...(2.15)}$$

The equations (2.13), (2.14) and (2.15) give the energy stored in a charged capacitor. The energy is stored in the capacitor at the expense of the energy of the source of e.m.f. and it resides in the capacitor in the form of electric field.

It may be pointed out that :

(i) The energy is stored in the capacitor at the expense of the energy of the source of *e.m.f.*

(ii) The energy resides in the capacitor in the form of electric field.

(iii) In case of alternating source of e.m.f., during one half cycle energy will be stored in the capacitor from the source and during the next half cycle, the same amount of energy is returned to the source. It is for this reason that average electric power of a capacitor is zero.

2.06. LC-OSCILLATIONS

Consider a charged capacitor of capacitance C. The energy is stored in the capacitor in the form of electric field between its two plates. If the charged capacitor is connected to an inductor L as shown in Fig. 2.09 (*a*), LC-oscillations are produced as explained below :

As soon as the capacitor is connected to inductor, it sends a current, thereby producing growing magnetic field inside the inductor. In turn, it produces induced e.m.f., which opposes the growth of current and hence the capacitor takes some finite time to discharge completely. When it discharges completely, the energy which was stored inside the capacitor in the form of electric field, now appears in the form of magnetic field inside the inductor [Fig. 2.09 (*b*)].

As soon as the discharge is complete, the current ceases to flow and the magnetic field lines linked with the inductor also start winding up. Owing to this, induced e.m.f. is produced in the inductor. The induced e.m.f. starts charging the capacitor but with opposite polarity. Now, the energy stored in the inductor in the form of magnetic field appears in the form of electric field between the plates of the capacitor as shown in Fig. 2.09 (*c*). In case the circuit does not possess any ohmic resistance, the capacitor will charge to the same initial value.

On getting fully charged, the capacitor will again discharge sending current through the inductor in opposite direction (as plates have opposite polarities) and therefore causing magnetic flux to link with inductor in the opposite direction as shown in Fig. 2.09 (*d*).

As before, on the complete discharge of capacitor, the magnetic flux linked with the inductor starts winding up. As a result, the induced e.m.f. so produced charges the capacitor, such that polarity of the plates is opposite to that in the previous case *i.e.* the same as it was in the beginning [Fig. 2.09 (*e*)].

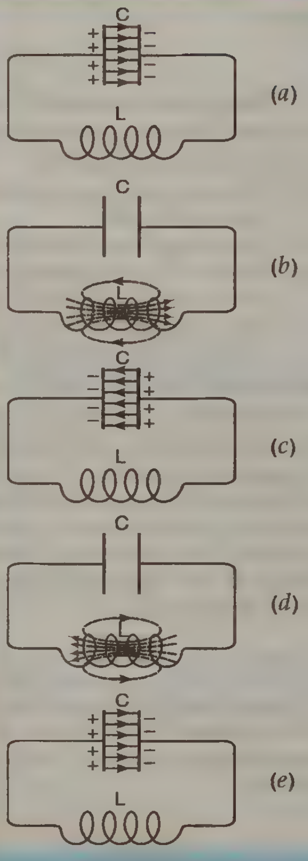


Fig. 2.09

The process, then, repeats itself indefinitely and the electromagnetic oscillations are produced as shown in Fig. 2.10. The oscillations are produced due to continuous conversion of energy in the form of electric field between the plates of the capacitor into magnetic field inside the inductor and vice versa. If the circuit does not have any resistance, no dissipation of energy will take place and the amplitude of oscillations produced will remain constant. Such oscillations are called ***undamped oscillations***. However, due to finite resistance of the circuit, the amplitude of oscillations goes on decreasing. It is because, a small amount of electric energy is dissipated in the form of heat energy during each oscillation. In other words, in practice, the ***damped oscillations*** are produced.

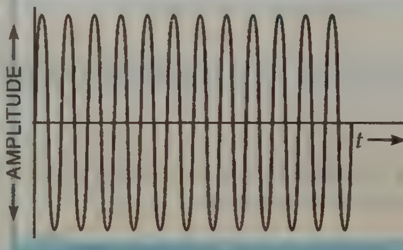


Fig. 2.10

The frequency of the LC-oscillations produced is given by

$$f = \frac{1}{2\pi\sqrt{\text{LC}}}$$

Mathematical treatment. Consider that a capacitor of capacitance C and itially charged to value q_0 is connected to an inductor of inductance L [Fig. 2.11]. nus,

$$\text{at } t = 0, \qquad q = q_0 \text{ and } \qquad I = 0.$$

As the capacitor is connected to inductor, it will start discharging through the ductor. Due to the varying current flowing through the inductor, induced e.m.f. ill be produced in it. Suppose that at any time t during the discharge of capacitor, arge left on capacitor is q, current in the circuit is I and the rate of change of rrent is $\frac{dI}{dt}$.

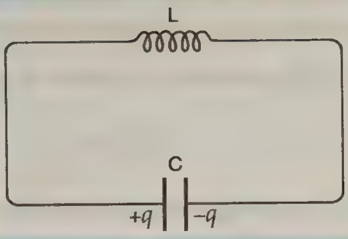


Fig. 2.11

Now, the induced e.m.f. across the inductor at any instant

$$= -L\frac{dI}{dt}$$

and the potential difference across the capacitor at that instant $= \frac{q}{C}$

Therefore, in the LC-circuit, at any instant

$$-L\frac{dI}{dt} = \frac{q}{C}$$

or $$L\frac{dI}{dt} + \frac{q}{C} = 0 \qquad \text{...(2.16)}$$

Key point

A varying magnetic field produces a varying electric field and in turn a varying electric field produces a varying magnetic field.

Now, $I = \frac{dq}{dt}$ so that $\frac{dI}{dt} = \frac{d^2q}{dt^2}$

Therefore, equation (2.16) becomes

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

or $$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \qquad \text{...(2.17)}$$

Let us substitute $\frac{1}{LC} = \omega^2$ ...(2.18)

Then, equation (2.17) becomes

$$\frac{d^2q}{dt^2} + \omega^2 q = 0 \qquad \text{...(2.19)}$$

The general solution of the differential equation (2.19) is of the form

$$q = A\cos\omega t = B\sin\omega t \qquad \text{...(2.20)}$$

To find A and B : The values of constants A and B are found by applying ne following boundary conditions :

(*i*) When $t = 0,\ q = q_0$

Setting this condition in equation (2.20), we have

$$q_0 = A\cos\omega\,(0) + B\sin\omega\,(0)$$

or $$A = q_0$$

(*ii*) The instantaneous value of current in the circuit is given by

$$I = \frac{dq}{dt} = \frac{d}{dt}(A\cos\omega t + B\,\omega\cos\omega t)$$

or $$I = -A\,\omega\sin\omega t + B\,\omega\cos\omega t \qquad \text{...(2.21)}$$

Now, when $t = 0, I = 0$

Setting this condition in equation (2.21), we have

$$0 = -A\,\omega\sin\omega\,(0) + B\,\omega\cos\omega\,(0)$$

or $$B = 0$$

Substituting for A and B in equation (2.20), we get

$$q = q_0\cos\omega t \qquad \text{...(2.22)}$$

Therefore, charge on the capacitor at any time is a harmonic function of time nd hence it keeps on oscillating. If f is the frequency of oscillation, then

$$\omega = 2\pi f \qquad \text{or} \qquad f = \frac{\omega}{2\pi}$$

Using equation (2.18), we have

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad \text{...(2.23)}$$

Just as in a harmonic oscillator, sum of kinetic and potential energies is constant ; in the LC-circuit, the sum of the energy stored in capacitor and the inductor at any instant is also constant *i.e.* the total energy of the LC-circuit does not depend on time. It can be shown as below :

Energy stored in capacitor at any instant $= \frac{1}{2}\frac{q^2}{C}$

$$= \frac{q_0^2}{2C}\cos^2 \omega t \qquad (\because q = q_0 \cos \omega t)$$

Also, energy stored in inductor at any instant $= \frac{1}{2} L I^2$

$$= \frac{1}{2} L \left(\frac{dq}{dt}\right)^2 = \frac{1}{2} L \left[\frac{d}{dt}(q_0 \cos \omega t)\right]^2$$

$$= \frac{q_0^2 L \omega^2 \sin^2 \omega t}{2} = \frac{q_0^2 L}{2}\left(\frac{1}{LC}\right)\sin^2 \omega t \qquad \left(\because \omega^2 = \frac{1}{LC}\right)$$

$$= \frac{q_0^2}{2C}\sin^2 \omega t$$

Therefore, total energy of the LC-circuit at any instant,

$$U = \frac{q_0^2}{2C}\cos^2 \omega t + \frac{q_0^2}{2C}\sin^2 \omega t = \frac{q_0^2}{2C} \quad \text{(constant)}$$

Note. The equation (2.22) can be used to find instantaneous and maximum value of current in the circuit. It follows that

$$I = \frac{dq}{dt} = q_0\, \omega \sin \omega t = I_0 \sin \omega t$$

Thus, maximum current, $I_0 = q_0\, \omega$

SOLVED NUMERICAL PROBLEMS

Type A. On Growth and Decay of Current in LR-circuit

Problem 2.01. An inductance of 2·5 henry is Connected to a battery of e.m.f. 3 volt through a resistance of 50 ohm. Calculate (*i*) time constant of circuit (*ii*) rate of change of current at the instant, the current begins to flow and (*iii*) the maximum value of the current.

Sol. Here, E = 3 volt ; L = 2·5 henry ; R = 50 ohm

(*i*) Time constant, $\frac{L}{R} = \frac{2 \cdot 5}{50} = 0 \cdot 05 \text{ s}$

(*ii*) For LR-circuit, $E - L\frac{dI}{dt} = IR$

$$\therefore \qquad \frac{dI}{dt} = \frac{E - IR}{L}$$

At the instant, when current begins to flow, I = 0. Therefore, rate of change of current at the instant the current begins to flow is given by

$$\therefore \qquad \frac{dI}{dt} = \frac{E-(0)R}{L} = \frac{E}{L} = \frac{3}{2 \cdot 5} = 1 \cdot 2 \text{ A s}^{-1}$$

(*iii*) Maximum current, $I_0 = \frac{E}{R} = \frac{3}{50} = 0 \cdot 06 \text{ A}$

Problem 2.02. A potential difference of 1 volt is applied to a coil of resistance 1 ohm and inductance 1 henry. (*i*) What is the current after 1 s ? (*ii*) How long does it take the current to reach half its final value ?

Sol. (*i*) Here, E = 1 volt ; R = 1 ohm ; L = 1 henry ; *t* = 1 s

Maximum value of current, $I_0 = \frac{E}{R} = \frac{1}{1} = 1 \text{ A}$

Let I be the value of current after 1 s.

Now, $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$$\therefore \qquad I = 1\left(1 - e^{-\frac{1}{1}\times 1}\right) = (1 - e^{-1}) = (1 - 0 \cdot 368)$$

$$= 0 \cdot 632 \text{ A}$$

(*ii*) Here, $I = \frac{I_0}{2}$

We know, $I = I_0 = \left(1 - e^{-\frac{R}{L}t}\right)$

$$\therefore \qquad \frac{I_0}{2} = I_0\left(1 - e^{-\frac{1}{1}t}\right)$$

or $\frac{1}{2} = \left(1 - e^{-t}\right)$ or $e^{-t} = \frac{1}{2}$ or $e^{t} = 2$

or $t = \log_e 2 = 2 \cdot 3036 \times \log_{10} 2 = 2 \cdot 3026 \times 0 \cdot 3010$

$= 0 \cdot 693 \text{ s}$

Problem 2.03. A steady current is flowing through a coil of resistance 20 ohm and inductance 0·1 henry. Find the time taken by the current to fall to 30% of its maximum value. What is the rate of change of current at this instant, if steady value of current through the coil is 2 ampere ?

Sol. Here, I_0 = 2 ampere ; L = 0·1 henry ; R = 20 ohm

Let *t* be the time, after which the current falls to 30% of its maximum value, *i.e.* it becomes

$$I = \frac{30}{1000} I_0 = 0 \cdot 3\, I_0$$

During decay, the value of current at time t is given by

$$I = I_0\, e^{-\frac{R}{L}t}$$

$$\therefore \quad 0 \cdot 3\, I_0 = I_0\, e^{-\frac{20}{0 \cdot 1}t} = I_0\, e^{-200\,t}$$

or $$e^{200\,t} = \frac{1}{0 \cdot 3} = 3 \cdot 3333$$

or $$200\,t = \log_e 3 \cdot 3333 = 2 \cdot 3026 \log_{10} 3 \cdot 3333$$

or $$t = \frac{2 \cdot 3026 \log_{10} 3 \cdot 3333}{200} = \frac{2 \cdot 3026 \times 0 \cdot 5229}{200}$$

$$= 6 \cdot 02 \times 10^{-3}\ \text{s}$$

When the current decays in a circuit (battery is not in ircuit),

$$IR = -L\frac{dI}{dt}$$

Therefore, the rate of decrease of current in the circuit is iven by

$$-\frac{dI}{dt} = \frac{R}{L}I = \frac{R}{L} \times 0 \cdot 3\, I_0 = \frac{20}{0 \cdot 1} \times 0 \cdot 3 \times 2$$

$$= 120\ \text{A s}^{-1}$$

Type B. On Growth and Decay of Charge in CR-circuit

Problem 2.04. If a circuit having a resistor of 2 megaohm n series with a capacitor of one microfarad is placed in eries with a cell of e.m.f. 2 volt, find the time after which he charge reaches 84·47% of its maximum.

Sol. Here, $R = 2$ megaohm $= 2 \times 10^6$ ohm ;

$C = 1$ microfarad $= 10^{-6}$ farad

Suppose that t is the time after which the charge reaches 84·47% of its maximum value *i.e.* the charge becomes

$$q = \frac{84 \cdot 47}{100} q_0 \quad \text{or} \quad \frac{q}{q_0} = \frac{86 \cdot 47}{100} = 0 \cdot 8647$$

We know, $$q = q_0\left(1 - e^{\frac{-t}{CR}}\right) \quad \text{or} \quad \frac{q}{q_0} = \left(1 - e^{\frac{-t}{CR}}\right)$$

$$\therefore \quad 0 \cdot 8647 = \left(1 - e^{-\frac{t}{2 \times 10^6 \times 10^{-6}}}\right) = \left(1 - e^{-t/2}\right)$$

or $$e^{-\frac{t}{2}} = 1 - 0 \cdot 8647 = 0 \cdot 1353$$

or $$-\frac{t}{2} = \log_e (0 \cdot 1353) = 2 \cdot 3026 \log_{10} (0 \cdot 1353)$$

$$= 2 \cdot 3026 \times (\bar{1} \cdot 1313)$$

or $$t = 2 \times 2 \cdot 3026 \times 0 \cdot 8687 = 4\ \text{s}$$

Problem 2.05. A capacitor of 1·443 μF capacitance after being charged, is shunted by a high resistance. If half of the charge leaks away in one minute, find the value of the resistance.

Sol. Here, $C = 1 \cdot 443\ \mu\text{F} = 1 \cdot 443 \times 10^{-6}$ F ;

$t = 1$ minute $= 60$ s

Charge on capacitor after 1 minute *i.e.* 60 s $= \dfrac{q_0}{2}$

We know, $$q = q_0\, e^{-\frac{t}{CR}}$$

or $$\frac{q_0}{2} = q_0\, e^{-\frac{60}{1 \cdot 443 \times 10^{-6} \times R}}$$

or $$\frac{1}{2} = e^{-\frac{60}{1 \cdot 443 \times 10^{-6} \times R}} \quad \text{or} \quad e^{\frac{60}{1 \cdot 443 \times 10^{-6} \times R}} = 2$$

or $$\frac{60}{R \times 1 \cdot 443 \times 10^{-6}} = \log_e 2 = 2 \cdot 303 \log_{10} (2)$$

$$= 2 \cdot 303 \times 0 \cdot 3010 = 0 \cdot 6931$$

or $$R = \frac{60}{1 \cdot 443 \times 10^{-6} \times 0 \cdot 6931} = 60 \times 10^6\ \Omega$$

Type C. On LC-oscillations

Problem 2.06. A radio can tune over the frequency range of a portion of MW broadcast band 800 kHz to 1200 kHz. If its LC-circuit has an effective inductance of 200 μH, what must be range of its variable capacitor ? *(Text Problem)*

Sol. Here, $L = 200\ \mu\text{H} = 200 \times 10^{-6}$ H

Frequency range of MW broadcast

$= 800$ kHz to 1200 kHz

$= 8 \times 10^5$ Hz to 12×10^{-5} Hz

For tuning the radio, the natural frequency of the LC-circuit should be equal to the frequency of the signal.

Now, $$f = \frac{1}{2\pi\sqrt{LC}}$$

or $$C = \frac{1}{4\pi^2 f^2 L}$$

Value of capacitor corresponding to lower limit of MW band *i.e.* $f = 8 \times 10^5$ Hz,

$$C_1 = \frac{1}{4\pi^2 (8 \times 10^5)^2 \times 200 \times 10^{-6}}$$

$$= 197 \cdot 9 \times 10^{-12}\ \text{F} \approx 98\ \text{pF}$$

Value of capacitor corresponding to upper limit of MW band *i.e.* $f = 12 \times 10^5$ Hz,

$$C_1 = \frac{1}{4\pi^2 (12 \times 10^5)^2 \times 200 \times 10^{-6}}$$

$$= 87 \cdot 95 \times 10^{-12}\ \text{F} \approx 88\ \text{pF}$$

Therefore, variable capacitor should be of range 88 pF to 198 pF.

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 2.01. Two circuits (A) and (B) connected to identical d.c. sources (e.m.f. = 10 V) differ greatly in their self-inductance. Circuit A has large self-inductance equal to 10 H, while the self-inductance of B is much smaller, equal to 0·05 H. The total external resistance in each circuit (which includes the resistance of the inductor itself) is 40 Ω.

(*a*) Are the steady current values in each circuit equal ? If so, what is the value ?

(*b*) Compare the times required for the currents in the two circuits to reach [1–(1 / *e*)] of their steady value.

(*c*) Which circuit requires greater energy consumption of the source to build up its current to the steady value ?

(d) After the steady state is reached, do the circuits dissipate the same power in the form of heat ? (*Text Problem*)

Ans. *Circuit A* *Circuit B*

$E = 10\,V; L = 10\,H; R = 40\,\Omega$ $E = 10\,V; L = 0{\cdot}05\,H; R = 40\,\Omega$

(*a*) Steady current, $I_0 = \dfrac{E}{R}$

Since the value of steady current is independent of the value of inductance (L) in the circuit, the steady current in each circuit is *same* and is equal to

$$I_0 = \frac{10}{40} = 0{\cdot}25\ \text{A}$$

(*b*) During growth : $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

Thus, time *t* required for current to reach the value $I_0(1 - 1/e)$ is given by

$$I_0\left(1 - \frac{1}{e}\right) = I_0\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{or} \quad e^{-\frac{R}{L}t} = \frac{1}{e} = e^{-1}$$

$$\text{or} \quad \frac{R}{L}t = 1 \quad \text{or} \quad t = \frac{L}{R}$$

$$\therefore \quad \frac{t_A}{t_B} = \frac{10/40}{0{\cdot}05/40} = 200$$

(*c*) Energy required to build up current I_0 in an inductor,

$$W = \frac{1}{2}LI_0^2$$

Therefore, $W_A = \dfrac{1}{2} \times 10 \times (0{\cdot}25)^2 = 312{\cdot}5 \times 10^{-3}\ \text{J}$

and $W_B = \dfrac{1}{2} \times 0{\cdot}05 \times (0{\cdot}25)^2 = 1{\cdot}5625 \times 10^{-3}\ \text{J}$

Hence, ***circuit A requires greater energy*** to build up current equal to I_0.

(*d*) Power dissipated on reaching the steady state $= I_0^2 R$

Since I_0 and R are same for both circuits, the two circuits will dissipate the *same power.*

Power dissipated in each circuit $= (0{\cdot}25)^2 \times 40 = \mathbf{2{\cdot}5\ W}$

Problem 2.02. Two capacitors 4 μF and 6 μF (in series), are connected through a resistance of 10 kΩ to a 18 V battery of negligible internal resistance [Fig. 2.12]. After a time of about 10 s, the battery is disconnected and the capacitors are allowed to discharge through the resistance. Determine the voltage across each capacitor after a time lapse of 48 milliseconds. (*Text Problem*)

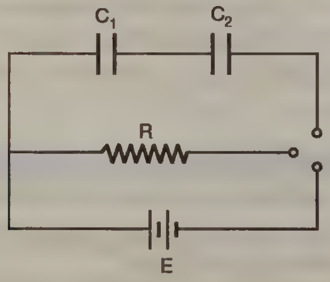


Fig. 2.12

Sol. Here, $C_1 = 4\,\mu F; C_2 = 6\,\mu F$

Therefore, total capacitance in series is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{6} = \frac{10}{24}$$

$$\text{or} \quad C = \frac{24}{10} = 2{\cdot}4\,\mu F = 2{\cdot}4 \times 10^{-6}\ \text{F}$$

Also, $R = 10\ \text{k}\Omega = 10^4\ \Omega$

Time constant of CR-circuit,

$$CR = 2{\cdot}4 \times 10^{-6} \times 10^4 = 2{\cdot}4 \times 10^{-2}\ \text{s}$$

As battery is connected to CR-circuit for 10 s, *i.e.* for a time very large as compared to time constant of the circuit ($CR = 2{\cdot}4 \times 10^{-2}$ s), the capacitor will get charged to the maximum value q_0 and voltage across its plates will become equal to 18 V *i.e.* equal to e.m.f. (E) of the battery.

Now, $q_0 = CE = 2{\cdot}4 \times 10^{-6} \times 18 = 4{\cdot}32 \times 10^{-5}$ C

During discharge, charge on the capacitor after time *t* is given by

$$q = q_0\, e^{-\frac{1}{CR}t}$$

Here, $t = 48$ millisecond $= 48 \times 10^{-3}$ s

$$\therefore \quad q = 4{\cdot}32 \times 10^{-5}\, e^{-\frac{1}{2{\cdot}4 \times 10^{-2}} \times 48 \times 10^{-3}}$$

$$= 4{\cdot}32 \times 10^{-5}\, e^{-2} = 4{\cdot}32 \times 10^{-5} \times 0{\cdot}135$$

$$= 5{\cdot}832 \times 10^{-6}\ \text{C}$$

Since capacitors are connected in series, each capacitor has the same charge *i.e.*

$$q = 5{\cdot}838 \times 10^{-6}$$

Therefore, voltage across C_1, $V_1 = \dfrac{q}{C_1} = \dfrac{5{\cdot}832 \times 10^{-6}}{4 \times 10^{-6}}$

$$= 1{\cdot}458\ \text{V}$$

and voltage across C_2, $V_2 = \dfrac{q}{C_2} = \dfrac{5{\cdot}832 \times 10^{-6}}{6 \times 10^{-6}}$

$$= 0{\cdot}972\ \text{V}$$

Problem 2.03. A 3 henry inductor is placed in series with a 10 ohm resistor. An e.m.f. of 30 volt is suddenly supplied to the combination. At 0·3 s (which is one inductive time constant) after start, find at which rate energy is stored in the magnetic field.

Sol. Here, $E = 30$ volt ; $L = 3$ henry ; $R = 10$ ohm ; $t = 0{\cdot}3$ s

Maximum current, $I_0 = \dfrac{30}{10} = 3$ ampere

From definition, after one inductive time constant *i.e.* at 0·3 s, $I = 0{\cdot}632\, I_0 = 0{\cdot}632 \times 3 = 1{\cdot}896$ A

Now, rate at which energy is stored in magnetic field,

$$\frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = LI\frac{dI}{dt}$$

But rate of increase of current during growth,

$$\frac{dI}{dt} = \frac{E - IR}{L}$$

Since at $t = 0{\cdot}3$ s, $I = 1{\cdot}896$ A, the rate of increase of current at 0·3 s,

$$\frac{dI}{dt} = \frac{30 - 1{\cdot}896 \times 10}{3} = \frac{30 - 18{\cdot}96}{3}$$

$$= \frac{11{\cdot}04}{3} = 3{\cdot}68\ \text{A s}^{-1}$$

$$\therefore \quad \frac{dW}{dt} = 3 \times 1{\cdot}896 \times 3{\cdot}68 = \mathbf{20{\cdot}93\ W}$$

Problem 2.04. An LC-circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC.

e resistance of the circuit is negligible. Let the instant at ich circuit is closed be $t = 0$.

(a) What is the total energy stored initially ? Is it con- ved during the LC oscillations ?

(b) What is the natural frequency of the circuit ?

(c) At what times is the energy stored : (i) completely ectrical (i.e. stored in the capacitor) ? (ii) completely gnetic (i.e. stored in the inductor) ?

(d) At what times is the total energy shared equally tween the inductor and the capacitor ?

(e) If a resistor is inserted in the circuit, how much ergy is eventually dissipated as heat ? *(Text Problem)*

Sol. Here, $L = 20 \text{ mH} = 20 \times 10^{-3}$ H ;

$C = 50 \ \mu\text{F} = 50 \times 10^{-6}$ F ;

Charge on the capacitor initially, $q_0 = 10 \text{ mC} = 10 \times 10^{-3}$ C

(a) Energy stored in the capacitor initially

$$= \frac{q_0^2}{2C} = \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}} = 1 \text{ J}$$

Since the LC-circuit has negligible resistance, the energy conserved in LC-oscillations.

(b) Natural frequency,

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = 159{\cdot}15 \text{ Hz}$$

(c) Time period of LC oscillations,

$$T = \frac{1}{f} = \frac{1}{159{\cdot}15} = 6{\cdot}28 \times 10^{-3} \text{ s}$$

(i) Energy stored is completely electrical at

$t = 0, T, T/2, 3T/2, \ldots$, where $T = 6{\cdot}28 \times 10^{-3}$ s

(ii) Energy stored is completely magnetic at

$t = T/4, 3T/4, 5T/4, \ldots$, where $T = 6{\cdot}28 \times 10^{-3}$ s

(d) The energy stored in the capacitor will become half that stored initially, if $q = \pm \dfrac{q_0}{\sqrt{2}}$

It is because, $W = \dfrac{q_0^2}{2C}$

i.e. $W \propto q_0^2$

During LC-oscillations, the charge on capacitor at any instant,

$$q = q_0 \cos \omega t$$

$$\therefore \quad \pm \frac{q_0}{\sqrt{2}} = q_0 \cos \omega t \quad \text{or} \quad \cos \omega t = \pm \frac{1}{\sqrt{2}}$$

Therefore, $\omega t = \pi/4, 3\pi/4, 5\pi/4, \ldots$

or $t = T/8, 3T/8, 5T/8, \ldots$, where $T = 6{\cdot}28 \times 10^{-3}$ s

(e) When resistance is introduced, whole of the energy will be dissipated in the form of heat. The introduction of resistance produces damped oscillations.

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. What is a transient current ?

Ans. The current which varies for a small finite time, hile growing to maximum or decaying to zero value, is alled transient current.

Q. 2.02. What is difference between a steady current nd d.c. ?

Ans. A steady current is that whose magnitude remains onstant. On the other hand, the current which always flows n one direction is called d.c. It does not reverse its direction vith time as a.c. does.

Q. 2.03. What are the dimensions of R/L ?

Ans. Reciprocal of time.

Q. 2.04. Write down the expression for the rate of rowth of current in an LR-circuit.

Ans. Refer to section 2.02. From equation (2.01), it ollows that

the rate of growth of current, $\dfrac{dI}{dt} = \dfrac{E - IR}{L}$

Q. 2.05. How does an inductor behave in the d.c. circuit after the steady state has been reached ?

Ans. After the steady state is reached, the inductor behaves just as a conducting wire.

Q. 2.06. A battery is connected to a series combination of an inductance (L) and a resistance R. On what factors does the value of maximum current depend in LR-circuit ?

Ans. The maximum value of current is given by

$$I_0 = \frac{E}{R}$$

It may be noted that it does not depend upon the value of inductance.

Q. 2.07. What are the dimensions of C R ?

Ans. Time.

Q. 2.08. Does the value of maximum charge on the capacitor in a CR-circuit depend on the values of both C and R ?

Ans. No, the value of maximum charge on the capacitor depends on the value of C. It is independent of the value of R.

Q. 2.09. Define time constant of an electrical circuit. *(I.S.C.E. 1998)*

Ans. The time constant of an electrical circuit is the time in which the current (in LR-circuit) or charge (in CR-circuit) grows to 1/eth of its maximum value.

Q. 2.10. Does the time constant of a CR-circuit or an LR-circuit depend on e.m.f. of the source ?

Ans. No, it does not depend on e.m.f. of the source. The time constant of a CR-circuit is CR and that of an LR-circuit is L/R.

Q. 2.11. Where does the energy reside in an inductor through which current has attained its maximum value ?

Ans. It resides inside the inductor in the form of magnetic field.

Q. 2.12. What are the dimensions of $\sqrt{LC}$?

Ans. Time.

Q. 2.13. What role does the resistance of inductor play in LC-circuit ?

Ans. Due to the resistance of the inductor, the LC-oscillations produced are damped one. It is because, during each oscillation, a part of electric energy is dissipated in the form of heat energy.

FAQ FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Define time constant in LR-circuit. How can it be increased ? *(H.S.S.C.E. 1994)*

Ans. For definition of time constant of LR-circuit, refer to section 2.02. The time constant of LR-circuit *i.e.* $\frac{L}{R}$ can be increased by increasing the value of L and decreasing the value of R.

Q. 2.02. Verify that L/R has the dimensions of time.

Ans. The instantaneous current in LR-circuit during growth is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

It follows that the factor $\frac{R}{L}t$ should be dimensionless *i.e.*

$$\frac{R}{L}t = \text{dimensionless}$$

Therefore, $\left[\frac{L}{R}\right] = \text{time}$

Q. 2.03. What factors determine the growth of current in LR-circuit ? Is it L or R or both ? Explain. *(H.S.S.C.E. 1994)*

Ans. The growth of current in LR-circuit is determined by both L and R. Practically, the current attains the maximum value in a time equal to $5\frac{L}{R}$. Therefore, lesser the value of L and greater the value of R, earlier the current will attain its maximum value.

Q. 2.04. Prove that total energy stored in an inductor is $\frac{1}{2}L I_{max}^2$, where I_{max} is the maximum current through the inductor. *(C.B.S.E. 1993)*

Ans. Refer to section 2.04.

Q. 2.05. Verify that CR has the dimensions of time.

Ans. The instantaneous charge on capacitor in CR-circuit during growth is given by

$$q = q_0\left(1 - e^{-\frac{1}{CR}t}\right)$$

It follows that the factor $\frac{1}{CR}t$ is dimensionless *i.e.*

$$\frac{1}{CR}t = \text{dimensionless}$$

Therefore, [C R] = time

Q. 2.06. Why does an LC-circuit produce oscillations ?

Ans. When a charged capacitor in an LC-circuit discharges through the inductor, the electric energy stored between the plates of capacitor appears as the magnetic energy inside the inductor. When the capacitor has discharged, the magnetic field linked with the inductor starts collapsing. Due to this, induced e.m.f. is produced in the inductor and the capacitor starts charging and ultimately the magnetic energy appears as the electric energy across the capacitor. This process repeats again and again, giving rise to LC-oscillations.

Q. 2.07. Calculate the wavelength of radio waves radiated out by a circuit consisting of 0·02 μF capacitor and 8 μH inductor in series. *(H.S.S.C.E. 1992)*

Ans. Here, $C = 0{\cdot}02\ \mu F = 0{\cdot}02 \times 10^{-6}\ F$;

$$L = 8\ \mu F = 8 \times 10^{-6}\ H$$

Now,
$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0{\cdot}02 \times 10^{-6} \times 8 \times 10^{-6}}}$$
$$= 3{\cdot}98 \times 10^{-5}\ Hz$$

Therefore, wavelength of the radio waves emitted,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3{\cdot}98 \times 10^5} = \mathbf{7{\cdot}54 \times 10^2\ m}$$

Q. 2.08. Why does an LC-circuit usually produce damped oscillations ?

Ans. An inductor possesses a small resistance also. Therefore, each time electric energy converts into magnetic energy during LC-oscillations, a small part of energy is dissipated as heat energy across the resistance of the inductor. As a result, the oscillations produced are damped in nature.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 2.01. Fig. 2.13 shows an inductor L and a resistance R connected in parallel to a battery through a switch. The resistance of R is the same as that of the coil that makes L. Two identical bulbs are put in each arm of the circuit.

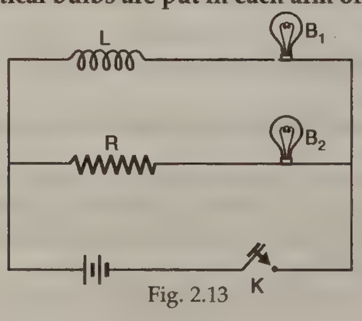


Fig. 2.13

(*i*) Which of the bulbs lights up earlier, when S is closed ? *(C.B.S.E. 1999 S, 1998)*

(*ii*) Will the bulbs be equally bright after some time ?

Ans. (*i*) The bulb B_2 will light up earlier. The reason is that current through bulb B_2 grows to maximum value instantaneously, while the current through bulb B_1 will take up some finite time $\left(\text{practically equal to } \frac{5L}{R}\right)$ to reach the maximum value.

(*ii*) After the steady state is reached, self-inductance will have no effect. Therefore, after some time, when steady state has been reached, the two bulbs will become equally bright.

Q. 2.02. An ideal inductor of 1 H is connected across a resistance of 100 Ω as shown in the Fig. 2.14.

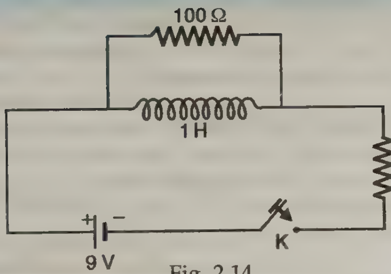


Fig. 2.14

(*i*) What is the current through the inductor as soon as ey K is closed ?

(*ii*) What is the potential difference across the 100 Ω resistance, when the current has attained a steady value ? (*I.S.C.E. 1994*)

Ans. **(*i*)** At the instant, the key K is closed, the current in the circuit begins to grow and the induced e.m.f. produced in the inductor opposes the flow of current through it. Thus, as soon as the key K is closed, **no current** flows through the inductor.

(*ii*) When the current attains steady value in the circuit, the inductive effect of the inductor ceases and it will short circuit the 100 Ω resistance. Since no current flows through the 100 Ω resistance, the potential difference across 100 Ω resistance will also be **zero.**

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Show that the current rises exponentially in a circuit containing inductance and resistance, when a steady e.m.f. is applied to it.
2. A certain LR-circuit has a time constant equal to L/R. Show that the time T that must elapse for the current to achieve half its final steady state value is

$$T = \frac{L}{R} \log_e 2$$

3. Derive the equation for decay of current in an LR-circuit connected to a source of constant e.m.f. What is meant by time constant ?
4. Show that during charging and discharging of capacitor through a resistance, the current always decreases from maximum to zero.
5. Derive an expression for the energy stored in an inductor. (*H.S.S.C.E. 1999 S*)
6. Show that the energy stored in an inductor L, when a current I is established through it, is $\frac{1}{2} L I^2$. (*H.S.S.C.E. 2002, 2001 ; C.B.S.E. 1990*)
7. Show that in an electric circuit consisting of inductance, the coefficient of self induction is numerically equal to twice the work done in establishing the magnetic flux associated with unit current in the circuit.
8. How is energy stored in an inductor and where does this energy reside ? Obtain an expression for this energy and give an example, where this energy is made use of.
9. Derive an expression for the energy stored in a charged capacitor C. Where does this energy reside ?
10. Prove that the potential energy of a charged capacitor is equal to half of the product of charge on it and potential difference between its plates. In what form does this energy reside ?
11. What happens, when a charged capacitor is discharged through a coil of inductance L and of negligible resistance ?
12. What do you understand by LC oscillations ? Discuss qualitatively, how these oscillations are produced.
13. A charged capacitor is connected to an inductor. Show that the discharge of capacitor through the inductor is oscillatory. Obtain the frequency of oscillations.

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Explain the meaning of transient current. Discuss the growth and decay of current in LR-circuit. (*C.B.S.E. 1990*)
2. (*a*) Derive an expression for the growth of current in a circuit containing L and R. Give the graph between time and rising current.
   (*b*) Draw a sketch to show the variation of current with time. How would the curve look like, if
   (*i*) R was replaced with R′ (R′ > R) ? (*ii*) E was replaced with E′ (E′ > E) ?
3. Discuss the charging and discharging of a capacitor in a CR-circuit. When will the time constant be 1 second ?
4. (*a*) Derive the equations for growth and decay of charge in CR-circuit connected to a constant e.m.f. What is meant by time constant ?
   (*b*) How does the current vary with time in such a circuit ?
   (*c*) If the resistance could be made negligibly small, which of the following will be correct ?
   (*i*) The capacitor gets charged almost instantaneously.
   (*ii*) The charging rate becomes gradual. (*iii*) There is no charging at all.

FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Growth and Decay of current in LR-circuit

1. What resistance must be connected in series with an inductor of 5 mH, so that the circuit has a time constant of 2×10^{-3} s ? [**Ans.** 2·5 Ω]
2. A cell of 1·5 V is connected across an inductor of 2 mH in series with a 2 Ω resistor. What is the rate of growth of current immediately after the circuit is switched on ? [**Ans.** 750 A s^{-1}]
3. A coil of resistance 20 ohm and inductance of 0·5 henry is applied d.c. of 200 volt. Calculate the rate of increase of current (*i*) at the instant of closing the switch (*ii*) at *t* = L/R s after the switch is closed. [**Ans.** (*i*) 400 A s^{-1} ; (*ii*) 147·2 A s^{-1}]
4. A coil of resistance 20 ohm and inductance 0·5 henry is connected to 100 volt d.c. battery. Calculate
   (*a*) the steady current
   (*b*) the rate of increase of current (*i*) at the instant of closing the switch (*ii*) at *t*= L/R s after the switch is closed.
   [**Ans.** (*a*) 5 A ; (*b*) (*i*) 200 A s^{-1}, (*ii*) 73·6 A s^{-1}]

5. A 2 volt battery of negligible internal resistance is applied to a coil of inductance 1 henry and resistance 10 ohm. Calculate the time required by current to attain a value half that of the steady current. **[Ans.** 0·0693 s**]**

6. A coil of inductance 8·4 mH and resistance 6 Ω is connected to a 12 V battery. Find the time, when the current in the coil will grow to 1·0 A. *(I.I.T. 1999)* **[Ans.** $9{\cdot}7 \times 10^{-4}$ s**]**

7. When a current of 10 A is flowing through a resistance of 20 Ω and inductance of 10 H, the battery is switched off. Find (*a*) current after 0·4 s, (*b*) the time this current takes to fall to 60% of its initial value. **[Ans.** (*a*) 4·5 A (*b*) 0·255 s**]**

Type B. On Growth and Decay of charge in CR-circuit

8. A charged capacitor of capacitance 40 μF is discharged through a 50 Ω resistor. Determine the time constant of the circuit. *(Text Problem)* **[Ans.** 2 ms**]**

9. A capacitor of 2 μF is connected to 20 volt supply through 1 megaohm resistance. How long will it take the capacitor to charge up to 63·2 % of its final charge ? Also calculate the maximum charge. **[Ans.** 2 s, 4×10^{-5} C**]**

10. A 0·18 μF capacitor is first charged and then discharged through a high resistance. If it takes 0·5 s for the charge to reduce to one fourth of its initial value, find the value of resistance. Use $\log_e 4 = 1{\cdot}386$. **[Ans.** 2×10^6 Ω**]**

11. A capacitor of capacitance 0·5 μF is discharged through a resistance of 10 megaohm. Find the time taken for half the charge on the capacitor to escape. **[Ans.** 3·466 s**]**

12. A capacitor of capacitance 0·1 mF and resistance of 10 megaohm is charged to a certain potential and then insulated. Find the time that the potential will take to fall to half its original value. **[Ans.** 693·15 s**]**

13. A capacitor charged to 10 V is being discharged through a resistance R. At the end of 1 s, the voltage across the capacitor is 5 V. What will be the voltage after 2 s ? *(C.B.S.E. 1997)***[Ans.** 2·5 V**]**

Type C. On Energy stored in a capacitor and Inductor

14. A capacitor having capacitance of 1 μF is charged with 0·01 coulomb of electricity. How much energy is stored in it ? **[Ans.** 50 J**]**

15. Three series capacitors of capacitances 2·0, 3·0 and 6·0 μF are charged by a 60 volt battery. Find the energy of the stored charge. **[Ans.** $1{\cdot}8 \times 10^{-3}$ J**]**

16. A coil has inductance of 5 henry and resistance 20 ohm. An e.m.f. of 100 volt is applied to it. What is the energy stored in the magnetic field, when the current has reached its final steady value ? **[Ans.** 62·5 J**]**

Type D. On LC-oscillations

17. A coil of inductance of 0·4 millihenry is connected to a capacitor of capacitance 400 pF. To what wavelength is the circuit tuned ? **[Ans.** 753·77 m**]**

18. Find the natural frequency of a circuit containing inductance of 100 μH and a capacity of 0·01 μF. To which wavelength, its response will be maximum ? For how long the oscillations will continue ? **[Ans.** 159·15 kHz, 1884·96 m ; ∞ **]**

19. A capacitor of 10 microfarad is connected in series with a resistance of $2{\cdot}2 \times 10^5$ ohm. Determine the time constant for the circuit. Can we think of oscillations in this circuit, such that the frequency is equal to the inverse of the time constant ? Justify your answer with proper explanation. **[Ans.** 2·2 s, No**]**

MISCELLANEOUS PROBLEMS

20. Two resistances, one of 100 Ω and another of 200 Ω and an inductance of 10 H are connected to 3 V battery through a key K as shown in Fig. 2.15. On closing the key, what is the initial potential drop across the inductance ? *(H.S.S.C.E. 2001)* **[Ans.** 1 V**]**

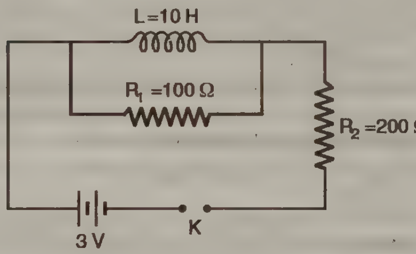


Fig. 2.15

21. A solenoid has an inductance of 10 henry and a resistance of 2 ohm. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach 1/4th of its maximum value ? *(I.I.T. 1996)* **[Ans.** 3·47 s**]**

22. A 20 μF capacitor is charged to 30 V of potential. The battery is then disconnected and a 200 mH of coil is connected across it, so that LC oscillations are set up. Calculate the frequency of the oscillations set up and the maximum current in the coil. **[Ans.** 79·6 Hz, 0·3 A**]**

23. An inductor of inductance 2·0 mH is connected across a charged capacitor of capacitance 5·0 μF and the resulting LC-circuit is set oscillating at its natural frequency. Let *q* denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that maximum value of charge on the capacitor is 200 μC.

(*a*) When *q* = 100 μC, what is the value of $\left|\dfrac{dI}{dt}\right|$?

(*b*) When *q* = 200 μC, what is the value of I ?

(*c*) Find the maximum value of I.

(*d*) When I is equal to one half its maximum value, what is the value of | *q* | ? *(I.I.T. 1998)*

[Ans. (*a*) 10^4 A s^{-1} ; (*b*) 0 ; (*c*) 2·0 A ; (*d*) 173·2 μC**]**

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Time constant $= \dfrac{L}{R}$ or $2 \times 10^{-3} = \dfrac{5 \times 10^{-3}}{R}$

or $R = \dfrac{5 \times 10^{-3}}{2 \times 10^{-3}} = \mathbf{2{\cdot}5\ \Omega}$

2. Now, $\dfrac{dI}{dt} = \dfrac{E - IR}{L}$

When the circuit is just switched on, I = 0.

$$\therefore \quad \frac{dI}{dt} = \frac{E}{L} = \frac{1{\cdot}5}{2 \times 10^{-3}} = \mathbf{750\ A\ s^{-1}}$$

3. (*i*) Rate of increase of current at the instant of closing the switch,

$$\frac{dI}{dt} = \frac{E}{L} = \frac{200}{0{\cdot}5} = \mathbf{400\ A\ s^{-1}}$$

(*ii*) Now, $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

Therefore, at $t = \dfrac{L}{R}$,

$$I = I_0\left(1 - e^{\frac{-R}{L} \times \frac{L}{R}}\right) = I_0\,(1 - e^{-1}) = 0{\cdot}632\, I_0$$

$$= 0{\cdot}632\,\frac{E}{R} = \frac{0{\cdot}632\times 200}{20} = 6{\cdot}32\text{ A}$$

Therefore, rate of increase of current at $t = \frac{L}{R}$,

$$\frac{dI}{dt} = \frac{E - IR}{L} = \frac{200 - 6{\cdot}32\times 20}{0{\cdot}5} = \mathbf{147{\cdot}2\ A\ s^{-1}}$$

4. **(a)** Now, $I_0 = \frac{E}{R} = \frac{100}{20} = \mathbf{5\ A}$

(b) Proceed as in problem no. 3.

5. Proceed as in solved problem 2.02 (*ii*)

6. Now, $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$ or $I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

or $1{\cdot}0 = \frac{12}{6}\left(1 - e^{-\frac{6}{8{\cdot}4\times 10^{-3}}t}\right)$ or $e^{-\frac{6\times 10^3}{8{\cdot}4}t} = 0{\cdot}5$

or $e^{\frac{6\times 10^3}{8{\cdot}4}t} = 2$ or $\frac{6\times 10^3}{8{\cdot}4}t = \log_e 2$

or $t = \frac{8{\cdot}4}{6\times 10^3}\times 2{\cdot}3026\log_{10} 2 = \frac{8{\cdot}4\times 2{\cdot}3026\times 0{\cdot}3010}{6\times 10^3}$

$= \mathbf{9{\cdot}7\times 10^{-4}\ s}$

7. **(a)** Now, $I = I_0\, e^{-\frac{R}{L}t}$ or $I = 10\, e^{-\frac{20}{10}\times 0{\cdot}4}$

or $I = 10\times e^{-0{\cdot}8}$ or $I = 10\times 0{\cdot}45 = \mathbf{4{\cdot}5\ A}$

(b) Now, $I_0\times\frac{60}{100} = I_0\, e^{-\frac{20}{10}t}$ or $e^{2t} = 1{\cdot}667$

or $2t = \log_e 1{\cdot}667$ or $t = \frac{1}{2}\times 2{\cdot}3026\log_{10} 1{\cdot}667$

or $t = \frac{1}{2}\times 2{\cdot}3026\times 0{\cdot}2219 = \mathbf{0{\cdot}255\ s}$

8. Time constant $= CR = 40\times 10^{-6}\times 50 = 2\times 10^{-3}\text{ s} = \mathbf{2\ ms}$

9. Now, $q = q_0\left(1 - e^{-\frac{1}{CR}t}\right)$

∴ $\frac{q_0\times 63{\cdot}2}{100} = q_0\left(1 - e^{-\frac{1}{2\times 10^{-6}\times 10^6}t}\right)$

or $0{\cdot}632 = 1 - e^{-\frac{t}{2}}$ or $e^{-\frac{t}{2}} = 1 - 0{\cdot}632 = 0{\cdot}368$

or $e^{\frac{t}{2}} = \frac{1}{0{\cdot}368} = 2{\cdot}717$ or $\frac{t}{2} = \log_e 2{\cdot}717$

or $t = 2\times 2{\cdot}3026\log_{10} 2{\cdot}717 = 2\times 2{\cdot}3026\times 0{\cdot}434 = \mathbf{2\ s}$

Also, maximum charge on capacitor,

$q_0 = CE = \mathbf{4\times 10^{-5}\ C}$

Aliter. The capacitor will charge to 63·2% of its final value in a time equal to CR *i.e.* $2\times 10^{-6}\times 10^{6} = \mathbf{2\ s}$

10. Now, $q = q_0\, e^{-\frac{1}{CR}t}$ or $\frac{q_0}{4} = q_0\, e^{-\frac{1}{0{\cdot}18\times 10^{-6}\times R}\times 0{\cdot}5}$

or $e^{\frac{2{\cdot}78\times 10^6}{R}} = 4$ or $\frac{2{\cdot}78\times 10^6}{R} = \log_e 4$

or $R = \frac{2{\cdot}78\times 10^6}{\log_e 4} = \frac{2{\cdot}78\times 10^6}{1{\cdot}386} = \mathbf{2\times 10^6\ \Omega}$

11. Proceed as in problem no. 10.

12. Now, $q = q_0\, e^{-\frac{1}{CR}t}$ or $\frac{q}{C} = \frac{q_0}{C}\, e^{-\frac{1}{CR}t}$

or $V = E\, e^{-\frac{1}{CR}t}$

Proceed as in problem no. 10 to find *t* by taking $V = \frac{E}{2}$.

13. Now, $V = E\, e^{-\frac{1}{CR}t}$ or $5 = 10\, e^{-\frac{1}{CR}\times 1}$

or $e^{-\frac{1}{CR}} = 0{\cdot}5$

Voltage after 2 s is given by

$$V = 10\, e^{-\frac{1}{CR}\times 2} = 10\times\left(e^{-\frac{1}{CR}}\right)^2 = 10\times(0{\cdot}5)^2 = \mathbf{2{\cdot}5\ V}$$

14. Now, $U = \frac{q_0^2}{2C} = \frac{(0{\cdot}01)^2}{2\times 10^{-6}} = \mathbf{50\ J}$

15. Now, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2{\cdot}0} + \frac{1}{3{\cdot}0} + \frac{1}{6{\cdot}0} = 1$

or $C_s = 1\ \mu F = 10^{-6}\text{ F}$

∴ $U = \frac{1}{2}C_s E^2 = \frac{1}{2}\times 10^{-6}\times(60)^2 = \mathbf{1{\cdot}8\times 10^{-3}\ J}$

16. Now, $I_0 = \frac{E}{R} = \frac{100}{20} = 5\text{ A}$

∴ $U = \frac{1}{2}L I_0^2 = \frac{1}{2}\times 5\times(5)^2 = \mathbf{62{\cdot}5\ J}$

17. Here, $L = 0{\cdot}4\text{ mH} = 0{\cdot}4\times 10^{-3}\text{ H}$;

$C = 400\text{ pF} = 400\times 10^{-12}\text{ F}$

Now, $f = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{0{\cdot}4\times 10^{-3}\times 4000\times 10^{-12}}}$$

$$= \frac{1}{2\pi\times 40\times 10^{-8}} = 3{\cdot}98\times 10^5\text{ Hz}$$

∴ $\lambda = \frac{c}{f} = \frac{3\times 10^8}{3{\cdot}98\times 10^5} = \mathbf{753{\cdot}77\ m}$

18. Proceed as in problem no. 17.
The oscillations will continue for infinitely long time, provided the circuit is non-resistive.

19. Time constant $= CR = 10\times 10^{-6}\times 2{\cdot}2\times 10^5 = \mathbf{2{\cdot}2\ s}$
A CR-circuit can not produce oscillations.

20. On closing the key, initially no current flows through inductor. Therefore, current will flow through the series combination of R_1 and R_2 and is given by

$$I = \frac{E}{R_1 + R_2} = \frac{3}{100 + 200} = 0{\cdot}01\text{ A}$$

Potential drop across L = potential drop across R_1
$= IR_1 = 0{\cdot}01\times 100 = \mathbf{1\ V}$

21. Here, $L = 10\text{ H}$; $R = 2\ \Omega$ and $E = 10\text{ V}$

∴ $I_0 = \frac{E}{R} = \frac{10}{2} = 5\text{ A}$

Maximum energy stored in the capacitor, $U_{max} = \frac{1}{2}L I_0^2$

Let I be the current, when energy stored in the capacitor will become $\frac{1}{4}$th of its maximum value *i.e.*

$\frac{1}{2}LI^2 = \frac{1}{4}\left(\frac{1}{2}L I_0^2\right)$ or $I = \frac{I_0}{2} = \frac{5}{2} = 2{\cdot}5\text{ A}$

$$\text{Now, } I = I_0\left(1-e^{-\frac{R}{L}t}\right) \quad \text{or} \quad 2\cdot5 = 5\left(1-e^{-\frac{2}{10}t}\right)$$

$$\text{or } (1-e^{-0\cdot2\,t}) = 0\cdot5 \quad \text{or} \quad e^{-0\cdot2\,t} = 0\cdot5$$

$$\text{or } e^{0\cdot2\,t} = 2 \quad \text{or} \quad 0\cdot2\,t = \log_e 2$$

$$\text{or } t = \frac{\log_e 2}{0\cdot2} = \frac{2\cdot3026\times\log_{10}2}{0\cdot2} = \frac{2\cdot3026\times0\cdot3010}{0\cdot2}$$

$= \mathbf{3\cdot47\ s}$

22. To find frequency of oscillations, proceed as in problem no. 17. It can be obtained that

$f = \mathbf{79\cdot6\ Hz}$

During LC-oscillations, charge on capacitor at any instant,

$$q = q_0 \cos \omega t$$

$$\text{Now, } I = \frac{dq}{dt} = \frac{d}{dt}(q_0 \cos \omega t) = q_0(-\sin \omega t)\,\omega$$

$$= -q_0\,\omega \sin \omega t$$

Therefore, maximum value of current,

$I_0 = q_0\,\omega = C\,E \times 2\pi f = 20 \times 10^{-6} \times 30 \times 2\pi \times 79\cdot6 = \mathbf{0\cdot3\ A}$

23. Here, $L = 2\cdot0\ mH = 2 \times 10^{-3}\ H$; $C = 5\cdot0\ \mu F = 5 \times 10^{-6}\ F$

$$\text{Now, } \omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2\times10^{-3}\times5\times10^{-6}}} = 10^4\ \text{rad s}^{-1}$$

Now, charge on capacitor at any instant during LC-oscillations,

$$q = q_0 \cos \omega t \quad \text{...(i)}$$

$$\therefore \quad I = \frac{dq}{dt} = -q_0\,\omega \sin \omega t \quad \text{...(ii)}$$

$$\text{and} \quad \frac{dI}{dt} = -q_0\,\omega^2 \cos \omega t \quad \text{...(iii)}$$

(a) To find $\frac{dI}{dt}$, when q = 100 μC :

Setting $q = 100\,\mu C$ and $q_0 = 200\,\mu C$ in equation (*i*), we hav

$$100 = 200 \cos \omega t \quad \text{or} \quad \cos \omega t = 0\cdot5$$

$$\text{Now, } \frac{dI}{dt} = q_0\,\omega^2 \cos \omega t \quad \text{(in magnitude)}$$

Here, $q_0 = 200\ \mu C = 200 \times 10^{-6}\ C$, $\omega = 10^4\ \text{rad s}^{-1}$ and $\cos \omega t = 0\cdot5$

$$\therefore \quad \frac{dI}{dt} = 200\times10^{-6}\times(10^4)^2\times0\cdot5 = \mathbf{10^4\ A\ s^{-1}}$$

(b) To find I, when q = 200 μC :

Setting $q = 200\,\mu C$ and $q_0 = 200\,\mu C$ in equation (*i*), we hav

$$200 = 200 \cos \omega t \quad \text{or} \quad \cos \omega t = 1 \quad \text{or} \quad \omega t = 0°$$

Now, $I = q_0\,\omega \sin \omega t$ (in magnitude)

$\therefore$ $I = 200 \times 10^{-6} \times 10^4 \sin 0° = \mathbf{0}$

(c) To find maximum value of I :

From equation (*ii*), maximum value of I is given by

$I_0 = q_0\,\omega$ (in magnitude)

$= 200 \times 10^{-6} \times 10^4 = \mathbf{2\ A}$

(d) To find q, when I = I₀/2 :

Now, $I = q_0\,\omega \sin \omega t$ (in magnitude)

$$\text{Setting } I = \frac{I_0}{2} \text{ or } I = \frac{q_0\,\omega}{2}, \text{ we have}$$

$$\frac{q_0\,\omega}{2} = q_0\,\omega \sin \omega t \quad \text{or} \quad \sin \omega t = 0\cdot5 \quad \text{or} \quad \omega t = 30°$$

Now, $q = q_0 \cos \omega t$

$\therefore$ $q = 200 \times 10^{-6} \cos 30° = 200 \times 10^{-6} \times 0\cdot866$

$= 173\cdot2 \times 10^{-6}\ C = \mathbf{173\cdot2\ \mu C}$

CHAPTER
3

Alternating Currents

3.01. ALTERNATING CURRENT AND E.M.F.

The electric current, whose magnitude changes with time and direction reverses riodically, is known as alternating current.

The electric mains supply sends current in a circuit, which may be represented a sine or cosine function of time. Therefore, the instantaneous value (value at any ne t) of alternating current is given by

$$I = I_0 \sin \omega t \qquad ...(3.01)$$

or

$$I = I_0 \cos \omega t, \qquad ...(3.02)$$

here I_0 is called ***current amplitude*** or ***peak (maximum) value of alternating current.*** T is time period of alternating current and f its frequency, then

$$\omega = \frac{2\pi}{T} = 2\pi f$$

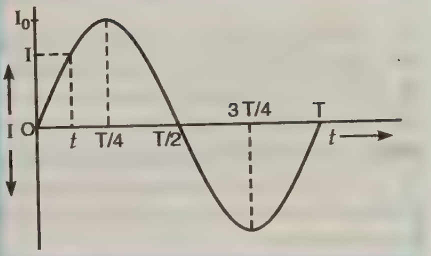


Fig. 3.01

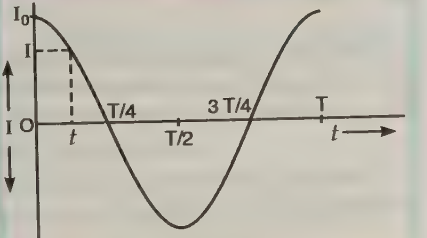


Fig. 3.02

It may be pointed out that instantaneous value of alternating current can be presented by equation (3.01) as a sine function or by equation (3.02) as a cosine nction of time t. Both the representations lead to the same result. Fig. 3.01 shows e instantaneous current I and current amplitude I_0 , when the equation (3.01) is sed to represent alternating current as a sine function of time. On the other hand, g. 3.02 represents I and I_0, when equation (3.02) is used to represent alternating urrent as a cosine function of time.

Similarly *the e.m.f. (or voltage), whose magnitude changes with time and direction verses periodically is known as alternating e.m.f.*

The instantaneous value of alternating e.m.f. may be represented by

$$E = E_0 \sin \omega t \qquad ...(3.03)$$

or

$$E = E_0 \cos \omega t \qquad ...(3.04)$$

Since alternating current varies continuously with time, its effect is measured y defining either the ***mean value*** (or ***average value***) of a.c. or by defining ***root mean quare value*** (or ***virtual value***) of a.c.

3.02. MEAN OR AVERAGE VALUE OF A.C.

It is that steady current, which when passed through a circuit for half the time period f the alternating current, sends the same amount of charge as is done by the alternating urrent in the same time through the same circuit.

It is denoted by I_m or I_a.

Relation between mean value and peak value of a.c. The instantaneous value f a.c. in a circuit is given by

$$I = I_0 \sin \omega t$$

The alternating current continuously changes with time. Suppose that the current through the circuit remains constant for an infinitesimally small time dt. Then, he small amount of charge that will pass through the circuit in time dt is given by

$$dq = I\,dt$$

or

$$dq = I_0 \sin \omega t\, dt$$

The amount of charge that will pass through the circuit in time T/2 (half time period of a.c.) can be obtained by integrating the above equation between the limits = 0 to $t = T/2$ *i.e.*

$$q = \int_0^{T/2} I_0 \sin \omega t\, dt = I_0 \int_0^{T/2} \sin \omega t\, dt = I_0 \left| -\frac{\cos \omega t}{\omega} \right|_0^{T/2} = -\frac{I_0}{\omega} \left| \cos \omega t \right|_0^{T/2}$$

$$= -\frac{I_0}{2\pi/T} \left| \cos \frac{2\pi}{T} \cdot \frac{T}{2} - \cos \frac{2\pi}{T} \cdot 0 \right| = -\frac{I_0 T}{2\pi} \left| \cos \pi - \cos 0 \right| = -\frac{I_0 T}{2\pi} \left| -1-1 \right|$$

or $$q = \frac{I_0 T}{\pi} \qquad ...(3.05)$$

If I_m is mean value of a.c., then by definition

$$q = I_m \cdot \frac{T}{2} \qquad ....(3.06)$$

From equations (3.05) and (3.06), we have

$$I_m \cdot \frac{T}{2} = \frac{I_0 T}{\pi}$$

or $$I_m = \frac{2 I_0}{\pi} = 0 \cdot 636 \, I_0 \qquad ...(3.07)$$

Thus, the mean or average value of alternating current during a half cycle is 0·636 times (or 63·6% of) its peak value. Similarly, it can be proved that mean value of alternating e.m.f. is

$$E_m = \frac{2 E_0}{\pi} = 0 \cdot 636 \, E_0 \qquad ...(3.08)$$

During the next half cycle, the mean value of a.c. will be equal in magnitude but opposite in direction. For this reason, the average value of a.c. over a complete cycle is always zero. It may be noted that as the average value of a.c. over the complete cycle of a.c. is zero, it is always defined over a half cycle of a.c.

3.03. ROOT MEAN SQUARE (RMS) OR VIRTUAL VALUE OF A.C.

It is that steady current, which when passed through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance and in the same time.

It is denoted by I_{rms} or I_v.

Relation between virtual value and peak value of a.c. The instantaneous value of a.c. passing through a resistance R is given by

$$I = I_0 \sin \omega t$$

The alternating current changes continuously with time. Suppose that the current through the resistance remains constant for an infinitesimally small time dt. Then, small amount of heat produced in the resistance R in time dt is given by

$$dH = I^2 R \, dt \, (I_0 \sin \omega t)^2 R \, dt = I_0^2 R \sin^2 \omega t \, dt$$

The amount of heat produced in the resistance in time T/2 (half time period) can be obtained by integrating the above equation between the limits $t = 0$ to $t = T/2$ *i.e.*

$$H = \int_0^{T/2} I_0^2 R \sin^2 \omega t \, dt = I_0^2 R \int_0^{T/2} \sin^2 \omega t \, dt = I_0^2 R \int_0^{T/2} \frac{1 - \cos 2\omega t}{2} dt$$

or $$H = \frac{I_0^2 R}{2} \left(\int_2^{T/2} dt - \int_0^{T/2} \cos 2\omega t \, dt \right) \qquad ...(3.09)$$

Now, $$\int_0^{T/2} dt = | t |_0^{T/2} = \frac{T}{2} - 0 = \frac{T}{2}$$

and $$\int_0^{T/2} \cos 2\omega t \, dt = \left| \frac{\sin 2\omega t}{2\omega} \right|_0^{T/2} = \frac{1}{2\omega} \left| \sin \frac{2\pi}{T} \cdot \frac{T}{2} - \sin \frac{2\pi}{T} \cdot 0 \right|$$

$$= \frac{1}{2\omega} | \sin \pi - \sin 0 | = \frac{1}{2\omega} | 0 - 0 | = 0$$

Substituting the values of the two integrals obtained above in equation (3.09), we have

$$H = \frac{I_0^2 R}{2} \left(\frac{T}{2} - 0 \right) = \frac{I_0^2 R}{2} \cdot \frac{T}{2} \qquad ...(3.10)$$

If I_v is virtual or r.m.s. value of a.c., then by definition,

$$H = I_v^2 R \cdot \frac{T}{2} \qquad ...(3.11)$$

From equations (3.10) and (3.11), we have

$$I_v^2 R \cdot \frac{T}{2} = \frac{I_0^2 R}{2} \cdot \frac{T}{2} \qquad \text{or} \qquad I_v^2 = \frac{I_0^2}{2}$$

$$\text{or} \qquad I_v = \frac{I_0}{\sqrt{2}} = 0{\cdot}707\, I_0 \qquad ...(3.12)$$

Thus, the r.m.s. or virtual value of a.c. is 0·707 times (or 70·7 % of) its peak value. imilarly, it can be proved that virtual value of alternating e.m.f. is

$$E_v = \frac{E_0}{\sqrt{2}} = 0{\cdot}707\, E_0 \qquad ...(3.13)$$

The r.m.s. or virtual value of a.c. is same even for a complete cycle of a.c. This act is made use of to construct ***hot wire instruments*** for measuring currents and oltages. Such ammeters and voltmeters record virtual values of alternating current nd voltage respectively.

3.04. PHASORS AND PHASOR DIAGRAM

We know that displacement of S.H.M. varies sinusoidally with time and its nstantaneous value is given by

$$y = r \sin \omega t$$

Further, the instantaneous value of the displacement (y) is equal to the rojection of the amplitude (r) on Y-axis.

In an a.c. circuit, the e.m.f. and current vary sinusoidally with time and may e mathematically represented as

$$E = E_0 \sin \omega t \qquad ...(3.14)$$

$$\text{and} \qquad I = I_0 \sin (\omega t + \phi) \qquad ...(3.15)$$

Here, ϕ is the phase angle between alternating e.m.f. and current.

Therefore, instantaneous values of alternating e.m.f. (E) and current (I) may e considered as the projections of e.m.f. amplitude (E_0) and current amplitude (I_0) espectively.

A quantity, which varies sinusoidally with time and represented as the projection of a otating vector, is called a ***phasor****.*

Therefore, the quantities, such as alternating e.m.f. and alternating current, may e called phasors.

The diagram, representing alternating e.m.f. and current (phasors) as the rotating ectors along with the phase angle between them, is called ***phasor diagram****.*

Let us draw phasor diagram for an a.c. circuit, in which e.m.f. and current vary according to the equations (3.14) and (3.15) as given above.

Draw two circles I and II of radii E_0 and I_0 (to the scale) respectively as shown in Fig. 3.03. Let $\angle AOX = \omega t$ and $\angle BOX = (\omega t + \phi)$. Then, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent rotating vectors having magnitudes E_0 and I_0 (the peak values of alternating e.m.f. and current) rotating in anticlockwise direction with the same angular frequency ω. Further, the projections OM (= E) and ON (= I) represent the phasors (instantaneous values of alternating e.m.f. and alternating current respectively) and the $\angle BOA = \phi$ represents the phase angle, by which the phasor I (alternating current) leads the phasor E (alternating e.m.f.). Fig. 3.03 is called phasor diagram for the alternating e.m.f. and current in an a.c. circuit, in which e.m.f. and current vary as given by equations (3.14) and (3.15). This phasor diagram may be simply represented as shown in Fig. 3.04. When one is interested only in the phase relationship, the phase diagram may be represented as shown in Fig. 3.05.

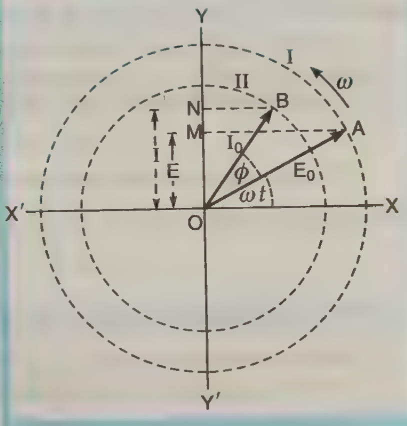


Fig. 3.03

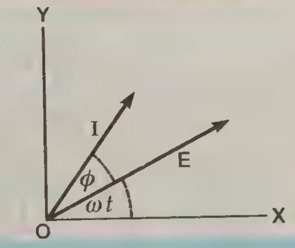


Fig. 3.04

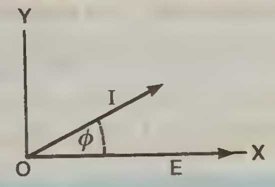


Fig. 3.05

3.05. A.C. THROUGH A RESISTOR

Consider that a pure ohmic resistor of resistance R (zero inductance) is connected to a source of an alternating e.m.f. [Fig. 3.06].

Let the instantaneous value of alternating e.m.f. be given by

$$E = E_0 \sin \omega t \qquad ...(3.16)$$

Now, the instantaneous e.m.f. due to source

= instantaneous value of potential drop across the resistance

If I is the current in the circuit at that instant, then

$$E = I\,R \qquad \text{or} \qquad I = \frac{E}{R}$$

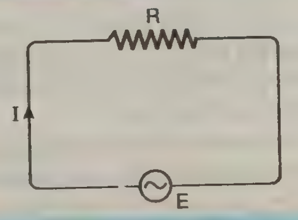


Fig. 3.06

Substituting for E, we have

$$I = \frac{E_0}{R} \sin \omega t$$

Now, $\frac{E_0}{R} = I_0$, the peak value of alternating current in the circuit. Therefore,

$$I = I_0 \sin \omega t \qquad \text{...(3.17)}$$

The equation (3.17) gives the instantaneous value of alternating current in the circuit containing R. From equations (3.16) and (3.17), it follows that the alternating current is ***in phase*** with the e.m.f., when a.c. flows through a resistance. This fact is graphically represented as shown in Fig. 3.07.

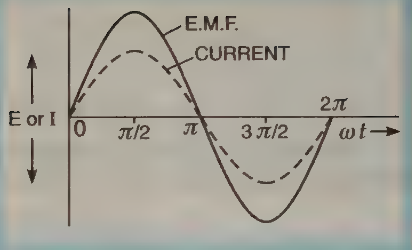


Fig. 3.07

Fig. 3.08 is the phasor diagram for alternating e.m.f. and current given by equations (3.16) and (3.17). As E and I are in phase with each other, both the phasors representing E and I have been drawn so as to make angle ωt with OX.

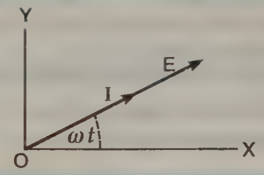


Fig. 3.08

3.06. A.C. THROUGH AN INDUCTOR

Consider that a pure inductor of inductance L (having no ohmic resistance) is connected to a source of an alternating e.m.f. [Fig. 3.09].

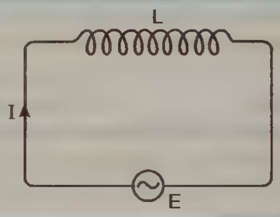


Fig. 3.09

Let the instantaneous value of alternating e.m.f. be given by

$$E = E_0 \sin \omega t \qquad \text{...(3.18)}$$

If I is the current through the circuit and $\frac{dI}{dt}$, the rate of change of current in the circuit at that instant, then the instantaneous induced e.m.f. produced across L,

$$e = -L\frac{dI}{dt}$$

Therefore, total instantaneous e.m.f. of the circuit $= E + \epsilon = E + \left(-L\frac{dI}{dt}\right)$

Now, total instantaneous e.m.f. in the circuit

= instantaneous potential drop across the circuit elements of the circuit

Since there is no circuit element across which potential drop may occur,

$$E + \left(-L\frac{dI}{dt}\right) = 0 \quad \text{or} \quad E = L\frac{dI}{dt}$$

or
$$dI = \frac{E}{L}dt = \frac{E_0}{L}\sin \omega t \, dt$$

or
$$\int dI = \frac{E_0}{L}\int \sin \omega t \, dt$$

or
$$I = \frac{E_0}{L}\left(\frac{-\cos \omega t}{\omega}\right) = -\frac{E_0}{\omega L}\cos \omega t = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left[\because \sin(\omega t - \pi/2) = \sin\{-(\pi/2 - \omega t)\} = -\sin(\pi/2 - \omega t) = -\cos \omega t\right]$$

Key point

The inductive reactance increases as the frequency of a.c. source is increased.

Now ωL has got units of resistance and it is called ***reactance of inductor*** or ***inductive reactance.*** It is denoted by X_L. Therefore,

$$I = \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Further, $\frac{E_0}{X_L} = I_0$, the peak value of alternating current in the circuit.

Therefore,
$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \text{...(3.19)}$$

The equation (3.19) gives the instantaneous value of current in the a.c. circuit containing L. From equations (3.18) and (3.19), it follows that the alternating current ***lags behind*** the e.m.f. by a phase angle $\pi/2$ (or e.m.f. ***leads*** the current by a phase angle $\pi/2$), when a.c. flows through an inductor. This fact is graphically represented as shown in Fig. 3.10.

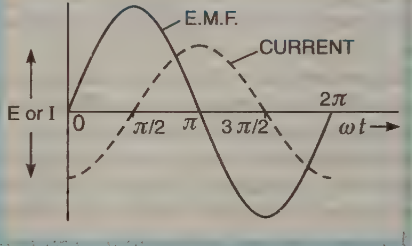


Fig. 3.10

Let us now draw phasor diagram, when a.c. flows through an inductor. The phasor representing E is drawn so as to make an angle ωt with OX in anticlockwise direction. As current lags behind the e.m.f. by phase angle $\pi/2$, the phasor representing I has been drawn so as to make angle $\pi/2$ in *clockwise direction* with the phasor representing E_v [Fig. 3.11].

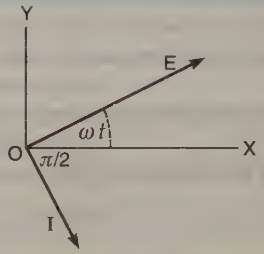


Fig. 3.11

It may be pointed out that

(*i*) the peak value of alternating current, $I_0 = \dfrac{E_0}{X_L}$

(*ii*) the inductive reactance X_L $(= \omega L = 2\pi f L)$ is zero for d.c. $(f = 0)$ and has nite value for a.c. In other words, *an inductor offers an easy path to d.c. and a resistive h to a.c.*

3.07. A.C. THROUGH A CAPACITOR

Consider that a capacitor of capacitance C is connected to a source of an ernating e.m.f. as shown in Fig. 3.12. Let the instantaneous value of alternating n.f. be given by

$$E = E_0 \sin \omega t \qquad \text{...(3.20)}$$

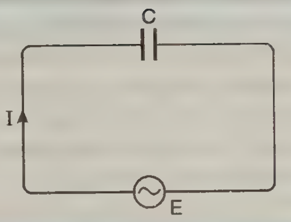


Fig. 3.12

Let I be current in the circuit and q be the charge over the capacitor at that stant. Then, the instantaneous potential difference across the capacitor $= \dfrac{q}{C}$

Now, the instantaneous e.m.f. due to source

= instantaneous potential difference across the capacitor

Therefore, $E = \dfrac{q}{C}$

or $q = C E = C E_0 \sin \omega t$

Differentiating both sides *w.r.t.* t, we have

$$\frac{dq}{dt} = \frac{d}{dt}(C E_0 \sin \omega t)$$

or $$I = C E_0 (\cos \omega t)(\omega) = \frac{E_0}{1/\omega C} \cos \omega t$$

or $$I = \frac{E_0}{1/\omega C}\left(\omega t + \frac{\pi}{2}\right) \qquad [\because \sin(\omega t + \pi/2) = \cos \omega t]$$

Key point

The capacitor reactance decreases, as frequency of a.c. source is increased.

Now, $1/\omega C$ has got units of resistance and is called ***reactance of capacitor*** or ***pacitive reactance***. It is denoted by X_C. Therefore,

$$I = \frac{E_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Further, $\dfrac{E_0}{X_C} = I_0$, the peak value of alternating current. Therefore,

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{...(3.21)}$$

The equation (3.21) gives instantaneous value of current in an a.c. circuit ntaining C. From equations (3.20) and (3.21), it follows that the alternating rrent ***leads*** the e.m.f. by phase angle $\pi/2$ (or e.m.f. ***lags behind*** the current by a hase angle $\pi/2$), when a.c. passes through a circuit containing capacitor. This fact graphically represented in Fig. 3.13.

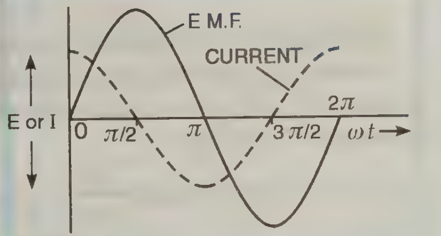


Fig. 3.13

Let us now draw phasor diagram, when a.c. flows through a circuit containing pacitor. The phasor representing E is drawn so as to make an angle ωt with OX anticlockwise direction. As current ***leads*** the e.m.f. by phase angle $\pi/2$, the phasor presenting I has been drawn so as to make an angle $\pi/2$ in *anticlock-wise direction* ith the phasor representing E [Fig. 3.14].

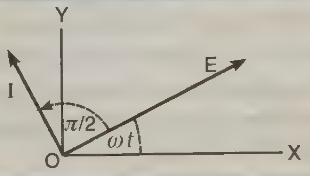


Fig. 3.14

It may be pointed out that

(*i*) the peak value of the alternating current, $I_0 = \dfrac{E_0}{X_C}$

(*ii*) the capacitive reactance $\left(X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}\right)$ is infinite for d.c. $(f = 0)$ and as a very small value for a.c. In other words, *a capacitor serves as a block for d.c. and fers an easy path to a.c.*

3.08. A.C. THROUGH LR-SERIES CIRCUIT

Consider that a source of an alternating e.m.f. is connected to a series ombination of an inductor of inductance L and a resistor of resistance R as shown Fig. 3.15.

Let E and I be the instantaneous values of e.m.f. and current in the LR-circuit and V_L and V_R be instantaneous values of the voltages across the inductor L and the resistor R respectively. Then,

$$V_L = I X_L \qquad \text{and} \qquad V_R = I R$$

Here, $X_L = \omega L$ is reactance due to the inductor.

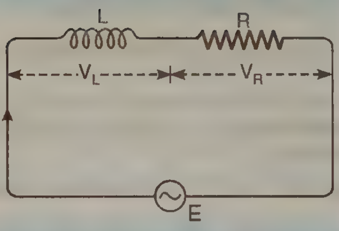


Fig. 3.15

In order to find the relation between E, V_L and V_R, let us draw phasor diagram for the current and voltages as explained below :

We know that when a.c. flows through a resistor, the voltage and the current are in phase with each other. Likewise, the phasors V_R (voltage across R) and V_I(current through R) are represented along OX [Fig. 3.16].

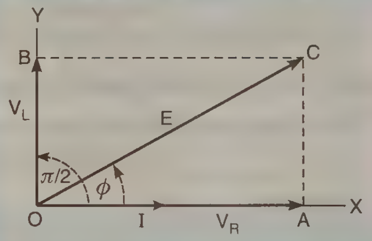


Fig. 3.16

Further, when a.c. flows through an inductor, voltage leads* the current by phase angle $\pi/2$. Since current through inductance L is also I and has been represented along OX, the phasor V_L (voltage across L) leading I by phase angle $\pi/2$ is represented along OY.

If OA and OB represent magnitudes of the phasors V_R and V_L respectively ; then OC, the resultant of OA and OB, represents E. Thus, from right angled Δ OAC, we have

$$OC = \sqrt{OA^2 + AC^2} = \sqrt{OA^2 + OB^2}$$

or

$$E = \sqrt{V_R{}^2 + V_L{}^2}$$

Substituting for V_R and V_L, we have

$$E = \sqrt{(I R)^2 + (I X_L)^2} = I\sqrt{R^2 + X_L{}^2}$$

or

$$I = \frac{E}{\sqrt{R^2 + X_L{}^2}} = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \qquad \text{...(3.22)}$$

The effective opposition offered by the series combination of L and R to a.c. is called ***impedance*** of LR-circuit. If Z is impedance of LR-circuit, then

$$I = \frac{E}{Z} \qquad \text{...(3.23)}$$

From equations (3.22) and (3.23), we have

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{R^2 + \omega^2 L^2} \qquad \text{...(3.24)}$$

The equation (3.24) gives the impedance of LR-circuit.

Key point

In LR-circuit, the value of phase angle (ϕ) between current and e.m.f. can be

$$0 < \phi < \pi/2$$

From the phasor diagram, it follows that in L R-circuit, e.m.f. ***leads*** the current by phase $\angle COA = \phi$ (as angle is traversed in *anticlockwise direction*). From right angled Δ OAC, we have

$$\tan\phi = \frac{AC}{OA} = \frac{V_L}{V_R} = \frac{I X_L}{I R}$$

or

$$\tan\phi = \frac{V_L}{V_R} = \frac{\omega L}{R} \qquad \text{...(3.25)}$$

The equation (3.25) gives the phase angle ϕ, by which alternating e.m.f. leads the current in an L R-circuit.

3.09. A.C. THROUGH CR-SERIES CIRCUIT

Consider that a source of an alternating e.m.f. is connected to a series combination of a capacitor of capacitance C and a resistor of resistance R as shown in Fig. 3.17.

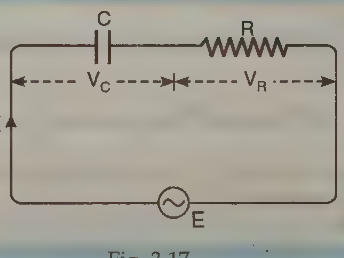


Fig. 3.17

Let E and I be the instantaneous values of e.m.f. and current in the CR-circuit and V_C and V_R be the instantaneous values of voltages across capacitor C and the resistor R respectively. Then,

$$V_C = I X_C \qquad \text{and} \qquad V_R = I R$$

Here, $X_C = \omega L$ is reactance due to the capacitor.

In order to find relation between E, V_C and V_R, let us draw the phasor diagram for the current and voltages as explained below :

---

*or the current lags behind the current voltage by phase angle $\pi/2$.

We know that when a.c. flows through a resistor, the voltage and the current re in phase with each other. Likewise, the phasors V_R (voltage across R) and I urrent through R) are represented along OX [Fig. 3.18].

Further, when a.c. flows through a capacitor, voltage lags behind* the current y phase angle $\pi/2$. Since current through the capacitor C is also I and has been epresented along OX, the phasor V_C (voltage across C) is represented along OY′. OA and OB represent the magnitudes of the phasors V_R and V_C respectively ; then C, the resultant of OA and OB, represents E. From right angled Δ OAC, we have

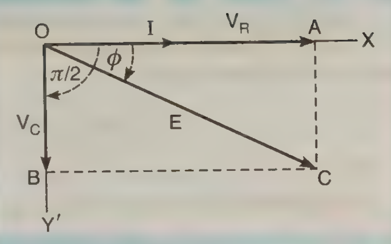


Fig. 3.18

$$OC = \sqrt{OA^2 + AC^2} = \sqrt{OA^2 + OB^2}$$

or
$$E = \sqrt{V_R{}^2 + V_C{}^2}$$

Substituting for V_R and V_C, we have

$$E = \sqrt{(I\,R)^2 + (I\,X_C)^2} = I\sqrt{R^2 + X_C{}^2}$$

or
$$I = \frac{E}{\sqrt{R^2 + X_C{}^2}} = \frac{E}{\sqrt{R^2 + 1/\omega^2 C^2}} \qquad ...(3.26)$$

The effective opposition offered by C and R to a.c. is called ***impedance*** of C R-ircuit. If Z is impedance of C R-circuit, then

$$I = \frac{E}{Z} \qquad ...(3.27)$$

From equations (3.26) and (3.27), we have

$$Z = \sqrt{R^2 + X_C{}^2} = \sqrt{R^2 + 1/\omega^2 C^2} \qquad ...(3.28)$$

Key point

In CR-circuit, the value of phase angle (ϕ) between current and e.m.f. can be
$-\pi/2 < \phi < 0$

The equation (3.28) gives impedance of C R-circuit.

From the phasor diagram, it follows that in C R-circuit, the alternating e.m.f. ***lags behind*** the current by phase $\angle\, COA = \phi$ (as angle ϕ is traversed in *clockwise* direction). From right angled Δ AOC, we have

$$\tan\phi = \frac{AC}{OA} = \frac{V_C}{V_R} = \frac{I\,X_C}{I\,R}$$

or
$$\tan\phi = \frac{X_C}{R} = \frac{1/\omega C}{R} \qquad ...(3.29)$$

The equation (3.29) gives the phase angle ϕ, by which alternating e.m.f. lags behind the current in a C R-circuit.

3.10. A.C. THROUGH LCR-SERIES CIRCUIT

Consider that a source of an alternating e.m.f. is connected to a series combination of an inductor of inductance L, a resistor of resistance R and a capacitor of capacitance C as shown in Fig. 3.19.

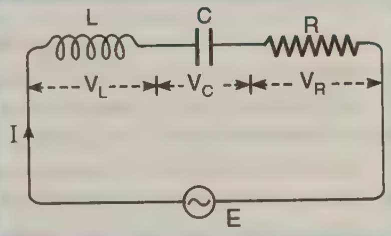


Fig. 3.19

Let E and I be the instantaneous values of e.m.f. and current in the LCR-circuit; and V_L, V_C and V_R be the instantaneous values of the voltages across inductor L, capacitor C and resistor R respectively. Then,

$$V_L = I\,X_L\,; \quad V_C = I\,X_C \quad \text{and} \quad V_R = I\,R$$

Here, $X_L = \omega\,L$ and $X_C = 1/\omega C$ are reactances due to inductor and capacitor respectively.

In order to find the relation between E, V_L, V_C and V_R, let us draw the phasor diagram for current and voltages as explained below :

In an a.c. circuit containing R only, V_R and I are in phase with each other. Therefore, both the phasors V_R and I are represented along OX [Fig. 3.20].

In an a.c. circuit containing L only, V_L leads I by phase angle $\pi/2$ and likewise the phasor V_L is represented along OY.

Finally, in an a.c. circuit containing C only, V_C lags behind I by phase angle $\pi/2$. Therefore, V_C is represented along OY ′.

Suppose that OA, OB and OC represent the magnitudes of the phasors V_R, V_L and V_C respectively.

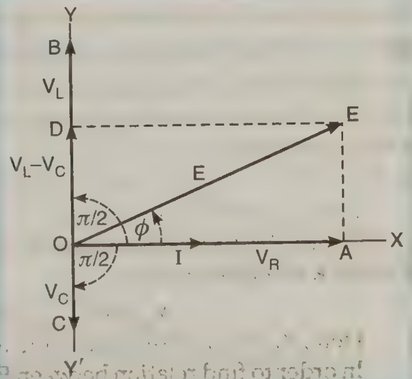


Fig. 3.20

---

*or the current leads the e.m.f. by phase angle $\pi/2$.

In case $V_L > V_C$ (as shown in figure), then OD represents $V_L - V_C$, the resultant of V_L and V_C. Finally, OE, the resultant of OA and OD represents E. Thus, from right angled Δ OAE, we have

$$OE = \sqrt{OA^2 + AE^2} = \sqrt{OA^2 + OD^2}$$

or
$$E = \sqrt{V_R{}^2 + (V_L - V_C)^2}$$

Substituting the values of V_R, V_L and V_C, we have

$$E = \sqrt{(I\,R)^2 + (I\,X_L - I\,X_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

or
$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad ...(3.30)$$

The effective opposition offered by L, C and R to a.c. is called ***impedance*** of LCR-circuit. If Z is impedance of LCR circuit, then

$$I = \frac{E}{Z} \qquad ...\ (3.31)$$

From equations (3.30) and (3.31), we have

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \qquad ...(3.32)$$

The equation (3.32) gives impedance of LCR-circuit.

Key point

In LCR-circuit, the value of phase angle (ϕ) between current and e.m.f. can be
$-\pi/2 < \phi < \pi/2$

From phasor diagram, it follows that in LCR-series circuit, E leads I (in case $V_L > V_C$ or $X_L > X_C$) by phase angle ϕ (as the angle ϕ is traversed in anticlockwise direction). From right angled Δ OAE, we have

$$\tan\phi = \frac{AE}{OA} = \frac{V_L - V_C}{V_R} = \frac{I\,X_L - I\,X_C}{I\,R}$$

or
$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R} \qquad ...(3.33)$$

The equation (3.33) gives the phase angle ϕ, by which alternating e.m.f. leads the current (in case $X_L > X_C$) in an LCR-circuit.

Let us study the phase relationship between current and e.m.f. in LCR-series circuit in the following cases :

(i) When ω **L > 1/**ω**C :** From equation (3.33), it follows that tan ϕ is positive *i.e.* ϕ is positive (between 0 and $\pi/2$). Hence, in such a case, e.m.f. leads the current.

(ii) When ω **L <** ω**C:** From equation (3.33), it follows that tanϕ is negative *i.e.* ϕ is negative (between 0 and $-\pi/2$). Hence, in such a case, e.m.f. lags behind the current.

(iii) When ω**L =** ω**C :** From equation (3.33), it follows that tan ϕ is zero *i.e.* ϕ is zero. Hence, in such a case, current and e.m.f. are in phase with each other.

In fact, when $\omega L = \omega C$, the impedance of the circuit becomes just equal to R (minimum). In other words, the LCR-series circuit will behave as a purely resistive circuit. Due to the minimum value of impedance (equal to R), the current in LCR-series circuit will be maximum and this condition is known as ***resonance***.

3.11. RESONANCE IN LCR-SERIES CIRCUIT (Series resonant circuit)

When an alternating source of e.m.f. is connected across the series combination of L, C and R, the current lags behind the e.m.f. (in case $X_L > X_C$) by phase angle ϕ, which is given by

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

Here, ω is the angular frequency of the a.c. source. Further, the impedance of the LCR-circuit is given by

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

The instantaneous value of current in the LCR-circuit is given by

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

It follows that in addition to the values of the circuit elements R, L and C, the m.s. value of current in the LCR-circuit depends upon the angular frequency ω f the a.c. source. It is because, the values of inductive and capacitive reactances e. ω L and $1/\omega$ C change with change in the angular frequency ω . When ω is low, nductive reactance ω L is low but capactive reactance $1/\omega$ C is high. As the value f ω is increased ; the inductive reactance increases, while the capacitive reactance ecreases. At a particular value of the angular frequency of the a.c. source, say ω_0, he inductive reactance and capacitive reactance will become just equal to each ther *i.e.*

$$\omega_0 L = 1/\omega_0 C \qquad \text{...(3.34)}$$

When it happens so, *i.e.* $\omega = \omega_0$, the impedence of the LCR-circuit is given by

$$Z \text{ (at } \omega_0) = \sqrt{R^2 + (\omega_0 L - 1/\omega_0 C)^2}$$

or

$$Z \text{ (at } \omega_0) = R \qquad (\because \omega_0 L = 1/\omega_0 C)$$

Thus, when angular frequency of a.c. source is equal to ω_0, the impedance of the LCR-circuit is minimum *i.e.* merely equal to the ohmic resistance of the circuit.

Also, when $\omega = \omega_0$, it follows that the value of current in LCR-circuit *i.e.*

$$I \text{ (at } \omega_0) = \frac{E}{R} \qquad \text{... (3.35)}$$

i.e. when angular frequency of a.c. source is equal to ω_0, the r.m.s. value of current in the LCR-circuit becomes maximum.

Key point

When resonance takes place in LCR-circuit,

(*i*) $\omega L = \dfrac{1}{\omega C}$,

(*ii*) impedance of the circuit becomes minimum and is equal to R,

(*iii*) current in the circuit becomes maximum and is equal to E/R ; and

(*iv*) current and e.m.f. are in phase with each other.

*A series LCR-circuit, which admits maximum current corresponding to a particular angular frequency ω_0 of the a.c. source, is called **series resonant circuit** and the angular frequency ω_0 is called the **resonant angular frequency**.*

Thus, in a series LCR-circuit, when the angular frequency of the a.c. source is equal to resonant angular frequency ω_0, impedence of the circuit becomes minimum and the r.m.s. value of current in the circuit becomes maximum. The equation (3.34), which gives the resonant angular frequency, is called ***condition for resonance*** in the LCR-circuit.

From equation (3.34), we have

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{...(3.36)}$$

If f_0 is the resonance frequency corresponding to resonant angular frequency ω_0, then

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \qquad \text{...(3.37)}$$

When the resonance takes place in a series LCR-circuit, the following is found to hold for the circuit :

1. At resonance, $\omega_0 L = 1/\omega_0 C$ and therefore $\tan\phi$ and hence ϕ becomes zero. Thus, when resonance takes place in series LCR-circuit, the current and e.m.f. are in phase with each other.

2. The impedance of the series LCR-circuit becomes minimum *i.e.* equal to R.

3. The current in the series LCR-circuit attains a maximum value, which depends only on the ohmic resistance of the circuit.

3.12. QUALITY FACTOR AND SHARPNESS OF RESONANCE

At resonant frequency ω_0 , the current in the LCR-series circuit becomes maximum. Fig. 3.21 shows the variation of the current amplitude with the angular frequency of the a.c. source for three different values of resistance R (the values of L and C are same in the three cases). It follows that the current versus frequency curve is quite flat for a large value of resistance (R_1) and becomes more and more sharp as the value of the resistance is decreased. Further, from these curves, it follows that when the resistance in the circuit is low, then with a little change in angular frequency, the current in the circuit changes by a large amount in comparison to that what happens in the case, when the resistance in the circuit is high. In other words, if the value of resistance in the LCR-series circuit is very low ; then a large current flows, when angular frequency of the a.c. source is close to the resonant frequency ω_0. Such an LCR-series circuit is said to be ***more selective*** or ***more sharp***. Therefore, for a sharp resonance in LCR-circuit, the ohmic resistance of the circuit should be low.

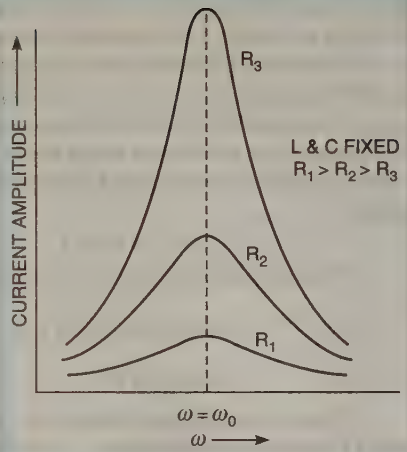


Fig. 3.21

The selectivity or sharpness of a resonant circuit is measured by ***Q-factor***, called the ***quality factor.***

The Q factor or quality factor of a resonant LCR-circuit is defined as ratio of the voltage drop across inductor (or capacitor) to the applied voltage. Thus,

$$Q = \frac{\text{voltage across L (or C)}}{\text{applied voltage}} \qquad ...(3.38)$$

When resonance occurs ; if I is instantaneous value of current through the circuit, then

voltage across L, $V_L = I\,X_L$ ...(3.39)

Since at resonant frequency, applied voltage is equal to potential drop across R, we have

applied voltage = I R ...(3.40)

Using equations (3.39) and (3.40), the equation (3.38) becomes

$$Q = \frac{I\,X_L}{I\,R} = \frac{\omega_0\,L}{R}$$

Since $\omega_0 = \frac{1}{\sqrt{LC}}$, we have

$$Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R}$$

or

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad ...(3.41)$$

It follows that Q-factor of LCR-series circuit will be large *i.e.* the circuit will have more selectivity or more sharpness, if R is low or L is large or C is low.

3.13. POWER OF AN A.C. CIRCUIT

In an a.c. circuit, both e.m.f. and current continuously change with time. Therefore, electric power (which is equal to product of voltage and current) can not be calculated as such. For an a.c. circuit, average power is calculated by defining the instantaneous power of the circuit.

The instantaneous power of an a.c. circuit is defined as the product of the instantaneous e.m.f. and the instantaneous current in it.

Suppose that the instantaneous e.m.f. and the current in an a.c. circuit are respectively given by

$$E = E_0 \sin \omega t$$

and

$$I = I_0 \sin (\omega t + \phi),$$

where ϕ is the phase angle by which current leads the e.m.f. in the a.c. circuit. Here, E_0 and I_0 are peak values of e.m.f. and current respectively ; and $2\pi/T$, where T is period of a.c.

If we assume that the values of e.m.f. and current in the a.c. circuit remain constant for a small time *dt*, then small amount of electrical energy consumed is given by

$$\begin{aligned} dW &= E\,I\,dt = E_0 \sin \omega t\; I_0 \sin (\omega t + \phi)\, dt \\ &= E_0 I_0 \sin \omega t\, (\sin \omega t \cos \phi + \cos \omega t \sin \phi)\, dt \\ &= E_0 I_0 (\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi)\, dt \qquad ...(3.42) \end{aligned}$$

Now, $\cos 2\omega t = 1 - 2\sin^2 \omega t$ or $\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$

Also, $\sin 2\omega t = 2 \sin \omega t \cos \omega t$ or $\sin \omega t \cos \omega t = \frac{\sin 2\omega t}{2}$

Substituting for $\sin^2 \omega t$ and $\sin \omega t \cos \omega t$ in equation (3.42), we have

$$dW = E_0 I_0 \left(\frac{1 - \cos 2\omega t}{2} \cos \phi + \frac{\sin 2\omega t}{2} \sin \phi \right) dt$$

$$= \frac{E_0 I_0}{2} (\cos \phi - \cos \phi \cos 2\omega t + \sin \phi \sin 2\omega t)\, dt$$

The electrical energy consumed in the circuit in time T (period of a.c.) can be tained by integrating the above equation between $t = 0$ to $t = T$ *i.e.*

$$W = \int_0^T \frac{E_0 I_0}{2} (\cos \phi - \cos \phi \cos 2\omega t + \sin \phi \sin 2\omega t)\, dt$$

or

$$W = \frac{E_0 I_0}{2} \left(\cos \phi \int_0^T dt - \cos \phi \int_0^T \cos 2\omega t\, dt + \sin \phi \int_0^T \sin 2\omega t\, dt \right) \quad ...(3.43)$$

Now, $\int_0^T dt = |t|_0^T = T - 0 = T$

It can be shown that $\int_0^T \cos 2\omega t\, dt = \int_0^T \sin 2\omega t\, dt$

Therefore, equation (3.43) becomes

$$W = \frac{E_0 I_0}{2} [\cos \phi\, (T) - \cos \phi\, (0) + \sin \phi\, (0)]$$

or

$$W = \frac{E_0 I_0 T}{2} \cos \phi$$

The average power of the a.c. circuit is given by

$$P_{av} = \frac{W}{T} = \frac{E_0 I_0 T}{2} \cos \phi \times \frac{1}{T} = \frac{E_0 I_0}{2} \cos \phi = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

or

$$P_{av} = E_v I_v \cos \phi \quad ...(3.44)$$

The average power of an a.c. circuit is also called ***true power*** of the circuit. urther, the product $E_v I_v$ is called the ***apparent power*** or ***virtual power*** and $\cos \phi$ s known as *power factor*. Thus,

average power = apparent power × power factor

Let us find the expression for average power in the following special cases :

(i) A.C. circuit having R only. For such a circuit, $\phi = 0$.

Therefore, $P_{av} = E_v I_v \cos 0 = E_v I_v (1) = E_v I_v$

(ii) A.C. circuit having L only. For such a circuit, $\phi = \pi/2$.

Therefore, $P_{av} = E_v I_v \cos \pi/2 = E_v I_v (0) = 0$

(iii) A.C. circuit containing C only. For such a circuit, $\phi = -\pi/2$

Therefore, $P_{av} = E_v I_v \cos(-\pi/2) = E_v I_v (0) = 0$

(iv) A.C. circuit containing L and R. For such a circuit, $\tan \phi = \frac{\omega L}{R}$; so that

$$\cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\therefore \quad P_{av} = E_v I_v \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

(v) A.C. circuit containing C and R. For such a circuit, $\tan \phi = \frac{1/\omega C}{R}$; so that

$$\cos \phi = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}} = \frac{1/\omega C}{R}$$

Key point

When an a.c. circuit contains resistor only, its power is maximum. When an inductor or a capacitor is connected in series to the resistor, power of the circuit always decreases.

$$\therefore \qquad P_{av} = E_v\, I_v \cdot \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

(vi) A.C. circuit containing L, C and R. For such a circuit, $\tan\phi = \dfrac{\omega L - 1/\omega C}{R}$; so that

$$\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\therefore \qquad P_{av} = E_v\, I_v \cdot \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

3.14. WATTLESS CURRENT

We know that average power of a.c. circuit is given by

$$P_{av} = E_v\, I_v \cos\phi,$$

where ϕ is the phase angle between r.m.s. value of e.m.f. and current in the a.c. circuit. Suppose that E_v leads I_v by phase angle ϕ as shown in Fig. 3.22. It can be assumed that I_v is the vector sum of two perpendicular components $I_v \cos\phi$ and $I_v \sin\phi$. Let us calculate power consumed due to each of the two components :

(i) Since phase angle between E_v and $I_v \cos\phi$ is zero, average power consumed in the circuit due to component $I_v \cos\phi$ is given by

$$P_{av}' = E_v\,(I_v \cos\phi)\cos 0° = E_u\, I_v \cos\phi$$

(ii) Since phase angle between E_v and $I_v \sin\phi$ is $\pi/2$, average power consumed in the circuit due to component $I_v \sin\phi$ is given by

$$P_{av}'' = E_v\,(I_v \sin\phi)\cos\frac{\pi}{2} = 0$$

Thus, the average power consumed in a.c. circuit is wholly due to the component $I_v \cos\phi$ of the alternating current and the component $I_v \sin\phi$ does not contribute to consumption of power in the a.c. circuit. As the component $I_v \sin\phi$ of the current does not contribute to the power, it is called the ***idle*** or ***wattless component*** of a.c.

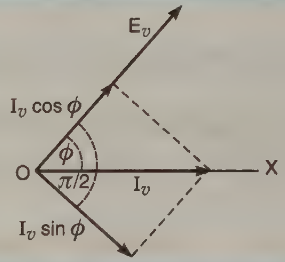


Fig. 3.22

3.15. ADVANTAGES AND DISADVANTAGES OF A.C. OVER D.C.

Advantages. 1. The generation of a.c. is found to be economical than that of d.c.

2. The alternating voltages can be easily stepped up or stepped down by using a transformer.

3. The alternating currents can be regulated by using a choke coil without any significant wastage of electrical energy.

4. The alternating voltages can be transmitted to distant places at a very small loss of the electric power.

5. Further, a.c. can be easily converted into d.c. by using rectifiers.

Disadvantages. 1. The a.c. supply is more fatal and dangerous than d.c. in terms of shock received due to them.

2. The alternating current always flows on the outer layer of the wire. It is called *skin effect*. Therefore, for use in a.c. circuits ; instead of a single thick wire, the wire for its use on a.c. supply is prepared by putting a number of thin wires together. As such, the alternating current flows through all the wires and the skin effect is avoided.

3. The alternating current cannot be used in electrolytic processes, such as electroplating, electrotyping, etc.

SOLVED NUMERICAL PROBLEMS

Type A. On Mean and R.M.S. Value of a.c.

Problem 3.01. The electric mains in a house is marked as 220 V, 50 c.p.s. Write down the equation for the instantaneous voltage.

Sol. Effective virtual value of voltage, $E_v = 220$ V ; frequency, $f = 50$ c.p.s.

Now, peak value of voltage,

$$E_0 = \sqrt{2}\,E_v = \sqrt{2} \times 220 = 311 \cdot 08 \approx 311 \text{ V}$$

Therefore, instantaneous value of voltage,

$$E = E_0 \sin 2\pi f t = 311 \sin (2\pi \times 50\, t)$$

or $$E = 311 \sin 100\, \pi t$$

Problem 3.02. The equation of alternating current for circuit is given by

$$I = 50 \cos 100\, \pi t$$

Find (*i*) frequency of a.c. applied (*ii*) mean value of rrent during positive half of the cycle (*iii*) virtual value current and (*iv*) the value of current 1/300 second after it as zero.

Sol. The value of current in an a.c. circuit is given by

$$I = I_0 \cos \omega t = I_0 \cos 2\pi f t$$

Here, $$I = 50 \cos 100\, \pi t$$

Comparing the two equations, we have

$$2\pi f t = 100\, \pi t \qquad ...(i)$$

and $$I_0 = 50 \text{ A} \qquad ...(ii)$$

Using these equations, we have

(*i*) Frequency, $f = \dfrac{100\,\pi t}{2\pi t} = 50$ **c.p.s.**

(*ii*) Mean value of a.c.

$$I_m = 0{\cdot}636\, I_0 = 0{\cdot}636 \times 50 = \mathbf{31{\cdot}8\ A}$$

(*iii*) Virtual value of a.c.,

$$I_v = 0{\cdot}707\, I_0 = 0{\cdot}707 \times 50 = \mathbf{33{\cdot}35\ A}$$

(*iv*) The value of current at time t (= 1/300 s) is given by

$$I = 50 \sin 100 \times \pi \times \frac{1}{300} = 50 \sin \frac{\pi}{3} = 50 \sin 60°$$

$$= 50 \times 0{\cdot}866 = \mathbf{43{\cdot}3\ A}$$

Type B. On a.c. circuit containing resistor

Problem 3.03. A sinusoidal voltage E = 200 sin 314 *t* is pplied to a resistor of 10 Ω resistance. Calculate (*i*) r.m.s. alue of the voltage (*ii*) r.m.s. value of the current, and *iii*) power dissipated as heat in watt. (*C.B.S.E. 1998*)

Sol. Here, R = 10 Ω

Instantaneous value of voltage,

$$E = 200 \sin 314\, t \qquad ...(i)$$

But instantaneous e.m.f. in a.c. circuit,

$$E = E_0 \sin \omega t \qquad ...(ii)$$

(*i*) Comparing the equations (*i*) and (*ii*), we have

$$E_0 = 200 \text{ V}$$

(*ii*) Therefore, r.m.s. value of voltage,

$$E_v = \frac{E_0}{\sqrt{2}} = \frac{200}{\sqrt{2}} = \mathbf{141{\cdot}4\ V}$$

The r.m.s. value of current,

$$I_v = \frac{E_v}{R} = \frac{141{\cdot}4}{10} = \mathbf{14{\cdot}14\ A}$$

(*iii*) Average power dissipated as heat,

$$P_{av} = E_v I_v = 141{\cdot}4 \times 14{\cdot}14 = \mathbf{2{,}000\ W}$$

Type C. On a.c. circuit containing inductor

Problem 3.04. A pure inductance of 1·0 H is connected across a 110 V, 70 Hz source. Find (*a*) reactance (*b*) current (*c*) peak value of current. (*I.S.C.E. 1997*)

Sol. Here, L = 1·0 H ; E_v = 110 V ; *f* = 70 Hz

(*a*) Now, $X_L = \omega L = 2\pi f L$

$$= 2 \times \frac{22}{7} \times 70 \times 1{\cdot}0 = \mathbf{440\ \Omega}$$

(*b*) Current through inductance,

$$I_v = \frac{E_v}{X_L} = \frac{110}{440} = \mathbf{0{\cdot}25\ A}$$

(*c*) Peak value of current,

$$I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 0{\cdot}25$$

$$= \mathbf{0{\cdot}354\ A}$$

Problem 3.05. The current through a coil of inductance 2 mH is represented by I = 0·2 sin 200 *t* ampere. Calculate the maximum value of the induced current. (*P.S.S.C.E. 2001*)

Sol. Here, L = 2 mH = 2×10^{-3} H ;

and $$I = 0{\cdot}2 \sin 200\, t \qquad ...(i)$$

The instantaneous value of alternating current is given by

$$I = I_0 \sin \omega t \qquad ...(ii)$$

Comparing equation (*i*) and (*ii*), we have

$$\therefore \quad \omega = 200 \text{ rad s}^{-1}$$

The inductive reactance,

$$X_L = \omega L = 200 \times 2 \times 10^{-3} = 0{\cdot}4\ \Omega$$

The induced e.m.f. (in magnitude) produced in the coil,

$$e = L \frac{dI}{dt} = 2 \times 10^{-3} \times \frac{d}{dt}(0{\cdot}2 \sin 200\, t)$$

$$= 2 \times 10^{-3} \times 0{\cdot}2 \times \cos 200\, t \times 200$$

$$= 0{\cdot}08 \cos 200\, t$$

Therefore, maximum value of induced e.m.f.,

$$e_0 = 0{\cdot}08 \text{ V}$$

Hence, maximum value of induced current,

$$I_0 = \frac{e_0}{X_L} = \frac{0{\cdot}08}{0{\cdot}4} = \mathbf{0{\cdot}2\ A}$$

Type D. On a.c. circuit containing capacitor

Problem 3.06. Find the virtual value of current through a capacitor of capacitance 10 μF, when connected to a source of 110 volt at 50 cycles supply. What is its reactance ?

Sol. Here, C = 10 μF = 10×10^{-6} F ; *f* = 50 cycles s^{-1} ;

E_v = 110 volt ;

Now, $$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= \mathbf{318{\cdot}3\ ohm}$$

Now, $$I_v = \frac{E_v}{X_C} = \frac{110}{318{\cdot}3} = \mathbf{0{\cdot}346\ A}$$

Type E. On LR–Circuit

Problem 3.07. A coil of inductance 4/π henry is joined in series with a resistance of 30 ohm. Calculate the current flowing in the circuit, when connected to a.c. mains of 200 volt and frequency 50 Hz.

Sol. Here, L = 4/π henry ; R = 30 ohm ;

E_v = 200 volt ; *f* = 50 Hz ;

Inductive reactance,

$$X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{4}{\pi} = 400\ \Omega$$

Now, impedance of L R-circuit,

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{(30)^2 + (400)^2} = \sqrt{160900} = 401{\cdot}1\ \Omega$$

Therefore, current in the circuit is given by

$$I_v = \frac{E_v}{Z} = \frac{200}{401{\cdot}1} = \mathbf{0{\cdot}499\ ampere}$$

Problem 3.08. When 100 volt d.c. is applied across a coil, a current of 1 ampere flows through it. When 100 volt a.c. at 50 cycle s^{-1} is applied to the same coil, only 0·5 ampere current flows. Calculate the resistance, the impedance and the inductance of the coil.

Sol. *In d.c. circuit* : The ohmic resistance of the coil,

$$R=\frac{E}{I}=\frac{100}{1}=\mathbf{100\ ohm}$$

In a.c. circuit : Impedance of the coil,

$$Z=\frac{E_v}{I_v}=\frac{100}{0\cdot5}=\mathbf{200\ ohm}$$

Thus, impedance of the coil for a.c. is greater than its ohmic resistance for d.c. It means that the coil has both inductive reactance X_L and the ohmic resistance R.

Now, $Z=\sqrt{R^2+X_L{}^2}$

or $X_L{}^2=Z^2-R^2=(200)^2-(100)^2$

$=40000-10000=30000$

or $X_L=\sqrt{3}\times100=173\cdot2$ ohm

But $X_L=2\pi fL$

$\therefore$ $L=\frac{X_L}{2\pi f}=\frac{173\cdot2}{2\times3\cdot14\times50}=\mathbf{0\cdot55\ henry}$

Problem 3.09. A coil of inductance 0·50 H and resistance 100 Ω is connected is a 240 V, 50 Hz a.c. supply. (*a*) What is the maximum current in the coil ? (*b*) What is the phase difference between the voltage maximum and current maximum ? (*Text Problem*)

Sol. Here, $L=0\cdot50$ H ; $R=100\ \Omega$; $f=50$ Hz ; $E_v=240$ V ;

(*a*) $$I_v=\frac{E_v}{\sqrt{R^2+\omega^2L^2}}=\frac{E_v}{\sqrt{R^2+(2\pi fL)^2}}$$

$$=\frac{240}{\sqrt{100^2+(2\pi\times50\times0\cdot50)^2}}$$

$$=\frac{240}{\sqrt{10000+24674}}=\frac{240}{186\cdot2}=1\cdot29\ A$$

Now, $I_0=\sqrt{2}\,I_v=\sqrt{2}\times1\cdot29=\mathbf{1\cdot824\ A}$

(*b*) For LR-circuit,

$$\tan\phi=\frac{\omega L}{R}=\frac{2\pi fL}{R}=\frac{2\pi\times50\times0\cdot50}{100}=1\cdot5707$$

or $\phi=\mathbf{57°31'}$ **(e.m.f. leads current)**

Problem 3.10. When an alternating voltage of 220 V is applied across a device X, a current of 0·5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit, but it lags behind the applied voltage by $\pi/2$ radians. (*a*) Name the devices X and Y. (*b*) Calculate the current flowing in the circuit, when same voltage is applied across the series combination of X and Y. (*C.B.S.E. 1997*)

Sol. (*a*) The current and voltage are in phase with each other, when alternating voltage is applied across a resistor. Hence, the **device X is a resistor.**

Obviously, $R=\frac{E_v}{I_v}=\frac{220}{0\cdot5}=440\ \Omega$

The current lags behind the voltage by phase angle $\pi/2$, when alternating voltage is applied across an inductor. Hence, the **device Y is an inductor.**

Obviously, $X_L=\frac{E_v}{I_v}=\frac{220}{0\cdot5}=440\ \Omega$

(*b*) Here, $E_v=220$ V ; $R=440\ \Omega$; $X_L=440\ \Omega$

If Z is impedance of LR-circuit, then

$$Z=\sqrt{R^2+X_L^2}=\sqrt{440^2+440^2}=440\sqrt{2}\ \Omega$$

Therefore, current in the LR-circuit,

$$I_v=\frac{E_v}{Z}=\frac{220}{440\sqrt{2}}=\mathbf{0\cdot354\ A}$$

Problem 3.11. An inductance coil has a reactance o 100 Ω. When a.c. signal of frequency 1000 Hz is applied t the coil, the applied voltage leads the current by 45 Calculate the self inductance of the coil. (*C.B.S.E. 1998 S*

Sol. Since voltage applied across the inductance lead the current by 45° (and not by 90°), the given inductance i not a pure inductance.

The given inductance behaves as a series combinatio of L and R. If Z is impedance of the given inductance and is the phase angle by which applied voltage leads the curren then

$$Z=\sqrt{R^2+X_L^2}\qquad ...(i$$

and $$\tan\phi=\frac{X_L}{R}\qquad ...(ii$$

Here, $Z=100\ \Omega$; $\phi=45°$

From equation (*i*), we have

$$\sqrt{R^2+X_L^2}=100\qquad ...(iii$$

Also, from equation (*ii*), we have

$$\tan45°=\frac{X_L}{R}\quad \text{or}\quad 1=\frac{X_L}{R}$$

or $R=X_L$

Substituting for R in equation (*iii*), we have

$$\sqrt{X_L^2+X_L^2}=100\quad \text{or}\quad \sqrt{2}\,X_L=100$$

or $X_L=70\cdot01\ \Omega$

If L is self inductance of the coil, then

$X_L=2\pi fL$

or $L=\frac{X_L}{2\pi f}=\frac{70\cdot71}{2\pi\times1000}=\mathbf{1\cdot1254\times10^{-2}\ H}$

Type F. On CR-circuit

Problem 3.12. A series circuit contains a resistor o 10 Ω, a capacitor and an ammeter of negligible resistance It is connected to a source of 220 V-50 Hz. If the reading o an ammeter is 2·0 A, calculate the reactance of the capacito (*P.S.S.C.E. 1998*

Sol. Here, $R=10\ \Omega$; $E_v=220$ V ; $f=50$ Hz ; $I_v=2\cdot0$ A ;

If Z is impedance of C R-circuit, then

$$Z=\frac{E_v}{I_v}=\frac{220}{2\cdot0}=110\ \Omega$$

But $Z=\sqrt{R^2+X_C^2}$

or $X_C=\sqrt{Z^2-R^2}=\sqrt{110^2-10^2}=\sqrt{120\times100}$

$=109\cdot54\ \Omega$

Problem 3.13. A circuit consists of a resistance 10 ohm and a capacitance 0·1 μF. If an alternating e.m.f. of 100 volt 50 Hz is applied, calculate the current in the circuit.

Sol. Here, $R=10\ \Omega$; $C=0\cdot1\ \mu F=0\cdot1\times10^{-6}$ F ;

$E_v=100$ volt ; $f=50$ Hz

The impedance of the C R-circuit is given by

$$Z = \sqrt{R^2 + (1/\omega C)^2} = \sqrt{R^2 + (1/2\pi f C)^2}$$

$$= \sqrt{(10)^2 + \left(1/2\pi \times 50 \times 0\cdot1 \times 10^{-6}\right)^2}$$

$$= \sqrt{(10)^2 + \left(10^5/\pi\right)^2} = 31831\ \Omega$$

Now, $I_v = \dfrac{E_v}{Z} = \dfrac{100}{31831} = 3{\cdot}142 \times 10^{-3}$ **A**

Problem 3.14. A 100 μF capacitor in series with a 40 Ω sistance is connected to a 110 V-60 Hz supply. (*a*) What is e maximum current in the circuit ? (*b*) What is the phase g between the current maximum and voltage maximum ? (*Text Problem*)

Sol. Here, $C = 100\ \mu F = 10^{-4}$ F ; $R = 40\ \Omega$;

$E_v = 110$ V ; $f = 60$ Hz

(*a*)
$$I_v = \frac{E_v}{\sqrt{R^2 + 1/\omega^2 C^2}} = \frac{E_v}{\sqrt{R^2 + 1/(2\pi f C)^2}}$$

$$= \frac{110}{\sqrt{40^2 + 1/(2\pi \times 60 \times 10^{-4})^2}}$$

$$= \frac{110}{\sqrt{1600 + 703\cdot62}} = \frac{110}{48} = 2\cdot292 \text{ A}$$

Now, $I_v = \sqrt{2}\, I_0 = \sqrt{2} \times 2\cdot292 = 3{\cdot}24$ **A**

(*b*) For C R-circuit,

$$\tan\phi = \frac{1/\omega C}{R} = \frac{1}{2\pi f C R}$$

$$= \frac{1}{2\pi \times 60 \times 10^{-4} \times 40} = 0\cdot6631$$

∴ **$\phi = 33°33'$ (e.m.f. lags behind the current)**

ype G. On LCR-circuit

Problem 3.15. A capacitor of capacitance 100 μF and coil of resistance 50 Ω and inductance 0·5 H are connected n series with a 110 V-50 Hz source. Calculate the r.m.s. value f the current in the circuit. (*C.B.S.E. 1998*)

Sol. Here, $C = 100\ \mu F = 100 \times 10^{-6} = 10^{-4}$ F ; $R = 50\ \Omega$;

$L = 0{\cdot}5$ H ; $E_v = 110$ V ; $f = 50$ Hz

Impedance of LCR-circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

or
$$Z = \sqrt{R^2 + (2\pi f L - 1/2\pi f C)^2}$$

$$= \sqrt{50^2 + \left(2\pi \times 50 \times 0\cdot5 - 1/2\pi \times 50 \times 10^{-4}\right)^2}$$

$$= \sqrt{50^2 + (157\cdot08 - 31\cdot83)^2} = \sqrt{50^2 + (125\cdot25)^2}$$

$$= \sqrt{2500 + 15687\cdot6}$$

or $Z = 134{\cdot}86\ \Omega$

If I_v is r.m.s. value of current in the circuit, then

$$I_v = \frac{E_v}{Z} = \frac{110}{134\cdot86} = \mathbf{0{\cdot}816\ A}$$

Problem 3.16. A 25 μF capacitor, 0·1 henry inductor and a 25·0 ohm resistor are connected in series with a source, whose e.m.f. is given by :

E = 310 cos 314 *t* volt

(*a*) What is the frequency of the e.m.f. ? (*b*) What is the reactance of the circuit ? (*c*) What is the impedance of the circuit ? (*d*) What is the current in the circuit ? (*e*) What is the phase angle of the current by which it leads or lags the applied e.m.f. ? (*f*) What is the expression for the instantaneous value of current in the circuit ? (*g*) What are the effective voltages across the capacitor, inductor and the resistor ? (*h*) What value of inductance will make the impedance of the circuit minimum ? (*Text Problem*)

Sol. Here, $L = 0{\cdot}1$ H ; $C = 25\ \mu F = 25 \times 10^{-6}$ F ; $R = 25\ \Omega$

Also, the e.m.f. of the source is given by

$$E = 310 \cos 314\, t \qquad \ldots(i)$$

The instantaneous value of alternative e.m.f. is given by

$$E = E_0 \cos 2\pi f t \qquad \ldots(ii)$$

Comparing the equations (*i*) and (*ii*), we have

(*a*) Frequency, $f = \dfrac{314}{2\pi} = \dfrac{314}{2 \times 3\cdot14} =$ **50 cycles s^{-1}**

(*b*) Inductive reactance, $X_L = 2\pi f L = 2 \times 3{\cdot}14 \times 50 \times 0{\cdot}1$

$= 31{\cdot}4\ \Omega$

Capacitive reactance, $X_C = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2 \times 3\cdot14 \times 50 \times 25 \times 10^{-6}} = 127\cdot4\ \Omega$$

Net reactance of the circuit, $X_C - X_L = 127{\cdot}4 - 31{\cdot}4 =$ **96 Ω**

As $X_C > X_L$, the net reactance is ***capacitive.***

(*c*) Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{(25)^2 + (96)^2} = \mathbf{99{\cdot}2\ \Omega}$$

(*d*) Also, from equations (*i*) and (*ii*), $E_0 = 310$ V

Therefore, virtual voltage, $E_v = \dfrac{E_0}{\sqrt{2}} = \dfrac{310}{\sqrt{2}} = 219{\cdot}2$ V

Hence, virtual current, $I_v = \dfrac{E_v}{Z} = \dfrac{219\cdot2}{99\cdot2} =$ **2·21 A**

(*e*) The phase angle ϕ is given by

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{96}{25} = 3\cdot84$$

or $\phi = 75\cdot4° = \dfrac{7\cdot54 \times \pi}{180} =$ **1·316 rad**

As $X_C > X_L$, the current ***leads*** the voltage by the angle ϕ as obtained above.

(*f*) The instantaneous current is given by

$$I = I_0' \cos(2\pi f t - \phi) = \frac{E_0}{Z} \cos(314\, t - 1\cdot316)$$

$$= \frac{310}{99\cdot2} \cos(314\, t - 1\cdot316)$$

or **$I = 3{\cdot}125 \cos(314\, t - 1{\cdot}316)$**

(*g*) Voltage across capacitor $= I_v X_C = 2{\cdot}21 \times 127{\cdot}4$

= 281·6 V

Voltage across inductor $= I_v X_L = 2{\cdot}21 \times 31{\cdot}4 =$ **69·4 V**

Voltage across resistor $= I_v R = 2{\cdot}21 \times 25 =$ **55·25 V** (*h*) For impedance of the circuit to be minimum,

$$\omega L = 1/\omega C$$

Therefore, the required value of inductance,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C}$$

$$= \frac{1}{(2\pi \times 50)^2 \times 25 \times 10^{-6}} = \mathbf{0{\cdot}405\ H}$$

Type H. On Resonance

Problem 3.17. A capacitor, a 5 Ω resistor and 101· 5 mH inductor are placed in series with a 50 Hz a.c. source. Calculate the capacitance of the capacitor, if the current is observed in phase with the voltage. (*C.B.S.E. 1992*)

Sol. When in an LCR-circuit, the current and e.m.f. are in phase with each other

$$\therefore \qquad \omega L = 1/\omega C$$

$$\text{or} \qquad C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}$$

Here, $f = 50$ Hz ; L = 101·5 mH = 101·5 × 10^{-3} H ;

Therefore, required capacitance of the capacitor,

$$C = \frac{1}{(2\pi \times 50)^2 \times 101{\cdot}5 \times 10^{-3}}$$

$$= 99{\cdot}8 \times 10^{-6}\ \text{F} \approx \mathbf{100\ \mu F}$$

Problem 3.18. A variable frequency 230 V alternating voltage source is connected across a series combination of L = 5 H, C = 80 μF and R = 40 Ω. Calculate (*a*) the angular frequency of the source which derives the circuit in resonance, (*b*) the impedance of the circuit and (*c*) amplitude of the current at resonance. (*C.B.S.E. 2001 S*)

Sol. Here, E_v = 230 V ; L = 5 H ;

C = 80 μF = 80 × 10^{-6} F and R = 40 Ω

(*a*) For an LCR-circuit to be in resonance,

$$X_L = X_C \quad \text{or} \quad 2\pi f L = 1/2\pi f C$$

$$\text{or} \qquad f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5 \times 80 \times 10^{-6}}}$$

$$= \mathbf{7{\cdot}96\ Hz}$$

(*b*) Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Since $X_L = X_C$, we have

$$Z = R = \mathbf{40\ \Omega}$$

(*c*) The peak value of e.m.f. of the source,

$$E_0 = \sqrt{2}\, E_v = 230\sqrt{2}\ \text{V}$$

Therefore, amplitude of the current at resonance

$$= \frac{E_0}{R} = \frac{230\sqrt{2}}{40} = \mathbf{8{\cdot}13\ A}$$

Problem 3.19. A 2 μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an a.c. source. What should be the frequency of this a.c. source, for which the current drawn in the circuit is maximum ? If the peak value of e.m.f. of the source is 200 V, find for maximum current, (*i*) the inductive and capacitive reactances of the circuit ; (*ii*) total impedance of the circuit ; (*iii*) peak value of current in the circuit ; (*iv*) the phase relation between voltages across inductor and resistor ; (*v*) the phase differen between voltages across inductor and capacitor. (*I.S.C.E. 199*

Sol. Here, C = 2 μF = 2 × 10^{-6} F ; R = 100 Ω ; L = 8 H ;

E_0 = 200 V

The current in the circuit becomes maximum, whe frequency of the a.c. source becomes equal to the resonar frequency f_0.

The resonant frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8 \times 2 \times 10^{-6}}} = \mathbf{39{\cdot}79\ H}$$

(*i*) X_L (or X_C) at resonant frequency

$$= 2\pi f_0 L = 2\pi \times 39{\cdot}79 \times 8 = \mathbf{2000\ \Omega}$$

(*ii*) Total impedance of the circuit at resonant frequenc is simply equal to ohmic resistance in the circuit *i.e.* **100 Ω**

(*iii*) Peak value of current, $I_0 = \frac{E_0}{Z} = \frac{E_0}{R} = \frac{200}{100} = \mathbf{2\ A}$

(*iv*) The voltages across inductor and resistor differ phase by **90°.**

(*v*) The voltages across inductor and capacitor differ phase by **180°.**

Problem 3.20. A series LCR-circuit is connected to a a.c. source (220 V-50 Hz) as shown in Fig. 3.23. If the re dings of the three voltmeter V_1, V_2 and V_3 are 65 V, 415 and 204 V respectively, calculate (*i*) the current in the circui (*ii*) the value of the inductor L ; (*iii*) the value of the capacit C ; and (*iv*) the value of C (for the same L) required produce resonance. (*I.S.C.E. 199*

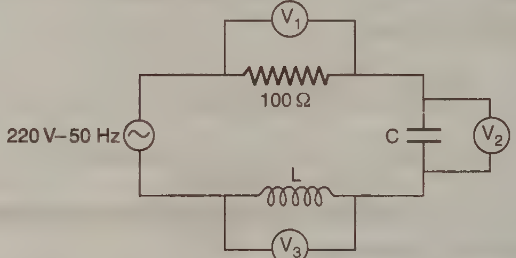


Fig. 3.23

Sol. Here, E_v = 220 V ; f = 50 Hz ; R = 100 Ω

V_R = 65 V ; V_C = 415 V ; V_L = 204 V ;

(*i*) Let I_v be r.m.s. value of current in the circuit. Ther

$$V_R = I_v R$$

$$\text{or} \qquad I_v = \frac{V_R}{R} = \frac{65}{100} = \mathbf{0{\cdot}65\ A}$$

$$(ii) \qquad V_L = I_v X_L$$

$$\text{or} \qquad X_L = \frac{V_L}{I_v} = \frac{204}{0{\cdot}65} = 313{\cdot}85\ \Omega$$

$$\text{Now,} \qquad X_L = 2\pi f L$$

$$\therefore \qquad L = \frac{X_L}{2\pi f} = \frac{313{\cdot}85}{2\pi \times 50} = \mathbf{1{\cdot}0\ H}$$

(*iii*) Again, $V_C = I_v X_C$

$$\text{or} \qquad X_C = \frac{V_C}{I_v} = \frac{415}{0{\cdot}65} = 638{\cdot}46\ \Omega$$

$$\text{Now,} \qquad X_C = \frac{1}{2\pi f C}$$

$$\therefore \quad C = \frac{1}{2\pi f X_C}$$

$$= \frac{1}{2\pi \times 50 \times 638\cdot 46} = 5 \times 10^{-6} = \mathbf{5\ \mu F}$$

(*iv*) If C′ is capacitance of the capacitor that will produce nance with inductor (L = 1·0 H), then

$$f = \frac{1}{2\pi\sqrt{LC'}}$$

$$\text{or} \quad C' = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (50)^2 \times 1\cdot 0}$$

$$= 10\cdot 1 \times 10^{-6}\ \text{F} = \mathbf{10\cdot 1\ \mu F}$$

Problem 3.21. Obtain the resonant frequency and actor of a series L C R-circuit with L = 3·0 H, C = 27 μF R = 7·4 Ω. *(Text Problem)*

Sol. Here, L = 3·0 H ; C = 27 μF = 27 × 10⁻⁶ F ;

$R = 7\cdot 4\ \Omega$;

Resonant angular frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3\cdot 0 \times 27 \times 10^{-6}}} = \mathbf{111\cdot 1\ rad\ s^{-1}}$$

$$\text{Q-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{7\cdot 4}\sqrt{\frac{3\cdot 0}{27 \times 10^{-6}}} = \mathbf{45\cdot 05}$$

pe I. On Power of a.c. circuit

Problem 3.22. Find the power consumed in a circuit ving a resistance of 30 ohm in series with an inductance 40 ohm in series with an a.c. with peak current of mpere and peak voltage of 220 volt.

Sol. Here, R = 30 ohm ; $X_L = 40$ ohm

Therefore, impedance of the LR-circuit,

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{900 + 1600}$$

$$= \sqrt{2500} = 50 \text{ ohm}$$

$$\text{Power factor, } \cos\phi = \frac{R}{Z} = \frac{30}{50} = 0\cdot 6$$

Now, $E_0 = 220$ volt ; $I_0 = 1$ ampere

$$\therefore \quad P_{av} = E_v I_v \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi = \frac{220 \times 1 \times 0\cdot 6}{\sqrt{2} \times \sqrt{2}} = \mathbf{66\ watt}$$

Problem 3.23. An alternating e.m.f. of 100 V (r.m.s.), Hz is applied across a capacitor of 10 μF and a resistor 100 Ω in series. Calculate (*a*) the reactance of the pacitor ; (*b*) the current flowing ; (*c*) the average power pplied. *(I.S.C.E. 1996)*

Sol. Here, $E_v = 100$ V ; $f = 50$ Hz ;

$C = 10\ \mu F = 10 \times 10^{-6} = 10^{-5}$ F ; R = 100 Ω

(*a*) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-5}}$$

$$= \mathbf{318\cdot 31\ \Omega}$$

(*b*) Current through the capacitor,

$$I_v = \frac{E_v}{X_C} = \frac{100}{318\cdot 31} = \mathbf{0\cdot 314\ A}$$

(*c*) Average power supplied,

$$P_{av} = E_v I_v \cos\phi = E_v I_v \cdot \frac{R}{\sqrt{R^2 + X_C{}^2}}$$

$$\text{or} \quad P_{av} = 100 \times 0\cdot 314 \times \frac{100}{\sqrt{100^2 + (318\cdot 31)^2}}$$

$$= \frac{3\cdot 14 \times 10^3}{333\cdot 65} = \mathbf{9\cdot 14\ W}$$

Problem 3.24. A 60 volt, 10 watt lamp is to be run on 100 volt-60 hertz a.c. mains. Calculate (*i*) the inductance of the choke required. (*ii*) If resistance is to be used instead of choke, what will be its value ? (*iii*) What will be the disadvantage in the latter case ?

Sol. Here, voltage marked on the lamp = 60 volt ;

power marked on the lamp = 10 watt

Therefore, current rating of the lamp

$$= \frac{\text{power}}{\text{voltage}} = \frac{10}{60} = \frac{1}{6} \text{ ampere}$$

$$\text{Resistance of the lamp, } R = \frac{60}{1/6} = 360 \text{ ohm}$$

When lamp is worked on 100 V a.c. :

$E_v = 100$ volt ; $f = 60$ Hz

Let Z be the impedance, which would maintain a current of 1/6 ampere through the lamp, when it is run on 100 volt a.c. Then,

$$Z = \frac{E_v}{1/6} = 100 \times 6 = 600 \text{ ohm}$$

(*i*) If L is the required inductance, then

$$Z = \sqrt{R^2 + (2\pi f L)^2}$$

$$\text{or} \quad L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} = \frac{\sqrt{(600)^2 - (360)^2}}{2\pi \times 60}$$

$$= \frac{\sqrt{960 \times 240}}{2\pi \times 60} = \frac{240 \times 2}{2\pi \times 60} = \frac{4}{\pi} = \mathbf{1\cdot 273\ henry}$$

(*ii*) Let R′ be the resistance connected in series to lamp (360 ohm), which will maintain a current of 1/6 ampere, when the lamp is worked on 100 volt a.c. Then,

$$R' + 360 = \frac{100}{1/6} = 600$$

$$\text{or} \quad R' = 600 - 360 = \mathbf{240\ ohm}$$

(*iii*) If resistance is used in place of inductance, the **electrical energy will be wasted** in the form of heat across the additional resistance (R′).

Problem 3.25. A 20 watt, 50 volt filament is connected in series to an a.c. mains of 250 volt-50 hertz. Calculate the value of the capacitor required to run the lamp.

Sol. Here, voltage marked on the lamp = 50 volt ;

power marked on the lamp = 20 watt

Therefore, current rating of the lamp,

$$I = \frac{\text{power}}{\text{voltage}} = \frac{20}{50} = 0\cdot 4 \text{ ampere}$$

$$\text{Resistance of the lamp, } R = \frac{50}{0\cdot 4} = 125 \text{ ohm}$$

When lamp is worked on 250 V a.c. :

$E_v = 250$ volt ; $f = 50$ hertz

Let Z be the impedance, which will maintain a current of 0·4 ampere, when the lamp is run on 250 volt a.c. Then,

$$Z = \frac{E_v}{0\cdot 4} = \frac{250}{0\cdot 4} = 625 \text{ ohm}$$

If C is the required capacitance, then

$$Z=\sqrt{R^2+(1/2\pi f C)^2}$$

or $$C=\frac{1}{2\pi f\sqrt{Z^2-R^2}}=\frac{1}{2\pi\times50\times\sqrt{(625)^2-(125)^2}}$$

$$=\frac{1}{2\pi\times50\times250\sqrt{6}}$$

$= 5{\cdot}198 \times 10^{-6}$ F

Problem 3.26. A circuit consists of a non-inductive resistance of 50 ohm, an inductance of 0·3 henry and a capacitor of 40 microfarad in series and supplied with a current of 200 volt-50 hertz. Find the impedance and power in the circuit. Take $\pi = 3{\cdot}142$.

Sol. Here, $E_v = 200$ volt ; $f = 50$ hertz ; R = 50 ohm ;
$L = 0{\cdot}3$ henry ; $C = 40\ \mu F = 40 \times 10^{-6}$ F ;

Now, inductive reactance, $X_L = 2\pi f L$
$= 2 \times 3{\cdot}142 \times 50 \times 0{\cdot}3 = 94{\cdot}26$ ohm

Capacitive reactance,

$$X_L=\frac{1}{2\pi f C}=\frac{1}{2\times3{\cdot}142\times50\times40\times10^{-6}}$$

$$=\frac{10^3}{2\times3{\cdot}142\times2}=\frac{250}{3{\cdot}142}=79{\cdot}57\text{ ohm}$$

The impedance of the LCR-circuit is given by

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$=\sqrt{(50)^2+(94{\cdot}26-79{\cdot}57)^2}$$

$$=\sqrt{(50)^2+(14{\cdot}69)^2}=\mathbf{52{\cdot}11\ ohm}$$

Now, $$I_v=\frac{E_v}{Z}=\frac{200}{52{\cdot}11}=3{\cdot}838\text{ ampere}$$

The power factor, $$\cos\phi=\frac{R}{Z}=\frac{50}{52{\cdot}11}=0{\cdot}9595$$

Therefore, average power,

$P_{av} = E_v I_v \cos\phi = 200 \times 3{\cdot}838 \times 0{\cdot}9595$
$= \mathbf{736{\cdot}5\ watt}$

Problem 3.27. A 200 V variable frequency a.c. sourc is connected to a series combination of L = 5 H, C = 80 μ and R = 40 Ω. Calculate (*a*) angular frequency of the sourc to get maximum current in the circuit, (*b*) the curren amplitude at resonance and (*c*) the power dissipation in th circuit. (*C.B.S.E. 200*

Sol. Here, $E_v = 200$ V ; L = 5 H ;
$C = 80\ \mu F = 80 \times 10^{-6}$ F and R = 40 Ω

(*a*) For maximum current (or resonance) in the circui

$$X_L = X_C$$

Let ω_0 be the angular frequency of the source th provides maximum current in the circuit. Then,

$$\omega_0 L=\frac{1}{\omega_0 C}\quad\text{or}\quad\omega_0=\frac{1}{\sqrt{LC}}$$

or $$\omega_0=\frac{1}{\sqrt{5\times80\times10^{-6}}}=\mathbf{50\ rad\ s^{-1}}$$

(*b*) At resonance, the impedance of LCR-circuit is equ to the ohmic resistance in the circuit. Therefore, r.m.s. value of current at resonance,

$$I_v=\frac{E_v}{R}=\frac{200}{40}=5\text{ A}$$

Therefore, current amplitude at resonance,

$$I_0=\sqrt{2}\,I_v=\sqrt{2}\times5=\mathbf{7{\cdot}07\ A}$$

(*c*) Now, $P_{av} = E_v I_v \cos\phi$

Since at resonance, the circuit is purely resistive, $\phi = 0$

Therefore, average power consumed in the circuit,

$P_{av} = 200 \times 5 \times \cos 0° = \mathbf{1000\ W}$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 3.28. In an LCR-circuit, a variable frequency 230 V source is connected to L = 5·0 H, C = 80 μF, R = 40 Ω.

(*a*) Determine the source frequency, which drives the circuit in resonance.

(*b*) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(*c*) Determine the r.m.s. potential drops across the three elements of the circuit. Show that the potential drop across the LC-combination is zero at the resonating frequency.

(*C.B.S.E. 1998 ; Text Problem*)

Sol. Here, L = 5·0 H ; $C = 80\ \mu F = 80 \times 10^{-6}$ F ; R = 40 Ω ;
$E_v = 230$ V

(*a*) The resonant angular frequency ω_0 is given by

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{5{\cdot}0\times80\times10^{-6}}}=\mathbf{50\ rad\ s^{-1}}$$

(*b*) At resonance, impedance of the LCR-circuit is simply equal to R.

Therefore, at resonance, Z = R = **40 Ω**

Therefore, r.m.s. value of current at resonance frequenc

$$I_v=\frac{E_v}{R}=\frac{230}{40}=5{\cdot}75\text{ A}$$

Hence, current amplitude at resonant frequency,

$$I_0=\sqrt{2}\,I_v=\sqrt{2}\times5{\cdot}75=\mathbf{8{\cdot}13\ A}$$

(*c*) R.M.S. potential drop across R
$= I_v R = 5{\cdot}75 \times 40 = \mathbf{230\ V}$

R.M.S. potential drop across L
$= I_v \omega_0 L = 5{\cdot}75 \times 50 \times 5{\cdot}0 = \mathbf{1437{\cdot}5\ V}$

R.M.S. potential drop across C

$$=I_v(1/\omega_0 C)=\frac{5{\cdot}75}{50\times80\times10^{-6}}=\mathbf{1437{\cdot}5\ V}$$

R.M.S. potential drop across LC
$= I_v(\omega_0 L - 1/\omega_0 C) = 5{\cdot}75\,(1437{\cdot}5 - 1437{\cdot}5) =$

Problem 3.29. A series LCR circuit with L = 0·12 H C = 480 nF, R = 23 Ω is connected to a 230 V variabl frequency supply.

(*a*) What is the source frequency for which current plitude is maximum ? Obtain this maximum value.

(*b*) What is the source frequency for which average wer absorbed by the circuit is maximum ? Obtain the ue of this maximum power. (*Text Problem*)

Sol. Here, $L = 0{\cdot}12\text{ H} ; C = 480\text{ nF} = 480 \times 10^{-9}\text{ F} ;$
$R = 23\ \Omega ; E_v = 230\text{ V}$

(*a*) Current amplitude is maximum at resonant quency. It is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0{\cdot}12 \times 480 \times 10^{-9}}}$$
$$= \mathbf{4166{\cdot}7\ rad\ s^{-1}}$$

$$\text{Now,} \qquad I_v = \frac{E_v}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\text{or} \qquad I_0 = \frac{\sqrt{2}\,E_v}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\therefore I_0\ (\text{at } \omega_0) = \frac{\sqrt{2}\,E_v}{R} = \frac{\sqrt{2} \times 30}{23} = \mathbf{14{\cdot}14\ A}$$
$$(\because \omega_0 L - 1/\omega_0 C = 0)$$

(*b*) Average power will also be maximum at resonant equency *i.e.* at **4166·7 rad s^{-1}**

$$P_{av} = E_v I_v . \cos\phi = E_v I_v \times \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\therefore P_{av}\ (\text{at } \omega_0) = E_v I_v \times \frac{R}{\sqrt{R^2 + 0^2}}$$

$$= E_v I_v = 230 \times \frac{14{\cdot}14}{\sqrt{2}} = \mathbf{2300\ W}$$

Problem 3.30. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, Hz supply. The resistance of the circuit is negligible.

(*a*) Obtain the current amplitude and r.m.s. values.

(*b*) Obtain the r.m.s. values of potential drops across each element.

(*c*) What is the average power transferred to the inductor ?

(*d*) What is the average power transferred to the capacitor ?

(*e*) What is the total average power absorbed by the circuit ? (*Text Problem*)

Sol. Here, $L = 80\text{ mH} = 80 \times 10^{-3}\text{ H} ;$
$C = 60\ \mu\text{F} = 60 \times 10^{-6}\text{ F} ; E_v = 230\text{ V} ; f = 50\text{ Hz}$

$$(a) \quad I_v = \frac{E_v}{\omega L - 1/\omega C} = \frac{E_v}{2\pi f L - 1/2\pi f C}$$
$$= \frac{230}{2\pi \times 50 \times 80 \times 10^{-3} - 1/2\pi \times 50 \times 60 \times 10^{-6}}$$
$$= \frac{230}{25{\cdot}13 - 53{\cdot}05} = \frac{230}{27{\cdot}92} = \mathbf{8{\cdot}24\ A}$$

[Take magnitude of (25·13 – 53·05)]

$$\therefore \quad I_0 = \sqrt{2}\,I_v = 2 \times 8{\cdot}24 = \mathbf{11{\cdot}65\ A}$$

(*b*) R.M.S. potential drop across L
$$= I_v\,\omega L = I_v \times 2\pi f L$$
$$= 8{\cdot}24 \times 2\pi \times 50 \times 80 \times 10^{-3} = \mathbf{207{\cdot}\ 1\ V}$$

R.M.S. potential drop across C
$$= I_v \frac{1}{\omega C} = I_v \frac{1}{2\pi f C}$$
$$= 8{\cdot}24 \times \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}} = \mathbf{437{\cdot}1\ W}$$

(*c*) As in inductor, e.m.f. leads current by $\pi/2$,
$$P_{av} = E_v I_v \cos\frac{\pi}{2} = \mathbf{0}$$

(*d*) As in capacitor, e.m.f. lags current by $\pi/2$,
$$P_{av} = E_v I_v \cos\left(-\frac{\pi}{2}\right) = \mathbf{0}$$

(*e*) Total average power absorbed = **0**

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. What is the frequency of direct current ? (*H.P.S.S.C.E. 1999 S*)

Ans. Zero.

Q. 3.02. The instantaneous current from a.c. source is = 5 sin 314 *t*. What is the peak value of current ? (*H.P.S.S.C.E. 2002*)

Ans. Since $I = I_0 \sin \omega t$, it follows that I_0 = **5 A.**

Q. 3.03. Define the mean value of an alternating urrent. (*P.S.S.C.E. 2001*)

Ans. Refer to section 3.02.

Q. 3.04. Define root mean square value of an alter-ating current. (*P.S.S.C.E. 2001 ; I.S.C.E. 1995*)

Ans. Refer to section 3.03.

Q. 3.05. What is the relation between peak value and oot mean sqare value of alternating e.m.f. ? (*H.S.S.C.E. 2001*)

$$\textbf{Ans. } I_{r.m.s.} = \frac{I_0}{\sqrt{2}}$$

Q. 3.06. The instantaneous current from an a.c. source is I = 6 sin 314 *t*. What is the r.m.s. value of the current ? (*C.B.S.E. 2000, 1996*)

Ans. Since $I = I_0 \sin \omega t$; it follows that $I_0 = 6$ A

$$\therefore I_{r.m.s.} = \frac{I_0}{\sqrt{2}} = \frac{6}{\sqrt{2}} = \mathbf{3\sqrt{2}\ A}$$

Q. 3.07. The electric mains in a house are marked 220 V, 50 Hz. Write down the equation for instantaneous voltage. (*H.S.S.C.E. 2002*)

Ans. Here, $f = 50$ Hz and $E_v = 220$ V

The instantaneous voltage is given by

$$E = E_0 \sin \omega t = \sqrt{2}\,E_v \sin 2\pi f t$$
$$\text{or} \qquad E = \sqrt{2} \times 200 \sin 2\pi \times 50\,t$$
$$= \mathbf{200\sqrt{2} \sin 100\pi t}$$

Q. 3.08. Which value of the current do you measure with an a.c. ammeter ?

Ans. Root mean square value of the current.

Q. 3.09. The divisions marked on the scale of an a.c. ammeter are not equally spaced. Why ?

Ans. An a.c. ammeter is constructed on the basis of heating effect of the electric current. Since the heat produced varies as the square of current (and not directly with current), the divisions marked on the scale are not equally spaced.

Q. 3.10. Why a d.c. voltmeter and d.c. ammeter cannot read a.c. ? *(P.S.S.C.E. 1998 S)*

Ans. It is because, average value of a.c. is zero over a complete cycle.

Q. 3.11. Show that resistance offered by an ideal inductor to the flow of direct current is zero. *(P.S.S.C.E. 1999 S)*

Or

What is the resistance offered by an inductance to d.c. ? *(H.S.S.C.E. 1992)*

Ans. For d.c., $f = 0$.

Therefore, $X_L = 2\pi f L = 2\pi (0) L = 0$

Q. 3.12. With reference to alternating currents and voltages, state any one fundamental difference between 'resistance' and 'reactance'. *(I.S.C.E. 1998)*

Ans. An ohmic resistor has fixed value of resistance, while reactance changes with change in frequency of a.c. supply.

Q. 3.13. The frequency of a.c. is doubled. What happens to X_L ?

Ans. Now, $X_L = 2\pi f L$

Therefore, on doubling the frequency of a.c. supply, X_L also becomes double.

Q. 3.14. Prove that an inductor offers an easy path to d.c. and a resistive path to a.c.

Ans. Inductive reactance, $X_L = \omega L = 2\pi f L$

It follows that X_L is zero for d.c. ($f = 0$) and has a finite value for a.c. Hence, an inductor offers an easy path to d.c. and a resistive path to a.c.

Q. 3.15. What is the phase relationship between current and voltage in an inductor ? *(H.S.S.C.E. 2002, 2001)*

Ans. The current lags behind the voltage by phase angle $\pi/2$.

Q. 3.16. What is meant by the statement that the current through an inductor lags e.m.f. across it by $\pi/2$?

Ans. It means that if at any instant, the e.m.f. is maximum (or minimum) in the circuit, then the current will become so after a time $T/4$, where T is period of a.c.

Q. 3.17. Sketch a graph showing the variation of inductive reactance with frequency of the applied voltage. *(H.S.S.C.E. 2002)*

Ans. Refer to Fig. 3.25.

Q. 3.18. What is capacitive reactance ? Give its SI unit. *(P.S.S.C.E. 1999 S)*

Ans. *The resistance offered by a capacitor in an a.c. circuit is called capacitive reactance.*

Its S.I. unit is **ohm.**

Q. 3.19. Find the reactance of a capacitance C at f Hz. *(I.S.C.E. 1997)*

Ans. Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

Q. 3.20. For a very high frequency a.c. supply, a capacitor behaves as a pure conductor. Why ?

Ans. Now, $X_C = \dfrac{1}{2\pi f C}$

When frequency of a.c. supply is high, X_C will be approxi-mately zero *i.e.* capacitor will behave as a conductor

Q. 3.21. A capacitor blocks d.c. but allows a.c. to pass through it. Explain, why. *(H.S.S.C.E. 2001*

Or

Discuss the behaviour of a capacitor in a.c. circuit. *(H.S.S.C.E. 1995*

Ans. Capacitve reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

It follows that capacitive reactance is infinite for d.c ($f = 0$) and has a finite value for a.c. In other words, a capacito serves as a block for d.c. and offers an easy path to a.c.

Q. 3.22. Sketch a graph showing the variation o reactance of a capacitor with frequency of the applie voltage. *(H.S.S.C.E. 2002, C.B.S.E. 2001)*

Ans. Refer to Fig. 3.26.

Q. 3.23. What do you mean by the impedance of LCR circuit ?

Ans. The effective resistance offered to the flow of curren by the LCR-circuit is called its impedance. It is given by

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

Q. 3.24. What is the phase difference between th voltage across an inductor and a capacitor in an a.c. circuit *(C.B.S.E. 1999*

Ans. 180°.

Q. 3.25. When are the voltage and current in LCR circuit in same phase ? *(H.S.S.C.E. 2002*

Ans. The voltage and current in an LCR-circuit are i same phase, when $X_L = X_C$.

Q. 3.26. When does a series LCR-circuit have minimun impedance ? *(I.S.C.E. 1996*

Ans. At resonance.

Q. 3.27. Give the phase difference between the applie a.c. voltage and the current in an LCR-circuit at resonance *(C.B.S.E. 1997*

Ans. It is zero.

Q. 3.28. If the frequency of the a.c. source in a LCR series circuit is increased, how does the current in the circuit change ? *(C.B.S.E. 1998 S*

Ans. With increase in frequency, current in a.c. circui first increases, attains a maximum value (at resonan frequency) and then decreases.

Q. 3.29. What is the power dissipation in an a.c. circui in which voltage and current are given by

$$E = 300 \sin (\omega t + \pi/2) \quad \text{and} \quad I = 5 \sin \omega t \ ?$$

(C.B.S.E. 2001

Ans. Here, $E_v = \dfrac{300}{\sqrt{2}}$ V ; $I_v = \dfrac{5}{\sqrt{2}}$ A and $\phi = \dfrac{\pi}{2}$

Average power dissipated in the circuit,

$$\therefore \ P_{av} = E_v I_v \cos\phi = \frac{300}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \cos\frac{\pi}{2} = 0$$

Q. 3.30. Define power factor.

(P.S.S.C.E. 2001, 1999 ; H.S.S.C.E. 2001, 1997 S

Ans. The cosine of the phase angle between alternating ent and e.m.f. in an a.c. circuit is called its power factor. , power factor = cos ϕ.

Q. 3.31. What are the maximum and minimum values ower factor of a.c. circuit ? *(P.S.S.C.E. 1999 S)*

Ans. The maximum value of power factor is 1 and the mum value is 0.

Q. 3.32. In an a.c. circuit, there is no power consum- n in an ideal inductor. Explain. *(C.B.S.E. 1998 ; P.S.S.C.E. 1998 S, 1991)*

Or

Prove mathematically that the average power over a plete cycle of alternating current through an ideal ıctor is zero. *(C.B.S.E. 1997)*

Ans. Average power of an a.c. circuit,

$$P_{av} = E_v I_v \cos\phi$$

For a.c. circuit containing an ideal inductor, $\phi = \pi/2$

$$\therefore \qquad P_{av} = E_v I_v \cos \pi/2 = E_v I_v (0) = 0$$

Q. 3.33. How much power is consumed in a (*i*) purely inductive and (*ii*) purely capacitive a.c. circuit ? *(C.B.S.E. 1992)*

Ans. The power consumed is zero in both the cases.

Q. 3.34. In a series LCR circuit, what is the value of power factor at resonance ? *(C.B.S.E. 1999 ; I.S.C.E. 1994)*

Ans. It is unity.

Q. 3.35. Write two advantages of alternating current (a.c.) over direct current (d.c.). *(P.S.S.C.E. 1999 S)*

Ans. Refer to section 3.15.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. Distinguish between 'average value' and s. value' of an alternating current. *(C.B.S.E. 1997)*

Ans. Refer to sections 3.02 and 3.03.

Q. 3.02. Calculate the r.m.s. value of alternating current wn in Fig. 3.24. *(C.B.S.E. 1998)*

Ans. Since the value of current has either a value of 2 – 2 A with the passage of time, the r.m.s. value of the nating current shown in Fig. 3.24 is also 2 A.

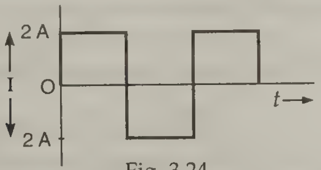


Fig. 3.24

Q. 3.03. Prove mathematically that the average value lternating current over one complete cycle is zero. *(C.B.S.E. 1997)*

Ans. Refer to section 3.02 :

The, average value of alternating current over one plete cycle,

$$I_a = \frac{1}{T}\int_0^T I_0 \sin\omega t\, dt = \frac{I_0}{T}\int_0^T \sin\omega t\, dt$$

$$= \frac{I_0}{T} - \cos\omega t\Big|_0^T = -\frac{I_0}{T}\Big|\cos\omega t\Big|_0^T$$

$$= -\frac{I_0}{T}\left[\cos\frac{2\pi}{T}\cdot T - \cos\frac{2\pi}{T}\cdot 0\right]$$

$$= -\frac{I}{T}(\cos 2\pi - \cos 0) = -\frac{I}{T}(1-1) = 0$$

Q. 3.04. A 110 V d.c. heater is used on an a.c. source, h that the heat produced is the same. What would be the .s. value of the alternating voltage ? *(I.S.C.E. 1996)*

Ans. Let E_v be the required value of r.m.s. voltage. Then, t produced in heater of resistance R in time T/2,

$$H_1 = \frac{E_v^2}{R}\cdot\frac{T}{2} \qquad \ldots(i)$$

Heat produced at 110 V d.c. in time T/2,

$$H_2 = \frac{E^2}{R}\cdot\frac{T}{2} = \frac{(110)^2}{T}\cdot\frac{T}{2} \qquad \ldots(ii)$$

Since $H_1 = H_2$, we have

$$\frac{(110)^2}{R}\cdot\frac{T}{2} = \frac{E_v^{\,2}}{R}\cdot\frac{T}{2} \qquad \text{or} \qquad E_v = 110 \text{ V}$$

Q. 3.05. The effective value of current in a 50 cycle a.c. circuit is 5·0 A. What is the value of the current $\frac{1}{300}$ s after it is zero ? *(P.S.S.C.E. 1994)*

Ans. Here, $I_v = 5{\cdot}0$ A ; $f = 50$ cycles s^{-1}

$$\text{Now,} \qquad I = I_0 \sin\omega t = \sqrt{2}\, I_v \sin 2\pi f t$$

$$= \sqrt{2} \times 5{\cdot}0 \sin 2\pi \times 50 \times \frac{1}{300}$$

$$= \sqrt{2} \times 5{\cdot}0 \sin\frac{\pi}{3} = \sqrt{2} \times 5{\cdot}0 \times \frac{\sqrt{3}}{2}$$

$$= 6{\cdot}124 \text{ A}$$

Q. 3.06. A coil has an inductive reactance of 160 ohm at frequency of 50 Hz. Calculate the self inductance of the coil. *(H.S.S.C.E. 1997 S)*

Ans. Here, $X_L = 160\ \Omega$; $f = 50$ Hz

$$\text{Now,} \qquad X_L = 2\pi f L \qquad \text{or} \qquad L = \frac{X_L}{2\pi f} = \frac{160}{2\pi \times 50}$$

$$= 0{\cdot}51 \text{ H}$$

Q. 3.07. An inductor of 1 H and negligible resistance is used on the mains of frequency 50 Hz. Find the reactance of the inductor. *(I.S.C.E. 1993 ; H.S.S.C.E. 1997)*

Ans. Inductive reactance, $X_L = 2\pi f L$

Here, $L = 1$ H ; $f = 50$ Hz

$$\therefore \qquad X_L = 2\pi \times 50 \times 1 = 100\,\pi \text{ H}$$

Q. 3.08. An inductor is in turn put across 220 V-50 Hz and 220 V-100 Hz supplies. Will the current flowing through it in the two cases be the same or different ? *(C.B.S.E. 1998)*

Ans. Now, $I_v = \frac{E_v}{X_L} = \frac{E_v}{2\pi f L}$

$$\text{Thus,} \qquad I_v \propto \frac{1}{f}$$

Thus, current flowing in the two cases will be **different**.

Q. 3.09. Calculate the capacitive reactance of 5 μF capacitor for a frequency of (*i*) 50 Hz, (*ii*) 10^6 Hz. (*H.S.S.C.E. 1995*)

Ans. Here, $C = 5\ \mu F = 5 \times 10^{-6}$ F

(*i*) For $f = 50$ Hz,

$$X_C = \frac{1}{2\pi \times 50 \times 5 \times 10^{-6}} = \mathbf{636{\cdot}6\ \Omega}$$

(*ii*) For $f = 10^6$ Hz,

$$X_C = \frac{1}{2\pi \times 10^{-6} \times 5 \times 10^{-6}} = \mathbf{0{\cdot}032\ \Omega}$$

Q. 3.10. A capacitor blocks d.c. Why ? (*H.S.S.C.E. 2001, 2000 ; P.S.S.C.E. 2001, 1993*)

Or

What is the capacitive reactance of a capacitor used in a circuit having d.c. e.m.f. ? Explain. (*P.S.S.C.E. 1992*)

Ans. The capacitive reactance is given by

$$X_C = \frac{1}{2\pi f C}$$

For d.c., $f = 0$

$$\therefore\ X_C\ (\text{for d.c.}) = \frac{1}{2\pi(0)C} = \infty$$

Thus, a capacitor offers infinite resistance to d.c. and hence acts as a perfect block for it.

Q. 3.11. An electric lamp connected in series with a variable capacitor and an a.c. source is glowing with some brightness. How will the brightness change on increasing the value of capacitance and why ? (*C.B.S.E. 1998*)

Ans. Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

When the capacitance (C) is increased, the capacitive reactance (X_C) will decrease. Due to decrease in the value of X_C, the current in the circuit will increase and hence brightness of the source will also increase.

Thus, a capacitor offers infinite resistance to d.c. and hence acts as a perfect block for it.

Q. 3.12. Distinguish between resistance, reactance and impedance for an a.c. circuit. (*C.B.S.E. 2000, 1999, 1994 ; H.S.S.C.E. 1999*)

Ans. Resistance. *The resistance of an a.c. circuit is the ohmic resistance offered by a conductor connected in the circuit.* It is due to the nature of the material (resistivity) of the conductor.

Reactance. *The reactance of an a.c. circuit is the resistance offered by an inductor or a capacitor connected in the circuit.* It arises because of the fact that the alternating e.m.f. and the current differ in phase by $\pi/2$, when a.c. flows through an inductor or a capacitor.

Impedance. *The impedance of an a.c. circuit is the effective resistance offered by the LR-circuit or CR-circuit or LCR-circuit.*

Q. 3.13. Explain the terms reactance and impedance as applied to components of an a.c. circuit. (*C.B.S.E. 2001 S*)

Ans. Refer to SAQ 3.12.

Q. 3.14. The frequency of a.c. source is doubled. How do R, X_L and X_C get affected ?

Ans. There is no effect of doubling the frequency of the a.c. source on R.

Since $X_L = 2\pi f L$, the value of X_L becomes double on doubling the frequency of a.c. source.

Further, $X_C = \frac{1}{2\pi f C}$. Therefore, the value of X becomes half on doubling the frequency of a.c. source.

Q. 3.15. Draw the graph showing the variation reactance of (*a*) a capacitor and (*b*) an inductor with th frequency of an a.c. circuit. (*C.B.S.E. 2002 ; 2000, 199 1998, 1995, 1994 ; H.S.S.C.E. 199*

(*a*) Reactance of a capacitor,

$$X_C = \frac{1}{2\pi f C} \quad \text{or} \quad X_C \propto \frac{1}{f}$$

Graph between f and X_C will be as shown in Fig. 3.25

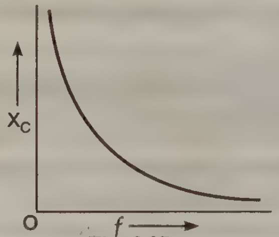


Fig. 3.25

(*b*) Reactance of an inductor,

$$X_L = 2\pi f L \quad \text{or} \quad X_L \propto f$$

Graph between f and X_L will be as shown in Fig. 3.26

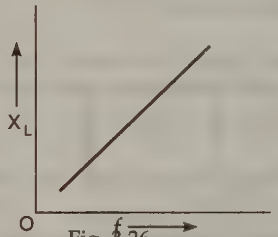


Fig. 3.26

Q. 3.26. An air core coil and an electric bulb a connected in series across a 220 V-50 Hz a.c. source. Th bulb glows with some brightness. How will the glow of th bulb be affected on introducing a capacitor in series in th circuit ? Justify your answer. (*C.B.S.E. 199*

Ans. The brilliance of the bulb depends upon the curre through the bulb. If R is resistance of the filament of the bu and L the inductance of the coil, then current through th bulb,

$$I_v = \frac{E}{\sqrt{R^2 + \omega^2 L^2}}$$

When capacitor is put in series, the current through th bulb becomes

$$I_v' = \frac{E}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Since $I'_v > I_v$, the bulb will glow more brightly on puttin the capacitor in series.

Q. 3.17. When an a.c. circuit with a series combinatio of inductance, capacitance and resistance is brought in resonance, the current in the circuit increases to a larg value. Why ?

Ans. In an LCR-series circuit, the current is given by

$$I_v = \frac{E_v}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

An LCR-circuit comes into resonance, when $\omega L = \frac{1}{\omega C}$.

In that case, the current is given by

$$I'_v = \frac{E_v}{R}$$

Since $R < \sqrt{R^2 + (\omega L - 1/\omega C)^2}$, it follows that $I'_v > I_v$.

Q. 3.18. When a capacitor is connected in series with a series LR- circuit, the alternating current flowing in the circuit increases. Explain, why. (*C.B.S.E. 1998*)

Ans. The impedance of LR-circuit,

$$Z_1 = \sqrt{R^2 + \omega^2 L^2}$$

Therefore, current in LR- circuit,

$$I_1 = \frac{E_v}{Z_1} = \frac{E_v}{\sqrt{R^2 + \omega^2 L^2}} \qquad \text{...}(i)$$

When a capacitor is connected in series with LR-circuit, then

$$Z_2 = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

Hence, current in the circuit,

$$I_2 = \frac{E_v}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), it follows that $I_2\ I_1$. Hence, when a capacitor is connected in series to an LR-circuit, current in the circuit increases.

Q. 3.19. Discuss resonance in a LCR-series a.c. circuit. (*H.S.S.C.E. 1994*)

Ans. Refer to section 3.11.

Q. 3.20. An inductor L of reactance X_L is connected in series with a bulb B to an a.c. source as shown in Fig. 3.27. Briefly explain how does the brightness of the bulb change, when (*a*) number of turns of the inductor is reduced and (*b*) a capacitor of reactance $X_C = X_L$ is included in series in the same circuit. (*C.B.S.E. 2002*)

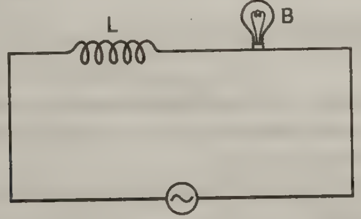


Fig. 3.27

Ans. (***a***) If R is resistance of the filament of bulb, the impedance of the circuit,

$$Z = \sqrt{R^2 + X_L^2}$$

When number of turns of the inductor is reduced, X_L and hence Z will decrease. This will result in increase of current in the circuit and hence the bulb will glow more brightly.

(***b***) When a capacitor of reactance X_C is connected in series, then impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Since $X_L = X_C$, $Z = R$ *i.e.* impedance of the circuit will be minimum (merely equal to the resistance of the filament of the bulb). Therefore, brightness of the bulb will become maximum.

Q. 3.21. What is the quality factor (Q) in an a.c. circuit ? (*C.B.S.E. 1992*)

Ans. Refer to section 3.12.

Q. 3.22. Explain sharpness of resonance. (*H.S.S.C.E. 1994*)

Ans. Refer to section 3.12.

Q. 3.23. What do you mean by power factor ? On what factors does it depend ? (*H.S.S.C.E. 1998*)

Ans. Power factor. *The cosine of the phase angle between alternating e.m.f. and current in an a.c. circuit is called the power factor of the circuit.*

If ϕ is the phase angle between alternating e.m.f. and current in an a.c. circuit, then

power factor of a.c. circuit,

$$\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

It follows that power factor of an a.c. circuit, in addition to the values of the circuit elements R, L and C, depends upon the frequency of the a.c. source.

Note. Also, true power (or average power)

= apparent power × power factor

Therefore, power factor $= \frac{\text{true power}}{\text{apparent power}}$

Thus, *power factor of an a.c. circuit may also be defined as the ratio of true power of the circuit to its apparent power.*

Q. 3.24. Explain the importance of power factor. (*P.S.S.C.E. 1997 S*)

Ans. The average power, $P = E_v I_v \cos\phi$

In the electric circuits operating some electric appliance, the power factor is kept as low as possible. It is done so that the power consumption in the circuit is low.

On the other hand, when power is to be transmitted, the power factor is made as large as possible.

Since $\qquad I_v = \frac{P}{E_v \cos\phi}$,

the large value of power factor will reduce current and hence dissipation of electric energy in the form of heat along the line wires will be low.

Q. 3.25. Which is more dangerous in use : a.c. or d.c. ? Explain, why.

Ans. The a.c. is more dangerous than d.c. of the same voltage. It is because, the peak value of a.c. is more than the indicated value. For example, a 220 V a.c. has a peak value of $220 \times \sqrt{2}$ V $\approx$ 311 V, whereas peak value of a 220 d.c. is same *i.e.* 220 V.

Q. 3.26. Why is a.c. more dangerous than d.c. for the same voltage ? (*H.S.S.C.E. 2001*)

Or

Why 220 V a.c. is more dangerous than 220 V d.c. ? (*P.S.S.C.E. 2001*)

Ans. Refer to SAQ 3.25.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 3.01. A choke coil in series with a lamp is connected to a d.c. line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes to change the lamp's brightness. Predict the corresponding observation, if the connection is to an a.c. line. *(Text Question)*

Ans. When constant d.c. is flowing, choke coil has no reactance. Therefore, even when iron core is inserted, the brightness of the lamp will not be affected. In case the lamp is connected to a.c. line, the insertion of iron core will increase the inductance of the choke coil and hence its reactance. Due to this, lesser current will flow and brightness of the lamp will be reduced.

Q. 3.02. A lamp is connected in series with a capacitor. Predict your observations for d.c. and a.c. connections. What happens in each case, if the capacitance of the capacitor is reduced ? *(Text Question)*

Ans. When a lamp is connected to d.c. line through the capacitor, it will be an open circuit. It is because, a capacitor acts as a block for d.c. ($X_C = \infty$). Therefore, lamp will not glow, even if C is reduced.

When the lamp is connected to a.c. through the capacitor, the lamp will glow. It is because, a capacitor has finite reactance for a.c. On reducing C, reactance due to capacitor $\left(X_C = I/\omega C\right)$ will increase. Since current becomes lesser, the lamp will shine less brightly.

Q. 3.03. In any a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit ? Is the same true for r.m.s. voltages ? *(Text Question)*

Ans. No, the applied instantaneous voltage is not equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit. It is because, voltages across different elements are not in phase. For the same reason, it is also not true for r.m.s. voltage.

Q. 3.04. An applied voltage signal consists of a superposition of a d.c. voltage and a.c. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the d.c. signal will appear across C and the a.c. signal across L . *(Text Question)*

Ans. We know, $X_L = \omega L$ and $X_C = 1/\omega C$

X_L has zero value for d.c. and high value for a.c., while X_C has infinite value for d.c. and low value for a.c. When applied voltage signal consisting of superposition of a d.c. voltage and an a.c. voltage, the inductor will oppose a.c., whereas it will allow easy path to d.c. Therefore, a.c. voltage will appear across L. For similar reason, d.c. voltage will appear across C.

Q. 3.05. Explain : voltages across L and C in series are 180° out of phase ; while for L and C in parallel, currents in L and C are 180° out of phase. *(Text Question)*

Ans. For given current I (= $I_0 \cos \omega t$), the expression for voltages across L and C in the LC-series circuit are

$$E = E_0 \cos\left(\omega t + \frac{\pi}{2}\right) \quad \text{and} \quad E = E_0 \cos\left(\omega t - \frac{\pi}{2}\right)$$

respectively.

It follows that the voltages are 180° out of phase, when L and C are in series.

For given voltage $E = E_0 \cos \omega t$, the expressions for current through L and C in the LC-parallel circuit are

$$I = I_0 \cos\left(\omega t - \frac{\pi}{2}\right) \text{ and } I = I_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

respectively. It follows that the, currents are 180° out of phase, when L and C are in parallel.

Q. 3.06. The total impedance of a circuit decreases, when a capacitor is added in series with L and R. Explain, why.

Ans. The impedance of LR-circuit is given by

$$Z_1 = \sqrt{R^2 + \omega^2 L^2}$$

When capacitor is added in series with L and R, impedance of the circuit becomes

$$Z_2 = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

Since $(\omega L - 1/\omega C) < \omega L$, it follows that $Z_2 < Z_1$.

Q. 3.07. For circuits used for transporting electric power, a low power factor implies large power loss in transm-ission. Why ? *(Text Question)*

Ans. We know, $P_{av} = E_v I_v \cos \phi$

or $$I_v = \frac{P_{av}}{E_v \cos \phi}$$

To supply a given power, low power factor *i.e.* low value of $\cos \phi$ means larger current will flow across the line wires. Since energy dissipation is $I_v^2 R$, large heat losses will occur in such a case.

Q. 3.08. Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain. *(Text Question)*

Ans. We know, $$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

By adjusting C, $1/\omega$ C can be made equal to L *i.e.* the value of Z can be made equal to (or approach) R, so that power factor becomes unity.

Q. 3.09. Why power factor correction is must in heavy machinery ? *(P.S.S.C.E. 2001, 1996)*

Ans. A heavy machinery requires a large power. We know that

$$P_{av} = E_v I_v \cos \phi$$

For given supply voltage, the required power can be supplied to the heavy machinery either by supplying large current or by improving power factor ($\cos \phi$). So that large current can be drawn by the machinery, thick wires have to be used. It will result in greater cost of installation. Therefore, in practice, power factor ($\cos \phi$) is corrected by decreasing ϕ. It is achieved by connecting a capacitor of suitable value. As the capacitive reactance and the inductive reactance are always out of phase with each other, the presence of capacitor increases the value of $\cos \phi$.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Prove mathematically that the average value of alternating current over one complete cycle is zero. (*C.B.S.E. 1999 S*)
2. Distinguish between the terms 'peak-value', 'mean value' and 'root mean square value' of an alternating current. How are they related to each other ?
3. Define peak value and root mean square value of alternating current. Derive an expression for the root mean square value of alternating current. (*H.P.S.S.C.E. 1999*)
4. Define virtual e.m.f. and find the relation between virtual e.m.f. and maximum e.m.f. in a.c. (*H.S.S.C.E. 1992*)
5. What is meant by root mean square value of alternating current ? Derive an expression for r.m.s. value of alternating current. (*H.P.S.S.C.E. 1998*)
6. An a.c. voltage $E = E_0 \sin \omega t$ is applied across an inductance L. Obtain an expression for the current I. (*P.S.S.C.E. 2001, C.B.S.E. 1990*)
7. Show that in an inductance, the voltage leads the current by $\pi/2$.
8. Derive an expression for impedance of a coil in an a.c. circuit. (*C.B.S.E. 1992*)
9. An alternating e.m.f. is applied across a capacitor. Show that current in it leads the applied e.m.f. by 90°. What is capacitive reactance of such a circuit ? Write down the units of capacitive reactance.
10. Discuss phase relationship between current and e.m.f. in an a.c. circuit containing a capacitor only. (*P.S.S.C.E. 1992*)
11. Derive the expression for the impedance of an a.c. circuit with an inductor 'L' and a resistor 'R' in series. (*C.B.S.E. 1993*)
12. An alternating source of e.m.f. is applied to an inductor and resistor in series. Investigate the phase relationship between current and e.m.f. What is the impedance of the circuit ?
13. Derive the expression for the impedance of an a.c. circuit with a capacitor and a resistor in series. (*C.B.S.E. 1993*)
14. An alternating source of e.m.f. is applied to a capacitor and resistor in series. Investigate the phase relationship between current and e.m.f. What is the impedance of the circuit ?
15. What do you mean by inductive reactance, capacitive reactance and impedance ? Show that a capacitor is a block for d.c. ; and inductor, a block for a.c.
16. What is impedance ? Derive a relation for it in an a.c. series LCR-circuit. Show it by a vector diagram. (*P.S.S.C.E. 1996 S*)
17. What is meant by an LCR-circuit ? Find the impedance of such a circuit. What is the phase relationship between the current and e.m.f. in such a circuit ?
18. How does the term electric 'resistance' differ from 'impedance' ? With the help of a suitable phasor diagram, obtain a relation for impedance in an a.c. series LCR-circuit. (*P.S.S.C.E. 1994*)
19. What do you mean by impedance of LCR-circuit ? Derive expression for it. What is the condition for resonance ? (*H.P.S.S.C.E. 1993*)
20. Discuss the phenomenon of resonance in a LCR series a.c. circuit. (*C.B.S.E. 1992*)
21. Explain the term resonance for a series LCR-circuit. Calculate the resonant frequency. (*H.S.S.C.E. 1998*)
22. Obtain condition for the resonance of LCR-circuit and show that resonance frequency is $\frac{1}{2\pi\sqrt{LC}}$. (*H.P.S.S.C.E. 1998 S*)
23. Draw a circuit diagram showing a series LCR-circuit and derive an equation for its resonant frequency. (*I.S.C.E. 1996*)
24. What do you mean by sharpness of resonance in a series resonant circuit ? Find expression for Q-factor of the circuit. (*H.S.S.C.E. 1992*)
25. Prove that an ideal resistor connected to an a.c. source dissipates power $\frac{V_{eff}^2}{R}$. (*C.B.S.E. 1996*)
26. Show mathematically that an ideal inductor does not consume any power in an a.c. circuit. (*P.S.S.C.E. 2002, 2001 ; C.B.S.E. 1990*)
27. Prove that an ideal inductor does not dissipate power in an a.c. circuit. (*C.B.S.E. 2001 S*)
28. Show mathematically that the average power supplied to an ideal capacitor by a source over a complete cycle of a.c. is zero. (*C.B.S.E. 2001 S*)
29. Prove that an ideal capacitor connected to an a.c. source does not dissipate power. (*H.S.S.C.E. 2001*)
30. Distinguish between the terms reactance and impedance of an a.c. circuit. Prove that an ideal capacitor connected to an a.c. source does not dissipate power. (*C.B.S.E. 1996*)
31. Derive an expression for the power in an LCR a.c. circuit. (*P.S.S.C.E. 2000, 1999 S, 1999, 1997 S ; H.S.S.C.E. 1998*)
32. Show that the average power transferred to an a.c. circuit is in general given by $P = V_{r.m.s.} I_{r.m.s.} R/Z$, where R is the resistance in the circuit. (*C.B.S.E. 1990*)
33. Find the expression for the true power and apparent power in an a.c. circuit. Determine the condition that a current in the circuit may be wattless.
34. Derive a relation for power in a.c. series LCR-circuit. What is power factor ? (*P.S.S.C.E. 1996*)

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Define mean value and root mean square value of alternating current. Derive an expression for root mean square value of alternating current. (*H.S.S.C.E. 2001*)
2. Derive the relations for average value and r.m.s. value (effective value) of a.c. (*P.S.S.C.E. 2000*)
3. Derive the relations for average and r.m.s. value of an alternating current. Which value of the current, do we measure with an a.c. ammeter ? (*P.S.S.C.E. 1998 S*)
4. An alternating e.m.f. is supplied to (*i*) a pure resistance, (*ii*) a pure inductance, (*iii*) a pure capacitance. Investigate the phase relationship between current and e.m.f. in each of the above cases.
5. How does the term ohmic resistance differ from inpedance ? With the help of a suitable phasor diagram, obtain the relation for impedance in an a.c. series LCR-circuit. (*P.S.S.C.E. 2002, 2000*)
6. Using phasor diagram, derive an expression for impedance of an a.c. circuit containing LCR in series. What do you mean by resonance of such a circuit ? (*P.S.S.C.E. 2002, 2000*)

7. A series circuit contains L, C and R as circuit elements. Discuss current-voltage relation, when a.c. is passed through such a circuit and hence find the expression for the impedance of the circuit and resonant frequency of the circuit. *(H.P.S.S.C.E. 2000)*
8. What is impedance of an electric circuit ? Derive an expression for impedance of an a.c. LCR series circuit. Also find the expression for resonant frequency. *(P.S.S.C.E. 2000)*
9. With the help of a phasor diagram, obtain a relation for impedance in an a.c. series LCR-circuit. What is meant by resonance of this circuit ? Find relation for resonant frequency of series LCR-circuit. *(P.S.S.C.E. 1999, 1995)*
10. Define an expression for the impedance of an a.c. circuit with an inductor L and a resistor R in series. Also obtain the expression for average power in this circuit. *(P.S.S.C.E. 2001)*
11. Deirve an expression for the impedance of an a.c. circuit with capacitor C and resistor R in series. Also obtain an expression for its average power in this circuit. *(P.S.S.C.E. 2001)*
12. Define power in an a.c. circuit and obtain expression for (*a*) instantaneous power (*b*) the average power over a complete cycle in case of a circuit containing inductance, resistance and capacitance. What is power factor and wattless current ?
13. Derive an expression between true power and virtual power of an a.c. circuit. How will you differentiate between true power and virtual power ? *(P.S.S.C.E. 2002)*
14. Derive an expression for the average power in a LCR-series circuit connected to a.c. supply. Hence define power factor. *(H.S.S.C.E. 2002)*
15. Find an expression for power of an a.c. circuit containing L, C and R. *(H.S.S.C.E. 2000)*
16. L, C and R are in series with an a.c. source. Calculate the power of such a circuit. Also obtain expression for resonant frequency. *(H.S.S.C.E. 1996)*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Equation for Instantaneous value of a.c.

1. A man is given a shock with 220 volt d.c. and thereafter with 220 volt (r.m.s.) a.c. Will he feel the same intensity of shock in the two cases ? Give reasons in support of your answer. [**Ans.** In case of 220 volt a.c., shock will be of greater intensity as peak value of e.m.f. is 311·1V]
2. The instantaneous current from an a.c. source is
$$I = 6 \sin 314\, t$$
What is the r.m.s. value of the current ? *(C.B.S.E. 1996)* [**Ans.** 4·24 A]
3. Find the time required for a 50 Hz alternating current to change its value from zero to the r.m.s. value. [**Ans.** $2{\cdot}5 \times 10^{-3}$ s]

Type B. On a.c. circuit containing resistor

4. A resistance of 20 Ω is connected to a source of alternating current rated 110 V, 50 Hz. Find (*a*) the r.m.s. of current, (*b*) the maximum instantaneous current in the resistor. [**Ans.** (*a*) 5·5 A, (*b*) 7·78 A]
5. A 100 ohm iron is connected to a 220 volt, 50 cycles wall plug. What is its (*i*) peak potential difference (*ii*) average potential difference and (*iii*) r.m.s. current ? [**Ans.** (*i*) 311·1 V ; (*ii*) 198·1 V ; (*iii*) 2·2 A]

Type C. On a.c. circuit containing inductor

6. A 100 Hz a.c. is flowing in a 14 millihenry coil. Find the reactance of the coil. *(H.S.S.C.E. 1998)* [**Ans.** 8·8 Ω]
7. At what ferquency will a 0·5 H inductor have a reactance of 1000 Ω ? *(H.S.S.C.E. 2002)* [**Ans.** 318·3 Hz]
8. Find the value of current through an inductor of 2·0 H and negligible resistance, when connected to an a.c. source of 150 V and 50 Hz. *(P.S.S.C.E. 1997)* [**Ans.** 0·239 A]
9. Find the maximum value of current, when inductance of one henry is connected to 200 volt, 50 cycles supply. *(P.S.S.C.E. 2000)* [**Ans.** 0·9 A]
10. An inductance of negligible resistance, whose reactance is 22 ohm at 200 hertz is connected to a 220 volt, 50 hertz power line. What is the value of the inductance and reactance ? [**Ans.** $1{\cdot}75 \times 10^{-2}$ H ; 5·5 Ω]

Type D. On a.c. circuit containing capacitor

11. Calculate the capacitive reactance of a capacitor of capacitance 1 microfarad for a frequency of $10^6/2\pi$ Hz. [**Ans.** 1 Ω]
12. One microfarad capacitor is joined to 200 volt, 50 hertz alternator. Find the r.m.s. current through the capacitor. [**Ans.** 0·0628 A]
13. A capacitor of 1 μF is connected to a source of a.c. having e.m.f. given by equation
$$E = 200 \cos 120\, \pi t$$
Find the value of r.m.s. current through the capacitor. [**Ans.** 0·0533 A]

Type E. On LR-circuit

14. A bulb of resistance 10 Ω, connected to an inductor of inductance L, is in series with an a.c. source marked 100 V – 50 Hz. If the phase angle between the voltage and current is $\pi/4$ radian, calculate the value of L. *(C.B.S.E. 2001)* [**Ans.** 0·032 H]
15. An a.c. source of 100 V (r.m.s.), 50 Hz is connected across a 20 ohm resistance and a 50 mH inductor in series. Calculate (*i*) impedance of the circuit and (*ii*) r.m.s. current in the circuit. *(C.B.S.E. 1993)* [**Ans.** (*i*) 25·43 Ω ; (*ii*) 3·93 A]
16. A current of 11 A flows through a coil, when connected to a 110 V d.c. When 110 V a.c. of 50 Hz is applied to the same coil, only 0·5 A current flows. Calculate the (*i*) resistance, (*ii*) impedance and (*iii*) inductance of the coil. *(C.B.S.E. 1992)* [**Ans.** (*i*) 10 Ω; (*ii*) 220 Ω ; (*iii*) 0·7 H]
17. A coil of inductance 0·5 H and resistance 100 Ω is connected to a 240 V, 50 Hz supply. What is the maximum current in the coil ? *(Pre-degree Kerala, 1994)* [**Ans.** 1·82 A (peak value)]
18. A 100 V, 50 Hz a.c. source is connected to a series combination of an inductance of 100 mH and a resistance of 25 ohm. Calculate the magnitude and phase of the current. *(C.B.S.E. 1991)* [**Ans.** 2·49 A, 51° 29′]
19. An alternating e.m.f. of 110 V is applied to a circuit containing a resistance of 40 Ω and an inductance L in series. The current is found to lag behind the voltage by an angle $\phi = \tan^{-1} 3/4$. Find the (*a*) inductive reactance ; (*b*) impedance of the circuit and (*c*) current flowing in the circuit. If the inductance has a value of 0·1 H, find the frequency of the applied e.m.f. *(I.S.C.E. 1995)* [**Ans.** (*a*) 30 Ω ; (*b*) 50 Ω ; (*c*) 2·2 A ; 47·75 Hz]
20. An a.c. circuit that contains an inductance and a resistance has an impedance of 50 ohm at 100 Hz and an impedance

of 100 ohm at 500 Hz. What are the values of inductance and the resistance of the circuit ?
[Ans. $28{\cdot}14 \times 10^{-3}$ H, 46·74 Ω]

21. When a series combination of a coil of inductance L and a resistor of resistance R is connected across a 12 V–50 Hz supply, a current of 0·5 A flows through the circuit. The current differs in phase from applied voltage by $\pi/3$ radian. Calculate the value of L and R. *(C.B.S.E. 1997)*
[Ans. 0·066 H, 12 Ω]

22. When a series combination of inductance and resistance is connected with a 10 V–50 Hz supply, a current of 1 A flows through the circuit. The voltage leads the current by a phase angle of $\pi/3$ radian. Calculate the values of inductance and resistance. *(C.B.S.E. 1999)*
[Ans. 0·028 H, 5 Ω]

23. A coil of negligible resistance and inductance 0·02 henry is connected in series with a wire of zero inductance and resistance 12 ohm. An alternating e.m.f. of 130 volt and 40 Hz is applied. Calculate the current, potential difference across the resistance and the angle of lag.
[Ans. 10 A, 120 V, 22° 44′]

24. An electric lamp, which runs at 100 volt d.c. and 10 ampere current, is to be run on 220 volt, 50 cycles a.c. mains. Calculate the inductance of the required choke coil.
[Ans. 0·0624 H]

25. An electric lamp marked 220 V d.c. consumes a current 10 A. It is connected to a 250 V – 50 Hz a.c. mains through a choke. Calculate the inductance of the choke required. *(P.S.S.C.E. 2002)* **[Ans.** 0·446 H]

26. An 80 V–800 W heater is to be operated on a 100 V–50 Hz supply. Calculate the inductance of the choke required. *(H.S.S.C.E. 2002)* **[Ans.** 0·031 H]

ype F. On CR-circuit

27. A series circuit contains a resistor of 20 Ω, a capácitor and an ammeter of negligible resistance. It is connected to a source of 220 V–50 Hz. If the reading of an ammeter is 2·5 A , calculate the reactance of the capacitor. *(P.S.S.C.E. 1999)* **[Ans.** 85·7 Ω]

28. What is the value of current in the a.c. circuit containing R = 10 Ω, C = 50 μF in series across 200 V, 50 Hz a.c. source ? *(C.B.S.E. 1993)* **[Ans.** 3·1 A]

29. An alternating current of 1·5 mA and angular frequency 300 rad s^{-1} flows through 10 kΩ resistor and a 0·5 μF capacitor in series. Find the r.m.s. voltage across the capacitor and impedence of the circuit. *(C.B.S.E. 1993)*
[Ans. 10 V, $1{\cdot}2 \times 10^4$ Ω]

30. When an electric derive X is connected to a 220 V, 50 Hz a.c. supply, the current is 0·5 A and is in same phase as the applied voltage. When another device Y is connected to the same supply, the electric current is again 0·5 A, but it leads the potential difference by $\pi/2$. (*a*) What are the devices X and Y ? (*b*) When X and Y are connected in series across the same source, what will be the current ?
(C.B.S.E. 1992)
[Ans. X–resistor, Y–capacitor, 0·354 A]

ype G. On LCR-circuit

31. A resistor of 50 ohm, an inductor of $20/\pi$ henry and a capacitor of $5/\pi$ microfarad are connected in series to a voltage source 230 V–50 Hz. Find the impedence of the cicruit. **[Ans.** 50 Ω]

32. A resistor of 12 Ω, a capacitor of reactance 14 Ω and a pure inductor of inductance 0·1 H are joined in series and placed across a 220 V–50 Hz a.c. supply. Calculate (*a*) the current in the circuit and (*b*) the phase angle between the current and voltage. Take $\pi = 3$. **[Ans.** (*a*) 11 A (*b*) 53·13°]

Type H. On Resonance

33. A series circuit with L = 0·12 H, C = 0·48 mF and R = 25 Ω is connected to a 220 V variable frequency power supply. At what frequency is the ciruit current maximum ?
(H.S.S.C.E. 2002) **[Ans.** 21 Hz]

34. A series LCR-circuit with L = 0·12 H, C = $4{\cdot}8 \times 10^{-7}$ F, and R = 23 ohm is connected to a variable frequency supply. At what frequency is the current maximum ?
(C.B.S.E. 1990) **[Ans.** 663·15 Hz]

35. A resistor, a capacitor of 100 μF capacitance and an inductor are in series with an a.c. source of frequency 50 Hz. If the current in the circuit is in phase with the voltage, calculate the inductance of the inductor used.
(C.B.S.E. 2000) **[Ans.** 0·1 H]

36. An inductor L, a capacitor 20 μF and a resistor 10 Ω are connected in series with an a.c. source of frequency 50 Hz. If the current is in phase with the voltage, calculate the inductance of the inductor. *(C.B.S.E. 2001)* **[Ans.** 0·507 H]

37. A capacitor, resitor of 5 Ω and an inductor of 50 mH are in series with an a.c. source marked 100 V, 50 Hz. It is found that voltage is in phase with the current. Calculate the capacitance of the capacitor and the impedance of the circuit. *(C.B.S.E. 1999)* **[Ans.** 202·64 μF, 5 Ω]

38. Find the capacitive reactance of a 10 μF capacitor at 1000 cycles s^{-1}. Calculate the inductance required to produce series resonance with the capacitor at this frequency. *(H.S.S.C.E. 2002)*
[Ans. 15·92 Ω, $2{\cdot}533 \times 10^{-3}$ H]

39. A 100 mH inductor, a 25 μF capacitor and a 15 Ω resistor are connected in series to a 120 V, 50 Hz a.c. source. Calculate (*i*) impedance of the circuit at resonance, (*ii*) current at resonance and (*iii*) resonant frequency.
(C.B.S.E. 2000) **[Ans.** (*i*) 15 Ω (*ii*) 8 A (*iii*) 100·66 Hz]

40. A variable frequency 230 V alternating voltage source is connected across a series combination L = 5·0 H, C = 80 μF and R = 40 Ω. Calculate (*a*) the angular frequency of the source which drives the circuit in resonance, (*b*) the amplitude of the current at resonant frequency and (*c*) r.m.s. potential drop across the inductor at resonating frequency. *(C.B.S.E. 2001 S)*
[Ans. (*a*) 50 rad s^{-1}, (*b*) 5·75 A (*c*) 1437·5 V]

41. A resistance of 2 Ω, a coil of inductance 0·01 H are connected with a capacitor and put across a 200 V and 50 Hz supply. Calculate (*i*) the capacitance of the capacitor, so that the circuit resonates and (*ii*) the current and voltage across the capacitor at resonance. (Take $\pi = 3$)
(C.B.S.E. 1999 S) **[Ans.** (*i*) 1·11 mF ; (*ii*) 100 A, 300·3 V]

Type I. On Power in a.c. circuit

42. A 100 ohm geyser is connected to 220 volt, 50 cycles s^{-1}. What is its (*i*) peak potential difference, (*ii*) average power delivered, (*iii*) peak power and (*iv*) energy delivered in 10 minutes ?
[Ans. (*i*) 311·1 V ; (*ii*) 484 W ; (*iii*) 967·8 W ; (*iv*) $2{\cdot}904 \times 10^5$ J]

43. A group of electric lamps having total power rating of 900 watt is supplied an a.c. voltage
$$E = 200 \cos(314\, t + 60°)$$
Find r.m.s. value of a.c. current. **[Ans.** 12·73 A]

44. An alternating voltage E = 200 sin 300 *t* is applied across a series combination of resistance of 10 Ω and an inductor of 800 mH. Calculate (*i*) impedance of the circuit, (*ii*) peak value of current in the circuit and (*iii*) power factor of the circuit. *(C.B.S.E. 1994)*
[Ans. (*i*) 240·2 Ω ; (*ii*) 0·833 A; (*iii*) 0·042]

45. An electric lamp runs at 120 volt a.c. and consumes 12 ampere. It is connected to 220 volt, 50 hertz a.c. Find

the value of inductance required and the power factor.
[**Ans.** 0·049 H ; 0·0545]

46. Find the capacitance of a capacitor, which when put in series with a resistance of 10 ohm makes the power factor equal to 0·5. Assume an 80 volt–100 Hz a.c. supply.
[**Ans.** $9 \cdot 19 \times 10^{-5}$ F]

47. A choke of 0·5 henry, a capacitor of 15 μF and resistance of 100 ohm are connected in series across 200 volt, 50 hertz main. Find (*i*) current in the circuit, (*ii*) power factor of the circuit. [**Ans.** (*i*) 1·752 A ; (*ii*)0·876]

48. A resistor of 12 Ω, a capacitor of reactance 14 Ω and an inductor of reactance 30 Ω are joined in series and placed across a 230 V, 50 Hz supply. Calculate (*a*) the current in circuit, (*b*) the phase angle between the current and the voltage and (c) power factor. (*H.S.S.C.E. 2002*)
[**Ans.** (*a*) 11·5 A (*b*) 53·13° (*c*) 0·6]

49. Calculate the value of an inductance, which should be connected in series with a capacitance of 5 μF, resistance of 10 ohm and a.c. source of 50 Hz, so that the power factor of the circuit is unity. [**Ans.** 2·026 H]

50. A series LCR-circuit with R = 10 Ω, L = 2 H and C = 25 μF is connected to a variable frequency 200 V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle? (*H.S.S.C.E. 2001*)
[**Ans.** 4000 W]

MISCELLANEOUS PROBLEMS

51. Obtain the answers to (*a*) and (*b*) in solved problem 3·09, if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence explain the statement that at ver high frequency, an inductor in a circuit nearly amounts t an open circuit. How does an inductor behave in a d. circuit after the steady state?
[**Ans.** (*a*) $1 \cdot 08 \times 10^{-2}$ A ; (*b*) ≈ 90°, inductor behaves like conductor in a d.c. circuit

52. Obtain the answers to (*a*) and (*b*) in solved problem 3·1 if the circuit is connected to a 110 V, 12 kHz supply. Henc explain the statement that a capacitor is a conductor at ver high frequencies. Compare this behaviour with that of capacitor in a d.c. circuit after the steady state.
[**Ans.** (*a*) 3·89 A ; (*b*) ≈ 0°, capacitor amount to an open path in a d.c.. circui

53. In an a.c. circuit, the potential difference across a inductance and resistance joined in series is respectivel 12 V and 16 V. Find the total potential difference across th circuit. [**Ans.** 20 V

54. A circuit containing a 80 mH inductor and a 60 μ capacitor in series is connected to a 230 V, 50 Hz suppl If the circuit has a resistance of 15 ohm, obtain the averag power transferred to each element of the circuit and th total power absorbed. (*H.S.S.C.E. 200*
[**Ans.** $P_{av}(R) = 790 \cdot 6$ W, $P_{av}(L) = 0$, $P_{av}(C) =$
$P_{av}(\text{total}) = 790 \cdot 6$ W

55. A coil has an inductance 0·7 henry and is joined in serie with a resistance of 220 ohm. Find the wattless componen of the current in the circuit, when an alternating e.m.f. o 220 volt at a frequency of 50 Hz is supplied to it.
[**Ans.** 0·5 A

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Refer to SAQ 3.25.

2. Proceed as in solved problem no. 3.02 (*iii*).

3. Now, $I = I_0 \sin \omega t$ ...(*i*)

Suppose that when $t = t'$, $I = I_v = \dfrac{I_0}{\sqrt{2}}$

Setting the above condition in equation (*i*), we get

$$\frac{I_0}{\sqrt{2}} = I_0 \sin \omega t' \quad \text{or} \quad \sin \omega t' = \frac{1}{\sqrt{2}}$$

$$\text{or } \omega t' = \frac{\pi}{4} \quad \text{or} \quad t' = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi / T} = \frac{T}{8}$$

$$\text{or } \quad t' = \frac{1}{8f} = \frac{1}{8 \times 50} = \mathbf{2 \cdot 5 \times 10^{-3}\ s} \qquad \left(\because T = \frac{1}{f}\right)$$

4. (*a*) $I_v = \dfrac{E_v}{R} = \dfrac{110}{20} = \mathbf{5 \cdot 5\ A}$

(*b*) $I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 5 \cdot 5 = \mathbf{7 \cdot 78\ A}$

5. (*i*) $E_0 = \sqrt{2}\, E_v = \sqrt{2} \times 220 = \mathbf{311 \cdot 1\ V}$

(*ii*) $E_{av} = \dfrac{2E_0}{\pi} = \dfrac{2 \times 311 \cdot 1}{\pi} = \mathbf{198 \cdot 1\ V}$

(*iii*) $I_v = \dfrac{E_v}{R} = \dfrac{220}{100} = \mathbf{2 \cdot 2\ A}$

6. Proceed as in solved problem no. 3.04 (*a*).

7. Proceed as in solved problem no. 3.04 (*a*).

8. Proceed as in solved problem no. 3.04 .

9. $X_L = 2\pi f \times L = 2\pi \times 50 \times 1 = 100\,\pi$ H

$$\therefore \quad I_v = \frac{E_v}{X_L} = \frac{200}{100\,\pi} = 0 \cdot 637 \text{ A}$$

Hence, $I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 0 \cdot 637 = \mathbf{0 \cdot 9\ A}$

10. Here, at $f = 200$ Hz, $X_L = 22\ \Omega$

Now, $X_L = \omega L = 2\pi f \times L$ or $22 = 2\pi \times 200 \times L$

$$\text{or} \quad L = \frac{22}{2\pi \times 200} = \mathbf{1 \cdot 75 \times 10^{-2}\ H}$$

When connected to 220 V – 50 Hz supply :

$X_L' = 2\pi f' \times L = 2\pi \times 50 \times 1 \cdot 75 \times 10^{-2} = \mathbf{5 \cdot 5\ \Omega}$

11. $$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times (10^6 / 2\pi) \times 10^{-6}} = \mathbf{1\ \Omega}$$

12. Proceed as in solved problem no. 3.06.

13. Here, $E = 200 \cos 120\,\pi t$

$\therefore E_0 = 200$ V and $\omega = 120\,\pi$

$$\text{Now, } X_C = \frac{1}{\omega C} = \frac{1}{120\,\pi \times 10^{-6}} = 2652 \cdot 6\ \Omega$$

$$\therefore \quad I_0 = \frac{E_0}{X_c} = \frac{200}{2652 \cdot 6} = 0 \cdot 0754 \text{ A}$$

$$\text{Hence, } I_v = \frac{I_0}{\sqrt{2}} = \frac{0 \cdot 0754}{\sqrt{2}} = \mathbf{0 \cdot 0533\ A}$$

14. $$\tan\phi = \frac{\omega L}{R} = \frac{2\pi f \times L}{R} \quad \text{or} \quad \tan\frac{\pi}{4} = \frac{2\pi \times 50 \times L}{10}$$

$$\text{or} \quad 1 = 10\,\pi L \quad \text{or} \quad L = \frac{1}{10\,\pi} = \mathbf{0 \cdot 032\ H}$$

15. Here, $E_v = 100$ V ; $f = 50$ Hz ; $R = 20\ \Omega$,
$L = 50$ mH $= 0 \cdot 05$ H

(*i*) $$Z = \sqrt{R^2 + (2\pi f L)^2} = \sqrt{(20)^2 + (2\pi \times 50 \times 0 \cdot 05)^2}$$
$$= \sqrt{400 + 246 \cdot 74} = \mathbf{25 \cdot 43\ \Omega}$$

(*ii*) $$I_v = \frac{E_v}{Z} = \frac{100}{25 \cdot 43} = \mathbf{3 \cdot 93\ A}$$

16. Proceed as in solved problem no. 3.08.

17. Proceed as in solved problem no. 3.09 (*a*).

18. Proceed as in solved problem no. 3.09.

19. Here, $E_v = 110$ V ; $R = 40\ \Omega$ and $\tan\phi = \dfrac{3}{4}$

(*a*) $\tan\phi = \dfrac{\omega L}{R} = \dfrac{X_L}{R}$

or $X_L = R \times \tan\phi = 40 \times \dfrac{3}{4} = \mathbf{30\ \Omega}$

(*b*) $Z = \sqrt{R^2 + {X_L}^2} = \sqrt{(40)^2 + (30)^2} = \mathbf{50\ \Omega}$

(*c*) $I_v = \dfrac{E_v}{Z} = \dfrac{110}{50} = \mathbf{2{\cdot}2\ A}$

Now, $X_L = 2\pi f L$

or $f = \dfrac{X_L}{2\pi L} = \dfrac{30}{2\pi \times 0{\cdot}1} = \mathbf{47{\cdot}75\ Hz}$

20. Now, $Z = \sqrt{R^2 + (2\pi f L)^2}$

At $f = 100$ Hz, $Z = 50\ \Omega$

$\therefore$ $50 = \sqrt{R^2 + (2\pi \times 100 \times L)^2}$

or $\sqrt{R^2 + (200\pi L)^2} = 50$ ...(*i*)

At $f = 500$ Hz, $Z = 100\ \Omega$

$\therefore$ $100 = \sqrt{R^2 + (2\pi \times 500 \times L)^2}$

or $\sqrt{R^2 + (1000\pi L)^2} = 100$ ...(*ii*)

From equations (*i*) and (*ii*), it can be obtained that

$\mathbf{L = 28{\cdot}14 \times 10^{-3}\ H}$ and $\mathbf{R = 46{\cdot}74\ \Omega}$

21. $I_v = \dfrac{E_v}{\sqrt{R^2 + (2\pi f L)^2}}$ or $0{\cdot}5 = \dfrac{12}{\sqrt{R^2 + (2\pi \times 50 \times L)^2}}$

or $\sqrt{R^2 + (100\pi L)^2} = 24$ ...(*i*)

Also, $\tan\phi = \dfrac{\omega L}{R} = \dfrac{2\pi f L}{R}$ or $\tan\dfrac{\pi}{3} = \dfrac{2\pi \times 50 \times L}{R}$

or $\dfrac{100\pi L}{R} = \tan 60°$ or $\dfrac{100\pi L}{R} = \sqrt{3}$

or $100\pi L = \sqrt{3}\,R$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$\sqrt{R^2 + (\sqrt{3}\,R)^2} = 24$ or $R = 12\ \Omega$

From equation (*ii*), we have

$L = \dfrac{\sqrt{3}\,R}{100\pi} = \dfrac{\sqrt{3} \times 12}{100\pi} = \mathbf{0{\cdot}066\ H}$

22. Proceed as in problem no. 21.

23. Here, $L = 0{\cdot}02$ H ; $R = 12\ \Omega$; $E_v = 130$ V ; $f = 40$ Hz

$$I_v = \frac{E_v}{\sqrt{R^2 + (2\pi f L)^2}} = \frac{130}{\sqrt{(12)^2 + (2\pi \times 40 \times 0{\cdot}02)^2}}$$

$$= \frac{130}{13} = \mathbf{10\ A}$$

Potential difference across R, $V_R = I \times R = 10 \times 12 = \mathbf{120\ V}$

Also, $\tan\phi = \dfrac{\omega L}{R} = \dfrac{2\pi f L}{R} = \dfrac{2\pi \times 40 \times 0{\cdot}02}{12} = 0{\cdot}4189$

or $\phi = \mathbf{22°\ 44'}$

24. ***When lamp is run on 100 V d.c. :***

If R is resistance of the lamp, then

$$R = \frac{E}{I} = \frac{100}{10} = 10\ \Omega$$

When lamp is run on 220 V a.c. :

Suppose that choke coil of inductance L is connected in series to the lamp so that current of 10 A (equal to its current rating) flows through it.

Now, $I_v = \dfrac{E_v}{\sqrt{R^2 + (2\pi f L)^2}}$

or $10 = \dfrac{220}{\sqrt{(10)^2 + (2\pi \times 50 \times L)^2}}$

or $\sqrt{100 + (100\pi L)^2} = 22$ or $100\pi L = \sqrt{(22)^2 - 100}$

or $L = \dfrac{\sqrt{(22)^2 - 100}}{100\pi} = \mathbf{0{\cdot}0624\ H}$

25. Proceed as in problem no. 24.

26. Here, $R = \dfrac{V^2}{P} = \dfrac{(80)^2}{800} = 8\ \Omega$

Now proceed as in problem no. 24.

27. Proceed as in solved problem no. 3.12.

28. Proceed as in solved problem no. 3.13.

29. Here, $I_v = 1{\cdot}5$ mA $= 1{\cdot}5 \times 10^{-3}$ A ; $\omega = 300$ rad s^{-1},

$R = 10\ \text{k}\Omega = 10^4\ \Omega$ and $C = 0{\cdot}5\ \mu F = 0{\cdot}5 \times 10^{-6}$ F

Now, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{300 \times 0{\cdot}5 \times 10^{-6}} = 6{\cdot}67 \times 10^3\ \Omega$

Therefore, r.m.s. voltage across the capacitor,

$V_C = I_v \times X_C = 1{\cdot}5 \times 10^{-3} \times 6{\cdot}67 \times 10^3 = \mathbf{10\ V}$

Also, $Z = \sqrt{R^2 + {X_C}^2} = \sqrt{(10^4)^2 + (6{\cdot}67 \times 10^3)^2}$

$= \mathbf{1{\cdot}2 \times 10^4\ \Omega}$

30. (*a*) When device X is connected to a.c. supply, the current and voltage are in phase with each other. Therefore, the device X is **resistor.** Its resistance is given by

$$R = \frac{220}{0{\cdot}5} = 440\ \Omega$$

When device Y is connected to a.c. supply, the current leads e.m.f. by phase angle $\pi/2$. Therefore, the device Y is **capacitor.** Its reactance is given by

$$X_C = \frac{220}{0{\cdot}5} = 440\ \Omega$$

(*b*) Now, $Z = \sqrt{R^2 + {X_C}^2} = \sqrt{(440)^2 + (440)^2} = 440\sqrt{2}\ \Omega$

$\therefore$ $I_v = \dfrac{E_v}{Z} = \dfrac{220}{440\sqrt{2}} = \mathbf{0{\cdot}354\ A}$

31. Here, $R = 50\ \Omega$; $L = \dfrac{20}{\pi}$ H ; $C = \dfrac{5}{\pi}\ \mu F = \dfrac{5}{\pi} \times 10^{-6}$ F ;

$E_v = 230$ V and $f = 50$ Hz

Now, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Here, $X_L = 2\pi f L = 2\pi \times 50 \times \dfrac{20}{\pi} = 2000\ \Omega$

and $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times (5/\pi \times 10^{-6})} = 2000\ \Omega$

$\therefore$ $Z = \sqrt{(50)^2 + (2000 - 2000)^2} = \mathbf{50\ \Omega}$

32. Here, $R = 12\ \Omega$; $X_C = 14\ \Omega$, $L = 0{\cdot}1$ H, $E_v = 220$ V ; $f = 50$ Hz

(*a*) Now, $X_L = 2\pi f L$

$= 2 \times 3 \times 50 \times 0{\cdot}1 = 30\ \Omega$ (Take $\pi = 3$)

$\therefore$ $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12)^2 + (30 - 14)^2} = 20\ \Omega$

Hence, $I_v = \dfrac{E_v}{Z} = \dfrac{220}{20} = \mathbf{11\ A}$

(*b*) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{30 - 14}{12} = 1{\cdot}3333$

or $\phi = \mathbf{53{\cdot}13°}$

33. Here, $L = 0{\cdot}12$ H ; $C = 0{\cdot}48$ mF $= 0{\cdot}48 \times 10^{-3}$ F,
$R = 25\ \Omega$, $E_v = 220$ V

Now, $I_v = \dfrac{E_v}{\sqrt{R^2 + (X_L - X_C)^2}}$

The current in the circuit will be maximum, when impedance of the circuit is minimum

i.e. $X_L = X_C$ or $2\pi f L = \dfrac{1}{2\pi f C}$

or $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0{\cdot}12 \times 0{\cdot}48 \times 10^{-3}}} = \mathbf{21\ Hz}$

34. Proceed as in problem no. 33.

35. Here, $C = 100\ \mu F = 10^{-4}$ F ; $f = 50$ Hz

The current in LCR-circuit is in phase with the voltage,

when $X_L = X_C$ or $2\pi f L = \dfrac{1}{2\pi f C}$

or $L = \dfrac{1}{4\pi^2 f^2 C} = \dfrac{1}{4\pi^2 \times (50)^2 \times 10^{-4}} = \mathbf{0{\cdot}1\ H}$

36. Proceed as in problem no. 35

37. Now, $C = \dfrac{1}{4\pi^2 f^2 L} = \dfrac{1}{4\pi^2 \times (50)^2 \times 50 \times 10^{-3}}$

$= 2{\cdot}0264 \times 10^{-4}$ F $= \mathbf{202{\cdot}64\ \mu F}$

Also, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Since $X_L = X_C$, $Z = R = 5\ \Omega$

38. $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 1000 \times 10 \times 10^{-6}} = \mathbf{15{\cdot}92\ \Omega}$

For finding the value of L, proceed as in problem no. 35.

39. Here, $L = 100$ mH $= 0{\cdot}1$ H ; $C = 25\ \mu F = 25 \times 10^{-6}$ F ;
$R = 15\ \Omega$ and $E_v = 120$ V

(*i*) At resonance, $Z = R = \mathbf{15\ \Omega}$

(*ii*) At resonance, $I_v = \dfrac{E_v}{Z} = \dfrac{120}{15} = \mathbf{8\ A}$

(*iii*) Resonant frequency,

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0{\cdot}1 \times 25 \times 10^{-6}}} = \mathbf{100{\cdot}66\ Hz}$$

40. Here, $E_v = 230$ V ; $L = 5{\cdot}0$ H ; $C = 80\ \mu F = 80 \times 10^{-6}$ F ;
$R = 40\ \Omega$

(*a*) For resonance in circuit,

$X_L = X_C$ or $\omega L = \dfrac{1}{\omega C}$ or $\omega = \dfrac{1}{\sqrt{LC}}$

or $\omega = \dfrac{1}{\sqrt{5{\cdot}0 \times 80 \times 10^{-6}}} = \mathbf{50\ rad\ s^{-1}}$

(*b*) Current at resonance, $I_v = \dfrac{E_v}{R} = \dfrac{230}{40} = \mathbf{5{\cdot}75\ A}$

(*c*) $V_L = I_v \times \omega L = 5{\cdot}75 \times 50 \times 5 = \mathbf{1437{\cdot}5\ V}$

41. Here, $R = 2\ \Omega$; $L = 0{\cdot}01$ H ; $E_v = 200$ V ; $f = 50$ Hz

(*i*) At resonance, $X_L = X_C$

or $C = \dfrac{1}{4\pi^2 f^2 L} = \dfrac{1}{4 \times (3)^2 \times (50)^2 \times 0{\cdot}01}$ (Take $\pi = 3$)

$= 1{\cdot}11 \times 10^{-3}$ F $= \mathbf{1{\cdot}11\ mF}$

(*ii*) At resonance, current through capacitor (or circuit),

$I_v = \dfrac{E_v}{Z} = \dfrac{200}{2} = \mathbf{100\ A}$

(*iii*) At resonance, voltage across capacitor,

$V_L = I_v \times X_C = I_v \times \dfrac{1}{2\pi f C}$

$= \dfrac{100}{2 \times 3 \times 50 \times 1{\cdot}11 \times 10^{-3}}$ ...... (Take $\pi = 3$)

$= \mathbf{300{\cdot}3\ V}$

42. Here, $R = 100\ \Omega$, $E_v = 220$ V ; $f = 50$ Hz

(*i*) $E_v = \sqrt{2}\,E_v = \sqrt{2} \times 200 = \mathbf{311{\cdot}1\ V}$

(*ii*) $P_{av} = E_v\, I_v \cos\phi = 220 \times \dfrac{220}{100} \times \cos 0° = \mathbf{484\ W}$

(*iii*) Peak power $= E_0\, I_0 = 311{\cdot}1 \times \dfrac{311{\cdot}1}{100} = \mathbf{967{\cdot}8\ W}$

(*iv*) Energy delivered in 10 minutes,

$W = P_{av} \times t = 484 \times 10 \times 60 = \mathbf{2{\cdot}904 \times 10^5\ J}$

43. Here, $P_{av} = 900$ W and $E = 200 \cos(314\, t + 60°)$

$\therefore E_0 = 200$ V, $\omega = 314$ rad s^{-1} and $\phi = 60°$

Now, $P_{av} = E_v\, I_v \cos\phi = \dfrac{E_0}{\sqrt{2}} \times I_v \cos\phi$

or $900 = \dfrac{200}{\sqrt{2}} I_v \cos 60°$ or $I_v = \dfrac{900\sqrt{2}}{200 \times \cos 60°} = \mathbf{12{\cdot}73\ A}$

44. Here, $R = 10\ \Omega$; $L = 800$ mH $= 0{\cdot}8$ H

and $E = 200 \sin 300\, t$

$\therefore E_0 = 200$ V and $\omega = 300$ rad s^{-1}

(*i*) Now, $X_L = \omega L = 300 \times 0{\cdot}8 = 240\ \Omega$

$\therefore Z = \sqrt{R^2 + X_L{}^2} = \sqrt{(10)^2 + (240)^2} = \mathbf{240{\cdot}2\ \Omega}$

(*ii*) $I_0 = \dfrac{E_0}{Z} = \dfrac{200}{240{\cdot}2} = \mathbf{0{\cdot}833\ A}$

(*iii*) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{10}{240{\cdot}2} = \mathbf{0{\cdot}042}$

45. To find L, proceed as in problem no. 24. It will be found that resistance of the lamp, $R = 10\ \Omega$ and $L = 0{\cdot}049$ H. Now, power factor,

$$\cos\phi = \frac{R}{\sqrt{R^2 + (2\pi f L)^2}} = \frac{10}{\sqrt{(10)^2 + (2\pi \times 50 \times 0{\cdot}049)^2}}$$

$= \mathbf{0{\cdot}0545}$

46. Here, $\cos\phi = 0{\cdot}5$, $R = 10\ \Omega$; $E_v = 80$ V ; $f = 100$ Hz

Now, $\cos\phi = \dfrac{R}{\sqrt{R^2 + X_C{}^2}}$ or $0{\cdot}5 = \dfrac{10}{\sqrt{(10)^2 + X_C{}^2}}$

or $\sqrt{100 + X_C{}^2} = 20$ or $X_C = \sqrt{300}$

or $\dfrac{1}{2\pi f C} = \sqrt{300}$

or $C = \dfrac{1}{2\pi f \times \sqrt{300}} = \dfrac{1}{2\pi \times 100 \times \sqrt{300}} = \mathbf{9{\cdot}19 \times 10^{-5}\ F}$

47. Here, $L = 0{\cdot}5$ H ; $C = 15\ \mu F = 15 \times 10^{-6}$ F ; $R = 100\ \Omega$,
$E_v = 200$ V ; $f = 50$ Hz

(*i*) $I_v = \dfrac{E_v}{\sqrt{R^2 + (X_L - X_C)^2}}$

Now, $X_L = 2\pi f L = 2\pi \times 50 \times 0{\cdot}5 = 1571\ \Omega$

and $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times 15 \times 10^{-6}} = 212{\cdot}21\ \Omega$

$\therefore I_v = \dfrac{200}{\sqrt{(100)^2 + (157{\cdot}1 - 212{\cdot}21)^2}} = \mathbf{1{\cdot}752\ A}$

(*ii*) Power factor,

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{100}{\sqrt{(100)^2 + (157{\cdot}1 - 212{\cdot}21)^2}}$$

$= \mathbf{0{\cdot}876}$

48. Here, $R = 12\ \Omega$; $X_C = 14\ \Omega$; $X_L = 30\ \Omega$, $E_v = 230$ V ; $f = 50$ Hz

Now, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12)^2 + (30-14)^2} = 20\ \Omega$

(a) $I_v = \dfrac{E_v}{Z} = \dfrac{230}{20} = \mathbf{11{\cdot}5\ A}$

(b) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{30-14}{12} = 1\cdot3333$

or $\qquad \phi = \mathbf{53{\cdot}13^\circ}$

(c) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{12}{20} = \mathbf{0{\cdot}6}$

49. Here, $C = 5\ \mu F = 5 \times 10^{-6}$ F ; $R = 10\ \Omega$; $f = 50$ Hz

The power factor of an a.c. circuit is unity, when the circuit is purely resistive *i.e.* $X_L = X_C$

or $L = \dfrac{1}{4\pi^2 f^2 C} = \dfrac{1}{4\pi^2 \times (50)^2 \times 5 \times 10^{-6}} = \mathbf{2{\cdot}026\ H}$

50. Here, $R = 10\ \Omega$; $L = 2$ H ; $C = 25\ \mu F = 25 \times 10^{-6}$ F ; $E_v = 200$ V

When the frequency of the a.c. supply equals the natural frequency of the circuit, $X_L = X_C$ *i.e.* $Z = R$

$\therefore\ P_{av} = E_v I_v = E_v \times \dfrac{E_v}{R} = \dfrac{200 \times 200}{10} = \mathbf{4000\ W}$

51. Here, $R = 100\ \Omega$; $L = 0{\cdot}50$ H ; $f = 10$ kHz $= 10^4$ Hz ; $E_v = 240$ V

(a) $I_v = \dfrac{E_v}{\sqrt{R^2 + (2\pi f L)^2}} = \dfrac{240}{\sqrt{(100)^2 + (2\pi \times 10^4 \times 0{\cdot}50)^2}}$

$= \dfrac{240}{31416{\cdot}1} = 7{\cdot}64 \times 10^{-3}$ A

Now, $I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 7{\cdot}64 \times 10^{-3} = \mathbf{1{\cdot}08 \times 10^{-2}\ A}$

(b) $\tan\phi = \dfrac{\omega L}{R} = \dfrac{2\pi f L}{R} = \dfrac{2\pi \times 10^4 \times 0{\cdot}50}{100} = 314{\cdot}16$

or $\qquad \phi = 89{\cdot}82^\circ \approx \mathbf{90^\circ}$

From the solved problem 3.09 (*a*), it follows that when $f = 50$ Hz, $I_v = 1{\cdot}29$ A

However, when frequency is very high ($\approx$ 10 kHz), the current drops to $1{\cdot}08 \times 10^{-2}$ A (nearly zero). Hence, at very high frequency in an a.c. circuit, an inductor nearly amounts to **an open circuit** (offers very high resistance).

For a d.c. supply, $f = 0$

$\therefore \qquad X_L = 2\pi f L = 2\pi \times 0 \times 0{\cdot}50 = 0$

Thus, in a d.c. circuit, inductor offers no resistance *i.e.* it **behaves like a conductor.**

52. Here, $R = 40\ \Omega$; $C = 100\ \mu F = 10^{-4}$ F ; $E_v = 110$ V ; $f = 12$ kHz $= 12 \times 10^3$ Hz

(a) $I_v = \dfrac{E_v}{\sqrt{R^2 + X_C^2}}$

Now, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 12 \times 10^3 \times 10^{-4}} = 0\cdot133\ \Omega$

$\therefore \qquad I_v = \dfrac{110}{\sqrt{(40)^2 + (0\cdot133)^2}} = 2\cdot75$ A

Hence, $I_0 = \sqrt{2}\, I_v = \sqrt{2} \times 2{\cdot}75 = \mathbf{3{\cdot}89\ A}$

(b) $\tan\phi = \dfrac{X_C}{R} = \dfrac{0\cdot133}{40} = 0\cdot0033$

or $\qquad \phi = 0{\cdot}2^\circ \approx \mathbf{0^\circ}$

In absence of capacitor,

current through the circuit $= \dfrac{E_v}{R} = \dfrac{110}{40} = 2\cdot75$ A

It follows that when frequency is very high ($\approx$ 12 kHz), current in the circuit is same, whether capacitor is connected in the circuit or not. In other words, at very high frequency capacitor **behaves like a conductor** in an a.c. circuit.

For d.c. supply, $f = 0$

$\therefore\ X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 0 \times 10^{-4}} = \infty$

Thus, in d.c. circuit, a capacitor amounts to **an open circuit** (offers infinite resistance).

53. Here, $V_L = 12$ V and $V_R = 16$ V

Let I_v be r.m.s. value of current in the circuit. Then,

$$X_L = \frac{V_L}{I_v} = \frac{12}{I_v} \quad \text{and} \quad R = \frac{V_R}{I_v} = \frac{16}{I_v}$$

$$\therefore \quad Z = \sqrt{R^2 + X_L^{\,2}} = \sqrt{(16/I_v)^2 + (12/I_v)^2}$$

$$= \frac{\sqrt{(16)^2 + (12)^2}}{I_v} = \frac{20}{I_v}$$

Hence, potential difference across the circuit,

$$E_v = I_v \times Z = I_v \times \frac{20}{I_v} = \mathbf{20\ V}$$

54. Here, $L = 80$ mH $= 0{\cdot}08$ H ; $C = 60\ \mu F = 60 \times 10^{-6}$ F ; $R = 15\ \Omega$; $E_v = 230$ V ; $f = 50$ Hz

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Now, $X_L = 2\pi f L = 2\pi \times 50 \times 0{\cdot}08 = 25{\cdot}13\ \Omega$

and $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times 60 \times 10^{-6}} = 53{\cdot}05\ \Omega$

$$\therefore \quad Z = \sqrt{(15)^2 + (25\cdot13 - 53\cdot05)^2} = 31\cdot7\ \Omega$$

Also, $I_v = \dfrac{E_v}{Z} = \dfrac{230}{31\cdot7} = 7\cdot26$ A

Now, average power transferred to R,

$$P_{av}(R) = V_R \times I_v \cos 0^\circ = (I_v R) \times I_v \times 1 = I_v^{\,2} R$$

$$= (7{\cdot}26)^2 \times 15 = \mathbf{790{\cdot}6\ W}$$

Average power transferred to L,

$$P_{av}(L) = V_L \times I_v \cos \pi/2 = \mathbf{0}$$

Average power transferred to C,

$$P_{av}(C) = V_C \times I_v \cos(-\pi/2) = \mathbf{0}$$

Total power absorbed by the circuit,

$$P_{av}\text{(total)} = 790{\cdot}6 + 0 + 0 = \mathbf{790{\cdot}6\ W}$$

Aliter. Total power absorbed by the circuit,

$$P_{av}\text{(total)} = E_v I_v \cos\phi = E_v I_v \times \frac{R}{Z}$$

$$= \frac{230 \times 7\cdot26 \times 15}{31\cdot7} = \mathbf{790{\cdot}1\ W}$$

55. Here, $L = 0{\cdot}7$ H ; $R = 220\ \Omega$; $E_v = 220$ V ; $f = 50$ Hz

Now, $X_L = 2\pi f L = 2\pi \times 50 \times 0{\cdot}7 = 220\ \Omega$

$$\therefore \quad I_v = \frac{E_v}{\sqrt{R^2 + X_L^{\,2}}} = \frac{220}{\sqrt{(220)^2 + (220)^2}} = 0\cdot707\ \text{A}$$

Also, $\tan\phi = \dfrac{X_L}{R} = \dfrac{220}{220} = 1$ $\qquad$ or $\qquad \phi = 45^\circ$

Therefore, wattless component of the current,

$$I_v \sin\phi = 0\cdot707 \sin 45^\circ = \mathbf{0{\cdot}5\ A}$$

CHAPTER
4

Electrical Machines & Device

4.01. TRANSFORMER

It is a device used for converting low alternating voltage at high current into high voltage at low current and vice-versa.

Principle. It works on the principle of mutual induction *i.e. if two coils are inductively coupled and when current or magnetic flux is changed through one of the two coils, then induced e.m.f. is produced in the other coil.*

Construction. It consists of two coils P and S wound on a soft iron core. The coils P and S are called primary coil and secondary coil respectively [Fig. 4.01]. The a.c. input is applied across the primary coil and the transformed output is obtained across the secondary coil. To minimise eddy currents, the soft iron core is laminated.

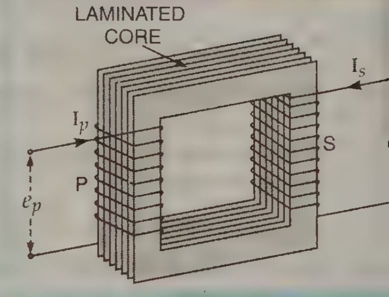


Fig. 4.01

The transformers are of following two types :

Step-up transformer. In a step-up transformer, the number of turns in secondary coil (N_s) is greater than the number of turns in primary coil (N_p) *i.e.* $N_s > N_p$. The primary coil is made of a thick insulated copper wire, while the secondary coil is made of a thin insulated wire. It converts a ***low voltage at high current*** into a ***high voltage at low current.***

Step-down transformer. The number of turns in secondary coil (N_s) of a step-down transformer is less than that in primary coil (N_p) *i.e.* $N_s < N_p$. In a step down transformer, the primary coil is made of a thin wire and the secondary coil is made of a thick wire. It converts a ***high voltage at low current*** into a ***low voltage at high current.***

Key point

A transformer cannot work on d.c.

Key point

In a step-up transformer, the increase
voltage is at the cost of current.

Theory. When an alternating e.m.f. is applied across the primary coil, the input voltage keeps on changing with time. Due to this, the magnetic flux through the primary coil also keeps on changing with time. The changing magnetic flux gets linked up with secondary coil through the laminated core, which in turn produces alternating e.m.f. across the secondary coil. The soft iron core is capable of coupling practically the whole of the magnetic flux produced in the primary coil with the secondary coil. If the magnetic field lines remain confined to the soft iron core, then all the field lines across the primary coil link up with each turn of secondary coil. Therefore, the magnetic flux linked with two coils are simply proportional to their number of turns. If ϕ_p and ϕ_s are magnetic flux linked with primary and secondary coils at any instant, then

$$\frac{\phi_s}{\phi_p} = \frac{N_s}{N_p} \qquad \text{...(4.01)}$$

or

$$\phi_s = \frac{N_s}{N_p}\,\phi_p$$

Differentiating both sides *w.r.t. t*, we have

$$\frac{d\phi_s}{dt} = \frac{d}{dt}\left(\frac{N_s}{N_p}\,\phi_p\right)$$

or

$$\frac{d\phi_s}{dt} = \frac{N_s}{N_p}\cdot\frac{d\phi_p}{dt} \qquad \text{...(4.02)}$$

According to Faraday's law of electromagnetic induction, the induced e.m.f. produced is given by

$$e = -\frac{d\phi}{dt}$$

Therefore, if e_p and e_s are the instantaneous values of induced e.m.f. produced in primary and secondary coils respectively, then

$$e_p = -\frac{d\varphi_p}{dt}$$

and $$e_s = -\frac{d\phi_s}{dt}$$

Using the above two expressions, equation (4.02) becomes

$$e_s = \frac{N_s}{N_p} = e_p \qquad \text{...(4.03)}$$

The ratio $\frac{N_s}{N_p} = k$ is called ***transformation ratio.***

For a step-up transformer, transformation ratio is greater than one, while for -down transformer, its value is less than one.

If we assume that there is no loss of energy, then

instantaneous output power = instantaneous input power

i.e. $e_s I_s = e_p I_p$

Here, I_p and I_s are respectively the values of current in primary and secondary at the instant, when the respective values of the voltage across the two coils $_p$ and e_p.

Therefore, $$\frac{e_s}{e_p} = \frac{I_p}{I_s}$$

Since in a step-up transformer, $e_s > e_p$, it follows that $I_s < I_p$. Thus, in accordance the law of conservation of energy, a step-up transformer increases the voltage ecreasing the current. Similarly, a step-down transformer decreases the voltage creasing the current. In other words, a transformer neither creates nor destroys lectric energy. It simply transforms the voltages and currents, obeying the law nservation of energy.

Need for laminated core. When a.c. input is applied across P- coil, the induced f. is produced in S-coil due to change of magnetic flux across it. In fact, magnetic changes through the soft iron core also and it produces induced e.m.f. in the core also. The induced e.m.f. developed in the iron core produces current in ore in the form of closed loops, called eddy currents. Since resistance of the core is quite small, the magnitude of eddy currents is quite large. As a result, ge amount of heat is produced and it may damage the insulation of the copper dings.

To avoid it, a laminated iron core is used in a transforer. The laminated iron is prepared by joining similar iron strips together after coating them with ish. As such, the induced e.m.f. produced in the core will cause eddy currents ch iron strip separately. Since, an iron strip is quite thin and possesses very large stance, the magnitude of eddy currents produced is quite small and hence only all amount of heat is produced.

Key point

The core of a transformer is made of soft iron. It is because, area of hysteresis loop for soft iron is small and as a result, dissipation of energy also becomes small.

4.02. ENERGY LOSSES IN ATRANSFORMER

In practice, a number of types of energy losses occur in a transformer. Due to the effeciency of a practical transformer is never 1 or 100%. The various types nergy losses, which occur in a transformer, are as given below :

(i) Flux losses. The coupling of the primary and secondary coils is never perfect whole of the magnetic flux produced in primary coil never gets linked up with secondary coil.

(ii) Copper losses. Due to resistance of the windings of primary and secondary s, some electrical energy is always converted into heat energy.

(iii) Iron losses. The varying magnetic flux produces eddy currents in the iron , which also leads to the wastage of energy in the form of heat. It is minimised using a laminated iron core.

(iv) Hysteresis losses. The alternating current flowing through the coils gnetises and demagnetises the iron core again and again over complete cycles. ring each cycle of magnetisation, some energy is lost due to hysteresis, the energy during a cycle of magnetisation being equal to area of the hysteresis loop (in gnitude). However, the loss of energy can be minimised by selecting the material ore, which has a narrow hysteresis loop.

(v) Humming losses. Due to the passage of alternating current, the core of the nsformer starts vibrating and produces humming sound. Thus, some part (may be very small) of the electrical energy is wasted in the form of humming sounds produced by the vibrating core of the transformer.

On account of the energy losses given above, the output power of a transformer is always less than the input power *i.e.*

$$e_s I_s < e_p I_p$$

The efficiency of the transformer is given by

$$\eta = \frac{e_s I_s}{e_p I_p} \qquad \text{...(4.05)}$$

Hence, in practice, the efficiency of a transformer is never 100%.

4.03. USE OF TRANSFORMER FOR LONG DISTANCE TRANSMISSION OF ELECTRIC ENERGY

When electrical energy has to be transmitted to a distant place, the resistance of line wires is considerable. Due to this, a good deal of electric power I^2R is dissipated as heat, where I is current through line wires and R is resistance. Also, a sufficient fall of potential I R occurs along the line wires. Hence, the effective voltage at the distant place will be much smaller than the actual value of the voltage supplied by the transmitting station. In addition to it, a few more difficulties are encountered, if transmission of electrical energy is done at low voltage (220 V). However, by the transmission of electrical energy at high voltage (say 22000 V), the difficulties encountered in the power transmission can be minimised. To understand the role of two factors which are important in power transmission, consider that 11000 watt of electric power is transmitted first at 220 V and then at 22000 V.

1. Loss of electric energy. When transmission of electric energy is done at 220 V, a current of $\frac{11000}{220}$ *i.e.* 50 A flows through the line wires. If R is resistance of line wires, energy equal to 50^2 R *i.e.* 2500 R joule will be dissipated per second as heat energy. On the other hand, when transmission is done at 22000 V, a current of $\frac{11000}{22000}$ *i.e.* 0·5 A only flows through the line wires. In this case, the electric energy dissipated per second as heat will be $0{\cdot}5^2$ R *i.e.* 0·25 R joule per second only.

Key point

The transmission of a.c. is done at hig voltage. As such, current through li wires and hence energy wasted in the for of heat across the wires becomes low.

Therefore, we conclude that *if transmission of electric energy is done at high voltage, the dissipation of energy is much reduced.*

2. Cost of transmission. If transmission of electric energy is done at 220 V, then as discussed above, to transmit electric power of 11000 W, the current capacity of line wires has to be 50 A and if transmission is done at 22000 V, the current capacity of line wires has to be only 0·5 A. Therefore, if transmission is done at low voltage, thick wires have to be used. Due to the use of thick line wires, the cost of transmission will increase. Further, stronger poles would be needed in order to support thicker line wires. It will further add to the cost of transmission. On the other hand, if the transmission is done at high voltage, the line wires required are of low current capacity *i.e.* thin wires and light poles may be used. Due to this, the cost of transmission will be low.

Therefore, it may be concluded that *if transmission is done at high voltage, the transmission is much economical from the point of view of the cost of line wires and poles.*

Keeping in view the above two factors, transmission of electric energy is done at high voltage and for this a step-up transformer is used. No doubt, there will be some loss of energy in transformer also, but the energy loss in a transformer is negligible as compared to the energy loss along the long line wires between two stations.

4.04. OTHER USES OF A TRANSFORMER

1. A step-down transformer is used for obtaining large current for electric welding.

2. A step-down transformer is used in induction furnace for melting the metals.

3. A step-up transformer is used for the production of X-rays.

4. Transformers are used in voltage regulators and stabilised power supplies.

5. Small transformers are used in radio sets, televisions, telephones, loud speakers, etc.

4.05. FLEMING'S RIGHT HAND RULE

We know that when a conductor is moved inside the magnetic field in a direction perpendicular to the direction of magnetic field, induced e.m.f. is produced across its two ends. The direction of flow of current can be found by applying Lenz's rule. The direction of flow of the induced current can also be found by applying Fleming's right hand rule, when the direction of motion of conductor inside the magnetic field and the direction of magnetic field acting on it are known.

Fleming's right hand rule. *It states that if the thumb, fore finger and the central finger of the right hand are kept perpendicular to each other [Fig. 4.02], so that the fore finger points in the direction of the field and the thumb in the direction of motion of the conductor, then the induced current flows in the direction of the central finger.*

It may be pointed out that in this rule, the word 'current' refers to ***conventional current.***

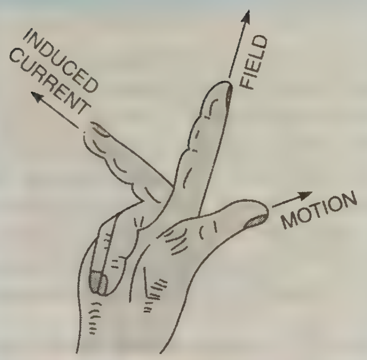


Fig. 4.02

Watch Out !

The word 'generator' is a misnomer. It generates nothing. It simply converts mechanical energy into electrical energy.

4.06. ALTERNATING CURRENT GENERATOR (A.C. DYNAMO)

An electric generator is a device used to convert mechanical energy into electrical energy.

Principle. It is based on the principle of the electromagnetic induction. *When a coil is rotated about an axis perpendicular to the direction of uniform magnetic field, an induced e.m.f. is produced across it.*

Construction. The a.c. generator consists of the following parts :

1. *Armature.* A rectangular coil ABCD consisting of a large number of turns of copper wire wound over a soft iron core is called the armature. The soft iron core is used to increase the magnetic flux.

2. *Field magnet.* It is usually a strong permanent magnet having concave poles. The armature is rotated between the two poles of a magnet, so that axis of the armature is perpendicular to magnetic field lines [Fig. 4.03].

3. *Slip rings.* The leads from the arms of the armature are connected to the two rings R_1 and R_2 separately. These rings help to provide movable contact and for this reason, they are called slip rings. As the armature and hence the leads rotate, the rings R_1 and R_2 also rotate about the central axis.

4. *Brushes.* The flexible metallic pieces B_1 and B_2, called brushes, are used to pass on the current from armature to the slip rings across which the external load resistance R is connected. As the slip rings rotate, the brushes provide movable contact by keeping themselves pressed against the rings.

Working. The working of the a.c. generator is illustrated with the help of five different positions of the armature ABCD at times $t = 0$, T/4, T/2, 3T/4 and T respectively [Fig. 4.04 (*a*)].

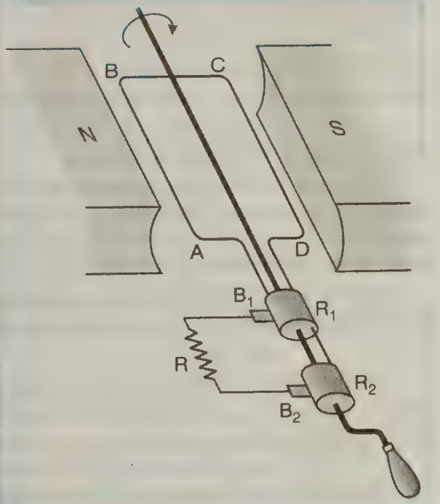


Fig. 4.03

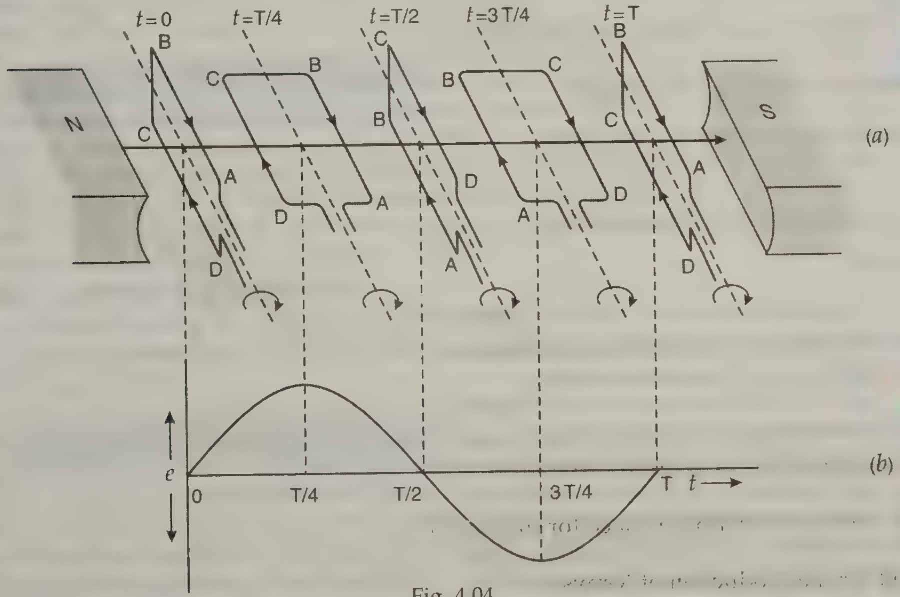


Fig. 4.04

Direction of flow of current. Initially at $t = 0$, the armature ABCD is vertical with arm AB up and CD down. During the motion of armature between $t = 0$

Key point

The direction of flow of induced current in the coil a generator is found by applying Fleming's right hand rule. It may be pointed out that Fleming's left hand rule does not apply to induced currents. It is used to find the direction of force on a current carrying conductor.

to $t = T/2$, the arm AB moves down and CD moves up. The application of Fleming's right hand rule tells that the current in the armature will flow in the direction DCBA (from the end B to A in arm AB and the end D to C in arm CD). On the other hand, during the motion of armature between $t = T/2$ to $t = T$, the motion of the two arms is just opposite to that what happens during $t = 0$ to $t = T/2$, *i.e.* the arm AB moves up and CD moves down. Therefore, during this interval, the current in the armature will flow in the direction ABCD.

Magnitude of induced e.m.f. Whenever the armature is vertical, its arms AB and CD momentarily move parallel to the field *i.e.* the rate of change of magnetic flux through the armature becomes zero. Accordingly, the induced e.m.f. $e\left(=-\dfrac{d\phi}{dt}\right)$ corresponding to the vertical positions of the armature is also zero $\left(\text{as } \dfrac{d\phi}{dt}=0\right)$ *i.e.* at time $t = 0$, T/2 and T, the induced e.m.f. produced is zero. Whenever the armature is horizontal, the arms AB and CD move normally to the direction of magnetic field lines and hence they cut or intercept the magnetic field lines with maximum speed*. Hence, the rate of change of magnetic flux is fastest or maximum corresponding to horizontal positions. Thus, at times $t = T/4$ and $3\,T/4$, the induced e.m.f. produced is maximum.

Key point

1. When a coil is rotated in magnetic field, induced e.m.f. produced is maximum when the magnetic field lines are perpendicular to its plane. It is because, near this position, the rate of intercepting the field lines is maximum.

2. The induced e.m.f. produced is zero (minimum), when the magnetic field lines are parallel to the plane of coil. It is because, near this position, the rate of intercepting the field lines is practically zero.

Therefore, output e.m.f. across the load resistance R during a complete rotation of the armature will vary as shown in Fig. 4.04 (*b*). The expression for instantaneous e.m.f. developed across R is given by

$$e = e_p \sin \omega t,$$

which can be deduced as below :

Expression for instantaneous e.m.f. produced. Fig. 4.05 shows the armature of the a.c. generator consisting of n turns and placed in uniform magnetic field B, such that its axis of rotation is perpendicular to the direction of the field. Initially, at $t = 0$, the coil is vertical with arm AB up and CD down.

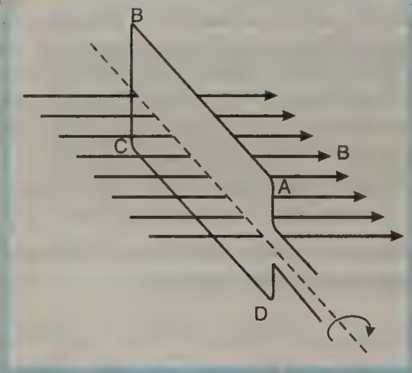


Fig. 4.05

Fig. 4.06 shows the position of the coil at any time t *i.e.* its plane makes an angle θ with the vertical. If ω is uniform angular speed of the coil, then

$$\theta = \omega t$$

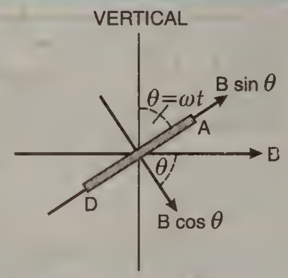


Fig. 4.06

In the position of the coil as shown in Fig. 4.06, the component of the field normal to the plane of the coil is B cos θ. If A is area of the coil and n, the number of turns in the coil, then magnetic flux linked with the coil in this position is given by

$$\phi = n\,B\cos\theta\,(A) = n\,B\,A\cos\theta = n\,B\,A\cos\omega t \qquad \text{...(4.06)}$$

The effect of number of turns n in the coil is to increase the magnetic flux linked with the coil to n times.

Differentiating both sides of the equation (4.06) *w.r.t.* time, we get

$$\frac{d\phi}{dt} = \frac{d}{dt}(n\,B\,A\cos\omega t) = n\,B\,A\frac{d}{dt}(\cos\omega t) = n\,B\,A\,(-\sin\omega t)\,(\omega)$$

$$= n\,B\,A\,\omega\sin\omega t$$

If e is the instantaneous induced e.m.f. produced at this instant, then

$$e = -\frac{d\phi}{dt} = -(-n\,B\,A\,\omega\sin\omega t)$$

or

$$e = n\,B\,A\,\omega\sin\omega t \qquad \text{...(4.07)}$$

The equation (4.07) gives the instantaneous induced e.m.f. in the coil. It follows that $n\,B\,A\,\omega$ is the maximum value of the e.m.f., say e_0. Therefore, we have

$$e = e_0 \sin \omega t \qquad \text{...(4.08)}$$

Thus, the induced e.m.f. produced is a function of time and therefore as the coil passes through different positions with the passage of time, e.m.f. produced changes with time as given by equation (4.08). The e.m.f. would be zero for $\omega t = 0$ (when coil is vertical) ; positive maximum for $\omega t = \pi/2$ (when the coil is horizontal), zero for $\omega t = \pi$ (when coil is vertical) ; negative maximum for $\omega t = 3\,\pi/2$ (when the coil is horizontal) and again zero for $\omega t = \pi/2$ (when the coil is vertical and returns to its initial position). Thus, graphically, e.m.f. will vary with time as shown in Fig. 4.04 (*b*).

---

*It may be pointed out that speed of rotation of the armature is uniform. But when armature is horizontal, its speed of cutting the magnetic field lines is maximum.

4.07. D.C. GENERATOR

It is a device, which converts mechanical energy into electrical energy in the form of irectional e.m.f. or current, called the direct current.

Principle. It is based on the principle of electromagnetic induction.

When a coil is rotated about an axis perpendicular to the direction of uniform magnetic , an induced e.m.f. is produced across it.

Construction. Fig. 4.07 shows the essential parts of a d.c. generator. In principle, construction of d.c. dynamo is the same as that of a.c. dynamo. The main erence is that the slip rings are replaced by the arrangement known as ***split ring mutators*** in order to obtain a unidirectional continuous current through the out- load resistance.

The split rings are two halves R_1 and R_2 of the same metallic ring insulated from h other as well from the axle of the dynamo. Each of the two rings is connected he either end of the armature coil. During the rotation of the armature, the leads m arms AB and CD of the armature remain connected to the rings R_1 and R_2 pectively. The rings are so arranged that during a half revolution of the coil, each g remains in contact with a particular brush but during next half revolution, the gs interchange their contacts and establish contact with the other brushes. By this angement, the current in the outer circuit remains in the same direction.

Working. To start with, suppose that ring $R_1$is in contact with brush B_1 and g R_2 is in contact with brush B_2. If the coil rotates in clockwise direction, then ring the first half of revolution, the direction of current in the armature coil will ABCD (by applying Fleming's right hand rule) *i.e.* in the outer load resistance it will enter from the brush B_2 and flow towards B_1as shown in Fig. 4.07. After f the revolution, R_1 establishes contact with brush B_2, while R_2 comes in contact th B_1. The direction of current in the armature coil is now DCBA and in the outer d resistance R, the current again flows from B_2 to B_1 *i.e.* in the same direction, in ich it was flowing during first half of the revolution [Fig. 4.08]. The strength of current is, however, not constant and during each half cycle, it varies from zero maximum and maximum to zero in the same direction [Fig. 4.09]. Such a current alled pulsating direct current. Thus, the current produced is unidirectional, but ctuates between maximum and zero values. As such, it is inconvenient for actical purposes.

To overcome this difficulty, we take two coils whose planes are inclined to each ner at an angle of 90° and divide the ring mounted on the same spindle into four rts. The leads of each coil are connected to the two diametrically opposite parts the split ring. In such a case, either of the coils will give the current curve of the me type but with the difference that when the current in one reaches a maximum lue, the current curve given by the other has zero value. However, the brushes w receive the current from one set of commutator segments only for quarter riod and that also when the value of the current is comparatively large. So, the sultant e.m.f. in the output circuit is less varying as shown by the thick curve in g. 4.10. Similarly, if we use four coils inclined at an angle of 45° to each other and vide the ring into eight parts, we get still less fluctuating current. It is, thus, clear at by taking a large number of coils with double the number of commutator ring gments, the final current curve will be practically parallel to the time axis and, erefore, will be constant in strength.

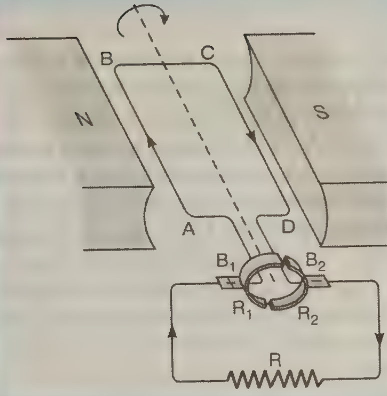


Fig. 4.07

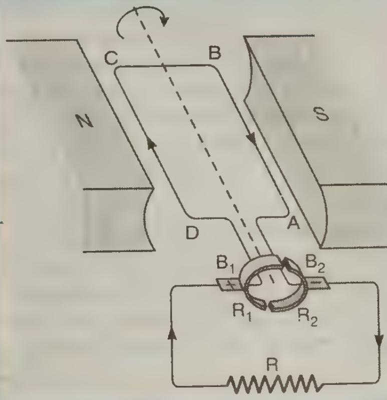


Fig. 4.08

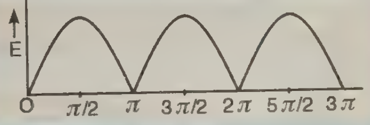


Fig. 4.09

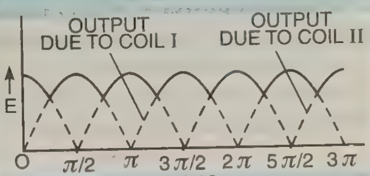


Fig. 4.10

4.08. SIMPLE D.C. MOTOR

An electric motor is a device for converting electrical energy into mechanical energy.

Principle. It is based on the fact that a current carrying coil experiences a torque a magnetic field.

Construction. A simple d.c. motor consists of an armature ABCD (a coil of a rge number of turns) wound on an iron core. It is placed in the magnetic field roduced due to the flow of current through the field windings [Fig. 4.11]. The ends f the armature are connected to two split rings R_1 and R_2 (two halves of a single ng). The two split rings alternately come in contact with carbon brushes, B_1 and $_2$ during the rotation of the armature. A battery is connected across the two brushes, o as to send d.c. through the armature.

Working. Suppose that initially the armature is horizontal, such that the split ing R_1 connected to arm AB is in contact with brush B_1, while the split ring R_2 onnected to arm CD is in contact with brush B_2. As the positive pole of the battery

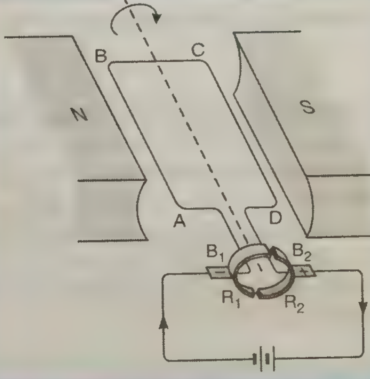


Fig. 4.11

is connected to brush B_2, the conventional current flows through the armature in the direction DCBA. According to Fleming's left hand rule, an upward force equal to $nBIl$ acts on arm AB, while an equal downward force $nBIl$ acts on arm CD. The two equal and opposite forces acting on the arms AB and CD of the armature constitute a torque and as a result of it, the armature starts rotating in clockwise direction. As such, during rotation of the coil, the perpendicular distance between the forces on the two arms goes on decreasing and becomes zero, when the armature becomes vertical and as a result of it, the armature tends to come to rest on reaching the vertical position. However, due to inertia, the armature is carried past the vertical position. As it happens, the split ring R_1 comes in contact with brush B_2, while the split ring R_2 assumes contact with brush B_1. Due to this, the direction of the flow of current through the armature gets reversed *i.e.* the current starts flowing in the direction ABCD. Since the directions of motion of arms AB and CD also get reversed (earlier arm AB was moving up and CD down, but now arm AB moves down, while CD moves up), the direction of rotation of the armature remains unchanged leading to an uninterrupted rotatory motion of the armature in one direction.

Back e.m.f. The armature of a motor continuously rotates inside the magnetic field. As the magnetic flux linked with the armature changes during its rotation, an induced e.m.f., say e is produced in the armature. According to Lenz's law, this induced e.m.f. opposes the applied battery e.m.f. and it is called ***back e.m.f.*** If E is the applied battery e.m.f. and R is the resistance of the armature, then current through the armature is given by

$$I = \frac{\text{effective e.m.f.}}{\text{resistance of the armature}} = \frac{E-e}{R} \qquad \text{...(4.09)}$$

It may be pointed out that when electric motor is just switched on, the magnitude of back e.m.f. is quite small due to the low speed of the motor. As such, the battery e.m.f. sends current through the armature, which may be large enough to burn the motor by breaking the insulation of the coils. To avoid damage to the motor on this account, a ***motor starter*** is used in series with the motor.

The motor starter also saves the motor from damage, if the electric supply fails by chance, while the motor is running. It is because, the magnitude of back e.m.f. is very large at break.

Efficiency of an electric motor. The efficiency of a motor decreases both due to the production of back e.m.f. and the production of heat in the armature coil.

The efficiency of an electric motor is defined as the ratio of the output power to the input power.

Thus, $$\eta = \frac{\text{output power}}{\text{input power}} \qquad \text{...(4.10)}$$

Now, input power = battery e.m.f. × current through armature = E I,
where I is current through the armature and is given by the equation (4.09).

Heat produced per second in the armature = $I^2 R$

Therefore, output power = $EI - I^2R = (E - IR) \times I$

From equation (4.09), we have

$$E - IR = e$$

Substituting for (E – I R) in above equation, we have

output power = eI

Substituting for output power and input power in equation (4.10), we have

$$\eta = \frac{eI}{EI} = \frac{e}{E} \qquad \text{...(4.11)}$$

Thus, *efficiency of a motor is equal to the ratio of the back e.m.f. produced to the applied battery e.m.f.*

Condition for maximum efficiency. From equation (4.11), it follows that efficiency of a motor will be 1 (maximum), if back e.m.f. setup in the armature is equal to applied battery e.m.f. In that event, the current through the armature will be zero and the motor will not run at all. Therefore, condition for maximum efficiency of the motor is obtained by finding the condition, so that output power is maximum.

Now, output power = $eI = \dfrac{e(E-e)}{R}$

he output power varies with back e.m.f. Therefore, output power will be ıum, if

$$\frac{d}{de}\left[\frac{e(E-e)}{R}\right]=0 \text{ or } \frac{d}{de}[eE-e^2]=0 \text{ or } E-2e=0$$

r

$$e=\frac{E}{2}$$

hus, *an electric motor will be maximum efficient i.e. the motor will deliver maximum power, when the back e.m.f. set up in the armature is half the value of the applied e.m.f.*

Vhen the motor is delivering maximum output power, then

$$\eta=\frac{E/2}{E}=\frac{1}{2} \text{ (or 50\%)}$$

Thus, *while delivering maximum output power, the motor has an efficiency of 50%.*

.09 MOTOR STARTER

motor starter is basically a variable resistor connected in series with the motor.

Vhen a motor is switched on and it has not attained its full speed, the back (*e*) produced will be very small. The current flowing through the armature is by

$$I=\frac{E-e}{R},$$

e R is the resistance of the armature and E is the e.m.f. applied to the motor. esistance of armature is usually kept small, so as to reduce copper losses. fore, due to low resistance of the armature and small back e.m.f. produced at art of the motor, current through the armature will be quite large, when the r is just switched on. Due to this, motor sometimes gets burnt due to breaking insulation of the coils. In order to save the motor from getting damaged on ccount, a motor starter is used.

Key point

When motor is just switched on, back e.m.f. produced in the armature is practically zero and hence a large current flows through it, which may cause damage to its windings. The motor starter introduces a resistance in the circuit, so as to keep the current low, when the motor is just switched on. When motor attains its full speed, it withdraws the resistance. Then, current is maintained at safe value by the back e.m.f.

Fig. 4.12 shows the essential parts of a motor starter and the lay out of its series ections to the motor M. To start the motor, the handle H of the starter is brought position A. As such, the entire resistance R of the motor starter comes in series the armature of the motor and only a safe low current flows through the ture. The electromagnet E gets magnetised due to the passage of current through l it attracts the iron piece I attached to the handle. Due to this, the resistance of notor starter decreases gradually. As the motor picks up speed, the back e.m.f. ncreases gradually. The result is that the current through the armature always ins within the safe value. Ultimately, the handle reaches the position D and the er resistance gets out of the circuit. At this stage, the motor attains its full speed he current through the armature is controlled by the back e.m.f.

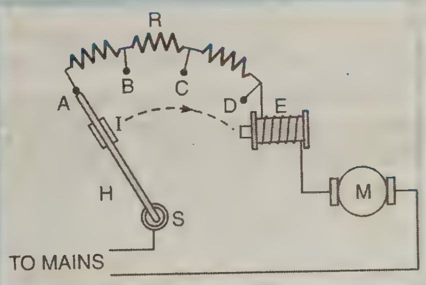


Fig. 4.12

When the motor is switched off, the electromagnet loses its magnetism and equently the handle gets released and goes to OFF-position due to spring action e spring S attached to the handle.

In case supply current fails suddenly, a large back e.m.f. will be produced at oreak and it may burn the motor. The motor also gets burnt in case the supply age increases due to certain reason. To save the motor in these situations, the or starter is designed accordingly.

4.10. CHOKE COIL

A choke coil is an electrical appliance used for controlling current in an a.c. circuit out wasting electrical energy in the form of heat.

Principle. A choke coil is based upon the principle that *when a.c. flows through nductor, the current lags behind the e.m.f. by a phase angle* $\pi/2$.

Construction. A choke coil is basically an inductance. It consists of a large nber of turns of insulated copper wire wound over a soft iron core. In order to imise loss of electrical energy due to production of eddy currents, a laminated core is used [Fig. 4.13].

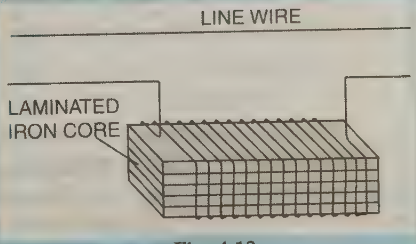


Fig. 4.13

In practice, a *low frequency choke coil* is made of insulated copper wire wound soft iron core, while a *high frequency choke coil* has air as core material.

Working. The choke coil is put in series with the electrical device, such as rescent tube requiring a low value of current. The inductive reactance decreases current. Since the alternating e.m.f. leads the current by phase angle $\pi/2$, the average power consumed by the choke coil is given by

$$P_{av} = E_v\, I_v \cos\frac{\pi}{2} = 0$$

However, a practical inductance possesses a small resistance *i.e.* a practical inductance may be treated as a series combination of inductance L and a small resistance r. Therefore, a practical inductance will consume a small average power given by

$$P_{av} = E_v\, I_v \times \frac{r}{\sqrt{r^2 + \omega^2 L^2}}$$

Here, $\frac{r}{\sqrt{r^2 + \omega^2 L^2}}$ is power factor ($\cos\phi$) for practical inductance, which is a series combination of L and r.

Uses. In a.c. circuits, a choke coil is used to control the current in place of a resistance. If a resistance is used to control the current, the electrical energy will be wasted in the form of heat. A choke coil decreases the current without wasting electrical energy in the form of heat.

SOLVED NUMERICAL PROBLEMS

Type A. On Transformer

Problem 4.01. A power transmission line feeds input power at 2300 V to a step down transformer having 4000 turns in its primary. What should be the number of turns in the secondary to get output power at 230 V ?

(*C.B.S.E. 1997*)

Sol. Here, $e_p = 2300$ V ; $N_p = 4000$; $e_s = 230$ V

Now, $$\frac{e_s}{e_p} = \frac{N_s}{N_p}$$

$$\therefore \quad N_s = N_p \times \frac{e_s}{e_p} = 4000 \times \frac{230}{2300} = \mathbf{400}$$

Problem 4.02. How much current is drawn by the primary of a transformer connected to 220 V supply, when it delivers power to a 110 V and 550 W refrigerator ?

Sol. Here, $e_p = 220$ V ; $e_s = 110$ V ; $e_s I_s = 550$ W

Now, $e_p I_p = e_s I_s$

or $$I_p = \frac{e_s I_s}{e_p} = \frac{550}{220} = \mathbf{2{\cdot}5\ A}$$

Problem 4.03. How much current is drawn by the primary coil of a transformer, which steps down 220 V to 22 V to operate a device with an impedance of 220 Ω ?

(*H.S.S.C.E. 2001*)

Sol. Here, $e_p = 220$ V ; $e_s = 22$ V

Impedance of the device connected across secondary,

$Z = 220\ \Omega$

Therefore, current in secondary coil,

$$I_s = \frac{e_s}{Z} = \frac{22}{220} = 0 \cdot 1\ \text{A}$$

If I_p is current in primary coil, then

$$e_p I_p = e_s I_s$$

or $$I_p = \frac{e_s}{e_p} \times I_s = \frac{22}{220} \times 0 \cdot 1 = \mathbf{0{\cdot}01\ A}$$

Problem 4.04. In an ideal transformer, number of turns in the primary and secondary are 200 and 1000 respectively. If the power input to the primary is 10 kW at 200 V, calculate (*a*) output voltage and (*b*) current in parimary.

(*C.B.S.E. 2002*)

Sol. Here $N_p = 200$; $N_s = 1000$; $e_p = 200$ V

Power input to primary, $e_p I_p = 10$ kW $= 10^4$ W

(*a*) Now, $$e_s = \frac{N_s}{N_p} \times e_p = \frac{1000}{200} \times 200 = \mathbf{1000\ V}$$

(*b*) $$I_p = \frac{10^4}{e_p} = \frac{10^4}{200} = \mathbf{50\ A}$$

Problem 4.05. When a voltage of 120 V is impresse across the primary of a transformer, the current in th primary is 1·85 A. Find the voltage across the secondar when it delivers 150 mA. The transformer has an efficienc of 95%.

Sol. Here, $e_p = 120$ V ; $I_p = 1 \cdot 85$ A ;

$I_s = 150$ mA $= 150 \times 10^{-3}$ A ; $\eta = 95\% = 0 \cdot 95$

Now, $$\eta = \frac{e_s I_s}{e_p I_p}$$

$$\therefore \quad e_s = \frac{\eta\, e_p I_p}{I_s} = \frac{0 \cdot 95 \times 120 \times 1 \cdot 85}{150 \times 10^{-3}} = \mathbf{1406\ V}$$

Problem 4.06. In a step-up transformer, the transfo mation ratio is 100. The primary voltage is 220 volt and inpu is 1100 watt. The number of turns in primary is 10 Calculate (*i*) the number of turns in the secondary (*ii*) th current in the primary, (*iii*) the voltage across the secondar and (*iv*) the current in the secondary.

Sol. Here, transformation ratio, $k = 100$; $N_p = 100$

(*i*) Now, $$k = \frac{N_s}{N_p}$$

$$\therefore \quad N_s = k \times N_p = 100 \times 100 = \mathbf{10{,}000}$$

(*ii*) Input power, $e_p I_p = 1100$ watt ; $e_p = 220$ volt

$$\therefore \quad I_p = \frac{1100}{e_p} = \frac{1100}{220} = \mathbf{5\ A}$$

(*iii*) Also, $$k = \frac{e_s}{e_p}$$

$$\therefore \quad e_s = k\, e_p = 100 \times 200 = \mathbf{22{,}000\ V}$$

(*iv*) Also, $$\frac{e_s}{e_p} = \frac{I_p}{I_s}$$

$$\therefore \quad I_s = \frac{e_p}{e_s} \times I_p = \frac{220}{22{,}000} \times 5 = \mathbf{0{\cdot}05\ A}$$

Problem 4.07. 11 kW of electric power can be trans- ted to a distant station at (*i*) 220 V, or (*ii*) 22,000 V. Which he two modes of transmission should be preferred and y ? Support your answer with possible calculations. (*C.B.S.E. 1998*)

Sol. Here, $e_p\,I_p = 11\text{ kW} = 11{,}000\text{ W}$

(i) When power is transmitted at $e_s = 220$ V :

If I_s is current in secondary coil, then

$$e_s\,I_s = e_p\,I_p$$

or $$I_s = \frac{11{,}000}{220} = 50\text{ A}$$

If R is resistance of transmission lines, then electrical ergy dissipated as heat I_s^2

$$= 50^2\,R = 2500\,R\text{ watt}$$

(*ii*) When power is transmitted at *e_s = 22,000 V.*

If I_s is current in secondary coil, then

$$e_s'\,I_s' = e_p'\,I_p'$$

or $$I_s' = \frac{e_p\,I_p}{e_s'} = \frac{11{,}000}{22{,}000} = 0{\cdot}5\text{ A}$$

Therefore, electrical energy dissipated as heat

$$= (I_s')^2\,R = (0{\cdot}5)^2\,R = 0{\cdot}25\,R\text{ watt}$$

Since electrical energy dissipated as heat is much less, en transmission is done at 22,000 V ; transmission should done at **22,000 V.**

pe B. On a.c. generator

Problem 4.08. A rectangular coil of dimensions 0 m × 0·5 m consisting of 2000 turns rotates about axis parallel to its long side, making 2100 volutions per minute in a field of 0·1 T. What is e maximum e.m.f. induced in the coil ? Also find e instantaneous e.m.f, when the coil is at 30° to the eld.

Sol. Here, $B = 0{\cdot}1\text{ T}\,;\ n = 2000\,;$

area of the coil, $A = 0{\cdot}10\text{ m} \times 0{\cdot}5\text{ m} = 0{\cdot}05\text{ m}^2$

angular velocity, $\omega = 2100$ r.p.m.

$$= \frac{2\,\pi \times 2100}{60} = 220\text{ rad s}^{-1}$$

Maximum e.m.f. induced in the coil,

$$e_0 = n\,B\,A\,\omega = 2100 \times 0{\cdot}1 \times 0{\cdot}05 \times 220$$
$$= \mathbf{2200\ V}$$

The e.m.f. induced in the coil, when it makes an angle *t* with the vertical is given by

$$e = e_0 \sin \omega t$$

Here, the angle, which the coil makes with the vertical,

$$\omega t = 90° - 30° = 60°$$

$$\therefore \quad e = 2200 \sin 60° = \mathbf{1905{\cdot}3\ V}$$

Problem 4.09. An a.c. generator consists of a coil of 00 turns and cross-sectional area of 3 m², rotating at a onstant angular speed of 60 rad s⁻¹ in a uniform magnetic ield 0·04 T. The resistance of the coil is 500 Ω. Calculate (*a*) naximum current drawn from the generator and (*b*) naximum power dissipation in the coil. (*C.B.S.E. 2002*)

Sol. Here, $n = 100\,;\ A = 3\text{ m}^2\,;\ \omega = 60\text{ rad s}^{-1}\,;\ B = 0{\cdot}04\text{ T}$

(*a*) Maximum e.m.f. produced in the coil,

$$e_0 = n\,B\,A\,\omega = 100 \times 0{\cdot}04 \times 3 \times 60 = 720\text{ V}$$

Since resistance of the coil is 50 Ω, the maximum current rawn from the generator,

$$I_0 = \frac{e_0}{R} = \frac{720}{500} = \mathbf{1{\cdot}44\ A}$$

(*b*) Maximum power dissipation in the coil,

$$e_0\,I_0 = 720 \times 1{\cdot}44 = 1036{\cdot}8\text{ W}$$

Problem 4.10. An a.c. generator consists of a coil of 50 turns and area 25 m² rotating at an angular speed of 60 rad s⁻¹ in a uniform magnetic field B = 0·30 T between two fixed pole pieces. The resistance of the circuit including that of the coil is 500 Ω.

(*a*) What is the maximum current drawn from the generator ?

(*b*) What is the flux through the coil, when the current is zero ? What is the flux, when the current is maximum ?

(*c*) Would the generator work, if the coil were stationary and instead of the pole pieces rotated together with the same speed as above ? (*Text Problem*)

Sol. Here, $n = 50\,;\ A = 2{\cdot}5\text{ m}^2\,;\ \omega = 60\text{ rad s}^{-1}\,;$

$$B = 0{\cdot}30\text{ T}\,;\ R = 500\ \Omega$$

(*a*) Maximum e.m.f. produced,

$$e_0 = n\,B\,A\,\omega = 50 \times 0{\cdot}30 \times 2{\cdot}5 \times 60 = 2250\text{ V}$$

Therefore, maximum current drawn from the generator,

$$I_0 = \frac{e_0}{R} = \frac{2250}{500} = 4{\cdot}5\text{ A}$$

(*b*) The current is zero, when coil is vertical. In this position, flux through the coil is maximum. On the other hand, the current is maximum, when coil is horizontal. In this position, flux through the coil is minimum.

(*c*) Yes. For generation of electricity, there should be relative motion between magnetic field and the coil.

Type C. On Motor

Problem 4.11. The back e.m.f. of a d.c. motor delivering 5 kW of mechanical power is 200 V, when operating on a 220 V line. Determine the armature current and the motor resistance. (*I.S.S.C.E. 1997*)

Sol. Here, $E = 220\text{ V}\,;\ e = 200\text{ V}\,;$

Let R be motor resistance and I be the armature current. Since mechanical output power is 5 kW *i.e.* 5000 W,

$$e\,I = 5000 \quad \text{or} \quad 200\,I = 5000 \quad \text{or} \quad \mathbf{I = 25\ A}$$

Also, $$I = \frac{E - e}{R}$$

$$\therefore \quad R = \frac{E - e}{I} = \frac{220 - 200}{25} = \frac{20}{25} = \mathbf{0{\cdot}8\ \Omega}$$

Problem 4.12. A small d.c. motor operating at 200 V draws a current of 5·0 A at its full speed of 3000 r.p.m. The resistance of the armature of the motor is 8·5 Ω. Determine the back e.m.f. of the motor. Obtain the power input, power output (mechanical) and the efficiency of the motor. (*Text Problem*)

Sol. Here, $R = 8{\cdot}5\ \Omega\,;\ E = 200\text{ V}\,;\ I = 5\text{ A}$

When the rotor of motor rotates, the back e.m.f. *e* is produced. Therefore, current drawn by motor,

$$I = \frac{E - e}{R}$$

so that $$e = E - I\,R = 200 - 5 \times 8{\cdot}5 = \mathbf{157{\cdot}5\ V}$$

Power input $= E\,I = 200 \times 5 = \mathbf{1000\ W}$

Power output $= E\,I - I^2\,R = 200 \times 5 - 5^2 \times 8{\cdot}5$
$= \mathbf{787{\cdot}5\ W}$

$$\text{Efficiency of the motor} = \frac{\text{power output}}{\text{power input}} = \frac{787{\cdot}5}{1000}$$
$$= \mathbf{0{\cdot}7875 \text{ or } 78{\cdot}75\%}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & Curious Students

Problem 4.13. At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is 100 m^3 s^{-1}. If the turbine-generator efficiency is 60%, estimate the electric power available from the plant ($g = 9{\cdot}8$ m s^2). *(Text Problem)*

Sol. Here, $h = 300$ m ; $g = 9{\cdot}8$ m s^{-2} ; $= 100$ m^3 s^{-1} ; volume of water flowing per second

Now, density of water, $\rho = 1000$ kg m^{-3} ;

Therefore, mass of water flowing per second

$$= 100 \times 1000 = 10^5 \text{ kg s}^{-1}$$

Potential energy of water made use of per second

$$= 10^5 \times 9{\cdot}8 \times 300 = 2{\cdot}94 \times 10^8 \text{ J s}^{-1} = 2{\cdot}94 \times 10^8 \text{ W}$$

Efficiency of turbine = 60%

Therefore, electric power available from the plant

$$= \frac{2{\cdot}94 \times 10^8 \times 60}{100} = 1{\cdot}764 \times 10^8 \text{ W}$$

$$= \mathbf{176{\cdot}4\ MW}$$

Problem 4.14. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two-wire line carrying power is 0·5 Ω per km. The town gets power from the line through a 4000–220 V step-down transformer at a sub-station in the town. (*a*) Estimate the line power loss in the form of heat. (*b*) How much power must the plant supply, assuming there is negligible power loss due to leakage ? (*c*) Characterize the step-up transformer at the plant. *(Text Problem)*

Sol. Here, length of wire line $= 15 \times 2 = 30$ km ;

resistance of wire line, $R = 30 \times 0{\cdot}5 = 15\ \Omega$

voltage at which power is transmitted = 4000 V ;

power transmitted, $P = 800 \text{ kW} = 8 \times 10^5$ W

Now, $P = e_s I_s$

$$\therefore \quad I_s = \frac{P}{e_s} = \frac{8 \times 10^5}{4000} = 200 \text{ A}$$

(*a*) Line power loss $I_s^2 R = (200)^2 \times 15 = 6 \times 10^5$ W

$$= \mathbf{600\ kW}$$

(*b*) Power that must be supplied by the plant

$$= 800 + 600 = \mathbf{1400\ kW}$$

(*c*) Voltage drop across wire line $= 15 \times 200 =$ **3000**

The plant generates power at 440 V and it has to stepped up, so that after a voltage drop of 3000 V across w line, the power at 4000 V is received at the sub-station in town. Therefore, the step up transformer at the plan 440 V – (4000 + 3000) V *i.e.* **440 V – 7000 V.**

Problem 4.15. A circular coil of radius 8·0 cm and turns rotates about its vertical diameter with an angu speed of 50 rad s^{-1} in a uniform horizontal magnetic fi of magnitude 3·0 × 10^{-2} T. Obtain the maximum and average e.m.f. induced in the coil. If the coil forms a clos loop of resistance 10Ω, how much power is dissipated as h ? What is the source of this power ? *(Text Proble*

Sol. Here, radius of coil, $r = 8{\cdot}0 \text{ cm} = 8 \times 10^{-2}$ m ;

number of turns, $n = 20$; $\omega = 50$ rad s^{-1} ; $B = 3{\cdot}0 \times 10^{-}$

and area of coil, $A = \pi r^2 = \pi \times (8 \times 10^{-2})^2 = 64\,\pi \times 10^{-4}$

Now, $e = n\,B\,A\,\omega \sin \omega\, t$

$$\therefore \quad e_0 = n\,B\,A\,\omega = 20 \times 3{\cdot}0 \times 10^{-2} \times 64\,\pi \times 10^{-4} \times$$

$$= \mathbf{0{\cdot}6032\ V}$$

The average value of an alternating e.m.f. is zero ove complete cycle. Therefore,

$$e_{av} = 0$$

The average power consumed by a resistor,

$$P_{av} = \frac{e_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = \frac{e_{max}\, I_{max}}{2} = \frac{e_{max}^2}{2R}$$

Here, $R = 10\ \Omega$

$$\therefore \quad P_{av} = \frac{(0 \cdot 6032)^2}{2 \times 10} = \mathbf{0{\cdot}0182\ W}$$

The external agent which keeps the coil rotating is source of this power.

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. Write the principle of a transformer. *(H.S.S.C.E. 1997 S ; P.S.S.C.E. 1992)*

Ans. Mutual induction.

Q. 4.02. Why cannot transformer work on d.c. ? *(H.S.S.C.E. 2001)*

Or

A transformer cannot be used to step up d.c. voltage. Why ? *(P.S.S.C.E. 1999 S)*

Ans. If d.c. voltage is applied across the primary coil of a transformer, then the magnetic flux linked with the coil will not vary with time and hence no induced e.m.f. will be produced in the secondary coil.

Q. 4.03. Does a step up transformer contradict the principle of conservation of energy ? *(H.S.S.C.E. 1993)*

Ans. No, it does not.

Q. 4.04. Give two reasons for power loss in a transformer.

Ans. Magnetic flux loss and hysteresis loss.

Q. 4.05. What is iron loss in a transformer and hov can be reduced ? *(P.S.S.C.E. 2000, 1999, 1998*

Ans. Due to production of eddy currents in the iron co the loss of electrical energy (in the form of heat energy known as iron loss. It is reduced by making use of a lamina iron core.

Q. 4.06. What is copper loss in a transformer ? *(P.S.S.C.E. 19*

Ans. Refer to section 4.02.

Q. 4.07. What causes the core of a transformer to heated up under operation ? *(I.S.C.E. 19*

Ans. Due to the production of eddy currents in the c of the transformer.

Q. 4.08. Why is the core of a transformer laminate Explain. *(P.S.S.C.E. 2002, 1999, 19*

Ans. The core of a transformer is laminated, so as reduce loss of energy due to eddy currents. The magnitu of the eddy currents set up is considerably minimised, wh the core is laminated.

Q. 4.09. A transformer is used to step down a.c. voltage. at appliance will you use to step down a d.c. voltage ?

Ans. A pure resistance can be used to step down d.c. tage.

Q. 4.10. Why is the core of a transformer made of a gnetic material of high permeability ? (*C.B.S.E. Sample Q. Paper*)

Ans. The transformer works on the principle of mutual duction. In case the core has a high value of magnetic rmeability, the magnetic field lines will crowd through the re and the linkage of magnetic flux between the two coils ll also be more.

Q. 4.11. State the principle on which induction coil is nstructed.

Ans. It is constructed on the principle of mutual duction.

Q. 4.12. A capacitor is used in the primary circuit of an duction coil. Why ? (*Text Question*)

Ans. At the break, a large induced e.m.f. is produced. case capacitor is not connected, sparking will take place. ut when capacitor is used, the large induced e.m.f. produced break is used up in charging the capacitor and no sparking kes place.

Q. 4.13. Name two useful devices based on the phenomenon of mutual induction.

Ans. Transformer and induction coil.

Q. 4.14. State the principle of a.c. generator. (*H.S.S.C.E. 1995*)

Ans. Refer to section 4.06.

Q. 4.15. From where does the electric energy come in a generator ?

Ans. The mechanical energy used to rotate a coil inside magnetic field is converted into electric energy.

Q. 4.16. At which position of the rotating coil in the magnetic field, the induced e.m.f. is maximum ?

Ans. The induced e.m.f. is maximum, when the plane of the coil is parallel to magnetic field lines.

Q. 4.17. Write down principle of a d.c. motor. (*P.S.S.C.E. 2000, 1999 ; H.S.S.C.E. 2001, 1998 S, 1997*)

Ans. It is based on the principle that a current carrying coil placed inside a magnetic field experiences a torque.

Q. 4.18. Is the motor starter a variable R or L or C ?

Ans. Motor starter is a variable resistor.

Q. 4.19. What is the function of a choke coil in a fluorescent tube ? (*H.S.S.C.E. 1999 S ; I.S.C.E. 1996*)

Ans. The choke coil decreases current in the circuit without wasting electrical energy in the form of heat energy.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS
With Answers/Hints

Q. 4.01. What are the factors which reduce the effi-iency of the transformer ?

Ans. Copper losses due to the heating of the primary and econdary coils ; iron losses due to the eddy currents set up n core ; hysteresis loss due to the loss of energy in nagnetising and demagnetising the iron core and the leakage f magnetic flux are the various reasons, which reduce the fficiency of the transformer.

Q. 4.02. How are the energy losses reduced in a trans-ormer ? (*P.S.S.C.E. 1999*)

Or

What are the factors which help in increasing the efficiency of a transformer ?

Ans. The efficiency of a transformer can be increased by using (*i*) laminated core to minimise the loss of energy due o eddy currents and (*ii*) by selecting a material for the core, whose B-H curve has lesser area in order to minimise hysteresis loss.

Q. 4.03. The core of a transformer is made of a material whose hysteresis loop is narrow. Why ?

Ans. A transformer works on a.c. supply. Therefore, during its operation, the iron core of the transformer is taken over the complete cycle of magnetisation again and again. In taking the core over a complete cycle of magnetisation, the energy spent per unit volume of the core is equal to area of the hysteresis loop. Therefore, to minimise the loss of energy, the core of the transformer is made of a material, whose hysteresis loop is narrow.

Q. 4.04. Discuss the use of transformer for long distance transmission of electrical energy. (*P.S.S.C.E. 2001*)

Ans. Refer to section 4.03.

Q. 4.05. Can a transformer be helpful in reducing the cost of transmission of electrical energy to long distances ? Explain.

Ans. By using a step-up transformer, the cost of transmission of electrical energy to long distances can be reduced. For details, refer to section 4.03.

Q. 4.06. A transformer of 100% efficiency has 500 turns in the primary and 10,000 turns in the secondary coil. If the primary is connected to 220 V mains supply, what is the voltage across the secondary coil ? (*I.S.C.E. 1994*)

Ans. For a 100% efficient transformer,

$$e_s = e_p \times \frac{N_s}{N_p}$$

Here, $N_p = 500$; $N_s = 100{,}000$ and $e_p = 220$ V

$$\therefore \quad e_s = 220 \times \frac{10{,}000}{500} = \mathbf{4400\ V}$$

Q. 4.07. State the factors on which the induced e.m.f. in a coil rotating in a uniform magnetic field depends. (*P.S.S.C.E. 1991*)

Ans. The induced e.m.f. produced in a coil rotated inside a magnetic field is given by

$$e = n\,\mathrm{B}\,\mathrm{A}\,\omega \sin \omega t$$

Therefore, induced e.m.f. depends upon number of turns of the coil ; area of coil ; its angular speed of rotation and the strength of the magnetic field.

Q. 4.08. State and explain Fleming's right hand rule. (*P.S.S.C.E. 1999*)

Ans. Refer to section 4.05.

Q. 4.09. Although direction of current through the armature reverses after each half cycle, yet it keeps on rotating in one direction. Why ?

Ans. Refer to section 4.08 (working of d.c. motor).

Q. 4.10. What is back e.m.f. in a d.c. motor ? *(I.S.C.E. 1997)*

Ans. Refer to section 4.08.

Q. 4.11. When a motor is at its full speed, a lesser current flows through the armature. Why ?

Ans. Refer to section 4.08.

Q. 4.12. What is the use of a starter ?

Ans. When a motor is just switched on, the back e.m.f. is zero and hence, due to a large current through the armature, the motor may get burnt. A starter is a variable resistance. When it is used in series with the motor, the effective resistance (resistance of the armature and starter resistance) becomes quite large. Therefore, when the motor is switched on, a small current will flow even in the absence of back e.m.f.

Q. 4.13. Why is choke preferred to rheostat in controlling a.c. supply ? *(H.P.S.S.C.E. 1998 S)*

Or

Why is a choke coil preferred over a resistor to adjust current in an a.c. circuit ? *(I.S.C.E. 1993)*

Ans. If a rheostat is used in controlling a.c. supply, electrical energy will be wasted in the form of heat energy across the resistance wire (windings of the rheostat). However, a.c. supply can be controlled with a choke without any wastage of energy. This is because, power factor (cos ϕ) for a resistance is one and it is zero for an inductance.

Q. 4.14. An ideal inductor consumes no electric powe in an a.c. circuit. Explain. *(H.P.S.S.C.E. 199*

Ans. Refer to section 4.10.

Q. 4.15. Why choke coil cannot be used in d.c ? *(H.S.S.C.E. 2001, 1997*

Ans. In a.c. circuit, a choke coil (inductance) offers rea tance (resistance) to the flow of current, which is given by

$$X_L = \omega L = 2\pi f L,$$

where f is frequency of a.c.

The frequency of d.c. supply is zero. Therefore, reactanc due to inductance in d.c. circuits,

$$X_L = 2\pi(0) L = 0$$

Hence, choke coil cannot be used in d.c. circuits.

Q. 4.16. A radio frequency choke is air-cored, wherea an audio frequency choke is iron-cored. Give reasons fo this difference. *(C.B.S.E. 199*

Ans. The inductive reactance,

$$X_L = \omega L = 2\pi f (\mu_0 \mu_r n^2 l A)$$

It follows that at radio frequency (f = very high), X will be very large, even when the choke is air-cored ($\mu_r = 1$ and at audio frequency (f = quite low), X_L will be larg provided the choke is iron-cored (μ_r =large). Hence, a radi frequency choke is air-cored and an audio frequency chok is iron-cored.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 4.01. Fig. 4.14 below shows three different orientations of a circular coil rotating in the magnetic field between the poles of a horse shoe magnet.

(*a*) Determine the direction of induced current in the coil, if the rotation is anticlockwise as viewed by the reader.
(*b*) In which orientation during rotation (with uniform angular speed) is the induced e.m.f. greatest ?

(Text Question)

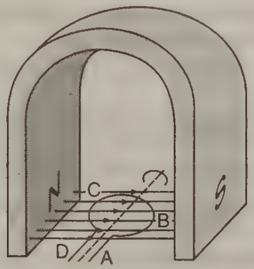


Fig. 4.14 (*a*)

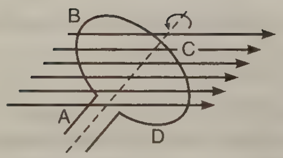


Fig. 4.14 (*b*)

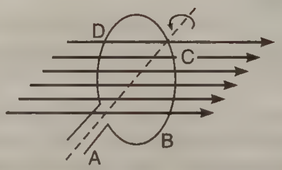


Fig. 4.14 (*c*)

Ans. (*a*) In position of coil shown in Fig. 4.14 (*a*), induced current will flow along ABCD.

In position of coil shown in Fig. 4.14 (*b*), induced current will flow along ABCD.

As the plane of the loop is normal to the field ; no current will flow, when the coil is in position as shown in Fig. 4.14 (*c*).

(*b*) Greatest induced e.m.f. is produced corresponding t orientation as shown in Fig. 4.14 (*a*). It is because, when co is horizontal, $\frac{d\phi}{dt}$ is maximum.

Q. 4.02. Why is the e.m.f. zero, when maximu number of magnetic lines of force pass through th coil ?

Ans. In the vertical position of the coil, the magnetic flu linked with the coil is maximum. But as the coil starts rotatin from this position, the rate of change of magnetic flux is zer

Since $e = -\frac{d\phi}{dt}$, the induced e.m.f. produced is zero.

Q. 4.03. When the motor of an electric refrigerat starts, the lights in the house become dim momen tarily. Why ?

Ans. When the electric motor of the refrigerator starts, draws maximum current from the a.c. mains. It is because, ju at the start of the motor, the back e.m.f. is zero. Due to the larg current drawn by the motor, the current through the light (connected in parallel) decreases and they become dim However, as the armature of the motor rotates, back e.m.f. produced and the motor draws current less than that draw in the beginning. Then, the lights recover their brilliance.

Q. 4.04. Can a capacitor of suitable capacitance repla a choke coil in an a.c. circuit ? Explain.

Ans. In an a.c. circuit, reactance offered by capacitor given by

$$X_C = \frac{1}{2\pi f C}$$

Since the alternating e.m.f. lags the current by a pha angle $\pi/2$ the average power consumed in a capacitor wi e zero. Thus, capacitor will also decrease current in a.c. circuit vithout consuming power just as the choke coil does and ence it can replace the choke coil in a.c. circuit.

Q. 4.05. Write the working principle of a starter used along with a choke in a fluorescent tube.

(C.B.S.E. 1998 S)

Ans. A starter used along with a choke in a fluorescent ube is a thermal switch in the form of a bimetallic strip. When the circuit is switched on, a high current passes through the thermal switch and raises the electrodes at the two ends of the fluorescent tube to incandescence. Due to rise in temperature inside the starter, the bimetallic strip bends and the contact is instantaneously broken. Due to this, a high potential difference (≈ 1000 V) is induced between the electrodes of the tube, which allows a discharge to pass through the mercury vapours enclosed in the tube.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. With the help of a labelled diagram, explain the working principle of a step-up transformer. *(C.B.S.E. 2002)*
2. Explain the principle and theory of a transformer. Why is the efficiency of a transformer always less than one ? *(H.P.S.S.C.E. 1995 ; P.S.S.C.E. 1994)*
3. Discuss the principle, working and use of a transformer for long distance transmission of electrical energy. *(P.S.S.C.E. 1997)*
4. What is the principle of a transformer ? Explain its theory and its application for long distance transmission of electrical energy. *(P.S.S.C.E. 1996 S)*
5. Draw a labelled diagram of an a.c. generator. Write the principle on which it works. *(C.B.S.E. 2002)*
6. Explain on what factors the induced e.m.f. in a coil rotating in a uniform magnetic field depends. Explain, why there is sparking when an electrical circuit is broken, but not when it is made. *(C.B.S.E. Sample Q. Paper)*
7. A rectangular coil of n turns and area A is held in a uniform magnetic field B. If the coil is rotated at a steady angular speed ω, deduce an expression for the induced e.m.f. in the coil at any instant of time.
8. State law of electromagnetic induction. A rectangular loop of N turns of length `$a$' and width `b' is rotated with an angular velocity ω in a uniform magnetic field of induction B. Show that induced e.m.f. is given by
   $$e = \omega \, N \, b \, a \, B \sin \omega \, t$$
9. Draw a labelled diagram of a d.c. generator and describe its construction briefly. *(P.S.S.C.E. 2002)*
10. Briefly explain the use of a starter for operating a d.c. motor. *(H.S.S.C.E. 2001)*
11. Explain the use of a motor starter. *(P.S.S.C.E. 2001, 1991)*
12. Describe principle, construction and uses of a choke coil. *(H.S.S.C.E. 1996)*
13. Explain the construction and working of a choke. Where is it used ? Explain, why is it preferred to resistance in a.c. circuits. *(C.B.S.E. Sample Q. Paper)*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Give the principle, construction and theory of a transformer. *(P.S.S.C.E. 1999 S ; H.S.S.C.E. 1999 S)*
2. Describe the principle construction and working of a transformer. Why is core of a transformer laminated ? *(H.P.S.S.C.E. 1997 ; C.B.S.E. 1991)*
3. Explain with the help of a diagram, the principle, construction and working of a step-up transformer. Why is its core laminated ? *(H.S.S.C.E. 2002 ; C.B.S.E. 1995)*
4. Explain the princple, construction and working of a step-down tranformer. Can it be used with a d.c. circuit ? *(H.S.S.C.E. 2002)*
5. Describe the principle, construction and working of a transformer. What are the energy losses in it and how we reduce them ? *(H.P.S.S.C.E. 2002 ; P.S.S.C.E. 2002, 2000, 1999 ; H.S.S.C.E. 1993)*
6. Explain the principle and working of a transformer. How is it useful for transmission of electrical energy ? *(P.S.S.C.E. 2000, 1999, 1998)*
7. Explain with the help of a labelled diagram, the construction and working of an a.c. generator. *(H.S.S.C.E. 2002)*
8. Draw a labelled diagram of a.c. generator. State its principle and explain its working. *(H.P.S.S.C.E. 1996)*
9. Describe the principle, construction and working of an a.c. generator. *(P.S.S.C.E. 2001 ; H.S.S.C.E. 1999, 1997, 1996)*
10. Draw a labelled diagram of an a.c. generator. Give principle, construction and working of an a.c. generator. *(H.P.S.S.C.E. 2002 ; C.B.S.E. 1994)*
11. Give the construction, principle and working of an a.c. dynamo (a.c. generator). *(P.S.S.C.E. 1999 S ; H.P.S.S.C.E. 1999 S ; H.S.S.C.E. 1992)*
12. Give the principle, construction and working of d.c. generator. *(C.B.S.E. 1994)*
13. Explain the construction, principle and working of a d.c. motor. Find the expression for its efficiency. *(P.S.S.C.E. 2001 ; H.S.S.C.E. 1993)*
14. Draw a neat diagram of a d.c. motor and explain its working. What is the function of starter ? *(H.S.S.C.E. 2002)*
15. Draw a labelled diagram of a d.c. motor. State its principle and explain its working. *(C.B.S.E. Sample Q. Paper)*
16. What is choke coil ? Describe its construction, working and uses. Why it cannot be used in a d.c. circuit ? *(P.S.S.C.E. 2001)*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Transformer

1. An a.c. voltage of 200 V is applied to the primary of a transformer and voltage of 2000 V is obtained from the secondary. Calculate the ratio of the currents through the primary and secondary coils. *(C.B.S.E. Sample Q. Paper)* **[Ans.** 10]
2. In an ideal transformer, the number of turns in primary and secondary coils are 2000 and 100 respectively. If maximum voltage in primary is 120 V, what is maximum voltage in secondary ? *(H.P.S.S.C.E. 1999 S)* **[Ans.** 6 V]

3. The output voltage of an ideal transformer, connected to a 240 V a.c. mains, is 24 V. When this transformer is used to light a bulb with rating 24 V – 24 W, calculate the current in the primary coil of the circuit. (*C.B.S.E. 2000*) [**Ans.** 0·1 A]

4. A step-up transformer is used on a 120 volt line to provide a potential difference of 2400 volt at 2 ampere current. If primary has 1000 turns, find the number of turns in secondary and current in the primary coil. [**Ans.** 20,000 ; 40 A]

5. How much current is drawn by the primary coil of a transformer, which steps down 220 V to 44 V to operate a device with an impedance of 440 Ω ? (*H.S.S.C.E. 1998*) [**Ans.** 0·02 A]

6. A step-up transformer operates on a 200 volt line and supplies a load of 2 ampere. The ratio of primary and secondary windings is 1 : 5. Determine the secondary voltage, primary current and power output. Assume efficiency to be 100%. [**Ans.** 1000 V, 10 A, 2000 W]

7. A step-down transformer is used at 220 volt line to provide a current of 0·5 ampere to a 15 watt bulb. If the secondary has 20 turns, find the current and number of turns in the primary coil. [**Ans.** 0·0682 A, 147]

8. A transformer has an efficiency of 80%. It works at 4 kilowatt and 100 volt. If the secondary voltage is 240 volt, calculate the primary and secondary currents. (*C.B.S.E. 2001*) [**Ans.** 40 A ; 13·33 A]

9. Calculate the current drawn by the primary of a transformer, which steps down 200 V to 20 V to operate a device of resistance 20 Ω. Assume the efficiency of the transformer to be 80%. (*C.B.S.E. 2001*) [**Ans.** 0·125 A]

10. A step-down tranformer converts a voltage of 2200 V into 220 V in the transmission line. Number of turns in primary coil is 5000. Efficiency of transformer is 90% and its ouput power is 8 kW. Calculate (*a*) number of turns in secondary coil and (*b*) input power. [**Ans.** (*a*) 500 (*b*) 8·889 kW]

Type B. On a.c. generator

11. A closely wound rectangular coil of 200 turns and size 0·3 m × 0·1 m is rotating in a magnetic field of induction 0·005 Wb m^{-2} with a frequency of 1800 r.p.m. about an axis normal to the field. Calculate the maximum value o induced e.m.f. [**Ans.** 5·655 V

12. A rectangular coil 25 cm × 10 cm and having 700 turn rotates about an axis at right angles to a magnetic field o 20 gauss at 1500 revolutions per minute. Calculate th maximum value of the e.m.f. generated in the coil. [**Ans.** 5·5 V

13. A flat coil of 500 turns, each of area 50 cm^2, rotates in uniform magnetic field of 0·14 Wb m^{-2} at an angular spee of 150 rad s^{-1}. The coil has a resistance of 5 Ω. The induce e.m.f. is applied to an external resistance of 10 Ω. Calculat the peak current through the resistance. (*C.B.S.E. 1995* [**Ans.** 3·5 A

14. An a.c. generator consists of a coil of 2000 turns each of are 80 cm^2 and rotating at an angular speed of 200 r.p.m. in uniform magnetic field of 4·8 × 10^{-2} T. Calculate the pea and r.m.s. value of e.m.f. induced in the coil. (*P.S.S.C.E. 2001*) [**Ans.** 16·085 V, 11·375 V

15. A rectangular coil of dimension 40 cm × 25 cm havin 1000 turns rotates in a uniform magnetic field of strengt 0·08 Wb m^{-2} about an axis perpendicular to the field. I the coil makes 300 revolutions per minute, find the valu of instantaneous e.m.f. when the plane of the coil make angle of (*i*) 0° (*ii*) 45° (*iii*) 90° with magnetic lines of force [**Ans.** (*i*) 251·33 V ; (*ii*) 177·72V ; (*iii*) zerc

Type C. On Motor

16. An electric motor operating at 200 V draws a current c 5 A at its full speed. The resistance of the armature is 1 Ω. Calculate the back e.m.f. and efficiency of the motor. [**Ans.** 150 V, 75 %

17. A motor runs at 220 V. The resistance of the armature i 11 Ω. If the back e.m.f. produced is 198 V, when at ful speed ; then calculate the current through armature, whe (*i*) motor is just switched on and (*ii*) motor is at full speed [**Ans.** (*i*) 20 A (*ii*) 2 A

18. A motor develops back e.m.f. of 210 V, when it is run a 240 V. If the armature current is 5 A, calculate (*a*) resistanc of the armature, (*b*) the mechanical output power and (*c* the power dissipated in the motor. [**Ans.** (*a*) 6 Ω, (*b*) 1050 W (*c*) 150 W

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $\dfrac{I_p}{I_s}=\dfrac{e_s}{e_p}=\dfrac{2000}{200}=10$

2. Proceed as in solved problem no. 4.01.

3. Proceed as in solved problem no. 4.02.

4. Here, $e_p = 120$ V, $e_s = 2400$ V ; $I_s = 2$ A, $N_p = 1000$

$$\text{Now, } N_s = N_p \times \frac{e_s}{e_p} = \frac{1000\times 2400}{120} = \mathbf{20{,}000}$$

$$\text{Also, } I_p = I_s \times \frac{e_s}{e_p} = \frac{2\times 2400}{120} = \mathbf{40\ A}$$

5. Proceed as in solved problem no. 4.03.

6. Here, $e_p = 200$ V ; $I_s = 2$ A ; $\dfrac{N_p}{N_s}=\dfrac{1}{5}$

$$\text{Now, } e_s = e_p \times \frac{N_s}{N_p} = \frac{e_p}{N_p/N_s} = \frac{200}{1/5} = \mathbf{1000\ V}$$

$$\text{Also, } I_p = I_s \times \frac{e_s}{e_p} = \frac{2\times 1000}{200} = \mathbf{10\ A}$$

Output power = $e_s I_s$ = 1000 × 2 = **2000 W**

7. Here, $e_p = 220$ V ; $I_s = 0\cdot5$ A ; $e_s I_s = 15$ W, $N_s = 20$

$$\text{Now, } I_p = \frac{e_s I_s}{e_p} = \frac{15}{220} = \mathbf{0\cdot0682\ A}$$

$$\text{Also, } e_s = \frac{e_s I_s}{e_p} = \frac{15}{0\cdot5} = 30\ \text{V}$$

$$\therefore\ N_p = N_s \times \frac{e_p}{e_s} = \frac{20\times 220}{30} = \mathbf{147}$$

8. Here, $\eta = 80\%$, $e_p I_p = 4$ kW $= 4000$ W ;
$e_p = 100$ V ; $e_s = 240$ V

$$\text{Now, } I_p = \frac{e_p I_p}{e_p} = \frac{4000}{100} = \mathbf{40\ A}$$

$$\text{Also, } \eta = \frac{e_s I_s}{e_p I_p} \quad \text{or} \quad \frac{80}{100} = \frac{240\times I_s}{4000}$$

$$\text{or } I_s = \frac{80\times 4000}{100\times 240} = \mathbf{13\cdot33\ A}$$

9. Here, $\eta = 80\%$, $e_p = 200$ V ; $e_s = 20$ V and Z = 20 Ω

$$\therefore\ I_s = \frac{e_s}{Z} = \frac{20}{20} = 1\ \text{A}$$

Now, $\eta = \dfrac{e_s I_s}{e_p I_p}$ or $\dfrac{80}{100} = \dfrac{20 \times 1}{200 \times I_p}$

or $I_p = \dfrac{20 \times 1 \times 100}{200 \times 80} = \mathbf{0{\cdot}125\ A}$

10. Here, $e_p = 2200$ V ; $e_s = 220$ V, $N_p = 5000$,
$e_s I_s = 8$ kW $= 8000$ W and $\eta = 90\%$

(a) Now, $N_s = N_p \times \dfrac{e_s}{e_p} = \dfrac{5000 \times 220}{2200} = \mathbf{500}$

(b) Also, $\eta = \dfrac{e_s I_s}{e_p I_p}$ or $\dfrac{90}{100} = \dfrac{e_s I_s}{e_p I_p}$

or $e_p I_p = \dfrac{e_s I_s \times 100}{90} = \dfrac{8000 \times 100}{90} = 8889 \text{ W} = \mathbf{8{\cdot}889\ kW}$

11. Proceed as in solved problem no. 4.08.

12. Proceed as in solved problem no. 4.08.

13. Here, $n = 500$; $A = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$,
$B = 0{\cdot}14 \text{ Wb m}^{-2}$ and $\omega = 150 \text{ rad s}^{-1}$
Now, $e_0 = n B A \omega = 500 \times 0{\cdot}14 \times 50 \times 10^{-4} \times 150 = 52{\cdot}5$ V
Resistance of the coil, $R = 5\ \Omega$
and external resistance, $R_L = 10\ \Omega$
Therefore, peak current through external resistance,

$$I_0 = \frac{e_0}{R + R_L} = \frac{52{\cdot}5}{5 + 10} = \mathbf{3{\cdot}5\ A}$$

14. Here, $n = 2000$; $A = 80 \text{ cm}^2 = 80 \times 10^{-4} \text{ m}^2$;
$B = 4{\cdot}8 \times 10^{-2}$ T

and $\omega = 200$ r.p.m. $= \dfrac{2\pi \times 200}{60} = \dfrac{20\pi}{3} \text{ rad s}^{-1}$

Now, $e_0 = n B A \omega = 2000 \times 4{\cdot}8 \times 10^{-2} \times 80 \times 10^{-4} \times \dfrac{20\pi}{3}$
$= \mathbf{16{\cdot}085\ V}$

Also, $e_{r.m.s.} = \dfrac{e_0}{\sqrt{2}} = \dfrac{16{\cdot}085}{\sqrt{2}} = \mathbf{11{\cdot}375\ V}$

15. Proceed as in solved problem no. 4.08.

16. Now, $I = \dfrac{E - e}{R}$ or $5 = \dfrac{200 - e}{10}$

or $e = 200 - 50 = \mathbf{150\ V}$
Input power $= E\,I = 200 \times 5 = 1000$ W
Output power $= E\,I - I^2 R = 200 \times 5 - (5)^2 \times 10 = 750$ W

$\therefore \quad \eta = \dfrac{750}{1000} \times 100 = \mathbf{75\ \%}$

Aliter. $\eta = \dfrac{e}{E} \times 100 = \dfrac{150}{200} \times 100 = \mathbf{75\ \%}$

17. Here, $E = 220$ V ; $e = 198$ V ; $R = 11\ \Omega$
(i) When motor is just switched on, $e = 0$

$\therefore \quad I = \dfrac{E - e}{R} = \dfrac{220 - 0}{11} = \mathbf{20\ A}$

(ii) When motor is at full speed,

$I = \dfrac{E - e}{R} = \dfrac{220 - 198}{11} = \mathbf{2\ A}$

18. **(a)** Now, $I = \dfrac{E - e}{R}$ or $5 = \dfrac{240 - 210}{R}$

or $R = \mathbf{6\ \Omega}$
(b) Output power $= E\,I - I^2 R = 240 \times 5 - (5)^2 \times 6 = \mathbf{1050\ W}$
(c) Power dissipated in the motor $= I^2 R = (5)^2 \times 6 = \mathbf{150\ W}$
Aliter. Input power $= E\,I = 240 \times 5 = 1200$
Output power $= e\,I = 210 \times 5 = \mathbf{1050\ W}$
Therefore, power dissipated $= 1200 - 1050 = \mathbf{150\ W}$

Competitive Examination File (Unit IV)

REVISION AT A GLANCE

Chapter 1. Electromagnetic induction

☐ **Magnetic flux.** *The number of magnetic field lines crossing a surface normally is called magnetic flux (ϕ) linked with the surface.*

Mathematically : $\phi = \vec{B} . \vec{A} = BA \cos \theta$,

where B is the magnetic field, A is the area of the surface and θ is the angle between the direction of the magnetic field and normal to the surface.

Unit. In SI, unit of magnetic flux is **weber (Wb)**

1 weber = 10^8 maxwell

☐ **Electromagnetic induction.** *It is the phenomenon of production of e.m.f. in a coil, when the magnetic flux linked with the coil is changed.*

The e.m.f. so produced is called ***induced e.m.f.*** and the current so produced is called ***induced current***.

☐ **Faraday's laws of electromagnetic induction :**

First law. *Whenever magnetic flux linked with a circuit (a loop of wire or a coil or an electric circuit in general) changes, induced e.m.f. is produced.*

Second law. *The induced e.m.f. lasts as long as the change in the magnetic flux continues.*

Third law. *The magnitude of the induced e.m.f. is directly proportional to the rate of change of the magnetic flux.*

Mathematically : Induced e.m.f., $e = -\frac{d\phi}{dt} = -\frac{\phi_2 - \phi_1}{t}$

☐ **Lenz's law.** It states that *the induced current produced in a circuit always flows in such a direction that it opposes the change or the cause that produces it.*

Lenz's law can be used to find the direction of induced current.

☐ **Motional E.M.F.** When a conductor of length l moves with a velocity v in a magnetic field B, so that magnetic field is perpendicular to both the length of the conductor and its direction of motion, the magnetic Lorentz force on the conductor gives rise to e.m.f.

Mathematically : $e = Blv$

☐ **Eddy currents.** *The currents induced in the body of a conductor, when the magnetic flux linked with the conductor changes, are called eddy currents (or Focault's currents).*

The direction of the eddy currents set up in the conductor can be found by applying ***Lenz's law*** or ***Fleming's right hand rule***.

☐ **Self induction.** *The phenomenon according to which an opposing induced e.m.f. is produced in a coil as a result of change in current or magnetic flux linked with it, is called self induction.*

☐ **Coefficient of self induction.** *The coefficient of self induction or simply self-inductance (L) of a coil is numerically equal to the magnetic flux (ϕ) linked with the coil, when a unit current flows through it.*

Mathematically : $\phi = LI$

The self-inductance of a coil is also numerically equal to the induced e.m.f. produced in the coil, when the rate of change of current in the coil is unity.

Mathematically : $e = -L\frac{dI}{dt}$

Unit : In SI, the unit of self-inductance is **henry (H).**

The self-inductance of a coil is said to be one henry, if a rate of change of current of 1 ampere per second induces an e.m.f. of 1 volt in it.

☐ **Self-inductance of a long solenoid.** The self-inductance of a long solenoid of length l, area of cross-section A and number of turns per unit length n is given by

$$L = \mu_0 n^2 l A$$

☐ **Mutual induction.** *The phenomenon according to which an opposing induced e.m.f. is produced in a coil as a result of change in current or magnetic flux linked with a neighbouring coil is called mutual induction.*

☐ **Coefficient of mutual induction.** *The coefficient of mutual induction or simply mutual inductance (M) of the two coils is numerically equal to the magnetic flux (ϕ) linked with one coil, when a unit current flows through the neighbouring coil.*

Mathematically : $\phi = MI$

The mutual inductance of two coils is also numerically equal to the induced e.m.f. produced in one coil, when the rate of change of current is unity in the other coil.

Mathematically : $e = -M \frac{dI}{dt}$

Unit. In SI, the unit of mutual inductance is **henry (H).**

The mutual inductance of two coils is said to be one henry, if a rate of change of current of 1 ampere per second in one coil induces *ı e.m.f. of 1 volt in the neighbouring coil.*

$$\textbf{1 henry} = \frac{\textbf{1 volt}}{\textbf{1 ampere second}^{-1}}$$

☐ **Mutual inductance of two long solenoids.** When over a solenoid S_1 of length l, area of cross-section A and number f turns per unit length n_1, another solenoid S_2 of same length and number of turns per unit length n_2 is wound, then mutual ductance between the two solenoids (current is passed through solenoid S_1) is given by

$$M = \mu_0 n_1 n_2 l A$$

ome Useful Facts :

1. The induced e.m.f. in a coil can be produced in three ways, namely : by changing magnetic field B, by changing area f the coil inside the field or by changing inclination of the coil with the magnetic field.

2. When a conductor is moved inside a magnetic field, the e.m.f. produced across its two ends is due to magnetic Lorentz rce experienced by the electrons inside the conductor.

3. The eddy currents have both ***practical useful applications*** as well as ***undesirable effects***.

Chapter 2. Transient current

☐ **Growth of current in LR-circuit.** The instantaneous current is given by

$$I = I_0 \left(1 - e^{-\frac{R}{L}t}\right)$$

☐ **Decay of current in LR-circuit.** The instantaneous current is given by

$$I = I_0 e^{-\frac{R}{L}t}$$

Both growth and decay of current in an LR-circuit are exponential in nature.

☐ **Inductive time constant.** $\frac{L}{R}$ has dimensions of time and it is called ***inductive time constant*** of LR-circuit.

The inductive time constant of an LR-circuit is defined as the time, in which current grows from zero to 0·632 of its maximum alue during growth or decays from maximum value to 0·368 of its maximum value during decay.

The current takes practically a time equal to $\frac{5L}{R}$ to grow from zero to maximum value (during growth) or to decay om maximum value to zero (during decay).

☐ **Growth of charge in CR-circuit.** The instantaneous charge on the capacitor is given by

$$q = q_0 \left(1 - e^{-\frac{t}{CR}}\right)$$

Also potential difference across the two plates of the capacitor and current in the CR-circuit at any instant are respectively iven by

$$V = E \left(1 - e^{-\frac{t}{CR}}\right)$$

and $$I = I_0 e^{-\frac{t}{CR}}$$

☐ **Decay of charge in CR-circuit.** The instantaneous charge on the capacitor, potential difference across its plates and ne current in the circuit are respectively given by

$$q = q_0 e^{-\frac{t}{CR}}$$

$$V = E e^{-\frac{t}{CR}}$$

and $$I = -I_0 e^{-\frac{t}{CR}}$$

☐ **Capacitive time constant.** CR has dimensions of time and it is called ***capacitive time constant*** of circuit.

The capacitive time constant of a CR-circuit is defined as the time, in which charge grows from zero to 0·632 of its maximum value during growth or decays from maximum to 0·368 of its maximum value during decay.

☐ **Energy stored in an inductor.** When current flows through an inductor, the energy stored in it is given by

$$U = \frac{1}{2} L I^2,$$

where I_0 is final value of current in the inductor. The energy is stored inside an inductor in the form of magnetic field.

☐ **Energy stored in a capacitor.** Energy stored in a capacitor, when it is charged, is given by

$$U = \frac{1}{2} C E^2 = \frac{1}{2} \frac{q_0^2}{C} = \frac{1}{2} q_0 E$$

where E is maximum value of potential difference set up across the two plates of the capacitor.

The energy is stored inside the capacitor in the form of electric field between the plates of the capacitor.

☐ **LC-oscillations.** When a charged capacitor discharges through an inductor, LC-oscillations are produced. The frequency of the oscillations is given by

$$f = \frac{1}{2\pi \sqrt{LC}}$$

Some Useful Facts :

1. In a CR-circuit, current always decays from maximum value to zero during both the growth and decay of charge in a CR- circuit.

2. In an inductance or in a capacitor, energy is stored at the expense of the energy of the source of e.m.f.

Chapter 3. Alternating current

☐ **Alternating current.** *The current, whose magnitude changes with time and the direction reverses periodically is known as alternating current.*

It can be represented by a sine or cosine function of time.

The instantaneous value (value at any time *t*) of alternating current is given by

$$I = I_0 \sin \omega t \qquad \text{or} \qquad I = I_0 \cos \omega t$$

Here, I_0 is called ***peak value*** of a.c. and $\omega = \frac{2\pi}{T} = 2\pi f$

Similarly, the alternating e.m.f. is represented as

$$E = E_0 \sin \omega t \qquad \text{or} \qquad E = E_0 \cos \omega t$$

☐ **Mean or Average value of a.c.** *It is that steady current, which when passed through a circuit for half the time period of the alternating current sends the same amount of charge as is done by the alternating current in the same time through the same circuit.*

Mathematically : $I_m \text{ (or } I_a) = \frac{2 I_0}{\pi} = 0 \cdot 636\, I_0$

☐ **Root mean square (r.m.s.) or virtual value of a.c.** *It is that steady current, which when passed through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance and in the same time.*

Mathematically : $I_{r.m.s.} \text{ (or } I_v) = \frac{I_0}{\sqrt{2}} = 0 \cdot 707\, I_0$

☐ **a.c. through a resistor.** The alternating e.m.f. is ***in phase*** with the current, when a.c. flows through a resistor.

☐ **a.c. through an inductor.** The alternating e.m.f. ***leads*** the current by a phase angle $\pi/2$, when a.c. flows through an inductor.

☐ **Inductive reactance.** *It is the opposition offered by the inductor to the flow of alternating current through it.*

Mathematically : $X_L = \omega L = 2\pi f L$

The inductive reactance is zero for d.c. ($f = 0$) and has a finite value for a.c.

☐ **a.c. through a capacitor.** The alternating e.m.f. ***lags behind*** the current by a phase angle $\pi/2$, when a.c. flows through a capacitor.

☐ **Capacitive reactance.** *It is the opposition offered by the capacitor to the flow of alternating current through it.*

Mathematically : $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

The capacitive reactance is infinite for d.c. ($f = 0$) and has a finite value for a.c.

☐ **Phasor diagram.** *A diagram representing alternating voltages and currents (of same frequency) as vectors along with the phase angle between them is called a phasor diagram.*

☐ **a.c. through a series LR-circuit.** The alternating e.m.f. ***leads*** the current by a phase angle ϕ given by

$$\tan\phi = \frac{\omega L}{R}$$

Impedance of LR-circuit, $Z = \sqrt{R^2 + \omega^2 L^2}$

☐ **a.c. through a series CR-circuit.** The alternating e.m.f. ***lags behind*** the current by a phase angle ϕ is given by

$$\tan\phi = \frac{1/\omega C}{R}$$

Impedance of CR-circuit, $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

☐ **a.c. through a series LCR-circuit.** The alternating e.m.f. ***leads/lags behind*** the current by a phase angle ϕ given by

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

The e.m.f. leads the current, if $\omega L > 1/\omega C$ and it lags behind the current, if $\omega L < 1/\omega C$.

Impedance of LCR-circuit, $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

☐ **Resonant series LCR-circuit.** When the frequency of a.c. supply is such that the inductive reactance and capacitive ctance become equal ($X_L = X_C$), the impedance of the series LCR-circuit is merely equal to the ohmic resistance in the circuit. such, the current in the circuit becomes maximum. Such a series LCR-circuit is called ***resonant series LCR-circuit*** and frequency of the a.c. supply is called ***resonant frequency*** (f_0). The resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

☐ **Q-factor.** It indicates the sharpness of the resonance.

Mathematically : Q-factor $= \frac{R}{2\pi f_0 L}$

The higher value of Q-factor indicates sharper peak in the current.

☐ **Average power of an a.c. circuit.** It is given by

$$P_{av} = E_v I_v \cos\phi,$$

ere $E_v I_v$ is called ***apparent power*** and $\cos\phi$ is called ***power factor.***

For a.c. circuit having R only ; L only ; C only ; L and R ; C and R ; and L, C and R ; the power factor is equal to 1 ; 0 ; 0 ;

$\frac{R}{{}^2 + \omega^2 L^2}$; $\frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$ and $\frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$ respectively.

☐ **Wattless current.** The component $I_v \sin\phi$ of alternating current is called ***wattless*** or ***idle current.***

me Useful Facts :

1. The average value of a.c. is zero over a complete cycle of a.c.
2. The peak value of a.c. in a circuit containing inductor is $\frac{E_0}{X_L}$ and in a circuit containing capacitor is $\frac{E_0}{X_C}$.
3. An inductor offers an easy path to d.c. and a resistive path to a.c.
4. A capacitor serves as a block for d.c. and an easy path to a.c.

Chapter 4. Electrical machines and devices

☐ **Transformer.** *It is a device used to change low alternating voltage at high current into high voltage at low current and vice-sa.* It is based on the phenomenon of mutual induction. For an ideal transformer,

$$\frac{e_s}{e_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

Here, k is called ***transformation ratio.***

For a ***step up transformer,*** $k > 1$ *i.e.* $e_s > e_p$, $I_s < I_p$ and $N_s > N_p$.

For a ***step down transformer,*** $k < 1$ *i.e.* $e_s < e_p$, $I_s > I_p$ and $N_s < N_p$.

In practice, efficiency of a transformer,

$$\eta = \frac{e_s\, I_s}{e_p\, I_p}$$

☐ **Induction coil.** *It is a device used to obtain high potential difference from a low d.c. potential difference.*

It is based upon the phenomenon of mutual induction.

☐ **Alternating current generator.** *It is a device used to obtain a supply of alternating e.m.f. by converting rotational mechanical energy into electrical energy.* It is based on the phenomenon of electromagnetic induction.

The instantaneous value of e.m.f. produced is given by

$$e = n\, B\, A\, \omega \sin \omega\, t,$$

where n is number of turns of the coil ; A is the area of coil and ω is angular frequency of rotation of the coil inside a magnetic field strength B.

☐ **d.c. motor.** *It is a device used to convert electrical energy in the form of d.c. supply into mechanical energy of rotation.*

The current through the armature of the motor is given by

$$I = \frac{E - e}{R},$$

where E is applied d.c. e.m.f., e is back e.m.f. produced and R is resistance of the armature.

☐ **Motor starter.** *It is a variable resistor, which is used to protect the motor from the possible damage, during its operation.*

☐ **Choke coil.** *It is an inductor used to control the flow of current in an a.c. circuit.*

Practically, average power comsumed by a choke coil is zero.

Some Useful Facts :

1. In a transformer, if we gain in voltage, we lose in current and vice-versa.
2. The transmission of electric energy is done at high voltage, so as to reduce the dissipation of energy.
3. In an induction coil, although the potential difference across the terminals reverses regularly, yet a large effective potential difference is obtained in one direction.
4. A practical inductance consumes a small electrical energy due to its small ohmic resistance.

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Electromagnetic induction

Problem 1.01. A current of 10 A is flowing in a long straight wire situated near a rectangular circuit, whose two sides of length 0·2 m are parallel to the wire. One of them is at a distance of 0·05 m and the other at a distance of 0·1 m from the wire. The wire is in the plane of the rectangle. Find the magnetic flux through the rectangular circuit. If the current decays uniformly to zero in 0·02 s, find the e.m.f. induced in the circuit and indicate the direction in which the induced current flows. *(Roorkee, 1994)*

Sol. Fig. 1.01 shows a rectangular circuit ABCD, whose two sides AD and BC, each of length = 0·02 m are at distances $r_1 = 0{\cdot}5$ m and $r_2 = 0{\cdot}1$ m from the long straight wire carrying current I = 10 A.

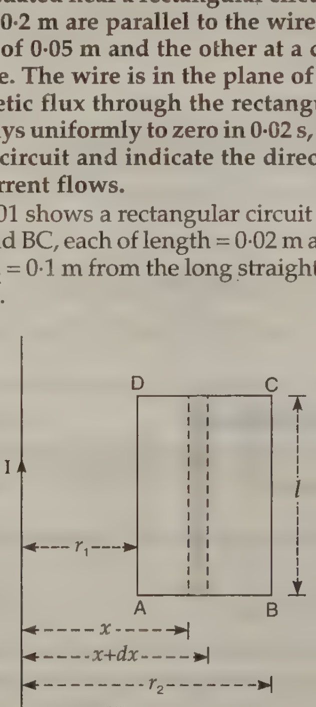


Fig. 1.01.

The magnetic field due to the current carrying wire decreases, as the distance from the wire increases. If B is magnetic field at a distance x from the wire, then the magnetic flux linked with the shaded strip,

$$d\phi = B \times \text{area of shaded strip}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I}{x} \times l\, dx$$

Therefore, magnetic flux linked with the rectangular circuit,

$$\phi = \int_{r_1}^{r_2} d\phi = \frac{\mu_0}{4\pi} \cdot 2Il \int_{r_1}^{r_2} \frac{1}{x}\, dx = \frac{\mu_0}{4\pi} \times 2Il \left| \log_e x \right|_{r_1}^{r_2}$$

$$= \frac{\mu_0}{4\pi} \cdot 2Il \left| \log_e r_2 - \log_e r_1 \right| = \frac{\mu_0}{4\pi} \cdot 2Il \log_e \frac{r_2}{r_1}$$

Setting values of I, l, r_1 and r_2, we have

$$\phi = 10^{-7} \times 2 \times 10 \times 0 \cdot 2 \log_e \frac{0 \cdot 1}{0 \cdot 05} = 4 \times 10^{-7} \log_e 2$$

$$= 4 \times 10^{-7} \times 0{\cdot}693$$

or $\boldsymbol{\phi = 2{\cdot}772 \times 10^{-7}}$ **Wb**

When the current changes to zero in time 0·02 s, the final magnetic flux linked with the coil becomes zero. If e is induced e.m.f. produced, then

$$e = -\frac{d\phi}{dt} = -\frac{0 - 2 \cdot 772 \times 10^{-7}}{0 \cdot 02} = 1 \cdot 386 \times 10^{-5}$$

$$= \mathbf{13{\cdot}86}\ \boldsymbol{\mu}\mathbf{V}$$

Problem 1.02. A pair of parallel horizontal conducting ls of negligible resistance shorted at one end is fixed on able. The distance between the rails is L. A conducting assless rod of resistance R can slide on the rails ctionlessly. The rod is tied to a massless string which sses over a pulley fixed to the edge of the table. A mass tied to the other end of the string, hangs vertically. A nstant magnetic field B exists perpendicular to the table. the system is released from rest. Calculate

(*a*) the terminal velocity achieved by the rod and

(*b*) the acceleration of the mass at the instant, when the elocity of the rod is half the terminal velocity.

(I.I.T. 1997)

Sol. (*a*) Fig. 1.02 shows a conducting rod AB of length and resistance R sliding with a velocity v along Y-axis, while constant magnetic field B exists along Z- axis.

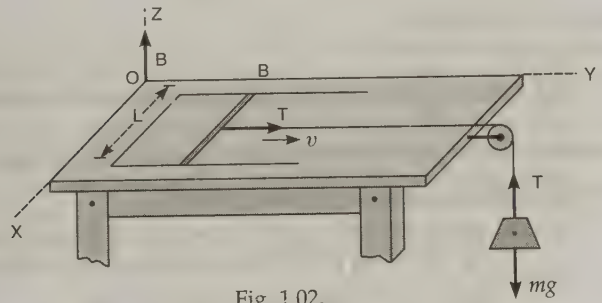


Fig. 1.02.

The e.m.f. induced in the rod,

$$e = \mathrm{B\,L}\,v$$

The induced current flowing through the rod,

$$\mathrm{I} = \frac{e}{\mathrm{R}} = \frac{\mathrm{B\,L}\,v}{\mathrm{R}}$$

The direction of flow of the current through the rod AB vill be such that the induced e.m.f. produced would oppose he motion of the rod along OY. Thus, the rod will experience orce along negative Y-axis. The magnitude of the force is iven by

$$\mathrm{F} = \mathrm{B\,I\,L} = \mathrm{B} \times \frac{\mathrm{B\,L}\,v}{\mathrm{R}} \times \mathrm{L}$$

$$= \frac{\mathrm{B}^2\,\mathrm{L}^2\,v}{\mathrm{R}}$$

The force acting on the rod equals the tension in the tring. If the rod moves with acceleration a along OY, then

$$m\,a = m\,g - \mathrm{T} = m\,g - \mathrm{F} = m\,g - \frac{\mathrm{B}^2\,\mathrm{L}^2\,v}{\mathrm{R}}$$

or $$a = g - \frac{\mathrm{B}^2\,\mathrm{L}^2\,v}{\mathrm{MR}} \qquad ...(i)$$

If the rod acquires the terminal velocity v_0, then $a = 0$. Setting $v = v_0$ and $a = 0$ in equation (*i*), we have

$$0 = g - \frac{\mathrm{B}^2\,\mathrm{L}^2\,v}{m\mathrm{R}}$$

or $$v_0 = \frac{m\,g\,\mathrm{R}}{\mathrm{B}^2\,\mathrm{L}^2}$$

(*b*) The acceleration of the rod, when it moves with velocity $v = \frac{1}{2}v_0$ *i.e.* $v = \frac{m\,g\,\mathrm{R}}{2\,\mathrm{B}^2\,\mathrm{L}^2}$ is given by

$$a = g - \frac{\mathrm{B}^2\,\mathrm{L}^2}{m\,\mathrm{R}}\,\frac{m\,g\,\mathrm{R}}{2\,\mathrm{B}^2\mathrm{L}^2} = g - 0{\cdot}5\,g = 0{\cdot}5\,g$$

Problem 1.03. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown in Fig. 1.03. A horizontal metallic bar L of mass 0·2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0·6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0·76 W and 1·2 W respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2.

(I.I.T. 1994)

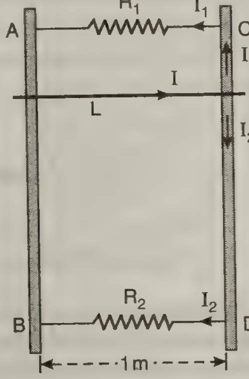


Fig. 1.03.

Thought Process

1. The metallic bar will attain terminal velocity, when the weight (m g) of the bar and the magnetic Lorentz force (B I l) on it become equal and opposite.

2. The induced e.m.f. produced is equal to potential difference across resistance R_1 or across R_2.

Sol. Let e be the induced e.m.f. and I be the induced current in the metallic bar. The bar slides down under the action of the following two forces :

(*i*) The weight $m\,g$ of the bar acting vertically downwards and

(*ii*) The force on the bar $\mathrm{B\,I}\,l$ due to magnetic field acting vertically upwards.

The bar will attain terminal velocity, when

$$\mathrm{B\,I}\,l = m\,g$$

Here, $\mathrm{B} = 0{\cdot}6\ \mathrm{T}$; $m = 0{\cdot}2$ kg and $l = 1$ m

$$\therefore \quad 0{\cdot}6 \times \mathrm{I} \times 1 = 0{\cdot}2 \times 9{\cdot}8$$

or $$\mathrm{I} = \frac{0{\cdot}2 \times 9{\cdot}8}{0{\cdot}6 \times 1} = 3{\cdot}266\ \mathrm{A}$$

If v is the terminal velocity attained, then

$$e = \mathrm{B}\,v\,l \qquad ...(i)$$

Suppose that part I_1 of the induced current flows through resistance R_1 and the remaining part I_2 flows through R_2. Then,

$$I_1 + I_2 = I$$

or $$I_1 + I_2 = 3{\cdot}266\ \mathrm{A} \qquad ...(ii)$$

Further, power dissipated in resistance R_1,

$$e\,I_1 = 0{\cdot}76 \qquad ...(iii)$$

and power dissipated in resistance R_2, $e\,I_2 = 1{\cdot}2$ ...(*iv*)

Dividing the equation (*iii*) by (*iv*), we have

$$\frac{e\,I_1}{e\,I_2} = \frac{0{\cdot}76}{1{\cdot}2} = 0{\cdot}633$$

or $\quad I_1 = 0{\cdot}633\, I_2$ ...(*v*)

Substituting for I_1 in equation (*ii*), we have

$$0{\cdot}633\, I_2 + I_2 = 3{\cdot}266 \text{ or } I_2 = 2 \text{ A}$$

From equation (*v*), we have

$$I_1 = 0{\cdot}633 \times 2 = 1{\cdot}266 \text{ A}$$

Substituting for I_1 in equation (*iii*), we have

$$e = 1{\cdot}266 = 0{\cdot}76 \text{ or } e = 0{\cdot}6 \text{ V}$$

Substituting for ε in equation (*i*), we have

$$0{\cdot}6 = 0{\cdot}6 \times v \times 1 \text{ or } v = \mathbf{1\ m\ s^{-1}}$$

Now, induced e.m.f. produced is also equal to potential difference across resistance R_1 or across R_2, we have

$$I_1 R_1 = I_2 R_2 = \varepsilon$$

$$\therefore \quad R_1 = \frac{e}{I_1} = \frac{0{\cdot}6}{1{\cdot}266} = \frac{9}{19} = \mathbf{0{\cdot}474\ \Omega}$$

and $\quad R_2 = \dfrac{e}{I_2} = \dfrac{0{\cdot}6}{2} = 0{\cdot}3\ \Omega$

Problem 1.04. A metal bar AB can slide on two parallel thick metallic rails separated by a distance *l*. A resistance R and an inductance L are connected to the rails as shown in Fig. 1.04. A long straight wire carrying a constant current

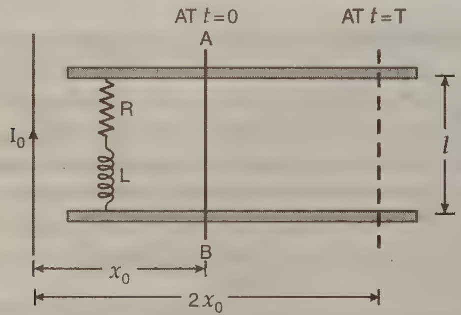


Fig. 1.04

I_0 is placed in the plane of the rails and perpendicular to them as shown in the figure. The bar AB is held at rest at a distance x_0 from the long wire. At $t = 0$, it is made to slide on the rails away from the wire. Answer the following questions :

(*a*) Find the relation among I, $\dfrac{dI}{dt}$ and $\dfrac{d\phi}{dt}$, where I is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(*b*) It is observed that at time $t = T$, the metal bar AB is at a distance $2\,x_0$ from the long wire and the resistance R carries a current I_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.

(*c*) The bar is suddenly stopped at time T. The current through resistance R is found to be $I_1/4$ at time 2 T. Find the value of L/R in terms of the other given quantities.

(*I.I.T. 2002*)

Thought Process

1. When the bar AB moves away, magnetic flux linked with the circuit increases due to increase in enclosed area of the circuit. As a result, induced e.m.f. $\dfrac{d\phi}{dt}$ *is produced in the circuit due to rate of change of magnetic flux linked with the circuit. Also induced e.m.f.* $-L\dfrac{dI}{dt}$ *is produced in the inductance due to rate of current through it. The induced e.m.f. in the inductance will oppose the e.m.f.* $\dfrac{d\phi}{dt}$.

2. When the bar AB is suddenly stopped, the current in LR-circuit decays according to the relation,

$$I = I_0\, e^{-\frac{R}{L}t}$$

Sol. (*a*) Let $\dfrac{d\phi}{dt}$ be rate of change of magnetic flux linked with the circuit ; I, the current and $\dfrac{dI}{dt}$, rate of change of current in the circuit at any instant. Then,

$$\frac{d\phi}{dt} + \left(-L\frac{dI}{dt}\right) = IR$$

$$\text{or} \quad \frac{d\phi}{dt} = IR + L\frac{dI}{dt} \qquad ...(i)$$

It is the required relation between I, $\dfrac{dI}{dt}$ and $\dfrac{d\phi}{dt}$.

(*b*) From equation (*i*), we have

$$d\phi = IR\,dt + L\,dI \quad \text{or} \quad d\phi = R\,dq + L\,dI$$

$$\text{or} \quad dq = \frac{1}{R}[d\phi - L\,dI] \qquad ...(ii)$$

Let ϕ_1 be magnetic flux linked with the circuit at time $t = 0$ (when bar AB is at a distance x_0 from the straight current) and ϕ_2 be magnetic flux linked with the circuit at time $t = T$ (when bar AB is at a distance $2x_0$ from the straight conductor. At $t = 0$, current through the circuit is zero and at time $t = T$, current through the circuit is I_1.

Integrating both sides of equation (*ii*) within proper limits, we have

$$\int_0^q dq = \frac{1}{R}\left[\int_{\phi_1}^{\phi_2} d\phi - L\int_0^{I_1} dI\right]$$

$$\text{or} \quad |\,q\,|_0^q = \frac{1}{R}\left[\,|\,\phi\,|_{\phi_1}^{\phi_2} - L\,|\,I\,|_0^{I_1}\right]$$

$$\text{or} \quad q = \frac{1}{R}[(\phi_2 - \phi_1) - L\,I_1] \qquad ...(iii)$$

To evaluate $\phi_2 - \phi_1$ **:** Suppose that the bar AB is at a distance x from the straight conductor at any time t. Then, magnetic field at the bar AB at time t,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\,I_0}{x} = \frac{\mu_0\, I_0}{2\pi x}$$

Here, I_0 is current passed through the straight conductor. Further, suppose that the conductor moves through an tesimally small distance dx in time dt. Then, small ge in magnetic flux linked with the circuit,

$$d\phi = B\,(l\,dx) = \frac{\mu_0\, I_0\, l}{2\pi x}\, dx$$

Therefore, change in magnetic flux, when the bar moves initial distance x_0 to distance $2\,x_0$ from the straight uctor,

$$\phi_2 - \phi_1 = \int_{x_0}^{2x_0} \frac{\mu_0\, I_0\, l}{2\pi x}\, dx$$

$$= \frac{\mu_0\, I_0\, l}{2\pi} \int_{x_0}^{2x_0} \frac{1}{x}\, dx = \frac{\mu_0\, I_0\, l}{2\pi} \left| \log_e x \right|_{x_0}^{2x_0}$$

$$= \frac{\mu_0\, I_0\, l}{2\pi} (\log_e 2\,x_0 - \log_e x_0) = \frac{\mu_0\, I_0\, l}{2\pi} \log_e 2$$

Substituting for $\phi_2 - \phi_1$ in equation (*iii*), we get

$$q = \frac{1}{R}\left[\frac{\mu_0\, I_0\, l}{2\pi} \log_e 2 - L\, I_1\right]$$

(*c*) When the bar is suddenly stopped at time T, current through the LR-circuit is I_1. It, then, decays with time according to relation,

$$I = I_1\, e^{-\frac{R}{L}t} \qquad \text{...(iv)}$$

At time 2 T, current in the circuit is found to be $I_1/4$. Thus, when $t = 2\,T - T = T$, $I = I_1/4$

Setting the above condition in equation (*iv*), we have

$$\frac{I_1}{4} = I_1\, e^{-\frac{R}{L}T} \qquad \text{or} \qquad e^{\frac{R}{L}T} = 4$$

$$\text{or} \quad \frac{R}{L}T = \log_e 4 \qquad \text{or} \qquad \frac{L}{R} = \frac{T}{\log_e 4}$$

Chapter 2. Transient current

Problem 2.01. An inductor of inductance L = 400 mH resistors of resistances R_1 = 2 Ω and R_2 = 2 Ω are nected to a battery of e.m.f. E = 12 V as shown in Fig. The internal resistance of the battery is negligible. The ch S is closed at time t = 0. What is the potential drop ss L as a function of time ? After the steady state is hed, the switch is opened. What is the direction and the nitude of current through R_1 as a function of time ?

(*I.I.T. 2001*)

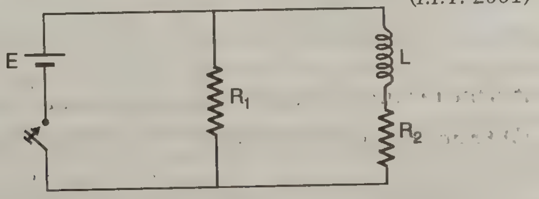


Fig. 2.01

Sol. Here, $R_1 = R_2 = 2\,\Omega$; L = 400 mH = 0·4 H

and $E = 12\,V$

When at t = 0, switch S is closed : Current equal to $\frac{E}{R_1}$

$\frac{12}{2}$ *i.e.* 6 A will flow through resistance R_1, while current eries combination of L and R_2 will grow according to the ation,

$$I = I_0\left(1 - e^{-\frac{R_2}{L}t}\right) \qquad \text{...(i)}$$

ere $I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\,A$

Therefore, equation (*i*) becomes

$$I = 6\left(1 - e^{-\frac{2}{0\cdot4}t}\right) = 6\left(1 - e^{-5t}\right)$$

Hence, potential drop across L at any time t,

$$V = L\frac{dI}{dt} = L\frac{d}{dt}\left[6\,(1 - e^{-5t})\right] = 0\cdot4 \times 6\left[0 - 6^{-5t}\,(-5)\right]$$

or $\qquad V = 12\, e^{-5t}$

When after steady state, switch is opened : The current rough the inductance decays through series combination resistances R_1 and R_2 with time according to the relation,

$$I = I_0\, e^{-\frac{R_1 + R_2}{L}t}$$

$$\text{or} \qquad I = 6\, e^{-\frac{2+2}{0\cdot4}t} = 6\, e^{-10\,t}$$

The direction of current through R_1 will be from its lower to upper end.

Problem 2.02. In the circuit shown in Fig. 2.02, the battery is an ideal one with e.m.f. V. The capacitor is initially uncharged. The switch S is closed at time t = 0.

(*a*) Find the charge Q on the capacitor at time t.

(*b*) Find the current in AB at time t. What is the limiting value as $t \to \infty$?

(*I.I.T. 1998*)

Sol. (*a*) Here, at $t = 0$, charge on capacitor, Q = 0

When the switch S is closed, the current begins to flow in the circuit and the capacitor starts charging. Let at any time t, Q be charge on the capacitor and I be the current in the circuit from the battery. The current divides in various parts of the circuit in accordance with Kirchhoff's first law as shown in the figure.

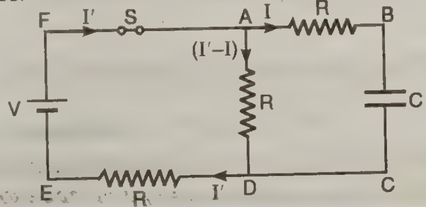


Fig. 2.02

In the closed part FABCDEF of the circuit : According to Kirchhoff's second law, we have

$$V = I\,R + \frac{Q}{C} + I'\,R$$

$$\text{or} \qquad I'\,R = V - \frac{Q}{C} - I\,R \qquad \text{...(i)}$$

In the closed part ABCDA of the circuit : According to Kirchhoff's second law, we have

$$I\,R + \frac{Q}{C} - (I' - I)\,R = 0$$

$$\text{or} \qquad 2\,I\,R + \frac{Q}{C} - I'\,R = 0 \qquad \text{...(ii)}$$

From equation (*i*), substituting for I′ R in equation (*ii*), we have

$$2IR + \frac{Q}{C} - \left(V - \frac{Q}{C} - IR\right) = 0$$

or $$V = \frac{2Q}{C} + 3IR \qquad ...(iii)$$

Setting $I = \frac{dQ}{dt}$, equation (*iii*) becomes

$$V = \frac{2Q}{C} + 3R\frac{dQ}{dt}$$

or $$\frac{dQ}{dt} = \frac{V}{3R} - \frac{2Q}{3CR} = -\frac{2}{3CR}\left(Q - \frac{VC}{2}\right)$$

or $$\frac{dQ}{Q - VC/2} = -\frac{2}{3CR}\,dt$$

Integrating both sides of the above equation within proper limits, we have

$$\int_0^Q \frac{dQ}{Q - VC/2} = -\frac{2}{3CR}\int_0^t dt$$

or $$\left|\log_e\left(Q - \frac{VC}{2}\right)\right|_0^Q = -\frac{2}{3CR}\,|\,t\,|_0^t$$

or $$\log_e\left(Q - \frac{VC}{2}\right) - \log_e\left(-\frac{VC}{2}\right) = -\frac{2}{3CR}(t - 0)$$

or $$\log_e \frac{Q - VC/2}{-VC/2} = -\frac{2}{3CR}t$$

or $$\log_e\left(1 - \frac{2}{VC}Q\right) = -\frac{2}{3CR}t$$

or $$1 - \frac{2}{VC}Q = e^{-\frac{2}{3CR}t}$$

or $$Q = \frac{VC}{2}\left(1 - e^{-\frac{2}{3CR}t}\right)$$

It gives the charge on the capacitor at any time *t*.

(*b*) Now, current in the closed part ABCDA,

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[\frac{VC}{2}\left(1 - e^{-\frac{2}{3CR}t}\right)\right]$$

$$= \frac{VC}{2}\left(0 - e^{-\frac{2}{3CR}t}\right) \times \left(-\frac{2}{3CR}\right)$$

$$= \frac{V}{3R}e^{-\frac{2}{3CR}t}$$

The current through branch AB of the circuit *i.e.* (I′ – I) can be found by using equation (*ii*). Dividing both sides of equation (*ii*) by R, we have

$$2I + \frac{Q}{CR} - I' = 0$$

or $$I' - I = I + \frac{Q}{CR}$$

Substituting for I and Q, we have

$$I' - I = \frac{V}{3R}e^{-\frac{2}{3CR}t} + \frac{1}{CR} \times \frac{VC}{2}\left(1 - e^{-\frac{2}{3CR}t}\right)$$

$$= \frac{V}{3R}e^{-\frac{2}{3CR}t} + \frac{V}{2R} - \frac{V}{2R}e^{-\frac{2}{3CR}t}$$

or $$I' - I = \frac{V}{2R} - \frac{V}{6R}e^{-\frac{2}{3CR}t}$$

It gives the current through branch AB of the circuit at any time *t*.

The limiting value of current through AB, as *t* → ∞,

$$I' - I = \underset{t \to \infty}{\text{Lt}}\left[\frac{V}{2R} - \frac{V}{6R}e^{-\frac{2}{3CR}t}\right]$$

$$= \frac{V}{2R} - \frac{V}{6R}e^{-\frac{2}{3CR}\times\infty} = \frac{V}{2R} - 0 = \frac{V}{2R}$$

Chapter 3. Alternating currents

Problem 3.01. A lamp having a hot resistance of 25 Ω is not allowed to pass current more than 5 A. Find the value of the inductance, which must be used in series with the lamp, which is supplied by an a.c. of maximum r.m.s. 325 V at 50 Hz. (*I.S.M. Dhanbad, 1993*)

Sol. Here, R = 25 Ω ; E_v = 325 V ; *f* = 50 Hz

Since the lamp cannot allow current more than 5 A to pass through it, it follows that I_v = 5 A. Let L be the required inductance that should be used in series with the lamp.

Now, $$I_v = \frac{E_v}{\sqrt{R^2 + (\omega L)^2}}$$

∴ $$5 = \frac{325}{\sqrt{25^2 + (2\pi \times 50 \times L)^2}}$$

or $$25^2 + (314{\cdot}16\,L)^2 = \left(\frac{325}{5}\right)^2 = 65^2$$

or $$314{\cdot}16\,L = \sqrt{65^2 - 25^2} \quad \text{or} \quad L = 0{\cdot}191\ \text{H}$$

Problem 3.02. An LCR circuit has L = 10 m H, R = 3 Ω and C = 1 μF connected in series to a source of 15 cos ω t volt. Calculate the current amplitude and the average power dissipated per cycle at a frequency that is 10% lower than the resonant frequency. (*Roorkee, 1988*)

Sol. Here, L = 10 mH = 10^{-2} H ; R = 3 Ω ;

C = 1 μF = 10^{-6} F

If ω_0 is resonant angular frequency, then

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 10^{-6}}}$$

$$= 10^4\ \text{rad s}^{-1}$$

If ω is angular frequency 10% lower than ω_0, then

$$\omega = \omega_0 - \frac{\omega_0 \times 10}{100} = 10^4 - \frac{10^4 \times 10}{100} = 9 \times 10^3\ \text{rad s}^{-1}$$

The values of inductive and capacitive reactances at gular frequency ω are given by

$$X_L = \omega L = 9 \times 10^3 \times 10^{-2} = 90\ \Omega$$

$$\text{and } X_C = \frac{1}{\omega C} = \frac{1}{9 \times 10^3 \times 10^{-6}} = 111 \cdot 11\ \Omega$$

Therefore, impedance of LCR-circuit at angular equency ω,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (90 - 111 \cdot 11)^2}$$

$$= 21{\cdot}32\ \Omega$$

Now, applied voltage, $E_v = 15 \cos \omega t$

It follows that $E_0 = 15$ V

Therefore, current amplitude, $I_0 = \dfrac{E_0}{Z} = \dfrac{15}{21 \cdot 32} = \mathbf{0{\cdot}704\ A}$

$$\text{Average power, } P = E_v\, I_v \cos \phi = \frac{1}{2} E_0\, I_0 \left(\frac{R}{Z}\right)$$

$$= \frac{1}{2} \times 15 \times 0 \cdot 704 \times \frac{3}{21 \cdot 32} = \mathbf{0{\cdot}743\ W}$$

Problem 3.03. A 750 Hz, 20 V source is connected to a esistance of 100 W, an inductance of 0·1803 H and a apacitance of 10μF all in series. Calculate the time in which he resistance (thermal capacity = 2 J °C^{-1}) will get heated y 10°C. *(Roorkee, 1989)*

Sol. Here, R = 100 Ω ; L = 0·1803 H ;

$C = 10\ \mu F = 10^{-5}$ F ; $E_v = 20$ V ; $f = 750$ Hz

$$\text{Now,} \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{100^2 + \left(2\pi \times 750 \times 0 \cdot 1803 - \frac{1}{2\pi \times 750 \times 10^{-5}}\right)^2}$$

$$= \sqrt{100^2 + (849 \cdot 64 - 21 \cdot 22)^2} = \sqrt{100^2 + 828 \cdot 42^2}$$

$$= 834{\cdot}43\ \Omega$$

Average power, $P = E_v\, I_v \cos \phi$

$$= E_v \left(\frac{E_v}{Z}\right) \frac{R}{Z} = E_v^{\,2} \times \frac{R}{Z^2}$$

$$= \frac{20^2 \times 100}{(834 \cdot 43)^2} = 0 \cdot 0574\ \text{W}$$

If t is the required time, then

$P \times t$ = thermal capacity × rise in temperature

or $0{\cdot}0574 \times t = 2 \times 10$

or $\quad t = \dfrac{2 \times 10}{0 \cdot 0574} = \mathbf{348{\cdot}4\ s}$

Chapter 4. Electric machines and devices

Problem 4.01. A very small circular loop of area 5×10^{-4} m^2, resistance 2 Ω and negligible inductance is nitially coplanar and concentric with a much larger fixed ircular loop of radius 0·1 m. A constant current of 1 A is assed in the bigger loop and the smaller loop is rotated with angular velocity ω rad s^{-1} about a diameter. Calculate i) the flux linked with the smaller loop, (*ii*) induced e.m.f. nd (*iii*) induced current in the smaller loop, as a function of time. *(Roorkee, 1992)*

Sol. Fig. 4.01 shows two concentric circular loops of radii a_1 and a_2, such that $a_1 > a_2$. The current $I_1 = 1$ A is passed through the larger loop and the smaller loop is rotated with angular velocity ω about its diameter X′OX.

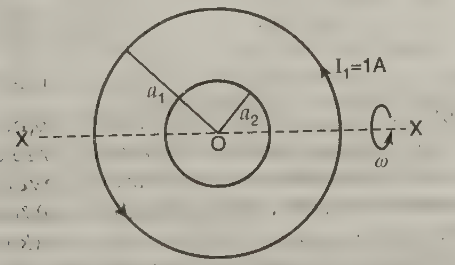


Fig. 4.01.

Also, radius of larger loop, $a_1 = 0{\cdot}1$ m

and area of smaller loop,

$$A_2 = \pi a_2^2 = 5 \times 10^{-4}\ \text{m}^2.$$

Magnetic field produced by the larger loop,

$$B_1 = \frac{\mu_0 I_1}{2 a_1} = \frac{4\pi \times 10^{-7} \times 1}{2 \times 0 \cdot 1}$$

$$= 2\pi \times 10^{-6}\ \text{Wb m}^{-2}$$

(*i*) Initially, the loops are coplanar. Therefore, at $t = 0$, the magnetic field produced due to larger loop will be perpendicular to the plane of the smaller loop. At any time t, the plane of the smaller loop will turn through angle θ (= ω t), so that magnetic flux linked with the smaller loop at any time t,

$$\phi_2 = B_1 A_2 \cos \omega t$$

$$\text{or} \quad \omega_2 = 2\pi \times 10^{-6} \times 5 \times 10^{-4} \cos \omega t$$

$$= \boldsymbol{\pi \times 10^{-9} \cos \omega t\ \text{Wb}}$$

(*ii*) If e_2 is induced e.m.f. produced in smaller loop at any time t, then

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(\pi \times 10^{-9} \cos \omega t)$$

$$= \pi \times 10^{-9}(-\sin \omega t) \times \omega$$

$$\text{or} \quad e_2 = \pi \times 10^{-9} \sin \omega t\ \text{V}$$

(*iii*) If I_2 is induced current in the smaller loop at any time t, then

$$I_2 = \frac{e_2}{R_2} = \frac{\pi \times 10^{-9}\ \omega \sin \omega t}{2}$$

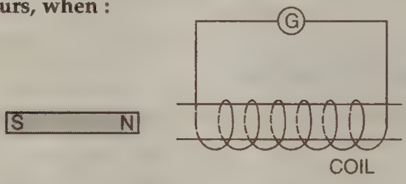
$$= \boldsymbol{5\pi \times 10^{-10}\ \omega \sin \omega t\ \text{A}}$$

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

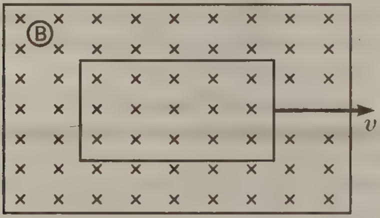
Chapter 1. Electromagnetic induction

PART I. TEXT-BASED QUESTIONS

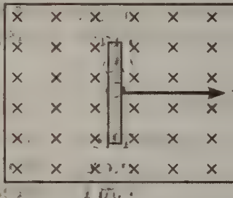
1. **The SI unit of magnetic flux is :**
   (A) gauss (B) oersted
   (C) tesla (D) weber
2. **In electromagnetic induction, the induced e.m.f. is independent of**
   (A) change of flux. (B) time.
   (C) number of turns in the coil.
   (D) resistance of the coil.
3. **Magnitude of e.m.f. produced in a coil, when a magnet is inserted into it ; does not depend upon :**
   (A) number of turns in the coil.
   (B) resistance of the coil.
   (C) magnetic moment of magnet.
   (D) speed of the magnet.
4. **A magnet is moved towards a coil, first quickly and then slowly. The induced e.m.f. produced is**
   (A) larger in first case. (B) smaller in first case.
   (C) equal in both cases.
   (D) larger or smaller, depending upon resistance of the coil.
5. **Refer to the figure. Deflection in the galvanometer (G) occurs, when :**



   (A) the magnet is pushed into the coil.
   (B) the magnet is rotated into the coil.
   (C) the magnet is stationary at the centre of the coil.
   (D) the number of turns in the coil is reduced.
   (Karnataka Entrance, 1992, 1990)
6. **Faraday's law of electromagentic induction is related to the law of conservation of**
   (A) charge (B) energy
   (C) mass (D) angular momentum.
   (C.E.T. 1998)
7. **A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere. The current induced in the loop is**



   (A) $\frac{B l v}{R}$ clockwise (B) $\frac{B l v}{R}$ anticlockwise
   (C) $\frac{2 B l v}{R}$ anticlockwise
   (D) zero *(I.I.T. 1989)*
8. **A square shaped coil of side 10 cm and number of turns 500 is placed perpendicular to magnetic flux lines, which is changing at the rate of 1·0 T s^{-1}. The e.m.f. induced in the coil is**
   (A) 0·1 V (B) 0·5 V
   (C) 1·0 V (D) 5·0 V *(C.P.M.T. 19*
9. **A magnetic field of 2×10^{-2} T acts at right angles to a of area 100 cm^2 with 50 turns. The average e.m.f. indu in the coil is 0·1 V, when it is removed from the field in time t. The value of t is**
   (A) 0·1 s (B) 0·01 s
   (C) 1 s (D) 10 s *(C.B.S.E. 19*
10. **Magnetic flux ϕ (in weber) in a closed circuit of resista 10 Ω varies with time t (in seconds) according to equation : $\phi = 5t^2 - 4t + 1$.**
    The magnitude of the induced e.m.f. in the circui t = 0·2 s is :
    (A) 0·4 V (B) – 0·4 V
    (C) – 2 V (D) 2 V
11. **Lenz's law is a consequence of the law of conservation**
    (A) charge only. (B) momentum only.
    (C) energy only. (D) energy and momentum.
    (Pre-degree Kerala, 1
12. **The direction of induced e.m.f. is given by**
    (A) Fleming's left hand rule.
    (B) Fleming's right hand rule.
    (C) Lenz's law. (D) Biot Savart's law.
13. **A straight conductor of length 0·4 m is moved with a sp of 7 m s^{-1} perpendicular to a magnetic field of inten 0·9 Wb m^{-2}. The induced e.m.f. across the conductor i**
    (A) 5·04 V (B) 1·26 V
    (C) 2·52 V (D) 25·2 V *(Roorkee, 1*
14. **A piece of wire is passed through the gap between the p of a magnet in 0·1 s. An e.m.f. of 4×10^{-3} V is induce the wire. The magnetic flux between the poles of magnet is**
    (A) 10 Wb (B) 4×10^{-4} Wb
    (C) 4×10^{-2} Wb (D) 0·1 Wb
15. **A conductor of length l is moved perpendicular to unif magnetic field B with velocity v as shown in figure. e.m.f. induced across the length of the conductor is**
    (A) zero
    (B) B l v
    (C) B l v/2
    (D) B l^2 v/2 *(C.P.M.T. 1993)*



16. **An aeroplane in which the distance between the tip the wings is 50 m is flying horizontally with a spee 360 km h^{-1} over a place, where the vertical componen earth's magnetic field is $2{\cdot}0 \times 10^{-4}$ T. The poten difference between the tips of the wings would be**
    (A) 0·1 V (B) 1·0 V
    (C) 0·2 V (D) 0·01 V *(C.P.M.T. 19*
17. **A car moves on a plane road. The induced e.m.f. in the connecting the two wheels is maximum, when it mov**
    (A) eastwards at the equator.
    (B) westwards at the equator.
    (C) eastwards in the latitude 45°.
    (D) at the poles.
18. **The two rails of a railway track insulated from each ot and the ground are connected to millivoltmeter. What is reading of the millivoltmeter, when a train passes speed of 180 km h^{-1} along the track, given that the vert**

component of earth's magnetic field is 0·2 × 10^{-4} Wb m^{-2} and rails are separated by 1 m ?

(A) 10^{-2} V　(B) 10 mV
(C) 1 V　(D) 1 mV　*(I.I.T. 1981)*

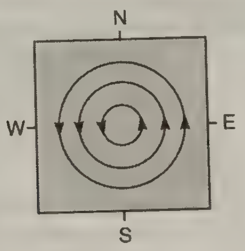
19. **Two rails of a railway track 1 m apart and insulated from each other and the ground are connected to a millivoltmeter. What is the reading on the millivoltmeter, when a train travels at a speed of 20 m s^{-1} along the track. Given that the horizontal component of earth's magnetic field is 0·2 × 10^{-4} T.**

(A) 4 mV　(B) 0·4 mV
(C) zero　(D) 10 mV

20. **Eddy currents are**

(A) induced currents due to a changing magnetic flux.
(B) induced currents due to a high magnetic flux.
(C) induced currents in a inhomogeneous material.
(D) unstable currents in a conductor.
(Karnataka Entrance, 1991)

21. **When a sheet of metal is placed in magnetic field, which changes from zero to a maximum value, induced currents are set up in the direction shown in the diagram. What is the direction of the magnetic field ?**



(A) Into the plane of paper.
(B) Out of the plane of the paper.
(C) East to west.　(D) North to south.

22. **An induction furnace works on the principle of**

(A) self induction　(B) mutual induction
(C) eddy currents　(D) Faraday's laws *(C.E.T. 1999)*

23. **The SI unit of inductance, the henry, can be written as**

(A) weber ampere^{-1}　(B) volt second ampere^{-1}
(C) joule ampere^{-2}　(D) ohm second.　*(I.I.T. 1998)*

24. **When the current in a coil changes from 2 A to 4 A in 0·05 s, an e.m.f. of 8 V is induced in the coil. The coefficient of self-induction of the coil is**

(A) 0·1 H　(B) 0·2 H
(C) 0·4 H　(D) 0·8 H
(M.N.R. 1993, 1980 ; C.P.M.T. 1991)

25. **The current in a self-inductance L = 40 mH is to be increased uniformly from 1 A to 11 A in 4 millisecond. The e.m.f. induced in the inductor during the process is**

(A) 100 V　(B) 0·4 V
(C) 40 V　(D) 440 V　*(C.B.S.E. 1990)*

26. **When the current through a solenoid increases at a constant rate, the induced current**

(A) is constant and is in the direction of inducing current.
(B) is constant and is opposite to the direction of the inducing current.
(C) increases with time and is in the direction of the inducing current.
(D) increases with time and opposite to the direction of the inducing current.　*(M.N.R. 1992)*

27. **When the number of turns and length of a solenoid are doubled keeping the area of cross-section same, the inductance**

(A) is halved　(B) remains the same
(C) is doubled　(D) becomes four times.
(A.F.M.C. 1995)

28. **A solenoid has 2,000 turns wound over a length of 0·3 m. The area of its cross-section is 1·2 × 10^{-3} m^2. Around its central part, a coil of 300 turns is wound. If an initial current of 2 A is reversed in 0·25 s, the induced e.m.f. produced in the coil is**

(A) 6 × 10^{-4} V　(B) 4·8 × 10^{-2} V
(C) 6 × 10^{-2} V　(D) 48 V.

29. **The mutual inductance between two coils depends on**

(A) medium between the coils only.
(B) separation between two coils only.
(C) both (A) and (B).
(D) none of (A) and (B).　*(C.P.M.T. 1993)*

30. **Two coils have a mutual inductance 0·005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where I_0 = 10 A and $\omega = 100\,\pi$ rad s^{-1}. The maximum value of e.m.f. in the second coil is**

(A) $2\,\pi$　(B) $5\,\pi$
(C) $6\,\pi$　(D) $12\,\pi$　(C.B.S.E. 1998)

PART II. THOUGHT-BASED QUESTIONS

31. **A copper ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of the ring. The acceleration of the falling magnet is**

(A) equal to that due to gravity.
(B) less than that due to gravity.
(C) more than that due to gravity.
(D) depends on the diameter of the ring and the length of the magnet.　*(C.B.S.E. 1996 ; M.N.R. 1987)*

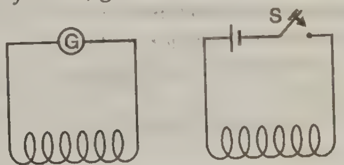
32. **A circular loop of radius R, carrying current I, lies in XY-plane with its centre at origin. The total magnetic flux through XY-plane is**

(A) directly proportional to I
(B) directly proportional to R
(C) inversely proportional to R
(D) zero.　*(I.I.T. 1999)*

33. **In electromagnetic induction, the induced charge is independent of :**

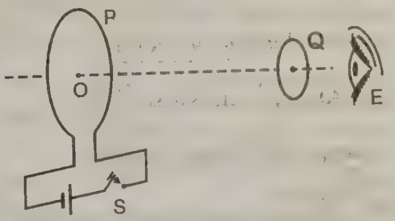
(A) time.　(B) change of flux.
(C) resistance of the coil. (D) none of the above.

34. **The switch (S) is closed and opened in the circuit periodically. Then, galvanometer (G) shows**



(A) no deflection.
(B) a steady deflection to the left.
(C) a steady deflection to the right.
(D) periodic deflections.　*(Karnataka Entrance, 1990)*

35. **As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is**

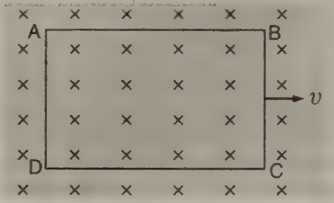


closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q_1} flows in Q. The switch remains

closed for a long time. When S is opened, a current I_{Q_2} flows in Q. Then the direction of I_{Q_1} and I_{Q_2} (as seen by E) are

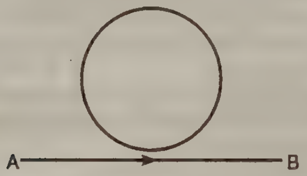
(A) both clockwise (B) both anticlockwise
(C) respectively clockwise and anticlockwise
(D) respectively anticlockwise and clockwise. (*I.I.T. 2002*)

36. **A metallic square loop ABCD is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. Electric field is induced :**



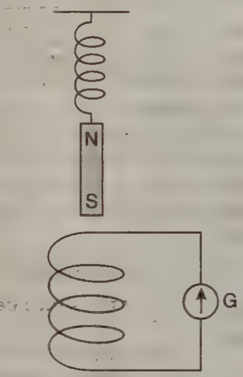
(A) in AD, but not in BC
(B) in BC, but not in AD
(C) neither in AD nor in BC
(D) in both AD and BC. (*I.I.T. 2001*)

37. **An electron moves along the line AB, which lies in the same plane as a circular loop of conducting wires. What will be the direction of the current induced in the loop, if any ?**



(A) No current will be induced.
(B) The current will be clockwise.
(C) The current will be anticlockwise.
(D) The current will change direction as the electron passes by.

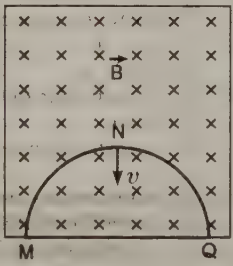
38. **A magnet NS is suspended from a spring and while it oscillates, the magnet moves in and out of the coil C. The coil is connected to a galvanometer G. Then, as the magnet oscillates,**



(A) G shows deflection to the left and right with constant amplitude.
(B) G shows deflection on one side.
(C) G shows no deflection.
(D) G shows deflection to the left and right, but the amplitudes steadily decreases.

(*Karnataka Entrance, 1989*)

39. **A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ as shown in the figure. At the position MNQ, the speed of the ring is v. Then, the potential difference developed across the ring is**



(A) zero.
(B) $\frac{1}{2} B v R^2$ and M is at higher potential.
(C) $\pi R B v$ and Q is at higher potential.
(D) $2 R B v$ and Q is at higher potential. (*I.I.T. 1996*)

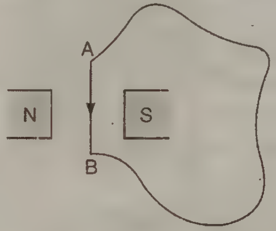
40. **A metal sheet is placed in front of the pole of a very strong magnet. A force is needed to**

(A) hold the sheet in its position, if the metal is magnetic.
(B) hold the sheet in its position, if the metal is non-magnetic.
(C) move the sheet away with a uniform velocity, if the metal is magnetic.
(D) move the sheet away with a uniform velocity, if the metal is non-magnetic.

41. **A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant, uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following :**

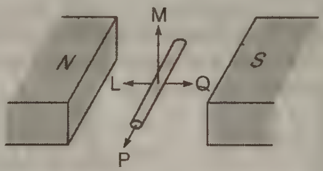
(A) The entire rod is at the same electric potential.
(B) There is an electric field in the rod.
(C) The electric potential is highest at the centre of the rod and decreases towards its ends.
(D) The electric potential is lowest at the centre of the rod and increases towards its ends. (*I.I.T. 1998*)

42. **In what way should the conductor AB be moved in a magnetic field, such that the current flows as shown in the figure ?**



(A) Normally into the plane of the paper.
(B) Vertically downward.
(C) Vertically upwards. (D) None of the above.

43. **A potential difference will be induced between the ends of the conductor shown in the figure, when the conductor moves along**



(A) P (B) Q
(C) L (D) M (*A.I.I.M.S. 1982*)

44. **Dimensions of coefficient of self induction is**

(A) $[M L^2 T^2 A^{-2}]$ (B) $[M L^2 T^{-3} A^{-2}]$.
(C) $[M L^2 T^{-3}]$ (D) none of the above.

(*B.H.U. 1997*)

45. **The self-inductance of a straight conductor is**

(A) zero. (B) infinity.
(C) very large. (D) very small.

46. **In an inductor of self inductance L = 2 mH, current changes with time according to relation $I = t^2 e^{-t}$. At what time, e.m.f. is zero ?**

(A) 4 s (B) 3 s
(C) 2 s (D) 1 s (*C.B.S.E. 2001*)

47. **If N is the number of turns in a coil, the value of self-inductance varies as**

(A) N^0 (B) N
(C) N^2 (D) N^{-2} (*C.B.S.E. 1993*)

A short-circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be
(A) halved (B) the same
(C) doubled (D) quadrupled (I.I.T. 2002)

A small square loop of wire of side l is placed inside a large square loop of wire of side L (L >> l). The loops are co-planar and their centres coincide. The mutual inductance of the system is proportional to
(A) l/L (B) l^2/L
(C) L/l (D) L^2/l (I.I.T. 1998)

Two pure inductors each of self-inductance L are connected in parallel but are well separated from each other. Then, the total inductance is
(A) L (B) 2 L
(C) $\frac{\text{L}}{2}$ (D) $\frac{\text{L}}{4}$

A coil is wound on a former of rectangular cross- section. If all the linear dimensions of the former are increased by a factor of 2 and the number of turns per unit length of the coil remain unchanged, the self-inductance of the coil increases by a factor of
(A) 2 (B) 4
(C) 8 (D) 16 (A.I.I.M.S. 1980)

52. A constant current is passed through a solenoid. If an iron rod is inserted along the axis of the solenoid, which of the following quantities will increase ?
(A) Rate of production of heat due to Joule heating.
(B) Magnetic field at the centre of the solenoid.
(C) Magnetic flux linked with the solenoid.
(D) Self inductance of the solenoid.

53. Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current, which increases with time. In response, loop-B
(A) remains stationary.
(B) is attracted by the loop-A.
(C) is repelled by the loop-A.
(D) rotates about its C.M., with C.M. fixed.
(I.I.T. 1999)

Chapter 2. Transient current

RT I. TEXT-BASED QUESTIONS

In an LR-circuit, the current rises to final value slowly, when
(A) L is small. (B) L is large.
(C) L is zero. (D) none of the above.

An inductance of 2 H and a resistance of 10 Ω are connected to a battery of 5 V. The time constant of the circuit is
(A) 20 s (B) 0·2 s
(C) 5 s (D) 100 s

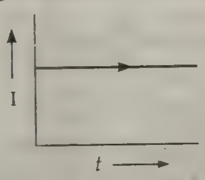
In Q. No. 2, the final steady current is
(A) 0·5 A (B) 2·5 A
(C) 2 A (D) 0·25 A

In Q. No. 2, the initial rate of increase of current is
(A) 0·5 A s^{-1} (B) 2·5 A s^{-1}
(C) 2 A s^{-1} (D) 0·25 A s^{-1}

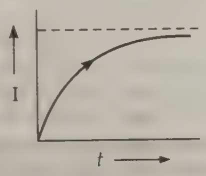
In Q. No. 2, the rate of increase of current in the circuit at the instant current is 0·3 A, is
(A) 0·2 A s^{-1} (B) 0·015 A s^{-1}
(C) 1 A s^{-1} (D) 0·05 A s^{-1}

A coil of inductance 8 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1·0 A at the time (approximately)
(A) 500 s (B) 20 s
(C) 35 ms (D) 1 ms (I.I.T. 1999)

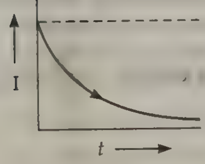
In an LR-circuit, the current grows with time as shown in figure :



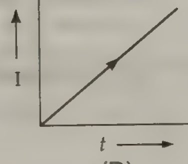
(A)



(B)

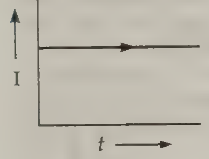


(C)

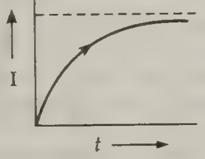


(D)

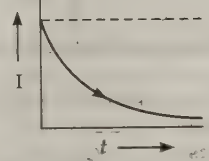
8. In an LR-circuit, the variation of voltage across the inductance with time during growth is :



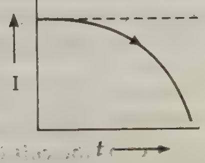
(A)



(B)



(C)



(D)

9. Which of the following has dimensions same as that of time ?
(A) L C (B) $\frac{\text{R}}{\text{L}}$
(C) $\frac{\text{L}}{\text{R}}$ (D) $\frac{\text{C}}{\text{L}}$
(C.E.T. 1999 ; C.B.S.E. 1996)

10. A 10 μF capacitor charged to 100 V is discharged through 10^{-6} Ω resistor. The time taken for half the charge to leak is :
(A) 693 s (B) 6·93 s
(C) 0·693 s (D) 13·86 s
Given that $\log_e 2 = 0·693$

11. The time constant of CR-circuit is
(A) 1/C R (B) C/R
(C) C R (D) R/C (C.B.S.E. 1992)

12. The dimensions of CR is that of
(A) time. (B) inverse time.
(C) square of time. (D) square of inverse time.
(C.B.S.E. 1995)

13. An inductor may store energy in
(A) its electric field. (B) its coils.
(C) its magnetic field.
(D) both in electric and magnetic fields. (C.B.S.E. 1990)

14. In Q. No. 2, when the steady state is reached, the energy stored in the inductance in the form of magnetic field is
(A) 0·25 J (B) 6·25 J
(C) 2 J (D) 0·0625 J

15. A 100 mH coil carries a current of 1 A. Energy stored in the form of magnetic field is
(A) 0·5 J (B) 1 J
(C) 0·05 J (D) 0·1 J *(C.B.S.E. 1992)*

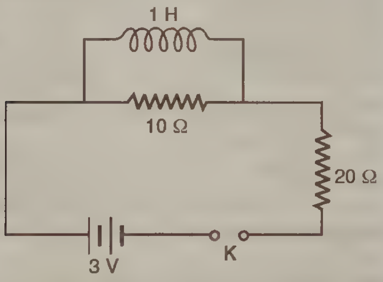
16. If a capacitor of capacitance C is connected in series with an inductor of inductance L, then the angular frequency is
(A) $\sqrt{LC}$ (B) LC
(C) $\sqrt{L/C}$ (D) $1/\sqrt{LC}$ *(A.F.M.C. 1997)*

17. An LC-circuit cannot produce oscillations, when
(A) capacitance is large. (B) inductance is large.
(C) resistance is large. (D) none of the above.

PART II. THOUGHT-BASED QUESTIONS

18. An inductor and a resistance are connected to a battery. Which of the following quantities is not zero just after the connection ?
(A) current in the circuit.
(B) e.m.f. produced in the inductor.
(C) energy stored in the inductor.
(D) power delivered by the battery.

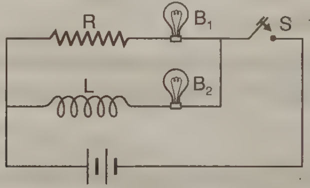
19. In the circuit shown, what is the final value of current in 10 Ω resistance on inserting the plug in the key K ?



(A) 0·3 A (B) 0·15 A
(C) 0·1 A (D) zero.

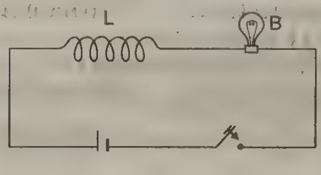
20. In the circuit shown in last question, what is the final value of current in 20 Ω resistance, on inserting the plug in the key K ?
(A) 0·3 A (B) 0·15 A
(C) 0·1 A (D) zero.

21. Figure shows two bulbs B_1 and B_2, resistor R and an inductor L. When the switch S is turned off,



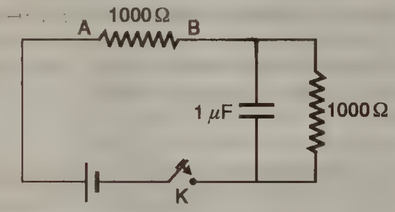
(A) both B_1 and B_2 die out promptly.
(B) both B_1 and B_2 die out with some delay.
(C) B_1 dies out promptly, but B_2 with some delay.
(D) B_2 dies out promptly, but B_1 with some delay.
(C.P.M.T. 1989)

22. In the circuit shown, the bulb will become suddenly bright, if



(A) contact is made or broken.
(B) contact is made. (C) contact is broken.
(D) will not become bright at all. *(C.B.S.E. 1*

23. When the key K is pressed at time $t = 0$, then which o following statement about the current I in the resisto of the given circuit is true ?

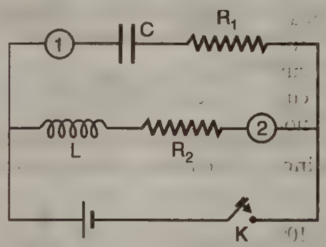


(A) I = 1 mA at all t.
(B) I = 2 mA at all t.
(C) I oscillates between 1 mA and 2 mA.
(D) at $t = 0$, I = 2 mA and with time, it goes to 1 mA.
(A.F.M.C. 1996 ; C.B.S.E. 1

24. A coil of wire having finite inductance and resistance a conducting ring placed co-axially within it. The co connected to a battery at time $t = 0$, so that a ti dependent current $I_1(t)$ starts flowing through the co $I_2(t)$ is the current induced in the ring and $B(t)$ is magnetic field at the axis of the coil due to $I_1(t)$, then function of time ($t > 0$), the product $I_2(t)\,B(t)$
(A) increases with time.
(B) decreases with time.
(C) does not vary with time.
(D) passes through a maximum. *(I.I.T. 2*

25. L, C and R represent the physical quantities inducta capacitance and resistance respectively. The combina which have the dimensions of frequency are
(A) $\frac{1}{RC}$ (B) $\frac{RL}{C}$
(C) $\frac{R}{LC}$ (D) $\frac{C}{L}$ *(B.I.T. 1*

26. In a circuit given in the figure, 1 and 2 are ammeters. If K is pressed to complete the circuit, the readin ammeters is



(A) zero in both 1 and 2.
(B) maximum in both 1 and 2.
(C) zero in 1, maximum in 2.
(D) maximum in 1, zero in 2. *(C.B.S.E. 1*

27. In an oscillating circuit, the restoring force is must. I LC-circuit, the restoring force is provided by
(A) a capacitor (B) an inductance
(C) a resistance (D) none of the above.

Chapter 3. Alternating current

PART I. TEXT-BASED QUESTIONS

1. **A normal domestic electric supply is an alternating current, whose average value is**
(A) zero.
(B) half the peak value.
(C) the peak value multiplied by $\frac{\pi}{2}$.
(D) the peak value divided by $\frac{\pi}{2}$.
(Karnataka Entrance, 1992)

2. **The mean value of a.c. is related to peak value of a.c. by the equation**
(A) $I_m = \sqrt{2}\, I_0$ (B) $I_0 = \sqrt{2}\, I_m$
(C) $I_m = \frac{2 I_0}{\pi}$ (D) $I_0 = \frac{2 I_m}{\pi}$

3. **The r.m.s. value of a.c. is related to peak value of a.c. by the equation**
(A) $I_{r.m.s.} = \sqrt{2}\, I_0$ (B) $I_0 = \sqrt{2}\, I_{r.m.s.}$
(C) $I_{r.m.s.} = \frac{2 I_0}{\pi}$ (D) $I_0 = \frac{2 I_{r.m.s.}}{\pi}$

4. **The equation of an a.c. voltage is**
$V = 200 \sin 50\, \pi\, t$.
Then, the r.m.s. value of voltage is
(A) $100\sqrt{2}$ V (B) $200\sqrt{2}$ V
(C) 100 V (D) 400 V
(Pre-degree Kerala, 1989)

5. **An electric lamp is connected to 220 V-50 Hz supply. The peak voltage in the circuit is**
(A) 110 V (B) 220 V
(C) 311 V (D) 320 V *(A.F.M.C. 1996)*

6. **An electric bulb rated 220 V is connected to 220 V-5 Hz source. Then, the bulb**
(A) does not glow. (B) glows intermittently.
(C) glows continuously. (D) fuses.

7. **An inductor**
(A) allows a.c. to pass, but blocks d.c.
(B) allows d.c. to pass, but blocks a.c.
(C) allows both a.c. and d.c. to pass.
(D) blocks d.c.

8. **In an a.c. circuit containing inductance,**
(A) current lags the voltage by 90°.
(B) current lags the voltage by 180°.
(C) current leads the voltage by 90°.
(D) none of the above. *(Karnataka Entrance, 1984)*

9. **The inductive reactance of an inductance of $\frac{1}{\pi}$ H at 50 Hz is**
(A) 100 Ω (B) $\frac{50}{\pi}$ Ω
(C) $\frac{\pi}{50}$ Ω (D) $50\,\pi$ Ω

10. **An alternating e.m.f. is applied to a pure inductance, such that inductive reactance is 10 Ω. If the frequency of a.c. is doubled, the reactance will become**
(A) 5 Ω (B) 10 Ω
(C) 15 Ω (D) 20 Ω

11. **When a.c. flows through a capacitor, then the current**
(A) leads e.m.f. by $\frac{\pi}{2}$. (B) lags e.m.f by $\frac{\pi}{2}$.
(C) is in phase with e.m.f.
(D) none of the above.

12. **A capacitor**
(A) offers easy path to a.c., but blocks d.c.
(B) offers easy path to d.c., but blocks a.c.
(C) offers easy path to both a.c. and d.c.
(D) blocks a.c.

13. **The reactance of a capacitor of capacitance C is X. If both the frequency and capacitance be doubled, then new reactance will be**
(A) X (B) 2 X
(C) 4 X (D) X/4 *(C.B.S.E. 2001)*

14. **An alternating e.m.f. is applied to a pure capacitance, such that the capacitive reactance is 10 Ω. If the frequency of a.c. is doubled, the reactance will become**
(A) 5 Ω (B) 10 Ω
(C) 15 Ω (D) 20 Ω

15. **An a.c. voltage source $E = 200\sqrt{2} \sin 100\, t$ is connected across a circuit containing an a.c. ammeter and a capacitor of capacitance 1 μF. The reading of ammeter is**
(A) 10 mA (B) 20 mA
(C) 40 mA (D) 80 mA
(C.P.M.T. 1991 ; N.C.E.R.T. 1984)

16. **In an a.c. circuit, the current**
(A) is always in phase with the e.m.f.
(B) always leads the e.m.f.
(C) always lags the e.m.f.
(D) any of the above, depending upon the elements (L, C or R) of the circuit.

17. **A 220 V-50 Hz a.c. source is connected to an inductance of 0·2 henry and a resistance of 20 ohm in series. What is the current in the circuit ?**
(A) 10 A (B) 5 A
(C) 33·3 A (D) 3·33 A
(Karnataka Entrance, 1991)

18. **In an LCR-series a.c. circuit, the current**
(A) is always in phase with the voltage.
(B) always lags the generator voltage.
(C) always leads the generator voltage.
(D) may lag behind or lead the voltage.
(C.E.T. 1998 ; C.P.M.T. 1992)

19. **The impedance of an a.c. (LCR) circuit is given by**
(A) $\sqrt{R^2 - (X_C + X_L)^2}$ (B) $\sqrt{R^2 + (X_C - X_L)^2}$
(C) $\sqrt{R^2 - (X_C - X_L)^2}$ (D) $\sqrt{R^2 + (X_C + X_L)^2}$
(Karnataka Entrance, 1984)

20. **For the current in LCR-circuit to be maximum,**
(A) $\omega^2 = L\,C$ (B) $\omega^2 = \frac{1}{L\,C}$
(C) $\omega = \frac{1}{L\,C}$ (D) $\omega = L\,C$

21. **A 1 mH inductance and 10 μF capacitance, when connected in series to an a.c. source, possess equal reactance. The angular frequency of the a.c. source is**
(A) 10^4 (B) 100
(C) 10 (D) $200\,\pi$

22. **In Q. 21, the value of the reactance of either of them is**
(A) 0·1 Ω (B) 0·01 Ω
(C) $0{\cdot}2\,\pi$ Ω (D) 10 Ω

23. **An a.c. circuit using an inductor and a capacitor in series has a maximum current. If L = 0·5 H and C = 8 μF, then the angular frequency of a.c. voltage will be**
(A) 500 (B) 5×10^5
(C) 4000 (D) 5000 *(C.P.M.T. 1991)*

24. In an LCR-circuit, the capacitance is changed from C to 4 C. For the same resonant frequency, the inductance should be changed from L to
(A) 2 L (B) L/2
(C) 4 L (D) L/4

25. An LCR-circuit is connected to a source of a.c. current under resonant condition. The phase difference between the applied voltage and the current in the circuit is
(A) $\pi/4$ (B) $\pi/2$
(C) π (D) zero *(C.E.T. 1999)*

26. For the current in LCR-circuit to be maximum,
(A) $X_L = 0$ (B) $X_C = 0$
(C) $X_L = X_C$ (D) $R = X_L + X_C$

27. The average power of an a.c. circuit is
(A) $E_v I_v$ (B) $E_v I_v \cos\phi$
(C) $E_v I_v \sin\phi$ (D) zero.

28. In an a.c. circuit, with phase voltage V and current I, the power dissipated is
(A) $\frac{1}{2}VI$ (B) $\frac{1}{\sqrt{2}}VI$
(C) VI
(D) depends on the phase angle between V and I *(C.B.S.E. 1997)*

29. The average power dissipation in a pure inductance is
(A) zero (B) $\frac{1}{2}LI^2$
(C) $\left(\frac{1}{2}LI^2\right)^2$ (D) $\sqrt{\frac{1}{2}LI^2}$

30. In an a.c. circuit, E and I are given by
$$E = 100 \sin 100\,t \quad \text{(in volt)}$$
and $$I = 100 \sin(100\,t + \pi/3) \quad \text{(in mA)}$$
The power dissipated in the circuit is
(A) 10^4 W (B) 10 W
(C) 2500 W (D) 5·0 W *(C.P.M.T. 1991)*

31. A current $I = I_0 \sin(\omega t + \pi/2)$ flows in a circuit across which an alternating potential $E = E_0 \sin \omega t$ is applied. The power consumed in the circuit is
(A) $E_0 I_0/2$ (B) $E_0 I_0$
(C) E (D) zero
(C.E.T. 1998 ; similar C.B.S.E. 1995)

32. The minimum and maximum values of power factor in an a.c. circuit are
(A) 0 and 1 (B) 1 and ∞
(C) 0 and ∞ (D) 0·1 and 1

33. An a.c. supply may be used directly for all these except for one. Identify the one, for which it cannot be used :
(A) Heating (B) Lighting
(C) Transforming voltage
(D) Electroplating. *(Karnataka Entrance, 1992, 1989)*

PART II. THOUGHT-BASED QUESTIONS

34. An alternating current is given by
$$I = I_1 \sin \omega t + I_2 \cos \omega t.$$
The r.m.s. value of the current is given by
(A) $(I_1 + I_2)/\sqrt{2}$ (B) $(I_1 + I_2)/2$
(C) $\sqrt{I_1^2 + I_2^2}/\sqrt{2}$ (D) $\sqrt{I_1^2 + I_2^2}/2$

35. In a.c. circuits, the a.c. meters measure
(A) r.m.s. values. (B) peak values.
(C) mean values. (D) mean square values.

36. Hot wire ammeters are used for measuring
(A) both a.c. and d.c. (B) only a.c.
(C) only d.c. (D) neither a.c. nor d.c.

37. In an electrical circuit carrying an alternating current, moving coil galvanometer cannot be used, because
(A) the net magnetic field is too high.
(B) the net magnetic field is zero.
(C) there is too much energy loss.
(D) the voltage is too high. *(Karnataka Entrance, 1987*

38. Ohm's law (E = I R)
(A) can never be applied to a.c.
(B) can be applied to a.c. in the same manner as to d.c.
(C) can be applied to a.c., but after replacing R by (impedance).
(D) none of the above.

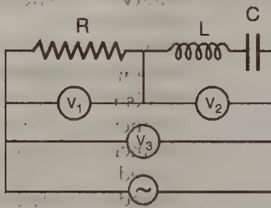
39. When 100 V d.c. is applied across a solenoid, a current o 1·0 A flows in it. When 100 V a.c. is applied across the sam coil, the current drops to 0·5 A . If the frequency of a. source is 50 Hz, the impedance and inductance of solenoi are
(A) 200 Ω and 0·55 H (B) 100 Ω and 0·86 H
(C) 200 Ω and 1·0 H (D) 100 Ω and 0·93 H.
(C.P.M.T. 1990

40. The reactance of an a.c. circuit is zero. Then, the circui contains
(A) an inductor, but no capacitor.
(B) a capacitor, but no inductor.
(C) both an inductor and a capacitor.
(D) neither an inductor nor a capacitor.

41. In a series resonant circuit, the a.c. voltage across resistanc R, inductance L and capacitance C are 5 V, 10 V and 10 respectively. The a.c. voltage applied to the circuit will be
(A) 20 V (B) 10 V
(C) 5 V (D) 25 V
(Karnataka Entrance, 1994

42. A wire of resistance R is connected in series with a inductor of reactance ωL. Then quality factor of LR-circui is
(A) $R/\omega L$ (B) $\omega L/R$
(C) $\frac{R}{\sqrt{R^2 + \omega^2 L^2}}$ (D) $\frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$ *(C.B.S.E. 200*

43. A resistance R , an inductance L , a capacitance C an voltmeters V_1, V_2 and V_3 are connected to an alternatin source of e.m.f. as shown. When the frequency of th source is increased, then at resonant frequency, the readin of voltmeter V_3 is equal to
(A) that of V_1.
(B) that of V_2.
(C) both of V_1 and V_2.
(D) neither of V_1 nor of V_2.



44. An inductor is connected to an a.c. source of e.m.f. If th energy stored inside the inductor in the form of magneti field changes from maximum to minimum value in 5 m then the frequency of the source is
(A) 20 Hz (B) 50 Hz
(C) 200 Hz (D) 500 Hz

45. In a.c. circuit, the power is consumed only in
(A) inductance. (B) capacitance.
(C) resistance. (D) all the given above.

46. The current flowing in a coil is 3 A and the powe consumed is 108 W. If the a.c. source is of 120 V-50 Hz, th resistance in the circuit is
(A) 24 Ω (B) 36 Ω
(C) 12 Ω (D) 6 Ω *(A.F.M.C. 1998*

. For a series LCR-circuit, the power loss at resonance is

(A) $\dfrac{V^2}{\omega L - 1/\omega C}$ (B) $\dfrac{V^2}{\omega C}$

(C) $I^2 \omega C$ (D) $I^2 R$ (C.B.S.E. 2002)

. The power in a.c. circuit is given by

$$P = E_v I_v \cos\phi$$

The value of power factor cos ϕ in series LCR-circuit at resonance is

(A) zero (B) 1

(C) 1 / 2 (D) $1/\sqrt{2}$ (C.P.M.T. 1993)

49. Wattless current is possible, only in

(A) resistive circuit. (B) non-resistive circuit.

(C) LR-circuit. (D) LCR-circuit.

Chapter 4. Electrical machines and devices

RT I. TEXT-BASED QUESTIONS

. A transformer is used to change

(A) mechanical energy into electrical energy.

(B) electrical energy into mechanical energy.

(C) high voltage d.c. into low voltage d.c.

(D) high voltage a.c. into low voltage a.c.

(Karnataka Entrance, 1991)

. A transformer steps up or steps down

(A) a.c. only (B) d.c. only

(C) either a.c. or d.c. (D) a.c. mixed with d.c.

(Karnataka Entrance, 1988)

. The primary winding of a transformer has 500 turns, whereas its secondary has 5000 turns. The primary is connected to an a.c. supply 20 V-50 Hz. The secondary will have an output of

(A) 200 V-50 Hz (B) 200 V-500 Hz

(C) 2 V-50 Hz (D) 2 V-5 Hz

(C.B.S.E. 1997 ; A.F.M.C. 1995)

. A transformer has 200 windings in the primary and 400 windings in the secondary. The primary is connected to an a.c. supply of 110 V and a current of 10 A flows in it. The voltage across the secondary and the current in it respectively are

(A) 55 V, 20 A (B) 440 V, 5 A

(C) 220 V, 10 A (D) 220 V, 5 A

. A step-up transformer operates on a 230 V line and supplies a current of 2 A . The ratio of primary and secondary windings is 1 : 25. The primary current is

(A) 12·5 A (B) 50 A

(C) 8·8 A (D) 25 A

(C.B.S.E. 1998 ; A.I.I.M.S. 1989)

. A step-down transformer is used to reduce 220 V supply to 11 V. The primary draws 5 A of current, while the secondary 90 A. The efficiency of the transformer is

(A) 20% (B) 40%

(C) 70% (D) 90%

. The core used in transformers and other electromagnetic devices is laminated, so as

(A) to increase the magnetic field.

(B) to increase the magnetic flux.

(C) to reduce the magnetism in the core.

(D) to reduce eddy current losses in the core.

(N.C.E.R.T. 1984)

. Power is transmitted from a power station at a very high a.c. voltage, because

(A) the rate of transmission is faster at high voltage.

(B) it is more economical due to less power loss.

(C) power can not be generated at low voltage.

(D) theft of power is not possible at high voltage.

. In an induction coil, the coefficient of mutual inductance is 4 H. If the current of 5 A in the primary coil is cut off in 1/1500 s, the e.m.f. at the terminals of the secondary coil will be

(A) 15 kV (B) 60 kV

(C) 10 kV (D) 30 kV (N.C.E.R.T. 1984)

. In an induction coil, the secondary e.m.f. is :

(A) zero during break of the circuit.

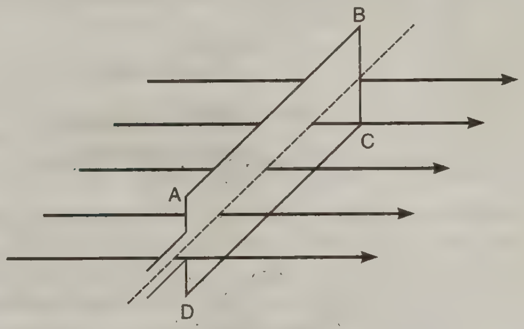
(B) very high during make of the circuit.

(C) zero during make of the circuit.

(D) very high during the break of the circuit.

(Karnataka Entrance, 1994)

11. A rectangular coil ABCD is rotated anticlockwise with a uniform angular velocity about the axis shown in the figure. The axis of rotation of the coil as well as the magnetic field B are horizontal. The induced e.m.f. in the coil would be maximum, when the plane of the coil



(A) is vertical.

(B) makes an angle of 45° with the magnetic field.

(C) is parallel to the magnetic field.

(D) makes an angle of 30° with the magnetic field.

(C.P.M.T. 1988)

12. A dynamo works on the principle of :

(A) electromagnetic induction.

(B) induced current. (C) induced magnetism.

(D) Faraday's effect. (A.F.M.C. 1998)

13. Split ring commutators are used in which of the following ?

(A) a.c. dynamo (B) d.c. dynamo

(C) a.c. motor (D) In all the above.

14. If the speed of rotation of a dynamo is doubled, then the induced e.m.f. will

(A) become half. (B) become double.

(C) become four times. (D) remain unchanged.

(A.I.I.M.S. 1999)

15. A dynamo develops 0·5 ampere at 6 volt. The power produced is

(A) 3 watt (B) 12 watt

(C) 0·083 watt (D) none of these.

(Karnataka Entrance, 1992, 1990)

16. In a region of uniform magnetic induction $B = 10^{-2}$ T, a circular coil of radius 30 cm and resistance π^2 ohm is rotated about an axis, which is perpendicular to the direction of B and which forms a diameter of coil. If the coil rotates at 200 r.p.m., the amplitude of alternating current induced in the coil is

(A) $4\pi^2$ mA (B) 30 mA

(C) 6 mA (D) 200 mA (C.B.S.E. 1990)

17. A motor

(A) converts electrical energy into mechanical energy.

(B) converts mechanical energy into electrical energy.

(C) converts electrical energy into magnetic energy.
(D) produces mechanical energy.
(Karnataka Entrance, 1990)

18. **The device that does not work on the principle of mutual induction is :**
(A) induction coil. (B) motor.
(C) tesla coil. (D) transformer.
(Karnataka Entrance, 1994)

19. **As motion of an armature increases, back e.m.f.**
(A) increases. (B) decreases.
(C) remains the same.
(D) may increase or decrease, depending on the nature of the armature.

20. **Radio frequency choke is air-cored to**
(A) keep frequency low.
(B) keep frequency high.
(C) keep inductive reactance low.
(D) keep inductive reactance high.

21. **A coil and an electric bulb are connected in series with an a.c. source. On introducing a soft iron bar inside the coil, the intensity of the bulb will**
(A) increase. (B) decrease.
(C) fluctuate. (D) remain the same.

22. **A motor starter is a :**
(A) fixed resistance. (B) a variable resistance.
(C) variable inductance. (D) variable capacitor.

PART II. THOUGHT-BASED QUESTIONS

23. **The number of turns in the primary and the secondary coils of a transformer are 1000 and 3000 respectively. If 80 V a.c. is applied to the primary coil of a transformer, then the potential difference per turn of the secondary coil will be**
(A) 240 V (B) 2400 V
(C) 0·24 V (D) 0·08 V. *(C.P.M.T. 1990)*

24. **The magnitude of the e.m.f. across the secondary of a transformer does not depend upon**
(A) the magnitude of the e.m.f. applied across the primary.
(B) the number of turns in the primary.
(C) the number of turns in the secondary.
(D) the resistance of the primary and the secondary.

25. **Soft iron is used in many parts of electrical machines for**
(A) low hysteresis loss and low permeability.
(B) low hysteresis loss and high permeability.
(C) high hysteresis loss and low permeability.
(D) high hysteresis loss and high permeability.

26. **The essential difference between an a.c. dynamo and d.c. dynamo is that**
(A) a.c. dynamo has an electromagnet, but d.c. dynamo has a permanent magnet.
(B) a.c. dynamo has slip rings, but d.c. dynamo has commutators.
(C) a.c. dynamo has coil would on soft iron, but the coil in d.c. dynamo is wound on copper.
(D) a.c. dynamo generates much high voltage.

27. **A dynamo is sometimes said to generate electricity. It actually acts as a source of—**
(A) charge. (B) magnetism.
(C) e.m.f. (D) energy.

28. **As the speed of an electric fan increases, current**
(A) increases. (B) decreases.
(C) remains the same.
(D) becomes maximum, when speed is maximum.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

Chapter 1. Electromagnetic induction

1. (D) 2. (D) 3. (B) 4. (A) 5. (A & D) 6. (B) 7. (D) 8. (D) 9. (A) 10. (D)
11. (C) 12. (B & C) 13. (C) 14. (B) 15. (B) 16. (B) 17. (D) 18. (D) 19. (C) 20. (A)
21. (A) 22. (C) 23. (A, B, C & D) 24. (B) 25. (A) 26. (B) 27. (C) 28. (B) 29. (C)
30. (B) 31. (B) 32. (D) 33. (A) 34. (D) 35. (D) 36. (D) 37. (C) 38. (D) 39. (D)
40. (A, C & D) 41. (B) 42. (A) 43. (D) 44. (A) 45. (A) 46. (C) 47. (C) 48. (B)
49. (B) 50. (C) 51. (C) 52. (B, C & D) 53. (C)

Chapter 2. Transient current

1. (B) 2. (B) 3. (A) 4. (B) 5. (C) 6. (D) 7. (B) 8. (C) 9. (C) 10. (B)
11. (C) 12. (A) 13. (C) 14. (A) 15. (C) 16. (D) 17. (C) 18. (B) 19. (D) 20. (B)
21. (C) 22. (A) 23. (D) 24. (D) 25. (A) 26. (D) 27. (B)

Chapter 3. Alternating current

1. (D) 2. (C) 3. (B) 4. (A) 5. (C) 6. (B) 7. (B) 8. (A) 9. (A) 10. (D)
11. (A) 12. (A) 13. (D) 14. (A) 15. (B) 16. (D) 17. (D) 18. (D) 19. (B) 20. (B)
21. (A) 22. (D) 23. (A) 24. (D) 25. (D) 26. (C) 27. (B) 28. (D) 29. (A) 30. (C)
31. (D) 32. (A) 33. (D) 34. (C) 35. (A) 36. (B) 37. (B) 38. (C) 39. (A) 40. (C & D)
41. (C) 42. (B) 43. (A) 44. (B) 45. (C) 46. (C) 47. (D) 48. (B) 49. (B)

Chapter 4. Electrical machines and devices

1. (D) 2. (A) 3. (A) 4. (D) 5. (B) 6. (D) 7. (D) 8. (B) 9. (D) 10. (D)
11. (C) 12. (A) 13. (B) 14. (B) 15. (A) 16. (C) 17. (A) 18. (B) 19. (A) 20. (C)
21. (B) 22. (B) 23. (D) 24. (D) 25. (B) 26. (B) 27. (C) 28. (B)

UNIT
V

LECTROMAGNETIC WAVES

Electromagnetic Waves

eems we have strong reason to conclude that light itself (including radiant heat, and
er radiations, if any) is an electromagnetic disturbance in the form of waves propaga-
through field according to electromagnetic laws." **– James Clerk Maxwell**

CHAPTER 1

Electromagnetic Waves

1.01 DISPLACEMENT CURRENT AND ORIGIN OF ELECTROMAGNETIC WAVES

The origin of electromagnetic waves is connected to the concept of ***displacement ent.***

When a capacitor is allowed to charge in an electric circuit [Fig. 1.01], the current s through the connecting wires. As the capacitor charges, charge accumulates ne two plates of the capacitor and as a result, a changing electric field is produced ss the gap between its two plates. If a small compass needle is placed in the gap veen the two plates of the capacitor, it shows deflection indicating the existence agnetic field. The conduction current* that flows through the connecting wires not pass across the gap between the two plates of the capacitor. However, xwell assumed the flow of current across the gap between the plates of the acitor also, so as to explain the existence of magnetic field between the plates. amed it as displacement current and attributed its origin to the varying electric d between the plates of the capacitor. Thus, according to Maxwell, the varying tric field across the plates of the capacitor gives rise to magnetic field.

Key point

The origin of displacement current is due to varying electric field between the two plates of a charged capacitor.

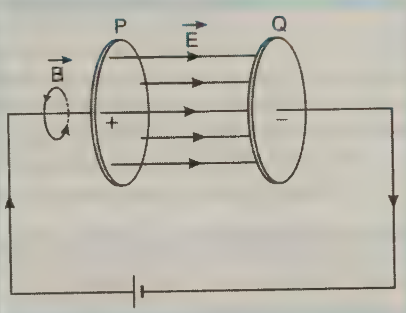


Fig. 1.01

Maxwell's concept of displacement current makes a continuous flow of current uch an electric circuit. Thus, through the connecting wires, there is flow of duction current I_C (no displacement current) and through the gap across the tes of the capacitor, there is flow of displacement current I_D (no conduction rent). In the light of this, Ampere's circuit law $\left(\oint \vec{B}\cdot\vec{dl}=\mu_0 I\right)$ was modified to

$$\oint \vec{B}\cdot\vec{dl}=\mu_0\,(I_C+I_D) \qquad \text{...(1.01)}$$

It is called ***modified Ampere's circuital law*** or ***Ampere-Maxwell's circuital law***.

The displacement current arising due to varying electric field is given by

$$I_D=\varepsilon_0\,\frac{d\phi_E}{dt} \qquad \text{...(1.02)}$$

In a circuit, the conduction current and the displacement currents are always ual. Therefore, modified Ampere-circuital law may be expressed as

$$\oint \vec{B}\cdot\vec{dl}=\mu_0\left(I_C+\varepsilon_0\,\frac{d\phi_E}{dt}\right) \qquad \text{...(1.03)}$$

Key point

When a capacitor is connected across a battery, through the connecting wires, there is flow of conduction current, while through the gap between the plates of capacitor, there is flow of displacement current.

The following inferences can be drawn from the above discussion :

1. *The conduction and displacement currents are individually discontinuous, **but the** o currents together possess the property of continuity through any closed electric circuit.*

2. *The displacement current is precisely equal to the conduction current, when **the two** present in different parts of the circuit.*

3. *The displacement current arises due to rate of change of electric flux (or electric field) ween the two plates of the capacitor.*

4. *Just as the conduction current is, the displacement current is also the source of gnetic field.*

The conduction current produces magnetic field due to charges in motion, nereas the displacement current produces magnetic field due to the time rate of ange of electric field. Thus, in the space between the plates of the capacitor, both e electric and magnetic fields exist. The time varying electric field between the

Key point

The conduction and displacement currents are entirely different from each other. However, displacement current produces magnetic field in the same manner, as the conduction current does.

---

* The current due to flow of charge is called conduction current.

plates of the capacitor is directed perpendicular to the plane of the plates and the time varying magnetic field produced by the electric field is along the perimeter of a circle in a plane parallel to the plates of the capacitor [Fig. 1.02]. Thus, the time varying electric field (E) and magnetic field (B) are perpendicular to each other.

The two mutually perpendicular time varying electric and magnetic fields may be represented by

$$E = E_0 \sin \omega (t - x/c) \quad ...(1.04)$$

and

$$B = B_0 \sin \omega (t - x/c) \quad ...(1.05)$$

respectively. The two fields combine to constitute electromagnetic wave, which propagates in space in a direction perpendicular to the directions of both the fields.

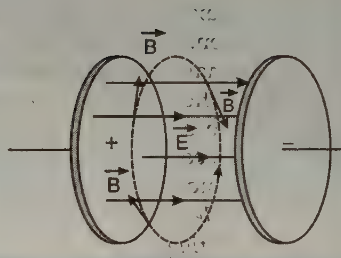


Fig. 1.02

1.02. MAXWELL'S EQUATIONS

In fact, while attempting to express the laws of electricity and magnetism, namely ***Gauss's law in electrostatics, Gauss's law in magnetism, Faraday's law of electromagnetic induction*** and ***Ampere's circuital law*** in terms of electric and magnetic fields and their sources, Maxwell noticed an inconsistency in Ampere's circuital law and an asymmetry in the laws of electromagnetism. The concept of displacement current removed the inconsistency in Ampere's circuital law and the laws of electromagnetism also became symmetrical. The above said four basic laws of physics were stated by Maxwell in the form of four equations, called ***Maxwell's equations***. These equations predict that the time and space dependent electric and magnetic fields propagate as transverse waves, called ***electromagnetic waves***, which possess velocity equal to that of light.

Key point

Maxwell's equations are mathematica formulation of Gauss's law in electrostatics, Gauss's law in magnetism, Faraday's law of electromagnetic inductio and Ampere's circuital law.

The four Maxwell's equations are as given below :

1. Gauss's law in electrostatics. It states that *the total electric flux through any closed surface is always equal to* $\frac{1}{\varepsilon_0}$ *times the net charge enclosed by the surface.*

Mathematically, $$\oint \vec{E} \cdot \vec{dl} = \frac{q}{\varepsilon_0} \quad ...(1.06)$$

The equation (1.06) is called ***Maxwell's first equation.***

2. Gauss's law in magnetism. It states that *the net magnetic flux crossing any closed surface is always zero.*

Mathematically, $$\oint \vec{B} \cdot \vec{dS} = 0 \quad ...(1.07)$$

A direct consequence of Gauss's law in magnetism is that an isolated magnetic monopole does not exist. The equation (1.07) is called ***Maxwell's second equation.***

3. Faraday's law of electromagnetic induction. It states that *the induced e.m.f. produced in a circuit is numerically equal to rate of change of magnetic flux through it.*

Mathematically, $$\varepsilon = -\frac{d\phi_B}{dt}$$

The negative sign indicates that the induced e.m.f. so produced opposes the rate of change of magnetic flux.

Since e.m.f. can be defined as the line integral of electric field, the above relation may be expressed as

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt} \quad ...(1.08)$$

In this form, the law states that *the line integral of electric field along a closed path is equal to rate of change of magnetic flux through the surface bounded by that closed path.*

The equation (1.08) is called ***Maxwell's third equation.***

4. Maxwell-Ampere's circuital law. It states that *the line integral of magnetic field along a closed path is equal to* μ_0 *times the total current (sum of conduction and displacement currents) threading the surface bounded by that closed path.*

Mathematically, $$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right) \quad ...(1.09)$$

The equation (1.09) is called ***Maxwell's fourth equation.***

1.03. ELECTROMAGNETIC WAVES

Maxwell gave the idea of electromagnetic waves, while Hertz and other scientists produced and studied these waves experimentally. The following is a brief account of the history of electromagnetic waves :

In 1865, Maxwell predicted the existence of electromagnetic waves on the basis hese equation. According to him, an accelerated charge produces a sinusoidal* e varying magnetic field, which in turn produces a sinusoidal time varying ctric field. The two fields so produced are mutually perpendicular and are rces of each other. The mutually perpendicular time varying electric and gnetic fields constitute electromagnetic waves, which propagate in space in a ection perpendicular to the directions of both the fields.

Thus, *the electromagnetic waves consist of sinusoidally time varying electric and gnetic fields acting at right angles to each other as well as at right angles to the direction ropagation of waves.*

Key point

Electromagnetic waves are transverse in nature.

The velocity of electromagnetic in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \, \varepsilon_0}} \qquad \text{...(1.10)}$$

ere μ_0 (= $1{\cdot}257 \times 10^{-6}$ T m A^{-1}) and ε_0 (= $8{\cdot}854 \times 10^{-12}$ C^2 N^{-1} m^{-2}) are resctively the absolute permeability and absolute permittivity of the free space. The ocity of electromagnetic waves in free space (vacuum) is equal to velocity of ht in vacuum *i.e.* 3×10^8 m s^{-1}.

1.04. PROPERTIES OF ELECTROMAGNETICWAVES

A few important properties of electromagnetic waves are as given below :

1. Electromagnetic waves propagate in the form of varying electric and gnetic fields, such that the two fields are perpendicular to each other and also the direction of propagation of the wave. In other words, electromagnetic waves transverse in nature.

2. Electromagnetic waves are produced by accelerated charges.

3. Electromagnetic waves do not require any material medium for their opagation.

4. In free space, electromagnetic waves travel with a velocity,

$$c = \frac{1}{\sqrt{\mu_0 \, \varepsilon_0}} = 3 \times 10^8 \text{ m s}^{-1}$$

with a velocity equal to that of light in free space.

In a material medium , velocity of electromagnetic waves is given by

$$v = \frac{1}{\sqrt{\mu \, \varepsilon}},$$

here μ and ε are absolute permeability and absolute permittivity of that medium.

Key point

The frequency of electromagnetic waves is its inherent characteristic. When an electromagnetic wave travels from one medium to another, its wavelength changes but frequency remains unchanged.

5. The electromagnetic waves obey the principle of superposition.

6. The variations in the amplitudes of electric and magnetic fields in the ectromagnetic waves always take place at the same time and at the same point in e space. Thus, the ratio of the amplitudes of electric and magnetic fields is always nstant and it is equal to velocity of the electromagnetic waves. Mathematically,

$$\frac{E}{B} = c$$

7. The energy in electromagnetic waves is divided equally between the electric nd magnetic field vectors.

8. The electric vector is responsible for optical effects due to an electromagnetic ave. For this reason, electric vector is called ***light vector.***

1.05. TRANSVERSE NATURE OF ELECTROMAGNETICWAVES

The electromagnetic waves consists of sinusoidally time varying electric and agnetic fields acting at right angles to each other as well as at right angles to the irection of propagation of waves. The two mutually perpendicular time varying lectric and magnetic fields may be represented by

$$E = E_0 \sin \omega \, (t - x/c)$$

and

$$B = B_0 \sin \omega \, (t - x/c)$$

espectively. The two fields combine to constitute electromagnetic wave, which ropagate in space in a direction perpendicular to the directions of both the fields.

Key point

The ratio ω/c gives the magnitude of propagation vector for an electromagnetic wave.

---

* Sine function of time.

Fig. 1.03 shows the propagation of a plane electromagnetic wave with velocity c along X-axis. The electric field vector is along Y-axis, while the magnetic field vector along Z-axis. Since the electric and magnetic fields are perpendicular to each other and to the dirction of wave propagation, the electromagnetic waves are transverse in nature.

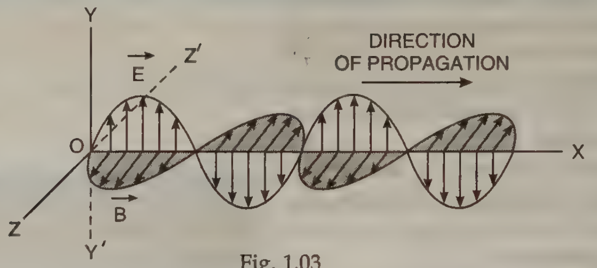


Fig. 1.03

Key point

The direction of propagation of an electromagnetic wave is given by the cross product of electric field and magnetic field vectors *i.e.* by $\vec{E} \times \vec{B}$.

As said earlier, the velocity of electromagnetic waves in free space (vacuum) is given by

$$c = \frac{1}{\sqrt{\mu_0 \, \varepsilon_0}}$$

The velocity of electromagnetic waves is also given as the ratio of the amplitudes of the time varying electric and magnetic fields *i.e.*

$$c = \frac{E_0}{B_0} \qquad ...(1.11)$$

1.06. ELECTROMAGNETIC SPECTRUM

Hertz and other scientists proved that it is possible to produce electromagnetic waves of wavelengths ranging from several kilometres to 6×10^{-3} m. The visible light is found to be electromagnetic waves of very short wave length produced by atoms and molecules, which behave as natural oscillators. By the end of last century, electromagnetic radiation ranging from several kilometres to 6 mm and from 10^{-6} m (infra-red) to 10^{-8} m (ultra-violet) became known. Then, X-rays were discovered, which were found to be electromagnetic waves of even shorter wavelength. The study of radioactive phenomenon led to the discovery of γ-rays, which are electro-magnetic waves of wavelength even shorter than X-rays. Then, originally existing gaps in the electromagnetic spectrum got filled on the production of corresponding electromagnetic waves experimentally.

Key point

All the types of electromagnetic waves travel with the same speed in free space.

The orderly distribution of electromagnetic waves (according to wavelength or frequency) in the form of distinct groups having widely differing properties is called electromagnetic spectrum.

Main parts of electromagnetic spectrum. The electromagnetic spectrum may be divided into the following main parts :

1. γ-rays. These waves are of nuclear origin and range from 3×10^{19} to 5×10^{20} Hz. γ-rays are highly energetic radiations and are mainly emitted by radioactive substances.

2. X-rays. X-rays were discovered by the German physicist W. Rontgen. These rays are of atomic origin and are produced, when a target of an element having high atomic number is bombarded by fast moving electrons. X-rays have frequencies in the range 10^{16} to 3×10^{19} Hz. X-rays possess a high penetrating power.

3. Ultra-violet rays. Ultra-violet rays were discovered by Ritter in 1801. The ultra-violet rays are part of solar spectrum. They can be produced by the arcs of mercury and iron. They can also be obtained by passing discharge through hydrogen and xenon. The frequency of ultra-violet rays lies in the range of 8×10^{14} to 10^{16} Hz.

4. Visible light. It forms a very narrow part of the electromagnetic spectrum and its frequency ranges from 4×10^{14} to 8×10^{14} Hz. The visible light is emitted due to the atomic excitation. Human eye is sensitive to only visible part of the electromagnetic spectrum.

5. Infra-red rays. Infra-red rays were discovered by Herschell. Infra-red rays are heat radiation and therefore all hot bodies are the sources of infra-red rays. About 60% of the solar radiation is infra-red in nature. The frequency range of infra-red rays is $10^{13} - 4 \times 10^{14}$ Hz.

6. Microwaves. The microwaves are produced by oscillating electronic circuits. e frequency of microwaves lies between $10^{19} - 3 \times 10^{11}$ Hz. The microwaves are ed in radar and other communication systems.

7. Radiowaves. Like microwaves, radiowaves are also produced by oscillating ctronic circuits. The frequency of radiowaves varies from a few Hz to 10^{19} Hz. e radiowaves are used as carrier waves in radio broadcasting and TV nsmission.

Electromagnetic waves are classified according to the type of the excitation. erlapping in certain parts of the spectrum means that the corresponding wavegths can be produced by two methods. For example, 10^{-4} m waves can be oduced both with the help of an artificial oscillator and in the process of thermal diation. It may be pointed out that the physical properties of electromagnetic ves are determined by the wavelength and not by the method of their excitation. e following table gives a brief account of the electromagnetic spectrum :

Electromagnetic Spectrum

ame	Frequency range (Hz)	Wavelength range (m)	How produced
-rays	3×10^9 to 5×10^{20}	6×10^{-13} to 10^{-10}	Nuclei of atoms.
-rays	10^{16} to 3×10^{19}	10^{-10} to 3×10^{-8}	Bombardment of high Z target by electrons.
ltra-Violet rays (UV)	8×10^{14} to 10^{16}	3×10^{-8} to 4×10^{-7}	Excitation of atoms and spark.
isible light	4×10^{14} to 8×10^{14}	4×10^{-7} to 8×10^{-7}	Excitation of atoms, spark and arc flame.
nfra-Red rays (IR)	10^{12} to 4×10^{14}	8×10^{-7} to 3×10^{-4}	Excitation of atoms and molecules.
icrowaves	5×10^9 to 10^{12}	3×10^{-4} to 6×10^{-2}	Oscillating currents in special vacuum tubes.
adiowaves	3×10^3 to 3×10^{11}	10^{-3} to 10^5	Oscillating circuits.

Human eye is sensitive to only visible part of the electromagnetic spectrum. ifferent types of detectors are used to detect and study the different parts of the ectrum.

The following table gives the wavelength range of different colours of visible ght :

Visible Spectrum

Colour	Wavelength (m)	Colour	Wavelength (m)
Violet	$3{\cdot}9 \times 10^{-7}$ to $4{\cdot}5 \times 10^{-7}$	Yellow	$5{\cdot}7 \times 10^{-7}$ to $5{\cdot}9 \times 10^{-7}$
Blue	$4{\cdot}5 \times 10^{-7}$ to 5×10^{-7}	Orange	$5{\cdot}9 \times 10^{-7}$ to $6{\cdot}2 \times 10^{-7}$
Green	5×10^{-7} to $5{\cdot}7 \times 10^{-7}$	Red	$6{\cdot}2 \times 10^{-7}$ to $7{\cdot}8 \times 10^{-7}$

1.07. RADIOWAVES AND MICROWAVES

Radiowaves are produced by oscillating electric circuits having an inductor nd a capacitor. Like radiowaves, microwaves are also produced by such electric rcuits. An oscillating circuit produces time varying magnetic field, which then roduces time varying electric field. The two time varying fields constitute adiowaves, which are basically electromagnetic waves. The microwaves have omparatively smaller wavelength range as compared to the radiowaves.

Properties. 1. Radiowaves and microwaves are electromagnetic waves and ravel with a speed of 3×10^8 m s^{-1}.

2. Radiowaves obey laws of reflection and refraction.

3. Radiowaves get diffracted from obstacles coming in their path. Since vavelength of radiowaves is quite large, the size of such obstacles should also be arge.

4. Microwaves produce heat, when absorbed by matter.

Applications. 1. Radiowaves are used for wireless communication purposes. hey are used to transmit radio and TV signals.

2. Radiowaves are used in radioastronomy.

3. Microwaves are used in radar systems.

4. Microwaves are used in long distance telephone communication systems.

5. Microwave ovens are used for cooking.

6. Microwaves are used in the study of atomic and molecular structure.

1.08 INFRA-RED RAYS

Infra-red rays were discovered by Herschell. Infra-red rays are heat radiation and therefore all hot bodies are the sources of infra-red rays. About 60% of the solar radiation is infra-red in nature. The following are a few sources to produce infra-red rays :

Watch out !

Short wavelength infra-red radiation is often called an infra-red light, though it cannot excite vision !

1. ***Nernst lamp.*** The filament of Nernst lamp is made from the mixture of zirconium, thorium and cerium. When a current is passed through such a filament, then at a temperature of about 1200 K , infra-red rays are emitted.

2. ***Globar.*** It is basically a rod of silicon carbide, which when heated to a temperature of about 900 K by passing current, produces infra-red rays.

3. ***LASER.*** It is used to produce monochromatic infra-red rays. For example, He – Ne LASER gives infra-red rays of wavelength $0{\cdot}69 \times 10^{-6}$ m , $1{\cdot}19 \times 10^{-6}$ m and $3{\cdot}39 \times 10^{-6}$ m. CO_2 LASER provides infra-red rays of wavelength $10{\cdot}6 \times 10^{-6}$ m.

To detect infra-red rays, thermocouples, thermopiles, bolometers, photoconducting cells are used.

Properties. 1. Infra-red rays are electromagnetic waves and travel with a speed of 3×10^8 m s^{-1}.

2. Infra-red rays obey laws of reflection and refraction.

3. Infra-red rays can produce interference and can be polarised.

4. When allowed to fall on matter, infra-red rays produce an increase in tempera-ture.

5. Infra-red rays affect a photographic plate.

6. When absorbed by molecules, the energy of infra-red rays gets converted into molecular vibrations.

7. They are scattered less as compared to the visible light by the atmosphere. Hence, infra-red rays can travel through longer distances through atmosphere under the conditions of smoke, fog, etc. Nitrogen and oxygen gases are found to be transparent medium to all the wavelengths of infra-red rays.

Applications. 1. Infra-red rays from the sun keep the earth warm and hence help to sustain life on earth.

2. The coal deposits in the interior of earth are the result of conversion of forest wood into coal due to infra-red rays.

3. Infra-red rays are used in solar water heaters and cookers.

4. Infra-red rays photographs are used for weather forecasting.

5. Infra-red rays are used for taking photographs during the conditions of fog, smoke, etc.

6. Infra-red rays absorption spectra is used in the study of molecular structure and then to check the purity of the chemicals.

7. Infra-red rays are used for producing dehydrated fruits.

8. Infra-red rays are used to provide electrical energy to satellites by using solar cells.

9. Infra-red rays are used to treat muscular strains.

1.09. ULTRA-VIOLET RAYS

Ultra-violet rays were discovered by Ritter in 1801. The ultra- violet rays are part of solar spectrum. They can be produced by the arcs of mercury and iron. They can also be obtained by passing discharge through hydrogen and xenon.

Properties. 1. Ultra-violet rays are electromagnetic waves and travel with the speed of 3×10^8 m s^{-1}.

2. They also obey the laws of reflection and refraction.

3. They can also undergo interference and can be polarised.

4. When allowed to fall on metals, they cause the emission of photoelectrons.

5. They can affect a photographic plate.

6. Ultra-violet rays can cause flourescence in certain materials.

7. Ultra-violet rays can not pass through glass but quartz, fluorite and rock salt are transparent to them.

8. Ultra-violet rays possess the property of synthesizing vitamin D, when the skin is exposed to the sunlight.

Applications. 1. Ultra-violet rays are used for checking the mineral samples by making use of its property of causing flourescence.

2. Ultra-violet absorption spectra is used in the study of molecular structure.

3. Ultra-violet rays destroy bacteria and hence it is used for sterilizing surgical ruments.

4. As ultra-violet rays can cause photoelectric effect, they are used in burglars m.

1.10. X-RAYS

The discovery of X-rays was accidentally made by a German professor Rontgen 895. In laboratory, X-rays can be produced by using Coolidge X-ray tube.

Properties. X-rays have the following properties :

1. X-rays are electromagnetic waves of very short wavelength ranging from Å to 10 Å.

2. X-rays travel in vacuum with the speed of light $(3 \times 10^8 \text{ m s}^{-1})$, as they are electromagnetic waves.

3. They are not deviated by electric and magnetic fields.

4. They affect the photographic plate very intensely.

5. They ionise the gas through which they pass.

6. They cause fluorescence in substances like zinc sulphide, barium platino-nide, calcium tungstate, etc.

7. Like light, X-rays can also cause photoelectric effect.

8. They travel in straight line and while doing so, they cast the shadows of the ects falling in their path.

9. X-rays can undergo reflection, refraction, interference, diffraction and larisation.

10. X-rays can penetrate the materials that are opaque to visible or ultra-violet ht. They can easily pass through paper, thin sheet of metals, wood, flesh, etc, but ey cannot penetrate denser objects such as bones, heavy metals, etc.

11. They have injurious effect on human bodies. Exposure of human body to rays causes reddening of the skin. The long exposures result into surface sores.

12. When X-rays fall on certain metals, secondary X-rays are produced, which e characteristic of the metal. The secondary X- rays are accompanied by fast oving electrons.

Applications. X-rays have been found to be of great use in variety of fields as ted below :

1. ***Surgery.*** X-rays are used in surgery for the detection of fractures, diseased gans, foreign matter like bullets and formation of bones or stones in the human ody and observing the progress of healing bones. They are also used to diagnose e diseases.

2. ***Radiotherapy.*** Controlled X-ray exposures are used to cure untractable skin iseases and malignant growths. Soft X-rays are used, if the affected parts are uperficial, while hard X-rays are used for deep seated organs.

3. ***Engineering.*** X-rays are used in engineering for detecting faults, cracks, flaws nd gas pockets in finished metal products. They are used for the testing of weldings, astings and moulds. They are also used to detect cracks in the engines of cars and eroplanes.

4. ***Detective departments.*** They are used by detective departments for detection f explosives, opium and other contraband goods and are used by custom epartment for detecting gold and silver in the body of smugglers. They are also sed in mints, where coins are made. They are also used for distinguishing real iamonds, gems, etc from artificial ones.

5. ***Industry.*** X-rays are used in industry for detecting the pearls in oysters, for esting the uniformity of insulating materials and homogeneity of timber. They are lso used to examine the defect in rubber tyres, golf and tennis balls, wood, etc.

6. ***Scientific research.*** X-rays have been used in investigating the structures of rystals, constitution and properties of atoms and arrangement of atoms and nolecules in the complex substance.

In addition to the above mentioned uses, X-rays are also used in detecting hange in old oil paintings.

1.11. GAMMA RAYS

Gamma rays were discovered by Rutherford. They are emitted from the nuclei of some radioactive elements and are electromagnetic waves of very short wave length. Gamma rays can be detected with Geiger Counter.

Properties. 1. γ-rays are electromagnetic waves and have velocity equal to that of light.

2. γ-rays are highly penetrating. They can penetrate through several centimetres thick iron and lead blocks.

3. They have got small ionising power.

4. They can produce fluorescence in a substance like willemite.

5. They can affect a photographic plate.

6. γ-rays are not deflected by electric and magnetic fields.

7. γ-rays knock out electrons from the surface on which they fall.

8. γ-rays produce heating effect in the surface exposed to them.

Applications. 1. γ-rays are used in ***radiotherapy***. In hospitals, γ-rays are used to treat cancer and tumours.

2. In food industry, soft γ-rays are used to kill microorganisms. It helps to preserve the foodstuffs for a prolonged time.

3. γ-rays are used to produce nuclear reactions.

1.12. PROPAGATION OF ELECTROMAGNETIC WAVES IN ATMOSPHERE

The sun is main source of electromagnetic radiation and it sends out electromagnetic waves of different wavelengths towards the earth. As the electromagnetic waves propagate through the earth's atmosphere, a major part of it is absorbed by the atmosphere. Most of the infra-red radiation is absorbed by the atmosphere and the atmosphere gets heated. The visible light is only slightly absorbed. The electromagnetic radiation from the sun is quite rich in ultra-violet radiation. Owing to its small wavelength (high energy), ultra-violet radiation is harmful to plants and living cells. However, the ozone layer absorbs most of the ultra-violet radiation and other harmful radiations of lower wavelengths. The ozone layer converts the ultra-violet radiation to infra-red, which further heats up the atmosphere and the earth's surface.

1.13. GREEN HOUSE EFFECT

The sun is source of energy. It emits energy in the form of visible light, infra-red and ultra-violet radiations. The behaviour of atmosphere is different towards different types of radiations. Whereas the ultra violet radiation and other low wavelength waves are absorbed by the ozone layer, a large part of the infra-red radiation is not allowed by the atmosphere to pass through it. The earth's atmosphere is transparent to visible light. Therefore, only visible light and a part of infra-red radiations reach the earth's surface. These radiations keep the earth's surface warm even at night due to the green house effect of the atmosphere, as explained below :

The earth gets heated to only a low temperature due to the solar energy reaching its surface. At such a low temperature, the energy emitted from the earth lies mostly in the infra-red region. Since earth's atmosphere is not transparent to infra-red radiations, these radiations are reflected back. The low lying clouds and heavy gases like CO_2 present in the atmosphere reflect infra-red radiation back towards the earth's surface. Due to this, the earth's atmosphere becomes richer in infra-red radiation. As this radiation is absorbed by the objects readily, they get heated in this process. This phenomenon is called the ***green house effect.***

It may be pointed out that the ultra-violet radiation and other low wavelength waves coming from sun and stars are absorbed by ozone layer. Had ozone layer not absorbed them, these radiations would have proved damaging to plants and living cells.

Key point ✓

The ozone layer absorbs the ultra-violet rays from the sun and thus prevents them from producing harmful effect on living organisms on the earth. Further, it traps the infra-red rays and prevents them from escaping the surface of earth. It helps to keep the earth's atmosphere warm.

SOLVED NUMERICAL PROBLEMS

TYPE A. On Speed, Frequency and Wavelength of Electromagnetic Waves

Problem 1.01. Green light of mercury has a wavelength $5{\cdot}5 \times 10^{-5}$ cm. (*a*) What is the frequency in MHz and period in μs in vacuum ? (*b*) What is the wavelength in glass, if refractive index of glass is 1·5 ? Given, $c = 3 \times 10^8$ m s^{-1}.

(Text Problem)

Sol. Here, wavelength, $\lambda = 5{\cdot}5 \times 10^{-5}$ cm $= 5{\cdot}5 \times 10^{-7}$ m

velocity of light, $c = 3 \times 10^8$ m s^{-1}

(*a*) If ν is the frequency, then

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{5{\cdot}5 \times 10^{-7}}\ \text{Hz} = \frac{3 \times 10^8}{5{\cdot}5 \times 10^{-7} \times 10^6}$$

$$= \mathbf{5{\cdot}45 \times 10^8\ MHz}$$

$$\text{Time period, } T = \frac{1}{\nu} = \frac{\lambda}{c} = \frac{5{\cdot}5 \times 10^{-7}}{3 \times 10^8} = 1{\cdot}8 \times 10^{-15}\ \text{s}$$

$$= 1{\cdot}8 \times 10^{-15} \times 10^6 = \mathbf{1{\cdot}8 \times 10^{-19}\ \mu s}$$

(*b*) Now, refractive index,

$$\mu = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in glass}} = \frac{c}{v}$$

Therefore, velocity of light in glass,

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m s}^{-1}$$

The wavelength of light in glass,

$$\lambda' = \frac{v}{\nu} = v \cdot \mathrm{T} \qquad \left(\because \nu = \frac{1}{\mathrm{T}}\right)$$

$$= 2 \times 10^8 \times 1.8 \times 10^{-15}$$

$$= \mathbf{3.6 \times 10^{-7}\ m}$$

Note. The, wavelength of light in glass can also be determined by using the releation, $\lambda' = \dfrac{\lambda}{\mu}$

Problem 1.02. Electromagnetic waves travel in a medium with a speed of 2×10^8 m s^{-1}. The relative permeability of the medium is 1. Find the relative permittivity.

Sol. Here, $v = 2 \times 10^8$ m s^{-1} and $\mu_r = 1$

The speed of electromagnetic waves in a medium is given by

$$v = \frac{1}{\sqrt{\mu\,\varepsilon}}, \qquad \text{...}(i)$$

where μ and ε are absolute permeability and absolute permittivity of the medium.

Now, $\mu = \mu_0\,\mu_r$ and $\varepsilon = \varepsilon_0\,\varepsilon_r$

Therefore, equation (*i*) becomes

$$v = \frac{1}{\sqrt{\mu_0\ \mu_r \times \varepsilon_0\ \varepsilon_r}} = \frac{1}{\sqrt{\mu_0\ \varepsilon_0}} \times \frac{1}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$$

$$\text{or} \quad \varepsilon_r = \frac{c^2}{v^2\ \mu_r} = \frac{(3 \times 10^8)^2}{(2 \times 10^8)^2 \times 1} = 2.25$$

Problem 1.03. The electric field of a plane electromagnetic wave in vacuum is represented by

$$\mathbf{E_x = 0,\ E_y = 0.5 \cos\,[2\,\pi \times 10^8\,(t - x/c)]}$$

and $\quad \mathrm{E}_z = 0$.

(*a*) What is the direction of propagation of electromagnetic waves ?

(*b*) Determine the wavelength of the wave.

(*c*) Compute the component of associated magnetic field. (*C.B.S.E. 1992*)

Sol. (*a*) The equation :

$$\mathrm{E}_y = 0.5 \cos\,[2\,\pi \times 10^8\,(t - x/c)] \qquad \text{...}(i)$$

indicates that the electromagnetic waves are propagating along the **positive direction of X-axis.**

(*b*) Comparing the equation (*i*) with the equation in standard form *i.e.*

$$\mathrm{E}_y = \mathrm{E}_0 \cos \omega\,(t - x/c)$$

we have

$$\omega = 2\,\pi \times 10^8 \quad \text{or} \quad 2\,\pi\,\nu = 2\,\pi \times 10^8 \quad \text{or} \quad \nu = 10^8 \text{ s}^{-1}$$

$$\text{Now,} \quad \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{10^8} = \mathbf{3\ m}$$

(*c*) The associated magnetic field is perpendicular to both the electric field and the direction of propagation. Since wave is propagating along X-axis and electric field is along Y-axis, magnetic field must be along Z-axis. Hence, the components of associated magnetic field are

$$\mathbf{B_x = 0, \quad B_y = 0}$$

$$\text{and} \quad \mathrm{B}_z = \mathrm{B}_0 \cos\,[2\,\pi \times 10^8\,(t - x/c)]$$

$$= \frac{\mathrm{E}_0}{c} \cos\,[2\,\pi \times 10^8\,(t - x/c)]$$

$$\text{or} \quad \mathbf{B_z = \frac{0.5}{3 \times 10^8} \cos\,[2\,\pi \times 10^8\,(t - x/c)]}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 1.04. A plane electromagnetic wave propagating in the X-direction has a wavelength of 6 mm. The electric field is in the Y-direction and its maximum magnitude is 33 V m^{-1}. Write suitable equations for the electric and magnetic fields as a function of *x* and *t*. (*Text Problem*)

Sol. For the electromagnetic wave propagating along X-direction, electric and magnetic fields will be along Y-direction and Z-direction and are given by

$$\mathrm{E}_y = \mathrm{E}_0 \sin \omega\,(t - x/c)$$

$$\text{and} \quad \mathrm{B}_z = \mathrm{B}_0 \sin \omega\,(t - x/c)$$

$$\text{Here,} \quad \lambda = 6 \text{ mm} = 6 \times 10^{-3} \text{ m} \quad \text{and} \quad \mathrm{E}_0 = 33 \text{ V m}^{-1}$$

$$\text{Now,} \quad \omega = 2\,\pi\,\nu = \frac{2\,\pi\,c}{\lambda} = \frac{2\,\pi \times 3 \times 10^8}{6 \times 10^{-3}} = \pi \times 10^{11} \text{ rad s}^{-1}$$

$$\text{and} \quad \mathrm{B}_0 = \frac{\mathrm{E}_0}{c} = \frac{33}{3 \times 10^8} = 1.1 \times 10^{-7}\ \mathrm{T}$$

Hence, $\mathbf{E_y = 33 \sin \pi \times 10^{11}\,(t - x/c)}$

and $\mathbf{B_z = 1.1 \times 10^{-7} \sin \pi \times 10^{11}\,(t - x/c)}$

Problem 1.05. Find the amplitude of the electric field in a parallel beam of light of intensity 8 W m^{-2}.

Sol. The intensity of a plane electromagnetic wave is given by

$$\mathrm{I} = \frac{1}{2}\,\varepsilon_0\,\mathrm{E}_0^{\,2}\,c$$

$$\text{or} \quad \mathrm{E}_0 = \sqrt{\frac{2\mathrm{I}}{\varepsilon_0\,c}} = \sqrt{\frac{2 \times 8}{8.85 \times 10^{-12} \times 3 \times 10^8}} = \mathbf{77.6\ N\ C^{-1}}$$

Problem 1.06. The wave emitted by any atom or molecule must have some finite total length. For sodium light, this length (called coherence length) is 2.4 cm. Deduce (*i*) the number of oscillations in this length (*ii*) the coherence time. Given, $\lambda = 5.9 \times 10^{-7}$ cm, $c = 3.0 \times 10^8$ m s^{-1}. (*Text Problem*)

Sol. Here, the coherence length, $l = 2.4$ cm $= 0.024$ m

wavelength, $\lambda = 5.9 \times 10^{-7}$ cm $= 5.9 \times 10^{-9}$ m

(*i*) Therefore, the number of oscillations in coherence length *l* is given by

$$\nu = \frac{l}{\lambda} = \frac{0.024}{5.9 \times 10^{-9}} = \mathbf{4.068 \times 10^6\ Hz}$$

(*ii*) The coherence time,

$$t = \frac{l}{c} = \frac{0.024}{3 \times 10^8} = \mathbf{8 \times 10^{-11}\ s}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. Is displacement current, like conduction current, a source of magnetic field ?

Ans. Yes, the displacement current is also a source of magnetic field like the conduction current.

Q. 1.02. The charging current for a capacitor is 0·25 A. What is the displacement current across its plates ? *(C.B.S.E. 1996)*

Ans. The displacement current is always equal to conduction current and therefore it is also equal to 0·25 A.

Q. 1.03. State modified Ampere's circuital law.

Ans. The modified Ampere's circuital law states that

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (I_C + I_D) = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

Here, the letters have their usual meanings.

Q. 1.04. Write down Maxwell's equation for steady electric field. *(H.S.S.C.E. 1993)*

Ans. Maxwell's equation for steady electric field are

(*i*) $\oint \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$ (*ii*) $\oint \vec{E} \cdot \vec{dl} = 0$

Q. 1.05. What are electromagnetic waves ?

Ans. *The transverse time varying electric and magnetic fields propagating in space in a direction perpendicular to the directions of both the electric and magnetic fields are said to constitute electromagnetic waves.*

Q. 1.06. Write an expression for speed of electromagnetic waves in free space. *(H.S.S.C.E. 1998)*

Ans. The speed of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \, \varepsilon_0}}$$

Q. 1.07. Write the relationship between amplitudes of electric and magnetic fields in free space for an electromagnetic wave. *(C.B.S.E. 1995, 1994)*

Ans. The amplitudes of the electric and magnetic fields in free space are related by $E/B = c$, where c is speed of electromagnetic waves in free space.

Q. 1.08. Give the ratio of velocities of light rays of wavelengths 4000 Å and 8000 Å in vacuum. *(C.B.S.E. 1997)*

Ans. Velocity of light rays of different wavelength in vacuum is same and hence the ratio of their velocities is 1.

Q. 1.09. The wavelength of electromagnetic radiation is doubled. What will happen to the energy of the photon ? *(C.B.S.E. 1993)*

Ans. The frequency and hence energy will become half, when the wavelength of the electromagnetic radiation is doubled.

Q. 1.10. State the properties of electromagnetic waves. *(P.S.S.C.E. 2002)*

Ans. Refer to section 1.04.

Q. 1.11. What is electromagnetic spectrum ? *(H.S.S.C.E. 1997)*

Ans. *The orderly distribution of electromagnetic waves in the form of distinct groups having widely differing properties is called electromagnetic spectrum.*

Q. 1.12. Name the main parts of electromagnetic spectrum.

Ans. Gamma rays, X-rays, ultra-violet rays, visible light, infra-red rays, heat radiations, microwaves, radio waves.

Q. 1.13. Which of the following belong to the electromagnetic spectrum :

α-rays, β-rays, γ-rays, cathode rays, X-ray, ultra-violet rays, microwaves, ultrasonic waves, radiowaves, infra-red rays ? Arrange them in order of increasing frequency. *(I.S.C.E. 1995)*

Ans. The following belong to electromagnetic spectrum and are arranged in order of increasing frequency :

Radiowaves, microwaves, infra-red rays, ultra-violet rays, X-rays and γ-rays.

Q. 1.14. Name any six electromagnetic waves in increasing order of their frequencies. *(H.S.S.C.E. 2002)*

Ans. Radiowaves, infra-red rays, visible light, ultra-violet rays, X-rays and γ-rays.

Q. 1.15. Arrange the given electromagnetic radiations in the descending order of their frequencies : Infra-red, X-rays, ultra-violet and gamma rays. *(C.B.S.E. 2002)*

Ans. Gamma rays, X-rays, ultra-violet and infra-red.

Q. 1.16. Write the following radiations in an ascending order in respect of their frequencies : X-rays, microwaves, ultra-violet rays and radiowaves. *(C.B.S.E. 1998)*

Ans. Radiowaves, microwaves, ultra-violet rays and X-rays.

Q. 1.17. Rewrite the following radiations in a descending order of wavelength values : Infra-red rays, radiowaves, γ-rays, microwaves. *(C.B.S.E. 1998)*

Ans. Radio waves, microwaves, infra-red rays and γ-rays.

Q. 1.18. Write the following radiations in a descending order of frequencies : red light, X-rays, microwaves, radiowaves. *(C.B.S.E. 1998)*

Ans. X-rays, red light, microwaves and radiowaves.

Q. 1.19. Arrange the following radiations in the descending order of wavelength : X-ray, infra-red ray, red light, yellow light, radio waves. *(H.P.S.S.C.E. 1995)*

Ans. Radio waves, infra-red ray, red light, yellow light, γ-rays.

Q. 1.20. What is common between different types of e.m. radiations ?

Ans. All types of e.m. radiations are transverse in nature and travel with the same speed (= 3×10^8 m s^{-1}) in vacuum.

Q. 1.21. Which of the following has the lowest frequencies : microwaves, ultra-violet rays and X-rays ? *(C.B.S.E. 1996)*

Ans. Microwaves.

Q. 1.22. Write the frequency limit of visible range of electromagnetic spectrum in kHz. *(C.B.S.E. 1998, 1990)*

Or

What is the approximate range of visible spectrum ? *(H.S.S.C.E. 1999 S)*

Ans. The wavelength range of visible part of electromagnetic spectrum is 3900 Å to 7600 Å.

The corresponding frequency range of visible spectrum $\times 10^{14}$ Hz to $7{\cdot}7 \times 10^{14}$ Hz.

Q. 1.23. What is the ratio of speed of infra-red rays and ra-violet rays in vacuum ? (*C.B.S.E. 2001*)

Ans. It is 1.

Q. 1.24. What is the ratio of speed of gamma rays and diowaves in vacuum ? (*C.B.S.E. 2001*)

Ans. It is 1.

Q. 1.25. Radiowaves and gamma rays both are trans-rse in nature and electromagnetic in character and have the me speed in vacuum. In what respects are they different ? (*I.S.C.E. 1997*)

Ans. Radiowaves have atomic orgin, while the γ-rays are nuclear origin. Further, owing to their very small wave ngth, γ-rays are highly peneterating in comparison to diowaves.

Q. 1.26. What is the main difference between charac-ristic X- rays and γ-rays ? (*I.S.C.E. 1994*)

Ans. X-rays are emitted, when orbital electron jumps om some outer shell to the inner shell in case of an atom of heavy element (Z = high), while γ-rays are emitted by dioactive nuclei.

Q. 1.27. How does the frequency of a beam of ultra-iolet light change, when it goes from air into glass ? (*C.B.S.E. 2001*)

Ans. There is no effect on the frequency of ultraviolet light.

Q. 1.28. Why are microwaves used in RADAR ? (*C.B.S.E. 1999*)

Or

Which part of the electromagnetic spectrum is used in operating a RADAR ? (*C.B.S.E. 1996*)

Ans. Microwaves are e.m. waves of very short wavelength. Such waves are used in RADAR due to the reason that they can travel in a particular direction in the form of a beam.

Q. 1.29. Name the electromagnetic radiations used for viewing objects through haze and fog. (*H.S.S.C.E. 1997*)

Ans. Infra-red rays.

Q. 1.30. State two applications of X-rays. (*P.S.S.C.E. 2002*)

Ans. Refer to section 1.10.

Q. 1.31. The small ozone layer on the top of strato-sphere is crucial for human survival. Why ? (*P.S.S.C.E. 2000 ; H.P.S.S.C.E. 2000 ; C.B.S.E. 1996*)

Ans. The ultraviolet radiation from the sun is harmful to living cells and plants. The ozone layer absorbs ultraviolet radiation and prevents it from reaching the earth. It also keeps the earth warm by trapping infra-red radiation.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What is Maxwell's displacement current ? (*H.S.S.C.E. 1994 ; C.B.S.E. 1994*)

Ans. The displacement current is produced, when in an lectric circuit (say across the plates of a charged capacitor), he electric field changes with time. Mathematically,

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

Q. 1.02. A variable-frequency a.c. source is connected o a capacitor. Will the displacement current increase or decrease with increase in frequency ?

Ans. If we increase ω, the reactance of the capacitor will decrease and consequently conduction current will increase. Since displacement current is equal to conduction current, the displacement current will increase with increase in frequency of a.c. source.

Q. 1.03. Why is it that induced electric fields due to changing magnetic flux are more readily observable than the induced magnetic fields due to changing electric fields ?

Ans. The induced magnetic fields produced due to displacement currents are of magnitude too small to be observed. The effect can be increased by increasing the displacement current. For example, in an a.c. circuit, it can be increased by increasing ω. On the other hand, the induced electric fields due to changing magnetic flux can be easily increased by simply taking a coil of large turns and hence, it is readily observable.

Q. 1.04. What does an electromagnetic wave constist of ? On what factors does its velocity in vacuum depend ? (*P.S.S.C.E. 1994, 1992*)

Ans. An electromagnetic wave consists of electric and magnetic fields varying both in space and time. The two fields are perpendicular to each other and to the direction of propagation of wave. The velocity of electromagnetic wave in vacuum depends upon the values of its absolute permeability (μ_0) and permittivity (ε_0).

Q. 1.05. State the principle of production of e.m. waves. What is the value of velocity of these waves ? (*P.S.S.C.E. 1996*)

Ans. The accelerated charge produces electric and magnetic fields, which vary both in space and time. These varying electric and magnetic fields give rise to electromagnetic waves.

The velocity of e.m. waves in free space is 3×10^8 m s^{-1}.

Q. 1.06. State any four properties of electromagnetic waves. (*P.S.S.C.E. 2000, 1999, 1997, 1996 S ; C.B.S.E. 1995*)

Ans. The e.m. spectrum consists of a number of distinct parts. But the radiations from all the parts possess the following common properties :

1. They are transverse in nature.

2. They do not require any material medium for propagation.

3. They travel with the same speed of 3×10^8 m s^{-1} in vacuum.

4. They consist of mutually transverse varying electric and magnetic fields.

Q. 1.07. What is the wave length of a television station, which transmits vision on 500 MHz ? Given $c = 3 \times 10^8$ m s^{-1}. (*I.S.C.E. 1993*)

Ans. Here, $\nu = 500$ MHz $= 500 \times 10^6$ Hz ; $c = 3 \times 10^8$ m s^{-1}

Now,
$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{500 \times 10^6} = 0{\cdot}8 \text{ m}$$

Q. 1.08. Why can light travel in vacuum, whereas sound can not do so ?

Ans. Light waves are electromagnetic in nature. They do not require a material medium for propagation and can travel in vacuum. On the other hand, sound waves require a material medium for propagation. They are mechanical waves and cannot travel in vacuum.

Q. 1.09. Radio waves diffract pronouncedly around building; while light waves, which are also electromagnetic waves, do not. Why ?

Ans. For a wave to suffer diffraction, the wavelength should be of the order of size of the obstacle. The wavelength of radio waves (short radio waves) is of the order of the size of the building and the other obstacles coming in their path and hence they easily get diffracted. The wavelength of light waves is very small and they are not diffracted by the buildings.

Q. 1.10. Give one use of each of the following :

(*i*) infra-red rays (*ii*) gamma rays (*iii*) microwaves and (*iv*) ultraviolet radiations. *(C.B.S.E. 1992)*

Ans. (*i*) Infra-red rays photographs are used for weather forecasting.

(*ii*) Gamma rays are used in nuclear reactions.

(*iii*) Microwaves are used in RADAR.

(*iv*) Ultra-violet rays are used for sterilizing surgical instruments.

Q. 1.11. What are microwaves ? Give their any one use *(H.S.S.C.E. 1998)*

Ans. Microwaves are electromagnetic in nature and thei wavelength ranges from 10^{-3} m to 0·3 m. These waves are used in radars and in the study of atomic and molecula structure.

Q. 1.12. If the earth did not have atmosphere, would its average surface temperature be higher or lower than what it is now ? *(P.S.S.C.E. 1993 ; C.B.S.E. 1990)*

Ans. The infrared radiation emitted by earth are retained by the earth's atmosphre due to the green house effect and this keeps the earth warm. If the earth did not have atmosphere, its average temperature would have been low.

Q. 1.13. Discuss the significance of green house effect in the atmosphere. *(C.B.S.E. 1999 S)*

Or

What do you know about green house effect ? *(H.P.S.S.C.E. 1993)*

Or

Explain the green house effect. *(P.S.S.C.E. 1999 ; H.P.S.S.C.E. 1999, 1994)*

Or

What do you understand by green house effect ? Explain it. *(H.S.S.C.E. 1995)*

Ans. Refer to section 1.13.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. Static crashes are heard on radio, when a lightning flash occurs-even if the lightning occurs far away. Why does this happen ?

Ans. A lightning flash involves tremendous electrical fields and currents, which oscillate between the earth and the clouds or between two groups of clouds. In this electrical activity, many charges oscillate and produce a wide variety of electromagnetic waves. The flashes of light we see are emitted by atoms during this intense activity. Those electromagnetic waves, which have frequencies in radio-wave range, interfere with radio waves. Since light and radiowaves travel with the same speed, they arrive at the same time as does the light.

Q. 1.02. What would be the order of magnitude of the minimum frequency of electromagnetic waves that could be used to detect the presence of (*a*) the planet Venus, (*b*) an aircraft 50 m long (*c*) a bird 0·1 m long ? From what sources of electromagnetic radiation, would you be able to generate radiation of these wavelengths ?

Ans. In order to use a wave phenomenon to detect the presence of some object, the wavelength of the waves used must be comparable to or smaller than the dimensions of the object to be detected.

(*a*) Venus is about 10^7 m in diameter. The frequency of electromagnetic waves of wavelength 10^7 m is given by

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{10^7} = 30\text{ Hz}$$

The waves of frequency 30 Hz correspond to very low andio-frequency radio waves. Practically, it will not be possible to detect Venus by employing such waves, because of the following reasons :

1. It would be almost impossible to get much power into such a low frequency wave.

2. Such a radio wave would be absorbed completely in the upper atmosphere.

3. Even if we could send such a radiowave to Venus and receive an echo, its beam would be so broad that we will not be able to pinpoint the direction of the Venus.

(*b*) The waves of wavelength 50 m required to detect an aircraft possess frequency,

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{50} = 6\times10^6\text{ Hz} = 6\text{ MHz}$$

This frequency is higher than the frequency of radiowaves used by A.M. broadcasting stations. Primitive radars operated at frequencies about 20 times this.

(*c*) A 0·1 m long bird will require frequency,

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{0\cdot1} = 3000\text{ MHz}$$

It is very close to the popular radar frequency. Indeed, radars occasionally detect birds.

Q. 1.03. Suggest reasons, why (*a*) food in metal containers cannot be cooked in a microwave oven, (*b*) an empty glass container does not get hot in a microwave oven.

Ans. In a microwave oven, the frequency of microwaves is selected to match the resonance frequency of water molecules, so that the energy from the waves is transferred efficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water.

(*a*) The atoms of the metallic container are set into forced vibrations by the microwaves. Due to this, energy of the microwaves is not efficiently transferred to the metallic container. Owing to this, food in metallic containers cannot be cooked in a microwave oven.

(*b*) The molecules of the glass container do not respond ne frequency of microwaves. Due to this, energy from the rowaves is not transferred to the glass container and hence oes not get hot in a microwave oven.

Q. 1.04. Optical and radio telescopes are built on the ground, but X-ray astronomy is possible only from llites orbiting the earth. Why ?

Ans. The earth's atmosphere is transparent to visible light radio waves, but absorbs X-rays. Therefore, X-ray onomy is possible only from satellites orbiting the earth.

Q. 1.05. Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction ?

Ans. Scientists estimate that in case of global nuclear war, the clouds produced will cover probably whole of the sky. In that case, solar radiation would be prevented from reaching the earth and it will result in, what they call as *nuclear winter* on the earth.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Why did Maxwell introduce the concept of displacement current ? Explain. (*H.S.S.C.E. 1992*)
2. Give a brief history of electromagnetic waves. (*P.S.S.C.E. 2001, 1999 S, 1991 ; H.S.S.C.E. 1997*)
3. What are electromagnetic waves ? Explain in brief.
4. What do you understand by electromagnetic waves ? Give its properties. (*H.P.S.S.C.E. 2002*)
5. What are electromagnetic waves ? Give its any four properties. (*P.S.S.C.E. 2001*)
6. Write down the important properties of electromagnetic waves.
7. What is meant by electromagnetic spectrum ? Give its four uses. (*P.S.S.C.E. 1999 S*)
8. Write one property and one use each of infra-red rays, ultra-violet rays and radiowaves. (*H.S.S.C.E. 2002*)
9. Give two uses each of the following : (*i*) Gamma rays (*ii*) Infra-red rays (*iii*) Ultra-violet radiation. (*P.S.S.C.E. 2001*)
10. Write down some important applications of different parts of electromagnetic spectrum.
11. Show that electromagnetic waves are transverse in nature. (*H.S.S.C.E. 2001*)
12. Explain the 'green house effect' of earth's atmosphere. (*H.S.S.C.E. 2002, 2001 ; P.S.S.C.E. 1991*)
13. What do you understand by green house effect ? (*H.S.S.C.E. 2002*)
14. What is the role of ozone layer in the atmosphere ? (*H.S.S.C.E. 2002*)

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What are electromagnetic waves ? What is the principle of their production ? Give a brief account of the history of electromagnetic waves and write down the important properties of electromagnetic waves.
2. What is electromagnetic spectrum ? Name the main parts of the electromagnetic spectrum giving the frequency range and the source of production in each case.
3. How are infra-red rays, ultra-violet rays and X-rays produced ? Give their important properties and applications.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

pe A. On Speed, Frequency and Wavelength of Electromagnetic waves

1. The wavelength of X-rays is 1 Å. Calculate its frequency. [**Ans.** 3×10^{18} Hz]
2. A radar transmitter generates waves, whose frequency is 3×10^9 Hz. What is the wavelength of the waves ? [**Ans.** 0·1 m]
3. Electromagnetic waves of frequency 5×10^{14} Hz are passed through a liquid. The wavelength of the waves in liquid is measured to be $4{\cdot}5 \times 10^{-7}$ m. Calculate (*i*) the wavelength of e.m. waves in vacuum, (*ii*) velocity of e.m. waves in the liquid and (*iii*) refractive index of the liquid. Given, velocity of e.m. waves in vacuum = 3×10^8 m s^{-1}. [**Ans.** (*i*) 6×10^{-7} m, (*ii*) $2{\cdot}25 \times 10^8$ m s^{-1} and (*iii*) 1·33]
4. If absolute permittivity and permeability of free space are $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ C^2 N^{-1} m^{-2} and $\mu_0 = 4\pi \times 10^{-7}$ N s^2 C^{-2}. Calculate the velocity of electromagnetic waves. [**Ans.** 3×10^8 m s^{-1}]
5. In a plane e.m. wave, the electric field oscillates sinusoidally with a frequency of 2×10^{10} Hz and amplitude 48 V m^{-1}.
   (*a*) What is the wavelength of the wave ? (*C.B.S.E. 1990*)
   (*b*) What is the amplitude of the oscillating magnetic field ? [**Ans.** (*a*) $1{\cdot}5 \times 10^{-2}$ m (*b*) $1{\cdot}6 \times 10^{-7}$ T]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{1 \times 10^{-10}} = 3 \times 10^{18}$ Hz

2. $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3 \times 10^9} = 0{\cdot}1$ m

3. (*i*) $\lambda_{vacuum} = \frac{c}{\nu} = \frac{3 \times 10^8}{5 \times 10^{14}} = 6 \times 10^{-7}$ m

(*ii*) $v = \nu \lambda_{liquid} = 5 \times 10^{14} \times 4{\cdot}5 \times 10^{-7}$

$= 2{\cdot}25 \times 10^8$ m s^{-1}

(*iii*) $\mu = \frac{c}{v} = \frac{3 \times 10^8}{2{\cdot}25 \times 10^8} = 1{\cdot}33$

4. Now, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\therefore \quad c = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8{\cdot}85 \times 10^{-12}}} = 3 \times 10^8$ m s^{-1}

5. $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1{\cdot}5 \times 10^{-2}$ m

$B_0 = \frac{E_0}{\;} = \frac{48}{\;} = 1{\cdot}6 \times 10^{-7}$ T

Competitive Examination File (Unit V)

REVISION AT A GLANCE

Chapter 1. Electromagnetic waves

☐ **Conduction current.** *It is the current, which arises due to flow of electrons through the connecting wires in an electric circuit.*

☐ **Displacement current.** *It is the current, which arises due to time rate of change of electric flux (ϕ_E) in some part of the electric circuit.* It is given by

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt},$$

where ε_0 is absolute permittivity of free space.

When a capacitor is charged or a charged capacitor is allowed to discharge, the electric flux between the plates of the capacitor changes with time and it gives rise to displacement current between the plates. The current that flows through the connecting wires is called the conduction current.

☐ **Modified Ampere circuital law.** It states that *the line integral of magnetic field $\vec{B}$ over a closed path is equal to μ_0 times the sum of the conduction current (I_C) and displacement current (I_D) threading the closed path.*

Mathematically : $\oint \vec{E} \cdot \vec{dl} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$

It is also known as ***Ampere-Maxwell's circuital law.***

☐ **Maxwell's equations.** Following four equations, which describe the laws of electromagnetism are called Maxwell's equation :

1. $\oint \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$ ***(Gauss's law in electrostatics)***
2. $\oint \vec{B} \cdot \vec{dS} = 0$ ***(Gauss's law in magnetism)***
3. $\oint \vec{B} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$ ***(Faraday's law of electromagnetic induction)***
4. $\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_B}{dt} \right)$ ***(Ampere-Maxwell's circuital law)***

☐ **Electromagnetic waves.** *The electric and magnetic fields varying in space and time and propagating through space, such that the two fields are perpendicular to each other and perpendicular to the direction of propagation, constitute electromagnetic waves.*

1. The electromagnetic waves are transverse in nature.

2. The velocity of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m s}^{-1}$$

3. The ratio of the amplitudes of electric and magnetic fields is constant and it is equal to velocity of electromagnetic waves in free space.

4. The electric vector of an electromagnetic wave is responsible for its optical effect. For this reason, the electric vector is also called ***light vector.***

☐ **Electromagnetic spectrum.** *The orderly distribution of electromagnetic waves (according to wavelength or frequency) in the form of distinct groups having widely differing properties is called electromagnetic spectrum.*

The main parts of electromagnetic spectrum are namely : γ-rays, X-rays, ultra-violet rays, visible light, infra-red rays, heat radiation, microwaves and radio waves.

☐ **Green house effect.** *The infra-red radiation trapped by the atmosphere, so as to keep the earth's surface warm, is called green house effect.*

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Problem 1.01. A capacitor of 2 μF is charged to 20 V and then suddenly short-circuited by a coil of negligible resistance and of inductance 8 μH. Calculate (*i*) the maximum amplitude and (*ii*) the frequency of the resulting oscillation.

Sol. Here, 2 mF = 2×10^{-6} F, L = 8 mH = 8×10^{-6} H and E = 20 V

(*i*) Maximum charge on capacitor, $q_0 = \text{C E}$

$$= 2 \times 10^{-6} \times 20 = 40 \times 10^{-6} \text{ C}$$

When a capacitor is discharged through an inductance, the oscillatory discharge is described by the equation,

$$q = q_0 \sin \omega t$$

The current in LC-circuit during oscillatory discharge is given by

$$\text{I} = \frac{dq}{dt} = \frac{d}{dt}(q_0 \sin \omega t) = q_0\, \omega \cos \omega t$$

Therefore, maximum amplitude of current during oscillatory discharge,

$$\text{I}_0 = q_0\, \omega\, q_0 \times \frac{1}{\sqrt{\text{LC}}} = \frac{40 \times 10^{-6}}{\sqrt{8 \times 10^{-6} \times 2 \times 10^{-6}}} = \mathbf{10\ A}$$

(*ii*) Frequency of oscillations,

$$f = \frac{1}{2\pi\sqrt{\text{LC}}} = \frac{1}{2\pi\sqrt{8 \times 10^{-6} \times 2 \times 10^{-6}}}$$

$$= \mathbf{3{\cdot}98 \times 10^4\ Hz}$$

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Electromagnetic Waves

PART I. TEXT-BASED QUESTIONS

1. **The SI unit of displacement current is**
(A) H (B) A
(C) F m^{-1} (D) C

2. **An electromagnetic radiation of frequency ν, wavelength λ, travelling with velocity c in air, enters a glass slab of refractive index μ. The frequency, wavelength and velocity of light in the glass slab will be respectively :**
(A) $\frac{\nu}{\mu}, \frac{\lambda}{\mu}$ and $\frac{c}{\mu}$ (B) $\nu, \frac{\lambda}{\mu}$ and $\frac{c}{\mu}$
(C) $\nu, 2\lambda$ and $\frac{c}{\mu}$ (D) $\frac{2\nu}{\mu}, \frac{\lambda}{\mu}$ and c
(C.B.S.E. 1997)

3. **The speed of electromagnetic waves is independent of**
(A) wavelength (B) frequency
(C) intensity (D) medium, in which it travels.

4. **Electromagnetic waves are produced due to**
(A) a charge at rest. (B) a moving charge.
(C) an accelerating charge.
(D) none of the these.

5. **The speed of electromagnetic waves in vacuum is equal to**
(A) $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ (B) $\sqrt{\varepsilon_0 \mu_0}$
(C) $\varepsilon_0 \mu_0$ (D) $\frac{1}{\varepsilon_0 \mu_0}$

6. **Dimensions of $\varepsilon_0 \mu_0$ is**
(A) $L T^{-1}$ (B) $L^{-1} T$
(C) $L^2 T^{-2}$ (D) $L^{-2} T^2$

7. **The dimensions of $\frac{1}{2} \varepsilon_0 E^2$ (ε_0 : permittivity of free space ; E : electric field) is**
(A) $M L T^{-1}$ (B) $M L^2 T^{-2}$
(C) $M L^{-1} T^{-2}$ (D) $M L^2 T^{-1}$ *(I.I.T. 2000)*

8. **If ε_0 and μ_0 are the electric permittivity and magnetic permeability in free space, ε and μ are the corresponding quantities in a medium, then index of refraction of the medium is**
(A) $\sqrt{\frac{\varepsilon_0 \mu}{\varepsilon \mu_0}}$ (B) $\sqrt{\frac{\varepsilon}{\varepsilon_0}}$
(C) $\sqrt{\frac{\varepsilon_0 \mu_0}{\varepsilon \mu}}$ (D) $\sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}}$ *(C.B.S.E. 1997)*

9. **An electric charge in uniform motion produces**
(A) an electric field only.
(B) a magnetic field only.
(C) both electric and magnetic fields.
(D) no such field at all.
(C.P.M.T. 1990, 1983 ; M.N.R. 1988)

10. **Which of the following statements is a wrong one ? Electromagnetic waves**
(A) are produced by accelerating charges.
(B) are transverse in nature.
(C) travel with the same speed irrespective of the nature of the medium.
(D) travel with velocity of light in vacuum.

11. **If $\vec{E}$ and $\vec{B}$ represent electric and magnetic field vectors of the electromagnetic waves, then the direction of propagation of the electromagnetic waves is that of**
(A) $\vec{E}$ (B) $\vec{B}$
(C) $\vec{E} \times \vec{B}$ (D) $\vec{B} \times \vec{E}$ *(C.B.S.E. 2002)*

12. **Among the following, longest wave is**
(A) radio wave. (B) γ-ray.
(C) microwave. (D) X-ray. *(C.P.M.T. 1982)*

13. **Which of the following has minimum wavelength ?**
(A) X-rays (B) Ultraviolet rays
(C) γ-rays (D) Cosmic rays *(C.B.S.E. 2002)*

14. **Which has the longest wavelength ?**
(A) Radio waves (B) X-rays
(C) Ultra-violet rays (D) Infra-red rays

15. **Infrared spectrum lies between**
(A) radio wave and microwave region.
(B) microwave and visible region.
(C) visible and ultra-violet region.
(D) ultra-violet and X-ray. *(C.E.T. 1998 ; N.C.E.R.T. 1980)*

16. **The frequency of γ-rays, X-rays and ultraviolet rays are a, b and c respectively. Then,**
(A) $a > b > c$ (B) $a < b < c$
(C) $a = b = c$ (D) $a > c > b$ *(C.B.S.E. 2000)*

17. **Red light differs from blue light in its**
(A) speed. (B) frequency.
(C) intensity. (D) amplitude.

18. **Which of the following statements is wrong ?**
(A) Ultra-violet rays have a wavelength longer than infra-red rays.
(B) Infra-red rays travel with the same velocity as visible light.
(C) Infra-red can be focussed by a lens and can be reflected by a mirror just as visible light.
(D) Infra-red rays have more heating power than visible light rays. *(N.C.E.R.T. 1976)*

19. **Which of the following represents an infra-red wavelength ?**
(A) 10^{-4} cm (B) 10^{-5} cm
(C) 10^{-6} cm (D) 10^{-7} cm *(N.C.E.R.T. 1975)*

20. **Ozone layer is present in**
(A) troposphere (B) stratosphere
(C) ionosphere (D) mesosphere.

21. **Ozone layer blocks the radiation of wave-length :**
(A) less than 3×10^{-7} m.
(B) equal to 3×10^{-7} m.
(C) more than 3×10^{-7} m.
(D) none of these. *(C.B.S.E. 1999)*

22. **Ozone layer above earth's atmosphere will**
(A) prevent infra-red radiation from sun.
(B) prevent infra-red rays reflected from earth from escaping earth's atmosphere.
(C) prevent ultra-violet rays from sun.
(D) reflect back radio waves. *(Pre-degree Kerala, 1992)*

23. **What is the cause of 'Green house effect' ?**
(A) Infra-red rays (B) Ultra-violet rays
(C) X-rays (D) Radiowaves. *(C.B.S.E. 2002)*

24. **Ozone layer in atmosphere is useful, because it**
(A) stops ultra-violet radiation
(B) absorbs pollutent gases
(C) stops green house effect
(D) stops increase in temperature of atmosphere.
(C.B.S.E. 2001)

RT II. THOUGHT-BASED QUESTIONS

5. **The displacement current flows in the dielectric of a capacitor, when the potential difference across its plates**
(A) is increasing with time.
(B) is decreasing with time.
(C) becomes zero.
(D) attains a constant value.

6. **The displacement current is produced across the gap between the two plates of a capacitor, when the charge on the capacitor**
(A) is zero. (B) decreases.
(C) increases. (D) does not change.

7. **If an electromagnetic wave propagating through vacuum is described by**

$$E = E_0 \sin (kx - \omega t) \; ; \; B = B_0 \sin (kx - \omega t),$$

then

(A) $E_0 k = B_0 \omega$ (B) $E_0 B_0 = \omega k$
(C) $E_0 \omega = B_0 k$ (D) $E_0 B_0 = \omega / k$

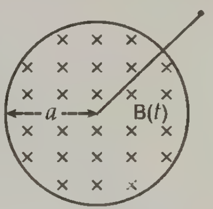
3. **Which of the following is independent of wavelength ?**
(A) k (B) ω
(C) ωk (D) k / ω

9. **A magnetic field can be produced by**
(A) a charge at rest only.
(B) a moving cahrge only.
(C) a changing electric field.
(D) both by (B) and (C).

Which of the following pairs of space and time varying E and B fields would generate a plane electromagnetic wave travelling along the Z-direction ?
(A) E_x, B_y (B) E_y, B_x
(C) E_x, B_z (D) E_z, B_x

31. **In a plane electromagnetic wave, which of the following have zero average value ?**
(A) magnetic field (B) magnetic energy
(C) electric field (D) electric energy.

32. **A uniform but time-varying magnetic field B(t) exists in a circular region of radius a and directed into the plane of the paper as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region**



(A) is zero
(B) decreases as $1/r$
(C) increases as r
(D) decreases as $1/r^2$.

(I.I.T. 2000)

33. **Let [ε_0] denote the dimensional formula of the permittivity of the vacuum and [μ_0] that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current,**
(A) $[\varepsilon_0] = M^{-1} L^{-3} T^2 I$ (B) $[\varepsilon_0] = M^{-1} L^{-3} T^4 I^2$
(C) $[\mu_0] = M L T^{-2} I^{-2}$ (D) $[\mu_0] = M L^2 T^{-1} I$

(I.I.T. 1998)

34. **Finger prints on a piece of paper may be detected by sprinkling flourescent powder on the paper and then looking it into**
(A) dark light (B) sun-light
(C) Infra-red light (D) ultra-violet light

ANSWERS TO MULTIPLE CHOICE QUESTIONS

apter 1. Electromagnetic waves

1. (B)	2. (B)	3. (C)	4. (C)	5. A)	6. (D)	7. (C)	8. (D)	9. (C)	10. (C)
1. (C)	12. (A)	13. (C)	14. (A)	15. (B)	16. (A)	17. (B)	18. (A)	19. (A)	20. (B)
1. (A)	22. (B & C)	23. (A)	24. (A)	25. (A & B)	26. (B & C)	27. (A)	28. (D)	29. (D)	30. (A & B)
1. (A & C)	32. (B)	33. (B & C)	34. (D)						

UNIT
VI

OPTICS

1. **Refraction of Light**
2. **Dispersion**
3. **Optical Instruments**
4. **Huygens' Principle**
5. **Interference of Light**
6. **Diffraction of Light**
7. **Polarisation of Light**

> "Our ether, does it really exist ? I do not believe that more precise observations could ever reveal more than relative displacements."
> **– Henri Poincare**

CHAPTER

1

Refraction of Light

1.01. REFRACTION

In a homogeneous medium, light travels along a straight path. But when a ray of light travels from one transparent medium into another, it bends while crossing the surface separating the two media.

The phenomenon of change in path of light as it goes from one medium to another is called refraction.

When the ray of light passes from an optically rarer medium to a denser medium, it bends towards the normal and when it travels from a denser medium to a rarer medium, it bends away from the normal. Fig. 1.01 shows a surface XY separating two transparent media – air (a) and glass (b). The ray of light travelling along AO in air (optically rarer medium) enters glass (optically denser medium) at point O. From its straight path AO, it bends towards the normal NON′ and travels along the path OB. The rays AO and OB are called ***incident ray*** and ***refracted ray*** respectively. The angle between the incident ray and the normal denoted by i is called ***angle of incidence,*** while the angle between refracted ray and the normal denoted by r is called ***angle of refraction.***

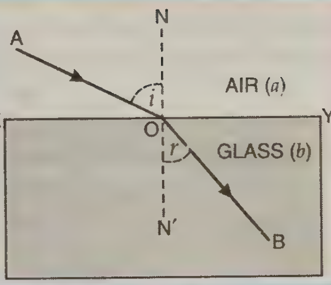


Fig. 1.01

1.02. LAWS OF REFRACTION

The phenomenon of refraction takes place according to the following two laws :

1. *The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray – all lie in the same plane.*

2. *The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for the two given media.* This constant is denoted by and is called the ***relative refractive index*** of medium b (in which refracted ray travels) *w.r.t.* medium a (in which incident ray travels).

$$\therefore \qquad \frac{\sin i}{\sin r} = {}^a\mu_b \qquad \text{...(1.01)}$$

This law is also called **Snell's law** of refraction.

The value of relative refractive index of a medium b *w.r.t.* a depends on

(*i*) *the nature of the pair of media a and b* and

(*ii*) *the wavelength* (colour) *of the light*. The refractive index of a medium is greater for violet light (shorter wavelength) than that for red light (longer wavelength).

If medium a is vacuum (or air), then the constant ratio given by equation (1.01) is called the ***absolute refractive index*** of medium b.

The absolute refractive index of a medium can be defined in another way also. It is found that light travels with maximum velocity c ($= 3 \times 10^8$ m s^{-1}) in vacuum and in any other transparent medium, light travels with velocity v, such that $v < c$.

The ratio of the velocity of light in vacuum to the velocity of light in a medium is called absolute refractive index of the medium.

Thus, absolute refractive index of the medium,

$$\mu = \frac{\text{velocity of light in vacuum } (c)}{\text{velocity of light in medium } (v)} \qquad \text{...(1.02)}$$

A medium having a larger value of refractive index is called ***optically denser,*** while the one having a smaller value is said to be ***optically rarer.***

Note. The refractive index of a medium is also denoted by the letter 'n'.

Key point ✓

When light travels from air to a medium of refractive index μ, its wavelength decreases by a factor μ *i.e.* becomes λ/μ, while the frequency remains the same.

1.03. PRINCIPLE OF REVERSIBILITY OF LIGHT

It states that *if light after suffering any number of reflections and refractions has its final path reversed, it travels back along the same path in the opposite direction.*

Explanation. Consider a ray of light incident along AO at point O on the interface XY separating rarer medium a (air) from the denser medium b (water). After refraction, the ray of light proceeds along path OB. Let i and r be angle of incidence and angle of refraction respectively. If a plane mirror M is placed so that the refracted ray OB is normal to it, then it will be reflected back along BO. At point O, it will suffer refraction, so as to emerge along OA as shown in Fig. 1.02. Thus, if the path of light is reversed, it retraces its whole path in the opposite direction.

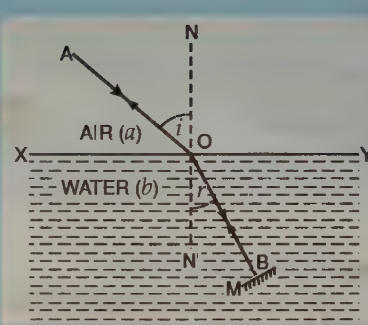


Fig. 1.02

The principle of reversibility of light leads to a very useful result as obtained below :

When light travels from medium a to medium b, then the relative refractive index of medium b *w.r.t.* medium a is given by

$$^{a}\mu_{b} = \frac{\sin i}{\sin r} \qquad \text{...(1.03)}$$

On reversing the path, the light travels from medium b to medium a and likewise r is the *angle of incidence* and i is the corresponding angle of refraction. Therefore, relative refractive index of medium a *w.r.t.* medium b is given by

$$^{b}\mu_{a} = \frac{\sin r}{\sin i} \qquad \text{...(1.04)}$$

Multiplying the equations (1.03) and (1.04), we have

$$^{a}\mu_{b} \times {}^{b}\mu_{a} = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1$$

or
$$^{a}\mu_{b} = \frac{1}{^{b}\mu_{a}} \qquad \text{...(1.05)}$$

It follows that *the refractive index of the medium b w.r.t. the medium a is equal to the reciprocal of the refractive index of the medium a w.r.t. the medium b.*

1.04. LATERAL SHIFT

Consider a glass slab bounded by parallel faces XY and X′Y′ and of thickness t as shown in Fig. 1.03. A ray of light travelling in air along the path AO gets incident on the face XY at the angle of incidence i. On refraction into glass (denser medium), it bends towards normal NON′ and travels along the path OQ inside the glass, such that r is the angle of refraction.

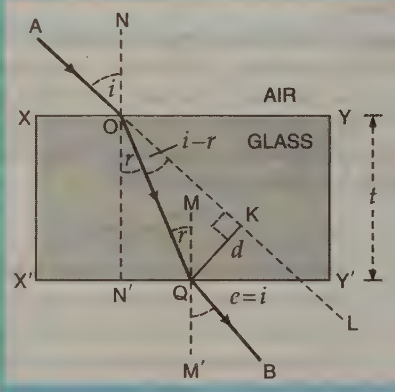


Fig. 1.03

Key point

When white light passes through a glass slab having parallel faces, it is dispersed into its constituent colours also, as it undergoes refraction at the first face. However, the constituent colours recombine to produce white light, when refraction takes place at the second face of the glass slab.

Since the two faces of the glass slab are parallel, the refracted ray OQ strikes the surface X′Y′ at the angle of incidence r. On refraction into air, the ray of light bends away from the normal MOM′ and travels along the path QB in the air, such that i is the angle of refraction. Obviously, the emergent ray QB is parallel to the incident ray AOL. From the point Q, drop QK perpendicular to the initial path AOL. Then, QK = d, is called the ***lateral shift*** produced by the glass slab in the path of the light.

Thus, *the perpendicular distance between the incident ray and the emergent ray, when the light is incident obliquely on a parallel sided refracting slab, is called lateral shift.*

In right angled ΔOKQ, $\angle KOQ = i - r$

$$\therefore \qquad \sin(i-r) = \frac{QK}{OQ} = \frac{d}{OQ}$$

or
$$d = OQ \sin(i-r)$$

From right angled ΔON′Q, we have

$$\cos r = \frac{ON'}{OQ} = \frac{t}{OQ}$$

or
$$OQ = \frac{t}{\cos r}$$

Substituting for OQ, we have

$$d = \frac{t}{\cos r}\sin(i-r) \qquad \text{..(1.06)}$$

It may be pointed out that the lateral shift produced by a glass slab increases with

(*i*) *increase in the thickness of the glass slab,*
(*ii*) *increase in the value of the angle of incidence* and
(*iii*) *increase in the refractive index of the slab.*

1.05. REFRACTION THROUGH MULTIPLE REFRACTING MEDIA

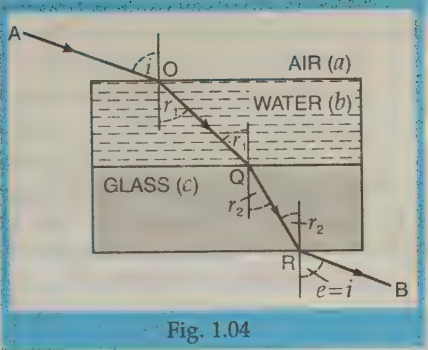


Fig. 1.04

Consider a compound plate of two media *b* (water) and *c* (glass) bounded by parallel faces and placed in the medium *a* (air) as shown in Fig. 1.04. Suppose that a ray of light AO travelling in medium *a* is incident at point O on the surface separating the two media *a* and *b*. It is refracted along OQ in medium *b*, such that *i* is the angle of incidence and r_1, the angle of refraction. Then,

$$ {}^a\mu_b = \frac{\sin i}{\sin r_1} \qquad \text{...(1.07)}$$

Now, the ray of light OQ meets the surface separating the two media *b* and *c* at angle of incidence r_1. It is, then, refracted along QR in medium *c*, such that r_2 is the angle of refraction. Then,

$$ {}^b\mu_c = \frac{\sin r_1}{\sin r_2} \qquad \text{...(1.08)}$$

Finally, the ray of light QR strikes the surface separating the media *c* and *a* at point R, such that r_2 is the angle of incidence. The ray of light, then emerges along path RB in medium *a*. As the various interfaces are parallel, the angle of emergence will be equal to *i*. Therefore,

$$ {}^c\mu_a = \frac{\sin r_2}{\sin i} \qquad \text{...(1.09)}$$

Multiplying the corresponding sides of the equations (1.07), (1.08) and (1.09), we have

$$ {}^a\mu_b \times {}^b\mu_c \times {}^c\mu_a = \frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i} = 1$$

or $$ {}^a\mu_b \times {}^b\mu_c = \frac{1}{{}^c\mu_a} \qquad \left(\because \frac{1}{{}^c\mu_a} = {}^a\mu_c \right)$$

or $$ {}^a\mu_c = {}^a\mu_c \times {}^b\mu_c \qquad \text{...(1.10)}$$

1.06. REAL AND APPARENT DEPTHS (NORMAL SHIFT)

As a consequence of refraction, the depth of an object lying inside an optically denser medium appears to be less than its real depth.

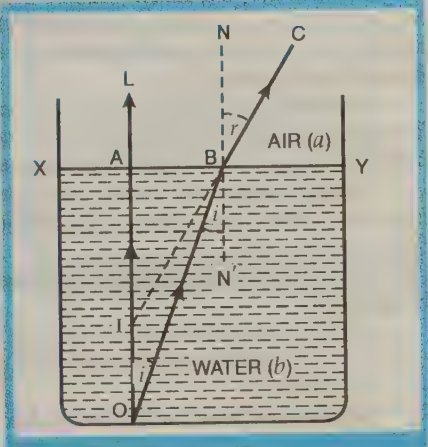


Fig. 1.05

Suppose that the surface XY separates medium *a* (air) and medium *b* (water or any other transparent liquid). Let O be a point object lying inside the liquid [Fig. 1.05]. A ray of light OA incident along normal to the surface of the liquid, will be refracted as such along AL. Another ray of light OB, on refraction into air, bends away from the normal along BC. The two refracted rays appear to come from the point I. Therefore, the object, though lying at depth AO, appears to be at depth AI. Accordingly, AO the ***real depth*** and AI is called ***apparent depth*** of the object O.

Suppose that ∠ OBN′ = *i* and ∠ NBC = *r*. Then, refractive index of medium *a* (air) *w.r.t.* medium *b* (water) is given by

$$ {}^b\mu_a = \frac{\sin i}{\sin r}$$

Since $$ {}^a\mu_b = \frac{1}{{}^b\mu_a}, \text{ we have}$$

$$ {}^a\mu_b = \frac{\sin r}{\sin i} \qquad \text{...(1.11)}$$

Now, ∠ AOB = ∠ OBN′ = *i*

Therefore, from the right angled Δ OAB, we have

$$ \sin i = \frac{AB}{OB}$$

Also, ∠ AIB = ∠ NBC = *r*

Therefore, from the right angled Δ IAB, we have

$$ \sin r = \frac{AB}{IB}$$

Substituting for sin *i* and sin *r* in equation (1.11), we have

$$ {}^a\mu_b = \frac{AB/IB}{AB/OB} = \frac{OB}{IB}$$

Since the aperture of the eye is very small, the two rays AL and BC will enter the eye only, if point B lies very close to the point A. In such a situation,

OB ≈ OA, the real depth of the object

and IB ≈ IA, the apparent depth of the object

Therefore, $${}^a\mu_b = \frac{OA}{IA} = \frac{\text{real depth}}{\text{apparent depth}} \qquad ...(1.12)$$

The object appears to be raised from its real position O to its apparent position I. The distance OI through which the position of the object appears to be raised, is called ***normal shift*** and it is denoted by *d*. Therefore,

normal shift, $$d = OA - IA$$

$$= OA\left(1 - \frac{IA}{OA}\right) = OA\left(1 - \frac{1}{OA/IA}\right)$$

or $$d = t\left(1 - \frac{1}{{}^a\mu_b}\right) \qquad ...(1.13)$$

It may be pointed out that the normal shift in the position of the object depends upon

(*i*) *the real depth of the object i.e. the thickness of the refracting medium* and

(*ii*) *the refractive index of the refracting medium.*

Key point

The apparent shift (*d*) in the position of an object placed in a denser medium depends upon the value of its refractive index (μ). The higher is the value of μ, greater is the apparent shift *d*.

1.07. TOTAL INTERNAL REFLECTION

Consider a surface XY separating the rarer medium *a* (air) from the denser medium *b* (water) as shown in Fig. 1.06. A ray of light OA from the object O in denser medium incident normally on the surface of separation, gets refracted into the rarer medium as such along AL. Another ray of light incident along the oblique path OA_1 is refracted away from the normal along the path A_1B_1 in the rarer medium. As the angle of incidence is increased, the angle of refraction (in air) also goes on increasing, till for a certain angle of incidence C, called the ***critical angle***, the refracted ray grazes along the surface of separation *i.e.* angle of refraction becomes just 90°.

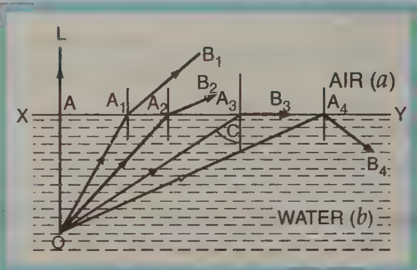


Fig. 1.06

If the angle of incidence is further increased, there will be no refracted ray. A ray, such as OA_4 incident at an angle greater than the critical angle, gets reflected along A_4B_4 according to the laws of reflection. This phenomenon is called ***total internal reflection.***

Therefore, *total internal reflection may be defined as the phenomenon of reflection of light that takes place, when a ray of light travelling in a denser medium gets incident at the interface of the two media at an angle greater than the critical angle for that pair of media.*

Critical angle. *The critical angle for a pair of media may be defined as the angle of incidence in the denser medium, for which the angle of refraction in the rarer medium is 90°.*

Thus, for the total internal reflection to take place, following conditions should be obeyed :

1. *The ray incident on the interface should travel in optically denser medium.*
2. *The angle of incidence should be greater than the critical angle for the given pair of media.*

Key point

For total internal reflection to take place, the ray of light should travel from optically denser to rarer medium and the angle of incidence must be greater than the value of critical angle for the pair of two media.

Relation between refractive index and critical angle. According to Snell's law, refractive index of rarer medium (*a*) *w.r.t.* denser medium (*b*),

$${}^b\mu_a = \frac{\sin i}{\sin r} \qquad ...(1.14)$$

When the ray of light is incident at critical angle *i.e.*

when $i = C, r = 90°$

Setting the above condition, equation (1.14) becomes

$${}^b\mu_a = \frac{\sin C}{\sin 90°} = \sin C$$

But $${}^b\mu_a = \frac{1}{{}^a\mu_b}$$

Therefore, refractive index of medium *b w.r.t.* medium *a* is given by

$${}^a\mu_b = \frac{1}{\sin C} \qquad ...(1.15)$$

It may be pointed out that there is practically no loss of light in total reflection ; vhile in reflection from a mirror, a small part of light is absorbed. In view of this, otally reflecting prisms* are used in optical instruments in order to change the path of light, whenever required.

1.08. PRACTICAL APPLICATIONS OF TOTAL INTERNAL REFLECTION

The phenomenon of total internal reflection is made use of in a number of ptical instruments. Moreover, it can be used in explaining many daily life bservations as discussed below :

1. Totally reflecting prisms. An isosceles right angled glass prism having angles 45°-90°- 45° can be used to deviate a ray of light through 90° or 180°. Such a glass prism deviates the ray of light through 90° or 180° making use of the phenomenon of total internal reflection and likewise such prisms are called ***totally reflecting prisms.***

For glass ($^a\mu_g = 1{\cdot}5$), the value of critical angle is about 42°. When a ray of light is incident normally on its any refracting surface, it passes through that face as such and comes to incident on the other face of the prism at a angle of incidence of 45°. Since the value of the angle of incidence is greater than 42°, the ray of light undergoes total internal reflection. By making light to fall on a *side face* or the *hypotenuse face*, the ray of light can be deviated through 90° or 180° as explained below :

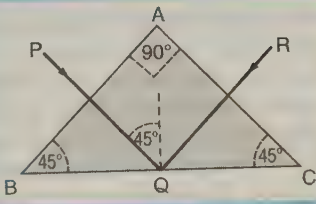


Fig. 1.07

(a) To deviate a ray of light through 90°. Consider that a ray of light from point P is incident normally on the side face AB of an isosceles right angled glass prism [Fig. 1.07]. The ray of light passes into the glass prism along PQ as such** and comes to meet the hypotenuse face BC of the prism at point Q. Since the angle of incidence is 45° *i.e.* greater than the critical angle for glass-air interface, it suffers total internal reflection at point Q on face BC. After total internal reflection, the ray of light comes to meet the side face AC at right angle and is refracted out of the prism as such along QR. It follows that the path of the ray has been deviated by the prism through 90°.

A totally reflecting prism is used to deviate the path of the ray of light through 90°, when it is inconvenient to view the direct light. In Michelson's method to find velocity of light, the direct light from the octagonal mirror is avoided from direct viewing by making use of totally reflecting prism.

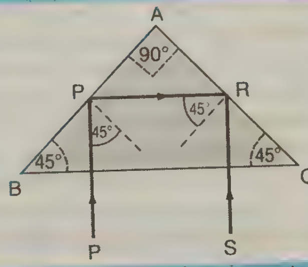


Fig. 1.08

(b) To deviate a ray of light through 180°. Fig. 1.08 shows a ray of light incident normally on the hypotenuse face BC of the 45°-90°-45° glass prism. The ray of light is refracted into the prism as such along PQ, suffers total internal reflection firstly at point Q on side face AB and then at point R on the second side face AC of the prism. When the ray of light comes to meet the hypotenuse face BC at right angles to it, it is refracted out of the prism as such along the path RS. The path of the ray of light has been turned through 180° due to two total internal reflections.

By deviating a ray of light through 180°, totally reflecting prisms are used in prism binoculars to produce an erect image† or to correct for lateral inversion.

The totally reflecting prisms have following advantages over the silvered plane mirrors :

(i) A reflecting prism does not require any silvering.

(ii) A silvered plane mirror always absorbs some light. In a good mirror, the reflection of light is about 90-95%. However, in a reflecting prism, almost 100% reflection is secured.

(iii) A thick plane mirror causes multiple reflections. Therefore, alongwith the image produced due to the first reflection at the silvered surface, several other images are produced due to subsequent multiple reflections. Obviously, the image formed is not very bright. Since in a totally reflecting prism, only one image is formed, hence the image obtained is very bright.

(iv) With the constant use of the mirror, its silvered surface may go bad. Then, the mirror will produce an image of a very poor quality. A reflecting prism is free from such a trouble.

(v) Due to imperfect and non-uniform silvering of the mirror, the image obtained with a mirror is often a distorted one.

---

* Refer to Section 1.08.

**When a ray of light is incident normally, $i = 0°$. According to Snell's law, r is also $0°$.

†For details, refer to SAQ 1.18.

2. Mirage. It is an optical illusion observed in deserts and coal-tarred roads on a hot day. The object such as a tree appears inverted and the observer gets the impression as if the inverted image has been formed by a pool of water. This phenomenon is known as mirage.

Due to intense heat, the surface of earth becomes quite hot and the temperature of air near the surface of earth is maximum. The temperature of the other layers of the air goes on decreasing as one goes up. Therefore, density as well as refractive index of the air increases slightly for higher layers. Thus, a ray of light travelling from point O of a tree [Fig. 1.09], passes through air of gradually decreasing refractive index and is therefore refracted more and more away from the normal and accordingly the angle of incidence goes on increasing. At a layer, when the angle of incidence becomes greater than the critical angle, total internal reflection takes place. Then, the ray of light starts traversing layers of increasing refracting indices and goes on bending more and more towards the normal. Ultimately, when the ray reaches the eye of the observer, it appears to be coming from the point I. Hence, the inverted image of the tree produces the impression of reflection from a pool of water.

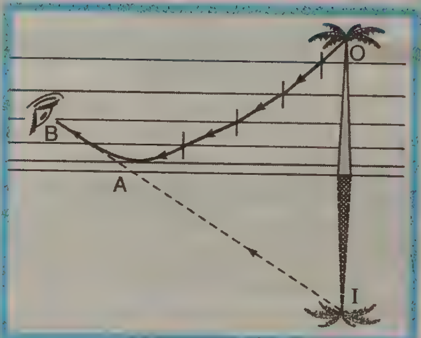


Fig. 1.09

3. The brilliance of diamonds and other precious stones. The refractive index of diamond is 2·47 and the critical angle for diamond-air interface is 23° only. Due to the low value of critical angle, a diamond can be cut so as to have a large number of faces, such that a ray of light on entering it from one face undergoes repeated total internal reflections from other faces. As a result, the faces through which the light emerges, shine very brilliantly.

4. Optical fibres. In 1870, John Tyndall, a British physicist, demonstrated that light could be made to follow curved path along a thin stream of water coming out of a water tank. This effect was made use of in illuminated fountains. The light follows curved path as it suffers a series of total internal reflections.

Optical fibres are used to transmit light from one place to the other along curved path in a more effective manner. The optical fibres consist of thousands of strands of a very fine quality glass or quartz of refractive index about 1·7 or so. The thickness of a strand is about 10^{-6} cm. The strands are coated with a layer of some material of lower refractive index (about 1·5). The ends of the strands are polished and clamped firmly after aligning them carefully.

When light is incident at a small angle at one end, it gets refracted into the strands (or fibres) and gets incident on the interface of the fibres and the coating. The angle of incidence being greater than the critical angle, the ray of light undergoes total internal reflections [Fig. 1.10]. It suffers the internal reflections again and again*, till the angle of incidence remains greater than the critical angle for the fibre material *w.r.t.* coating. Each fibre acts as a pipe and such a bundle of fibres can be used to convey images along paths of any shape. It may be pointed that the optical fibre does not bend light. Instead, light follows the zig-zag path through the fibres.

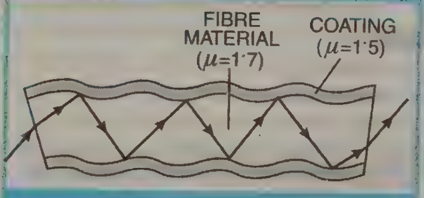


Fig. 1.10

Uses. Optical fibres have been found to be of great use. A few applications of optical fibres are as listed below :

(i) The optical fibres are used in the field of communication and the computers. They are used for transmitting and receiving electrical signals, which are converted into light.

(ii) The optical fibres are used for transmitting the optical signals and the two dimensional pictures.

(iii) The optical fibres are used for making medical investigations. Using an optical fibre, a doctor can visually examine the inside of the stomach, intestine, etc. It is called ***endoscopy*** in medical science.

(iv) The optical fibres in the form of photometric sensors are used for measuring the blood flow in the heart.

(v) The optical fibres in the form of refractometers are used to determine the refractive indices of liquids.

(vi) The optical fibre sensors have been used to measure temperature and pressure.

1.09. SPHERICAL REFRACTING SURFACE

The portion of a refracting medium, whose curved surface forms the part of a sphere, is called spherical refracting surface.

---

*A ray of light inclined at 30° to the axis of the fibres undergoes 22700 reflections per metre.

The spherical refracting surfaces are of two types :

(*a*) **Convex spherical refracting surface.** The spherical refracting surface convex towards the rarer medium is called convex spherical refracting surface [Fig. 1.11].

(*b*) **Concave spherical refracting surface.** The spherical refracting surface concave towards the rarer medium is called concave spherical refracting surface [Fig. 1.12].

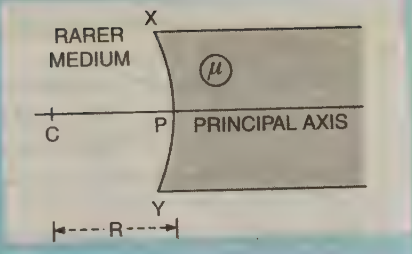


Fig. 1.11

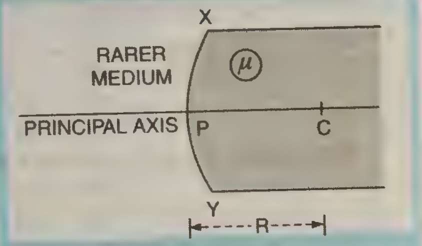


Fig. 1.12

Following terms are related to the study of refraction from spherical refracting surfaces :

1. Pole. *The centre of the spherical refracting surface is called its pole.* It is denoted by P.

2. Centre of curvature. *The centre of the sphere of which the curved surface forms a part is called its centre of curvature.* It is denoted by C.

3. Radius of curvature. *The radius of the sphere of which the curved surface forms a part is called its radius of curvature.* It is denoted by R.

4. Aperture. *The diameter of the spherical refracting surface is called its aperture.* The line joining the points X and Y represents the aperture of the spherical refracting surface.

5. Principal axis. *The line passing through the pole and the centre of curvature of the spherical refracting surface is called its principal axis.*

1.10. NEW CARTESIAN SIGN CONVENTIONS AND ASSUMPTIONS

As in the case of spherical mirrors, the refraction from spherical refracting surfaces will be discussed using the *new* ***Cartesian sign conventions.***

New Cartesian sign conventions. 1. *All the distances are measured from the pole of the spherical refracting surface.*

2. *The distances measured in the direction of the incident light are taken positive and those measured in the direction opposite to the direction of incident light are taken as negative.*

To study refraction from the spherical refracting surfaces, we make the following assumptions :

Assumptions. 1. The aperture of the refracting surface is small.

2. The object is a point object situated on the principal axis.

3. The incident and the refracted rays make small angles with the principal axis.

1.11. REFRACTION AT CONVEX SPHERICAL SURFACE

Consider a convex spherical refracting surface of absolute refractive index μ_2 placed in an optically rarer medium of absolute refractive index μ_1, such that the convex surface is towards the rarer medium. Let P be its pole, C the centre of curvature and PC, the principal axis of the convex spherical refracting surface.

(*a*) When object lies in the rarer medium and image formed is real. Let O be a point object in the rarer medium and situated on the principal axis. A ray of light OP incident normally on the refracting surface along the principal axis, passes undeviated into the denser medium. Another ray of light OA incident at point A at an angle of incidence i, gets refracted towards the normal CAN′. Let r be the angle of refraction. The two rays meet at a point I on the principal axis and therefore I is the real image of the point object O as shown in Fig. 1.13.

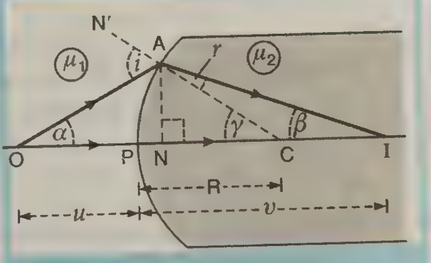


Fig. 1.13

Let $\angle AOP = \alpha$; $\angle AIP = \beta$ and $\angle ACP = \gamma$

From the point A, drop AN perpendicular to the principal axis of the spherical refracting surface. From triangle AOC, we have

$$i = \alpha + \gamma \qquad \text{...(1.16)}$$

Since aperture of the spherical refracting surface is small, the point A will be close to the point P and hence angles α, β and γ will be small. As such, these angles may be replaced by their tangents.

Therefore, equation (1.16) may be written as

$$i = \tan\alpha + \tan\gamma \qquad \text{...(1.17)}$$

From right angled triangles ANO and ANC, we have

$$\tan\alpha = \frac{AN}{NO} \quad \text{and} \quad \tan\gamma = \frac{AN}{NC}$$

Substituting for tan α and tan α in equation (1.17), we have

$$i = \frac{AN}{NO} + \frac{AN}{NC} \qquad \text{...(1.18)}$$

Again, as aperture of the refracting surface is small, point N will be close to point P, the pole of the refracting surface. Therefore,

$NO \approx PO$ and $NC \approx PC$

Therefore, equation (1.18) becomes

$$i = \frac{AN}{PO} + \frac{AN}{PC} \qquad ...(1.19)$$

Now, from triangle ACI, $\gamma = r + \beta$

or $$r = \gamma - \beta$$

Since angles β and γ are small, we have

$$r = \tan\gamma - \tan\beta \qquad ...(1.20)$$

From right angled triangles ANC and ANI, we have

$$\tan\gamma = \frac{AN}{NC} \approx \frac{AN}{PC}$$

and $$\tan\beta = \frac{AN}{NI} \approx \frac{AN}{PI}$$

Substituting for $\tan\beta$ and $\tan\gamma$ in equation (1.20), we have

$$r = \frac{AN}{PC} - \frac{AN}{PI} \qquad ...(1.21)$$

If μ is relative refractive index of the denser medium (μ_2) *w.r.t.* the rarer medium (μ_1), then

$$\mu = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

or $$\mu_1 \sin i = \mu_2 \sin r$$

Since the angles i and r are also small, the above equation becomes

$$\mu_1 i = \mu_2 r$$

From equations (1.19) and (1.21), substituting the values of i and r, we have

$$\mu_1\left(\frac{AN}{PO} + \frac{AN}{PC}\right) = \mu_2\left(\frac{AN}{PC} - \frac{AN}{PI}\right)$$

or $$\frac{\mu_1}{PO} + \frac{\mu_1}{PC} = \frac{\mu_2}{PC} - \frac{\mu_2}{PI}$$

or $$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC} \qquad ...(1.22)$$

Applying new cartesian sign conventions :

$PO = -u$ (∵ distance of object is *against* incident light)

$PI = +v$ (∵ distance of image is *along* incident light)

$PC = +R$

(∵ distance of centre of curvature is *along* incident light)

Therefore, equation (1.22) becomes

$$\frac{\mu_1}{-u} + \frac{\mu_2}{+v} = \frac{\mu_2 - \mu_1}{+R}$$

or $$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R} \qquad ...(1.23)$$

The equation (1.23) connects u, v and R to the absolute refractive indices of the material of the refracting surface and that of the rarer medium.

The above formula can be expressed in another form as given below :

Dividing both sides of equation (1.23) by μ_1, we have

$$-\frac{1}{u} + \frac{\mu_2/\mu_1}{v} = \frac{\mu_2/\mu_1 - 1}{R}$$

Since $\mu_2/\mu_1 = \mu$, we have

$$-\frac{1}{u} + \frac{\mu}{v} = \frac{\mu_1 - 1}{R}$$

***(b)* When object lies in the rarer medium and image formed is virtual.** When an object is situated in rarer medium very close to the convex spherical refracting surface, its virtual image is formed in the rarer medium.

Let O be a point object in rarer medium and situated on the principal axis and close to the pole of the refracting surface. A ray of light OP incident normally on the refracting surface along the principal axis, passes undeviated into the denser medium. Another ray of light OA incident at point A at an angle of incidence i, gets refracted towards the normal CAN′. Let r be the angle of refraction. The two rays do not meet but appear to come from the point I on the principal axis, and therefore I is the virtual image of the point object O [Fig. 1.14].

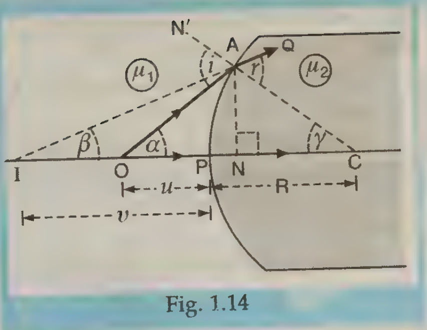


Fig. 1.14

Let $\angle AOP = \alpha$; $\angle AIP = \beta$ and $\angle ACP = \gamma$

From the point A, drop AN perpendicular to the principal axis of the spherical refracting surface.

From triangle AOC,

$$i = \alpha + \gamma$$

or $$i = \tan\alpha + \tan\gamma = \frac{AN}{NO} + \frac{AN}{NC}$$

As aperture of the refracting surface is small, it follows that

$$NO \approx PO \,;\, NC \approx PC$$

$$\therefore \quad i = \frac{AN}{PO} + \frac{AN}{PC} \qquad \text{...(1.24)}$$

From triangle AIC, $r = \beta + \gamma$

Now, angles β and γ are small. Therefore, we may write

$$r = \tan\beta + \tan\gamma = \frac{AN}{NI} + \frac{AN}{NC}$$

Again, as the aperture of the refracting surface is small,

$$NI \approx PI \,;\, NC \approx PC$$

$$\therefore \quad r = \frac{AN}{PI} + \frac{AN}{PC} \qquad \text{...(1.25)}$$

If μ is relative refractive index of the denser medium (μ_2) *w.r.t.* the rarer medium (μ_1), then

$$\mu = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

or $$\mu_1 \sin i = \mu_2 \sin r$$

Since the angles i and r are small, we have

$$\mu_1 i = \mu_2 r$$

From equations (1.24) and (1.25), substituting the values of i and r in the above equation, we have

$$\mu_1\left(\frac{AN}{PO} + \frac{AN}{PC}\right) = \mu_2\left(\frac{AN}{PI} + \frac{AN}{PC}\right)$$

or $$\frac{\mu_1}{PO} - \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC} \qquad \text{...(1.26)}$$

From new cartesian sign conventions :

$PO = -u$ ($\because$ distance of object is *against* incident light)

$PI = -v$ ($\because$ distance of image is *against* incident light)

$PC = +R$

($\because$ distance of centre of curvature is *along* incident light)

Therefore, equation (3.26) becomes

$$\frac{\mu_1}{-u} - \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{+R}$$

or $$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R} \qquad \text{...(1.27)}$$

The above relation can be expressed in another form as follows :

Dividing both sides of equation (1.27) by μ_1, we have

$$-\frac{1}{u} + \frac{\mu_2/\mu_1}{v} = \frac{\mu_2/\mu_1 - 1}{R}$$

or $$-\frac{1}{u} + \frac{\mu}{v} = \frac{\mu - 1}{R}$$

(c) When object lies in denser medium. Let O be a point object in denser medium and situated on the principal axis. A ray of light incident normally on the refracting surface along the principal axis, passes undeviated. Another ray of light OA incident at point A at an angle of incidence i ; after refraction, bends away from the normal CAN′, such that r is angle of refraction. The two rays appear to come from point I on the principal axis and therefore I is the virtual image of the point object O as shown in Fig. 1.15.

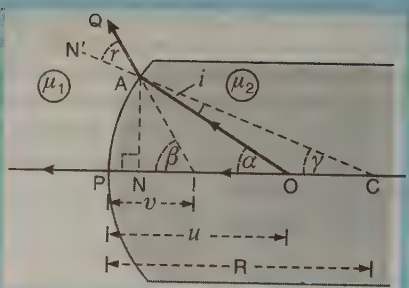


Fig. 1.15

Let $\angle AOP = \alpha$; $\angle AIP = \beta$

and $\angle ACP = \gamma$

From the point A, drop AN perpendicular to the principal axis of the spherical refracting surface. From triangle AOC, we have

$$\alpha = i + \gamma$$

or $$i = \alpha - \gamma$$

Since angles α and γ are small, we may write

$$i = \tan \alpha - \tan \gamma = \frac{AN}{NO} - \frac{AN}{NC}$$

Again, as aperture of the refracting surface is small, NO ≈ PO and NC ≈ PC. Therefore, the above equation becomes

$$i = \frac{AN}{PO} - \frac{AN}{PC} \qquad ...(1.28)$$

Now, from triangle ACI, $\beta = r + \gamma$ or $r = \beta - \gamma$

As said earlier, as the angles β and γ are small, they can be replaced by their tangents.

$$\therefore \qquad r = \tan \beta - \tan \gamma = \frac{AN}{NI} - \frac{AN}{NC} \qquad ...(1.29)$$

Since NI ≈ PI and NC ≈ PC, we have

$$r = \frac{AN}{PI} - \frac{AN}{PC}$$

If μ is relative refractive index of denser medium (μ_2) *w.r.t.* rarer medium (μ_1), then

$$\mu = \frac{\mu_2}{\mu_1} = \frac{\sin r}{\sin i} \quad \text{or} \quad \mu_2 \sin i = \mu_1 \sin i$$

Since i and r are also small, we have

$$\mu_2 i = \mu_1 r$$

From equations (1.28) and (1.29), substituting for i and r in the above equation, we have

$$\mu_2 \left(\frac{AN}{PO} - \frac{AN}{PC} \right) = \mu_1 \left(\frac{AN}{PI} - \frac{AN}{PC} \right)$$

or $$\frac{\mu_2}{PO} - \frac{\mu_2}{PC} = \frac{\mu_1}{PI} - \frac{\mu_1}{PC}$$

or $$\frac{\mu_2}{PO} - \frac{\mu_1}{PI} = \frac{\mu_2 - \mu_1}{PC} \qquad ...(1.30)$$

Applying new cartesian sign conventions :

PO = − u (∵ distance of object is *against* the incident ray)

PI = − v (∵ distance of image is *against* the incident ray)

PC = − R

(∵ distance of centre of curvature is *against* incident ray)

Therefore, equation (1.30) becomes

$$\frac{\mu_2}{-u} - \frac{\mu_1}{-v} = \frac{\mu_2 - \mu_1}{-R}$$

or $$-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R} \qquad ...(1.31)$$

The above relation may be expressed in another form as given below :

Dividing both sides of the equation (1.31) by μ_1 and replacing $\frac{\mu_2}{\mu_1}$ by μ, we have

$$-\frac{\mu_2/\mu_1}{u} + \frac{1}{v} = \frac{1 - \mu_2/\mu_1}{R}$$

$$-\frac{\mu}{u} + \frac{1}{v} = \frac{1 - \mu}{R}$$

Note. 1. In case, the object is situated in rarer medium so that the incident ray travels in rarer medium and the refracted ray travels in denser medium, then as obtained in section 1.11 (*a*) and 1.11 (*b*), the relation between u, v, R, μ_1 and μ_2 is

$$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R} \qquad ...(1.32)$$

2. In case, the object is situated in denser medium, so that the incident ray travels in denser medium and refracted ray in rarer medium, then as obtained in section 1.11 (*c*), the relation between u, v, R, μ_1 and μ_2 is

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R} \qquad ...(1.33)$$

It may be noted that the equation (1.33) can be obtained from equation (1.32) by simply interchanging μ_1 and μ_2.

3. The factor $\frac{\mu_2-\mu_1}{R}$ is called power of the spherical refracting surface. It gives the measure of the degree to which the refracting surface can converge or diverge the rays of light passing through it.

1.12. REFRACTION AT CONCAVE SPHERICAL SURFACE

Consider a concave spherical refracting surface of absolute refractive index μ_2 placed in an optically rarer medium of absolute refractive index μ_1. Let P be pole ; C, the centre of curvature and PC, the principal axis of the spherical refracting surface.

Let O be a point object in the rarer medium and situated on the principal axis. A ray of light OP incident normally on the refracting surface along the principal axis, passes undeviated into the denser medium. Another ray of light OA incident at the point A at an angle of incidence i is refracted towards the normal CAN′. Let r be the angle of refraction. The two rays AQ and OP appear to come from the point I on the principal axis and therefore I is the virtual image of the point object O as shown in Fig. 1.16.

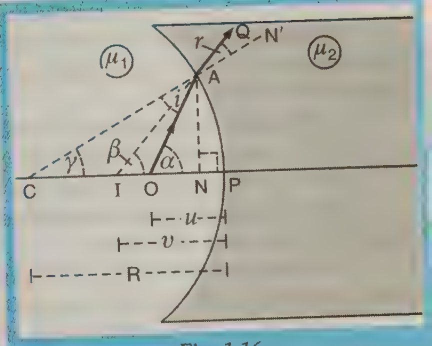


Fig. 1.16

From triangle AOC, $\alpha=i+\gamma$

or $\quad i=\alpha-\gamma$

Since angles α and γ are small, they can be replaced by their tangents. Therefore

$$i=\tan\alpha-\tan\gamma=\frac{AN}{NO}-\frac{AN}{NC}$$

As aperture of the spherical refracting surface is small, NO ≈ PO and NC ≈ PC.

$$\therefore \qquad i=\frac{AN}{PO}-\frac{AN}{PC} \qquad ...(1.34)$$

From triangle AIC, $\beta=r+\gamma$

or $\quad r=\beta-\gamma$

Again angles β and γ are small. Therefore,

$$r=\tan\beta-\tan\gamma=\frac{AN}{NI}-\frac{AN}{NC}$$

As aperture of the spherical refracting surface is small, NI = PI and NC ≈ PC.

$$\therefore \qquad r=\frac{AN}{PI}-\frac{AN}{PC} \qquad ...(1.35)$$

If μ is the relative refractive index of the denser medium (μ_2) *w.r.t.* the rarer medium (μ_1), then

$$\mu=\frac{\mu_2}{\mu_1}=\frac{\sin i}{\sin r} \quad \text{or} \quad \mu_1\sin i=\mu_2\sin r$$

Since the angles i and r are small, we have

$$\mu_1 i=\mu_2 r$$

From equations (1.34) and (1.35), substituting the values of i and r in the above equation, we have

$$\mu_1\left(\frac{AN}{PO}-\frac{AN}{PC}\right)=\mu_2\left(\frac{AN}{PI}-\frac{AN}{PC}\right)$$

or

$$-\frac{\mu_1}{PO}+\frac{\mu_2}{PI}=\frac{\mu_2-\mu_1}{PC} \qquad ...(1.36)$$

From new cartesian sign conventions :

$$PO=-u,\ PI=-v,\ PC=-R$$

Therefore, equation (1.36) becomes

$$-\frac{\mu_1}{-u}+\frac{\mu_2}{-v}=\frac{\mu_2-\mu_1}{-R}$$

or
$$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R} \qquad ...(1.37)$$

The above relation can be expressed in another form as given below :

Dividing both sides of equation (1.37) by μ_1 and setting $\frac{\mu_2}{\mu_1}=\mu$, we have

$$-\frac{1}{u}+\frac{\mu}{v}=\frac{\mu-1}{R}$$

Note. In case, the object is situated in the denser medium so that the incident ray travels in the denser medium and the refracted ray travels in the rarer medium, then the relation between u, v, R, μ_1 and μ_2 can be obtained by interchanging μ_1 and μ_2 and the relation will be same as given by equation (1.33).

1.13. LENS MAKER'S FORMULA

The lens maker's formula relates the focal length of a lens to the refractive index of its material and the radii of curvature of its two surfaces. This formula is used to manufacture a lens of particular focal length from the glass of given refractive index. For this reason, it is called ***lens maker's formula.***

To derive lens maker's formula, we make the following assumptions and adopt new Cartesian sign conventions :

Assumptions. 1. *The lens is thin, so that the distances measured from the poles of two surfaces of the lens can be taken as to be the same as the distances measured from the optical centre of the lens.*

2. *The aperture of the lens is small.*

3. *The object is a point object situated on the principal axis.*

4. *The incident and refracted rays make small angles with the principal axis.*

New cartesian sign conventions. 1. All the distances are to be measured from the optical centre of the lens.

2. *The distances measured in the direction of the incident ray are taken as positive and those measured in the direction opposite to the incident ray are taken as negative.*

Consider a convex lens made of material of absolute refractive index μ_2 and placed in an optically rarer medium of absolute refractive index μ_1. Let P_1 and P_2 be the poles ; C_1 and C_2, the centres of curvature and R_1 and R_2, the radii of curvature of the two surfaces XP_1Y and XP_2Y of the lens respectively. Let C be the optical centre of the lens.

Suppose that O is a point object placed on the principal axis of the lens [Fig. 1.17].

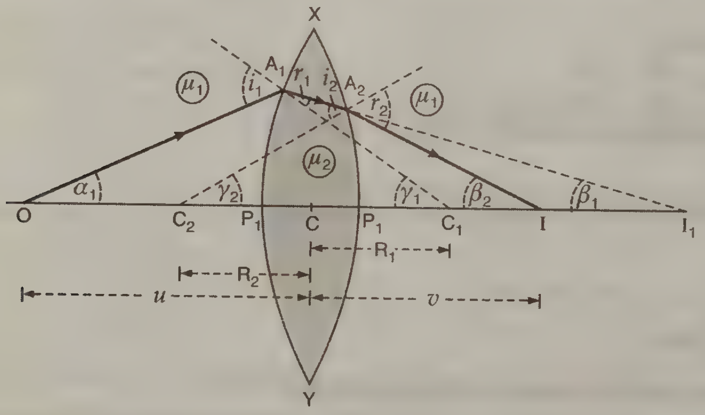


Fig. 1.17

The surface XP_1Y forms the real image I_1 (assuming that material of the lens extends beyond the face XP_1Y as such). It can be obtained* that

$$\frac{\mu_1}{P_1O}+\frac{\mu_1}{P_1I_1}=\frac{\mu_2-\mu_1}{P_1C_1} \qquad ...(1.38)$$

---

*Refer to section 1.11. The equation (1.38) is same as the equation (1.22) obtained in section 1.11.

Since lens is thin, the point P_1 lies very close to the optical centre C of the lens. herefore, we may write

$$P_1O \approx CO\,;\, P_1I_1 \approx CI_1$$

and $$P_1C_1 \approx CC_1$$

Therefore, equation (1.38) becomes

$$\frac{\mu_1}{CO} + \frac{\mu_2}{CI_1} = \frac{\mu_2 - \mu_1}{CC_1} \qquad \text{...(1.39)}$$

Let us now consider refraction from the other surface XP_2Y of the convex lens. Actually, material of lens does not extend beyond the surface XP_2 Y. Therefore, before he refracted ray from point A_1 could meet the principal axis, it will suffer refraction t the point A_2 on the second surface XP_2 Y and the ray of light will actually come o meet the principal axis at the point I. Thus, point I may be considered as the real mage of the virtual object I_1 (placed in the material of the lens) as formed due to efraction from the surface XP_2Y of the lens. It can be deduced that

$$-\frac{\mu_2}{P_2I_1} + \frac{\mu_1}{P_2I} = \frac{\mu_2 - \mu_1}{P_2C_2} \qquad \text{...(1.40)}$$

Again as the lens is thin, we may write

$$P_2I_1 \approx CI_1,\; P_2I \approx CI \quad \text{and} \quad P_2C_2 \approx CC_2$$

Therefore, equation (1.40) may be written as

$$-\frac{\mu_2}{CI_1} + \frac{\mu_1}{CI} = \frac{\mu_2 - \mu_1}{CC_2} \qquad \text{...(1.41)}$$

Adding equations (1.39) and (1.41), we have

$$\frac{\mu_1}{CO} + \frac{\mu_2}{CI_1} - \frac{\mu_2}{CI_1} + \frac{\mu_1}{CI} = \frac{\mu_2 - \mu_1}{CC_1} + \frac{\mu_2 - \mu_1}{CC_2}$$

or $$\frac{\mu_1}{CO} + \frac{\mu_1}{CI} = (\mu_2 - \mu_1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right) \qquad \text{...(1.42)}$$

Applying new cartesian sign conventions :

$CO = -u$ (∵ distance u is *against* the incident ray)

$CI = +v$ (∵ distance v is *along* the incident ray)

$CC_1 = +R_1$

(∵ radius of curvature R_1 is *along* the incident ray)

and $CC_2 = -R_2$

(∵ radius of curvature R_2 is *against* the incident ray)

Therefore, equation (1.42) becomes

$$\frac{\mu_1}{-u} + \frac{\mu_1}{+v} = (\mu_2 - \mu_1)\left(\frac{1}{+R_1} + \frac{1}{-R_2}\right)$$

or $$-\frac{\mu_1}{u} + \frac{\mu_1}{+v} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Dividing both sides of the above equation by μ_1, we have

$$-\frac{1}{u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since $\frac{\mu_2}{\mu_1} = \mu$, we have

$$-\frac{1}{u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{...(1.43)}$$

First principal focus and the focal length. *First principal focus is the position of the object on the principal axis of the lens, for which the image is formed at infinity.* It is denoted by point F_1 [Fig. 1.18].

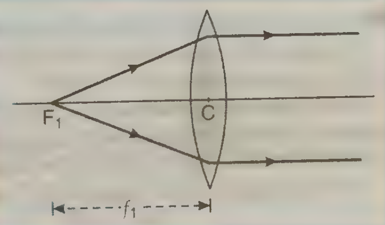


Fig. 1.18

The distance between optical centre and first principal focus is called ***first principal focal length*** *of the lens.* It is denoted by f_1. Thus,

if $$u = CF_1 = -f_1,\ \text{then } v = \infty$$

Setting the above condition in equation (1.43), we have

$$-\frac{1}{-f_1} + \frac{1}{\infty} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(1.44)$$

Second principal focus and the focal length. *Second principal focus is the position of the image on the principal axis of the lens, when the object lies at infinity.* It is denoted by point F_2 as shown in Fig. 1.19.

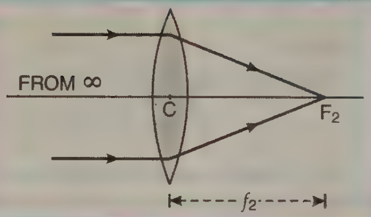


Fig. 1.19

The distance between optical centre and second principal focus is called ***second principal focal length*** *of the lens.* It is denoted by f_2.

Thus, if $u=\infty, v=CF_2=+f_2$

Setting this condition in equation (1.43), we have

$$-\frac{1}{\infty}+\frac{1}{+f_2}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or $$\frac{1}{f_2}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(1.45)$$

From the equations (1.44) and (1.45), it follows that two principal focal lengths of a lens are equal *i.e.*

$$f_1=f_2=f \text{ (say)}$$

Therefore, the equation (1.44) or (1.45) may be written as

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad ...(1.46)$$

The equation (1.46) is called ***lens maker's formula.***

Note. 1. The lens maker's formula deduced for a convex lens is applicable to both the convex and the concave lenses.

2. The focal length of a convex lens is taken as positive and that of a concave lens as negative.

3. For a convex lens, $R_1=+$ ve and $R_2=-$ ve ; while for a cocave lens, $R_1=-$ ve and $R_2=+$ ve.

Key point ✓

The focal length of a lens depends upon the value of refractive index of its material and radii of curvative of its two surfaces.

Key point ✓

1. Focal length of a lens (made of material having refractive index μ) increases, when it is placed in a medium of refractive index μ' ($\mu'<\mu$) and it still behaves as the lens of same type.

2. In case lens is placed in a medium of refractive index greater than that of the material of lens, the nature of lens changes (a converging lens behaves as a diverging lens and vice-versa).

1.14. GAUSSIAN FORM OF LENS EQUATION (From lens maker's formula)

The relation connecting the distances of the object and image with the focal length of a lens is called ***Gaussian form of lens equation*** or ***simply lens equation.*** Let us obtain this relation directly from the lens maker's formula.

Refer to equation (1.43) obtained while deriving the lens maker's formula. It was obtained that

$$-\frac{1}{u}+\frac{1}{v}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Also, from lens maker's formula, we have

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Therefore, from the above two equations, we have

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad ...(1.47)$$

Note 1. The lens equation and the mirror formula obtained by applying new cartesian sign conventions are not same.

2. In case of a lens, the real image is formed on the other side of the real object ; while the virtual image is formed on the same side the object is. Therefore, in case of a lens, *the distance of the real image from the lens is taken as positive and that of virtual image is taken as negative.*

1.15. TO DERIVE LENS EQUATION (Convex Lens)

In case of a convex lens, the image formed may be real or virtual. Let us derive the lens equation in the two cases :

(a) **When image formed is real.** Consider that an object AB is placed at a point beyond F on the principal axis of a convex lens [Fig. 1.20]. Its real image A′B′ will be formed on the other side of the object as shown in the figure.

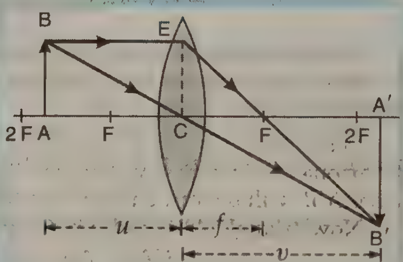


Fig. 1.20

Now, triangles ABC and A′B′C are similar. Therefore,

$$\frac{CA}{CA'}=\frac{AB}{A'B'} \qquad ...(1.48)$$

Also, triangles CEF and A′B′F are similar. Therefore,

$$\frac{CF}{FA'} = \frac{CE}{A'B'}$$

Since CE = AB, the above equation becomes

$$\frac{CF}{FA'} = \frac{AB}{A'B'} \qquad ...(1.49)$$

From equations (1.48) and (1.49), we have

$$\frac{CF}{CA'} = \frac{CF}{FA'}$$

or

$$\frac{CA}{CA'} = \frac{CF}{CA' - CF} \qquad ...(1.50)$$

According to new Cartesian sign conventions :

$$CA = -u\ ; CA' = +v \text{ and } CF = +f$$

Therefore, equation (1.50) becomes

$$\frac{-u}{+v} = \frac{+f}{+v-(+f)} \quad \text{or} \quad -\frac{u}{v} = \frac{f}{v-f}$$

or

$$-uv + uf = vf \quad \text{or} \quad -vf + uf = uv$$

Dividing both sides by $u\,v\,f$, we have

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

It is the required relation between u, v and f.

***(b)* When image formed is virtual.** Consider that an object AB is placed between optical centre C and focus F on the principal axis of a convex lens [Fig. 1.21]. Its virtual image A′B′ will be formed on the same side of the object as shown in the figure.

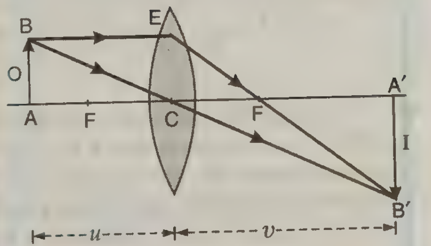


Fig. 1.21

Now, triangles ABC and A′B′C are similar. Therefore,

$$\frac{CF}{CA'} = \frac{AB}{A'B'} \qquad ...(1.51)$$

Also, triangles CEF and A′B′F are similar. Therefore,

$$\frac{CF}{FA'} = \frac{CE}{A'B'}$$

Since CE = AB, the above equation becomes

$$\frac{CF}{FA'} = \frac{CF}{A'B'} \qquad ...(1.52)$$

From equations (1.51) and (1.52), we have

$$\frac{CA}{CA'} = \frac{CF}{FA'}$$

or

$$\frac{CA}{CA'} = \frac{CF}{CA' + CF}$$

According to new Cartesian sign conventions :

$$CA = -u\ ; CA' = -v \text{ and } CF = +f$$

Therefore, equation (1.53) becomes

$$\frac{-u}{-v} = \frac{+f}{-v+(+f)} \quad \text{or} \quad \frac{u}{v} = \frac{f}{-v+f}$$

or

$$-uv + uf = vf \quad \text{or} \quad -vf + uf = uv$$

Dividing both sides by $u\,v\,f$, we have

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

It is the required relation between u, v and f.

Key point

When a convex lens produces a real image of an object, the minimum distance between the object and the image is equal to four times its focal length.

It is because, the lens equation gives a real solution for u and v only if focal length f is greater than one-fourth of the distance between the object and image.

16. TO DERIVE LENS EQUATION (Concave lens)

In case of a concave lens, image formed is always virtual in nature. Consider that an object AB is placed on the principal axis of a concave lens [Fig. 1.22]. Its virtual image A′B′ will be formed on the same side of the object as shown in the figure.

Now, triangle ABC and A′B′C are similar. Therefore,

$$\frac{CA}{CA'} = \frac{AB}{A'B'} \qquad ...(1.54)$$

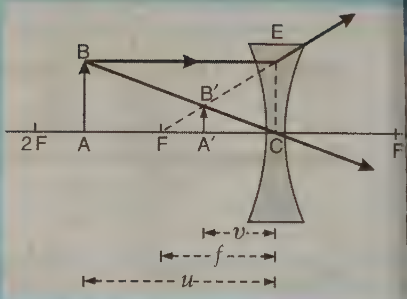


Fig. 1.22

Also, triangles CEF and A′B′F are similar. Therefore,

$$\frac{CF}{FA'}=\frac{CE}{A'B'}$$

Setting CE = AB, the above equation becomes

$$\frac{CF}{FA'}=\frac{AB}{A'B'} \qquad \text{...(1.55)}$$

From equations (1.54) and (1.55), we have

$$\frac{CA}{CA'}=\frac{CF}{FA'}$$

or

$$\frac{CA}{CA'}=\frac{CF}{CF-CA'} \qquad \text{...(1.56)}$$

Applying new cartesian sign conventions : CA = $-u$; CA′ = $-v$ and CF = $-f$

Therefore, equation (1.56) becomes

$$\frac{-u}{-v}=\frac{-f}{-f-(-v)} \quad \text{or} \quad \frac{u}{v}=\frac{f}{-f+v}$$

or $\quad -uf+uv=-vf \quad$ or $\quad -vf+uf=uv$

Dividing both sides by uvf, we have

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

It is the required relation between u, v and f.

1.17. LINEAR MAGNIFICATION

The ratio of the size of the image formed by a lens to the size of the object is called linear magnification produced by the lens. It is denoted by m.

If O and I are the sizes of object and image respectively, then

$$m=\frac{I}{O} \qquad \text{...(1.57)}$$

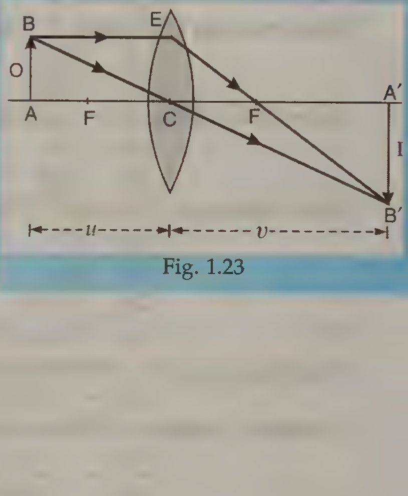


Fig. 1.23

Expression for linear magnification. Consider a convex lens of focal length f. Let C be optical centre and F be its principal focus. Suppose that an object AB is placed on the principal axis of the convex lens. Then, the real image A′B′ of the object AB is formed by the convex lens as shown in Fig. 1.23.

Let u and v be the distances of the object AB and image A′B′ from the optical centre (C) of the lens.

Now, triangles A′B′C and ABC are similar.

Therefore, $$\frac{A'B'}{AB}=\frac{CA'}{CA} \qquad \text{...(1.58)}$$

Applying new cartesian sign conventions, we have

A′B′ = $-$I (∵ height is *downwards*)

AB = +O (∵ height is *upwards*)

CA′ = $+v$ (∵ image distance is *along* incident light)

CA = $-u$ (∵ object distance is *against* incident light)

Therefore, equation (1.58) becomes

$$\frac{-I}{+O}=\frac{+v}{-u} \quad \text{or} \quad \frac{I}{O}=\frac{v}{u} \qquad \text{...(1.59)}$$

From equations (1.57) and (1.58), we have

$$m=\frac{I}{O}=\frac{v}{u} \qquad \text{...(1.60)}$$

Now, for a lens,

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Multiplying both sides of the lens equation by u, we have

$$-1+\frac{u}{v}=\frac{u}{f} \quad \text{or} \quad \frac{u}{v}=1+\frac{u}{f}=\frac{f+u}{f}$$

or

$$\frac{v}{u}=\frac{f}{f+u} \qquad \text{...(1.61)}$$

Also, multiplying both sides of the lens equation by v, we have

$$-\frac{v}{u}+1=\frac{v}{f} \quad \text{or} \quad \frac{v}{u}=1-\frac{v}{f}$$

or

$$\frac{v}{u}=\frac{f-v}{f} \qquad ...(1.62)$$

From equation (1.60), (1.61) and (1.62), we have

$$m=\frac{\text{I}}{\text{O}}=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f} \qquad ...(1.63)$$

Note. The expression (1.63) also holds for linear magnification produced by a ncave lens.

1.18. IMAGE FORMED BY A CONVEX LENS

(*i*) When object lies at infinity. Consider that an object lies at infinity [Fig. 1.24]. en, $u = \infty$

From the lens equation, we have

$$-\frac{1}{\infty}+\frac{1}{v}=\frac{1}{f} \quad \text{or} \quad v=f \qquad \text{(at focus)}$$

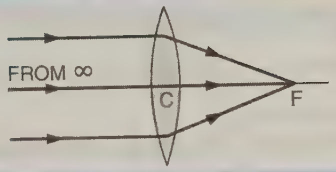


Fig. 1.24

As focal length of the convex lens is positive, v is also positive. Therefore, real nage will be formed at focus on the other side of the lens.

Now, magnification, $m=\frac{v}{u}=\frac{v}{\infty}=0$ (point in size)

Therefore, when object lies *at infinity*, image will be formed *at F* of the lens nd it will be *real, inverted* and *point in size*.

(*ii*) When object lies beyond 2 F. Consider that the object AB lies beyond 2 F s shown in Fig. 1.25. Then distance of the object from the optical centre of the lens s negative and $|u| > 2f$.

Now, $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ or $v=\frac{uf}{u+f}$

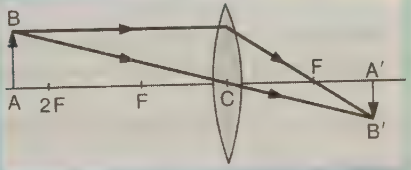


Fig. 1.25

Since $|u| > 2f$, it follows that v will be positive and will have a value between and $2f$ (image between F and 2 F on the other side of lens).

Since $m=\frac{v}{u}$, the value of m will be negative and $|m| < 1$ (real image and maller in size).

Therefore, when object lies *beyond 2 F*, the image will be formed *between F and F* and will be ***real, inverted*** and ***smaller in size***.

(*iii*) When object lies at 2 F. in such a case, $u = -2f$. From the lens equation, we have

$$v=\frac{uf}{u+f}$$

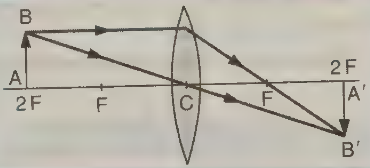


Fig. 1.26

Setting $v = -2f$, we obtain

$$v=2f \qquad \text{(image at 2 F)}$$

Since $m=\frac{v}{u}$, the value of m will be equal to -1 (real image and equal in size).

Therefore, when object lies at *at 2 F*, the image will be formed *at 2 F* and will be *real, inverted and equal in size* [Fig. 1.26].

(*iv*) When object lies between F and 2 F. When object lies between F and 2 F [Fig. 1.27], $f < |u| < 2f$. From the lens equation, we have

$$v=\frac{uf}{u+f}$$

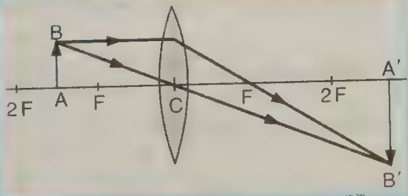


Fig. 1.27

Since $f < |u| < 2f$, v will be positive and will have a value greater then $2f$ (image beyond 2 F on the other side of the lens).

Since $m=\frac{v}{u}$, m will be negative and $|m| > 1$ (real image and larger in size).

Therefore, when object lies *between F and 2 F*, the image will be formed *beyond* 2 F and will be **real, inverted** and **larger in size**.

(v) When object lies at F. When object lies at F, $u = -f$. From the lens equation, we have

$$-\frac{1}{-f}+\frac{1}{v}=\frac{1}{f} \quad \text{or} \quad \frac{1}{v}=0 \quad \text{or} \quad v=\infty \qquad \text{(image at infinity)}$$

Now, $$m=\frac{v}{u}=\frac{\infty}{-f}=-\infty \qquad \text{(real image and highly magnified)}$$

Therefore, when object lies *at F*, the image will be formed *at infinity* and will be *real, inverted* and *highly magnified* [Fig. 1.28].

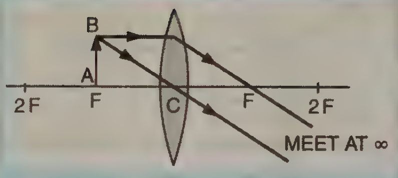


Fig. 1.28

(vi) When object lies between F and C. For object lying between F and C [Fig. 1.29], $|u| < f$. From the lens equation, we have

$$v=\frac{uf}{u+f}$$

Since $|u| < f$, v will be negative and $|v| > f$.

From $m=\frac{v}{u}$, it follows that m will be positive and greater than 1 (virtual and magnified image).

Therefore, when object lies *between F and C*, image will be formed *beyond F* on the same side of the lens. Further, the image will be *virtual, erect* and *magnified*.

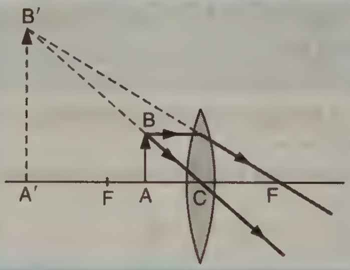


Fig. 1.29

1.19. IMAGE FORMED BY A CONCAVE LENS

Consider that an object AB is placed on the principal axis of a concave lens as shown in Fig. 1.30. For a concave lens, both u and f are negative. From the lens equation, we have

$$v=\frac{uf}{u+f}$$

Since both u and f are negative, it follows that v will be negative and $|v| < |f|$ (image on same side of the lens and between F and C).

From $m=\frac{v}{u}$, it follows that m will be positive and less than 1 (virtual image and smaller in size).

It may be pointed out that irrespective of the position of the object, a concave lens always produces a *virtual* and *erect image on the same side of the lens,* which is *smaller in size*.

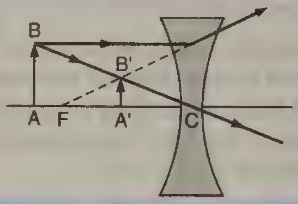


Fig. 1.30

Key point

The image produced by a concave lens is always virtual and smaller in size.

1.20. POWER OF LENS

The ability of a lens, to converge or diverge the rays of light incident on it, is called the **power of the lens.**

The power of a lens is measured as the reciprocal of its focal length in metre.

$$P=\frac{1}{f\text{ (metre)}} \qquad \text{...(1.64)}$$

If $f = 1$ m, then

$$F=\frac{1}{1\text{ m}}=\mathbf{1\ m^{-1}=1\ dioptre\ (D)}$$

The power of a lens is said to be one dioptre, if its focal length is one metre.

The power of a *converging lens* is *positive* and that of a *diverging lens* is *negative.*

Since the power of a lens is just reciprocal of its focal length, it characterises its focal properties (such as the distance at which image is formed, nature and size of the image, etc).

According to the lens maker's formula for a lens,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Since $P=\frac{1}{f}$, we have

$$P=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{...(1.65)}$$

Here, R_1 and R_2 are to be measured in *metre.*

Key point

The power of a thick lens is greater than that of a thin lens.

1.21. EQUIVALENT LENS (Two thin lenses placed in contact)

A single lens, which will form the image of a given object at the same point as is formed by the component lenses (combination of two or more lenses), is called an equivalent lens.

Consider that two thin lenses L_1 and L_2 of focal lengths f_1 and f_2 are placed in contact with each other. Suppose that a point object O lies on the principal axis of the two lenses. Fig. 1.31 shows the formation of the final image I of the object O in two steps. In the first step, the lens L_1 produces I′ as the real image of the object O. If $C_1 O = u$ and $C_1 I' = v'$, then for lens L_1, we have

$$-\frac{1}{u}+\frac{1}{v'}=\frac{1}{f_1} \qquad \text{...(1.66)}$$

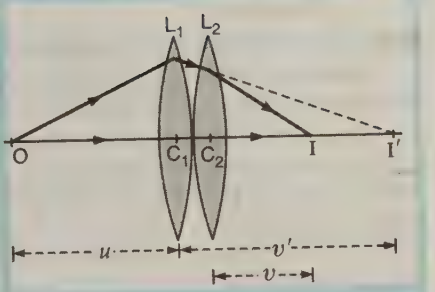


Fig. 1.31

In the second step, the lens L_2 produces the image I of the image I′, which acts as virtual object for it.

As the two lenses are thin,

$$C_2 I' \approx C_1 I' = +v'$$

The positive sign has been taken for the reason that the distance of image I′ (virtual object for lens L_2) from C_2 is measured in the direction of incident ray. If $C_2 I = v$, then the focal length of the lens L_2 is given by

$$-\frac{1}{v'}+\frac{1}{v}=\frac{1}{f_2} \qquad \text{...(1.67)}$$

Adding the equations (1.66) and (1.67), we have

$$-\frac{1}{u}+\frac{1}{v'}-\frac{1}{v'}+\frac{1}{v}=\frac{1}{f_1}+\frac{1}{f_2}$$

or

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f_1}+\frac{1}{f_2} \qquad \text{...(1.68)}$$

If a single lens of focal length f produces the image I of the object O exactly as the lenses L_1 and L_2 together do, then

$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad \text{...(1.69)}$$

From the equations (1.68) and (1.69), we have

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \qquad \text{...(1.70)}$$

Here, f is called ***focal length of the equivalent lens.***

Thus, *reciprocal of the focal length of an equivalent lens is equal to sum of the reciprocals of the focal lengths of the two lenses placed in contact.*

Discussion. 1. If the focal lengths f_1 and f_2 are in metre, then equation (1.70) becomes

$$P = P_1 + P_2 \qquad \text{...(1.71)}$$

Here, P_1 and P_2 are powers of the two lenses and P is the power of the equivalent lens.

In general,

$$P = P_1 + P_2 + P_3 + \ldots\ldots\ldots\ldots,$$

where P_1, P_2, P_3 .... are powers of component lenses placed in contact.

2. When two lenses are placed in contact, then image produced by first lens acts as the object for the second lens, which further produces image of this image. Obviously, if m_1 and m_2 are magnifications produced by the first and the second lens respectively, then total magnification produced by the combination is given by

$$m = m_1 \times m_2 \qquad \text{...(1.72)}$$

In general, if $m_1, m_2, m_3, \ldots\ldots\ldots\ldots\ldots$ are magnifications produced by the component lenses, then magnification produced by the combination is given by

$$m = m_1 \times m_2 \times m_3 \ldots\ldots\ldots$$

3. Two lenses may also be combined by placing them at a finite distance apart. It can be proved that if two lenses of focal lengths f_1 and f_2 are placed at a distance x apart, then focal length of the equivalent lens is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2} \qquad \text{...(1.73)}$$

Key point

When a number of thin lenses are placed in contact,

(*i*) power of the equivalent lens is equal to sum of the powers of the individual lenses and

(*ii*) magnification produced is equal to the product of the magnifications produced by the component lenses.

4. The two lenses may be combined so as :

(*i*) to produce magnified image,

(*ii*) to produce an erect image and

(*iii*) to minimise the lens defects, such as chromatic aberration or spherical aberration.

SPHERICAL ABERRATION IN LENSES

Spherical aberration. *The inability of a lens of large aperture to bring all the rays in a wide beam falling on it, to focus at a single point, is called* ***spherical aberration.***

Cause of spherical aberration. The rays of light travelling parallel to the principal axis after refraction from a convex lens meet at a single point only if a narrow beam of light falls on it or if the lens is of small aperture. In case, a wide beam of light falls on a convex lens of large aperture, the rays after refraction through the lens do not come to focus at a single point. In a wide beam of the light, the rays of light which travel close to the principal axis are called ***paraxial rays,*** while the rays which travel quite away from the principal axis are called ***marginal rays.*** It is found that the marginal rays get focussed relatively close to the optical centre of the lens as compared to the paraxial rays.

Fig. 1.32 shows the focussing of the marginal and paraxial rays in case of a convex lens, while Fig. 1.33 shows the focussing of such rays in case of a concave lens. It follows that the focus for marginal rays (F_m) is closer to the optical centre than the focus for paraxial rays (F_p) is. Therefore, a lens of large aperture is unable to bring a parallel beam of light to a sharp focus at a point on its principal axis.

Thus spherical aberration is a defect in a lens of wide aperture due to its different focal lengths for the light refracted from different portions of its aperture. It follows that focal length of the lens for marginal rays (f_m) is less than that for paraxial rays (f_p). The difference in focal lengths for paraxial and marginal rays gives the measure of ***axial*** or ***longitudinal spherical aberration.*** Thus,

$$\text{longitudinal spherical aberration} = f_p - f_m \qquad ...(1.74)$$

Further, if a screen is moved between the points F_m and F_p, the size of the circular image (focus of parallel beam of light) goes on varying. However, it is minimum in the position AB. This circular image in position AB is called *circle of least confusion*. It gives the best position for the focus of the parallel beam of light. The variation in the size of the circular image is called ***lateral spherical aberration*** and the radius of the circle of least confusion gives the measure of lateral spherical aberration.

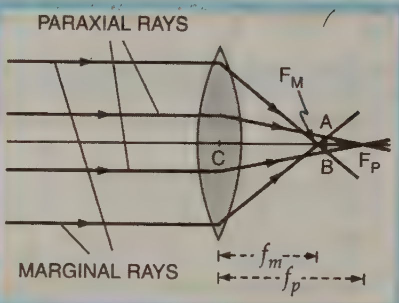


Fig. 1.32

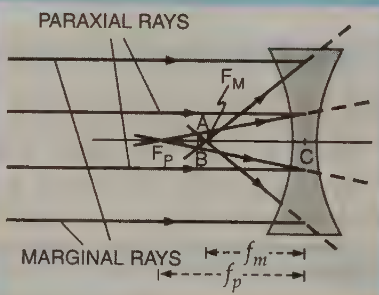


Fig. 1.33

Spherical aberration can be minimised by the following methods :

1. ***By using stops.*** Spherical aberration in a lens of wide aperture can be minimised by using stops so as to either allow paraxial rays or marginal rays to fall on the lens.

2. ***By placing the lens so that the total deviation is shared equally by the two surfaces of the lens.*** For example, in case of a plano-convex lens, spherical aberration is minimum, when its curved surface faces incident or emergent rays, whichever is more parallel to the principal axis.

3. ***By using two plano-convex lenses.*** The spherical aberration can be minimised by using two plano-convex lenses and placing them apart at a distance equal to the difference in their focal lengths.

SOLVED NUMERICAL PROBLEMS

Type A. On Laws of refraction

Problem 1.01. The refractive index of glass is 1·5. What is the speed of light in glass ? Speed of light in vacuum is $3 \cdot 0 \times 10^8$ m s^{-1}.

Sol. Here, $\mu = 1{\cdot}5$; $c = 3{\cdot}0 \times 10^8$ m s^{-1}

$$\text{Speed of light in glass, } v = \frac{c}{\mu} = \frac{3{\cdot}0 \times 10^8}{1{\cdot}5}$$

$$= 2{\cdot}0 \times 10^8 \text{ m s}^{-1}$$

Problem 1.02. Monochromatic light of wavelength 589 nm is incident from air on water surface. What are wavelength, frequency and speed of (*a*) reflected and (*b*) refracted light ? Refractive index of water is 1·33.

Sol. Here, $\mu = 1{\cdot}33$; wavelength of light in air,

$$\lambda_a = 589 \text{ nm} = 589 \times 10^{-9} \text{ m ;}$$

We know, velocity of light in air, $c = 3 \times 10^8$ m s^{-1}

(*a*) Wavelength of light on reflection remains unchanged. Therefore, wavelength of light after reflection,

$$= \mathbf{589 \text{ nm} = 589 \times 10^{-9} \text{ m}}$$

Since after reflection, light travels in the same medium (air),

velocity of light after reflection = $\mathbf{3 \times 10^8}$ **m s^{-1}**

Frequency of light after reflection,

$$\nu = \frac{c}{\lambda_a} = \frac{3\times10^8}{589\times10^{-9}} = \mathbf{5{\cdot}0934\times10^{14}\ Hz}$$

(b) On refraction, wavelength of light changes but its equency remains unchanged.

Therefore, after refraction, frequency of light,

$$\nu = \mathbf{5{\cdot}0934\times10^{14}\ Hz}$$

Let λ_w be the wavelength and v, velocity of light in water fter refraction).

Now, $$v = \frac{c}{\mu} = \frac{3\times10^8}{1\cdot33} = \mathbf{2{\cdot}256\times10^8\ m\ s^{-1}}$$

Also, $$\lambda_w = \frac{v}{\nu} = \frac{2\cdot256\times10^8}{5\cdot0934\times10^{14}}$$

$$= \mathbf{422{\cdot}93\times10^{-19}\ m = 422{\cdot}93\ nm}$$

ype B. On Lateral shift

Problem 1.03. A ray of light is incident at an angle of 0° on one face of a rectangular glass slab of thickness ·1 m and refractive index 1·5. Calculate the lateral shift roduced. *(Karnataka, 1983)*

Sol. Here, $i = 60°\ ;\ \mu = 1\cdot5\ ;\ t = 0\cdot1$ m

Now $$\mu = \frac{\sin i}{\sin r}$$

or $$\sin r = \frac{\sin i}{\mu} = \frac{\sin 60°}{1\cdot5} = \frac{0\cdot866}{1\cdot5} = 0\cdot5773$$

or $$r = 35°16'$$

Now, lateral shift, $$d = \frac{t}{\cos r}\sin(i - r)$$

$$\therefore \quad d = \frac{0\cdot1}{\cos 35°\,16'} = \sin(60° - 35°\,16')$$

$$= \frac{0\cdot1}{\cos 35°\,16'}\sin 24°\,44' = \frac{0\cdot1\times0\cdot4185}{0\cdot8164}$$

$$= \mathbf{0{\cdot}0513\ m}$$

Type C. On Compound refraction

Problem 1.04. Refractive indices of water and glass are 4/3 and 3/2 respectively. A ray of light travelling in water is incident on the water-glass interface at 30°. Calculate the angle of refraction. *(Karnataka, 1983)*

Sol. Here, refractive index of glass, ${}^a\mu_g = 4/3$;
refractive index of water, ${}^a\mu_w = 3/2$
Now, ${}^a\mu_w \times {}^w\mu_g = {}^a\mu_g$
Therefore, refractive index of glass *w.r.t.* water,

$${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{3/2}{4/3} = 1\cdot125$$

Further, the angle of incidence, $i = 30°$
If r is the angle of refraction, then

$$\frac{\sin i}{\sin r} = 1\cdot125$$

or $$\sin r = \frac{\sin 30°}{1\cdot125} = \frac{0\cdot5}{1\cdot125} = 0\cdot4444$$

or $$\mathbf{r = 26°\ 23'}$$

Type D. On Real and Apparent depths

Problem 1.05. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised, if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table ? Refractive index of glass = 1·5. Does the answer depend on the location of the slab ? *(Text Problem)*

Sol. The normal shift in the position of the pin is given by

$$d = t\left(1 - \frac{1}{\mu}\right)$$

Here, real thickness of glass slab,

$$d = 15\left(1 - \frac{1}{1\cdot5}\right) = 15\left(\frac{1\cdot5 - 1}{1\cdot5}\right) = \mathbf{5\ cm}$$

The answer **does not depend** upon the location of the slab.

Problem 1.06. Velocity of light in glass is 2×10^8 m s^{-1} and that in air is 3×10^8 m s^{-1}. By how much would an ink dot appear to be rasied, when covered by a glass plate 6·0 cm thick ? *(P.S.S.C.E. 1999, 1997 S)*

Sol. Here, velocity of light in air, $c = 3\times10^8$ m s^{-1}
velocity of light in glass, $v = 2\times10^8$ m s^{-1}
Therefore, refractive index of glass *w.r.t.* air,

$$\mu = \frac{c}{v} = \frac{3\times10^8}{2\times10^8} = 1\cdot5$$

Normal shift in the position of ink dot, $d = t\left(1 - \frac{1}{\mu}\right)$

Here, thickness of the glass plate, $t = 6\cdot0$ cm

$$\therefore\ d = 6\cdot0\left(1 - \frac{1}{1\cdot5}\right) = \frac{6\cdot0\times0\cdot5}{1\cdot5} = \mathbf{2{\cdot}0\ cm}$$

Problem 1.07. A tank is filled with water to a height of 12·5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9·4 cm. What is the refractive index of water ? If water is replaced by a liquid of refractive index 1·63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again ? *(H.P.S.S.C.E. 2000 ; Text Problem)*

Sol. ***When tank is filled with water :***

Real depth = 12·5 cm ; apparent depth = 9·4 cm ;

$$\therefore \quad \mu = \frac{\text{real depth}}{\text{apparent depth}} = \frac{12\cdot5}{9\cdot4} = \mathbf{1{\cdot}33}$$

When tank is filled with the liquid :

Real depth = 12·5 cm ; $\mu = 1\cdot63$;

Therefore, apparent depth $$= \frac{\text{real depth}}{\mu} = \frac{12\cdot5}{1\cdot63}$$

$$= 7\cdot67\text{ cm}$$

Therefore, distance through which microscope has to be moved = 9·4 – 7·67 = **1·73 cm**

Problem 1.08. A mark is made at the bottom of a beaker and a microscope is focussed on it. The microscope is then raised through 0·015 m. To what height water must be poured into the beaker to bring the mark again into focus ? Given, ${}^a\mu_w = 4/3$. *(Karnataka, 1984)*

Sol. Here, $${}^a\mu_w = \frac{4}{3}$$

The microscope was focussed directly on the mark over the beaker. On raising the microscope through 0·015 m, the mark no longer remains in focus. It will again come into focus, if the mark gets raised exactly through 0·015 m on pouring water into the beaker. Therefore,

normal shift, $d = 0\cdot015$ m

If t is the height upto which water must be poured into the beaker, then

$$\text{normal shift, } d = t\left(1 - \frac{1}{{}^a\mu_w}\right)$$

or $$0{\cdot}015 = t\left(1 - \frac{1}{4/3}\right) = t\left(1 - \frac{3}{4}\right)$$

or $$t = 0{\cdot}06 \text{ m}$$

Problem 1.09. What is the apparent position of an object below a rectangular block of glass 6 cm thick [Fig. 1.34], if a layer of water 4 cm thick is on top of the glass ? Given, ${}^a\mu_g = 3/2$ and ${}^a\mu_w = 4/3$.

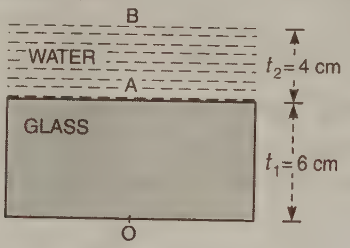


Fig. 1.34

Sol. Here, ${}^a\mu_g = 1{\cdot}5$, ${}^a\mu_w = 1{\cdot}33$

The object O will appear to be raised due to normal shifts produced by water and glass.

The normal shift in the position of the object due to glass,

$$d_1 = t_1\left(1 - \frac{1}{{}^a\mu_g}\right) = 6\left(1 - \frac{1}{3/2}\right) = 2 \text{ cm}$$

The normal shift in the position of the object due to water,

$$d_2 = t_2\left(1 - \frac{1}{{}^a\mu_w}\right) = 4\left(1 - \frac{1}{4/3}\right) = 1 \text{ cm}$$

Therefore, object will appear to be displaced in upward direction through the distance,

$$d = d_1 + d_2 = 2 + 1 = 3 \text{ cm}$$

Type E. On Total internal reflection

Problem 1.10. A glass slab is immersed in water. Find the critical angle at glass-water interface. Given, ${}^a\mu_g = 1{\cdot}5$ and ${}^a\mu_w = 1{\cdot}33$. *(P.S.S.C.E. 1996)*

Or

Find the critical angle for a glass-water interface, if refractive index of glass is 3/2 and that of water is 4/3. *(Karnataka, 1986)*

Sol. Here, refractive index of glass = 1·5 ;
refractive index of water = 1·33

Therefore, the refractive index of glass *w.r.t.* water

$$= \frac{1{\cdot}5}{1{\cdot}33} = 1{\cdot}1278$$

If C is critical angle for glass-water interface, then

$$\frac{1}{\sin C} = 1{\cdot}1278 \text{ or } \sin C = \frac{1}{1{\cdot}1278} = 0{\cdot}8867$$

or $$C = 62°27'$$

Problem 1.11. Calculate the critical angle for a glass-air surface, if a ray of light which is incident in air on the surface is deviated through 15°, when its angle of incidence is 40°. *(I.S.C.E. 1993)*

Sol. Fig. 1.35 shows a ray of light AB incident on a glass-air surface at an angle of incidence, $i = 40°$. The ray is refracted along BC, so that deviation in its path *i.e.* ∠ QBC = 15°.

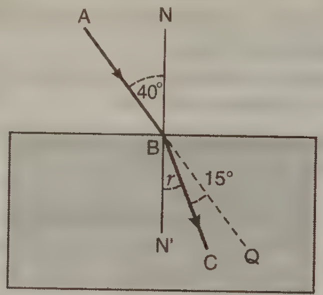


Fig. 1.35

Obviously, angle of refraction,

$$r = \angle CBN' = 40° - 15° = 25°$$

If μ is refractive index of glass *w.r.t.* air, then

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 40°}{\sin 25°} = \frac{0{\cdot}6428}{0{\cdot}4226} = 1{\cdot}52$$

If C is critical angle for glass-air surface, then

$$\mu = \frac{1}{\sin C} \text{ or } \sin C = \frac{1}{\mu} = \frac{1}{1{\cdot}52} = 0{\cdot}6579$$

or $$C = 41{\cdot}1°$$

Problem 1.12. A liquid of refractive index 1·5 is poured into a cylindrical jar of radius 20 cm upto a height of 20 cm. A small bulb is lighted at the centre of the bottom of the jar. Find the area of the liquid surface through which the light of the bulb passes into air. *(I.S.C.E. 1998)*

Sol. The point L shows the position of bulb at the bottom of jar containing liquid ($\mu = 1{\cdot}5$) up to the height h (= 20 cm). The light will emerge out through the largest area *i.e.* through a circle of maximum radius OA = OB = r, if on refraction at liquid-air interface, it comes out parallel to the surface [Fig 1.36].

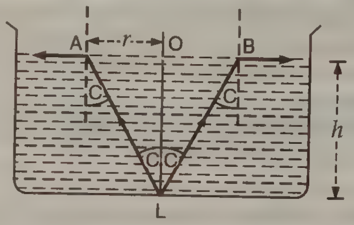


Fig. 1.36

It will happen so, if the angle of incidence for the rays of light incident at the points on the circumference of the circle of radius r is equal to the critical angle for liquid-air interface. If C is the critical angle, then

$$\mu = \frac{1}{\sin C}$$

or $$\sin C = \frac{1}{\mu} = \frac{1}{1{\cdot}5} = 0{\cdot}6667$$

or $$C = 41{\cdot}8°$$

From the right angled Δ AOL, we have

$$\tan C = \frac{OA}{OL} = \frac{r}{h} \quad \text{or} \quad r = h \tan C$$

or $$r = 20 \times \tan 41{\cdot}8° = 20 \times 0{\cdot}8941 = 17{\cdot}882 \text{ cm}$$

Therefore, the required surface area

$$= \pi r^2 = \pi \times (17\cdot882)^2 = \mathbf{1004\cdot6\ cm^2}$$

Problem 1.13. A narrow beam of light is incident normally on one face of an equiangular glass prism of refractive index 1·45. When the prism is immersed in a suitable liquid, the ray makes a grazing emergence along the other face. Find the refractive index of this liquid and draw a diagram showing the path of rays. (*I.S.C.E. 1997*)

Sol. Fig. 1.37 shows an equiangular glass prism ABC, such that ∠A = 60°.

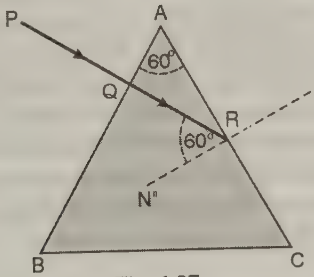


Fig. 1.37

When a narrow beam of light is incident normally on the face AB of the prism along PQ, it will proceed as such. It will come to meet the face AC at point R, so that the angle C of incidence inside the prism *i.e.* ∠ PRN′ = 60°as shown in the figure.

Since the beam of light makes a grazing emergence along the face AC, the ∠PRN′ = 60°corresponds to critical angle C for the material of prism, when placed in the liquid. If $^l\mu_g$ is refractive index of the material of the prism *w.r.t.* liquid, then

$$^l\mu_g = \frac{1}{\sin C} = \frac{1}{\sin 60°} = \frac{1}{0\cdot866} = 1\cdot155$$

Let $^a\mu_l$ be refractive index of the liquid. Then,

$$^l\mu_g = \frac{^a\mu_g}{^a\mu_l}$$

or $$^a\mu_l = \frac{^a\mu_g}{^l\mu_g} = \frac{1\cdot45}{1\cdot155} = \mathbf{1\cdot255}$$

Type E. On optical fibre

Problem 1.14. Fig. 1.38 shows a cross-section of a light pipe made of glass fibre of refractive index 1·68. The outer coating of the pipe is made of a material of refractive index 1·44. What is the range of angles of the incident rays with the axis of the pipe for which total internal reflection inside the pipe takes place as shown ? (*Text Problem*)

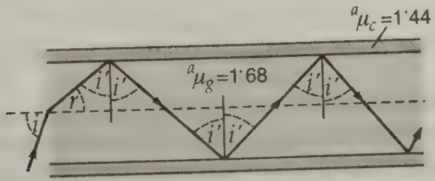


Fig. 1.38

Sol. Here, refractive index of the glass fibre *w.r.t.* air,

$$^a\mu_g = 1\cdot68$$

and refractive index of the outer coating material *w.r.t* air,

$$^a\mu_c = 1\cdot44$$

Let C be the critical angle for the fibre material *w.r.t.* the material of the outer coating. Then, refractive index of glass fibre *w.r.t.* the coating material is given by

$$^c\mu_g = \frac{1}{\sin C} \qquad ...(i)$$

Also, $$^c\mu_g = \frac{^a\mu_g}{^a\mu_c} = \frac{1\cdot69}{1\cdot44} = 1\cdot167 \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\sin C = \frac{1}{1\cdot67} = 0\cdot8569$$

or $$C = 58\cdot97°$$

Suppose that a ray of light travelling in air makes an angle *i* with the axis of the pipe and is refracted at angle *r* [Fig. 1.38]. Further, suppose that the refracted ray comes to incident on the glass fibre-coating interface at angle *i′*. The total reflection will occur, if angle *i′* is equal to or greater than the critical angle C (= 58·97°) for the glass fibre and coating interface. Thus, *r* will be maximum, when *i′* = C.

Therefore, the maximum value of *r* is given by

$$r = 90° - 58\cdot97° = 31\cdot03°$$

Now, refractive index of glass-fibre *w.r.t.* air is given by

$$^a\mu_g = \frac{\sin i}{\sin 31\cdot03°} = 1\cdot68$$

or $\sin i = 1\cdot68 \times \sin 31\cdot03° = 1\cdot68 \times 0\cdot5155 = 0\cdot8660$

or $i = 60°$

This is the maximum value of *i*. Thus, all rays having ***angle of incidence between* 0° *to* 60°** will suffer total internal reflection.

Type F. On Refraction from Spherical refracting surfaces

Problem 1.15. What curvature must be given to the bounding surface of a refracting medium (μ = 1·5) for the virtual image of an object in the adjacent medium (μ = 1) at 10 cm to be formed at a distance of 40 cm ?

Sol. Let R be the radius of curvature of the refracting surface. When object lies in rarer medium,

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Here, $\mu_1 = 1$; $\mu_2 = 1\cdot5$; $u = -10$ cm ; $v = -40$ cm

$$\therefore \quad -\frac{1}{-10} + \frac{1\cdot5}{-40} = \frac{1\cdot5 - 1}{R} \quad \text{or} \quad \frac{0\cdot5}{R} = \frac{1}{10} - \frac{1\cdot5}{40}$$

or **R = + 8 cm (convex)**

Problem 1.16. A small point object is placed in air at a distance of 60 cm from the convex spherical refracting surface of refractive index 1·5. If radius of curvature of the spherical surface is 25 cm, calculate the position of the image and the power of the refracting surface.

Sol. When object is placed in air, the relation between *u*, *v* and R is given by

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

Here, $u = -60$ cm ; R = + 25 cm ; $\mu_1 = 1$; $\mu_2 = 1\cdot5$

$$\therefore \quad -\frac{1}{-60} + \frac{1\cdot5}{v} = \frac{1\cdot5 - 1}{+25}$$

or $$\frac{1\cdot5}{v} = \frac{0\cdot5}{25} - \frac{1}{60} = \frac{1}{300}$$

∴ $\mathbf{v = +\,450\ cm}$

As *v* is positive, the image will be formed on the *other side* of the object *i.e.* a ***real image*** will be formed inside the refracting denser medium.

Power of the refracting surface, $P=\frac{\mu_2-\mu_1}{R}$

As $\quad R=+25\text{ cm}=+0\cdot25\text{ m}$, we have

$$P=\frac{(1\cdot5-1)}{+0\cdot25}=\frac{0\cdot5}{0\cdot25}=\mathbf{2\ dioptre}$$

Problem 1.17. A spherical surface of radius of curvature R separates air (refractive index 1·0) from glass (refractive index 1·5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and PO = OQ. Find the distance of object from the spherical surface. *(I.I.T. 1998)*

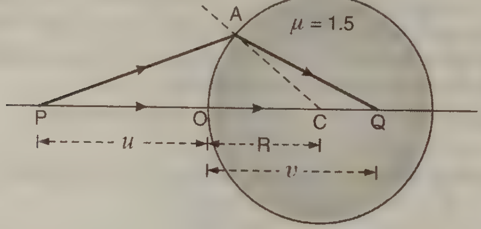


Fig. 1.39

Ans. Here, $\quad \mu=1\cdot5$; PO = OQ = x (say)

As shown in Fig. 1.39, refraction takes place from rarer to denser medium. Therefore,

$$-\frac{1}{u}+\frac{1}{v}=\frac{\mu-1}{R}$$

It follows that

$$u=-x\,;\,v=x$$

$$\therefore \quad -\frac{1}{x}+\frac{1\cdot5}{x}=\frac{1\cdot5-1}{R}$$

or $\quad \frac{0\cdot5}{R}=\frac{2\cdot5}{x}$

or $\quad x=\mathbf{5\,R}$

Problem 1.18. A small piece of paper stuck on a glass sphere of 5 cm radius is viewed through the glass from the position directly opposite. Find the position of the image. Refractive index of glass is 1·5.

Sol. The piece of paper stuck at point O will serve as the object. Its virtual image will be formed by the sphere as shown in the Fig. 1.40.

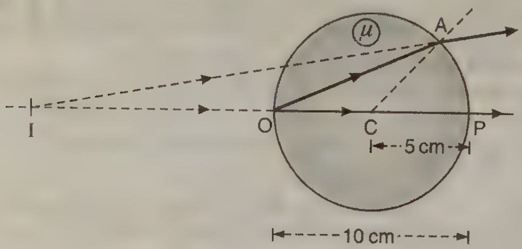


Fig. 1.40

When object lies in denser medium *i.e.* incident ray travels in the denser medium, the relation between u, v and R is given by

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R}$$

Here, $\mu_1=1$; $\mu_2=1\cdot5$; $u=-10$ cm and $R=-5$ cm

$$\therefore \quad -\frac{1\cdot5}{-10}+\frac{1}{v}=\frac{1-1\cdot5}{-5}$$

or $\quad \frac{3}{20}+\frac{1}{v}=\frac{1}{10} \quad$ or $\quad \frac{1}{v}=\frac{1}{10}-\frac{3}{20}=-\frac{1}{20}$

or $\quad v=\mathbf{-\,20\ cm}$

The negative sign shows that the image will be formed on the *same side* of the object *i.e.* a ***virtual*** image will be formed in the rarer medium as shown in the figure.

Problem 1.19. Fig. 1.41 shows a solid glass sphere of radius 5 cm that has a small air bubble O trapped at a distance 2 cm from the centre C. The refractive index of the material of glass is 1·5. Find the apparent position of the bubble where it will appear, when seen through the surface of the sphere from an outside point E. *(I.S.C.E. 1998)*

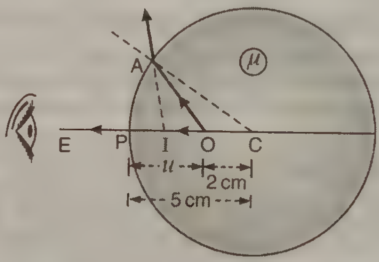


Fig. 1.41

Sol. The course of rays giving the apparent position I of the air bubble O trapped inside the glass sphere will be as shown in the figure.

Here, $\quad \mu=1\cdot5$; $R=-5$ cm ; OC = 2 cm

$\therefore \quad u=-\text{PO}=-(\text{PC}-\text{OC})=-(5-2)=-3$ cm

Let PI = v be the distance of the apparent position of air bubble from the surface of glass sphere.

As the incident ray travels inside the glass, we have

$$-\frac{\mu}{u}+\frac{1}{v}=\frac{1-\mu}{R}$$

$\therefore \quad -\frac{1\cdot5}{-3}+\frac{1}{v}=\frac{1-1\cdot5}{-5} \quad$ or $\quad \frac{1}{2}+\frac{1}{v}=\frac{1}{10}$

or $\quad \frac{1}{v}=\frac{1}{10}-\frac{1}{2}=-\frac{4}{10}$

or $\quad v=\mathbf{-\,2\cdot5\ cm}$

Problem 1.20. A small air bubble within a glass sphere of radius 0·02 m appears to be 0·01 m below the surface, when looked along a diameter containing the bubble. Find the real depth of the bubble along the line of sight. Given, μ = 1·54.

Sol. Let O be the air bubble inside the glass sphere. Due to refraction, it will appear to be at point I as shown in Fig. 1.42.

Here, R = – 0·02 m, μ = 1·54, v = – 0·01 m

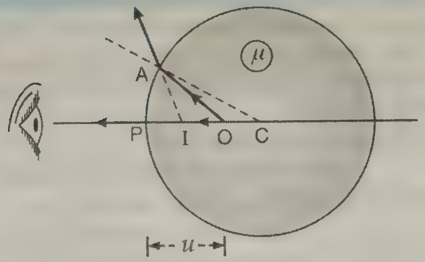


Fig. 1.42

Let u be the real depth of the air bubble.

As the incident ray travels inside the glass, we have

$$-\frac{\mu}{u}+\frac{1}{v}=\frac{1-\mu}{R}$$

or $$-\frac{1\cdot 54}{u}+\frac{1}{-0\cdot 01}=\frac{1-1\cdot 54}{-0\cdot 02}$$

or $$\frac{1\cdot 54}{u}=-27-100=-127$$

or $$u=-\frac{1\cdot 54}{127}=-\mathbf{0\cdot 0121\ m}$$

Problem 1.21. An empty spherical flask of diameter 15 cm is placed in water of refractive index 4/3. A parallel beam of light strikes the flask. Where does it get focussed, when observed from within the flask ?

Sol. Fig. 1.43 shows a spherical flask placed inside water. As such, the flask inside water behaves as a concave spherical surface. The centre of the flask is the centre of curvature of the concave spherical surface. A parallel beam of light striking on the flask will diverge and the beam of light will appear to come from point I as shown in the figure.

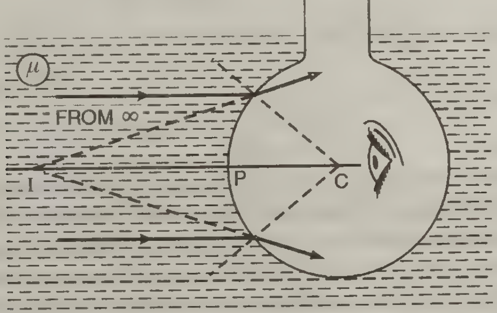


Fig. 1.43

Here, $u=-\infty\,;\ R=+\frac{15}{2}\text{ cm}\,;\ \mu=\frac{4}{3}\,;$

As incident ray travels in denser medium, we have

$$-\frac{\mu}{u}+\frac{1}{v}=\frac{1-\mu}{R}$$

$$\therefore \quad -\frac{4/3}{-\infty}+\frac{1}{v}=\frac{1-4/3}{15/2}$$

or $$v=\frac{-45}{2}=-\mathbf{22\cdot 5\ cm}$$

As v is negative, the image will be formed on the same side of the object *i.e.* the side from which the parallel beam of light is incident on the flask. Therefore, the parallel beam of light will appear to come from point I as shown in the figure.

Type G. On Lens Maker's formula

Problem 1.22. A binoconvex lens with both faces of the same radius of curvature is to be manufactured from a glass of refractive index 1·55. What should be the radius of curvature for the focal length of the lens to be 20 cm ? (*H.S.S.C.E. 2002*)

Sol. Here, μ = 1·55 ; f = + 20 cm ;

Suppose that R_1 = + R. Then, R_2 = – R

Now, $$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\therefore \quad \frac{1}{+20}=(1\cdot 55-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=0\cdot 55\times\frac{2}{R}$$

or R = 40 × 0·55 = **22·0 cm**

Problem 1.23. Find the radius of curvature of the convex surface of a plano convex lens, whose focal length is 0·3 m and the refractive index of the material of the lens is 1·5. (*I.C.S.E. 1998*)

Ans. Here, μ = 1·5 ; f = 0·3 m

Suppose that radius of curvature of one surface is R_1 = R

Then, radius of curvature of the other surface will be

$R_2 = -\infty$

Now, $$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or $$\frac{1}{0\cdot 3}=(1\cdot 5-1)\left(\frac{1}{R}-\frac{1}{-\infty}\right)$$

or $0\cdot 5\left(\frac{1}{R}\right)=\frac{1}{0\cdot 3}$ or **R = 0·15 m**

Problem 1.24. The radius of curvature of either face of a convex lens is equal to its focal length. What is the refractive index of its material ?

Sol. Let f be the focal length of the lens.

Therefore, $$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here $R_1=+f\,;\ R_2=-f$

$$\therefore \quad \frac{1}{f}=(\mu-1)\left(\frac{1}{+f}-\frac{1}{-f}\right)$$

or $2(\mu-1)=1$ or μ = **1·5**

Problem 1.25. A double convex lens has radii 20 cm. The refractive index of glass is 1·5. Compute the focal length of this lens in air and when immersed in carbon disulphide of refractive index = 1·63.

Sol. Here, ${}^a\mu_g$ = 1·5 ; ${}^a\mu_c$ = 1·63 ;

R_1 = + 20 cm, R_2 = – 20 cm

Focal length of the lens in air is given by

$$\frac{1}{f_{air}}=({}^a\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1\cdot 5-1)\left(\frac{1}{20}-\frac{1}{-20}\right)=0\cdot 5\times\frac{1}{10}=\frac{1}{20}$$

or f_{air} = **20 cm**

If f_c is focal length of the lens, when immersed in carbon disulphide and ${}^c\mu_g$, the refractive index of glass *w.r.t.* carbon disulphide, then

$$\frac{1}{f_c} = ({}^{c}\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now, $${}^{c}\mu_g = \frac{{}^{a}\mu_g}{{}^{a}\mu_c} = \frac{1\cdot5}{1\cdot63}$$

$$\therefore \quad \frac{1}{f_c} = \left(\frac{1\cdot5}{1\cdot63} - 1\right)\left(\frac{1}{20} - \frac{1}{-20}\right) = -\frac{0\cdot13}{1\cdot63} \times \frac{1}{10}$$

or $$f_c = -\frac{1\cdot63 \times 10}{0\cdot13} = -\mathbf{125{\cdot}4\ cm}$$

Problem 1.26. A convex lens of focal length 0·2 m and made of glass ($\mu = 1{\cdot}50$) is immersed in water ($\mu = 1{\cdot}33$). Find the change in the focal length of the lens.

(C.B.S.E. 1991 ; P.S.S.C.E. 1991)

Sol. Here, $f_{air} = 0{\cdot}2$ m ; ${}^{a}\mu_g = 1{\cdot}50$

Now, $$\frac{1}{f_{air}} = \left({}^{a}\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \quad \frac{1}{0\cdot2} = (1\cdot50 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{or} \quad \frac{1}{R_1} - \frac{1}{R_2} = 10$$

Let f_w be the focal length of the lens, when immersed in water and ${}^{w}\mu_g$ be refractive index of glass *w.r.t.* water. Then,

$${}^{w}\mu_g = \frac{{}^{a}\mu_g}{{}^{a}\mu_w} = \frac{1\cdot50}{1\cdot33} = 1\cdot128$$

Now, $$\frac{1}{f_w} = ({}^{w}\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1\cdot128 - 1) \times 10 = 1\cdot28$$

or $$f_w = \frac{1}{1\cdot28} = \mathbf{0{\cdot}78\ m}$$

Therefore, change in focal length,

$$f_w - f_{air} = 0{\cdot}78 - 0{\cdot}2 = \mathbf{0{\cdot}58\ m}$$

Problem 1.27. Explain what happens, when a convex lens of refractive index 1·2 is immersed in a liquid of refractive index 1·3.

Ans. Let f be focal length of lens. Then,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1\cdot2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= 0\cdot2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{0\cdot2 \times f} \qquad ...(i)$$

If f' is focal length of lens, when immersed in liquid, then

$$\frac{1}{f'} = (\mu' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

where $\mu' = \frac{1\cdot2}{1\cdot3}$ is the refractive index of the material of lens *w.r.t.* liquid.

$$\therefore \quad \frac{1}{f'} = \left(\frac{1\cdot2}{1\cdot3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{-1}{13}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Using equation (*i*), we have

$$\frac{1}{f'} = \frac{-1}{13} \times \frac{1}{0\cdot2 \times f} \quad \text{or} \quad f' = -\mathbf{2{\cdot}6}\,f$$

Type H. On Lens equation

Problem 1.28. A beam of light converges to point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge, if the lens is (*a*) a convex lens of focal length 20 cm (*b*) a concave lens of focal length 20 cm ?

Sol. (*a*) Here, $f = +20$ cm ;

$u = +12$ cm (distance of virtual object)

From the lens equation we, have

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{+20} + \frac{1}{+12} = \frac{3+5}{60} = \frac{8}{60}$$

or $$v = +7\cdot5 \text{ cm}$$

As v is positive, the beam will still converge (after refraction through the lens) at a point at distant **7·5 cm** from the lens.

(*b*) Here, $f = -20$ cm ; $u = +12$ cm

Again, from the lens equation,

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-20} + \frac{1}{+12} = \frac{-3+5}{60} = \frac{2}{60} = +30 \text{ cm}$$

As v is positive, the beam will still converge (after refraction through the lens) at a point at distance **30 cm** from the lens.

Problem 1.29. A convergent beam of light passes through a diverging lens of focal length 0·2 m and comes to focus at distance 0·3 m behind the lens. Find the position of the point at which the beam would converge in the absence of the lens. *(C.B.S.E. Question Paper)*

Sol. Here, $f = -0\cdot2$ m (diverging lens)

$v = +0\cdot3$ m

($\because$ image distance is along the incident light)

From the lens equation,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or $$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{+0\cdot3} - \frac{1}{-0\cdot2} = \frac{1}{0\cdot3} + \frac{1}{0\cdot2} = \frac{50}{6}$$

or $$u = +0\cdot12 \text{ m}$$

In the absence of lens, the beam would converge at a point at a distance **0·12 m** from the present position of the lens.

Problem 1.30. A needle 10 cm long is placed along the axis of a convex lens of focal length 10 cm such that the middle point of the needle is at a distance of 20 cm from the lens. Find the length of the image of the needle.

(Karnataka, 1992)

Sol. Here, the focal length of the lens, $f = +10$ cm

Suppose that the needle AB of length 10 cm is placed along the axis of the lens [Fig. 1.44], so that the distance of the centre of the needle from the lens *i.e.* CO = 20 cm

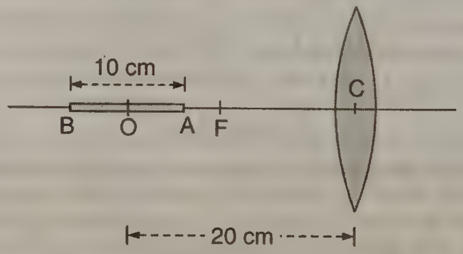


Fig. 1.44

For the end of the needle A :

$$u = -CA = -(CO - OA)$$
$$= -(20 - 5) = -15 \text{ cm}$$

Let v_1 be the distance, at which the image of the end A is formed. Then,

$$\frac{1}{v_1} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{-15}$$

or $\quad v_1 = 30$ cm

For the end of the needle B :

$$u = -CB = -(CO + OB) = -(20 + 5) = -25 \text{ cm}$$

Let v_2 be the distance at which the image of the end B is formed. Then,

$$\frac{1}{v_2} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} + \frac{1}{-25}$$
$$v_2 = 16 \cdot 67 \text{ cm}$$

Therefore, length of the image of the needle,

$$v_1 - v_2 = 30 - 16 \cdot 67 = \mathbf{13{\cdot}33 \text{ cm}}$$

Problem 1.31. A double convex lens has 10 cm and 15 cm as its two radii of curvatures. The image of an object, placed 30 cm from the lens, is formed at 20 cm from the lens on the other side. Find the refractive index of the material of the lens. What will be the focal length ? What will be the focal length of the lens, if it is immersed in water of refractive index 1·33 ? *(I.S.C.E. 1995)*

Sol. Here, $u = -30$ cm ; $v = 20$ cm ; $R_1 = 10$ cm ; $R_2 = -15$ cm

If f is focal length of the lens, then

$$\frac{1}{f} = -\frac{1}{u} + \frac{1}{v} = -\frac{1}{-30} + \frac{1}{20} = \frac{1}{30} + \frac{1}{20} = \frac{5}{60}$$
$$f = \frac{60}{5} = \mathbf{12 \text{ cm}}$$

Let ${}^a\mu_g$ be the refractive index of the material of the lens *w.r.t.* air. Then,

$$\frac{1}{f} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or
$$\frac{1}{12} = ({}^a\mu_g - 1)\left(\frac{1}{10} - \frac{1}{-15}\right)$$
$$= ({}^a\mu_g - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = ({}^a\mu_g - 1) \times \frac{1}{6}$$

or
$${}^a\mu_g = 1 + \frac{6}{12} = \mathbf{1{\cdot}5}$$

If f_w is focal length of the lens, when immersed in water, then

$$\frac{1}{f_w} = ({}^w\mu_g - 1) = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, ${}^w\mu_g$ is refractive index of glass *w.r.t.* water.

$$\therefore \quad \frac{1}{f_w} = \left(\frac{{}^a\mu_g}{{}^a\mu_g} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= \left(\frac{1 \cdot 5}{1 \cdot 33} - 1\right)\left(\frac{1}{10} - \frac{1}{-15}\right) = \frac{0 \cdot 17}{1 \cdot 33} \times \frac{1}{6}$$

or
$$f_w = \frac{1 \cdot 33 \times 6}{0 \cdot 17} = \mathbf{46{\cdot}94 \text{ cm}}$$

Type I. On Linear magnification

Problem 1.32. A convex lens of focal length 0·12 m produces an image, which is three times as long as the object. Find the distance between the object and the lens for a real image. *(Karnataka, 1986)*

Sol. Here, $f = +0{\cdot}12$ m,

As image formed is real *i.e.* inverted, $m = -3$

Now, $\quad m = \dfrac{f}{u + f}$

$$\therefore \quad -3 = \frac{0 \cdot 12}{u + 0 \cdot 12} \quad \text{or} \quad u + 0{\cdot}12 = -0{\cdot}04$$

or $\quad \mathbf{u = -0{\cdot}16 \text{ m}}$

Problem 1.33. A convex lens is to be used to throw on a screen 10 m from the lens, a magnified image of an object. If the magnification is to be 19, find the focal length of the lens. *(Karnataka, 1984)*

Sol. As the convex lens forms the image on the screen, the image formed is real *i.e.* inverted.

$$\therefore \quad m = -19$$

Further, as the image lies on the other side of the object, the distance of the image is positive *i.e.*

$$v = +10 \text{ cm}$$

Now $\quad m = \dfrac{f - v}{f}$

$$\therefore \quad -19 = \frac{f - 10}{f} \quad \text{or} \quad -19f = f - 10$$

or $\quad \mathbf{f = 0{\cdot}5 \text{ m}}$

Problem 1.34. The image obtained with a convex lens is erect and its length is four times the length of the object. If the focal length of the lens is 20 cm, calculate the object and image distance. *(H.S.S.C.E. 1996, 1992 ; P.S.S.C.E. 1996)*

Sol. As the image formed is erect and hence virtual, the magnification produced by the lens is positive *i.e.* $m = +4$

Also $\quad f = +20$ cm

Now, $\quad m = \dfrac{f}{u - f}$

$$\therefore \quad 4 = \frac{20}{u + 20} \quad \text{or} \quad u + 20 = 5$$

or $\quad \mathbf{u = -15 \text{ cm}}$

Again, $\quad m = \dfrac{f - v}{f} \quad \therefore \quad 4 = \dfrac{20 - v}{20}$

or $\quad v = 20 - 80 = \mathbf{-60 \text{ cm}}$

Problem 1.35. A needle placed 45 cm from a lens forms an image on a screen placed 90 cm on the other side of the lens. Identify the type of the lens and determine its focal length. What is the size of the image, if the size of needle is 15·0 cm ? *(P.S.S.C.E. 2000, 1999, 1996 S ; C.B.S.E. 1990)*

Sol. Here, O = 5 cm ; $u = -45$ cm ; $v = +90$ cm

Now, for a lens, $\quad -\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$

$$\therefore \quad -\frac{1}{-45} + \frac{1}{90} = \frac{1}{f} \quad \text{or} \quad \frac{1}{f} = \frac{1}{90} + \frac{1}{45} = \frac{3}{90}$$

or $\quad \mathbf{f = +30 \text{ cm}}$

As f is positive, it is a **convex lens**.

Also, $m = \frac{I}{O} = \frac{v}{u}$

∴ $\frac{I}{5} = \frac{+90}{-45}$ or $I = -10$ cm

The negative sign shows that the image formed is *inverted*.

Problem 1.36. An object of size 3·0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens, if the object is moved farther from the lens ?

Sol. Here, $O = +3{\cdot}0$ cm ; $u = -14$ cm ; $f = -21$ cm

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

∴ $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-21} + \frac{1}{-14} = -\frac{1}{21} - \frac{1}{14}$

or $v = -8{\cdot}4$ **cm**

As v is negative, the image is **formed on the same side of the object** and like wise, the image is **virtual**.

Now, $m = \frac{I}{O} = \frac{v}{u}$

∴ $I = O \times \frac{v}{u} = 3{\cdot}0 \times \frac{-8{\cdot}4}{-14} = +1{\cdot}8$ **cm**

The positive sign shows that the image formed is *erect*.

When *the object is moved away* from the lens, *the virtual image moves away from the lens and towards the focus of the lens* (but never beyond the focus) and progressively *decreases in size*.

Type J. On Power of lens

Problem 1.37. A convex lens is made of glass of refractive index 1·5. If radius of curvature of the each of its two surfaces is 20 cm, find the ratio of the power of the lens, when placed in air to its power, when immersed inside a liquid of refractive index 1·25.

Sol. Here, $R_1 = 20$ cm $= 0{\cdot}20$ m ; $R_2 = -20$ cm $= -0{\cdot}2$ m,

When lens is placed in air : If P_1 is power of the lens when placed in air, then

$$P_1 = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1{\cdot}5 - 1)\left(\frac{1}{0{\cdot}2} - \frac{1}{-0{\cdot}2}\right)$$
$$= 0{\cdot}5\,(5 + 5) = 5\text{ D}$$

When lens is placed inside the liquid : If μ' is refractive index of the material of the lens *w.r.t.* the liquid, then power of the lens, when placed inside the liquid is given by

$$P_2 = (\mu' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $\mu' = \frac{1{\cdot}5}{1{\cdot}25} = 1{\cdot}2$

∴ $P_2 = (1{\cdot}2 - 1)\left(\frac{1}{0{\cdot}2} - \frac{1}{-0{\cdot}2}\right) = 0{\cdot}2\,(5 + 5) = 2\text{ D}$

Hence, $\frac{P_1}{P_2} = \frac{5}{2} = 2{\cdot}5$

Type K. On Equivalent lens

Problem 1.38. Two thin converging lenses of focal lengths 15 cm and 30 cm are held in contact with each other. Calculate power and focal length of the combination.

(C.B.S.E. 1994)

Sol. Here, $f_1 = 15$ cm ; $f_2 = 30$ cm

Let f be the focal length of the combination. Then,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{15} + \frac{1}{30} = \frac{1}{10}$$

or $f = 10$ cm $=$ **0·1 m**

Power, or combination, $P = \frac{1}{f\text{ (in metre)}} = \frac{1}{0{\cdot}1} = $ **10 D**

Problem 1.39. Two lenses of powers – 1·5 and + 2·75 D are kept in contact. Find the focal length of the combination.

(Karnataka 1986)

Sol. Here, $P_1 = -1{\cdot}5$ D ; $P_2 = +2{\cdot}75$ D

Now, power of the combination, $P = P_1 + P_2$

∴ $P = -1{\cdot}5 + 2{\cdot}75 = 1{\cdot}25$ D

Therefore, focal length of the combination (in metre),

$$f = \frac{1}{P} = \frac{1}{1{\cdot}25} = \mathbf{0{\cdot}8\ m}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For **a**mbitious, **b**rilliant & **c**urious Students

Problem 1.39. A transparent cube of side 210 mm contains a small air bubble. Its apparent distance, when viewed through one face of the cube, is 100 mm and when viewed through the opposite face is 40 mm. What is the actual distance of the bubble from the second face and what is the refractive index of the material of the cube ?

(Karnataka, 1982)

Sol. Let O be the air bubble inside the transparent cube. The side of the cube is 210 mm. If the distance of the bubble from the face I is x, then its distance from the face II will be $(210 - x)$. Suppose that the bubble appears at I_1, when viewed from the face I and at I_2, when viewed from the face II [Fig. 1.45]. Let μ be the refractive index of the material of the cube.

When viewed from face I :

Here, the real depth$= x$; apparent depth$=100$ mm

$$\therefore \quad \mu = \frac{x}{100} \qquad \ldots(i)$$

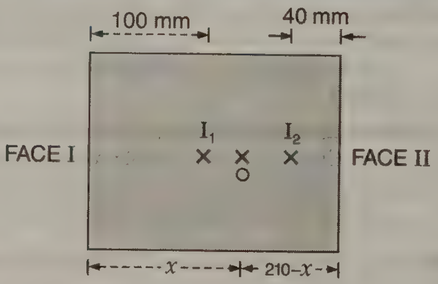


Fig. 1.45

When viewed from face II :

Here, real depth $= (210 - x)$; apparent depth $= 40$ mm

$$\therefore \quad \mu = \frac{210 - x}{40} \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{x}{100} = \frac{210 - x}{40} \quad \text{or} \quad x = 150 \text{ mm}$$

Therefore, distance of the bubble from the face II

$$= 210 - 150 = \mathbf{60\ mm}$$

From equation (*i*), we have

$$\mu = \frac{x}{100} = \frac{150}{100} = \mathbf{1{\cdot}5}$$

Problem 1.40. One face of a glass cube of side 0·06 m is silvered. An object is placed at a distance of 0·07 m from the face opposite to the silvered face. Looking from the object side, the image of the object appears to be 0·11 m behind the silvered face. Calculate the refractive index of the material of glass. (*Karnataka, 1994*)

Sol. Let ABCD be the section of the glass cube of each side 0·06 m. Its face AB is silvered and an object O is placed at a distance of 0·07 m from the face CD, the face opposite to the silvered face AB. Looking from the object side, the silvered face will appear raised, say as A′B′. Let *x* be the depth of the apparent silvered face A′B′ below the face CD [Fig. 1.46.]

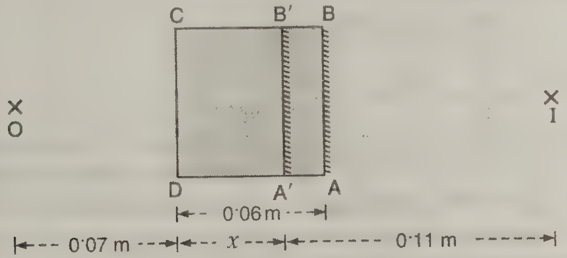


Fig. 1.46

Since the image I of the object appears at a distance of 0·11 m behind the apparent silvered face A′B′,

$$0{\cdot}07 + x = 0{\cdot}11$$

or $$x = 0{\cdot}11 - 0{\cdot}07 = 0{\cdot}04 \text{ m}$$

Thus, the real depth of the silvered face, DA = 0·06 m and the apparent depth of the silvered face,

$$DA' = x = 0{\cdot}04 \text{ m}$$

Therefore, refractive index of the glass,

$$\mu = \frac{DA}{DA'} = \frac{0{\cdot}06}{0{\cdot}04} = \mathbf{1{\cdot}5}$$

Problem 1.41. A ray of light incident on the horizontal surface of a glass slab at 70° just grazes the adjacent vertical surface after refraction. Compute the critical angle and refractive index of the glass. (*Karnataka, 1991*)

Sol. A ray of light PA incident at an angle of 70° on the horizontal surface X_1Y_1 gets refracted along AB into the glass slab at angle of refraction *r* (say) and then comes to incident at the point B on the vertical surface Y_1Y_2. The ray of light will graze along the surface Y_1Y_2, if the angle of incidence at point B on the vertical surface Y_1Y_2 equals the critical angle C for the glass-air interface [Fig. 1.47].

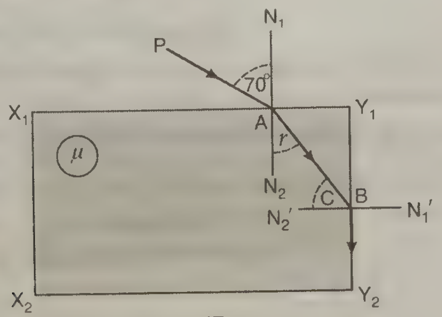


Fig. 1.47

Since N_1N_2 and $N_1'N_2'$ are perpendicular to each other,

$$r + C = 90°$$

or $$r = 90° - C$$

For refraction at point A, we have

$$\frac{\sin 70°}{\sin r} = \mu \quad \text{or} \quad \sin r = \frac{\sin 70°}{\mu}$$

$$\text{or } \sin(90° - C) = \frac{\sin 70°}{\mu}$$

$$\text{or} \quad \cos C = \frac{\sin 70°}{\mu} \qquad \ldots(i)$$

Also, for refraction at point B, we have

$$\frac{1}{\sin C} = \mu$$

$$\text{or} \quad \sin C = \frac{1}{\mu} \qquad \ldots(ii)$$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{\sin C}{\cos C} = \frac{1}{\mu} \times \frac{\mu}{\sin 70°}$$

$$\text{or} \quad \tan C = \frac{1}{\sin 70°} = \frac{1}{0{\cdot}9397} = 1{\cdot}0642$$

$$\text{or} \quad C = \mathbf{46° 47'}$$

$$\text{Now,} \quad \mu = \frac{1}{\sin C} = \frac{1}{\sin 46°47'} = \frac{1}{0{\cdot}7288} = \mathbf{1{\cdot}372}$$

Problem 1.42. A right angle prism (45°-90°-45°) of refractive index *μ* has a plate of refractive index *μ*, ($\mu_1 < \mu$) cemented to its diagonal face. The assembly is in air. A ray is incident at face AB as shown in Fig. 1.48.

(*a*) Calculate the angle of incidence at face AB for which the ray strikes the diagonal face at critical angle.

(*b*) Assuming $\mu = 1{\cdot}352$, calculate the angle of incidence at face AB for which the refracted ray passes through the diagonal face undeviated. (*I.I.T. 1996*)

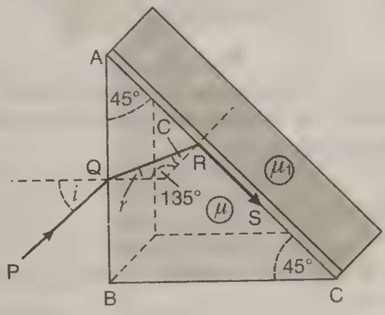


Fig. 1.48

Sol. (*a*) When a ray of light PQ is incident at angle *i* on face AB, it is refracted along QR inside the prism as shown in the figure. If *r* is angle of refraction, then for refraction at point Q,

$$\mu = \frac{\sin i}{\sin r} \qquad \ldots(i)$$

Let C be the critical angle for the ray travelling inside the prism (refractive index = *μ*) along QR to the plate (refractive index = μ_1) cemented to the diagonal face of the prism. Then,

$$\frac{\mu}{\mu_1} = \frac{1}{\sin C}$$

or $\sin C = \frac{\mu_1}{\mu}$ ...(*ii*)

From $\triangle$QRK, it follows that

$$r = 180° - (135° + C) = 45° - C$$

Substituting for r in equation (*i*), we have

$$\mu = \frac{\sin i}{\sin (45° - C)}$$

or $\sin i = \mu \sin (45° - C)$

$$= \mu (\sin 45° \cos C - \cos 45° \sin C)$$

or $\sin i = \mu \left(\frac{1}{\sqrt{2}} \sqrt{1 - \sin^2 C} - \frac{1}{\sqrt{2}} \sin C \right)$

Using equation (*ii*), we have

$$\sin i = \frac{\mu}{\sqrt{2}} \left[\sqrt{1 - \left(\frac{\mu_1}{\mu} \right)^2} - \frac{\mu_1}{\mu} \right] = \frac{\sqrt{\mu^2 - \mu_1^2} - \mu_1}{\sqrt{2}}$$

or $i = \sin^{-1} \left[\frac{\sqrt{\mu^2 - \mu_1^2} - \mu_1}{\sqrt{2}} \right]$

(*b*) The refracted ray QR will pass through the diagonal face AC undeviated, if it comes to be incident on the diagonal face at 90° [Fig. 1.49].

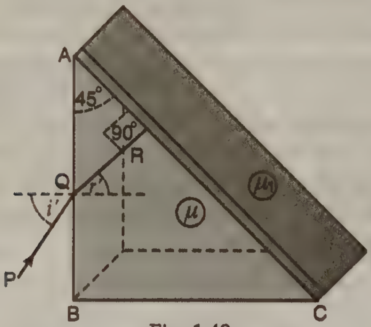


Fig. 1.49

Suppose that it happens so, when the ray of light is incident at angle i' on the face AB. If r' is the corresponding angle of refraction, then

$$r' = 180° - (90° + 45°) = 45°$$

For refraction at face AB,

$$\mu = \frac{\sin i'}{\sin r'} \quad \text{or} \quad \sin i' = \mu \sin r'$$

or $\sin i' = 1{\cdot}352 \sin 45° = 1{\cdot}352 \times \frac{1}{\sqrt{2}} = 0{\cdot}9562$

or $i' = 73°$

Problem 1.43. A beam of light strikes a glass sphere of 20 cm diameter converging towards a point 40 cm behind the pole of the spherical surface. Find the position of the image, if the refractive index of glass is 1·5.

Sol. A beam of light converging at point O is intercepted by the glass sphere of diameter of 20 cm, such that PO = 40 cm. Therefore, point O acts as the virtual object, when image is formed due to refraction by the sphere [Fig. 1.50].

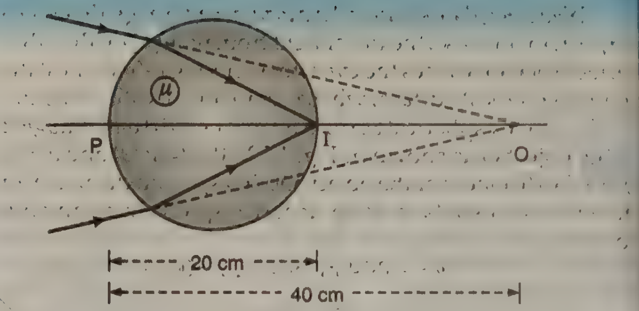


Fig. 1.50

Here, $u = +40$ cm ; $R = +\frac{20}{2} = +10$ cm ;

$$\mu_1 = 1 ; \quad \mu_2 = 1{\cdot}5$$

As the object lies in rarer medium (incident ray travels in rarer medium), we have

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$\therefore \quad -\frac{1}{40} + \frac{1{\cdot}5}{v} = \frac{1{\cdot}5 - 1}{10} = \frac{1}{20}$$

or $\frac{3}{2v} = \frac{1}{20} + \frac{1}{40} = \frac{3}{40}$

or $v = 20$ cm

Since diameter of the sphere is also 20 cm, it will converge the rays of light just at the opposite (other) end of the diameter and hence image will be formed at point I as shown in the figure.

Problem 1.44. The greatest thickness of a plano-convex lens, when viewed normally through its plane surface, appears to be 8/300 m and when viewed normally through its curved surface, appears to be 16/500 m. If the actual thickness is 0·04 m, find its (*i*) refractive index, (*ii*) radius of curvature and (*iii*) focal length.

Sol. *When the plano-convex lens is viewed through its plane surface* : In this case, the ray of light from point P on the other surface (convex) reaches the eye after suffering refraction at the plane surface of the plano-convex lens as shown in Fig. 1.51.

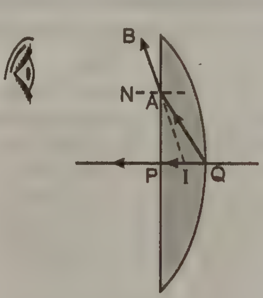


Fig. 1.51

Due to normal shift, the point P appears to be at I. Thus, the actual thickness of the lens, x = PQ = 0·04 m

and the apparent thickness of the lens, $y = \text{IQ} = \frac{8}{300}$ m

If μ is refractive index of the material of the lens, then

$$\mu = \frac{\text{real thickness } (x)}{\text{apparent thickness } (y)} = \frac{0{\cdot}04}{8/300} = 1{\cdot}5$$

When the plano-convex lens is viewed through the curved urface : In this case, the ray of light from point Q on the other urface (plane) reaches the eye after suffering refraction at the onvex spherical surface, whose centre of curvature is point . The convex spherical surface forms the virtual image I f the object Q as shown in Fig. 1.52. If u (= P Q) and v (= P) are the distances of the object and image from the pole P of he spherical surface, then according to Cartesian sign onventions,

$$u = -0{\cdot}04 \text{ m}\ ;\ v = -16/500 \text{ m}$$

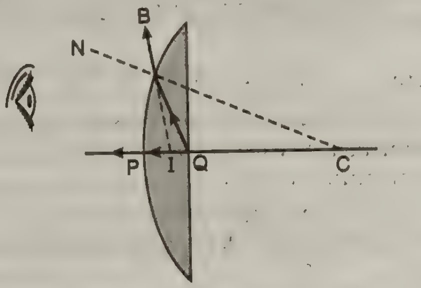


Fig. 1.52

Let R (= P C) be the radius of curvature of the convex spherical surface of the lens.

As the incident ray travels in the denser medium, we have

$$-\frac{\mu}{u} + \frac{1}{v} = \frac{1-\mu}{R}$$

$$\therefore\ -\frac{1{\cdot}5}{-0{\cdot}04} + \frac{1}{-16/500} = \frac{1-1{\cdot}5}{R}$$

or $$\frac{0{\cdot}5}{R} = -\frac{1{\cdot}5}{0{\cdot}04} + \frac{500}{16} = -37{\cdot}5 + 31{\cdot}25 = -6{\cdot}25$$

or $$\mathbf{R = -0{\cdot}08\ m}$$

According to lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the given plano-convex lens,

$$R_1 = \infty\ ;\ R_2 = R = -0{\cdot}08 \text{ m}$$

$$\therefore\ \frac{1}{f} = (1{\cdot}5 - 1)\left(\frac{1}{\infty} - \frac{1}{-0{\cdot}08}\right)$$

$$= 0{\cdot}5 \times \left(0 + \frac{1}{0{\cdot}08}\right) = \frac{1}{0{\cdot}16}$$

or $$\mathbf{f = 0{\cdot}16\ m}$$

Problem 1.45. What is the minimum distance between an object and the real image formed by a convex lens ?

Ans. For a convex lens, we have

$$-\frac{1}{\mu} + \frac{1}{v} = \frac{1}{f} \quad \text{...}(i)$$

Suppose that an object is placed at a distance x from the convex lens and its real image is formed on the other side of the lens at a distance v from it [Fig. 1.53].

Now, $u = -x\ ;\ v = v$ and $f = f$

Therefore, equation (*i*) becomes

$$-\frac{1}{-x} + \frac{1}{v} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{x} = \frac{x-f}{f\,x}$$

or $$v = \frac{f\,x}{x-f} \quad \text{...}(ii)$$

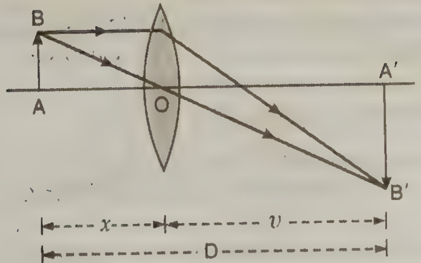


Fig. 1.53

If D is the distance between the object and the image, then

$$v = D - x \quad \text{...}(iii)$$

From equations (*ii*) and (*iii*), we have

$$D - x = \frac{f\,x}{x-f}$$

or $$D = x + \frac{f\,x}{x-f} \quad \text{...}(iv)$$

Differentiating both sides of the equation (*iv*) *w.r.t.* x, we have

$$\frac{dD}{dx} = 1 + \frac{f}{x-f} + f\,x(-1)(x-f)^{-2}$$

$$= 1 + \frac{f}{x-f} - \frac{f\,x}{(x-f)^2}$$

For D to be minimum, $\frac{dD}{dx} = 0$

i.e. $$1 + \frac{f}{x-f} - \frac{f\,x}{(x-f)^2} = 0 \quad \text{or} \quad \frac{x}{x-f} = \frac{f\,x}{(x-f)^2}$$

or $$\frac{f}{x-f} = 1$$

or $$x = 2f$$

Putting $x = 2f$ in equation (*iv*), we have

$$D = 2f + \frac{f \times 2f}{2f - f} = 2f + 2f = 4f$$

Problem 1.46. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Sol. Let O and I represent the positions of the object and the image (screen) ; and L_1 and L_2, the two positions of the lens [Fig. 1.54].

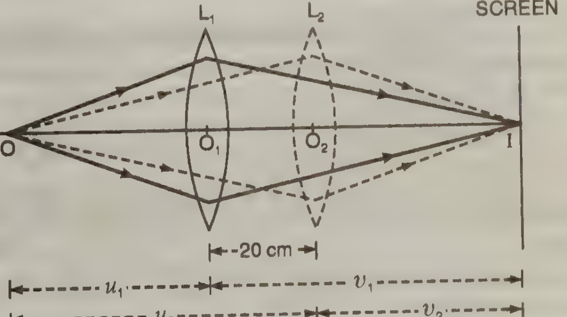


Fig. 1.54

Here, $OI = 90$ cm ; $O_1 O_2 = 20$ cm

When the lens is in position L_1 : Suppose that the distance of the object from the lens is x. Then, the distance of the image from the lens will be $(90 - x)$. If u_1 and v_1 are the distances of the object and image from the point O_1, then according to Cartesian sign conventions,

$$\mu_1 = -x \; ; \; v_1 = +(90 - x)$$

If f is the focal length of the lens, then

$$\frac{1}{f} = -\frac{1}{u_1} + \frac{1}{v_1} = -\frac{1}{-x} + \frac{1}{(90-x)}$$

or $$\frac{1}{f} = \frac{1}{x} + \frac{1}{(90-x)} \qquad ...(i)$$

When the lens is in position L_2 : If u_2 and v_2 are distances of the object and image from the point O_2, then

$$u_2 = -(x+20) \; ; \; v_2 = +\{90 - (x - 20)\} = +(70 - x)$$

From the lens equation, we have

$$\frac{1}{f} = -\frac{1}{u_2} + \frac{1}{v_2} = -\frac{1}{-(x+20)} + \frac{1}{(70-x)}$$

or $$\frac{1}{f} = \frac{1}{(x+20)} + \frac{1}{(70-x)} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{1}{x} + \frac{1}{(90-x)} = \frac{1}{(x+20)} + \frac{1}{(70-x)}$$

On solving for x, we get

$$x = 35 \text{ cm}$$

Substituting for x in equation (*i*), we have

$$\frac{1}{f} = \frac{1}{35} + \frac{1}{(90-35)} = \frac{1}{35} + \frac{1}{55} = \frac{55+35}{35 \times 55}$$

or $$f = \frac{55 \times 35}{90} = 21{\cdot}4 \text{ cm}$$

Problem 1.47. An object is placed at a distance of 1·5 m from a screen and a convex lens is interposed between them. The magnification produced is 4. What is the focal length of the lens ? (*Karnataka, 1984*)

Sol. As the image of the object is formed by the convex lens on the screen, the image formed is real and inverted *i.e.* $m = -4$.

Suppose that the object is placed at a distance x from the convex lens. As the distance between the object and the screen is 1·5 m, the distance of the image from the lens is $(1{\cdot}5 - x)$. According to Cartesian sign conventions,

$$u = -x \; ; \; v = +(1{\cdot}5 - x)$$

Now, $$m = \frac{v}{u}$$

$\therefore$ $$-4 = \frac{(1{\cdot}5 - x)}{-x} \quad \text{or} \quad 4x = 1{\cdot}5 - x \quad \text{or} \quad x = 0{\cdot}3 \text{ m}$$

$\therefore$ $$u = -0{\cdot}3 \text{ m} \quad \text{and} \quad v = 1{\cdot}5 - 0{\cdot}3 = 1{\cdot}2 \text{ m}$$

If f is the focal length of the lens, then

$$\frac{1}{f} = -\frac{1}{u} + \frac{1}{v} = -\frac{1}{-0{\cdot}3} + \frac{1}{1{\cdot}2} = \frac{5}{1{\cdot}2}$$

or $$f = 0{\cdot}24 \text{ m}$$

Problem 1.48. Two thin lenses, when in contact, produce a combination of power +10 dioptres. When they are 0·25 m apart, the power reduces to + 6 dioptres. Find the focal lengths of the two lenses. (*I.I.T. 1997*)

Sol. Let f_1 and f_2 (in metre) be the focal lengths of the two lenses and P_1 and P_2 be their respective powers. Then,

$$P_1 = \frac{1}{f_1} \quad \text{and} \quad P_2 = \frac{1}{f_2}$$

When lenses are placed in contact : If f is focal length of the combination of the two lenses, then

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

If P is the power of the lens, then the above relation becomes

$$P = P_1 + P_2$$

Here, $P = +10$ dioptres

$\therefore$ $$P_1 + P_2 = 10 \qquad ...(i)$$

When lenses are placed some distance apart : If f' is the focal length of the combination of two lenses, when placed a distance x apart, then

$$\frac{1}{f'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

If P′ is power of the combination, then the above relation becomes

$$P' = P_1 + P_2 - x P_1 P_2$$

Here, $P' = +6$ diopters ; $x = 0{\cdot}25$ m

$\therefore$ $$P_1 + P_2 - 0{\cdot}25 P_1 P_2 = 6 \qquad ...(ii)$$

Using equation (*i*), we have

$$10 - 0{\cdot}25 P_1 P_2 = 6$$

or $$P_1 P_2 = \frac{10-6}{0{\cdot}25} = 16 \qquad ...(iii)$$

Now, $$P_1 P_2 = \sqrt{(P_1 + P_2)^2 - 4 P_1 P_2}$$

Using equations (*i*) and (*iii*), we have

$$P_1 - P_2 = \sqrt{10^2 - 4 \times 16} = \sqrt{36}$$

or $$P_1 - P_2 = 6 \qquad ...(iv)$$

From equations (*i*) and (*iv*), we obtain

$$P_1 = 8 \text{ dioptres and } P_2 = 2 \text{ dioptres}$$

$\therefore$ $$f_1 = \frac{1}{P_1} = \frac{1}{8} = \mathbf{0{\cdot}125\ m} \quad \text{and} \quad f_2 = \frac{1}{P_2} = \frac{1}{2} = \mathbf{0{\cdot}5\ m}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What is refraction ? (*H.S.S.C.E. 2001*)

Ans. *The phenomenon of change in path of light as it goes from one medium to another is called refraction.*

Q. 1.02. Give the ratio of velocities of two light waves travelling in vacuum and having wavelengths 4000 Å and 8000 Å. (*C.B.S.E. 1999 S*)

Ans. In vacuum, light of all the wavelengths travel with the same velocity *i.e.* 3×10^8 m s^{-1}.

Q. 1.03. Is the ratio of frequencies of ultraviolet rays and infra-red rays in glass more than, less than or equal to one ? (*C.B.S.E. 1997*)

Ans. It is one.

Q. 1.04. For the same angle of incidence, the angles of refraction in three different media A, B and C are 15°, 25° and 35° respectively. In which medium will the velocity of light be minimum ? (*C.B.S.E. 1993*)

Ans. For the same value of the angle of incidence i, the refractive indices of three media will be

$$\mu_A = \frac{\sin i}{\sin 15°}\,;\ \mu_B = \frac{\sin i}{\sin 25°} \quad \text{and} \quad \mu_C = \frac{\sin i}{\sin 35°}$$

Since $\mu = \frac{c}{v}$, the velocity of light will be minimum in the medium, whose refractive index is maximum.

As μ_A is maximum, velocity of light will be minimum in medium A.

Q. 1.05. Can absolute refractive index of a medium be less than unity ?

Ans. No. It is because velocity of light is maximum in vacuum (or air).

Q. 1.06. State Snell's law of refraction of light. *(P.S.S.C.E. 2000, 1998 S)*

Ans. Refer to section 1.02.

Q. 1.07. Can relative refractive index of a medium *w.r.t.* another be less than unity ?

Ans. Yes. For example, the relative refractive index of water *w.r.t.* glass is less than 1.

Q. 1.08. For what angle of incidence, the lateral shift produced by parallel sided glass plate is zero ?

Ans. For $i = 0$.

Q. 1.09. For what angle of incidence, the lateral shift produced by a parallel sided glass plate is maximum ?

Ans. For $i = 90°$, lateral shift is maximum and is equal to thickness of the glass plate.

$$\text{For } i = 90°, d = \frac{t}{\cos r} \sin(90° - r) = \frac{t}{\cos r} \times \cos r = t$$

Q. 1.10. What are the factors on which the normal shift depends ?

Ans. Thickness of the refracting medium and its refractive index.

Q. 1.11. Write the conditions for total internal reflection to take place.

(P.S.S.C.E. 2000, 1994, 1991 ; H.S.S.C.E. 1999 S ;
H.P.S.S.C.E. 1998 S ; C.B.S.E. 1994, 1993)

Ans. Conditions for total internal reflection. For total internal reflection to take place, following conditions should be obeyed :

1. *The ray of light incident on the interface of the two refracting media should travel in an optically denser medium.*
2. *The angle of incidence should be greater than the critical angle for the given pair of media.*

Q. 1.12. Define critical angle for total internal reflection. *(P.S.S.C.E. 2000, 1999, 1998 S)*

Ans. *The angle of incidence in denser medium, for which angle of refraction in rarer medium is 90°, is called critical angle.*

Q. 1.13. What is critical angle ? Give one application of total internal reflection. *(H.S.S.C.E. 2002 ; C.B.S.E. 1992 S)*

Ans. Critical angle. Refer to VSQ 1.12.

Applications. Refer to section 1.08.

Q. 1.14. What is critical angle for a material of refractive index $\sqrt{2}$? *(H.P.S.S.C.E. 1993)*

Ans. $\frac{1}{\sin C} = \mu = \sqrt{2}$ or $\sin C = \frac{1}{\sqrt{2}}$ or $C = 45°$

Q. 1.15. A substance has critical angle of 45° for yellow light. What is its refractive index ?. *(H.S.S.C.E. 2000 ; I.S.C.E. 1997)*

Ans. $\mu = \frac{1}{\sin C} = \frac{1}{\sin 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

Q. 1.16. For glass-air interface, the critical angle is C. Will the critical angle for glass-water interface be greater or less than C ?

Ans. For glass-air interface, $\sin C = \frac{1}{{}^a\mu_g}$

If C′ is critical angle for glass-water interface, then

$$\sin C' = \frac{1}{{}^w\mu_g}$$

Since ${}^w\mu_g < {}^a\mu_g$; $\sin C' > \sin C$ or $C' > C$

Q. 1.17. Can total internal reflection take place, when light travels from rarer to denser medium ?

Ans. No.

Q. 1.18. What is optical fibre ? *(H.S.S.C.E. 1998)*

Ans. Refer to section 1.08.

Q. 1.19. State the principle of optical pipe.

(P.S.S.C.E. 2002, 2000, 1999)

Or

Name the physical principle on which the working of optical fibres is based. *(I.S.C.E. 1995)*

Ans. The working of optical fibre is based on total internal reflection.

Q. 1.20. What is a lens ?

Ans. A lens is a transparent refracting medium bounded by two spherical surfaces (or by one spherical and one plane surface).

Q. 1.21. Define focus and principle focus of a lens. *(P.S.S.C.E. 1998 S)*

Ans. Focus. *The point, at which a parallel beam of light after refraction through the lens, comes to meet or appears to come from, is called focus.*

Principal focus. *The point on the principal axis, at which a parallel beam of light travelling parallel to principal axis after refraction through the lens, comes to meet or appears to come from.*

Q. 1.22. Can a lens be used in the medium of which it is made of ?

Ans. No.

Q. 1.23. What type of lens is an air bubble inside water ?

Ans. An air bubble inside water behaves as a diverging lens.

Q. 1.24. What is the focal length and power of a plane glass plate ?

Ans. The focal length of a glass plate is infinite and its power is zero.

Q. 1.25. A lens made of glass is immersed in water. Will its power increase or decrease ?

Ans. Its power will decrease.

Q. 1.26. What is lens maker's formula ? *(H.P.S.S.C.E. 1999)*

Or

Why is the lens maker's formula called so ?

Ans. The values of the radii of curvature of the two spherical surfaces, which a lens of required focal length should have, can be calculated from this formula. Then, by grinding the two surfaces of a piece of glass, its two surfaces can be given the calculated value of the radii of curvature. Then, the lens so produced will possess the required focal length.

Q. 1.27. An object is placed at the focus of a concave lens. Where will its image be formed ? *(C.B.S.E. 1999 S)*

Ans. The rays will appear to come from infinity. Therefore, a virtual image will be formed at infinity.

Q. 1.28. A convex lens of focal length 10 cm is combined with a concave lens of focal length 15 cm. Find the focal length of the combination. (*Pre-degree Kerala, 1994*)

Ans. $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{10}+\frac{1}{-15}=\frac{1}{30}$ or $F = 30$ cm

Q. 1.29. Why is the power of a lens measured as the reciprocal of its focal length ?

Ans. A lens is said to possess more power, if it can focus the rays of light quite close *i.e.* if its focal length is small. For this reason, the power of the lens is measured as the reciprocal of its focal length.

Q. 1.30. What is the unit of power of a lens ? (*H.P.S.S.C.E. 2002*)

Ans. Dioptre.

Q. 1.31. The image of an object formed by a lens on th screen is not in sharp focus. Suggest a method to get a clea focussing of the image on the screen without disturbing th position of the object, the lens or the screen. (*C.B.S.E. 1998*

Ans. The focal length of a lens depends upon th refractive index of the material of the lens, which in tur depends upon the wavelength of light. Therefore, clear image of the object can be obtained by using light of suitable wavelength.

Q. 1.32. What are the uses of putting two lenses in contact with each other ?

Ans. Uses. 1. To increase the magnification of the image.

2. To produce erect image, if the image formed by a lens is inverted.

3. To minimise spherical aberration.

4. To increase the field of view.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. A light of wavelength 6000 Å in air, enters a medium with refractive index 1·5. What will be frequency and wavelength of light in the medium ? (*I.I.T. 1997*)

Ans. Here, wavelength of light in air,

$$\lambda = 6000\ \text{Å} = 6 \times 10^{-7}\ \text{m};$$

refractive index of medium, $\mu = 1{\cdot}5$

Frequency of the light does not change, when light travels from air to a refracting medium.

$$\therefore \qquad \nu' = \nu = \frac{c}{\lambda} = \frac{3\times10^8}{6\times10^{-7}} = 5 \times 10^{14}\ \text{s}^{-1}$$

Wavelength of the light in medium,

$$\lambda' = \frac{\lambda}{\mu} = \frac{60}{1{\cdot}5} = 4000\ \text{Å}$$

Q. 1.02. A ray of light of frequency of 5 × 10⁴ Hz is passed through a liquid. The wavelength of light measured inside the liquid is found to be 450 × 10⁻⁹ m. Calculate refractive index of the liquid. (*H.P.S.S.C.E. 1998 S*)

Ans. Here, $\nu = 5 \times 10^{14}$ Hz ;

wavelength of light inside liquid, $\lambda = 450 \times 10^{-9}$ m

Therefore, velocity of light in liquid,

$$v = \nu\lambda = 5 \times 10^{14} \times 450 \times 10^{-9} = 225 \times 10^8\ \text{m s}^{-1}$$

Hence, refractive index of liquid,

$$\mu = \frac{c}{v} = \frac{3\times10^8}{2{\cdot}25\times10^8} = 1{\cdot}33$$

Q. 1.03. The refractive index of glass is 1·5 and that of water is 1·3. If the speed of light in water is 2·25 × 10⁸ m s⁻¹, what is the speed of light in glass ? (*I.S.C.E. 1996*)

Ans. Here, ${}^a\mu_g = 1{\cdot}5$; ${}^a\mu_g = 1{\cdot}3$

Let v_1 and v_2 be speeds of light in glass and water respectively. If c is speed of light in air, then

$$\frac{c}{v_1} = 1{\cdot}5 \quad \text{and} \quad \frac{c}{v_2} = 1{\cdot}3$$

$$\therefore \qquad \frac{c}{v_2} \times \frac{v_1}{c} = \frac{1{\cdot}3}{1{\cdot}5}$$

$$\text{or} \qquad v_1 = \frac{1{\cdot}3}{1{\cdot}5} \times v_2 = \frac{1{\cdot}3}{1{\cdot}5} \times 2{\cdot}25 \times 10^8$$

$$= 1{\cdot}95 \times 10^8\ \text{m s}^{-1}$$

Q. 1.04. A ray of monochromatic light travelling in vacuum with speed c, wavelength λ and frequency ν, enters into a medium of refractive index 1·5. What will be its new speed, wavelength and frequency ? (*I.S.C.E. 1995*)

Ans. Let v be speed of light in the medium of refractive index $\mu = 1{\cdot}5$. Then,

$$\mu = \frac{c}{v}$$

$$\text{or} \qquad v = \frac{c}{\mu} = \frac{c}{1{\cdot}5}$$

The frequency of light in the medium will remain ν **(unchanged).**

If λ' is the wavelength of light in the medium, then

$$v = \nu\lambda'$$

$$\text{or} \qquad \lambda' = \frac{v}{\nu} = \frac{v}{c/\lambda} = \frac{\lambda}{c/v} = \frac{\lambda}{\mu}$$

$$\text{or} \qquad \lambda' = \frac{\lambda}{1{\cdot}5}$$

Q. 1.05. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. Find the angle of deviation for the ray of light. (*I.I.T. 1997*)

Ans. When the ray of light PQ is incident normally on the face AB of the prism ABC (∠A = 30°), it travels unrefracted along path QR and comes to incident on the face AC of the prism at an angle of incidence, $i = 30°$ [Fig. 1.55].

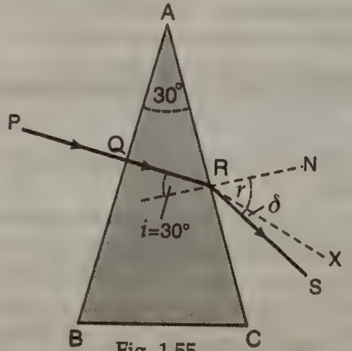


Fig. 1.55

If r is the angle of refraction, then

$$\frac{1}{\mu}=\frac{\sin i}{\sin r} \quad \text{or} \quad \frac{1}{\sqrt{2}}=\frac{\sin 30°}{\sin r}$$

$$\text{or} \quad \sin r=\sqrt{2}\sin 30° \quad \text{or} \quad \sin r=\sqrt{2}\times\frac{1}{2}=\frac{1}{\sqrt{2}}$$

$$\text{or} \quad r=45°$$

The angle of deviation,

$$\delta=\angle NRS-\angle NRX=r-i=45°-30°=15°$$

Q. 1.06. How will you explain twinkling of stars ? (*C.B.S.E. 1992 ; Text Question*)

Ans. The light from stars undergoes refraction continuously before it reaches earth. Due to this, the apparent position of a star is slightly different than its actual position. Due to the variation in atmospheric conditions, (change in temperature and density), this apparent position of the star constantly keeps on changing. The fluctuating apparent positions of the star give rise to the twinkling effect.

Q. 1.07. The sun is seen a little before it rises and for a short while after it sets. Explain, why. (*C.B.S.E. 2002, 2000*)

Or

Watching the sunset on a beach, one can see the sun for several minutes after it has 'actually set'. Explain. (*C.B.S.E. 1992 ; Text Question*)

Or

How does refraction of light affect the length of the day ?

Ans. This effect is due to atmospheric refraction of light. At the time of sunrise or sunset, the sun is just near the horizon. The line XY represents the horizon for an observer O on the surface of the earth as shown in Fig. 1.56. Near the surface of earth, air is a little denser than the outer space. The refractive index of the air near the surface of earth is 1·00029 *w.r.t.* the outer space. Therefore, a ray of light travelling from the sun towards earth, travels through air, which becomes more and more optically denser. Due to this, it undergoes refraction and therefore gradually deviates towards the normal. As a result, to the observer O on the surface of earth, the sun appears at S′. The apparent shift in the direction of the sun is only 0·5° and the time difference between the actual sunset and the apparent sunset is about 2 minutes.

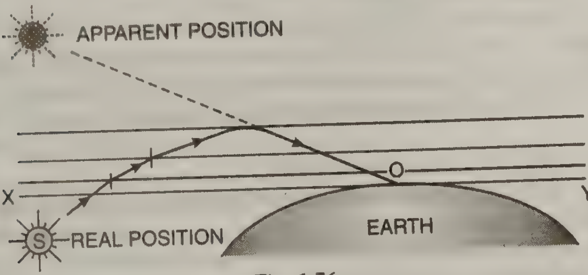


Fig. 1.56

Q. 1.08. Discuss refraction through a glass-slab and show that emergent ray is parallel to the incident ray but displaced. (*H.P.S.S.C.E. 1996*)

Ans. Refer to section 1.04.

Q. 1.09. A coin placed at the bottom of a tank appears to be raised, when water is poured into the tank. Why ?

Or

Why does a tank filled with water appear shallow ?

Or

Does the apparent depth of a tank of water change, if viewed obliquely ? If so, does the apparent depth increase or decrease ? (*Text Question*)

Ans. Refer to Fig. 1.05. It is because of the normal shift in the position of the object due to the phenomenon of refraction. When the rays of light from the coin in the denser medium fall on the interface, the rays of light get away from the normal due to refraction. The point from which the refracted rays appear to come gives the apparent position of the coin. As the rays appear to come from a point above the coin, the coin appears to be raised.

Q. 1.10. A stick partially immersed obliquely under water appears to be bent. Explain, why.

Ans. The different points on the stick, which has been dipped in the water obliquely, are at different depths. Since the normal shift depends upon the depth of the object, the different points on the stick undergo different amounts of normal shifts and hence the stick appears bent.

Q. 1.11. What is total internal reflection ? Under what conditions does it take place ? (*H.S.S.C.E. 2002, 1996 ; P.S.S.C.E. 2000, 1999 ; H.P.S.S.C.E. 1999 S ; C.B.S.E. 1993*)

Ans. Refer to section 1.07.

Q. 1.12. Define critical angle. Write the conditions for total internal reflection to take place. (*H.P.S.S.C.E. 1998 S*)

Ans. Refer to section 1.07.

Q. 1.13. State the conditions for producing total internal reflection. (*C.B.S.E. 2002, 1994 ; P.S.S.C.E. 1994, 1991*)

Ans. Refer to section 1.07.

Q. 1.14. Derive the relation between refractive index of a medium and its critical angle. (*C.B.S.E. 1993*)

Ans. Refer to section 1.07.

Q. 1.15. A ray of light while travelling from a denser to rarer medium undergoes total internal reflection. Derive the expression for the critical angle in terms of the speed of light in the two media. (*C.B.S.E. 1996*)

Ans. Refer to section 1.07. It can be proved that

$$\sin C=\frac{1}{\mu}$$

If c and v are speeds of light in vacuum and the medium, then $\mu=\frac{c}{v}$.

Therefore, $\sin C=\frac{v}{c}$

Q. 1.16. Refractive index of glass is 1·5. Find the speed of light in glass, if its speed in air is 3×10^8 m s^{-1}. Also find critical angle for glass. (*H.S.S.C.E. 1997 ; I.S.C.E. 1995*)

Ans. Speed of light in glass,

$$v=\frac{c}{\mu}=\frac{3\times10^8}{1\cdot5}=2\times10^8 \text{ m s}^{-1}$$

If C is critical angle for glass, then

$$\mu=\frac{1}{\sin C} \quad \text{or} \quad \sin C=\frac{1}{\mu}=\frac{1}{1\cdot5}=0\cdot6667$$

$$\text{or} \quad C=41\cdot8°$$

Q. 1.17. Calculate the speed of light in a medium whose critical angle is 45°. (*C.B.S.E. 2002*)

Ans. Now, $\mu=\frac{1}{\sin C}=\frac{1}{\sin 45°}=\sqrt{2}$

Also, $\mu = \frac{c}{v}$ or $v = \frac{c}{\mu} = \frac{3 \times 10^8}{\sqrt{2}}$

$= 2{\cdot}12 \times 10^8 \text{ m s}^{-1}$

Q. 1.18. Fig. 1.57 shows object PQ in front of a right angled (45°-90°-45°) glass prism. The critical angle of glass is 42°. Redraw this figure tracing the complete path of rays from P and Q into and out of the prism.

(I.S.C.E. 1994 ; Text Question)

Or

Show with a ray diagram, how an image is produced in total reflecting prism. *(I.S.C.E. 1996)*

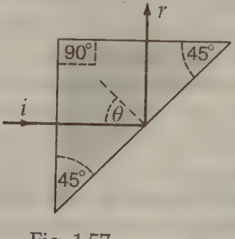


Fig. 1.57

Ans. The path of the two rays from points P and Q travelling normal to the hypotenuse face AC of the 45°-90°-45° glass prism will be as shown in Fig. 1.57. The two rays will undergo total internal reflection, firstly at the side face AB and then at side face BC. As the angle of incidence at the two faces is 45° *i.e.* greater than the critical angle (42°) for the glass, the ray of light undergoes total internal reflection. The detailed description of the total internal reflections at the two side faces has been given in section 1.08. The final image P′ Q′ will be an inverted one as shown in the figure.

Q. 1.19. Fig. 1.58 shows a triangular prism of glass. A ray incident normal on one face is totally reflected. What can you conclude about the minimum value of index of refraction of glass ? *(C.B.S.E. 1995)*

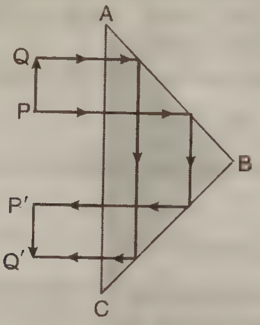


Fig. 1.58

$\theta = 90°/2 = 45°$

Ans. From Fig. 1.58, it follows that

As the light incident at 45° undergoes total internal reflection, the value of critical angle for the material of prism must be less than or equal to 45° *i.e.* maximum value of the critical angle C is 45°.

Since $\mu = \frac{1}{\sin C}$, corresponding to maximum value of C, the value of μ is minimum.

Hence, $\mu_{min} = \frac{1}{\sin 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

Q. 1.20. Draw ray diagram to show how a right angled isosceles prism can be used to (*a*) deviate the ray through 180°, (*b*) deviate the ray through 90° and (*c*) invert the ray.

Ans. For Q. 1.20. (*a*) and (*b*), refer to section 1.08.

For Q. 1.20. (*c*), refer to SAQ 1.19.

Q. 1.21. What are the advantages of total reflecting prism over a plane mirror ?

Ans. Advantages. 1. It does not require silvering. **2.** The reflection of light is 100%. **3.** Multiple reflections do not take place in a reflecting prism. Due to this, only one image is formed, which is very bright. **4.** A reflecting prism is free from the trouble arising out of vanishing of the silvering of mirror with the passage of time.

Q. 1.22. How do you explain the mirage effect produced in very hot deserts ? *(H.P.S.S.C.E. 1993)*

Or

On a hot summer day in desert, one sees the reflected image of the distant objects. Explain, why.

Ans. It is an optical illusion, called mirage. For details, refer to section 1.08.

Q. 1.23. Show through a labelled diagram the formation of mirage in deserts. *(C.B.S.E. 1991 S)*

Ans. Refer to Fig. 1.09.

Q. 1.24. Why does a diamond sparkle ? Give reasons. *(P.S.S.C.E. 2001)*

Ans. Refer to section 1.08.

Q. 1.25. Why is the diamond brilliant ? Is it a source of light also ? *(P.S.S.C.E. 2002 ; 1998 S)*

Ans. For brilliance of diamond, refer to section 1.08.

No, a diamond is not a source of light.

Q. 1.26. What are optical fibres ? Write in short about optical pipe. *(P.S.S.C.E. 1995)*

Ans. Refer to section 1.08.

Q. 1.27. What are optical fibres ? Give their one use. *(H.S.S.C.E. 2002 ; H.P.S.S.C.E. 2000 ; C.B.S.E. 1992)*

Ans. Refer to section 1.08.

Use. The optical fibres are used in medical investigations. Using an optical fibre, a doctor can examine the inside of the stomach and the intestine. It is called *endoscopy*.

Q. 1.28. What is the main use of optical fibres ? *(H.S.S.C.E. 1999)*

Ans. Refer to section 1.08.

Q. 1.29. Explain with a ray diagram, how optical fibres transmit signals. *(C.B.S.E. 1993)*

Ans. Refer to Fig 1.10.

Q. 1.30. A luminous object is placed below a glass slab of thickness *t*. A circular opaque disc is placed vertically above the object on the surface of the glass-slab. If C is critical angle for glass-air interface, then predict the minimum radius of the opaque disc so that the object cannot be seen from any uncovered portion of the glass-slab.

Ans. The maximum radius of the circle through which the light can be received from the luminous object is given by

$r = t \tan C,$

where C is the critical angle for glass-air interface and t is the thickness of the glass slab. For details, refer to solved problem 1.12 and its Fig. 1.36.

Obviously, if an opaque disc of radius $r\ (= t \tan C)$ is placed on the slab, then the luminous object will not be seen from any uncovered portion of the glass slab.

Q. 1.31. When observed from under water, all the objects above the surface can be seen within a cone of 97°. Explain, why.

Ans. Again, refer to solved problem 1.12. It follows that the objects above the surface will be visible in a cone of semivertical angle C, where C is the critical angle for the water-air interface.

Now, $$\mu = \frac{1}{\sin C}$$

or $$\sin C = \frac{1}{\mu} = \frac{1}{1.33} = 0.7519$$

or $$C = 48.75°$$

Therefore, required angle of the cone = $48.75° \times 2 \approx 97°$

Q. 1.32. A fish in a tank sees the outside world as if it is at the vertex of a cone, such that the circular base of the cone is at the surface of the water. If the depth of the fish is *d* and the critical angle for water-air interface is C, what is the radius of the circle ?

Ans. Refer to solved problem 1.12. The radius of the circular base of the cone is given by

$$r = d \tan C$$

Q. 1.33. For a spherical surface, when refraction takes place from a rarer medium (μ_1) to a denser medium (μ_2), we write

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R},$$

where the symbols have their usual meanings. Now suppose that the object is placed in the denser medium and refraction takes place from the denser to the rarer medium. Then, rewrite the above equation.

Ans. When the object is placed in the denser medium and refraction takes place from the denser to the rarer medium, the equation will be

$$-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

It can be obtained from the given equation by simply interchanging μ_1 and μ_2.

Q. 1.34. Can a convergent lens in one medium behave as a divergent lens in some other medium ?

Ans. Yes. A convex lens made of glass behaves as a convergent lens when placed in air or water. However, when the same lens is immersed in carbon disulphide ($\mu = 1.63$), it behaves as a divergent lens. In other words, when a convergent lens is placed inside a transparent medium of refractive index greater than that of the material of the lens, it behaves as a divergent lens.

Q. 1.35. A thin converging lens has a focal length *f* in air. If it is completely immersed in a liquid, briefly explain how the focal length of the lens will vary ? (*C.B.S.E. 2001*)

Ans. The focal length of the lens in air is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(i)$$

If μ' is refractive index of the material of the lens *w.r.t.* liquid, then focal length of the lens, when placed in liquid is be given by

$$\frac{1}{f'} = (\mu' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{f'}{f} = \frac{(\mu - 1)}{(\mu' - 1)}$$

Since $\mu' < \mu$, $f' > f$ *i.e.* focal length of the lens will **increase** on immersing it in a liquid.

Q. 1.36. The refractive index of the material of a concave lens is μ_1. It is immersed in a medium of refractive index μ_2. A parallel beam of light is incident on the lens. Trace the path of rays, when $\mu_2 < \mu_1$. (*C.B.S.E. 2001 S*)

Ans. Since the refractive index of the material of the concave lens is greater than that of the medium in which it is placed, it will still behave as a concave lens. Since focal length of the lens increases, its diverging action will decrease. For course of rays, refer to Fig. 1.68.

Q. 1.37. A concave mirror and a convex lens are held in water. What change, if any, do you expect in their respective focal lengths as compared to their values in air ? (*C.B.S.E. 2000, 1995*)

Ans. The focal length of a concave mirror has nothing to do with the medium in which it is placed. Hence, it will remain ***unchanged.***

However, the focal length of a convex lens would change, when held in water. It *increases* due to decrease in the value of relative refractive index of the material of lens *w.r.t.* water.

Q. 1.38. A convex lens (refractive index μ_L) is immersed in a medium (refractive index μ_M). Will it behave as a convergent or divergent lens, if (*i*) $\mu_L > \mu_M$ (*ii*) $\mu_L < \mu_M$? (*Pre-degree Kerala, 1991*)

Ans. (*i*) *When* $\mu_L > \mu_M$: The convex lens will still behave as a convex lens. It is because, ${}^{M}\mu_{L} = \frac{\mu_L}{\mu_M}$ is greater than 1 and the lens maker's formula gives a positive value for the focal length of the lens.

(*ii*) *When* $\mu_L < \mu_M$: The convex lens will behave as a concave lens. It is because, ${}^{M}\mu_{L}$ is less than 1 and hence focal length of the lens has a negative value.

Q. 1.39. Explain, why an air bubble inside a transparent liquid behaves like a diverging lens.

Ans. An air bubble acts as a convex lens made of air and placed inside water. Therefore, the air bubble acts as a lens, whose focal length is given by

$$\frac{1}{f} = ({}^{g}\mu_a - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, ${}^{g}\mu_a < 1$ *i.e.* ${}^{g}\mu_a - 1$ is negative. Since the factor $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ is positive, the value of *f* will be negative. Hence, the air bubble inside a transparent liquid behaves as a diverging lens.

Q. 1.40. What changes in the focal length of a (*a*) concave mirror and (*b*) convex lens occur, when the incident violet light on them is replaced by red light ? (*C.B.S.E. 2002*)

Ans. (*a*) No change in the focal length of concave mirror.

(*b*) For a lens, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Since $\mu_r < \mu_v$, the focal length of convex lens will **increase** on the replacing violet light by red light.

Q. 1.41. A single thin lens of focal length 30 cm is placed in front of the lens of a camera, the camera lens is set at

infinity. Where should you place an object to get a clear picture on the film with the extra lens ?

Ans. To get a clear picture with the camera set at infinity, the light entering the camera lens must be in the form of a parallel beam. The extra lens placed in front of camera lens will issue parallel rays, if the object to be photographed is placed at the focus of the extra lens *i.e.* at a distance of 30 cm from the extra lens.

Q. 1.42. A camera lens of focal length 5 cm is mounted on a fine screw thread for adjusting the focus. What range of movement is needed, if the camera is to be able to take sharp photographs at 1 m as well as from great distances ?

Ans. For the camera lens : $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

or $$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}+\frac{1}{5}+\frac{1}{u}$$

For very distant object, $u=\infty$

$$\therefore \qquad \frac{1}{v}=\frac{1}{5}+\frac{1}{\infty}=\frac{1}{5} \quad \text{or} \quad v=5\text{ cm}$$

For an object at 1 m from the camera,

$$u=-1\text{ m}=-100\text{ cm}$$

$$\therefore \qquad \frac{1}{v}=\frac{1}{5}+\frac{1}{-199}=\frac{20-1}{100}=\frac{19}{100} \quad \text{or} \quad v=5{\cdot}26\text{ cm}$$

Since in both the cases, image must be cast on the film, the lens must have a range of movement of 5·26 – 5 *i.e.* **0·26 m.**

Q. 1.43. A concave lens produces a virtual and diminished image independent of the location of the object. Explain, why.

Ans. From the lens equation $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, we have

$$v=\frac{u\,f}{u+f}$$

The focal length of a concave lens is negative. Since distance of a real object from the lens is also negative, it follows that v will also be negative. Therefore, a concave lens forms the image on the same side of the object. In other words, the image formed is always *virtual.*

Again, $$m=\frac{f}{u+f}$$

Since both f and u are negative, the value of m is positiv and always *less than 1.*

Hence, a concave lens produces a virtual and diminishe image independent of the position of the object.

Q. 1.44. Draw a ray diagram showing the formation o image by a concave lens. Discuss the nature of the image. (*C.B.S.E. 1991*

Ans. For ray diagram, refer to Fig. 1.30.

The image is always virtual, erect, diminished and i formed on the same side of the object.

Q. 1.45. Define power of a lens. Give its units. (*H.P.S.S.C.E. 2001 ; H.S.S.C.E. 1994*)

Or

What is meant by power of a lens ? What is one dioptre ? (*H.P.S.S.C.E. 1994 ; H.S.S.C.E. 1994*)

Ans. Refer to section 1.20.

Q. 1.46. Draw a diagram to illustrate spherical aberration in a convex lens. (*I.S.C.E. 1995*)

Ans. Refer to Fig. 1.32.

Q. 1.47. Why do lenses of large aperture suffer from spherical aberration ?

Ans. In case of a lens of large aperture, the behaviour of the paraxial and the marginal rays are markedly different from each other. The two types of rays come to focus at different points on the principal axis of the lens. However, in case of a lens of small aperture, the two types of rays come to focus quite close to each other.

Q. 1.48. Explain, how the spherical aberration can be minimised.

Ans. The spherical aberration can be minimised either by allowing only paraxial rays to fall on the lens or by allowing the marginal rays to fall. The paraxial rays can be obstructed by using a stop *i.e.* circular disc. On the other hand, the marginal rays can be obstructed by using a circular aperture.

Q. 1.49. Red light is incident on a thin converging lens of focal length *f*. Briefly explain how the focal length of the lens will change, if red light is replaced with blue light. (*C.B.S.E. 2001*)

Ans. Refer to SAQ 1·39.

TECHIE STUFF –CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. When does Snell's law in refraction fail ?

Ans. Snell's law fails, when light is incident normally on the surface of a refracting medium. It is because, when light is incident normally, angle of incidence is zero *i.e.* $i=0$ and hence $r=0$. The ratio $\frac{\sin i}{\sin r}$ no longer remains constant. Instead, it becomes indeterminant.

Q. 1.02. A pile 4 m high stands in the lake, such that it protrudes by 1 m above the surface of water. Determine the length of the shadow of the pile on the bottom of lake, when the sunrays make an angle of 45° with the water surface. Take refractive index of water, $\mu = 4/3$.

Ans. Suppose that the pile AB of height 4 m stands in the lake, such that its portion BQ = 3 m is inside water and the portion AQ = 1 m is above the water. The sunrays making an angle of 45° with the water surface will produce the images BQ′ and Q′ S′ (= QS) of the portions BQ and QA [Fig. 1.59].

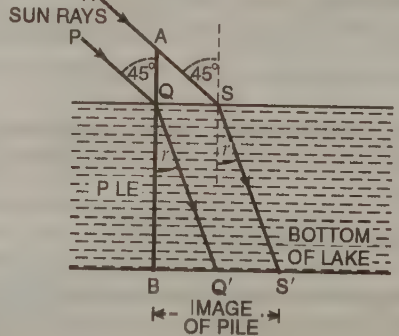


Fig. 1.59

Therefore, total length of the shadow of the pile,

$$BS' = BQ' + Q'S' = BQ' + QS$$

Since in ΔAQS, $\angle QAS = \angle ASQ = 45°$,

$$QS = AQ = 1 \text{ m}$$

If r is angle of refraction, then

$$\mu = \frac{\sin i}{\sin r}$$

or $$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{4/3} = \frac{3}{4} \times 0{\cdot}7071 = 0{\cdot}5303$$

or $$r = 32°\text{-}2'$$

From right angled $\Delta QBQ'$

$$BQ' = BQ \tan r = 3 \times \tan(32°\text{-}2') = 3 \times 0{\cdot}6257$$
$$= 1{\cdot}8771 \approx 1{\cdot}88 \text{ m}$$

Therefore, length of the shadow of the pile on the bottom of lake = 1·88 + 1 = **2·88 m**

Q. 1.03. Does refraction of light mean a change in the direction of light propagation ?

Ans. No. The direction of propagation of light can change on reflection from a mirror also.

The change in direction of light propagation, when light goes from one medium to another, is called refraction.

Q. 1.04. If a plane glass-slab is placed on letters of different colours, then red coloured letters appear more raised up. Why ?

Ans. The refractive index of glass is minimum for red light. Because of this, the normal shift,

$$d = t\left(1 - \frac{1}{\mu}\right)$$

is maximum for red light and the red coloured letters appear more raised up.

Q. 1.05. A beam of light is converging towards a point on a screen. A plane parallel plate of glass is introduced in the path of this converging beam. How will the point of convergence be shifted ? Draw the ray diagram. *(I.I.T. 1974)*

Ans. Fig. 1.60 shows a glass plate introduced in the path of a beam of light converging to point O.

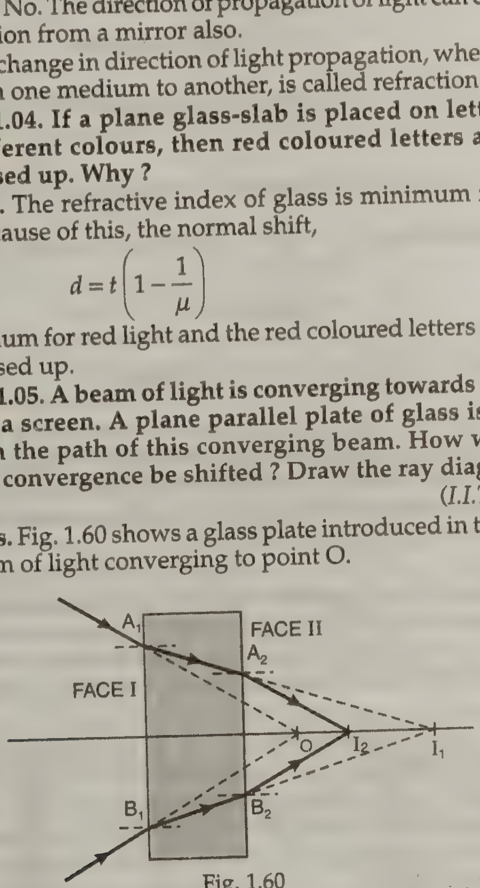


Fig. 1.60

The face I of the glass plate will refract the rays of light at points A_1 and B_1, towards the normal so that the two rays will appear to meet at point I_1. However, due to refraction at the face II of the glass plate at points A_2 and B_2, the rays will be refracted away from the normals so that the beam actually converges to point I_2. The ray $A_2 I_2$ is parallel to A_1O and it is laterally shifted through a distance,

$$d = t\left(1 - \frac{1}{\mu}\right),$$

where t is thickness of the glass plate between the faces I and II.

Q. 1.06. The moon moves across the sky at approximately 0·5° per hour.

(a) When it passes in front of a star, what do you see ? Discuss.

(b) What you would see, if the moon had an atmosphere like the earth ?

Ans. (*a*) Light travels along a straight path from star to the earth through vacuum. Therefore, until the solid edge of the moon cuts off the light, the star will be seen. It will remain fixed *w.r.t.* other stars and its brightness will also remain constant.

(*b*) If the moon had an atmosphere (shown by dotted circle around the moon) and the light from star S traversed its path through it, then light from the star would be refracted as shown in Fig. 1.61 and the star would seem to shift from S to S′. Thus, as the moon will move towards the line of sight of the star, the star would seem to move ahead of the moon. Due to scattering of the blue part of the visible spectrum from moon's atmosphere, it will appear red and dim, before the solid edge of the moon cuts off its light.

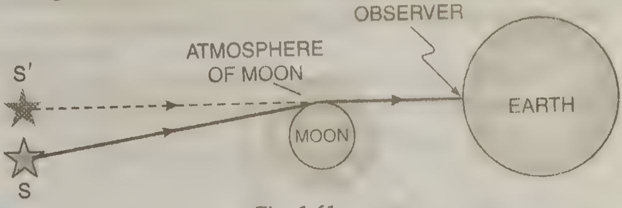


Fig. 1.61

Q. 1.07. The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to diamond cutter ?

Ans. The refractive index of diamond ($\mu = 2{\cdot}42$) is much greater than that of the ordinary glass ($\mu = 1{\cdot}5$). Since $\sin C = \frac{1}{\mu}$, the critical angle for diamond is much less and is about 24°. Therefore, if angle of incidence remains between 90° to 24°, the light will undergo internal reflection again and again, till it remains between 24° and 90°. For ordinary glass, due to low value of its refractive index, such a range for angle of incidence is quite small. Because of a large range of angle of incidence available, a diamond cutter is able to cut the diamond, so as to have a large number of faces. Due to a large number of internal reflections taking place from these faces (before the ray of light finally comes out), the diamond exhibits brilliance.

Q. 1.08. An empty test tube dipped into water in a beaker appears silvery, when viewed from a suitable direction. Why ?

Ans. The ray of light incident on the water-air interface suffers total internal reflection, in case the angle of incidence is greater than the critical angle. Therefore, if the tube is viewed from a suitable direction (so that the angle of incidence is greater than the critical angle), the rays of light incident on the tube undergo total internal reflection. As a result, the test tube appears as highly polished *i.e.* silvery.

Q. 1.09. A fish swims in a glass tank. A person, whose eye is above the level of water, sees two fishes. Draw a ray diagram to illustrate this.

Ans. Fig. 1.62 shows a fish O inside the water. A person whose eye is above sea water sees two fishes due to refraction of light taking place from the surface of water and from the vertical wall of the glass tank. The image I_1 of the fish is seen due to refraction of the light at point A on the surface of water and another image I_2 is seen due to refraction at point B on the vertical wall of the glass tank.

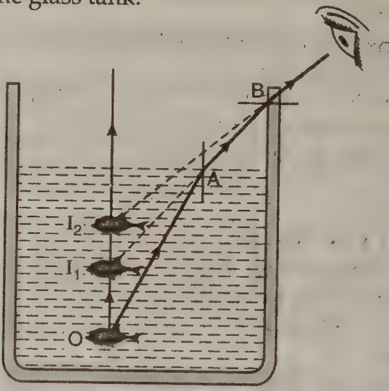


Fig. 1.62

Q. 1.10. To a fish under water viewing obliquely a fisherman standing on the bank of a lake, does the man look taller or shorter than what he actually is ?

(Text Question)

Ans. The man would look taller than what he actually is. Fig. 1.63 shows a man AB standing on the bank of the lake.

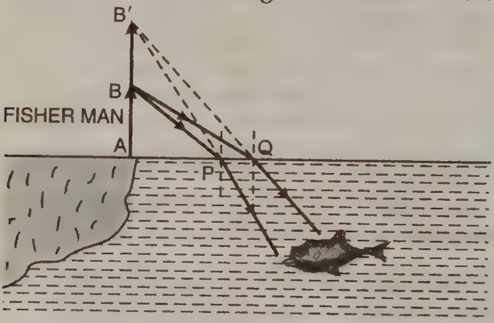


Fig. 1.63

The rays of light BP and BQ from the head (B) of the fisherman, on refraction at the water-air interface, bend towards the normals at points P and Q and appear to come from point B′. Therefore, to the fish, the fisherman will appear as AB′ *i.e.* taller than what he actually is. It may be pointed out that as the aperture of the eye of the fish is very small, the points P and Q will actually be very close to each other.

Q. 1.11. What is the relation between refractive indices μ, μ_1, μ_2, if the behaviour of light rays is as shown in Figs. 1.64 and 1.65 ? *(I.I.T. 1979)*

Ans. In Fig. 1.64 : The focal length of the lens is given by

$$\frac{1}{f}=\left(\frac{\mu}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

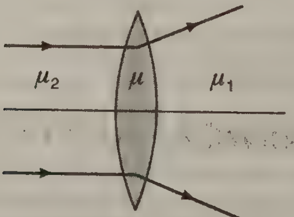


Fig. 1.64

As the rays pass through the lens undeviated, $f=\infty$

$$\therefore \left(\frac{\mu}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{\infty}=0$$

Since for a double convex lens, $\frac{1}{R_1}-\frac{1}{R_2}$ is positive, it follows that

$$\frac{\mu}{\mu_1}-1=0$$

or $\quad \frac{\mu}{\mu_1}=1 \quad$ or $\quad \mu=\mu_1$

In Fig. 1.65 : The focal length of the lens is given by

$$\frac{1}{f}=\left(\frac{\mu}{\mu_2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

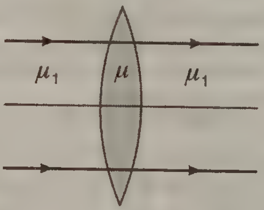


Fig. 1.65

As the lens behaves as a diverging lens, f is negative. Since the factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is positive, it follows that

$\frac{\mu}{\mu_2}-1$ is negative *i.e.* $\frac{\mu}{\mu_2}-1<0$

or $\quad \frac{\mu}{\mu_2}<1 \quad$ or $\quad \mu<\mu_2$

Q. 1.12. A concave lens made of a material of refractive index μ_g is immersed in a medium of refractive index μ_1, (*i*) greater than, (*ii*) equal to and (*iii*) less than μ_g. When a parallel beam of light is incident on the lens, trace the path of the emergent rays in each case.

(C.B.S.E. 2000 ; I.I.T. 1973)

Ans. When a parallel beam of light is incident on the lens of refractive index μ_g immersed in a medium of refractive index μ_l, the course of rays will be as shown in Fig. 1.66, Fig. 1.67 and 1.68, when $\mu_g>\mu_l$; $\mu_l=\mu_g$ and $\mu_l<\mu_g$ respectively.

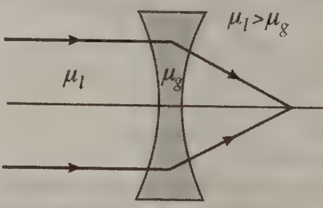


Fig. 1.66

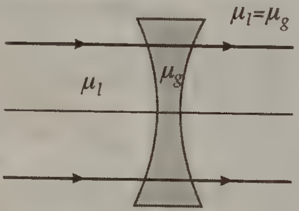


Fig. 1.67

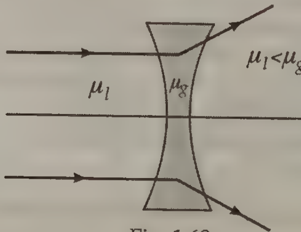


Fig. 1.68

When $\mu_1 > \mu_g$, the lens behaves as converging lens (nature of the lens changes). On the other hand, when $\mu_l = \mu_g$, the lens surface becomes indistinguishable from the medium in which the lens is immersed.

Q. 1.13. The lens shown in Fig. 1.69 is made of two different materials. A point object is placed on the axis. How many images the object will it form ? *(I.I.T. 1975)*

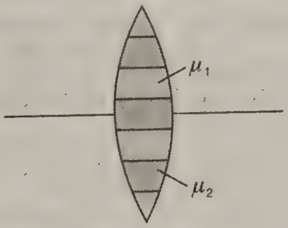


Fig. 1.69

Ans. According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the lens is made of two different materials of refractive indices say μ_1 and μ_2, it will possess two focal lengths given by

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Hence, such a lens will form two images of an object.

Q. 1.14. Although the surfaces of a goggle lens are curved, it does not have any power. Explain, why.

Or

Why is that sun-glasses (goggles), which have curved surfaces, do not have any power ?

Ans. The two surfaces of the goggle lens are parallel *i.e.* possess same radii of curvature. Since one surface is convex and other concave, the powers of the two surfaces is equal but of opposite signs.

The total power of the goggle lens (P′) is the algebraic sum of the powers of the individual surfaces *i.e.*

$$P' = P_1 + P_2 = P + (-P) = 0$$

Q. 1.15. A portion of a lens is broken. Will we get a complete image of an object with such a lens ?

Ans. Yes. But the intensity of the image will be lesser.

Q. 1.16. A virtual image, we always say, cannot be caught on a screen. Yet when we see a virtual image, we are obviously bringing it on to the `screen' (*i.e.* the retina) of our eye. Is there a contradiction ? *(Text Question)*

Ans. The virtual image produced by a spherical mirror or a lens acts as a virtual object for the eye-lens, which produces a real image on the retina. Hence, there is no such contradiction.

Q. 1.17. The image of a small electric bulb fixed at the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose ? *(Text Question)*

Ans. For a given fixed distance (3 m) between the object and its real image, the lens equation does not give a real solution for u and v, if f is greater than one-fourth of the distance between object and the image.

Therefore, the maximum focal length of the lens

$$= \frac{3 \cdot 0}{4} = \mathbf{0 \cdot 75\ m}$$

Q. 1.18. The image of a candle formed by a convex lens is obtained on a screen. Will the full size of image be obtained, if the lower half of the lens is painted black and completely opaque ? Illustrate your answer with a ray diagram. *(I.I.T. 1977)*

Ans. Yes, the image formed will still be of full size. The ray diagram showing the formation of image is as shown in Fig. 1.70. It may be pointed out that the intensity of the image will be lesser, when the lower half of the lens is painted black. It is because, light gathering power of a lens depends upon its diameter.

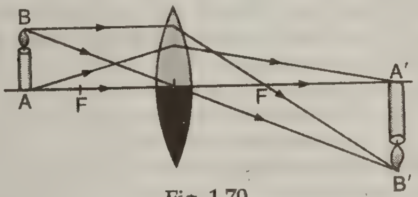


Fig. 1.70

Q. 1.19. An ant is approaching a convex lens with a uniform speed upto first focus. How would the speed of the image of the ant formed by lens change ?

Ans. From the lens equation, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, we have

or $$v = \frac{u\,f}{u + f}$$

Differentiating both sides *w.r.t.* t, we have

$$\frac{dv}{dt} = \frac{(u + f) \times f\,\frac{du}{dt} - u\,f \times \left(\frac{du}{dt} + 0\right)}{(u + f)^2}$$

or $$\frac{dv}{dt} = \frac{f^2}{(u + f)^2} \cdot \frac{du}{dt} \qquad \ldots(i)$$

If u and u are distances of the ant and its image from the optical centre of the lens, then $\frac{du}{dt}$ and $\frac{dv}{dt}$ represent speed of the ant and its image respectively.

Now, for a convex lens, f is positive and u is negative.

Therefore, the factor $\frac{f^2}{(u + f)^2} > 1$. Then, from equation (*i*), it follows that $\frac{dv}{dt} > \frac{du}{dt}$ *i.e.* the image of the ant moves with a speed greater than that of the ant.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain the phenomenon of refraction at a plane surface separating two transparent media and show that $\mu = \frac{c}{v}$, where letters have their usual meanings.
2. Define refractive index of a material. Distinguish between the terms absolute and relative refractive index of a medium. How are they related ? Give the expression for refractive index in terms of velocities of light.

3. Prove the relation : ${}^a\mu_b = \frac{1}{{}^b\mu_a}$
4. What is meant by lateral shift ? Explain. For a given slab, what is the quantity that determines lateral shift ? Obtain the expression for lateral shift for refraction through a parallel sided glass slab.
5. Discuss refraction through a glass-slab and show that emergent ray is parallel to the incident ray but displaced. *(H.P.S.S.C.E. 1996)*
6. State generalized Snell's law for refraction through multiple parallel media. Prove the relation :
$${}^a\mu_c = {}^a\mu_b \times {}^b\mu_c$$
7. Deduce the relation : $\mu = \dfrac{\text{real depth}}{\text{apparent depth}}$
8. What is normal shift ? Explain. Obtain the expression for normal shift. Indicate, how this helps to determine refractive index of a medium.
9. With the help of a ray diagram explain the phenomenon of total internal reflection. Obtain the relation between critical angle and the refractive index of the medium. *(C.B.S.E. 2001 S)*
10. Define total internal reflection and critical angle. Find the relation between critical angle and refractive index. *(P.S.S.C.E. 1993 ; Karnataka, 1989 S)*
11. What do you understand by the phenomenon of total internal reflection ? Give two conditions for it to take place. Derive the relation connecting refractive index and critical angle for a given pair of media. *(C.B.S.E. 2001)*
12. Explain the phenomenon of total internal reflection. What are the conditions of the phenomenon ? Explain the meaning of the critical angle. *(C.B.S.E. 1993)*
13. State the conditions for total internal reflection of light to take place at an interface separating two transparent media. Hence derive the expression for the critical angle in terms of the speeds of light in the two media. *(C.B.S.E. 2000)*
14. What is total internal reflection ? Define critical angle and give its relation with refractive index. Give one application of total internal reflection. *(H.S.S.C.E. 1997 S)*
15. What are optical fibres ? How are light waves propagated in them ? *(C.B.S.E. 1992 S)*
16. How do optical fibres transmit light without significant absorption ? Mention one practical application of optical fibres. *(C.B.S.E. 1991)*
17. Explain the following : (*i*) Brilliance of diamond (*ii*) Optical fibre.
18. What is total internal reflection? How is it applied to optical fibres ? *(H.P.S.S.C.E. 2001)*
19. What are optical fibres? Explain with the help of a diagram. Write their any two uses. *(P.S.S.C.E. 1999 S)*
20. Derive a relation between the focal length of a convex lens and its radii of curvature. *(H.S.S.C.E. 1998)*
21. Derive a relation between the focal length of a lens and its radii of curvature. *(P.S.S.C.E. 1991)*
22. Derive lens formula from curved surface formulae. *(H.S.S.C.E. 1993)*
23. What is lens formula ? Derive it for a convex lens. *(P.S.S.C.E. 1995, 1992, H.S.S.C.E. 1993)*
24. Draw a ray diagram to show how the image is formed, when the object is placed between f and $2f$ distances from a convex lens. Deduce the relation between the distances of the object and the image from the lens and the focal length of the lens under this condition. *(C.B.S.E. 1998)*
25. Draw a ray diagram to show the formation of the image of an object placed between the optical centre and focus of a convex lens. Deduce the relationship between the object distance, image distance and focal length under the conditions stated. *(C.B.S.E. 1998)*
26. Draw a ray diagram to show the formation of image of an object placed between f and $2f$ distances from a thin concave lens. Deduce the relationship between the object distance, image distance and focal length under this condition. *(C.B.S.E. 1998)*
27. Define linear magnification produced by a lens. Hence derive expression for it. Also define the power of a lens and its unit. *(H.P.S.S.C.E. 1998 S)*
28. Define linear magnification produced by a lens. Derive expression for it in terms of (*i*) object and image distance, (*ii*) object distance and focal length and (*iii*) image distance and focal length.
29. Explain the method of obtaining the image of an extended object due to refraction through a thin convex lens. Draw a diagram to indicate the formation of a real diminished image.
30. Give the position and nature of image of an extended object for different distances from a convex lens. How do you explain this from lens formula ?
31. What is power of a lens ? Give and define unit of the power of a lens. How is power of a lens related to the power of its refracting surface ?
32. What is an equivalent lens ? Obtain an expression for the effective focal length of two thin convex lenses placed in contact coaxially with each other. *(Karnataka, 1989)*
33. What is an equivalent lens ? Obtain an expression for the focal length of a combination of two thin lenses placed in contact.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What is the total internal reflection ? What are the conditions under which this takes place ? Discuss two important applications of this phenomenon. *(H.S.S.C.E. 2001)*
2. Explaining the 'critical angle of incidence', establish relationship between the critical angle of incidence and refractive index of the denser medium, considering the rarer medium to be air of refractive index unity. How can a right angled prism be used to invert the path of a beam of light ? *(C.B.S.E. Sample Question Paper)*
3. (*a*) How do you explain the 'mirage' effect produced in very hot deserts ?
(*b*) Describe with ray diagrams, the applications of total reflection prisms. *(H.P.S.S.C.E. 1993)*
4. Discuss refraction from a denser to rarer medium at a convex spherical refractive surface. *(H.S.S.C.E. 2001)*
5. Derive the expression :
$$-\frac{\mu_1}{\mu} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R},$$
when refraction occurs from rarer to denser medium at convex spherical refracting surface ($\mu_1 < \mu_1$). *(H.S.S.C.E. 2001 ; H.P.S.S.C.E. 2000 ; P.S.S.C.E. 1996 ; Karnataka, 1992)*
6. Prove that when refraction occurs from rarer to denser medium at a convex spherical refracting surface,
$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$
(H.P.S.S.C.E. 1997 ; P.S.S.C.E. 1995 S)

7. Derive the equation relating u, v and R in case of refraction of refractive index μ_1 from a medium of refractive index μ_2. State the assumption made in your derivation. *(I.S.C.E. 1993)*
8. Establish a relation for refraction at a single spherical surface, when the ray travels from rarer to denser medium and real image is formed. *(H.S.S.C.E. 1997)*
9. By stating the sign-conventions and assumptions used, derive the relation between distance of object, distance of image and radius of curvature of a convex spherical surface, when refraction takes place from optically denser to optically rarer medium. *(H.S.S.C.E. 1998 S, P.S.S.C.E. 1993)*
10. Derive Lens Maker's formula for convex surface, when ray travels from a rarer to denser medium. *(H.S.S.C.E. 2001)*
11. By stating the sign-conventions and assumptions used, derive the relation between distance of object, distance of image and radius of curvature of convex spherical surface, when refraction takes place from optically rarer to optically denser medium and the image formed is virtual. *(P.S.S.C.E. 1999 S)*
12. Stating the new Cartesian sign conventions, derive the relation between u, v and R, when refraction takes place from a concave spherical refracting surface. *(P.S.S.C.E. 1999 S)*
13. By stating the sign conventions and assumptions used, derive the relation between distance of object, distance of image and radius of curvature of a convex spherical surface, when refraction takes from optically denser to optically rarer medium, when image formed is real. *(P.S.S.C.E. 1999 S ; H.S.S.C.E. 1998 S)*
14. Derive the lens-maker's formula in case of a double convex lens. State the assumptions made and convention of signs used. *(C.B.S.E. 2001 S)*
15. Derive an expression for lens maker's formula. *(P.S.S.C.E. 1998 S)*
16. Derive lens maker's formula for a thin convex lens. *(P.S.S.C.E. 1998 S, 1998)*
17. Derive a relation between focal length of a double convex lens and its radii of curvatures. *(P.S.S.C.E. 2002, 2000)*
18. Derive expression for lens maker's formula *i.e.*

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right),$$

where the letters have their usual meanings. *(H.S.S.C.E. 1996, 1995, C.B.S.E. 1996, P.S.S.C.E. 1991)*

19. Derive lens maker's formula for a thin biconvex lens. *(C.B.S.E. 1996)*
20. Derive lens maker's formula. *(H.S.S.C.E. 1995)*
21. Define power and magnification of a lens. Assuming formula for refraction at a spherical surface, derive lens maker's formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$ *(Karnataka, 1991 S)*

22. Draw a ray diagram to show the formation of image of an object placed between the optical centre and focus of a thin convex lens. Write two characteristics of the image formed. Using this diagram, derive the relation between object distance u, image distance v and focal length f of the convex lens. Draw the graph showing the variation of v with u. *(C.B.S.E. 1999 S)*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Laws of Refraction

1. The refractive index for water is 4/3, for glass it is 3/2. A ray of light travelling in water is incident at an angle of 30° upon a water-glass interface. Calculate the angle of refraction into glass. **[Ans.** 26·4°]

Type B. On Real and Apparent depths

2. Calculate the index of refraction of a liquid from the following into glass :
   Reading for the bottom of an empty beaker : 11·324 cm
   Reading for the bottom of the beaker, when partially filled with the liquid : 11·802 cm
   Reading for the upper level of the liquid in the beaker : 12·895 cm **[Ans.** 1·437]
3. While determining the refractive index of a liquid experimentally, the microscope was focussed at the bottom of a beaker, when its reading was 3·965 cm. On pouring liquid upto a height of 2·537 cm inside the beaker, the reading of the refocussed microscope was 3·348 cm. Find the refractive index of the liquid. **[Ans.** 1·321]
4. The bottom of a container is a 4·0 cm thick glass (μ = 1·5) slab. The container contains two immiscible liquids A and B of depths 6·0 and 8·0 cm. respectively. What is the apparent position of a scratch on the outer surface on the bottom of the glass slab, when viewed through the container ? Refractive indices of A and B are 1·4 and 1·3 respectively. *(Text Problem)* **[Ans.** 4·89 cm (displaced upwards)]

Type C. On Total Internal reflection

5. Find the critical angle for a ray of light going from paraffin oil to air. Given that the refractive index of paraffin oil with respect to air is 1·44. *(H.S.S.C.E. 2002)* **[Ans.** 43·98°]
6. What is the critical angle for a ray going from glass into water ? The refractive indices of glass and water are 1·62 and 1·32 respectively. **[Ans.** 54·6°]
7. Velocity of light in a liquid is $1{\cdot}5\times10^8$ m s^{-1} and in air, it is 3×10^8 m s^{-1}. If a ray of light passes from this liquid to air, calculate the value of critical angle. *(P.S.S.C.E. 1999)* **[Ans.** 30°]
8. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out ? Refractive index of water is 1·33. Consider the bulb to be a point source. *(Text Problem)* **[Ans.** 26149·3 cm^2]

Type D. On Optical Fibre

9. Monochromatic light is incident on the plane interface AB between two media of refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) at an angle of incidence θ (just greater than the critical angle for the two media *i.e.* $\theta \geq$ C) so that total internal reflection takes place. Now, a transparent slab DEFG of uniform thickness and of refractive index μ_3 ($\mu_3 < \mu_2$) is introduced on the interface as shown in Fig. 1.71. Show that for any value of μ_3, all light will ultimately be reflected back into medium of refractive index μ_2.

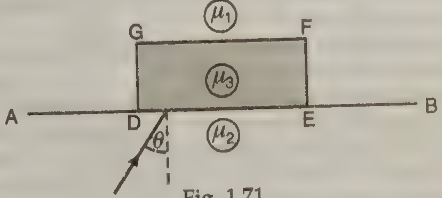


Fig. 1.71

Type E. On Refraction from Spherical refracting surfaces

10. A convex refracting surface of radius of curvature 20 cm separates two media of refractive indices 4/3 and 1·60. An object is placed in the first medium ($\mu = 4/3$) at a distance of 200 cm from the refracting surface. Calculate the position of the image formed. (*P.S.S.C.E. 2001*)
[**Ans.** at 240 cm in rarer medium]

11. The radius of curvature of convex surface is 10 cm and if an object lies at a distance of 20 cm from it in the rarer medium, find the position of the image assuming that refractive index of the rarer medium is 1·0, while that of the denser medium is 2·0.
[**Ans.** at 40 cm (in denser medium)]

12. One end of a cylindrical rod is grounded to a hemispherical surface of radius R = 20 mm. It is immersed in water of refractive index 1·33. If the refractive index of the rod is 1·50, find the position of the image of an object placed on the axis of the rod inside water at 10 cm from the pole.
[**Ans.** at 31·25 cm (inside water)]

13. A small object is enclosed in a sphere of glass 7 cm in radius. It is situated 1 cm from the centre and is viewed from the side to which it is nearest. Where will it appear to be, if refractive index is 1·4 ? Also find the apparent position of the object as seen by an eye looking along diameter through greatest thickness of the glass.
[**Ans.** 5·68 cm, 8·485 cm]

14. A small air bubble is entrapped in a sphere with diameter 6 cm and refractive index 1·5. When looked along the diameter, the bubble appears to be 1 cm inside the sphere. Find true location of the bubble. [**Ans.** 1·286 cm]

15. A point object is placed in air at distance of 40 cm from a concave refracting surface of refractive index 1·5. If radius of curvature of the spherical surface is 20 cm, calculate the position of the image. [**Ans.** at 30 cm (in air)]

16. A concave spherical surface of refractive index 3/2 is immersed in water of refractive index 4/3. If a point object lies in water at a distance of 10 cm from the pole of the refracting surface, calculate the position of the image. Given that radius of curvature of the spherical surface is 18 cm.
[**Ans.** at 10·52 cm (in water)]

17. A small object is 2 cm below the concave meniscus of water in a test tube. The radius of the meniscus is 5 mm and μ for water is 4/3. Find the nature and position of the image. [**Ans.** virtual image, at 0·75 cm (inside water)]

Type F. On Lens Maker's formula

18. The radii of curvature of a double convex lens are 15 cm and 30 cm and its refractive index is 1·5. Calculate its focal length. (*H.S.S.C.E. 1999*) [**Ans.** 20 cm]

19. The radii of curvature of a double convex lens are 30 cm and 60 cm and its refractive index is 1·5. Calculate its focal length. (*C.B.S.E. 1992*) [**Ans.** 40 cm]

20. The radii of curvature of the two faces of a convex lens are 0·1 m and 0·15 m respectively. If the focal length of the lens is 0·12 m, find the refractive index of the material of the lens. (*Karnataka, 1983*) [**Ans.** 1·5]

21. A thin double convex lens has surfaces of radii of curvature 30 cm each. Find the refractive index of the material of lens, if the focal length is 25 cm. [**Ans.** 1·6]

22. A plano-convex lens ($\mu = 1·5$) has a curved surface of radius 15 cm. What is its focal length ? [**Ans.** 30 cm]

23. The radius of curvature of the convex face of a plano-convex lens is 12 cm and the refractive index of its material is 1·5. Find the focal length of the lens. [**Ans.** 24 cm]

24. The radius of curvature of each face of a biconcave lens made of glass of refractive index 1·5 is 30 cm. Calculate the focal length of the lens in air. (*C.B.S.E. 1996*)
[**Ans.** – 30 cm]

25. The radii of curvature of the two surfaces of a convexo-concave lens are 15 cm and 30 cm. Find the focal length of the lens, if the refractive index of its material is 1·6.
[**Ans.** 50 cm]

26. The focal length of a glass convex lens in air is 15 cm. Calculate its focal length, when it is totally immersed in water. Given ${}^a\mu_w = 4/3$ and ${}^a\mu_g = 1·5$. (*P.S.S.C.E. 2001*)
[**Ans.** 60 cm]

27. The focal length of a concavo-convex lens of radii of curvature 5 cm and 10 cm is 20 cm. What will be its focal length in water ($\mu = 4/3$) ? [**Ans.** – 80 cm]

28. If the refractive index from air to glass is 3/2 and that from air to water is 4/3, find the ratio of the focal length of glass lens in water and in air. [**Ans.** 4]

29. A convex lens of refractive index 1·5 is immersed in a liquid of refractive index : (*i*) 1·6 ; (*ii*) 1·3 ; (*iii*) 1·5. What changes happen to the focal length of the lens in the three cases ? (*Pre-degree Kerala, 1992*)
[**Ans.** (*i*) $-8\,f_{air}$; (*ii*) $+3·25\,f_{air}$; (*iii*) ∞]

Type G. On Lens equation

30. An object is placed at 0·06 m from a convex lens of focal length 0·10 m. Calculate the position and nature of the image. (*H.S.S.C.E. 2002*) [**Ans.** At 15 cm, virtual]

31. An object is kept 0·2 m from a convex lens of focal length 0·15 m. Find the position of the image. (*C.B.S.E. 1993*)
[**Ans.** 0·6 m]

32. A converging beam of light forms a sharp image on a screen. A lens is placed in the path of the beam at 10 cm from the screen. It is found that screen has to be moved 8 cm further away from the lens to obtain a sharp image. Find the focal length and nature of the lens.
[**Ans.** – 22·5 cm (concave lens)]

33. An object is placed at a distance of 75 cm from a screen. Where should a convex lens of focal length 12 cm be placed so as to obtain real image of the object on the screen ?
[**Ans.** 15 cm or 60 cm]

34. The image of a needle placed 45 cm from a lens is formed on a screen placed 90 cm on the other side of the lens. Find the displacement of the image, if the object is moved by 5·0 cm away from the lens. (*P.S.S.C.E. 2000*)
[**Ans.** 15 cm (towards the lens)]

35. If the distance between the screen and object is 50 cm and the displacement of the lens is 10 cm, calculate the focal length of the lens. [**Ans.** 12 cm]

Type H. On Linear magnification

36. An object is placed at a distance 5 cm from a convex lens of focal length 10 cm. Calculate the position and magnification of the image. (*C.B.S.E. 1992*) [**Ans.** – 10 cm, 2]

37. Where should an object placed from a converging lens of focal length 20 cm, so as to obtain a real image of magnification 2 ? (*C.B.S.E. 2001*) [**Ans.** at 30 cm]

38. At what distance should an object be placed from a convex lens of a focal length 10 cm, so as to obtain an image twice the size of the object ?
[**Ans.** at 15 cm (for real image), at 5 cm (for virtual image)]

39. At what distance should an object be placed from a convex lens of focal length 15 cm to obtain an image three times the size of the object ?
[**Ans.** at 20 cm (for real image), at 10 cm (for virtual image)]

40. A lens placed at a distance of 20 cm from an object produces a virtual image 2/3 the size of the object. Find the position of image, kind of lens and its focal length.
[**Ans.** at 13·3 cm on the same side as object,
– 40 cm (concave lens)]

41. The image obtained with a convex lens is erect and its length is three times the length of the object. If the focal length of the lens is 15 cm, calculate the object and image distance. (*P.S.S.C.E. 1991*) [**Ans.** – 10 cm, – 30 cm]

42. An object is placed at a distance of 150 cm from a screen and a convex lens placed in between produces an image magnified 4 times on the screen. Calculate the focal length and the position of the lens.
[**Ans.** 24 cm, at 120 cm from screen and 30 cm from the object]

43. A lens of focal length 12 cm produces a virtual image, whose linear dimensions are 1/3 that of the object. What kind of a lens is it ? Determine the position of the object and the image. [**Ans.** diverging, 24 cm, 8 cm]

Type I. On Power of lens

44. A lens has a power of + 5 D in air. If completely immersed in water, what will be its power ? Given, refractive index of glass = 3/2, refractive index of water = 4/3.
[**Ans.** 1·25 D]

45. Find the ratio of the powers of a glass lens in water and in air. Given, refractive index of glass = 1·50, refractive index of water = 1·33. [**Ans.** 0·256]

46. A glass convex lens has a power of + 10 D. When this lens is totally immersed in a liquid, it acts as concave lens of focal length 50 cm. Calculate the refractive index of the liquid. Given ${}^a\mu_g = 1{\cdot}5$. (*P.S.S.C.E. 2001*) [**Ans.** 1·67]

Type J. On Equivalent lens

47. What is the focal length of the combination of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm ? Is the system a converging or a diverging lens ? Ignore thickness of the lenses.
[**Ans.** – 60 cm (diverging)]

48. A concave lens is placed in contact with a convex lens of focal length 25 cm. The combination produces a real image at a distance of 80 cm, when the object is at a distance of 40 cm. What is the focal length of the concave lens ?
(*Pre-degree Kerala, 1992*) [**Ans.** – 400 cm]

49. Two thin lenses of focal lengths + 10 cm and – 5 cm are kept in contact. What is the (*i*) focal length and (*ii*) power of combination ? (*C.B.S.E. 1993*) [**Ans.** – 10 cm, – 10 D]

50. Two lenses of focal lengths 10 cm and – 20 cm are placed in contact. What is the total power of the combination ?
(*P.S.S.C.E. 1999*) [**Ans.** 5 D]

51. A convex lens of focal length 30 cm and a concave lens of focal length 60 cm are placed in contact. Find the power of the combination. (*Karnataka, 1985*) [**Ans.** 1·67 D]

52. Two lenses of powers 15·5 D and – 5·5 D are placed in contact. What is the focal length of the combination.
(*P.S.S.C.E. 1999*) [**Ans.** 0·1 m]

53. Two thin lenses of power + 5 D and – 3 D are in contact. What is the focal length of the combination ?
(*C.B.S.E. 2001 S*) [**Ans.** 0·5 m]

54. A concave lens is kept in contact with a convex lens of focal length 20 cm. The combination works as a convex lens of focal length 50 cm. Find the power of the concave lens.
(*P.S.S.C.E. 2002*) [**Ans.** – 3 D]

55. Two lenses of powers + 15 D and – 5 D are in contact with each other forming a combination lens. (*a*) what is the focal length of this combination ? (*b*) An object of size 3 cm is placed at 30 cm from this combination of lenses. Calculate the position and size of the image formed. (*C.B.S.E. 2002*)
[**Ans.** (*a*) 10 cm (*b*) 15 cm, – 1·5 cm (inverted)]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Here, ${}^a\mu_w = \dfrac{4}{3}$ and ${}^a\mu_g = \dfrac{3}{2}$

$$\therefore \quad {}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

$$\text{Now, } \sin r = \frac{\sin i}{{}^w\mu_g} = \frac{\sin 30°}{9/8} = 0{\cdot}444 \quad \text{or} \quad r = 26{\cdot}4°$$

2. Real depth = 12·895 – 11·324 = 1·571 cm
Apparent depth = 12·895 – 11·802 = 1·093 cm

$$\text{Now, } \mu = \frac{\text{real depth}}{\text{apparent depth}} = \frac{1{\cdot}571}{1{\cdot}093} = \mathbf{1{\cdot}437}$$

3. Proceed as in problem no. 2.

4. Total normal shift, $d = d_1 + d_2 + d_3$

$$\text{or} \quad d = t_1\left(1 - \frac{1}{{}^a\mu_g}\right) + t_2\left(1 - \frac{1}{{}^a\mu_A}\right) + t_3\left(1 - \frac{1}{{}^a\mu_B}\right)$$

$$= 4{\cdot}0\left(1 - \frac{1}{1{\cdot}5}\right) + 6{\cdot}0\left(1 - \frac{1}{1{\cdot}4}\right) + 8{\cdot}0\left(1 - \frac{1}{1{\cdot}3}\right)$$

$= 1{\cdot}33 + 1{\cdot}71 + 1{\cdot}85$

= 4·89 cm (displaced upwards)

5. $\sin C = \dfrac{1}{\mu} = \dfrac{1}{1{\cdot}44} = 0{\cdot}6944$

$\therefore$ $C = \mathbf{43{\cdot}98°}$

6. Proceed as in solved problem no. 1.10.

7. $\mu = \dfrac{3 \times 10^8}{1{\cdot}5 \times 10^8} = 2$

Now, proceed as problem no. 5.

8. Proceed as in solved problem no. 1.12.

9. Let C be critical angle, when light is refracted at the interface AB between two media of refractive indices μ_1 and μ_2. Then,

$$\sin \theta \geq \sin C = \frac{\mu_1}{\mu_2}$$

On introducing slab DEFG :

Case (i) When $\mu_3 < \mu_1$:

$$\text{Then, } \frac{\mu_3}{\mu_2} < \frac{\mu_1}{\mu_2}$$

Since $\dfrac{\mu_3}{\mu_2} < \dfrac{\mu_1}{\mu_2}$, it follows that when light travels from medium of refractive index μ_2 to μ_3, the value of critical angle will be less than C. Therefore, if light is incident at angle θ ($\geq$C), it will go back into medium μ_2 after suffering total internal reflection.

Case (ii) When $\mu_3 > \mu_1$:
Since $\mu_3 > \mu_1$, $\theta <$ C and hence the ray of light will be refracted into medium μ_3 as shown in Fig. 1.72.

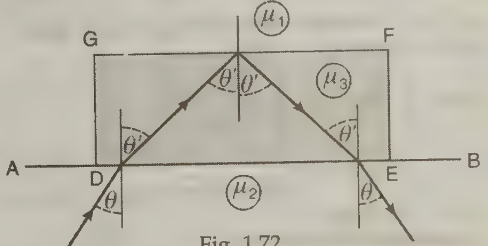


Fig. 1.72

The angle of refraction θ' is given by

$$\frac{\sin \theta}{\sin \theta'} = \frac{\mu_3}{\mu_2} \quad \text{or} \quad \sin \theta' = \frac{\mu_2}{\mu_3} \sin \theta$$

$$\text{or} \quad \sin \theta' \geq \frac{\mu_2}{\mu_3} \cdot \frac{\mu_1}{\mu_2} \quad \text{or} \quad \sin \theta' \geq \frac{\mu_1}{\mu_3}$$

Thus, angle θ' is greater than critical angle for pair of media μ_2 and μ_3 and therefore total internal reflection will take place at face FG. After undergoing refraction at face DE, the ray of light will return back into the medium μ_2.

10. $$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$$

$R = 20$ cm, $\mu_1 = 4/3$, $\mu_2 = 1{\cdot}60$, $u = -200$ cm

$$\therefore\ -\frac{4/3}{-200}+\frac{1{\cdot}60}{v}=\frac{1{\cdot}60-4/3}{20}$$

or **$v = 240$ cm (in denser medium)**

11. Proceed as in problem no. 10.

12. $$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$$

$R = 20$ mm $= 2$ cm, $\mu_1 = 1{\cdot}33$, $\mu_2 = 1{\cdot}5$ and $u = -10$ cm

$$\therefore\ -\frac{1{\cdot}33}{-10}+\frac{1{\cdot}5}{v}=\frac{1{\cdot}5-1{\cdot}33}{2}$$

or **$v = -31{\cdot}25$ cm (inside water)**

13. ***When object is viewed from the side to which it is nearest*** :

$u = -(7-1) = -6$ cm, $R = -7$ cm, $\mu = 1{\cdot}4$

$$\text{Now, } -\frac{\mu}{u}+\frac{1}{v}=\frac{1-\mu}{R} \quad\text{or}\quad \frac{-1{\cdot}4}{-6}+\frac{1}{v}=\frac{1-1{\cdot}4}{-7}$$

$$\text{or}\quad \frac{1}{v}=\frac{0{\cdot}4}{7}-\frac{1{\cdot}4}{6} \quad\text{or}\quad v=-5{\cdot}68\text{ cm}$$

When object is viewed through the greatest thickness of glass :

$v = -(7+1) = -8$ cm, $R = -7$ cm, $\mu = 1{\cdot}4$ cm

It will be obtained that

$v = -8{\cdot}485$ cm

14. $v = -1$ cm, $R = -6/2 = -3$ cm, $\mu = 1{\cdot}5$

$$-\frac{\mu}{u}+\frac{1}{v}=\frac{1-\mu}{R} \quad\text{or}\quad \frac{-1{\cdot}5}{u}+\frac{1}{-1}=\frac{1-1{\cdot}5}{-3}$$

$$\text{or}\quad \frac{1{\cdot}5}{u}=-1-\frac{0{\cdot}5}{3} \quad\text{or}\quad u=-1{\cdot}286\text{ cm}$$

15. $v = -40$ cm, $R = -20$ cm, $\mu = 1{\cdot}5$

$$-\frac{1}{u}+\frac{\mu}{v}=\frac{\mu-1}{R} \quad\text{or}\quad -\frac{1}{-40}+\frac{1{\cdot}5}{v}=\frac{1{\cdot}5-1}{-20}$$

or **$u = -30$ cm (in air)**

16. $u = -10$ cm, $R = -18$ cm, $\mu_1 = 4/3$ and $\mu_2 = 3/2$

$$\text{Now,}\quad -\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$$

$$\text{or}\quad -\frac{4/3}{-10}+\frac{3/2}{v}=\frac{3/2-4/3}{-18}$$

or **$v = -10{\cdot}52$ cm (in water)**

17. $u = -2$ cm, $R = 5$ mm $= 0{\cdot}5$ cm, $\mu = 4/3$

$$\text{Now,}\quad -\frac{\mu}{u}+\frac{1}{v}=\frac{1-\mu}{R} \quad\text{or}\quad \frac{-4/3}{-2}+\frac{1}{v}=\frac{1-4/3}{0{\cdot}5}$$

or **$v = -0{\cdot}75$ cm (virtual image inside water)**

18. $R_1 = +15$ cm, $R_2 = -30$ cm, $\mu = 1{\cdot}5$

$$\text{Now,}\quad \frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1{\cdot}5-1)\left(\frac{1}{15}-\frac{1}{-30}\right)$$

or **$f = 20$ cm**

19. Proceed as in problem no. 18.

20. Proceed as in problem no. 18.

21. Proceed as in problem no. 18.

22. $\mu = 1{\cdot}5$; $R_1 = 15$ cm, $R_2 = -\infty$

$$\text{Now,}\quad \frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1{\cdot}5-1)\left(\frac{1}{15}-\frac{1}{-\infty}\right)$$

or **$f = 30$ cm**

23. Proceed as in problem no. 22.

24. $R_1 = -30$ cm ; $R_2 = 30$ cm ; $\mu = 1{\cdot}5$

$$\text{Now,}\quad \frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1{\cdot}5-1)\left(\frac{1}{-30}-\frac{1}{30}\right)$$

or **$f = -30$ cm**

25. $R_1 = 15$ cm, $R_2 = 30$ cm, $\mu = 1{\cdot}6$

$$\text{Now,}\quad \frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1{\cdot}6-1)\left(\frac{1}{15}-\frac{1}{30}\right)$$

or **$f = 50$ cm**

26. Proceed as in solved problem no. 1.26.

27. $R_1 = -5$ cm, $R_2 = -10$ cm, $f_{air} = -20$ cm

$$\text{Now,}\quad \frac{1}{f_{air}}=\left({}^a\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\text{or}\quad -\frac{1}{20}=\left({}^a\mu_g-1\right)\left(\frac{1}{-5}-\frac{1}{-10}\right)$$

or ${}^a\mu_g = 1{\cdot}5$

When lens is immersed in water :

$$\frac{1}{f_w}=\left(\frac{{}^a\mu_g}{{}^a\mu_w}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=\left(\frac{1{\cdot}5}{4/3}-1\right)\left(\frac{1}{-5}-\frac{1}{-10}\right)$$

or **$f_w = -80$ cm**

28. Proceed as in solved problem no. 1.26 and then find the ratio f_{air}/f_w.

29. $$\frac{1}{f_{air}}=\left({}^a\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=(1{\cdot}5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\text{or}\quad \left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{2}{f_{air}}$$

(i) Here, ${}^a\mu_g = 1{\cdot}6$

$$\frac{1}{f_l}=\left(\frac{{}^a\mu_g}{{}^a\mu_l}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{1{\cdot}5}{1{\cdot}6}-1\right)\times\frac{2}{f_{air}}$$

or **$f_l = -8\,f_{air}$**

(ii) Here, ${}^a\mu_l = 1{\cdot}3$

$$\frac{1}{f_l}=\left(\frac{{}^a\mu_g}{{}^a\mu_l}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{1{\cdot}5}{1{\cdot}3}-1\right)\times\frac{2}{f_{air}}$$

or **$f_l = +3{\cdot}25\,f_{air}$**

(iii) Here, ${}^a\mu_l = 1{\cdot}5$

$$\frac{1}{f_l}=\left(\frac{{}^a\mu_g}{{}^a\mu_l}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{1{\cdot}5}{1{\cdot}5}-1\right)\times\frac{2}{f_{air}}$$

or $f_l = \infty$

30. $u = -0{\cdot}06$ m $= -6$ cm, $f = 0{\cdot}10$ m $= 10$ cm

$$\text{Now,}\quad -\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad\text{or}\quad -\frac{1}{-6}+\frac{1}{v}=\frac{1}{10}$$

or **$v = -15$ cm (virtual image)**

31. Proceed as in problem no. 31.

32. $u = +10$ cm, $v = +(10 + 8) = +18$ cm

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{10} + \frac{1}{18} = \frac{1}{f}$

or $\quad f = -22{\cdot}5$ **cm (concave lens)**

33. Here, $f = 12$ cm,
distance between object and image (screen) = 75 cm
If v is the distance of image from the lens, then

$$u = -(75 - v)$$

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{-(75-v)} + \frac{1}{v} = \frac{1}{12}$

or $v^2 - 75\,v + 900 = 0$

$$\text{or } v = \frac{75 \pm \sqrt{(75)^2 - 4 \times 1 \times 900}}{2}$$

$$= \frac{75 \pm 45}{2} = 60 \text{ cm or } 15 \text{ cm}$$

Therefore, distance of object from the lens,
$|u| = 75 - 60$ or $75 - 15$ *i.e.* **15 cm** or **60 cm**

34. Here, $u = -45$ cm, $\quad v = 90$ cm

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{-45} + \frac{1}{90} = \frac{1}{f}$

When object is moved away from the lens by 5 cm :

$u' = -(45 + 5) = -50$ cm

Now, $-\frac{1}{u'} + \frac{1}{v'} = \frac{1}{f}$ or $-\frac{1}{-50} + \frac{1}{v'} = \frac{1}{30}$

or $\quad v' = 75$ cm

Therefore, distance through which image moves,

$v - v' = 90 - 75 =$ **15 cm (towards the lens)**

35. Distance between object and image (screen) = 50 cm

In first position of object : If v is the distance of image from the lens, then

$$u = -(50 - v)$$

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{-(50-v)} + \frac{1}{v} = \frac{1}{f}$

$$\text{or} \quad \frac{1}{50-v} + \frac{1}{v} = \frac{1}{f} \qquad \ldots(i)$$

In second position of object : When lens is moved towards the screen through 10 cm, distance of image from the lens becomes $v' = (v - 10)$. Likewise, distance of object from the lens becomes

$$u' = -\{50 - (v - 10)\} = -(60 - v)$$

Now, $-\frac{1}{u'} + \frac{1}{v'} = \frac{1}{f}$ or $-\frac{1}{-(60-v)} + \frac{1}{v-10} = \frac{1}{f}$

$$\text{or} \quad \frac{1}{60-v} + \frac{1}{v-10} = \frac{1}{f} \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$\frac{1}{50-v} + \frac{1}{v} = \frac{1}{60-v} + \frac{1}{v-10}$ or $v = 30$ cm

Substituting for v in equation (*i*), we get

$$f = \mathbf{12\ cm}$$

36. Here, $u = -5$ cm, $f = 10$ cm

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{-5} + \frac{1}{v} = \frac{1}{10}$

or $\quad v = -10$ **cm**

$$m = \frac{v}{u} = \frac{-10}{-5} = \mathbf{2}$$

37. Proceed as in solved problem no. 1.32.

38. ***When image formed is real :***

$m = -2, \quad f = 10$ cm

Now, $m = \frac{f}{u+f}$ or $-2 = \frac{10}{u+10}$ or $u =$ **– 15 cm**

When image formed is virtual :

$m = +2, \quad f = 10$ cm

Now, $m = \frac{f}{u+f}$ or $2 = \frac{10}{u+10}$ or $u =$ **– 5 cm**

39. Proceed as in problem no. 38.

40. Here, $u = -20$ cm, $\quad m = +\frac{2}{3}$ (virtual image)

Now, $m = \frac{f}{u+f}$ or $\frac{2}{3} = \frac{f}{-20+f}$

or $\quad f = -40$ **cm (concave lens)**

Also, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{-20} + \frac{1}{v} = \frac{1}{-40}$

or $\quad v = -13{\cdot}3$ **cm**

41. Here, $m = +3, f = 15$ cm

Now, $m = \frac{f}{u+f}$ or $3 = \frac{15}{u+15}$ or $u =$ **– 10 cm**

Also, $m = \frac{f-v}{f}$ or $3 = \frac{15-v}{15}$ or $v =$ **– 30 cm**

42. As the image is produced on the other side of the object, it must be real in nature.

$\therefore \quad m = -4$

Also, distance between object and image (screen) = 150 cm
If v is the distance of image from the lens, then

$$u = -(150 - v)$$

Now, $m = \frac{v}{u}$ or $-4 = \frac{v}{-(150-v)}$

or $\quad v = 120$ cm

$\therefore \quad u = = (150 - 120) = -30$ cm

i.e. lens should be placed at 30 cm from the object and at 120 cm from the screen.

Again, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{30} + \frac{1}{120} = \frac{1}{f}$

or $\quad f =$ **24 cm**

43. The virtual image produced by a convex lens is always magnified. Since $m = 1/3$ (diminished), the lens cannot be a convex lens. Hence, it is **diverging** in nature *i.e.*

$$f = -12 \text{ cm}$$

Now, $m = \frac{f}{u+f}$ or $\frac{1}{3} = \frac{-12}{u+(-12)}$

or $\quad u =$ **– 24 cm**

Also, $m = \frac{f-v}{f}$ or $\frac{1}{3} = \frac{-12-v}{-12}$ or $v =$ **– 8 cm**

44. Focal length of lens, $f = \frac{1}{P} = \frac{1}{5} = 0{\cdot}2$ m

Now, $\frac{1}{f} = \left({}^a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{1}{0{\cdot}2} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{1}{R_1} - \frac{1}{R_2} = 10$

When lens is immersed in water :

$$\frac{1}{f_{water}} = \left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3/2}{4/3} - 1\right) \times 10$$

or $\quad f_{water} = 0{\cdot}8$ m

Therefore, power of lens, when immersed in water,

$$f_{water} = \frac{1}{0{\cdot}8} = \mathbf{1{\cdot}25\ D}$$

45. $\frac{1}{f_{air}} = \left({}^a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= (1\cdot50 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $P_{air} = 0\cdot5\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(*i*)

Also, $\frac{1}{f_{water}} = \left(\frac{{}^a\mu_g}{{}^a\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{1\cdot50}{1\cdot33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $P_{water} = 0\cdot128\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$\therefore \quad \frac{P_{water}}{P_{air}} = \mathbf{0\cdot256}$

46. Here, P = + 10 D

$\therefore \quad f = \frac{1}{P} = \frac{1}{10} = 0\cdot1 \text{ m} = 10 \text{ cm}$

Now, $\frac{1}{f_{air}} = \left({}^a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{1}{10} = (1\cdot5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{5}$

When lens is immersed in liquid : $f_l = -50$ cm

Now, $\frac{1}{f_l} = \left(\frac{{}^a\mu_g}{{}^a\mu_l}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{1}{-50} = \left(\frac{1\cdot5}{{}^a\mu_g} - 1\right) \times \frac{1}{5}$

or $\mathbf{{}^a\mu_l = 1\cdot67}$

47. Here, $f_1 = 30$ cm, $f_2 = -20$ cm

Now, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{-20}$

or $\mathbf{f = -60 \text{ cm}}$ **(diverging)**

48. ***For the combination :***

$u = -40$ cm ; $v = 80$ cm

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{-40} + \frac{1}{80} = \frac{1}{f}$

or $f = 80/3$ cm

Also, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ or $\frac{3}{80} = \frac{1}{25} + \frac{1}{f_2}$

or $\mathbf{f_2 = -400 \text{ cm}}$

49. (*a*) $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} + \frac{1}{-5}$ or $\mathbf{f = -10 \text{ cm}}$

(*b*) $P = \frac{1}{f \text{ (in metre)}} = \frac{1}{-0\cdot1} = \mathbf{-10 \text{ D}}$

50. Proceed as in problem no. 49.

51. Proceed as in problem no. 49.

52. Power of combination,

$P = P_1 + P_2 = 15\cdot5 + (-5\cdot5) = 10 \text{ D}$

Therefore, focal length of combination,

$f = \frac{1}{P} = \frac{1}{10} = \mathbf{0\cdot1 \text{ m}}$

53. Proceed as in problem no. 52.

54. Here, $f = 50$ cm, $f_1 = 20$ cm

Now, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ or $\frac{1}{50} = \frac{1}{20} + \frac{1}{f_2}$

or $f_2 = -\frac{100}{3} \text{ cm} = -\frac{1}{3} \text{ m}$

Therefore, power of concave lens,

$P_2 = \frac{1}{f_2 \text{(in metre)}} = \frac{1}{-1/3} = \mathbf{-3 \text{ D}}$

55. (*a*) Power of combination,

$P = P_1 + P_2 = 15 + (-5) = 10 \text{ D}$

Therefore, focal length of combination,

$f = \frac{1}{P} = \frac{1}{10} = 0\cdot1 \text{ m} = \mathbf{10 \text{ cm}}$

(*b*) Here, $u = -30$ cm

Now, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ or $-\frac{1}{-30} + \frac{1}{v} = \frac{1}{10}$

or $\mathbf{v = 15 \text{ cm}}$

Also $m = \frac{f}{u + f} = \frac{10}{-30 + 10} = -\frac{1}{2}$

Therefore, size of image $= 3 \times \left(-\frac{1}{2}\right)$

$= \mathbf{-1\cdot5 \text{ cm}}$ **(inverted)**

CHAPTER

2

Dispersion

2.01. PRISM

A prism is the portion of transparent refracting medium bound by two plane surfaces ieeting each other along a straight edge.

The two plane surfaces are called *refracting faces* and the line along which the wo faces meet is called the *refracting edge* of the prism [Fig. 2.01]. The angle between he two refracting faces is called the *angle of the prism* and is usually denoted by A.

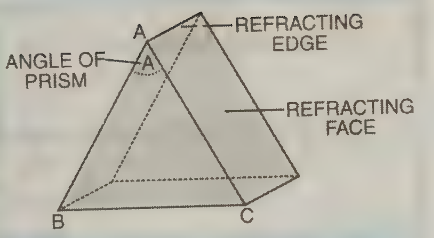


Fig. 2.01

2.02. REFRACTION THROUGH A PRISM

Consider that a ray of light PQ is incident on the refracting face AB of the prism t point Q as shown in Fig. 2.02. According to the laws of refraction, at point Q, the ay of light PQ is refracted along QR towards the normal NQ. The refracted ray QR gain suffers refraction at point R on the second refracting face AC and emerges out f the prism along RS away from the normal N′R drawn at the point R. The rays PQ, QR and RS are called ***incident ray, refracted ray*** and ***emergent ray*** respectively.

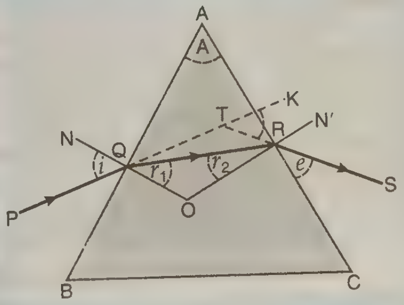


Fig. 2.02

Also, $\angle PQN = i$ (***angle of incidence***), $\angle SRN' = e$ (***angle of emergence***), $\angle RQO = r_1$ (angle of refraction at face AB) and $\angle QRO = r_2$ (angle of refraction at face AC). Produce SR backwards, so as to meet the ray PQ at point T, when produced. Then,

KTS = δ is called ***angle of deviation.***

Since $\angle TQO = i$ and $\angle RQO = r_1$, we have

$$\angle TQR = i - r_1$$

Also, $\angle TRO = e$ and $\angle QRO = r_2$. Therefore,

$$\angle TRQ = e - r_2$$

Now, in triangle TQR, the side QT has been produced outwards. Therefore, the exterior angle δ should be equal to the sum of the interior opposite angles *i.e.*

$$\delta = \angle TQR + \angle TRQ = (i - r_1) + (e - r_2)$$

or

$$\delta = (i + e) - (r_1 + r_2) \qquad \text{...(2.01)}$$

In triangle QRO, the sum of the angles is 180°. Therefore,

$$r_1 + r_2 + \angle QOR = 180° \qquad \text{...(2.02)}$$

In quadrilateral AQOR, each of the angles AQO and ARO is 90°. Since the sum of the four angles of a quadrilateral is four right angles, the sum of the remaining two angles should be 180° *i.e.*

$$A + \angle QOR = 180° \qquad \text{...(2.03)}$$

From the equations (2.02) and (2.03), we have

$$r_1 + r_2 = A \qquad \text{...(2.04)}$$

Substituting for $(r_1 + r_2)$ in equation (2.01), we have

$$\delta = (i + e) - A$$

or

$$A + \delta = i + e \qquad \text{...(2.05)}$$

Thus, when a ray passes through a prism, *the sum of the angle of prism and the angle of deviation is equal to the sum of the angle of incidence and the angle of emergence.*

To prove $\mu = \dfrac{\sin (A + \delta_m)/2}{\sin A/2}$. The deviation produced by a prism depends on :

(*i*) *angle of prism,* (*ii*) *material of prism* and (*iii*) *the angle of incidence.*

If the angle of incidence is increased gradually, the angle of deviation first decreases, attains a minimum value (δ_m) and then again starts increasing [Fig. 2.03]. When angle of deviation is minimum, the prism is said to be in ***minimum deviation position.*** A prism can deviate the incident ray through the same angle δ for two different values i_1 and i_2 of the angle of incidence. But when it is placed in minimum deviation position, there is only one angle of incidence corresponding to the angle

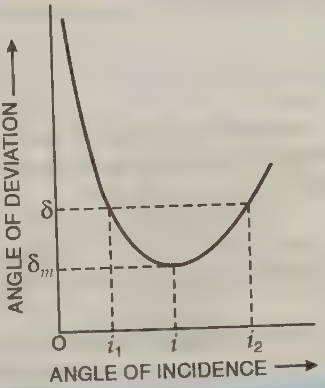


Fig. 2.03

of minimum deviation. In other words, *there is only one angle of incidence, for which the angle of deviation is minimum.*

It can be proved that when the prism is placed in minimum deviation position, the prism lies symmetrically *w.r.t.* incident ray and the emergent ray, *i.e.* the angle of incidence is equal to the angle of emergence. Further, in minimum deviation position, the angle of refraction at the first face and that at the second face are also equal and the refracted ray passes parallel to the base of prism.

Therefore, when $\delta = \delta_m$; (in minimum deviation position),

$$e = i \quad \text{and} \quad r_2 = r_1 = r \text{ (say)}$$

Setting $r_1 = r_2 = r$ in equation (2.04), we obtain

$$r + r = A$$

or $$r = A/2$$

Also, setting $\delta = \delta_m$ and $e = i$ in equation (2.05), we have

$$A + \delta_m = i + i$$

or $$i = (A + \delta_m)/2$$

The refractive index of the material (${}^a\mu_g$ or simply μ) of the prism is given by

$$\mu = \frac{\sin i}{\sin r}$$

$\therefore$ $$\mu = \frac{\sin (A + \delta_m)/2}{\sin A/2} \qquad \text{...(2.06)}$$

Key point

There are two values of the angle of incidence, for which a ray passing through a prism deviates through a given angle.

Key point

When a prism, made of material of refractive index μ is placed in a medium of refractive index μ' ($\mu' < \mu$), the prism still deviates the light towards its base. But the angle of deviation decreases.

2.03. DEVIATION THROUGH A PRISM OF SMALL ANGLE

The deviation produced by a prism depends upon the angle of prism. In case of a prism of small angle, the deviation produced is small.

Refer to Fig. 2.02. When a ray of light PQ is incident on the refracting face AB of the prism at the angle of incidence i, it gets refracted along QR making angle of refraction equal to r_1. If μ is the refractive index of the material of the prism, then from Snell's law

$$\mu = \frac{\sin i}{\sin r_1} \qquad \text{...(2.07)}$$

If the ray of light is incident at a small angle (≈ 10° or less) *i.e.* angle i is small, then r_1 will also be small. When the angles i and r_1 are small, $\sin i \approx i$ and $\sin r_1 \approx r_1$.

Therefore, from equation (2.07), we have

$$\mu = \frac{i}{r_1}$$

or $$i = \mu\, r_1$$

The refracted ray QR comes to be incident on the other refracting face AC of the prism at angle r_1 inside the prism and gets refracted out of the prism along RS, making angle of emergence equal to e.

Therefore, for refraction at face AC, we have

$$\mu = \frac{\sin e}{\sin r_2}$$

If the prism is of small angle, then the angles e and r_2 will also be small. For such a prism, the above equation becomes

$$\mu = \frac{e}{r_2}$$

or $$e = \mu\, r_2$$

Again, for a prism of small angle, angle of deviation δ for a ray of light is practically the same as the angle of minimum deviation. Therefore, for a prism of small angle,

$$A + \delta = i + e$$

or $$\delta = (i + e) - A$$

Substituting for i and e, we have

$$\delta = (\mu\, r_1 + \mu\, r_2) - A = \mu\,(r_1 + r_2) - A$$

When the prism is placed in minimum deviation position,

$$r_1 + r_2 = A$$

$\therefore$ $$\delta = \mu A - A$$

or $$\delta = A\,(\mu - 1) \qquad \text{...(2.08)}$$

The equation (2.08) gives the deviation produced by a prism of small angle, provided the ray of light is incident such that the angle of incidence is small. It may be pointed out that in such a case, *the deviation suffered by the ray is independent of the angle of incidence.*

2.04. DISPERSION

If a beam of white light is made to fall on one face of a prism, the light emerging from the other face of the prism consists of seven colours namely ***violet* (V)**, ***indigo* (I)**, ***blue* (B)**, ***green* (G)**, ***yellow* (Y)**, ***orange* (O)** and ***red* (R)** as shown in Fig. 2.04.

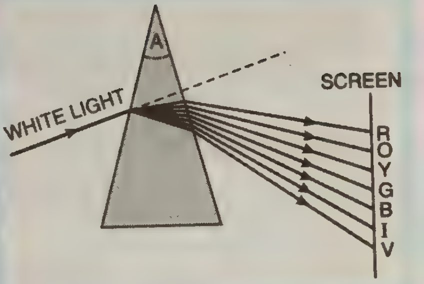


Fig. 2.04

The seven constituent colours of white light can be remembered by the word **VIBGYOR.** The deviation suffered by the violet colour is maximum, while that suffered by the red light is minimum.

This phenomenon of splitting up of white light into its constituent colours is called ***dispersion of light****.*

Further, *the band of seven colours produced on the screen is called* ***spectrum****.*

Cause of dispersion. The different colours of light are due to different wavelengths. The wavelength of violet light is smaller than that of red light.

Cauchy obtained expression for the refractive index of a material in terms of the wavelength of the light. It is given by

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}, \qquad \text{...(2.09)}$$

where a, b and c are constants for the material. Since wavelength of violet light is smaller than that of red light; from this formula, it follows that the refractive index of the material for violet light is more than that for red light *i.e.*

$$\mu_v > \mu_r$$

For a small angled prism, we have

$$\delta = \mathrm{A}\,(\mu - 1)$$

Since $\mu_v > \mu_r$, the violet light will suffer greater deviation than that suffered by the red light *i.e.*

$$\delta_v > \delta_r \qquad \text{...(2.10)}$$

The other colours will deviate through angles having values between δ_r and δ_v. Thus, white light consisting of different colours will emerge from the glass prism in different directions due to their different wavelengths, which is termed as dispersion.

Key point ✓

A prism disperses white light into constituent colours due to the fact that light of different wavelengths undergo unequal deviations, while passing through the prism. It is because, refractive index of the material of a prism depends upon the wave length of light.

Note. The refractive index of a material for yellow light (μ) is practically equal to the mean of the refractive indices for the violet and red lights *i.e.*

$$\mu = \frac{\mu_v + \mu_r}{2}$$

For this reason, yellow light is called ***mean light.***

2.05. ANGULAR DISPERSION AND DISPERSIVE POWER

Angular dispersion. The angular dispersion produced in a ray of white light by a prism may be defined for any two colours.

The angular dispersion *for any two colours* may be defined as the *difference in deviations suffered by the two colours in passing through the prism.*

Fig. 2.05 represents the deviations produced in the path of violet, yellow and red rays of light by a prism of small angle. It follows that the constituent colours of white light disperse in an angle θ. The angle θ between the extreme violet and red rays of light is called *angular dispersion* produced by the prism for violet and red lights.

Thus, angular dispersion for violet and red rays is given by

$$\theta = \delta_v - \delta_r$$

Let μ_v and μ_r be the refractive indices of the material of the prism for the violet and red colours.

If A is the angle of prism, then

$$\delta_v = \mathrm{A}\,(\mu_v - 1)$$

and

$$\delta_r = \mathrm{A}\,(\mu_r - 1)$$

∴

$$\delta_v - \delta_r = \mathrm{A}\,(\mu - 1) - \mathrm{A}\,(\mu_v - 1)$$

or

$$\delta_v - \delta_r = \mathrm{A}\,(\mu_v - \mu_r) \qquad \text{...(2.11)}$$

The equation (2.11) gives the angular dispersion for violet and red rays.

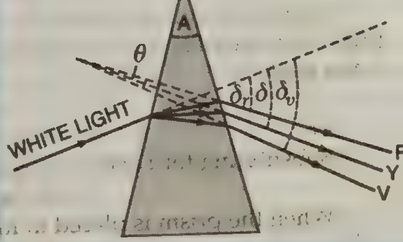


Fig. 2.05

Dispersive power. *The dispersive power of a material for any two colours may be defined as the ratio of the angular dispersion for these two colours to the deviation suffered by mean light.* It is denoted by ω.

If $\delta_v - \delta_r$ is the angular dispersion for violet and red lights and δ, the deviation suffered by mean light, then

$$\omega = \frac{\delta_v - \delta_r}{\delta} \qquad ...(2.12)$$

If μ is the refractive index of the material of the prism for mean light and δ, the deviation suffered by it, then

$$\delta = A(\mu - 1)$$

Substituting for $\delta_v - \delta_r$ and δ in equation (2.12), we have

$$\omega = \frac{A(\mu_v - \mu_r)}{A(\mu - 1)}$$

or

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1} \qquad ...(2.13)$$

The above equation gives dispersive power of the material of the prism for violet and red lights.

Note. 1. It may be pointed out that the violet light, owing to its small wavelength, is harmful to the eyes. Because of this, angular dispersion and the dispersive power of a material are measured for *blue* (in place of violet) and *red lights* in experimental spectroscopy. Therefore,

$$\omega = \frac{\mu_b - \mu_r}{\mu - 1}$$

2. The Fraunhoffer lines F, D and C in the solar spectrum possess wavelengths 4861 Å, 5890 Å and 6563 Å respectively and lie in blue, yellow and red regions of the spectrum. Sometimes, refractive indices of the materials of prisms for blue, mean and red light *i.e.* μ_b, μ and μ_r are respresented by μ_F, μ_D and μ_C respectively.

2.06. DISPERSION WITHOUT DEVIATION

Two prisms of different materials and different suitable angles can be combined so as to produce only dispersion and no deviation in the path of the light of a particular colour.

Condition for no deviation. Suppose that two prisms, one made of crown glass and the other of flint glass are placed together keeping their bases opposite to each other. Let A and A′ be the angles of the crown glass and flint glass prisms respectively. Let $\mu_v{}'$, μ and μ_r be the refractive indices of the crown glass for violet, mean and red lights ; while $\mu_v{}'$, μ and $\mu_r{}'$ be the corresponding values for the flint glass. The angles and materials of the two prisms are chosen, so as to produce dispersion without deviation in the path of mean light as shown in Fig. 2.06.

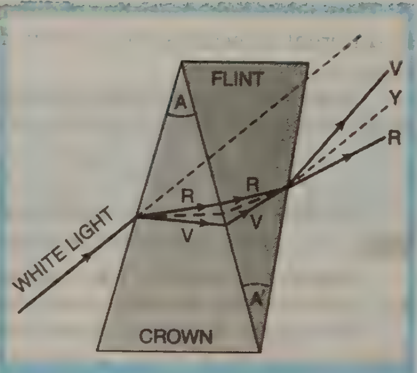


Fig. 2.06

The deviation suffered by mean light through crown glass prism,

$$\delta = A(\mu - 1)$$

Also, the deviation suffered by mean light through flint glass prism,

$$\delta' = A'(\mu - 1)$$

So that the combination does not produce any deviation,

$$\delta + \delta' = 0$$

or $A(\mu' - 1) + A'(\mu' - 1) = 0$

or

$$\frac{A'}{A} = -\frac{\mu - 1}{\mu' - 1} \qquad ...(2.14)$$

The negative sign indicates that the two prisms have to be placed in ***opposite manner***. The equation (2.14) is ***condition for no deviation.***

Key point

When two prisms (of different materials and angles of prism) are combined so as to produce dispersion without deviation, the condition given by equation (2.14) applies to no deviation in the path of mean light.

Net angular dispersion. The angular dispersion produced by the crown glass prism,

$$\delta_v - \delta_r = A(\mu_v - \mu_r)$$

Also, the angular dispersion produced by the flint glass prism,

$$\delta_v{}' - \delta_r{}' = A'(\mu_v{}' - u_r{}')$$

$$\text{Net angular dispersion} = (\delta_v - \delta_r) + (\delta_v{}' - \delta_r{}')$$

$$= A(\mu_v - \mu_r) + A'(\mu_v{}' - \mu_r{}')$$

$$= A\left[(\mu_v - \mu_r) + \frac{A'}{A}(\mu_v{}' - \mu_r{}')\right] \qquad ...(2.15)$$

From equation (2.14), substituting for $\frac{A'}{A}$, we have

$$\text{net angular dispersion} = A\left[(\mu_v-\mu_r)-\frac{(\mu-1)}{(\mu'-1)}\cdot(\mu_v'-\mu_r')\right]$$

$$= A(\mu-1)\left(\frac{\mu_v-\mu_r}{\mu-1}-\frac{\mu_v'-\mu_r'}{\mu'-1}\right)$$

$$= \delta(\omega-\omega') \qquad ...(2.16)$$

The equation (2.15) or (2.16) gives the ***net angular dispersion*** produced.

Discussion. 1. In equation (2.16), since $\omega' > \omega$, the factor $\omega - \omega'$ and hence $(\omega - \omega')$ is negative*. It implies that *the order of the colours in the spectrum due combination is opposite to that due to crown glass prism alone.*

2. Since $(\mu' - 1) > (\mu - 1)$, from equation (2.14), it follows that

$$A > A'$$

. to produce dispersion without deviation, *the angle of crown glass prism is greater an that of flint glass prism.*

Watch out !

When two prisms are combined to as to produce dispersion without deviation, the order of constituent colour in emergent light is opposite to that inside the first prism.

2.07. DEVIATION WITHOUT DISPERSION

When white light passes through a prism, it suffers deviation as well as ispersion. However the two prisms of different material can be combined so as o produce deviation in the path of light, without producing any deviation. The two risms, one made of crown glass and the other of flint glass, are placed together eeping their bases opposite to each other.

Condition for no dispersion. Let A and A′ be the angles of the crown glass nd flint glass prisms respectively. Let μ_v, μ and μ_r be the refractive indices of the rown glass for violet, mean and red light, while μ_v', μ' and μ_r' be the corresponding alues for the flint glass. The angles and the materials of the two prisms are chosen o as to produce deviation without dispersion [Fig. 2.07].

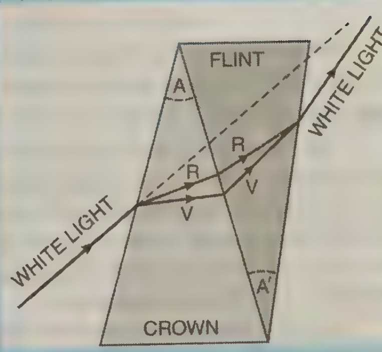


Fig. 2.07

The angular dispersion produced by crown glass prism,

$$\delta_v - \delta_r = A(\mu_v - \mu_r)$$

Also, the angular dispersion produced by flint glass prism,

$$\delta_v' - \delta_r' = A'(\mu_v' - \mu_r')$$

So that the combination does not produce any dispersion,

$$(\delta_v - \delta_r) + (\delta_v' - \delta_r') = 0$$

or $$A(\mu_v - \mu_r) + A'(\mu_v' - \mu_r') = 0 \qquad ...(2.17)$$

or $$\frac{A'}{A} = -\frac{\mu_v - \mu_r}{\mu_v' - \mu_r'} \qquad ...(2.18)$$

The *negative sign* indicates that the two prisms should be placed in *opposite nanner.* The equation (2.18) is called the ***condition for no dispersion*** or ***condition or achromatism.*** It can be put in another form as obtained below :

Multiplying and dividing the first term of the equation (2.17) by $(\mu - 1)$ and he second term by $(\mu' - 1)$, we have

$$A(\mu-1)\cdot\frac{\mu_v-\mu_r}{\mu-1} + A'(\mu'-1)\cdot\frac{\mu_v'-\mu_r'}{\mu'-1} = 0 \qquad ...(2.19)$$

But $\quad A(\mu - 1) = \delta \quad$ and $\quad A'(\mu' - 1) = \delta'$.

the deviations suffered by mean light through crown and flint glass prisms respectively. Also,

$$\frac{\mu_v-\mu_r}{\mu-1} = \omega \quad \text{and} \quad \frac{\mu_v'-\mu_r'}{\mu'-1} = \omega'$$

the dispersive powers of the crown and flint glasses respectively.

Therefore, the equation (2.19) becomes

$$\delta\,\omega + \delta'\,\omega' = 0$$

or $$\frac{\omega}{\omega'} = -\frac{\delta'}{\delta} \qquad ...(2.20)$$

Hence, *for no dispersion, the materials and the angles of the two prisms should be chosen, so that their dispersive powers are in inverse ratio of the deviations suffered by mean light through them.*

---

*The dispersive power of flint glass is greater than that of crown glass *i.e.* $\omega' > \omega$

Net deviation. The deviation suffered by mean light through crown glass prism,

$$\delta = A(\mu - 1)$$

Also, the deviation suffered by mean light through flint glass prism,

$$\delta' = A'(\mu' - 1)$$

$$\text{Net deviation} = \delta + \delta'$$

$$= A(\mu - 1) + A'(\mu' - 1) \qquad \text{...(2.21)}$$

$$= A(\mu - 1)\left[1 + \frac{A'(\mu' - 1)}{A(\mu - 1)}\right]$$

From equation (2.18), substituting the value of $\frac{A'}{A}$, we have

$$\text{net deviation} = A(\mu - 1)\left[1 - \frac{(\mu_v - \mu_r)}{\mu'_v - \mu'_r} \cdot \frac{\mu' - 1}{\mu - 1}\right] = A(\mu - 1)\left(1 - \frac{\omega}{\omega'}\right)$$

or

$$\text{net deviation} = \delta\left(1 - \frac{\omega}{\omega'}\right) \qquad \text{...(2.22)}$$

The equation (2.21) or (2.22) gives *net deviation* produced.

Discussion. 1. In equation (2.22), since $\omega' > \omega$, $1 - \frac{\omega}{\omega'}$ will be positive. Hence, it follows that *the combination will produce deviation in the same direction as produced by the crown glass prism.*

2. Since $\omega' > \omega$, from equation (2.20), it follows that $\delta > \delta'$ *i.e.*

$$A(\mu - 1) > A'(\mu' - 1)$$

Since $\mu' - 1 > \mu - 1$, it follows that $A > A'$ *i.e. to produce deviation without dispersion, the angle of crown glass prism has to be greater than that of the flint glass prism.*

2.08. SCATTERING OF LIGHT

Light travels in the form of electromagnetic radiation. *Emission, absorption* and *scattering* are three processes by which the radiation interacts with the matter. In emission, an accelerated charge radiates energy ; while in absorption, the electric charge gains energy at the expense of the incident electromagnetic radiation. The processes of emission and absorption have been discussed, while studying the emission and absorption spectra.

When an electromagnetic radiation is incident on an electric charge at rest, the charged particle gets accelerated along the direction of the electric field of the incident radiation. Since the electric charge is at rest, it does not experience any force due to the magnetic field of the electromagnetic radiation. On being accelerated by the electric field, the electric charge emits radiation in all the directions, and this process is called ***scattering***.

The light, while passing through atmosphere, is found to scatter also. The amount of scattering suffered by different wavelengths of light is different. Lord Rayleigh studied the scattering of light by the matter in detail. He found that the intensity of a particular wavelength in the scattered light depends greatly on its wavelength and ultimately he formulated the following law :

Key point

The blue light (having smaller wavelength) is scattered more than red light (having larger wavelength), when light travels through the atmosphere.

The intensity of the light corresponding to a wavelength in the scattered light varies inversely as the fourth power of the wavelength. If λ is a wavelength in scattered light, then

$$\text{amount of scattering} \propto \frac{1}{\lambda^4}$$

It is called ***Rayleigh's law of scattering***.

Many optical phenomena can be explained on the basis of Rayleigh's scattering of light.

Illustrations. A few illustrations of Rayleigh's scattering of light are as given below :

1. ***The blue colour of sky.*** When light from the sun travels through the earth's atmosphere, the different wavelengths get scattered from their path through different amounts obeying Rayleigh's law of scattering. Since, the wavelength of the blue colour is approximately half the wavelength of red colour, the scattering

of blue light is about 2^4 times *i.e.* 16 times more than that of red light. Due to this, blue colour predominates and the sky appears blue.

2. ***At sunrise or sunset the sun looks almost reddish.*** The reason is that at the time of sun set or sun rise (when sun is near horizon), the light from the sun has to traverse larger thickness of atmosphere than what it covers, when the sun is overhead as shown in Fig. 2.08.

Due to this, more of the blue light is scattered away and the sun appears reddish.

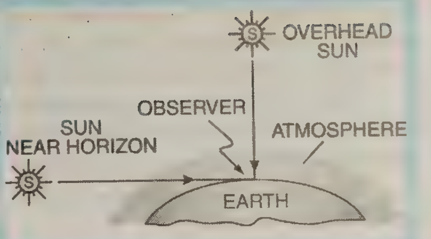


Fig. 2.08

3. ***Red light is used in danger signals.*** So that the danger signals can be seen distinctly upto large distances, the light used in the danger signals should not get scattered much, while passing through the atmosphere. Since, the red colour is scattered through a small amount due to its longer wavelength, the danger signals make use of red light.

4. ***The clouds are generally white.*** It is because of the reason that larger particles like dust and water drops present in the atmosphere do not scatter light according to Rayleigh law of scattering. Dust particles and water drops scatter light of all colours, almost equally and hence clouds are white in colour.

2.09. THE RAINBOWS

Sometimes, during a light drizzle or immediately after rain, concentric coloured arcs are seen in the sky, when the sun is at the back of the observer.

The coloured arcs or the spectrum of white light from the sun in the form of bows seen immediately after rain by an observer with his back towards the sun is called a ***rainbow.***

The formation of rainbows is due the dispersion of white light from the sun due to the phenomenon of total internal reflection from the water droplets suspended in the air. Ordinarily, two rainbows, called the ***primary rainbow*** and the ***secondary rainbow,*** are seen. However, the common centre of both the rainbows lies on the line joining the sun and the observer.

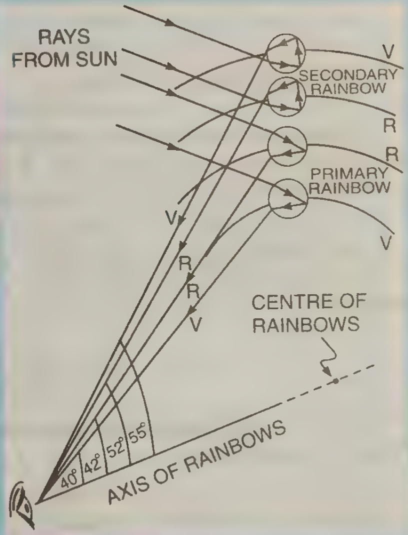


Fig. 2.09

Primary rainbow. The primary rainbow has violet colour on the inner edge and the red colour on the outer edge of the rainbow as shown in Fig. 2.09. It is formed, when the light from the sun after undergoing dispersion from the water droplets reaches the observer after suffering one internal reflection. It can be mathematically proved that an observer receives intense red light in a direction making an angle of 42° and intense violet (or blue) light in a direction making an angle of 40° with the line joining the sun and the observer. This line is called the *axis of the rainbow.*

Secondary rainbow. The secondary rainbow has red colour on the inner edge and the violet colour on the outer edge of the rainbow *i.e.* the sequence of colours in the secondary rainbow is just reverse of that in the primary rainbow. It is formed, when the light from the sun undergoing dispersion from the suspended water droplets, reaches the observer after suffering two internal reflections. The calculations show that the violet light is received by the observer in a direction making an angle of 55° and red light in a direction making an angle of 52° with the axis of the rainbow.

It is found that the region of the sky between the two rainbows is comparatively darker and the regions below the primary rainbow and above the secondary rainbow are comparatively brighter than the rest of the sky.

Two observers cannot see the same rainbow. For a primary rainbow to be seen by an observer, the rays emerging out from water droplets must subtend a mean angle of $41^\circ\left(=\frac{40^\circ+42^\circ}{2}\right)$ at the observer's eye and for a secondary rainbow, a mean angle of $53\cdot5^\circ\left(=\frac{52^\circ+55^\circ}{2}\right)$. The positions of the water droplets, which send such rays, depend upon the position of the observer. Hence, two observers at different positions do not see the rainbow formed by the same set of raindrops.

SOLVED NUMERICAL PROBLEMS

Type A. On Refraction through a prism

Problem 2.01. A ray of light incident at 49° on the face of an equilateral prism passes symmetrically. Calculate the refractive index of the material of the prism.

Sol. As the prism is an equilateral one, A = 60°. Further, as the ray of light passes symmetrically, when incident at 49°, the prism is in minimum deviation position.

We know that when the prism is placed in minimum deviation position,

$$r = \frac{A}{2} = \frac{60}{2} = 30° \;;\text{ Also, } i = 49°$$

$$\therefore \qquad \mu = \frac{\sin i}{\sin r} = \frac{\sin 49°}{\sin 30°} = \frac{0{\cdot}7547}{0{\cdot}5} = \mathbf{1{\cdot}51}$$

Problem 2.02. A ray of light suffers minimum deviation, while passing through a prism of refractive index 1·5 and refracting angle 60°. Calculate the angle of deviation and angle of incidence. *(P.S.S.C.E. 1997)*

Sol. Here, $\mu = 1{\cdot}5 \;;\; A = 60°$

$$\mu = \frac{\sin (A + \delta_m)/2}{\sin A/2}$$

or $$1{\cdot}5 = \frac{\sin (60° + \delta_m)/2}{\sin 60°/2}$$

or $(60° + \delta_m)/2 = 1{\cdot}5 \times \sin 30°$

or $\sin (60° + \delta_m)/2 = 1{\cdot}5 \times 0{\cdot}5 = 0{\cdot}75$

or $(60° + \delta_m)/2 = 48{\cdot}6°$

or $\delta_m = 48{\cdot}6° \times 2 - 60° = \mathbf{37{\cdot}2°}$

Also, angle of incidence,

$$i = (A + \delta_m)/2\; (60° + 37{\cdot}2°)/2 = \mathbf{48{\cdot}6°}$$

Problem 2.03. A prism of refractive index 1·53 is placed in water of refractive index 1·33. If the angle of the prism is 60°, calculate the angle of minimum deviation in water. *(P.S.S.C.E. 1998)*

Sol. Here, $A = 60° \;;\; {}^a\mu_g = 1{\cdot}53 \;;\; {}^a\mu_w = 1{\cdot}33$

Relative refractive index of the material of prism *w.r.t.* water,

$${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1{\cdot}53}{1{\cdot}33} = 1{\cdot}15$$

Let δ_m be the angle of minimum deviation on placing the prism in water. Then,

$${}^w\mu_g = \frac{\sin (A + \delta_m)/2}{\sin A/2}$$

$$\therefore \qquad 1{\cdot}15 = \frac{\sin (60° + \delta_m)/2}{\sin 60°/2}$$

or $\sin (60° + \delta_m)/2 = 1{\cdot}15 \times \sin 30°$

or $\sin (60° + \delta_m)/2 = 1{\cdot}15 \times 0{\cdot}5 = 0{\cdot}575$

or $(60° + \delta_m)/2 = 35{\cdot}1°$

or $\delta_m = 35{\cdot}1° \times 2 - 60° = \mathbf{10{\cdot}2°}$

Problem 2.04. An equilateral glass prism (μ = 1·6) is immersed in water (μ = 1·33). Calculate the angle of deviation produced for a ray of light incident at 40° on one face of the prism. *(Karnataka, 1990)*

Sol. Refer to Fig. 2.10. Since it is an equilateral glass prism, A = 60° ;

refractive index of glass, ${}^a\mu_g = 1{\cdot}6$;

refractive index of water, ${}^a\mu_w = 1{\cdot}33$;

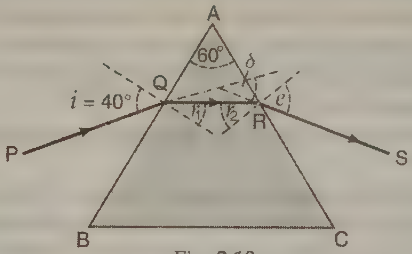


Fig. 2.10

Therefore, refractive index of glass *w.r.t.* water,

$${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1{\cdot}6}{1{\cdot}33} = 1{\cdot}203$$

For refraction at face AB :

The angle of incidence, $i = 40°$

Let r_1 be the angle of refraction at face AB.

Then, $${}^w\mu_g = \frac{\sin i}{\sin r_1}$$

or $$1{\cdot}203 = \frac{\sin 40°}{\sin r_1}$$

or $$\sin r_1 = \frac{\sin 40°}{1{\cdot}203} = \frac{0{\cdot}6428}{1{\cdot}203} = 0{\cdot}5343$$

or $r_1 = 32°\,18'$

For refraction at face AC :

Let r_2 be the angle of refraction at the second face AC.

We know, $r_1 + r_2 = A$

$\therefore$ $r_2 = A - r_1 = 60° - 32°\,18' = 27°\,42'$

Let e be the angle of emergence. For refraction at face AC, we have

$${}^w\mu_g = \frac{\sin e}{\sin r_2}$$

or $\sin e = {}^w\mu_g \times \sin r_2 = 1{\cdot}203 \times \sin 27°\,42'$

$= 1{\cdot}203 \times 0{\cdot}4648 = 0{\cdot}5592$

or $e = 34°$

Now, the angle of deviation,

$$\delta = i + e - A = 40° + 34° - 60° = \mathbf{14°}$$

Type B. On Angular dispersion and Dispersive power

Problem 1.05. Find the angle of dispersion between the red and violet colours produced by a flint glass prism of refracting angle 60°. Refractive indices of the prism for red and violet colours are 1·622 and 1·663 respectively. *(P.S.S.C.E. 1991)*

Sol. Here, $A = 60° \;;\; \mu_r = 1{\cdot}622 \;;\; \mu_v = 1{\cdot}663$

Now, angle of dispersion between red and violet colours,

$$\delta_v - \delta_r = A(\mu_v - \mu_r) = 60° \times (1{\cdot}663 - 1{\cdot}622) = \mathbf{2{\cdot}46°}$$

Problem 2.06. A prism with a refracting angle of 60° gives angles of minimum deviation 53°, 51° and 52° for blue, red and yellow light respectively. What is the dispersive power of the material of the prism ? *(I.S.C.E. 1996)*

Sol. Here, $A = 60° \;;\; \delta_b = 53° \;;\; \delta_r = 51° \;;\; \delta = 52°$

Dispersive power of the material of prism,

$$\omega = \frac{\delta_b - \delta_r}{\delta} = \frac{53° - 51°}{52°} = \frac{2°}{52°} = \mathbf{0{\cdot}0385}$$

Problem 2.07. Calculate the dispersive power of crown and flint glass prisms from the following data : For

rown glass : $\mu_b = 1\cdot522$, $\mu_r = 1\cdot514$; for flint glass : $\mu_b = 1\cdot662$, $_r = 1\cdot644$. *(Karnataka, 1991)*

Sol. *For crown glass :* $\mu_b = 1\cdot522$; $\mu_r = 1\cdot514$

Therefore, refractive index for the mean light,

$$\mu = \frac{\mu_b + \mu_r}{2} = \frac{1\cdot522 - 1\cdot514}{2} = 1\cdot518$$

Also, dispersive power of crown glass,

$$\omega = \frac{\mu_b - \mu_r}{\mu - 1} = \frac{1\cdot522 - 1\cdot514}{1\cdot518 - 1} = \mathbf{0\cdot0154}$$

For flint glass : $\mu_b = 1\cdot622$; $\mu_r = 1\cdot644$

$$\therefore \qquad \mu = \frac{\mu_b - \mu_r}{2} = \frac{1\cdot662 + 1\cdot644}{2} = 1\cdot653$$

$$\text{Also,} \quad \omega = \frac{\mu_b - \mu_r}{\mu - 1} = \frac{1\cdot662 - 1\cdot644}{1\cdot653 - 1} = \mathbf{0\cdot0276}$$

Problem 2.08. In a certain spectrum produced by a glass prism of dispersive power 0·031, it was found that the refractive index for the red ray is 1·645, whereas the refractive index for the blue ray is 1·665. What is the refractive index for the yellow ray ? *(I.S.C.E. 1995)*

Sol. Here, $\omega = 0\cdot031$; $\mu_r = 1\cdot645$; $\mu_b = 1\cdot655$

If μ is refractive index of glass for yellow ray, then

$$\omega = \frac{\mu_b - \mu_r}{\mu - 1}$$

$$\text{or} \qquad \mu = 1 + \frac{\mu_b - \mu_r}{\omega} = 1 + \frac{1\cdot665 - 1\cdot645}{0\cdot031}$$

$$= 1 + 0\cdot645 = \mathbf{1\cdot645}$$

Type C. On Dispersion without deviation

Problem 2.09. Determine the angle of flint glass prism, which should be combined with a crown glass prism of 5° so as to give dispersion but no deviation.

Given, for crown glass : $\mu_v = 1\cdot523$, $\mu_r = 1\cdot515$; for flint glass : $\mu_v{}' = 1\cdot688$, $\mu_r{}' = 1\cdot650$.

Sol.

Crown glass — *Flint glass*

$\mu_v = 1\cdot523$; $\mu_r = 1\cdot515$; A = 5° $\mu_v{}' = 1\cdot688$; $\mu_r{}' = 1\cdot650$; A' = ?

So that the combination of the two prisms does not produce any deviation,

$$A' = -\frac{A(\mu - 1)}{\mu' - 1}$$

$$\text{Now,} \qquad \mu = \frac{\mu_v + \mu_r}{2} = \frac{1\cdot523 + 1\cdot515}{2} = 1\cdot519$$

$$\text{and} \qquad \mu' = \frac{\mu_v{}' + \mu_r{}'}{2} = \frac{1\cdot688 + 1\cdot650}{2} = 1\cdot669$$

$$\therefore \qquad A' = -\frac{5(1\cdot519 - 1)}{1\cdot669 - 1} = -\frac{5 \times 0\cdot519}{0\cdot669} = \mathbf{-3\cdot88°}$$

The negative sign shows that the two prisms are to be placed in opposite manner.

Type D. On Deviation without Dispersion

Problem 2.10. Find the angle of a prism of dispersive power 0·021 and refractive index 1·53 to form an achromatic combination with prism of angle 4·2° and dispersive power 0·045 having refractive index 1·65. Find the resultant deviation.

Sol. Here, $\omega = 0\cdot021$; $\mu = 1\cdot53$; $\omega' = 0\cdot045$;
$\mu' = 1\cdot65$; $A' = 4\cdot2°$

For no dispersion, $\omega\,\delta + \omega'\delta' = 0$

or $\omega\,A(\mu - 1) + \omega' A'(\mu - 1) = 0$

$$\text{or} \qquad A = -\frac{\omega' A'(\mu - 1)}{\omega(\mu - 1)} = -\frac{0\cdot045 \times 4\cdot2 \times (1\cdot65 - 1)}{0\cdot021 \times (1\cdot53 - 1)}$$

$$= \mathbf{-11\cdot04°}$$

$$\text{Net deviation,}\ \delta + \delta' = A(\mu - 1) + A'(\mu' - 1)$$

$$= -11\cdot04\,(1\cdot53 - 1) + 4\cdot2\,(1\cdot65 - 1)$$

$$= -11\cdot04 \times 0\cdot53 + 4\cdot2 \times 0\cdot65$$

$$= -5\cdot85 + 2\cdot73 = \mathbf{3\cdot12°}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 2.11. A ray of light is incident on the face AB of a glass prism ABC having the vertex angle A equal to 40°. The face AC is silvered and a ray of light is incident on the face AB, such that it retraces its path. If refractive index of the glass is 1·54, calculate the angle of incidence on the face AB.

Sol. Here, $A = 40°$; $\mu = 1\cdot54$

Suppose that the ray of light PQ is incident on the face AB at angle i, such that the angle of refraction is r_1. The ray will retrace its path, if the refracted ray QR falls on the silvered surface normally, as shown in Fig. 2.11.

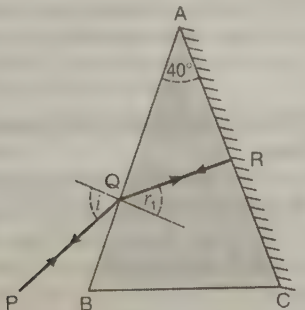


Fig. 2.11.

Since the Δ ARQ is a right angled triangle, it follows that

$$r_1 = 40°$$

$$\text{Now,} \qquad \mu = \frac{\sin i}{\sin r_1}$$

$$\text{or} \qquad \sin i = \mu \sin r_1 = 1\cdot54 \sin 40°$$

$$= 1\cdot54 \times 0\cdot6428 = 0\cdot9899$$

$$\text{or} \qquad \mathbf{i = 81°50'}$$

Problem 2.12. At what angle should a ray of light be incident on the face of a prism of refracting angle 60°, so that it just suffers internal reflection at the other face ? The refractive index of the prism is 1·524.

Sol. A ray of light incident on face AB will just suffer

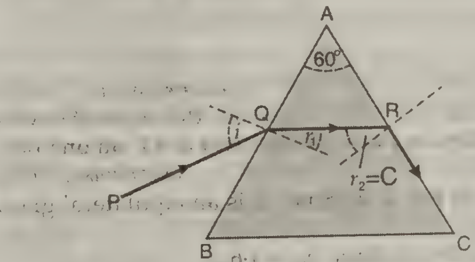


Fig. 2.12.

internal reflection at the other face AC, if it gets incident on face AC at critical angle for the material of the prism [Fig. 2.12]. If critical angle for the material of prism is C, then

$$\sin C = \frac{1}{\mu} = \frac{1}{1{\cdot}524} = 0{\cdot}6562$$

or $\quad C = 41°$

Now, for a prism, we have

$$r_1 + r_2 = A$$

Since $r_2 = C$, we have

$$r_1 + C = A \quad \text{or} \quad r_1 = A - C$$

Setting the values of A and C, we obtain

$$r_1 = 60° - 41° = 19°$$

At face AB : $\quad \dfrac{\sin i}{\sin r_1} = \mu \quad$ or $\quad \dfrac{\sin i}{\sin 19°} = 1{\cdot}524$

or $\quad \sin i = 1{\cdot}524 \times \sin 19° = 1{\cdot}524 \times 0{\cdot}3256 = 0{\cdot}4962$

or $\quad i = 29{\cdot}75°$

Problem 2.13. The dispersive powers of crown and flint glasses are 0·03 and 0·05 respectively. If difference in the refractive indices of blue and red colour is 0·015 for crown glass and 0·022 for flint glass, calculate the angles of the two prisms for a deviation of 2° without dispersion.

Sol.

Crown glass	*Flint glass*
$\mu_b - \mu_r = 0{\cdot}015\ ;\ \omega = 0{\cdot}03$	$\mu_b' - \mu_r' = 0{\cdot}022\ ;\ \omega' = 0{\cdot}05$

Also, net deviation, $\delta + \delta' = 2°$

Now, net deviation, $\delta + \delta' = A(\mu - 1) + A'(\mu' - 1)$

$\therefore \quad 2° = A(\mu - 1) + A'(\mu' - 1)$

or $\quad \dfrac{A(\mu_b - \mu_r)(\mu - 1)}{\mu_b - \mu_r} + \dfrac{A'(\mu_b' - \mu_r')(\mu' - 1)}{\mu_b' - \mu_r'} = 2°$

or $\quad \dfrac{A(\mu_b - \mu_r)}{\omega} + \dfrac{A'(\mu_b' - \mu_r')}{\omega'} = 2°$

or $\quad \dfrac{A \times 0{\cdot}015}{0{\cdot}03} + \dfrac{A' \times 0{\cdot}022}{0{\cdot}05} = 2°$

$$\frac{1}{2}A + \frac{11}{25}A' = 2° \qquad \ldots(i)$$

For no dispersion,

$$A(\mu_b - \mu_r) + A'(\mu_b' - \mu_r') = 0$$

$\therefore \quad A \times 0{\cdot}015 + A' \times 0{\cdot}022 = 0$

or $\quad A = -\dfrac{22}{15}A' \qquad \ldots(ii)$

In equation (*i*), substituting for A, we get

$$\frac{1}{2} \times \left(-\frac{22}{15}A'\right) + \frac{11}{25}A' = 2° \quad \text{or} \quad -\frac{22}{75}A' = 2°$$

or $\quad A' = -\dfrac{2 \times 75}{22} = -\dfrac{75}{11} = -6{\cdot}82°$

The negative sign shows that the two prisms have to be placed in the opposite manner.

From equation (*ii*), we have

$$A = -\frac{22}{15} \times \left(-\frac{75}{11}\right) = 10°$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Is the angle of minimum deviation produced by a prism different for different wavelengths ?

Ans. Yes.

Q. 2.02. A glass prism is immersed in water. What happens to the value of angle of minimum deviation ?

Ans. When a prism is immersed in water,

$${}^{w}\mu_g = \frac{\sin(A + \delta_m)/2}{\sin A/2}$$

Since the value of ${}^{w}\mu_g$ is less than ${}^{a}\mu_g$ (the refractive index of glass w.r.t. air), the value of angle of minimum deviation decreases on immersing the prism in water.

Q. 2.03. What is dispersion of light ? (*P.S.S.C.E. 1997*)

Ans. The splitting of white light into its constituent colours on passing through a prism is called dispersion of light.

Q. 2.04. Out of red and blue lights, for which colour is the refractive index of glass greater ? (*C.B.S.E. 1999 S*)

Ans. $\mu_b > \mu_r$.

Q. 2.05. Refractive indices of glass for lights of yellow, green and red colours are μ_y, μ_g and μ_r respectively. Rearrange these symbols in an increasing order of values. (*C.B.S.E. 1997*)

Ans. Refractive indices of glass for lights of yellow, green and red colours in the order of increasing values are μ_r, μ_y and μ_g.

Q. 2.06. What is dispersive power ? (*I.S.C.E. 1996*)

Or

What do you mean by dispersive power of the material of a prism ? (*I.S.C.E. 1994*)

Ans. The dispersive power of a material for light of any two colours is defined as the ratio of the angular dispersion for these colours to the deviation suffered by mean light.

Q. 2.07. On what factors does the dispersive power of a prism depend ?

Ans. The refractive index of the material of prism and the choice of the two colours for which the dispersive power is to be measured.

Q. 2.08. What is the effect of high dispersive power of the material of a thin prism on the spectrum ?

Ans. The angular dispersion will be large.

Q. 2.09. Does dispersive power of the material of a prism depend on the shape, size and angle of the prism ? Explain.

Ans. The dispersive power of the material of the prism is independent of the shape (angle of prism) or size of the prism. It depends only on the refractive index of the material of the prism.

Q. 2.10. People usually prefer light coloured dresses during summer and dark dresses during winter. Why ? (*Text Question*)

Or

Explain, why one prefers even in summer a black umbrella to a white one.

Ans. White clothes reflect all the light that falls on it, whereas black (or dark) clothes absorb all the light falling on it. The light absorbed by the dark clothes is converted into heat. For this reason, dark dresses are used in winters to keep warm.

Q. 2.11. Why is a rainbow formed in the sky ?

Ans. Rainbow is formed due to the dispersion of sun s, when they fall on the suspended tiny droplets of water nich act as prisms of small angles). The rainbow will be ble to an observer having sun at his back.

Q. 2.12. What is the essential condition for observing a rainbow ?

Ans. The observer must stand with his back towards the sun.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Draw a properly labelled graph between the gle of incidence and the angle of deviation for a prism d show the point of minimum deviation. (*I.S.C.E. 1996*)

Or

For a ray of monochromatic light passing through a ss prism, draw a graph to show the variation of the angle deviation (δ) with the angle of incidence (*i*). When does e ray suffer minimum deviation ? (*I.S.C.E. 1993*)

Or

How does the angle of deivation vary with the angle incidence in case of a prism ? What is the angle of mini- um deviation ? (*C.B.S.E. 1990 S*)

Ans. Refer to Fig. 2.03.

Q. 2.02. A monochromatic ray of light is made to fall a normal 60° prism under minimum deviation condition. hat is the relation between the angle of incidence and the gle of emergence ? (*I.S.C.E. 1998*)

Ans. When light is incident on a prism placed in the inimum deviation position, the refracted ray travels parallel the base of the prism. Consequently, the prism lies mmetrically *w.r.t.* incident and emergent rays. Hence, in inimum deviation position, angle of incidence and angle of nergence are equal.

Q. 2.03. A thin prism of 2° angle gives a deviation of . What is the value of refractive index of the material of e prism ? (*P.S.S.C.E. 1999*)

Ans. Here, $A = 2° \,;\, \delta = 1°$

Now, for a prism of small angle,

$$\delta = A(\mu - 1)$$

or $$\mu = 1 + \frac{\delta}{A} = 1 + \frac{1}{2} = 1{\cdot}5$$

Q. 2.04. A glass prism has a minimum angle of devia- on δ_m in air. State with reason, how the angle of minimum eviation will change, if the prism is immersed in a liquid f refractive index greater than 1. (*C.B.S.E. 2002*)

Ans. We know that for a glass prism placed in air,

$$\mu = \frac{\sin(A + \delta_m)/2}{\sin A/2}$$

When prism is immersed in a liquid of refractive index reater than 1, its refractive index *w.r.t.* liquid will become less han μ. From the above relation, it follows that angle of ninimum deviation will decrease.

Q. 2.05. What is dispersion of light ? Define dispersive ower of a prism. (*P.S.S.C.E. 1997*)

Ans. Refer to sections 2.04 and 2.05.

Q. 2.06. Explain, why white light is dispersed when assing through a prism.

(*H.S.S.C.E. 2002 ; P.S.S.C.E. 2001 ; C.B.S.E. 1995*)

Ans. The velocity of light in a material medium depends upon its colour (wavelength) *i.e.* the refractive index of a material is different for different colours. If we incident a ray of white light on a prism ; on emerging, the different colours are deviated through different angles. Due to this, white light splits into its constituent colours and the phenomenon is called dispersion.

Q. 2.07. Rays of red and blue light are incident on a given prism. Explain, which will have the larger value of δ_m, the angle of minimum deviation. (*I.S.C.E. 1994*)

Ans. For a prism of small angle, the angle of deviation is given by

$$\delta = A(\mu - 1)$$

$$\therefore \quad \delta_r = A(\mu_r - 1) \quad \text{and} \quad \delta_b = A(\mu_b - 1)$$

Since $\mu_b > \mu_r$, it follows that $\delta_b > \delta_r$.

Thus, blue light will have larger value of the angle of minimum deviation.

Q. 2.08. Why does the sky appear blue ?

(*H.P.S.S.C.E. 2002 ; H.S.S.C.E. 2000, 1999 S ; P.S.S.C.E. 2001, 1999 S, 1996 ; A.I.S.S.C.E. 1992 S*)

Ans. According to Rayleigh's scattering law, when the light traverses the atmosphere, the intensity of light scattered is inversely proportional to the fourth power of the wavelength of the light. Therefore the scattering of blue light is prominent due to its small wavelength. Due to this, sky looks blue.

Q. 2.09. Why does sky look blue ? What will it look like on moon ? (*P.S.S.C.E. 1996*)

Ans. For blue colour of sky, refer to SAQ 2.08.

On moon, there is no atmosphere. Therefore, in the absence of scattering of light, the atmosphere will appear white.

Q. 2.10. Sun looks reddish at sunset. Why ?

Or (*C.B.S.E. 2002*)

Why the rising sun is red in colour ? (*P.S.S.C.E. 1999*)

Or

Why the rising sun is red in colour (or at sunset) ?

(*P.S.S.C.E. 2001, 1998 S ; C.B.S.E. 2000 ; H.S.S.C.E. 2000, 1999 S*)

Ans. Refer to illustration no. 2 of section 2.08.

Q. 2.11. Eye is more sensitive to yellow colour. Why are then danger signals red ? (*H.P.S.S.C.E. 2002, 1998*)

Ans. The scattering (Rayleigh scattering) of light is inversely proportional to the fourth power of the wavelength of light. So, the scattering of red light is much less than the yellow light and accordingly, the red light signal can be seen upto a longer distance without much loss in its intensity. For this reason, the danger signals are red in colour.

Q. 2.12. Why are danger signals red in colour ? Give reasons. (*P.S.S.C.E 2001*)

Ans. Refer to SAQ 2.11.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 2.01. When does a ray incident on a prism deviate away from the base ?

Ans. When the prism is immersed in a transparent liquid of refractive index greater than that of the material of the prism, the incident ray will deviate away from the base of the prism.

Q. 2.02. Two identical prisms are placed in the path of white light, such that their bases are parallel [Fig. 2.13]. What will be the nature of the emergent light ?

Ans. The first prism will disperse the white light into its constituent colours as shown in the figure. If the bases of the two identical prisms (same material and same value of the angles of prisms) are parallel, a broad beam of coloured light will emerge parallel to the incident beam of white light.

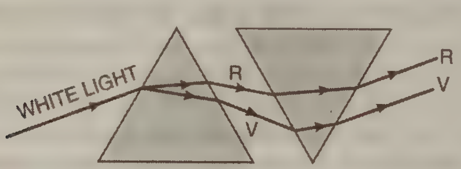


Fig. 2.13

Q. 2.03. Explain the prismatic action of a convex lens.

Ans. A convex lens may be considered to be made of a number of prisms, the prisms being placed one above the other. The bases of all the prisms are towards the centre of the lens.

As the two surfaces of the lens are curved, the angles of all these prisms are not same. The portion of the lens near the edge behaves as the prism having the maximum value of the angle of prism, while the portion at the middle behaves as a prism of zero angle. Since the angle of deviation produced by a prism depends upon the angle of prism, the prismatic part of the lens near its edge deviates the light maximum, while that at the centre does not produce any deviation in the path of the light as shown in Fig. 2.14. The result is that all the rays after refraction from the lens meet at one point, called the focus of the lens. Thus, a convex lens converges light due to the prismatic action of the lens. In addition to produce deviation in the path of light, a thick lens disperses white light in the same manner as the prism does.

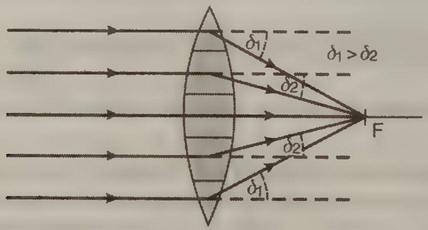


Fig. 2.14

Q. 2.04. Show that the limiting value of the angle of prism is twice its critical angle. (*Karnataka, 1991*)

Ans. Refer to section 2.02 and Fig. 2.03 :

Angle of prism, $A = r_1 + r_2$

Therefore, limiting (maximum) value of angle of prism,

$$A_{max} = (r_1)_{max} + (r_2)_{max} \quad ...(i)$$

Now, for $r_1 = (r_1)_{max},\ i = 90°$

But, when the angle *i* becomes equal to 90°, $(r_1)_{max}$ mu be equal to the critical angle *i.e.*

$$(r_1)_{max} = C$$

Similarly, $(r_2)_{max} = C$

Substituting for $(r_1)_{max}$ and $(r_2)_{max}$ in equation (*i*), we g

$$A_{max} = C + C = 2C$$

i.e. limiting value of the angle of prism is equal to twic the critical angle.

Q. 2.05. A ray of white light shows no dispersion o emerging from a glass plate (slab). Is there n dispersion inside the glass plate as well ?

Ans. The light disperses into different colours on enterin the glass slab *i.e.* dispersion takes place inside a glass slab als But when it emerges from its other parallel face, the differe colours again combine to form white light.

Q. 2.06. A beam of white light passing through hollow prism gives no spectrum. Explain. (*I.I.T. 198*

Ans. Consider a hollow prism as shown in the Fig. 2.1 Here, ABC and A′B′C′ represent the outer and inner surface of the prism. The three faces of the prism simply act as th three glass slabs having parallel faces.

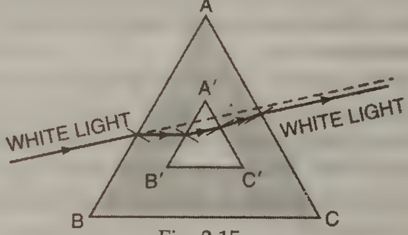


Fig. 2.15

When a ray of light is incident on the face AB, it will pas through a glass slab formed between the surfaces AB an A′B′. A ray of light will not disperse but it will get laterall displaced. The similar effect will take place, when the ray fall on the inner surface A′C′. Thus, the ray of light will come ou of the prism without being dispersed but again gettin laterally displaced.

Q. 2.07. What does a welder protect against, when h wears a mask ? (*Text Question*

Ans. The ultraviolet light coming out of welding arc i harmful to eyes. The mask has a filter that absorbs ultraviole light.

Q. 2.08. Do materials always have the same colou whether viewed by reflected light or throug transmitted light ?

Ans. It is not necessary. A material may reflect one colou strongly and transmit another colour.

Q. 2.09. How would a blue object appear under sodiu lamp light ?

Ans. An object looks blue in white light, because i absorbs all colours except the blue. If it is placed in sodium light (yellow), it will absorb the yellow light. Hence, the objec will appear black.

Q. 2.10. What colour do you observe, when white ligh passes through a blue and yellow filter ? (*Text Question*

Ans. Green colour with a tinge of blue and yellow wil be observed.

Q. 2.11. Occasionally one sees an aircraft in the sky shortly after sunset or shortly before sunrise and is prised to see it appear to be very bright, more like a star planet than like an aircraft as seen in daylight. Explain, y.

Ans. Shortly after sunset or before sunrise, an observer the surface of earth does not receive the sunlight directly m the sun. The light reflected from the aircraft is received the observer as shown in Fig. 2.16. In the reflected sunlight, e spacecraft appears bright against the darkening ckground of the early morning or the evening sky.

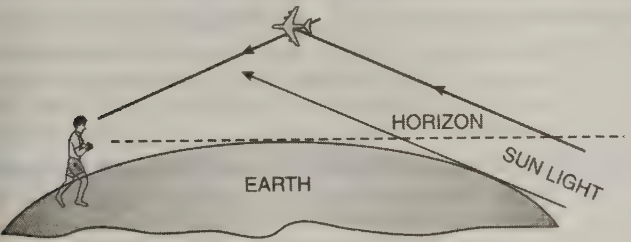


Fig. 2.16

Q. 2.12. Artificial satellites can often be seen as bright objects high in the sky long after sunset. What must the minimum altitude of a satellite moving above the rth's equator to be still visible directly overhead two hours ter sunset ?

Ans. At sunset, the sunlight will be tangent to the earth's rface. Further, in two hours, the earth will rotate through 30°. Therefore, satellite will come in the path of sunlight, if its orbit is at a distance BS = h (say) above the earth's equator [Fig. 2.17]. If OA = OB = R is radius of earth, then from right angled triangle OAB, we have

$$\cos 30° = \frac{OA}{OS} = \frac{OA}{OB + BS} = \frac{R}{R + h}$$

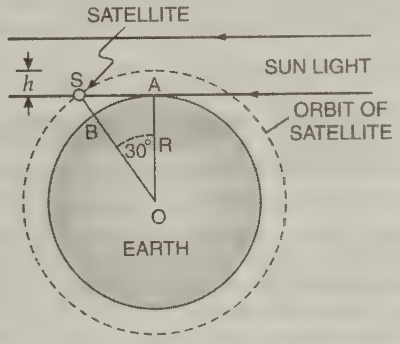


Fig. 2.17

or $$h = \frac{R}{\cos 30°} - R = \frac{R}{0{\cdot}866} - R$$

$$= 0{\cdot}155\ R$$

Q. 2.13. Why are 'haloes' (rings) observed sometimes round the sun or moon ?

Ans. When the sun or the moon is seen through a thin veil of the high cloud, haloes are formed. A large number of icy crystals are present in the clouds. The haloes are formed due to the reflection of light from the icy crystals.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Show that in case of a prism : A + δ = i + e, where the symbols have their usual meanings. *(H.P.S.S.C.E. 1997)*
2. What is meant by angle of minimum deviation ? Explain. Indicate in a graph, how the angle of deviation changes with the angle of incidence. State the condition under which angle of deviation is minimum.
3. Discuss the phenomenon of refraction through a prism. Prove the formula $\delta = A(\mu - 1)$. *(H.S.S.C.E. 1992)*
4. Derive an expression for the angle of deviation of a small prism in terms of the refractive index and the angle of the prism. *(I.S.C.E. 1996)*
5. What is dispersion of light ? Explain it with a ray diagram. Also explain the cause of dispersion of light. *(P.S.S.C.E. 2002, 1999 S)*
6. Explain the meanings of dispersion, angular dispersion and dispersive power. *(P.S.S.C.E. 1997)*
7. What is dispersion of light ? Define dispersive power of a prism. *(P.S.S.C.E. 1997)*
8. What is dispersion and angular dispersion ? Derive expression for the angular dispersion produced by a prism of small angle.
9. Explain the terms angular dispersion and dispersive power. How are the two related ? *(H.S.S.C.E. 2001, P.S.S.C.E. 1997)*
10. Define angular dispersion and dispersive power. Derive an expression for the dispersive power of a material in terms of refractive indices.
11. Explain, how two thin prisms can be combined to produce dispersion without deviation of the mean light. Obtain an expression for the resultant dispersion.
12. What do you understand by deviation without dispersion ? Obtain the condition for it. *(P.S.S.C.E. 1995)*
13. Write a short notes on rainbow. *(H.P.S.S.C.E. 1993)*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Derive the expression for the angle of deviation for a ray of light passing through an equilateral prism of refracting angle A. *(C.B.S.E. 1993)*
2. State and prove prism formula. *(H.P.S.S.C.E. 1997 ; H.S.S.C.E. 1993)*
3. Prove the relation A + δ = i + e for a prism. What does this relation become, when prism is placed in minimum deviation ? *(H.S.S.C.E. 2001)*
4. Discuss the phenomenon of refraction through a prism and prove that for a prism :

$$\mu = \frac{\sin (A + \delta_m)/2}{\sin A/2},$$

where A is angle of prism, μ is refractive index of the material of prism and δ_m is the angle of minimum deviation. *(P.S.S.C.E. 2001)*

5. Derive the relation : $\mu = \dfrac{\sin (A + \delta_m)/2}{\sin A/2}$ for refraction through a prism. *(H.S.S.C.E. 2002, 2000, 1993 ; Karnataka, 1993)*
6. Derive an expression for the refractive index of the material of a prism in terms of angle of prism and the angle of minimum deviation. *(Karnataka 1994, 1992, 1990, 1988 ; I.S.C.E. 1993)*

7. Derive the relation for refractive index of the material of prism, when the prism is placed in minimum deviation position. (*P.S.S.C.E. 2002, 2000, 1998 S*)
8. Derive an expression for the refractive index of the material of the prism in terms of the refracting angle and the angle of minimum deviation. (*I.S.C.E. 1993*)
9. Derive the formula used for determining the refractive index of the material of a glass prism using a spectrometer. (*I.S.C.E. 1997*)
10. Discuss the phenomenon of refraction through a prism. Prove that $\delta = A(\mu - 1)$ for a small angled prism, where the symbols have their usual meanings. (*P.S.S.C.E. 2000*)
11. What is rainbow ? Differentiate between a primary rainbow and secondary rainbow with a diagram. Why two observers do not see the same rainbow ? (*P.S.S.C.E. 2001, 1995*)
12. Explain the formation of colours in a rainbow. What is a secondary rainbow ? (*C.B.S.E. 1992 S*)
13. Explain in brief the formation of primary and secondary rainbows.

FAQs FREQUENTLY ASKED NUMERICAL QUESTIONS

For Practice

Type A. On Refraction through a prism

1. A ray of light is inclined to one face of the prism at an angle of 50°. If the angle of the prism is 60° and the ray is deviated through an angle of 42° ; find the angle which the emergent ray makes with the second face of the prism. [**Ans.** 28°]
2. Calculate the refractive index of a prism, if the angle of the prism is 60° and the angle of minimum deviation is 50°. [**Ans.** 1·638]
3. A ray of light incident on an equilateral glass prism shows minimum deviation of 30°. Calculate the speed of light through the glass prism. (*C.B.S.E. 2001*) [**Ans.** $2{\cdot}12 \times 10^8$ m s^{-1}]
4. The refracting angle of a prism is 60° and the refractive index of the material of the prism is 1·632. Calculate the angle of minimum deviation. (*Karnataka, 1983*) [**Ans.** 49·38°]
5. Calculate the value of angle of minimum deviation, which a ray of light, passing through a prism of angle 60° and refractive index 1·5 will suffer. What is the corresponding angle of incidence ? [**Ans.** 37·18° ; 48·59°]
6. A ray of light is incident on the face of a prism placed in minimum deviation position. If the angle of prism is 60° and deviation produced is 34°, find the angle of incidence and refractive index. [**Ans.** 47° ; 1·463]
7. What should be the refracting angle of a prism of refractive index 1·5 in order that the ray incident on the face at an angle of 50° may suffer minimum deviation ? [**Ans.** 61·42°]
8. A ray of light is incident on the glass prism at an angle of 40°. Find the angle of emergence, if the angle of prism is 60° and refractive index is 1·5. [**Ans.** 58·48°]
9. A ray of light passes through an equilateral glass prism, such that the angle of incidence is equal to the angle of emergence. If the angle of emergence is 3/4 times the angle of prism, calculate the refractive index of the glass prism. (*C.B.S.E. 2000*) [**Ans.** 1·414]
10. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. By rotating the prism, the minimum angle of deviation is measured to be 40°. What is the refractive index of the prism ? If the prism is placed in water (${}^a\mu_w = 1{\cdot}33$), predict the new minimum angle of deviation of a parallel beam of light. The refracting angle of prism is 60°. [**Ans.** 1·532 ; 10·34°]
11. A glass prism is immersed in water. If the angle of prism is 70°, calculate the angle of minimum deviation for a ray of light while passing through the prism in water. Given ${}^a\mu_g = 1{\cdot}62$ and ${}^a\mu_w = 1{\cdot}33$. (*P.S.S.C.E. 2001*) [**Ans.** 18·64°]
12. A ray of light falls normally on the face of a glass prism having a refractive indices of 1·5. Find the angle of prism, if the ray just fails to emerge from the second face. [**Ans.** 41·8°]
13. An equilateral prism of refractive index 1·45 is surrounded by water of refractive index 1·33. A narrow beam of light is incident normally on one face of the prism. Find the direction of the emergent ray. [**Ans.** 70·72°]

Type B. On Angular dispersion and Dispersive power

14. White light is passed through a glass prism of angle 5°. If the refractive indices for red and blue colours are 1·641 and 1·659 respectively, calculate the angle of dispersion between these colours. (*P.S.S.C.E. 1999 S*) [**Ans.** 0·09°]
15. Using a spectrometer, the following data are obtained for a crown glass prism and a flint glass prism :

	Crown glass prism	*Flint glass prism*
Angle of prism :	A = 72·0°	A′ = 60·0°
Minimum deviation angle :		
	$\delta_b = 54{\cdot}6°, \delta_r = 53{\cdot}0°$,	$\delta_b' = 52{\cdot}8°, \delta_r' = 50{\cdot}6°$,
	$\delta = 54{\cdot}0°$	$\delta' = 51{\cdot}9°$

Compare the dispersive powers of the two varieties of glass prisms. (*Text Problem*) [**Ans.** 0·7]

16. The refractive indices of crown and flint glasses for blue and red rays are 1·523, 1·513 and 1·773, 1·743 respectively. Calculate the dispersive powers of the two glasses. (*Pre-degree Kerala, 1992*) [**Ans.** 0·0193, 0·0396]

Type C. On Dispersion without deviation

17. A combination of two prisms, one of flint and other of crown glass produces dispersion without deviation. The angle of the flint glass prism is 15°. Calculate the angle of crown glass prism and the angular separation of the red and violet rays on emergence from the spectroscope. (μ for crown glass = 1·52, μ for flint glass = 1·65, ω for crown glass = 0·02, ω for flint glass = 0·03). [**Ans.** 18·75°, 0·0975°]
18. The refractive indices of crown and flint glass for C, D and F lines are given below :

Glass	μ_C	μ_D	μ_F
Crown	1·527	1·530	1·535
Flint	1·790	1·795	1·806

Calculate the dispersive powers of two glasses. If the angle of crown glass prism is 9°, calculate the angle of the flint glass prism, which is combined with crown glass prism to produce dispersion without deviation. [**Ans.** 0·015, 0·02, 6°]

Type D. On Deviation without dispersion

19. A crown glass prism of refracting angle 6° is to be achromatised for red and blue light using a flint glass prism. Find the angle of the second prism and also the mean deviation from the following data :

Refractive index for crown glass : $\mu_r = 1{\cdot}520$; $\mu_b = 1{\cdot}531$

Refractive index for flint glass : $\mu_r' = 1{\cdot}662$; $\mu_b' = 1{\cdot}684$.

[**Ans.** 3°, 1·134°]

20. The dispersive powers of crown and flint glasses are 0·03 and 0·05 respectively. If difference in the refractive indices of blue and red colour is 0·015 for crown glass and 0·022 for flint glass, calculate the angles of the two prisms for a deviation of 2° without dispersion. **[Ans.** 10·02°, 6·83°]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $i = 90° - 50° = 40°$; $A = 60°$, $\delta = 42°$

   Now, $e = A + \delta - i = 60° + 42° - 40° = 62°$

   Therefore, angle which emergent ray makes with the second face of the prism = 90° – 62° = **28°**

2. $$\mu = \frac{\sin (A+\delta_m)/2}{\sin A/2} = \frac{\sin (60° + 50°)/2}{\sin 30°}$$
   $$= \frac{\sin 55°}{\sin 30°} = \frac{0\cdot 8192}{0\cdot 5} = \mathbf{1\cdot 638}$$

3. $$\mu = \frac{\sin (A+\delta_m)/2}{\sin A/2} = \frac{\sin (60° + 30°)/2}{\sin 30°}$$
   $$= \frac{\sin 45°}{\sin 30°} = \frac{0\cdot 707}{0\cdot 5} = 1\cdot 414$$

   Speed of light through glass prism,
   $$v = \frac{c}{\mu} = \frac{3 \times 10^8}{1\cdot 414} = \mathbf{2\cdot 12 \times 10^8\ m\ s^{-1}}$$

4. $$\mu = \frac{\sin (A+\delta_m)/2}{\sin A/2} \quad \text{or} \quad 1\cdot 632 = \frac{\sin (60° + \delta_m)/2}{\sin 30°}$$

   or $\sin (60° + \delta_m)/2 = 1\cdot 632 \times 0\cdot 5 = 0\cdot 816$

   or $(60° + \delta_m)/2 = 54\cdot 69°$

   or $\delta_m = 54\cdot 69° \times 2 - 60° = \mathbf{49\cdot 38°}$

5. To find δ_m, proceed as in problem no. 4. It will be found that $\delta_m = 37\cdot 18°$

   Then, angle of incidence,
   $$i = (A + \delta_m)/2 = (60° + 37\cdot 18°)/2 = \mathbf{48\cdot 59°}$$

6. To find μ, proceed as in problem no. 2.

   Angle of incidence,
   $$i = (A + \delta_m)/2 = (60° + 34°)/2 = \mathbf{47°}$$

7. $(A + \delta_m)/2 = 50°$, $\mu = 1\cdot 5$

   $$\text{Now, } \mu = \frac{\sin (A+\delta_m)/2}{\sin A/2} \quad \text{or} \quad 1\cdot 5 = \frac{\sin 50°}{\sin A/2}$$

   or $\sin A/2 = \dfrac{0\cdot 7660}{1\cdot 5} = 0\cdot 5107$ or $A/2 = 30\cdot 71°$

   or $A = \mathbf{61\cdot 42°}$

8. ***For refraction at first face :***
   $$\mu = \frac{\sin i}{\sin r_1}$$

   or $\sin r_1 = \dfrac{\sin i}{\mu} = \dfrac{\sin 40°}{1\cdot 5} = \dfrac{0\cdot 6428}{1\cdot 5} = 0\cdot 4285$

   or $r_1 = 25\cdot 37°$

   For refraction at second face :

   Now, $A = (r_1 + r_2)$

   or $r_2 = A - r_1 = 60° - 25\cdot 37° = 34\cdot 63°$

   If e is angle of emergence, then
   $$\mu = \frac{\sin e}{\sin r_2} \quad \text{or} \quad \sin e = \mu \sin r_2 = 1\cdot 5 \times \sin 34\cdot 63°$$

   or $\sin e = 1\cdot 5 \times 0\cdot 5683 = 0\cdot 8525$ or $e = \mathbf{58\cdot 48°}$

9. As the angle of incidence is equal to angle of emergence, prism is in minimum deviation position.

   Here, $e = i = \dfrac{3}{4} A = \dfrac{3}{4} \times 60° = 45°$

   Now, $i = (A + \delta_m)/2$

   or $\delta_m = 2i - A = 2 \times 45° - 60° = 30°$

   $$\text{Also, } \mu = \frac{\sin (A+\delta_m)/2}{\sin A/2} = \frac{\sin (60° + 30°)/2}{\sin 30°}$$
   $$= \frac{\sin 45°}{\sin 30°} = \frac{0\cdot 707}{0\cdot 5} = \mathbf{1\cdot 414}$$

10. $${}^a\mu_g = \frac{\sin (A+\delta_m)/2}{\sin A/2} = \frac{\sin (60° + 40°)/2}{\sin 30°}$$
    $$= \frac{\sin 50°}{\sin 30°} = \frac{0\cdot 7660}{0\cdot 5} = \mathbf{1\cdot 532}$$

    When prism is placed in water : Let μ' be refractive index of the material of prism w.r.t. water and δ_m' be angle of minimum deviation. Then,
    $$\mu' = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1\cdot 532}{1\cdot 33} = 1\cdot 152$$
    $$\text{Now, } \mu' = \frac{\sin (A + \delta_m')/2}{\sin A/2}$$

    or $1\cdot 152 = \dfrac{\sin (60° + \delta_m')/2}{\sin 30°}$

    or $\sin(60° + \delta_m')/2 = 1\cdot 152 \times 0\cdot 5 = 0\cdot 576$

    or $(60° + \delta_m')/2 = 35\cdot 17°$

    or $\delta_m' = 35\cdot 17° \times 2 - 60° = \mathbf{10\cdot 34°}$

11. Proceed as in problem no. 10.

12. The ray of light incident normally on the face AB of the prism will pass as such and will come to meet the face AB at point R as shown in Fig. 2.18.

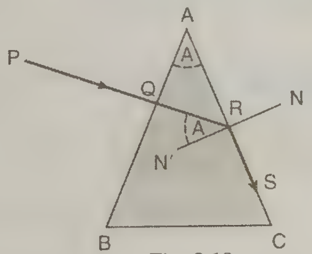


Fig. 2.18

    Obviously, ∠QRN′ = A, angle of prism. Since the ray just fails to emerge from the face AC, angle A equals the critical angle for the material of lens.

    Thus, $\mu = \dfrac{1}{\sin A}$ or $\sin A = \dfrac{1}{1\cdot 5} = 0\cdot 6667$

    or $A = \mathbf{41\cdot 8°}$

13. Let μ' be refractive index of material of prism w.r.t. water.

    Then, $\mu' = \dfrac{1\cdot 45}{1\cdot 33} = 1\cdot 09$

    As explained in problem no. 12, when a ray of light is incident normally on one face of the prism, it comes to incident on the second face at an angle equal to angle of prism. If e is angle of emergence, then,
    $$\mu' = \frac{\sin e}{\sin A} \quad \text{or} \quad \sin e = \mu' \sin A = 1\cdot 09 \times \sin 60°$$

    or $\sin e = 1\cdot 09 \times 0\cdot 8660 = 0\cdot 9439$ or $e = \mathbf{70\cdot 72°}$

14. Angle of dispersion,
    $$\theta = A(\mu_b - \mu_r) = 5° (1\cdot 659 - 1\cdot 641) = \mathbf{0\cdot 09°}$$

15. Dispersive power of crown glass,

$$\omega = \frac{\delta_b - \delta_r}{\delta} = \frac{54\cdot 6^\circ - 53\cdot 0^\circ}{54\cdot 0^\circ} = 0\cdot 0296$$

Dispersive power of flint glass,

$$\omega' = \frac{\delta_b' - \delta_r'}{\delta'} = \frac{52\cdot 8^\circ - 50\cdot 6^\circ}{51\cdot 9^\circ} = 0\cdot 0424$$

$$\therefore \qquad \frac{\omega}{\omega'} = \frac{0\cdot 0296}{0\cdot 0424} = \mathbf{0\cdot 7}$$

16. ***For crown glass :*** $\mu_b = 1{\cdot}523$, $\mu_r = 1{\cdot}513$

Refractive index for mean light,

$$\mu = \frac{\mu_b + \mu_r}{2} = \frac{1\cdot 523 + 1\cdot 513}{2} = 1\cdot 518$$

$$\therefore \qquad \omega = \frac{\mu_b - \mu_r}{\mu - 1} = \frac{1\cdot 523 - 1\cdot 513}{1\cdot 518 - 1} = \mathbf{0\cdot 0193}$$

For flint glass : In the same manner, it can be obtained that $\omega' = \mathbf{0{\cdot}0396}$

17. For no deviation, $A(\mu - 1) = A'(\mu' - 1)$

$$\text{or} \qquad A' = \frac{A(\mu - 1)}{(\mu' - 1)} = \frac{15^\circ(1\cdot 65 - 1)}{(1\cdot 52 - 1)} = \mathbf{18\cdot 75^\circ}$$

$$\text{Net angular dispersion} = \delta(\omega - \omega')$$
$$= A(\mu - 1)(\omega - \omega')$$
$$= 15^\circ(1\cdot 65 - 1)(0\cdot 03 - 0\cdot 02) = \mathbf{0\cdot 0975^\circ}$$

18. ***For crown glass :***

$$\omega = \frac{\mu_F - \mu_C}{\mu_D - 1} = \frac{1\cdot 535 - 1\cdot 527}{1\cdot 530 - 1} = \mathbf{0\cdot 015}$$

For flint glass :

$$\omega' = \frac{\mu_F' - \mu_C'}{\mu_D' - 1} = \frac{1\cdot 806 - 1\cdot 790}{1\cdot 795 - 1} = \mathbf{0\cdot 02}$$

For no deviation,

$$A' = \frac{A(\mu_D - 1)}{(\mu_D' - 1)} = \frac{9^\circ(1\cdot 530 - 1)}{(1\cdot 795 - 1)} = \mathbf{6^\circ}$$

19. For no dispersion, $A(\mu_b - \mu_r) = A'(\mu_b' - \mu_r')$

$$\text{or} \qquad A' = \frac{A(\mu_b - \mu_r)}{(\mu_b' - \mu_r')} = \frac{6^\circ(1\cdot 531 - 1\cdot 520)}{(1\cdot 684 - 1\cdot 662)} = \mathbf{3^\circ}$$

Net deviation $= A(\mu - 1) - A'(\mu' - 1)$

$$= 6^\circ\left[\frac{1\cdot 531 + 1\cdot 520}{2} - 1\right] - 3^\circ\left[\frac{1\cdot 684 + 1\cdot 662}{2} - 1\right]$$

$$= 6^\circ(1\cdot 5255 - 1) - 3^\circ(1\cdot 673 - 1) = \mathbf{1\cdot 134^\circ}$$

20. ***For crown glass :*** $\omega = 0{\cdot}03$, $\mu_b - \mu_r = 0{\cdot}015$

For flint glass : $\omega' = 0{\cdot}05$, $\mu_b' - \mu_r' = 0{\cdot}022$

For no dispersion, $A(\mu_b - \mu_r) = A'(\mu_b' - \mu_r')$

$$\text{or} \qquad A \times 0\cdot 015 = A' \times 0\cdot 022$$

$$\text{or} \qquad A = \frac{22}{15}A' \qquad \ldots(i)$$

Net deviation $= A(\mu - 1) - A'(\mu' - 1)$,

$$= \frac{A(\mu - 1)(\mu_b - \mu_r)}{\mu_b - \mu_r} - \frac{A'(\mu' - 1)(\mu_b' - \mu_r')}{\mu_b' - \mu_r'}$$

$$= \frac{A(\mu_b - \mu_r)}{\omega} - \frac{A'(\mu_b' - \mu_r')}{\omega'}$$

$$\text{or } 2^\circ = \frac{22\,A'}{15} \times \frac{0\cdot 015}{0\cdot 03} - \frac{A' \times 0\cdot 022}{0\cdot 05}$$

$$= 0\cdot 733\,A' - 0\cdot 44\,A'$$

or $A' = \mathbf{6{\cdot}83^\circ}$

From equation (*i*), we have

$$A = \frac{22}{15} \times 6\cdot 83^\circ = \mathbf{10\cdot 02^\circ}$$

CHAPTER 3

Optical Instruments

3.01. SIMPLE MICROSCOPE

A convex lens of *short focal length* can be used to see magnified image of a small object and is called ***a magnifying glass*** or a ***simple microscope***.

Principle. When a small object is placed between optical centre and focus of a convex lens, its *virtual, erect and magnified* image is formed on the *same side* of the lens. The lens is held close to eye and the distance of the object is adjusted, till the image is formed at *the least distance of distinct vision* from the eye. For a normal eye, the least distance of distinct vision is 25 cm.

Working. Let AB be an object placed between the points F and C. Its virtual image A′B′ will be formed on the same side of the object. If the distance of the object is changed from the lens, then the distance of the image also changes. The position of object AB is so adjusted that the image A′B′ is formed at the least distance of distinct vision (D) as shown in Fig. 3.01.

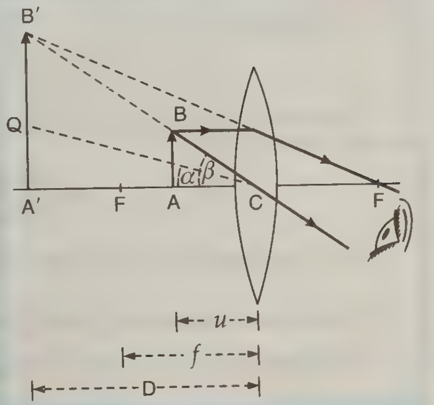


Fig. 3.01

Key point

A simple microscope is a convex lens of short focal length.

Magnifying power (*When image is formed at D*). *It is defined as the ratio of the angle subtended by the image at the eye and the angle subtended by the object seen directly, when both lie at the least distance of distinct vision.* It is also called *angular magnification* produced by the simple microscope. It is denoted by M.

Let $\angle A'CB' = \beta$ be the angle subtended by the image at the eye. Cut A′Q equal to AB and join QC. Then, $\angle A'CQ' = \alpha$ is the angle subtended by the object at the eye, when it is placed at the least distance of distinct vision.

By definition, magnifying power of the simple microscope is given by

$$M = \frac{\beta}{\alpha}$$

In practice, angles α and β are small. Therefore, angles α and β can be replaced by their tangents *i.e.*

$$M = \frac{\tan\beta}{\tan\alpha} \qquad ...(3.01)$$

From right angled Δ CA′Q, $\tan\alpha = \frac{A'Q}{CA'} = \frac{AB}{CA'}$ $\qquad (\because A'Q = AB)$

Also, from right angled Δ CAB, $\tan\beta = \frac{AB}{CA}$

Substituting for $\tan\alpha$ and $\tan\beta$ in equation (3.01), we have

$$M = \frac{AB/CA}{AB/CA'} = \frac{CA'}{CA} \qquad ...(3.02)$$

If CA = u, the distance of object AB from the lens and CA′ = v, the distance of image A′B′ from the lens, then

$$M = \frac{v}{u} \qquad ...(3.03)$$

If f is focal length of the lens acting as simple microscope, then

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or

$$\frac{v}{u} = 1 - \frac{v}{f} \qquad ...(3.04)$$

From equations (3.03) and (3.04), we have

$$M = 1 - \frac{v}{f}$$

Applying new cartesian sign conventions :

$$v = -D\,;\ f = +f$$

$\therefore$ $$M = 1 - \frac{-D}{+f}$$

or $$M = 1 + \frac{D}{f} \qquad ...(3.05)$$

From equation (3.05), it follows that *lesser is the focal length of the convex lens used as simple microscope, greater is the value of the magnifying power obtained.*

Further, the ***positive*** value of magnifying power of a simple microscope tells that image formed is *erect and hence* virtual.

Magnifying power (*When image is formed at infinity*). *It is defined as the ratio of the angle formed by the image (situated at infinity) at the eye to the angle formed by object at the eye, when situated at least distance of distinct vision. It is denoted by M.*

Draw a line A′B′ = AB and perpendicular to principal axis at a distance CA′ = D (least distance of distinct vision). Join B′C′. Then ∠B′CA′ = α is the angle formed by object at the eye, when situated at distance D [Fig. 3.02].

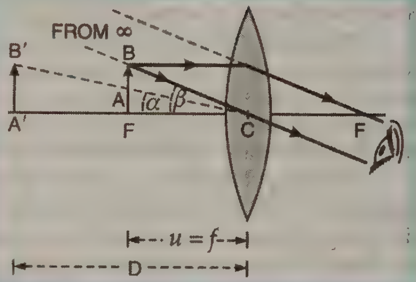


Fig. 3.02

The angle formed by the image situated at infinity at the eye is same as the angle formed by the object AB at the eye. Thus, ∠BCA = β is the angle formed by the image at the eye,

By definition,

$$M = \frac{\beta}{\alpha}$$

In practice, angles α and β are small. Therefore, angles α and β can be replaced by their tangents *i.e.*

$$M = \frac{\tan\beta}{\tan\alpha} \qquad ...(3.06)$$

From right angled Δ B′ AC, $\tan\alpha = \frac{A'B'}{CA'} = \frac{AB}{CA'}$ $\quad (\because A'B' = AB)$

Also, from right angled Δ BAC, $\tan\beta = \frac{AB}{CF}$

Substituting for tan α and tan β in equation (3.06), we have

$$M = \frac{AB}{CF} \times \frac{CA'}{AB} = \frac{CA'}{CF}$$

Applying new cartesian sign conventions :

$$CF = -f\,;\, CA' = -D$$

$\therefore$ $$M = \frac{-D}{-f}$$

or $$M = \frac{D}{f} \qquad ...(3.07)$$

The above equation gives magnifying power of a simple microscope, when image is formed at infinity.

Uses. 1. Jewellers and watch makers make use of convex lens of short focal length to obtain a magnified view of the fine jewellery work and the small components of the watches.

2. In science laboratories, a magnifying glass is used to see slides and to read the vernier scales attached to the instruments.

3. The use of magnifying glass enables us to place the object close to eye, making it appear bright and yet clearly visible. In position AB, object lies close to the eye. In absence of lens, the object will not be clearly visible.

Note. 1. The equation (3.05) gives magnifying power of a simple microscope, when the image is formed at least distance of distinct vision *i.e.* when the image is seen distinctly. If the object is brought closer to the lens, the image also moves closer to the eye. No doubt, the magnifying power of the simple microscope increases, but the image can not be seen distinctly. On the other hand, if the object is moved away from the lens (up to its focus), the magnifying power decreases. The virtual image of the object can be seen, till the distance of the object does not increase the focal length of the lens. When the object lies at the focus, the image is formed at infinity and the magnifying power of the simple microscope is given by equation (3.07). From equations (3.05) and (3.07), it follows that magnifying power of the simple microscope is one less, when the image is formed at infinity. However, the viewing of the image is more comfortable.

2. The magnification produced by a lens is given by

$$m = \frac{I}{O} = \frac{v}{u}$$

The magnifying power of the simple microscope and the magnification produced by the lens are different from each other. The two become equal, only when the image is formed at the least distance of distinct vision.

3.02. COMPOUND MICROSCOPE

A compound microscope is used to see extremely small objects.

It consists of two lenses. A lens of ***short aperture*** and ***short focal length*** facing the object is called ***object lens**** and another lens of ***short focal length*** but ***large aperture*** is called ***eye lens***†. The two lenses are placed coaxially at the two ends of a tube. To focus over an object, the distance of the object lens from the object is adjusted with the help of rack and pinion arrangement.

Key point

In a compound microscope, object lens is of short focal length and short operature, while eye lens has short focal length and large operature.

Principle. When a small object is placed just outside the focus of the object lens, its real, inverted and magnified image is produced on the other side of the lens between F and 2 F. The image produced by object lens acts as object for the eye lens. The distance of object from the object lens is so adjusted that the final image is formed at the least distance of distinct vision from the eye.

Working. When a small object is placed just outside the focus of the object lens, its real, inverted and magnified image is produced on the other side of the lens between F and 2 F. The image produced by object lens acts as object for the eye lens. The distance of object from the object lens is so adjusted that the final image is formed at the least distance of distinct vision from the eye.

Let AB be an object placed just outside the focus F_o of the object lens. Its virtual image A′B′ is formed on the other side of the lens. The image A′B′ lies between focus F_e and optical centre C′ of the eye lens [Fig. 3.03] and it acts as object for the eye lens. Using the rack and pinion arrangement, the distance between object lens and the object AB is adjusted, till its virtual and magnified image A′′B′′ is formed on the same side at the least distance of distinct vision.

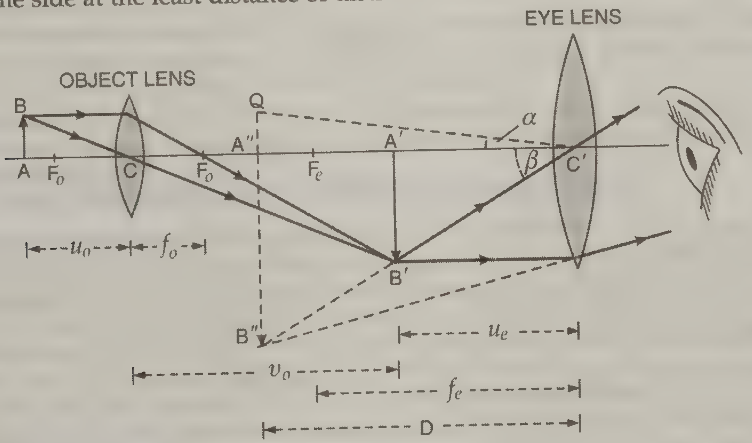


Fig. 3.03

Magnifying power (***When image is formed at D***). *It is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object seen directly, when both are placed at the least distance of distinct vision.*

Let $\angle A''C'B'' = \beta$ be the angle subtended by the final image at the eye. Cut A′′Q equal to AB and join QC′. Then, $\angle A''C'Q = \alpha$, the angle subtended by the object at the eye, when situated at the least distance of distinct vision. By definition, magnifying power of the compound microscope,

$$M = \frac{\beta}{\alpha}$$

Since the angles α and β are small, they can be replaced by their tangents *i.e.*

$$M = \frac{\tan \beta}{\tan \alpha} \qquad \text{...(3.08)}$$

---

*In practice, to eliminate chromatic aberration, a combination of two lenses in contact is used. It is called *objective*.

†In place of an eye lens, a combination of two lenses at certain distance apart satisfying certain conditions (to minimize chromatic and spherical aberrations) is used. It is called *eye piece*.

From the right angled Δ C′A″Q, $\tan\alpha = \frac{A''Q}{C'A''} = \frac{AB}{C'A''}$ $\quad (\because A''Q = AB)$

Also, from the right angled D C′A″B″, $\tan\beta = \frac{A''B''}{C'A''}$

Substituting the values of tan α and tan β in equation (3.08), we have

$$M = \frac{A''B''/C'A''}{AB/C'A''} = \frac{A''B''}{AB}$$

Multiplying and dividing by A′B′, we have

$$M = \frac{A''B''}{AB} \times \frac{A'B'}{A'B'} = \frac{A'B'}{AB} \times \frac{A''B''}{A'B'}$$

or
$$M = m_o \times m_e, \qquad ...(3.09)$$

where $\frac{A'B'}{AB} = m_o$, the magnification produced by object lens

and $\frac{A''B''}{A'B'} = m_e$, the magnification produced by eye lens.

To find m_o : Let CA = u_o, the distance of object AB from object lens ; and CA′= v_o, the distance of image A′B′ from object lens.

By definition, linear magnification produced by object lens,

$$m_o = \frac{v_o}{u_o} \qquad ...(3.10)$$

To find m_e : Let C′A′ = u_e, the distance of object A′B′ from eye lens and C′A″ = v_e ,the distance of image A″B″ from eye lens

Again, by definition, linear magnification produced by eye lens

$$m_e = \frac{v_e}{u_e} \qquad ...(3.11)$$

Now, for the eye lens,

$$-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$$

or
$$\frac{v_e}{u_e} = 1 - \frac{v_e}{f_e} \qquad ...(3.12)$$

From equations (3.11) and (3.12), we have

$$m_e = 1 - \frac{v_e}{f_e}$$

Applying new cartesian sign conventions :

$$v_e = -D ; f_e = +f_e$$

∴
$$m_e = 1 - \frac{-D}{+f_e}$$

or
$$m_e = 1 + \frac{D}{f_e} \qquad ...(3.13)$$

Using equations (3.10) and (3.13), the equation (3.09) becomes

$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) \qquad ...(3.14)$$

In practice, the focal length of the object lens is very short and the object AB is placed just outside the focus of object lens.

∴
$$u_o \approx f_o \qquad \text{(in magnitude)}$$

So that the final image is formed at least distance of distinct vision from the eye, the image A′B′ must lie between optical centre and focus of the eye lens. Since, the focal length of the eye lens is also small, the distance of the image A′B′ from the object lens is nearly equal to the length of the microscope tube *i.e.*

$$v_o \approx L$$

According to new cartesian sign conventions :

$$u_o \approx -f_o \text{ and } v_o \approx L$$

Key point

In a compound microscope, object lens and eye lens are placed at a fixed distance apart. The microscope is focussed on an object by changing distance of the object lens from it with the help of rack and pinion arrangement.

Substituting for u_o and v_o in equation (*vii*), we have

$$M = -\frac{L}{f_o}\left(1+\frac{D}{f_e}\right) \qquad ...(3.15)$$

From the above expression, it follows that a compound microscope will have large magnifying power, if both the object lens and the eye lens are of small focal length. In practice, focal length of object lens is smaller than that of eye lens *i.e.* $f_o < f_e$. Further the ***negative*** value of magnifying power of compound microscope tells that final image formed is ***inverted.***

Magnifying power (*When image is formed at infinity*). The magnifying power of a compound microscope,

$$M = m_o \times m_e$$

If u_o is distance of object from the object lens and v_o, then distance of image from it, then as obtained earlier,

$$m_o = \frac{v_o}{u_o}$$

Further, as discussed in case of a simple microscope ; it can be obtained that

$$m_e = \frac{D}{f_e},$$

where f_e is focal length of the eye lens.

Therefore, magnifying power of a compound microscope is given by

$$M = \frac{v_o}{u_o} \times \frac{D}{f_e} \qquad ...(3.16)$$

3.03. TELESCOPE

A telescope is an instrument for seeing distant objects clearly and magnified, though apparently. It is of two types :

1. Astronomical telescope. A telescope used to see heavenly bodies is called *astronomical telescope*. It produces a virtual and inverted image. As such bodies are round, the inverted image does not affect the observation.

2. Terrestrial telescope. For observing objects on earth, it would be necessary to have an erect image. A telescope used for observing objects on earth is called a terrestrial telescope.

3.04. ASTRONOMICAL TELESCOPE (Refracting Type)

An astronomical telescope is used to see heavenly objects.

An astronomical refracting telescope consists of two lens systems. The lens system facing the object is called ***objective***. It has large aperture and is of ***large focal length*** (f_o). The other lens system is called ***eye-piece***. It has ***small aperture*** and is ***of short focal length*** (f_e). The objective and the eye-piece are mounted coaxially in two metallic tubes. The tube holding the eye-piece can be made to slide into the tube holding the objective with the help of rack and pinion arrangement.

Key point ✓

In a telescope, object lens is of large focal length and large operature, while eye lens has short focal length and short operature.

Principle. The objective forms the real and inverted image of the distant object at its focal plane. The position of the eye-piece is adjusted, till the final image is formed ***at least distance of distinct vision.*** In case, position of the eye-piece is so adjusted that final image is formed at infinity, the telescope is said to be in ***normal adjustment.*** Let us now discuss the formation of the final image in the two cases :

(*a*) When final image is formed at infinity (Normal adjustment). When a parallel beam of light rays from the distant object falls on the objective, its real and inverted image A′B′ is formed on the other side of the objective [Fig. 3.04]. If the

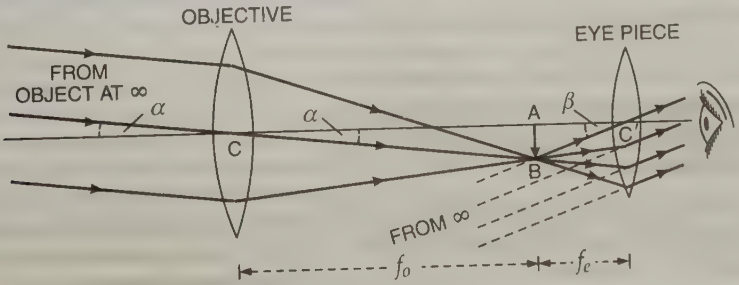


Fig. 3.04

position of eye-piece is adjusted, so that the image A′B′ lies at its focus, then the final highly magnified image will be formed at infinity.

Magnifying power. *Magnifying power of a telescope in normal adjustment is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object seen directly, when both the object and the image lie at infinity.* It is also called *angular magnification* of the telescope and is denoted by M.

As the object is at a very large distance, the angle subtended by it at the eye is practically the same as that subtended by it at the objective.

Thus, $\angle A'CB' = \alpha$ may be considered as the angle subtended by object at the eye. Let $\angle A'C'B' = \beta$.

Then, by definition, $M = \dfrac{\beta}{\alpha}$

Since the angles α and β are small, $\alpha \approx \tan\alpha$ and $\beta \approx \tan\beta$. Therefore,

$$M = \frac{\tan\beta}{\tan\alpha} \qquad \text{...(3.17)}$$

From right angled $\Delta\, CA'B'$, $\tan\alpha = \dfrac{A'B'}{CA'}$

and from right angled $\Delta\, C'A'B'$, $\tan\beta = \dfrac{A'B'}{C'A'}$

Substituting for $\tan\alpha$ and $\tan\beta$ in equation (3.17), we have

$$M = \frac{A'B'/C'A'}{A'B'/CA'} = \frac{CA'}{C'A'} \qquad \text{...(3.18)}$$

Applying new cartesian sign conventions :

$CA' = +f_o$ ($\because$ distance of A′B′ from object lens is *along* the incident light)

$C'A' = -f_e$ ($\because$ distance of A′B′ from eye lens is *against* incident light)

Substituting for CA′ and C′A′ in equation (3.18), we have

$$M = -\frac{f_o}{f_e} \qquad \text{...(3.19)}$$

It follows that the magnifying power of a telescope in normal adjustment will be large, if objective is of large focal length and the eye-piece is of short focal length.

Further, when telescope is in normal adjustment, the distance between the two lenses is equal to sum of their focal lengths ($f_o + f_e$).

Further, the ***negative*** value of the magnifying power of the telescope tells that final image formed is ***inverted*** **and** ***real.***

(*b*) When final image is formed at least distance of distinct vision. When a parallel beam of light rays from the distant object falls on the objective, its real and inverted image A′B′ is formed on the other side of the objective. The position of eye-piece is adjusted so that the final image A″B″ is formed at least distance of distinct vision. The course of the rays will be as shown in Fig. 3.05.

Key point ✓

When a telescope is in normal adjustment, distance between the object lens and eye lens is equal to the sum of their focal lengths.

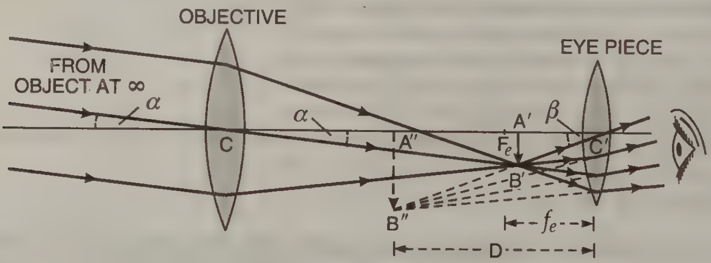


Fig. 3.05

Magnifying power. *Magnifying power of a telescope is defined as the ratio of the angle subtended at the eye by the image formed at the least distance of distinct vision to angle subtended at the eye by the object lying at infinity, when seen directly.*

Again, as the object is at a very large distance, the angle α subtended by it at the objective is practically the same as that subtended by it at the eye. Therefore, if $\angle A''C'B'' = \beta$, then

$$M = \frac{\beta}{\alpha}$$

Again, as angle α and β are small, they can be replaced by their tangents. erefore,

$$M = \frac{\tan\beta}{\tan\alpha} \qquad \text{...(3.20)}$$

From right angledΔ CA′B′, $\tan\alpha = \frac{A'B'}{CA'}$

and from right angled Δ C′A′B′, $\tan\beta = \frac{A'B'}{C'A'}$

In equation (3.20), substituting for tan α and tan β, we have

$$M = \frac{A'B'/C'A'}{A'B'/CA'} = \frac{CA'}{C'A'} \qquad \text{...(3.21)}$$

Now, $\quad CA' = f_o$ and $C'A' = u_e$

In equation (3.21), subsituting for CA′ and C′A′, we get

$$M = \frac{f_o}{u_e} \qquad \text{...(3.22)}$$

For eye lens, we have

$$-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e} \quad \text{or} \quad \frac{1}{u_e} = -\frac{1}{f_e} + \frac{1}{v_e}$$

or
$$\frac{1}{u_e} = -\frac{1}{f_e}\left(1 - \frac{f_e}{v_e}\right)$$

Substituting for $\frac{1}{u_e}$ in equation (3.22), we have

$$M = -\frac{f_o}{f_e}\left(1 - \frac{f_e}{v_e}\right)$$

Applying new cartesian sign conventions :

$$f_o = +f_o\,;\ v_e = -D\,;\ f_e = +f_e$$

$\therefore$
$$M = -\frac{+f_o}{+f_e}\left(1 - \frac{+f_e}{-D}\right)$$

or
$$M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) \qquad \text{...(3.23)}$$

Therefore, a refracting telescope will have large magnifying power, if the object ens is of large focal length and eye lens is of short focal length. Further, the ***negative*** alue of magnifying power of the telescope tells that the final image formed is ***nverted*** and ***real.***

Key point ✓

In a telescope, the distance between object lens and eye lens depends upon the position of the object. The telescope is focussed on a distant object by varying the distance between the two lenses with the help of rack and pinion arrangement.

Note. 1. From the equation (3.19) and (3.23), it follows that the magnifying power of a telescope is greater, when the image is formed at least distance of distinct vision than in the case, when the telescope is in normal adjustment.

2. It may be pointed out that the image formed by a telescope is actually not magnified. A telescope is used to increase the ***visual angle***. A distant object, even very large in size forms a very small angle α at the eye. By producing the image close to the eye (at least distance of distinct vision), the telescope increases the angle (β), in which the image appears to be formed.

3.05. REFLECTING TYPE TELESCOPE

An astronomical telescope used to see the distant stars should possess ***large light gathering power*** in order to obtain a bright image of even a faint star and ***high resolving power*** so as to observe its minute and finer details. Both the light gathering power and the resolving power for a telescope will be large, if the objective of the telescope is of large aperture. However, it cannot be achieved in a refracting type astronomical telescope. It is because, a convex lens of large aperture free from spherical and chromatic aberrations cannot be designed.

Watch out !

The image produced by a telescope is inverted laterally also !

Reflecting type telescope was designed by Newton in order to overcome the drawbacks of refracting type telescope. In a reflecting type telescope, a concave mirror of large aperture is used as objective in place of a convex lens. It possesses a large light gathering power and a high resolving power. Due to this, it enables us to see even faint stars and observe their minute details.

Fig. 3.06 shows the various parts of the reflecting type telescope and the course of rays showing the formation of the final image of the distant star. The concave mirror of large aperture and large focal length is fixed at one end of the tube T_1, whose other end is kept open to allow the light from the distant star to fall on the spherical concave mirror. Before the reflected rays of light meet to form the real, inverted and diminished image of the star at the focal plane of the concave mirror, they are intercepted and reflected by a plane mirror (M) inclined at an angle of 45° to the axis of the tube T_1. The plane mirror deviates the rays and the real image is formed in front of the *eye-piece* fitted in a side metal tube T_2 with its axis at a right angle to the axis of the tube T_1. The eye-piece only acts as magnifier and produces the final virtual and magnified image of the star enabling eye to see it distinctly.

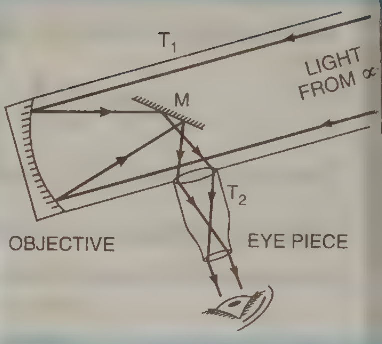


Fig. 3.06

If f_o is focal length of the concave spherical mirror used as objective and f_e, the focal length of the eye-piece, then magnifying power* of the reflecting telescope is given by

$$M = \frac{f_o}{f_e}$$

Further, if D is diameter of the objective and d, the diameter of the pupil of the eye, then brightness ratio is given by

$$B = \frac{D^2}{d^2}$$

A concave mirror of very large diameter can be easily manufactured and such a telescope may be used to observe even very faint stars.

Advantages of a reflecting type telescope. A reflecting type telescope has following advantages over a refracting type telescope.

1. Since objective is not a lens, the image formed is free from chromatic aberration.

2. Spherical aberration can be minimised by making use of a parabolic mirror in place of concave spherical mirror as objective. It enables to produce a very sharp and distinct image.

3. A concave spherical mirror of large aperture can be easily manufactured and hence due to very large light gathering power, it enables to see even very faint stars.

4. Also, in a reflecting type telescope, the objective (a concave spherical mirror) of large aperture provides a very high resolving power so as to see even the minute and fine details of the distant stars.

3.06. SOME IMPORTANT FEATURES OF A TELESCOPE

A telescope should have *high magnifying power, high resolving power* and *large light gathering power* for the reasons explained below :

1. High magnifying power. A telescope having high magnifying power produces apparently a magnified image and hence enables to see the finer details of the distinct object. When in normal adjustment (final image formed at infinity), the magnifying power of a telescope is given by

$$M = \frac{f_o}{f_e},$$

where f_o and f_e are respectively the focal length of objective and eye-piece. Hence, *for high value of magnifying power, objective should be of large focal length, while eye-piece should be of small focal length.*

2. High resolving power. A telescope of high resolving power has ability to show the images of two very near parts of an object as separate. Therefore, a telescope of high resolving power also enables to observe the finer details of the distant object. The angular limit of resolution (reciprocal of resolving power) is given by

$$\theta = \frac{1 \cdot 22\,\lambda}{D},$$

where D is diameter of objective and λ is wavelength of the light used. Hence, *a*

---

* The magnifying power of a reflecting type telescope is also negative. It is because, focal length of a concave mirror (f_o) is negative.

scope will have small value of angular limit of resolution and hence high resolving power, jective of large diameter and light of short wavelength are used to see the distant object.

3. Large light gathering power. A telescope of large light gathering power ects a large amount of light from the distant object and hence the image formed l be very bright. The light gathering power and hence the brightness of the image lirectly proportional to the area of the objective *i.e.*

$$\text{brightness} \propto \frac{\pi D^2}{4},$$

ere D is diameter of the objective of the telescope.

The brightness ratio of a telescope is defined as the ratio of the light gathered by the scope to that gathered by the unaided eye from the distant object.

If D is diameter of the telescope objective and *d*, the diameter of the pupil of eye, then brightness ratio is given by

$$B = \frac{\pi D^2 / 4}{\pi d^2 / 4} = \frac{D^2}{d^2}$$

Therefore, for large light gathering power and hence large brightness ratio, the telescope ective should be of large aperture.

3.07. COMPARISON OF REFRACTING AND REFLECTING TYPE TELESCOPES

There are following points of difference between the two types of telescopes :

Refracting type telescope	Reflecting type telescope
The objective is a converging lens system.	1. The objective is a concave spherical mirror.
It suffers from chromatic aberration and hence the image obtained is multicoloured and blurred.	2. As the objective is not a lens, it is free from chromatic aberration.
It suffers from spherical aberration also. It also results in a blurred image.	3. The spherical aberration is minimised by using a parabolic mirror in place of the concave spherical mirror as objective.
As a lens of large aperture cannot be manufactured easily, its light gathering power is low and hence cannot be used to see faint stars.	4. A concave spherical mirror of large aperture can be easily made. Such a telescope will possess high light gathering power and hence can be used to see even faint stars.
The largest refracting type telescope at Yerkes has objective of diameter 1 m or about 40 inches.	The reflecting type telescope at Mount Polomar, U.S.A. is of 200 inch objective, while one in U.S.S.R. is of 234 inch objective.

The above points of comparison lead to the conclusion that in astronomical esearch, a reflecting type telescope is more advantageous than a refracting type elescope and hence they are widely used all over the world.

3.08. PRISM BINOCULARS

To increase the magnifying power of an astronomical telescope, it is necessary use an objective of large focal length. As a result, it will require a telescope tube f large length.

Prism binoculars is an indigenous arrangement for using a short tube and at ne same time making it possible to use objective of large focal length. The instrument ssentially consists of a pair of astronomical telescopes and totally reflecting prisms Fig. 3.07]. The use of prisms not only cuts short the length of the tube but produces n erect and magnified image free from lateral inversion along with a wide field f view.

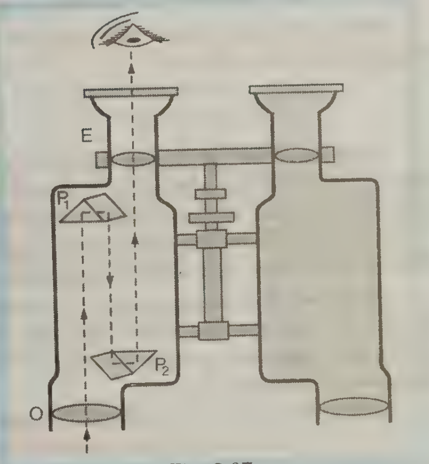


Fig. 3.07

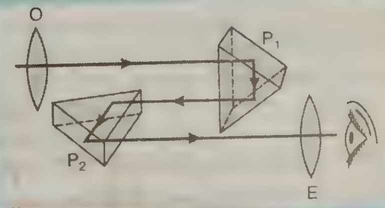


Fig. 3.08

Theory. Two totally reflecting right-angled prisms P_1 and P_2 with their efracting edges perpendicular to each other are arranged between the objective O nd the eye-piece E of each of the two telescopes. The objective produces a real nd inverted image but the prism P_1 renders it erect [Fig. 3.08]. However, the mage becomes laterally inverted. The image is corrected for lateral inversion by he second prism P_2. The reversal of the image by the two internal reflection in each prism is further explained in Fig. 3.09. Here, *a* and *b* represent the incident rays and *'* and *b'* the reflected rays in reverse order. If the incident ray *a* is above *b*, the eflected ray *a'* is below *b'*, so that the image is inverted. If *a* is to the left of *b* as seen rom the direction of the source, then *a'* is to the right of *b'*, when viewed from the same position.

The magnifying power of a telescope can be increased by using an objective of large focal length as the magnification is equal to the ratio of the focal length of the objective to that of the eye-piece. In prism binoculars, the arrangement of prisms enables us to use an objective of large focal length, as the rays of light traverse the distance between the objective and the eye-piece thrice before converging to produce the image. This not only increases the magnification but also makes the telescope short and compact. Further, as the distance between the objectives of the two telescopes is greater than between the two eyes, *the field of view is wider and objects present a better three-dimensional view, when seen through it.*

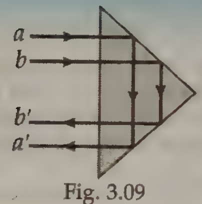


Fig. 3.09

SOLVED NUMERICAL PROBLEMS

Type A. On Simple microscope

Problem 3.01. Calculate the magnifying power of a magnifying glass of 5 cm focal length. Distance of distinct vision = 25 cm.

Sol. Here, $f = 5$ cm, D = 25 cm ;

Now, $$M = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 1 + 5 = 6$$

Problem 3.02. A converging lens of focal length 6·25 cm is used as a magnifying glass. If the near point of the observer is 25 cm from the eye and the lens is held close to the eye, calculate (*i*) the distance of the object from the lens and (*ii*) the angular magnification.

Find the angular magnification, when the final image is formed at infinity. (*I.S.C.E. 1993*)

Sol. Here, $f = 6{\cdot}25$ cm ; $v = -D = -25$ cm

(*i*) Let u be the distance of object from the lens. Then,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or $$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{6{\cdot}25} = -\frac{1}{25} - \frac{4}{25} = \frac{-5}{25}$$

or $u = -5$ **cm**

(*ii*) Now, angular magnification,

$$M = 1 + \frac{D}{f} = 1 + \frac{25}{6{\cdot}25} = 1 + 4 = 5$$

When the final image is formed at infinity, the angular magnification is given by

$$M = \frac{D}{f} = \frac{25}{6{\cdot}25} = 4$$

Problem 3.03. A figure divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass of focal length 10 cm held close to the eye.

(*a*) What is the magnification produced by the lens ? How much is the area of each square in the virtual image ?

(*b*) At what distance should the lens be held from the figure in order to view the squares distinctly with the maximum possible magnification ? What is the magnification in this case ? (*Text Problem*)

Sol. Here, size of each square in the figure,

$O = 1$ mm^2 ; $u = -9$ cm ; $f = 10$ cm

(*a*) The magnification produced by the lens, $M = \frac{v}{u}$

For a lens, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\therefore \quad -\frac{1}{-9} + \frac{1}{v} = \frac{1}{10} \quad \text{or} \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{9}$$

or $v = -90$ cm

$$\therefore \quad M = \frac{-90}{-9} = 10$$

Now, each square in the figure has area 1 mm^2 *i.e.* 1 m × 1mm. Since the lens produces a linear magnification of 1 size of the each square in the image will appear as 10 mm 10 mm *i.e.* **100 mm^2.**

(*b*) Maximum magnifying power is obtained, when th virtual image is formed at the least distance of distinct vision

For a lens, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Here $v = -D = -25$ cm ; $f = +10$ cm

$$\therefore \quad -\frac{1}{u} + \frac{1}{-25} = \frac{1}{10} \quad \text{or} \quad \frac{1}{u} = -\frac{1}{25} - \frac{1}{10} = -\frac{7}{50}$$

or $$u = -\frac{50}{7} = -7{\cdot}14 \text{ cm}$$

When the object is placed so that the image is formed a least distance of distinct vision ; magnifying power is given by

$$M = \left(1 + \frac{D}{f}\right) = 1 + \frac{25}{10} = 3{\cdot}5$$

Problem 3.04. Refer to solved problem 3.03 (*a*). At what distance should the lens be held from the figure in order to view the squares distinctly with the maximum possible magnification ?

(*b*) What is the magnification (image size/object size) in this case ?

(*c*) Is the magnification equal to the magnifying power in this case ? Explain. (*Text Problem*)

Sol. (*a*) Maximum magnifying power is obtained, when the virtual image is formed at the least distance of distinct vision.

For a lens, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Here, $v = -25$ cm ; $f = +10$ cm ;

$$\therefore \quad -\frac{1}{u} + \frac{1}{-25} = \frac{1}{10} \quad \text{or} \quad \frac{1}{u} = -\frac{1}{25} - \frac{1}{10} = -\frac{7}{50}$$

or $$u = -\frac{50}{7} = 7{\cdot}14 \text{ cm}$$

(*b*) Magnification, $m = \frac{v}{u} = \frac{-25}{-50/7} = 3{\cdot}5$

(*c*) When the object is placed, so that the image is formed at least of distinct vision ; magnifying power is given by

$$M = \frac{D}{u} = \left(1 + \frac{D}{f}\right)$$

Since $f = +10$ cm, we have

$$M = \left(1 + \frac{D}{f}\right) = 1 + \frac{25}{10} = 3{\cdot}5$$

Thus, the magnifying power and the magnification of age are equal in this case.

For explanation, refer to the note given in section 3.01.

ype B. On Compound microscope

Problem 3.05. A compound microscope has a magni- ation of 30. The focal length of its eye-piece is 5 cm. ssuming the final image to be formed at least distance of stinct vision (25 cm), calculate the magnification produced the objective. *(C.B.S.E. 1991)*

Sol. Here, $M = -30$; (magnifying power is negative)

$$f_e = 5 \text{ cm} ; D = 25 \text{ cm} ;$$

Now, $$M = m_o \times m_e = m_o \left(1 + \frac{D}{f_e}\right)$$

$$\therefore \quad -30 = m_o \left(1 + \frac{25}{5}\right) \quad \text{or} \quad m_o = -5$$

Problem 3.06. The focal lengths of the objective and ye-piece of a compound microscope are 4 cm and 6 cm espectively. If an object is placed at a distance of 6 cm from e objective, what is the magnification produced by the icroscope ? Distance of distinct vision = 25 cm.

Sol. Here, $f_o = 4$ cm ; $f_e = 6$ cm ; $D = 25$ cm ; $u_o = -6$ cm ;

Suppose that the image of the object is formed by the bject lens at a distance v_o from it. Then, the distance v_o is iven by

$$-\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$$

or $$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{+4} + \frac{1}{-6} = \frac{1}{12}$$

or $$v_o = 12 \text{ cm}$$

Now, $$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

$$\therefore \quad M = \frac{12}{-6}\left(1 + \frac{25}{6}\right) = -2 \times \frac{31}{6} = -10{\cdot}33$$

Problem 3.07. The focal length of the eye piece and objective of a compound microscope are 5 cm and 1 cm respectively and the length of the tube is 20 cm. Calculate the magnifying power of the microscope, when the final image is formed at infinity. The value of least distance of distinct vision is 25 cm. *(I.S.C.E. 1998)*

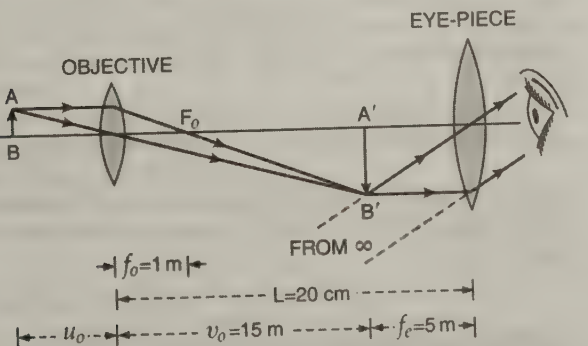


Fig. 3.10

Sol. Fig. 3.10 shows the formation of the final image of the object at infinity. When the object AB is placed at distance CA = u_o from the objective, its real image A′B′ is formed on the other side of the lens at a distance CA′ = v_o from it. The final image will be formed at infinity, if the image lies at distance C′A′ = f_e = 5 cm from the eye lens. Since length of the microscope tube is L = 20 cm, it follows that

$$v_o = L - f_e = 20 - 5 = 15 \text{ cm}$$

For object lens : $$-\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$$

Here, $$f_o = 1 \text{ cm} ; v_o = 15 \text{ cm}$$

$$\therefore \quad \frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{15} - \frac{1}{1} = -\frac{14}{15}$$

or $$u_o = -\frac{15}{14} \text{ cm}$$

When the final image is formed at infinity, the magnifying power of the microscope is given by

$$M = \frac{v_o}{u_o} \times \frac{D}{f_e}$$

Here, $$D = 25 \text{ cm} ; f_e = 5 \text{ cm}$$

$$\therefore \quad M = \frac{15}{-15/14} \times \frac{25}{5} = -70$$

The negative sign shows that the image formed is *inverted* and *real*.

Type C. On Astronomical telescope

Problem 3.08. An astronomical telescope, consists of two thin lenses set 36 cm apart and has a magnifying power 8. Calculate the focal length of the lenses. *(C.B.S.E. 1997)*

Sol. Here, $$f_o + f_e = 36 \text{ cm} \qquad ...(i)$$

$$M = -8 \quad \text{(magnifying power is negative)}$$

Now, $$M = -\frac{f_o}{f_e}$$

$$\therefore \quad -8 = -\frac{f_o}{f_e}$$

or $$f_o = 8 f_e \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$8 f_e + f_e = 36 \quad \text{or} \quad f_e = 4 \text{ cm}$$

Again $$f_o = 8 f_e = 8 \times 4 = 32 \text{ cm}$$

Problem 3.09. The focal length of the objective of an astronomical telescope is 75 cm and that of the eye-piece is 5 cm. If the final image is formed at the least distance of distinct vision from the eye, calculate the magnifying power of the telescope.

Sol. Here, $f_o = 75$ cm ; $f_e = 5$ cm

We know, D = 25 cm

Now, $$M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = -\frac{75}{5}\left(1 + \frac{5}{25}\right)$$

$$= -15 \times 1{\cdot}2 = -18$$

Problem 3.10. A small telescope has an objective lens of focal length 140 cm and eye-piece of focal length 5·0 cm. What is the magnifying power of the telescope for viewing distant objects, when (*a*) the telescope is in normal adjustment and (*b*) the final image is formed at least distance of distinct vision ?

Sol. Here, $f_o = 140$ cm ; $f_e = 5{\cdot}0$ cm

(*a*) When telescope is in normal adjustment,

$$M = -\frac{f_o}{f_e} = -\frac{140{\cdot}0}{5{\cdot}0} = -28$$

(*b*) When image is formed at least distance of distinct vision, then

$$M = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right) = -\frac{140}{5\cdot0}\left(1+\frac{5\cdot0}{25}\right)$$

$$= -28 \times (1+0\cdot2) = -33\cdot6$$

Problem 3.11. A telescope objective of focal length 1 m forms a real image of the moon 0·92 cm in diameter. Calculate the diameter of the moon taking its mean distance from the earth to be 38 × 10⁴ km. If the telescope uses an eye piece of 5 cm focal length, what would be the distance between the two lenses for (*i*) the final image to be formed at infinity, (*ii*) the final image (virtual) at 25 cm from the eye.

(*I.S.C.E. 1997*)

Sol. Here, $f_o = 1$ m ; $f_e = 5$ cm $= 0\cdot05$ m

Size of the image of the the moon,

$$I = 0\cdot92 \text{ cm} = 0\cdot92 \times 10^{-2} \text{ m}$$

Mean distance of the moon from earth,

$$u_o = 38 \times 10^4 \text{ km} = 38 \times 10^{-7} \text{ m}$$

Let d be the diameter of the moon.

Refer to Fig. 3.05 or 3.06. Both moon and its image form the same angle *i.e.* α at the objective.

Angle subtended by moon at the objective, $\alpha = \frac{d}{u_o}$ ...(*i*)

and angle subtended by image of the moon at the objective,

$$\alpha = \frac{I}{f_o} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{d}{u_o} = \frac{I}{f_o}$$

or

$$d = \frac{I}{f_o} \times u_o = \frac{0\cdot92 \times 10^{-2}}{1} \times 38 \times 10^7$$

$$= 3{\cdot}496 \times 10^6 \text{ m} = \mathbf{3496\ km}$$

(*i*) When the final image is formed at infinity, then distance between the two lenses is equal to sum of their focal lengths *i.e.*

$$L = f_o + f_e = 1 + 0\cdot05 = 1\cdot05 \text{ cm} = \mathbf{105\ cm}$$

(*ii*) For the image formed by eye piece :

$$f_e = 5 \text{ cm} ;\ v = -D = -25 \text{ cm}$$

If u_e is the distance of the image of moon formed by objective from eye piece, then

$$-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e} \text{ or } \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = -\frac{6}{25}$$

or

$$u_e = -\frac{25}{6} = -4\cdot17 \text{ cm}$$

Therefore, distance between the two lenses,

$$L' = f_o + |\,u_e\,| = 100 + 4\cdot17 = \mathbf{104{\cdot}17\ cm}$$

Type D. On Reflecting type telescope

Problem 3.12. A reflecting type telescope has a concave reflector of radius of curvature 120 cm. Calculate the focal length of eye piece to secure a magnification of 20.

(*H.P.S.S.C.E. 1998*)

Sol. Here, M = –20 (magnifying power is negative),

Here, radius of curvature of concave reflector,

$$R = 120 \text{ cm}$$

Therefore, focal length of concave reflector,

$$f_o = \frac{R}{2} = -\frac{120}{2} = -60 \text{ cm}$$

Now, magnifying power, $M = \frac{f_o}{f_e}$

or

$$f_e = \frac{f_o}{M} = \frac{-60}{-20} = \mathbf{3\ cm}$$

Type E. On Light gathering power of a telescope

Problem 3.13. The lens of our eye has a diameter of 0·8 cm. How much fainter star can be seen through the 508 cm objective telescope at Mount Polomar in U.S.A. ?

Sol. Here, $d = 0\cdot8$ cm ; D = 508 cm

The brightness ratio, $B = \frac{D^2}{d^2} = \frac{(508)^2}{(0\cdot8)^2} = \mathbf{403225}$

The telescope can see a star 403225 times fainter than the faintest star that can be seen with naked eye.

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 3.14. A compound microscope consists of an objective lens of focal length 2·0 cm and an eye-piece of focal length 6·25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (*a*) least distance of distinct vision (25 cm) and (*b*) infinity ? (*Text Problem*)

Sol. Here, $f_o = 2\cdot0$ cm ; $f_e = 6\cdot25$ cm ;

(a) To obtain the final image at least distance of distinct vision : Refer to Fig. 3.03. If AB is the object, then final image A″B″ is formed at the distance of distinct vision D.

For eye lens : $-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$

Here, $v_e = -25$ cm ; $f_e = +6\cdot25$ cm

$$\therefore \quad -\frac{1}{u_e} + \frac{1}{-25} = \frac{1}{6\cdot25}$$

or

$$\frac{1}{u_e} = -\frac{1}{25} - \frac{4}{25} = -\frac{5}{25}$$

or

$$u_e = -5 \text{ cm}$$

For object lens : $-\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$

Now, CC′ = 15 cm ;

$$v_o = CA' = CC' - (u_e) = 15 - |-5| = 10 \text{ cm} ;\ f_o = +2 \text{ cm}$$

$$\therefore \quad -\frac{1}{u_o} + \frac{1}{+10} = \frac{1}{+2}$$

or

$$\frac{1}{u_o} = \frac{1}{10} - \frac{1}{2} = -\frac{4}{10}$$

$$u_o = \mathbf{-2{\cdot}5\ cm}$$

(b) To obtain the final image at infinity : Final image will be formed at infinity, if the object lens forms the image of the object at the focus of eye lens, *i.e.* at a distance of 6·25 cm.

Therefore, $v_o = +(15 - 6{\cdot}25) = +8{\cdot}75$ cm

Now, $$-\frac{1}{u_o}+\frac{1}{v_o}=\frac{1}{f_o}$$

∴ $$-\frac{1}{u_o}+\frac{1}{+8{\cdot}75}=\frac{1}{2{\cdot}0}$$

or $$\frac{1}{u_o}=\frac{1}{8{\cdot}75}-\frac{1}{2{\cdot}0}=\frac{2-8{\cdot}75}{8{\cdot}75\times 2}$$

or $$u_o=-\frac{8{\cdot}75\times 2}{6{\cdot}75}=-2{\cdot}59\text{ cm}$$

Problem 3.15. A refracting telescope has an objective of focal length 1 m and an eye-piece of focal length 0 cm. A real image of the sun 10 cm in diameter is formed n a screen 24 cm from the eye-piece. What angle does the un subtend at the objective ? *(Text Problem)*

Sol. Here, $f_o = 1$ m = 100 cm ; $f_e = 20$ cm ; D = 24 cm ;

Refer to Fig. 3.05 : Final size of the image of the sun,

A′′B′′ = 10 cm

If A′B′ is the size of the image formed by object lens, then ngle formed by the sun at the objective is given by

$$\tan\alpha=\frac{A'B'}{f_o}$$

To find A′B′ : As shown in Fig. 3.04, A′B′ acts as object or eye-piece and its *virtual image* A′′B′′ is formed at a istance of 24 cm. Therefore,

$$\frac{A''B''}{A'B'}=1+\frac{D}{f_e}$$

or $$\frac{10}{A'B'}=1+\frac{24}{20}=\frac{44}{20}\quad\text{or}\quad A'B'=\frac{50}{11}\text{ cm}$$

∴ $$\tan\alpha=\frac{50}{11\times 100}=\frac{1}{22}=\mathbf{0{\cdot}0455\ radian}$$

Problem 3.16. (*a*) A giant refracting telescope at observatory has an objective lens of focal length 15 m. If an eye-piece of focal length 1·0 cm is used, what is the angular magnification of the telescope ?

(*b*) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens ? The diameter of the moon is 3·48 × 10⁶ m and the radius of the lunar orbit is 3·8 × 10⁸ m. *(Text Problem)*

Sol. Here, $f_o = 1$ m ; $f_e = 1{\cdot}0$ cm $= 0{\cdot}01$ m ;

(*a*) Angular magnification $= -\dfrac{f_o}{f_e}=\dfrac{-0{\cdot}5}{0{\cdot}01}=\mathbf{-1500}$

(*b*) Angle subtended by the moon,

$$\alpha=\frac{\text{diameter of moon}}{\text{radius of lunar orbit}}$$

or $$\alpha=\frac{3{\cdot}48\times10^6}{3{\cdot}8\times10^8}\qquad\text{...}(i)$$

Angle subtended by the image I of the moon as formed by the objective,

$$\alpha=\frac{\text{size of image}}{f_o}$$

or $$\alpha=\frac{I}{15}\qquad\text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{I}{15}=\frac{3{\cdot}48\times10^6}{3{\cdot}8\times10^8}$$

or $$I=\frac{3{\cdot}48\times15}{3{\cdot}8}\times10^{-2}=\mathbf{0{\cdot}137\ m}$$

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. Which is the natural optical instrument ?

Ans. Eye is called the natural optical instrument. It is ecause, eye has the capability of automatic adjustment of perture and focussing.

Q. 3.02. What is range of vision for normal eye ?

Ans. Eye can see up to considerably large distance (infinite) without any physical strain. However, if the objects are held close to the eye, strain is felt. The value of least distance of distinct vision for a normal eye is 25 cm.

Q. 3.03. What should be the position of an object relative to a biconvex lens, so that it behaves like a magnifying lens ? *(C.B.S.E. 1990 S)*

Ans. Object should be placed between the optical centre and focus of the lens at such a point that its image is formed at least distance of distinct vision.

Q. 3.04. What is the magnification produced by a single convex lens used as simple microscope in normal use ? *(I.S.S.C.E. 1995)*

Ans. $M = 1+\dfrac{D}{f}$

Q. 3.05. Why do we prefer a magnifying glass of smaller focal length ?

Ans. For a magnifying glass, $M=\left(1+\dfrac{D}{f}\right)$

It will be large, if the focal length of the magnifying lens is small.

Q. 3.06. Name a few uses of magnifying glass.

Ans. Refer to section 3.01.

Q. 3.07. Define magnifying power of compound microscope. *(P.S.S.C.E. 2001)*

Ans. Refer to section 3.02.

Q. 3.08. Why must both the objective and the eye-piece of a compound microscope have short focal lengths ?

Ans. The magnifying power of a compound microscope is given by

$$M=-\frac{L}{f_o}\left(1+\frac{D}{f_e}\right),$$

where the letters have their usual meanings. Obviously, M will have a large value, if both f_e and f_o are small.

Q. 3.09. Two lenses of focal lengths 5 cm and 50 cm are to be used for making a telescope. Which will you use for the objective ? *(I.S.C.E. 1997)*

Ans. In a telescope, objective is a lens of large aperture and large focal length. Therefore, lens of larger focal length *i.e.* 50 cm will be used as objective.

Q. 3.10. What is normal adjustment ?

Ans. A telescope is said to be in normal adjustment, if the final image is formed at infinity.

Q. 3.11. What is the distance between object and eye lens, when the telescope is on normal adjustment ?

Ans. Length of the telescope in normal adjustment

$$= f_o + f_e$$

Q. 3.12. Express the angular magnification of an astronomical telescope in terms of the focal lengths of the objective and the eye-piece. *(I.S.C.E. 1994)*

Ans. $M = -\frac{f_o}{f_e}$ (when final image is formed at ∞)

and $M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

(when final image is formed at D)

Q. 3.13. How does magnifying power of a telescope change on decreasing the aperture of its objective lens ? Justify your answer. *(C.B.S.E. 1998)*

Ans. Magnifying power of the telescope is independen of the aperture of the objective lens.

Q. 3.14. If a telescope is inverted, will it serve as a microscope ?

Ans. No.

Q. 3.15. What is a prism binoculars ?

Ans. A prism binoculars basically consists of a pair o astronomical telescopes with relatively quite small length o the telescope tubes in comparison to that of an ordinary telescope. The use of prisms cuts short the length of the telescope tubes.

Q. 3.16. Telescope lens usually has large aperture. Why ?

Ans. The object lens of a large aperture in a telescope provides greater resolving power and produces a bright image due to greater light gathering.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. Draw a labelled diagram showing course of rays for a simple microscope. *(H.P.S.S.C.E. 1998 S)*

Or

Draw a neat labelled diagram for a simple microscope or magnifying glass. *(I.S.C.E. 1996)*

Ans. Refer to Fig. 3.01.

Q. 3.02. Using a ray diagram, show the image formation by a simple microscope. What is the nature of the image formed ? *(C.B.S.E. 1990 S)*

Ans. Refer to Fig. 3.01.

The image formed is virtual in nature.

Q. 3.03. An object is first seen in red light and then in violet light through a simple microscope. In which case is the magnifying power of simple microscope greater ? *(C.B.S.E. 1999 S)*

Ans. The magnifying power of the simple microscope will be greater, when the object is seen in violet light.

For details, refer to conceptual SAQ 3.05 (*a*).

Q. 3.04. Draw a ray diagram to show how an image is formed by a compound microscope.

(C.B.S.E. 2002, 1998, 1997)

Or

Draw the ray diagram of a compound micro-scope.

(H.P.S.S.C.E. 2001 ; P.S.S.C.E. 1999 S ; H.S.S.C.E. 2001, 1998 S)

Or

Draw a labelled ray diagram showing formation of image in a compound microscope.

(C.B.S.E. 2001, 1999, 1992 S, 1990 ; H.P.S.S.C.E. 1995)

Ans. Refer to Fig. 3.03.

Q. 3.05. Show the working of a compound microscope with the help of a ray diagram. *(P.S.S.C.E. 1998 S)*

Ans. Refer to Fig. 3.03.

Q. 3.06. Draw a labelled ray diagram showing the formation of image in a compound microscope. Also define the magnifying power of compound microscope. *(P.S.S.C.E. 2001)*

Ans. Ray diagram. Refer to fig. 3.03.

Magnifying power. Refer to section 3.02.

Q. 3.07. Draw a labelled ray diagram to show the formation of image of an object in a compound microscope. Write the expression for its magnifying power. *(C.B.S.E. 1998 S)*

Ans. For ray diagram, refer to Fig. 3.03.

Magnifying power, $M = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$,

where the letters have their usual meanings.

Q. 3.08. Draw a labelled ray diagram showing the formation of image of a distant object using an astronomical telescope in the normal adjustment.

(H.S.S.C.E. 2001 ; C.B.S.E. 2000)

Or

Draw a ray diagram to show formation of image by a refracting type astronomical telescope.

(C.B.S.E. 2002, 1999, 1998, 1997, 1996 ; H.S.S.C.E. 1997 S)

Ans. Refer to Fig. 3.04 or 3.05.

Q. 3.09. Draw a ray diagram showing how the final image of a distant object is formed using an astronomical telescope in the near point position. *(C.B.S.E. 2001)*

Ans. Refer to Fig. 3.05.

Q. 3.10. Draw the course of rays in an astronomical telescope, when the final image is formed at infnity. Also define the magnifying power of the astronomical telescope in this position. *(P.S.S.C.E. 2001)*

Ans. Course of rays. Refer to Fig. 3.04

Magnifying power. Refer to section 3.04 (*a*).

Q. 3.11. Draw the course of rays in an astronomical telescope, when the final image is formed at the least distance of distinct vision. Also define the magnifying power of the telescope in this position. *(P.S.S.C.E. 2001)*

Ans. Course of rays. Refer to Fig. 3.05

Magnifying power. Refer to section 3.04 (*b*).

Q. 3.12. Draw the diagram of an astronomical telescope, when the final image is formed at the distance of distinct vision. Write the formula for magnifying power in the above noted case. *(H.S.S.C.E. 1995)*

Ans. Refer to Fig. 3.05.

Magnifying power, $M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

Q. 3.13. Draw a ray diagram to illustrate image formation by a Newtonian type reflecting telescope. *(C.B.S.E. 2001, 2000, 1999)*

Or

Draw a neat labelled ray diagram for a simple astronomical telescope using a reflector. *(I.S.C.E. 1994)*

Ans. Refer to Fig. 3.06.

Q. 3.14. List some advantages of a reflecting telescope.

Ans. Advantages. 1. The image formed is free from chromatic aberration.

2. The spherical aberration can also be minimised by using a parabolic mirror as objective.

3. The image formed is very bright due to its large light gathering power. As such, it enables us to see even very faint stars.

Q. 3.15. Distinguish between magnifying power and resolving power of a telescope. *(C.B.S.E. 1992 S)*

Ans. Refer to sections 3.06.

Q. 3.16. On what factors does (*i*) magnifying power and (*ii*) resolving power of a compound microscope depend ? *(C.B.S.E. 1998)*

Ans. (*i*) Magnifying power, $M = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$

Therefore, magnifying power of a compound microscope depends upon the focal lengths of the objective and eye piece of the microscope.

(*ii*) Resolving power $= \frac{2\mu \sin \theta}{\lambda}$

Therefore, resolving power of a microscope depends upon the wavelength of light used to illuminate the object, refractive index of the glass forming the objective, and the semi-vertical angle of the cone, in which the rays from the object enter the objective.

Q. 3.17. On what factors does (*i*) magnifying power and (*ii*) resolving power of a refracting type astronomical telescope depend ? *(C.B.S.E. 1998)*

Ans. (*i*) Magnifying power, $M = -\frac{f_o}{f_e}$

Therefore, magnifying power of a telescope depends upon the focal lengths of the objective and the eye-piece of the telescope.

(*ii*) Resolving power $= \frac{D}{1 \cdot 22\, \lambda}$

Therefore, resolving power of a telescope depends upon the wavelength of light in which the object is seen and the diameter (aperture) of the objective.

Q. 3.18. How will the magnifying power of a refracting type astronomical telescope be affected on increasing for its eye piece (*i*) the focal length, and (*ii*) the aperture ? Justify your answer. *(C.B.S.E. 1997)*

Ans. (*i*) Refer to SAQ 3.17. Since $M \propto \frac{1}{f_e}$, the magnifying power of the telescope will decrease on increasing the focal length of the eye piece.

(*ii*) The resolving power of the telescope depends on the aperture of its objective. It does not depend on the aperture of the eye-piece.

Q. 3.19. What do you mean by light gathering power of a telescope ?

Ans. Refer to section 3.06.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 3.01. What for are the optical instruments used ?

Ans. The optical instruments are used (*i*) to produce the image of the close lying objects at the least distance of distinct vision, (*ii*) to increase visual angle and hence to produce magnification and (*iii*) to improve the resolving power of the eye.

Q. 3.02. The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense, then, does a magnifying glass provide angular magnification ? *(Text Question)*

Ans. It is true that the angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. The angular magnification produced by the magnifying glass lies in the fact as follows :

In case the magnifying glass is not used, the object to be seen has to be placed at a distance of 25 cm, whereas the magnifying glass allows us to place the object much closer to eye. Obviously, the angle formed by the object at the eye, when it is placed closer to eye will be much larger than the angle formed by it when placed at a distance of 25 cm from it. It is in this sense that a magnifying lens produces angular magnification.

Q. 3.03. When observed from the earth, the angular diameter of the solar disc is 32′. Determine the diameter of the image of the sun formed by a convex lens of focal length 25 cm.

Ans. Here, $f = 25$ cm

and angular diameter of the sun,

$$\theta = 32' = \frac{\pi}{180} \times \frac{32}{60} \text{ rad}$$

The image of solar disc will be formed in the focal plane of the lens [Fig. 3.11]. The diameter (D) of the image of solar disc will subtend at the optical centre of the lens an angle equal to the angular diameter of the sun. Therefore,

$$D = f\,\theta = 25 \times \frac{\pi}{180} \times \frac{32}{60}$$

$$= 0 \cdot 233 \text{ cm}$$

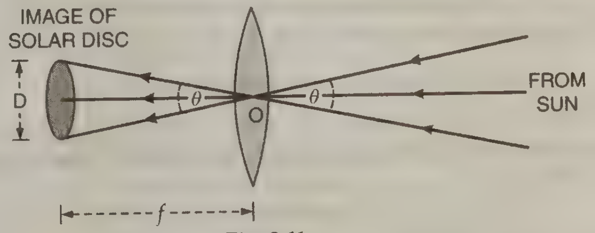


Fig. 3.11

Q. 3.04. In viewing through a magnifying glass, one usually positions one's eye very close to the lens. Does angular magnification change, if the eye is moved back ? *(Text Question)*

Ans. Refer to Fig. 3.01. While viewing through a magnifying glass, the eye has to be placed very close to the lens. It is essential, otherwise the angle subtended at the eye by the object and its virtual image will not be equal.

If the eye is moved back, the angular magnification decreases a little. It is because, now the angle subtended at the eye by the image is less than that subtended by the image at the lens. (The angle subtended by the object at the eye is also less than that subtended at the lens, but the difference is very small).

Q. 3.05. Microscopes in which object is illuminated by ultraviolet light can give higher magnifications than the microscopes that use visible light.

(*a*) How do you explain this ?

(*b*) How are the images seen, if no visible light is used ?

(*c*) Since glass is opaque to ultraviolet light, how can such a microscope be made ?

Ans. (*a*) The magnification produced by a microscope,

$$M = -\frac{L}{f_o}\left(1+\frac{D}{f_e}\right)$$

Focal length of a lens decreases with decrease of wavelength. Since wavelength of ultraviolet light is less than that of visible light, the focal lengths of the object and the eye lenses will be lesser, when ultraviolet light is used. Hence, magnification produced by the microscope will increase on using ultraviolet light.

(*b*) In a microscope using ultraviolet light, the images are produced on a fluorescent screen or on a photographic plate.

(*c*) The lenses used in the microscope must be transparent to the ultraviolet light. Quartz is transparent down to 2100 Å, while fluorite down to about 1200 Å. However, air absorbs electromagnetic waves strongly, whose wavelength is below 1800 Å. Therefore, lenses made of quartz or fluorite may be used in ultraviolet light of wavelength above 1800 Å.

Q. 3.06. What is difference between magnification and angular magnification produced by a lens ?

Ans. The magnification produced by a lens is the ratio of the size of the image to the size of the object. It can be easily seen that it is also equal to the ratio of the distance of the image from the lens to the distance of the object from the lens. Thus,

magnification produced, $m = \frac{v}{u}$

On the other hand, angular magnification produced by a lens is defined as the ratio of the angle subtended by image at the eye to the angle subtended by object seen directly, when both lie at the least distance of distinct vision. It is also called ***magnifying power*** of the lens. It can be easily verified that

angular magnification produced $= \frac{D}{u}$,

where D is least distance of distinct vision. It follows that magnification becomes equal to the angular magnification, when

$$v = D$$

Q. 3.07. Magnifying power of a microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power ? *(Text Question)*

Ans. A lens of short focal length is not easy to manufacture. Further, a lens of short focal length would be quite thick at centre and such a lens (due to its prismatic action) disperses the white light and a multicoloured image is formed. This defect in a lens is called chromatic aberration.

In practice, an ordinary convex lens can be used to produce a magnifying power of 3 or so. However, by designing an achromat (a combination of two lenses having no chromatic aberration), one may get a magnifying power of 10 or so.

Q. 3.08. When viewing through a compound microscope, our eye should be positioned not on the eye-piece but a short distance away from it for best viewing. Why ? How much should be that short distance between the eye and eye-piece ? *(Text Question)*

Ans. The eye-piece produces the image of the object lens itself. This image is called 'eye ring'. An important point about the eye ring is that all the rays from the object and refracted from the object lens pass through it. The eye would receive all the rays from object, if it is placed at the position of the eye ring, provided area of the pupil of the eye is greater than or atleast equal to the area of the eye ring. In case eye is placed close to the eye-piece, then it would not collect all the rays from the object. Moreover, the field of view will be reduced.

Q. 3.09. Why is a reflecting type telescope preferred in astronomy ?

Ans. A lens of large aperture and at the same time free from lens-aberrations cannot be manufactured. Therefore, the lens used cannot be of very large aperture. Obviously, light gathering power of such a telescope will be very low and hence it will not enable us to see faint stars.

The spherical aberration can be eliminated by making use of a parabolic mirror. Since mirrors of large apertures can be easily designed, reflecting type telescope will have large light gathering power.

Q. 3.10. A convex and concave lens having focal lengths of 30 cm and 10 cm respectively are placed at a distance 20 cm from each other. Where should a source of light be placed so that the system of the two lenses will produce a beam of parallel rays ?

Ans. Suppose that convex lens L_1 (f_1 = 30 cm) and concave lens L_2 (f_2 = 10 cm) are placed at a distance 20 cm apart as shown in Fig. 3.12.

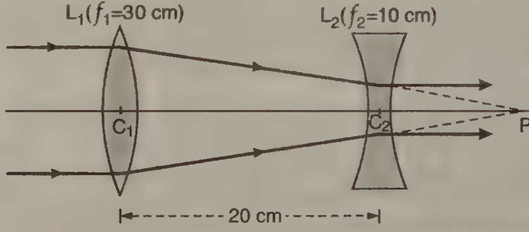


Fig. 3.12

Suppose that the beam of light after falling on thre convex lens converges at point P.

As the beam of light is rendered parallel by the concave lens,point P is focus of the concave lens.

$\therefore \quad C_1P = C_1C_2 + C_2P$

$= 20 + 10 = 30$ cm

Since focal length of the convex lens is also 30 cm, the point P is focus of convex lens. Thus, the beam of light after falling on the convex lens is coverging at its focus. Hence, a parallel beam of light is falling on the vovex lens, which requires that the source of light must be situated at infinity.

Q. 3.11. What is the special point about prism binoculars, though it is a pair of astronomical telescopes ?

Ans. An astronomical telescope will have large magnifying power, only if it makes use of an object lens of large focal length. In that case, the length of the telescope will become very large. Moreover, in a telescope, image formed is laterally inverted.

Prism binoculars is an arrangement, which not only cuts short the length of the tube, but produces an erect and highly magnified image without any lateral inversion.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain, how a convex lens held close to an eye may be used as a simple microscope and calculate the magni-fication obtained.
2. Explain the working of a simple microscope and find an expression for its magnifying power. *(H.S.S.C.E. 2002)*
3. Describe a simple microscope and prove that smaller the focal length of the lens forming a simple microscope, greater is its magnifying power.
4. Derive an expression for the angular magnification of a simple microscope. *(H.P.S.S.C.E. 1999 ; I.S.C.E. 1996)*
5. Explain the working of a simple microscope and show that its magnification m is given by $m = 1 + \frac{D}{f}$, where f = focal length and D = distance of distinct vision.*(C.B.S.E. 1992)*
6. Draw a ray diagram to show image formation in a compound microscope. Find an expression for its magnifying power. *(H.S.S.C.E. 2002)*
7. Derive a relation for the magnifying power of a compound microscope. *(P.S.S.C.E. 1997)*
8. Define magnifying power of a compound microscope and derive a relation for it. *(P.S.S.C.E. 1996 S)*
9. Draw a ray diagram to show image formation in an astronomical telescope. Find an expression for its magnifying power. *(H.S.S.C.E. 2002)*
10. Define magnifying power of an astronomical telescope and derive a general relation for it. *(H.S.S.C.E. 1999 S ; P.S.S.C.E. 1996)*
11. Draw a labelled diagram of an astronomical telescope forming the final image at the near point. Write down the formula for its magnifying power. *(C.B.S.E. 1993)*
12. Draw a ray diagram to show image formation by a Newtonian reflecting telescope. Give its two advantages. *(H.S.S.C.E. 2002)*
13. Give the construction and working of a reflecting type telescope. Also trace the course of rays through such a telescope. What are the advantages of the reflecting type telescope over the others ?
14. Draw a labelled ray diagram for a reflecting type telescope. Write four advantages of a reflecting type telescope over a refracting type telescope. On what factors does its resolving power depend ? *(C.B.S.E. 1998, 1997)*
15. Bring out the difference between a refracting and a reflecting type telescope.
16. Describe the working of a prism binocular with a suitable diagram. Discuss its advantages over an ordinary astronomical telescope.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. With the help of a ray diagram, explain the working principle of a compound microscope. *(P.S.S.C.E. 1996 ; C.B.S.E. 1993)*
2. Draw a ray diagram showing the image formation in a compound microscope. Find an expression for its magnifying power. *(H.S.S.C.E. 1998)*
3. With the help of ray diagram, illustrate the formation of the final image of an object in a compound microscope. Derive an expression for its magnifying power. *(P.S.S.C.E. 1999, 1998 S ; H.S.S.C.E. 1998 ; H.P.S.S.C.E. 1996)*
4. Draw the course of rays through a compound microscope. Obtain an expression for the magnifying power of a microscope. *(H.S.S.C.E. 2002 ; H.P.S.S.C.E. 2002)*
5. You are given two convex lenses of short aperture having focal lengths 4 cm and 8 cm respectively. Which one of these will you use as an objective and which one as an eye piece for constructing a compound microscope ? Draw a ray diagram to show the formation of the image of a small object due to a compound microscope. Derive an expression for its magnifying power. *(C.B.S.E. 2001 S)*
6. Explain the construction and working of a compound microscope. Derive an expression for its magnifying power. *(P.S.S.C.E. 2002, 2000)*
7. Draw a labelled diagram to illustrate the action of a compound microscope in normal use. Define angular magnification and use it to find the magnification, when the final image is formed at the least distance of distinct vision. *(I.S.C.E. 1995)*
8. Define (*i*) magnifying and (*ii*) resolving power of a telescope. Find the magnifying power of a microscope, when final image is formed at the distance of distinct vision. *(H.S.S.C.E. 1995)*
9. Explain the construction and working of an astronomical telescope. Derive the expression for its magnifying power in normal adjustment. *(P.S.S.C.E. 2000)*
10. Describe construction and working of an astronomical telescope. Calculate its magnifying power, when the image is formed at the least distance of distinct vision. *(H.S.S.C.E. 1996)*
11. What is an astronomical telescope ? Give its ray diagram. Find out the expression for its magnifying power, when the final image is formed at the least distance of distinct vision. *(H.P.S.S.C.E. 1999 S)*
12. Draw the course of rays in an astronomical telescope and find a relation for its magnifying power, when the final image is formed at distance of distinct vision. *(P.S.S.C.E. 1994, 1992)*

13. Draw a neat labelled ray diagram to show how the image of a distant object is formed by an astronomical refracting telescope in normal adjustment. Obtain an expression for its magnifying power. *(C.B.S.E. 1999 ; I.S.C.E. 1994)*
14. You are given two convex lenses of focal lengths 80 mm and 800 mm. Which one will you use as an objective and which one as an eye-piece for constructing an astronomical telescope ? Trace the course of rays through the two lenses to show the formation of final image of a distant object. Derive an expression for the magnifying power of the telescope for normal adjustment. *(C.B.S.E. 1995)*
15. Draw a labelled diagram of a telescope and explain its working. Give an expression for its magnifying power. *(C.B.S.E. 1992)*
16. Calculate the magnifying power of compound microscope and astronomical telescope. *(H.P.S.S.C.E. 1993)*

FREQUENTLY ASKED NUMERICAL QUESTIONS

For Practice

Type A. On Simple Microscope

1. A converging lens of power 25 D is used as a simple microscope. Calculate the magnifying power, if the distance of distinct vision is 25 cm. [**Ans.** 7·25]
2. A figure divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass of focal length 10 cm. At what distance should the lens be held from the figure, so that the virtual image of each square in the figure is to have an area of 6·25 mm^2 ? Will you be able to see squares distinctly with your eye very close to the magnifier ? *(Text Problem)* [**Ans.**– 6 cm, No]
3. A man with normal near point (25 cm) reads a book with small print using a magnifying glass – thin convex lens of focal length 5 cm. *(a)* What are the closest and the farthest distances at which he can read the book when viewing through the magnifying glass ? *(b)* What is the maximum and minimum angular magnifications possible using the above magnifying glass ? [**Ans.** *(a)* 4·17 cm, 5 cm *(b)* 6, 5]

Type B. On Compound Microscope

4. The focal lengths of the objective and eye-piece of a compound microscope are 4 cm and 6 cm respectively. If an object is placed at a distance of 6 cm from the objective, find the distance between the objective and the eye-piece. Distance of distinct vision is 25 cm. [**Ans.** 16·84 cm]
5. The total magnification produced by a compound microscope is 20, while that produced by the eye-piece alone is 5. When the microscope is focussed on a certain object, the distance between objective and eye-piece is 14 cm. Find the focal length of objective and eye-piece, if distance of distinct vision is 20 cm. [**Ans.** 2 cm, 5 cm]
6. An angular magnification of 30 X is desired using an objective of focal length 1·25 cm and an eye-piece of focal length 5 cm. How will you set up the compound microscope ? [**Ans.** Placing two lenses 11·67 cm apart and object at a distance of 1·5 cm from objective]

Type C. On Astronomical Telescope

7. In a simple form of an astronomical telescope, the focal length of the objective lens is 75 cm and that of eye-piece 5 cm. Calculate the magnifying power and the distance between the two lenses, when the final image of the distant object is seen at a distance of 25 cm from the eye. [**Ans.** – 18, 79·17]
8. The magnifying power of an astronomical telescope is 5. When it is set for normal adjustment, the distance between the two lenses is 24 cm. Calculate the focal length of its eye-piece and that of its objective. [**Ans.** 4 cm ; 20 cm]
9. A telescope has an objective lens of focal length 140 cm and an eye-piece of focal length 5 cm. If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens ? Also calculate the height of the final image of the tower, if it is formed at 25 cm. [**Ans.** 4·67 cm, 28 cm]

Type D. On Reflecting type Telescope

10. A reflecting type telescope has a large spherical mirror for its objective with radius of curvature equal to 80 cm. What is the magnifying power of telescope, if eye-piece used has a focal length of 1·6 cm. *(Text Problem)* [**Ans.** – 25]

Type E. On Light gathering power of a Telescope

11. The objective of telescope A has a diameter 3 times that of the objective of telescope B. How much greater amount of light is gathered by A as compared to B ? [**Ans.** 9]
12. The lens of our eye has a diameter of 8 mm. How much fainter objects can be seen through a telescope of 120 cm aperture as compared to the faintest naked eye star ? [**Ans.** 22500]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $f=\frac{1}{P}=\frac{1}{25}=0\cdot04\,\text{m}=4\,\text{cm}$

   Now, $m=1+\frac{D}{f}=1+\frac{25}{4}=7\cdot25$

2. Linear magnification, $m=\sqrt{\frac{6\cdot25}{1}}=2\cdot5$

   Now, $m=\frac{f}{u+f}$ or $2\cdot5=\frac{10}{u+10}$ or $u=-6$ **cm**

   Also, $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ or $-\frac{1}{6}+\frac{1}{v}=\frac{1}{10}$

   or $v=-15$ cm

   The image will be found at 15 cm from the lens. Since least distance of distinct vision is 25 cm, the squares will **not be seen distinctly**, when eye is held close to the magnifier.

3. ***(a) The closest distance (u) of book from the*** magnifying lens corresponds to the position, when image is formed at distance of distinct vision.

   From the relation : $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, we have

   $-\frac{1}{u}+\frac{1}{-25}=\frac{1}{5}$ or $u=-4\cdot17$ **cm**

   The farthest distance (u') of the book from the magnifying lens corresponds to the position, when image is formed at infinity.

   From the relation : $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, we have

   $-\frac{1}{u'}+\frac{1}{\infty}=\frac{1}{5}$ or $u'=-5$ **cm**

   (b) Angular magnification $=\frac{D}{u}$

   Thus, maximum angular magnification $=\frac{25}{4\cdot17}=\mathbf{6}$

   and minimum angular magnification $=\frac{25}{5}=\mathbf{5}$

4. Refer to Fig. 3.03. The distance between objective and eye-piece,

$$L = v_o + |u_e| \qquad ...(i)$$

To find v_0:

$u_o = -6$ cm, $f_o = 4$ cm

Now, $-\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$ or $-\frac{1}{-6} + \frac{1}{v_o} = \frac{1}{4}$

or $v_o = 12$ cm

To find u_e: $v_e = -25$ cm, $f_e = 6$ cm

Now, $-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$ or $-\frac{1}{u_e} + \frac{1}{-25} = \frac{1}{6}$

or $u_e = -4.84$ cm

From equation (*i*), we have

$L = 12 + 4.84 = \mathbf{16.84\ cm}$

5. Magnification produced by a compound microscope,

$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

Here, $M = -20$ (final image is inverted)
and $m_e = 5$ (magnification produced by eye-piece)
Therefore, magnification produced by objective,

$$m_o = \frac{M}{m_e} = -\frac{20}{5} = -4$$

Now, $m_e = 1 + \frac{D}{f_e}$ or $5 = 1 + \frac{20}{f_e}$ or $f_e = \mathbf{5\ cm}$

Also, $m_e = \frac{f_e}{u_e + f_e}$ or $5 = \frac{5}{u_e + 5}$ or $u_e = -4$ cm

Refer to Fig. 3.03, The distance between objective and eye-piece,

$v_o + |u_e| = 14$ or $v_o + 4 = 14$ or $v_o = 10$ cm

Now, magnification produced by objective,

$$m_o = \frac{v_o}{u_o}$$

or $-4 = \frac{10}{u_o}$ or $u_o = -2.5$ cm

Also, $-\frac{1}{u_o} + \frac{1}{v_o} = \frac{1}{f_o}$ or $-\frac{1}{2.5} + \frac{1}{10} = \frac{1}{f_o}$

or $f_o = \mathbf{2\ cm}$

6. Here, $M = -30$, $f_o = 1.25$ cm $f_e = 5$ cm
Magnification produced by eye-piece,

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Also, $m_e = \frac{f_e}{u_e + f_e}$ or $6 = \frac{5}{u_e + 5}$

or $u_e = -4.17$ cm

Magnification produced by objective,

$$m_o = \frac{M}{m_e} = \frac{-30}{6} = -5$$

Now, $m_o = \frac{f_o}{u_o + f_o}$ or $-5 = \frac{1.25}{u_o + 1.25}$

or $u_o = \mathbf{-1.5\ cm}$

Also, $m_o = \frac{f_o - v_o}{f_o}$ or $-5 = \frac{1.25 - v_o}{1.25}$

or $v_o = 7.5$ cm

Refer to Fig. 3.03. The distance between objective and eye-piece,

$L = v_o = |u_e| = 7.5 + 4.17 = \mathbf{11.67\ cm}$

Therefore, objective and eye-piece should be placed at 11·67 cm apart and object at a distance of 1·5 cm from the objective.

7. Here, $f_o = 75$ cm, $f_e = 5$ cm

Now, $M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = -\frac{75}{5}\left(1 + \frac{5}{25}\right) = \mathbf{-18}$

Refer to Fig. 3.05. The distance between the objective and eye-piece,

$$L = f_o + |u_e|$$

Now, $-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$ or $-\frac{1}{u_e} + \frac{1}{-25} = \frac{1}{5}$

or $u_e = -4.17$ cm

$\therefore$ $L = 75 + 4.17 = \mathbf{79.17\ cm}$

8. Magnification produced by a telescope in normal adjustment,

$$M = -\frac{f_o}{f_e} \quad \text{or} \quad -5 = -\frac{f_o}{f_e}$$

or $f_o = 5 f_e$

Also, distance between two lenses (when telescope is in normal adjustment),

$L = f_o + f_e$ or $24 = f_o + f_e$

or $24 = 5 f_e + f_e$ or $f_e = \mathbf{4\ cm}$

Also, $f_o = 5 \times 4 = \mathbf{20\ cm}$

9. Here, $f_o = 140$ cm, $f_e = 5$ cm
Let α be angle formed by the object at the objective lens. Then,

$$\tan\alpha = \frac{\text{height of the object}}{\text{distance of the object}} = \frac{100}{3 \times 100} = \frac{1}{30}$$

Refer to Fig. 3.05. Let A′B′ be the height of image produced by objective lens. The image is formed at the focus of objective lens and the angle formed by it at the objective lens is also equal to α. Therefore,

$$\tan\alpha = \frac{A'B'}{f_o}$$

or $\tan\alpha = \frac{A'B'}{140}$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$\frac{A'B'}{140} = \frac{1}{30}$ or $A'B' = \frac{140}{30} = \mathbf{4.67\ cm}$

Magnification produced by eye-piece,

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Therefore, height of final image of the tower,

$A''B'' = m_e \times A'B' = 6 \times 4.67 = \mathbf{28.0\ cm}$

10. Proceed as in solved problem 3.12.

11. $\frac{\text{Light gathered by telescope A}}{\text{Light gathered by telescope B}}$

$$= \left(\frac{\text{diameter of the objective of telescope of A}}{\text{diameter of the objective of telescope of B}}\right)^2$$

$= (3)^2 = \mathbf{9}$

12. Here, $d = 8$ mm $= 0.8$ cm, $D = 120$ cm
The brightness ratio,

$$B = \frac{D^2}{d^2} = \frac{(120)^2}{(0.8)^2} = \mathbf{22{,}500}$$

CHAPTER
4

Huygen's Principle

4.01. WAVEFRONT

According to wave theory of light, a source of light sends out disturbance in all directions. In a homogeneous medium, the disturbance reaches all those particles of the medium in phase, which are located at the same distance from the source of light and hence at any instant, all such particles must be vibrating in phase with each other.

The locus of all the particles of the medium, which at any instant are vibrating in the same phase, is called the ***wavefront****.*

Key point

A spherical wavefront is produced by point source, while a cylindrical wavefron by a line source of light. The plan wavefront is produced, when the source light is situated far-off *i.e.* at infinity.

Depending upon the shape of the source of light, wavefront can be of the following types :

1. Spherical wavefront. A spherical wavefront is produced by a point source of light. It is because, the locus of all such points, which are equidistant from the point source, is a sphere [Fig. 4.01 (*a*)].

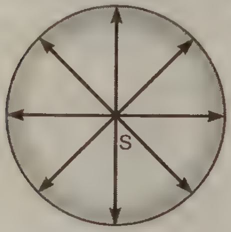


Fig. 4.01 (*a*)

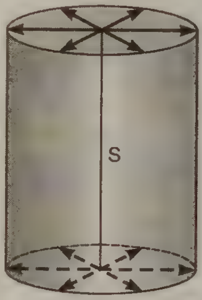


Fig. 4.01 (*b*)

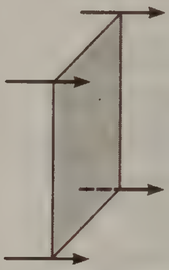


Fig. 4.01 (*c*)

Key point

A ray of light is always normal to th wavefront.

2. Cylindrical wavefront. When the source of light is linear in shape (such as a slit), a cylindrical wavefront is produced. It is because, all the points, which are equidistant from the linear source, lie on the surface of a cylinder [Fig. 4.01 (*b*)].

3. Plane wavefront. A small part of a spherical or a cylindrical wavefront originating from a distant source will appear plane and hence it is called a plane wavefront Fig. 4.01 (*c*).

Ray of light. *An arrow drawn normal to the wavefront and pointing in the direction of propagation of disturbance represents a ray of light.* A ray of light is the path along which light travels. In Fig. 4.01, thick arrows represent the rays of light.

Figs. 4.02 (*a*), 4.02 (*b*) and 4.02 (*c*) represent the wavefronts of a parallel beam, a converging beam and a diverging beam respectively. The wavefront representing a parallel beam is a plane wavefront, while the wavefront representing a converging or a diverging beam of light is spherical in shape.

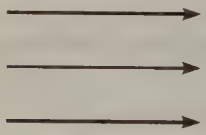


Fig. 4.02 (*a*)

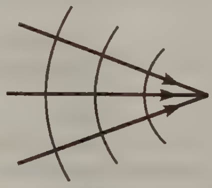


Fig. 4.02 (*b*)

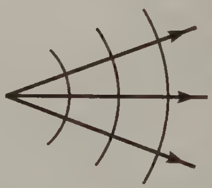


Fig. 4.02 (*c*)

4.02. HUYGENS' PRINCIPLE

Huygens' principle is a geometrical construction, which is used to determine the new position of a wavefront at a later time from its given position at any instant.

In other words, the principle gives a method to know as to how light spreads out in the medium.

Key point

Huygens' principle is simply a geometrical construction to find the position of wavefront at a later time.

Huygens' principle is based on the following assumptions :

1. *Each point on the given or primary wavefront acts as a source of secondary wavelets, sending out disturbance in all directions in a similar manner as the original source of light does.*

2. *The new position of the wavefront at any instant (called secondary wavefront) is the envelope of the secondary wavelets at that instant.*

The above two assumptions are known as ***Huygens' principle*** or ***Huygens' construction.***

Explanation. Consider a point source of light. Let AB be a section of primary spherical wavefront at any time t. In order to find the new position of wavefront at time $t + \Delta t$, consider a number of points a, b, c, d, ... on the primary wavefront [Fig. 4.03]. These points act as the sources of secondary wavelets. In time Δt, light travels a distance $c\,\Delta t$. Taking points a, b, c, d, ... as the centres, draw spheres[*] each of radius $c\,\Delta t$. These spheres are called secondary wavelets originating from points on primary wavefront. Further, draw the envelope of the secondary wavelets. Parts A′B′ and A″B″ of the envelope are termed as forward secondary wavefront and backward secondary wavefront respectively. It may be pointed out that the effective part of the secondary wavelets is the portion which lies on the forward secondary wavefront. Huygens assumed that the portion of the secondary wavelets, which lies on the backward secondary wavefront, does not exist at all. However, many years later, Voigt and Kirchhoff mathematically proved that the contribution of a wavelet in any direction making angle θ with the normal to the wavelet is proportional to $\frac{1}{2}(1+\cos\theta)$. Obviously, for the portion of the wavelet, which lies just on the back of the secondary wave-front, θ is equal to 180° and likewise the factor $\frac{1}{2}(1+\cos\theta)$ is equal to zero. Therefore, the forward secondary wavefront A′B′ is the new position of the wavefront at time $t + \Delta t$.

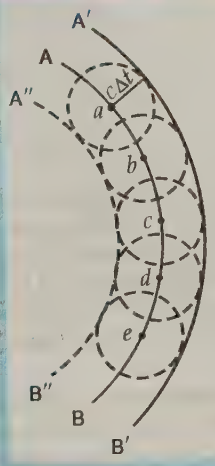


Fig. 4.03

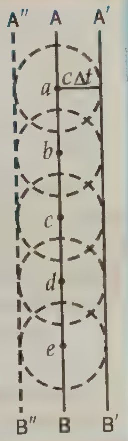


Fig. 4.04

Key point

The effective part of the second wavelets is the front portion of seconda wavefront. The countribution of wavelets on the back of the secondary wavefront is zero.

The same procedure is adopted for finding the new position of a cylindrical or a plane wavefront. In Fig. 4.04, AB represents the primary plane wavefront, while A′B′ represents the new position of wavefront at a time Δt later.

4.03. LAWS OF REFLECTION ON WAVE THEORY

Let XY be a reflecting surface and PA be a plane wavefront just incident on the surface. The normals LA and MP to the wavefront represent incident rays. If AN is normal to the reflecting surface at A, then $\angle$ NAL = i is angle of incidence [Fig. 4.05]. First of all, the plane wavefront[**] reaches point A on the reflecting surface. The wavefront will reach points between A and P′ later in time but in this order, so that it will reach point P′ in the last. When the disturbance from point P just reaches point P′ on the reflecting surface, the secondary wavelet from point A on the reflecting surface will grow into a sphere of radius say AA′, such that AA′ = PP′.

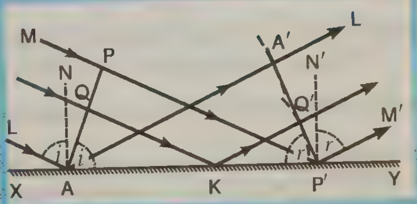


Fig. 4.05

To find the reflected wavefront (new position of wavefront after reflection from the surface XY) ; with point A as centre, draw a sphere of radius AA′ = PP′. From point P′, draw a plane P′A′ tangent to the sphere. Then, P′A′ represents reflected wavefront and normals A′L′ and P′M′ to the reflected wavefront P′A′ represent reflected rays. Draw P′N′ normal to the reflecting surface at point P′. Then, $\angle$ M′P′N′ = r is the angle of reflection.

To prove the laws of reflection : Consider any point Q on the incident wavefront. Suppose that when disturbance from point P on incident wavefront reaches point P′, the disturbance from point Q reaches Q′ via point K on the reflecting surface. Since P′A′ represents the reflected wavefront, the time taken by light to travel from any point on incident wavefront to the corresponding point[†] on the reflected wavefront should always be same. If c is velocity of light, then time taken by light to go from the point Q to Q′ is given by

$$t = \frac{QK}{c} + \frac{KQ'}{c} \qquad \text{...(4.01)}$$

---

[*] A sphere is a three dimensional geometrical figure. On a piece of apper *i.e.* in a plane, spherical wavefronts can be represented by circles only.

[**] To represent the plane wavefront, one will draw a straight line through the point A.

[†] For points A, P and Q on incident wavefront PA, the points A′, P′ and Q′ respectively are the corresponding points on the reflected wavefront P′A′.

In right angled ΔAQK, ∠ PAP′= i

$$\therefore \qquad QK = AK \sin i$$

Also, in right angled ΔKQ′P′, ∠ A′P′A = r

$$\therefore \qquad KQ' = KP' \sin r$$

In equation (4.01), substituting for QK and KQ′, we have

$$t = \frac{AK \sin i}{c} + \frac{KP' \sin r}{c}$$

or
$$t = \frac{AK \sin i + (AP' - AK) \sin r}{c} \qquad (\because KP' = AP' - AK)$$

or
$$t = \frac{AP' \sin r + AK (\sin i - \sin r)}{c} \qquad ...(4.02)$$

For rays from different points on the incident wavefront, the values of AK are different. The rays from different points on incident wavefront will take the same time to reach the corresponding points on the reflected wavefront, if t, given by equation (4.02), is independent of AK. So that the equation (4.02) is independent of AK, it follows that AK (sin i – sin r) should be zero *i.e.*

$$AK (\sin i - \sin r) = 0 \text{ or } (\sin i - \sin r) = 0 \text{ or } \sin i = \sin r$$

or
$$i = r$$

i.e. the angle of incidence is equal to the angle of reflection.

Further, as the incident wavefront (PA), the reflecting surface (XY) and the reflected wavefront (P′A′) are all perpendicular to the plane of the paper, therefore, *the incident ray* (LA or MP′), *normal* (AN) and *the reflected ray* (AA′L′ or P′M′), which are respectively perpendicular to AP, XY and A′P′, *all lie in the same plane.*

Hence, the laws of reflection are proved on the basis of the wave theory.

4.04. REFRACTION ONTHE BASIS OFWAVETHEORY

Let XY be a plane surface separating air from a denser medium and PA be a ne wavefront just incident on it. The normals LA and MP to the incident wavefront represent incident rays. If AN is normal to the refracting surface at point A, then ∠ NAL = i is the angle of incidence as shown in Fig. 4.06.

First of all, the wavefront reaches point A. From the points located between points A to P, the wavefront will reach points between A to P′ later in time but in the same order, so that it will reach point P′ in the last. Thus, different points on the surface XY will become sources of secondary wavelets at different instants of time. When the disturbance from point P on incident wavefront just reaches the point P′on the surface XY, the secondary wavelet from point A on the surface will grow into a sphere (inside the denser medium) of radius say A A′, such that

$$\frac{AA'}{v} = \frac{PP'}{c} \qquad \text{or} \qquad AA' = \frac{v}{c} + PP',$$

where v and c are the velocities of light in denser medium and air respectively.

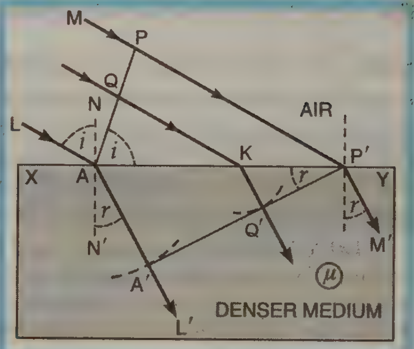


Fig. 4.06

To find the refracted wavefront (new position of the wave point after refraction) ; with point A as centre, draw a sphere of radius $AA' = \frac{v}{c} \times PP'$. From point P′, draw a tangent plane P′A′to the sphere. Then, P′A′ represents refracted wavefront and lines A A′L′ and P′M′ normals to the refracted wavefront represent refracted rays. Also, ∠N′A A′ = r is the angle of refraction.

To prove the laws of refraction : Consider any point Q on the incident wavefront. Suppose that when disturbance from point P on incident wavefront reaches point P′ on the refracted wavefront, the disturbance from point Q reaches point Q′ via point K on the surface of separation of air and denser medium. Since P′A′ represents the refracted wavefront, the time taken by light to travel from a point on incident wavefront to the corresponding point on refracted wavefront should be constant. Now, time taken by light to go from Q to Q′ will be

$$t = \frac{QK}{c} + \frac{KQ'}{v} \qquad ...(4.03)$$

In right angled Δ AQK, ∠ QAK = i

$$\therefore \qquad QK = AK \sin i$$

Also, in right angled Δ P′Q′K, ∠ Q′P′K = r

$$\therefore \qquad KQ' = KP' \sin r$$

In equation (4.03), substituting for QK and KQ′, we have

$$t=\frac{\text{AK}\sin i}{c}+\frac{\text{KP}'\sin r}{v}$$

or
$$t=\frac{\text{AK}\sin i}{c}+\frac{(\text{AP}'-\text{AK})\sin r}{v}\qquad(\because \text{KP}'=\text{AP}'-\text{AK})$$

or
$$t=\frac{\text{AP}'}{v}\sin r+\text{AK}\left(\frac{\sin i}{c}-\frac{\sin r}{v}\right)\qquad\text{...(4.04)}$$

For rays from different points on the incident wavefront, the values of AK are different. The rays from different points on incident wavefront will take the same time to reach the corresponding points on the refracted wavefront, if t, given by equation (4.04), is independent of AK. It will happen so, if

$$\frac{\sin i}{c}-\frac{\sin r}{v}=0\qquad\text{or}\qquad\frac{\sin i}{\sin r}=\frac{c}{v}$$

But $\frac{c}{v}=\mu$ (constant) is called the ***refractive index*** of the denser medium *w.r.t.* air.

$$\therefore\qquad\frac{\sin i}{\sin r}=\mu\text{ (constant)}$$

i.e. when a ray of light is incident on a refracting surface, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for the given pair of media.

Further, as explained in last section, it follows that *the incident ray* (LA or MP′), *the normal* (AN) *and the refracted ray* (AA′L′ or P′M′) *all lie in the same plane.*

Hence, the laws of refraction are proved on the basis of the wave theory.

4.05. BEHAVIOUR OF A LENS, PRISM AND CONCAVE MIRROR

It is interesting and useful to investigate the behaviour of lenses, prisms and spherical mirrors, when a plane wavefront is incident on them.

1. Behaviour of a prism. Consider a prism ABC of small angle and a plane wavefront PB incident on the prism [Fig. 4.07]. The lines LB and MP (normals to the incident wavefront PB) represent incident rays. It follows that the rays from different parts of the wavefront travel different thicknesses of the material of the prism. Light from point B travels totally inside the material of prism, while light from point P travels totally in the air. The refracted wave front will be the locus of all those points to which light from different points on incident wavefront reaches in phase. Suppose that the light from point B reaches the corresponding point say C on refracted wavefront by travelling along path BC = x (say) inside the material of the prism and the light from point P reaches the corresponding point say Q on the refracted wavefront by travelling in air. Let the length of path between points P and Q *i.e.* PA + AQ be equal to y. As the time taken along the two paths should be same,

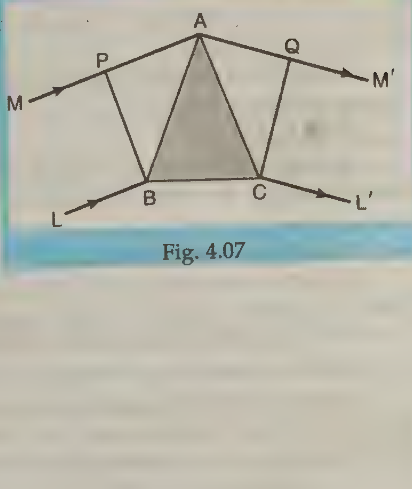


Fig. 4.07

$$\frac{x}{v}=\frac{y}{c}\qquad\text{or}\qquad y=\frac{c}{v}x,$$

where c is velocity of light in air and v is velocity of light in the material of the prism. Since v is less than c, y is greater than x. Out of the total path y, light covers path PA in air before it gets incident at point A and the remaining path (y – PA) *i.e.* AQ is covered after emerging from the prism. Hence, the emerging wavefront is of the shape CQ. The lines CL′ and QM′ (normal to the emergent wavefront CQ) represent emergent rays. It explains, why a ray of light passing through a prism is deviated towards the base of the prism.

2. Behaviour of a lens. Consider a plane wavefront AB incident on a double convex lens. The rays of light from points A and B travel totally inside the air, while the ray of light from point O travels partly in air and partly inside the material of the lens. Again, the refracted wavefront will be the locus of all those points to which light from different points on incident wavefront reaches in phase. As explained in case of behaviour of prism, it can be argued that the refracted wavefront will be a converging wavefront A′P$_2$ B′ [Fig. 4.08], such that

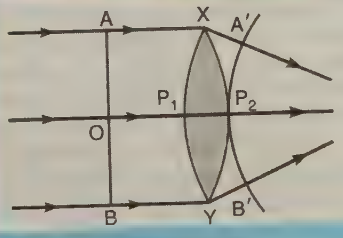


Fig. 4.08

$$\frac{\text{AX}+\text{XA}'}{c}=\frac{\text{OP}_1}{c}+\frac{\text{P}_1\text{P}_2}{v}=\frac{\text{BY}+\text{YB}'}{c}$$

Therefore, the wavefront after refraction through the lens is rendered into a converging wavefront as shown in the figure.

3. Behaviour of a spherical mirror. Consider a plane wavefront AB incident on a concave spherical mirror [Fig. 4.09]. Just as discussed above, the refracted wavefront will be a converging wavefront in nature, such that

$$AX + XA' = OP + PO' = BY + YB'$$

Since distance OP is greater than AX or BY, consequently distance PQ′ is less than XA′or YB′. Therefore, the incident plane wavefront will be rendered into a converging spherical wavefront after reflection from the concave mirror as shown in the figure.

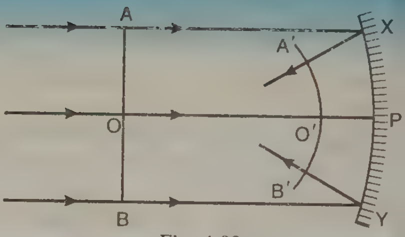


Fig. 4.09

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. What is ether medium ?

Ans. Ether is a hypothetical medium possessing very low density and very large value of volume elasticity and filling the whole space.

Q. 4.02. What is a wavefront ?

(H.S.S.C.E. 2001 ; P.S.S.C.E. 1997 S)

Or

Define wavefront.

(P.S.S.C.E. 2002 ; H.P.S.S.C.E. 1999 S ; H.S.S.C.E. 1995)

Or

What do you understand by a wavefront ?

(I.S.C.E. 1997, 1996, 1993)

Ans. Refer to section 4.01.

Q. 4.03. What is a ray of light ? *(P.S.S.C.E. 1998)*

Or

Define a ray of light. *(P.S.S.C.E. 1997)*

Ans. An arrow drawn normal to the wavefront and pointing in the direction of propagation of disturbance is called a ray of light.

Q. 4.04. What is the shape of the wavefront diverging from a point of source ? *(I.S.C.E. 1993)*

Ans. Spherical.

Q. 4.05. What is the shape of wavefront emitted by a light source in the form of a narrow slit ?

(H.S.S.C.E. 2002)

Ans. Cylindrical in shape.

Q. 4.06. Name the shape of a wave front originating from (*i*) a point source (*ii*) a line source.

(C.B.S.E. 2000 ; I.S.C.E. 1996)

Ans. (*i*) Spherical wave front (*ii*) Cylindrical wavefront.

Q. 4.07. What is the geometrical shape of the wavefront in each of the following cases ? (*a*) Light diverging from a point source. (*b*) Light emerging out of a convex lens, when a point source is placed at its focus. (*c*) The portion of the wavefront of light from a distant star intercepted by the earth. *(Text Question)*

Ans. (*a*) Spherical in shape. (*b*) Plane wavefront. (*c*) It is also a plane wavefront.

Q. 4.08. Sketch the wavefronts corresponding to converging rays. *(C.B.S.E. 1998)*

Ans. Refer to Fig. 4.02 (*b*).

Q. 4.09. What is the geometrical shape of the wavefront of the light diverging from a point source ?

(C.B.S.E.1999 S)

Or

Sketch the wavefronts corresponding to diverging rays. *(C.B.S.E. 1998)*

Ans. Refer to Fig. 4.02 (*c*).

Q. 4.10. How is a wavefront to the direction of corresponding rays ?

Ans. The wavefront is perpendicular to the direction of rays.

Q. 4.11. State Huygens' principle. *(P.S.S.C.E. 1996 S)*

Ans. Refer to section 4.02.

Q. 4.12. What is the phase difference between any two points on a wavefront ? *(H.S.S.C.E. 1994)*

Ans. Zero.

Q. 4.13. Can the phenomenon of rectilinear propagation, reflection and refraction be explained on the basis of wave nature of light ?

Ans. Yes.

Q. 4.14. What are the drawbacks of wave theory of light ?

Ans. The wave theory of light is based on the existence of all prevading ether medium. However, Michelson and Morley's experiment proved that the ether medium did not exist at all. It led to the failure of the wave theory of light.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 4.01. Why ether was called a hypothetical medium ?

Ans. The two properties of ether *i.e.* very low value of density and very large value of volume elasticity are contradictory in nature. A medium having such properties could not be conceived, when wave theory was profounded. For this reason, ether is called a hypothetical medium.

Q. 4.02. Why is the contribution of the wavelets lying on back of secondary wavefront zero ?

Ans. According to Voigt and Kirchhoff, the contribution of a wavelet in any direction making an angle θ with the normal to the wavelet is proportional to $\frac{1}{2}$ $(1 + \cos\ \theta)$.

For wavelet lying on the back of secondary wavefront, $\theta = 180°$ and like wise the factor

$$\frac{1}{2}(1+\cos\theta)=\frac{1}{2}(1+\cos 180°)=\frac{1}{2}(1-1)=0$$

Q. 4.03. What is the shape of wavefront orginating from (*i*) a point source and (*ii*) a line source ?

(I.S.C.E. 1998 S ; C.B.S.E. 1998)

Ans. (*i*) The wave front originating from a point source is *spherical* in shape. It is because, the locus of all such points, which are equidistant from the point source, is a sphere.

(ii) The wave front originating from a line source is cylindrical in shape. It is because, all the points, which are equidistant from the linear source, lie on the surface of a cylinder.

Q. 4.04. What is meant by a wavefront ? What is the shape of the wavefront of a beam of parallel rays ? *(I.S.C.E. 1998 S)*

Ans. The locus of all the particles of the medium, which at any instant are vibrating in the same phase, is called the wavefront.

Corresponding to a beam of parallel rays of light, the wavefront is plane in shape.

Q. 4.05. What is the geometrical shape of wavefront of light emerging out of a convex lens ; when a point source is placed at its focus ? *(C.B.S.E. 1999 S)*

Ans. When a point source is placed at the focus of a convex lens, it renders the light into the form of a parallel beam. The rays of light are always perpendicular to the wavefront. Therefore, the light emerging out of the convex lens will be in the form of a plane wavefront.

Q. 4.06. Explain, how a plane wavefront gets reflected from a concave mirror.

Ans. Refer to section 4.05.

Q. 4.07. A plane wavefront is incident on a prism. Draw the shape of the refracted wavefront.

Ans. Refer to section 4.05 and Fig. 4.06.

Q. 4.08. Discuss the refraction of a plane wavefront from a convex lens.

Ans. Refer to section 4.05.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 4.01. Is the speed of light in glass independent of the colour of light ? If not, which of the two colours (red and violet) travels slower in a glass prism ?

Ans. No, speed of light is not independent of the colour (wavelength) of the light. The violet colour travels slower than the red light in a glass prism.

Q. 4.02. When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency as the incident frequency. Explain, why. *(Text Question)*

Ans. Reflection and refraction (scattering in general) are the results of interaction of incident light with the atoms, which may be viewed as oscillators. When incident light falls on the atoms, the forced oscillations of the frequency of incident light are produced. Since, the frequency of light emitted by a charged oscillator is equal to its own frequency of oscillation, the frequency of reflected or refracted light is same as that of incident light.

Q. 4.03. When light travels from a rarer to a denser medium, it loses some speed. Does the reduction in speed imply a reduction in the energy carried by the light waves ? *(Text Question)*

Ans. No, decrease in speed of light does not imply reduction in the energy carried by the light wave. It is because, energy carried by a wave does not depend upon its speed of propagation. Instead, it depends upon its amplitude.

Q. 4.04. In what respects is Huygens' wave theory of light similar to Maxwell's electromagnetic theory ? In what respect are the two theories different ?

Ans. The two theories are similar in the respect that according to both the theories, light is a wave motion. On the other hand, the two theories are different in the respect that whereas according to electromagnetic theory, light does not require any medium for its propagation; a material medium is necessary for the propagation of light waves according to wave theory. For this, Huygens assumed the existence of all prevading ether medium.

Q. 4.05. In the wave picture of light, intensity of light is determined by the square of amplitude of wave. What determines the intensity of light in the photon picture of light ? *(Text Question)*

Ans. In the wave picture of light, intensity of light at a point is determined by the number of photons incident per unit area around that point.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is wavefront and Huygens' principle ? Explain.
2. What are the two assumptions on which Huygens' principle is based ? Explain Huygens' geometrical construction for wavefronts. *(H.P.S.S.C.E. 1996)*
3. *(a)* State the postulates of Huygens' wave theory.
   (b) Draw the type of wavefront that corresponds to a beam of light *(i)* coming from a very far-off source and *(ii)* diverging from a point source. *(C.B.S.E. 2001)*
4. Verify law of reflection using Huygens' wave theory. *(C.B.S.E. 1998, 1992)*
5. Describe the phenomenon of refraction from Huygens' wave theory. *(H.S.S.C.E. 2000)*
6. Using Huygens' principle and drawing the sketches of wavefronts, show how a parallel beam of light is reflected from a polished surface and hence verify that $\angle i = \angle r$. *(C.B.S.E. 1999 S)*
7. Deduce the laws of reflection on the basis of Huygens' principle. *(H.P.S.S.C.E. 2002 ; P.S.S.C.E. 2001 ; C.B.S.E. 1992)*
8. Using Huygens' wave theory, prove that for reflection of light, angle of incidence and angle of reflection are equal. *(C.B.S.E. 1995)*
9. Verify Snell's law of refraction using Huygens' wave theory. *(H.S.S.C.E. 1999 S ; CB.S.E. 1999, 1998)*
10. Deduce Snell's law of refraction using Huygens' wave theory. *(C.B.S.E. 2002)*
11. Derive Snell's law of refraction using Huygens' principle. *(P.S.S.C.E. 1996)*
12. Using Huygens' wave theory, derive Snell's law. *(C.B.S.E. 1993)*
13. Using Huygens' principle and drawing the sketches of wavefronts, show how a parallel beam of light is refracted from the surface of separation of a rarer medium to a denser medium. Verify Snell's law. *(C.B.S.E. 1999 S)*
14. Deduce the laws of refraction on the basis of Huygens' principle. *(C.B.S.E. 1992)*
15. Explain the action of a prism, a convex lens and a concave mirror, when a plane wavefront is incident on it.

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. State Huygens' principle and use it to describe the process of reflection of a parallel beam of light incident on a plane mirror obliquely with the help of a labelled diagram. Prove that the angle of incidence is equal to the angle of reflection. *(I.S.C.E. 1998)*
2. What is a wavefront ? What is the shape of the wavefront diverging from a point source ? State Huygens' principle and apply it to show that, for a parallel beam incident on a reflecting surface, the angle of incidence is equal to the angle of reflection. *(I.S.C.E. 1993)*
3. State Huygens' principle. Using this principle, illustrate how a parallel beam of light is reflected from a plane mirror. Prove the law of reflection. *(I.S.C.E. 1994)*
4. State Huygens' principle for the propagation of light and prove the laws of refraction of light on its basis. *(P.S.S.C.E. 2001)*
5. State Huygens' principle and derive the laws of refractio on its basis. *(P.S.S.C.E. 2002 ; 2000, 1999, 1997 ; H.P.S.S.C.E. 197*
6. State Huygens' principle and wavefront. Define refractiv index of a medium on the basis of wave theory. *(H.S.S.C.E. 199*
7. State Huygens' principle. Use it to derive Snell's law c refraction for a parallel beam incident on the plane surfac of a refracting medium. *(I.S.C.E. 1996*
8. Huygens' principle is the basis of wave theory of ligh What are the two assumptions it is based on ? Deduce th laws of refraction of light using Huygens' principle. *(Pre-degree Kerala, 199*

CHAPTER

5

Interference of Light

5.01. INTERFERENCE OF LIGHT

When a source of light emits energy, the distribution of energy is uniform in the medium. But when two sources of light lie close to each other and emit light of *same wavelength* and preferably of *same amplitude*, then due to superposition of waves from the two sources, the distribution of light energy no longer remains uniform.

*The phenomenon of non-uniform distribution of energy in the medium due to superposition of two light waves is called **interference** of light.*

At some points in the medium, the intensity of light is maximum (***constructive interference***); while at some other points, the intensity is minimum (***destructive interference***). In case the two waves are of the same amplitude, then at points of destructive interference, the intensity of the light is zero *i.e.* complete darkness results. Usually, on the interference of the light waves, we obtain alternate bright and dark bands of light called ***interference fringes***.

It may be clearly understood that in interference of light, energy is neither created nor destroyed. The energy that disappears at the points of destructive interference appears at the points of constructive interference and vice-versa. Thus, in interference of light, merely redistribution of light energy takes place.

Key point

Due to interference of two light waves, redistribution of energy takes place in the medium.

5.02. YOUNG'S DOUBLE SLIT EXPERIMENT

In 1801, Thomas Young demonstrated the interference of light experimentally. His experiment gave a deadly blow to the corpuscular theory of light as the experiment led to the conclusion that light has wave nature.

A source of monochromatic light illuminates a rectangular narrow slit S, which acts as the line source for illuminating two narrow slits S_1 and S_2. The two slits lie very close to each other and at equal distances from the source S [Fig. 5.01]. The wave-fronts from the slits S_1 and S_2 spread out in all directions and in the region they superpose on the screen, alternate ***bright and dark fringes*** are observed. At the centre O of screen, the intensity of light is maximum and it is called ***central maximum***.

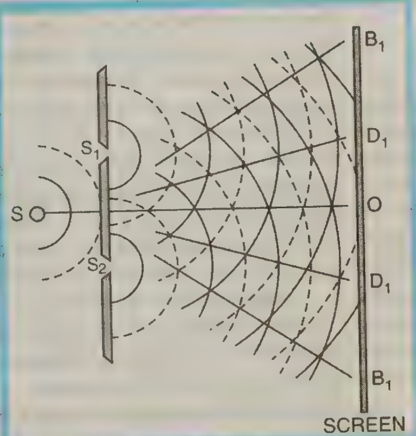


Fig. 5.01

Explanation. According to Huygens' principle, the two slits S_1 and S_2 send out waves of equal wavelengths and of same amplitude in all directions. As the paths S_1O and S_2O are equal, the light waves from the two slits S_1 and S_2 reach point O *in phase i.e.* either crest of one wave falls on the crest of the other wave or trough of one falls on the trough of the other. Hence, intensity of light at point O becomes maximum. It is called ***central maximum***.

As we move above the centre O of the screen (or below the point O), we come across points D_1, B_1, D_2, B_2, ... (the points D_2, B_2, ... are not shown in the figure).

At points D_1, D_2, ... , the crest of one wave falls on the trough of the other and vice-versa. The resultant amplitude and hence the resultant intensity of light becomes minimum at these points due to ***destructive interference*** of light. These points correspond to positions of ***dark fringes*** on the screen.

At points B_1, B_2, the crest of one wave falls on the crest of the other or trough of one on the trough of the other. Due to this, the resultant amplitude and hence resultant intensity of light at these points becomes maximum due to ***constructive interference*** of light. These points correspond to the position of ***bright fringes*** on the screen.

Further, in Young's double slit experiments, the following facts can also be verified :

1. *The interference pattern disappears, if one of the two slits is closed.* It shows that interference pattern is due to superposition of waves from the two slits.

2. *The interference pattern obtained is not a permanent one and the positions of maximum and minimum intensity do not remain fixed, if instead of two slits illuminated with a source, two independent sources are used.* It shows that for producing a sustained interference pattern, two sources should be obtained from a single source. Such sources are called ***coherent sources.***

5.03. SUSTAINED INTERFERENCE

The interference pattern, in which the positions of maximum and minimum of intensity of light remain fixed all along on the screen, is called sustained or permanent interference pattern.

For producing sustained interference, following conditions should be satisfied :

Conditions for sustained interference. 1. *The two sources should emit the light waves continuously.*

2. *The light waves should be of same wavelength.*

3. *The light waves emitted should preferably be of the same amplitude.* If the amplitudes of the waves are equal, the dark fringes are completely dark (zero intensity of light).

4. *The waves emitted by the two sources of light should either be in phase or should have a constant phase difference.*

5. *The two sources of light must lie very close to each other.* If it is not so, then the path difference between the light waves reaching a particular point will be very large. As we shall see, in such a case, maxima and minima will lie very close to each other and may result in overlapping.

6. *The two light sources should be very narrow.* A broad source of light is equivalent to a large number of narrow sources lying together. Each set of two sources will, therefore, give its own interference pattern and their overlapping will result in general illumination.

Key point

The sustained interference is observed only, when two coherent sources of light are used.

5.04. COHERENT SOURCES

In practice, a sustained interference pattern cannot be obtained by using two independent sources of light. It is because of the following reasons :

1. *Two independent sources of light cannot emit waves continuously.*

2. *The waves emitted by two independent sources of light do not have same phase or a constant phase difference.* It is because light is emitted by each excited atom of the source, which are billions in number. An excited atom emits light in a time of the order of 10^{-8} second. Therefore, if we have two independent sources, then the waves from two points in two sources will have a definite phase relationship only for 10^{-8} second and accordingly interference pattern formed by them will last on the screen only for this much time. The next bursts of waves coming from the two points in the two sources may be having an altogether different phase relationship and consequently the interference pattern may get shifted to some other place on the screen. These changes in positions of maxima and minima of intensity of light will occur about 10^8 times in one second. As the changes are too fast to be followed by the human eye, we shall not observe any interference pattern and there will be general illumination over the screen.

In order to obtain sustained interference pattern, the phase difference between the light waves arriving at a point on the screen from the two sources must remain constant with time. This can be made possible only if the two sources are derived from a single source. Such sources are called ***coherent sources.*** A change in phase, if any, will affect the two sources equally. Moreover, light waves from the two coherent sources are of *same wavelength* and *amplitude.*

Therefore, two sources are said to be coherent, if they emit light waves of same frequency or wavelength and of a stable phase difference.

When we make use of two coherent sources, the first four conditions for producing sustained interference pattern automatically get satisfied. In practice, two coherent sources may be two images of a source (both real or both virtual) or a source and its image. In case of a Lloyd's mirror, a source and its image act as the two coherent sources, while in case of a Fresnel's biprism, two virtual images of a source act as the two coherent sources.

5.05. PRINCIPLE OF SUPERPOSITION OF LIGHTWAVES

It states that *when two or more wave trains of light travelling in a medium superpose upon each other, then the resultant displacement at any instant is equal to the vector sum* *the displacements due to individual waves.* If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ...$ be the displacements ue to different waves, then resultant displacement of the particle is given by

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$$

If crest of one wave falls on the trough of the other, the resultant amplitude will ecrease. However, when crests or troughs of one wave superpose the crests or oughs respectively of the other wave, the resultant amplitude increases. The terference of light waves can be easily explained on the basis of the principle of uperposition of waves.

5.06. CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

A source of monochromatic light S illuminates two narrow slits S$_1$ and S$_2$. The vo illuminated slits act as the two coherent sources. Further, the two slits lie very ose to each other and at equal distance from the source S [Fig. 5.02]. The avefronts from S$_1$ and S$_2$ spread out in all directions and in the region they uperpose on the screen, alternate bright and dark fringes are observed. At the centre of the screen, the intensity of light is maximum and it is called *central maximum*.

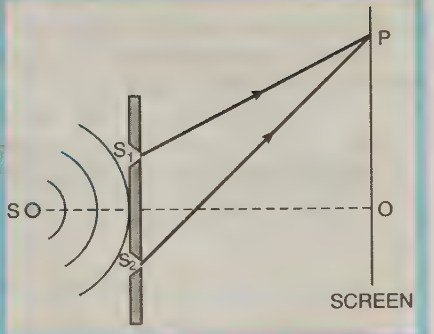


Fig. 5.02

Condition for maximum and minimum. Let the displacements of the waves om the sources S$_1$ and S$_2$ at point P on the screen at any time t be given by

$$y_1 = a_1 \sin \omega t$$

and $y_2 = a_2 \sin (\omega t + \phi)$,

where ϕ is the constant phase difference between the two waves. By the uperposition principle, the resultant displacement at point P is given by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi) = a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

or $$y = (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t \quad ...(5.01)$$

Let $$a_1 + a_2 \cos \phi = \mathrm{A} \cos \theta \quad ...(5.02)$$

and $$a_2 \sin \phi = \mathrm{A} \sin \theta \quad ...(5.03)$$

Then, equation (5.01) becomes

$$y = \mathrm{A} \cos \theta \sin \omega t + \mathrm{A} \sin \theta \cos \omega t$$

or $$y = \mathrm{A} \sin (\omega t + \theta)$$

Hence, resultant displacement at point P is simple harmonic wave having mplitude A and phase difference θ with the harmonic wave from the source S$_1$. n order to know the amplitude of the resultant simple harmonic wave, we proceed s below :

Squaring and adding both sides of the equations (5.02) and (5.03), we obtain

$$\mathrm{A}^2 \cos^2 \theta + \mathrm{A}^2 \sin^2 \theta = (a_1 + a_2 \cos \phi)^2 + a_2^2 \sin^2 \phi$$

or $$\mathrm{A}^2 = a_1^2 + a_2^2 (\cos^2 \phi + \sin^2 \phi) + 2\, a_1 a_2 \cos \phi$$

or $$\mathrm{A}^2 = a_1^2 + a_2^2 + 2\, a_1 a_2 \cos \phi$$

The intensity of light is directly proportional to the square of amplitude of the vave. For the sake of simplicity, we assume that intensity of light is equal to the quare of the amplitude. Therefore, intensity of light at point P on the screen is given y

$$\mathrm{I} = a_1^2 + a_2^2 + 2\, a_1 a_2 \cos \phi \quad ...(5.04)$$

Constructive interference. From equation (5.04), it follows that the intensity f light at point P will be maximum, if

$$\cos \phi = +1 \quad \text{or} \quad \phi = 0, 2\pi, 4\pi, ...$$

or $$\phi = 2\,n\pi, \quad \text{where } n = 0, 1, 2, ... \quad ...(5.05)$$

It is the *condition for constructive interference* between the two light waves *n terms of phase difference* between the waves. The two light waves from sources $_1$ and S$_2$ reach point P after covering unequal paths *i.e.* the two waves possess *path ifference*. The above condition for constructive interference can also be expressed n terms of path difference between the two waves.

In a harmonic wave, the time in which the phase of the vibrating particle hanges by 2π, the wave travels a distance equal to wavelength λ. In other words, path difference equal to wavelength λ is equivalent to a phase difference of 2π. herefore, if the two waves from the sources S$_1$ and S$_2$ reaching point P have a path ifference x, then phase difference between the two waves will be

$$\phi = \frac{2\pi}{\lambda} x \quad ...(5.06)$$

Using equation (5.06), the condition for constructive interference *i.e.* equation (5.05) becomes

$$\frac{2\pi}{\lambda}x = 2n\pi$$

or $$x = n\lambda, \quad \text{where } n = 0, 1, 2, 3, \ldots\ldots\ldots \qquad \ldots(5.07)$$

It is the ***condition for constructive interference*** between two light waves ***in terms of path difference*** between them. From equations (5.05) and (5.07), it follows that *the point P will be the seat of maximum intensity of light (bright fringe), if between the two light waves reaching point P, the phase difference is an even integral multiple of π or the path difference is an integral multiple of λ.*

Destructive interference. From equation (5.04), it follows that the intensity of light at point P will be minimum, if

$$\cos\phi = -1 \quad \text{or} \quad \phi = \pi, 3\pi, 5\pi, \ldots\ldots\ldots$$

or $$\phi = (2n+1)\pi, \text{ where } n = 0, 1, 2, \ldots\ldots\ldots \qquad \ldots(5.08)$$

Also, from equations (5.06) and (5.08), we have

$$\frac{2\pi}{\lambda}x = (2n+1)\pi$$

or $$x = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, \ldots\ldots\ldots \qquad \ldots(5.09)$$

The equations (5.08) and (5.09) are the ***conditions for destructive interference in terms of phase difference*** and ***path difference*** respectively. From these conditions, it follows that *the point P will be seat of minimum intensity of light, if between the two light waves reaching point P, the phase difference is an odd integral multiple of π or the path difference is an odd integral multiple of $\frac{\lambda}{2}$.*

A few important results. The expressions for intensity of light in the following cases are found to be of immense use in the study of the phenomenon of interference of light :

1. ***Ratio of intensity of light at maxima and minima.*** From equation (5.04), it follows that

$$\mathrm{I}_{max} = a_1^2 + a_2^2 + 2a_1a_2 \times 1 = (a_1 + a_2)^2$$

and $$\mathrm{I}_{min} = a_1^2 + a_2^2 + 2a_1a_2 \times (-1) = (a_1 - a_2)^2$$

Therefore, $$\frac{\mathrm{I}_{max}}{\mathrm{I}_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} \qquad \ldots(5.10)$$

2. ***Ratio of intensity of light due to two sources.*** The intensity of light due to a slit (source of light) is directly proportional to the width of the slit. Therefore, if w_1 and w_2 are widths of the two slits S_1 and S_2 ; and I_1 and I_2 are intensities of light due to the respective slits on the screen, then

$$\frac{w_1}{w_2} = \frac{\mathrm{I}_1}{\mathrm{I}_2} \qquad \ldots(5.11)$$

Since intensity of light is directly proportional to the square of the amplitude of the light wave,

$$\frac{\mathrm{I}_1}{\mathrm{I}_2} = \frac{a_1^2}{a_2^2} \qquad \ldots(5.12)$$

From equations (5.11) and (5.12), we have

$$\frac{w_1}{w_2} = \frac{a_1^2}{a_2^2} \qquad \ldots(5.13)$$

5.07. THEORY OF INTERFERENCE FRINGES

Consider that two coherent sources of light S_1 and S_2 are placed at a distance d apart and a screen is placed at a distance D from the plane of the two sources. Let P be a point on the screen at a distance y from the centre of the screen [Fig. 5.03]. If x is path difference between the light waves reaching point P from the sources S_1 and S_2, then

$$x = S_2P - S_1P$$

In right angled $\Delta\, S_2\,BP$, we have

$$S_2P^2 = S_2B^2 + BP^2 = D^2 + \left(y + \frac{d}{2}\right)^2$$

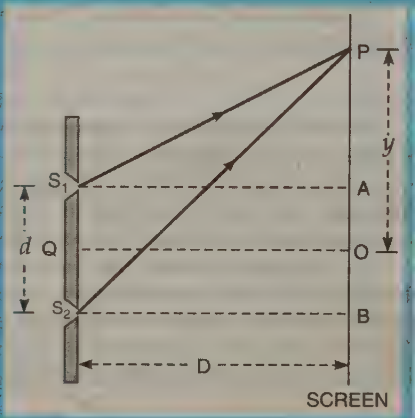


Fig. 5.03

Also, in right angled ΔS_1AP, we have

$$S_1P^2 = S_1A^2 + D^2 + \left(y - \frac{d}{2}\right)^2$$

$$\therefore \quad S_2P^2 - S_1P^2 = \left(D^2 + \left(y + \frac{d}{2}\right)^2\right) - \left(D^2 + \left(y - \frac{d}{2}\right)^2\right)$$

or $(S_2P + S_1P)(S_2P - S_1P) = \left(y + \frac{d}{2}\right)^2 - \left(y - \frac{d}{2}\right)^2$

Since $S_2P - S_1P = x$ (the path difference between the two light waves), the above equation becomes

$$(S_2P + S_1P) \,.\, x = 4y \,.\, \frac{d}{2} = 2\,y\,d \quad \text{or} \quad x = \frac{2\,y\,d}{S_2P + S_1P}$$

In practice, point P lies very close to O. Therefore, $S_2P \approx S_1P \approx D$.

Hence, $$x = \frac{2\,y\,d}{D + D} = \frac{2\,y\,d}{2\,D}$$

or $$x = \frac{y\,d}{D} \qquad \text{...(5.14)}$$

Position of maximum and minimum on the screen. From equations (5.07) and (5.14), it follows that point P will be centre of a bright fringe, if

$$\frac{y\,d}{D} = n\,\lambda$$

or $$y = \frac{n\,D\,\lambda}{d}, \quad \text{where } n = 0, 1, 2, \text{..........} \qquad \text{...(5.15)}$$

Suppose that the values of y for $n = 0, 1, 2, 3,$ .... n are $y_0, y_1, y_2,$ .... y_n respectively. Then, the values of $y_0, y_1, y_2,$ .... y_n can be obtained by putting $n = 0, 1, 2,$.......... n in equation (5.15).

For $n = 0$: $y_0 = 0$, *the distance of central bright fringe from point O*

For $n = 1$: $y_1 = \frac{D\lambda}{d}$, *the distance of first bright fringe from point O*

For $n = 2$: $y_2 = \frac{2D\lambda}{d}$, *the distance of second bright fringe from point O*

... ...

... ...

For $n = n$: $y_n = \frac{nD\lambda}{d}$, *the distance of nth bright fringe from point O*

From the equations (5.09) and (5.14), it follows that point P will be centre of a dark fringe, if

$$\frac{y\,d}{D} = (2\,n + 1)\,\frac{\lambda}{2}$$

or $$y = \frac{(2\,n + 1)\,D\,\lambda}{2\,d}, \quad \text{where } n = 0, 1, 2, \qquad \text{...(5.16)}$$

Suppose that the values of y for $n = 0, 1, 2,$ .... n are $y_0', y_1', y_2',$ .... y_n' respectively. Then, the values of $y_0', y_1', y_2',$ .... y_n' can be obtained by putting $n = 0, 1, 2,$ .... n in equation (5.16).

For $n = 0$: $y_0' = \frac{D\lambda}{2d}$, *the distance of first dark fringe from point O.*

For $n = 1$: $y_1' = \frac{3D\lambda}{2d}$, *the distance of second dark fringe from point O.*

For $n = 2$: $y_2' = \frac{5D\lambda}{2d}$, *the distance of third dark fringe from point O.*

... ...

... ...

For $n = n$: $y_n' = \frac{(2n+1)D\lambda}{2d}$, *the distance of (n + 1)th dark fringe from point O.*

Fringe width. The spacing between any two consecutive bright fringes is equal to the width of a dark fringe. Therefore, width of the dark fringe,

$$y_n - y_{n-1} = \frac{n\,D\,\lambda}{d} - \frac{(n-1)\,D\,\lambda}{d} = \frac{D\,\lambda}{d} \qquad ...(5.17)$$

The spacing between the two consecutive dark fringes is equal to the width of a bright fringe. Therefore, width of the bright fringe

$$y_n' - y_{n-1}' = \frac{(2n+1)\,D\,\lambda}{2d} - \frac{[(2n-1)+1]\,D\,\lambda}{2d} = \frac{D\,\lambda}{d} \qquad ..(5.18)$$

From equations (5.17) and (5.18), it follows that in Young's double slit experiment, the bright and dark fringes are of the same width. The fringe width is denoted by β. Therefore,

$$\beta = \frac{D\,\lambda}{d} \qquad ...(5.19)$$

The width of central bright fringe is equal to the distance between the first dark fringes formed on the two sides of the centre of the screen. Therefore, the width of the central bright fringe,

$$\beta_0 = 2\,y_0' = 2 \times \frac{D\,\lambda}{2\,d}$$

or

$$\beta_0 = \frac{D\,\lambda}{d} \qquad ...(5.20)$$

Key point

In the interference pattern produced by two narrow slits, the fringe width increases

(*i*) on using light of smaller wavelength,

(*ii*) on increasing distance between slits and the screen,

(*iii*) on decreasing distance between the two slits and

(*iv*) on replacing the medium between the slits and the screen by a medium of higher refractive index.

Key point

In the interference pattern produced by two narrow slits, the width of all the fringes (bright, dark and central) is same.

In case, the two sources emit light of same amplitude a, then the bright fringe will be of intensity

$$I_{max} = (a + a)^2 = 4\,a^2$$

On the other hand, the dark fringes will be of intensity

$$I_{min} = (a - a)^2 = 0$$

Therefore, all the interference fringes will be of equal widths, such that the dark fringes will be totally dark and the bright fringes will be of constant intensity ($4\,a^2$) as shown in Fig. 5.04. The bright fringes are formed at points $B_1, B_2, ...$, while the dark fringes are formed at the points $D_1, D_2, ...$ on both the sides of the central bright fringe formed at the centre of the screen.

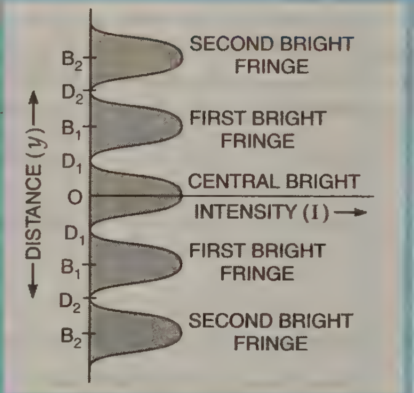


Fig. 5.04

Discussion. 1. As expression for β is independent of n, all the fringes are of same width.

2. As β is directly proportional to λ, the fringes produced by light of shorter wavelength will be narrower as compared to those produced by light of longer wavelength.

3. For appreciable fringe width, D should be large and d small.

4. The positions of nth bright and (n + 1)th dark fringes are respectively given by

$$y_n = \frac{n\,D\,\lambda}{d} \quad \text{and} \quad y_n' = \frac{(2n+1)\,D\,\lambda}{2\,d}$$

The bright and dark fringes will be distinct, if y_n and y_n' are large *i.e.* d is small. For this reason, the two slits should lie very close to each other.

5. As all the dark and bright fringes are of the same width, the angular width of a fringe is given by

$$\theta = \frac{\beta}{D} = \frac{D\,\lambda/d}{D}$$

or

$$\theta = \frac{\lambda}{d}$$

6. On immersing the apparatus in a liquid (say water), the wavelength of light decreases $\left(\lambda' = \frac{\lambda}{\mu}\right)$. From equation (5.19), it follows that when the apparatus is immersed in a liquid, the fringe width will decrease.

Key point

In the interference pattern produced by two narrow slits, the intensity of all the bright fringes is same.

5.08. INTERFERENCE DUE TO THIN FILMS (Qualitative Only)

A thin film of a liquid (soap film or a film of oil spread over water surface) appears bright or dark, when viewed in a monochromatic light.

This effect can be explained in terms of interference phenomenon.

Consider a thin transparent film of thickness t and refractive index μ. A monochromatic ray of light AB incident at angle i on the upper surface X_1Y_1 of the film, is partly reflected along BC and partly refracted into the material of the film along BD, such that r is angle of refraction [Fig. 5.05]. At point D, the ray of light is partly reflected along DF and partly transmitted out of the medium along DE. As the two surfaces of the film are parallel, the transmitted ray DE will be parallel to the incident ray AB. The ray of light will suffer reflections and refractions at points F, H, etc. in a similar manner. When viewed from the same side, the film appears dark or bright due to the interference of the reflected rays BC and FG. The rays BC and FG are said to interfere in *reflected system*. On the other hand, the film appears dark or bright due to interference of the transmitted rays DE and HI. The rays DE and HI are said to interefere in *transmitted system.*

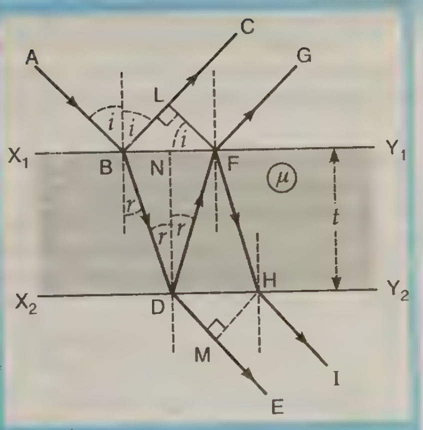


Fig. 5.05

Condition for film to appear bright or dark. The geometrical path difference between the rays BC and FG is given by

$$x = \mu\,(BC + DF) - BL$$

From simple geometry, it can be proved that

$$x = 2\,\mu\,t\cos r \qquad \text{...(5.21)}$$

Reflected system. Due to reflection of the ray of light BC at the surface of denser medium (thin liquid film), an additional phase difference of π or a path difference of $\frac{\lambda}{2}$ is introduced. As a result, the film will *appear dark*, when path difference x is an integral multiple of λ *i.e.* when

$$2\,\mu\,t\cos r = n\,\lambda, \qquad \text{where } n = 1, 2, 3, \text{..........} \qquad \text{...(5.22)}$$

In case film is of negligible thickness *i.e.*, $t << \lambda$, then net path will be just $\frac{\lambda}{2}$ (the path difference introduced due to reflection from denser medium) and hence film will appear dark.

The film will *appear bright*, when

$$2\,\mu\,t\cos r = (2\,n + 1)\,\frac{\lambda}{2}, \qquad \text{where } n = 0, 1, 2, \text{..........} \qquad \text{...(5.23)}$$

Transmitted system. When the film is viewed from the other side *i.e.* in transmitted system, there is no additional path difference between the two rays DE and HI. It is because, both the rays come after reflection at rarer medium at points D and F.

Therefore, the film will *appear bright*, when

$$2\,\mu\,t\cos r = n\,\lambda, \qquad \text{where } n = 1, 2, 3, \text{..........} \qquad \text{...(5.24)}$$

and the film will appear dark, when

$$2\,\mu\,t\cos r = (2\,n + 1)\,\frac{\lambda}{2}, \qquad \text{where } n = 0, 1, 2, \text{..........} \qquad \text{...(5.25)}$$

In case film is of negligible thickness, then there will be no path difference and hence the film will appear bright.

Note. It may be pointed out that conditions for film to appear bright or dark in the reflected light are just reverse of those in the transmitted light. Therefore, the film which appears bright in reflected system will appear dark in transmitted light and vice-versa.

5.09. COLOURS INTHIN FILMS

As discussed in last section ; in monochromatic light, a thin film may appear bright and dark depending upon the path difference between the rays of light coming after reflection from upper and lower surfaces of the film. However, in white light, a soap film or a thin film of oil spread over water surface appears to be coloured as explained below :

The path difference $2\,\mu\,t\cos r$ between the rays of light coming after reflection from the upper and bottom surfaces of a film depends upon μ, t, λ and r.

When a particular part of film is looked from a particular position of the eye, the parameters t and r get fixed. Since μ depends upon λ, the light of only a particular wavelength will satisfy the condition for the film to appear bright.

Hence, only a few wavelengths will be observed with maximum intensity, a few others having values in the neighbourhood will appear with lesser intensity and some others which satisfy the condition for film to appear dark will be missing.

Thus, the particular portion of the film appears of a particular colour, when seen from a particular position of the eye, when instead of monochromatic light, white light is incident on it.

Key point ✓

Thin soap or oil films appear coloured due to the phenomenon of interference, when light falls on such films.

5.10. INTERFERENCE AND CONSERVATION OF ENERGY

Consider that the two sources of light are emitting light waves of amplitude a_1 and a_2 respectively. If there is no interference between the light waves from the two sources, then the intensity of light at any point on the screen will be

$$I = I_1 + I_2$$

or
$$I = a_1^2 + a_2^2 \quad \text{...(5.26)}$$

In case the light from two sources intereferes, then maxima and minima will be formed. The intensities of light at maxima and minima are given by

$$I_{max} = (a_1 + a_2)^2 \quad \text{and} \quad I_{min} = (a_1 - a_2)^2$$

Therefore, average intensity of light on the screen due to interference is given by

$$I_{av} = \frac{I_{max} + I_{min}}{2} = \frac{(a_1 + a_2)^2 + (a_1 - a_2)^2}{2}$$

or
$$I_{av} = a_1^2 + a_2^2 \quad \text{...(5.27)}$$

From equations (5.26) and (5.27), it follows that when the interference takes place, the light energy which disappears at the regions of destructive interference appears at the regions of constructive interference so that the average intensity of light remains the same. In other words, in the phenomenon of interference, neither energy is created at the regions of constructive interference nor it is destroyed at the regions of destructive interference. Hence, the law of conservation of energy is being obeyed in the phenomenon of interference of light.

Key point

The phenomenon of interference does not violate the law of conservation of energy. The light energy, which disapears at the positions of minima appears at the positions of maxima.

SOLVED NUMERICAL PROBLEMS

Type A. On Intensities of maxima and minima

Problem 5.01. If the two slits in Young's experiment have widths in the ratio 4 : 9, then find the ratio of intensity at the maximum to the intensity at the minimum in the interference pattern. *(H.P.S.S.C.E. 2002)*

Sol. The intensity of light due to a slit is directly proportional to the width of the slit. Therefore,

$$\frac{I_1}{I_2} = \frac{w_1}{w_2}$$

Since $\frac{w_1}{w_2} = \frac{4}{9}$, we have

$$\frac{I_1}{I_2} = \frac{4}{9} \quad \text{...(i)}$$

If a_1 and a_2 are amplitudes of the waves from the two slits, then

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \quad \text{...(ii)}$$

From equations (*i*) and (*ii*), we have

$$\frac{a_1^2}{a_2^2} = \frac{4}{9} \quad \text{or} \quad \frac{a_1}{a_2} = \frac{2}{3} \quad \text{or} \quad a_2 = 1{\cdot}5\, a_1$$

Now, $$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(a_1 + 1{\cdot}5\, a_1)^2}{(a_1 - 1{\cdot}5\, a_1)^2} = \frac{25}{1} = \mathbf{25 : 1}$$

Type B. On Young's double slit experiment

Problem 5.02. In a Young's double slit experiment, the fringe width obtained is 0·6 cm, when light of wavelength 4800 Å is used. If the distance between the screen and the slit is reduced to half, what should be the wavelength of light used to obtain fringes 0·0045 m wide? *(P.S.S.C.E. 1997)*

Sol. Here, $\lambda = 4800 \text{ Å} = 4800 \times 10^{-10}$ m ;
$\beta = 0{\cdot}6 \text{ cm} = 0{\cdot}6 \times 10^{-2}$ m

Now, $$\beta = \frac{D}{d}\lambda$$

or $$\frac{d}{D} = \frac{\lambda}{\beta} = \frac{4800 \times 10^{-10}}{0{\cdot}6 \times 10^{-2}} = 8 \times 10^{-5} \quad \text{...(i)}$$

When the distance between the screen and the slits is reduced to half of its initial value :

Here, $\beta' = 0{\cdot}0045$ m ; $D' = \frac{D}{2}$ and $d' = d$

Let λ' be the wavelength of the light source.

Then, $$\beta' = \frac{D'\lambda'}{d'}$$

or $$\lambda' = \beta' \frac{d'}{D'} = 0{\cdot}0045 \frac{d}{D/2} = 0{\cdot}009 \frac{d}{D} \quad \text{...(ii)}$$

From equations (*i*) and (*ii*), we have

$$\lambda' = 0{\cdot}009 \times 8 \times 10^{-5} = 7{\cdot}2 \times 10^{-7} \text{ m} = \mathbf{7200 \text{ Å}}$$

Problem 5.03. In a Young's double slit experiment, the slits are 1·5 mm apart. When the slits are illuminated by a monochromatic source and the screen is kept 1 m apart from the slits, width of 10 fringes is measured as 3·93 mm. Calculate the wavelength of light used. What will be the width of 10 fringes, when the distance between the slits and the screen is increased by 0·5 m ? The source of light used remains the same. *(Karnataka, 1995)*

Sol. Here, $d = 1{\cdot}5 \text{ mm} = 1{\cdot}5 \times 10^{-3}$ m ; D = 1 m ;
and width of 10 fringes = 3·93 mm

Therefore, fringe width,

$$\beta = \frac{3{\cdot}93}{10} = 0{\cdot}393 \text{ mm} = 0{\cdot}393 \times 10^{-3} \text{ m}$$

Now, $$\lambda = \beta \frac{d}{D} = 0{\cdot}393 \times 10^{-3} \times \frac{1{\cdot}5 \times 10^{-3}}{1}$$
$$= \mathbf{5{\cdot}895 \times 10^{-7} \text{ m}}$$

When the distance between the screen and the slits is changed :

Here, $D' = 1 + 0{\cdot}5 = 1{\cdot}5$ m ; $\lambda = 5{\cdot}895 \times 10^{-7}$ m ;
$d = 1{\cdot}5 \times 10^{-3}$ m ;

Therefore, width of 10 fringes = 10 β

$$= 10 \times \frac{D'}{d}\lambda = 10 \times \frac{1\cdot 5}{1\cdot 5 \times 10^{-3}} \times 5\cdot 895 \times 10^{-7}$$

$$= 5\cdot 895 \times 10^{-3} \text{ m}$$

Problem 5.04. Two slits in Young's experiment are 0·02 cm apart. The interference fringes for light of wavelength 6000 Å are formed on a screen 80 cm away. Calculate the distance of the fifth bright fringe.

Sol. Here, $d = 0\cdot 02$ cm ; D = 80 cm ;
$\lambda = 6000 \times 10^{-8}$ cm $= 6 \times 10^{-5}$ cm ;

For nthe bright fringe, $y_n = \frac{n D \lambda}{d}$

For fifth maximum, $n = 5$

$$\therefore \qquad y_5 = \frac{5 \times 80 \times 6 \times 10^{-5}}{0\cdot 02} = \mathbf{1\cdot 2 \text{ cm}}$$

Problem 5.05. In a Young's double slit experiment, the slits are 0·03 cm apart and the screen is placed 1·5 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1 cm. Determine the wavelength of light used. *(H.S.S.C.E. 2002 ; C.B.S.E. 1995, 1993 ; P.S.S.C.E. 1993 ; Roorkee, 1986)*

Sol. Here, $d = 0\cdot 03$ cm $= 3 \times 10^{-4}$ m ; D = 1·5 m ;

Now, distance of fourth bright fringe,

$$y_4 = 1 \text{ cm} = 10^{-2} \text{ m}$$

But $\quad y_4 = \frac{4 D \lambda}{d}$

or $\quad \lambda = \frac{y_4 \times d}{4 D} = \frac{10^{-2} \times 3 \times 10^{-4}}{4 \times 1\cdot 5} = \mathbf{5 \times 10^{7} \text{ m}}$

Note. Alternatively, one may proceed as below :
Distance between central and fourth bright fringe
= width of four dark fringes = 4 β

Problem 5.06. Two slits 0·125 mm apart are illuminated by light of wavelength 4500 Å. The screen is one metre away from the plane of the slits. Find the separation between the second bright fringes on both sides of the central maximum. *(P.S.S.C.E. 1996 S)*

Sol. Here, $d = 0\cdot 125$ mm $= 0\cdot 125 \times 10^{-3}$ m ;
$\lambda = 4500$ Å $= 4500 \times 10^{-10}$ m ; D = 1 m

The distance of nth bright fringe from the central maximum, $y_n = n \frac{D}{d} \lambda$

Therefore, distance of 2nd bright fringe from the central maximum,

$$y_2 = 2 \times \frac{1}{0\cdot 125 \times 10^{-3}} \times 4500 \times 10^{-10} \text{ m} = 7\cdot 2 \times 10^{-3} \text{ m}$$

Hence, separation between the second bright fringes formed on the both sides of the central maximum,

$$2\,y_2 = 2 \times 7\cdot 2 \times 10^{-3} = 14\cdot 4 \times 10^{-3} \text{ m} = \mathbf{144 \text{ m}}$$

Note. Alternatively, one may proceed as below :
Separation between the second bright fringes on both sides of the central maximum
= width of four dark fringes = 4 β

Problem 5.07. In a Young's double slit experiment, the fringes are formed at a distance of 1 m from double slit of separation 0·12 mm. Calculate the distance of 3rd dark band from the centre of the screen. Given, λ = 6000 Å.

Sol. Here, $d = 0\cdot 12$ mm $= 0\cdot 012$ cm ; D = 1 m = 100 cm ;
$\lambda = 6000$ Å $= 6000 \times 10^{-8}$ cm

For $(n + 1)$th dark fringe, $y_n' = (2n + 1)\frac{D\lambda}{2}$

For third dark fringe, $\quad n = 2$

$$\therefore \; y_2' = (2 \times 2 + 1) \times \frac{100}{2 \times 0\cdot 012} \times 6000 \times 10^{-8} = \mathbf{1\cdot 25 \text{ cm}}$$

Problem 5.08. In a Young's double slit experiment, the slits are 0·2 mm apart and the screen is 1·5 m away. It is observed that the distance between the central bright fringe and fourth dark fringe is 1·8 cm. Find the wavelength of light used. *(P.S.S.C.E. 1993, 1992)*

Sol. Here, $d = 0\cdot 2$ mm $= 2 \times 10^{-4}$ m ; D = 1·5 m ;

Now, distance of fourth dark fringe,

$$y_3' = 1\cdot 8 \text{ cm} = 1\cdot 8 \times 10^{-2} \text{ m}$$

But $\quad y_3' = \frac{7 D \lambda}{2 d}$

or $\quad \lambda = \frac{2\, y_3'\, d}{7 D} = \frac{2 \times 1\cdot 8 \times 10^{-2} \times 2 \times 10^{-4}}{7 \times 1\cdot 5}$

$$= \mathbf{6\cdot 86 \times 10^{-7} \text{ m}}$$

Problem 5.09. In a Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2·0 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1·33 ? *(C.B.S.E. 1991)*

Sol. *In air* : $\beta = 2\cdot 0$ mm $= 2\cdot 0 \times 10^{-3}$ m ;
$\lambda = 6000$ Å $= 6 \times 10^{-7}$ m ;

Now, $\quad \beta = \frac{D\lambda}{d}$

or $\quad \frac{D}{d} = \frac{\beta}{\lambda} = \frac{2\cdot 0 \times 10^{-3}}{6 \times 10^{-7}}$

In liquid : Let λ' be wavelength of light in liquid and β' be the corresponding fringe width. Then,

$$\lambda' = \frac{\lambda}{\mu} = \frac{6 \times 10^{-7}}{1\cdot 33}$$

Now, $\quad \beta' = \frac{D\lambda'}{d} = \frac{2\cdot 0 \times 10^{-3}}{6 \times 10^{-7}} \times \frac{6 \times 10^{-7}}{1\cdot 33} = 1\cdot 5 \times 10^{-3}$ m

$$= \mathbf{1\cdot 5 \text{ mm}}$$

Problem 5.10. A double slit is illuminated by light of wavelength 6000 Å. The slits are 0·1 cm apart and the screen is placed one metre away. Calculate (*a*) the angular position of the 10th maximum in radian and (*b*) separation of the two adjacent minima. *(P.S.S.C.E. 1996)*

Sol. Here, $\quad \lambda = 6000$ Å $= 6000 \times 10^{-10}$m ;
$d = 0\cdot 1$ cm $= 10^{-3}$ m ; D = 1 m ;

(*a*) The angular position of nth maximum,

$$\theta_n = \frac{y_n}{D} = \frac{n D \lambda}{d} \Big/ D = \frac{n\lambda}{d}$$

Therefore, angular position of 10th maximum,

$$\theta_{10} = \frac{10 \times 6000 \times 10^{-10}}{10^{-3}} = \mathbf{0\cdot 006 \text{ radian}}$$

(*b*) Separation between the two adjacent minima *i.e.* fringe width,

$$\beta = \frac{D\lambda}{d} = \frac{1 \times 6000 \times 10^{-10}}{10^{-3}} = 0\cdot 6 \times 10^{-3} \text{ m}$$

$$= \mathbf{0\cdot 6 \text{ mm}}$$

Type C. On Interference due to thin film

Problem 5.11. Using light of wavelength 6000 Å, it is found that in a thin film of air, 7 fringes occur between two points. Deduce the difference of the film thickness between these points. *(H.S.S.C.E. 1997 S)*

Sol. Let t be the difference of thickness of the air film between two points.

Then,

$$2\,\mu\,t\cos r = n\,\lambda$$

Here, for air film, $\mu = 1; n = 7;$

$$\lambda = 6000\text{ Å} = 6000 \times 10^{-10}\text{ m}$$

For normal incidence of light, $r = 0$

$\therefore$ $2 \times 1 \times t\cos 0° = 7 \times 6000 \times 10^{-10}$

or $t = \mathbf{2{\cdot}1 \times 10^{-6}\text{ m}}$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 5.12. White light may be considered to have λ from 4000 Å to 7500 Å. If an oil film has thickness 10^{-4} cm, deduce the wavelength in the visible region for which the reflection along the normal direction will be (*i*) weak (*ii*) strong. Take μ of the oil as 1·4.

Sol. Here, $\mu = 1{\cdot}4\,;\, t = 10^{-4}$ cm

For normal incidence (i and hence $r = 0°$), the path difference is given by

$$2\,\mu\,t = 2 \times 1{\cdot}4 \times 10^{-4}\text{ cm}$$
$$= 2{\cdot}8 \times 10^{-4} \times 10^{8} = 28000\text{ Å}$$

(*i*) Condition for weak reflection (destructive interference) is given by

$$2\,\mu\,t = n\,\lambda$$

or

$$\lambda = \frac{2\,\mu\,t}{n} = \frac{28000}{n}$$

The value of n should be such, so that λ lies between 3900 Å and 7600 Å. This will be possible, if

$$\lambda = \frac{28000}{4} = \mathbf{7000\text{ Å}} \qquad (\text{when } n = 4)$$

$$\lambda = \frac{28000}{5} = \mathbf{5600\text{ Å}} \qquad (\text{when } n = 5)$$

$$\lambda = \frac{28000}{6} = \mathbf{4667\text{ Å}} \qquad (\text{when } n = 6)$$

$$\lambda = \frac{28000}{7} = \mathbf{4000\text{ Å}} \qquad (\text{when } n = 7)$$

The other values of n are not allowed as for those values of n, λ does not lie within the given wavelength range from 3900 Å to 7600 Å. Hence, the above values of λ cause weak reflection.

(*ii*) For strong reflection, we have

$$2\,\mu\,t = (2\,n + 1)\,\lambda/2$$

$$\therefore \qquad \lambda = \frac{2(2\,\mu\,t)}{2\,n+1} = \frac{2 \times 28000}{2\,n+1} = \frac{56000}{2\,n+1}$$

Now,

$$\lambda = \frac{56000}{9} = \mathbf{6222\text{ Å}} \qquad (\text{when } n = 4)$$

$$\lambda = \frac{56000}{11} = \mathbf{5091\text{ Å}} \qquad (\text{when } n = 5)$$

$$\lambda = \frac{56000}{13} = \mathbf{4308\text{ Å}} \qquad (\text{when } n = 6)$$

Hence, only the above values of n will cause strong reflection. For other values of n, the wavelength will not be within the given wavelength range.

Problem 5.13. In a certain region A to B in a thin film, we get 10 fringes with light of wavelength 4358 Å. How many fringes will be observed in the same region with wavelength 5893 Å ?

Sol. Let n_1 and n_2 be the number of fringes observed between A and B. If t is the change in thickness of the film and λ_1 and λ_2 are wavelengths, then for normal incidence

$$2\,\mu\,t = n_1\,\lambda_1 = n_2\,\lambda_1$$

$$\therefore \qquad n_2 = \frac{n_1\,\lambda_1}{\lambda_2}$$

Here, $n_1 = 10\,;\, \lambda_1 = 4358\text{ Å} = 4358 \times 10^{-10}\text{ m}\,;$
$\lambda_2 = 5893 \times 10^{-10}\text{ m}$

Hence,

$$n_2 = \frac{10 \times 4358 \times 10^{-10}}{5893 \times 10^{-10}} = \mathbf{7{\cdot}4}$$

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. What is inference of light ? *(H.S.S.C.E. 2002)*

Ans. *The phenomenon of non-uniform distribution of energy in the medium due to superposition of two light waves is called interference of light.*

Q. 5.02. Define coherent sources of light. *(P.S.S.C.E. 1998 S)*

Or

What are coherent sources of light ? *(C.B.S.E. 2002, 2001, 1998 ; P.S.S.C.E. 2002, 2000, 1999 S ; H.S.S.C.E. 2002, 2001, 1997 S, 1992 ; I.S.C.E. 1994)*

Ans. Two sources obtained from a single source of light are called coherent sources.

Q. 5.03. Can two independent sources of light be coherent ?

Ans. No.

Q. 5.04. State the essential condition for two light waves to be coherent. *(H.S.S.C.E. 2002)*

Ans. 1. The light waves should be of same wavelength.

2. The two light waves should either be in phase or should have a constant phase difference.

Q. 5.05. What happens to the interference pattern, if the phase difference between the two sources continuously varies ?

Ans. If the phase difference between the two sources continuously varies, the positions of minima and maxima will also vary. Such an interference pattern will not be a sustained one and it will not be observed.

Q. 5.06. Two slits in Young's double slit experiment are illuminated by two different sodium lamps emitting light of same wavelength. Do you observe any interference pattern on the screen ? *(I.I.T. 1984)*

Ans. No. It is because, the phase difference between the light waves from the two independent sources keeps on changing continuously.

Q. 5.07. Why should we have narrow sources to produce good interference fringes ? *(H.S.S.C.E. 1993)*

Ans. It is because, a broad source is equivalent to a large number of narrow sources lying close to each other. Due to overlapping of interference patterns due to different pairs of narrow sources, the interference pattern obtained is not a distinct one.

Q. 5.08. What is the main condition to produce interference of light ? *(H.P.S.S.C.E. 1998 S)*

Or

State the most essential condition for observing interference of light. *(I.S.C.E. 1996)*

Ans. The two sources of light must be coherent sources.

Q. 5.09. What is the principle of superposition of light waves ?

Ans. It states that when two or more wave trains of light travelling in a medium superpose upon each other, then the resultant displacement at any instant is equal to the vector sum of the displacement due to individual waves.

Q. 5.10. What happens to the interference pattern, when one of the slits is closed ?

Ans. The interference pattern will disappear.

Q. 5.11. What is the relation between path difference and phase difference ?

Ans. Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

Q. 5.12. State the condition for constructive inference. *(H.S.S.C.E. 2002 ; H.P.S.S.C.E. 1998)*

Ans. Path difference, $x = n\lambda$, where $n = 0, 1, 2, 3....$

Q. 5.13. State the condition for destructive interference. *(H.S.S.C.E. 2002 ; C.B.S.E. 1992)*

Ans. Path difference, $x = (2n+1)\frac{\lambda}{2}$, where $n = 0, 1, 2....$

Q. 5.14. Widths of two slits in a Young's experiment are in the ratio 4 : 1. What is the ratio of the amplitudes of light waves from them ?

Ans. Here, $\frac{w_1}{w_2} = \frac{4}{1}$

Now, $\frac{a_1^2}{a_2^2} = \frac{w_1}{w_2} = \frac{4}{1}$

Therefore $\frac{a_1}{a_2} = \frac{2}{1}$

Q. 5.15. What is the ratio of slit widths, when the amplitudes of light waves from them have a ratio $\sqrt{2} : 1$?

Ans. Here, $\frac{a_1}{a_2} = \frac{\sqrt{2}}{1}$

Now, $\frac{w_1}{w_2} = \frac{a_1^2}{a_2^2} = \left(\frac{\sqrt{2}}{1}\right)^2 = \frac{2}{1}$

Q. 5.16. If the two slits in Young's double-slit experiment have width ratio 4 : 1, deduce the ratio of intensity at maxima and minima in the interference pattern. *(C.B.S.E. 1999 S)*

Ans. Here, $\frac{w_1}{w_2} = \frac{a_1^2}{a_2^2} = \frac{4}{1}$ or $\frac{a_1^2}{a_2^2} = \frac{2}{1}$ or $a_1 = 2\,a_2$

If I_1 and I_2 are intensities of light at maxima and minima respectively, then

$$\frac{I_1}{I_2} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)} = \frac{(2\,a_2 + a_2)^2}{(2\,a_2 - a_2)^2} = \frac{9}{1}$$

Q. 5.17. If the separation between two slits is decreased in Young's double slit experiment, keeping the screen position fixed, what will happen to the fringe width ? *(I.S.C.E. 1995)*

Ans. We know that $\beta = \frac{D\lambda}{d}$

When the two slits are moved closer (d is decreased), the fringe width will increase.

Q. 5.18. Why is interference pattern not detected, when two coherent sources are far apart ? *(H.S.S.C.E. 2001)*

Ans. As the distance between two sources is increased, the fringe width decreases. When the sources are far apart, the fringe width practically becomes zero.

Q. 5.19. What happens to fringe width, when the separation between the sources is increased ? *(Text Question)*

Ans. We know that $\beta = \frac{D\lambda}{d}$. The fringe width decreases, when the separation d between the two sources in the Young's double slit experiment is increased.

Q. 5.20. What is the effect of slit width and wavelength of light source on fringe width of the fringes formed by Young's double slit experiment ? *(C.B.S.E. 1995 S)*

Ans. Again, $\beta = \frac{D\lambda}{d}$

It follows that when slit width (distance between slits) is increased, fringe width will decrease.

On the other hand, when wavelength is increased, the fringe width will increase.

Q. 5.21. In Young's double slit experiment, the separation of the slits is doubled and the distance between the slits and the screen is halved. How will it affect the fringe width ?

Ans. In the interference pattern, the fringe width is given by

$$\beta = \frac{D\lambda}{d}$$

When D is halved and d is doubled, β becomes one-fourth. *i.e.* the fringe width reduces to one-fourth of its previous value.

Q. 5.22. Is the law of conservation of energy obeyed by the interference ?

Ans. Yes.

Q. 5.23. When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation. *(H.P.S.S.C.E. 1998 S)*

Ans. A low flying aircraft reflects the TV signal. Due to interference between the direct signal and the reflected signal, the picture on TV screen shows a slight shaking.

Q. 5.24. Can white light produce interference ? What is its nature ?

Ans. Yes, white light can produce interference. However, only first few coloured fringes can be detected and beyond that the fringes of different colours overlap to produce general illumination, white in colour.

Q. 5.25. When a thin transparent film is placed just in front of one of the two slits in the Young's double slit experiment using white light, what change results in the fringe system ? *(I.S.C.E. 1997)*

Ans. The entire fringe pattern will get displaced.

Q. 5.26. Why do the oil films on the surface of water appear to be coloured ?

Ans. The oil films on the surface of water appear to be coloured due to the interference of white light reflected from the two surfaces of the film.

Q. 5.27. Why does an excessively thin film appear black in reflected light ? *(H.S.C.C.E. 1992)*

Ans. In case of an excessively thin film $(t \approx 0)$, the path difference is just $\frac{\lambda}{2}$. The path difference of $\frac{\lambda}{2}$ occurs due to the reason that one of the two rays suffers reflection at denser medium. As the path difference between the two rays is $\frac{\lambda}{2}$, the film appears dark.

Q. 5.28. Bubbles of colourless soap solution appear coloured in sunlight. Why ?

Ans. It is due to interference of white light from the thin film of the soap bubbles.

Q. 5.29. Thin films such as soap bubbles or a thin layer of oil on water show beautiful colours, when illuminated by white light. Explain the observation. *(Text Problem)*

Ans. Light waves reflected from the upper and lower surfaces of a thin film interfere with each other. The coloured interference fringes are produced due to the reason that the conditions for constructive and destructive interference depend on wavelength of the light used.

Q. 5.30. Define the term `coherence' for light waves. *(I.S.C.E. 1998)*

Ans. When the light waves interfering with each other have a phase difference that remains constant with time, they are called coherent.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 5.01. What are the conditions for two sources to be coherent ? *(P.S.S.C.E. 1995 ; C.B.S.E. 1990)*

Or

State the conditions, which must be satisfied for two light sources to be coherent. *(H.S.S.C.E. 1998)*

Ans. 1. Two sources should emit light waves continuously.

2. The light waves emitted by the two sources should be of the same wavelength.

3. The phase difference between light waves emitted by the two sources should not change with passage of time.

Q. 5.02. What is the difference between coherent and incoherent sources of light ?

Ans. Two sources of light, which fulfil the conditions given in SAQ 5.01, are called coherent sources. On the other hand, if the two sources emitting light do not fulfil these conditions, they are called incoherent sources.

Q. 5.03. State the necessary conditions for the sustained interference.

(C.B.S.E. 2002, 1998 S ; P.S.S.C.E. 1998 S)

Ans. Refer to section 5.03.

Q. 5.04. State the three conditions for sustained interference to take place. Explain, why these conditions are necessary.

Ans. Refer to section 5.03.

Q. 5.05. Consider interference between waves from two sources of intensities I and 4 I. Find intensities at points, where phase difference is (*i*) $\frac{\pi}{2}$ (*ii*) π. *(Roorkee ; 1987)*

Ans. Here, $I_1 = a_1^2 = I$ and $I_2 = a_2^2 = 4\,I$

Resultant light intensity,

$$I' = a_1^2 + a_2^2 + 2\,a_1\,a_2 \cos\phi$$
$$= I + 4\,I + 2\sqrt{I}\,\sqrt{4\,I}\cos\phi = 5\,I + 4\,I\cos\phi$$

(*i*) When $\phi = \frac{\pi}{2} : I' = 5\,I + 4\,I\cos\frac{\pi}{2} = 5\,I + 4\,I \times 0 = \mathbf{5\,I}$

(*ii*) When $\phi = \pi : I' = 5\,I + 4\,I\cos\pi = 5\,I + 4\,I\,(-1) = \mathbf{I}$

Q. 5.06. Consider interference between two sources of intensities I and 4 I. Obtain intensity at a point, where the phase difference is $\frac{\pi}{2}$. *(C.B.S.E. 1995 S)*

Ans. Refer to SAQ 5.05 (*i*).

Q. 5.07. Find the ratio of intensities at two points X and Y on a screen in Young's double slit experiment, where waves from S_1 and S_2 have path difference of (*i*) 0 and (*ii*) $\lambda/4$. *(C.B.S.E. 1993)*

Ans. Resultant light intensity, $I = a_1^2 + a_2^2 + 2\,a_1\,a_2\cos\phi$

(*i*) Path difference equal to zero at point X corresponds to phase difference $\phi = 0°$ at that point.

$$\therefore \quad I_X = a_1^2 + a_2^2 + 2\,a_1\,a_2\cos 0° = a_1^2 + a_2^2 + 2\,a_1\,a_2$$

(*ii*) Path difference equal to $\frac{\lambda}{4}$ at point Y corresponds to phase difference $\phi = \frac{\pi}{2}$ at that point

$$\therefore \quad I_Y = a_1^2 + a_2^2 + 2\,a_1\,a_2\cos\frac{\pi}{2} = a_1^2 + a_2^2$$

$$\therefore \quad \frac{I_X}{I_Y} = \frac{a_1^2 + a_2^2 + 2\,a_1\,a_2}{a_1^2 + a_2^2}$$

In case, $a_1 = a_2 = a$ (say), then

$$\frac{I_X}{I_Y} = \frac{a^2 + a^2 + 2\,a \times a}{a^2 + a^2} = \frac{2}{1}$$

Q. 5.08. Two light waves of amplitudes a_1 and a_2 interfere with each other. Find the ratio of the intensities of a maxima to that of a minima.

Ans. When two light waves of amplitudes a_1 and a_2 differing in phase by ϕ interfere, the intensity of the resultant light is given by

$$I = a_1^2 + a_2^2 + 2\,a_1\,a_2\cos\phi$$

The intensity of light will be maximum, when $\phi = 0$.

$$\therefore \quad I_{max} = a_1^2 + a_2^2 + 2\,a_1\,a_2\cos 0$$
$$= a_1^2 + a_2^2 + 2\,a_1\,a_2 \times 1 = (a_1 + a_2)^2$$

On the other hand, intensity of light will be minimum, when $\phi = \pi$.

$$\therefore \quad I_{min} = a_1^2 + a_2^2 + 2\,a_1\,a_2 \cos \pi$$
$$= a_1^2 + a_2^2 + 2\,a_1\,a_2 \times (-1) = (a_1 - a_2)^2$$

Hence, $$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

Q. 5.09. What will be the intensities of the maxima and ninima, when the light waves interfering with each other re of the same amplitude ? Show that the intensity of the naxima is four times the intensity of light due to each of he two slits.

Ans. In case, the two light waves are of same amplitude *.e.*

$$a_1 = a_2 = a \text{ (say), then}$$
$$I_{max} = (a + a)^2 = 4\,a^2$$
and $$I_{min} = (a - a)^2 = 0$$

Now, intensity of light due to each slit, $I = a^2$

It follows that $I_{max} = 4\,I$ *i.e.* maximum intensity in the interference pattern is four times the intensity of light due to each slit.

Q. 5.10. Show that in Young's experiment, the dark and bright fringes are equally spaced.

Ans. Refer to section 5.07.

Q. 5.11. Draw the graph showing the variation of intensity in the interference pattern in Young's double slit experiment. (*C.B.S.E. 2002, 1998 S*)

Ans. Refer to Fig. 5.04.

Q. 5.12. What is the effect on the interference pattern in Young's double-slit experiment due to each of the following operations ? (*a*) The widths of the slits are increased equally. (*b*) The whole apparatus is kept in a denser medium. (*C.B.S.E. 1998 S*)

Ans. (*a*) As the widths of the slits are increased equally, the fringe width decreases. It is because,

$$\beta \propto \frac{1}{d}$$

(*b*) When the apparatus is immersed in denser medium of refractive index μ, the wavelength of the light used decreases. Since $\beta \propto \lambda$, the fringe width decreases.

Q. 5.13. Why is no interference pattern observed, when two coherent sources are (*i*) infinitely close to each other ? (*ii*) far apart from each other ? (*C.B.S.E. 1998*)

Ans. We know, $\beta = \dfrac{D\,\lambda}{d}$

(*i*) When the two sources are quite close to each other (*d* quite small), sufficiently wide fringes are formed. In case the two sources are infinitely close to each other, general illumination will take place.

(*ii*) When the sources are moved far apart (*d* very large), the fringe width will be very small and they will not be separately visible.

Q. 5.14. What is the effect on the interference fringes in a Young's double slit experiment due to each of the following operations ?

(*a*) The screen is moved away from the plane of the slits. (*C.B.S.E. 2001 S*)

(*b*) The source is replaced by another source of shorter wavelength.

(*c*) The separation between the two slits is decreased. (*C.B.S.E. 2000, 1999*)

(*d*) The monochromatic source is replaced by a source of white light. (*C.B.S.E. 2001 S*)

Ans. We know that $\beta = \dfrac{D\,\lambda}{d}$

(*a*) When the screen is moved away from the plane of slits, D increases. Hence, fringe width will increase.

(*b*) The decrease in λ will decrease fringe width.

(*c*) When separation between the two sources is decreased, *d* decreases. Hence, fringe width will increase.

(*d*) The different colours of white light will produce different interference patterns but the central bright fringes due to all colours are at the same position. Therefore, central bright fringe is white in colour. Since blue colour has the lowest λ, the fringe closest on either side of the central white fringe is blue and the farthest is red. Beyond a few fringes, no clear fringe pattern is visible.

Q. 5.15. How does the central fringe appear, when white light is used in a biprism experiment ? Explain your answer. (*I.S.C.E. 1994*)

Ans. Refer to SAQ 5.14 (*d*)

Q. 5.16. Imagine a situation in which Young's double slit apparatus is completely immersed in water. What will be the change in fringe width as compared to the case, when the apparatus lies in air ?

Ans. When apparatus is in air, $\beta_{air} = \dfrac{D\,\lambda}{d}$

When apparatus is immersed in water, the wavelength of light decreases to $\dfrac{\lambda}{\mu}$.

Therefore, $$\beta_{water} = \frac{D\,\lambda}{\mu\,d}$$

$$\therefore \quad \frac{\beta_{water}}{\beta_{air}} = \frac{D\,\lambda}{\mu\,d} \times \frac{d}{D\,\lambda} = \frac{1}{\mu}$$

Q. 5.17. What will be the effect on the fringes formed in Young's double slit experiment, if (*i*) the apparatus is immersed in water, (*ii*) white light is used instead of monochromatic light ? (*P.S.S.C.E. 1999 S, 1993 ; C.B.S.E. 1999*)

Ans. (*i*) Refer to SAQ 5.16.

(*ii*) Refer to SAQ 5.14 (*d*).

Q. 5.18. Two slits are 1 mm apart and the same slits are 1 m from a screen. Find out fringe separation, when light of wavelength 500 nm is used. (*H.S.S.C.E. 1996*)

Ans. Here, $d = 1\text{ mm} = 10^{-3}\text{ m}$; $D = 1\text{ m}$;

$$\lambda = 500\text{ nm} = 500 \times 10^{-9}\text{ m}$$

$$\therefore \quad \beta = \frac{D\,\lambda}{d} = \frac{1 \times 500 \times 10^{-9}}{10^{-3}} = 0{\cdot}5 \times 10^{-3}\text{ m}$$
$$= 0{\cdot}5\text{ mm}$$

Q. 5.19. In a Young's double slit experiment, the distance between the slits and the screen is 1 m. If the distance between the slits is 5 mm, the fringe width is found to be 0·1 mm. Calculate the wavelength of the light used. (*H.S.S.C.E. 1998*)

Ans. Here, $d = 5\text{ mm} = 5 \times 10^{-3}$; $\beta = 0{\cdot}1\text{ mm} = 10^{-4}\text{ m}$;

$$D = 1\text{ m}$$

Now, $$\beta = \frac{D\,\lambda}{d}$$

or $$\lambda = \frac{\beta d}{D} = \frac{10^{-4} \times 5 \times 10^{-3}}{1} = 5 \times 10^{-7}\ \text{m}$$

Q. 5.20. Draw a labelled diagram to show how interference can take place in thin transparent films by reflected light. *(I.S.C.E. 1997)*

Ans. Refer to Fig. 5.05.

Q. 5.21. Explain, how the law of conservation of energy is obeyed in the phenomenon of interference of light. *(P.S.S.C.E. 1999 S)*

Ans. Refer to section 5.10.

Q. 5.22. When two light waves interfere at some point to produce darkness, what becomes of the light energy ?

Ans. Refer to section 5.10.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 5.01. Why is interference pattern not detected, when the two coherent sources are far apart ?

Ans. The fringe width in the interference pattern is inversely proportional to the separation between the sources $\left(\beta \propto \frac{1}{d}\right)$. As the distance between the sources is increased, the fringe width goes on decreasing and at very large separation, it becomes too small to be detected.

Q. 5.02. If the two point-like coherent sources are placed infinitely close to each other, what happens to the interference pattern ?

Ans. The fringe width is inversely proportional to the separation between the sources. Therefore, when the coherent sources are placed very close to each other, the fringe width becomes very large and for infinitely close sources, even a single fringe may occupy the screen. As a result, the interference pattern may not be observed.

Q. 5.03. Can two electric bulbs, point-like and having filaments of the same material placed close together, produce interference ? Explain.

Ans. The two independent sources, such as two electric bulbs, even point-like, cannot act as coherent sources. So the sustained interference cannot be produced. The emission of light in the bulbs is due to millions of atoms in which electrons jump from higher orbit to the lower orbit. This process takes place in a time of about 10^{-8} s. So, the phase difference can remain constant for about 10^{-8} s only. In other words, in one second, the phase will change 10^8 times approximately. Such a fast variation in the interference pattern cannot be detected even with the most superior camera available so far. Therefore, the interference will be produced, but it will go undetected.

Q. 5.04. What change is observed in interference pattern of Young's double slit experiment, if one of the two slits is painted so that it transmits half the light intensity of the other ?

Ans. ***When the slit is not painted*** :

Here, $a_1^2 = a_2^2 = \text{I}$

or $a_1 = a_2 = \sqrt{\text{I}}$

Therefore, $\text{I}_{max} = (\sqrt{\text{I}} + \sqrt{\text{I}})^2 = \mathbf{4\ I}$

and $\text{I}_{min} = (\sqrt{\text{I}} - \sqrt{\text{I}})^2 = \mathbf{0}$

When one of the slits is painted :

Here, $a_1^2 = \text{I}\ ;\ a_2^2 = \frac{\text{I}}{2}$

or $a_1 = \sqrt{\text{I}}$ and $a_2 = \sqrt{\frac{\text{I}}{2}}$

$$\therefore \quad \text{I}_{max} = \left(\sqrt{\text{I}} + \sqrt{\frac{\text{I}}{2}}\right)^2 = \text{I}\left(1 + \frac{1}{\sqrt{2}}\right)^2 = \mathbf{2{\cdot}915\ I}$$

$$\text{and} \quad \text{I}_{min} = \left(\sqrt{\text{I}} + \sqrt{\frac{\text{I}}{2}}\right)^2 = \text{I}\left(1 - \frac{1}{\sqrt{2}}\right)^2 = \mathbf{0{\cdot}086\ I}$$

Therefore, on painting one of the two slits, intensity of maxima will *decrease* from 4 I to 2·915 I and the intensity of minima will *increase* from 0 to 0·086 I.

Q. 5.05. One method of producing two in-phase point sources of light for interference is to form two images of a point source by means of the two halves of a lens, which has been split along a diameter [Fig. 5.06].

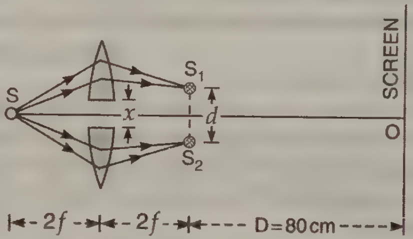


Fig. 5.06

(*a*) How must the two halves of a 5 cm focal length lens be placed to form two real image sources 0·24 cm apart and 20 cm from the source ?

(*b*) A screen is placed perpendicular to the principal axis and 80 cm from the image sources. What is the width of central maxima formed on the screen, if the wavelength of the light used is 5·4 × 10⁻⁵ cm ?

Ans. (*a*) As one lens, the two halves will form a single image on the principal axis of a point source located on the principal axis. As the two halves of the lens are separated, each half will produce an image *w.r.t.* its own principal axis. If the two halves are separated by a distance x, the principal axis of each half is now $x/2$ from the original axis through the source S. The source will then be at a distance $x/2$ off the principal axis of each half of the lens. Since the two images S_1 and S_2 are to be produced at a distance of 20 cm from the source S, $u = v = 20$ cm. As the focal length of the lens 5 cm, the images S_1 and S_2 will be produced at 20 cm from the source, if $u = v = \frac{20}{2} = 10$ cm *i.e.* the two halves should be placed at a distance of 10 cm from the source. As such, the image produced by each half of the lens will be $x/2$ on the other side of its principal axis. If d is distance between the two images S_1 and S_2, then

$$d = 4 \times \frac{x}{2} = 2x$$

Here, $d = 0{\cdot}24$ cm

$\therefore$ $2\,x = 0{\cdot}24$ cm or $x = \mathbf{0{\cdot}12}$ **cm**

(*b*) Now, width of the central maxima, $\beta_0 = \dfrac{D\lambda}{d}$

Here, $D = 80$ cm ; $d = 0{\cdot}24$ cm and $\lambda = 5{\cdot}4 \times 10^{-5}$ cm

$$\therefore \quad \beta_0 = \frac{80 \times 5{\cdot}4 \times 10^{-5}}{0{\cdot}24} = \mathbf{1{\cdot}8 \times 10^{-2}\ cm}$$

Q. 5.06. What will happen to the interference pattern in Young's experiment, if the source is not exactly on he centre line between the slits ?

Ans. In the normal position of the source *w.r.t.* the double slits [Fig. 5.07], the central bright fringe results at point O.

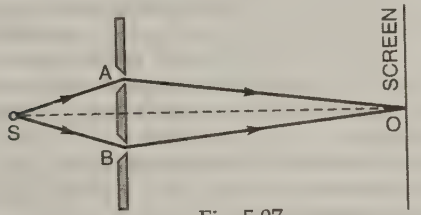


Fig. 5.07

It is because, the paths traversed by light along SAO and SBO are equal.

When the source is moved upward to position S′, the central fringe will occur at the point O′ on the screen [Fig. 5.08], where the paths of light S′AO′ and S′BO′ are equal.

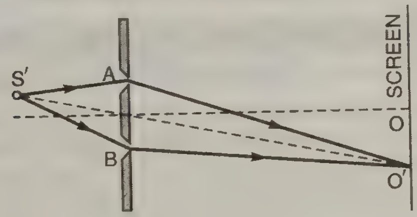


Fig. 5.08

Therefore, when the source of light is not exactly on the centre line and is moved upwards, the central bright fringe will move downwards.

Q. 5.07. Can you suggest reasons, why no interference is seen, when light reflects from the two surfaces of a windowpane ?

Ans. Suppose that the glass window pane is viewed in white light and the glass pane is say 0·2 cm thick. For light of wave-length 6000 Å, the path difference between the light reflected from the two surfaces will be $2\,\mu\,t$ *i.e.* $2 \times 1{\cdot}5 \times 0{\cdot}2 =$ 6·0 cm *i.e.* $\dfrac{0{\cdot}6}{6000 \times 10^{-8}} = 10{,}000$ wavelengths. In reflected system, the light will interfere destructively and will appear dark. It can be calculated that corresponding to light of wavelength 5999·7 Å, the path difference will be 10,000·5 wavelengths. Light of this wavelength will interfere constructively in reflected system. Thus, many wavelengths evenly distributed over the entire spectrum would be reflected and the net result would be, not coloured interference fringes, but a white reflected light.

If monochromatic light were used, we might obtain interference fringes. However, the surfaces of most [illegible] glass are too far from being optically flat to prod interference fringes.

Q. 5.08. Why does the colour of the oil film on the surface of water continuously change ?

Ans. The location of the bright and dark fringes produced by thin oil films depends upon the thickness of the film. The thickness of the oil film on the surface of water continuously varies and as a result, the position of the coloured fringes also varies. This appears as a variation in the colour of the oil films.

Q. 5.09. If a thin soap film is arranged vertically, the coloured horizontal interference bands move downwards and at the same time change their width. After some time, a rapidly growing dark spot appears at the top of the film, which bursts shortly afterwards. Explain, why.

Ans. In a thin soap film, the coloured bands are produced due to interference of light waves reflected from the upper and lower surfaces of the soap film. The path difference between the light waves reflected from the two surfaces depends upon the thickness of the film at the point of reflection and the wavelength of the light waves. Therefore, each of the horizontal interference bands corresponds to some definite thickness of the film.

In a soap film arranged vertically, water flows gradually from upper part to the lower part of the film. Due to this, the lower part gradually grows thicker, while the upper part becomes thinner. The points corresponding to some definite thickness are gradually displaced and the respective interference bands move downwards with them. After some time, the film in the upper part becomes of the thickness equal to one quarter of the shortest wavelength of the incident light. At this point of the film, the light waves of the wavelengths reflected from the two surfaces of the film will interfere destructively and the film will appear dark at that point in the reflected light.

Q. 5.10. Lenses are often coated with a thin film to reduce the intensity of reflected light.

(*a*) If the index of refraction of the coating is 1·3, what is the smallest thickness that will give minimum reflection of yellow light (λ = 5800 Å) ?

(*b*) Such lenses often show a faint purple colour by reflected light. Why ?

Ans. (*a*) When light falls on a lens ($\mu = 1{\cdot}5$) coated with a thin film ($\mu = 1{\cdot}3$), a part of light gets partially reflected from the film and partially from the lens. The path difference between the light rays reflected from film and glass depends upon the thickness of the film. Since in both the cases, reflection takes place from denser medium, no extra path difference (or phase difference) is created. A film of thickness $\lambda/4$ will give a total path difference of $\lambda/2$ and the two reflected light waves will interfere destructively. In this manner, a thin film coated on the lens helps to reduce the intensity of the reflected light.

The wavelength of yellow light inside the film,

$$\lambda = \frac{\lambda_{air}}{\mu_{film}} = \frac{5800}{1{\cdot}3} = 4462\ \text{Å}$$

For minimum reflection of yellow light, the thickness of film is given by

$$t = \frac{\lambda}{4} = \frac{4462}{4} = \mathbf{1115{\cdot}5\text{Å}}$$

(*b*) If the film is of proper thickness to produce destructive interference of yellow light, red light and violet light will be some what reflected. The reflected light will, therefore, have a reddish or purple tinge.

Q. 5.11. An observer sees a green fringe passing through a given point in an oil film. Would other obser-ver looking at the same point necessarily see the green fringe there ? Explain by writing down the necessary relation.

Ans. The path difference between the two rays is $2\mu t \cos r$. Therefore, for the given oil film, μ and t are constant and the path difference depends only on angle of refraction, which in turn depends on angle of incidence. It is not possible that two different observers will look at a point in the same direction and therefore, for the two observers, the path difference will not be same. Hence, the other observer may not necessarily see green fringe there.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What are coherent sources of light ? Two independent sources of light cannot be coherent. Explain. Why ? *(P.S.S.C.E. 2001)*
2. Two sources of intensity I_1 and I_2 undergo interference in Young's double slit experiment. Show that
$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2,$$
where a_1 and a_2 are the amplitudes of disturbance for two sources S_1 and S_2. *(C.B.S.E. 2001 S)*
3. What are coherent sources of light ? Deduce an expression for the intensity at a point on the screen in Young's double slit experiment. *(C.B.S.E. 2001)*
4. Describe Young's double slit experiment. How can it be used to determine the wavelength of a light source ? Give three examples of interference.
5. What do you understand by fringe width ? Derive an expression for fringe width in the interference pattern. *(H.P.S.S.C.E. 1994)*
6. Light after passing through two adjacent narrow slits falls on a screen. Find the expression for fringe width. *(H.S.S.C.E. 1994)*
7. Derive the expression for the fringe width in Young's double slit interference. *(H.S.S.C.E. 2001 ; P.S.S.C.E. 1999 ; C.B.S.E. 1997)*
8. Prove that the fringe width of both the bright and dark fringes in interference is same in Young's double slit experiment. *(H.P.S.S.C.E. 2000, 1999 S)*
9. Prove that the law of conservation of energy is obeyed during interference of light. *(P.S.S.C.E. 2002, 2001, 2000)*
10. Consider a monochromatic ray incident on a film of uniform thickness t and refractive index μ. Derive the condition for a dark fringe, when viewed from the same side of the film. *(I.S.C.E. 1997)*
11. Discuss the conditions for film to appear bright and dark, when viewed from the other side of the film.
12. Explain colour formation in this film. *(H.S.S.C.E. 2001)*

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Draw a schematic labelled diagram for the experimental determination of the wavelength of light by Young's interference experiment. Derive the formula used. *(I.S.C.E. 1994)*
2. Deduce the conditions of maxima and minima in Young's double slit experiment and find an expression for fringe width.
3. State the necessary conditions for sustained interference pattern. Derive an expression for fringe width using Young's double slit method for interference of light. *(P.S.S.S.C.E. 2000)*
4. Derive an expression for fringe width using Young's double slit method for interference of light. What will happen, if the distance between the two slits becomes nearly zero ? *(P.S.S.S.C.E. 2002, 2000)*
5. Derive an expression for fringe width in Young's double slit experiment for interference of light. What will be the effect on the fringes formed in this experiment, if whole of the apparatus is immersed in water ? *(P.S.S.C.E. 2001)*
6. A part of Young's double slit experimental set-up has been shown in the Fig. 5.09.

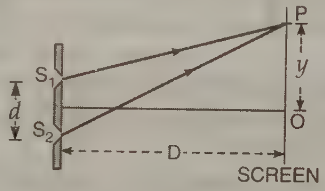


Fig. 5.09

Find the path difference and hence the phase difference of the waves arriving at P from the slits S_1 and S_2. State the condition for the waves arriving at P from the two slits to interfere constructively. Find the fringe width in terms of D, d and the wavelength λ.

FREQUENTLY ASKED NUMERICAL QUESTIONS

For Practice

Type A. On Slit width and Intensities of maxima and minima

1. If the two slits in Young's experiment have width ratio 1 : 4, deduce the ratio of intensity at maxima and minima in interference pattern. *(H.P.S.S.C.E. 1995)* [**Ans.** 9 : 1]
2. Two coherent sources whose intensity ratio is 16 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system. *(H.S.S.C.E. 2001)* [**Ans.** 25 : 9]
3. Two coherent sources whose intensity ratio is 81 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system. *(C.B.S.E. 1995)* [**Ans.** 25 : 16]
4. In Young's interference experiment, the ratio of intensity at the maxima and minima in the interference pattern is 25 : 9. What will be the ratio of widths of the two slits ? *(P.S.S.C.E. 1999 S)* [**Ans.** 16 : 1]

ype B. On Young's double slit experiment

5. In a Young's double slit experiment, interference fringes were produced on a screen placed at 1·5 m from the two slits 0·3 mm apart and illuminated by light of 6400 Å. Find the fringe width. *(H.S.S.C.E. 2002)* [**Ans.** 3·2 mm]

6. In a Young's double slit experiment, light has a frequency of 6×10^{-14} s^{-1}. The distance between the centres of adjacent bright fringes is 0·75 mm. If the screen is 1·5 m away, what is the distance between the slits ? [**Ans.** 0·1 cm]

7. In Young's double slit experiment, the two slits are 0·5 mm apart. The screen is placed 1 m away from the slits. The distance of 11th fringe from the first fringe is 1·0 cm. Calculate the wavelength of light used. *(H.S.S.C.E. 1999 S)* [**Ans.** 5000 Å]

8. Light of wavelength 5000 Å is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 10 cm, find the distance between the two slits. *(P.S.S.C.E. 2002)* [**Ans.** 0·1 mm]

9. Green light of wavelength 5100 Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find slit separation. *(H.P.S.S.C.E. 1993)* [**Ans.** 0·51 mm]

10. In Young's experiment, the distance of screen from the two slits is 1·0 m. When a light of wavelength 6000 Å is allowed to fall on the slits, the width of the fringes obtained on the screen is 2·0 mm. Determine (*i*) distance between the two slits (*ii*) the width of the fringe, if the wavelength of incident light is 4800 Å. [**Ans.** 3×10^{-4} m, $1·6\times 10^{-3}$ m]

11. The fringe width in a Young's double slit interference pattern is $2·4 \times 10^{-4}$ m, when red light of wavelength 6400 Å is used. By how much will it change, if blue light of wavelength 4000 Å is used ? [*H.S.S.C.E. 2002*] [**Ans.** $0·9 \times 10^{-4}$ m (decrease)]

12. In Young's double slit experiment, the fringe width obtained is 0·6 cm, when light of wavelength 4000 Å is used. If the distance between the screen and the slit is reduced to half, what should be the wavelength of light used to obtain fringes 0·5 cm wide ? *(P.S.S.C.E. 1999)* [**Ans.** 6666·7 Å]

13. In Young's double slit experiment, the width of the fringes obtained with light of wavelength 4500 Å is 1·8 mm. If the distance between the two slits is 0·3 mm, find the distance of the screen from the slits. What will be the fringe width, if the entire apparatus is immersed in water of refractive index 4/3. [**Ans.** 1·2 m, 1·35 mm]

14. In Young's double slit experiment, the slits are separated by 0·28 mm and the screen is 1·4 m away. The distance between the central bright fringe and the fourth bright fringe is 1·2 cm. Find the frequency of light used. Use the standard value of velocity of light. *(P.S.S.C.E. 1997 S)* [**Ans.** 5×10^{14} Hz]

15. In a Young's double slit experiment, the slits are separated by 0·56 mm and the screen is placed 2·8 m away. The distance between the central bright fringe and the fifth bright fringe is measured to be 1·5 cm. Determine the wavelength of light used in the experiment. *(H.S.S.C.E. 1998 S)* [**Ans.** 6×10^{-7} m]

16. The two slits in Young's double slit experiment are separated by a distance of 0·03 cm. When light of wavelength 5000 Å falls on the slits, an interference pattern is produced on the screen 1·5 m away. Find the distance of 4th bright fringe from the central maximum. *(C.B.S.E. 1995)* [**Ans.** 1 cm]

17. In Young's double slit experiment, light of wavelength 5000 Å is used. The third bright band on the screen is formed at a distance of 1 cm from the central bright band. If the screen is at a distance of 1·5 m from the centre of the narrow slits, calculate the separation between the slits. *(P.S.S.C.E. 1994)* [**Ans.** $2·25\times 10^{-4}$ m]

18. Two parallel slits used for Young's interference experiment are 0·5 mm apart. The screen on which fringes are projected is 1·5 m from the slits. How far is the third dark fringe from the central bright one ? Wavelength of light used is 6000 Å. [**Ans.** 0·45 cm]

19. In Young's double-slit experiment, two slits are separated by 3 mm distance and illuminated by light of wavelength 480 nm. The screen is at 2 m from the plane of the slits. Calculate the separation between the 8th bright fringe and the 3rd dark fringe observed with respect to the central bright fringe. *(C.B.S.E. 2001)* [**Ans.** 1·76 mm]

20. In a Young's double slit experiment, red light of wavelength 6000 Å is used and the *n*th bright fringe is obtained at a point P on the screen. Keeping the same setting, the source is replaced by green light of 5000 Å and now (*n* + 1)th bright fringe is obtained at the point P. Calculate the value of *n*. *(H.S.S.C.E. 2002)* [**Ans.** 5]

21. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 8·3 mm. A second light produces an interference pattern in which the bright fringes are separated by 7·6 mm. Find the wavelength of the second light. *(I.S.C.E. 1994)* [**Ans.** 576·87 nm]

22. In Young's double slit experiment, light of wavelength 6000 Å is used to get an interference pattern on a screen. The fringe width changes by 1·5 mm, when the screen is brought towards the double slit by 50 cm. Find the distance between the two slits. *(H.S.S.C.E. 2002)* [**Ans.** 0·2 mm]

23. In young's double slit experiment, the slits are 0·2 mm apart. The interference fringes for light of wavelength 6000 Å are formed on a screen distant 1·5 m from the slits. Calculate (*i*) the angular position of the third maxima, (*ii*) the angular position of the fifth minima and (*iii*) the fringe width. [**Ans.** (*i*) 0·009 rad (*ii*) 0·0135 rad (*iii*) 4·5 mm]

Type C. On Colour of thin films

24. Find the minimum thickness of a film which will strongly reflect the light of wavelength 589 nm, when incident normally on it. The refractive index of the material of the film is 1·25. [**Ans.** 117·8 nm]

25. Light of wavelength 6000 Å is incident on a thin film of refractive index 1·5, such that angle of refraction into the film in 60°. Calculate the smallest thickness of the film, which will make it appear dark by reflection. [**Ans.** 4×10^{-7} m]

26. A soap film of refractive index 4/3 is illuminated by white light incident at an angle 45°. The transmitted light is examined in a spectroscope and a bright band is found to be at 6000 Å. Find the minimum thickness of the film. [**Ans.** $2·654 \times 10^{-7}$ m]

MISCELLANEOUS PROBLEMS

27. The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width, find the ratio of the maximum to the minimum intensity in the interference pattern. [**Ans.** 9]

28. Two coherent sources of intensity ratio β interfere. Prove that in interference pattern,

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\beta}}{1+\beta}$$

29. Two sources of intensities I and 4 I are used in an interference experiment. Obtain intensities at points, where the waves from two sources superimpose with a phase difference of (*i*) 0 (*ii*) $\pi/2$ and (*iii*) π. (*Roorkee, 1991, 1987*) **[Ans.** (*i*) 9 I ; (*ii*) 5 I ; (*iii*) I]

30. Find the maximum intensity in case of interference of n identical waves each of intensity I_0, if the interference is (*a*) coherent and (*b*) incoherent. (*I.I.T. 1986*) **[Ans.** (*a*) $n^2 I_0$; (*b*) $n I_0$]

31. A double slit arrangement produces interference fringes for sodium light ($\lambda = 5890$ Å) that are 0·40° apart. What is the angular fringe separation, if the entire arrangement is immersed in water ? Given that μ for water= 4/3. (*Rooekee, 1988*) **[Ans.** 0·30°]

32. In a Young's double slit experiment, the angular width of a fringe formed on a distant screen is 0·12°. If the wavelength of light used is 4800 Å, find the distance between the slits. **[Ans.** $2{\cdot}29 \times 10^{-4}$ m]

33. The optical path of a monochromatic light is the same, if it goes through 2·0 cm of glass or 2·25 cm of water. If the refractive index of water is 4/3, what is the refractive index of glass ? **[Ans.** 1·5]

34. Monochromatic light of wavelength 6000 Å is used in a young's double slit experiment. One of the slits is covered by a transparent film of thickness $1{\cdot}2 \times 10^{-5}$ m having refractive index 1·4. How many fringes will shift due to the introduction of the film ? **[Ans.** 8]

35. In Young's double slit experiment, the central bright fringe produced by light of wavelength 5600 Å shifts to the position of 5th bright fringe, when a thin transparent film of refractive index 1·28 is introduced in the path of light from one of the two slits. Find the thickness of the film. **[Ans.** 10^{-5} m]

36. A beam of light consisting of two wavelength 4800 Å and 6000 Å is used to obtain interference fringes in a double slit experiment. The distance between the slits is 1·8 mm and the distance of screen from the plane of slits is 1·2 m. (*a*) Find the distance of the fifth bright fringe from the centre of the screen for the wavelength 6000 Å. (*b*) What is the least distance from the centre of the screen, where the bright fringes due to both the wavelengths coincide ? **[Ans.** (*a*) 2 mm (*b*) 1·6 mm]

37. A double-slit apparatus is immersed in a liquid of refractive index 1·33. It has slit separation of 1 mm and distance between the plane of the slits and screen is 1·33 m. The slits are illuminated by a parallel beam of light, whose wavelength in air is 6300 Å. (*a*) Calculate the fringe-width. (*b*) One of the slits of the appratus is covered by a thin glass sheet of refractive index 1·53. Find the smallest thickness of the sheet to bring the adjacent minimum on the axis. (*I.I.T. 1996*) **[Ans.** (*a*) 0·63 mm ; (*b*) $1{\cdot}579 \times 10^{-6}$ m]

38. Find the ratio of the intensity at the centre of a bright fringe to the intensity at the point one quarter of the distance between two fringes from the centre. **[Ans.** 2]

39. In a Young's double slit experiment, two slits are 1·5 mm apart and a screen is placed at a distance 1·2 m from the plane of the slits. The slits are illuminated with light of wavelength 6000 Å. Find the minimum distance from the central maximum for which the intensity is half of the maximum intensity. **[Ans.** $1{\cdot}2 \times 10^{-4}$ m]

40. A thin film is 4000 Å thick. White light from an extended source falls normally on the film. If the refractive index of film is 1·5, what wavelength within the visible spectrum will be intensified in the reflected beam, if visible range of the spectrum is 3900 Å to 7800 Å ? **[Ans.** 4800 Å]

41. White light is incident on a soap film at an angle $\sin^{-1} 4/5$ and the reflected light on examination by a spectroscope shows dark bands. The consecutive dark bands correspond to wavelengths 6100 Å and 6000 Å. If the refractive index of the film is 4/3, calculate the thickness. **[Ans.** $1{\cdot}716 \times 10^{-5}$ m]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Proceed as in solved problem no. 5.01 or VSQ 5.16.

2. Here, $\dfrac{I_1}{I_2} = \dfrac{a_1^2}{a_2^2} = \dfrac{16}{1}$ or $\dfrac{a_1}{a_2} = \dfrac{4}{1}$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(4+1)^2}{(4-1)^2} = \frac{25}{9}$$

3. Proceed as in problem no. 2.

4. Here, $\dfrac{I_{max}}{I_{min}} = \dfrac{(a_1+a_2)^2}{(a_1-a_2)^2} = \dfrac{25}{9}$ or $\dfrac{a_1+a_2}{a_1-a_2} = \dfrac{5}{3}$

or $\dfrac{(a_1+a_2)+(a_1-a_2)}{(a_1+a_2)-(a_1-a_2)} = \dfrac{5+3}{5-3}$ or $\dfrac{a_1}{a_2} = \dfrac{4}{1}$

Now, $\dfrac{w_1}{w_2} = \dfrac{a_1^2}{a_2^2} = \dfrac{16}{1}$

5. Now, $\beta = \dfrac{D\lambda}{d} = \dfrac{1{\cdot}5 \times 6400 \times 10^{-10}}{0{\cdot}3 \times 10^{-3}}$

$= 3{\cdot}2 \times 10^{-3}$ m = **3·2 mm**

6. Here, $\lambda = \dfrac{c}{v} = \dfrac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7}$ m,

$\beta = 0{\cdot}75$ mm $= 0{\cdot}75 \times 10^{-3}$ m

Now, $d = \dfrac{D\lambda}{\beta} = \dfrac{1{\cdot}5 \times 5 \times 10^{-7}}{0{\cdot}75 \times 10^{-3}} = 10^{-3}\text{m} = \mathbf{0{\cdot}1\ cm}$

7. The distance of 11th fringe from the first fringe is equal to width of 10 fringes.

$\therefore$ $10\,\beta = 1{\cdot}0$ cm or $\beta = 0{\cdot}1$ cm $= 10^{-3}$ m,

$d = 0{\cdot}5$ mm $= 0{\cdot}5 \times 10^{-3}$ m, D = 1 m

Now, $\lambda = \dfrac{\beta d}{D} = \dfrac{10^{-3} \times 0{\cdot}5 \times 10^{-3}}{1} = 5 \times 10^{-7}$ m

= **5000 Å**

8. Proceed as in problem no. 7.

9. Proceed as in problem no. 7.

10. (*i*) $d = \dfrac{D\lambda}{\beta} = \dfrac{1{\cdot}0 \times 6000 \times 10^{-10}}{2{\cdot}0 \times 10^{-3}} = \mathbf{3 \times 10^{-4}\ m}$

(*ii*) $\beta = \dfrac{D\lambda}{d} = \dfrac{1{\cdot}0 \times 4800 \times 10^{-10}}{3 \times 10^{-4}} = \mathbf{1{\cdot}6 \times 10^{-3}\ m}$

11. ***When red light is used :***

$\beta = \dfrac{D\lambda}{d}$ or $2{\cdot}4 \times 10^{-4} = \dfrac{D \times 6400 \times 10^{-10}}{d}$

or $\dfrac{D}{d} = \dfrac{2{\cdot}4 \times 10^{-4}}{6400 \times 10^{-10}}$

When blue light is used :

$$\beta' = \frac{D\lambda'}{d} = \frac{2\cdot4\times10^{-4}\times4000\times10^{-10}}{6400\times10^{-10}}$$

$$= 1{\cdot}5\times10^{-4}\text{ m}$$

$$\therefore\quad \beta-\beta' = 2{\cdot}4\times10^{-4}-1{\cdot}5\times10^{-4}$$

$$= \mathbf{0{\cdot}9\times10^{-4}\ m\ (decrease)}$$

12. Proceed as in solved problem no. 5.02.

13. Proceed as in solved problem no. 5.09.

14. Here, $d = 0{\cdot}28\text{ mm} = 0{\cdot}28\times10^{-3}\text{ m}$,

$y_4 = 1{\cdot}2\text{ cm} = 1{\cdot}2\times10^{-2}$,

$D = 1{\cdot}4\text{ m}$

Now, $y_4 = \dfrac{4D\lambda}{d}$ or $\lambda = \dfrac{y_4\times d}{4D}$

or $$\lambda = \frac{1\cdot2\times10^{-2}\times0\cdot28\times10^{-3}}{4\times1\cdot4} = 6\times10^{-7}\text{m}$$

$$\therefore\quad \nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{6\times10^{-7}} = \mathbf{5\times10^{14}\ Hz}$$

15. Proceed as in problem no. 14.

16. $$y_4 = \frac{4D\lambda}{d} = \frac{4\times1\cdot5\times5000\times10^{-10}}{0\cdot03\times10^{-2}}$$

$$= 10^{-2}\text{ m} = \mathbf{1\ cm}$$

17. Proceed as in problem no. 16.

18. $$y_3' = \frac{5D\lambda}{2d} = \frac{5\times1\cdot5\times6000\times10^{-10}}{2\times0\cdot5\times10^{-3}} = 4{\cdot}5\times10^{-3}\text{ m}$$

$$= \mathbf{0{\cdot}45\ cm}$$

19. $$y_8 - y_3' = \frac{8D\lambda}{d} - \frac{5D\lambda}{2d} = \frac{11D\lambda}{2d}$$

$$= \frac{11\times2\times480\times10^{-9}}{2\times3\times10^{-3}} = 1{\cdot}76\times10^{-3}\text{ m}$$

$$= \mathbf{1{\cdot}76\ mm}$$

20. Let y be distance of point P from the centre of screen.

When red light (λ) is used :

As nth bright fringe is formed at point P,

$$y = \frac{nD\lambda}{d} = \frac{nD\times6000\times10^{-10}}{d} \qquad ...(i)$$

When blue light (λ') is used :

As $(n+1)$th bright fringe is formed at point P,

$$y = \frac{(n+1)D\lambda'}{d} = \frac{(n+1)D\times5000\times10^{-10}}{d} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{nD\times6000\times10^{-10}}{d} = \frac{(n+1)D\times5000\times10^{-10}}{d}$$

or $6n = 5(n+1)$ or $\mathbf{n = 5}$

21. Now, $\beta = \dfrac{D\lambda}{d}$

When D and d are kept fixed,

$$\frac{\beta}{\beta'} = \frac{\lambda}{\lambda'} \quad\text{or}\quad \lambda' = \frac{\lambda\beta'}{\beta} = \frac{630\times7\cdot6}{8\cdot3} = \mathbf{576{\cdot}87\ nm}$$

22. Now, $\beta = \dfrac{D\lambda}{d}$

Since λ and d are constant, change in fringe width,

$$d\beta = \frac{(dD)\lambda}{d}$$

or $$d = \lambda\frac{dD}{d\beta} = \frac{6000\times10^{-10}\times50\times10^{-2}}{1\cdot5\times10^{-3}}$$

$$= 2\times10^{-4}\text{ m} = \mathbf{0{\cdot}2\ mm}$$

23. (*i*) The angular position of nth maximum,

$$\theta_n = \frac{y_n}{D} = \frac{nD\lambda}{d}\Big/D = \frac{n\lambda}{d}$$

$$\therefore\quad \theta_3 = \frac{3\times6000\times10^{-10}}{0\cdot2\times10^{-3}} = \mathbf{0{\cdot}009\ rad}$$

(*ii*) The angular position of the fifth minima,

$$\theta_5' = \frac{y_5'}{D} = \frac{9D\lambda}{2d}\Big/D = \frac{9\lambda}{2d}$$

$$= \frac{9\times6000\times10^{-10}}{2\times0\cdot2\times10^{-3}} = \mathbf{0{\cdot}0135\ rad}$$

(*iii*) $$\beta = \frac{D\lambda}{d} = \frac{1\cdot5\times6000\times10^{-10}}{0\cdot2\times10^{-3}}$$

$$= 4{\cdot}5\times10^{-3}\text{ m} = \mathbf{4{\cdot}5\ mm}$$

24. In reflected system, film appears bright (strongly reflected), when

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2}$$

For the thickness of film to be minimum, $n = 0$ *i.e.*

$$2\mu t\cos r = \lambda/2$$

For normal incidence, $i = 0$ and consequently $r = 0$.

$$\therefore\quad 2\mu t = \lambda/2$$

or $$t = \frac{\lambda}{4\mu} = \frac{589}{4\times1\cdot25} = \mathbf{117{\cdot}8\ nm}$$

25. For film to appear dark in reflected system,

$$2\mu t\cos r = n\lambda$$

For the thickness of film to be smallest, $n = 1$

i.e. $2\mu t\cos r = \lambda$

or $$t = \frac{\lambda}{2\mu\cos r} = \frac{6000\times10^{-10}}{2\times1\cdot5\cos60^\circ} = \mathbf{4\times10^{-7}\ m}$$

26. In transmitted system, film appears bright, when

$$2\mu t\cos r = n\lambda$$

For the thickness of film to be minimum, $n = 1$

i.e. $2\mu t\cos r = \lambda$

or $$t = \frac{\lambda}{2\mu\cos r} = \frac{\lambda}{2\sqrt{\mu^2\cos^2 r}} = \frac{\lambda}{2\sqrt{\mu^2(1-\sin^2 r)}}$$

$$= \frac{\lambda}{2\sqrt{\mu^2-\sin^2 i}} \qquad (\because\ \mu\sin r = \sin i)$$

$$= \frac{6000\times10^{-10}}{2\sqrt{(4/3)^2-\sin^2 45^\circ}} = \frac{6000\times10^{-10}}{2\sqrt{16/9-1/2}}$$

$$= \mathbf{2{\cdot}654\times10^{-7}m}$$

27. Let $2a$ and a be amplitudes of light from the two slits.

Then, $$\frac{I_{max}}{I_{min}} = \frac{(2a+a)^2}{(2a-a)^2} = \mathbf{9}$$

28. Here, $\dfrac{I_1}{I_2} = \beta$ or $\dfrac{a_1^2}{a_2^2} = \beta$ or $\dfrac{a_1}{a_2} = \sqrt{\beta}$

or $a_1 = \sqrt{\beta}\,a_2$

$$\therefore\quad I_{max} = (a_1+a_2)^2 = (\sqrt{\beta}\,a_2+a_2)^2$$

$$= (\beta+1+2\sqrt{\beta})\,a_2^2$$

$$I_{min} = (a_1-a_2)^2 = (\sqrt{\beta}\,a_2-a_2)^2$$

$$= (\beta+1-2\sqrt{\beta})\,a_2^2$$

$$\therefore \quad \frac{I_{max}-I_{min}}{I_{max}+I_{min}}$$

$$= \frac{(\beta+1+2\sqrt{\beta})\,a_2^2-(\beta+1-2\sqrt{\beta})\,a_2^2}{(\beta+1+2\sqrt{\beta})\,a_2^2+(\beta+1-2\sqrt{\beta})\,a_2^2}$$

$$= \frac{4\sqrt{\beta}}{2(\beta+1)} = \frac{2\sqrt{\beta}}{1+\beta}$$

29. Here, $I_1 = I$, $I_2 = 4I$

Resultant intensity is given by

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(i) When $\phi = 0$:

$$I' = I + 4I + 2\sqrt{I \times 4I}\cos 0 = I + 4I + 4I = 9I$$

(ii) When $\phi = \pi/2$:

$$I' = I + 4I + 2\sqrt{I \times 4I}\cos\pi/2 = I + 4I + 0 = 5I$$

(iii) When $\phi = \pi$:

$$I' = I + 4I + 2\sqrt{I \times 4I}\cos\pi = I + 4I - 4I = I$$

30. ***(a) When interference is coherent :***

When two waves of intensities I_1 and I_2 having a phase difference ϕ interfere, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \qquad ...(i)$$

The intensity will be maximum, when

$$\phi = 0 \quad \text{or} \quad \cos\phi = 1$$

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2$$

In case, n identical waves each of intensity I_0 interfere,

$$I_{max} = \left(\sqrt{I_0}+\sqrt{I_0}+\sqrt{I_0}+\ldots\ldots n \text{ times}\right)^2 = (n\sqrt{I_0})^2 = n^2 I_0$$

(b) When interference is incoherent :

In such a case, phase difference ϕ between the two waves varies randomly with time. Since the average value of $\cos\phi$ over a complete cycle is zero, the equation (i) becomes

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \times 0 = I_1 + I_2$$

In case, n identical waves each of intensity I_0 interfere,

$$I_{max} = I_0 + I_0 + I_0 + \ldots\ldots\ldots n \text{ times} = n\,I_0$$

31. If fringes of width β are produced on a screen held at a distance D from the slits, then angular fringe width

$$\theta = \frac{\beta}{D} = \frac{D\lambda}{d}\Big/ D = \frac{\lambda}{d}$$

When the apparatus is immersed in water :

Wavelength of light inside water,

$$\lambda' = \frac{\lambda}{\mu}$$

Therefore, angular fringe width,

$$\theta' = \frac{\lambda'}{d} = \frac{\lambda}{\mu d} = \frac{\beta\,\theta}{\mu} = \frac{0{\cdot}40^\circ}{4/3} = 0{\cdot}30^\circ$$

32. Angular width, $\theta = \dfrac{\lambda}{d}$

$$\therefore \quad \frac{\lambda}{d} = 0{\cdot}12^\circ = \frac{0{\cdot}12^\circ}{180^\circ} \times \pi \text{ rad}$$

$$\text{or} \quad d = \frac{\lambda \times 180^\circ}{0{\cdot}12^\circ \times \pi} = \frac{4800\times10^{-10}\times180^\circ}{0{\cdot}12^\circ\times\pi}$$

$$= 2{\cdot}29 \times 10^{-4}\text{ m}$$

33. If x_1 and x_2 are distances travelled by light in glass and water respectively, then

$$^{a}\mu_g \times x_1 = {}^{a}\mu_w \times x_2 \quad \text{or} \quad {}^{a}\mu_g \times 2{\cdot}0 = \frac{4}{3}\times 2{\cdot}25$$

$$\text{or} \quad {}^{a}\mu_g = \frac{4\times2{\cdot}25}{3\times2{\cdot}0} = 1{\cdot}5$$

34. Suppose that thickness of the transparent film is t. When the light travels through the film, its optical path is $\mu\,t$ (as air is replaced by material of refractive index μ). Thus, when light travels in the transparent film, the optical path changes by $\mu\,t - t$ *i.e.* $(\mu - 1)\,t$.

One fringe shifts, when optical path changes by one wavelength. Therefore, number of fringes shifted due to introduction of the film,

$$n = \frac{(\mu-1)\,t}{\lambda} = \frac{(1{\cdot}4-1)\times1{\cdot}2\times10^{-5}}{6000\times10^{-10}} = 8$$

35. Proceed as in problem no. 34.

36. **(a)** $$y_5 = \frac{5\,D\lambda}{d} = \frac{5\times1{\cdot}2\times6000\times10^{-10}}{1{\cdot}8\times10^{-3}}$$

$$= 2\times10^{-3}\text{m} = 2\text{ mm}$$

(b) Suppose that n_1th bright fringe of wavelength λ_1 (= 4800 Å) coincides with n_2th bright fringe of wavelength λ_2 (= 6000 Å). Then,

$$\frac{n_1\,D\lambda_1}{d} = \frac{n_2\,D\lambda_2}{d} \quad \text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$$

$$\text{or} \quad \frac{n_1}{n_2} = \frac{6000}{4800} = \frac{5}{4}$$

Since n_1 and n_2 are integers, the minimum values of n_1 and n_2 satisfying the above equation are 5 and 4 respectively. The distance of 5th bright fringe for wavelength 4800 Å (or 4th bright fringe for wavelength 6000 Å) is given by

$$y_5 = \frac{5\,D\lambda}{d} = \frac{5\times1{\cdot}2\times4800\times10^{-10}}{1{\cdot}8\times10^{-3}}$$

$$= 1{\cdot}6\times10^{-3}\text{ m} = 1{\cdot}6\text{ mm}$$

37. **(a)** When apparatus is immersed in a liquid of refractive index μ,

$$\beta' = \frac{D\lambda'}{d} = \frac{D}{d}\left(\frac{\lambda}{\mu}\right) = \frac{1{\cdot}33\times6300\times10^{-10}}{1\times10^{-3}\times1{\cdot}33}$$

$$= 0{\cdot}63\times10^{-3}\text{ m} = 0{\cdot}63\text{ mm}$$

(b) Refractive index of glass sheet w.r.t. water,

$$\mu' = \frac{1{\cdot}53}{1{\cdot}33} = 1{\cdot}15$$

Suppose that one of the slit is covered with the glass sheet of thickness t. When light travels through the sheet, the optical path travelled is $\mu' t$ and hence the optical path changes by $\mu'\,t - t$ *i.e.* $(\mu' - 1)\,t$. The adjacent (first) minimum will shift on axis (centre of screen), if the change in optical path is $\dfrac{\lambda'}{2}$. Thus,

$$(\mu'-1)\,t = \frac{\lambda'}{2} \quad \text{or} \quad (\mu'-1)\,t = \frac{\lambda}{2\mu}$$

$$\text{or} \quad t = \frac{\lambda}{2\mu(\mu'-1)} = \frac{6300\times10^{-10}}{2\times1{\cdot}33\,(1{\cdot}15-1)}$$

$$= 1{\cdot}579\times10^{-6}\text{ m}$$

38. If ϕ is phase difference, when two waves of amplitudes a_1 and a_2 superpose each other at a point, then

$$I = a_1^2 + a_2^2 + 2\,a_1\,a_2\cos\phi$$

In case, two waves have same amplitude a,

$$I = a^2 + a^2 + 2\,a^2\cos\phi = 2\,a^2\,(1+\cos\phi)$$

let I_1 be intensity at the centre of a bright fringe. A bright fringe is formed, when ϕ is equal to 0.

$\therefore \quad I_1 = 2a^2(1+\cos 0) = 4a^2$

The points at the centre of the two bright fringes differ in phase by 2π. Therefore, points at distance one quarter of the distance between two bright fringes will differ in phase by $\frac{2\pi}{4}$ *i.e.* $\pi/2$.

If I_2 is intensity at such a point, then

$$I_2 = 2a^2(1+\cos \pi/2) = 2a^2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{4a^2}{2a^2} = 2$$

39. Refer to problem no. 38.

The intensity at the bright fringe $= 4a^2$

Let ϕ be phase difference between the light waves, when the intensity is one half of the maximum intensity (*i.e.* bright fringe). Then,

$$2a^2(1+\cos\phi) = \frac{1}{2}(4a^2) \quad \text{or} \quad \cos\phi = 0$$

$$\text{or} \qquad \phi = \pi/2$$

The first bright fringe is located at a distance $\frac{D\lambda}{d}$ from the central maximum and their centres differ in phase by 2π. Therefore, distance of the point from the central maximum differing in phase by $\pi/2$ is given by

$$y = \frac{1}{4}\left(\frac{D\lambda}{d}\right) = \frac{1\cdot 2\times 6000\times 10^{-10}}{4\times 1\cdot 5\times 10^{-3}} = \mathbf{1{\cdot}2\times 10^{-4}\ m}$$

40. Since light is incident normally, $r = 0$

For film to appear bright in reflected system,

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2}, \quad \text{where } n = 0, 1, 2 \ldots\ldots$$

$$\text{or} \qquad 2\mu t = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{4\mu t}{2n+1}$$

$$\text{or} \qquad \lambda = \frac{4\times 1\cdot 5\times 4000}{(2n+1)} = \frac{24000}{(2n+1)}\ \text{Å}$$

For $n = 0,\ \lambda = \frac{24000}{2\times 0+1} = 24000$ Å,

For $n = 1,\ \lambda = \frac{24000}{2\times 1+1} = 8000$ Å,

For $n = 2,\ \lambda = \frac{24000}{2\times 2+1} = 4800$ Å,

For $n = 3,\ \lambda = \frac{24000}{2\times 3+1} = 3428{\cdot}6$ Å

Since visible range of spectrum is from 3900 Å to 7800 Å, the light of wavelength **4800 Å** will be intensified.

41. Here, $\sin i = 4/5$

$$\therefore \quad \sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3} = 0\cdot 6$$

$$\text{or} \quad \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0\cdot 6)^2} = 0\cdot 8$$

For film to appear dark in reflected system,

$2\mu t\cos r = n\lambda$, where $n = 1, 2, 3\ldots\ldots$

Suppose that n and $(n+1)$ th dark bands correspond to wavelengths 6100 Å and 6000 Å respectively. Then,

$$2\times\frac{4}{3}\times t\times 0\cdot 8 = n\times 6100\times 10^{-10} \qquad \ldots(i)$$

$$\text{and} \quad 2\times\frac{4}{3}\times t\times 0\cdot 8 = (n+1)\times 6000\times 10^{-10} \qquad \ldots(ii)$$

From equation (*i*) and (*ii*), we have

$$n\times 6100\times 10^{-10} = (n+1)\times 6000\times 10^{-10}$$

$$\text{or} \qquad n = 60$$

Substituting for n in equation (*i*), we have

$$2\times\frac{4}{3}\times t\times 0\cdot 8 = 60\times 6100\times 10^{-10}$$

$$\text{or} \qquad t = \mathbf{1{\cdot}716\times 10^{-5}\ m}$$

CHAPTER
6

Diffraction of Light

6.01. DIFFRACTION OF LIGHT

Light travels in a straight line. As a result, light casts the shadow of the object coming in its path. When a narrow slit AB is placed in the path of the light, only the part A′B′ of the screen should get illuminated and no light should enter the regions A′X and B′Y of the screen as shown in Fig. 6.01. On the other hand, when an obstacle AB is placed, then its distinct geometrical shadow A′B′ would be obtained on the screen [Fig. 6.02.].

It happens so only when the size of the slit or the obstacle is large. However, if the size of the slit or the obstacle is made small, then light enters in the geometrically prohibited regions, showing thereby that the light bends round the corners of the slit or the obstacle.

*The phenomenon of bending of light round the sharp corners and spreading into the regions of the geometrical shadow is called **diffraction**.*

The light waves are diffracted only when the size of the obstacle is comparable to the wavelength of the light.

In principle, all the types of wave motion exhibit diffraction effect. The diffraction of radiowaves and sound waves is readily observed in practice.

Diffraction of light is of two types :

1. Fresnel diffraction. *It is the type of diffraction that takes place at a slit, when the source of light lies at a finite distance from it.* Since the source of light is close to the slit, the wave front is either spherical (in case of point source) or cylindrical (in case of a line source) in nature. The screen is also at a finite distance from the slit.

2. Fraunhofer diffraction. *It is the type of diffraction that takes place at a slit, when a plane wavefront is incident on it and the wavefront emerging from the slit is also plane.* Obviously, both the source and the screen should be at infinite distance from the narrow slit.

However, in the laboratory, to make a plane wave-front incident on the slit, a convex lens is used between the source and the slit. Further, to bring the emergent (diffracted) wave front on the screen, again a convex lens is used between the slit and the screen.

Frensel gave the exact explanation of the phenomenon of diffraction. According to him, diffraction is due to the interference of secondary wavelets originating from the portion of the primary wavefront, which is not blocked by the slit or the obstacle.

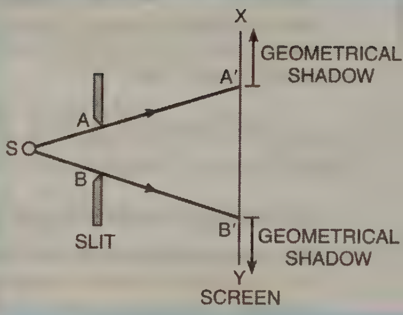


Fig. 6.01

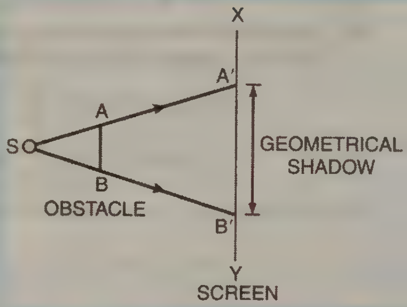


Fig. 6.02

Key point ✓

The diffraction of light is observed, when size of the obstacle is of the order of wavelength of light.

6.02. DIFFRACTION AT A SINGLE SLIT (Fraunhofer Diffraction)

Consider that a monochromatic source of light S, emitting light waves of wavelength λ, is placed at the principal focus of the convex lens L_1. A parallel beam of light *i.e.* a plane wave-front gets incident on a narrow slit AB of width *a* as shown in Fig. 6.03. The diffraction pattern is obtained on a screen lying at a distance D from

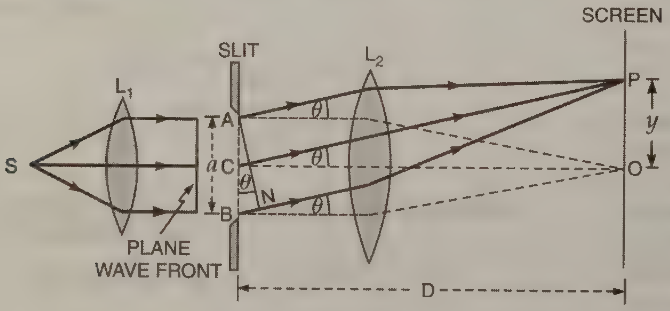


Fig. 6.03

he slit and at the focal plane of the convex lens L_2. The diffraction pattern is found o have central maxima at the centre O of the screen, which is flanked on both the sides by a number of dark and bright bands, called secondary maxima and minima. The diffraction pattern is obtained on the screen, which lies at the focal plane of the convex lens L_2. According to rectilinear propagation of light, a bright image of the slit is expected at the centre O of the screen. But in practice, we get a diffraction pattern *i.e.* a central maximum at the centre O flanked by a number of dark and bright fringes called *secondary maxima* and *minima* on either side of the point O. It is found that

(*i*) *the width of the central maximum is twice as that of a secondary maximum and*

(*ii*) *the intensity of the secondary maxima goes on decreasing with the order of maxima.*

These observations can be explained on the basis of the phenomenon of diffraction as below :

Central maximum. According to Huygens' principle, when light falls on the slit, it becomes a source of secondary wavelets. These wavelets are initially in phase and spread out in all directions. Let C be the centre of the slit AB. The wavelets which fall on the lens in a direction parallel to CO meet at point O in the same phase. This is because, the wavelets are initially in phase and their optical paths between the slit and the point O are also equal. Therefore, the wavelets reinforce each other and give rise to the ***central maximum*** at point O.

Positions and widths of secondary maxima and minima. Consider a point P on the screen at which wavelets travelling in a direction making angle θ with CO are brought to focus by the lens. The wavelets from different parts of the slit do not reach point P in phase, although they are initially in phase. It is because, they cover unequal distances in reaching the point P. The wavelets from points A and B will have a path difference equal to BN. From right angled Δ ANB, we have

$$BN = AB \sin\theta$$

or
$$BN = a \sin\theta \qquad \text{...(6.01)}$$

Suppose that the point P on the screen is at such a distance from the centre of the screen that BN = λ and the angle $\theta = \theta_1$. Then, equation (6.01) gives

$$\lambda = a \sin\theta_1$$

or
$$\sin\theta_1 = \frac{\lambda}{a} \qquad \text{...(6.02)}$$

Such a point on the screen will be the position of ***first secondary minimum***. It is because, if the slit is assumed to be divided into two equal parts, then wavelets from the corresponding points of the two halves of the slit will have a path difference of $\frac{\lambda}{2}$ *i.e.* wavelets from the two halves will reach the point P on the screen in opposite phase so as to produce minimum.

Similarly, if the point P on the screen is at such a distance from the centre of the screen that BN = 2λ and angle $\theta = \theta_2$, then from equation (6.01), we have

$$2\lambda = a \sin\theta_2$$

or
$$\sin\theta_2 = \frac{2\lambda}{a}$$

Such a point on the screen will be the position of ***second secondary minimum***. In this case, the slit may be assumed to be divided into four equal parts and wavelets from the corresponding points of the two adjacent parts of the slit will possess path difference $\frac{\lambda}{2}$. As the wavelets reach in opposite phase, minimum is produced at the point P on the screen.

In general, for *n*th minimum at point P,

$$\sin\theta_n = \frac{n\lambda}{a} \qquad \text{...(6.03)}$$

If y_n is the distance of the *n*th minimum from the centre of the screen and D is the distance between the slit and the screen, then from right angled Δ COP, we have

$$\tan\theta_n = \frac{OP}{CO}$$

or
$$\tan\theta_n = \frac{y_n}{D} \qquad \text{...(6.04)}$$

In case θ_n is small,

$$\sin \theta_n \approx \tan \theta_n$$

Therefore, from equations (6.03) and (6.04), we have

$$\frac{y_n}{D} = \frac{n\lambda}{a} \qquad \text{...(6.05)}$$

or

$$y_n = \frac{n D \lambda}{a}$$

The width of the secondary maximum,

$$\beta = y_n - y_{n-1} = \frac{n D\lambda}{d} - \frac{(n-1) D\lambda}{a}$$

$$\beta = \frac{D\lambda}{a} \qquad \text{...(6.06)}$$

Since β is independent of n, all the secondary maxima are of the same width β.

On the other hand, if point P on the screen is at such a distance from the point O that BN $= \frac{3\lambda}{2}$ and angle $\theta = \theta_1'$, then from equation (6.01), we have

$$\sin \theta_1' = \frac{3\lambda}{2a}$$

Such a point on the screen will be the position of ***first secondary maximum***. In this case, the slit may be assumed to be divided into three equal parts ; the wavelets from the first two parts will reach point P in opposite phase, cancelling the effect of each other. The wavelets from the third part of the slit remain uncancelled and produce the first maximum at point P.

Similarly, corresponding to BN $= \frac{5\lambda}{2}$ and angle $\theta = \theta_2'$, the ***second secondary maximum*** is produced, such that

$$\sin \theta_2' = \frac{5\lambda}{2a}$$

In this case, the slit may be assumed to be divided into five equal parts ; the wavelets from first four parts cancel the effect of each other, while the wavelets from the fifth part produce the second secondary maximum at point P.

In general, for nth maximum at point P,

$$\sin \theta_n' = \frac{(2n+1)\lambda}{2a} \qquad \text{...(6.07)}$$

If y_n' is the distance of nth maximum from the centre of the screen, then the angular position of the nth maximum is given by

$$\tan \theta_n' = \frac{y_n'}{D} \qquad \text{...(6.08)}$$

In case θ_n' is small, $\sin \theta_n' \approx \tan \theta_n'$

Therefore, from equations (6.07) and (6.08), we have

$$y_n' = \frac{(2n+1) D\lambda}{2a} \qquad \text{...(6.09)}$$

Key point

In diffraction pattern produced by a single slit, all secondary maxima and minima are of same width and the central maximum in twice as wide as any other secondary maximum or minimum.

The width of the secondary minimum,

$$\beta' = y_n' - y'_{n-1} = \frac{(2n+1)\lambda}{2a} - \frac{\{2(n-1)-1\}\lambda}{2a}$$

or

$$\beta' = \frac{D\lambda}{a} \qquad \text{...(6.10)}$$

Since β' is independent of n, all the secondary minima are of the same width β'.

Further, from equations (6.06) and (6.10), it follows that $\beta = \beta'$ *i.e. all the secondary maxima and minima are of the same width.*

Width of central maximum. The central maximum extends upto distance y_1 (the distance of first secondary minimum) on both sides of the centre of the screen.

Therefore, width of the central maximum, $\beta_0 = 2\, y_1$

In equation (6.04), setting $n = 1$, we have

$$y_1 = \frac{D\lambda}{a}$$

Therefore, $$\beta_0 = \frac{2\,D\lambda}{a} \qquad \text{...(6.11)}$$

It follows that $\beta_0 = 2\beta$ *i.e. the central maximum is twice as wide as any other secondary maximum or minimum.*

The angular position (θ_1) of the first secondary minimum is called the ***half angular width*** of the central maximum. It is given by equation (6.02).

Therefore, angular spread of light in the form of central maximum takes place in the angle $\pm\,\theta_1$.

From the above discussion, it follows that in diffraction from a single slit, the secondary maxima and minima are of equal width. However, the intensity of light of a secondary maximum goes on decreasing with the order of the maximum. It is because, the intensity of central maximum is due to wavelets from all parts of the slit, the first secondary maximum is due to wavelets from one third part of the slit only (first two parts send wavelets in opposite phase), the second secondary maximum is due to wavelets from the fifth part only (first four parts send wavelets in opposite phase) and so on. Hence, *intensity of secondary maxima goes on decreasing* as shown in Fig. 6.04.

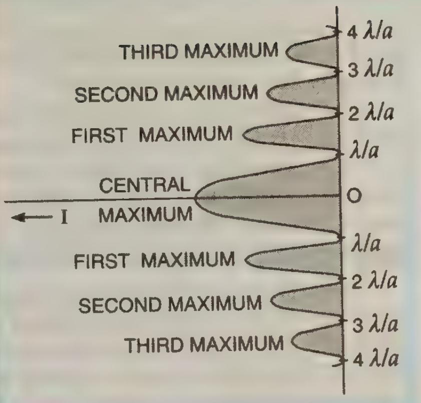


Fig. 6.04

Key point

In the diffraction pattern produced by a single slit, intensity of secondary maxima goes on decreasing with the order of the maximum.

Discussion. 1. From equations (6.06) and (6.11), it follows that the width of a secondary maximum or the central maximum is directly proportional to λ. Therefore, greater the wavelength of the light used, greater is the width of the maximum.

2. From equation (6.06), it follows that the width of a secondary maximum is inversely proportional to a, the width of the slit. Therefore, as the width of the slit is increased, the secondary maximum will become narrower. In case of a sufficiently wide slit, the secondary maxima and minima become invisible and only the central maximum is obtained, which corresponds to the sharp image of the slit. In other words, *the distinct diffraction pattern will be obtained only in the case of a narrow slit.*

6.03. FRESNEL DISTANCE

The distance of the screen from the slit, so that spreading of light due to diffraction from the centre of screen is just equal to size of the slit, is called ***Fresnel distance***. It is denoted by Z_F.

The diffraction pattern of a slit consists of secondary maxima and minima on the two sides of the central maximum. Therefore, one can say that on diffraction from a slit, light spreads on the screen in the form of central maximum. The angular position of first secondary minimum is called *half* ***angular width*** of the central maximum and it is given by

$$\theta_1 = \frac{\lambda}{a} \qquad \text{(provided } \theta_1 \text{ is small)}$$

If the screen is placed at a distance D from the slit, then the linear spread of the central maximum is given by

$$y_1 = D\,\theta_1 = \frac{D\,\lambda}{a}$$

It is, in fact, the distance of first secondary minimum from the centre of the screen. It follows that as the screen is moved away (D is increased), the linear size of the central maximum *i.e.* spread of light on the screen goes on increasing.

From the definition of Fresnel distance,

when $D = Z_F$, $\quad y_1 = a \qquad$ (size of the slit)

Setting this condition in the above equation, we have

$$a = \frac{Z_F\lambda}{a}$$

or $$Z_F = \frac{a^2}{\lambda} \qquad \text{...(6.12)}$$

It follows that if screen is placed at a distance beyond Z_F, the spreading of light due to diffraction will be quite large as compared to the size of the slit.

6.04. INTERFERENCE AND DIFFRACTION (Points of difference)

Following are the main points of difference between interference and diffraction :

1. Interference is the result of interaction of light coming from two different wavefronts originating from two coherent sources, whereas diffraction pattern is the result of interaction of light coming from different parts of the same wavefront.

2. In interference, the fringes may or may not be of the same width, while in diffraction, the fringes are always of varying width.

3. In interference, the fringes of minimum intensity are perfectly dark (provided light waves are of same amplitude), whereas in diffraction, the fringes of minimum intensity are not perfectly dark.

4. In interference, all the bright fringes are of same intensity ; while in diffraction, the bright fringes are of varying intensity.

5. In the interference pattern, there is a good contrast between bright and dark fringes. On the other hand, in diffraction pattern, the contrast between bright and dark fringes is comparatively poor.

6.05. RAYLEIGH'S CRITERION OF LIMITING RESOLUTION

The optical instruments are based on the laws of geometrical optics. Therefore, the image of a point object produced by an optical instrument must be a geometric point. In practice, the image of a bright geometric point produced by a lens is never a geometric point. It is always a circular patch of light with a bright disc at the centre surrounded by alternate dark and bright rings. It arises because of diffraction of light.

If two bright point objects lying very close to each other are seen with a naked eye or with the help of an optical instrument (a microscope or a telescope), then they may or may not be seen as two separate distinct objects due to the overlapping of their diffraction patterns.

Key point

Two objects are said to be just resolved, if the separation between the central maxima of their diffraction patterns is just equal to distance between the central maximum of one object and first secondary minimum of the other object.

The smallest separation (linear or angular) between the two point objects at which they appear just separated is called ***the limit of resolution of an optical instrument*** *and the reciprocal of the limit of resolution is called its* ***resolving power****.*

Whether the two objects are seen as two separate point objects or not, depends on the separation between the centres of the bright discs of the images of the two objects. Therefore, when two objects are seen with a naked eye or with the help of an optical instrument the two objects may be just resolved, well resolved or unresolved as explained below :

1. The two objects are said to be ***just resolved,*** if the separation between the central maxima of the two objects is just equal to the distance between the central maximum and the first minimum of any of the two objects [Fig. 6.05].

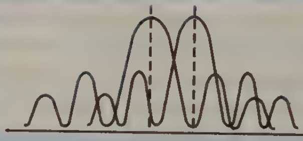


Fig. 6.05

2. The two objects are said to be ***well resolved,*** if the separation between the central maxima of the two objects is greater than the distance between the central maximum and the first minimum of any of the two objects [Fig. 6.06].

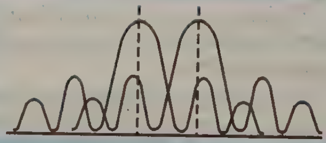


Fig. 6.06

3. The two objects are said to be ***unresolved,*** if the separation between the central maxima of the two objects is less than the distance between the central maximum and the first minimum of any of the two objects [Fig. 6.07].

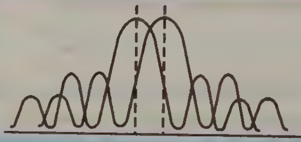


Fig. 6.07

These results are called ***Rayleigh's criterion of limiting resolution.***

Therefore, it follows that diffraction limits the resolving power of an optical instrument.

6.06. RESOLVING POWER OF A MICROSCOPE

Resolving power of a microscope is defined as the reciprocal of the least separation between two close objects, so that they appear just separated, when seen through the microscope.

Consider that a point object is illuminated with the light of wavelength λ and is seen through a microscope. The rays of light scattered from the object enter the objective of the microscope in a cone of semi-vertical angle θ [Fig. 6.08]. The least separation between the two objects, so that they appear just separated is given by

$$d = \frac{\lambda}{2\,\mu \sin \theta}, \qquad \text{...(6.13)}$$

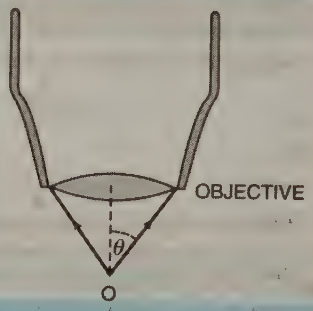


Fig. 6.08

where μ is refractive index of the medium between the objective of the microscope and the object.

The least distance given by equation (6.13) is called the *limit of resolution* of the microscope.

From definition, the resolving power of the microscope is given by

$$\text{resolving power of microscope, } \frac{1}{d} = \frac{2\,\mu \sin \theta}{\lambda} \qquad \text{...(6.14)}$$

From the above expression, the following conclusions can be drawn :

1. The resolving power of a microscope increases with increase in the value of the refractive index of the medium between its objective and the object.

For this reason, ***oil immersion objective microscopes*** are used to achieve high solving power.

2. The resolving power of a microscope increases with decrease in the value the wavelength of the light used to illuminate the object.

Since wavelength of ultraviolet light is less than that of the visible light, the icroscopes employing ultraviolet light for illuminating the objects are used to hieve high resolving power. Such microscopes are called ***ultra microscopes.***

Still higher resolving power is obtained in ***electron microscope.***

6.07. RESOLVING POWER OF A TELESCOPE

Resolving power of a telescope is defined as the reciprocal of the smallest angular paration between two distant objects, so that they appear just separated, when seen through e telescope.

Consider that two distant objects are observed through a telescope, whose jective is of diameter D. Let λ be the wavelength of the light, in which the objects e observed. The smallest angular separation between the two objects, so that they pear just separated is found to be

$$d\theta = \frac{1 \cdot 22\,\lambda}{D} \qquad ...(6.15)$$

From the definition of the resolving power of the telescope, we have

$$\text{resolving power of telescope, } \frac{1}{d\theta} = \frac{D}{1 \cdot 22\,\lambda} \qquad ...(6.16)$$

From the above expression, it follows that the resolving power of a telescope creases, when objective of larger diameter is used or light of smaller wavelength used to see the objects.

Key point

The resolving power of a microscope or a telescope increases, when light of smaller wavelength is used.

SOLVED NUMERICAL PROBLEMS

ype A. On Diffraction from a single slit

Problem 6.01. Light of wavelength 6000 Å from a istant source falls on a slit 0·5 mm wide. What is the istance between the two dark bands on each side of the entral bright band of the diffraction pattern observed on screen placed 2·0 m from the slit ? (*H.S.S.C.E. 2002*)

Sol. Here, $a = 0 \cdot 5 \text{ mm} = 0 \cdot 5 \times 10^{-3} \text{ m}$;

$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10} \text{ m}$; $D = 2 \cdot 0 \text{ m}$;

The distance between the two dark bands on each side the central band is equal to width of the central bright band e. $\frac{2D\lambda}{a}$.

Therefore, required distance

$$= \frac{2D\lambda}{a} = \frac{2 \times 2 \cdot 0 \times 6000 \times 10^{-10}}{0 \cdot 5 \times 10^{-3}}$$

$$= 4 \cdot 8 \times 10^{-3} \text{ m} = 4 \cdot 8 \text{ mm}$$

Problem 6.02. Light of wavelength 5900 ×10^{-10} m falls ormally on a slit of width 11·8 × 10^{-5} cm and the resulting raunhofer diffraction is received on a screen. Calculate the ngular position of the first minimum. Also find the ngular width of the central maximum. (*I.S.C.E. 1997*)

Sol. Here, $\lambda = 5900 \times 10^{-10}$ m ;

$a = 11 \cdot 8 \times 10^{-5} \text{ cm} = 11 \cdot 8 \times 10^{-7} \text{ m}$,

The angular position of first minimum is given by

$$\sin\theta_1 = \frac{\lambda}{a} = \frac{5900 \times 10^{-10}}{11 \cdot 8 \times 10^{-7}} = 0 \cdot 5 \quad \text{or} \quad \theta_1 = 30°$$

The angular width of central maximum,

$$2\theta_1 = 2 \times 30° = 60°$$

Problem 6.03. A slit of width *a* is illuminated by mono-hromatic light of wavelength 650 nm at normal incidence. Calculate the value of *a*, when (*a*) the first minimum falls at an angle of diffraction of 30°, (*b*) the first maximum falls at an angle of diffraction of 30°. (*C.B.S.E. 2002*)

Sol. (*a*) For first minimum at angle θ_1, we have

$$\sin\theta_1 = \frac{\lambda}{a}$$

Here, $\lambda = 650 \text{ nm} = 600 \times 10^{-9} \text{ m}$; $\theta_1 = 30°$

$$\therefore \quad a = \frac{\lambda}{\sin\theta_1} = \frac{650 \times 10^{-9}}{\sin 30°} = \frac{650 \times 10^{-9}}{0 \cdot 5}$$

$$= \mathbf{1 \cdot 3 \times 10^{-6} \text{ m}}$$

(*b*) For first maximum at angle θ_1', we have

$$\sin\theta_1' = \frac{3\lambda}{2a}$$

Here, $\lambda = 650 \times 10^{-9} \text{ m}$; $\theta_1' = 30°$

$$\therefore \quad a = \frac{3\lambda}{\sin\theta_1'} = \frac{3 \times 650 \times 10^{-9}}{2 \sin 30°} = \frac{3 \times 650 \times 10^{-9}}{2 \times 0 \cdot 5}$$

$$= \mathbf{1 \cdot 95 \times 10^{-6} \text{ m}}$$

Type B. On Fresnel distance

Problem 6.04. Light of wavelength 600 nm is incident on an aperture of size 2 mm. Calculate the distance upto which the ray of light can travel, such that its spread is less than the size of the aperture. (*C.B.S.E. 1996*)

Sol. Here, $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$;

$a = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$$\text{Now, required distance, } Z_F = \frac{a^2}{\lambda} = \frac{(2 \times 10^{-3})^2}{600 \times 10^{-9}} = \mathbf{6 \cdot 67 \text{ m}}$$

Type C. On Resolving power

Probelm 6.05. A telescope is used to resolve two stars separated by 4·6 × 10^{-6} radian. If the wavelength of light used is 5460 Å, what should be the aperture of the objective of the telescope ?

Sol. Here, $d\theta = 4{\cdot}6 \times 10^{-6}$ rad ;

$\lambda = 5460$ Å $= 5460 \times 10^{-10}$ m ;

The aperture (D) of the telescope is given by

$$D = \frac{1{\cdot}22\,\lambda}{d\theta} = \frac{1{\cdot}22 \times 5460 \times 10^{-10}}{4{\cdot}6 \times 10^{-6}}$$

$$= \mathbf{0{\cdot}1448\ m}$$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 6.06. In a simple slit diffraction experiment, first minimum for red light (660 nm) coincides with first maximum of some other wavelength λ'. Find the value of λ'.

Sol. The angular position of first minimum for red light of wavelength λ_r is given by

$$\sin\theta_1 = \frac{\lambda_r}{a} \qquad ...(i)$$

The angular position of first maximum for light of wavelength λ' is given by

$$\sin\theta_1{}' = \frac{3\,\lambda'}{2\,a} \qquad ...($$

As the first minimum for red light coincides with the fir maximum for wavelength λ',

$$\sin\theta_1 = \sin\theta_1{}'$$

Therefore, from equation (*i*) and (*ii*), we have

$$\frac{3\,\lambda'}{2\,a} = \frac{\lambda_r}{a} \quad \text{or} \quad \lambda' = \frac{2\,\lambda_r}{3}$$

$$\text{or} \qquad \lambda' = \frac{2 \times 660}{3} = \mathbf{440\ nm}$$

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 6.01. What is diffraction of light ? (*H.S.S.C.E. 2002*)

Ans. *The phenomenon of bending of light round the sharp corners and its spreading into regions of the geometrical shadow is called diffraction.*

Q. 6.02. Does phenomenon of diffraction help us to decide about the longitudinal or transverse nature of light ?

Ans. No.

Q. 6.03. What are the reasons to believe that light is a wave motion ? (*P.S.S.C.E. 2000*)

Ans. The phenomena of interference, diffraction and polarisation establish the wave nature of light.

Q. 6.04. State the condition for diffraction of light to occur. (*C.B.S.E. 2001*)

Ans. The size of the obstacle should be of the order of the wavelength of light used.

Q. 6.05. What should be the approximate slit size to observe light diffraction with it ? (*H.P.S.S.C.E. 1994*)

Ans. The size of the slit should be of the order of the wavelength of the light used.

Q. 6.06. What should be the order of size of obstacle/aperture for diffraction of light ? (*H.S.S.C.E. 1995*)

Ans. The diffraction in light is observed only, when the size of the obstacle is of the same order as the wavelength of the light.

Q. 6.07. Yellow light is used in a single slit diffraction experiment with slit width of 0·6 mm. If yellow light is replaced by X-rays, how will the diffraction pattern be affected ? (*I.I.T. 1999*)

Ans. Since wavelength of X-rays is very small as compared to the size of the slit, no diffraction pattern will be observed.

Q. 6.08. What is the condition for first minimum in cas of diffraction due to a single slit ? (*H.S.S.C.E. 199*

Ans. The angular position of first minimum, $\theta_1 = \dfrac{\lambda}{a}$.

Q. 6.09. What is the basic difference between inte ference and diffraction of light ? (*P.S.S.C.E. 199*

Ans. Interference is the result of superposition of ligh waves from two sources, whereas diffraction takes place du to interference of light waves from the different parts of th same source of light.

Q. 6.10. A diffraction grating has 5000 lines per cm What is its grating element ? (*I.S.C.E. 199*

Ans. Grating element $= \dfrac{1}{5000} = \mathbf{2 \times 10^{-4}\ cm}$

Q. 6.11. Define the resolving power of an optica instrument. (*I.S.C.E. 1994 : C.B.S.E. 199*

Ans. *The smallest separation (linear or angular) between th two points objects at which they appear just separated is called th* ***limit of resolution*** of an optical instrument.

The reciprocal of the limit of resolution is called its ***resolvin power.***

Q. 6.12. Define resolving power of a telescope. (*P.S.S.C.E. 1999 S*

Ans. Refer to SAQ. 6.15.

Q. 6.13. How does resolving power of a telescop change on decreasing the aperture of its object lens ? Justif your answer. (*C.B.S.E. 199*

Ans. Resolving power of a telescope $= \dfrac{D}{1{\cdot}22\,\lambda}$

Therefore, on decreasing aperture (D) of objective len the resolving power of telescope decreases.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 6.01. Why is diffraction of sound waves easier to observe than diffraction of light waves ? (*C.B.S.E. 2002, 1997*)

Or

Why can sound waves be diffracted more easily tha light waves ? (*P.S.S.C.E. 1999 S, 1992*

Or

Diffraction is common in sound but not common in light waves. Why ? (*H.S.S.C.E. 1994*)

Ans. For diffraction to take place, the size of the obstacle/aperture should be of the order of the wavelength of the waves. Since the wavelength of light ($\approx 10^{-6}$ m) is very small as compared to the size of the obstacle/aperture around us, light cannot be diffracted readily. On the other hand, as wavelength of sound is of the order of such objects, it gets diffracted easily.

Q. 6.02. Radiowaves diffract pronouncedly around buildings ; while light waves, which are electromagnetic waves, do not. Why ? (*H.P.S.S.C.E. 1998 S*)

Ans. For a wave to undergo diffraction, its wavelength should be of the size of the obstacle. The radiowaves (particularly short radiowaves) have wavelengths of the order of the size of the building and other obstacles coming in their way and hence they easily get diffracted.

Since wavelength of the light waves is very small, they are not diffracted by buildings.

Q. 6.03. Coloured spectrum is seen, when we look through a muslin cloth. Why ? (*H.P.S.S.C.E. 1997*)

Ans. Muslin cloth is made of very fine threads and as such fine slits are formed. White light passing through these slits gets diffracted giving rise to coloured spectrum. The central maximum will be white while the secondary maxima will be coloured. It is because, position of all the secondary maxima (except central) depends upon wavelength of the light.

Q. 6.04. Coloured spectrum is observed, when we see through muslin cloth. But no such spectrum is observed, when we see through coarse cloth. Why ?

Ans. Refer to SAQ 6.03.

In a coarse cloth, the slits formed due to threads are very wide and hence no such spectrum is seen.

Q. 6.05. A slit of width *a* is illuminated by monochromatic light at normal incidence. Draw the intensity distribution curve observed on the screen due to diffraction. (*C.B.S.E. 1993*)

Or

Draw the intensity pattern obtained in a single slit diffraction experiment. (*I.S.C.E. 1994*)

Ans. Refer to Fig. 6.04.

Q. 6.06. What two main changes in diffraction pattern of a single slit will you observe, when the monochromatic source of light is replaced by a source of white light ? (*C.B.S.E. 1997*)

Ans. 1. In each diffraction order, the diffracted image of the slit gets dispersed into component colours of white light.

2. In higher order spectra, the dispersion is more and it results in overlapping of different colours.

Q. 6.07. Prove that the central maxima is twice as wide as any other secondary maxima in the diffraction of a single slit. (*H.P.S.S.C.E. 1999*)

Ans. Refer to section 6.02.

Q. 6.08. In a single slit diffraction pattern, how is the width of central bright maximum changed, when (*a*) the slit width is decreased, (*b*) the distance between the slit and the screen is increased and (*c*) light of smaller wavelength is used. Justify your answer. (*C.B.S.E. 2002, 1994*)

Ans. The width of central maximum is given by

$$\beta_0 = \frac{2\,D\,\lambda}{a}, \qquad \ldots(i)$$

where the letters have their usual meanings.

(*a*) *Effect of slit width.* From equation (*i*), it follows that $\beta_0 \propto \frac{1}{a}$. Therefore, as the slit width is decreased, the width of central maximum will ***increase.***

(*b*) *Effect of distance between slit and screen.* From equation (*i*), it follows that $\beta_0 \propto D$. Therefore, as the distance between slit and the screen is increased, the width of central maximum will also *increase*.

(*c*) *Effect of wavelength of light.* From equation (*i*), it follows that $\beta_0 \propto \lambda$. Therefore, as the light of smaller wavelength is used, the width of central maximum will *decrease*.

Q. 6.09. In the diffraction of a single slit experiment, how would the width of and the intensity of central maximum change, if (*a*) slit width is halved and (*b*) visible light of longer wavelength is used ? (*C.B.S.E. 2001*)

Ans. (*a*) The intensity of central maximum will become one fourth.

(*b*) The intensity of central maximum will decrease.

Q. 6.10. Give two differences between fringes formed in single slit diffraction and Young's double slit experiment. (*C.B.S.E. 1996*)

Ans. 1. In Young's experiment, all the bright fringes formed are of the same intensity, whereas in single slit diffraction experiment, the bright fringes are of varying intensity.

2. In Young's experiment, fringes of minimum intensity are perfectly dark, whereas in slit diffraction experiment, fringes of minimum intensity are not perfectly dark.

Q. 6.11. Give two points of difference between the phenomenon of interference and diffraction of light.
(*H.P.S.S.C.E. 2001 ; P.S.S.C.E. 1999 S, C.B.S.E. 1995*)

Ans. Refer to section 6.04.

Q. 6.12. Give three points of difference between the phenomenon of interference and diffraction of light.
(*P.S.S.C.E. 2002, 2001, 1999 S*)

Or

Distinguish between interference and diffraction of light. (*P.S.S.C.E. 1999 ; H.S.S.C.E. 1999 ; C.B.S.E. 1994 ; H.P.S.S.C.E. 1993 ; P.S.S.C.E. 1993*)

Ans. Refer to section 6.04.

Q. 6.13. Define resolving power of a compound microscope. On what factors does it depend ? (*P.S.S.C.E. 2001*)

Ans. *Resolving power of a microscope is defined as the reciprocal of the least separation between two close objects, so that they appear just separated, when seen through the microscope.*

The resolving power of a microscope depeds upon the wavelength (λ) of the light used and the value of the semi-vertical angle (θ), the object makes at the objective of the microscope. (Refer to Fig. 6.08).

Q. 6.14. How does the resolving power of a compound microscope change on (*i*) decreasing the wave length of light used, and (*ii*) decreasing the diameter of its object lens ? (*C.B.S.E. 1998*)

Ans. Resolving power of microscope $= \dfrac{2\,\mu \sin\theta}{\lambda}$

(*i*) On decreasing wavelength of light, resolving power will increase.

(*ii*) On decreasing diameter of objective lens, semi-vertical angle θ will decrease and hence resolving power will decrease.

Q. 6.15. Define resolving power of a telescope. On what factors does it depend ? (*P.S.S.C.E. 2001*)

Ans. *Resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects, so that they appear just separated, when seen through the telescope.*

The resolving power of a telescope depends upon the wavelength (λ) of the light used and the diameter or aperture (D) of the objective of the telescope.

Q. 6.16. Define (*i*) resolving power and (*ii*) magnifying power of a telescope. (*C.B.S.E. 1991*)

Ans. (*i*) Resolving power, Refer to SAQ. 6.15.

(*ii*) Magnifying power, Refer to section 3.04 – Chapter 3.

Q. 6.17. How will the resolving power of a compound microscope be affected, when (*i*) the frequency of light used to illuminate the object is increased, and (*ii*) the focal length of the objective is increased. Justify your answer in each case. (*C.B.S.E. 1997*)

Ans. Resolving power of microscope

$$= \frac{2\,\mu \sin\theta}{\lambda} = \frac{2\,\nu\,\mu \sin\theta}{c} \qquad (\because c = \nu\lambda)$$

(*i*) When the frequency of light is increased, resolving power will increase.

(*ii*) When focal length of objective is increased, distance between object and objective will increase. It will lead to a decrease in the value of angle θ and hence resolving power will decrease.

Q. 6.18. Define resolving power of a telescope. How does diffraction limit its resolving power ? (*C.B.S.E. 1998 S*)

Ans. Resolving power. Refer to SAQ 6.15.

For remaining part of SAQ 6.18, refer to section 6.05.

Q. 6.19. Calculate the resolving power of a telescope, whose objective lens has an aperture of 1·0 m for the wavelength of light 500 nm ? (*C.B.S.E. 1998 S*)

Ans. Here, D = 1·0 m ; λ = 500 nm = 500×10^{-9} m

Resolving power of telescope

$$= \frac{D}{1\cdot22\,\lambda} = \frac{1\cdot0}{1\cdot22 \times 500 \times 10^{-9}} = \mathbf{1{\cdot}64 \times 10^6}$$

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 6.01. Why do we fail to observe the diffraction from a wide slit illuminated by monochromatic light ?

Ans. When the size of the slit is large, the central maximum is small in size and the variation in the intensity of the other maxima and minima is so small that they cannot be distinguished. That is why, we fail to observe the diffraction pattern from a wide slit illuminated by monochromatic light.

Q. 6.02. Why are diffraction effects more prominent through a slit formed by two blades than through a slit formed by two fingers ?

Ans. The diffraction pattern can be observed by forming a slit with the help of two blades or two fingers. However, the diffraction pattern will be observed clearly only, if the slit formed is fine and the edges of the blades/fingers are parallel. Since such an adjustment can be nicely done in case of two blades, the diffraction effects are more prominent through the slit formed by two blades than through the slit formed by two fingers.

Q. 6.03. Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other, even though they can converse easily ? (*C.B.S.E. 1990 ; Text Question*)

Ans. For pronounced diffraction effect, the size of the obstacle should be comparable to wavelength of the waves. The size of partition is very large as compared to the wavelength of light and hence it is not diffracted and the two students cannot see each other.

The wavelength of sound waves is of the order of the height of partition. It causes sound to diffract and hence they can converse with each other.

Q. 6.04. What happens to the diffraction of a single slit, when the slit is turned through angle θ, instead of holding the slit perpendicular to the path of the light ? Explain.

Ans. When the slit is rotated from position AB to A′B′ through angle θ [Fig. 6.09], the path difference between the rays reaching the centre of screen remains zero. It is because, the extra path AA′ covered by the light to travel from source of light to the top edge of the slit is compensated for by the extra path BB′, the light travels from the bottom edge of the slit to the screen. As a result, the central maximum does not move.

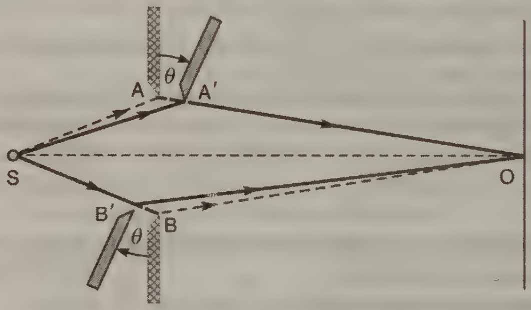


Fig. 6.09

It follows that the effective width of the slit decreases, when the slit is turned through angle θ. If a is the width of the slit, then on rotating through angle θ, its width decreases to $a\cos\theta$. Since the slit becomes narrower, the diffraction pattern becomes wider.

Q. 6.05. In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band ? (*C.B.S.E. 2001 S ; Text Question*)

Ans. The width of central maximum $= \dfrac{2\,D\,\lambda}{a}$

If the width of the slit (a) is made double of the original, the width of central diffraction band will reduce to half.

The intensity of the central diffraction band will become four times.

Q. 6.06. When a tiny circular obstacle is placed in the path of light from a distance source, a bright spot is een at the centre of the shadow of the obstacle. Explain, hy. *(Text Question)*

Ans. The central bright spot is produced due to the onstructive interference of waves from the edge of the ircular obstacle.

Q. 6.07. In what way is diffraction from each slit related to the interference pattern in Young's double slit xperiment ?

Ans. The intensity of interference fringes in Young's ouble slit experiment is superposed by the diffraction attern of each slit.

Q. 6.08. Stars are often photographed through a blue filter. What is the advantage of this ?

Ans. The resolving power of a telescope

$$= \frac{1 \cdot 22\,D}{\lambda},$$

where D is diameter of the telescope's objective.

If light of a shorter wavelength is used, the resolving power of the telescope will be more and like wise the stars which subtend a smaller angle at the earth, will also be resolved and seen distinctly as separate ones. The use of blue filter allows only blue light (shorter wavelength) from the star to enter the telescope.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain diffraction of light due to a single slit and formation of pattern of fringes. *(H.S.S.C.E. 1997)*
2. Discuss the formation of maxima and minima by diffraction at a single slit. *(H.S.S.C.E. 1997 S)*
3. What is diffraction of light ? Explain with a diagram the intensity distribution in the diffraction pattern due to a single slit. *(Karnataka, 1994)*
4. Explain Fraunhoffer diffraction at a single slit and derive relation for linear width of central maximum. *(P.S.S.C.E. 1999 S, 1999, 1998)*
5. Prove that in diffraction from a slit, the secondary maxima and minima are of same width.
6. Derive an expression for the width of the central maxima for diffraction of light at a single slit. How does this width change with increase in width of the slit ? *(C.B.S.E. 1998)*
7. Show that the central maximum in the single slit diffraction is twice as wide as the secondary maximum and the pattern becomes narrow as the width of the slit is increased.
8. How does diffraction limit the resolving power of an optical instrument ? *(C.B.S.E. 1990)*
9. Explain with reasons, how the resolving power of a compound microscope will change, when (*i*) frequency of the incident light on the objective lens is increased (*ii*) focal length of the objective lens is increased and (*iii*) aperture of the objective lens is increased. *(C.B.S.E. 2002)*
10. Explain with reason, how the resolving power of an astronomical telescope will change when (*i*) frequency of the incident light on the objective lens is increased, (*ii*) focal length of the objective lens is increased and (*iii*) aperture of the objective lens is halved. *(C.B.S.E. 2002)*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What is meant by diffraction ? Draw a graph to show the relative intensity distribution for a single slit diffraction pattern. Obtain an expression for the diffraction of the first minimum in the diffraction pattern. *(I.S.C.E. 1993)*
2. Discuss Fraunhoffer diffraction at a single slit. Also derive the relation for linear width of central maximum. *(P.S.S.C.E. 2001)*
3. Explain diffraction at a single slit and derive relation for linear width of maxima. *(P.S.S.C.E. 2000)*
4. What is diffraction of light ? Explain the formation of maxima and minima due to diffraction from a rectangular narrow slit. Show that the intensity of the central maximum is inversely proportional to the width of the slit.
5. What is a diffraction grating ? Explain, how does it produce diffraction spectrum.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Diffraction from a slit

1. A slit of width 0·15 cm is illuminated by light of wavelength 5×10^{-5} cm and a diffraction pattern is obtained on a screen 2·1 m away. Calculate the width of the central maxima. [**Ans.** 1·4 mm]
2. The light of wavelength 600 nm is incident normally on a slit of width 3 mm. Calculate the linear width of central maximum on a screen kept 3 m away from the slit. *(C.B.S.E. 2001)* [**Ans.** 1·2 mm]
3. Light of wavelength 600 nm is incident on a single slit of width 0·5 mm at normal incidence. Calculate the separation between two dark bands on either side of the central maximum, if the diffraction pattern is observed on a screen placed at 2 m from the slit. *(C.B.S.E. 2002)* [**Ans.** 4·8 mm]
4. Red light of wavelength 6500 Å from a distant source falls on a slit 0·5 mm wide. What is the distance between two dark bands on each side of central bright band of diffraction pattern observed on a screen placed 1·8 m from the slit ? *(H.S.S.C.E. 2001 ; C.B.S.E. 1990)* [**Ans.** 4·68 mm]
5. A screen is placed 2 m away from a narrow slit. Find the slit width, if the first minimum lies 5 mm on either side of the central maximum, when plane waves of 5×10^{-7} m are incident on the slit. [**Ans.** 0·2 mm]
6. A slit 4·0 cm wide is irradiated with microwaves of wavelength 2·0 cm. Find the angular spread of central maxima assuming incidence normal to the plane of the slit. *(H.S.S.C.E. 1993)* [**Ans.** 60°]
7. (*i*) Light of wavelength $5 \cdot 4 \times 10^{-5}$ cm passes through a slit 0·12 cm wide and forms a diffraction pattern on a screen 2·7 m away. What is the width of the central maxima ?

(*ii*) If the apparatus is immersed in a liquid of refractive index 1·35, what would be the width of the central fringe ? **[Ans.** (*i*) 2·43 mm ; (*ii*) 1·8 mm]

8. Determine the angular separation between central maximum and first order maximum of the diffraction pattern due to a single slit of width 0·25 mm, when light of wavelength 5890 Å is incident on it normally. (*C.B.S.E. 1993*) **[Ans.** $3{\cdot}534 \times 10^{-3}$ rad]
9. A slit of width *a* is illuminated by red light of wavelength 6200 Å. For what value of *a*, will the first minimum fall at an angle of diffraction of 30° ? **[Ans.** $1{\cdot}24 \times 10^{-6}$ m]
10. A slit of width *a* is illuminated by a light of wavelength 5500 Å. What will be the value of *a*, when (*a*) the first minimum falls at an angle of diffraction of 30°, (*b*) the first maximum falls at an angle of diffraction of 30° ? (*C.B.S.E. 1998 ; H.S.S.C.E. 1992*) **[Ans.** (*a*) $1{\cdot}1 \times 10^{-6}$ m, (*b*) $1{\cdot}65 \times 10^{-6}$ m]

Type B. On Fresnel distance

11. Calculate the distance which a beam of light of wavelength 500 nm can travel without significant broadening, if the diffracting aperture is 3 mm wide. (*C.B.S.E. 1990*) **[Ans.** 18 m]
12. Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400 nm. (*Text Problem*) **[Ans.** 40 m]

Type C. On Resolving power

13. What is the minimum angular separation between two stars, if a telescope is used to observe them with an objective of aperture 0·2 m ? The wavelength of light used is 5900 Å. **[Ans.** $3{\cdot}6 \times 10^{-6}$ rad]
14. A telescope has an objective of diameter 0·6 m. The telescope is directed to view two distant almost point sources of light (*e.g.* two stars of a binary). The sources are roughly at the same distance of 10^4 light years along the line of sight but separated by a distance of 10^{10} m. Will the telescope resolve the two objects ? Assume that mean wavelength of light is 6000 Å. (*Text Problem*) **[Ans.** No]

MISCELLANEOUS PROBLEMS

15. (*a*) Two slits are made 1 mm apart and screen is placed 1 m away. What is the fringe separation, when blue-green light of wavelength 500 nm is used ?
(*b*) What should be the width of each slit to obtain 10 maxima of the double slit pattern, within the central maximum of the single slit pattern ? (*Text Problem*) **[Ans.** (*a*) 0·5 mm (*b*) 0·2 mm]
16. Two spectral lines of sodium D_1 and D_2 have wavelengths of approximately 5890 Å and 5896 Å. A sodium lamp sends incident plane wave on a slit of width 2×10^{-6} m. A screen is located 2 m from the slit. Find the spacing between the first maxima of two sodium lines as measured on the screen. **[Ans.** 0·9 mm]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Here, $a = 0{\cdot}15 \text{ cm} = 0{\cdot}15 \times 10^{-2}$ m,

$$\lambda = 5 \times 10^{-5} \text{ cm} = 5 \times 10^{-7} \text{ m}, \ D = 2{\cdot}1 \text{ m}$$

Now, $$\beta = \frac{2D\lambda}{a} = \frac{2 \times 2\cdot1 \times 5 \times 10^{-7}}{0\cdot15 \times 10^{-2}} = 1\cdot4 \times 10^{-3} \text{m}$$

$= \mathbf{1{\cdot}4 \text{ mm}}$

2. Proceed as in problem no. 1.
3. Proceed as in solved problem no. 6.01.
4. Proceed as in solved problem no. 6.01.
5. $y_1' = 5 \text{ mm} = 5 \times 10^{-3}$ m or $\dfrac{D\lambda}{a} = 5 \times 10^{-3}$ m

or $$a = \frac{D\lambda}{5 \times 10^{-3}} = \frac{2 \times 5 \times 10^{-7}}{5 \times 10^{-3}} = 2 \times 10^{-4} \text{m}$$

$= \mathbf{0{\cdot}2 \text{ mm}}$

6. Proceed as in solved problem no. 6.02.
7. (*i*) Here, $\lambda = 5{\cdot}4 \times 10^{-5} \text{ cm} = 5{\cdot}4 \times 10^{-7}$ m,

$$a = 0{\cdot}12 \text{ cm} = 0{\cdot}12 \times 10^{-2} \text{ m}, \ D = 2{\cdot}7 \text{ m}$$

Now, $$\beta_0 = \frac{2D\lambda}{a} = \frac{2 \times 2\cdot7 \times 5\cdot4 \times 10^{-7}}{0\cdot12 \times 10^{-2}}$$

$$= 2\cdot43 \times 10^{-3} \text{m} = \mathbf{2{\cdot}43 \text{ mm}}$$

(ii) When apparatus is immersed in liquid :

$$\beta_0' = \frac{2D\lambda'}{a} = \frac{2D\lambda}{a\mu} = \frac{\beta_0}{\mu} = \frac{2\cdot43}{1\cdot35} = \mathbf{1{\cdot}8 \text{ mm}}$$

8. Here, $\lambda = 5890 \text{ Å} = 5890 \times 10^{-10}$ m,

$a = 0{\cdot}25 \text{ mm} = 0{\cdot}25 \times 10^{-3}$ m

Angular separation between central and first order aximum,

$$\theta_1' = \frac{3\lambda}{2a} = \frac{3 \times 5890 \times 10^{-10}}{2 \times 0\cdot25 \times 10^{-3}} = \mathbf{3{\cdot}534 \times 10^{-3} \text{ rad}}$$

9. Proceed as in solved problem no. 6.03 (*a*).
10. Proceed as in solved problem no. 6.03.
11. $$Z_F = \frac{a^2}{\lambda} = \frac{(3 \times 10^{-3})^2}{500 \times 10^{-9}} = \mathbf{18 \text{ m}}$$
12. $$Z_F = \frac{a^2}{\lambda} = \frac{(4 \times 10^{-3})^2}{400 \times 10^{-9}} = \mathbf{40 \text{ m}}$$
13. $$d\theta = \frac{1\cdot22\lambda}{D} = \frac{1\cdot22 \times 5900 \times 10^{-10}}{0\cdot2} = \mathbf{3{\cdot}6 \times 10^{-6} \text{ rad}}$$
14. Smallest angular separation that can be resolved by the telescope,

$$d\theta = \frac{1\cdot22\lambda}{D} = \frac{1\cdot22 \times 6000 \times 10^{-10}}{0\cdot6} = 1{\cdot}22 \times 10^{-6} \text{ rad}$$

Distance of stars = 10^4 light years

$= 10^4 \times 9{\cdot}46 \times 10^{15}$ m

Angle subtended by two stars separated by a transverse distance of 10^{10} m,

$$d\theta' = \frac{10^{10}}{10^4 \times 9\cdot46 \times 10^{15}} = 1{\cdot}057 \times 10^{-10} \text{ rad}$$

Since $d\theta' << d\theta$, the two stars of the binary **cannot be resolved** by the telescope.

15. (*a*) $$\beta = \frac{D\lambda}{d} = \frac{1 \times 500 \times 10^{-9}}{1 \times 10^{-3}} = 5 \times 10^{-4} \text{m} = \mathbf{0{\cdot}5 \text{ mm}}$$

(*b*) Let *a* be width of the slit.

The width of central maximum of diffraction pattern,

$$\beta_0 = \frac{2D\lambda}{a}$$

Fringe width of interference pattern,

$$\beta = \frac{D\lambda}{d} = \frac{D\lambda}{1 \times 10^{-3}}$$

Since $\beta_0 = 10\,\beta$, we have

$$\frac{2\,D\lambda}{a} = 10 \times \frac{D\lambda}{1 \times 10^{-3}}$$

or $a = 0{\cdot}2 \times 10^{-3}\text{ m} = \mathbf{0{\cdot}2\ mm}$

16. The distance of first secondary maxima from the centre of the screen,

$$y_1{}' = \frac{3\,D\lambda}{2\,a}$$

For sodium D_1 line :

$$y_1{}' = \frac{3 \times 2 \times 5890 \times 10^{-10}}{2 \times 2 \times 10^{-6}} = 0 \cdot 8835\text{ m}$$

For sodium D_2 line :

$$y_1{}' = \frac{3 \times 2 \times 5896 \times 10^{-10}}{2 \times 2 \times 10^{-6}} = 0 \cdot 8844\text{ m}$$

Therefore, spacing between the first maxima for D_1 and D_2 lines

$$= 0{\cdot}8844 - 0{\cdot}8835 = 9{\cdot}0 \times 10^{-4}\text{ m} = \mathbf{0{\cdot}9\ mm}$$

CHAPTER
7

Polarisation of Light

7.01. TRANSVERSE NATURE OF LIGHT

In general, the waves are of two types :

1. ***Longitudinal waves.*** *The waves, in which the particles oscillate along the direction of propagation of the waves, are called longitudinal waves.*

2. ***Transverse waves.*** *The waves, in which the direction of oscillation of particles is perpendicular to the direction of propagation of the waves, are called transverse waves.*

Both types of these waves exhibit the phenomena of reflection, refraction, diffraction and interference but *polarisation* of the waves can only be exhibited by the transverse waves. This is the only phenomenon, where the two types of waves essentially differ from one another.

Key point

The phenomenon of polarisation establishes that light is transverse in nature.

To understand the polarisation of the waves, let us first consider the following mechanical experiment :

Two vertical slits S_1 and S_2 are placed parallel to each other and a string is passed through them. The end B of the string is fixed and the end A is given jerks in any direction perpendicular to its length. Then, transverse wave, whose vibrations are parallel to the length of the slit, will only pass through the slit S_1. If the slit S_2 is parallel to S_1, then amplitude of vibrations will remain unaffected [Fig. 7.01].

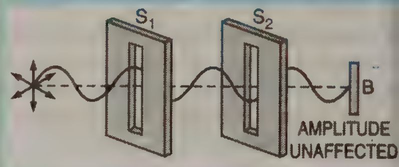


Fig. 7.01

On the other hand, when the slit S_2 is at right angle to the slit S_1, no vibrations will pass through it *i.e.* amplitude of vibrations will become zero [Fig. 7.02.].

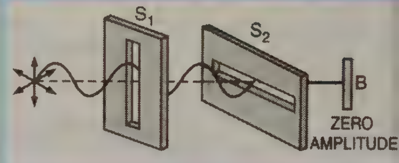


Fig. 7.02

If instead of producing transverse vibrations, we produce longitudinal vibrations in the string at the end A, the vibrations will pass through the slit S_2 irrespective of the fact that the slits S_1 and S_2 are parallel to or at right angles to each other.

From the above mechanical experiment, it follows that *if on rotating the slit S_2, the amplitude of vibration in the string decreases and becomes zero, when the two slits are at right angles, then the wave motion is transverse in nature.* On the other hand, *if the amplitude of vibration remains unaffected, the wave motion is a longitudinal one.*

A Nicol prism or tourmaline crystal cut parallel to its crystallographic axis behaves in a similar manner for light waves as the slit does in case of the wave motion produced in the string. When ordinary light is incident normally on a pair of such crystals C_1 and C_2, the intensity of emergent light is maximum, when the axis of the two crystals are parallel [Fig. 7.03.] and minimum, when the axis of the two crystals become at right angles to each other [Fig. 7.04.]. In view of the mechnical experiment discussed earlier, it follows that *light is a transverse wave motion.*

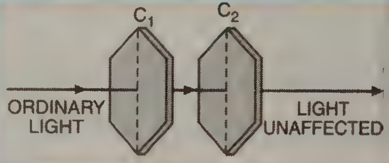


Fig. 7.03

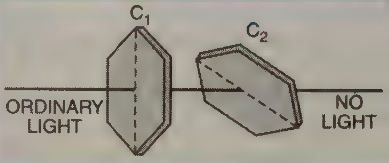


Fig. 7.04

According to Maxwell, light is electromagnetic in nature. An electromagnetic wave consists of varying electric and magnetic fields, such that the two fields are mutually perpendicular to each other and to the direction of propagation of waves. The optical phenomena *i.e.* phenomena concerning light are described by the *electric field vector* in electromagnetic waves. Therefore, transverse nature of light may be attributed to the vibrations of electric field vector in a direction perpendicular to the direction of propagation of light. In ordinary or unpolarised light, *the vibrations of electric field vector are regularly or symmetrically distributed in a plane perpendicular to the direction of the propagation of the light.*

In an ordinary ray of light, the vibrations are in all the directions but perpendicular to the direction of propagation of the light. Such a ray of light is called a ray of *ordinary* or *unpolarised light*. It is schematically represented as shown in Fig. 7.05. The arrows represent vibrations in the plane of the paper, while the dots represent vibrations in a direction perpendicular to the plane of the paper.

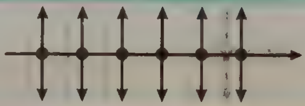


Fig. 7.05

We know that light is electromagnetic wave motion. An electromagnetic wave consists of varying electric and magnetic fields, such that the two fields are mutually perpendicular to each other and to the direction of propagation of waves. The optical phenomena are described by the *electric field vector* in electromagnetic waves. Therefore, transverse nature of light may be attributed to the vibrations of electric field vector in a direction perpendicular to the direction of propagation of light.

7.02. POLARISATION OF LIGHT

The phenomenon, due to which the vibrations of light are restricted in a particular plane, is called the polarisation of light.

When ordinary light *i.e.* unpolarised light passes through a tourmaline crystal, out of all the vibrations which are symmetrical about the direction of propagation, only those pass through it, which are parallel to its crystallographic axis AB. Therefore, on emerging through the crystal, the vibrations no longer remain symmetrical about the direction of propagation but are confined to a single plane as shown in Fig. 7.06.

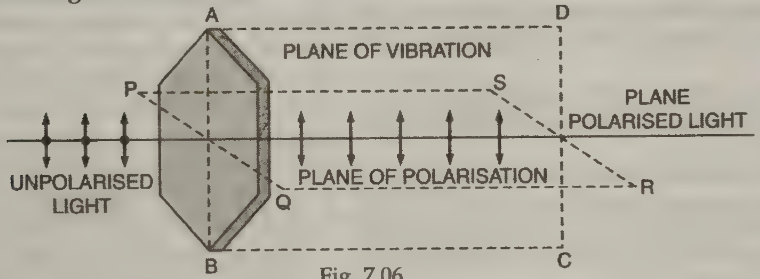


Fig. 7.06

Key point

The vibrations in plane polarised light are perpendicular to the plane of polarisation.

Plane of vibration. *The plane (ABCD), which contains the vibrations of plane polarised light, is called the plane of vibration.*

Plane of polarisation. *The plane (PQRS) perpendicular to the plane of vibrations is called the plane of polarisation.*

Plane polarished light. It *may be defined as the light, in which the vibrations of the light (vibrations of the electric vector) are restricted to a particular plane.*

According to the electromagnetic theory of light, the electric vector acts as light vector. Therefore, in a plane polarised light, the electric vector vibrates along a fixed line in a plane perpendicular to the direction of propagation.

In a plane polarised light, the vibrations are restricted to a fixed plane, so that vibrations are perpendicular to the direction of propagation of light. Fig. 7.07 (*a*) represents plane polarised light having vibrations in the plane of the paper and Fig. 7.07 (*b*) represents the plane polarised light having vibrations in a plane perpendicular to the plane of the paper.

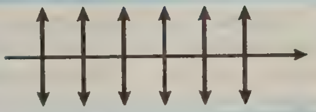


Fig. 7.07 (*a*)

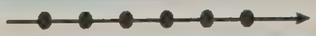


Fig. 7.07 (*b*)

7.03. TO DETECT PLANE POLARISED LIGHT

The naked eye or the polariser alone cannot make distinction between the unpolarised light and the plane polarised light. Another such crystal used to analyse the nature of light is called ***analyser****.

The tourmaline crystal or Nicol prism used to produce plane polarised light is called ***polariser***. If the polariser is rotated in the path of the ordinary light, the intensity of the light transmitted from the polariser remains unchanged. It is because, in each orientation of the polariser, the plane polarised light is obtained, which has vibrations in a direction parallel to the axis of the crystal in that orientation.

If the analyser is rotated in the path of the light transmitted from the polariser, so that the axes of the polariser and the analyser are parallel to each other, then the intensity of light is found to remain unaffected (or maximum) as shown in Fig. 7.08.

Key point

A polariser alone cannot distinguish between an unpolarised and plane polarised light.

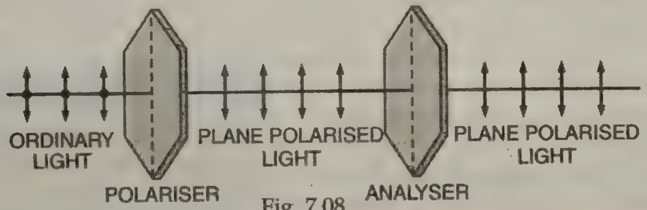


Fig. 7.08

---

*In Fig. 7.03 or 7.04, the crystal C_1 acts as a polariser and the crystal C_2 acts as an analyser.

On the other hand, the intensity of light becomes minimum, when the axis of the polariser and the analyser are perpendicular to each other [Fig. 7.09]. In the position shown in Fig. 7.09, the polariser and the analyser are said to be in *crossed positions.*

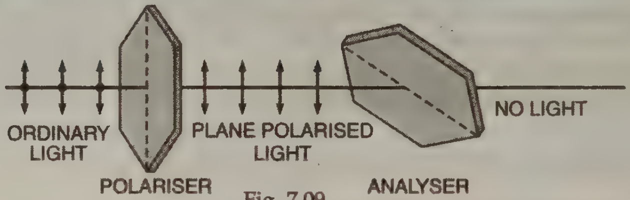


Fig. 7.09

7.04. POLARISATION BY REFLECTION

The simplest method to produce plane polarised light is by reflection.

Malus found that an ordinary beam of light on reflection from a transparent medium becomes partially polarised. The degree of polarisation increases as the angle of incidence is increased. At a particular value of angle of incidence, the reflected beam becomes completely polarised. This angle of incidence is called the ***polarising angle*** for that medium.

Therefore, *the angle of incidence, at which when ordinary light is incident on transparent refracting medium, the reflected light becomes maximum rich in plane polarised light is called polarising angle.* It is denoted by p.

The value of polarising angle varies with the wavelength of light and hence complete polarisation can be obtained only with monochronic light. In Fig. 7.10, the ordinary light is incident along AB on the surface separating air from a medium of reflective index μ. The completely plane polarised light is obtained along the path BC of reflected light, when light is incident at polarising angle p. The plane polarised light has vibrations perpendicular to the plane of paper. The light travelling along the path BD (refracted light) is unpolarised light.

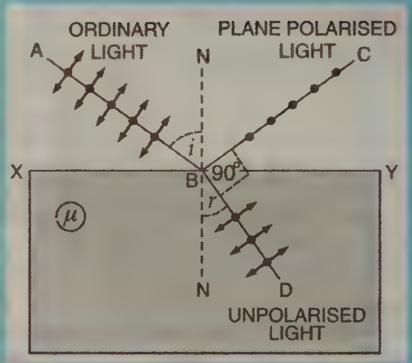


Fig. 7.10

Explanation. The vibrations perpendicular to the plane of the paper always remain parallel to the reflecting surface, whatever be the angle of incidence, and hence the condition of reflection remains the same, even if the angle of incidence is changed. The other vibrations make different angles with the reflecting surface as the angle of incidence is changed.

When light is incident at the ***polarising angle,*** the vibrations perpendicular to the plane of the paper or parallel to the reflecting surface are reflected along BC, while the other vibrations (making some angle with the reflecting surface) are transmitted and are not reflected. The reflected light is, therefore, completely *polarised in the plane of incidence.*

Key point

The light reflected from a transparent refracting medium is always plane polarised. However, the plane of vibration becomes maximum rich in such vibrations, when light is incident at polarising angle.

7.05. BREWSTER'S LAW

It states that *when light is incident at polarising angle at the interface of a refracting medium, the refractive index of the medium is equal to the tangent of the polarising angle.*

If p is polarising angle and μ, the refractive index of the refracting medium, then according to Brewster's law,

$$\mu = \tan p \qquad ...(7.01)$$

When light is incident at polarising angle, the reflected light is found to be completely plane polarised.

Refer to Fig. 7.10. When light is incident at polarising angle p on a refracting medium of refractive index μ, let r be the angle of refraction. Then, according to Snell's law,

$$\mu = \frac{\sin p}{\sin r} \qquad ...(7.02)$$

From equations (7.01) and (7.02), we have

$$\frac{\sin p}{\sin r} = \tan p$$

or $$\frac{\sin p}{\sin r} = \frac{\sin p}{\cos p}$$

or $\sin r = \cos p$ or $\sin r = \sin(90° - p)$ or $r = 90° - p$

or $r + p = 90°$

Now, angle of reflection *i.e.* $\angle$ NBC = p, the angle of incidence. Since $r + p = 90°$, it follows that $\angle$ CBD = 90° *i.e.* reflected ray BC is perpendicular to refracted ray BD.

Key point

When light is incident on a transparent refracting surface at polarising angle, the reflected and refracted rays are perpendicular to each other.

Hence, *when a ray of light is incident at polarising angle, the reflected ray is at right angle to the refracted ray.*

7.06. POLARISATION BY SCATTERING

When a beam of light passes through a medium, it gets scattered from the particles constituting the medium, provided the size of the particles is of the order of the wavelength of the light. The scattered light viewed in a direction perpendicular to the direction of the beam of light is found to be plane polarised. It gives us a method to produce plane polarised light by scattering.

Consider a beam of unpolarised light propagating along X-axis. The vibrations are confined in YZ-plane [Fig. 7.11]. After being scattered from a particle O, the light scattered along Y-axis has only those vibrations, which are perpendicular to Y-axis but parallel to Z-axis. On the other hand, the light scattered along Z-axis is found to contain only those vibrations, which are perpendicular to Z-axis and parallel to Y-axis. Thus, the scattered light, both along X-axis and Z-axis, is plane polarised light.

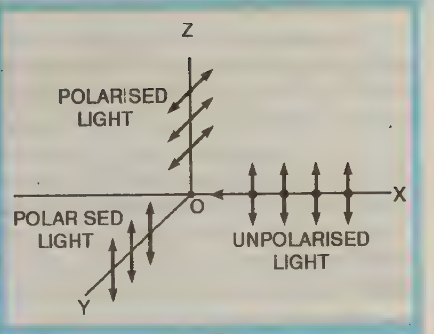


Fig. 7.11

7.07. LAW OF MALUS

It states that *when a completely plane polarised light beam is incident on an analyser, the intensity of the emergent light varies as the square of the cosine of the angle between the planes of transmission of the analyser and the polariser.*

Consider that the plane of polariser and the plane of analyser are inclined at an angle θ as shown in Fig. 7.12. Further, suppose that the plane polarised light of intensity I_0 and amplitude a is incident on the polariser. The amplitude of the incident light can be resolved into two components :

(*i*) The component $a \cos \theta$ along the plane of transmission of the analyser and

(*ii*) The component $a \sin \theta$ in a direction perpendicular to the plane of the analyser.

In case, there is no loss of light due to absorption, while traversing the analyser, then intensity of light transmitted from the analyser is given by

$$I = (a \cos \theta)^2 = a^2 \cos^2 \theta$$

But $a^2 = I_0$, the intensity of incident plane polarised light. Therefore,

$$I = I_0 \cos^2 \theta$$

or

$$I \propto \cos^2 \theta \qquad \text{...(7.03)}$$

It is called *law of Malus.*

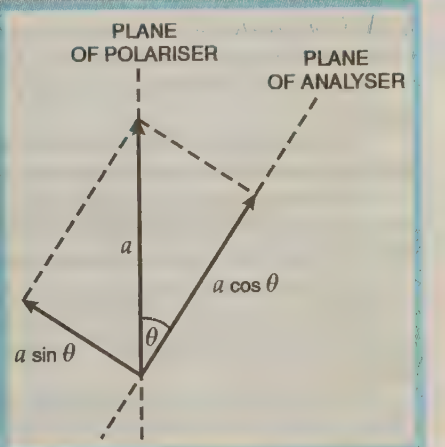


Fig. 7.12

Discussion. 1. When $\theta = 0°$ or $180°$ so that $\cos \theta = \pm 1$,

$$I = I_0$$

Therefore, *when polariser and analyser are parallel, the intensity of light transmitted from the analyser is same as that falls on it from polariser.*

2. When $\theta = 90°$ so that $\cos \theta = 0$,

$$I = 0$$

Therefore, *when the polariser and analyser are perpendicular, the intensity of light transmitted from the analyser is zero i.e. minimum.*

3. If we plot a graph between intensity of light and the angle between polariser and analyser, it will be shown in Fig. 7.13.

4. In case, the light incident on analyser is unpolarised, then

$$I = \frac{1}{2} I_0$$

It is because, in unpolarised light, the light vector vibrates randomly in a plane perpendicular to the direction of propagation and therefore to determine the intensity of transmitted light, average value of $\cos^2 \theta$ has to be substituted in equation (7.02). It can be shown that the average value of $\cos^2 \theta$ *i.e.* $\overline{\cos^2 \theta} = \frac{1}{2}$.

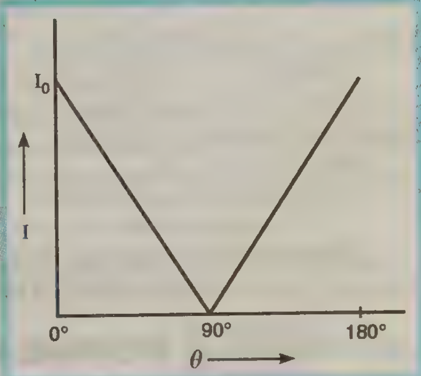


Fig. 7.13

7.08. POLAROIDS

A Nicol prism or a tourmaline crystal cannot be used to produce plane polarised beam of light of large cross-section. It is because of the reason that very large crystals of calcite and tourmaline are not available in practice.

Polaroids are thin and large sheets of crystalline polarising material (made artificially) capable of producing plane polarised beams of large cross-section.

In 1852, Herapath discovered that the synthetic small needle-shaped crystals of iodosulphate of quinine possess the property of polarising the light. These crystals were called *herapathite.* These crystals are not stable and cannot bear even a slight strain. Since such crystals cannot be used as such, a polaroid sheet is prepared from the suspension of these crystals in nitrocellulose. To impart stability, its thin sheet is mounted between the two sheets of glass or celluloid.

A polaroid sheet can also be obtained from a sheet of polyvinyl alcohol. When such a sheet is subjected to a large strain, the molecules get oriented in the direction of the applied strain. When the sheet is impregnated with iodine, the material becomes *dichroic**. The polaroid sheet so obtained is called *H-polaroid.*

If the stretched sheet of polyvinyl alcohol is heated in the presence of a dehydrating agent such as HCl, it becomes strongly dichroic as well as very stable. Such a polaroid sheet is called ***K-polaroid***.

Each polaroid sheet is enclosed between thin glass plates so as to provide mechanical support. Further, a polaroid has a characteristic plane called *transmission plane.* When unpolarised light falls on a polaroid, only the vibrations parallel to the transmission plane get transmitted [Fig. 7.14].

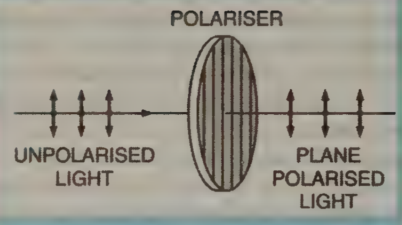


Fig. 7.14

If a polariod P_1 is placed in front of a source of light, then light coming out of it is plane polarised. When another polaroid P_2 is placed so that their transmission planes are parallel [Fig. 7.15], the plane polarised light produced by the polaroid P_1 also passes through it. However, when the polaroid P_2 is rotated through 90°, so that the transmission planes of the two polaroids become perpendicular to each other, then no light comes out [Fig. 7.16]. The two polaroids are said to be *crossed*.

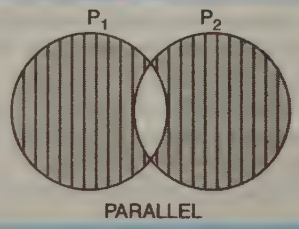


Fig. 7.15

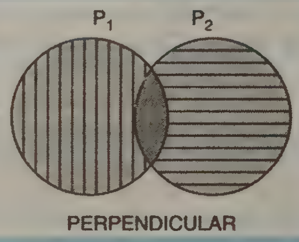


Fig. 7.16

Uses. The polaroids are widely used and a few important uses are as listed below :

1. *In sun glasses.* Polaroids are used in sun glasses. Such goggles are more efficient than those made of coloured glasses. They protect the eyes from the glare.

2. *In wind shields in automobiles.* The wind shield of an automobile is made of polaroid. Such a wind shield protects the eyes of the driver of the automobile from the dazzling light of the approaching vehicles.

For the same reasons, the covers of head lights are made of polaroids.

3. *In window panes.* The polaroids are used in window panes of a train and especially of an aeroplane. They help to control the light entering through the windows.

4. *In three dimensional motion pictures.* The pictures taken by a stereoscopic camera, when seen with the help of polaroid spectacles, create three dimensional effect.

In addition to the above uses, polaroids are used as filters in photography to produce and detect the plane polarised light in the laboratory and to study the optical properties of the metals.

SOLVED NUMERICAL PROBLEMS

TYPE A. ON BREWSTER'S LAW

Problem 7.01. A ray of light is incident on the surface of a glass plate of refractive index 1·536 at the polarising angle. Calculate the angle of refraction.

Sol. Here, $\mu = 1{\cdot}536$

If p is the polarising angle, then

$$\tan p = \mu = 1{\cdot}536 \quad \text{or} \quad p = 56°\,56'$$

If r is the angle of refraction, then

$$r = 90° - p = 90° - 56°56' = \mathbf{33°4'}$$

Problem 7.02. For a given medium, the polarising angle is 60°. What will be the critical angle for this medium ?

(H.P.S.S.C.E. 1996)

Sol. Here, $p = 60°$

Now, $\mu = \tan p = \tan 60° = \sqrt{3}$

If C is the critical angle, then $\mu = \dfrac{1}{\sin C}$

$$\text{or} \qquad \sin C = \frac{1}{\mu} = \frac{1}{\sqrt{3}} = 0{\cdot}5774 \quad \text{or} \quad \mathbf{C = 35°16'}$$

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 7.03. Two polaroids are crossed to each other. Now, one of them is rotated through 60°. What percentage of incident unpolarised light will pass through the system ?

Sol. Let I_0 be intensity of the incident unpolarised light on the system. As the incident light is unpolarised, the intensity of light on being transmitted from the first polaroid will become

$$I = I_0\,\overline{\cos^2\theta} = \frac{I_0}{2} \qquad \left(\because \overline{\cos^2\theta} = \frac{1}{2}\right)$$

---

* A calcite crystal splits the incident ray of light into two refracted rays called *ordinary ray* and *extraordinary ray.* This phenomenon is called *double refraction.*

The tourmaline crystal is also a doubly refracting crystal with an additional property that is absorbs ordinary and extraordinary rays unequally. Such crystals are called *dichroic crystals* and this phenomenon is called *dichroism* (*selective absorption*).

If θ is the angle between the transmission planes of the wo polaroids, then intensity of light on passing through the econd polaroid is given by

$$I' = I\cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

Since the two polaroids were initially crossed, the angle etween the transmission planes of the two polaroids on otating through 60° will become

$$\theta = 90° - 60° = 30°$$

$$\therefore \qquad I' = \frac{I_0}{2} \times \cos^2 30° = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = 0{\cdot}375\, I_0$$

Hence, percentage of incident light that passes through he system,

$$\frac{I'}{I_0} \times 100 = 0{\cdot}375 \times 100 = \mathbf{37{\cdot}5\%}$$

Problem 7.04. Two polaroids are placed at 90° to each other. What happens, when (N – 1) more polaroids are inserted between them ? Their axis are equally spaced. How does the transmitted intensity behave for large N ?

Sol. When (N – 1) poleroids are inserted between two polaroids, the total number of polaroids becomes (N + 1). The axes of all the polaroids are equally spaced. If θ is angle between the axes of two conservative polaroids, then

$$\theta + \theta + \theta \text{ .... N times} = 90° \quad \text{or} \quad N\theta = \frac{\pi}{2}$$

$$\text{or} \qquad \theta = \frac{\pi}{2N}$$

According to Malus law, the intensity of light on passing through a pair of polaroid is proportional to $\cos^2\theta$. Before the light passes out of the last polaroid, this change in intensity will be repeated N times. If I_0 is intensity of the incident light and I, the intensity of light after passing through all the polaroids, then

$$I = I_0 (\cos^2\theta)^N = I_0 (\cos\theta)^{2N}$$

$$\text{or} \qquad I = I_0 \left(\cos\frac{\pi}{2N}\right)^{2N}$$

When N is very large, angle $\theta \left(= \frac{\pi}{2N}\right)$ approaches zero and hence $\cos\frac{\pi}{2N}$ approaches 1. Therefore, when N is very large, I will approach I_0.

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 7.01. Sound waves cannot be polarised. Why ?

Ans. It is because, sound waves are longitudinal in nature.

Q. 7.02. What evidence is there that sound is not electromagnetic in nature ? (C.B.S.E. 1995)

Ans. Because, sound waves can not be polarised.

Q. 7.03. Will ultrasonic waves show any polarisation ? Give reasons for your answer. (I.S.C.E. 1995)

Ans. No. It is because, ultrasonic waves are longitudinal in nature.

Q. 7.04. What information does one obtain from polarisation about the nature of light ?

Ans. Only a transverse wave motion can be polarised. Since light waves can be polarised, they must be transverse in nature.

Q. 7.05. Can electromagnetic waves be polarised ?

Ans. Yes.

Q. 7.06. Does the value of wavelength of light have any role in polarisation ?

Ans. No. The electromagnetic waves of all wavelengths can be polarised.

Q. 7.07. What is plane polarised light ? (I.S.C.E. 1997 ; C.B.S.E. 1992)

Ans. The light in which the vibrations of the light (vibrations of electric field vector) are restricted in a particular plane is called plane polarised light.

Q. 7.08. Which plane is defined as the plane of polarisation in a plane polarised electromagnetic wave ? (I.S.C.E. 1998)

Ans. The plane perpendicular to the plane of vibrations in a plane polarised light is called plane of polarisation.

Q. 7.09. Can you detect by the naked eye, whether a given light is polarised or not ?

Ans. No.

Q. 7.10. Define the polarisation angle for polarisation by reflection. (P.S.S.C.E. 2002)

Ans. *The angle of incidence, at which when ordinary light is incident on transparent refracting medium, the reflected light becomes maximum rich in plane polarised light, is called polarising angle.*

Q. 7.11. State Brewster's law.
(H.S.S.C.E. 2000, 1998 S, 1996 ; P.S.S.C.E. 1998 S ; I.S.C.E. 1996, 1995)

Ans. Refer to section 7.05.

Q. 7.12. State the law connecting the angle of polarisation with the refractive index of a medium.

Ans. Brewster's law connects the angle of polarisation and refractive index of the medium.

For statement, refer to section 7.05.

Q. 7.13. A ray of light is incident on a medium at polarising angle. What is the angle between the reflected and refracted rays ? (I.S.C.E. 1997)

Ans. 90°.

Q. 7.14. When light is polarised by reflection, what is the plane of vibration of the electric field vector in polarised light ?

Ans. The electric field vector is perpendicular to the plane of incidence.

Q. 7.15. The polarising angle of a medium is 60°. What is the refractive index of the medium ? (C.B.S.E. 2000, 1999)

Or

A ray of light strikes a glass plate at an angle of incidence 60°. If the reflected and refracted rays are perpendicular to each other, find the index of refraction of glass.

Ans. When the reflected ray and the refracted rays are perpendicular to each other, then angle of incidence is equal to polarising angle.

Therefore, $\mu = \tan p = \tan 60° = \sqrt{3}$

Q. 7.16. A ray of light falls on a transparent slab of $\mu = \sqrt{3}$. What is the value of angle of incidence, if the refracted light is completely polarised ? (*P.S.S.C.E. 1999*)

Or

What is the polarising angle of a medium of refractive index $\sqrt{3}$? (*C.B.S.E. 2000, 1999*)

Ans. Refer to VSQ 7.15.

Here, $i = p = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = \mathbf{60°}$

Q. 7.17. What is a polaroid ? (*P.S.S.C.E. 2002, 2000 ; I.S.C.E. 1995*)

Ans. A polaroid is a thin and large sheet of a crystalline polarising material capable of producing plane polarised beams of light of large cross-section.

Q. 7.18. Write simple uses of polaroids. (*P.S.S.C.E. 1999 S ; H.S.S.C.E. 1997*)

Ans. Polaroids are used in sunglasses, in wind shields of automobiles, in window panes, in making three dimensional motion pictures, etc.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 7.01. Differentiate between polarised and unpolarised light. How are these represented ? (*P.S.S.C.E. 1997*)

Ans. For distinction between polarised and unpolarised light, refer to section 7.01.

For representation, refer to Fig. 7.05 (for unpolarised light) and Fig. 7.07 (*a*) or (*b*) (for plane polarised light).

Q. 7.02. Explain what is meant by : (*i*) plane polarised light, (*ii*) polarising angle. (*I.S.C.E. 1995*)

Ans. (*i*) Refer to section 7.02. (*ii*) Refer to section 7.04.

Q. 7.03. How can one distinguish between an unpolarised light beam and a linearly polarised beam using a polaroid ? (*C.B.S.E. 1996*)

Ans. When a polaroid is rotated in the path of unpolarised light, the intensity of the light transmitted from the polaroid remains undiminished. However, when the polaroid is rotated in the path of plane polarised light, its intensity will vary from maximum (when the vibrations of the plane polarised light are parallel to the axis of the polaroid) to minimum (when the direction of vibrations becomes perpendicular to the axis of the crystal).

Q. 7.04. Explain polarisation by reflection. (*Pre-degree Kerala, 1992*)

Ans. Refer to section 7.04.

Q. 7.05. Draw a sketch showing the incident, reflected and transmitted rays, when light is incident at the polarizing angle on a glass slab. (*I.S.C.E. 1994*)

Ans. Refer to Fig. 7.10.

Q. 7.06. Define (*i*) critical angle (*ii*) polarising angle. What is the relation between the two ? (*P.S.S.C.E. 1999 S, 1999, 1996 S*)

Ans. ***Critical angle (C).*** It is the angle of incidence in denser medium, at which the ray of light after undergoing refraction at the surface of separation of the two media, becomes parallel to the surface of separation.

Polarising angle (p). Refer to section 7.04.

The relation between the polarising angle and the critical angle is given by

$$\tan p = \frac{1}{\sin C}$$

Q. 7.07. The critical angle between a given transparent medium and air is denoted by C. A ray of light in air medium enters this transparent medium at an angle of incidence equal to polarising angle *p*. Deduce a relation for the angle of refraction in terms of C. (*I.S.C.E. 1998*)

Ans. According to Brewster's law, when a ray of light is incident at polarising angle,

$$\mu = \tan p \qquad \ldots(i)$$

If C is critical angle for the transparent medium and air, then

$$\mu = \frac{1}{\sin C} \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$\tan p = \frac{1}{\sin C} \qquad \ldots(iii)$$

If *r* is angle of refraction, then

$$P = 90° - r$$

or $$\tan p = \tan(90° - r) = \cot r = \frac{1}{\tan r} \qquad \ldots(iv)$$

From equations (*iii*) and (*iv*), we have

$$\tan r = \sin C \quad \text{or} \quad r = \tan^{-1}(\sin C)$$

Q. 7.08. Draw the graph showing the variation of intensity of polarised light transmitted by the analyser. (*C.B.S.E. 2001*)

Or

Sketch the graph showing the dependence of intensity of transmitted light on the angle of rotation between a polariser and an anlyser. (*C.B.S.E. 1998 S*)

Ans. For sketch of the graph, refer to Fig. 7.12.

Q. 7.09. Name one device for producing polarised light. Draw a graph showing the dependence of intensity of transmitted light on the angle between polariser and analyser. (*C.B.S.E. 1991*)

Ans. Nicol prism can be used to produce plane polarised light. For graph, refer to Fig. 7.12.

Q. 7.10. What are polaroids ? Explain their one use. (*P.S.S.C.E. 2000, 1998*)

Ans. Refer to section 7.08.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 7.01. What is the basic difference between a source of light and a source of radio waves ?

Ans. In a visible light source, the atoms act as oscillators producing oscillations independent of each other. Due to this, oscillations produced are randomly oriented (unpolarised).

In a radio wave source, the charges oscillate in a plane perpendicular to the direction of propagation of radio waves (polarised).

Q. 7.02. In light waves, the vibrations are perpendicular to the direction of propagation of light. But it is said that light requires no material medium for its pro-pagation, then what vibrates transversely in light waves ?

Ans. Light is electromagnetic in nature and it travels in the form of transverse varying electric and magnetic fields. The optical phenomena are described by the electric field vector of the electromagnetic waves. Hence, the electric field vector vibrates transversely in light waves.

Q. 7.03. Does the value of polarising angle depend upon colour (wavelength) of incident light ?

Ans. The refractive index of a material depends on the wavelengths of the light. As polarising angle depends on the refractive index, it also depends on wavelength of the light.

Q. 7.04. How do sunglasses reduce the glare of intense light ? Explain the physical principle involved.

Ans. The sunglasses are made of polaroids. When ordinary light passes through sunglasses, the ordinary light is rendered into plane polarised light. Due to this, the glare of the intense light gets reduced.

Q. 7.05. Define elliptically polarised light.

(C.B.S.E. 1992)

Ans. When two plane polarised waves are superimposed, the resultant light vector rotates in a plane perpendicular to the direction of propagation. If the resultant light vector has constant magnitude, then the tip of the light vector traces a circle and the light is said to be circularly polarised. However, if the magnitude of the resultant light vector varies periodically during its rotation, the tip of the light vector traces an ellipse and the light is said to be elliptically polarised.

Q. 7.06. What do you mean by the term crossed polaroids ?

Ans. Two polaroids are said to be crossed, when the planes of polarisation of the two polaroids are perpendicular to each other.

Q. 7.07. Two polaroids are placed at 90° to each other and the transmitted intensity is zero. What happens, when one more polaroid is placed between these two, bisecting the angle between them ?

Ans. When polaroids are placed as stated, the electric field is reduced by a factor of cos 45° and again by a factor of cos 45° by the second and third polaroids. In other words, the electric field gets reduced by a factor of $\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$.

Since, intensity of light waves varies as the square of the magnitude of electric field vector, the intensity of light gets reduced by a factor of $\left(\frac{1}{2}\right)^2$ *i.e.* $\frac{1}{4}$.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What do you understand by polarisation of light ? Describe an experiment to demonstrate transverse nature of light. *(H.P.S.S.C.E. 2001)*
2. Describe an experiment to show that light waves are transverse in nature. *(H.S.S.C.E. 1999 ; C.B.S.E. 1999, 1994, 1993)*
3. A ray of light is incident at an angle of incidence p on the surface of separation between the air and a medium of refractive index μ, such that the angle between the reflected and refracted rays is 90°. Obtain the relation between p and μ. *(C.B.S.E. 1998 S)*
4. What is the phenomenon of polarisation ? Derive the relation connecting the polarising angle of a medium and its refractive index. *(C.B.S.E. 2002)*
5. What is plane polarised light ? Explain a method of producing plane polarised light. *(Karnataka,1994)*
6. Give a method for producing a beam of plane polarised light. Show how you will detect the presence of plane polarised light. Give one practical use of polarised light. *(I.S.C.E. 1996)*
7. State and prove Brewster's law of polarisation. *(H.S.S.C.E. 2002 ; P.S.S.C.E. 2001, 2000, 1999)*
8. State Brewster's law. Show that the reflected ray and the refracted ray are perpendicular to each other, when the angle of incidence is equal to polarising angle.
9. Show that when a light ray is incident at the polarising angle, the reflected ray is at right angles to the refracted ray. *(I.S.C.E. 1995 ; Karnataka, 1995, 1993)*
10. What is a plaroid ? How is plane polarised light obtained with its help ? How will you use it to distinguish between unpolarised light and plane polarised light ? *(C.B.S.E. 2001 S)*
11. What are polaroids ? Write their four uses. *(P.S.S.C.E. 2001)*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. (*a*) Describe an experiment to demonstrate the transverse nature of light.
   (*b*) Differentiate between polarised and unpolarised light. How are they represented ? *(P.S.S.C.E. 2001)*
2. What do you understand by polarisation of light ? What information you get about the nature of light from the study of polarisation ? How would you obtain plane polarised light by reflection ? What do you mean by plane of polarisation and plane of vibration ?
3. What do you mean by polarisation of light ? How polarised and unpolarised light is represented ? How will you prove that light is transverse wave in nature ? *(P.S.S.C.E. 2002)*

FAQs FREQUENTLY ASKED NUMERICAL QUESTIONS

For Practice

Type A. On Brewster's law

1. Refractive index of water is 1·33. Calculate the angle of polarisation for light reflected from the surface of a lake. **[Ans. 53·06°]**
2. The refractive index of a medium is $\sqrt{3}$. What is the angle of refraction, if the unpolarised light is incident on it at the polarising angle of the medium ? *(C.B.S.E. 2002)* **[Ans. 30°]**

3. A ray of light is incident on the surface of glass plate of refractive index 1·5 at the polarising angle. What is the angle of refraction ? (*P.S.S.C.E. 1995*) [**Ans.** 33° 41′]
4. A ray of light strikes a glass plate at an angle of incidence 57°. If the reflected and refracted rays are perpendicular to each other, find the index of refraction of glass. [**Ans.** 1·54]

Type B. On Law of Malus

5. Two polaroids are oriented with their planes of transmission making an angle of 30° with that of preceeding sheet. What fraction of incident unpolarised light is transmitted ? [**Ans.** 37·5%]
6. An unpolarised beam of light is incident on a group of four polarising sheets, which are arranged in such a way that the characteristic direction of each polarising sheet makes an angle of 30° with that of the preceeding sheet. What fraction of light is transmitted ? $\left[\textbf{Ans. } \dfrac{27}{128}\right]$

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $\tan p = \mu = 1{\cdot}33$ or $p = 53{\cdot}06°$
2. Proceed as in solved problem no. 7.01.
3. Proceed as in solved problem no. 7.01.
4. Since reflected and refracted rays are perpendicular,

$$p = i = 57°$$

$$\therefore \quad \mu = \tan p = \tan 57° = 1{\cdot}54$$

5. Proceed as in solved problem no. 7.03.
6. As obtained in solved problem no. 7.03, the intensity of unpolarised light transmitted through the polarising sheets arranged with their planes of transmission making an anlge of 30° is given by

$$I' = \frac{I_0}{2}\cos^2 30° = \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{I_0}{2}\times\frac{3}{4}$$

Therefore, intensity of transmitted light through the third polarising sheet,

$$I'' = I'\cos^2 30° = \frac{I_0}{2}\times\frac{3}{4}\times\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{I_0}{2}\times\left(\frac{3}{4}\right)^2$$

Finally, intensity of light transmitted through the fourth polarising sheet,

$$I = I''\cos^2 30° = \frac{I_0}{2}\times\left(\frac{3}{4}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{I_0}{2}\times\left(\frac{3}{4}\right)^3$$

$$= \frac{27\,I_0}{128}$$

$$\text{or} \quad \frac{I}{I_0} = \frac{27}{128}$$

Competitive Examination File (UnitVI)

REVISION AT A GLANCE

Chapter 1. Refraction of Light

☐ **Refraction.** *The phenomenon of change in path of light as it goes from one medium to another is called refraction.*

☐ **Laws of refraction. 1.** *The incident ray, the normal to the refracting surface of the point of incidence and the refracted ray all in the same plane.*

2. *The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for any two given media.* It is called *ell's law.*

Mathematically : $\frac{\sin i}{\sin r} = {}^a\mu_b$

Here, ${}^a\mu_b$ is called ***relative refractive index*** of medium b (in which refracted ray travels) *w.r.t.* medium a (in which incident y travels).

☐ **Absolute refractive index.** *The absolute refractive index of a medium is defined as the ratio of the velocity of light in vacuum) to the velocity of light in the medium (v).*

Mathematically : Absolute refractive index, $\mu = \frac{c}{v}$

☐ **Lateral shift.** *The perpendicular distance between the incident and the emergent rays, when the light is incident obliquely on a rallel sided slab of a refracting material is called lateral shift.*

Mathematically : Lateral shift, $d = \frac{t}{\cos r} \sin (i - r)$

☐ **Refraction from a compound plate.** When a compound plate of two refracting slabs of materials b and c (each havin arallel refracting surfaces) is placed in a medium a, then the emergent ray is always parallel to the incident ray and th efractive indices of the three media are related to each other as below :

$${}^a\mu_b \times {}^b\mu_c = {}^a\mu_c$$

☐ **Real and apparent depth.** When an object is placed in an optically denser medium, the apparent depth of the object s always less than its real depth.

Mathematically : 1. $\frac{\text{Real depth}}{\text{Apparent depth}} = {}^a\mu_b$

2. Normal shift, $d = t\left(1 - \frac{1}{{}^a\mu_b}\right)$

☐ **Total internal reflection.** *The phenomenon of reflection of light that takes place when a ray of light travelling in a denser medium ets incident at the interface of the two media at an angle greater than the critical angle for that pair of media.*

Mathematically : ${}^a\mu_c = \frac{1}{\sin C}$

Here, ${}_a\mu^b$ is refractive index of the denser medium b *w.r.t.* the rarer medium a and C is critical angle.

☐ **Spherical refracting surface.** *The portion of a refracting medium, whose curved surface forms the part of a sphere, is called pherical refracting surface.*

When object is situated in the rarer medium, the relation between μ_1 (refractive index of rarer medium), μ_2 (refractive ndex of the spherical refracting surface) and R (radius of curvature) with the object and image distances is given by

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

When the object is situated in denser medium, the relation between μ_1, μ_2 R, u and v can be obtained by interchanging $_1$ and μ_2. In that case, the relation becomes

$$-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Power of spherical refracting surface : $P = \frac{\mu_2 - \mu_1}{R}$

Here, R is measured in **metre.**

☐ **Lens maker's formula.** The relation connecting the focal length of the lens with the radii of curvature of its two surfaces and the refractive index of the material of the lens is called lens maker's formula.

Mathematically : $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

The focal length of a convex lens is taken as positive, while that of concave lens is taken as negative.

☐ **Lens equation.** The relation between the focal length, the object and image distances is called lens equation.

Mathematically : $-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

☐ **Linear magnification.** *The ratio of the image (formed by the lens) to the size of the object is called linear magnification produced by the lens.*

Mathematically : $m=\frac{I}{O}=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$

☐ **Power of a lens.** *It is defined as the reciprocal of the focal length of the lens in metre.*

Mathematically : 1. $P=\frac{1}{f}$

2. $P=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

In the above two formulae ; f, R_1 and R_2 are measured in **metre.**

☐ **Two thin lenses placed in contact.** When two lenses of focal lengths f_1 and f_2 are placed in contact, the focal length of the combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

Power of the equivalent lens : $P=P_1+P_2$

Magnification produced by equivalent lens : $m=m_1\times m_2$

☐ **Spherical aberration.** *The inability of a lens of large aperture to bring all the rays in a wide beam of light falling on it to focus a single point is called spherical aberration.*

Some Useful Facts :

1. Snell's law fails, when light is incident normally on a refracting medium.
2. Lateral shift is maximum (equal to thickness of refracting slab), when the angle of incidence is 90°.
3. The total internal reflection takes place only when the incident ray travelling in denser medium falls on the interface of the two media at an angle of incidence greater than the critical angle.
4. The power of a spherical refracting surface or a lens gives the measure of the degree to which it can converge or diverge the rays of light passing through it.
5. The focal length and power of a spherical refracting surface of a lens depend upon the refractive index of the surrounding medium also in addition to the refractive index of the material and the radii of curvature of its two surfaces.

Chapter 2. Dispersion

☐ **Refraction through a prism.** *A prism is the portion of a transparent refracting medium bounded by two plane surfaces meeting each other along a straight edge.*

A ray of light incident on one face of the prism suffers refraction successively at the two surfaces and then emerges out of it.

Mathematically : 1. $A=r_1+r_2$

2. $A+\delta=i+e$

3. When prism is placed in minimum deviation position,

$$\mu=\frac{\sin(A+\delta_m)/2}{\sin A/2}$$

4. When angle of prism is small, $\delta=A(\mu-1)$

☐ **Dispersion.** *The phenomenon of splitting up of white light into its constituent colours is called dispersion.*

Spectrum. *The band of seven colours obtained on the screen is called spectrum.*

Pure spectrum. *A spectrum, in which the constituent colours have sharp boundaries and are distinctly visible, is called the pure spectrum.*

☐ **Angular dispersion.** *The angular dispersion for any two colours is defined as the difference in deviations suffered by the two colours in passing through the prism.*

Mathematically : Angular dispersion, $\theta=\delta_v-\delta_r$

☐ **Dispersive power.** *The dispersive power of the material of a prism for any two colours may be defined as the ratio of angular dispersion for these two colours to the deviation suffered by the mean light.*

Mathematically : Dispersive power, $\omega = \dfrac{\delta_v - \delta_r}{\delta} = \dfrac{\mu_v - \mu_r}{\mu - 1}$

☐ **Dispersion without deviation.** Two prisms of suitable materials and angles of prism can be combined, so as to produce spersion without deviation.

Condition for no deviation : $\dfrac{A'}{A} = -\dfrac{\mu - 1}{\mu' - 1}$

$$\text{Net angular dispersion} = \delta\,(\omega - \omega')$$

☐ **Deviation without dispersion.** Two prisms of suitable materials and angles of prism can also be combined, so as to oduce deviation without dispersion.

Condition for no dispersion : **1.** $\dfrac{A'}{A} = -\dfrac{\mu_v - \mu_r}{\mu_v{}' - \mu_r{}'}$

2. $\dfrac{\omega}{\omega'} = -\dfrac{\delta'}{\delta}$

$$\text{Net deviation} = \delta\left(1 - \frac{\omega}{\omega'}\right)$$

☐ **Chromatic aberration.** *The inability of a lens to bring the light of different colours to focus at a single point is called chromatic erration.*

Mathematically : Chromatic aberration, $f_r - f_v = \omega \times f$

☐ **Spectrometer.** *It is an optical instrument to obtain and study the pure spectrum.*

☐ **Rayleigh's law of scattering.** *It states that the intensity of the light of wavelength λ in the scattered light varies inversely as e fourth power of its wavelength.*

☐ **Rainbow.** *The coloured arcs (or the spectrum of white light from the sun in the form of bows), seen immediately after the rain an observer with his back towards the sun, are called rainbows.*

ome Useful Facts :

1. A prism always deviates a ray of light towards its base. However, the ray of light is deviated away from the base, hen the prism is placed inside a denser medium.

2. The white light gets dispersed, while passing through a prism due to the fact that white light consists of light of different avelength.

3. When two prisms are combined to produce dispersion without deviation, the order of the colours in the spectrum ue to the combination is opposite to that due to the crown glass prism.

4. The chromatic aberration in a lens is because of the fact that it has different focal lengths for light of different colours.

5. The line emission spectrum is due to atoms, while the band emission spectrum is due to the molecules of an element.

6. The pure spectrum can be obtained by placing the prism in minimum deviation postion.

7. When light from the sun traverses the atmosphere, the scattering of the blue light is approximately sixteen times as rge as that of red light.

Chapter 3. Optical Instruments

☐ **Simple microscope.** A convex lens of small focal length is called a simple microscope or a magnifying glass.

The magnifying power of a microscope is defined as the ratio of the angle subtended by the image at the eye and the angle subtended y the object seen directly, when both lie at the least distance of distinct vision.

Mathematically : $M = \left(1 + \dfrac{D}{f}\right)$

Here, D is the least distance of distinct vision.

☐ **Compound microscope.** A compound microscope is a two lens system (object lens and eye lens of focal lengths f_o nd f_e). Its magnifying power is very large, as compared to the simple microscope.

Mathematically : $M = \dfrac{v_o}{u_o}\left(1 + \dfrac{D}{f_e}\right) = -\dfrac{L}{f_o}\left(1 + \dfrac{D}{f_e}\right)$

Here, u is distance of the object from the object lens and v ($\approx$ L, the length of the tube of microscope) is the distance at which the object lens forms the image of the object.

☐ **Astronomical telescope.** It is a two lens system and is used to observe distant heavenly objects. It is called refracting ype astronomical telescope.

☐ **Normal adjustment.** *When the final image is formed at infinity, the telescope is said to be in normal adjustment.*

Magnifying power of a telescope in normal adjustment is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object seen directly, when both the object and the image lie at infinity.

Magnifying power in normal adjustment, $M = -\frac{f_o}{f_e}$

When the final image is formed at least distance of distinct vision,

magnifying power of the telescope, $M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

☐ **Reflecting type telescope.** In a reflecting type telescope, the objective is a concave spherical mirror of large apertur in place of a convex lens.

The expression for magnifying power of a reflecting type telescope is same as that for refracting type astronomica telescope.

☐ **Prism binoculars.** It is a system of two astronomical telescopes making use of object lenses of very large focal length As such, the length of the telescopes would become very large. However, the length of the telescope tubes is cut short b making use of totally reflecting prisms.

Some Useful Facts :

1. The optical instruments are used as an aid to the human eye so as to produce higher magnification, greater resolvin power and to produce the image of close lying objects at least distance of distinct vision.
2. Eye is a natural optical instrument. It has a unique feature of automatic adjustment of its aperture and focussing.
3. The magnifying power and magnification produced by a lens are different from each other.
4. The magnifying power of a telescope is perceived as the increase in the visual angle of the image.

Chapter 4. Huygens' Principle

☐ **Wavefront.** *The locus of the points in the medium, which at any instant are vibrating in the same phase, is called wavefront Ray of light is the line drawn perpendicular to the wave front.*

☐ **Huygens' Principle. 1.** *Each point on a given (or primary) wave front acts as a source of secondary wavelets, sending ou disturbance in all directions in a similar manner as the original source of light does.*

2. *The new position of the wave front at any instant (called secondary wave front) is the envelope of the secondary wavelets at tha instant.*

It is also called ***Huygens' construction*** and it is used to find the new position of the wavefront at a later time.

Some Useful Facts :

1. The time taken for light to travel from one wave front to another is the same along any ray.
2. The contribution of a wavelet in any direction making angle θ with the normal to the wavelet is proportional t $\frac{1}{2}(1+\cos\theta)$. Since for a wavelet, which lies on the back of the secondary wavefront, θ is 180°, the contribution from such wavelet is $\frac{1}{2}(1+\cos 180°) = 0$.

Chapter 5. Interference of Light

☐ **Interference.** *The phenomenon of non-uniform distribution of energy in the medium due to superposition of two light waves called interference of light.*

☐ **Coherent sources.** *Two sources are said to be coherent, if they emit light waves of same wavelength (or frequency) and of stable phase difference.*

☐ **Young's double slit experiment.** Consider that two coherent sources are separated by a distance d so as to produc interference fringes on a screen held at a distance D from the plane of the slits. When the slits are illuminated with monochromatic light of wavelength λ, then on the two sides of the central bright fringe, alternate dark and bright fringe are formed. Let ϕ be the phase difference and x be the path difference between the two light waves reaching a point P on th screen.

1. ***Condition for maximum intensity*** **at point P :**

Phase difference, $\phi = 2n\pi$ or path difference, $x = n\lambda$, where $n = 0, 1, 2$ ....

2. ***Condition for minimum intensity*** **at point P :**

Phase difference, $\phi = (2n+1)\pi$ or path difference, $x = (2n+1)\frac{\lambda}{2}$ where $n = 0, 1, 2$ ....

3. ***Ratio of the maximum and minimum intensity :*** $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$

Here, a_1 and a_2 are amplitudes of the two light waves.

Also, ratio of intensity of light due to the two slits of widths w_1 and w_2 is given by

$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}$$

4. (a) *Distance of nth bright fringe from the centre of the screen :* $y_n = \frac{n D \lambda}{d}$

(b) *Angular position of the nth bright fringe :* $\theta_n = \frac{y_n}{D} = \frac{n\lambda}{d}$

5. (a) *Distance of nth dark fringe from the centre of the screen :* $y_n' = \frac{(2n+1) D\lambda}{2d}$

(b) *Angular position of the nth dark fringe :* $\theta_n' = \frac{y_n'}{D} = \frac{(2n+1)\lambda}{2d}$

6. *Fringe width :* $\beta = \frac{D\lambda}{d}$

me Useful Facts :

1. The intensity of light is directly proportional to the square of the amplitude of the wave.
2. In Young's double slit experiment, the bright and dark fringes are of equal widths.
3. The fringe width decreases, when Young's double slit experiment is immersed in a liquid.
4. The colour of thin films is due to interference between the two beams of light ; one reflected from the upper surface the film and the other emerging after reflection from the lower surface of the film.

Chapter 6. Diffraction of Light

☐ **Diffraction.** *The phenomenon of bending of light round the sharp corners and spreading into the regions of the geometrical shadow called diffraction.*

☐ **Diffraction from a slit.** A narrow slit of width a is placed at a distance D from the screen. When the slit is illuminated th a monochromatic light of wavelength λ, then alternate bright and dark bands of light are formed on both the sides of e central maximum.

1. (a) *Angular position of the nth secondary minimum :* $\theta_n = \frac{n\lambda}{a}$

(b) *Distance of the nth secondary maximum from the centre of the screen :* $y_n = \frac{n D\lambda}{a}$

2. (a) *Angular positions of the nth secondary maximum :* $\theta_n' = \frac{(2n+1)\lambda}{2a}$

(b) *Distance of the nth secondary maximum from the centre of the screen :* $y_n' = \frac{(2n+1) D\lambda}{2a}$

3. (a) *Width of a secondary maximum or minimum :* $\beta = \frac{D\lambda}{a}$

(b) *Width of the central maximum :* $\beta_0 = \frac{2D\lambda}{a}$

4. ***Half angular width of central maximum.*** The angular position of first secondary minimum is known as half angular idth of the central maximum. Thus,

$$\text{half angular width of central maximum}, \ \theta_1 = \frac{\lambda}{a}$$

5. ***Fresnel distance.*** *It is defined as the distance of the screen from the slit beyond which, the spreading of light due to diffraction ecomes quite large as compared to the size of the slit.* It is denoted by Z_F.

Mathematically : $Z_F = \frac{a^2}{\lambda}$

6. **Limit of resolution.** *The smallest separation (linear or angular) between the two point objects at which they appear just eparated, is called the limit of resolution.*

Linear limit of resolution of a microscope : $d = \frac{\lambda}{2\mu \sin\theta}$

Here, θ is semi-vertical angle of the cone formed by the object at the objective of the microscope and μ is the refractive ndex of the medium between the object and the objective.

Angular limit of resolution of a telescope : $d\theta = \frac{1 \cdot 22\,\lambda}{D}$

Here, D is diameter of the objective of the telescope.

7. **Resolving power.** *It is defined as the reciprocal of the limit of resolution.*

Resolving power of a microscope, $\frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$

Resolving power of a telescope, $\frac{1}{d\theta} = \frac{D}{1{\cdot}22\,\lambda}$

Some Useful Facts :

1. Diffraction pattern is the result of the interaction of light from the different parts of the same wavefront.
2. In diffraction pattern, the central maximum is twice as wide as any other secondary maximum.
3. All the secondary maxima are not of same intensity.

Chapter 7. Polarisation of Light

☐ **Polarisation.** *The phenomenon due to which vibrations of light are restricted in a particular plane is called polarisation.*

☐ **Plane polarised light.** *It is the light in which the vibrations of the light (vibrations of the electric vector) are restricted i particular plane.*

☐ **Plane of polarisation.** *The plane perpendicular to the plane containing the vibrations of plane polarised light is called plane polarisation.*

☐ **Brewster's law.** It states that *when light is incident at polarising angle, the reflected and refracted rays are perpendicular each other.*

Mathematically : $\mu = \tan p$

Here, p is the polarising angle for a medium of refractive index μ.

☐ **Law of Malus.** It states that *when a completely plane polarised light beam is incident on an analyser, the intensity of the emerge light varies as the square of the cosine of the angle between the planes of transmission of the analyser and the polariser.*

Mathematically : $I = I_0 \cos^2 \theta$

Here, I_0 is the intensity of incident plane polarised light and θ is the angle between the planes of transmission of analys and the polariser.

☐ **Polaroid.** *The thin and large sheets of crystalline polarising material capable of producing plane polarised beams of light of lar cross-section.*

☐ **Optical activity.** *The property of a substance of rotating the plane of vibration of the plane-polarised light, when passed throu it, is called optical activity.*

☐ **Specific rotation.** *The specific rotation of an optically active solution is numerically equal to the amount of rotation produc by the solution of length 1 m and concentration 1 kg m^{-3}.* It is denoted by S.

Mathematically : $S = \frac{\theta}{l\,c}$

Here, l, c and θ stand for length of the solution, concentration of the solution and the rotation produced in the plane polarisation of the light by the solution.

Some Useful Facts :

1. The naked eye cannot distinguish between ordinary light and the plane polarised light.
2. When a polariser is rotated in the path of ordinary light, the intensity of light remains undiminished. The intensity light transmitted from the polariser changes, in case the light falling on the polariser is plane polarised light. As such, t polariser is said to be acting as an analyser.

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Refraction of Light

Problem 1.01. The XY-plane is the boundary between two transparent media [Fig. 1.01]. Medium-I with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium-II with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium-I given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\,\hat{k}$ is incident on the plane separation. Find the unit vector in the direction of th refracted ray in medium-II. *(I.I.T. 199*

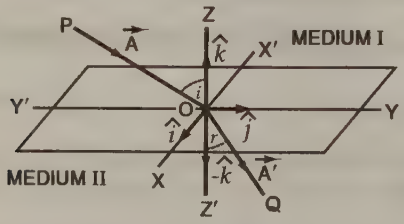


Fig. 1.01

Thought Process

1. *The angle of incidence (angle between incident r and Z-axis is given by)*

$$\cos i = \frac{\vec{A}\cdot(-\hat{k})}{|\vec{A}|}$$

Here, $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\,\hat{k}$

2. If r is the angle of refraction, then

$$\frac{\sin i}{\sin r}=\frac{\sqrt{3}}{\sqrt{2}}$$

3. Suppose that the refracted ray is represented by the vector

$$\vec{A}'=a\,\hat{i}+b\,\hat{j}+c\,\hat{k}$$

On refraction, the plane containing the refracted ray and the normal is the same plane, which contains the incident ray and the normal (only the inclination of the path of ray with the normal i.e. Z-axis changes). Therefore the values of a and b in vector $\vec{A}'$ must be the same as those in vector $\vec{A}$ i.e. $a=6\sqrt{3}$ and $b=8\sqrt{3}$. Hence, the vector representing the refracted ray is

$$\vec{A}'=6\sqrt{3}\,\hat{i}+8\sqrt{3}\,\hat{j}+c\,\hat{k}$$

4. The angle of refraction is given by

$$\cos r=\frac{\vec{A}'(-\hat{k})}{|\vec{A}'|}$$

Sol. The vector representing incident ray,

$$\vec{A}=6\sqrt{3}\,\hat{i}+8\sqrt{3}\,\hat{j}-10\,\hat{k}$$

$$\therefore\quad |\vec{A}|=\sqrt{(6\sqrt{3})^2+(8\sqrt{3})^2+(-10)^2}$$

$$=\sqrt{108+192+100}=20$$

Now, $$\cos i=\frac{\vec{A}.(-\hat{k})}{|\vec{A}|}$$

$$=\frac{(6\sqrt{3}\,\hat{i}+8\sqrt{3}\,\hat{j}-10\,\hat{k}).(-\hat{k})}{20}$$

$$=\frac{0+0+10}{20}=\frac{1}{2}$$

or $$i=60^\circ$$

From Snell's law, $$\frac{\sin i}{\sin r}=\frac{\sqrt{3}}{\sqrt{2}}$$

or $$\sin r=\frac{\sqrt{2}}{\sqrt{3}}\sin i=\frac{\sqrt{2}}{\sqrt{3}}\sin 60^\circ=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{2}$$

or $$\sin r=\frac{1}{\sqrt{2}}\quad\text{or } r=45^\circ$$

Now, the vector representing the refracted ray is given by

$$\vec{A}'=6\sqrt{3}\,\hat{i}+8\sqrt{3}+c\,\hat{k}$$

and $$\cos r=\frac{\vec{A}'(-\hat{k})}{|\vec{A}'|}$$

$$\therefore\quad \cos 45^\circ=\frac{(6\sqrt{3}\,\hat{i}+8\sqrt{3}\,\hat{j}+c\,\hat{k}).(-\hat{k})}{\sqrt{(6\sqrt{3})^2+(8\sqrt{3})^2+c^2}}$$

$$=\frac{0+0-c}{\sqrt{108+192+c^2}}$$

or $$\frac{1}{\sqrt{2}}=\frac{-c}{\sqrt{300+c^2}}\quad\text{or}\quad\frac{1}{2}=\frac{c^2}{300+c^2}$$

or $$c=\sqrt{300}=\pm 10\sqrt{3}$$

Since the refracted travels downwards, we choose

$$c=-10\sqrt{3}$$

$$\therefore\quad \vec{A}'=6\sqrt{3}\,\hat{i}+8\sqrt{3}\,\hat{j}-10\sqrt{3}\,\hat{k}$$

Also, $$|\vec{A}'|=\sqrt{(6\sqrt{3})\,\hat{i}+(8\sqrt{3})\,\hat{j}\,(-10\sqrt{3})\,\hat{k}}$$

$$=\sqrt{108+142+300}=10\sqrt{6}$$

If $\hat{n}$ is unit vector along refracted ray, then

$$\hat{n}=\frac{\vec{A}'}{|\vec{A}'|}=\frac{6\sqrt{3}\,\hat{i}+8\sqrt{3}\,\hat{j}-10\sqrt{3}\,\hat{k}}{10\sqrt{6}}$$

$$=\frac{3}{5\sqrt{2}}\,\hat{i}+\frac{4}{5\sqrt{2}}\,\hat{j}-\frac{1}{\sqrt{2}}\,\hat{k}$$

Problem 1.02. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1·5 is then placed close to the mirror in the space between the object and the mirror. Find the position of the final image formed. You may take the distance of the near surface of the slab from the mirror to be 1 cm. (I.I.T. 1980)

Thought Process

In the absence of the glass slab, image of the object is formed at distance u, which can be calculated by using mirror formula. When the glass slab is introduced, the reflected ray forming the image gets laterally displaced through a distance,

$$d=t\left(1-\frac{1}{\mu}\right)$$

Therefore, the distance of final image from the pole of the mirror = v + d

---

Sol. Here, $u=-\ 21$ cm ; $f=\frac{R}{2}=\frac{-10}{2}=-5$ cm ;

When the glass slab is not placed : Suppose that the image of the object AB is formed at a distance v from the pole of the mirror. Then,

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-5}=\frac{1}{-21}=-\frac{16}{105}$$

or $$v=\frac{-105}{16}=-6{\cdot}56\text{ cm}$$

The negative sign shows that the image formed is inverted and real. The image A′ B′ will be formed at a distance of6·56 cm from the pole of the mirror as shown in Fig. 1.02.

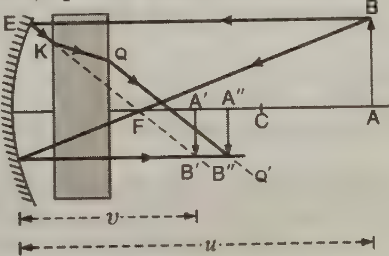


Fig. 1.02.

When the glass slab is introduced : The ray of light reflected from point E will suffer refraction at point K. It will follow the path KQ inside the glass slab and on emerging from the slab, it will travel along QQ′. Since the ray of light reflected from point E gets laterally displaced from path EKB′ to QQ′, the image will shift from A′ B′ to A′B′ *i.e.* through distance A′ A″ ≈ d given by

$$d = t\left(1 - \frac{1}{\mu}\right)$$

Here, $t = 3$ and $\mu = 1{\cdot}5$

$$\therefore \quad d = 3\left(1 - \frac{1}{1{\cdot}5}\right) = 1 \text{ cm}$$

Hence, the distance of final image from the mirror,

$$v + d = 6{\cdot}56 + 1 = \mathbf{7{\cdot}56\ cm}$$

Problem 1.03. A rectangular glass block of thickness 10 cm and refractive index 1·5 is placed over a small coin. A beaker is filled with water of refractive index 4/3 to a height of 10 cm and is placed over a small block.

(*a*) Find the apparent position of the object, when it is viewed at normal incidence.

(*b*) If the eye is slowly moved away from the normal ; at a certain position, the coin is found to disappear due to total internal reflection. At what surface does it happen and why ? (*I.I.T. 1975*)

Sol. Fig. 1.03 shows a glass slab ABCD of thickness t_1 = 10 cm placed over the coin. The beaker PQRS, filled with water to height t_2 = 10 cm, is placed over the glass slab. A ray of light OL from the coin gets refracted away from the normal LN_1 at the glass-water interface AQRD along the path LM. On reaching the water-air interface PS, it is refracted away from the normal MN_2 along MN.

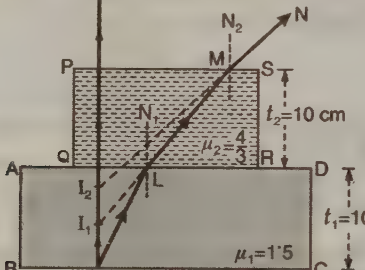


Fig. 1.03.

(*a*) The coin appears to be raised from the position O to I_1 due to normal shift produced due to refraction from glass to water ; and from position I_1 to I_2 due to normal shift produced due to refraction from water to air. If ${}^{w}\mu_g$ is refractive index of glass *w.r.t.* water, then the total normal shift in the position of the coin,

$$OI_2 = OI_1 + I_1 I_2 = t_1\left(1 - \frac{1}{{}^{w}\mu_g}\right) + t_2\left(1 - \frac{1}{\mu_2}\right)$$

$$\text{Now,} \quad {}^{w}\mu_g = \frac{\mu_2}{\mu_1} = \frac{1{\cdot}5}{4/3} = 9/8$$

$$\therefore \quad OI_2 = 10\left(1 - \frac{1}{9/8}\right) + 10\left(1 - \frac{1}{4/3}\right)$$

$$= 1{\cdot}11 + 2{\cdot}5 = 3{\cdot}11 \text{ cm}$$

Therefore, depth of the coin below the water surfac

$$= 20 - 3{\cdot}61 = \mathbf{16{\cdot}39\ cm}$$

(*b*) Of the two interfaces AQRD and PS, that interf will make the coin disappear, for which the value of criti angle is lesser. Let C_1 and C_2 be the values of the critical an for the glass-water interface and water-air interfa respectively.

$$\text{Then,} \quad \sin C_1 = \frac{1}{{}^{w}\mu_g} = \frac{1}{\mu_1/\mu_2} = \frac{\mu_2}{\mu_1} = \frac{4/3}{1{\cdot}5} = 0{\cdot}888$$

$$\text{or} \quad C_1 = 62°47'$$

$$\text{Also,} \quad \sin C_2 = \frac{1}{{}^{a}\mu_w} = \frac{1}{\mu_2} = \frac{1}{4/3} = 0{\cdot}75 \text{ or } C_2 = 48°$$

Since $C_2 < C_1$, the total internal reflection will take pl from the water-air interface first as the eye is slowly mov away. Hence, the coin disappears from the view due to to internal reflection at the water surface.

Problem 1.04. A plano-convex lens has thickness 4 c When placed on a horizontal table with the curved surfa in contact with it, the apparent depth of the bottom-m point of the lens is found to be 3 cm. If the lens is invert such that the plane face is in contact with the table, t apparent depth of the centre of plane face of the lens found to be $\frac{25}{8}$ cm. Find the focal length of the lens.

(*M.N.R. 1990 ; I.I.T. 19*

[Ans. 75 c

Hint. Proceed as in solved problem 1.44–Chapter 1.

Problem 1.05. A quarter cylinder of radius R a refractive index 1·5 is placed on a table. A point object P kept at a distance of *m* R from it. Find the value of *m* which a ray from P will emerge parallel to the table shown in Fig. 1.04. (*I.I.T. 199*

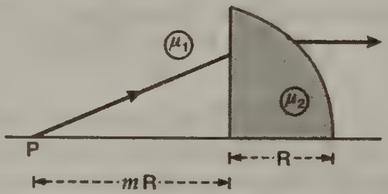


Fig. 1.04.

Sol. Due to refraction from the plane surface of t quarter cylinder, the virtual image I_1 of the object P is form as shown in Fig. 1.05.

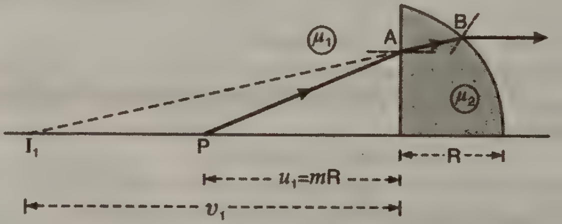


Fig. 1.05.

For the curved surface of the cylinder, I_1 acts as the re object. As the ray emerges parallel to the table, the final ima is formed at infinity.

Refraction from plane surface : Suppose that the ima I_1 is formed at a distance v_1 from the plane surface of t quarter cylinder. As the object is situated in rarer mediu we have

$$-\frac{1}{u_1}+\frac{\mu}{v_1}=\frac{\mu-1}{R_1}$$

Here, $v_1=-m\,R\,;\,R_1=\infty\,;\,\mu=1\cdot5$

$$\therefore\ -\frac{1}{-m\,R}+\frac{1\cdot5}{v_1}=\frac{1\cdot5-1}{\infty}\ \text{ or }\ \frac{1\cdot5-1}{\infty}=-\frac{1}{m\,R}$$

or $v_1=-1\cdot5\,m\,R$

Refraction from curved surface : *As the refracted ray travels in denser medium, we have*

$$-\frac{\mu}{u_2}+\frac{1}{v_2}=\frac{1-\mu}{R_2}$$

Here, $u_2=-(v_1+R)=-(1\cdot5\,m\,R+R)\,;$

$v_2=\infty,\ R_2=-R\,;\,\mu=1\cdot5$

$$\therefore\ -\frac{1\cdot5}{-(1\cdot5\,m\,R+R)}+\frac{1}{\infty}=\frac{1-1\cdot5}{-R}$$

or $$\frac{1\cdot5}{1\cdot5\,m\,R+R}=\frac{0\cdot5}{R}$$

$$1\cdot5\,m\,R+R=\frac{1\cdot5}{0\cdot5}R=3\,R\ \text{ or }\ 1\cdot5\,m\,R=2\,R$$

or $$m=\frac{2}{1\cdot5}=\frac{4}{3}$$

Problem 1.06. The distance between two point sources of light is 24 cm. Find out where would you place a converging lens of focal length 9 cm, so that the images of both sources are formed at the same point.

(Roorkee, 1988)

Sol. Fig. 1.06 shows a convex lens of focal length *f* placed between two objects I_1 and I_2 placed 24 cm apart. The lens produces a real image I_1 of the object S_1, while a virtual image I_2 of the object S_2.

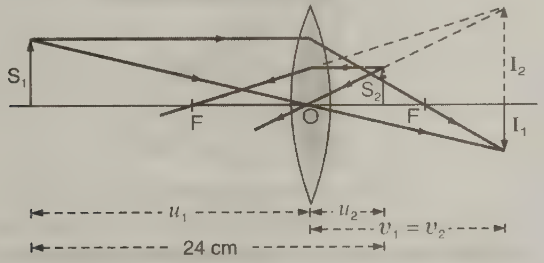


Fig. 1.06,

For source S_1 : Suppose that the object S_1 lies at a distance *x* from the lens. If u_1 and v_1 denote the distances of the object S_1 and the image I_1 from the lens, then

$$\frac{1}{f}=-\frac{1}{u_1}+\frac{1}{v_1}$$

Here, $u_1=-x$ and $f=+9$ cm

$$\therefore\ \frac{1}{v_1}=\frac{1}{f}+\frac{1}{u_1}=\frac{1}{9}+\frac{1}{-x}=\frac{1}{9}-\frac{1}{x}\qquad\text{...}(i)$$

For source S_2 : If u_2 and v_2 are distances of the object S_2 and image I_2 from the lens, then

$$\frac{1}{f}=-\frac{1}{u_2}+\frac{1}{v_2}\qquad\text{...}(ii)$$

Here, $u_2=-(24-x)$ and $f=+9$ cm

Also, as the image I_2 is formed on the same side of the lens and coincides with the image I_1,

$$v_2=-v_1\quad\text{or}\quad\frac{1}{v_2}=-\frac{1}{v_1}$$

Setting the values of u_2, *f* and $\frac{1}{v_2}$ in equation (*ii*), we have

$$\frac{1}{9}=-\frac{1}{-(24-x)}-\frac{1}{v_1}$$

or $$\frac{1}{v_1}=-\frac{1}{9}+\frac{1}{(24-x)}\qquad\text{...}(iii)$$

From equations (*i*) and (*iii*), we have

$$-\frac{1}{9}+\frac{1}{(24-x)}=\frac{1}{9}-\frac{1}{x}$$

or $$\frac{1}{(24-x)}+\frac{1}{x}=\frac{2}{9}$$

or $$x^2-24x+108=0$$

or $$x=\frac{-(-24)\pm\sqrt{(-24)^2-4\times1\times108}}{2}$$

$$=\frac{24\pm\sqrt{576-432}}{2}=\frac{24\pm12}{2}=\textbf{18 cm or 6 cm}$$

Therefore, the convex lens may be placed between the two sources, such that from one source, its distance is 18 cm and from the other source, the distance is 6 cm.

Problem 1.07. A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in Fig. 1.07. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principle axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid. *(I.I.T. 2001)*

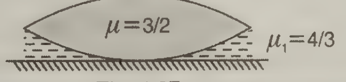


Fig. 1.07.

Thought Process

1. The radii of curvature of the two surfaces of the planoconcave lens of water (or liquid) formed between binconvex lens and plane mirror are –R and ∞, where R is radius of curvature of the surface of binconvex lens.

2. The object can coincide with its image, only if the rays of light retrace their paths i.e. they fall normally on the plane mirror. It will happen so, if the object lies at the focus of the combined lens system.

Sol. Let R be the radius of curvature of the each of the two surfaces of the biconvex lens made of material of refractive index $\mu(=3/2)$. Then, its focal length is given by

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=\left(\frac{3}{2}-1\right)\times\frac{2}{R}=\frac{1}{R}$$

When space between lens and plane mirror is filled with water :

Let f_1 be focal length of the planoconcave lens of water formed between the lens and the plane mirror. If μ_1 is refractive index of water, then

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{(4/3 - 1)}{R} = -\frac{1}{3R}$$

If F is focal length of the combined lens system, then

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f_1} \quad \text{or} \quad \frac{1}{15} = \frac{1}{R} - \frac{1}{3R} \quad \text{or} \quad R = 10 \text{ cm}$$

When space between lens and plane mirror is filled with liquid :

Let f_2 be focal length of the plano concave lens of liquid formed between the lens and the plane mirror. If μ_2 is refractive index of the liquid, then

$$\frac{1}{f_2} = (\mu_2 - 1)\left(-\frac{1}{R} - \frac{1}{\infty}\right) = -\frac{(\mu_2 - 1)}{R}$$

If F′ is focal length of the combined lens system, then

$$\frac{1}{F'} = \frac{1}{f} + \frac{1}{f_2} \quad \text{or} \quad \frac{1}{25} = \frac{1}{R} - \frac{(\mu_2 - 1)}{R}$$

$$\text{or} \quad \frac{1}{25} = \frac{1}{10} - \frac{(\mu_2 - 1)}{10} \quad \text{or} \quad \frac{(\mu_2 - 1)}{10} = \frac{1}{10} - \frac{1}{25} = \frac{3}{50}$$

$$\text{or} \quad \mu_2 = 1 + \frac{3}{5} = 1{\cdot}6$$

Problem 1.08. A pin is placed 10 cm in front of convex lens of focal length 20 cm made of material of refractive index 1·5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual ? *(I.I.T. 1978)*

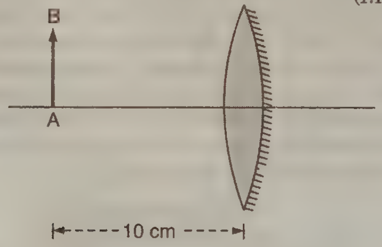


Fig. 1.08.

Thought Process

Fig. 1.08 shows a pin AB placed at a distance of 10 cm from the convex lens of focal length, f = 20 cm. The farther face of the lens of radius of curvature R_2 = 22 cm is silvered. When a ray of light from the pin falls on the silvered lens, first of all its image is formed due to refraction by the convex lens (f = 20 cm), then due to reflection by the farther silvered surface acting as a concave mirror $\left(f_m = \frac{R_2}{2}\right)$ *and finally again due to refraction (the rays reflected from the silvered surface are again refracted) by the lens (f = 20 cm).*

If F is the equivalent focal length of the system (lens, concave mirror and lens), then

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f_m} + \frac{1}{f}$$

Sol. Here, $f = 20$ cm ; $f_m = \frac{R_2}{2} = \frac{22}{2} = 11$ cm ;

$$\therefore \quad \frac{1}{F} = \frac{1}{f} + \frac{1}{f_m} + \frac{1}{f} = \frac{1}{20} + \frac{1}{11} + \frac{1}{20} = \frac{21}{10}$$

$$\text{or} \quad F = \frac{110}{21} \text{ cm}$$

As the system reflects back and converges the rays, it acts as a concave mirror of focal length $F = \frac{110}{21}$ cm.

For a concave mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{F}$$

Here, $u = -10$ cm ; $F = -\frac{110}{21}$ cm ;

$$\therefore \quad \frac{1}{v} = \frac{1}{F} - \frac{1}{u} = \frac{1}{-110/21} - \frac{1}{-10}$$

$$= -\frac{21}{110} + \frac{1}{10} = \frac{-10}{110}$$

or $v = -11$ **cm**

The negative sign tells that the image will be formed on the side as the pin is. Therefore, the image formed is **real** and **inverted.**

Problem 1.09. The radius of curvature of the convex face of a plano-convex lens is 12 cm and its refractive index is 1·5.

(*a*) Find the focal length of the lens.

(*b*) The plane surface of the lens is now silvered [Fig. 1.09]. At what distance from the lens will parallel rays incident on the convex surface converge ?

(*c*) Calculate the image distance, when a point object is placed on the axis 20 cm from the lens. *(I.I.T. 1979)*

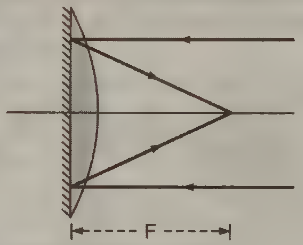


Fig. 1.09.

Sol. (*a*) Here, $R_1 = +12$ cm ; $R_2 = \infty$; $\mu = 1{\cdot}5$;

If *f* is focal length of the plano-convex lens, then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1{\cdot}5 - 1)\left(\frac{1}{12} \cdot \frac{1}{+\infty}\right) = \frac{1}{24}$$

or $f = 24$ **cm**

(*b*) When the plane surface of the plano-convex is silvered, it acts as a combination* of a plano-convex lens (f = 24 cm), a plane mirror ($f_m = \infty$) and a plano-convex lens (f = 24 cm). If F is effective focal length of the system, then

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f_m} + \frac{1}{f} = \frac{1}{24} + \frac{1}{\infty} + \frac{1}{24} = \frac{1}{12}$$

or F = 12 cm

As the system reflects back and converges the rays, it acts as a concave mirror of focal length F = 12 cm.

For a concave mirror,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{F}$$

---

*For more details, refer to Thought Process of problem no. 1.08.

Here, $u = -\infty$; F = – 12 cm

$$\therefore \quad \frac{1}{v} = \frac{1}{F} - \frac{1}{u} = \frac{1}{-12} - \frac{1}{-\infty} = -\frac{1}{12}$$

or $v = -12$ cm

Hence, the parallel rays will converge at a distance of 12 m from the silvered surface.

(*c*) Here, $u = -20$ cm ; F = – 12 cm

$$\therefore \quad \frac{1}{v} = \frac{1}{F} - \frac{1}{u} = \frac{1}{-12} - \frac{1}{-20} = -\frac{1}{30}$$

or $v = -30$ cm

The negative sign shows that the image formed is real. Therefore, a real image will be formed at a distance of 30 cm rom the silvered surface.

Problem 1.10. A point object is placed at a distance of 0·3 m from a convex lens (focal length 0·2 m) cut into two halves, each of which is displaced by 0·5 mm as shown in Fig. 1.10. Find the position of the image. If more than one image is formed, find their number and the distances between them. [Ans. Two images, 1·5 mm]

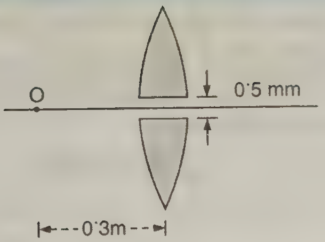


Fig.1.10.

Hint. The two halves of the lens will act as two separate lenses and hence will form two images.

To calculate distance between the two images, refer to problem 1.05-Competitive Examination File (Unit VI).

Note. For such one typical problem, refer to conceptual SAQ 5.05-Chapter 5 (Unit VI) also.

Chapter 2. Dispersion

Problem 2.01. A ray of light is incident at an angle of 60°on the first face of a prism of an angle 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray [Fig. 2.01]. Show that the emergent ray is perpendicular to the face through which it emerges. Also calculate the refractive index of the material of the prism. (*Karnataka, 1993 ; I.I.T. 1978*)

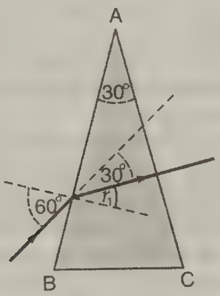


Fig. 2.01.

Sol. Here, the angle of prism, A = 30° ;

The angle of incidence at the first face, $i = 60°$;

The angle of deviation, $\delta = 30°$

If e is angle of emergence, then

$$A + \delta = i + e$$

or $e = A + \delta - i = 30° + 30° - 60° = 0°$

Therefore, as shown in the figure, the emergent ray will be perpendicular to the second face of the prism.

If r_1 is angle of refraction at the first face, then from the figure, it follows that $r_2 = 30°$.

$$\text{Now,} \quad \mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = 1{\cdot}732$$

Problem 2.02. A glass prism of angle 72° and refractive index 1·66 is immersed in water of refractive index 1·33. Find the angle of minimum deviation for a parallel beam of light passing through the prism. (*I.I.T. 1977*)

Sol. Here, A = 72° ; $^a\mu_g = 1{\cdot}66$; $^a\mu_w = 1{\cdot}33$;

$$\text{Now,} \quad {}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1{\cdot}66}{1{\cdot}33} = 1{\cdot}248$$

If δ_m is the angle of minimum deviation, when the prism is immersed in water, then

$${}^w\mu_g = \frac{\sin (A + \delta_m)/2}{\sin A/2}$$

or $\sin (A + \delta_m)/2 = {}^w\mu_g \sin A/2 = 1{\cdot}248 \times \sin 72°/2$

$= 1{\cdot}248 \times 0{\cdot}5878 = 0{\cdot}7336$

or $(A + \delta_m)/2 = 47°11'$

or $\delta_m = 2 \times (47°11') - A = 94°22' - 72° = 22°\ 22'$

Problem 2.03. A prism of refractive index μ_1 and another prism of refractive index μ_2 are struck together without a gap as shown in Fig. 2.02. The angles of the prisms are shown. μ_1 and μ_2 depend on λ, the wavelength of light according to

$$\mu_1 = 1{\cdot}20 + \frac{10{\cdot}8 \times 10^4}{\lambda^2} \text{ and } \mu_2 = 1{\cdot}45 + \frac{10{\cdot}8 \times 10^4}{\lambda^2},$$

where λ is in nm.

(*a*) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(*b*) For light of wavelength λ_0, find the angle of incidence *i* on the face AC, such that the deviation produced by the combination of prisms is minimum. (*I.I.T. 1998*)

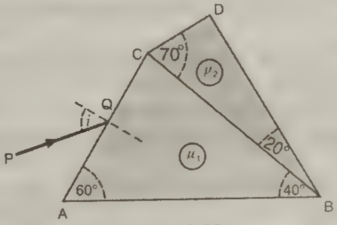


Fig. 2.02.

Thought Process

1. If rays incident at any angle on the interface BC pass through without bending at the interface for wavelength λ_0, then μ_1 and μ_2 must have the same value for that wavelength i.e.

$$1{\cdot}20 + \frac{1{\cdot}80 \times 10^4}{l_0^2} = 1{\cdot}45 + \frac{1{\cdot}80 \times 10^4}{l_0^2}$$

2. *For wavelength λ_0, $\mu_1 = \mu_2$. Therefore, the combination of the two prisms acts as a single prism, whose base angles are 60° each. Therefore, vertex angle E must also be 60°[Fig. 2.03].*

3. *In the minimum deviation position, the refracted ray QR is parallel to the base AB of the prism. Also, the angle of refaction,*

$$r = \frac{\textit{angle of prism}}{2}$$

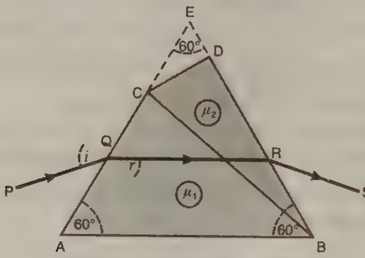


Fig. 2.03.

Sol. (*a*) If the ray incident at the interface BC passes undeviated for the wavelength λ_0, then

$$1\cdot20+\frac{10\cdot8\times10^4}{\lambda_0^2}=1\cdot45+\frac{1\cdot80\times10^4}{\lambda_0^2}$$

or $$\frac{9\cdot0\times10^4}{\lambda_0^2}=0\cdot25$$

or $$\lambda_0=\sqrt{\frac{9\cdot0\times10^4}{0\cdot25}}=6\cdot0\times10^2=\mathbf{600\ nm}$$

(*b*) Now, refractive index of the material of the prism for wavelength λ_0,

$$\mu=1\cdot20+\frac{1\cdot80\times10^4}{(600)^2}=1\cdot20+0\cdot3=1\cdot50$$

If the ray of light suffers minimum deviation, when incident at angle *i*, then angle of refraction,

$$r=\frac{60°}{2}=30°$$

From the relation : $\mu=\dfrac{\sin i}{\sin r}$, , we have

$$\sin i=\mu\sin r=1{\cdot}50\sin 30°=1{\cdot}50\times0{\cdot}5=0{\cdot}75$$

$\therefore\ \ i = \mathbf{48°\text{-}36'}$

Problem 2.04. The refractive indices of the crown glas for blue and red light are 1·51 and 1·49 respectively an those of the flint glass are 1·77 and 1·73 respectively. A isosceles prism of angle 60° is made of crown glass. A bean of white light is incident at a small angle on this prism. Th other flint glass isosceles prism is combined with the crow glass prism such that there is no deviation of the inciden light. Determine the angle of the flint glass prism. Calculat the net dispersion of the combined system. (*I.I.T. 2001*

Sol. Let A_1 and A_2 be angles of crown and flint glas prism respectively.

For crown glass prism : $\mu_b = 1{\cdot}51$; $\mu_r = 1{\cdot}49$ and $A_1 = 6$

Refractive index for mean light,

$$\mu=\frac{\mu_b+\mu_r}{2}=\frac{1\cdot51+1\cdot49}{2}=1\cdot50$$

For flint glass prism : $\mu_b' = 1{\cdot}77$; $\mu_r' = 1{\cdot}73$

Refractive index for mean light,

$$\mu'=\frac{\mu_b'+\mu_r'}{2}=\frac{1\cdot77+1\cdot73}{2}=1\cdot75$$

For no deviation,

$$A(\mu-1)+A'(\mu'-1)=0$$

or $$6°(1{\cdot}50-1)+A'(1{\cdot}75-1)=0$$

or $$A'=-\frac{6°\times0\cdot50}{0\cdot75}=\mathbf{-4°}$$

Net dispersion $= A(\mu_b-\mu_r)+A'(\mu_b'-\mu_r')$

$= 6°(1{\cdot}51-1{\cdot}49)+(-4°)(177-1{\cdot}73)$

$= \mathbf{-0{\cdot}04°}$

Chapter 3. Optical Instruments

Problem 3.01. A compound microscope is used to enlarge an object kept at a distance of 0·03 m from its objective, which consists of several convex lenses and has focal length 0·02 m. If a lens of focal length 0·1 m is removed from the objective, find out the distance by which the eye-piece of the microscope must be moved to refocus the image. (*Roorkee, 1991*)

Sol. Let F be focal length of the objective consisting of several convex lenses.

Here, F = 0·02 m ; *u* = – 0·03 m ;

If the image is formed at a distance *v* from the objective, then

$$\frac{1}{v}=\frac{1}{F}+\frac{1}{u}=\frac{1}{0\cdot02}+\frac{1}{-0\cdot03}=\frac{0\cdot03-0\cdot02}{0\cdot02\times0\cdot03}$$

or $$v=\frac{0\cdot02\times0\cdot03}{0\cdot01}=0\cdot06\ \text{m}$$

When one lens of focal length $f = 0{\cdot}1$ is removed, suppose that the new focal length of the objective becomes F′. Then,

$$\frac{1}{F}=\frac{1}{F'}+\frac{1}{f}$$

or $$\frac{1}{F'}=\frac{1}{F}-\frac{1}{f}=\frac{1}{0\cdot02}-\frac{1}{0\cdot1}=\frac{4}{0\cdot1}$$

or $$F'=\frac{0\cdot1}{4}=0\cdot025\ \text{m}$$

If the new position of the image is at a distance v' from the objective, then

$$\frac{1}{v'}=\frac{1}{F'}+\frac{1}{u}=\frac{1}{0\cdot025}+\frac{1}{-0\cdot03}=\frac{0\cdot03-0\cdot025}{0\cdot025\times0\cdot03}$$

or $$v'=\frac{0\cdot025\times0\cdot03}{0\cdot005}=0\cdot15\ \text{m}$$

Therefore, the distance through which the eye-piece of the microscope should be moved,

$$v'-v=0{\cdot}15-0{\cdot}06=\mathbf{0{\cdot}09\ m}$$

Problem 3.02. In a compound microscope, the objective and eye-piece have focal lengths of 0·95 cm and 5 cm respectively and are kept at a distance of 20 cm. The last image is formed at a distance of 25 cm from the eye-piece. Calculate the position of the object and the total magnification. (*Roorkee, 1988*)

Sol. Here, $f_o = 0{\cdot}95$ cm ; $f_e = 5$ cm ;

Distance between the objective and the eye-piece = 20 cm

Refer to Chapter 3 – Fig. 3.03 : For the eye-piece, the image A′ B′ (formed by objective) acts as the object and its age A'' B'' is formed at the least distance of distinct vision. e distance u_e of the image A' B' (objective for eye-piece) m the eye-piece is given by

$$-\frac{1}{u_e}+\frac{1}{v_e}=\frac{1}{f_e}$$

Here, $v_e = -25$ cm

$$\therefore \quad \frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}=\frac{1}{-25}-\frac{1}{5}=-\frac{6}{25}$$

or $u_e = -\frac{25}{6}$ cm

The distance of the object u_o from the objective is given

$$-\frac{1}{u_o}+\frac{1}{v_o}=\frac{1}{f_o}$$

If the distance of the image A' B' from the object is v_o, en

$$v_o + |\, u_e \,| = 20$$

or $v_o = 20 - |u_e| = 20 + -\frac{25}{6} = \frac{95}{6}$ cm

$$\therefore \quad \frac{1}{u_o}=\frac{1}{v_o}-\frac{1}{f}=\frac{1}{95/6}-\frac{1}{95}-\frac{1}{0\cdot95}=-\frac{94}{95}$$

or $u_o = -\frac{95}{94}$ cm

Magnification produced by the microscope,

$$M=\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)=\frac{95/6}{-95/94}\left(1+\frac{25}{2}\right)=-94$$

The negative sign shows that the final image formed is al and inverted.

Problem 3.03. A telescope has an objective of focal ngth 50 cm and eye-piece of focal length 5 cm. The least istance of distinct vision is 25 cm. The telescope is ocussed for distinct vision on a scale 200 cm away from the bjective. Calculate (*a*) the separation between the bjective and eye-piece (*b*) the magnification produced. *(I.I.T. 1980)*

Sol. (*a*) Here, $f_e = 5$ cm ; $f_o = 50$ cm ;
$u_o = -200$ cm ;

***Image formed by the objective*:** Refer to Chapter 3 – Fig. 3.05. If the objective forms the image A' B' at a distance v_o from the objective, then

$$\frac{1}{v_o}=\frac{1}{f_o}+\frac{1}{u_o}=\frac{1}{50}+\frac{1}{-200}=\frac{3}{200}$$

or $v_o = +\frac{200}{3}$ cm

***Image formed by the eye-piece*:** For eye-piece, the image A' B' acts as object and its virtual image A' B' is produced by the eye-piece at the least distance of distinct vision. If the image A' B' is at a distance u_e from the eye-piece, then

$$-\frac{1}{u_e}+\frac{1}{v_e}=\frac{1}{f_e}$$

Here, $v_e = -D = -25$ cm ; $f_e = 5$ cm

$$\therefore \quad \frac{1}{u_e}=\frac{1}{v_e}-\frac{1}{f_e}=\frac{1}{-25}-\frac{1}{5}=-\frac{6}{25}$$

or $u_e = -\frac{25}{6}$ cm

Therefore, separation between objective and eye-piece,

$$CC' = |v_o| + |u_e| = \frac{200}{3}+\frac{25}{6}=\frac{425}{6}=\mathbf{70{\cdot}83\ cm}$$

(*b*) Now, $m_o = \frac{v_o}{u_o} = \frac{200/3}{-200} = -\frac{1}{3}$

and $m_e = \frac{v_e}{u_e} = \frac{-25}{-25/6} = 6$

Magnification produced by the telescope,

$$m = m_o \times m_e = -\frac{1}{3} \times 6 = -2$$

The negative sign shows that final image formed is real and inverted.

Chapter 5. Interference of Light

Problem 5.01. In a two slit experiment with onochromatic light, fringes are obtained on a screen laced at some distance from the slits. If the screen is moved y 5 × 10^{-2} towards the slits, the change in fringe width is × 10^{-5}. If the distance between the slits is 10^{-3} m, calculate e wavelength of the light used. *(Roorkee, 1992)*

Thought Process

From the expression for fringe width $\left(\beta = \frac{D\lambda}{d}\right)$*, it follows that for a given slit separation (d) and the wavelength of light (λ), the fringe width (β) varies directly with the distance of screen (D) from the slits. Therefore, if D is decreased by Δ D, the fringe width will decrease by Δ β, such that*

$$\Delta\beta = \frac{\Delta D\lambda}{d}$$

Sol. Here, $d = 10^{-3}$ m; $\Delta D = 5 \times 10^{-2}$ m
$\Delta\beta = 3 \times 10–5$ m ;

Now, $\Delta\beta = \frac{\Delta D\lambda}{d}$

or $\lambda = \frac{\Delta\beta}{\Delta D} \times \frac{3\times10^{-5}}{5\times10^{-2}} \times 10^{-3} = 6 \times 10^{7}$ m

Problem 5.02. In a Young's double slit experiment, the angular width of a fringe formed on a distant screen is 0·1°. The wavelength of light used is 6000 Å. What is the spacing between the slits ? *(Roorkee, 1983)*

Hint. $\theta = \frac{\beta}{D} = \frac{D\lambda/d}{D} = \frac{\lambda}{d}$

It can be obtained that $d = 3{\cdot}44 \times 10^{-4}$ m

Problem 5.03. A beam of light consisting of two wavelengths 6500 Å and 5200 Å is used to obtain interference fringes in a Young's double slit experiment. The distance between the slits is 2 mm and distance between the plane of the slits and the screen is 120 cm.

(*a*) Find the distance of third bright fringe on the screen from the central maximum for the wavelength 6500 Å.

(*b*) What is the least distance from the central maximum, when the bright fringes due to both the wavelengths coincide ?

(I.I.T. 1985)

Sol. Here, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$;
$D = 120 \text{ cm} = 1{\cdot}2 \text{ m}$;
$\lambda = 6500 \text{ Å} = 6{\cdot}5 \times 10^{-7} \text{ m}$; $\lambda' = 5200 \text{ Å} = 5{\cdot}2 \times 10^{-7} \text{ m}$

(*a*) The distance of *n*th bright fringe from the central maximum,

$$y_n = \frac{n\,D\,\lambda}{d}$$

Therefore, distance of the third bright fringe ($n = 3$) for wavelength λ ($= 6{\cdot}5 \times 10^{-7}$ m),

$$y_3 = \frac{3 \times 1{\cdot}2 \times 6{\cdot}5 \times 10^{-7}}{2 \times 10^{-3}} = 1{\cdot}17 \times 10^{-3} \text{ m} = \mathbf{1{\cdot}17 \text{ mm}}$$

(*b*) Suppose that the *n*th bright fringe due to wavelength λ coincides with the n'th bright fringe due to wavelength λ'.

Then, $\dfrac{n\,D\,\lambda}{d} = \dfrac{n'\,D\,\lambda'}{d}$

or $\quad n\lambda = n'\lambda' \quad$ ...(*i*)

Since $\quad \lambda > \lambda', n < n'$.

The two bright fringes will coincide at the least distance, if n and n' differ by 1 *i.e.* $n' = n + 1$

Therefore, equation (*i*) becomes

$$n\lambda = (n+1)\lambda'$$

or $\quad n \times 6{\cdot}5 \times 10^{-7} = (n+1) \times 5{\cdot}2 \times 10^{-7}$

or $\quad 5n = 4(n+1)$

or $\quad n = 4$

Therefore, required distance,

$$y_4 = \frac{4\,D\,\lambda}{d} = \frac{4 \times 1{\cdot}2 \times 6{\cdot}5 \times 10^{-7}}{2 \times 10^{-3}} = 1{\cdot}56 \times 10^{-3} \text{ m}$$

$= \mathbf{1{\cdot}56 \text{ mm}}$

Problem 5.04. A point source S emitting light of wavelength 600 nm is placed at a very small height *h* above a flat reflecting surface AB [Fig. 5.01]. The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance D from it.

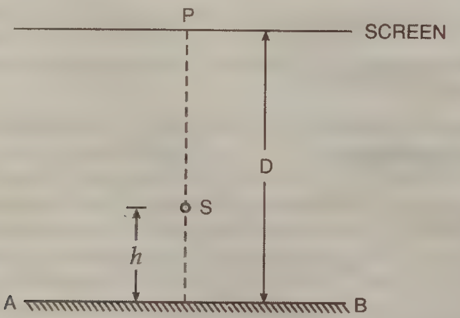


Fig.5.01

(*a*) What is the shape of the interference fringes on the screen ?

(*b*) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P.

(*c*) If the intensity at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes maximum. (*I.I.T. 2002*)

Sol. (*a*) The locus of point around point P, where light reaches directly from source S or after reflection from the reflecting surface AB after covering equal path is circle. Hence, interference fringes are **circular** in shape.

(*b*) Let I_1 and I_2 be intensities of light reaching the screen directly from the source and after reflection from the reflecting surface. Then,

$$I_2 = \frac{36}{100} I_1 \quad \text{or} \quad \frac{I_2}{I_1} = 0{\cdot}36$$

$$\text{or} \quad \frac{a_2^2}{a_1^2} = 0{\cdot}36 \quad \text{or} \quad a_2 = 0{\cdot}6\,a_1$$

$$\text{Now,} \quad \frac{I_{min}}{I_{max}} = \frac{(a_1 - a_2)^2}{(a_1 + a_2)^2} = \frac{(a_1 - 0{\cdot}6\,a_1)^2}{(a_1 + 0{\cdot}6\,a_1)^2}$$

$$= \left(\frac{0{\cdot}4}{1{\cdot}6}\right)^2 = \mathbf{\frac{1}{16}}$$

(*c*) Now, the source of light lies a distance *h* above the reflecting surface. Therefore, its image will lie a distance *h* below it. Hence, path difference between the light waves reaching the screen from the source and its image will be 2 *h*. Suppose that the source of light is shifted through a distance *x* from the reflecting surface. Then, path difference between the light waves reaching the screen from the source and its image will be $2(h \pm x)$. It follows that due to shifting of the source, an extra path difference,

$$\Delta x = 2(h \pm x) - 2h = \pm 2x$$

will be produced. The intensity of light at the point P will be again maximum, if

$$2x = \lambda \quad \text{or} \quad x = \frac{\lambda}{2} = \frac{600}{2} = \mathbf{300 \text{ nm}}$$

Problem 5.05. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0·45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10·4 μm and refractive index 1·5. The interference pattern is observed on a screen placed 1·5 m from the slits as shown in Fig. 5.02.

(*a*) Find the location of the central maximum (bright fringe with zero path difference) on the Y-axis.

(*b*) Find the light intensity of point O relative to the maximum fringe intensity. (*I.I.T. 1999*)

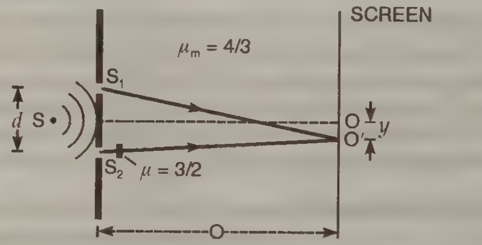


Fig.5.02

Thought Process

1. *Since light from two sources does not travel in the same optical medium, the path difference between the light from the two sources has to be calculated from the optical paths.*

2. *When both the slits are uncovered, the central maximum is formed at the centre of the screen. When the lower slit S_2 is covered with a glass plate, the path of light from lower slit S_2 upto the centre of the screen will be a little more than that from upper slit S_1. Therefore, the point O' on the screen, where*

the light from the two slits possesses zero path difference, will lie a little below the point O.

3. If ϕ is the phase difference between the light rays reaching point O from S_1 and S_2, then intensity of light at point O,

$$I = I_{max}\cos^2\frac{\phi}{2},$$

where I_{max} is intensity at point O.

Sol. (*a*) Fig. 5.02 shows the Young's double slit xperiment arrangement in a medium of refractive index μ_m. The lower slit S_2 is covered with a glass sheet of thickness t nd refractive index μ_g. The point O is centre of the screen. ince $\mu_g > \mu_m$, the length of the optical path S_1O will be less han that of S_2O. Let O′ be the point on the screen at a distance below the point O, such that the lengths of the optical paths f light from slits S_1 and S_2 to point O′ are equal.

Now, optical path of light from slit S_1 to O′

$$= \mu_m S_1O'$$

and optical path of light from slit S_2 to O′

$$= \mu_m S_2O' - \mu_m t + \mu_g t$$

For the central bright fringe to be formed at point O′, the two optical paths have to be equal *i.e.*

$$\mu_m S_1O' = \mu_m S_2O' - \mu_m t + \mu_g t$$

$$\text{or} \quad \mu_m (S_1O' - S_2O') = (\mu_g - \mu_m)\, t \qquad ...(i)$$

If d is distance between the two slits and D, the distance of the screen from the slits, then

$$S_1O' - S_2O' = \frac{y\,d}{D}$$

Therefore, equation (*i*) becomes

$$\mu_m\left(\frac{y\,d}{D}\right) = (\mu_g - \mu_m)\,t$$

Here, $\mu_m = \frac{4}{3}$; $\mu_g = 1{\cdot}5$;

$d = 0{\cdot}45$ mm $= 0{\cdot}45 \times 10^{-3}$ m ;

D = 1·5 m ; $t = 10{\cdot}4\ \mu$m $= 10{\cdot}4 \times 10^{-6}$ m

$$\therefore \quad \frac{4}{3}\left(y \times \frac{0\cdot45\times10^{-3}}{1\cdot5}\right) = \left(1\cdot5 - \frac{4}{3}\right)\times 10\cdot4\times10^{-6}$$

$$= \frac{0\cdot5\times10\cdot4\times10^{-6}\times3\times1\cdot5}{4\times3\times0\cdot45\times10^{-3}}$$

$$\text{or} \quad y = \frac{0\cdot5\times10\cdot4\times10^{-6}\times3\times1\cdot5}{4\times3\times0\cdot45\times10^{-3}}$$

$= \mathbf{4{\cdot}33 \times 10^{-3}}$ **m**

(*b*) The refractive index of glass plate *w.r.t.* medium,

$${}^m\mu_g = \frac{1\cdot5}{4/3} = \frac{9}{8}$$

Therefore, path difference between light paths S_1O and S_2O,

$$x = {}^m\mu_g\, t - t = \frac{9}{8}t - t = \frac{t}{8}$$

Phase difference between the light reaching point O from S_1 and S_2,

$$\phi = x \times \frac{2\pi}{\lambda} = \frac{t}{8} \times \frac{2\pi}{\lambda} = \frac{\pi t}{4\lambda}$$

Here, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m ; $t = 10{\cdot}4 \times 10^{-6}$ m

$$\therefore \quad \phi = \frac{\pi\times10\cdot4\times10^{-6}}{4\times600\times10^{-9}} = \frac{13\,\pi}{3}$$

Therefore, intensity of light at point O,

$$I = I_{max} = \cos^2\frac{\phi}{2} = I_{max}\cos^2\frac{13\,\pi}{6} = I_{max}\cos^2\frac{\pi}{6}$$

$$= I_{max}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}I_{max}$$

Problem 5.06. A glass plate of refractive index 1·5 coated with a thin layer of thickness t and refractive index 1·8. Light of wavelength λ travelling in air is incident normally on the layer and the two reflected rays interfere. Write the condition for their constructive interference. If λ = 648 nm, obtain the least value of t for which the rays interfere constructively. (*I.I.T. 2000*)

Sol. Here, refractive index of glass plate, $\mu = 1{\cdot}5$

and refractive index of the coating layer, $\mu' = 1{\cdot}8$

When the rays of light are reflected from the glass surface and the surface of the coating layer, the optical path difference between the two rays,

$$\Delta x = 2\,\mu'\,t,$$

where t is thickness of the coating layer.

Since $\mu' > \mu$, the rays of light suffering reflection from coating layer (optically denser medium) undergoes a phase change of π (or path difference of $\lambda/2$). On the other hand, there is no change in phase of the ray of light suffering reflection from glass plate (optically rarer medium). Therefore, net path difference in the path of two rays,

$$\Delta x' = \Delta x + \frac{\lambda}{2} = 2\,\mu'\,t + \frac{\lambda}{2}$$

For constructuve interference between the two rays of light,

$$\Delta x' = n\,\lambda \quad \text{or} \quad 2\,\mu'\,t + \frac{\lambda}{2} = n\,\lambda$$

$$\text{or} \quad \mathbf{2\,\mu'\,t} = \left(n - \frac{1}{2}\right)\lambda\text{, where } n = 1, 2, 3, ...$$

It is the condition for constructive interference between the rays of light.

The least value of the thickness of the coating layer corresponds to $n = 1$, *i.e.*

$$2\,\mu'\,t_{min} = \frac{\lambda}{2} \quad \text{or} \quad t_{min} = \frac{\lambda}{4\,\mu'}$$

Here, λ = 648 nm and $\mu' = 1{\cdot}8$

$$\therefore \quad t_{min} = \frac{648}{4\times1\cdot8} = \mathbf{90\ nm}$$

Problem 5.07. A vessel ABCD of 10 cm width has two small slits S_1 and S_2 sealed with identical glass plates of equal thickness. The distance between the slits is 0·8 mm. POQ is the line perpendicular to the plane AB and passing through O, the middle point of S_1 and S_2. A monochromatic light source is kept at S, 40 cm below P and 2 m from the vessel, to illuminate the slits as shown in Fig. 5.03. Calculate the position of the central bright fringe on the other wall CD with respect to the line OQ. Now a liquid is poured into

the vessel and filled upto OQ. The central bright fringe is found to be at Q. Calculate the refractive index of the liquid.

(I.I.T. 2001)

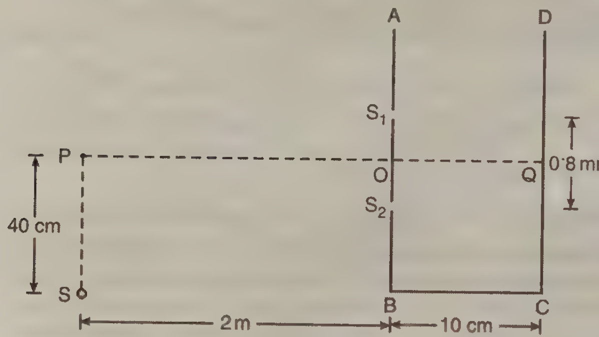


Fig. 5.03

Sol. Here, $S_1S_2 = 0{\cdot}8$ mm $= 0{\cdot}08$ cm ; PS = 40 cm,

OP = 2 m = 200 cm and OQ = 10 cm

Suppose that central maximum is formed at a point K on the wall CD, such that QK = y. The liquid waves from the source S illuminate the slits S_1 and S_2 and then reach point K as shown in Fig. 5.04.

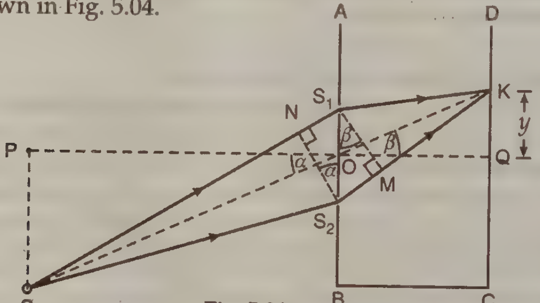


Fig. 5.04

For point K to be the position of central maximum,

$$(SS_1 + S_1K) - (SS_2 + S_2K) = 0$$

or $\quad SS_1 - SS_2 = S_2K - S_1K \qquad \text{...}(i)$

From points S_1 and S_2, drop S_1M and S_2N perpendiculars to S_2K and SS_1 respectively.

Then, $SS_1 - SS_2 \approx S_1N \quad$ and $\quad S_2K - S_1K \approx S_2M$

Therefore, equation (*i*) becomes

$$S_1N = S_2M \qquad \text{...}(ii)$$

Let $\angle POS = \alpha$. Then $\angle S_1S_2N \approx \alpha$

Hence, from right angled $\Delta\, S_1NS_2$, we have

$$S_1N = S_1S_2 \sin\alpha = 0{\cdot}08 \sin\alpha$$

Also from right angled Δ OPS, we have

$$\sin\alpha \approx \tan\alpha = \frac{PS}{OP} = \frac{40}{200} = 0{\cdot}2$$

$\therefore \quad S_1N = 0{\cdot}08 \times 0{\cdot}2 = 0{\cdot}016$ cm

Similarly, it can be obtained that

$$S_2M = S_1S_2 \sin\beta = S_1S_2 \tan\beta = 0{\cdot}8 \times \frac{QK}{OQ} = 0{\cdot}08 \times \frac{y}{10}$$

Substituting for S_1N and S_2M in equation (*ii*), we have

$$0{\cdot}016 = 0{\cdot}08 \times \frac{y}{10} \quad \text{or} \quad y = \frac{0{\cdot}016 \times 10}{0{\cdot}08} = 2 \text{ cm}$$

When liquid is poured into vessel upto OQ : On puring liquid upto OQ, the light waves from source S will reach the point Q as shown in Fig. 5.05.

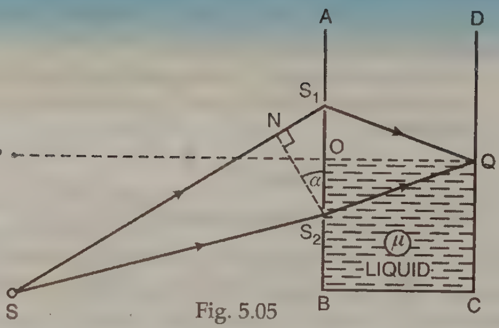


Fig. 5.05

Light waves from S_1 travel in air, while those from S_2 travel inside the liquid. Therefore, optical path difference between S_1Q and S_2Q is equal to $(\mu - 1)\, S_1Q$. Further, as $S_1S_2 << OQ$, $S_1Q \approx OQ$ and hence optical path difference between light waves reaching point Q from sources S_1 and S_2 is $(\mu - 1)$ OQ.

For point Q to be the position of central maximum,

$S_1N = (\mu - 1)$ OQ $\quad$ or $\quad 0{\cdot}016 = (\mu - 1) \times 10$

or $\quad \mu = 1{\cdot}0016$

Problem 5.08. In the Fig. 5.06, S is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0·10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0·5 mm. The distance along the axis from S to L_1 and L_2 is 0·15 m, while that from L_1 and L_2 to O is 1·30 m. The screen at O is normal to SO.

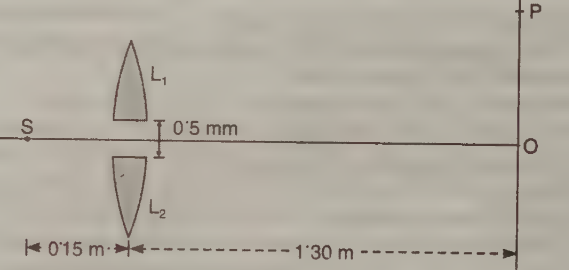


Fig. 5.06.

(*a*) If the third intensity maximum occurs at the point P on the screen, find the distance OP .

(*b*) If the gap between L_1 and L_2 is reduced from its original value of 0·5 mm, will the distance OP increase, decrease or remain the same ? *(I.I.T. 1993)*

Thought Process

The two parts L_1 and L_2 of the lens are placed to form the gap AB = 0·5 mm. As the source S lies between F and 2 F of the lens, the two parts L_1 and L_2 will form real images S_1 ans S_2 of the source S on the other side of the lens as shown in Fig. 5.07. The two images S_1 and S_2 act as two coherent sources and produce interference pattern on the screen.

Here, the distance of screen from the slits,

$$D = 1{\cdot}30\, v \quad \text{(in metre)},$$

where the distance v of the images S_1 and S_2 from the lens can be found by applying uv-relation for a lens. Further, the distance between the slits, $d = S_1 + S_2$. The value of d can be found from the similar triangles SAB and $SS_1\, S_2$.

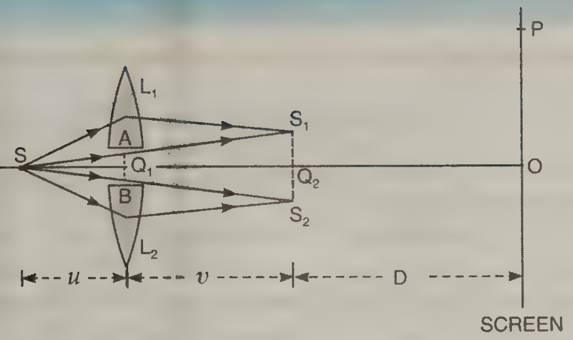


Fig. 5.07.

Sol. (*a*) Here, $u = -0{\cdot}15$ m ; $f = 0{\cdot}10$ m ;

Now, $$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{0{\cdot}01} + \frac{1}{-0{\cdot}15} = \frac{1}{0{\cdot}30}$$

or $$v = 0{\cdot}30 \text{ m}$$

$$\therefore \quad D = 1{\cdot}30 - 0{\cdot}30 = 1{\cdot}0 \text{ m}$$

Since triangles SAB and S_1 and S_2 are similar,

$$\frac{SQ_1}{SQ_2} = \frac{AB}{S_1S_2}$$

Now, $SQ_1 = u$; $SQ_2 = u + v$;

$AB = 0{\cdot}5$ mm $= 0{\cdot}5 \times 10^{-3}$ m and $S_1 S_2 = d$

$$\therefore \quad \frac{u}{u+v} = \frac{0{\cdot}5 \times 10^{-3}}{d}$$

or $$d = \frac{u+v}{u} \times 0{\cdot}5 \times 10^{-3} = \frac{0{\cdot}15 + 0{\cdot}30}{0{\cdot}15} \times 0{\cdot}5 \times 10^{-3}$$

$$= 1{\cdot}5 \times 10^{-3} \text{ m}$$

The distance of third intensity maximum is given by

$$y_3 = \frac{3 D \lambda}{d}$$

Here, $\lambda = 500$ nm $= 500 \times 10^{-9}$ m

$$\therefore \quad OP = y_3 = \frac{3 \times 1{\cdot}0 \times 500 \times 10^{-9}}{1{\cdot}5 \times 10^{-3}} = 10^{-3} \text{ m} = \mathbf{1\ mm}$$

(*b*) As the gap AB between the two parts L_1 and L_2 is decreased, the value of distance *d* will decrease. From the expression for y_3, it follows that OP will *increase*.

Chapter 6. Diffraction of Light

Problem 6.01. Angular width of central maximum in Fraunhofer diffraction pattern of a slit is measured. The is illuminated by light of wavelength 6000 Å. When the is illuminated by light of another wave-length, the gular width decreases by 30%. Calculate the wavelength this light. The same decrease in the angular width of ntral maximum is obtained, when the original apparatus mmersed in a liquid. Find refractive index of the liquid.

(*I.I.T. 1996*)

Sol. If *a* is width of the slit and λ, the wavelength of light ed, then

angular width of the central maximum,

$$2\theta_1 = \frac{2\lambda}{a} \qquad \ldots(i)$$

Suppose that when light of wavelength λ' is used, the dth of the central maximum becomes $2\theta_1'$. Then,

$$2\theta_1' = \frac{2\lambda'}{a} \qquad \ldots(ii)$$

Since the width of central maximum decreases by 30% *i.e.* becomes 70% on using light of wavelength λ' in place of λ,

$$2\theta_1' = \frac{70}{100} \times 2\theta_1 = 0{\cdot}70 \times 2\theta_1$$

or $$2\theta_1' = 0{\cdot}70\left(\frac{2\lambda}{a}\right) \qquad \ldots(iii)$$

From equations (*ii*) and (*iii*), we have

$$\frac{2\lambda'}{a} = 0{\cdot}70\left(\frac{2\lambda}{a}\right)$$

or $$\lambda' = 0{\cdot}70 \times \lambda = 0{\cdot}70 \times 6000 = \mathbf{4200\ Å}$$

The same change in angular width of the central maximum will be obtained on immersing the apparatus in a liquid, if the refractive index μ of the liquid is such that

$$\lambda' = \frac{\lambda}{\mu} \quad \text{or} \quad \mu = \frac{\lambda}{\lambda'} = \frac{6000}{4200} = \mathbf{1{\cdot}429}$$

Chapter 7. Polarisation of Light

Problem 7.01. A beam of plane polarised light falls rmally on a polariser (cross-sectional area 3 × 10^{-4} m²), ich rotates about the axis of the ray with an angular locity of 31·4 rad s^{-1}. Find the energy of light passing rough the polariser per revolution and the intensity of ergent beam, if flux of energy of the incident ray is 10^{-3}

(*Roorkee, 1994*)

Thought Process

As the polariser is continuously rotating, the angle θ between the direction of electric vector and the optical axis of the polariser also varies continuously from 0 to 2π. Therefore, in the expression for intensity of light transmitted from the polariser ($I = I_0 \cos^2\theta$), the average value of $\cos^2\theta$ has to be substituted. In other words,

$$I = I_0 \overline{\cos^2\theta}$$

Now, average value of $\cos^2\theta$ over complete cycle

i.e. $$\overline{\cos^2\theta} = \frac{1}{2}$$

Therefore, $$I = I_0 \times \frac{1}{2} = \frac{I_0}{2}$$

Sol. Here, flux of energy of the incident ray 10^{-3} W ; cross-sectional area of the polariser, A = 3 × 10^{-4} m² and angular velocity of the polariser, $\omega = 31{\cdot}4$ rad s^{-1}

Now, intensity of incident light,

$$I_0 = \frac{\text{flux of energy of the incident ray}}{\text{area of polariser}}$$

$$= \frac{10^{-3}}{3 \times 10^{-4}} \text{ W m}^{-2}$$

Therefore, intensity of emergent beam,

$$I = \frac{I_0}{2} = \frac{10^{-3}}{2 \times 3 \times 10^{-4}} = \frac{5}{3} \text{ W m}^{-2}$$

Further, energy of the light passing through the polariser per revolution, W = I × A × T,
where T is time taken by the polariser to complete one revolution.

Therefore,

$$E = I \times A \times \frac{2\pi}{\omega} = \frac{5}{3} \times 3 \times 10^{-4} \times \frac{2 \times 3 \cdot 14}{31 \cdot 4} = 10^{-4} \text{ J}$$

Problem 7.02. Unpolarised light of intensity 32 W m^{-2} passes through three polarisers, such that transmission axis of last polariser is crossed with the first. If intensity of emerging light is 3 W m^{-2}, what is the angle between the transmission axis of the first two polarisers ? At what angle will the intensity of transmitted light be maximum ?

Sol. Let θ_{12} be the angle between the transmission axes of the first two polarisers P_1 and P_2 and θ_{23}, the angle between the transmission axes of polarisers P_2 and P_3. Then,

$$\theta_{12} + \theta_{23} = 90° \qquad \text{...(}i\text{)}$$

Intensity of unpolarised light, $I_0 = 32 \text{ W m}^{-2}$

Let I_1, I_2 and I_3 be the intensities of light on passin through the first, second and third polariser respectivel Then,

$$I_1 = \frac{1}{2} I_0 = \frac{1}{2} \times 32 = 16 \text{ W m}^{-2}$$

According to Malus law, intensity of light on passin through second polariser,

$$I_2 = I_1 \cos^2 \theta_{12}$$

or $$I_2 = 16 \cos^2 \theta_{12} \qquad \text{...(}ii$$

and intensity of light on passing through third polarise

$$I_3 = I_2 \cos^2 \theta_{23} = [16 \cos^2 \theta_{12}] \cos^2 (90° - \theta_{12})$$

or $$I_3 = 16 \cos^2 \theta_{12} \sin^2 \theta_{12} = 4 \sin^2 2\,\theta_{12}$$

Since $I_3 = 3 \text{ W m}^{-2}$, we have

$$4 \sin^2 2\,\theta_{12} = 3 \quad \text{or} \quad \sin 2\,\theta_{12} = \frac{\sqrt{3}}{2}$$

or $$2\,\theta_{12} = 60° \quad \text{or} \quad \theta_{12} = 30°$$

For transmitted light (I_3) to be maximum,

$$\sin^2 2\,\theta_{12} = 1 \quad \text{or} \quad \sin 2\,\theta_{12} = 1$$

or $$2\,\theta_{12} = 90° \text{ or } \quad \theta_{12} = 45°$$

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Refraction of Light

ART I. TEXT-BASED QUESTIONS

1. **Velocity of light is maximum in**
(A) diamond (B) water
(C) glass (D) vacuum *(A.F.M.C. 1996)*

2. **The refractive index of water is 1·33. What will be the speed of light in water ?**
(A) 3×10^8 m s^{-1} (B) $2{\cdot}26 \times 10^8$ m s^{-1}
(C) 4×10^8 m s^{-1} (D) $1{\cdot}33 \times 10^8$ m s^{-1}
(C.B.S.E. 1996)

3. **Light travels through a glass plate of thickness t and having a refractive index μ. If c is the velocity of light in vacuum, the time taken by the light to travel this thickness of glass is**
(A) $t \mu c$ (B) $\frac{tc}{\mu}$
(C) $\frac{t}{\mu c}$ (D) $\frac{\mu t}{c}$ *(C.B.S.E. 1996)*

4. **Time taken by the sun light to pass through a glass slab of 4 mm thickness, whose refractive index is 1·5, is**
(A) 2×10^{-1} s (B) 2×10^{8} s
(C) 2×10^{-11} s (D) 2×10^{11} s
(A.F.M.C. 1997 ; C.B.S.E. 1993)

5. **Green light of wavelength 5460 Å is incident on an air-glass interface. If the refractive index of glass is 1·5, the wavelength of light in glass would be (Given velocity of light in air, $c = 3 \times 10^8$ m s^{-1})**
(A) 3640 Å (B) 5460 Å
(C) 4861 Å (D) None of the above.
(C.B.S.E. 1993, similar Karnataka Entrance, 1991)

6. **A beam of monochromatic light is refracted from vacuum into a medium of refractive index 1·5. The wavelength of refracted light will be**
(A) same.
(B) dependent on intensity of refracted light.
(C) larger. (D) smaller. *(C.B.S.E. 1992)*

7. **A glass slab of thickness 4 cm contains the same number of waves as 5 cm of water, when both are traversed by the same monochromatic light. If the refractive index of water is 4/3, what is the refractive index of glass ?**
(A) 5/3 (B) 5/4
(C) 16/15 (D) 1·5
(Karnataka Entrance, 1990)

8. **The lateral shift produced in a parallel sided glass slab depends on**
(A) the angle of incidence.
(B) the thickness of the glass slab.
(C) the refractive index of the material of the slab.
(D) all these. *(Karnataka Entrance, 1991)*

9. **The refractive index of air *w.r.t.* glass is 2/3. The refractive index of diamond *w.r.t.* air is 12/5. Then the refractive index of glass w.r.t. diamond will be**
(A) 5/8 (B) 8/9
(C) 5/18 (D) 18/5 *(C.B.S.E. 1993)*

10. **If ${}^i\mu_j$ represents refractive index, when a light ray goes from medium i to medium j, then the product ${}^2\mu_1 \times {}^3\mu_2 \times {}^4\mu_3$ is equal to**
(A) ${}^3\mu_1$ (B) ${}^3\mu_2$
(C) ${}^4\mu_1$ (D) ${}^4\mu_2$ *(C.B.S.E. 1990)*

11. **A transparent cube contains a small air bubble. Its apparent distance is 2 cm, when seen through one face and 5 cm when seen through other face. If the refractive index of the material of the cube is 1·5, the real length of the edge of the cube must be**
(A) 7 cm (B) 7·5 cm
(C) 10·5 cm (D) 4·67 cm
(C.B.S.E. 2000 ; C.P.M.T. 1992)

12. **A ray of light from a sodium lamp undergoes total internal reflection. The critical angle will be smallest, when it travels from**
(A) water to glass. (B) glass to air.
(C) glass to water. (D) water to air.
(Karnataka Entrance, 1987, 1985)

13. **Total internal reflection takes place, when light travels from**
(A) water to glass. (B) glass to diamond.
(C) water to air (D) air to mercury.
(C.E.T. 1998)

14. **The critical angle of light passing from glass to air is minimum for**
(A) red. (B) green.
(C) yellow. (D) violet.
(Karnataka Entrance, 1993)

15. **A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If μ is refractive index of the medium w.r.t. air, select the possible value(s) of μ from the following :**
(A) 1·3 (B) 1·4
(C) 1·5 (D) 1·6 *(I.I.T. 1998)*

16. **The sun becomes visible before the actual sunrise and remains visible even after the actual sunset. It is because of**
(A) scattering of light. (B) diffraction of light.
(C) refraction of light. (D) dispersion of light.

17. **The twinkling of stars is due to**
(A) periodic bursts of light from the star.
(B) interference between light from the sun and star.
(C) partial absorption of light in the atmosphere.
(D) refractive index fluctuation in the atmosphere.
(A.F.M.C. 1995)

18. **In optical fibres, the following principle is used :**
(A) Scattering (B) Successive reflections
(C) Refraction (D) Total internal reflection.
(Karnataka Entrance, 1994)

19. **Transmission of light in optical fibre is due to**
(A) scattering (B) diffraction
(C) polarisation
(D) multiple total internal reflection *(C.B.S.E. 2001)*

20. **Large aperture of lens results in**
(A) chromatic aberration.
(B) spherical aberration.
(C) curvature. (D) astigmatism.

21. **The spherical aberration in a lens may be minimised by designing the lens so that the deviation of rays is**
(A) minimum.
(B) minimum at first surface.
(C) minimum at second surface.
(D) equal at both surfaces.

22. **An equi-convex lens has focal length equal to the radius of curvature. Its refractive index is**

(A) 1·4 (B) 1·6
(C) 1·3 (D) 1·5 (*Karnataka, 1993 S*)

23. **What is the focal length of a double convex lens for which radius of curvature of the surfaces is 40 cm ($^a\mu_g = 1{\cdot}5$) ?**
(A) 50 cm (B) 40 cm
(C) – 30 cm (D) – 40 cm (*C.P.M.T. 1992*)

24. **A plano convex lens is made of refractive index 1·6. If the radius of curvature of the curved surface is 60 cm, then focal length of the lens is :**
(A) 50 cm (B) 100 cm
(C) 200 cm (D) 400 cm (*C.B.S.E. 1999*)

25. **If in a plano-convex lens, radius of curvature of convex surface is 10 cm and the focal length of the lens is 30 cm, the refractive index of the material of the lens will be**
(A) 1·5 (B) 1·66
(C) 1·33 (D) 3 (*M.N.R. 1988*)

26. **A thin symmetrical convex lens of refractive index 1·5 and radius of curvature 0·3 m is put in water of refractive index 4/3 . Its focal length is**
(A) 0·15 m (B) 0·30 m
(C) 0·60 m (D) 1·20 m (*Roorkee, 1992*)

27. **A concave lens of glass, refractive index 1·5, has both surfaces of the same radius of curvature R. On immersion in a medium of refractive index 1·75, it will behave as**
(A) convergent lens of focal length 3·05 R.
(B) convergent lens of focal length 3·0 R.
(C) divergent lens of focal length 3·5 R.
(D) divergent lens of focal length 3·0 R. (*I.I.T. 1999*)

28. **Focal length of a convex lens of refractive index 1·5 is 2 cm. Focal length of lens, when immersed in a liquid of refractive index of 1·25 will be**
(A) 10 cm (B) 7·5 cm
(C) 5 cm (D) 2·5 cm (*C.B.S.E. 1993*)

29. **A camera is focussed to take a close up picture of an insect at a distance of 5 cm from the camera lens. If the film is 10 cm away from the lens, what must be the focal length of the lens, expressed in cm ?**
(A) 3·3 cm (B) 10 cm
(C) 5 cm (D) 2 cm
(*Karnataka Entrance, 1992*)

30. **The graph drawn with object distance along abscissa and image distance as ordinate for a convex lens is**
(A) a straight line. (B) a circle.
(C) a parabola. (D) a rectangular hyperbola.
(*Karnataka Entrance, 1993*)

31. **Two plano-convex lenses, each having focal length of 0·4 m, are pressed against each other at their plane faces. This forms a double convex lens. At what distance from this lens must an object be placed to obtain a real, inverted image with magnification one ?**
(A) 0·8 m (B) 0·4 m
(C) 0·2 m (D) 1·6 m
(*Karnataka Entrance, 1989 ; N.C.E.R.T. 1980*)

32. **A film projector magnifies a 100 cm^2 film strip on a screen. If the linear magnification is 4, the area of the magnified film on the screen is**
(A) 1600 cm^2 (B) 400 cm^2
(C) 800 cm^2 (D) 200 cm^2
(*C.P.M.T. 1991 ; N.C.E.R.T. 1975*)

33. **The focal length of a convex lens is 30 cm and the size of image is quarter of the object. Then, the object distance is**
(A) 150 cm (B) 90 cm
(C) 60 cm (D) 30 cm (*A.F.M.C. 1995*)

34. **The distance between the object and the screen is D. A convex lens forms the image of the object on the screen in two positions separated by a distance *a*. The ratio between the size of the two images will be**
(A) D/*a* (B) D^2/*a*^{2}
(C) $\sqrt{D/a}$ (D) (D – *a*)^2/(D + *a*)^2
(*A.F.M.C. 1998*

35. **For a given distance between an object and an imag screen, magnification is 3 for a certain position of conve lens. On moving the lens through 0·2 m towards screen magnification is 1/3. Focal length of the lens is**
(A) 0·075 m (B) 0·01 m
(C) 1 m (D) 0·15 m (*Karnataka, 1995*

36. **An object is placed at a distance of *f* / 2 from a convex lens The image will be**
(A) at 3 *f* / 2, real and inverted.
(B) at 2 *f*, virtual and erect.
(C) at 2 *f*, real and inverted.
(D) at one of the foci, virtual and double its size.
(*C.P.M.T. 1989*

37. **Which of the following form(s) a virtual and erect imag for all positions of the object ?**
(A) convex lens (B) concave lens
(C) convex mirror (D) concave mirror
(*I.I.T. 1996*

38. **Two thin lenses of focal lengths f_1 and f_2 are placed in contact. The focal length of the composite lens will be**
(A) $(f_1 + f_2)/2$ (B) $(f_1 + f_2)/f_1 f_2$
(C) $\sqrt{f_1 f_2}$ (D) $f_1 f_2/(f_1 + f_2)$
(*A.I.I.M.S. 1998*

39. **Two lenses having powers + 6 D and – 4 D are placed in contact. The power of the combination is**
(A) – 2 D (B) – 4 D
(C) + 4 D (D) + 2 D (*A.F.M.C. 1989*)

40. **If a convex lens of focal length 80 cm and a concave lens of focal length 50 cm are combined together, what will be their resulting power ?**
(A) + 6·5 D (B) – 6·5 D
(C) + 7·5 D (D) – 7·5 D (*C.B.S.E. 1996*)

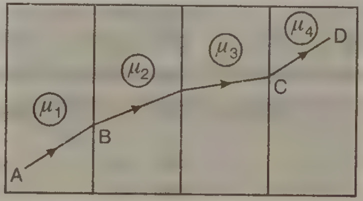
41. **Two lenses of power + 12 D and – 2 D are placed in contact. What will be the focal length of the combination ?**
(A) 10 cm (B) 12·5 cm
(C) 16·6 cm (D) 8·33 cm (*M.N.R. 1987*)

42. **A convex lens of power +6 D is placed in contact with a concave lens of power – 4 D. What will be the nature and focal length of this combination ?**
(A) concave, 25 cm (B) convex, 50 cm
(C) concave, 20 cm (D) convex, 100 cm
(*M.N.R. 1993*)

43. **A convex lens A of focal length 20 cm and a concave lens of focal length 5 cm are kept along the same axis with a distance *d* between them. If a parallel beam of light falling on A leaves B as a parallel beam, then the distance *d* (in cm) will be**
(A) 25 (B) 15
(C) 30 (D) 50
(*M.N.R. 1990 ; I.I.T. 1985*)

PART II. THOUGHT-BASED QUESTIONS

44. **A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in figure. The**



surfaces of all media are parallel. If the emerged ray CD is parallel to the incident ray AB, we must have

(A) $\mu_1 = \mu_2$ (B) $\mu_2 = \mu_3$
(C) $\mu_3 = \mu_4$ (D) $\mu_4 = \mu_1$ (*C.B.S.E. 2001*)

45. If a glass rod is put in a beaker containing a colourless liquid, the glass rod immediately seems to disappear. It is so, because

(A) the liquid and the glass have the same colour.
(B) the glass and the liquid have the same density.
(C) the glass and the liquid have the same refractive index.
(D) the glass reflects the light transmitted by the liquid.
(*Karnataka Entrance, 1987*)

46. A plane glass slab is kept over various coloured letters. The letter which appears least raised is

(A) blue (B) violet
(C) red (D) green (*B.H.U. 1998*)

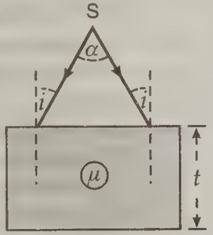
47. Air has refractive index 1·0003. The thickness of air column, which will have one more wavelength of yellow light (6000 Å) than in the same thickness of vacuum, is

(A) 2 mm (B) 2 cm
(C) 2 m (D) 2 km (*C.P.M.T. 1989*)

48. The wavelength of light diminishes μ times in a medium. A diver from inside water (μ = 1·33) looks at an object, whose natural colour is green. He sees the object as

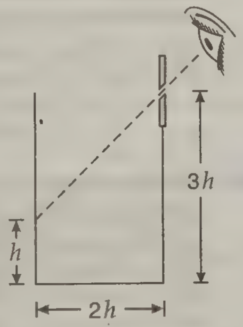
(A) green (B) blue
(C) yellow (D) red. (*C.P.M.T. 1990*)

49. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index μ, then the divergence angle of the emergent beam is



(A) zero
(B) α
(C) $\sin^{-1}(t/\mu)$
(D) $2\sin^{-1}(t/\mu)$ (*I.I.T. 2000*)

50. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is 3 h and its radius h. When the beaker is filled with a liquid up to a height 2 h, he can see the lower end of the rod. Then, the refractive index of the liquid is



(A) 5/2
(B) $\sqrt{5/2}$
(C) $\sqrt{3/2}$
(D) 3/2 (*I.I.T. 2002*)

51. A beam of light is converging towards a point I on a screen. A parallel plate of glass (thickness in the direction of beam t, refractive index μ) is introduced in the path of the beam. The convergence point is shifted by

(A) $t\left(1-\frac{1}{\mu}\right)$ away. (B) $t\left(1+\frac{1}{\mu}\right)$ away.

(C) $t\left(1-\frac{1}{\mu}\right)$ nearer. (D) $t\left(1+\frac{1}{\mu}\right)$ nearly.

(*M.N.R. 1987*)

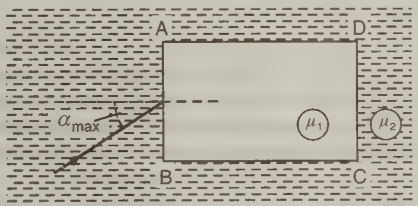
52. Light passing from air to glass is refracted, as is light passing from glass to air. However, when you look out of a window at the view outside, the light does not seem to have been distorted. This is because

(A) the angle of refraction is too small to observe.
(B) light incident upon the glass is partially reflected and this tends to mask the effect of refraction.
(C) the emergent ray is parallel to the incident ray and only lateral displacement occurs.
(D) the window pane is too thin for refraction to occur.
(*Karnataka Entrance, 1988*)

53. A person swimming at the bottom of a swimming pool looks up to the diving board. The board

(A) appears nearer. (B) appears farther.
(C) appears at the correct position.
(D) is not seen at all. (*Karnataka, 1992 S*)

54. A rectangular glass slab ABCD of refractive index μ_1 is immersed in water of refractive index μ_2 ($\mu_1 > \mu_2$). A ray of light is incident at the surface AB of the slab as shown.



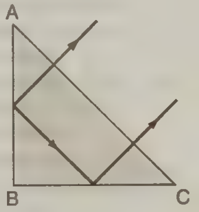
The maximum value of the angle of incidence α_{max}, such that the ray comes out only from the other surface CD is given by

(A) $\sin^{-1}\left[\frac{\mu_1}{\mu_2}\cos\left(\sin^{-1}\frac{\mu_2}{\mu_1}\right)\right]$

(B) $\sin^{-1}\left[\mu_1\cos\left(\sin^{-1}\frac{1}{\mu_2}\right)\right]$

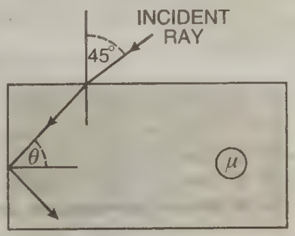
(C) $\sin^{-1}\frac{\mu_1}{\mu_2}$ (D) $\sin^{-1}\frac{\mu_2}{\mu_1}$

55. A ray of light falls on a right angled prism ABC (AB = BC) and travels as shown in the figure. The minimum refractive index of prism material should be



(A) 1·33
(B) $\sqrt{2}$
(C) 1·5
(D) 1·5 /1·33
(*Karnataka, 1992 S*)

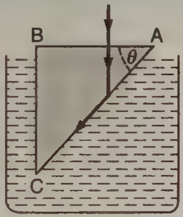
56. For the given incident ray as shown in the figure, the condition of total internal reflection of the ray will be satisfied, if the refractive index of block will be



(A) $(\sqrt{3}+1)/2$
(B) $(\sqrt{2}+1)/2$
(C) $\sqrt{3/2}$
(D) $\sqrt{7/6}$

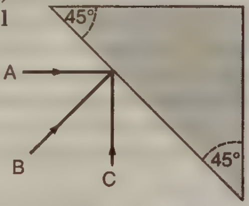
(*C.B.S.E. 2002*)

57. **A glass prism of refractive index 1·5 is immersed in water (μ = 4/3). A light beam incident normally on the face AB is totally reflected to reach the BC, if**



(A) $\sin\theta \geq 8/9$
(B) $2/3 < \sin\theta < 8/9$
(C) $\sin\theta \leq 2/3$
(D) none of the above.

58. **A, B and C in the diagram represent rays of light incident upon a face of a right-angled prism. Before emerging from the prism, which ray (or rays) will experience total internal reflection ?**



(A) A
(B) B
(C) C
(D) none of these.
(Karnataka Entrance, 1992, 1989)

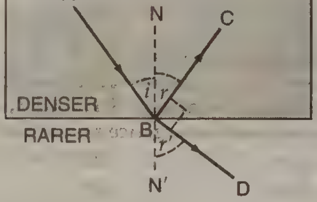
59. **A glass prism with refractive index 2 has refracting angle of 30°. One of the refracting face of the prism is polished. A beam of monochromatic light will retrace its path, if its angle of incidence over the refracting face of the prism is**
(A) 60° (B) 45°
(C) 30° (D) 0° *(B.H.U. 1999)*

60. **Refractive index of water is 5/3. A light source is placed in water at a depth of 4 m. Then what must be the maximum radius of disc placed on water surface so that the light of source can be stopped ?**
(A) 3 m (B) 4 m
(C) 5 m (D) ∞ *(C.B.S.E. 2001)*

61. **A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and fish is 12 cm below the surface of water, then the radius of the circular horizon is**
(A) 16 cm^2 (B) 9 cm^2
(C) 12 cm^2 (D) 13·6 cm^2 *(N.C.E.R.T. 1980)*

62. **A diver in swimming pool wants to signal his distress to a person lying on the edge of the pool by flashing his water proof flash light.**
(A) He must direct the beam vertically upward.
(B) He has to direct the beam horizontally.
(C) He has to direct the beam at an angle to the vertical, which is slightly less than the critical angle of incidence for total reflection.
(D) He has to direct the beam at an angle to the vertical, which is slightly more than the critical angle of incidence for total reflection. *(N.C.E.R.T. 1972)*

63. **A ray of light from denser medium strikes a rarer medium at an angle of incidence *i*. The reflected and refracted rays make an angle 90° with each other. The angles of reflection and refraction are *r* and *r′* as shown in the figure. The critical angle is**



(A) $\sin^{-1}(\tan r)$ (B) $\sin^{-1}(\tan i)$
(C) $\sin^{-1}(\tan r')$ (D) $\tan^{-1}(\sin i)$ *(I.I.T. 198*

64. **An air bubble inside the water behaves as :**
(A) a convex lens. (B) a concave lens.
(C) a plano convex lens. (D) a concave mirror.
(A.F.M.C. 1998 ; Karnataka Entrance, 1991, 198

65. **A lens behaves as a converging lens in air and a divergi lens in water. The refractive index of the material of t lens is**
(A) equal to unity. (B) equal to 1·33.
(C) between unity and 1·33.
(D) greater than 1·33. *(C.P.M.T. 1991 ; N.C.E.R.T. 197*

66. **A hollow double concave lens is made of very th transparent material. It can be filled with air or either two liquids L_1 and L_2 having refractive indices μ_1 and respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a paral beam of light, if it is filled with**
(A) air and placed in air.
(B) air and immersed in L_1.
(C) L_1 and immersed in L_2.
(D) L_2 and immersed in L_1. *(I.I.T. 200*

67. **A double convex lens of refractive index μ_1 is immers in a liquid of refractive index μ_2. The lens will act as**
(A) diverging lens, if $\mu_1 > \mu_2$.
(B) diverging lens, if $\mu_1 < \mu_2$.
(C) converging lens, if $\mu_1 > \mu_2$.
(D) converging lens, if $\mu_1 < \mu_2$. *(I.I.T. 199*

68. **A converging lens is used to form an image on a scree When the upper half of the lens is covered by an opaq screen,**
(A) half the image will disappear.
(B) complete image will be formed.
(C) intensity of image will decrease.
(D) intensity of image will increase.
(B.H.U. 1998 ; I.I.T. 198

69. **In order to include the effect of thickness *t* of the len following four modifications are suggested in the le equation. Which one accounts for the modificati correctly in your opinion ?**

(A) $-\frac{1}{u}+\frac{1}{v}=\frac{t}{f}$ (B) $-\frac{t}{u}+\frac{1}{v}=\frac{1}{f}$

(C) $-\frac{1}{u}+\frac{t}{v}=\frac{1}{f}$ (D) $-\frac{1}{u+t}+\frac{1}{v-t}=\frac{1}{f}$

70. **A beam of light is incident on a convex lens parallel to i principal axis. As one moves away from the optical cent of the lens on its other side, the intensity of light**
(A) continously increases.
(B) continuously decreases.
(C) first increases and then decreases.
(D) first decreases and then increases.

71. **A body is located on a wall. Its image of equal size is to b obtained on a parallel wall with the help of a convex len The lens is placed at a distance *d* ahead of second wa Then the required focal length will be**
(A) only $d/4$ (B) only $d/2$
(C) more than $d/4$ but less than $d/2$
(D) loss than $d/4$ *(C.B.S.E. 200*

72. **A lens is placed between a source of light and a wall. forms images of areas A_1 and A_2 on the wall for its tw different positions. The area of the source of light is**

(A) $\sqrt{A_1 A_2}$ (B) $\frac{A_1 + A_2}{2}$

(C) $\frac{A_1 - A_2}{2}$ (D) $\frac{1}{A_1} + \frac{1}{A_2}$ *(C.B.S.E. 1995)*

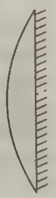
3. A luminous object is placed at a distance of 30 cm from the convex lens of focal length 20 cm. On the other side of the lens, at what distance from the lens, a convex mirror of radius of curvature 10 cm be placed in order to have an upright image of the object coincident with it ?
(A) 12 cm (B) 30 cm
(C) 50 cm (D) 60 cm *(C.B.S.E. 1998)*

74. The two surfaces of a convex lens are of same radius of curvature. It is cut into two equal parts by a plane perpendicular to its principal axis. If the power of the lens was originally D, then the power of the either portion will be
(A) D/4 (B) D/2
(C) D (D) 2 D

75. The two surfaces of a convex lens are of same radius of curvature. It is cut into two equal parts by a plane containing its principal axis. If the power of the lens was originally D, then the power of the either portion will be
(A) D/4 (B) D/2
(C) D (D) 2 D

76. A spherical surface of radius of curvature R separates air (refractive index 1·0) from glass (refractive index 1·5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and PO = OQ. The distance PO is equal to
(A) 5 R (B) 3 R
(C) 2 R (D) 1·5 R *(I.I.T. 1998)*

77. The plane face of the plano-convex lens of focal length 20 cm is silvered as shown in the figure. What type of mirror will it behave like and what will be its focal length (f) ?



(A) $f = 20$ cm (convex)

(B) $f = 20$ cm (concave)

(C) $f = 10$ cm (convex)

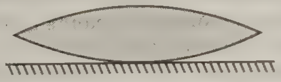
(D) $f = 10$ cm (concave)

78. A plano-convex lens is made of a material of refractive index $\mu = 1{\cdot}5$. The radius of curvature of curved surface of the lens is 20 cm. If its plane surface is silvered, the focal length of the silvered lens will be
(A) 10 cm (B) 20 cm
(C) 40 cm (D) 80 cm *(C.B.S.E. 2000)*

79. A plano convex lens acts like a concave mirror of focal length 20 cm, when its plane surface is silvered ; like a concave mirror of focal length 7 cm, when its convex surface is silvered. The refractive index of the lens material is
(A) 1·45 (B) 1·50
(C) 1·54 (D) 1·70 *(A.F.M.C. 1998)*

80. A thin lens of focal length f and its aperture has diameter d. It forms an image of intensity 3. Now the central part of aperture upto diameter $d/2$ is blocked by an opaque paper. The focal length and image intensity would change to
(A) $f/2$, I/2 (B) f, I/4
(C) $3f/4$, I/2 (D) f, 3 I/4 *(C.P.M.T. 1989)*

81. A convex lens is placed in contact with a mirror as shown in figure. If the space between them is filled with water, its power will



(A) decrease. (B) increase.
(C) remain unchanged.
(D) can increase or decrease depending on the focal length.

82. A beam of parallel rays is brought to focus by a plano-convex lens. A thin concave lens of same focal length is joined to the first lens. The effect of this is that
(A) the focal point shifts away from the lens by a small distance.
(B) the focus shifts to infinity.
(C) the focus remains undisturbed.
(D) the focus moves sidewards.

83. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical abberation
(A) is absent.
(B) is smaller, if the curved surface of the lens faces the object.
(C) is smaller, if the plane surface of the lens faces the object.
(D) is the same, whichever side of the lens faces the object. *(I.I.T. 1998)*

84. The 'f-number' of a given camera is 8. It means that
(A) the dimater of the stop of the camera is 0·08 m.
(B) the focal length of the lens of the camera is 8·0 m.
(C) the diameter of the stop is 0·125 of the focal length of the lens.
(D) none of the above. *(Dhanbad 1994)*

Chapter 2. Dispersion

ART I. TEXT-BASED QUESTIONS

1. A glass prism placed in a homogeneous transparent medium will deviate an incident ray
(A) always towards its base.
(B) always away from its base.
(C) towards its base, only if the medium has a refractive index greater than that of glass.
(D) towards the base, only if the medium has a refractive index less than that of glass.
(Karnataka Entrance, 1988)

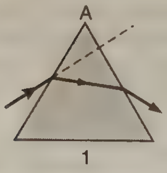
2. A ray of light passes through an equilateral prism, such that the angle of incidence is equal to the angle of emergence and the latter is equal to $\frac{3}{4}$ th the angle of prism. The angle of deviation is
(A) 45° (B) 39°
(C) 20° (D) 30° *(M.N.R. 1988)*

3. The refractive index of an equilateral prism is $\sqrt{3}$. What is the angle of minimum deviation ?
(A) 45° (B) 60°
(C) 37° (D) 30°
(C.B.S.E. 1999 ; Karnataka Entrance, 1992, 1988)

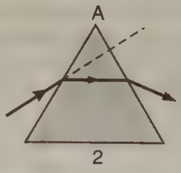
4. Angle of minimum deviation for a prism of refractive index 1·5 is equal to the angle of prism. The angle of prism is
(A) 62° (B) 41°
(C) 82° (D) 31°
(Given that cos 41° = 0·75)

5. Yellow light is refracted through a prism producing minimum deviation. If i_1 and i_2 denote the angle of incidence and emergence respectively for the prism, then
(A) $i_1 = i_2$ (B) $i_1 > i_2$
(C) $i_1 < i_2$ (D) $i_1 + i_2 = 90°$

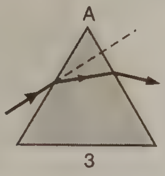
6. **The figures represent three cases of a ray passing through a prism of refractive edge A. The case corresponding to minimum deviation is**



1



2



3

(A) 1 (B) 2
(C) 3 (D) None of these.

7. **The refracting angle of a prism is A and the refractive index of the prism is cot A/2. The angle of minimum deviation is**
(A) 180° – 3A (B) 180° + 2A
(C) 90° – A (D) 180° – 2A (C.P.M.T. 1992)

8. **A prism splits a beam of white light into its seven constituent colours. This is so, because**
(A) phase of different colours is different.
(B) amplitude of different colours is different.
(C) energy of different colours is different.
(D) velocity of different colours is different.
(C.E.E.T. 1991)

9. **Refractive indices of a prism material for two wavelengths are 1·66 and 1·64. The dispersive power is**
(A) 0·02 (B) 0·012
(C) 0·01 (D) 0·03 (Karnataka, 1993 S)

10. **When a glass prism is placed inside water, its dispersive power**
(A) decreases. (B) increases.
(C) remains the same.
(D) may increase or decrease depending on the angle of prism.

11. **During dispersion of white light by prism placed in air,**
(A) only angular spread takes place.
(B) only angular deviation takes place.
(C) both angular deviation and angular spread take place.
(D) either angular spread or angular deviation for the mean colour takes place depending on the angle of the prism.
(Karnataka, 1995)

12. **A thin prism P_1 with angle 4° and made from glass of refractive index 1·54 is combined with another prism P_2 made from glass of refractive index 1·72 to produce dispersion without deviation. The angle of prism P_2 is**
(A) 5·33° (B) 4°
(C) 3° (D) 2·6° (I.I.T. 1990)

13. **The focal length of a converging lens is measured for violet, green and red colours. It is f_v, g_f and f_r respectively. We will get**
(A) $f_v = f_g$ (B) $f_g > f_r$
(C) $f_v < f_r$ (D) $f_v > f_r$ (C.B.S.E. 1997)

14. **If f_v and f_r are the focal lengths of a convex lens for violet and red light respectively and F_v and F_r are the respective focal lengths of a concave lens, then we must have**
(A) $f_v > f_r$ and $F_v < F_r$ (B) $f_v < f_r$ and $F_v < F_r$
(C) $f_v > f_r$ and $F_v > F_r$ (D) $f_v > f_r$ and $F_v > F_r$
(C.B.S.E. 1996)

15. **For achromatic combination of lenses, if we use two lenses of focal lengths f and f' and dispersive powers ω and ω' respectively, then**
(A) $\omega' = 2\omega, f' = 2f$ (B) $\omega' = 2\omega, f' = -2f$
(C) $\omega' = 2\omega, f' = f/2$ (D) $\omega' = 2\omega, f' = -f/2$
(C.P.M.T. 1982)

16. **Which of the following statement is correct ?**
(*i*) **Line spectra is characteristic of the atom.**
(*ii*) **Band spectra is characteristic of the molecule**
(A) only (*i*) is correct. (B) only (*ii*) is correct.
(C) Both (*i*) and (*ii*) are correct.
(D) Both (*i*) and (*ii*) are incorrect.

17. **Fraunhofer spectrum is a**
(A) line absorption spectrum.
(B) band absorption spectrum.
(C) line emission spectrum.
(D) band emission spectrum. (Karnataka, 19

18. **Which of the following phenomena is not explained Huygens' construction of wave-front ?**
(A) Refraction (B) Reflection
(C) Diffraction (D) Origin of spectra.
(C.B.S.E. 19

19. **The blue colour of the sky is due to the phenomenon**
(A) reflection (B) refraction
(C) scattering (D) dispersion (A.F.M.C. 19

20. **The sky appears blue, because**
(A) red light is absorbed.
(B) blue light is scattered the most.
(C) blue light is absorbed.
(D) it is its natural colour. (A.F.M.C. 19

21. **At sunrise or sunset, the sun looks more red than at m day, because**
(A) the sun is coldest at these times.
(B) of the effects of reflection and refraction.
(C) the sun is hottest at these times.
(D) of the scattering of light.
(A.F.M.C. 1995 ; C.E.E.T. 19

22. **Rainbows are formed by :**
(A) reflection and diffraction
(B) refraction and scattering
(C) dispersion and total internal reflection
(D) interference only. (C.B.S.E. 200

23. **Consider the following phenomena : Interferen dispersion, diffraction and reflection. Which phenome are involved in the formation of rainbow ?**
(A) reflection (B) diffraction
(C) interference and diffraction
(D) dispersion and reflection. (C.E.T. 199

PART II. THOUGHT-BASED QUESTIONS

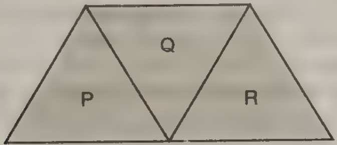
24. **If the light propagating along a straight line bends by small but fixed angle, it may be due to**
(A) reflection (B) refraction
(C) dispersion (D) diffraction

25. **Light appears to travel in straight lines, since**
(A) it is not absorbed by the atmosphere.
(B) it is reflected by the atmosphere.
(C) its wavelength is very small.
(D) its velocity is very large. (C.P.M.T. 199

26. **A ray is incident at an angle of incidence i on one surfa of a prism of small angle A and emerges normally from th opposite surface. If the refractive index of the material the prism is μ, the angle of incidence i is nealy equal to**
(A) A / μ (B) A / 2 μ
(C) μ A (D) μ A / 2 (C.B.S.E. 199

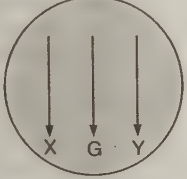
27. **A narrow beam of white light passes through a glass sla having parallel faces. Then,**
(A) the beam inside the slab remains as white light.
(B) the emergent beam is white light.
(C) the beam inside the slab undergoes dispersion.
(D) the glass slab never causes dispersion.

28. **A given ray of light suffers minimum deviation in an equilateral prisms P. Additional prisms Q and R of identical shape and of the same material as P are now added as shown in the figure. The ray will suffer**



(A) greater deviation
(B) no deviation
(C) same deviation as before
(D) total internal reflection (*I.I.T. 2001*)

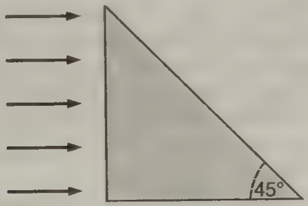
29. **The figure shows the view through the eye-piece of a prism spectrometer with its slit illuminated by the source of light emitting three wavelengths corresponding to yellow (Y), green (G) and unknown colour (X). The colour X may be**



(A) red
(B) orange
(C) pink
(D) none of the above.

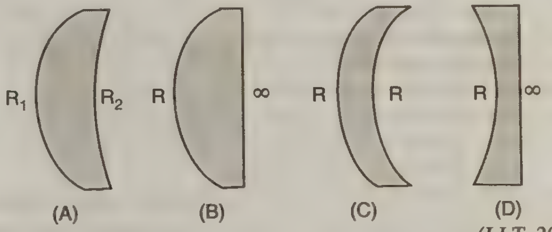
(*C.P.M.T. 1989*)

30. **A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1·39, 1·44 and 1·47 respectively. The prism will**



(A) separate part of the red colour from the green and blue colours.
(B) separate part of the blue colour from the red and green colours.
(C) separate all the three colours from one another.
(D) not separate even partially any colour from the other two colours. (*I.I.T. 1989*)

31. **Which one of the following spherical lenses does not exhibit dispersion ? The radii of curvature of the surfaces of the lenses are as given in the figure.**



(*I.I.T. 2002*)

32. **In case of a lens, which of the following quantities depend on the wavelengths of the incident light ?**
(A) Focal length (B) Radii of curvature
(C) Power (D) Chromatic aberration.

33. **A converging achromat is to be made by placing two lenses in contact. The proper choice is**
(A) convex lens of crown, concave lens of flint glass.
(B) convex lens of flint, concave lens of crown glass.
(C) both the lenses should be convex lenses of crown glass.
(D) both the lenses should be convex lenses of flint glass.

34. **In which one of the following regions of the electromagnetic spectrum will the vibrational motion of molecules give rise to absorption spectrum ?**
(A) ultraviolet (B) microwaves
(C) infrared (D) radiowaves.
(*S.C.R.A. 1994*)

35. **One cannot see through fog, because**
(A) fog absorbs light.
(B) the refractive index of fog is infinity.
(C) light suffers total internal reflection at the droplets in fog.
(D) light is scattered by the droplets in fog.
(*C.P.M.T. 1983*)

36. **The sky would appear red instead of blue, if**
(A) atmospheric particles scatter blue light more than red light.
(B) atmospheric particles scatter all colours equally.
(C) atmospheric particles scatter red light more than blue light.
(D) the sun was much hotter.
(*Karnataka Entrance, 1987, 1985*)

Chapter 3. Optical Instruments

PART I. TEXT-BASED QUESTIONS

1. **The focal length of the normal eye-lens is approximately**
(A) 2 cm (B) 25 cm
(C) 1 m (D) infinite

2. **A normal eye is unable to see objects closer than 25 cm (least distance of distinct vision). It is because,**
(A) the focal length of the eye-lens is 25 cm.
(B) the distance between the eye-lens and retina is 25 cm.
(C) the eye-lens cannot decrease its focal length beyond a limit.
(D) the eye-lens cannot decrease its distance from the retina beyond a limit.

3. **The final image produced by a simple microscope is**
(A) real and erect. (B) real and inverted.
(C) virtual and erect. (D) virtual and inverted
(*C.E.T. 1999*)

4. **If the least distance of distinct vision is 25 cm, then a convex lens of focal length 5 cm acts as a magnifier of magnifying power**
(A) 5 (B) less than 5
(C) 6 (D) more than 6.

5. **The focal length of the objective of a microscope is**
(A) greater than the focal length of eye-piece.
(B) less than the focal length of the eye- piece.
(C) equal to focal length of the eye-piece.
(D) arbitrary. (*M.N.R. 1994*)

6. **In a compound microscope, the intermediate image is**
(A) virtual, erect and magnified.
(B) real, erect and magnified.
(C) real, inverted and magnified.
(D) virtual, erect and reduced. (*I.I.T. 2000*)

7. **For relaxed eye, the magnifying power of a microscope is**
(A) $\frac{v_o}{u_o} \times \frac{D}{f_e}$ (B) $\frac{v_o}{u_o} \times \frac{f_e}{D}$
(C) $\frac{u_o}{v_o} \times \frac{D}{f_e}$ (D) $\frac{u_o}{v_o} \times \left(-\frac{D}{f_e}\right)$
(C.B.S.E. 1998)

8. **To increase the magnifying power of a telescope, one should increase**
(A) the focal length of the objective.
(B) the focal length of the eye-piece.
(C) aperture of the objective.
(D) aperture of the eye-piece.

9. **The convex lenses of focal lengths 0·3 m and 0·05 m are used to make a telescope. In normal adjustment, the distance between them is equal to**
(A) 0·35 m (B) 0·25 m
(C) 0·175 m (D) 0·15 m

10. **An observer looks at a tree of height 15 m with a telescope of magnifying power 10. To him, tree appears**
(A) 10 times taller. (B) 15 times taller.
(C) 10 times nearer. (D) 15 times nearer.
(N.C.E.R.T. ; C.P.M.T. 1975)

11. **An astronomical telescope of ten fold angular magnification has a length of 44 cm. The focal length of the object is**
(A) 4 cm (B) 40 cm
(C) 44 cm (D) 440 cm *(C.B.S.E. 1997)*

12. **The resolving power of a telescope depends upon**
(A) the focal length of the eye lens.
(B) the focal length of the object lens.
(C) the length of the telescope.
(D) the diameter of the object lens. *(N.C.E.R.T. 1983)*

13. **Large aperture telescopes are used for**
(A) greater resolution.
(B) greater magnification.
(C) reducing lens aberration.
(D) ease of manufacturer. *(C.P.M.T. 1989)*

14. **For observing a cricket match, binocular is preferred to terrestrial telescope for the reason that**
(A) the binocular is very easy to handle.
(B) the binocular provides three dimensional vision.
(C) the binocular produces image free of chromatic aberration.
(D) the binocular produces erect image.

15. **A reflecting telescope utilizes**
(A) a concave mirror. (B) a convex mirror.
(C) a prism. (D) a plano convex lens.
(C.E.T. 1998

PART II. THOUGHT-BASED QUESTIONS

16. **When we see an object, the image formed on the retina i**
(A) real (B) inverted
(C) virtual (D) erect.

17. **The distances of the human eye-lens from the retina is x. For a normal eye, the maximum focal length of the eye-len is**
(A) $x/2$ (B) x
(C) less than x (D) greater than x

18. **When we view objects lying at different distances from th eye, which of the following remain constant ?**
(A) The focal length of the eye-lens.
(B) The radii of curvature of the eye-lens.
(C) The object distance from the eye-lens.
(D) The image distance from the eye-lens.

19. **Myopia is due to**
(A) older age.
(B) change in focal length.
(C) shortening of eye ball.
(D) elongation of eye ball. *(A.F.M.C. 1996*

20. **When the length of a microscope tube is increased, it magnifying power**
(A) decreases. (B) increases.
(C) does not change. (D) cannot be predicted.
(M.N.R. 1986

21. **Magnifying power of a telescope is m. If the focal lengt of the eye-piece is doubled, then its magnifying power wil become**
(A) $2m$ (B) $m/2$
(C) $\sqrt{2}\,m$ (D) $3m$

22. **A fly is sitting on the objective of a telescope pointed t the moon. What effect is expected in the photograph of th moon taken through the telescope ?**
(A) The entire field of vision is blocked.
(B) There is an image of the fly on the photograph.
(C) There is no effect at all.
(D) The image of the moon is of lesser intensity.

23. **A telescope is focussed on the moon. If a tiny drop of in falls on its objective, then**
(A) moon will appear black.
(B) field of the view will be dark.
(C) there will be a dark spot in the field of view.
(D) brightness of the image will be slightly reduced.
(C.E.T. 1999

Chapter 4. Huygens' Principle

PART I. TEXT-BASED QUESTIONS

1. **Light has the following wave property :**
(A) Transverse. (B) Longitudinal.
(C) Sometimes longitudinal, sometimes transverse.
(D) Neither transverse nor longitudinal.
(Karnataka Entrance, 1994)

2. **Which of the following produces a plane wavefront ?**
(A) Point source (B) Line source
(C) Extended source (D) None of the above.

3. **As a plane wavefront propagates, its radius of curvature**
(A) decreases. (B) increases.
(C) first increases and then decreases.
(D) remains infinity.

4. **A plane wavefront is propagating in a medium. Which of the following is true ?**
(A) It propagates parallel to itself.
(B) It cannot propagate in the medium.
(C) It changes to spherical wavefront.
(D) It changes to cylindrical wavefront.

5. **Which of the following is a correct statement ? The ray o light**
(A) is always tangential to the wavefront.
(B) is always normal to the wavefront.
(C) does not exist in the Huygens' principle.
(D) may be tangential or normal to the wavefront.

6. **Which of the following does not support the wave nature of light ?**
(A) Interference (B) Diffraction
(C) Polarisation (D) Photoelectric effect.
(A.F.M.C. 1988)

7. **Huygens' wave theory of light cannot explain**
(A) diffraction. (B) interference.

(C) polarisation. (D) photoelectric effect. (A.F.M.C. 1993)

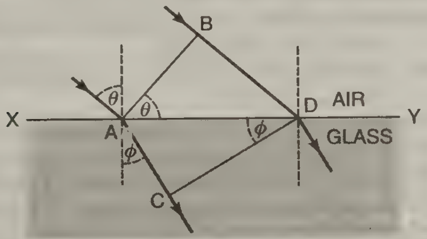
8. **Though quantum theory of light can explain a number of phenomena observed with light, it is necessary to retain the wave nature of light to explain the phenomenon of**
(A) photoelectric effect. (B) diffraction.
(C) Compton effect. (D) black body radiation.
(N.C.E.R.T. 1984)

9. **Huygens' concept of secondary waves**
(A) allows us to find the focal length of a thick lens.
(B) is a geometrical method to find a wave front.
(C) is used to determine the velocity of light.
(D) is used to explain polarisation. (A.F.M.C. 1998)

10. **Huygens' principle of secondary waves is used to**
(A) obtain the new position of wavefront geometrically.
(B) explain principle of superposition of waves.
(C) explain interference phenomenon.
(D) explain polarisation.

11. **Which one of the following phenomena is not explained by Huygens' construction of wave-front ?**
(A) Refraction (B) Reflection
(C) Diffraction (D) Origin of spectra.
(C.B.S.E. 1992)

12. **In the adjoining figure, a wavefront AB moving in air is incident on a plane glass surface XY. Its position CD, after refraction through a glass slab, is shown also along with the normals drawn at A and D. The refractive index of glass *w.r.t.* air will be equal to**



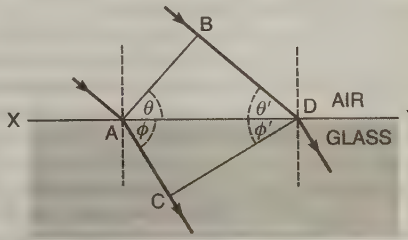
(A) $\frac{BD}{AC}$ (B) $\frac{AB}{CD}$

(C) $\frac{BD}{AD}$ (D) $\frac{AC}{AD}$ (C.P.M.T. 1986)

PART II. THOUGHT-BASED QUESTIONS

13. **Two points are situated at the same distance from a source of light but in opposite direction. The phase difference between the light waves from the two points will be**
(A) π (B) zero
(C) $\pi/2$ (D) none of the above.

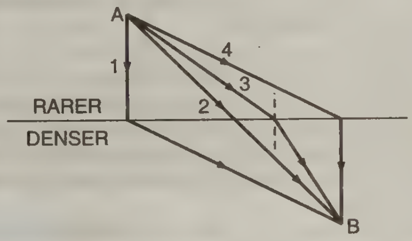
14. **In the adjoining diagram, a wave front AB moving in air is incident on a plane glass surface XY. Its position CD after refraction through a glass slab is shown along with the normals drawn at A and D. The refractive index of glass *w.r.t.* air ($^a\mu_g$) will be equal to**



(A) $\frac{\sin\theta}{\sin\theta'}$ (B) $\frac{\sin\theta}{\sin\phi'}$

(C) $\frac{\sin\phi}{\sin\theta}$ (D) $\frac{AB}{CD}$ (C.P.M.T. 1988)

15. **A ray travels the path from point A to B from a rarer to a denser medium. Of the four paths shown in the figure, which is the path for least time ?**



(A) 1 (B) 2
(C) 3 (D) 4

Chapter 5. Interference of Light

PART I. TEXT-BASED QUESTIONS

1. **One important similarity between sound and light waves is that both**
(A) can pass through even in the absence of any medium.
(B) are transverse waves.
(C) travel at the same speed in air.
(D) can show interference effects.
(*Karnataka Entrance, 1989*)

2. **Monochromatic light means**
(A) a beam of the same colour.
(B) a cluster of beam of different colours.
(C) a beam of white colour.
(D) none of these.

3. **The interference phenomenon can take place**
(A) in all waves.
(B) in transverse waves only.
(C) in longitudinal waves only.
(D) in standing waves only. (M.N.R. 1994)

4. **Two waves of the same frequency and same intensity are superimposed in the same phase. The intensity of the resultant wave at the central point will be**
(A) equal to that of the individual wave.
(B) twice that of the individual wave.
(C) three times that of the individual wave.
(D) four times that of the individual wave. (C.E.T. 1999)

5. **Two waves of the same frequency and intensity superimpose with each other in opposite phases. After the superposition, the**
(A) intensity increases by 4 times.
(B) intensity increases by 2 times.
(C) frequency increases by 4 times.
(D) none of these. (A.F.M.C. 1995)

6. **Ratio of intensities of two waves are given by 4 : 1. Then ratio of the amplitudes of the two waves is**
(A 2 : 1 (B) 1 : 2
(C) 4 : 1 (D) 1 : 4 (C.B.S.E. 1993)

7. **In Young's experiment, the ratio of maximum and minimum intensities in the fringe system is 9 : 1. The ratio of amplitudes of coherent sources is :**
(A) 9 : 1 (B) 3 : 1
(C) 2 : 1 (D) 1 : 1 (N.C.E.R.T 1990)

8. **Two waves have intensities in the ratio of 1 : 9. If these waves produce inteference, then the ratio of maximum and minimum intensities is**
(A) 3 : 1 (B) 4 : 1
(C) 9 : 1 (D) 16 : 1 *(A.F.M.C. 1998)*

9. **The intensity of light issuing out of two slits in Young's experiment is in the ratio 1 : 4. The intensity of the minimum to the maximum will be in the ratio**
(A) 1 : 2 (B) 1 : 4
(C) 1 : 9 (D) none of the above.

10. **The light waves from two coherent sources of intensity I interfere. At the minima, intensity of light is zero. The intensity of light at the maxima is**
(A) 4 I (B) I
(C) 2 I (D) I^2

11. **In Young's double slit experiment, the separation between the slits is halved and distance between the slits and screen is doubled. The fringe width is**
(A) unchanged. (B) halved.
(C) doubled. (D) quadrupled.
(I.I.T. 1981 ; similar Karnataka, 2000)

12. **How is interference pattern in double slit experiment affected, if a source of blue light is used in place of yellow light producing the same intensity ?**
(A) The fringe width will decrease.
(B) The fringe width will increase.
(C) The fringes will become brighter.
(D) The fringes will become fainter. *(C.B.S.E. 1992)*

13. **In a certain double slit experiment set up, interference fringes of width 1·0 mm each are observed with light of wavelength 5000 Å . Keeping the set up unaltered, if the source is replaced with one having a wavelength 6000 Å, the fringe width will be**
(A) 0·8 mm (B) 1·0 mm
(C) 1·2 mm (D) 1·44 mm *(C.E.T. 1998)*

14. **In a Young's experiment, two coherent sources are placed 0·90 mm apart and the fringes are observed 1 m away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of monochromatic light used would be**
(A) 60×10^{-4} cm (B) 10×10^{-4} cm
(C) 10×10^{-5} cm (D) 6×10^{-5} cm *(C.B.S.E. 1992)*

15. **In Young's double slit experiment, we get 15 fringes per cm on the screen, while using light of wavelength 5896 Å (sodium lamp). How many fringes per cm we get with light of wavelength 7370 Å ?**
(A) 18 (B) 15
(C) 12 (D) 10 *(A.F.M.C. 1998)*

16. **For constructive interference to take place between two monochromatic light waves of wavelength λ, the path difference should be**
(A) $(2n - 1)\lambda/4$ (B) $(2n - 1)\lambda/2$
(C) $n\lambda$ (D) $(2n + 1)\lambda/2$ *(M.N.R. 1992)*

PART II. THOUGHT-BASED QUESTIONS

17. **Four light waves are expressed as**
(i) $y_1 = a_1 \sin \omega t$, (ii) $y_2 = a_2 \sin 2\omega t$,
(iii) $y_3 = a_3 \sin (\omega t + \phi)$ and
(iv) $y_4 = a_4 \sin 2(\omega t + \phi)$.
The interference is possible between
(A) (*i*) and (*ii*) (B) (*i*) and (*iii*)
(C) (*ii*) and (*iv*) (D) (*iii*) and (*iv*).

18. **In double slit experiment, the two slits are illuminated with two independent identical sources of light. What happens to the interference pattern ?**
(A) No interference pattern is produced.
(B) The intensity of bright fringes gets doubled.
(C) The intensity of bright fringes becomes four times.
(D) Two sets of interference patterns are produced.

19. **The contrast in the fringes in any interference patter depends on**
(A) fringe width. (B) wavelength.
(C) intensity ratio of the sources.
(D) distance between the slits. *(Roorkee, 1992*

20. **In double slit experiment, when one of the two slits i covered :**
(A) the bright fringes become fainter.
(B) the fringe width decreases.
(C) there is a bright slit and no interference pattern i produced.
(D) there is uniform illumination all over the screen.

21. **In a Young's double slit experiment, two slits of equa widths are arranged symmetrically w.r.t. the source. Th intensity of the central bright fringe is I. If one of the slit is closed, the intensity at the centre of the screen will be**
(A) I (B) I/4
(C) I/2 (D) 2 I *(B.H.U. 1998*

22. **Two coherent monochromatic light beams of intensitie I and 4 I are superposed. The maximum and minimun possible intensities in the resulting beams are**
(A) 5 I and I (B) 9 I and I
(C) 5 I and 3 I (D) 9 I and 3 I *(I.I.T. 1982*

23. **In the Young's double slit experiment, the interferenc pattern is found to have an intensity ratio between brigh and dark fringes as 9. This implies that**
(A) the intensities at the screen due to two slits are 5 unit and 4 units respectively.
(B) the intensities at the screen due to slits are 4 units anc 1 unit respectively.
(C) the amplitude ratio is 3.
(D) the amplitude ratio is 2. *(I.I.T. 1982*

24. **Two beams of light having intensities I and 4I interfere t produce a fringe pattern on a screen. The phase differenc between the beams is $\pi/2$ at point A and π at point B. Ther the difference between the resultant intensities at A anc B is**
(A) 2 I (B) 4 I
(C) 5 I (D) 7 I *(I.I.T. 2001*

25. **In a double slit experiment, instead of taking slits of equa widths, one slit is made twice as wide as the other. Then in the interference pattern,**
(A) the intensities of both the maxima and the minim increase.
(B) the intensities of the maxima increases and the minim has zero intensity.
(C) the intensity of the maxima decreases and that of th minima increases.
(D) the intensity of the maxima decreases and the minim has zero intensity. *(I.I.T. 2000*

26. **In a Young's double slit experiment, 12 fringes are observe to be formed in a certain segment of the screen, when ligh of wavelength 600 nm is used. If the wavelength of ligh is changed to 400 nm, number of fringes observed in th same segment of the screen is given by**
(A) 12 (B) 18
(C) 24 (D) 30 *(I.I.T. 2001*

27. **Interference was observed in interference chamber, wher air was present. Now, the chamber is evacuated and if th same light is used, a careful observer will see**
(A) no interference.
(B) interfertence with bright bands.
(C) interference with dark bands.
(D) interference, in which width of the fringe will be slightl increased. *(C.B.S.E. 1993*

28. **In Young's double slit experiment ; if a thin glass plate is placed in the path of one of the interfering beams, then**
(A) fringe width decreases.
(B) fringe width increases.
(C) fringe pattern is shifted.
(D) fringe pattern is unaffected. *(Karnataka, 1992 S)*

29. **In an ideal double-slit experiment, when a glass plate (refractive index 1·5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred pareviously remains unchanged. The minimum thickness of the glass plate is**
(A) 2λ (B) $2\lambda/3$
(C) $\lambda/3$ (D) λ. *(I.I.T. 2002)*

30. **What happens, if monochromatic light used in Young's double slit experiment is replaced by white light ?**
(A) The bright fringes become white.
(B) The central fringe is white and all other are coloured.
(C) All fringes are coloured.
(D) No fringes are observed.

31. **White light is used to obtain interference fringes in Young's double slit experiment. Then,**
(A) the central fringe will be white.
(B) the fringe next to the central fringe will be violet.
(C) the fringe next to the central fringe will be red.
(D) no dark fringe will be observed.

32. **In a Young's double slit experiment, the fringe width is found to be 0·4 mm. If the whole apparatus is immersed in water of refractive index 4/3 without disturbing the geometrical arrangement, the new fringe width will be**
(A) 0·3 mm (B) 0·4 mm
(C) 0·53 mm (D) 540 microns.
(C.B.S.E. 1991)

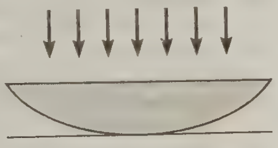
33. **In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If separation between the slits is 10^{-3} m, the wave- length of light used is**
(A) 6000 Å (B) 5000 Å
(C) 3000 Å (D) 4500 Å *(Roorkee, 1992)*

34. **White light is used to illuminate the two slits in a Young's double slit experiment. The separation between the slits is d and the screen is at a distance D ($>>d$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of the missing wavelengths are**

(A) $\lambda = \dfrac{d^2}{D}$ (B) $\lambda = \dfrac{2d^2}{D}$

(C) $\lambda = \dfrac{d^2}{3D}$ (D) $\lambda = \dfrac{2d^3}{3D}$ *(I.I.T. 1984)*

35. **A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat glass plate as shown in figure. The observed interference fringes from this combination shall be**
(A) straight.
(B) circular.
(C) equally spaced.
(D) having fringe spacing, which increases as we go outwards.



(I.I.T. 1999)

36. **In a Fresnel biprism experiment, the two positions of lens give separation between the slits as 16 cm and 9 cm respectively. What is the actual distance of separation ?**
(A) 12·5 cm (B) 12 cm
(C) 13 cm (D) 14 cm
(B.H.U. 1999 ; C.B.S.E. 1998)

37. **Which of the following is conserved, when light waves interfere ?**
(A) Intensity (B) Energy
(C) Amplitude (D) Momentum.

38. **Colours appear on a thin soap film and soap bubbles due to the phenomenon of**
(A) interference. (B) scattering.
(C) diffraction. (D) dispersion.
(C.B.S.E. 1999 ; A.F.M.C. 1997, 1995 ; N.C.E.R.T. 1990)

Chapter 6 Diffraction of Light

PART I. TEXT-BASED QUESTIONS

1. **Bending of light about the corner of a sharp edged obstacle is called**
(A) deviation. (B) dispersion.
(C) polarisation. (D) diffraction. *(A.F.M.C. 1995)*

2. **While both light and sound show wave character, diffraction is much harder to observe in light. This is because :**
(A) light does not require a medium.
(B) wavelength of light is far smaller.
(C) waves of light are transverse.
(D) speed of light is far greater. *(Karnataka Entrance, 1994)*

3. **The frequency of e.m. wave which is best suited to observe a particle of radius 3×10^{-4} cm, is of the order of**
(A) 10^{15} (B) 10^{13}
(C) 10^{14} (D) 10^{12} *(C.B.S.E. 1993)*

4. **The width of diffraction fringes varies :**
(A) directly as the distance between slit and the screen.
(B) inversely as the wavelength.
(C) directly as the width of the slit.
(D) none of the above.

5. **In the diffraction pattern due to single slit of width a with incident light of wavelength λ with angle of diffraction θ, the condition for the first minimum is**
(A) $\lambda \sin\theta = a$ (B) $a \cos\theta = \lambda$
(C) $a \sin\theta = \lambda$ (D) $\lambda \cos\theta = a$
(Karnataka, 1990 S)

6. **The first diffraction minima due to a single slit diffraction is at $\theta = 30°$ for a light of wavelength 5000 Å . The width of slit is**
(A) 5×10^{-5} cm (B) 10×10^{-5} cm
(C) $2{\cdot}5 \times 10^{-5}$ cm (D) $1{\cdot}25 \times 10^{-5}$ cm
(C.P.M.T. 1985)

7. **A single slit diffraction pattern is obtained using a beam of red light. What happens, if the red light is replaced by blue light ?**
(A) There is no change in the diffraction pattern.
(B) Diffraction fringes become narrow and crowded together.
(C) Diffraction fringes become broader and farther apart.
(D) Diffraction pattern disappears.
(C.P.M.T. 1984 ; A.I.I.M.S. 1982)

8. **Two points separated by a distance of 0·1 mm can just be inspected in a microscope, when light of wavelength 6000 Å is used. If the light of wavelength 4800 Å is used, the limit of resolution will become**
(A) 0·8 mm (B) 0·12 mm
(C) 0·1 mm (D) 0·08 mm *(C.P.M.T. 1988)*

9. **The limit of resolution of a 100 cm telescope for light of wavelength $5{\cdot}5 \times 10^{-7}$ m is**
(A) 0·14″ (B) 0·3″
(C) 1′ (D) 1° *(B.H.U. 1993)*

10. **The limit of resolution of one metre radio telescope for radio waves of wavelength 10 cm is about**
(A) 7° (B) few seconds of arc.
(C) 18° (D) one minute of arc.
(Karnataka Entrance, 1992)

PART II. THOUGHT-BASED QUESTIONS

11. **Ray optics is valid, when characteristic dimensions are**
(A) much smaller than the wavelength of light.
(B) much larger than the wavelength of light.
(C) of the same order as the wavelength of light.
(D) of the order of one millimetre.
(Pre-Medical/Dentral, 1994)

12. **The interference phenomenon differs from diffraction in that unlike diffraction,**
(A) interference fringes are of varying intensity.
(B) interference cannot be observed with white light.
(C) interference minima may be perfectly dark.
(D) interference fringes are of unequal width.

13. **Yellow light is used in a single slit diffraction experiment with slit width of 0·6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal**
(A) that the central maximum is narrower.
(B) more number of fringes.
(C) less number of fringes.
(D) no diffraction pattern. *(I.I.T. 1999)*

14. **A beam of light of wavelength 600 nm from a distant source falls on a single slit 1·00 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is**
(A) 1·2 cm (B) 1·2 mm
(C) 2·4 cm (D) 2·4 mm *(I.I.T. 1994)*

15. **A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is**
(A) 0 (B) $\pi/2$
(C) π (D) 2π *(I.I.T. 1998)*

16. **The amplitude modulated waves (AM) used in ratio broadcasting bend appreciably round the corners of a 1 m × 1 m wall but the frequency modulated waves (FM) bend only negligibly. If the average wavelengths of AM and FM waves are λ_1 and λ_2, then**
(A) $\lambda_1 > \lambda_2$ (B) $\lambda_1 = \lambda_2$
(C) $\lambda_1 < \lambda_2$ (D) cannot be predicted.

17. **The resolution limit of the eye is 1 minute. At a distance x km from the eye, two persons stand with a lateral separation of 3 m. For the two persons to be just resolved by the naked eye, x should be**
(A) 10 km (B) 15 km
(C) 20 km (D) 30 km *(C.P.M.T. 1990)*

18. **Diameter of human eye lens is 2 mm. What will be the minimum distance between two points to resolve them, which are situated at a distance of 50 m from eye. The wavelength of light is 5000 Å.**
(A) 2·32 m (B) 4·28 mm
(C) 1·25 cm (D) 12·48 cm *(C.B.S.E. 2002)*

19. **Scattering of light is a direct consequence of**
(A) interference. (B) dispersion.
(C) diffraction. (D) reflection.

20. **The aperture of the largest telescope in the world is of the order of 5 m. If the separation between the moon and the earth is 4×10^5 km and the wavelength of visible light is of the order of 5000 Å, then the minimum separation between objects on the surface of the moon, which can be just resolved is**
(A) 1 m approximately.
(B) 10 m approximately.
(C) 50 m approximately.
(D) 200 m approximately. *(C.P.M.T. 1989)*

Chapter 7 Polarisation of Light

PART I. TEXT-BASED QUESTIONS

1. **The phenomenon that requires transverse wave nature of light is**
(A) iffraction. (B) polarisation.
(C) interference. (D) dispersion.
(Karnataka, 1988)

2. **Light waves are transverse, because they**
(A) get reflected. (B) get refracted.
(C) get polarised. (D) do not get polarised.
(A.F.M.C. 1996)

3. **Waves that cannot be polarized are**
(A) longitudinal. (B) transverse.
(C) electromagnetic. (D) light.
(A.F.M.C. 1997 ; Karnataka Entrance, 1988, 1984)

4. **When a polaroid is rotated, the intensity of light is not found to vary. The incident light may be**
(A) completely plane polarised.
(B) partially plane polarised.
(C) unpolarised. (D) None of the above.

5. **The plane of vibration and the plane of polarisation of a beam of light**
(A) are identical to each other.
(B) are orthogonal to each other.
(C) make an angle, which depends on the colour of the light.
(D) rotate with respect of each other along the path of the beam. *(Karnataka Entrance, 1986)*

6. **Polarised light can be produced by :**
(A) dispersion. (B) scattering.
(C) interference. (D) diffraction.
(A.F.M.C. 1998)

7. **When the light is incident at the polarising angle, which of the following is completely polarised ?**
(A) Reflected light. (B) Refracted light.
(C) Both reflected and refracted light.
(D) Neither reflected nor refracted light.

8. **At the polarising angle, the angle between reflected and refracted rays is**
(A) 0° (B) 90°
(C) 180° (D) 60° *(Karnataka, 1991 S)*

9. **A ray of light is incident on the surface of a glass plate at an angle of incidence equal to Brewster's angle p. If μ is refractive index of glass *w.r.t.* air, then angle between the reflected and the refracted rays is**
(A) $90° + p$ (B) $\sin^{-1}(\mu \cos p)$
(C) 90° (D) $90° - \sin^{-1}\left(\dfrac{\sin p}{\mu}\right)$
(C.P.M.T. 1989)

10. The expression relating polarising angle and refractive index is
(A) $\mu \sin p = 1$ (B) $\mu \cot p = 1$
(C) $\mu \tan p = 1$ (D) $\mu \cos p = 1$
(Karnataka Entrance, 1989, 1984)

11. A beam of the light is partially reflected and partially refracted from a surface of refractive index $\sqrt{3}$. The angle between the reflected and refracted lights is 90°. The angle of incidence must be
(A) 50° (B) 60°
(C) 78° (D) 75°
(Karnataka Entrance, 1994)

12. Which of the following materials may be used to produce polaroids ?
(A) Tourmaline (B) Calcite
(C) Quartz
(D) Crystals of quinine iodosulphate.

13. The property of rotating the plane of polarisation is known as
(A) optical activity. (B) optical rotation.
(C) dichroism. (D) specific rotation.
(Karnataka, 1991 S)

14. The specific rotation of an optically active substance of length l cm and concentration c, with an optical rotation θ, is
(A) $\frac{l\,c}{10\,\theta}$ (B) $\frac{\theta\,l\,c}{10}$
(C) $\frac{10\,\theta}{l\,c}$ (D) $\frac{10}{\theta\,l\,c}$
Here, in the unit of specific rotation, the length of solution is expressed in decimetre (dm).
(Karnataka Entrance, 1987, 1985, 1984)

15. A decimetre length of solution with concentration 500 kg m^{-3} rotates the plane of polarization by 24°. What is the specific rotation of the solution ?
(A) 48° dm^{-1} g^{-1} cm^3 (B) 30° dm^{-1} g^{-1} cm^3
(C) 12° dm^{-1} g^{-1} cm^3 (D) 50° dm^{-1} g^{-1} cm^3
(Karnataka Entrance, 1990)

16. The specific rotation of a liquid of length 10 cm, concentration 2 g cm^{-3} is 40° dm^{-1} g^{-1} cm^3. The angle of rotation is
(A) 10° (B) 70°
(C) 80° (D) 90°
(Karnataka Entrance, 1984)

PART II. THOUGHT-BASED QUESTIONS

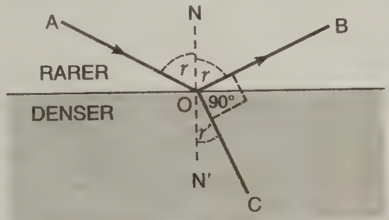
17. When a polaroid is rotated, the intensity of light varies and corresponding to a particular orientation, intensity reduces to zero. It shows that incident light is
(A) completely plane polarised.
(B) partially plane polarised.
(C) unpolarised.
(D) None of the above.

18. When a polaroid is rotated, the intensity of light varies but never reduces to zero. It shows that the incident light is
(A) completely polarised.
(B) partially plane polarised.
(C) unpolarised.
(D) None of the above.

19. Suppose p is polarising angle for a transparent medium and the speed of light in the medium is v. Then, according to Brewster's law
(A) $p = \cot^{-1} v/c$ (B) $p = \cos^{-1} v/c$
(C) $p = \sin^{-1} v/c$ (D) $p = \text{cosec}^{-1} v/c$
(Karnataka Entrance, 1987)

20. If the speed of light and the polarising angle for a given medium are c and p respectively, then from Brewster's law, we find
(A) $c = v \text{ cosec } p$ (B) $c = v \tan p$
(C) $c = v \cos p$ (D) $c = v \cot p$
(Karnataka Entrance, 1985)

21. A ray of light from a rarer medium strikes a denser medium as shown in figure. The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r'. The critical angle would be



(A) $\sin^{-1} (\tan r)$ (B) $\tan^{-1} (\sin r)$
(C) $\sin^{-1} (\tan r')$ (D) $\tan^{-1} (\sin r')$
(C.B.S.E. 1996)

22. In the propagation of light waves, the angle between the direction of propagation and the plane of polarisation is
(A) 0° (B) 45°
(C) 90° (D) 180° *(C.P.M.T. 1978)*

ANSWERS TO MULTIPLE CHOICE QUESTIONS

Chapter 1 Refraction of Light

1. (D) **2.** (B) **3.** (D) **4.** (C) **5.** (A) **6.** (D) **7.** (A) **8.** (D) **9.** (A) **10.** (C)
11. (C) **12.** (B) **13.** (C) **14.** (D) **15.** (B) **16.** (C) **17.** (D) **18.** (D) **19.** (D) **20.** (B)
21. (D) **22.** (D) **23.** (B) **24.** (B) **25.** (C) **26.** (D) **27.** (A) **28.** (C) **29.** (A) **30.** (D)
31. (B) **32.** (A) **33.** (A) **34.** (D) **35.** (A) **36.** (D) **37.** (B & C) **38.** (D) **39.** (D) **40.** (D)
41. (A) **42.** (B) **43.** (B) **44.** (D) **45.** (C) **46.** (C) **47.** (A) **48.** (B) **49.** (B) **50.** (B)
51. (A) **52.** (C) **53.** (B) **54.** (A) **55.** (B) **56.** (C) **57,** (A) **58.** (B) **59.** (D) **60.** (A)
61. (D) **62.** (C) **63.** (A) **64.** (B) **65.** (C) **66.** (D) **67.** (B & C) **68.** (B & C) **69.** (D)
70. (C) **71.** (B) **72.** (A) **73.** (C) **74.** (B) **75.** (C) **76.** (A) **77.** (D) **78.** (C) **79.** (C)
80. (D) **81.** (B) **82.** (B) **83.** (B) **84.** (C)

Chapter 2 Dispersion

1. (D) **2.** (D) **3.** (B) **4.** (C) **5.** (A) **6.** (B) **7.** (D) **8.** (D) **9.** (D) **10.** (A)
11. (C) **12.** (C) **13.** (C) **14.** (D) **15.** (B) **16.** (C) **17.** (A) **18.** (D) **19.** (C) **20.** (B)
21. (D) **22.** (C) **23.** (D) **24.** (A & B) **25.** (C) **26.** (C) **27.** (B & C) **28.** (C) **29.** (D) **30.** (A)
31. (C) **32.** (A, C & D) **33.** (A) **34.** (B) **35.** (D) **36.** (C)

Chapter 3 Optical Instruments

1. (A) 2. (C) 3. (C) 4. (C) 5. (B) 6. (C) 7. (A) 8. (A) 9. (A) 10. (C)
11. (B) 12. (D) 13. (A) 14. (B) 15. (A) 16. (A & B) 17. (B) 18. (D) 19. (D) 20. (B)
21. (B) 22. (D) 23. (D)

Chapter 4 Huygens' Principle

1. (A) 2. (D) 3. (D) 4. (A) 5. (B) 6. (D) 7. (D) 8. (B) 9. (B) 10. (A)
11. (D) 12. (A) 13. (B) 14. (B) 15. (C)

Chapter 5 Interference of Light

1. (D) 2. (A) 3. (A) 4. (D) 5. (D) 6. (A) 7. (C) 8. (B) 9. (C) 10. (A)
11. (D) 12. (A) 13. (C) 14. (D) 15. (C) 16. (C) 17. (B & C) 18. (A) 19. (C) 20. (C)
21. (B) 22. (B) 23. (B & D) 24. (B) 25. (C) 26. (B) 27. (D) 28. (C) 29. (A) 30. (B)
31. (A, B & D) 32. (A) 33. (A) 34. (A & C) 35. (A) 36. (B) 37. (B) 38. (A)

Chapter 6 Diffraction of Light

1. (D) 2. (B) 3. (C) 4. (A) 5. (C) 6. (B) 7. (B) 8. (A) 9. (A) 10. (A)
11. (B) 12. (C) 13. (D) 14. (B) 15. (D) 16. (A) 17. (A) 18. (C) 19. (C) 20. (C)

Chapter 7 Polarisation of Light

1. (B) 2. (C) 3. (A) 4. (C) 5. (B) 6. (B) 7. (A) 8. (B) 9. (C) 10. (B)
11. (B) 12. (D) 13. (A) 14. (C) 15. (A) 16. (C) 17. (A) 18. (B) 19. (A) 20. (B)
21. (C) 22. (A)

UNIT
VII

DUAL NATURE OF MATTER & RADIATION

1. **Particle Nature of Radiation**
2. **Wave Nature of Matter**

"...... the whole procedure was an act of despair because a theoretical interpretation had to be found at any price, no matter how high that might be." **– Max Planck**

CHAPTER

1

Particle Nature of Radiation

1.01. DUAL NATURE OF RADIATION

The phenomena such as interference, diffraction and polarisation were success- ly explained on the basis of wave nature of light. On the other hand, photoelectric fect, Compton effect, etc can be explained on the basis of quantum nature of diation. For instance, in photoelectric effect, when a photon of radiation strikes netal surface, it gives up all its energy to a single electron in an atom and the ctron is knocked out of the metal. It carries a part of the energy of the incident oton. In Compton effect, when an X-ray photon is incident on a free electron, the ctron recoils along a definite direction with some energy depending upon the rection along which the incident photon is scattered. From these two effects, it pears as if *a particle (photon of radiation) is colliding against another particle (electron).* ence, it became necessary to assume that in photoelectric effect and Compton fect, *radiation exhibits particle nature.*

> Quotable Quote
>
> *"If one does not feel dizzy when discussing the implications of Planck's constant h it means that one does not know what one is talking about."* – **Neils Bohr**

The various phenomena concerning radiation can be divided into three parts :

1. *The phenomena such as interference, diffraction, polarisation, etc in which teraction of radiation takes places with radiation itself.* Such phenomena can be plained on the basis of ***electromagnetic (wave) nature of radiation only.***

2. *The phenomena such as photoelectric effect, Compton effect, etc in which interaction radiation takes place with matter.* Such phenomena can be explained on the basis ***quantum (particle) nature of radiation.***

3. *The phenomena such as rectilinear propagation, reflection, refraction, etc in which e interaction of radiation takes place neither with itself, nor with matter.* Such enomena can be explained on the basis of either of the two natures of the diation.

It may be pointed out that in a particular experiment, radiation has a particular ture *i.e. either it possesses wave nature or particle nature.*

1.02. PHOTON

A photon is a packet of energy. It possesses energy given by

$$E = h\nu,$$

here h ($= 6{\cdot}62 \times 10^{-34}$ J s) is the Planck's constant and ν is the frequency of the oton. If λ is the wavelength of the photon, then

$$c = \nu\lambda$$

Here, $c = 3 \times 10^{8}$ m s^{-1} is velocity of light.

Therefore, $$E = h\nu = \frac{hc}{\lambda} \qquad \text{...(1.01)}$$

Energy of a photon is usually expressed in **electron volt (eV).**

$$\mathbf{1\ eV = 1{\cdot}6 \times 10^{-19}\ J}$$

The bigger units are keV and MeV.

$$\mathbf{1\ keV = 1{\cdot}6 \times 10^{-16}\ J \text{ and } 1\ MeV = 1{\cdot}6 \times 10^{-13}\ J}$$

> **Key point** ✓
>
> A photon is not a material particle. It is a quanta of energy.

1.03. PHOTOELECTRIC EFFECT

Hallwach discovered that an insulated zinc plate connected to a gold leaf ectroscope and charged negatively lost its charge, when a beam of ultraviolet light as directed on the plate. In order to explain this observation, Hallwach suggested at the metal surface loses negative charge due to ejection of electrons from its rface by the ultraviolet light. The effect was termed as ***photoelectric effect.***

The phenomenon of ejection of electrons from a metal surface, when light of sufficiently gh frequency falls upon it, is known as the photoelectric effect.

The electrons so emitted were called ***photoelectrons.*** J.J.Thomson showed that e photoelectrons were not different from the ordinary electrons.

> **Key point** ✓
>
> Photoelectric effect demonstrates particle nature of radiation.

1.04. EXPERIMENTAL STUDY OF PHOTOELECTRIC EFFECT

In 1900, Lenard studied the photoelectric effect experimentally. Fig. 1.01 shows the experimental arrangement used to study the photoelectric effect.

The apparatus consists of an evacuated glass tube fitted with two electrodes. The electrode E is called the *emitting electrode* and the other electrode C is called the *collecting electrode*. A varying potential difference can be applied across the two electrodes.

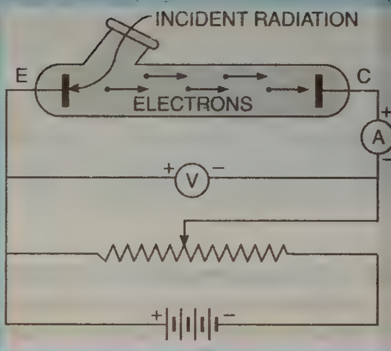


Fig. 1.01

When a suitable radiation is incident on the electrode E, electrons are ejected from it. The electrons, which have sufficient kinetic energy, reach the electrode C despite its negative polarity. The potential difference between the two electrodes acts as the retarding potential. As the collecting electrode is made more and more negative, fewer and fewer electrons will reach the cathode and the photo-electric current recorded by the ammeter will fall. In case the retarding potential equals V_0, called the ***stopping potential,*** no electron will reach the cathode and the current will become zero. In such a case, the work done by the stopping potential is equal to the maximum kinetic energy of the electrons *i.e.*

$$e\,V_0 = \frac{1}{2} m\, v_{max}^2 \qquad \text{...(1.02)}$$

As *e*, the charge on electron, is constant,

$$V_0 \propto \frac{1}{2} m\, v_{max}^2$$

Thus, the stopping potential gives the estimate of the maximum kinetic energy, with which the photoelectrons may be emitted.

Key point

The stopping potential (in volt) is numeri cally equal to the kinetic energy of photo electrons in eV.

Laws of photoelectric emission. Various experimental observations led to various conclusions, which became known as the laws of photoelectric emission, as explained below :

1. *The emission of photoelectrons takes place only when the frequency of the incident radiation is above a certain critical value, characteristic of that metal.* The critical value of frequency is known as the *threshold frequency* for the metal of the emitting electrode.

Suppose that when light of certain frequency is incident over a metal surface, the photoelectrons are emitted. To make the photoelectric current zero, a particular value of the stopping potential will be needed. If we go on reducing the frequency of incident light, the value of the stopping potential will also go on decreasing [Fig. 1.02]. At a certain value of frequency ν_0, the photoelectric current will become zero, even when no retarding potential is applied. This frequency ν_0 corresponds to the threshold frequency for the metal surface. The emission of photoelectrons does not take place till the frequency of incident light is below this value.

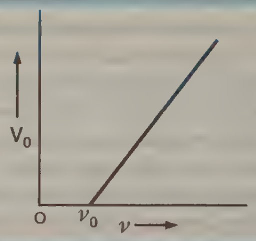


Fig. 1.02

Watch out !

If light were waves (not photons), it wil take about an year to eject a photoelectror out of the metal surface !

2. *The emission of photoelectrons starts as soon as light falls on metal surface.*

It has been found that the time lag between the incidence of photon and the emission of electron is less than 10^{-8} s.

3. *The maximum kinetic energy with which an electron is emitted from a metal surface is independent of the intensity of the light and depends only upon its frequency.*

When light of frequency ν (greater than threshold frequency ν_0) is incident, photoelectrons will be emitted. If we use incident light of different intensities (L_1, L_2), but of same frequency (ν), we shall obtain the graphs between the retarding potential and the photoelectric current as shown in Fig. 1.03. The graph shows that the value of stopping potential is the same for light of different intensities. Therefore, it follows that the maximum kinetic energy of the emitted photoelectrons is independent of the intensity of light.

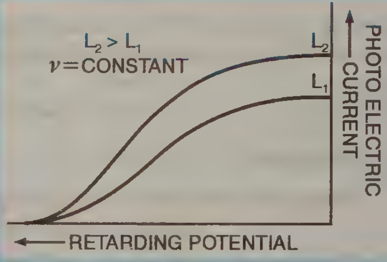


Fig. 10.3

On the other hand, if light of different frequencies (ν_1, ν_2), but of same intensity (L) is incident, we shall obtain the graphs between the retarding potential and the photoelectric current as shown in Fig. 1.04.

As the stopping potential has got different values, it follows that maximum kinetic energy with which the photoelectrons are emitted depends upon the frequency of light. Further, $V_0^{(2)} > V_0^{(1)}$ implies that if the frequency of incident light is increased, the maximum kinetic energy of the photoelectrons also increases.

4. *The number of photoelectrons emitted i.e. the photoelectric current, is independent of the frequency of the incident light and depends only upon its intensity.*

The fact that the number of photoelectrons emitted is independent of the frequency of the light can be easily followed from Fig. 1.04 and that the number of photoelectrons emitted depends upon the intensity of light is revealed by Fig. 1.03.

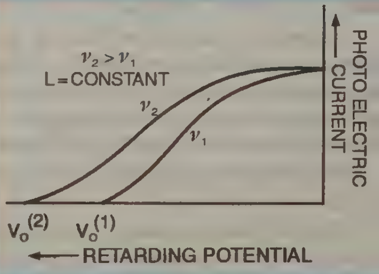


Fig. 1.04

From Fig. 1.03, it follows that as the intensity of light is increased ($L_2 > L_1$), the umber of photoelectrons emitted also increases.

1.05. EINSTEIN'S PHOTOELECTRIC EQUATION

To explain photoelectric effect, Einstein postulated that the energy carried by photon of radiation of frequency ν is $h\nu$. According to him, the emission of a hotoelectron was the result of the interaction of a single photon with an electron, which the photon is completely absorbed by the electron. We know that to remove n electron from a metal, a certain minimum amount of energy ω, called ***work unction*** of the metal, is required. Thus, when a photon of energy $h\nu$ is absorbed y an electron, an amount of energy at least equal to ω (provided $h\nu > \omega$) is used up n liberating the electron free and the difference $h\nu - \omega$ becomes available to the ectron as its maximum kinetic energy. Thus,

Watch out !

Brighter light means more photons, not more energetic photons !

$$\frac{1}{2} m v_{max}^2 = h\nu - \omega$$

or $$h\nu = \omega + \frac{1}{2} m v_{max}^2 \qquad ...(1.03)$$

Here, m is the mass of electron and v_{max} is the maximum velocity of the hotoelectrons. In fact, most of the electrons possess kinetic energy less than the aximum value as they lose a part of their kinetic energy due to collisions in scaping from the metal. Further, the work function* of the metal is a characteristic f the metal and does not depend upon the nature of the incident radiation. It is ometimes also called the ***threshold energy*** of the metal. If ν_0 is the frequency which orresponds to threshold energy of the metal, then

$$\omega = h\nu_0$$

Here, ν_0 is called the ***threshold frequency***. Putting $\omega = h\nu_0$ in equation (1.03), ve obtain

$$h\nu = h\nu_0 + \frac{1}{2} m v_{max}^2 \qquad ...(1.04)$$

The above relation is called the ***Einstein's photoelectric equation***. Since a high requency radiation consists of photons of high energy, while one of high intensity onsists of a large number of photons ; from Einstein's relation, it follows that :

1. the photoelectrons will possess positive kinetic energy and hence will be jected out of surface, only if $\nu > \nu_0$, *i.e.* if the frequency of incident radiation is greater han the threshold frequency for the metal.

2. The number of photoelectrons ejected will be large, when intense radiation s incident. This is because, an intense radiation will contain a large number of photons and photoelectric effect is a *one photon-one electron phenomenon i.e.* one photon s capable of ejecting only one electron.

3. the photoelectrons will come out of the metal surface with greater value of naximum kinetic energy $\left(\frac{1}{2} m v_{max}^2 = h\nu - h\nu_0\right)$, as the frequency of incident radiation is increased.

Key point ✓

There is no effect of frequency of incident light on the number of emitted photoelectrons.

Key point ✓

Photoelectric effect is a one photon – one electron phenomenon. One photon cannot eject more than one photoelectron.

Key point ✓

There is no effect of intensity of light on the kinetic energy of the emitted photoelectrons.

1.06. EXPERIMENTAL DETERMINATION OF PLANCK'S CONSTANT

R.A. Millikan was able to verify Einstein's photoelectric equation with the help of an experimental arrangement as shown in Fig. 1.05.

The drum D carries three cylindrical blocks. The surfaces of the blocks are coated with an alkali metal. The blocks can be cleaned with the help of knife edge K. When a beam of monochromatic light falls on the alkali metal, photoelectrons are emitted. The photoelectrons emitted reach the electrode C. The electrode C is usually kept negative *w.r.t.* the cylindrical blocks. Therefore, the photoelectrons will be repelled and only fast moving electrons will be able to reach the electrode C. The negative potential of electrode C is kept increasing, until even the fastest moving photoelectrons are repelled back. If $\frac{1}{2} m v_{max}^2$ is the kinetic energy of the fastest photoelectron and V_0 is the stopping potential, then

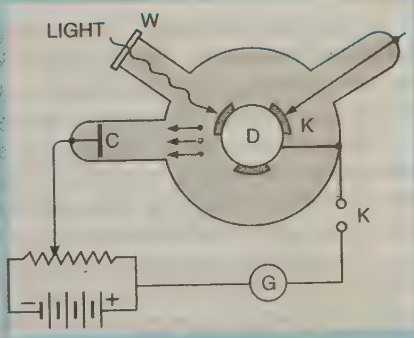


Fig. 1.05

$$\frac{1}{2} m v_{max}^2 = e\,V_0 \qquad ...(1.05)$$

Here, e is charge on an electron.

---

* Work functions of cesium, potassium, sodium, zinc and nickel are 1·8 eV, 2·3 eV, 2·5 eV, 3·4 eV and 5·9 eV respectively.

Millikan plotted a graph between the frequency (ν) of incident light and the stopping potential (V_0) over a wide range of frequencies. The graph was a straight line as shown in Fig. 1.06.

Let us consider Einstein's photoelectric equation,

$$h\nu = h\nu_0 + \frac{1}{2} m v_{max}^2$$

$$\therefore \qquad \frac{1}{2} m v_{max}^2 = h\nu - h\nu_0 \qquad \text{...(1.06)}$$

From equations (1.05) and (1.06), we obtain

$$e V_0 = h\nu - h\nu_0$$

or

$$V_0 = \frac{h\nu}{e} - \frac{h\nu_0}{e}$$

It is an equation of a straight line having slope $\frac{h}{e}$ and makes an intercept $-\frac{h\nu_0}{e}$ on the stopping potential axis.

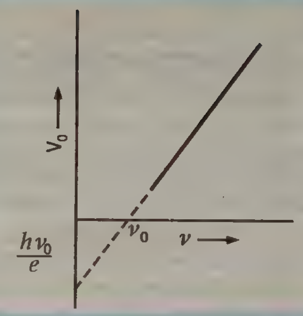


Fig. 1.06

Hence, experimental graph obtained between V_0 and ν is in agreement with Einstein's photoelectric equation.

To find the value of *h*. The slope of the graph between V_0 and ν can be measured. On multiplying with e (the electronic charge), the value of h can be obtained. It was found that

$$h = 6{\cdot}625 \times 10^{-34} \text{ J s}$$

Note. From this experimental study, one can also find the work function of the alkali metal coated on the block. By definition, when light of threshold frequency (ν_0) is incident, photoelectrons just come out but fail to reach the electrode C *i.e.* no stopping potential is required. Therefore,

when $\nu = \nu_0$, $V_0 = 0$

i.e. the intercept of the straight line graph on ν-axis gives the threshold frequency ν_0, which when multiplied with h, gives the work function of the alkali metal.

1.07. PHOTOELECTRIC CELLS

A photoelectric cell is an arrangement which converts light energy into electrical energy.

Photoelectric cells are of the following three types :

1. ***Photoemissive cells*** 2. ***Photovoltaic cells*** 3. ***Photoconductive cells.***

A photoemissive cell may be of ***vacuum type*** or ***gas filled type***. Let us discuss the working of a photoemissive cell.

Photoemissive cell. It consists of two electrodes, a cathode C and an anode A enclosed in a highly evacuated glass bulb [Fig. 1.07]. The cathode C is a semi-cylindrical plate coated with a photosensitive material, such as a layer of cesium deposited on silver oxide. The anode A is in the form of a wire, so that it does not obstruct the path of the light falling on the cathode.

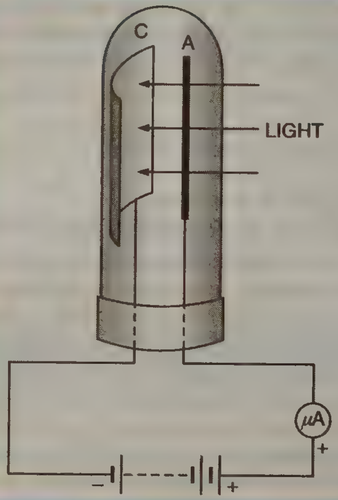


Fig. 1.07

When light of frequency above the threshold frequency for the cathode surface is incident on the cathode, photoelectrons are emitted. If a potential difference of about 10 V is applied between the anode and cathode, the photoelectrons are attracted towards the anode and the microammeter connected in the circuit will record the current.

The chief advantages of this type of photocell are that there is no time-lag between the incident light and the emission of photoelectrons and that the photoelectric current is proportional to the intensity of light. The cell is extremely acccurate in response. Hence, it is used in television and photometry.

The current may be increased by a factor of about 5 by filling the tube with an inert gas at a pressure of a few mm of mercury. When the potential difference between the electrodes exceeds the ionisation potential of the gas, the emitted photoelectrons ionise the gas atoms and now a larger current flows.

The gas filled photo cell cannot be used for registering very rapid changes of illumination, because it takes only 10^{-4} s for the ions to recombine and the current cannot follow the changes in such a short time. Hence, the current cannot maintain proportionality with the changing intensity of light. However, it is quite good for use in a film projector, as it can follow the changes in light intensity from the sound track.

1.08. APPLICATIONS OF PHOTOELECTRIC CELLS

Photoelectric cells have a variety of applications in industries and daily life. few important applications of photoelectric cells are as given below :

1. It is used for the reproduction of sound from the sound track recorded on ne edge of the cinema films.
2. It is used in a television studio to convert the light and shade of the object nto electric currents for transmission of picture.
3. It is used in a photographic camera for the automatic adjustment of aperture.
4. It is used to compare the illuminating powers of two sources of light and o measure the illumination of a surface.
5. It is used for automatic counting of the number of persons entering a hall, stadium, etc.
6. It is used for automatic switching of street lights and traffic-signals.
7. It is used for raising a fire alarm in the event of accidental fire in buildings, actories, etc.
8. It is used in burglar-alarms for houses, banks and treasuries.
9. A photo cell is also used in industries to locate flaws in metal sheets.
10. It is also used to control the temperature during a chemical reaction and hat of a furnace.
11. A photocell can be used for determining the opacity of solids and liquids.

SOLVED NUMERICAL PROBLEMS

Type A. On Energy of photon

Problem 1.01. If a light wave of wavelength 4950Å is iewed as a continuous flow of photons, what is the energy of ach photon in eV? Given, Planck's constant, $h = 6{\cdot}6\times10^{-34}$ J s, $=3\times10^{8}\,\text{m s}^{-1}$. *(I.S.C.E. 1998)*

Sol. Here, $\lambda = 4950\,\text{Å} = 4950\times10^{-10}$ m ; $h = 6{\cdot}6\times10^{-34}$ J s ; $c = 3\times10^{8}$ m s^{-1}

Energy of photon, $$E = \frac{hc}{\lambda} = \frac{6{\cdot}6\times10^{-34}\times3\times10^{8}}{4950\times10^{-10}} = 4{\cdot}0\times10^{-19}\,\text{J}$$

$$= \frac{4{\cdot}0\times10^{-19}}{1{\cdot}6\times10^{-19}} = \mathbf{2{\cdot}5\ eV}$$

Problem 1.02. Calculate the number of photons in 6·62 J of radiation energy of frequency 10^{12} Hz. Given, $h = 6{\cdot}62\times10^{-34}$ J s. *(P.S.S.C.E. 1998)*

Sol. Here, $E = 6{\cdot}62$ J ; $\nu = 10^{12}$ Hz ; $h = 6{\cdot}62\times10^{-34}$ J s

Energy of a photon, $h\nu = 6{\cdot}62\times10^{-34}\times10^{12} = 6{\cdot}62\times10^{-22}$ J

Therefore, number of photons in the given amount of radiation energy,

$$n = \frac{E}{h\nu} = \frac{6{\cdot}62}{6{\cdot}62\times10^{-22}} = 10^{22}$$

Problem 1.03. Calculate the energy of a photon, whose (*i*) frequency is 1000 kHz, (*ii*) wavelength is 5890 Å, (*iii*) wavelength is 1 Å. Also express the energy of the photon in eV in each case. Given, 1 eV = $1{\cdot}6\times10^{-19}$ J, $h = 6{\cdot}62\times10^{-34}$ J s and $c = 3\times10^{8}$ m s^{-1}.

Sol. The energy of a photon is given by

$$E = h\nu = \frac{hc}{\lambda},$$ where c is velocity of light.

Here, $h = 6{\cdot}62\times10^{-34}$ J s ; $c = 3\times10^{8}$ m s^{-1} ;
1 eV $= 1{\cdot}6\times10^{-19}$ J ;

(i) When v = 1000 kHz = 10^{6} Hz :

$$\therefore \quad E = h\nu = 6{\cdot}62\times10^{-34}\times10^{6} = \mathbf{6{\cdot}62\times10^{-28}\ J}$$

$$= \frac{6{\cdot}62\times10^{-28}}{1{\cdot}6\times10^{-19}} = \mathbf{4{\cdot}14\times10^{-9}\ eV}$$

(ii) When v = 5890 Å = 5890 × 10^{-10} m :

$$\therefore \quad E = \frac{hc}{\lambda} = \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{5890\times10^{-10}} = \mathbf{3{\cdot}37\times10^{-19}\ J}$$

$$= \frac{3{\cdot}37\times10^{-19}}{1{\cdot}6\times10^{-19}} = \mathbf{2{\cdot}11\ eV}$$

(iii) When λ = 1 Å = 10^{-10} m :

$$\therefore \quad E = \frac{hc}{\lambda} = \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{10^{-10}} = \mathbf{1{\cdot}99\times10^{-16}\ J}$$

$$= \frac{1{\cdot}99\times10^{-16}}{1{\cdot}6\times10^{-19}} = \mathbf{1243{\cdot}8\ eV}$$

Problem 1.04. (*a*) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0·66 Å. What is the maximum energy of a photon in the radiation ? *(P.S.S.C.E. 1995, 1992 ; C.B.S.E. 1990)*

(*b*) From your answer to (*a*), guess what order of accelerating voltage (for electrons) is required in such a tube.

Sol. Here, $\lambda = 0{\cdot}66\,\text{Å} = 0{\cdot}66\times10^{-10}$ m,
Now, $h = 6{\cdot}62\times10^{-34}$ J s and $c = 3\times10^{8}$ m s^{-1}

(*a*) The maximum energy of photon is given by

$$E = \frac{hc}{\lambda} = \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{0{\cdot}66\times10^{-10}} = \mathbf{3{\cdot}01\times10^{-15}\ J}$$

$$= \frac{3{\cdot}01\times10^{-15}}{1{\cdot}6\times10^{-16}} = \mathbf{18{\cdot}81\ keV}$$

(*b*) To produce electrons of energy 18·81 keV, accelerating potential of 18·81 kV *i.e.* of the order of **20 kV** is required.

Problem 1.05. In an accelerator experiment on high energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10·2 BeV into two γ-rays of equal

energy. What is the wavelength associated with each γ-ray ? (BeV =10^9 eV). *(Text Problem)*

Sol. Here, energy of two γ-rays = 10·2 BeV

Therefore, energy of one γ-ray $= \frac{10 \cdot 2}{2} = 5 \cdot 1$ BeV

$= 5 \cdot 1 \times 10^9 \text{ eV} = 5 \cdot 1 \times 10^9 \times 1 \cdot 6 \times 10^{-19}$ J

Now, $\lambda = \frac{hc}{E} = \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{5 \cdot 1 \times 10^9 \times 1 \cdot 6 \times 10^{-19}}$

$= \mathbf{2 \cdot 434 \times 10^{-16}}$ **m**

Type B. On work function and threshold wavelength

Problem 1.06. Light of wavelength 3500 Å is incident on two metals A and B. Which metal will yield photoelectrons, if their work functions are 4·2 eV and 1·9 eV respectively ? *(C.B.S.E. 1991)*

Sol. Here, $\lambda = 3500 \text{ Å} = 3500 \times 10^{-10}$ m

The energy of incident light,

$$h\nu = \frac{hc}{\lambda} = \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{3500 \times 10^{10}} \text{ J}$$

$$= \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{3500 \times 10^{-10} \times 1 \cdot 6 \times 10^{-19}} \text{ eV}$$

$= 3 \cdot 546$ eV

The incident light will eject electrons from a metal surface, if its energy is greater than the work function of that metal. Accordingly, **metal B** will yield photoelectrons.

Problem 1.07. A metal has threshold wavelength of 6000 Å . Calculate (*i*) threshold frequency (*ii*) the work function of metal in eV. Given, *h* = 6·62 × 10^{-34} J s and *e* = 1·6 × 10^{-19} C.

Sol. Here, $\lambda_0 = 6000 \text{ Å} = 6000 \times 10^{-10}$ m ;
$h = 6 \cdot 62 \times 10^{-34}$ J s ; $e = 1 \cdot 6 \times 10^{-19}$ C

(*i*) Threshold frequency,

$$\nu_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = \mathbf{5 \times 10^{14}} \textbf{ Hz}$$

(ii) Work function, $\omega = h\nu_0 = 6 \cdot 62 \times 10^{-34} \times 5 \times 10^{14}$
$= 3 \cdot 31 \times 10^{-19}$ J

$$= \frac{3 \cdot 31 \times 10^{-19}}{1 \cdot 6 \times 10^{-19}} = \mathbf{2 \cdot 069} \textbf{ eV}$$

Type C. On Einstein's photoelectric equation

Problem 1.08. If radiation of wavelength 5000 Å is incident on a surface of work function 1·2 eV, find the value of stopping potential. *(P.S.S.C.E. 1999, 1998 S)*

Sol. Here, work function, $\omega = 1 \cdot 2 \text{ eV} = 1 \cdot 2 \times 1 \cdot 6 \times 10^{-19}$
$= 1 \cdot 92 \times 10^{-19}$ J

Wavelength of incident radiation,
$\lambda = 5000 \text{ Å} = 5000 \times 10^{-10}$ m

If photoelectrons are emitted with maximum velocity v_{max}, then

$$\frac{1}{2} m v_{max}^2 = h\nu - \omega = \frac{hc}{\lambda} - \omega$$

$$= \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} - 1 \cdot 92 \times 10^{-19}$$

$= 3 \cdot 97 \times 10^{-19} - 1 \cdot 92 \times 10^{-19} = 2 \cdot 05 \times 10^{-19}$ J

If *e* is the charge on electron and V_0 is the stopping potential, then

$$e V_0 = \frac{1}{2} m v_{max}^2 = 2 \cdot 05 \times 10^{-19} \text{J}$$

or $V_0 = \frac{2 \cdot 05 \times 10^{-19}}{e} = \frac{2 \cdot 05 \times 10^{-19}}{1 \cdot 6 \times 10^{-19}} = \mathbf{1 \cdot 28} \textbf{ V}$

Problem 1.09. A metal has a work function of 2·0 e It is illuminated by monochromatic light of wavelengt 500 nm. Calculate : (*i*) the threshold wavelength, (*ii*) th maximum energy of photoelectrons, (*iii*) the stoppin potential. Given, Planck's constant, *h* = 6·6 × 10^{-34} J s, charg on electron, *e* = 1·6 × 10^{-19} C and 1 eV = 1·6 × 10^{-19} J. *(I.S.C.E. 199*

Sol. Here, $h = 6 \cdot 6 \times 10^{-34}$ J s ; $e = 1 \cdot 6 \times 10^{-19}$ C ;
1 eV = $1 \cdot 6 \times 10^{-19}$ J
$\omega = 2 \cdot 0 \text{ eV} = 2 \cdot 0 \times 1 \cdot 6 \times 10^{-19} = 3 \cdot 2 \times 10^{-19}$ J
$\lambda = 500 \text{ nm} = 500 \times 10^{-9}$ m ;

(*i*) If λ_0 is the threshold wavelength, then

$$\omega = \frac{hc}{\lambda_0}$$

or $\lambda_0 = \frac{hc}{\omega} = \frac{6 \cdot 6 \times 10^{-34} \times 3 \times 10^8}{3 \cdot 2 \times 10^{-19}}$

$= 618 \cdot 75 \times 10^{-9} \text{ m} = \mathbf{618 \cdot 75} \textbf{ nm}$

(*ii*) The maximum energy of photoelectrons,

$$\frac{1}{2} m v_{max}^2 = h\nu - \omega = \frac{hc}{\lambda} - \omega$$

$$= \frac{6 \cdot 6 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}} - 3 \cdot 2 \times 10^{-19}$$

$= 3 \cdot 96 \times 10^{-19} - 3 \cdot 2 \times 10^{-19} = \mathbf{0 \cdot 76 \times 10^{-19}}$

(*iii*) The stopping potential is given by

$$e V_0 = \frac{1}{2} m v_{max}^2 = 0 \cdot 76 \times 10^{-19}$$

or $V_0 = \frac{0 \cdot 76 \times 10^{-19}}{e} = \frac{0 \cdot 76 \times 10^{-19}}{1 \cdot 6 \times 10^{-19}} = \mathbf{0 \cdot 475} \textbf{ V}$

Problem 1.10. Ultra-violet light of wavelength 2271 from a 100 W mercury source irradiates a photocell mad of molybdenum metal. If the stopping potential is 1·3 calculate the work function of the metal. How would th photocell respond to a high intensity (≈ 10^5 W m^{-2}) re light of wavelength 6328 Å produced by a He-Ne laser ? *(Text Problem*

Sol. Here, $V_0 = 1 \cdot 3 \text{ V} ; \lambda = 2271 \text{ Å} = 2271 \times 10^{-10}$ m

We know, $h\nu = h\nu_0 + \frac{1}{2} m v_{max}^2$

Now, $\frac{1}{2} m v_{max}^2 = e V_0 ; h\nu = \frac{hc}{\lambda}$ and $h\nu_0 = \omega$

$\therefore \quad \frac{hc}{\lambda} = \omega + e V_0$

or $\omega = \frac{hc}{\lambda} - e V_0$

Taking $h = 6 \cdot 62 \times 10^{-34}$ J s ;
$e = 1 \cdot 6 \times 10^{-19}$ C and $c = 3 \times 10^8 \text{ m s}^{-1}$, we have

$$\omega = \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{2271 \times 10^{-10}} - 1 \cdot 6 \times 10^{-19} \times 1 \cdot 3$$

$= 8 \cdot 745 \times 10^{-19} - 2 \cdot 08 \times 10^{-19} = 6 \cdot 665 \times 10^{-19}$ J

$$= \frac{6 \cdot 665 \times 10^{-19}}{1 \cdot 6 \times 10^{-19}} = \mathbf{4 \cdot 166} \textbf{ eV}$$

Threshold wavelength is given by

$$\lambda_0 = \frac{hc}{\omega} = \frac{6\cdot62\times10^{-34}\times3\times10^{8}}{6\cdot665\times10^{-19}}$$
$$= 2\cdot98\times10^{-7}\ \text{m} = 2980\ \text{Å}$$

Since wavelength 6328 Å produced by He-Ne laser and ncident on the photocell is greater than λ_0, the photocell *will not respond.*

Problem 1.11. Monochromatic radiation of wavelength 640·2 nm (1 nm = 10^{-9} m) from a lamp neon irradiates a photosensitive material made of cesium or tungsten. The stopping voltage is measured to be 0·54 V. The source is replaced by an iron source and its 427·2 nm line irradiates the same photocell. Predict the new stopping voltage. *(Text Problem)*

Sol. *For neon lamp* : $\lambda = 640\cdot2\ \text{nm} = 640\cdot2\times10^{-9}\ \text{m}$;

$$V_0 = 0\cdot54\ \text{volt}$$

Now, $$h\nu = h\nu_0 + \frac{1}{2}m v_{max}^2$$

or $$\frac{hc}{\lambda} = \omega + e V_0$$

or $$\omega = \frac{hc}{\lambda} - eV_0 = \frac{6\cdot62\times10^{-34}\times3\times10^{8}}{640\cdot2\times10^{-9}} - 1\cdot6\times10^{-19}\times0\cdot54$$
$$= 3\cdot102\times10^{-19} - 0\cdot864\times10^{-19}\ \text{m};$$
$$= 2\cdot238\times10^{-19}\ \text{J}$$

For iron source : $\lambda = 427\cdot2\ \text{nm} = 427\cdot2\times10^{-9}\ \text{m}$;

$$\omega = 2\cdot238\times10^{-19}\ \text{J}$$

Again, $\omega = \frac{hc}{\lambda} - eV_0$ or $eV_0 = \frac{hc}{\lambda} - \omega$

or $$V_0 = \frac{1}{e}\left(\frac{hc}{\lambda} - \omega\right) = \frac{1}{1\cdot6\times10^{-19}}\times\left(\frac{6\cdot62\times10^{-34}\times3\times10^{8}}{427\cdot2\times10^{-9}} - 2\cdot238\times10^{-19}\right)$$
$$= \frac{1}{1\cdot6\times10^{-19}}(4\cdot649\times10^{-19} - 2\cdot238\times10^{-19})$$
$$= \frac{2\cdot411\times10^{-19}}{1\cdot6\times10^{-19}} = \mathbf{1\cdot507\ V}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 1.12. If 5% of the energy supplied to an incandescent light bulb is radiated as visible light, how many visible light photons are emitted by a 100 watt bulb ? Assume the average wavelength of all visible photons to be 5600 Å . Given, $h = 6\cdot625\times10^{-34}$ J s .

Sol. Here, $\lambda = 5600\ \text{Å} = 5600\times10^{-10}\ \text{m}$;

$$h = 6\cdot625\times10^{-34}\ \text{J s};$$

$$\text{Energy of one photon} = \frac{hc}{\lambda} = \frac{6\cdot625\times10^{-34}\times3\times10^{8}}{5600\times10^{-10}}$$
$$= 3\cdot549\times10^{-19}\ \text{J}$$

A 100 watt bulb has to be supplied 100 J of energy per second.

Therefore, energy radiated as visible light by the bulb per second $= \frac{100\times5}{100} = 5$ J

Therefore, number of visible light photons emitted per second as visible light

$$= \frac{5}{3\cdot549\times10^{-19}} = \mathbf{1\cdot41\times10^{-19}}$$

Problem 1.13. Estimating the following two numbers should be interesting. The first number will tell you, why radio engineers do not need to worry much about photons. The second number tells you, why our eye can never `count photons', even in barely detectable light. Find :

(*i*) The number of photons emitted per second by a MW transmitter of 10 kW power emitting radiowaves of wavelength 500 m.

(*ii*) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive ($\approx 10^{-10}$ W m^{-2}). Take the area of the pupil to be about 0·4 cm^2 and the average frequency of white light to be about 6×10^{14} Hz. *(Text Problem)*

Sol. (*i*) Here, power or energy transmitted per second,

$$E = 10\ \text{kW} = 10^4\ \text{W};\ \lambda = 500\ \text{m};$$

Now, energy of one photon

$$= \frac{hc}{\lambda} = \frac{6\cdot62\times10^{-34}\times3\times10^{8}}{500}\ \text{J}$$

Therefore, number of photons emitted by the transmitter per second

$$= \frac{10^4\times500}{6\cdot62\times10^{-34}\times3\times10^{8}} = \mathbf{2\cdot518\times10^{31}}$$

(*ii*) Minimum intensity of white light = 10^{-10} W m^{-2}

Area of the pupil = 0·4 cm^2 = $0\cdot4\times10^{-4}$ m^2

Therefore, light energy falling on the pupil per second

$$= 10^{-10}\times0\cdot4\times10^{-4} = 0\cdot4\times10^{-14}\ \text{W}$$

Frequency of white light, $\nu = 6\times10^{14}$ Hz

Therefore, energy of white light photon,

$$h\nu = 6\cdot62\times10^{-34}\times6\times10^{14}\ \text{J}$$

Therefore, number of photons entering pupil of our eye per second

$$= \frac{0\cdot4\times10^{-14}}{6\cdot62\times10^{-34}\times6\times10^{14}} = \mathbf{10^4}$$

Problem 1.14. Light of intensity 10^{-5} W m^{-2} falls on a sodium photo cell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate the time required for photoelectric emission in the wave picture of radiation. The work function for the metal is given to be 2 eV. What is the implication of your answer ? *(Text Problem)*

Sol. Since size of the atom is about 10^{-10} m, the effective area of an atom may be considered as 10^{-20} m^2 (roughly).

Area of each layer in sodium = 2 cm^2 = 2×10^{-4} m^2

Therefore, number of atoms in 5 layers of sodium

$$= \frac{2\times10^{-4}\times5}{10^{-20}} = 10^{17}$$

If we assume that sodium has one conduction electron per atom, then

number of electrons in 5 layers of sodium = 10^{17}

Intensity of incident light = 10^{-5} W m^{-2}

Therefore, energy incident per second on the surface of the photocell $= 10^{-5} \times 2 \times 10^{-4} = 2 \times 10^{-9}$ W

Since incident energy is absorbed by 5 layers of sodium, according to wave picture of radiation, the electrons present in all the 5 layers of sodium will share the incident energy equally.

Therefore, energy received by any one electron in the 5 layers of sodium

$$= \frac{2\times10^{-9}}{10^{17}} \text{ W} = 2\times10^{-26} \text{ J s}^{-1}$$

$$= \frac{2\times10^{-26}}{1\cdot6\times10^{-19}} = 1\cdot25 \times 10^{-7} \text{ eV s}^{-1}$$

Since work function of sodium is 2 eV, an electron will be ejected as soon as it gathers energy equal to 2 eV. Therefore

time required for photoemission $= \dfrac{2}{1\cdot25\times10^{-7}}$

$= 1\cdot6 \times 10^{7}$ s $\approx$ 0·5 yea

It is contrary to the observed fact that there is no tim lag between the incidence of light and the emission of photo electrons.

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. If radiation has both particle and wave properties, how can one decide which property to use in describing physical phenomenon ?

Ans. Refer to section 1.01.

Q. 1.02. Name a phenomenon, which illustrates the particle nature of light. *(H.S.S.C.E. 2002, 1999)*

Ans. Photoelectric effect.

Q. 1.03. On which factor does the energy carried by a quantum of light depend ? *(P.S.S.C.E. 1999, 1997 S)*

Ans. Energy of a photon, $E = h\nu$

Therefore, energy of a photon depends upon its frequency.

Q. 1.04. Calculate the frequency associated with a photon of energy $3{\cdot}3 \times 10^{-20}$ J. Given $h = 6{\cdot}6 \times 10^{-34}$ J s. *(H.S.S.C.E. 2002)*

Ans. $\nu = \dfrac{E}{h} = \dfrac{3\cdot3\times10^{-20}}{6\cdot6\times10^{-34}} = \mathbf{5 \times 10^{13}}$ **Hz**

Q. 1.05. What is the energy associated in joule with a photon of wavelength 4000 Å ? *(C.B.S.E. 1994)*

Ans. $E = \dfrac{hc}{\lambda} = \dfrac{6\cdot625\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}}$ J $=$ **1·656 J**

Q. 1.06. Which photon is more energetic : A red one or a violet one ? *(H.S.S.C.E. 2002, 1993 ; H.P.S.S.C.E. 1993)*

Ans. Violet photon is more energetic than red one. It is because, wavelength of violet light is less than that of red light.

Q. 1.07. Define an electron volt. *(P.S.S.C.E. 1999)*

Ans. The energy gained by an electron, when accelerated through a potential difference of 1 V, is called one electron volt (eV).

$$1\text{eV} = 1{\cdot}6 \times 10^{-19} \text{ J}$$

Q. 1.08. How many electron volts make one joule ? *(H.P.S.S.C.E. 1993)*

Ans. We know, 1 eV = $1{\cdot}6 \times 10^{-19}$ J

Therefore, $1\text{ J} = \dfrac{1}{1\cdot6\times10^{-19}} = \mathbf{6{\cdot}25 \times 10^{18}}$ **eV**

Q. 1.09. What is the frequency of a photon, whose energy is 66·3 eV ? Given, $h = 6{\cdot}673 \times 10^{-34}$ J s. *(P.S.S.C.E. 1997)*

Ans. Here, E = 66·3 eV = $66{\cdot}3 \times 1{\cdot}6 \times 10^{-19}$ J

Frequency of photon, $\nu = \dfrac{E}{h} = \dfrac{66\cdot3\times1\cdot6\times10^{-19}}{6\cdot673\times10^{-34}}$

$= \mathbf{1{\cdot}6 \times 10^{16}}$ **Hz**

Q. 1.10. Find the photon energy in eV for electromagnetic wave of wavelength (λ) 1 m. Given, $h = 6{\cdot}63 \times 10^{-34}$ J s $e = 1{\cdot}6 \times 10^{-19}$ C. *(H.S.S.C.E. 1993*

Ans. Here, $h = 6{\cdot}63 \times 10^{-34}$ J s ; $e = 1{\cdot}6 \times 10^{-19}$ C ; $\lambda = 1$ m

Energy of proton $= \dfrac{hc}{\lambda} = \dfrac{6\cdot63\times10^{-34}\times3\times10^{8}}{1}$

$= 19{\cdot}89\times10^{-26}\text{ J} = \dfrac{19\cdot89\times10^{-26}}{1\cdot6\times10^{-19}}$

$= \mathbf{1{\cdot}243 \times 10^{-6}}$ **eV**

Q. 1.11. What is photoelectric effect ? *(P.S.S.C.E. 1999 S ; C.B.S.E. 1992*

Ans. For definition, refer to section 1.03.

Q. 1.12. What are photoelectrons ?

Ans. When light of suitable wavelength falls on a metal surface, then the electrons so emitted are called photoelectrons.

Q. 1.13. Mention one physical process for the release of electrons from the surface of a metal. *(H.S.S.C.E. 2002*

Ans. By exposing the surface of metal to radiation of frequency greater than the threshold frequency for the metal

Q. 1.14. Do X-rays show the phenomenon of photoelectric effect ?

Ans. Yes, X-rays show photoelectric effect in sodium zinc, copper, etc. The energy of X-ray photon is quite large in comparison to that of visible light or ultraviolet light.

Q. 1.15. Why are alkali metals most suited for photoelectric emission ?

Ans. Alkali metals show photoelectric effect with visible light. The reason is that the work function of an alkali metal is quite low. Metals such as zinc, cadmium, etc. require much energetic radiation (such as ultraviolet light) to emit photoelectrons.

Q. 1.16. Define photoelectric work function. *(H.S.S.C.E. 1998)*

Ans. The minimum amount of energy required to eject an electron from a surface (without imparting any kinetic energy) is called work function of the surface.

Q. 1.17. What is threshold wavelength ?

Ans. *The maximum wavelength of incident light for a metal surface, above which the light cannot eject electrons, is called threshold wavelength for that metal surface.*

Q. 1.18. Two metals A and B have work functions 2 eV and 4 eV respectively. Which metal has a lower threshold wavelength for photoelectric effect ? *(C.B.S.E. 2002, 1996 ; H.S.S.C.E. 2002)*

Ans. We know, $\omega = \frac{hc}{\lambda_0}$ or $\lambda_0 \propto \frac{1}{\omega}$

It follows that for the metal B which has greater value of work function *i.e.* 4 eV, the threshold wavelength will be lower.

Q. 1.19. Define threshold frequency.

Ans. *The minimum frequency of incident light for a metal surface, below which the light cannot eject electrons, is called threshold frequency for that metal surface.*

Q. 1.20. Is photoelectric emission possible at all frequencies ? Give reasons for your answer. (*C.B.S.E. 1990*)

Ans. No, photoelectric emission is not possible at all frequencies. The incident photon of light can eject electrons, if it possesses energy greater than its work function. Since work function $\omega = h\,\nu_0$, photoelectric emission will take place, if frequency of incident light is greater than ν_0.

Q. 1.21. When radiation of frequency 10^{15} Hz is incident on a certain surface, no photoelectric emission takes place. What conculsion do you draw from this ?

Ans. It implies that frequency of 10^{15} Hz is below the threshold frequency for that metal surface.

Q. 1.22. The threshold frequency of a material is 2×10^{-14} Hz. What is its work function ? (*H.S.S.C.E. 1998 S*)

Ans. Here, $\nu_0 = 2 \times 10^{14}$ Hz

$$\therefore \quad \omega = h\,\nu_0 = 6{\cdot}62 \times 10^{-34} \times 2 \times 10^{14}$$
$$= 1{\cdot}324 \times 10^{-19}\ \text{J}$$
$$= \frac{1{\cdot}324 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = \mathbf{0{\cdot}8275\ eV}$$

Q. 1.23. For a photosensitive surface, work function is $3{\cdot}3 \times 10^{-19}$ J. Taking Planck's constant to be $6{\cdot}6 \times 10^{-34}$ J s, find threshold frequency. (*I.S.C.E. 1997*)

Ans. Here, $\omega = 3{\cdot}3 \times 10^{-19}$ J ; $h = 6{\cdot}6 \times 10^{-34}$ J s

$$\text{Threshold frequency, } \nu_0 = \frac{\omega}{h} = \frac{3{\cdot}3 \times 10^{-19}}{6{\cdot}6 \times 10^{-34}} = \mathbf{5 \times 10^{14}\ Hz}$$

Q. 1.24. Calculate the threshold frequency of photon for photoelectric emission from a metal of work function 0·1 eV. (*C.B.S.E. 1992*)

Ans. Here, $\omega = 0{\cdot}1\ \text{eV} = 0{\cdot}1 \times 1{\cdot}6 \times 10^{-19} = 1{\cdot}6 \times 10^{-20}$ J

Therefore, threshold frequency,

$$\nu_0 = \frac{\omega}{h} = \frac{1{\cdot}6 \times 10^{-20}}{6{\cdot}62 \times 10^{-34}} = \mathbf{2{\cdot}417 \times 10^{13} Hz}$$

Q. 1.25. Calculate the work function of a metal in eV, if its threshold wavelength is 6800 Å and $h = 6{\cdot}62 \times 10^{-27}$ erg s. (*H.S.S.C.E. 1996, 1995*)

Ans. Here, $\lambda_0 = 6800$ Å $= 6{\cdot}8 \times 10^{-7}$ m,

$$\therefore \quad \omega = \frac{hc}{\lambda_0} = \frac{6{\cdot}62 \times 10^{-34} \times 3 \times 10^{8}}{6{\cdot}8 \times 10^{-7}} = 2{\cdot}92 \times 10^{-9}\ \text{J}$$
$$= \mathbf{1{\cdot}825\ eV}$$

Q. 1.26. Work function of Na is 2·3 eV. Does sodium show photoelectric emission for light of wavelength 6800 Å ? (Given, $h = 6{\cdot}63 \times 10^{-34}$ J s) (*H.S.S.C.E. 1994*)

Ans. For $\lambda = 6800$ Å $= 6{\cdot}8 \times 10^{-7}$ m,

$$\text{energy of photon} = \frac{hc}{\lambda} = \frac{6{\cdot}63 \times 10^{-34} \times 3 \times 10^{8}}{6{\cdot}8 \times 10^{-7}}$$
$$= 2{\cdot}925 \times 10^{-19}\ \text{J} = 1{\cdot}828\ \text{eV}$$

Since the energy of photon is less than the work function for Na, photoelectric emission will *not* take place.

Q. 1.27. Does the threshold frequency depend on intensity of light ? (*C.B.S.E. 1982*)

Ans. No, threshold frequency does not depend on intensity of light.

Q. 1.28. How does the maximum kinetic energy of electrons emitted vary with the work function of the metal ? (*C.B.S.E. 2001*)

Ans. The maximum kinetic energy of emitted electrons decreases, as the work function of the metal increases.

Q. 1.29. If the maximum kinetic energy of electrons emitted by a photo cell is 5 eV, what is the stopping potential ? (*H.P.S.S.C.E. 2002 ; C.B.S.E. 2001, 1996*)

Ans. The stopping potential for electrons having maximum kinetic energy of 5 eV is 5 V.

Q. 1.30. The frequency (ν) of incident radiation is greater than threshold frequency (ν_0) in a photocell. How will the stopping potential vary, if frequency (ν) is increased, keeping other factors constant ? (*C.B.S.E. 2002*)

Ans. The kinetic energy of the emitted photoelectrons will increase.

Q. 1.31. If the intensity of incident radiation in a photocell is increased, how does the stopping potential vary ? (*C.B.S.E. 2002*)

Ans. There is no effect on stopping potential.

Q. 1.32. The maximum kinetic energy of photoelectrons emitted from a surface, when photons of energy 6 eV fall on it is 4 eV. What is the stopping potential (in volt) for the fastest photoelectrons ? (*I.I.T. 1999*)

Ans. The stopping potential (in volt) for fastest photoelectrons is numerically equal to their kinetic energy (in eV). Therefore,

stopping potential for fastest photoelectrons = **4 V**

Q. 1.33. State Einstein's photoelectric equation in mathematical form. (*H.S.S.C.E. 2002*)

Ans. $h\nu = h\nu_0 + \frac{1}{2} m\, v_{max}^2$,

where the letters have their usual meanings.

Q. 1.34. Can the phenomenon of photoelectric effect be explained on the basis of wave theory ?

Ans. No, it can not be explained on wave theory.

Q. 1.35. What determines the strength of photoelectric current ?

Ans. The photoelectric current increases with the intensity of incident light.

Q. 1.36. What is the effect on the velocity of the photoelectrons, if the wavelength of the incident light is decreased ? (*C.B.S.E. 1982*)

Ans. The velocity of the photoelectrons will increase on decreasing the wavelength of the incident light.

Q. 1.37. If the intensity of incident radiation on a metal surface is doubled, what happens to the kinetic energy of the electrons emitted ? (*C.B.S.E. 1993*)

Ans. There is no effect of intensity on kinetic energy of the emitted electrons.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. Define photoelectric effect and threshold frequency. *(H.S.S.C.E. 2002 ; Pre-degree Kerala, 1992)*

Ans. Photoelectric effect. Refer to section 1.03.

Threshold frequency. Refer to VSQ 1.17.

Q. 1.02. Define work function of a metal and photoelectric effect. *(H.P.S.S.C.E. 1998)*

Ans. Work function. Refer to VSQ 1.14.

Photoelectric effect. Refer to section 1.03.

Q. 1.03. Define photoelectric work function and threshold frequency. *(H.S.S.CE. 2002)*

Ans. Photoelectric work function. Refer to VSQ 1.14.

Threshold frequency. Refer to VSQ 1.17.

Q. 1.04. Two metals X and Y when illuminated with appropriate radiations emit photoelectrons. The work function of X is higher than that of Y. Which metal will have higher value of threshold frequency and why ? *(C.B.S.E. 2001 S)*

Ans. The work function of a metal is given by

$$\omega = h\nu_0,$$

where ν_0 is threshold frequency. Since work function of metal X is higher, it will also have higher value of threshold frequency.

Q. 1.05. It is harder to remove a free electron from copper than from sodium. Which metal has greater work function ? Which has higher threshold wavelength ?

Ans. As it is harder to remove an electron from copper than sodium, work function of copper is greater.

Now, $$\omega = h\nu_0 = \frac{hc}{\lambda_0} \quad \text{or} \quad \lambda_0 \propto \frac{1}{\omega}$$

As threshold wavelength is inversely proportional to the work function, its value will be more for sodium.

Q. 1.06. Visible light cannot eject photoelectrons from copper surface, whose work function is 4·4 eV. Why ? Prove it mathematically. *(P.S.S.C.E. 2001)*

Ans. Here, $$\omega = \frac{hc}{\lambda_0} = 4{\cdot}4 \text{ eV} = 4{\cdot}4 \times 1{\cdot}6 \times 10^{-19} \text{ J}$$

Therefore, threshold wavelength for copper surface,

$$\lambda_0 = \frac{hc}{\omega} = \frac{6{\cdot}62 \times 10^{-34} \times 3 \times 10^8}{4{\cdot}4 \times 1{\cdot}6 \times 10^{-19}}$$
$$= 2{\cdot}821 \times 10^{-7} \text{ m} = 2821 \text{ Å}$$

Since wavelength of visible light (3800 Å to 7800 Å) is greater than threshold wavelength for copper surface, it cannot eject electrons from copper.

Q. 1.07. Green light ejects electrons from a certain photo-sensitive surface, yellow light does not. What will happen in case of red and violet light ? Give reasons for your answer. *(C.B.S.E. 2001 S)*

Ans. As the green light ejects electrons from the surface, wavelength of green light is less than the threshold wavelength for the surface. Further, as the yellow light cannot eject photo-electrons, the wavelength of yellow light is greater than that threshold wavelength. The wavelength of red light is greater than that of yellow light while the wavelength of violet light is less than that of green light. Hence, red light cannot cause photoelectric emission, while photoelectric emission will take place for violet light.

Q. 1.08. Two metals X and Y have work functions 2 eV and 5 eV respectively. Which metal will emit electrons, when irradiated with light of wavelength 400 nm and why ? *(C.B.S.E. 1999)*

Ans. Here, λ = 400 nm = 400×10^{-9} m

Therefore, energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6{\cdot}62 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}$$
$$= 4{\cdot}965 \times 10^{-19} \text{ J}$$
$$= \frac{4{\cdot}965 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = 3{\cdot}1 \text{ eV}$$

Since work function of metal X is less than the energy of the incident light, photoelectrons will be emitted by this metal only.

Q. 1.09. Radiation of frequency 10^{15} Hz are incident on two photosensitive surfaces P and Q. Following observations are made :

(*i*) Surface P : Photoemission occurs but the photoelectrons have zero kinetic energy.

(*ii*) Surface Q : Photoemission occurs and photoelectrons have some kinetic energy.

Which of these has a higher work function ? If the incident frequency is slightly reduced, what will happen to photoelectrons emission in the two cases ? *(C.B.S.E. 1999 S)*

Ans. From Einstein's photoelectric equation, we may write

$$\omega = h\nu - \frac{1}{2} m v_{max}^2$$

For surface P, kinetic energy of photoelectrons is zero ; while for plate Q, the photoelectrons have some kinetic energy. It follows that work function of **surface P** is greater than that of surface Q.

If the incident frequency is slightly reduced, the energy of incident photon will become less than the work function for surface P. Hence, photoelectric emission **will not take place** in case of surface P.

Q. 1.10. State the laws of photoelectric emission. *(C.B.S.E. 1998 ; H.S.S.C.E. 2001, 1995)*

Or

Write down laws of photoelectric effect. *(H.S.S.C.E. 1997 S)*

Ans. Refer to section 1.04.

Q. 1.11. Photoelectric emission is an instantaneous process. Comment.

Ans. There is no time lag between the instant the photon of light falls on the metal surface and the instant a photoelectron is emitted from the metal surface. It can not be explained by assuming wave nature of radiation. If wave nature of radiation is assumed, then it will take quite a long time (about an year) for electron to come out of the metal surface.

Q. 1.12. Every metal has a definite work function. Why do all the photoelectrons not come out with the same energy, if incident radiation is monochromatic ? Why is there an energy distribution of photoelectrons ? *(P.S.S.C.E. 1996 ; Text Question)*

Ans. Work function of a metal is the minimum energy required to knock out an electron (from the conduction band). Since electrons are present in a continuous band of levels, different electrons require different amounts of energy to get out of the atom. For this reason, the electrons knocked off by a monochromatic radiation possess different energies.

Q. 1.13. Is it essential that each incident photon should eject a photoelectron ? Explain.

Ans. The photoelectric emission takes place, when the incident photon is absorbed by the electron in the atom. The energy equal to work function of metal is used up in ejecting photoelectron and the difference appears as the kinetic energy of the electron. Since one photon can be absorbed by one electron, a photon can eject only one electron.

Q. 1.14. What is the effect of (*i*) intensity and (*ii*) frequency of incident light on photoelectric current ? Explain. (*P.S.S.C.E. 1996*)

Ans. Refer to section 1.04.

Q. 1.15. Explain the terms cut off potential (stopping potential) and threshold frequency in photoelectric effect. (*P.S.S.C.E. 1996*)

Ans. Cut of potential. *The minimum value of opposing potential difference required so as to just stop the photoelectrons emitted with maximum kinetic energy, is called stopping potential.*

Threshold frequency. Refer to VSQ 1.17.

Q. 1.16. Define the term : (*a*) work function, (*b*) threshold frequency and (*c*) stopping potential with reference to photoelectric effect. (*C.B.S.E. 2002*)

Ans. (*a*) **Work function.** Refer to VSQ 1.14.

(*b*) **Threshold frequency.** Refer to VSQ 1.17.

(*c*) **Stopping potential.** Refer to SAQ 1.14.

Q. 1.17. A source of light is placed at a distance of 50 cm from a photocell and the cut off potential is found to be V_0. If the distance between the light source and photocell is made 25 cm, what will be the new cut off potential ? Justify your answer. (*C.B.S.E. 2001 S*)

Ans. On decreasing the distance of the source of light from the photocell from 50 cm to 25 cm, the intensity of light will become four times. Since there is no effect of intensity of light on the maximum kinetic energy of emitted photoelectrons, the stopping potential will remain the same *i.e.* V_0.

Q. 1.18. For photoelectric effect in a metal, the Fig. 1.08 shows the plot of cut off voltage versus frequency of incident radiation. Calculate (*i*) the threshold frequency and (*ii*) the work function for the given metal. (*C.B.S.E. 1995*)

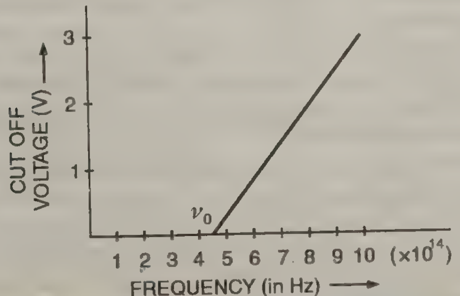


Fig. 1.08

Ans. (*i*) The threshold frequency (ν_0) is equal to the intercept made by the graph on frequency axis. Thus,

$$\nu_0 = \mathbf{4{\cdot}5 \times 10^{14}\ Hz}$$

(*ii*) Work function of the metal,

$$\omega = h\,\nu_0 = 6{\cdot}62 \times 10^{-34} \times 4{\cdot}5 \times 10^{14}$$
$$= 2{\cdot}98 \times 10^{-19}\ \text{J}$$
$$= \frac{2{\cdot}98 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = \mathbf{1{\cdot}86\ eV}$$

Q. 1.19. Draw a graph to show the variation of stopping potential with frequency of radiations incident on a metal plate. How can the value of Planck's constant be determined from this graph ? (*C.B.S.E. 1995*)

Ans. The graph between the radiation of stopping potential (or cut off voltage) and the frequency of the incident radiation is as shown in Fig. 1.08.

***To find h₀* :** From Einstein's photoelectric equation, we have

$$h\,\nu - h\,\nu_0 = \frac{1}{2}\,m\,v_{max}^2 = e\,V_0,$$

where V_0 is the stopping potential. Therefore,

$$h = \frac{e\,V_0}{(\nu - \nu_0)}$$

From the graph, by noting ν_0 and the value of frequency ν corresponding to a certain value of stopping potential, the value of Planck's constant can be found.

Q. 1.20. Draw a graph showing the variation of stopping potential with frequency of incident radiation in relation to photoelectric effect. Deduce an expression for the slope of this graph using Einstein's photoelectric equation. (*C.B.S.E. 1997*)

Ans. For graph, refer to Fig. 1.08.

From Einstein's photoelectric equation, we have

$$h\,\nu = h\,\nu_0 + \frac{1}{2}\,m\,v_{max}^2 = h\,\nu_0 + e\,V_0,$$

or
$$V_0 = \frac{h\,\nu}{e} - \frac{h\,\nu_0}{e} \quad \text{...}(i)$$

Since ν is plotted along X-axis and V_0 along Y-axis ; comparing the equation (*i*) with the equation of a straight line*, we have

slope of V_0 versus ν graph = h/e

Q. 1.21. Define the terms 'threshold frequency' and 'stopping potential' for photo-electric effect. Show graphically how the stopping potential for a given metal varies with frequency of the incident radiations. Mark threshold frequency on this graph. (*C.B.S.E. 1998*)

Ans. Threshold frequency. Refer to VSQ 1.17.

Stopping potential. Refer to SAQ 1.14.

For answer to the remaining part of SAQ 1.21, refer to Fig. 1.08.

Q. 1.22. Define the terms 'work function' and 'threshold frequency' for photoelectric effect. Show graphically how stopping potential for a given metal varies with frequency of incident radiation. What does the slope of this graph represent ? (*C.B.S.E. 1998*)

Ans. Work function. Refer to VSQ 1.14.

Threshold frequency. Refer to VSQ 1.17.

---

* the equation of a straight line having slope *m* and making intercept *c* on *y*-axis is $y = mx + c$

For graph between stopping potential and frequency of incident radiation, refer to Fig. 1.08.

Slope of the graph $= \frac{h}{e}$

For details refer to SAQ. 1.20.

Q. 1.23. State Einstein's photoelectric equation and explain the terms involved. *(H.S.S.C.E. 2002)*

Ans. Einstein's photoelectric equation :

$$h\nu = h\nu_0 + \frac{1}{2} m v_{max}^2$$

Here, ν is frequency of incident light, ν_0 is threshold frequency and v_{max} is the maximum velocity with which photoelectrons are emitted.

Q. 1.24. An increase in frequency of incident light increases the velocity with which a photoelectron is ejected. Explain, how.

Ans. According to Einstein's photoelectric equation,

$$h\nu = h\nu_0 + \frac{1}{2} m v_{max}^2$$

or $$\frac{1}{2} m v_{max}^2 = h(\nu - \nu_0)$$

As the frequency of incident light is increased, difference $\nu - \nu_0$ and hence the value of maximum kinetic energy of the photoelectrons goes on increasing.

Q. 1.25. For a photosensitive surface, threshold wavelength is λ_0. Does photoemission occur, if the wavelength (λ) of the incident radiation is (*a*) more than λ_0, (*b*) less than λ_0? Justify your answer. *(C.B.S.E. 2001 S)*

Ans. In terms of wavelength of the incident radiation, Einstein's photoelectric equation is

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \frac{1}{2} m v_{max}^2$$

(*a*) When $\lambda > \lambda_0$, $\frac{1}{2} m v_{max}^2$ is negative. Hence, photoemission will not occur, if $\lambda > \lambda_0$.

(*b*) When $\lambda < \lambda_0$, $\frac{1}{2} m v_{max}^2$ is positive. Therefore, when $\lambda < \lambda_0$, photoemission will occur.

Q. 1.26. Why does the maximum velocity of photoelectron not depend upon the intensity of radiation ? Explain.

Ans. The maximum kinetic energy of the photoelectrons is given by

$$\frac{1}{2} m v_{max}^2 = h\nu - h\nu_0$$

Thus, the maximum velocity of photoelectrons depends upon the frequency of the incident light and not on its intensity.

Q. 1.27. If the frequency of the incident radiation on the cathode of a photocell is doubled, how will the following change : (*i*) kinetic energy of the electrons, (*ii*) photoelectric current, (*iii*) stopping potential. *(C.B.S.E. 1999)*

Ans. From Einstein's photoelectric equation, we have

$$\frac{1}{2} m v_{max}^2 = h\nu - h\nu_0$$

(*i*) If the frequency of incident radiation is doubled, the difference $h\nu - h\nu_0$ and hence $\frac{1}{2} m v_{max}^2$ will increase *i.e.* kinetic energy of photoelectrons will *increase*.

(*ii*) The increase in frequency of the incident radiation has *no effect* on the photoelectric current (or the number of photoelectrons emitted).

(*iii*) If V_0 is the stopping potential, then

$$\frac{1}{2} m v_{max}^2 = e V_0$$

or $$e V_0 = h\nu - h\nu_0$$

Therefore, if the frequency of incident radiation is doubled, the stopping potential will ***increase***.

Q. 1.28. Explain the working of a photo cell. *(H.S.S.C.E. 1994)*

Ans. Refer to section 1.07.

Q. 1.29. What is a photoelectric cell ? State its two uses. *(P.S.S.C.E. 1999 S, 1999, 1992)*

Ans. Refer to sections 1.07 and 1.08.

Q. 1.30. Write four uses of photocell. *(P.S.S.C.E. 2002)*

Ans. Refer to section 1.08.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. When monochromatic radiation of wavelength 2000 Å falls upon a nickel plate, the latter acquires a positive charge. The wavelength is increased and at 3400 Å , however intense the incident radiation may be, the effect is found to cease. Explain it. *(Roorkee, 1983)*

Ans. When radiation of 2000 Å falls on nickel plate, it acquires positive charge due to emission of photoelectrons. It implies that the wavelength of the incident radiation is below the threshold wavelength of nickel.

When the incident radiation has wavelength equal to or greater than the threshold, no photoelectrons is emitted, howsoever intense the radiation may be. Since wavelength has been gradually increased to 3400 Å, when the effect is found to cease ; 3400 Å corresponds to threshold wavelength for the nickel surface.

Q. 1.02. In the photoelectric effect, there is a cut-off frequency. How does the photon picture explain this fact ? *(Roorkee, 1980)*

Or

Explain briefly, how classical theory could not explain the phenomenon of photoelectric effect.

Ans. To eject an electron from a metal surface, a minimum amount of energy, called the work function of the metal, is required. Classically, if incident radiation is a wave, its energy is shared by all the atoms in the metal surface and calculations show that it will take almost a year, when the requisite amount of energy may be accumulated by an electron. Hence, on classi-cal picture, photoelectric emission can not be instantaneous.

When the incident radiation is assumed to be consisting of photons, a photon of frequency ν_0 or more can possess the energy needed to eject an electron. If frequency of incident photon is less than ν_0, no photoelectric emission will take place. In a way, ν_0 is a cut-off frequency. As said earlier, the photo-electric emission will take place only, if the frequency of incident photon is equal to or greater than ν_0, the threshold frequency.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Describe photoelectric effect and state the laws of photoelectric emission. (*H.S.S.C.E. 2000*)
2. With reference to the photoelectric effect, define the terms 'work function' and 'threshold wavelength' for a given metal. On what factors will the following depend during photoelectric emission from a metal surface : (*i*) the magnitude of photoelectric current, and (*ii*) the velocity of ejected electrons ? (*I.S.C.E. 1998*)
3. Explain `stopping potential' and `threshold frequency' in photoelectric emission. Give an appropriate graph. (*I.S.C.E. 1996*)
4. What is photoelectric effect ? Write Einstein's photoelectric equation and use it to explain (*i*) independence of maximum energy of emitted photoelectrons from intensity of light, (*ii*) existence of a threshold frequency for emission of photo-electrons. (*C.B.S.E. 2001 S*)
5. State the dependence of work function on the kinetic energy of electrons emitted in a photo cell. If the intensity of incident light is doubled, what changes occur in the stopping potential and the photoelectric current ? (*C.B.S.E. 2000*)
6. Define threshold frequency, stopping potential and work function in photoelectric emission. Show that the existence of threshold frequency is an evidence in favour of the quantum theory. Bring out the difference in photoelectric effect produced by UV rays and that by visible light.
7. Explain Einstein's theory of photoelectric effect. (*H.S.S.C.E. 2001*)
8. What is photoelectric effect ? Derive Einstein's photoelectric equation. (*H.P.S.S.C.E. 2002*)
9. What is photoelectric effect ? Derive their laws from Einstein's photoelectric equation. (*P.S.S.C.E. 2002*)
10. Define the phenomenon of photoelectric effect. State the laws of photoelectric effect. Give Einstein's explanation of photoelectric effect. (*Karnataka, 1992 S*)
11. What is photoelectric cell ? Explain any one of the photoelectric cells. (*Karnataka, 1993 S*)
12. What is photoelectric cell ? How does it work ? Give its practical uses.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Discuss suitable experiments to study the laws of photoelectric emission. (*P.S.S.C.E. 2002, 2000, 1999, 1997 S*)
2. What is photoelectric effect ? Explain the effect of increase of (*i*) frequency (*ii*) intensity of the incident radiation on photoelectrons emitted by a photo-tube. (*C.B.S.E. 1994*)
3. Establish Einstein photoelectric equation. Use this equation to explain the laws of photoelectric emission. (*H.S.S.C.E. 1992 ; P.S.S.C.E. 1991*)
4. Discuss laws of photoelectric emission. How can these be explained on the basis of Einstein's photoelectric equation? (*P.S.S.C.E. 1996*)
5. What is photoelectric effect ? State its laws and derive these laws from Einstein's photoelectric equation. (*P.S.S.C.E. 2000, 1999, 1998 S*)
6. State the laws of photoelectric emission. Write Einstein's photoelectric equation. Explain in short these laws on the basis of Einstein's theory. (*P.S.S.C.E. 1995*)
7. Draw properly labelled graphs to show the following concerning photoelectric emission :
   (*a*) variation of photoelectron current with the intensity of incident radiation,
   (*b*) variation of photoelectron current with accelerating and retarding potential, and
   (*c*) variation of stopping potential with frequency of the incident radiation.
   From the graph, explain how the following can be determined :
   (*d*) Planck's constant,
   (*e*) the threshold frequency for the photosensitive material, and
   (*f*) the work function of the material. (*I.S.C.E. 1993*)
8. Write down Einstein's photoelectric equation and explain it. With the help of a neat diagram, describe briefly the construction and working of a photoemissive cell. Mention any four applications of photoelectric effect. (*Karnataka, 1994*)

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Energy of photon

1. Calculate the frequency associated with a photon of energy $3{\cdot}3 \times 10^{-20}$ J ($h = 6{\cdot}6 \times 10^{-34}$ J s). (*C.B.S.E. 1990*) **[Ans.** 5×10^{13} Hz**]**
2. Calculate the energy of a photon of green light of wavelength 5500 Å. Given, $h = 6{\cdot}62 \times 10^{-27}$ erg s. **[Ans.** 2·257 eV**]**
3. The wavelength of a spectral line is 4000 Å. Calculate its frequency and energy.
   Given, $c = 3 \times 10^8$ m s^{-1} and $h = 6{\cdot}6 \times 10^{-34}$ J s.
   (*H.S.S.C.E. 1992*) **[Ans.** $7{\cdot}5 \times 10^{14}$ Hz, 3·094 eV**]**
4. Calculate the frequency of a photon, whose energy is 7·5 eV. Given, 1 eV = $1{\cdot}6 \times 10^{-19}$ J and $h = 6{\cdot}62 \times 10^{-34}$ J s. **[Ans.** $1{\cdot}813 \times 10^{15}$ Hz**]**

Type B. On Threshold wavelength

5. Calculate the longest wavelength of the incident radiation, which will eject photoelectrons from a metal surface, whose work function is 3 eV. (*P.S.S.C.E. 2001*) **[Ans.** 4137·5 Å**]**
6. Calculate the threshold wavelength of photons, which can emit photoelectrons from cesium. Given that work function for cesium = 1·8 eV, $h = 6{\cdot}62 \times 10^{-34}$ J s. **[Ans.** 6896·8 Å**]**
7. Work function of a photosensitive metal is 1·875 eV. Calculate the wavelength of incident light, which will just cause the emission of photoelectrons. **[Ans.** 6620 Å**]**
8. The work function of sodium is 2·3 eV. Calculate the maximum wavelength in nm (nanometre) for the light that will cause photoelectrons to be emitted from sodium. (*M.N.R. 1980*) **[Ans.** 539·7 nm**]**
9. A metal has a threshold wavelength of 6000 Å. Calculate (*i*) threshold frequency (*ii*) work function of metal in eV.
   Given, $h = 6{\cdot}62 \times 10^{-34}$ J s, $e = 1{\cdot}6 \times 10^{-19}$ C.
   (*H.P.S.S.C.E. 1999 ; P.S.S.C.E. 1988*)
   [Ans. (*i*) 5×10^{14} Hz ; (*ii*) 2·07 eV**]**

10. Calculate the work function in electron volt for indium metal, given that photoelectric threshold wavelength is 6800 Å. Take $h = 6{\cdot}62 \times 10^{-27}$ erg s. **[Ans.** 1·825 eV**]**

11. A metal sheet is given a negative charge of 11·2 nC (nanocoulomb).
(*a*) How many photons of ultraviolet light is required to completely discharge the metal sheet ?
(*b*) What is the minimum amount of energy that must be absorbed by metal to effect this discharge ? Threshold frequency of metal = $4{\cdot}5 \times 10^{14}$ Hz, $h = 6{\cdot}6 \times 10^{-34}$ J s.
(*P.S.S.C.E. 1993*) **[Ans.** (*a*) 7×10^{10} ; (*b*) 1·856 eV**]**

Type C. On Einstein's photoelectric equation

12. Light of wavelength 5000 Å falls on a metal surface of work function 2·01 eV. Find the kinetic energy of photoelectrons.
(*P.S.S.C.E. 2001*) **[Ans.** 0·4725 eV**]**

13. Calculate the maximum kinetic energy of electrons emitted from a photosensitive surface of work function 3·2 eV, for the incident radiation of wavelength 300 nm.
(*C.B.S.E. 2002*) **[Ans.** 0·9375 eV**]**

14. Light of wavelength 5000 Å falls on a sensitive plate of work function 1·90 eV. Find the maximum kinetic energy of emitted photoelectrons in eV. Given, $h = 6{\cdot}62 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m s^{-1}. (*H.S.S.C.E. 1993*) **[Ans.** 0·5825 eV**]**

15. Calculate the kinetic energy of a photoelectron (in eV) emitted on shining light of wavelength $6{\cdot}2 \times 10^{-6}$ m on a metal surface. The work function of the metal is 0·1 eV.
(*C.B.S.C.E. 1992*) **[Ans.** 0·1 eV**]**

16. What is the energy of emitted photoelectrons, if light of frequency 10^{16} Hz is incident on a sodium target ? Work function of sodium = 2·5 eV. (*Pre-degree Kerala, 1992*)
[Ans. 38·875 eV**]**

17. The threshold wavelength of photosensitive metal is 5000 Å. Find the kinetic energy of the photoelectrons emitted by it, when radiation of wavelength 4000 Å is incident on it. Express it in eV. Given $h = 6{\cdot}625 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m s^{-1}, $e = 1{\cdot}6 \times 10^{-19}$ C. (*Karnataka, 1989 S*)
[Ans. 0·621 eV**]**

18. A sheet of silver is illuminated by monochromatic ultraviolet radiation of wavelength =1810 Å. What is the maximum energy of the emitted electron ? Threshold wavelength of silver is 2640 Å. **[Ans.** 2·156 eV**]**

19. Light of wavelength 5500 Å falls on a sensitive plate with work function 1·7 eV. Find (*a*) energy of photon, (*b*) energy of photoelectron and (*c*) stopping potential.
(*P.S.S.C.E. 2002*)
[Ans. (*a*) 2·257 eV, (*b*) 0·557 eV, (*c*) 0·557 V**]**

20. Light of wavelength 5000 Å falls on a sensitive plate with photoelectric work function equal to 1·90 eV. Find, (*i*) the energy of the photon in eV, (*ii*) the kinetic energy of the photoelectrons emitted and (*iii*) the stopping potential.
[*H.S.S.C.E. 1993*]
[Ans. (*i*) 2·483 eV ; (*ii*) 0·583 eV ; (*iii*) 0·583 V**]**

21. The photoelectric work function of a certain surface is 4 eV. What is the maximum velocity of the photoelectrons emitted by light of frequency 10^{15} Hz incident on that surface ? $h = 6{\cdot}625 \times 10^{-34}$ J s, 1 eV = $1{\cdot}602 \times 10^{-19}$ J.
(*Pre-degree Kerala, 1994*) **[Ans.** $2{\cdot}18 \times 10^5$ m s^{-1}

22. The work function of potassium is 2·3 eV. If the photoelectrons are emitted with maximum velocity of 10^4 m s^{-1} calculate the frequency of the incident radiation on the metal. Given that mass of electron, $m = 9{\cdot}1 \times 10^{-31}$ kg and and Planck's constant, $h = 6{\cdot}62 \times 10^{-34}$ J s.
[Ans. $1{\cdot}243 \times 10^{15}$ Hz**]**

23. Work function of sodium is 2·35 eV. What is the maximum wavelength of light that will cause photoelectrons to be emitted from the metal ? What will be the maximum energy of the photoelectrons, if radiation of 1000 Å falls on the metal surface ($h = 6{\cdot}625 \times 10^{-34}$ J s).
(*Karnataka, 1994 S*) **[Ans.** 5286 Å, 10·07 eV**]**

24. The energy of photoelectrons emitted from a photosensitive plate is 1·56 eV. If threshold wavelength is 2500 Å, calculate the wavelength of incident light. Given,
1 eV = $1{\cdot}6 \times 10^{-12}$ erg and $h = 6{\cdot}62 \times 10^{-27}$ erg s.
[Ans. 1902 Å**]**

25. Find the frequency of light, which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photoelectric effect begins in this metal at frequency of 6×10^{14} s^{-1}. Find the work function of this metal. (*H.P.S.S.C.E. 1996*)
[Ans. $13{\cdot}25 \times 10^{14}$ s^{-1}, 2·48 eV**]**

26. If photoelectrons are to be emitted from a potassium surface with a speed of 6×10^5 m s^{-1}, what frequency of radiation must be used ? Threshold frequency for potassium is $4{\cdot}22 \times 10^{14}$ Hz, $h = 6{\cdot}6 \times 10^{-34}$ J s and $m_e = 9{\cdot}1 \times 10^{-31}$ kg.
(*C.B.S.E. Sample Q. Paper*) **[Ans.** $6{\cdot}7 \times 10^{14}$ Hz**]**

MISCELLANEOUS PROBLEMS

27. Light of wavelength 2000 Å falls on an aluminium surface. In aluminium 4·2 eV are required to remove an electron. What is the kinetic energy in eV of (*a*) the fastest, (*b*) the slowest emitted photoelectrons ? What is (*c*) the stopping potential (*d*) cut off wavelength for aluminium ? Planck's constant, $h = 6{\cdot}6 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m s^{-1}.
(*M.N.R. 1986*)
[Ans. (*a*) 1·9875 eV ; (*b*) 0 ; (*c*) 1·9875 V ; (*d*) 2946 Å**]**

28. When light of wavelength 250 nm is incident on the cathode of a photocell, the stopping potential recorded is 4 V. If the wavelength of the incident light is increased to 300 nm, calculate the new stopping potential.
(*C.B.S.E. 2000*) **[Ans.** 3·17 V**]**

29. When a surface is irradiated with a light of wavelength 4950 Å, a photocurrent appears which vanishes, if a retarding potential greater than 0 ·6 V is applied across the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1·1 V. Find the work function of the emitting surface and the wavelength of second source. If the photoelectrons (after emission from the surface) are subjected to a magnetic field of 10 T, what changes will be observed in the above two retarding potentials ? (*Roorkee, 1989*)
[Ans. 1·9 eV, 4129 Å, No change**]**

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $\nu = \frac{E}{h} = \frac{3\cdot3\times10^{-20}}{6\cdot6\times10^{-34}} = \mathbf{5\times10^{13}\ Hz}$

2. $E = \frac{h\,c}{\lambda} = \frac{6\cdot62\times10^{-27}\times3\times10^{10}}{5500\times10^{-8}}$

$= 3\cdot61\times10^{-12}$ erg

$= \frac{3\cdot61\times10^{-12}}{1\cdot6\times10^{-12}} = \mathbf{2\cdot257\ eV}$

3. $\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{4000\times10^{-10}} = \mathbf{7\cdot5\times10^{14}\ Hz}$

$E = h\nu = 6\cdot6\times10^{-34}\times7\cdot5\times10^{14} = 4\cdot95\times10^{-19}$ J

$= \frac{4\cdot95\times10^{-19}}{1\cdot6\times10^{-19}} = \mathbf{3\cdot094\ eV}$

4. $\nu = \frac{E}{h} = \frac{7\cdot5\times1\cdot6\times10^{-19}}{6\cdot62\times10^{-34}} = \mathbf{1\cdot813\times10^{15}\ Hz}$

5. $\lambda_0 = \frac{h\,c}{E} = \frac{6\cdot62\times10^{-34}\times3\times10^{8}}{3\times1\cdot6\times10^{-19}}$

$= 4\cdot1375\times10^{-7}\ \mathrm{m} = \mathbf{4137\cdot5\ Å}$

6. Proceed as in problem no. 5.
7. Proceed as in problem no. 5.

8. $\lambda_0 = \frac{h\,c}{E} = \frac{6\cdot62\times10^{-34}\times3\times10^{8}}{2\cdot3\times1\cdot6\times10^{-19}}$

$= 5\cdot397\times10^{-7}\ \mathrm{m} = \mathbf{539\cdot7\ nm}$

9. (*i*) $\nu_0 = \frac{c}{\lambda_0} = \frac{3\times10^{8}}{6000\times10^{-10}} = \mathbf{5\times10^{14}\ Hz}$

(*ii*) $\omega = h\,\nu_0 = 6\cdot62\times10^{-34}\times5\times10^{14} = 3\cdot31\times10^{-19}$ J

$= \frac{3\cdot31\times10^{-19}}{1\cdot6\times10^{-19}} = \mathbf{2\cdot07\ eV}$

10. $\omega = \frac{h\,c}{\lambda_0} = \frac{6\cdot62\times10^{-27}\times3\times10^{10}}{6800\times10^{-8}}$

$= 2\cdot92\times10^{-12}\ \mathrm{erg} = \frac{2\cdot92\times10^{-12}}{1\cdot6\times10^{-12}} = \mathbf{1\cdot825\ eV}$

11. (*a*) $n = \frac{q}{e} = \frac{11\cdot2\times10^{-9}}{1\cdot6\times10^{-19}} = \mathbf{7\times10^{10}}$

(*b*) $\omega = h\,\nu_0 = 6\cdot6\times10^{-34}\times4\cdot5\times10^{14} = 2\cdot97\times10^{-19}$ J

$= \frac{2\cdot97\times10^{-19}}{1\cdot6\times10^{-19}} = \mathbf{1\cdot856\ eV}$

12. $\frac{1}{2}m\,v_{max}^2 = \frac{h\,c}{\lambda} - \omega$

$= \frac{6\cdot62\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}\times1\cdot6\times10^{-19}} - 2\cdot01$

$= 2\cdot4825\times2\cdot01 = \mathbf{0\cdot4725\ eV}$

13. Proceed as in problem no. 12.
14. Proceed as in problem no. 12.
15. Proceed as in problem no. 12.

16. $\frac{1}{2}m\,v_{max}^2 = h\nu - \omega = \frac{6\cdot62\times10^{-34}\times10^{16}}{1\cdot6\times10^{-19}} - 2\cdot5$

$= 41\cdot375 - 2\cdot5 = \mathbf{38\cdot875\ eV}$

17. $\frac{1}{2}m\,v_{max}^2 = \frac{h\,c}{\lambda} - \frac{h\,c}{\lambda_0} = h\,c\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

$= 6\cdot625\times10^{-34}\times3\times10^{8}$

$\times\left(\frac{1}{4000\times10^{-10}} - \frac{1}{5000\times10^{-10}}\right)$

$= 9\cdot94\times10^{-20}\ \mathrm{J} = \mathbf{0\cdot621\ eV}$

18. Proceed as in problem no. 17.

19. (*a*) $E = \frac{h\,c}{\lambda} = \frac{6\cdot62\times10^{-34}\times3\times10^{8}}{5500\times10^{-10}}$

$= 3\cdot61\times10^{-19}\ \mathrm{J} = \frac{3\cdot61\times10^{-19}}{1\cdot6\times10^{-19}} = \mathbf{2\cdot257\ eV}$

(*b*) $\frac{1}{2}m\,v_{max}^2 = \frac{hc}{\lambda} - \omega = 2\cdot257 - 1\cdot7 = \mathbf{0\cdot557\ eV}$

(*c*) V_0 = Maximum K.E. (in eV) = **0·557 V**

20. Proceed as in problem no. 19.

21. $\frac{1}{2}m\,v_{max}^2 = h\,\nu - \omega$

$= 6\cdot625\times10^{-34}\times10^{15} - 4\times1\cdot602\times10^{-19}$

$= 6\cdot625\times10^{-19} - 6\cdot408\times10^{-19} = 0\cdot217\times10^{-19}$ J

$\therefore\ v_{max} = \sqrt{\frac{2\times0\cdot217\times10^{-19}}{m}}$

$= \sqrt{\frac{2\times0\cdot217\times10^{-19}}{9\cdot1\times10^{-31}}} = \mathbf{2\cdot18\times10^{5}\ m\ s^{-1}}$

22. $h\,\nu = \omega + \frac{1}{2}m\,v_{max}^2$

$= 2\cdot3\times1\cdot6\times10^{-19} + \frac{1}{2}\times9\cdot1\times10^{-31}\times(10^4)^2$

$= 3\cdot68\times10^{-19} + 4\cdot55\times10^{-19} = 8\cdot23\times10^{-19}$ J

$\therefore\ \nu = \frac{8\cdot23\times10^{-19}}{6\cdot62\times10^{-34}} = \mathbf{1\cdot243\times10^{15}\ Hz}$

23. $\lambda_0 = \frac{h\,c}{\omega} = \frac{6\cdot625\times10^{-34}\times3\times10^{8}}{2\cdot35\times1\cdot6\times10^{-19}}$

$= 5\cdot286\times10^{-7}\ \mathrm{m} = \mathbf{5286\ Å}$

$\frac{1}{2}m\,v_{max}^2 = \frac{h\,c}{\lambda} - \omega$

$= \frac{6\cdot625\times10^{-34}\times3\times10^{8}}{1000\times10^{-10}\times1\cdot6\times10^{-19}} - 2\cdot35$

$= 12\cdot42 - 2\cdot35 = \mathbf{10\cdot07\ eV}$

24. $\frac{h\,c}{\lambda} = \frac{h\,c}{\lambda_0} + \frac{1}{2}m\,v_{max}^2$

$= \frac{6\cdot62\times10^{-27}\times3\times10^{10}}{2500\times10^{-8}} + 1\cdot56\times1\cdot6\times10^{-12}$

$= 7\cdot944\times10^{-12} + 2\cdot496\times10^{-12} = 10\cdot44\times10^{-12}$ erg

$\therefore\ \lambda = \frac{h\,c}{10\cdot44\times10^{-12}} = \frac{6\cdot62\times10^{-27}\times3\times10^{10}}{10\cdot44\times10^{-12}}$

$= 1\cdot902\times10^{-5}\ \mathrm{cm} = \mathbf{1902\ Å}$

25. $h\,\nu = h\,\nu_0 + e\,V_0$ or $\nu = \nu_0 + \frac{e\,V_0}{h}$

or $\nu = 6\times10^{-14} + \frac{1\cdot6\times10^{-19}\times3}{6\cdot62\times10^{-34}}$

$= 6\times10^{14} + 7\cdot25\times10^{14} = \mathbf{13\cdot25\times10^{14}\ s^{-1}}$

$$\omega = h\nu_0 = 6{\cdot}62 \times 10^{-34} \times 6 \times 10^{14} = 3{\cdot}97 \times 10^{-19}\ \text{J}$$

$$= \frac{3{\cdot}97\times10^{-19}}{1{\cdot}6\times10^{-19}} = \mathbf{2{\cdot}48\ eV}$$

26. $h\nu = h\nu_0 + \frac{1}{2} m v_{max}^2$

or $\nu = \nu_0 + \frac{1}{h}\left(\frac{1}{2} m v_{max}^2\right)$

$$= 4{\cdot}22\times10^{14} + \frac{1}{6{\cdot}6\times10^{-34}}\left[\frac{1}{2}\times 9{\cdot}1\times10^{-31}\times(6\times10^{5})^{2}\right]$$

$$= 4{\cdot}22 \times 10^{14} + 2{\cdot}48 \times 10^{14} = \mathbf{6{\cdot}7 \times 10^{14}\ Hz}$$

27. ***(a)*** $\frac{1}{2} m v_{max}^2 = \frac{hc}{\lambda} - \omega$

$$= \frac{6{\cdot}6\times10^{-34}\times3\times10^{8}}{2000\times10^{-10}} - 4{\cdot}2\times1{\cdot}6\times10^{-19}$$

$$= 9{\cdot}9 \times 10^{-19} - 6{\cdot}72 \times 10^{-19} = 3{\cdot}18 \times 10^{-19}\ \text{J}$$

$$= \frac{3{\cdot}18\times10^{-19}}{1{\cdot}6\times10^{-19}} = \mathbf{1{\cdot}9875\ eV}$$

(b) Slowest electrons emitted will have **zero** kinetic energy.

(c) V_0 = Maximum K.E. (in eV) = **1·9875 V**

(d) $\lambda_0 = \frac{hc}{\omega} = \frac{6{\cdot}6\times10^{-34}\times3\times10^{8}}{4{\cdot}2\times1{\cdot}6\times10^{-19}}$

$$= 2{\cdot}946 \times 10^{-7}\ \text{m} = \mathbf{2946\ Å}$$

28. $\frac{hc}{\lambda} = \omega + eV_0$ or $\omega = \frac{hc}{\lambda} - eV_0$

$$\therefore\ \omega = \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{250\times10^{-9}} - 1{\cdot}6\times10^{-19}\times4$$

$$= 7{\cdot}944 \times 10^{-19} - 6{\cdot}4 \times 10^{-19} = 1{\cdot}544 \times 10^{-19}\ \text{J}$$

Let V_0' be stopping potential, when wavelength of inci light is increased to λ' (= 300 nm).

$$\therefore\ eV_0' = \frac{hc}{\lambda'} - \omega$$

$$= \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{300\times10^{-9}} - 1{\cdot}544\times10^{-19}$$

$$= 6{\cdot}62 \times 10^{-19} - 1{\cdot}544 \times 10^{-19} = 5{\cdot}076 \times 10^{-19}\ \text{J}$$

or $V_0' = \frac{5{\cdot}076\times10^{-19}}{e} = \frac{5{\cdot}076\times10^{-19}}{1{\cdot}6\times10^{-19}} = \mathbf{3{\cdot}17\ V}$

29. Now, $\frac{hc}{\lambda} = \omega + eV_0$

or $\omega = \frac{hc}{\lambda} - eV_0$

$$= \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{4950\times10^{-10}} - 1{\cdot}6\times10^{-19}\times0{\cdot}6$$

$$= 4{\cdot}01 \times 10^{-19} - 0{\cdot}96 \times 10^{-19} = 3{\cdot}05 \times 10^{-19}\ \text{J}$$

$$= \frac{3{\cdot}05\times10^{-19}}{1{\cdot}6\times10^{-19}} = \mathbf{1{\cdot}9\ eV}$$

Let λ' be wavelength of second source. Then,

$$\frac{hc}{\lambda'} = \omega + eV_0 = 3{\cdot}05\times10^{-19} + 1{\cdot}6\times10^{-19}\times1{\cdot}1$$

$$= 3{\cdot}05 \times 10^{-19} + 1{\cdot}76 \times 10^{-19} = 4{\cdot}81 \times 10^{-19}\ \text{J}$$

$$\therefore\ \lambda' = \frac{hc}{4{\cdot}81\times10^{-19}}$$

$$= \frac{6{\cdot}62\times10^{-34}\times3\times10^{8}}{4{\cdot}81\times10^{-19}} = 4{\cdot}129\times10^{-7}\ \text{m}$$

$$= \mathbf{4129\ Å}$$

There is **no effect of magnetic field** on photoelect emission.

CHAPTER
2

Wave Nature of Matter

2.01. DE-BROGLIE WAVES (Matter waves)

Radiation behaves both as wave and particle. In 1924, Loius de-Broglie put rward a bold hypothesis that matter should also possess dual nature.

Quotable Quote

"Some say that the only thing quantum theory has going for it is unquestionably correct." – **Michio Kaku**

The following observations led him to the duality hypothesis for matter :

1. *The whole energy in this universe is in the form of matter and electromagnetic diation.*

2. *The nature loves symmetry. As the radiation has got dual nature, matter should also ssess dual nature.*

Thus, according to de-Broglie, a wave is associated with every moving particle. hese waves are called ***de-Broglie waves*** or ***matter waves.***

According the quantum theory of radiation, energy of a photon is given by

$$E = h\nu \qquad \text{...(2.01)}$$

Further, the energy of a relativistic particle is given by

$$E = \sqrt{m_0^2\, c^2 + p^2\, c^2}$$

Since photon is a particle of zero rest mass, setting $m_0 = 0$ in the above equation, e have

$$E = p\,c \qquad \text{...(2.02)}$$

Key point

A photon has zero rest mass but a photon of energy E possesses a mass equal to E/c^2 and momentum equal to E/c.

From equations (2.01) and (2.02), we have

$$p\,c = h\nu$$

or

$$p = \frac{h\nu}{c} = \frac{h\nu}{\upsilon\lambda} \qquad (\because c = \upsilon\,\lambda)$$

or

$$p = \frac{h}{\lambda}$$

Therefore, the wavelength of the photon is given by

$$\lambda = \frac{h}{p} \qquad \text{...(2.03)}$$

Key point

de-Broglie waves are matter waves *i.e.* waves altogether different from electromagnetic waves.

de-Broglie asserted that the equation (2.03) is completely a general formula and pplies to photons as well as other moving particles. The momentum of a particle f mass m moving with velocity υ is $m\,\upsilon$. Hence, de-Broglie wavelength is given by

$$\lambda = \frac{h}{m\,\upsilon} \qquad \text{...(2.04)}$$

This is called ***de-Broglie relation***. It connects the ***momentum***, which is *haracteristic of the particle*, with the *wavelength*, which is *characteristic of the wave*. From his equation, the following conclusions can be drawn :

(*i*) *Lighter the particle, greater is its de-Broglie wavelength.*

(*ii*) *The faster the particle moves, the smaller is its de-Broglie wavelength.*

(*iii*) *The de-Broglie wavelength of a particle is independent of the charge or nature of the particle.*

(*iv*) *The matter waves are not electromagnetic in nature. The electromagnetic waves are produced only by charged particles.*

Key point

When a particle exhibits wave nature, it is associated with a wave packet, rather than a wave.

The calculations show that the velocity of de-Broglie waves associated with a moving particle is greater than the velocity with which the particle moves. Therefore, t was difficult to conceive as to how the de-Broglie waves are associated with a particle. If a large number of waves of frequencies differing by very small amounts are superimposed, a wave packet is formed. The velocity of the wave packet is found to be the same as that of the particle. Therefore, a plausible solution to the problem is that a moving particle is associated with a ***wave packet*** [Fig. 2.01], rather than a wave.

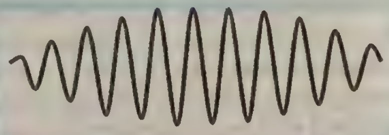


Fig. 2.01

If the velocity of the particle is comparable to the velocity of light, then mass of the particle is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Substituting for the mass m of the particle in equation (2.04), the de-Broglie wavelength of the particle, when moving with velocity comparable to velocity of light, is given by

$$\lambda = h\frac{\sqrt{1 - v^2/c^2}}{m_0\, v} \qquad ...(2.05)$$

2.02. DE-BROGLIE WAVELENGTH OF ELECTRON

Consider that an electron of mass m and charge e is accelerated through a potential difference V. If E is the energy acquired by the particle, then

$$E = e\,V$$

If v is the velocity of electron, then

$$E = \frac{1}{2} m\, v^2$$

or $$v = \sqrt{\frac{2\,E}{m}}$$

Now, de-Broglie wavelength of electron is given by

$$\lambda = \frac{h}{m\,v} = \frac{h}{m\sqrt{2\,E/m}}$$

or $$\lambda = \frac{h}{\sqrt{2\,m\,E}} \qquad ...(2.06)$$

Substituting the value of E, we get

$$\lambda = \frac{h}{\sqrt{2\,m\,e\,V}} \qquad ...(2.07)$$

Setting $m = 9{\cdot}1 \times 10^{-31}$ kg ; $e = 1{\cdot}6 \times 10^{-19}$ C and $h = 6{\cdot}62 \times 10^{-34}$ J s, we get

$$\lambda = \frac{12{\cdot}27}{\sqrt{V}} \times 10^{-10}\ \text{m}$$

or $$\lambda = \frac{12{\cdot}27}{\sqrt{V}}\ \text{Å} \qquad ...(2.08)$$

For example, the de-Broglie wavelength of electrons, when accelerated through a potential difference of 100 volt, will be

$$\lambda = \frac{12{\cdot}27}{\sqrt{100}} \approx 1{\cdot}227\ \text{Å}$$

Thus, the wavelength of de-Broglie waves associated with 100 eV electrons is of the order of the wavelength of X-rays, but the two are quite different from each other in nature.

Key point

A particle can behave both as a particl and wave but in a given situation, i behaves either as a particle or as a wave

Key point

As the accelerating voltage is increased de-Broglie wavelength of electron decreases.

Watch out !

Don't try to work out the momentum of high speed particle and hence its de-Broglie wavelength using its rest mass Mass increases significantly as the spee of the particle approaches the speed o light !

2.03. EXPERIMENTAL DEMONSTRATION OF WAVE NATURE OF ELECTRON

The wave nature of the material particles as predicted by de-Broglie was confirmed by Davisson and Germer (1927) in United States and by G.P. Thomson (1928) in Scotland. To demonstrate the wave nature of electrons, Davisson and Germer experiment is as discussed below :

Davisson and Germer experiment. Experimental arrangement used by Davisson and Germer is as shown in Fig. 2.02. Electrons from hot tungsten cathode (C) are accelerated by a potential difference V between the cathode and anode (A). A narrow hole in the anode renders the electrons into a fine beam of electrons and allows it to strike the nickel crystal. The electrons are scattered in all directions by the atoms in the crystal. The intensity of the electron beam scattered in a given direction is found by the use of a detector. By rotating the detector about an axis through the point O, the intensity of the scattered beam can be measured for different values of ϕ, the angle between incident and the scattered direction of electron beam. The graph is plotted between angle ϕ and the intensity of scattered electron beam.

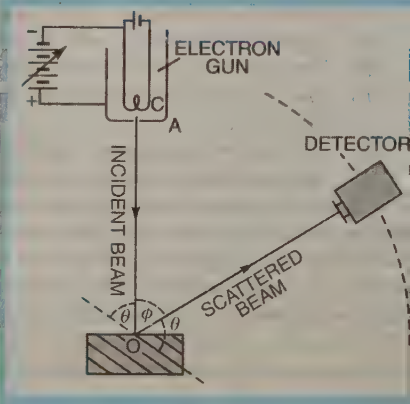


Fig. 2.02

ch graphs are plotted at different accelerating voltages. In each graph, the intensity electron beam in a given direction is proportional to the distance of the curve from e point O (the point of scattering) in that direction. The experimental curves otained by Davisson and Germer were as shown in Fig. 2.03.

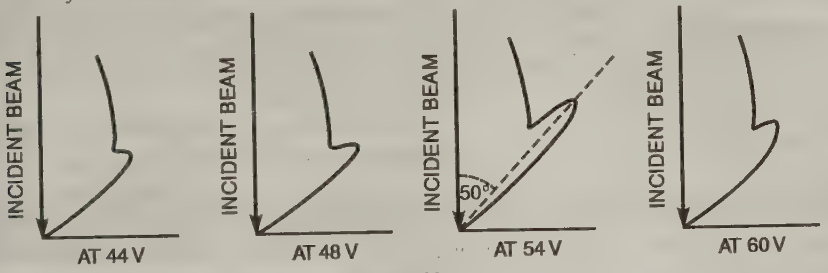


Fig. 2.03

From these experimental curves, following inferences can be drawn :

(*i*) Intensity of scattered electrons depends upon the angle of scattering ϕ.

(*ii*) Always a `bump' or a kink occurs in the curve at $\phi = 50°$, the angle which the scattered beam makes with the incident beam.

(*iii*) The size of the bump goes on increasing as the accelerating voltage is increased.

(*iv*) The size of the bump becomes maximum, when accelerating voltage is 54 volt.

(*v*) The size of the bump starts decreasing with further increase in accelerating voltage.

However, according to classical physics, there should be very little variation n the intensity of electron beam with the angle of scattering and the accelerating oltages.

Explanation. The selective reflection of the 54 volt electrons at an angle of 0°between incident and scattered beam can be termed as the diffraction of electrons rom the regularly spaced electrons of nickel crystal by virtue of their wave nature.

Key point

Since phenomenon of diffraction is exhibited by waves, electrons must be behaving as waves in Davisson and Germer experiment.

From Fig. 2.02, it follows that

$$\theta = \frac{1}{2}(180° - \phi)$$

For $\phi = 50°, \theta = \frac{1}{2}(180° - 50°) = \frac{1}{2} \times 130° = 65°$

According to Bragg's law, for first order diffraction maximum ($n = 1$), we have

$$2\,d \sin\theta = 1 \times \lambda$$

For nickel crystal, the distance between atomic planes, $d = 0{\cdot}91$ Å

Therefore, $\lambda = 2 \times 0{\cdot}91 \times \sin 65° = 1{\cdot}66$ Å

Now, wave nature of electron and hence de-Broglie hypothesis will be confirmed, if de-Broglie wavelength of electrons when accelerated through 54V comes out to be just 1·66 Å.

Putting V = 54 volt in equation (2.08), we get

$$\lambda = \frac{12{\cdot}27}{\sqrt{54}} = 1{\cdot}65 \text{ Å}$$

As the two results are in remarkable agreement, the experiment establishes the wave nature of an electron in particular and of a particle in general.

SOLVED NUMERICAL QUESTIONS

Type A. On de-Broglie wavelength of a particle

Problem 2.01. Calculate the de-Broglie wavelength for electron and proton, if their speed is 10^5 m s^{-1}. Given, mass of an electron = $9{\cdot}1 \times 10^{-31}$ kg, mass of proton = $1{\cdot}67 \times 10^{-27}$ kg and Planck's constant = $6{\cdot}62 \times 10^{-34}$ J s.

Sol. Here, $h = 6{\cdot}62 \times 10^{-34}$ J s

de-Broglie wavelength of electron :

Here, $m_e = 9{\cdot}1 \times 10^{-31}$ kg ; $v_e = 10^5$ m s^{-1}

$$\therefore \quad \lambda_e = \frac{h}{m_e\, v_e} = \frac{6{\cdot}62 \times 10^{-34}}{9{\cdot}1 \times 10^{-31} \times 10^5} = \mathbf{7{\cdot}27 \times 10^{-9}\ m}$$

de-Broglie wavelength of proton :

Here, $m_p = 1{\cdot}67 \times 10^{-27}$ kg ; $v_p = 10^5$ m s^{-1}

$$\therefore \quad \lambda_p = \frac{h}{m_p\, v_p} = \frac{6{\cdot}62 \times 10^{-34}}{1{\cdot}67 \times 10^{-27} \times 10^5} = \mathbf{3{\cdot}96 \times 10^{-12}\ m}$$

Problem 2.02. Find the de-Broglie wavelength associated with an electron moving with a velocity 0·5 *c* and rest mass = 9·1 × 10^{-31} kg.

Sol. Here, $m_0 = 9{\cdot}1 \times 10^{-31}$ kg ; $h = 6{\cdot}62 \times 10^{-34}$ J s ;

$$v = 0{\cdot}5\,c \quad \text{or} \quad v/c = 0{\cdot}5$$

Since electron is moving at a speed comparable to the velocity of light,

$$\lambda = \frac{h}{m\,v} = \frac{h\sqrt{1 - v^2/c^2}}{m_0\,v}$$

$$= \frac{6{\cdot}62 \times 10^{-34}\sqrt{1 - (0{\cdot}5)^2}}{9{\cdot}1 \times 10^{-31} \times 1{\cdot}5 \times 10^8}$$

$$= \frac{6{\cdot}62 \times 10^{-34}\sqrt{1 - 0{\cdot}25}}{9{\cdot}1 \times 10^{-31} \times 1{\cdot}5 \times 10^8}$$

$$= \mathbf{4{\cdot}2 \times 10^{-12}\ m}$$

Type B. On de-Broglie wavelength of an accelerate electron

Problem 2.03. Calculate the de-Broglie wavelengt of an electron of energy 400 eV. Given, Planck's constant = 6·6 × 10^{-34} J s, mass of electron = 9·1 × 10^{-31} kg 1 eV = 1·6 × 10^{-19} J *(H.P.S.S.C.E. 2000 ; I.S.C.E. 1993*

Sol. Here, $h = 6{\cdot}6 \times 10^{-34}$ J s ; $m = 9{\cdot}1 \times 10^{-31}$ kg ;

$$1\ \text{eV} = 1{\cdot}6 \times 10^{-19}\ \text{J}$$

$$E = 400\ \text{eV} = 400 \times 1{\cdot}6 \times 10^{-19} = 6{\cdot}4 \times 10^{-17}\ \text{J}$$

If λ is de-Broglie wavelength of electron, then

$$\lambda = \frac{h}{\sqrt{2\,m\,E}} = \frac{6{\cdot}6 \times 10^{-34}}{\sqrt{2 \times 9{\cdot}1 \times 10^{-31} \times 6{\cdot}4 \times 10^{-17}}}$$

$$= \frac{6{\cdot}6 \times 10^{-34}}{10{\cdot}8 \times 10^{-24}} = 0{\cdot}61 \times 10^{-10}\ \text{m}$$

$$= \mathbf{0{\cdot}61\ Å}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 2.04. Obtain the de-Broglie wavelength associated with thermal neutrons at room temperature (27°C). Hence, explain why a fast neutron beam needs to be thermalized with the environment, before it can be used for neutron diffraction experiments. *(Text Problem)*

Sol. Here, T = 27 + 273 = 300 K

Energy of neutron at temperature T, $E = \frac{3}{2}kT$

Taking $k = 1{\cdot}38 \times 10^{-23}$ J molecule^{-1} K^{-1}, we have

$$E = \frac{3}{2} \times 1{\cdot}38 \times 10^{-23} \times 300 = 6{\cdot}21 \times 10^{-21}\ \text{J}$$

Now, $\lambda = \frac{h}{\sqrt{2\,m\,E}}$

Taking mass of deutron, $m = 1{\cdot}675 \times 10^{-27}$ kg, we have

$$\lambda = \frac{6{\cdot}62 \times 10^{-34}}{\sqrt{2 \times 1{\cdot}675 \times 10^{-27} \times 6{\cdot}21 \times 10^{-21}}} = \frac{6{\cdot}62 \times 10^{-34}}{4{\cdot}56 \times 10^{-24}}$$

$$= \mathbf{1{\cdot}452 \times 10^{-10}\ m}$$

Thus, thermal neutrons have wavelength of the order of interatomic spacing(≈ 10^{-10} m)and therefore they are suitable for diffraction experiments. Fast neutrons will possess wavelength quite small as compared to interatomic spacing and are not suitable for diffraction experiments. Hence, for neutron diffraction experiments, fast neutron beam needs to be thermalized.

Problem 2.05. Crystal diffraction experiments can be performed using X-rays or electrons accelerated through appropriate voltage. Which probe has greater energy, an X-ray photon or the electron ? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of order of interatomic spacing in the lattice). Given, m_e = 9·11 × 10^{-31} kg. *(Text Problem)*

Sol. *For X-ray photon of wavelength of 1 Å :*

$$\lambda = 1\ Å = 10^{-10}\ \text{m}$$

$$E = \frac{hc}{\lambda} = \frac{6{\cdot}62 \times 10^{-34} \times 3 \times 10^8}{10^{-10}} = 1{\cdot}986 \times 10^{-15}\ \text{J}$$

$$= \frac{1{\cdot}986 \times 10^{-15}}{1{\cdot}6 \times 10^{-19}} = 12{\cdot}4 \times 10^3\ \text{eV} = 12{\cdot}4\ \text{keV}$$

For electron of wavelength 1 Å :

$$\lambda = 1\ Å = 10^{-10}\ \text{m}$$

Now $\lambda = \frac{h}{m\,v}$

or $m\,v = \frac{h}{\lambda} = \frac{6{\cdot}62 \times 10^{-34}}{10^{-10}} = 6{\cdot}62 \times 10^{-24}$ kg m s^{-1}

Energy of electron,

$$E = \frac{1}{2}m\,v^2 = \frac{(m\,v)^2}{2\,m}$$

$$= \frac{(6{\cdot}62 \times 10^{-24})^2}{2 \times 9{\cdot}11 \times 10^{-31}} = 2{\cdot}405 \times 10^{-17}\ \text{J}$$

$$= \frac{2{\cdot}405 \times 10^{-17}}{1{\cdot}6 \times 10^{-19}} = \mathbf{150{\cdot}5\ eV}$$

It follows that X-ray photon has greater energy.

Problem 2.06. Obtain the de-Broglie wavelength of a neutron of kinetic energy 150 eV. As said in conceptual problem 2.05, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable ? Explain. Given, m_n = 1·675 × 10^{-27} kg. *(Text Problem)*

Sol. Here, $m_n = 1{\cdot}675 \times 10^{-27}$ kg ;

$$E = 150\ \text{eV} = 150 \times 1{\cdot}6 \times 10^{-19}$$

$$= 2{\cdot}4 \times 10^{-17}\ \text{J}$$

Now, $\lambda = \frac{h}{m\,v} = \frac{h}{\sqrt{2\,m\,E}}$

Therefore, de-Broglie wavelength of neutron is given by

$$\lambda = \frac{6{\cdot}62 \times 10^{-34}}{\sqrt{2 \times 1{\cdot}675 \times 10^{-27} \times 2{\cdot}4 \times 10^{-17}}} = \frac{6{\cdot}62 \times 10^{-34}}{2{\cdot}835 \times 10^{-22}}$$

$$= 2{\cdot}335 \times 10^{-12}\ \text{m}$$

Since the interatomic spacing(≈ 10^{-10} m) is about 100 times greater than the de-Broglie wave length of 150 eV neutrons, they are not suitable for crystal diffraction experiments.

Problem 2.07. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other

ctors (such as numerical aperture, etc.) are taken to be ughly the same, how does the resolving power of an ectron microscope compare with that of an optical microope which uses yellow light ? (*Text Problem*)

Sol. Here, V=50 kV

Therefore, energy of electrons

$$E = 50 \text{ keV} = 50 \times 10^3 \times 1{\cdot}6 \times 10^{-19}$$
$$= 8{\cdot}0 \times 10^{-15} \text{ J}$$

Now, $\lambda = \dfrac{h}{\sqrt{2\,m\,E}}$

Taking $m = 9{\cdot}1 \times 10^{-31}$ kg, we have

$$\lambda = \frac{6{\cdot}62 \times 10^{-34}}{\sqrt{2 \times 9{\cdot}1 \times 10^{-31} \times 8{\cdot}0 \times 10^{-15}}} = \frac{6{\cdot}62 \times 10^{-34}}{1{\cdot}207 \times 10^{-22}}$$
$$= 5{\cdot}485 \times 10^{-12} \text{ m}$$

The resolving power of a microscope is inversely proportional to the wavelength of the radiation used. Since wavelength of the yellow light is 5990Å *i.e.* $5{\cdot}99 \times 10^{-7}$ m, it follows that the electron microscope will possess resolving power 10^5 times as large as that of the optical microscope.

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Is photon a wave or a particle ?

Ans. Photon is a particle. It possesses wave nature also.

Q. 2.02. What is the momentum of a photon of equency ν ? (*H.P.S.S.C.E. 1996*)

Ans. Momentum of photon of frequency ν, $p = \dfrac{h\nu}{c}$

Q. 2.03. Write down the relation between the energy nd the momentum of a photon. (*I.S.C.E. 1993*)

Ans. $p = \dfrac{E}{c}$, where *c* is the velocity of light.

Q. 2.04. What is the rest mass of a photon ? (*P.S.S.C.E. 1999 S ; H.P.S.S.C.E. 1993*)

Ans. Rest mass of a photon is zero.

Q. 2.05. What led to the discovery of matter waves ?

Ans. de-Broglie's concept of 'nature loves symmetry' led o the discovery of matter waves.

Q. 2.06. Is there any difference between light waves nd matter waves ?

Ans. Light waves are electromagnetic waves, whereas natter waves are not. Matter waves are associated with a noving particle.

Q. 2.07. Are matter waves electromagnetic ? Write de-Broglie wave equation. (*H.S.S.C.E. 1992*)

Ans. No, matter waves are not electromagnetic in nature.

de-Broglie relation : $\lambda = \dfrac{h}{m v}$

Q. 2.08. What information is derived from electron diffraction experiments ? (*I.S.C.E. 1997*)

Ans. A particle possesses wave nature also.

Q. 2.09. What is de-Broglie wavelength of a 3 kg object moving with a speed of 2 m s^{-1}? (*C.B.S.E. 1992*)

Ans. de-Broglie wavelength,

$$\lambda = \frac{h}{m v} = \frac{6{\cdot}62 \times 10^{-23}}{3 \times 2}$$
$$= 1{\cdot}1 \times 10^{-34} \text{ m}$$

Q. 2.10. Calculate the momentum of electrons ; if their wavelength is 2 Å. Given that Planck's constant, $h = 6{\cdot}626 \times 10^{-34}$ J s ; mass of electron, $m = 9{\cdot}1 \times 10^{-31}$ kg . (*H.P.S.S.C.E. 1998 S*)

Sol. Here, $\lambda = 2$ Å $= 2 \times 10^{-10}$ m ; $h = 6{\cdot}626 \times 10^{-34}$ J s ;
$m = 9{\cdot}1 \times 10^{-31}$ kg

Momentum of electron,

$$p = \frac{h}{\lambda} = \frac{6{\cdot}626 \times 10^{-34}}{2 \times 10^{-10}}$$
$$= 3{\cdot}313 \times 10^{-24} \text{ kg m s}^{-1}$$

Q. 2.11. A photon and an electron have the same wavelength. Which particle is moving faster ?

Ans. A photon travels with velocity of light, whereas velocity of electron is always less than that of light.

Q. 2.12. On what principle is electron microscope based ?

Ans. An electron microscope is based on the fact that the electrons can behave as waves of extremely small wavelength when accelerated through a potential difference of about 50 kV.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. What do you understand by the phrase 'dual nature of radiation' ? (*C.B.S.E. 1982, 1981*)

Ans. The phenomena such as interference, diffraction and polarisation can only be explained on the basis of wave nature of the radiation. On the other hand, the phenomena such as photoelectric effect and Compton effect can be explained by considering particle nature of radiation.

It was, therefore, concluded that the radiation has dual nature.

Q. 2.02. Derive de-Broglie's wave equation for material particles. (*H.P.S.S.C.E. 1993*)

Ans. Refer to section 2.01.

Q. 2.03. Find the momentum of an X-ray photon of wavelength λ. (*I.S.C.E. 1997*)

Ans. Refer to section 2.01.

Q. 2.04. The de-Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle, if its kinetic energy were K /4 ? (*I.S.C.E. 1993*)

Ans. If λ is the de-Broglie wavelength of the particle of kinetic energy K, then

$$\lambda = \frac{h}{\sqrt{2\,m\,K}}$$

Suppose that the de-Broglie wavelength of the particle becomes λ', when its kinetic energy is K/4.

Then, $$\lambda' = \frac{h}{\sqrt{2\,m\,K/4}} = 2\left(\frac{h}{\sqrt{2\,m\,K}}\right) = \mathbf{2\,\lambda}$$

Q. 2.05. For a given kinetic energy, which of the following has the smallest de-Broglie wavelength : electron, proton or α-particle ? *(Text Question)*

Ans. Let m_1, m_2, m_3 be masses and v_1, v_2, v_3 be velocities for electron, proton and α-particle respectively. As the three particles are of same kinetic energy, it follows that

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_3 v_3^2 = \text{E (say)}$$

$$\therefore \quad m_1 v_1 = \sqrt{2 m_1 \text{E}}, \; m_2 v_2 = \sqrt{2 m_2 \text{E}}$$

$$\text{and} \quad m_3 v_3 = \sqrt{2 m_3 \text{E}}$$

Obviously, $m_1 v_1 < m_2 v_2 < m_3 v_3$

Since de-Broglie wavelength, $\lambda = \frac{h}{m v}$, it follows that de-Broglie wave will be the smallest for α-particle.

Q. 2.06. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non zero velocities. What is the ratio of the de-Broglie wavelengths of the two particles ? *(I.I.T. 1999)*

Ans. Suppose that the particle of mass M (at rest) decays into two particles of masses m_1 and m_2, which start moving with velocities v_1 and v_2 respectively. Then, according to the principle of conservation of momentum,

$$m_1 v_1 + m_2 v_2 = \text{M} \times 0 \text{ or } m_1 v_1 = -m_2 v_2$$

$$\text{or} \quad |m_1 v_1| = |m_2 v_2|$$

Now, de-Broglie wavelength of a particle,

$$\lambda = \frac{h}{m v}$$

Since $|m_1 v_1| = |m_2 v_2|$, the two particles will have equal de-Broglie wavelengths *i.e.*

$$\frac{\lambda_1}{\lambda_2} = 1$$

Q. 2.07. A photon and electron have got the same de-Broglie wavelength. Which has greater total energy ? Explain. *(H.P.S.S.C.E. 1998)*

Or

An electron is having the de-Broglie wavelength same as the wavelength of a photon. Prove that energy of electron is greater than that of photon. *(H.P.S.S.C.E. 1995)*

Ans. Let λ be the common wavelength of electron and photon. If m and v are mass and velocity of the electron, then

$$\text{de-Broglie wavelength of electron, } \lambda = \frac{h}{m v}$$

$$\text{Energy of electron (mass } m\text{), } \text{E}_1 = m c^2$$

$$\text{Energy of the photon} \left(\text{wavelength, } \lambda = \frac{h}{m v} \right),$$

$$\text{E}_2 = \frac{h c}{\lambda} = h c \times \frac{m v}{h}$$

$$\text{or} \quad \text{E}_2 = m v c$$

$$\text{since} \quad c > v, \text{E}_1 > \text{E}_2$$

Q. 2.08. The de-Broglie wavelength of a photon is sam as the wavelength of an electron. Show that the K.E. o photon is 2 λ m c/h times the K.E. of the electron, where r is the mass of electron and c is the velocity of light.

Ans. If the photon and an electron having mass m an moving with velocity v possess the same de-brogli wavelength λ, then as obtained in SAQ 2.07,

$$\text{the energy of photon, } \frac{h c}{\lambda} = m v c$$

$$\text{and the energy of electron } = m c^2$$

The rest mass energy of photon is zero. Therefore, energ of photon refers to kinetic energy of photon. However, th energy of the electron *i.e.* $m c^2$ is the sum of its rest mass energ ($m_0 c^2$) and kinetic energy $\left(\frac{1}{2} m v^2\right)$. If E_1 and E_2 are kineti energies of photon and electron, then

$$\frac{\text{E}_1}{\text{E}_2} = \frac{m v c}{\frac{1}{2} m v^2} = \frac{2 c}{v}$$

$$\text{Now, de-Broglie wavlength, } \lambda = \frac{h}{m v}$$

$$\text{or} \quad v = \frac{h}{m \lambda}$$

Substituting for v in the above equation, we have

$$\frac{\text{E}_1}{\text{E}_2} = 2 c \Big/ \frac{h}{m \lambda} = \frac{2 \lambda m c}{h}$$

Q. 2.09. An electron and a proton have the same de Broglie wavelength. Which one of these has higher kineti energy ? *(I.S.C.E. 1994*

Or

A proton and an electron have same de-Broglie wavelength. Which of them moves fast and which possesses more kinetic energy ? Justify your answer. *(C.B.S.E. Sample Q. Paper*

Ans. Refer to SAQs 2.07 and 2.08. Photon has highe kinetic energy.

Q. 2.10. Why is the wave nature of matter not apparen in our daily observations ?

Ans. The value of de-Broglie wavelength associated wit a moving particle is given by

$$\lambda = \frac{h}{m v}$$

Consider a ball of 0·5 kg moving with a velocity o 10 m s^{-1}. It is an object from our daily observations. The de-Broglie wavelength of the ball will be

$$\lambda = \frac{6 \cdot 625 \times 10^{-34}}{0 \cdot 5 \times 10} = 1 \cdot 325 \times 10^{-34} \text{ m}$$

The value of λ is very small as compared to the size a ball of 0·5 kg would possess. Hence, wave nature of matte is not apparent in our daily observations.

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 2.01. Is the de-Broglie wavelength of a photon of an electromagnetic radiation equal to the wavelength of the radiation ? *(Text Problem)*

Ans. Yes, wavelength of a photon and de-Broglie wavelength of a photon are equal.

Q. 2.02. Prove that de-Broglie hypothesis of matter waves is in agreement with Bohr's theory.

Ans. According to Bohr's theory, *angular momentum of an electron in a stationary orbit is equal to integral multiple of* $\frac{h}{2\pi}$ *i.e.*

$$m v r = n \frac{h}{2\pi} \qquad ...(i)$$

This quantisation principle applies to particle nature of the electron. If electron behaves as a wave, then stationary orbit may be interpreted as the one, whose circumference contains integral multiple of de-Broglie wavelengths of electron. Therefore, if λ is de-Broglie wavelength of electron, then

$$2\pi r = n\lambda \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*), we get

$$\frac{m v r}{2\pi r} = \frac{n h}{2\pi} \cdot \frac{1}{n\lambda} \quad \text{or} \quad \lambda = \frac{h}{m v}$$

Hence, de-Broglie hypothesis is in agreement with Bohr's theory.

Q. 2.03. The extent of localisation of a particle is determined roughly by its de-Broglie wave. If an electron is localized within the nucleus (of size about 10^{-14} m) of an atom, what is its energy ? Compare this energy with the typical binding energies (of the order of a few MeV) in a nucleus and hence argue why electrons cannot reside in a nucleus. (*Text Question*)

Ans. The de-broglie wavelength of electron,

$$\lambda = \text{size of atom} = 10^{-14}\ \text{m}$$

Corresponding to this wavelength, the electron possesses energy,

$$E = \frac{h c}{\lambda} = \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^{8}}{10^{-14}} = 1 \cdot 986 \times 10^{-11}\ \text{J}$$

$$= \frac{198 \cdot 6 \times 10^{-13}}{1 \cdot 6 \times 10^{-13}} = 124 \cdot 1\ \text{MeV}$$

If the electrons possess energy of the order of 124 MeV, they will not be able to stay inside the nucleus. It is because Coulomb's force cannot provide such a high value of binding energy to the electrons. Hence, electrons cannot reside inside the nucleus.

Q. 2.04. A fast neutron beam needs to be thermalized with the environment, before it can be used for neutron diffraction experiments. Why ? (*Text Question*)

Ans. For diffraction experiment, the de-Broglie wavelength must be of the order of interatomic spacing *i.e.* 1 Å. An electron having energy 150 eV or so possesses de-Broglie wavelength of the order of 1 Å but a neutron having the same energy possesses wavelength quite small as compared to the interatomic spacing. Hence, the neutron beam cannot be used as such for diffraction experiments.

However, a neutron at room temperature, say 27°C, possesses wavelength of the order of interatomic spacing. Therefore, in order to study diffraction experiment with a neutron beam, it should be thermalized.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Describe de-Broglie concept of matter waves and obtain an expression for the momentum and wavelength of the particle.
2. Derive an expression for the de-Broglie wavelength of an electron moving under a potential difference of V volt. (*H.S.S.C.E. 1999*)
3. Show that the de-Broglie wavelength λ of electrons of energy E is given by the relation :
$$\lambda = \frac{h}{\sqrt{2 m E}} \qquad \text{(C.B.S.E. 1993)}$$
4. Explain de-Broglie concept of matter waves with the help of relevant expressions. Find the wavelength of an electron moving under a potential difference of V volt.

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What led de-Broglie to think that matter might have wave properties ? How was his hypothesis experimentally verified ?
2. Derive de-Broglie wavelength formula. How has the same been experimentally verified ?
3. Explain dual nature of radiation and matter and describe briefly Davisson and Germer experiment for qualitative verification of matter waves.
4. Describe Davisson and Germer experiment to depict the wave nature of electron. (*H.S.S.C.E. 1999 S*)

FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On de-Broglie wavelength of a particle

1. Calculate the wavelength of matter waves associated with a particle of mass 0·5 g moving with a velocity of 400 cm s^{-1}. **[Ans.** 3·31 × 10^{-29} cm]
2. Calculate the wavelength of de-Broglie wave associated with a ball of mass 1 kg travelling with a speed of 1 m s^{-1}. **[Ans.** 6·62 × 10^{-34} m]
3. Calculate the de-Broglie wavelength of electron and proton, if their speed is 10^5 m s^{-1}. Given mass of proton = 1·67 × 10^{-27} kg ; mass of electron = 9·1 × 10^{-31} kg and h = 6·62 × 10^{-34} J s. (*H.P.S.S.C.E. 1999 S*) **[Ans.** 7·27 × 10^{-9} m ; 3·96 × 10^{-12} m]
4. Is it possible to observe de-Broglie wave associated with a material particle of 10^{-4} g moving with the velocity of light ? Planck's constant is 6·6 × 10^{-27} erg s. **[Ans.** No]
5. The de-Broglie wavelength of an electron is 2 Å. Calculate its momentum. **[Ans.** 3·31 × 10^{-24} kg m s^{-1}]
6. Find the wavelength for a beam of neutrons, whose kinetic energy is 100 eV. Given that mass of neutron = 1·676 × 10^{-24} g and h = 6·62 × 10^{-27} erg s. **[Ans.**0·0286 Å]
7. What wavelength is predicted for a beam of electrons, whose kinetic energy is 100 eV ? Given, h = 6·6 × 10^{-34} J s. (*Pre-degree Kerala, 1992*) **[Ans.** 1·223 Å]
8. Find the de-Broglie wavelength of a 2 MeV proton. Given, mass of proton is 1·67 × 10^{-27} kg. **[Ans.** 2·025 × 10^{-14} m]

Type B. On de-Broglie wavelength of an accelerated electron

9. Calculate de-Broglie wavelength of an electron beam accelerated through a potential difference of 60 V. (*C.B.S.E. 1993*) **[Ans.** 1·584 Å]

10. Calculate the wavelength of de-Broglie wave associated with a beam of electrons, which traverse a potential difference of 300 V. Assume the initial speed of electrons to be zero. **[Ans.** 0·708 Å]

11. What voltage must be applied to an electron microscope to produce electrons of wavelength 0·4 Å ? **[Ans.** 940·96 V]

MISCELLANEOUS PROBLEMS

12. Find de-Broglie wavelength of neutron at 127°C. Given, Boltzmann's constant $k = 1{\cdot}38 \times 10^{-23}$ J molecule^{-1} K^{-1}, $h = 6{\cdot}625 \times 10^{-34}$ J s, mass of neutron $= 1{\cdot}66 \times 10^{-27}$ kg. **[Ans.** 1·263 Å]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $\lambda = \dfrac{h}{mv} = \dfrac{6\cdot 62 \times 10^{-27}}{0\cdot 5 \times 400}$ (Take $h = 6{\cdot}62 \times 10^{-27}$ erg s)

   $= \mathbf{3{\cdot}31 \times 10^{-29}}$ **cm**

2. Proceed as in problem no. 1.

3. *For electron :*

   $$\lambda = \frac{h}{mv} = \frac{6\cdot 62 \times 10^{-34}}{9\cdot 1 \times 10^{-31} \times 10^{5}} = \mathbf{7{\cdot}27 \times 10^{-9}\ m}$$

   For proton :

   $$\lambda = \frac{h}{mv} = \frac{6\cdot 62 \times 10^{-34}}{9\cdot 1 \times 10^{-27} \times 10^{5}} = \mathbf{3{\cdot}96 \times 10^{-12}\ m}$$

4. A material particle can not move with the velocity of light. Therefore, it is **not possible** to deserve de-Broglie wavelength associated with such a particle.

5. $$p = \frac{h}{\lambda} = \frac{6\cdot 62 \times 10^{-34}}{2 \times 10^{-10}} = \mathbf{3{\cdot}31 \times 10^{-24}\ kg\ m\ s^{-1}}$$

6. $$\lambda = \frac{h}{\sqrt{2mE}}$$

   $$= \frac{6\cdot 62 \times 10^{-27}}{\sqrt{2 \times 1\cdot 676 \times 10^{-24} \times 100 \times 1\cdot 6 \times 10^{-12}}}$$

   $= 2{\cdot}86 \times 10^{-10}$ cm $= \mathbf{0{\cdot}0286}$ **Å**

7. Take mass of electron as $9{\cdot}1 \times 10^{-31}$ kg and proceed as in problem no. 6.

8. Proceed as in problem no. 6.

9. $$\lambda = \frac{h}{\sqrt{2meV}}$$

   $$= \frac{6\cdot 62 \times 10^{-34}}{\sqrt{2 \times 9\cdot 1 \times 10^{-31} \times 1\cdot 6 \times 10^{-19} \times 60}}$$

   $= 1{\cdot}584 \times 10^{-10}$ m $= \mathbf{1{\cdot}584}$ **Å**

10. Proceed as in problem no. 9.

11. Proceed as in problem no. 9.

12. Energy of neutron at 127°C *i.e.* 400 K,

    $$E = \frac{3}{2} kT = \frac{3}{2} \times 1\cdot 38 \times 10^{-23} \times 400$$

    $$= 8\cdot 28 \times 10^{-21}\ \text{J}$$

    Now, $\lambda = \dfrac{h}{\sqrt{2mE}}$

    $$= \frac{6\cdot 62 \times 10^{-34}}{\sqrt{2 \times 1\cdot 66 \times 10^{-27} \times 8\cdot 28 \times 10^{-21}}}$$

    $= 1{\cdot}263 \times 10^{-10}$ m $= \mathbf{1{\cdot}263}$ **Å**

Competitive Examination File (Unit VII)

REVISION AT A GLANCE

Chapter 1. Particle Nature of Radiation

☐ **Photon.** It is a packet of energy. A photon of frequency ν possesses energy $h\,\nu$. The rest mass of a photon is zero.

☐ **Work function of a metal.** *The minimum energy, which must be supplied to the electron so that it can just come out of a metal surface, is called the work function of the metal.* It is denoted by ω.

☐ **Photoelectric effect.** *The phenomenon of ejection of electrons from a metal surface, when light of sufficiently high frequency falls upon it, is known as photoelectric effect.*

The electrons so emitted are called ***photoelectrons.***

☐ **Threshold frequency.** *The minimum frequency (ν_0) which the incident light must possess so as to eject photoelectrons from a metal surface, is called threshold frequency of the metal.*

Mathematically : Work function, $\omega = h\,\nu_0$

☐ **Laws of photoelectric effect. 1.** Photoelectric emission takes place from a metal surface, when the frequency of incident light is above its threshold frequency.

2. The photoelectric emission starts as soon as the light is incident on the metal surface.

3. The maximum kinetic energy with which an electron is emitted from a metal surface is independent of the intensity of light and depends upon its frequency.

4. The number of photoelectrons emitted is independent of the frequency of the incident light and depends only upon its intensity.

☐ **Cut off potential.** *It is that minimum value of negative potential* (V_0) *, which should be applied to the anode in a photo cell so that the photoelectric current becomes zero.*

Mathematically :

$$e\,V_0 = \frac{1}{2} m\,v_{max}{}^2,$$

where v_{max} is the maximum velocity with which photoelectrons are emitted.

☐ **Einstein's photoelectric equation.** When light of frequency ν is incident on a metal surface, whose work function is ω (*i.e.* $h\,\nu_0$), then the maximum kinetic energy $\left(\frac{1}{2} m\,v_{max}{}^2\right)$ of the emitted photoelectrons is given by

$$h\,\nu = h\,\nu_0 = \frac{1}{2} m\,v_{max}{}^2$$

It is called Einstein's photoelectric equation. It can explain all the laws of photoelectric emission.

☐ **Photoelectric cell.** *A photo cell is an arrangement, which produces electric current, when light falls on its cathode.*

Some Useful Facts.

1. Wave nature of light cannot explain the fact that the emission of photoelectrons takes place as soon as the light is incident on the metal surface.

2. Photoelectric effect is one-one phenomenon *i.e.* one photon of light is capable of ejecting one electron only.

Chapter 2. Wave Nature of Matter

☐ **de-Broglie hypothesis.** *Both radiation and matter have dual nature.*

A particle of momentum p is associated with de-Broglie wave of wavelength

$$\lambda = \frac{h}{p} = \frac{h}{m\,v}$$

The above relation is called de-Broglie relation and the wavelength of the wave associated is called ***de-Broglie wavelength of the particle.***

☐ **de-Broglie wavelength of electron.** An electron of kinetic energy E possesses de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2\,m\,E}}$$

If electron is accelerated through a potential difference V, so as to acquire kinetic energy E (= e V), then

$$\lambda = \frac{h}{\sqrt{2\,m\,e\,V}} = \frac{12{\cdot}27}{\sqrt{V}}\ \text{Å}$$

Some Useful Facts

1. A moving particle is associated with a wave packet, rather than a wave.

2. In case, the particle is moving with a speed (v) comparable to the speed of light (c), then in expression

$$\lambda = \frac{h}{m\,v},$$

m refers to relativistic mass of the particle *i.e.*

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Here, m_0 is the rest mass of the particle.

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Particle Nature of Radiation

Problem 1.01. Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6000 Å . *(Roorkee, 1983)*

Sol. Here, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m ;

power of source, P = 25 W = 25 J s^{-1} ;

Now, energy of a photon,

$$E = \frac{h\,c}{\lambda} = \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{6 \times 10^{-7}} = 3 \cdot 31 \times 10^{-19}\ \text{J}$$

Therefore, number of photons emitted per second,

$$n = \frac{P}{E} = \frac{25}{3 \cdot 31 \times 10^{-19}} = \mathbf{7{\cdot}553 \times 10^{19}}$$

Problem 1.02. Radiation of wavelength 180 nm ejects photo-electrons from a plate, when work function is 2·0 eV. If a uniform magnetic field of flux density 5·0 × 10^{-5} tesla (or weber metre^{-2} is applied parallel to the plate, what should be the radius of the path followed by electrons ejected normally from the plate with maximum energy ? *(Roorkee, 1988)*

Sol. Here,

$\omega = 2{\cdot}0$ eV $= 2{\cdot}0 \times 1{\cdot}6 \times 10^{-19} = 3{\cdot}2 \times 10^{-19}$ J ;

$\lambda = 180$ nm $= 180 \times 10^{-19}$ m

Therefore, energy of incident photon,

$$h\,v = \frac{h\,c}{\lambda} = \frac{6 \cdot 61 \times 10^{-34} \times 3 \times 10^8}{180 \times 10^{-9}} = 11{\cdot}03 \times 10^{-19}\ \text{J}$$

The maximum kinetic energy of emitted electrons,

$$\frac{1}{2} m\,v_{max}^{2} = h\,v - \omega = 11 \cdot 03 \times 10^{-19} - 3 \cdot 2 \times 10^{-9}$$

$$= 7{\cdot}83 \times 10^{-19}\ \text{J}$$

$$\therefore \quad v_{max} = \sqrt{\frac{2 \times 7{\cdot}83 \times 10^{-19}}{m}} = \sqrt{\frac{2 \times 7{\cdot}83 \times 10^{-19}}{9{\cdot}1 \times 10^{-31}}}$$

$$= 1{\cdot}31 \times 10^6\ \text{m s}^{-1}$$

If r is radius of the path followed by the electron ejected with maximum energy, then

$$B\,e\,v_{max} = \frac{m\,v_{max}^{2}}{r}$$

or $$r = \frac{m\,v_{max}}{B\,e}$$

Here, B = 5·0 × 10^{-5} T

$$r = \frac{9 \cdot 1 \times 10^{-31} \times 1 \cdot 31 \times 10^6}{5 \cdot 0 \times 10^5 \times 1 \cdot 6 \times 10^{-19}} = \mathbf{0{\cdot}149\ m}$$

Problem 1.03. A photon of wavelength 3310 Å falls a photocathode and an electron of energy 3 × 10^{-19} J ejected. If the wavelength of the incident photon is change to 5000 Å , the energy of the ejected electron is 0·972 × 10^{-19} Calculate the value of Planck's constant and threshol wavelength of the photocathode. Given, the velocity of lig = 3 × 10^8 m s^{-1}. *(I.S.T. Dhanbad, 199*

Hint. From Einstein's photoelectric equation :

$$\frac{h\,c}{\lambda_0} = \frac{h\,c}{\lambda} - \frac{1}{2} m\,v_{max}^{2}$$

For $\lambda = 3310$ Å ; $\frac{1}{2} m\,v_{max}^{2} = 3 \times 10^{-19}$ J ;

and for $\lambda = 5000$ Å, $\frac{1}{2} m\,v_{max}^{2} = 0 \cdot 972 \times 10^{-19}$ J.

Setting the above data in Einstein's photoelectr equation, we obtain two equations in h and λ_0. From the equations, the values of h and λ_0 can be obtained.

It will be found that

$$h = 6{\cdot}62 \times 10^{-34}\ \text{J s}^{-1}\ ;\ \lambda_0 = 6620\ \text{Å}.$$

Problem 1.04. A beam of light has three wavelength 4144 Å , 4972 Å and 6216 Å with a total intensity 3·6 × 10^{-3} W m^{-2} equally distributed among the thre wavelengths. The beam falls normally on an area 1·0 cm^2 of a clean metallic surface of work function 2·3 e Assume that there is no loss of light by reflection and th each energetically capable photon ejects one electro Calculate the number of photoelectrons liberated i 2 seconds. *(I.I.T. 198*

Sol. Here, work function of the metal,

$\omega = 2{\cdot}3$ e V $= 2{\cdot}3 \times 1{\cdot}6 \times 10^{-19}$ J

Now, $$\omega = \frac{h\,c}{\lambda_0}$$

$$\therefore \quad \lambda_0 = \frac{h\,c}{\omega_0} = \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{2 \cdot 3 \times 1 \cdot 6 \times 10^{-19}} = 5{\cdot}397 \times 10^{-7}$$

$$= 5397\ \text{Å}$$

The incident beam of light has wavelengths

$\lambda_1 = 4144$ Å, $\lambda_2 = 4972$ Å and $\lambda_3 = 6216$ Å

The wavelength $\lambda_3 = 6216$ Å will not cause photoelectr emission, as $\lambda_3 > \lambda_0$.

Now, total intensity of incident radiation

$= 3{\cdot}6 \times 10^{-3}$ W m^{-2}

Since total intensity is equally distributed among th three wavelengths, the intensity of each wavelength in th incident beam of light,

$$I = \frac{3\cdot6\times10^{-3}}{3} = 1\cdot2\times10^{-3}\ \text{W m}^{-2}$$

Area of surface on which the beam of light falls,

$$A = 1\cdot0\ \text{cm}^2 = 10^{-4}\ \text{m}^2$$

Therefore, energy incident for each wavelength in 2 s,

$$E = I\times A\times t = 1\cdot2\times10^{-3}\times10^{-4}\times2 = 10^{-7}\ \text{J}$$

Energy of a photon of wavelength

$$\lambda_1 = \frac{hc}{\lambda_1} = \frac{6\cdot62\times10^{-34}\times3\times10^8}{4144\times10^{-10}} = 4\cdot79\times10^{-19}\ \text{J}$$

Energy of a photon of wavelength

$$\lambda_2 = \frac{hc}{\lambda_2} = \frac{6\cdot62\times10^{-34}\times3\times10^8}{4972\times10^{-10}} = 3\cdot99\times10^{-19}\ \text{J}$$

Since each emergetically capable photon ejects one ectron, total number of photons emitted in 2 s,

$$n = \frac{E}{hc/\lambda_1} + \frac{E}{hc/\lambda_2}$$

$$= \frac{2\cdot4\times10^{-7}}{4\cdot79\times10^{-19}} + \frac{2\cdot4\times10^{-7}}{3\cdot99\times10^{-19}}$$

$$= 0\cdot5\times10^{12} + 0\cdot6\times10^{12} = \mathbf{1\cdot1\times10^{12}}$$

Problem 1.05. When a beam of 10·6 eV photons f intensity 2 W m^{-2} falls on a platinum surface of area 0^{-4} m^2 and work function 5·6 eV, 0·53% of the incident hotons eject photoelectrons. Find the number of hotoelectrons emitted per second and their minimum nd maximum energies (in eV). Take 1 eV = 1·6 × 10^{-19} J. *(I.I.T. 2000)*

Sol. Here, energy of photon,

$$E = 10\cdot6\ \text{eV} = 10\cdot6\times1\cdot6\times10^{-19}\ \text{J}$$

$$= 1\cdot696\times10^{-18}\ \text{J}$$

Energy incident per unit area per second,

$$E_{in} = 2\ \text{W m}^{-2} = 2\ \text{J s}^{-1}\ \text{m}^{-2}$$

Area of the platinum surface, $A = 10^{-4}$ m^2

Therefore, energy incident on platinum surface per econd,

$$E_{tot} = E_{in}\times A = 2\times10^{-4}\ \text{J s}^{-1}$$

Hence, number of photons incident on the whole latinum surface per second,

$$N = \frac{E_{tot}}{E} = \frac{2\times10^{-4}}{1\cdot696\times10^{-18}} = 1\cdot18\times10^{14}$$

Since 0·53% of the incident photons eject photoelectrons, umber of photoelectrons emitted from the platinum surface er second,

$$n = \frac{N\times0\cdot53}{100} = \frac{1\cdot18\times10^{14}\times0\cdot53}{100} = \mathbf{6\cdot254\times10^{11}}$$

The minimum energy of photoelectrons emitted is **zero.**

Also, the maximum energy of photoelectrons emitted,

$$E_{max} = E - \text{work function of platinum}$$

$$= 10\cdot6 - 5\cdot6 = \mathbf{5\cdot0\ eV}$$

Problem 1.06. Two metallic plates A and B, each of area 5 × 10^{-4} m^2, are placed parallel to each other at a separation of 1 cm. Plate B carries a positive charge of 33·7 × 10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each starts falling on plate A at *t* = 0 so that 10^{16} photons fall on it per square metre per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV. Determine

(*a*) the number of photoelectrons emitted upto *t* = 10 s.

(*b*) the magnitude of the electric field between the plates A and B at *t* = 10 s and

(*c*) the kinetic energy of the most energetic photoelectrons emitted at *t* = 10 s, when it reaches plate B.

Neglect the time taken by the photoelectrons to reach plate B. Take $\varepsilon_0 = 8\cdot85\times10^{-12}$ C^2 N^{-1} m^{-2}. *(I.I.T. 2002)*

Sol. Here, Area of each of the two plates,

$$A = 5\times10^{-4}\ \text{m}^2$$

Separation between the two plates, $d = 1$ cm $= 10^{-2}$ m

(*a*) Number of photons falling on plate A per second per unit area, N = 10^{16}

Therefore, number of photons falling on plate A in 10 s,

$$N_{tot} = N\times A\times t = 10^{16}\times5\times10^{-4}\times10 = 5\times10^{13}$$

Since one photoelectron is emitted for every 10^6 incident photons, number of photoelectrons emitted in 10 s,

$$n = \frac{N_{tot}}{10^6} = \frac{5\times10^{13}}{10^6} = \mathbf{5\times10^7}$$

(*b*) Charge on plate A after 10 s (*i.e.* after the emission of 5×10^7 photoelectrons),

$$q_A = +ne = 5\times10^7\times1\cdot6\times10^{-19} = 8\times10^{-12}\ \text{C}$$

Charge on plate B after 10 s (*i.e.* after collecting 5×10^7 photoelectrons),

$$q_B = \text{initial charge} + (-ne) = 33\cdot7\times10^{-12} - 8\times10^{-12}$$

$$= 25\cdot7\times10^{-12}\ \text{C}$$

Therefore, excess charge on plate B (difference of charge on plates B and A),

$$q = q_B - q_A = 25\cdot7\times10^{-12} - 8\times10^{-12} = 17\cdot7\times10^{-12}\ \text{C}$$

Hence, electric field between the two plates,

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q/A}{2\varepsilon_0} = \frac{q}{2\varepsilon_0 A} = \frac{17\cdot7\times10^{-12}}{2\times8\cdot85\times10^{-12}\times5\times10^{-4}}$$

$$= \mathbf{2\times10^3\ N\ C^{-1}}$$

(*c*) Kinetic energy possessed by photoelectrons emitted from plate A,

$$\frac{1}{2}m{v_{max}}^2 = 5 - 2 = 3\ \text{eV}$$

Energy gained by photoelectrons inside electric field between the two plates,

$$W = Ve = (E\,d)\,e\ (\text{in joule}) = E\,d\ (\text{in eV})$$

$$= 2\times10^3\times10^{-2} = 20\ \text{eV}$$

Hence, energy of photoelectrons on reaching plate B,

$$W' = W + \frac{1}{2}m{v_{max}}^2 = 20 + 3 = \mathbf{23\ eV}$$

Problem 1.07. The peak emission from a black body at a certain temperature occurs at a wavelength of 9000 Å. On increasing its temperature, the total radiation emitted is increased 81 times. At the initial temperature, when the peak radiation from the black body is incident on a metal surface, it does not cause any photoemission from the surface. After the increase of temperature, the peak radiation from the black body caused photoemission. To bring these photoelectrons to rest, a potential equivalent to the excitation energy between *n* = 2 to *n* = 3 Bohr levels of hydrogen atom is required. Find the work function of the metal. *(Roorkee, 1993)*

Sol. Suppose that the total energy radiated per second per unit area is E and temperature of the black body is T, when the wave length of maximum emission is λ_m (= 9000 Å × 10^7 m). Then,

$$E = \sigma T^4 \qquad ...(i)$$

and $\lambda_m T = b$

or $9 \times 10^{-7} T = b \qquad ...(ii)$

Further, suppose that when the temperature of the black body is increased to T′, the total energy emitted per second per unit area becomes E′ = 81 E and the wavelength of maximum emission becomes λ_m'. Then,

$$E' = \sigma T'^4$$

or $81 E = s T'^4 \qquad ...(iii)$

and $\lambda_m' T' = b \qquad ...(iv)$

Dividing equation (*iii*) by (*i*), we have

$$\frac{81 E}{E} = \frac{\sigma T'^4}{\sigma T^4}$$

or $T' = 3 T \qquad ...(v)$

From equations (*ii*) and (*iv*), we have

$$\lambda_m' T' = 9 \times 10^{-7} T$$

or $\lambda_m' \times 3 T = 9 \times 10^{-7} T$

or $\lambda_m' = 3 \times 10^{-7}$ m

The energy of the radiation corresponding to the wavelength of maximum emission λ_m',

$$h\nu' = \frac{hc}{\lambda_m'} = \frac{6 \cdot 62 \times 10^{-34} \times 3 \times 10^8}{3 \times 10^{-7}} = 6 \cdot 62 \times 10^{-19} \text{ J}$$

$$= \frac{6 \cdot 62 \times 10^{-19}}{1 \cdot 6 \times 10^{-19}} = 4 \cdot 14 \text{ eV}$$

K.E. of photoelectrons emitted,

$\frac{1}{2} m v^2$ = excitation energy between $n = 2$ to $n = 3$ Bohr levels

$$= 13 \cdot 6 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \text{eV} = 13 \cdot 6 \times \frac{5}{36} = 1 \cdot 89 \text{ eV}$$

According to Einstein's photoelectric equation,

$$h\nu' = \omega + \frac{1}{2} m v_{max}^2$$

Therefore, work function of metal surface,

$$\omega = h\nu' - \frac{1}{2} m v_{max}^2 = 4 \cdot 14 - 1 \cdot 89 = \mathbf{2{\cdot}25\ eV}$$

Chapter 2. Wave Nature of Matter

Problem 2.01. In a photoelectric effect set up, a point source of light of power 3·2 × 10^{-3} W emits monoenergetic photons of energy 5·0 eV. The source is located at a distance of 0·8 m from the centre of a stationary metallic sphere of work function 3·0 eV and of radius 8·0 × 10^{-3}. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photoelectrons are instantly swept away after emission.

(*a*) Calculate the number of photoelectrons emitted per second.

(*b*) Find the ratio of the wavelength of incident light to the de-Broglie wavelength of the fastest photoelectrons emitted. (*I.I.T. 1995*)

Sol. (*a*) Here, power of the source of light

$$= 3{\cdot}2 \times 10^{-3} \text{ W} = 3{\cdot}2 \times 10^{3} \text{ J s}^{-1}$$

The source of light emits photons of energy in all directions. The light energy falling per second on the metallic sphere of radius *r* situated at a distance R from the source of light,

$$E = \frac{\pi r^2}{4 \pi R^2} \times \text{power of the source of light}$$

Here, $r = 8{\cdot}0 \times 10^{-3}$ m and R = 0·8 m

$$\therefore \quad E = \frac{\pi \times (8 \cdot 0 \times 10^{-3})^2}{4 \pi \times (0 \cdot 8)^2} = 3{\cdot}2 \times 10^{-3} = 8 \times 10^{-8} \text{ J s}^{-1}$$

$$= \frac{8 \times 10^{-8}}{1 \cdot 6 \times 10^{-19}} = 5 \times 10^{11} \text{ eV s}^{-1}$$

Since each photon of incident light has energy 5 eV, the number of photons striking the metal sphere,

$$N = \frac{5 \times 10^{11}}{5} = 10^{11} \text{ s}^{-1}$$

Further, as the efficiency of photoelectron emission is one for every 10^6 incident photons, the number of photo-electrons emitted,

$$n = \frac{N}{10^6} = \frac{10^{11}}{10^6} = \mathbf{10^5\ s^{-1}}$$

(*b*) Now, work function of the metal sphere, ω = 3·0 eV

and energy of incident photon, $\frac{hc}{\lambda} = 5 \cdot 0$ eV

Therefore, maximum kinetic energy of the emitted photoelectrons,

$$E_{max} = \frac{1}{2} m v_{max}^2 = \frac{hc}{\lambda} - \omega = 5 \cdot 0 - 3 \cdot 0 = 2 \cdot 0 \text{ eV}$$

$$= 2{\cdot}0 \times 1{\cdot}6 \times 10^{-19} = 3{\cdot}2 \times 10^{-19} \text{ J}$$

If λ′ is de-Broglie wavelength of the fastest photoelectron, then

$$\lambda' = \frac{h}{\sqrt{2 m E_{max}}} \qquad ...(i)$$

The wavelength of incident photon of light is given by

$$\frac{hc}{\lambda} = 5 \cdot 0 \text{ eV} = 5 \cdot 0 \times 1 \cdot 6 \times 10^{-19} = 8 \cdot 10^{-19} \text{ J}$$

or $\lambda = \frac{hc}{8 \times 10^{-19}} \qquad ...(ii)$

Dividing equation (*ii*) by (*i*), we have

$$\frac{\lambda}{\lambda'} = \frac{hc}{8 \times 10^{-19}} \times \frac{\sqrt{2 m E_{max}}}{h} = \frac{c\sqrt{2 m E_{max}}}{8 \times 10^{-19}} \qquad ...(iii)$$

Now, $c = 3 \times 10^8$ m s^{-1} ; $m = 9{\cdot}1 \times 10^{-31}$ kg

and $E_{max} = 3{\cdot}2 \times 10^{-19}$ J

Setting the values of *c*, *m* and E_{max} in equation (*iii*), we have

$$\frac{\lambda}{\lambda'} = \frac{3 \times 10^8 \times \sqrt{2 \times 9 \cdot 1 \times 10^{-31} \times 3 \cdot 2 \times 10^{-19}}}{8 \times 10^{-19}}$$

$$= \frac{3 \times 10^8 \times 7 \cdot 63 \times 10^{-25}}{8 \times 10^{-19}} = \mathbf{286{\cdot}1}$$

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Particle Nature of Radiation

PART I. TEXT-BASED QUESTIONS

1. **Wavelength of the radiation of frequency 100 Hz is :**
(A) 2×10^6 m (B) 3×10^6 m
(C) 4×10^6 m (D) 5×10^6 m *(C.B.S.E. 1999)*

2. **The value of Planck's constant is**
(A) $6{\cdot}63 \times 10^{-34}$ J s^{-1} (B) $6{\cdot}63 \times 10^{-34}$ kg m s^{-1}
(C) $6{\cdot}63 \times 10^{-34}$ kg m^2 (D) $6{\cdot}63 \times 10^{-34}$ J s
(C.B.S.E. 2002)

3. **The unit of Planck's constant is**
(A) N m (B) eV
(C) J s^{-1} (D) J s
(Karnataka Entrance, 1991)

4. **Planck's constant has the dimensions of**
(A) linear momentum (B) angular momentum
(C) energy (D) power *(C.B.S.E. 2001)*

5. **The energy of a photon of light is 3 eV. Then the wavelength of photon must be :**
(A) 4125 nm (B) 412·5 nm
(C) 41250 nm (D) 4 nm *(C.B.S.E. 2000)*

6. **A photon has velocity c and frequency ν. Then, which of the following represents its wavelength ?**
(A) $\dfrac{hc}{E}$ (B) $\dfrac{h\nu}{c}$
(C) $\dfrac{h\nu}{c^2}$ (D) $h\nu$ *(C.B.S.E. 1996)*

7. **The wavelength of photons is proportional to (where ν is frequency)**
(A) ν (B) $\sqrt{\nu}$
(C) $1/\sqrt{\nu}$ (D) $1/\nu$ *(C.B.S.E. 1996)*

8. **The energy of a photon of wavelength λ is**
(A) $hc\lambda$ (B) hc/λ
(C) λ/hc (D) $h\lambda/c$ *(C.B.S.E. 1992)*

9. **Electron volt is a unit of**
(A) charge (B) momentum
(C) potential difference (D) energy
(Karnataka Entrance, 1988)

10. **Which of the following phenomena exhibits particle nature of light ?**
(A) Interference (B) Diffraction
(C) Polarisation (D) Photoelectric effect
(C.B.S.E. 2001)

11. **Photoelectrons are emitted, when**
(A) a zinc plate is heated.
(B) a zinc plate is hammered.
(C) a zinc plate is irradiated with ultra-violet light.
(D) a zinc plate is subjected to very high pressure.
(Karnataka Entrance, 1993, 1991, 1988)

12. **When ultra-violet radiation is incident on a surface, no photoelectrons are emitted. If a second beam causes photoelectrons to be ejected, it may consist of**
(A) radio waves (B) infra-red rays
(C) visible light rays (D) X-rays
(C.B.S.E. 2002 ; Karnataka Entrance, 1992, 1990)

13. **The minimum energy required to remove an electron is called**
(A) work function (B) kinetic energy
(C) stopping potential (D) potential energy
(A.F.M.C. 1995)

14. **Light from a bulb falls on the wooden piece but no photoelectrons are emitted. It is because, work function of wood is**
(A) much higher than the energy of photon.
(B) less than the energy of photon.
(C) equal to energy of photon.
(D) none of these. *(A.F.M.C. 1997)*

15. **As the intensity of incident light increases :**
(A) photoelectric current increases.
(B) photoelectric current decreases.
(C) kinetic energy of emitted photo-electrons increases.
(D) kinetic energy of emitted photo-electrons decreases.
(C.B.S.E. 1999 ; C.B.S.E. 1995)

16. **Which of the following statements is correct ?**
(A) The photocurrent increases with increasing frequency.
(B) The photocurrent is proportional to the applied voltage.
(C) The photocurrent increases with intensity of light.
(D) The stopping potential increases with intensity of incident light. *(C.B.S.E. 1997)*

17. **The threshold wavelength of sodium metal is 6780 Å . The work function of metal will be**
(A) 1·83 eV (B) 2·75 eV
(C) 2·95 eV (D) 3·25 eV *(A.F.M.C. 1998)*

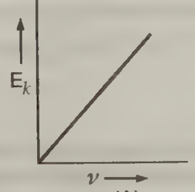
18. **The work function of a substance is 4·0 eV. The longest wavelength of light that causes photoelectron emission from this substance is approximately**
(A) 540 nm (B) 400 nm
(C) 310 nm (D) 220 nm *(I.I.T. 1998)*

19. **The photoelectric work function for a metal surface is 4·125 eV. The cut-off wavelength for this surface is :**
(A) 4125 Å (B) 2062·5 Å
(C) 3000 Å (D) 6000 Å *(C.B.S.E. 1999)*

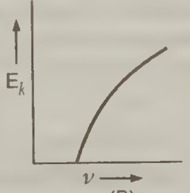
20. **The strength of photoelectric current depends upon**
(A) frequency of incident radiation.
(B) intensity of incident radiation.
(C) angle of incident radiation.
(D) distance between anode and cathode. *(M.N.R. 1981)*

21. **Consider light of given intensity and frequency falling on a substance that emits photoelectrons. The intensity is decreased to one-third its value and the frequency increased by three times. Consequently the velocity of the photoelectrons will**
(A) remain the same.
(B) increase or decrease depending on the exact values of the new intensity and frequency.
(C) decrease. (D) increase.
(Karnataka Entrance, 1987)

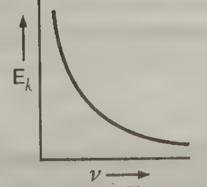
22. **The kinetic energy (E_k) of a photoelectron varies with the frequency (ν) of the incident radiation as which of the following graphs ?**



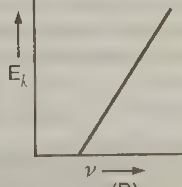
(A)



(B)



(C)



(D)

(C.B.S.E. 1996)

23. The cathode of a photoelectric cell is changed, such that the work function changes from ω_1 to ω_2 ($\omega_2 > \omega_1$). If the currents before and after change are I_1 and I_2, all other conditions unchanged, then (assuming $h\nu > \omega_2$),

(A) $I_1 = I_2$ (B) $I_1 < I_2$

(C) $I_1 > I_2$ (D) $I_1 < I_2 < 2I_1$. *(C.B.S.E. 1992)*

24. Photoelectrons are being obtained by irradiating zinc by a radiation of 3100 Å. In order to increase the kinetic energy of ejected photoelectrons,

(A) the intensity of radiation should be increased.

(B) the wavelength of radiation should be increased.

(C) the wavelength of radiation should be decreased.

(D) both wavelength and intensity of radiation should be increased. *(M.N.R. 1980)*

25. Einstein's photoelectric equation is expressed as

(A) $h\nu = \frac{1}{2}\omega_0 - m v^2$ (B) $h\nu = \omega_0 - \frac{1}{2} m v^2$

(C) $h\nu = \omega_0 + \frac{1}{2} m v^2$ (D) $h\nu = \frac{1}{2} m v^2 - \omega_0$

(C.E.T., 1998)

26. Einstein's work on photoelectric effect gives support to :

(A) $E = mc^2$ (B) $E = h\nu$

(C) $h\nu = \frac{1}{2} m v^2$ (D) $E = h/\lambda$ *(C.B.S.E. 2000)*

27. Photoelectric work function of a metal is 1 eV. Light of wavelength λ = 3000 Å falls on it. The photoelectrons come out with velocity

(A) 10 m s^{-1} (B) 10^2 m s^{-1}

(C) 10^4 m s^{-1} (D) 10^6 m s^{-1} *(C.B.S.E. 1990)*

28. Light of wavelength 5000 Å falls on a photo-sensitive plate with photoelectric work function of 1·9 eV. The kinetic energy of the photoelectrons emitted will be

(A) 0·58 eV (B) 2·48 eV

(C) 1·24 eV (D) 1·16 eV *(C.B.S.E. 1998)*

29. Light of two different frequencies, whose photons have energies 1 eV and 2·5 eV respectively successively illuminate a metal of work function 0·5 eV. The ratio of maximum speeds of the emitted electrons will be

(A) 1 : 5 (B)1 : 4

(C) 1 : 2 (D) 1 : 1 *(C.P.M.T. 1983)*

30. The maximum kinetic energy of photoelectrons emitted from a surface, when photons of energy 6 eV fall on it, is 4 eV. The stopping potential (in volt) is

(A) 2 (B) 4

(C) 6 (D) 10 *(I.I.T. 1997)*

31. The maximum velocity of an electron ejected from a photoelectric emitter, when radiation falls on the latter is found to be 2×10^6 m s^{-1}. Assuming the charge to mass ratio of electron (e/m) to be $1 \cdot 8 \times 10^{11}$ C kg^{-1}, the stopping potential is (in volt)

(A) 1·2 (B) 2·1

(C) 11·1 (D) 16·3

(Karnataka Entrance, 1987)

32. When radiation is incident on a photoelectron emitter, the stopping potential is determined to be 9 volt . Assuming the charge to mass ratio (e/m) for electron to be $1 \cdot 8 \times 10^{11}$ coulomb kg^{-1}, the maximum velocity of the ejected electron is found to be (in m s^{-1})

(A) 6×10^5 (B) 8×10^5

(C) 10×10^5 (D) $1 \cdot 8 \times 10^6$

(Karnataka Entrance, 1985)

33. A photo-sensitive material would emit electrons, if excited by photons beyond a threshold. To cross the threshold, you would increase

(A) intensity of light. (B) wavelength of light.

(C) frequency of light.

(D) the voltage applied to light source. *(M.N.R. 1983)*

34. Ultra-violet radiation of 6·2 eV falls on an aluminium surface (work function 4·2 eV). The kinetic energy (in joule) of the fastest electron emitted is approximately

(A) $3 \cdot 2 \times 10^{-21}$ (B) $3 \cdot 2 \times 10^{-19}$

(C) 4×10^{-17} (D) 3×10^{-15} *(M.N.R. 1987)*

35. When a light source is at a distance d from a photoelectric cell, the number of photoelectrons emitted from the cell is n. If the distance of light source and cell is reduced to half, then the number of photoelectrons emitted will become :

(A) $n/2$ (B) $2n$

(C) $4n$ (D) n *(C.B.S.E. 2001)*

36. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed 2 m away, the electrons emitted by the photo cathode ;

(A) each carry one quarter of their previous energy.

(B) each carry one quarter of their previous momenta.

(C) are half as numerous.

(D) are one quarter as numerous.

PART II. THOUGHT-BASED QUESTIONS

37. The mass of a photon at rest is

(A) zero (B) $1 \cdot 67 \times 10^{-35}$ kg

(C) 1 a.m.u. (D) 9×10^{-31} kg

(C.B.S.E. 1991 ; M.N.R. 1981)

38. Which one of the following statements about photon is incorrect ?

(A) Photon's rest mass is zero.

(B) Momentum of photon is $h\nu/c$.

(C) Photon's energy is $h\nu$.

(D) Photons exert no pressure. *(C.P.M.T. 1990)*

39. A radio transmitter operates at a frequency of 880 kHz and a power of 10 kW. The number of photons emitted per second is

(A) $1 \cdot 72 \times 10^{31}$ (B) $13 \cdot 27 \times 10^{24}$

(C) $13 \cdot 27 \times 10^{34}$ (D) $13 \cdot 27 \times 10^{44}$ *(C.B.S.E. 1996)*

40. The minimum light intensity that can be perceived by the eye is about 10^{-10} W m^{-2}. The number of photons of wavelength $5 \cdot 6 \times 10^{-7}$ m that must enter the pupil of area 10^{-4} m^2 per second for vision is approximately.

(Use $h = 6 \cdot 6 \times 10^{-34}$ J s)

(A) 3×10^2 photons (B) 3×10^3 photons

(C) 3×10^4 photons (D) 3×10^5 photons.

(N.C.E.R.T. 1982)

41. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a

(A) 50 W infra-red lamp.

(B) 1 W infra-red lamp.

(C) 50 W ultra-violet lamp.

(D) 1 W ultra-violet lamp. *(I.I.T. 1984, 1982)*

42. Photoelectric effect supports the quantum nature of light, because

(A) there is minimum frequency of light below which no photoelectrons are emitted.

(B) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on intensity.

(C) even when a metal surface is faintly illuminated, the photoelectrons leave the surface immediately.

(D) electric charge of the electrons is quantised.

(C.E.T. 1998 ; I.I.T. 1987)

In a photoemmisive cell, with exciting wavelength λ, the fastest electron has speed of v. If the exciting wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be

(A) $v\left(\frac{3}{4}\right)^{1/2}$ (B) $v\left(\frac{4}{3}\right)^{1/2}$

(C) less than $v\left(\frac{3}{4}\right)^{1/2}$ (D) greater than $v\left(\frac{4}{3}\right)^{1/2}$

(C.B.S.E. 1998)

When a monochromatic point source of light is at a distance of 0·2 m from a photoelectric cell, the cut off voltage and saturation current are respectively 0·6 V and 18·0 mA . If the same source is placed 0·6 m away from the photocell, then

(A) the stopping potential will be 0·2 V.

(B) the stopping potential will be 0·6 V.

(C) the saturation current will be 2·0 mA.

(D) the saturation current will be 6·0 mA. *(I.I.T. 1992)*

45. If the distance of 100 W lamp is increased from a photo cell, the saturation current I in the photo cell varies with distance d as

(A) $I \propto d^2$ (B) $I \propto d$

(C) $I \propto \frac{1}{d}$ (D) $I \propto \frac{1}{d^2}$ *(N.C.E.R.T. 1990)*

46. An image of the sun is formed by a lens of focal length 50 cm on a photosensitive surface of a photoelectric cell and it produces a current I. The lens is then replaced by another lens of same diameter but of focal length 25 cm. The photoelectric current produced will be

(A) I / 2 (B) 2 I

(C) I (D) 4 I

47. When a proton is accelerated (from rest) through a potential difference of one volt, the kinetic energy gained by it is roughly equal to

(A) 1 eV (B) 1837 eV

(C) $\frac{1}{1837}$ eV (D) none of the above

Chapter 2. Wave Nature of Matter

PART I. TEXT-BASED QUESTIONS

1. If c is velocity of light, then momentum of a photon of frequency ν is

(A) $h\nu/c^2$ (B) $h\nu/c$

(C) ν/c (D) $h\nu c^2$

2. A photon behaves as if it had a mass equal to

(A) $h\nu c^2$ (B) $c^2/h\nu$

(C) $\nu c^2/h$ (D) $h\nu/c^2$

(Karnataka Entrance, 1993)

3. The momentum of a photon of wavelength 5000 Å will be

(A) $1{\cdot}3 \times 10^{-27}$ kg m s^{-1}.

(B) $1{\cdot}3 \times 10^{-28}$ kg m s^{-1}.

(C) 4×10^{-29} kg m s^{-1}.

(D) 4×10^{-18} kg m s^{-1}. *(A.F.M.C. 1998)*

4. The energy of a photon is 3×10^{-19} J. Its momentum is

(A) 3×10^{-7} kg m s^{-1} (B) 10^{-27} kg m s^{-1}

(C) 9×10^{-11} kg m s^{-1} (D) 10^{-8} kg m s^{-1}

5. The de-Broglie wave corresponding to a particle of mass m and velocity v has a wavelength associated with it

(A) $\frac{h}{mv}$ (B) hmv

(C) $\frac{mh}{v}$ (D) $\frac{m}{hv}$ *(C.B.S.E. 1992)*

6. The de-Broglie wavelength of an electron moving with a speed of $6{\cdot}6 \times 10^5$ m s^{-1} is nearly equal to

(A) 10^{-11} m (B) 10^{-9} m

(C) 10^{-7} m (D) 10^{-5} m *(C.P.M.T. 1993)*

7. The wavelength of an electron moving with a velocity of 500 km s^{-1} is (take $m_e = 9 \times 10^{-31}$kg and $h = 6{\cdot}6 \times 10^{-34}$ J s)

(A) 2·9 mm (B) 3·0 mm

(C) 0·5 nm (D) 1·47 nm *(N.C.E.R.T. 1984)*

8. The following particles are moving with the same velocity. Then maximum de-Broglie wavelength will be for

(A) proton (B) α-particle

(C) neutron (D) β-particle *(C.B.S.E. 2002)*

9. The de-Broglie wavelength λ of a particle is related to its kinetic energy E as ;

(A) $\lambda \propto E$ (B) $\lambda \propto \frac{1}{E}$

(C) $\lambda \propto \sqrt{E}$ (D) $\lambda \propto \frac{1}{\sqrt{E}}$

10. An electron accelerated through a potential difference of V volt has a wavelength λ associated with it. Mass of proton is nearly 2000 times that of an electron. In order to have the same λ for proton, it must be accelerated through a potential difference (in volt) of

(A) V (B) 2000 V

(C) $\frac{V}{2000}$ (D) $\sqrt{2000}$ V

(N.C.E.R.T. 1975)

11. An electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum. Its final velocity will be

(A) $\frac{eV}{2m}$ (B) $\frac{eV}{m}$

(C) $\sqrt{\frac{2eV}{m}}$ (D) $\sqrt{\frac{eV}{m}}$ *(C.B.S.E. 1996)*

12. An electron of mass m, when accelerated through a potential difference V, has de-Broglie wavelength λ. The de-Broglie wavelength associated with a proton of mass M and accelerated through the same potential difference will be

(A) $\lambda\sqrt{\frac{M}{m}}$ (B) $\lambda\sqrt{\frac{m}{M}}$

(C) $\lambda\frac{M}{m}$ (D) $\lambda\frac{m}{M}$ *(C.B.S.E. 1995)*

13. An electron is accelerated through a potential difference of 150 V. The wavelength associated with it is :

(A) 100 Å (B) 1 Å

(C) 4·2 Å (D) 10 Å

14. An electron is accelerated through a potential difference of 100,000 V. The energy acquired by the electron is

(A) $0{\cdot}53 \times 10^{-17}$ J (B) $1{\cdot}6 \times 10^{-14}$ J

(C) $1{\cdot}6 \times 10^{-10}$ J (D) $1{\cdot}6 \times 10^{-34}$ J

15. An electron beam has a kinetic energy equal to 100 eV. Find the wavelength associated with the beam, if mass of

electron = $9{\cdot}1 \times 10^{-31}$ kg 1 eV = $1{\cdot}6 \times 10^{-19}$ J and Planck's constant = $6{\cdot}6 \times 10^{-34}$ J s.
(A) 1·22 Å (B) 6·3 Å
(C) 24·6 Å (D) 0·12 Å *(C.B.S.E. 1996)*

16. **A photon and an electron have the same wavelength. Then, the velocity of photon is**
(A) less than that of the electron.
(B) greater than that of the electron.
(C) equal to that of the electron.
(D) none of the above.

17. **If we consider electrons and photons of the same wavelength, then they will have the same**
(A) energy (B) velocity
(C) momentum (D) angular momentum.
(A.F.M.C. 1997)

18. **In X-ray diffraction experiment, distance between atomic lattice planes of diffraction grating is $2{\cdot}8 \times 10^{-10}$ m. Then the maximum wavelength of X-rays (in metre) is**
(A) 2·5 (B) $5{\cdot}6 \times 10^{-10}$
(C) 4×10^{-10} (D) $4{\cdot}6 \times 10^{-10}$ *(C.B.S.E. 2001)*

19. **One can study crystal structure by electron diffraction as well as by neutron diffraction. In order to have the same wavelength λ for the electron (mass m_e) and neutron (mass m_n), their velocities should be in the ratio (electron velocity/neutron velocity)**
(A) 1 (B) m_e / m_n
(C) $m_e\, m_n$ (D) m_n / m_e

20. **The magnification produced in an electron microscope is of the order of**
(A) 10 (B) 10^3
(C) 10^5 (D) 10^7

HT-BASED QUESTIONS

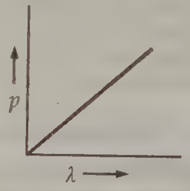
21. **The energy E and momentum p of a photon is given by $E = h\nu$ and $p = h/\lambda$. The velocity of photon will be**
(A) E/p (B) $E\,p$
(C) $(E/p)^2$ (D) $(E/p)^{1/2}$
(C.P.M.T., 1991 ; N.C.E.R.T. 1980)

22. **Neglecting variation of mass with velocity, the wavelength associated with an electron having the kinetic energy E is proportional to**
(A) $E^{1/2}$ (B) $E^{-1/2}$
(C) E (D) E^{-2} *(N.C.E.R.T. 1980)*

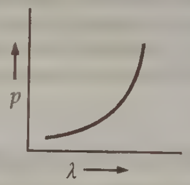
23. Consider a proton moving with kinetic energy E. de-Broglie wavelength is given by (c = speed of ligh
h = Planck's constant ; M = mass of proton ; q = charge
proton)
(A) $hc/\sqrt{2ME}$ (B) $h/q\sqrt{ME}$
(C) $h/\sqrt{2ME}$ (D) $\sqrt{2MEq}/hc$
(Karnataka Entrance, 19

24. **A particle of mass M at rest decays into two particles masses m_1 and m_2 having non-zero velocities. The ratio the de-Broglie wavelengths of the particles (λ_1/λ_2) is**
(A) m_1/m_2 (B) m_2/m_1
(C) 1 (D) $\sqrt{m_2}/\sqrt{m_1}$ *(I.I.T. 199*

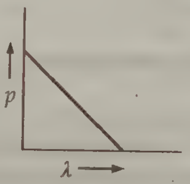
25. **Which of the following figures represent the variation particle momentum and the associated de-Brogl wavelength ?**



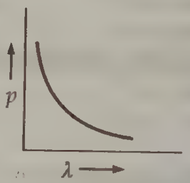
(A)



(B)



(C)



(D)

(A.I.I.M.S. 1982

26. **An electron microscope gives higher magnification than a optical microscope, because**
(A) the electrons have lesser wavelength as compared t light waves.
(B) the velocity of electrons is smoother than that of light
(C) the electrons have more energy than the light particles
(D) more powerful lenses are used in the electron microscope. *(C.E.T. 1998*

ANSWERS TO MULTIPLE CHOICE QUESTIONS

Chapter 1. Particle Nature of Radiation

1. (B)	2. (D)	3. (D)	4. (B)	5. (B)	6. (A)	7. (D)	8. (B)	9. (D)	10. (D)
11. (C)	12. (D)	13. (A)	14. (A)	15. (A)	16. (C)	17. (A)	18. (C)	19. (C)	20. (B)
21. (D)	22. (D)	23. (A)	24. (C)	25. (C)	26. (B)	27. (D)	28. (A)	29. (C)	30. (B)
31. (C)	32. (D)	33. (C)	34. (B)	35. (C)	36. (D)	37. (A)	38. (D)	39. (A)	40. (C)
41. (C & D)	42. (A, B & C)		43. (D)	44. (B & C)	45. (D)	46. (C)	47. (A)		

Chapter 2. Wave Nature of Matter

1. (B)	2. (D)	3. (A)	4. (B)	5. (A)	6. (B)	7. (D)	8. (D)	9. (D)	10. (C)
11. (C)	12. (B)	13. (B)	14. (B)	15. (A)	16. (B)	17. (C)	18. (B)	19. (D)	20. (C)
21. (A)	22. (B)	23. (C)	24. (C)	25. (D)	26. (A)				

UNIT
VIII

ATOMIC NUCLEUS

1. **Structure of Nucleus**
2. **Radioactivity**
3. **Nuclear Reactions**

> "It was almost as incredible as if you had fired a fifteen inch shell at a piece of tissue and it came back and hit you."
>
> **– Earnest Rutherford**

CHAPTER

1

Structure of Nucleus

1.01. RUTHERFORD'S ALPHA SCATTERING EXPERIMENT

Quotable Quote

"We have no right to assume that any physical laws exist, or if they have existed upto now, that they will continue to exist in a similar manner in the future."

– **Max Planck**

Rutherford and his two associates, Geiger and Marsden, studied the scattering of the α-particles from a thin gold foil in order to investigate the structure of the atom. An α-particle is a positively charged particle having a mass equal to that of helium atom and positive charge in magnitude equal to twice the charge on an electron. They are found to be emitted by many radioactive elements. The α-particle scattering experiment provided very useful information about the structure of the atom.

Experimental arrangement. The α-particles from $_{83}Bi^{214}$ (a radioactive source) contained in a lead cavity are collimated into a narrow beam with the help of a lead plate having a narrow slit as shown in Fig. 1.01. The narrow beam of α-particles then falls on a thin gold foil (about $2{\cdot}1 \times 10^{-7}$m thick). The α-particles scattered in different directions were detected with the help of an α-particle detector. The whole apparatus was arranged inside a vacuum chamber to prevent the scattering of α-particles from air molecules.

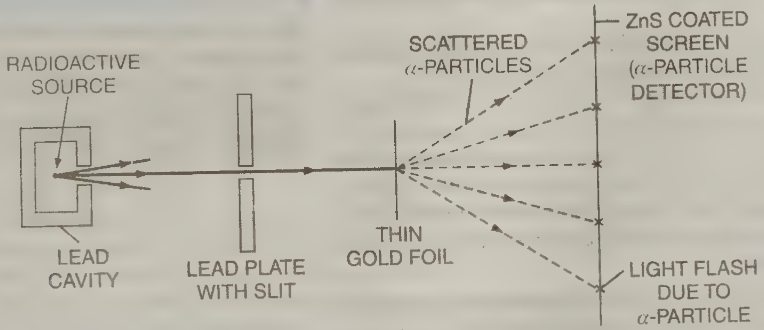


Fig. 1.01

Observations. 1. *Most of the α-particles were found to pass through the gold foil without any appreciable deflection.*

2. *The different α-particles, in passing through the gold foil, underwent different amounts of deflections. A large number of α-particles suffered fairly large deflections.*

3. *A very small number of α-particles (about 1 in 8000) practically retraced their paths or suffered deflection of nearly 180°.*

4. *The graph between the total number of α-particles $N(\theta)$ scattered through angle α and the scattering angle θ was found to be as shown in Fig. 1.01.*

The α-particles scattered from the gold foils are very heavy particles (about 7000 times heavier than electrons) and possess high initial speeds. Thomson's atom model cannot explain the scattering of α-particles through large angles and particularly through 180°. It is because, the light particles like electrons cannot deflect the massive α-particles through 180°.

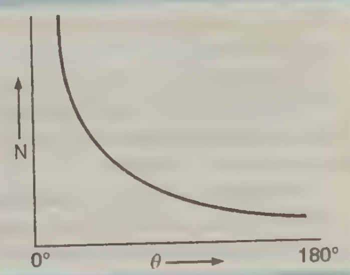


Fig. 1.02

Explanation. To explain the above experimental observations, Rutherford assumed that the scattering of α-particles from the gold atoms was due to Coulomb's interaction between the positive charge of the α-particle and the positive charge in the gold atom. On this basis, he theoretically calculated the number of α-particles that would be deflected through angle θ. The theoretical graph between N (θ) and θ was found to be in full agreement with the graph shown in Fig. 1.02 obtained experimentally. The experimental observations led Rutherford to the following conclusions :

1. Since most of the α-particles passed undeviated, *the atom has a lot of empty space in it.*

2. Since the fast and the heavy α-particles could be deflected even through 180°, *the whole of the positive charge and practically the entire mass of the atom is confined to an extremely small central core.* It was called ***nucleus.***

Since 1 in about 8000 α-particles is deflected through 180°, the size of the nucleus is about $\frac{1}{10,000}$th of the size of the atom.

3. The scattering of different α-particles through different angles was explained on the ***nuclear model of the atom*** as given below :

The α-particles such as a and a', which pass through the atom at a large distance from the nucleus, experience a small electrostatic force of repulsion due to the nucleus and hence undergo a very small deflection. The α-particles b and b', which pass through the atom at a close distance from the nucleus suffer a large deflection. The α-particles such as c, which travel towards the nucleus directly, slow down as they move and ultimately come to rest and then after being deflected through 180°, retrace their path [Fig. 1.03].

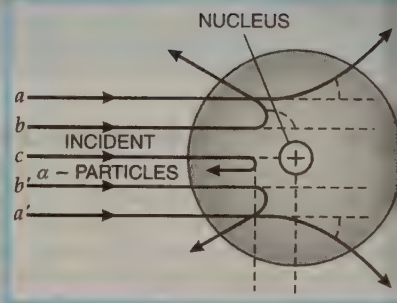


Fig. 1.03

1.02. CONCLUSIONS FROM RUTHERFORD'S EXPERIMENT

The following conclusions can be drawn from Rutherford's α-scattering experiment :

1. An atom may be regarded as a sphere of diameter 10^{-10}m but whole of the positive charge and almost the entire mass of the atom is concentrated in a small central core called nucleus, having a diameter of about 10^{-14} m.
2. The nucleus is surrounded by electrons. In other words, the electrons are spread over the remaining part of the atom, leaving plenty of empty space in the atom.
3. As the atom is electrically neutral, the total positive charge on the nucleus is equal to the total negative charge of the electrons in the atom.
4. Rutherford pointed out that the electrons inside the atom are not stationary for if they were at rest, they would be pulled into the positive nucleus due to the strong electrostatic force of attraction between the nucleus and the electrons. He assumed that the electrons are revolving around the nucleus in circular orbits, the necessary centripetal force being provided to them by the electrostatic force of attraction between the electrons and the nucleus.

Key point

An atom has a nearly continuous mas distribution in Thomson's atom model But according to Rutherford's aton model, the distribution of mass in the aton is highly non-uniform.

1.03. COMPOSITION OF NUCLEUS

The nucleus was first discovered in 1911 by Lord Rutherford and his associates by experiments on scattering of alpha particles by atoms. He found that the scattering results could be explained, if atoms consist of a *small, central, massive* and *positive core* surrounded by orbiting electrons. The experimental results indicated that the size of the nucleus is of the order of 10^{-14} metre and is thus 10,000 times smaller than the size of atom. The study of radioactivity revealed that nucleus is not a composite body but is made of ***nucleons.*** Further, it is practically responsible for the whole of the mass of the atom.

The Jargon

Protons and neutrons inside the nucleu are called nucleons.

In periodic table, elements are arranged according to their atomic numbers. The Zth element in the periodic table has Z protons in its nucleus and this number Z is known as ***atomic number.*** If there are N neutrons in the nucleus, then the number of protons and neutrons is given by

$$A = Z + N,$$

where A is known as ***mass number.*** It may be made clear that the nuclear mass is different from the mass number. Actually, mass number is the integer closest to the nuclear mass.

In nuclear physics, it is very convenient to represent nucleus of an element as ${}_ZX^A$, where Z and A are respectively the atomic number and mass number of an element, whose chemical symbol is X. For example, nucleus of sodium having 11 protons and 12 neutrons ($Z = 11$, $A = 23$) is represented as ${}_{11}Na^{23}$.

Magnetic moment of nucleus. In order to explain hyperfine structure of the spectral lines, in 1924, Pauli suggested angular momentum for the nucleus. The protons and neutrons were also considered as particles having orbital and spin motion.

When a charged particle like proton moves in a closed path, it possesses angular momentum and at the same time it produces magnetic field. The magnetic field at ge distances may be described as due to a magnetic dipole located at the centre the current loop (closed path of proton). Thus, the orbital and spin motion of otons produce extranuclear magnetic field, which can be assumed as due to gnetic dipoles located at the centre of the nucleus. Since a magnetic dipole ssesses magnetic moment, the nucleus also possesses magnetic moment.

Proton. Proton is a positively charged particle. The magnitude of charge on proton is 1·6 × 10^{-19} coulomb. The rest mass of proton is 1·6725× 10^{-27} kg. rther, the rest mass of proton (m_p) is 1836·1 times the rest mass of electron (m_e). has got intrinsic angular momentum (spin) equal to $\frac{1}{2}$. It also possesses a very all magnetic moment as compared to that of electron.

Neutron. Neutron possesses no charge and its rest mass (m_n) is 1836·6 times rest mass of electron. Neutron has got intrinsic angular momentum equal to that proton. Since magnetic moment of a particle is the effect of the rotating electric arge, the uncharged neutron might be expected to have no magnetic moment at . In fact, it also possesses a small magnetic moment.

Thus, we find that neutrons and protons have identical properties except that oton is a positively charged particle and neutron without any charge. As said rlier, protons and neutrons are collectively called nucleons.

Watch out !

The main drawback with neutrons is that : they have no charge. There is no way to artificially accelerate them !

The following gives a detailed account of the properties of neutron :

1. Neutron is constituent of nucleus and is an elementary particle having mass ual to 1·6748× 10^{-27} kg.
2. It is an uncharged particle and hence the electric and magnetic fields have effect on it.
3. Being neutral, it possesses very high penetrating power.
4. Also due to its neutral nature, it possesses very low ionising power.
5. Neutron is a stable particle within the nucleus. However, outside the ucleus, the neutron is an unstable particle. Its half life period is about 12 minutes. decays according to the following scheme :

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu},$$

here $\bar{\nu}$ is called ***antineutrino***.

6. The neutrons can be slowed down by passing them through heavy water, araffin wax, graphite, etc. This happens due to the elastic scattering of the neutrons. e scatterers are commonly known as ***moderators***.
7. The slow neutrons, also known as ***thermal neutrons***, are found to be more ficient in causing nuclear reactions.

1.04. ISOTOPES, ISOTONES AND ISOBARS

Isotopes. *The atoms of an element, which have the same atomic number but different ass numbers, are called isotopes*. In other words, the number of protons inside the ucleus or the number of electrons outside the nucleus of such atoms is the same ut the number of neutrons inside the nucleus is different. Thus, isotopes differ only s regards the number of neutrons inside their nuclei. As their atomic number is ame, they occupy the same position in the periodic table and hence they possess lentical chemical properties. Such elements cannot be separated by chemical means nd different techniques have been developed for their separation and to study their elative abundance. Some examples of isotopes are :

(*i*) ${}_8O^{16}$, ${}_8O^{17}$, ${}_8O^{18}$, (*ii*) ${}_{17}Cl^{35}$, ${}_{17}Cl^{37}$, (*iii*) ${}_{82}Pb^{206}$, ${}_{82}Pb^{207}$, ${}_{82}Pb^{208}$

Self-test Question

Values of atomic mass for most elements are close to whole number multiples of the mass of one hydrogen atom, but some are not (e.g. chlorine has an atomic mass of 35·5). Why ?

It may be pointed out that though the isotopes have identical chemical roperties, the nuclear properties are highly dissimilar. For example, out of three sotopes of sodium, ${}_{11}Na^{24}$ is found to decay by positron-emission, ${}_{11}Na^{23}$ is stable nd ${}_{11}Na^{22}$ is found to decay by beta-emission.

Self-test Question

Chlorine's proton number is 17. How many neutrons are there in each of its isotopes : Cl^{35} and Cl^{37} ? Which of the two isotopes is most abundant and by how much ?

Isotones. *The atoms whose nuclei have same number of neutrons are called isotones*. ecause of symmetry of nuclear forces in protons and neutrons, isotones and sotopes play very similar roles in nuclear forces.

Isobars. *The atoms, which have same mass number but different atomic numbers, are alled isobars*. Since isobars have same mass number, they have same number of ucleons (neutrons and protons) in their nuclei. The number of protons in their nuclei nd hence the number of electrons revolving around their nuclei are different in such toms. Therefore, the chemical properties of isobars are different. Some examples f isobars are :

(*i*) ${}_1H^3$ and ${}_2He^3$ (*ii*) ${}_3Li^7$ and ${}_4Be^7$ (*iii*) ${}_{18}Ar^{40}$ and ${}_{20}Ca^{40}$ (*iv*) ${}_{32}Ge^{76}$ and ${}_{34}Se^{76}$
No stable isobars are found among the higher elements.

1.05. SIZE OF NUCLEUS

The fact that nucleus has a finite size became evident from the Rutherford's alpha particle scattering experiments. The results of α-scattering experiment can be easily explained on the basis of the Coulomb's interaction between the α-particle and the nucleus of the gold atom. Further, the measurement of the distance of closest approach of the α-particle helped in estimating the size of the nucleus. The radius of gold nucleus was estimated to be about $4{\cdot}1\times 10^{-14}$m.

The size of the nucleus has been measured with the help of a variety of experiments involving the scattering of particles such as neutrons, protons, electrons, etc. In every case, it is found that *the volume of the nucleus is directly proportional to the number of nucleons (mass number) constituting the nucleus.* If R is the radius of the nucleus having mass number A, then

$$\frac{4}{3}\pi R^3 \propto A$$

or $$R \propto A^{1/3}$$

or $$R = R_0 A^{1/3}, \qquad \text{...(1.01)}$$

where $R_0 = 1{\cdot}1\times 10^{-15}$ is the range of nuclear force. It is also known as ***nuclear unit radius***. It is believed to be the average nucleon size. The value of R_0 is not definite. Electron scattering experiments predict slightly smaller value of R_0 than that predicted by neutron scattering experiment. It is because, in neutron scattering experiment, the charge on nucleus does not play any role as it does in electron scattering experiments. The different values of nuclear radius from these experiments indicate that the matter and charge are not distributed in a similar fashion inside the nucleus.

Self-test Question

High energy X-rays may have wa lengths as short as 10^{-11} m. What are implications for probing size of nucleus using such X-rays ?

Owing to the small size of the nucleus, fermi (F) is found to be a convenient unit of length in nuclear physics. It is defined as

$$\mathbf{1\ fermi\ (F) = 10^{-15}\ m}$$

1.06. NUCLEAR DENSITY

The expression for radius of nucleus can be used to compute the density of the nucleus. Let us find the density of the nucleus of an atom, whose mass number is A. If we neglect the mass of the orbital electrons, then

mass of the nucleus of the atom of mass number A

$$= A \text{ a.m.u.} = A \times 1{\cdot}660565 \times 10^{-27} \text{ kg}$$

If R is the radius of the nucleus, then

$$\text{volume of nucleus} = \frac{4}{3}\pi R^2 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^{\,3} A$$

Taking $R_0 = 1{\cdot}1 \times 10^{-15}$ m, we have

$$\text{volume of the nucleus} = \frac{4}{3}\pi (1{\cdot}1\times 10^{-15})^3 \times A \text{ m}^3$$

$$\text{Density of the nucleus, } \rho = \frac{\text{mass of nucleus}}{\text{volume of nucleus}} = \frac{A\times 1{\cdot}660565\times 10^{-27}}{\frac{4}{3}\pi(1{\cdot}1\times 10^{-15})^3\times A}$$

$$= 2{\cdot}97 \times 10^{17} \text{ kg m}^{-3} \qquad \text{(independent of A)}$$

Watch out !

A test tube full of bare nuclei will we heavier than the earth !

Discussion. 1. The density of the nuclei of all the atoms is same as it is independent of mass number.

2. The high density of the nucleus ($\approx 10^{17}$ kg m^{-3}) suggests the compactness of the nucleus. Such examples of high densities are met in the form of neutron stars.

3. The experiments reveal that the density of nucleus is maximum at the centre and falls to zero as we move radially outwards. In other words, the nucleus does not have sharp boundary. The effective value of radius of the nucleus is taken as the distance from its centre, where the density is half its value at the centre.

4. Recent high energy electron scattering experiments have proved that matter is not uniformly distributed inside the nucleus.

Key point

The density of nuclear matter is same all the nuclei (independent of Z).

Key point

The density of nuclear matter is extreme large as compared to the average dens of matter distributed in the atom.

1.07. FORCE BETWEEN NUCLEONS

The nucleus of an atom is a small sphere of diameter of about 10^{-14} metre and a number of protons and neutrons are packed inside it. Due to electrostatic interaction between protons (positively charged), the strong Coulomb's repulsive force will try to blow them apart, whereas the weak gravitational force between the cleons due to their rest masses will try to hold them together. In order to see, what ll actually happen, let us make an estimate of the magnitudes of Coulomb's and avitational forces between two protons inside a nucleus.

Magnitude of gravitational force. The gravitational force between two protons ch of mass m_p and separated by a distance r is given by

$$F_{grav} = G\frac{m_p\, m_p}{r^2}$$

Now, $m_p = 1{\cdot}67 \times 10^{-27}$ kg and $G = 6{\cdot}66 \times 10^{-11}\ \text{N m}^2\,\text{kg}^{-2}$

$$\therefore \quad F_{grav} = \frac{6{\cdot}66\times10^{-11}\times(1{\cdot}67\times10^{-27})^2}{r^2} = \frac{1{\cdot}86\times10^{-64}}{r^2}\ \text{N}$$

Magnitude of Coulomb's force. The Coulomb's or electrostatic force between vo protons each having charge q and located at the same distance r is given by

$$F_{elec} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\cdot q}{r^2}$$

Now $\quad \dfrac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N m}^2\,\text{C}^{-2}\ ;\ q = 1{\cdot}6\times10^{-19}\ \text{C}$

$$\therefore \quad F_{elec} = \frac{9\times10^9\times(1{\cdot}6\times10^{-19})^2}{r^2} = \frac{2{\cdot}304\times10^{-28}}{r^2}\ \text{N}$$

Hence, $\quad \dfrac{F_{elec}}{F_{grav}} = \dfrac{2{\cdot}304\times10^{-28}}{r^2}\times\dfrac{r^2}{1{\cdot}86\times10^{-64}} \approx 10^{36}$

Therefore, inside the nucleus, Coulomb's electrostatic repulsion between two rotons is about 10^{36} times the gravitational attraction between them.

Key point

Inside the nucleus, electrostatic force of repulsion between two protons is about 10^{36} times the gravitational force of attraction between them.

On the other hand, electrostatic force of attraction between a proton and an electron kept a certain distance apart is about 10^{42} times the gravitational force of attraction between them at the same distance.

1.08. NUCLEAR INSTABILITY

It follows that Coulomb's electrostatic repulsion between two protons is extremely large as compared to the gravitational force of attraction between them. Therefore, if Coulomb's repulsive and gravitational attractive forces are the only forces operating inside the nucleus, it cannot be stable. The stability of nuclus has been explained on the basis of the existence of a third type of force inside the nuclues, called ***nuclear force.*** It is such a strong attractive force that Coulomb's repulsive force between protons is unable to tear off the nucleus.

Key point

The extremely large magnitude of electrostatic force of repulsion between protons is the basic cause of nuclear instability.

The stability of a nucleus is determined by many factors. A brief account of a few such factors is as given below :

1. The stability of a nucleus is determined by the value of its binding energy per nucleon*. In general, higher the value of the binding energy per nucleon, more is the nucleus stable.

2. The stability of a nucleus is also determined by its neutron to proton ratio. In general, for stable nuclei, neutron proton ratio should be greater than 1.

Only the lightest nuclei tend to have equal number of protons and neutrons. As Z increases, the disruptive forces due to Coulomb's repulsion between the protons would tend to tear off the neucleus, unless some extra attractive forces were not brought into the nuclear structure. These extra attractive forces are provided by neutrons, whose number exceeds the number of protons by a larger and larger amount as Z increases.

Key point

Electrostatic repulsion would tear the nucleus apart. There has to be a force to bind all nucleons. Neutrons are not repelled electrostatically, but they would tend to drift about anywhere if not held tightly in the nucleus by a strong attractive force.

3. Apart from the high values of binding energy per nucleon and a proper neutron-proton ratio, the stability of a nuclon is determined by the consideration, whether it is an ***even-even nucleus*** (even Z and even A) or an ***even-odd nucleus*** (even Z and odd A) or ***odd-even nucleus*** (odd Z and even A) or an ***odd-odd nucleus*** (odd Z and odd A). It is found that an even-even nucleus is more stable, even if it has comparatively low value of binding energy per nucleon. An even-odd or an odd-even nucleus is found to be lesser stable, while the odd-odd nucleus is found to be least stable.

1.09. NUCLEAR FORCE

Inside the nucleus, Coulomb's repulsive force between two protons is about 10^{36} times as large as the gravitational force between them. In the presence of such a tremendous repulsive force between two protons, the small attractive gravitational force can do little in preventing them from getting away from each other. But we know that even a heavy nucleus containing a large number of protons is stable. It is, therefore, evident that a third type of force does exist between the nucleons and it is known as *nuclear force*. This force should be basically a very strong attractive force, so that even the large electrostatic force of repulsion between the protons may not be able to blow apart the protons.

---

*Refer to section 1.12.

Key point

Nuclear forces give rise to a negative p ential energy (binding energy), which observed as a loss in mass (mass defec

The exact force law obeyed by the nuclear force is not known but at a distance of about 10^{-14} m, nuclear force is estimated to be about 100 times as strong as the Coulomb's force between the two protons. Therefore, the relative strengths of the gravitational, Coulomb's and nuclear force are

$$F_g : F_e : F_n :: 1 : 10^{36} : 10^{38}$$

Properties. The nuclear forces are found to possess following properties :

1. ***Nuclear forces are strongest forces in nature.*** Nuclear forces are about 10^{38} times as strong as gravitational forces.

2. ***Nuclear forces are short range forces.*** The nuclear force between two nucleons becomes negligible, when the distance between them is greater than 4·2 fermi or $4{\cdot}2 \times 10^{-15}$ m. However, the range of nuclear force is taken as 1·5 fermi, where the magnitude of the nuclear force becomes about one-third of its maximum value. Thus, unlike gravitational and electromagnetic forces (which extend up to infinity), nuclear force is a short range force.

Key point

Nuclear forces are strongest attracti forces, which hold nucleons togeth inside the nucleus. They are short rang charge independent and non-centr forces.

3. ***Nuclear forces are basically strong attractive forces, but contain a small component of repulsive forces.*** The nuclear force comes into action as an attractive force, when two nucleons are at a distance of about 10^{-14} m (size of nucleus). As the distance is decreased, the attractive force increases rapidly and becomes maximum at a distance of about $1{\cdot}5 \times 10^{-15}$. If distance is decreased further, at a distance of $0{\cdot}5 \times 10^{-15}$ m, it changes suddenly into a repulsive force. Thus, over a large part of nucleous, the nuclear force is strong attractive and is strong repulsive only over a small part of the nucleus. This repulsive part saves the nucleus from collapsing.

4. ***Nuclear forces are saturated forces.*** A nucleon inside the nucleus does not experience nuclear force due to all other nucleons. It experiences force only due to its nearest neighbouring nucleons. A direct consequence of this property is that mean binding energy per nucleon for most of the nuclei has a constant value of 8 MeV per nucleon.

5. ***Nuclear forces are charge independent.*** Inside the nucleus, the attractive forces between two nucleons are of three types *i.e.* force between a neutron and a proton (*n-p*), between two neutrons (*n-n*) and between two protons (*p-p*).

Experiments indicate that the three types of forces *i.e.* *n-p*, *n-n* and *p-p* have the same magnitude *i.e.*

$$n\text{-}p \approx n\text{-}n \approx p\text{-}p$$

Thus, inside the nucleus, the force between nucleons is not affected by the charge of the proton and this property is known as *charge independence of nuclear forces.*

6. ***Nuclear forces are spin-dependent.*** The force between two nucleons having parallel spins is greater than the force between the two nucleons having anti-parallel spins and hence nuclear forces are spin dependent.

7. ***Nuclear forces are exchange forces.*** The nuclear force between the two nucleons is the result of the exchange of π mesons (π^0, π^+ and π^-) between them.

8. ***Nuclear forces are non-central forces.*** The force between two nucleons does not act along the line joining their centres and for this reason, it is called a *non-central force.*

1.10. MASS-ENERGY EQUIVALENCE

Einstein's mass-energy equivalence is a consequence of variation of mass with velocity. From the special theory of relativity, it follows that the kinetic energy of a moving particle is equal to c^2 times the increase in mass due to its motion. If m_0 is rest mass of the particle and m, the mass in motion, then kinetic energy of the particle,

$$T = (m - m_0)\, c^2$$

According to Einstein, the rest mass of the particle is due to an internal store of energy $m_0 c^2$, called the ***rest mass energy*** of the particle. Therefore, the total energy of a moving particle is given by

$$E = T + m_0 c^2 = (m - m_0)\, c^2 + m_0 c^2$$

or

$$E = m c^2 \qquad \text{...(1.02)}$$

Thus, a mass m is equivalent to an energy $m\,c^2$. This is the famous ***Einstein's -energy relationship*** showing the equivalence of mass and energy.

Discussion. The mass-energy equivalence relation leads to the following rtant conclusions :

1. The equation (1.02) may be written as

$$\Delta E = \Delta m\,c^2,$$

e Δm is the increase in mass corresponding to the increase in energy ΔE. It forms asis of nuclear fission and fusion relations. A mass Δm is equivalent to energy ^{2} and an energy ΔE is equivalent to a mass $\Delta E/c^2$. Thus, *special theory of relativity ves energies to all masses and mases to all energies.*

2. In classical mechanics, the laws of conservation of mass and energy are two rate principles independent of each other. The relation $E = m\,c^2$, leads to the cation of the two laws into one law *i.e.* ***law of conservation of relativistic energy.***

3. In classical mechanics, mass is considered something fundamental to er, while energy is a property of the matter acquired by virtue of its position figuration) or motion. The relation $E = m\,c^2$ puts an end to such a distinction veen mass and energy.

Self-test Question

How long could you run a 1000 MW power station on 1 kg of matter (assuming all of it is converted into useful energy ?

1.11. ATOMIC MASS UNIT

The atomic mass unit* (a.m.u.) is a very small unit of mass and it is found to ery convenient in nuclear physics.

Atomic mass unit is defined as $\frac{1}{12}$ *th of the mass of one* ${}_6C^{12}$ *atom.*

According to Avogadro's hypothesis, number of atoms in 12 g of ${}_6C^{12}$ is equal vogadro number. *i.e.* $6{\cdot}023 \times 10^{23}$.

Therefore, the mass of one carbon atom (${}_6C^{12}$) is $\dfrac{12}{6{\cdot}023 \times 10^{23}}$ *i.e.* $1{\cdot}992678 \times$ ^{26} kg.

Therefore, $\quad 1 \text{ a.m.u.} = \dfrac{1}{12} \times 1{\cdot}992678 \times 10^{-26}$ kg

or $\quad$ **1 a.m.u. = $1{\cdot}660565 \times 10^{-27}$ kg**

The mass of a hydrogen atom comes out to be 1·00794a.m.u. The masses of ny atoms are found to be close to integral multiple† of the mass of an hydrogen m. However, the atomic mass of the chlorine atom is 35·47 a.m.u. and is not egral multiple of the atomic mass of hydrogen. The measurement of atomic sses with the help of mass-spectrometer revealed that many elements possess topes. The chlorine is composed of two isotopes having atomic masses 34·98 ı.u. and 36·98 a.m.u. Their relative abundance is 75·4% and 24·6% respectively. e atomic mass of chlorine is taken weighted average of the masses of its two topes. Thus,

$$\text{atomic mass of chlorine atom} = \frac{75{\cdot}4 \times 34{\cdot}98 + 24{\cdot}6 \times 36{\cdot}98}{100} = 35{\cdot}47 \text{ a.m.u.}$$

On the atomic scale, mass of a proton is taken as

$$m_p = 1{\cdot}007275 \text{ a.m.u.} = 1{\cdot}007275 \times 1{\cdot}660565 \times 10^{-27} = 1{\cdot}67265 \times 10^{-27} \text{ kg}$$

d mass of a neutron is taken as

$$m_n = 1{\cdot}008665 \text{ a.m.u.} = 1{\cdot}008665 \times 1{\cdot}660565 \times 10^{-27} = 1{\cdot}67495 \times 10^{-27} \text{ kg}$$

Energy equivalent of a.m.u.

According to Einstein's mass-energy equivalence relation, the energy uivalent of mass m is given by

$$E = m\,c^2,$$

here c is speed of light.

Suppose that $\quad m = 1 \text{ a.m.u.} = 1{\cdot}660565 \times 10^{-27}$ kg

Since $\quad c = 2{\cdot}998 \times 10^8 \text{ m s}^{-1}$, the energy equivalent of 1 a.m.u. is given by

$$1 \text{ a.m.u.} = (1{\cdot}660565 \times 10^{-27} \text{ kg}) \times (2{\cdot}998 \times 10^8 \text{ m s}^{-1})^2 = 1{\cdot}4925 \times 10^{-10} \text{ J}$$

Since $\quad 1 \text{ MeV} = 1{\cdot}602 \times 10^{-13}$ J, we have

---

*Atomic mass unit is also denoted as u.

†Before 1960, atomic mass unit was defined as $\frac{1}{16}$ th of mass of an ${}_8O^{16}$ atom. On this basis, the atomic masses of many elements iffer considerably from the integral multiples of the mass of hydrogen atom.

$$1\ \text{a.m.u.} = \frac{1{\cdot}4925\times10^{-10}}{1{\cdot}602\times10^{-13}}$$

or $$\mathbf{1\ a.m.u. = 931{\cdot}5\ MeV}$$

Note. It can be calculated that the energy equivalents of the mass of an electron, proton and neutron are respectively

$$m_e = 0{\cdot}511\ \text{MeV},\ m_p = 938{\cdot}279\ \text{MeV and } m_n = 939{\cdot}573\ \text{MeV}$$

1.12. MASS DEFECT AND BINDING ENERGY

In general, the nucleus of an atom is stable. The protons and neutrons are held inside the nucleus by the strong attractive forces, called nuclear forces. Since the free protons would exert a very strong force of repulsion on each other, it is worthwhile to investigate as to how a nucleus is formed from free protons and neutrons.

Mass defect. It is found that the mass of a stable nucleus is always less than the total mass of the constituent nucleons. In other words, if a certain number of neutrons and protons are brought together to form a nucleus, the mass of the nucleons forming the nucleus decreases or *defects.*

The difference between the sum of the masses of the nucleons constituting a nucleus and the rest mass of the nucleus is known as mass defect. It is denoted by Δm.

Let us calculate the mass defect in case of the nucleus of an atom ${}_ZX^A$. The nucleus of the atom contains Z protons and (A–Z) neutrons. Therefore,

mass of the constituting nucleons $= Z\,m_p + (A - Z)\,m_n$

If $m_N({}_ZX^A)$ is mass of the necleus of the atom ${}_ZX^A$, then the mass defect is given by

$$\Delta m = [Z\,m_p + (A - Z)\,m_n] - m_N({}_ZX^A) \qquad ...(1.03)$$

The mass defect can also be expressed in another form as explained below :

Adding and subtracting the mass of Z electrons *i.e.* $Z\,m_e$ on the R.H.S. of equation (1.03), we have

$$\Delta m = [Z\,m_p + (A - Z)\,m_n + Z\,m_e] - m_N({}_ZX^A) - Z\,m_e$$
$$= [Z\,(m_p + m_e) + (A - Z)\,m_n] - [m_N({}_ZX^A) + Z\,m_e]$$

Now, $m_p + m_e = m({}_1H^1)$, mass of hydrogen atom and

$m_N({}_ZX^A) + Z\,m_e = m({}_ZX^A)$, mass of the atom ${}_ZX^A$

Therefore, $$\Delta m = [Z\,m({}_1H^1) + (A - Z)\,m_n] - m({}_ZX^A) \qquad ...(1.04)$$

Binding energy. When a nucleus is formed from the free nucleons, the decrease in mass of the nucleons is released as equivalent energy (in accordance with Einstein's mass-energy relation). The energy equivalent to mass defect is used in *binding* the nucleons and is called ***binding energy*** of the nucleus. In order to break the nucleus or to completely separate the nucleons from each other, an equal amount of work has to be done.

Thus, *the binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.* It may be measured as the work required to be done to separate the nucleons an infinite distance apart, so that they no longer interact with each other.

If Δm is mass defect of a nucleus, then according to Einstein's mass-energy relation,

binding energy of the nucleus $= \Delta m\, c^2$ (in joule)

Here, mass defect Δm has to be measured in kilogram. In case, mass defect is measured in a.m.u., then

binding energy of the nucleus $= \Delta m \times 931{\cdot}5$ (in MeV)

The binding energy of a nucleus may also be expressed in terms of masses of protons, neutrons and the nucleus as explained below :

From equation (1.03), we have

$$\text{binding energy} = [\{Z\,m_p + (A - Z)\,m_n\} - m_N({}_ZX^A)] \times 931{\cdot}5 \qquad ...(1.05)$$

Here, $m_N({}_ZX^A)$ is mass of the nucleus of the atom ${}_ZX^A$.

Also, using equation (1.04), we have

$$\text{binding energy} = [\{Z\,m({}_1H^1) + (A - Z)\,m_n\} - m({}_ZX^A)] \times 931{\cdot}5 \qquad ...(1.06)$$

Here, $m({}_1H^1)$ and $m({}_ZX^A)$ represent mass of hydrogen atom (${}_1H^1$) and the atom ${}_ZX^A$.

Key point

Nuelceons are bound together by the strong nuclear force. The binding energy of the nucleus may be termed as the work done against this binding force to pull the nucleons apart.

Watch out !

Mass defect and hence binding energy of hydrogen nucleus is zero. It is because, there is only one nucleon. Therefore, strong nuclear force is not involved and so no work need to be done to overcome it !

Watch out !

Binding energy is sometimes considered a negative quantity, since it is an energy deficit !

Binding energy per nucleon. *The binding energy per nucleon is the average energy ired to extract one nucleon from the nucleus.* It is obtained by dividing the binding gy of the nucleus by the number of nucleons it contains.

$$\text{Thus, binding energy per nucleon} = \frac{\text{binding energy}}{A} \qquad ...(1.07)$$

It may be pointed out that the binding energy per nucleon of a nucleus is more ortant than the total binding energy of the nucleus. The higher value of the ding energy per nucleon indicates comparatively greater stability of the nucleus.

Packing fraction. *The packing fraction of a nucleus is defined as the mass defect per eon of the nucleus.*

$$\text{Thus,} \qquad \text{packing fraction} = \frac{\text{mass defect}}{A} \qquad ...(1.08)$$

The packing fraction of a nucleus is closely related to its binding energy per leon.

1.13. BINDING ENERGY CURVE (Nuclear Stability)

Knowing the mass of the nucleus or the mass of the atom, the binding energy l binding energy per nucleon of a nucleus can be easily determined. The values hese two quantities for a few nuclei are given in the following table :

Nuclei	Binding energy	Binding energy per nucleon
$_1H^2$	2·22 MeV	1·11 MeV
$_2H^4$	28·32 MeV	7·08 MeV
$_3Li^7$	37·66 MeV	5·38 MeV
$_{17}Cl^{35}$	287·84 MeV	8·22 MeV
$_{26}Fe^{56}$	492·26 MeV	8·79 MeV
$_{83}Bi^{209}$	1640·26 MeV	7·85 MeV
$_{92}U^{235}$	1786·0 MeV	7·6 MeV

It follows that binding energy of $_3Li^7$ is greater than that of $_2He^4$, but the value its binding energy per nucleon is lesser. However, $_2He^4$ is found to be more stable an $_3Li^7$. Therefore, it may be concluded that the stability of a nucleus (in addition a few other factors*) depends upon binding energy per nucleon rather than the cal binding energy of the nucleus. Hence, the knowledge of binding energy per cleon of a nucleus is more important than its binding energy.

Fig. 1.04 shows the graph between binding energy per nucleon and mass mber of different nuclei. From the binding energy curve, the following inferences n be drawn :

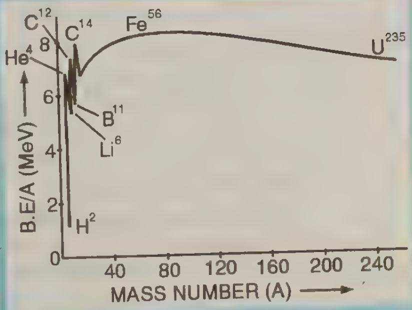


Fig. 1.04

1. The binding energy per nucleon for light nuclei, such as $_1H^2$, is very small.

2. The binding energy per nucleon increases rapidly for nuclei upto mass mber 20 and the curve possesses peaks corresponding to nuclei $_2He^4$, $_6C^{12}$ and O^{16}. The peaks indicate that these nuclei are more stable than those in their eighbourhood. It confirms the reason for extraordinary stability of α-particle.

3. After mass number 20, binding energy per nucleon increases gradually and r mass number between 40 and 120, the curve becomes more or less flat. The verage value of binding energy per nucleon in this region is about 8·5 MeV. For A 56 ($_{26}Fe^{56}$), the binding energy per nucleon is maximum and it is equal to 8·8 MeV.

4. After mass number 120, binding energy per nucleon starts decreasing and rops to 7·6 MeV for uranium. This low value of binding energy per nucleon in case f heavy nuclei is unable to have control over the Coulomb's repulsion between the rge number of protons. Such nuclei are unstable and are found to disintegrate by mitting α-particles. An α-particle is a nucleus of helium (2 protons and 2 neutrons). he emission of α-particle not only decreases Coulomb's repulsive force inside the ucleus but also increases the value of B.E./A of the nucleus due to its extraordinary table structure (α-particle has large binding energy). It is called ***α-decay***.

Sometimes, the heavy nuclei increase the value of their B.E./A by emitting an lectron. It is called ***β-decay***. Inside the nucleus, an electron does not exist. It is created t the time of β-decay due to conversion of a neutron into proton. The β-decay leads o increase in Coulomb's repulsive force, but it increases B.E./A and also improves he ***neutron-proton ratio***†.

---

*Refer to section 1.08.

†As discussed in section 1.08, it is a factor, which determines the stability of a nucleus.

All such nuclei, which undergo α and β-decay are called *radioactive nuclei.*

5. The binding energy per nucleon has a low value for both very light and very heavy nuclei. In order to attain higher value of binding energy per nucleon, the lighter nuclei may unite together to form a heavier nucleus (process of ***nuclear fusion***) or a heavier nucleus may split into lighter nuclei (process of ***nuclear fission***). In both the nuclear processes, the resulting nucleus acquires greater value of binding energy per nucleon along with the liberation of energy.

Watch out !

Once you reach iron in nuclear reaction it's all over. No more energy availabl Iron is ***cold and deed*** in terms of availabl nuclear energy !

SOLVED NUMERICAL PROBLEMS

Type A. On size of nucleus

Problem 1.01. Obtain approximately the ratio of the nuclear radii of the gold isotope $_{79}Au^{197}$ and the silver isotope $_{47}Ag^{107}$. *(Text Problem)*

Sol. We know that $R \propto A^{1/3}$

Therefore, $$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

Here, $A_1 = 197;\ A_2 = 107$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{197}{107}\right)^{1/3} = \mathbf{1\cdot226}$$

Type B. On Nuclear density

Problem 1.02. Assuming that protons and neutrons have equal masses, calculate how many times nuclear matter is denser than water. Given that nuclear radius is given by $R = 1\cdot2 \times 10^{-15}\ A^{1/3}$ metre and mass of a nucleon $= 1\cdot67 \times 10^{-27}$ kg.

Sol. Here, $R = 1\cdot2 \times 10^{-15}\ A^{1/3}$ m ;

mass of one necleon $= 1\cdot67 \times 10^{-27}$ kg ;

Let us calculate the density of the nucleus of mass number A.

Then, mass of the nucleus $= 1\cdot67 \times 10^{-27} \times A$ kg

Volume of the nucleus, V

$$= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\,(1\cdot2\times10^{-15}\ A^{1/3})^3\ \text{m}^3$$

Density of the nucleus,

$$\rho_{nuc} = \frac{1\cdot67\times10^{-27}\times A}{\frac{4}{3}\pi\,(1\cdot2\times10^{-15}\ A^{1/3})^3}$$

$$= 2\cdot307\times10^{17}\ \text{kg m}^{-3}$$

Now, density of water, $\rho_{wat} = 10^3$ kg m^{-3}

$$\frac{\rho_{nuc}}{\rho_{wat}} = \frac{2\cdot307\times10^{17}}{10^3} = \mathbf{2\cdot307 \times 10^{14}}$$

Type C. On Mass defect and Binding energy

Problem 1.03. Find out the binding energy per nucleon of an α-particle in MeV. It is given that the masses of α-particle, proton and neutron are respectively 4·00150 a.m.u., 1·00728 a.m.u. and 1·00867 a.m.u. *(I.S.C.E. 1998)*

Sol. An α-particle *i.e.* nuclues of helium, consists of two protons and two neutrons.

Here, $m_{He} = 4\cdot00150$ a.m.u. ; $m_p = 1\cdot00728$ a.m.u. ;
$m_n = 1\cdot00867$ a.m.u.

Therefore, mass of protons and neutrons constituting an α-particle,

$$2\,m_p + 2\,m_n = 2\times1\cdot00728 + 2\times1\cdot00867 = 4\cdot0319 \text{ a.m.u}$$

Mass defect, $\Delta m = (2\,m_p + 2\,m_n) - m_{He}$

$$= 4\cdot0319 - 4\cdot00150 = 0\cdot0304 \text{ a.m.u.}$$

Now, 1 a.m.u. = 931·5 MeV

Therefore, binding energy of α-particle

$$= \Delta m \times 931\cdot5 = 0\cdot0304 \times 931\cdot5 = \mathbf{28\cdot32\ Me}$$

Problem 1.04. The mass of deutron ($_1H^2$)nucleus 2·013553 a.m.u. If the masses of proton and neutron a 1·007275 a.m.u. and 1·008665 a.m.u. respectively, calcula the mass defect, the packing fraction, binding energy an binding energy per nucleon.

Sol. Here, $m_p = 1\cdot007275$ a.m.u. ; $m_n = 1\cdot008665$ a.m.u

and mass of $_1H^2$ nucleus, $m_N\,(_1H^2) = 2\cdot013553$ a.m.u

The deutron nucleus contains one proton and on neutron. Therefore,

mass of nucleons constituting deutron,

$$m_p + m_n = 1\cdot007275 + 1\cdot008665 = 2\cdot015940 \text{ a.m.u.}$$

Therefore, mass defect, $\Delta m = (m_p + m_n) - m_N(_1H^2)$

$$= 2\cdot015940 - 2\cdot013553 = \mathbf{0\cdot002387\ a.m.u.}$$

Packing fraction, $$\frac{\Delta m}{A} = \frac{0\cdot002387}{2} = \mathbf{0\cdot0011935\ a.m.u.}$$

Now, binding energy of deutron

$$= \Delta m \times 931\cdot5 = 0\cdot002387 \times 931\cdot5$$
$$= \mathbf{2\cdot22\ MeV}$$

Therefore, binding energy per nucleon of deutron

$$= 2\cdot22/2 = \mathbf{1\cdot11\ MeV}$$

Problem 1.05. Obtain the binding energy of the nucle $_{26}Fe^{56}$ and $_{83}Bi^{209}$ in units of MeV from the following data mass of hydrogen atom = 1·007825 a.m.u., mass of neutror = 1·008665 a.m.u.; mass of $_{26}Fe^{56}$ atom = 55·934939 a.m.u and mass of $_{83}Bi^{209}$ atom = 208·980388 a.m.u.

(C.B.SE. 1993, Pre-degree Kerala, 1991)

Also calculate binding energy per nucleon in the two cases. *(Text Problem)*

Sol. Here, mass of hydrogen atom,

$m(_1H^1) = 1\cdot007825$ a.m.u. $m_n = 1\cdot008665$ a.m.u

If $m(_ZX^A)$ represents the mass of the atom $_ZX^A$, then binding energy of the nucleus of the atom $_ZX^A$ is given by

$$\text{B.E.} = (_ZX^A)$$
$$= [\{Z\,m(_1H^1) + (A - Z)\,m_n\} - m(_ZX^A)] \times 931\cdot5$$

(*i*) Here, $m(_{26}Fe^{56}) = 55\cdot934939$ a.m.u. ;

$$Z = 26 \text{ and } A - Z = 56 - 26 = 30$$

$\therefore$ B.E. ($_{26}Fe^{56}$)

$= [\{26 \times 1\cdot007825 + 30 \times 1\cdot008665\} - 55\cdot934939] \times 931\cdot5$

$= [\{26\cdot20345 + 30\cdot25995\} - 55\cdot934939] \times 931\cdot5$

$= \mathbf{492\cdot26\ MeV}$

B.E. per nucleon of $_{26}Fe^{56} = 492\cdot26/56 = \mathbf{8\cdot79\ MeV}$

(*ii*) Here, $m(_{83}Bi^{209}) = 208\cdot980388$ a.m.u. ;

$$Z = 83 \text{ and } A - Z = 209 - 83 = 126$$

Therefore, B.E. ($_{83}Bi^{209}$)

$= [\{83 \times 1\cdot007825 + 126 \times 1\cdot008665\} - 208\cdot980388] \times 931\cdot5$

$= [\{83\cdot64975 + 127\cdot09179\} - 208\cdot980388] \times 931\cdot5$

$= \mathbf{1640\cdot26\ MeV}$

B.E. per nucleon of $^{83}Bi^{209} = 1640\cdot26/209 = \mathbf{7\cdot848\ MeV}$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. How many electrons, protons and neutrons re there in a nucleus of atomic number 11 and mass umber 24 ? (*I.I.T. 1982*)

Ans. Number of electrons or protons, Z = **11**
and number of neutrons, A – Z = 24 – 11 = **13**

Q. 1.02. What are the number of protons and the umber of netrons in a nucleus of ${}_{92}U^{238}$? (*I.S.C.E. 1993*)

Ans. Here, number of protons, Z = **12**
number of neutrons, A – Z = 238 – 92 = **146**

Q. 1.03. How many electrons, protons and neutrons are here in 16 g of ${}_8O^{16}$?

Ans. Number of atoms in 16 g of ${}_8O^{16}$ is equal to vogadro number, N = $6{\cdot}02 \times 10^{23}$. Therefore,
number of electrons or protons in 16 g of ${}_8O^{16}$,

$$N \times Z = 6{\cdot}02 \times 10^{23} \times 8 = \mathbf{4{\cdot}816 \times 10^{24}}$$

and number of neutrons in 16 g of ${}_8O^{16}$,

$$N \times (A - Z) = 6{\cdot}02 \times 10^{23} \times (16 - 8) = \mathbf{4{\cdot}816 \times 10^{24}}$$

Q. 1.04. Is a free neutron stable particle ?

Ans. A free neutron is not a stable particle.

Q. 1.05. Give an equation representing the decay of a free neutron. (*H.S.S.C.E. 1998*)

Ans. Equation representing decay of a free neutron.

$${}_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu}$$

Q. 1.06. What is meant by the term 'isotope' ? (*I.S.C.E. 1998*)

Ans. The atoms of an element, which have the same atomic number but different mass numbers, are called isotopes.

Q. 1.07. What are isotones ? (*P.S.S.C.E. 2002, 1999*)

Ans. The atoms, whose nuclei have same number of neutrons, are called isotones.

Q. 1.08. How is radius of nucleus related to mass number ?

Ans. The relation between radius and mass number is :

$$R = R_0 A^{1/3}$$

Q. 1.09. What will be the ratio of the radii of two nuclei of mass numbers A_1 and A_2? (*C.B.SE. 2000, 1995*)

Ans. Let R_1 and R_2 be radii of the nuclei of mass numbers A_1 and A_2 respectively. Then, from the relation $R = R_0 A^{1/3}$, it follows that

$$\frac{R_1}{R_2} = \frac{R_0 A_1^{1/3}}{R_0 A_2^{1/3}} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

Q. 1.10. What is the order of the magnitude of the density of nucleus ?

Ans. Nuclear density ≈ 10^{17} kg m^{-3}

Q. 1.11. What is the ratio of the nuclear densities of two nuclei having mass numbers in the ratio 1 : 4 ? (*C.B.SE.1998 S*)

Ans. Nuclear density is independent of the mass number of a nucleus. Since the two nuclei having mass numbers in the ratio 1 : 4 have same nuclear density, ratio of their nuclear densities is 1.

Q. 1.12. What holds nucleons together in a nucleus ? (*H.S.S.C.E. 2001*)

Ans. Inside the neclueons, the electrostatic force of repulsion between the protons is extremely large. They are held inside the nucleus due to nuclear force between them, which is basically a strong attractive force.

Q. 1.13. State Einstein's mass-energy relation. (*I.S.C.E. 1997*)

Ans. In a process, when a mass *m* is completely converted into energy, the energy obtained is given by

$$E = m c^2$$

Q. 1.14. Define atomic mass unit. (*H.S.S.C.E. 1997 S*)

Or

What is atomic mass unit ? (*I.S.C.E. 1995*)

Ans. 1 a.m.u. = $\frac{1}{12}$th of the mass of one ${}_6C^{12}$ atom

Q. 1.15. Express one atomic mass unit (1 a.m.u.) in kilogram. (*H.S.S.C.E. 1994*)

Ans. 1 a.m.u = **$1{\cdot}66 \times 10^{-27}$ kg**

Q. 1.16. How many joule are there in one MeV ? (*H.P.S.S.C.E. 1997*)

Ans. 1 MeV = 10^6 eV = $10^6 \times 1{\cdot}6 \times 10^{-19}$ = **$1{\cdot}6 \times 10^{-13}$ J**

Q. 1.17. Define mass defect. (*H.S.S.C.E. 2001*)

Ans. *The difference between the sum of the masses of the neucleons constituting a nucleus and the rest mass of the nucleus is known as mass defect.*

Q. 1.18. What do you mean by mass defect of a nucleus ? (*H.S.S.C.E. 1993*)

Ans. Refer to section 1.12.

Q. 1.19. Define binding energy of a nucleus. (*H.S.S.C.E. 2002 ; H.P.S.S.C.E. 2001*)

Ans. *The binding energy of a nucleus is equal to the amount of work done to separate the nucleons an infinite distance apart from each other, so that they no longer interact with each other.*

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. Draw a labelled schematic arrangement showing Rutherford's alpha particle scattering experiment. (*I.S.C.E. 1997*)

Ans. Refer to Fig. 1.01.

Q. 1.02. What conclusions are drawn from Rutherford scattering experiments ? (*I.S.C.E. 1995*)

Ans. Refer to section 1.02.

Q. 1.03. Nuclear atom model is required to explain the results of Rutherford experiment. Why ?

Ans. In Rutherford experiment, a few α-particles are scattered through large angles. Therefore, massive and positive part of atom should be concentrated in a small portion of the atom. This small positive part having almost the entire mass of the atom was called nucleus and therefore the main feature of any atom model should be the presence of the central part called nucleus.

Q. 1.04. Explain, how Rutherford's experiment on scattering of α-particles led to the estimation of the size of the nucleus. (*C.B.S.E. 1993*)

Ans. As most of the α-particles passed straight through gold foil and a very few were scattered back, it indicated that the central part of the atom must be positive and it should be a very small part of the atom. As only 1 in about 8000 α-particles retraced its path, the size of nucleus was estimated to be about $\frac{1}{10,000}$ th of the size of an atom.

Q. 1.05. The three stable isotopes of neon : Ne^{20}, Ne^{21} and Ne^{22} have respectively abundances of 90·51%, 0·27% and 9·22%. The atomic masses of the three isotopes are 19·99 a.m.u., 20·99 a.m.u. and 21·99 a.m.u. respectively. Obtain the average atomic mass of neon. *(Text Question)*

Sol. The average atomic mass of neon,

$$A = \frac{90 \cdot 51 \times 19 \cdot 99 + 0 \cdot 27 \times 20 \cdot 99 + 9 \cdot 22 \times 21 \cdot 99}{100}$$

$= 20 \cdot 177$ a.m.u.

Q. 1.06. Prove that nuclear density is same for all nuclei. *(P.S.S.C.E. 1999 S, 1992)*

Or

Show that the density of nuclear matter is constant. *(P.S.S.C.E. 1992)*

Ans. Refer to section 1.06.

Q. 1.07. Why is the density of nucleus more than that of the atom ?

Ans. The size of nucleus is of the order of 10^{-14} m and it is made of protons and neutrons only.

On the other hand, size of atom is of the order of 10^{-10} m. Whereas, the size of atom increases by a very large factor (10^4 times as large as the nucleus), its mass increases only by a small amount (additional mass is the mass of orbital electrons only). For this reason, the density of nucleus is very large as compared to that of the atom.

Q. 1.08. You are given two nuclides $_3X^7$ and $_3Y^4$.

(*i*) Are they the isotopes of the same element ? Why ?

(*ii*) Which one of the two is likely to be more stable ? Give reasons. *(C.B.S.E. 2000, 1995)*

Ans. **(*i*)** Yes, $_3X^7$ and $_3Y^4$ are isotopes of the same elements. It is because, an element is characterised by its atomic number. Since, both X and Y have atomic number 3, they represents the same element *i.e.* Li.

(*ii*) $_3X^7$ (or $_3Li^7$)is more stable than $_3Y^4$ (or $_3Li^4$)(. It is because, the greater number of neutrons in $_3Li^7$ results in greater attractive force between the nucleons so as to win over the Coulomb's repulsive force between the protons.

Q. 1.09. What are nuclear forces ?

Ans. Refer to section 1.09.

Q. 1.10. Give two important characteristics of nuclear forces. *(P.S.S.C.E. 2002, 1999 S, 1999 ; C.B.S.E. 1992)*

Ans. Refer to section 1.09.

Q. 1.11. What do you mean by the charge independent nature of nuclear forces ?

Ans. Refer to section 1.09.

Q. 1.12. Express 16 mg mass into equivalent energy in electron volt. *(P.S.S.C.E. 2000 ; H.S.S.C.E. 1992)*

Ans. Here $m = 16 \text{ mg} = 16 \times 10^{-3} \text{ g} = 16 \times 10^{-6} \text{ kg}$

$\therefore \quad E = m c^2 = 16 \times 10^{-6} \times (3 \times 10^8)^2 = 1 \cdot 44 \times 10^{12}$ J

$$= \frac{1 \cdot 44 \times 10^{12}}{1 \cdot 6 \times 10^{-19}} = 9 \times 10^{-30} \text{ eV}$$

Q. 1.13. Convert one a.m.u. in MeV. *(P.S.S.C.E. 1998 S)*

Or

Calculate the energy equivalent to one atomic mass unit (1 a.m.u.) in MeV. *(H.S.S.C.E. 1995)*

Ans. Refer to section 1.11.

Q. 1.14. What is mass defect ? What light does it thro on the binding energy of a nucleus ? *(I.S.C.E. 199*

Ans. Mass defect. For definition, refer to VSQ 1.17.

The binding energy of a nucleus is measured as th energy equivalent to the mass defect.

Q. 1.15. Why nuclei invariably have mass less tha the sum of the masses of the individual nucleons i them ?

Ans. To make a nucleus, protons and neutrons have t come together in a very small space of the order of 10^{-14} m The energy required to do so is provided by the nucleons a the expense of their masses. Due to this, the mass of th nucleus formed is always less than the sum of the masses o the consituent nucleons.

Q. 1.16. Draw a curve to show the variation of bindin energy per nucleon with mass number. Show the positio of H^2 and U^{238} on this curve. *(I.S.C.E. 1995*

Ans. Refer to Fig. 1.04.

Q. 1.17. Draw the graph showing variation of bindin energy per nucleon with mass number. Write two mai inferences drawn from the graph. *(C.B.S.E. 2002, 1994 ; H.S.S.C.E. 2002*

Ans. For graph between B.E/A and A, refer to Fig. 1,04

The following inferences can be drawn from thi graph :

1. The peaks corresponding to $_2He^4$, $_6C^{12}$ and $_8O^{16}$ indicate that these nuclei are more stable than those in thei neighbourhood.

2. B.E./A has a low value for both light nuclei (A < 20) and for very heavy nuclei (A > 210). The light nuclei tend t unite (process of fusion) and heavy nuclei tend to split (process of fission) in order to attain greater value of B.E./A and hence greater stability.

Q. 1.18. For greater stability, a nucleus should have greater value of binding energy per nucleon. Why ?

Ans. The B.E. of $_3Li^7$ is greater than that of $_2He^4$ but the value of its B.E./A is lesser. Since $_2He^4$ is found to be more stable than $_3Li^7$, it follows that the stability of a nucleus depends upon the higher value of its B.E./A rather than the higher value of its total B.E.

Q. 1.19. How will you correlate B.E. with the stability of a nucleus ? *(P.S.S.C.E. 1995)*

Ans. Refer to SAQ 1.18.

Q. 1.20. Draw a graph showing the variation of binding energy per nucleus with mass number of different nuclei. Mark the regions, where nuclei are (*i*) prone to fusion (*ii*) prone to fission and (*iii*) most stable. *(C.B.S.E. 1996)*

Ans. For graph between B.E./A and A, refer to Fig. 1.04.

(*i*) In the region A < 20, the B.E./A of the nuclei is quite low except for the nuclei $_2He^4$, $_6C^{12}$ and $_8O^{16}$. In an attempt to have greater value of B.E./A, the nuclei in the region A < 20 unite to form a heavier nucleus and therefore, the nuclei in this region are prone to nuclear fusion.

(*ii*) In the region A > 210, the B.E./A of the nuclei is again quite low. The nuclei in this region have a tendency to split so as to improve the value of their B.E./A. Hence, in region A > 210, the nuclei are prone to nuclear fission.

(*iii*) In the region 40 < A < 120, the nuclei are most stable. It is indicated by the flat shape of the graph. The value of B.E./A in this region is maximum *i.e.* about 8·8 MeV per nucleon.

Q. 1.21. Draw a diagram to show the variation of binding energy per necleon with mass number for different uclei. State with reason why light nuclei usually undergo uclear fusion. (*C.B.S.E. 2001*)

Ans. For binding energy per nucleon versus mass umber diagram, refer to Fig. 1.04.

For cause of light nuclei to undergo fusion, refer to SAQ .20. (*i*).

Q. 1.22. Draw a diagram to show the variation of binding energy per nucleon with mass number for different nuclei. State why heavy nuclei undergo fission. (*C.B.S.E. 2001*)

Ans. For binding energy per nucleon versus mass number diagram, refer to Fig. 1.04.

For cause of heavy nuclei to undergo fission, refer to SAQ 1.20. (*ii*).

abc TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. Is the probability of backward scattering (*i.e.* scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same or much greater than that predicted by Rutherford's model ? (*Text Question*)

Ans. Thomson's model predicts the backward scattering of α-particles much less than that predicted by Rutherford's model. The reason is that in Thomson's model, positive charge is uniformly distributed over whole of the nucleus instead of being concentrated in a small part.

Q. 1.02. The isotope $_8O^{16}$ has 8 protons, 8 neutrons and 8 electrons, while $_4Be^8$ has 4 protons, 4 neutrons and 4 electrons. Yet the ratio of their atomic masses is not exactly 2. Why ? (*Text Question*)

Ans. It is because of the fact that the mass of a nucleus is slightly less than the mass of the constituent nucleons. This decrease in mass is called mass defect. Since, the mass defect in case of $_8O^{16}$ is not exactly twice the mass defect in case of $_4Be^8$, the ratio of the atomic masses is not exactly 2.

Q. 1.03. The atomic mass of an element is the weighted average of the atomic masses of different isotopes of that element. This explains, why atomic masses of many elements show large departures from integer values. However, even if we consider masses of individual isotopes, they are not strictly integer multiples of the mass of a hydrogen atom. How do you account for this fact ? (*Text Question*)

Ans. It is because of following reasons :

1. The mass of a proton is not the same as that of a neutron.
2. The atomic mass also includes mass of electrons and an atom of mass number A has Z electrons (and not A electrons).
3. The mass of nucleus is slightly less than the mass of the constituent nucleons.

Q. 1.04. In heavy nuclei, number of neutrons is more than number of protons. Why ?

Ans. In heavy nuclei, the number of protons is very large. Due to this, the electrostatic force, which is a long range force, also becomes very large. To contain the nucleons inside the nucleus, the only attractive force is the nuclear force. But it is a short range force and can act only between neighbouring nucleons. A nucleus will be stable, if the number of neutrons is more than the number of protons so as to provide sufficient attractive force.

Q. 1.05. Binding energy per nucleon versus mass number curve for nuclei is shown in Fig. 1.05. W, X, Y, Z are four nuclei indicated on the curve. The following processes can possibly take place :

(*a*) Y → 2 Z (*b*) W → X + Z (*c*) W → 2 Y (*d*) X → Y + Z

Which of the above four processes, leads to the release of energy ? (*I.I.T. 1999*)

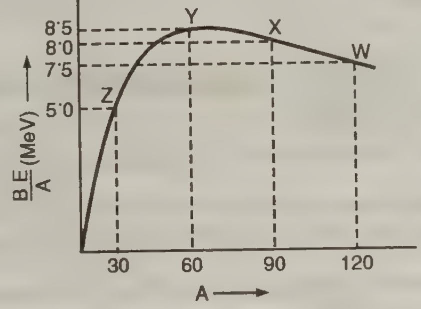


Fig. 1.05

Ans.(*a*) *For process* Y → 2 Z :

Binding energy before decay = 60 × 8·5 = 510 MeV

Binding energy after decay = 2 × (30 × 5·0) = 300 MeV

(*b*) ***For process* W → X + Z :**

Binding energy before decay = 120 × 7·5 = 900 MeV

Binding energy after decay = 90 × 80 + 30 × 5·0 = 870 MeV

(*c*) ***For process* W →2 Y :**

Binding energy before decay = 120 × 7·5 = 900 MeV

Binding energy after decay = 2 × (60 × 8·5) = 1020 MeV

(*d*) ***For process* X → Y + Z :**

Binding energy before decay = 90 × 8·0 = 720 MeV

Binding energy after decay = 60 × 8·5 + 30 × 5·0 = 660 MeV

Only the process (*c*) is accompanied with the increase in the binding energy. The larger the binding energy of the product nuclei formed, the larger would be the energy released. Hence, the process W → 2 Y leads to the release of the energy.

Q. 1.06. M_1 and M_2represent the masses of $_{10}Ne^{20}$ nucleus and $_{20}Ca^{40}$nucleus respectively. State, whether $M_1 = 2\ M_1$ or $M_2 > 2\ M_1$ or $M_2 < 2\ M_1$? (*P.S.S.C.E. 2001 ; I.I.T. 1998*)

Ans. Let m_p be the mass of a proton and m_n, the mass of a neutron. Since some energy is required to bind a nucleus, the mass of a nucleus is always less than the mass of constituent nucleons. Therefore,

$$M_1 < 10\ (m_p + m_n) \text{ and } M_2 < 20\ (m_p + m_n)$$

Since the binding energy per nucleon of $_{20}Ca^{40}$ is more than that of $_{10}Ne^{20}$, the difference $\dfrac{M_2 - 20\ (m_p - m_n)}{40}$ will be more than that of $\dfrac{M_1 - 10\ (m_p + m_n)}{20}$

i.e. $M_2 - 20\,(m_p + m_n) > \frac{40}{20}\,[M_1 - 10\,(m_p + n_n)]$ or $M_2 > 2\,M_1$

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain Rutherford's experiment on the scattering of alpha particles and state the significance of the results. *(C.B.S.E. 1996)*
2. Explain the observations and significance of Rutherford's alpha particle scattering experiment. *(P.S.S.C.E. 1994)*
3. Discuss Rutherford's experiment on scattering of alpha particles. What are its consequences ? *(P.S.S.C.E. 1996)*
4. Define `atomic number' and `mass number'. Distinguish between isotopes and isobars. Give examples.
5. Why is the density of the nucleus more than that of the atom ? Prove that the density of the nuclear matter is same for all nuclei.
6. Explain any three properties of the nucleus. *(P.S.S.C.E. 2001, 1997 S, 1997)*
7. Define atomic mass unit and electron volt. Derive the relation between them. *(P.S.S.C.E. 2001)*
8. Define electron-volt and atomic mass unit. Calculate the energy in joule equivalent to the mass of one proton. *(C.B.S.E. 1996)*
9. What do you mean by mass defect and packing fraction ? How do you account for mass defect ?
10. What is meant by the binding energy of the nucleus ? How is it related to mass defect ?

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What are nuclear forces ? Discuss the important properties. *(H.S.S.C.E. 1999 S)*
2. Define binding energy. How does binding energy per nucleon vary with mass number ? What is its significance ? *(H.S.S.C.E. 2001, 1998 S)*
3. Define binding energy of a nucleus. Draw a curve between mass number and average binding energy. Give the salient features of the curve. *(H.S.S.C.E. 1999)*
4. Define binding energy of a nucleus. Draw a curve between mass number and average binding energy. Disucss the binding energy curve. *(H.S.S.C.E. 2001)*

FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Size of nucleus

1. Calculate the radius of oxygen nucleus. Given that $R_0 = 1{\cdot}1 \times 10^{-15}$ m. **[Ans.** $2{\cdot}77 \times 10^{-15}$ m]
2. The radius of oxygen nucleus ($_8O^{16}$) is $2{\cdot}8 \times 10^{-15}$ m. Calculate the radius of the lead nucleus ($_{82}Pb^{205}$). **[Ans.** $6{\cdot}55 \times 10^{-15}$ m]

Type B. On Nuclear density

3. Compare the radii of two nuclei with mass numbers 1 and 27 respectively. Also compare their nuclear densities. *(P.S.S.C.E. 2001)* $\left[\textbf{Ans.}\ \frac{1}{3}, 1\right]$
4. Calculate the density of hydrogen nucleus in SI units. Given, $R_0 = 1{\cdot}1 \times 10^{-15}$ m and 1 a.m.u. = $1{\cdot}66 \times 10^{-27}$ kg. **[Ans.** $9{\cdot}354 \times 10^{17}$ kg m^{-3}]

Type C. On Mass defect and Binding energy

5. The mass of $_3Li^7$ nucleus is 0·042 a.m.u. less than the sum of masses of its nucleons. Find the binding energy per nucleon. **[Ans.** 5·586 MeV]
6. Calculate the binding energy an alpha-particle. Given : mass of proton = 1·0073 a.m.u., mass of neutron = 1·0087 a.m.u., mass of α-particle = 4·0015 a.m.u. *(I.S.C.E. 1994)* **[Ans.** 28·4 MeV]
7. Calculate the binding energy pernucleon of $_{17}Cl^{35}$ nucleus. Given that mass of $_{17}Cl^{35}$ nucleus = 34·98000 a.m.u., mass of proton = 1·007825 a.m.u., mass of neutron = 1·008665 a.m.u. and 1 a.m.u. = 931 MeV. *(C.B.S.E. 2002)* **[Ans.** 8·2 MeV]
8. Calculate the binding energy per nucleon of $_{20}Ca^{40}$ nucleus. Given mass of $_{20}Ca^{40}$ nucleus = 39·962589 a.m.u., mass of proton = 1·007825 a.m.u., mass of neutron = 1·008665 a.m.u. and 1 a.m.u. = 931 MeV. *(C.B.S.E. 2002)* **[Ans.** 8·55 MeV]
9. Calculate binding energy per nucleon of $_{26}F^{56}$. Given : $m_N(_{26}F^{56})$ = 55·934939 a.m.u. ; m(neutron) = 1·008665 a.m.u. ; m(proton) = 1·007825 a.m.u. *(C.B.S.E. 2002, 1995)* **[Ans.** 8·79 MeV]
10. Calculate the binding energy per nucleon of $_{83}Bi^{209}$. Given, $m_N(_{83}Bi^{209})$ = 208·980388 a.m.u. ; m(neutron) = 1·008665 a.m.u. and m(proton) = 1·007825 a.m.u. *(C.B.S.E. 1995)* **[Ans.** 7·844 MeV]
11. Calculate the (*i*) mass defect, (*ii*) binding energy and (*iii*) the binding energy per nucleon for a $_6C^{12}$ nucleus. Nuclear mass of $_6C^{12}$ = 12· 000000 a.m.u., mass of hydrogen nucleus = 1·007825 a.m.u. and mass of neutron = 1·008665 a.m.u.
[Ans. (*i*) 0· 09894 a.m.u. ; (*ii*) 92·1 MeV ; (*iii*) 7·675 MeV]
12. Calculate the binding energy of $_2He^4$, $_{17}Cl^{35}$ and $_{26}Fe^{56}$, if the masses of their respective nuclei are 4·002603 a.m.u. ; 34·9800 a.m.u. and 55·934939 a.m.u. respectively. Also, calculate binding energy per nucleon in each case. *(Text Problem)* **[Ans.** 7·07 MeV, 8·22 MeV ; 8·79 MeV]
13. The masses of $_{26}Fe^{56}$ and $_{83}Bi^{209}$ nuclei are 55·934939 and 208·980388 u respectively. Mass of a proton = 1·007825 u and mass of a neutron = 1·008665 u. Calculate the binding energies of these nuclei and show which nucleus is more stable. *(H.P.S.S.C.E. 1996)* **[Ans.** 492·0 MeV, 1639·38 MeV ; $_{26}Fe^{56}$ is more stable]

MISCELLANEOUS PROBLEMS

14. What energyis needed to split an alpha particle to pieces ? Given mass of α-particle is 4·0039 a.m.u., mass of proton = 1·007825 a.m.u., mass of neutron = 1·008665 a.m.u. and 1 a.m.u. = 931 MeV. *(C.B.S.E. 1990)* **[Ans.** 27·07 MeV]
15. The neutron separation energy is defined to be the energy required to remove a neutron from a nucleus. Obtain the

neutron separation energies of the nuclei $_{20}\text{Ca}^{41}$ and $_{13}\text{Al}^{27}$ from the following data :
m_n = 1·008665 a.m.u. ; $m(_{20}\text{Ca}^{40})$ = 39·962591 a.m.u. ; $m(_{20}\text{Ca}^{41})$ = 40·962278 a.m.u. ; $m(_{13}\text{Al}^{26})$ = 25·986895 a.m.u. and $m(_{13}\text{Al}^{27})$ = 26·981541 a.m.u. *(Text Problem)*

[Ans. 836 MeV , 13·06 MeV]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $R = R_0 A^{1/3} = 1\cdot1 \times 10^{-15} \times (16)^{1/3} = 1\cdot1 \times 10^{-15} \times 2\cdot52$
$= \mathbf{2\cdot77 \times 10^{-15}\ m}$

2. *For oxygen :*
$R = R_0 A^{1/3}$
or $R_0 = \dfrac{R}{A^{1/3}} = \dfrac{2\cdot8\times10^{-15}}{(16)^{1/3}} = \dfrac{2\cdot8\times10^{-15}}{2\cdot52}$
$= 1\cdot11\times10^{-15}\text{m}$
For lead :
$R = R_0 A^{1/3} = 1\cdot11 \times 10^{-15} \times (205)^{1/3}$
$= 1\cdot11 \times 10^{-15} \times 5\cdot90 = \mathbf{6\cdot55 \times 10^{-15}\ m}$

3. $\dfrac{R_1}{R_2} = \left(\dfrac{A_1}{A_2}\right)^{1/3} = \left(\dfrac{1}{27}\right)^{1/3} = \mathbf{\dfrac{1}{3}}$
Nuclear density is independent of mass number and is same for all nuclei. Therefore,
$\dfrac{\rho_1}{\rho_2} = \mathbf{1}$

4. $\rho = \dfrac{A\,(\text{in kg})}{4/3\times\pi R^3} = \dfrac{A\,(\text{in kg})}{4/3\times\pi R_0^3\,A}$
$= \dfrac{A\times1\cdot66\times10^{-27}}{4/3\times\pi\times(1\cdot1\times10^{-15})^3\times A}$
$= \mathbf{9\cdot354 \times 10^{17}\ kg\ m^{-3}}$

5. $\dfrac{\text{B.E.}}{A} = \dfrac{\Delta m\times931}{A} = \dfrac{0\cdot042\times931}{7} = \mathbf{5\cdot586\ MeV}$

6. $\text{B.E.} = \left[(2\,m_n + 2\,m_p) - m(_2\text{He}^4)\right]\times931$
$= [2 \times 1\cdot0087 + 2 \times 1\cdot0073 - 4\cdot0015] \times 931$
$= \mathbf{28\cdot4\ MeV}$

7. Proceeding as in problem no. 6, it can be obtained that
B.E. = 286·89 MeV
$\therefore\ \dfrac{\text{B.E.}}{A} = \dfrac{286\cdot89}{35} = \mathbf{8\cdot2\ MeV}$

8. Proceed as in problem no. 7.

9. Proceed as in problem no. 7.

10. Proceed as in problem no. 7.

11. $\Delta m = (6\,m_n + 6\,m_p) - m(_6\text{C}^{12})$
$= (6 \times 1\cdot008665 + 6 \times 1\cdot007825) - 12\cdot000000$
$= \mathbf{0\cdot09894\ a.m.u.}$
$\text{B.E.} = \Delta m \times 931 = 0\cdot09894 \times 931 = \mathbf{92\cdot1\ MeV}$
$\therefore\ \dfrac{\text{B.E.}}{A} = \dfrac{92\cdot1}{12} = \mathbf{7\cdot675\ MeV}$

12. Take m_n = 1·008665 a.m.u., m_p = 1·007825 a.m.u. and proceed as in problem no. 7.

13. By proceeding as in problem no. 7, find B.E. of $_{26}\text{Fe}^{56}$ and $_{83}\text{Bi}^{209}$ and then their $\dfrac{\text{B.E.}}{A}$. As $\dfrac{\text{B.E.}}{A}$ of $_{26}\text{Fe}^{56}$ is greater than that of $_{83}\text{Bi}^{209}$, $_{26}\text{Fe}^{56}$ is more stable.

14. Energy needed to split an α-particle is equal to its B.E.

15. ***For $_{20}$Ca^{41} nucleus :***
Neutron separation energy
$= \left[\{m(_{20}\text{Ca}^{40}) + m_n\} - m(_{20}\text{Ca}^{41})\right]\times931$
$= \left[(39\cdot962591 + 1\cdot008665) - 40\cdot962278\right]\times931$
$= \mathbf{8\cdot36\ MeV}$
Similarly, find neutron separation energy of $_{13}\text{Al}^{27}$ nucleus.

CHAPTER
2

Radioactivity

2.01. NATURAL RADIOACTIVITY

Natural radioactivity was discovered by Henri Becquerel in 1896. He found that a photographic plate gets blackened, when placed near double sulphates of potassium and uranium. Further observations in this respect led Becquerel to the conclusion that uranium emitted special kind of rays and such rays were called *Becquerel rays*. Pierre and Marrie Curie found that the radiation from pitchblende was four times stronger than uranium. This led to an intensive search for the source of stronger radiation. Finally, in 1898 they succeeded in discovering two new substances which they named polonium ($_{84}Po^{210}$) and radium ($_{88}Ra^{226}$).

The spontaneous transformation of an element into another with the emission of some particle (or particles) or electromagnetic radiation is called ***natural radioactivity***.

The substances capable of emitting radiations are called ***radioactive substances***. As a rule, radioactivity is displayed by the heavy nuclei occurring at the end of the periodic table beyond lead.

2.02. NUCLEAR RADIATION

Two years after the discovery of radioactivity in 1828, Earnest Rutherford engaged himself in measuring the intensity of the radiation emitted by a radioactive substance. The experimental arrangement made was as follows :

A radioactive substance was placed at a fixed distance from the detector. Foils of increasing thickness were placed between the radioactive substance and the detector, starting with very thin foils of paper and then aluminium foils. A graph between thickness of the foil and the intensity of radiation was plotted. The graph obtained was as shown in Fig. 2.01. The resulting curve shows three distinct portions :

(*i*) The portion AB of the curve represents the quick absorption of one component of the radiation which is less penetrating. This component was called *α-rays*.

(*ii*) After a particular thickness of foil, the α-rays get absorbed and along BC, the curve does not fall off with distance quickly. It, therefore, represents the absorption of a penetrating component of the radiation, which was called ***β- rays***.

(*iii*) A similar change takes place along CD showing the absorption of the third component of the radiation. Since the fall of curve with distance is very slow in this part of the graph, it corresponds to a highly penetrating component of the radiation. This component was given the name *γ-rays*.

In 1899, Rutherford analysed the radiation emitted by radioactive substances by studying the action of electric and magnetic fields on them. A small quantity of a radioactive substance is placed in a deep cavity made in a thick block of lead. The radiation striking the walls of the cavity are absorbed, while the remaining portion of the radiation is rendered into a narrow and a parallel beam. The lead cavity containing the radioactive substance is placed in an evacuated enclosure. A strong electric field is applied to the radiation beam by placing two charged parallel plates on either side of the beam [Fig. 2.02]. It is found that a part of the beam bends towards the negative plate through a small amount, thereby showing that they consist of heavy positively charged particles, called α-particles. Another part of the beam bends towards positive plate through a large amount and hence must be consisting of *light* and *negatively charged* particles, called *β-particles*. However, the remaining part of the beam goes straight without being affected by the electric field. They were given the name *gamma-rays* (γ-rays) and were found to be electromagnetic in nature.

Quotable Quote

"Whatever nature has in store for mankind, unpleasant as it may be, men must accept, for ignorance is never better than knowledge." – **Enrico Fermi**

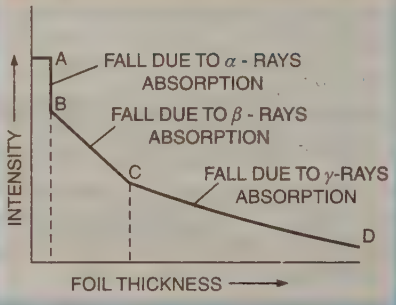


Fig. 2.01

Watch out !

The word *ray* can be used to describe radiated particles as particles as well as photons of electromagnetic radiation !

Key point ✓

Geiger counters are best for detecting α and β-rays. They are less efficient for detecting γ-rays due to the high penetrating and low ionizing powers of γ-rays.

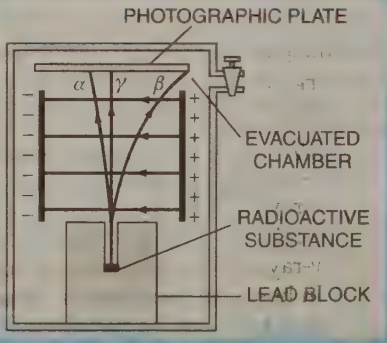


Fig. 2.02

The nature of the three types of radiation emitted by a radioactive substance can also be determined by applying magnetic field perpendicular to their path. Fig. 2.03 shows the deflection in the path of the radiation, when a strong magnetic field is applied perpendicular to the plane of the paper and in the inward direction. The application of the Fleming's left hand rule tells that the component which is deflected towards left consists of positively charged particles while the one deflected towards the right consists of negatively charged particles, recognised as α-rays and β-rays respectively. The third component *i.e.* γ-rays, however, remains undeflected showing that they do not have any charge.

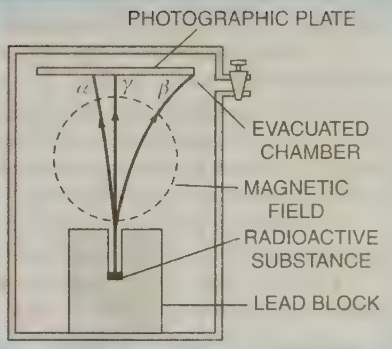


Fig. 2.03

Properties of α-rays. 1. α-particles have been identified as helium nuclei (atoms of helium that have lost two orbital electrons). Thus α-particles carry double the positive charge of proton and are four times as heavy.

2. The velocity of α-particles ranges from $1{\cdot}4 \times 10^7$ m s^{-1} to $2{\cdot}05 \times 10^7$ m s^{-1} depending upon the radioactive substance emitting them. In other words, energy of α-particles emitted from radioactive substance varies from 4·19 to 6·78 MeV.

3. Although α-particles possess large amount of energy, they are the least penetrating of the three types of rays. This is because they are massive particles and hence are easily stopped by an aluminium sheet only 0·02 mm thick or even by an ordinary sheet of paper.

Key point

α-particles are helium nuclei.

4. They have unusually great ionising power. Each α-particle produces thousands of ions before being absorbed.

5. At normal pressure in air, the range of α-particles varies from 3 to 8 cm depending on the initial kinetic energy of the particles.

6. α-particles produce fluroscence in substances like zinc sulphide and barium platinocyanide.

7. α-particles are deflected by electric and magnetic fields.

8. α-particles affect a photographic plate.

9. α-particles, while passing through metal foils, get scattered.

10. They can produce artificial radioactivity in certain elements and can produce nuclear reactions.

Properties of β-rays. 1. β-particles have been found to be streams of electrons.

2. The velocity of β-particles ranges from 33% to 99·8% of the velocity of light. These are the swiftest material particles known in physics. Although the velocities of these particles are enormous, their average energy is only 2 to 3 MeV, because of their extremely small mass.

Key point ✓

Electrons of nuclear origin are called β-particles.

3. β-particles can easily pass through a few millimetre thick aluminium sheets.

4. They can produce ionization in gases, but their ionizing power is quite small as compared to that of α-particles.

5. The range of β-particles in air is several metres.

Self-test Question ?

Explain why β-particles have a higher charge to mass ratio than α-particles.

6. They can also produce fluorescence in materials like zinc sulphide and barium platinocyanide.

7. β-particles are deflected by electric and magnetic fields.

8. They can affect a photographic plate.

9. They can produce artificial radioactivity.

Properties of γ-rays. 1. γ-rays are electromagnetic waves and have velocity equal to that of light.

2. γ-rays are highly penetrating. They can penetrate through several centimetres thick iron and lead blocks.

Key point ✓

γ-rays are high energy photons of electromagnetic waves having extremely small wavelengths.

3. They have got small ionising power.

4. They can produce fluorescence in a substance like willemite.

5. They can affect a photographic plate.

6. γ-rays are not deflected by electric and magnetic fields.

7. γ-rays knock out electrons from the surface on which they fall.

8. γ-rays produce heating effect in the surface exposed to them.

9. γ-rays can produce nuclear reactions.

10. In hospitals, hard γ-rays are used in radio-therapy.

2.03. LAWS OF RADIOACTIVE DECAY

Rutherford and Soddy studied the phenomenon of radioactivity in details and formulated the following laws, known as the laws of radioactive decay :

1. *Radioactivity is a spontaneous phenomenon and one cannot predict, when a particular atom* in a given radioactive sample will undergo disintegration.*

2. *When a radioactive atom disintegrates, either an α-particle (nucleus of helium) or a β-particle (electron) is emitted.*

The new atom so formed (called daughter atom) may emit a γ-ray photon, in case the nucleus is left in excited state on emitting the α or β-particle. Further, both α and β-particles are never emitted simultaneously. It may also be pointed out that a radioactive atom can never emit more than one α-particle or β-particle at a time.

Watch out !

All radioactive nculei have the same chance or probability of disintegration. However, a radioactive nucleus may undergo disintegration next moment, while some other may have to wait for billions of years to decay !

3. *The emission of an α-particle by a radioactive atom results in a daughter atom, whose atomic number is 2 units less and mass number is 4 units less than that of the parent atom.*

Due to α-decay, the transformation of the radioactive parent atom (${}_{Z}X^{A}$) into the daughter atom (${}_{Z-2}Y^{A-4}$) may be represented as below :

$$ {}_{Z}X^{A} \xrightarrow{\alpha-\text{decay}} {}_{Z-2}Y^{A-4} $$

4. *The emission of a β-particle by a radioactive atom results in a daughter atom, whose atomic number is 1 unit more but mass number is same as that of the parent atom.*

Due to β-decay, the transformation of the parent atom (${}_{Z}X^{A}$) into the daughter atom (${}_{Z+1}Y^{A}$) may be represented as below :

$$ {}_{Z}X^{A} \xrightarrow{\beta-\text{decay}} {}_{Z+1}Y^{A} $$

5. *The number of atoms disintegrating per second of a radioactive sample at any time is directly proportional to the number of atoms present at that time.* The rate of disintegration of the sample cannot be altered by changing the external factors, such as pressure, temperature, etc. It is known as ***radioactive decay law***.

Consider that a radioactive sample contains N_0 atoms initially ($t = 0$). As the time elapses, the atoms of the sample decrease due to disintegration. Suppose that after time t, the number of the atoms reduce to N and after time $t + dt$, the number of atoms further decrease to $N - dN$. Obviously, in the time interval between t and $t + dt$ *i.e.* equal to dt, the number of atoms decrease by $N - (N - dN)$ *i.e.* dN.

Therefore, rate of disintegration of atoms at time $t^{\dagger} = \dfrac{dN}{dt}$

According to radioactive decay law, the rate of disintegration at any time t is directly proportional to the number of atoms present at time t *i.e.*

$$ \frac{dN}{dt} \propto N $$

or $$ \frac{dN}{dt} = -\lambda N, \qquad \text{...(2.01)} $$

where the constant of proportionality λ is called ***decay constant*** of the radioactive sample. It is also known as ***disintegration constant*** or ***transformation constant***. Its value depends upon the nature of the radioactive sample. Further, the negative sign indicates that the number of the atoms of the sample decreases with the passage of time.

From equation (2.01), we have

$$ \frac{dN}{N} = -\lambda\, dt $$

Integrating, we have

$$ \int \frac{dN}{N} = -\lambda \int dt $$

or $$ \log_e N = -\lambda t + k, \qquad \text{...(2.02)} $$

where k is constant of integration and it can be evaluated from the following boundary condition :

When $t = 0, N = N_0$

On setting $t = 0$ and $N = N_0$, the equation (2.02) becomes

$$ \log_e N_0 = -\lambda \times 0 + k $$

or $$ k = \log_e N_0 $$

Substituting for k in equation (2.02), we get

$$ \log_e N = -\lambda t + \log_e N_0 $$

---

* It is better to say that the nucleus of an atom undergoes disintegration.

[†] In fact, it is the rate of disintegration of the atoms in the time interval between t and $t + dt$.

or $$\log_e \frac{N}{N_0} = -\lambda t \quad \text{or} \quad \frac{N}{N_0} = e^{-\lambda t}$$

or $$N = N_0 e^{-\lambda t} \qquad ...(2.03)$$

The equation (2.03) gives the number of atoms left after time t.

It follows that the number of atoms of a radioactive sample decreases xponentially with the passage of time. If we plot a graph between N (the number f atoms left behind at time t) and the time t, the graph will be as shown in Fig. 2.04. ollowing inferences can be drawn from the graph :

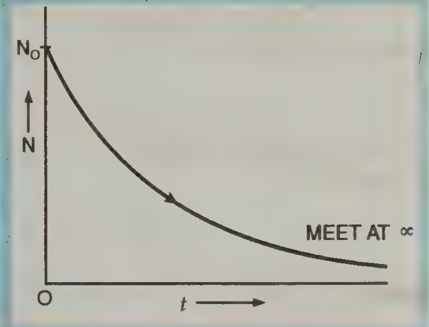


Fig. 2.04

1. Since the decay of radioactive sample obeys exponential law, the number f atoms decreases rapidly in beginning and the rate of decay becomes slower and lower, as the number of atoms decrease with passage of time.

2. The graph meets the time axis at infinity. Therefore, all the radioactive amples (irrespective of their nature) will always take infinite time to disintegrate ompletely. In this respect, all the radioactive samples seem to be identical. However, t is not true. Some radioactive substances are found to be more active than the thers.

3. The slope of the graph *i.e.* rate of disintegration of a radioactive sample is etermined by the value of its decay constant. A radioactive sample having the larger alue of decay constant λ will decay rapidly and in that case the graph between N nd t falls off rapidly with time.

2.04. RADIOACTIVE DECAY CONSTANT

The decay constant of a radioactive sample is a characteristic of the radioactive ubstance. The radioactive substances having large value of the decay constant, ecay rapidly in comparison to those having a small value of decay constant.

According to radioactive decay law,

$$\frac{dN}{dt} = -\lambda N \quad \text{or} \quad \lambda = \frac{-dN/dt}{N}$$

Hence, *radioactive decay constant of a substance (radioactive) may be defined as the atio of its instantaneous rate of disintegration to the number of atoms present at that time.*

Again, $$N = N_0 e^{-\lambda t}$$

If $$t = \frac{1}{\lambda},$$

then $$N = N_0 e^{-\lambda \frac{1}{\lambda}} = \frac{1}{e} N_0 = \frac{N_0}{2 \cdot 718} = 0 \cdot 368\, N_0$$

Hence, *radioactive decay constant of a substance may also be defined as the reciprocal f the time, after which the number of atoms of a radioactive substance decreases to 0·368 or 36·8%) of their number present initially.*

2.05. HALF-LIFE

The half-life of a radioactive substance is defined as the time during which the nuclei f half of the atoms of the radioactive substance will disintegrate. It is denoted by T.

Consider that a radioactive sample contains N_0 atoms at time $t = 0$. Then, the umber of atoms left behind after time t is given by

$$N = N_0 e^{-\lambda t} \qquad ...(2.04)$$

From the definition of half-life, it follows that

when $t = T$ (half-life of the sample), $N = \frac{N_0}{2}$

Setting the above condition in equation (2.04), we have

$$\frac{N_0}{2} = N_0\, e^{-\lambda T}$$

or $$e^{-\lambda T} = \frac{1}{2} \quad \text{or} \quad e^{\lambda T} = 2$$

or $$\lambda T = \log_e 2 = 2{\cdot}303 \log_{10} 2 = 2{\cdot}303 \times 0{\cdot}3010 = 0{\cdot}693$$

or $$T = \frac{0 \cdot 693}{\lambda} \qquad ...(2.05)$$

Thus, half-life of a radioactive substance is inversely proportional to its decay onstant and is a characteristic property of its nucleus. It cannot be altered by any nown method.

Key point

The half-life of a radioactive substance cannot be altered by any known method.

Self-test Question

The half-life of Rn^{220} is 55·5 s. Use equation (2.05) to show that the decay constant for radon is 0·0125 s^{-1}.

Note. There is another useful relation to express the fraction of atoms of a adioactive sample left behind after time t in terms of its half-life :

We know that after time T (half life), the number of atoms of the sample reduce to $\frac{N_0}{2}$ *i.e.* $\frac{N}{N_0}$ becomes equal to $\frac{1}{2}$. After another time equal to T, the value of $\frac{N}{N_0}$ will become $\frac{1}{2} \times \frac{1}{2}$ *i.e.* $\left(\frac{1}{2}\right)^2$. Therefore, it follows that after a time equal to n half lives of the sample,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

In general, after a time t $(= n\,T)$, the fraction of the radioactive atoms left behind is given by

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T} \qquad ...(2.06)$$

2.06. MEAN-LIFE OR AVERAGE-LIFE

The average-life of a radioactive substance is defined as the average time for which the nuclei of the atoms of the radioactive substance exist. It is denoted by T_a.

Obviously, the average-life of a radioactive sample may be calculated by finding the total life of all the atoms of the substance in a sample and then dividing it by the total number of atoms present in that sample.

Consider that initially $(t = 0)$, the number of atoms present in a sample of radioactive substance is N_0. Suppose that at time t, the number of atoms reduces to N and at time $t + dt$, the number of atoms becomes $N - dN$. Obviously, each of the dN atoms disintegration during the time interval between t and $t + dt$ has lived for a time t.

Therefore, the total life of dN atoms $= t\,dN$

and the total life of all the N_0 atoms $= \int_0^{N_0} t\,dN$

Hence, the average-life of the radioactive sample*, $T_a = \dfrac{\int_0^{N_0} t\,dN}{N_0} = \dfrac{N_0/\lambda}{N_0}$

or

$$T_a = \frac{1}{\lambda} \qquad ...(2.07)$$

Thus, the average-life of a radioactive substance is equal to the reciprocal of its decay constant. From the equations (2.05) and (2.07), we have

$$T = 0{\cdot}693\ T_a \qquad ...(2.08)$$

i.e. half-life of a radioactive substance is equal to 0·693 times its average-life.

2.07. ACTIVITY OF RADIOACTIVE SUBSTANCE

The activity of a radioactive substance may be defined as the rate at which the nuclei of its atoms in the sample disintegrate.

If a radioactive sample contains N atoms at any time t, then its activity at time t is defined as

$$R = -\frac{dN}{dt} \qquad ...(2.09)$$

The negative sign shows that with the passage of time, the activity of the radioactive substance decreases.

Key point

The activity of a radioactive substanc decreases as the number of undecaye nuclei decrease with time.

Since according to the radioactive decay law,

$$\frac{dN}{dt} = -\lambda N,$$

the equation (2.09) may be expressed as

$$R = \lambda N \qquad ...(2.10)$$

Since $N = N_0 e^{-\lambda t}$, we have

$$R = \lambda N_0 e^{-\lambda t}$$

or

$$R = R_0 e^{-\lambda t} \qquad ...(2.11)$$

---

* It can be proved that $\int_0^{N_0} t\,dN = \frac{N_0}{\lambda}$

Here, $\lambda N_0 = R_0$ is activity of the radioactive sample at time $t = 0$.

The equation (2.09) tells that the activity of a radioactive sample may be xpressed as *disintegrations per second*. The practical unit of activity of a radioactive ample is **curie (ci).**

The activity of a radioactive sample is called one curie, if it undergoes $3{\cdot}7 \times 10^{10}$ isintegrations per second. Thus,

$$\textbf{1 curie (ci)} = 3{\cdot}7 \times 10^{10} \text{ disintegrations s}^{-1}$$

Key point ✓

In terms of activity, the equation (2.06) can be expressed as

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

The **millicurie (mci)**, one thousandth of a curie and **microcurie (μci)**, equal to ne millionth of a curie are smaller units of radioactivity. Thus,

$$\textbf{1 millicurie (mci)} = 3{\cdot}7 \times 10^{7} \text{ disintegrations s}^{-1}$$

$$\textbf{1 microcurie } (\mu\textbf{ci}) = 3{\cdot}7 \times 10^{4} \text{ disintegrations s}^{-1}$$

There is also another unit of radioactivity, called **rutherford (rd).**

The activity of a radioactive sample is called one rutherford, if it undergoes 10^6 isintegrations per second. Thus,

$$\textbf{1 rutherford (rd)} = 10^{6} \text{ disintegrations s}^{-1}$$

The smaller units in this case are **millirutherford (mrd)** and **microrutherford urd).** Thus,

$$\textbf{1 millirutherford (mrd)} = 10^{3} \text{ disintegrations s}^{-1}$$

$$\textbf{1 microrutherford } (\mu\textbf{rd}) = 1 \text{ disintegration s}^{-1}$$

It follows that

$$\textbf{1 curie (ci)} = 3{\cdot}7 \times 10^{4} \text{ rutherford}$$

Self-test Question ?

Show that

1 curie = $3{\cdot}7 \times 10^4$ rutherford

2.08. ALPHA DECAY

Nuclei that contain more than about 210 nucleons are so large that the short ange nuclear force holding them together becomes insufficient to counterbalance he long range electrostatic repulsive force between the protons. Such nuclei may chieve greater stability by emitting alpha particles. An α-particle has high value of binding energy ($\approx$ 28 MeV). Therefore, on emission of α-particle, the binding nergy per nucleon increases and the residual nucleus tends towards greater stability.

In general, the α-decay can be represented as below :

$${}_ZX^A \longrightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + E_\alpha, \qquad \text{..(2.12)}$$

where E is the energy released in the process of α-decay.

Key point ✓

On the emission of an α-particle, binding energy per nucleon of the residual nucleus increases. It is because, though an α-particle consists small number of nucleons *i.e.* 4, its binding eneergy is quie large. Hence, by α-decay, a radioactive nucleus tends to become more stable.

If m_X, m_Y and m_{He} represent the masses of parent nucleus X, daughter nucleus Y and the α-particle respectively, then the energy released is given by

$$E = (m_X - m_Y - m_{He})\, c^2 \qquad \text{...(2.13)}$$

The energy carried by the α-particle in the process of α-decay can be calculated by applying principles of conservation of energy and momentum. Then, by taking he masses of parent nucleus X, daughter nucleus Y and the α-particle as A, A – 4 and 4 a.m.u. respectively, it can be proved that the kinetic energy of the α-particle s given by

$$E_\alpha = \frac{A-4}{A} E \qquad \text{...(2.14)}$$

In the decay of ${}_{86}Rn^{222}$, it is found that Q = 5·587 MeV

Therefore, kinetic energy of the α-particle,

$$E_\alpha = \frac{222-4}{222} \times 5{\cdot}587 = 5{\cdot}486 \text{ MeV}$$

Key point ✓

The energy of α-particles emitted by a radioactive substabce (α-emitter) has a unique value.

The value of kinetic energy of the α-particle is really found to be of the order of 4-9 MeV for various α-emitters. But the calculations tell that the nucleus of the α-emitter behaves as a potential barrier, which can be surmounted by an α-particle having energy of the order of 25 MeV. Classically, it is impossible to explain as to how an α-particle having insufficient energy will come out of the nucleus.

In 1928, Gamow, Congdon and Gurney explained the emission of α-particles in terms of the penetration of the nuclear potential barrier on the basis of quantum theory as explained below :

An α-particle does not exist in the nucleus as such. But just at the time of emission, two protons and two neutrons combine to form an α-particle. The α-particle is constantly in motion inside the nucleus with a speed* of about 10^7 m s^{-1}. Since size of the nucleus is about 10^{-14} m, the α-particle takes about 10^{-21} s to move across

Key point ✓

The nucleus of an α-emitter poses as a potential barrier of height several times the kinetic energy of an α-particle. On classical picture, an α-particle cannot come out of the nucleus. However, α-decay does occur through the process of tunneling of the nucleus, which is based on quantum mechanical considerations.

---

* Corresponding to the kinetic energy of α-particles in the decay of Rn^{222} *i.e.* 5·486 MeV, it can be shown that the speed of the α-particles is about $1{\cdot}62 \times 10^7$ m s^{-1}.

the nucleus. In other words, α-particle presents itself again and again before the potential barrier about 10^{21} times in a second. Quantum mechanically, even with insufficient kinetic energy, the α-particle has a very small but definite probability p of its crossing the potential barrier. But since the frequency of attempts ν (= 10^{21} s^{-1}) is very large, the probability of its escaping the potential barrier,

$$P = p \times \nu$$

becomes considerable and it penetrates the potential barrier. It is called ***tunneling of the nucleus.***

This is how the quantum theory explains the emission of α-particle from the nucleus, which otherwise has got insufficient energy to surmount the much higher nuclear potential barrier.

Watch out !

Classical physics is the old stuff. It believes in determinism : perfect laws of nature from which everything could ultimately be predicted. Quantum physics accepts uncertainity.

2.09. BETA DECAY

A nucleus is usually unstable due to its low value of binding energy per nucleon. Another reason of the unstability of the nucleus is either a higher or a lower neutron to proton ratio than that for the stable nuclei.

The β-decay occurs in case of nuclei, which possess a higher value of neutron to proton ratio than that for the stable nuclei. The β-particle is an electron of nuclear origin. Since an electron does not exist inside the nucleus, it is assumed to be formed by the conversion of a neutron into a proton and electron. Since neutron number decreases by one and the proton number increases by one, β-decay helps to decrease the neutron to proton ratio. In the beginning, the creation of electron inside the nucleus was supposed to be according to the following equation :

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0 \qquad ...(2.15)$$

Key point

The radioactive nuclei, which have neutron to proton ratio greater than that for stable nuclei undergo β-decay. By β-decay, such nuclei tend to decrease their neutron to proton ratio.

Therefore, the β-decay may be taking place according to the equation

$$_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + E_\beta, \qquad ...(2.16)$$

where E_β is the energy with which β-particle is emitted. The calculations show that β-decay is energetically possible (E_β =+ve), only if mass of the parent atom (whose nucleus is $_ZX^A$) is greater than the mass of the daughter atom (whose nucleus is $_{Z+1}Y^A$). For example, $_{83}Bi^{210}$ decays to $_{84}Po^{210}$ by emitting β-particle of energy 1·17 MeV *i.e.*

$$_{83}Bi^{210} \longrightarrow {}_{84}Po^{210} + {}_{-1}e^0 + 1{\cdot}17 \text{ MeV}$$

From the above equation, it follows that in the β-decay of $_{83}Bi^{210}$, the energy of the electrons should always be equal to 1·17 MeV. This is because according to the above decay scheme, electron is the only particle, which leaves the nucleus and it should carry whole of the energy. However, the energy of electrons in β-decay of $_{83}Bi^{210}$ is found to vary continuously between zero to maximum value 1·17 MeV, called *end point energy* as shown in Fig. 2.05.

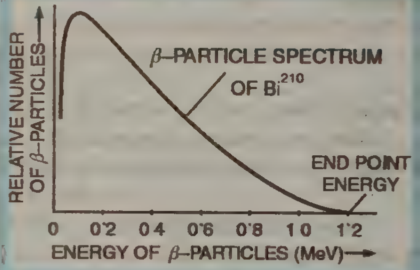


Fig. 2.05

Further, if electron is created inside the nucleus in accordance with the equation (2.15), then spin conservation principle is also violated as it is known that each of the three particles, *i.e.* neutron, proton and electron, possesses spin equal to $\frac{1}{2}$. Thus, if β-decay takes place according to the equation (2.16), then

(*i*) experimentally observed continuous energy spectrum of β-decay cannot be explained.

(*ii*) spin conservation principle will be violated.

Key point

The particle neutrino was postulated in order to explain the continuous energy spectrum of β-decay and to keep the spin conservation principle in tact.

These problems were overcome by Pauli in 1930. According to him, when a neutron disintegrates into a proton and electron, an uncharged particle of zero rest mass (practically) and spin $\frac{1}{2}$, called ***antineutrino*** ($\bar{\nu}$)) is also produced. The decay of neutron takes place as given below :

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu} \qquad ...(2.17)$$

The existence of neutrino was experimentally observed in 1956 by Reines and Cohean.

Also, the transformation describing the β-decay will be as below :

$$_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + E_\beta \qquad ...(2.18)$$

Thus, in β-decay, electron and antineutrino come out of the nucleus and share the end point energy of β-emission in all proportions with each other, so that when antineutrino does not get any energy, all the energy is carried by the electron and in case antineutrino grabs whole of the energy, then electron is left without any

Key point

The energy of β-particles emitted by a radioactive substance (β-emitter) can have any value between zero and end-point energy.

energy. Further, the decay scheme suggested by equation (2.17), keeps the spin conservation principle intact.

2.10. GAMMA DECAY

We know that in an atom electrons revolve in different shells around the nucleus associated with definite amount of energy. These are called ***atomic energy levels*** and transitions between these energy levels account for the emission of visible and X-ray spectra. It has been investigated that similar energy states called ***nuclear energy levels*** exist inside the atomic nuclei. The transitions between two nuclear energy levels also result in the emission of energy. The frequency of the energy emitted in nuclear transitions falls in γ-ray region.

Key point

X-rays are high-energy photons, like γ-rays. The only difference between the two is the way in which they are produced. X-rays are emitted by the excited atoms, whereas γ-rays are emitted by the excited nuclei.

During radioactive decay, the emission of α or β-particles usually leaves the residual nucleus in an excited state, which is found to go to some other excited state or ground state by the emission of γ-rays, whose energy is equal to the difference of energy between the two nuclear energy levels. For example, it is found that in the decay of ${}_{27}\mathrm{Co}^{60}$, the emission of β-particle is immediately followed by emission of γ-ray photons of energy 1·17 MeV and 1· 33 MeV [Fig. 2·06]. The usual β-decay of ${}_{27}\mathrm{Co}^{60}$ into ${}_{28}\mathrm{Ni}^{60}$ may be written as

$${}_{27}\mathrm{Co}^{66} \longrightarrow {}_{28}\mathrm{Ni}^{60*} + {}_{-1}e^{0}$$

The asterisk mark (*) on the symbol for nickel indicates that it is left in an excited state having energy greater than that of the ground state. The excited nucleus of nickel comes to ground state by first emitting γ-ray of energy 1·17 MeV and then of energy 1·33 MeV. Thus, transition from excited state to ground state may be represented by

$${}_{28}\mathrm{Ni}^{60*} \longrightarrow {}_{28}\mathrm{Ni}^{60} + \mathrm{E}_{\gamma}$$

By studying the energies of the γ-rays emitted, the energy levels of a nucleus can be obtained.

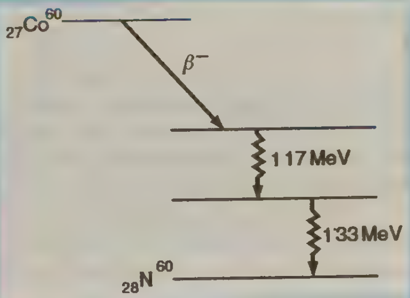


Fig. 2.06

Key point

The energy of γ-rays emitted by a radioactive substance can have unique and discrete values.

2.11. ARTIFICIAL RADIOACTIVITY

Apart from naturally occurring radioactive substance, the radioactivity can also be induced by artificial means through nuclear transmission. The phenomenon of artificial radioactivity was discovered by Joliot and Curie, when they bombarded aluminium with α-particles obtained from polonium. They found that even when the source of α-radiations was removed, the aluminium piece continued giving off some type of radiation. Further investigations revealed that the radiation consisted of positrons. Also, it was found that the rate of emission of the radiation was found to decrease exponentially as is the case with natural radioactivity. Hence the phenomenon was termed as artificial radioactivity. The nuclear transformation in the above described case proceeds as follows :

$${}_{2}\mathrm{He}^{4} + {}_{13}\mathrm{Al}^{27} \longrightarrow {}_{15}\mathrm{P}^{30} + {}_{0}n^{1}$$

The isotope ${}_{15}\mathrm{P}^{30}$ is unstable and radioactive. It decays as follows :

$${}_{15}\mathrm{P}^{30} \longrightarrow {}_{14}\mathrm{Si}^{30} + {}_{1}e^{0} + \gamma$$

The half-life period of radioactive phosphorus is found to be 2·2 minutes. Soon, other scientists started investigations and found that not only the α-particles but other projectiles like proton, deutrons and even neutrons can induce artificial radioactivity.

2.12. RADIO ISOTOPES

The isotopes of an element, capable of emitting radiation just as radioactive elements do, are called radioisotopes.

Usually, the radioisotopes are artificially produced. For example, ${}_{6}\mathrm{C}^{11}$ and ${}_{6}\mathrm{C}^{13}$ are radioisotopes of ${}_{6}\mathrm{C}^{12}$; and ${}_{11}\mathrm{Na}^{22}$ and ${}_{11}\mathrm{Na}^{24}$ are radioisotopes of ${}_{11}\mathrm{Na}^{23}$. The chemical properties of the radioisotopes of an element are same and they differ only in their atomic weights. A few common radioisotopes are *Iodine-131, Phosphorus-33, Carbon-14, Cobalt-60, Sulphur-35 and Sodium-24*. The half lives of radioisotopes are usually small. A large number of radioisotopes are produced by $(n - \gamma)$ reaction. Such reactions are achieved by bombarding stable nuclides with slow neutrons. Following are a few examples :

$${}_{53}\mathrm{I}^{127} + {}_{0}n^{1} \longrightarrow {}_{53}\mathrm{I}^{128*} + \gamma$$

Here, ${}_{53}\mathrm{I}^{128*}$ is radioisotope of iodine.

$${}_{27}\mathrm{Co}^{59} + {}_{0}n^{1} \longrightarrow {}_{27}\mathrm{Co}^{60*} + \gamma$$

$${}_{15}\mathrm{P}^{31} + {}_{0}n^{1} \longrightarrow {}_{15}\mathrm{P}^{32*} + \gamma$$

Radioisotope of carbon *i.e.* $_6C^{14*}$ is continuously produced in the atmosphere due to neutrons in cosmic rays bombarding nitrogen nuclei :

$$_7N^{14} + {_0n^1} \longrightarrow {_6C^{14*}} + {_1H^1}$$

There are many other nuclear reactions which lead to the production of radioisotopes.

Radioisotopes have proved to be of great use in a number of fields and are produced on a large scale these days. Before studying the other uses of radioisotopes, it is necessary to understand their use as *tracers*.

Tracer. The use of radioisotopes as tracers has provided solution to many problems, particularly those connected with the life processes :

During the intake of different elements and compounds, the biological organisms absorb them differently. Further, the exact distribution of the elements and their function in the various parts of the organisms cannot be known clearly. In tracer technique, a radioisotope is made to enter the organism along with the elements and compounds, whose absorption, functioning and distribution to other parts of the organism is to be studied. The radioisotope acts as a *tag* or a *label* for the element or compound under study. By detecting the radiation emitted by the radioactive isotope from the different parts of the body, the details regarding the absorption and function of the compounds by the organism are found out.

Uses of radioisotopes. The radioisotopes have been used successfully for research in fields of biology, physiology, chemistry, physics, agriculture and other sciences in addition to industry. Following are a few broad uses of radioisotopes in different fields :

1. ***In medical science.*** The radioisotopes have been used to study the deficiency of red blood cells in human body (by using *Iron-59*), pumping action of heart (by using *Sodium-24* and *Iodine-131*), functioning of organs like kidney, liver under normal and diseased conditions (by using *Iodine-131*). Radioisotopes have been used to detect brain tumours and relieve the patient from severe heart pain. Further, *Cobalt-60* and *Gold-198* are used for the treatment of the cancer, *Phosphorus-32* for skin diseases and *Iodine-131* for the treatment of thyroid gland.

2. ***In agriculture.*** The radioisotopes are used to study the function of fertilisers for different plants. They have also been used for developing new species of a plant by causing genetic mutations.

3. ***In industry.*** The radioisotopes are used to study wear and tear of piston rings and of gears in engines and then in the study of suitable lubricant for their prevention. They have been used to study the underlying earth layers during oil-exploration operations. They are also used to detect flaws in metal structure and for different treatments of alloys such as quenching, annealing and case hardening. They are used for uniform mixing of lubricants and greases.

4. ***In nuclear research.*** Radioisotopes have been used in the study of nuclear disintegration of elements.

5. ***In food preservation.*** By exposing the vegetables and other food-stuffs to radiations from radioisotopes, their *shelf-life* can be increased.

6. ***In radioactive dating.*** *Carbon-14* has been used for dating the culture history or for estimating the age* of some geological formation.

2.13. RADIOACTIVE DATING

An interesting practical application of radioactivity has been developed in recent years in the shape of estimating the age of earth, old rocks, deep sea sediments and other geological formations. This method was first developed by Rutherford and Jolly in 1913.

Uranium-lead dating is used to know the age of earth, while carbon dating is used to estimate the time that has elapsed after the death of a once living organism. The real secret of dating the age of earth or some geological formation lies in the fact that radioactive isotopes have got very long half lives as compared to that of their decay products, till stable element is formed.

Self-test Question ?

All uranium on earth is older than the earth itself. U^{238} has a half-life of around 4·5 billion years ; U^{235} has a much shorter half-life of $7{\cdot}1 \times 10^8$ years. Is it any surprise that U^{238} is the more abundant series ?

Radioactive carbon dating. Carbon-14 has been used for dating the cultural history or for estimating the age of some geological formation.

The isotope of carbon $_6C^{14}$ has got a half life of 5568 years and is produced in the atmosphere by bombardment of $_7N^{14}$ nuclei by neutrons in cosmic rays. Living plants and animals absorb carbon in themselves from the carbon dioxide they take from atmosphere. Thus, radioactive isotope ${}_6C^{14}$ is found in living plants and animals in the same proportion as it is found in the atmosphere. But when an organism dies, the intake of ${}_6C^{14}$ stops and the ${}_6C^{14}$ already depcsited decays, with a half life of 5568 years.

---

* For details, refer to section 2·13-Radioactive dating.

Suppose that the number of ${}_6C^{14}$ atoms in an organism was N_0, when it met its death at time $t = 0$. Today *i.e.* after a time t after its death, the remains of that once living organism are found to contain the atoms of ${}_6C^{14}$ and ${}_6C^{12}$ equal to say N_{14} and N_{12} respectively. Obviously, the present number of ${}_6C^{12}$ atoms together with the number of ${}_6C^{14}$ atoms must be equal to N_0, the number ${}_6C^{14}$ atoms present at $t = 0$ *i.e.*

$$N_0 = N_{12} + N_{14} \qquad ...(2.19)$$

According to radioactive decay law, we have

$$N_{14} = N_0 e^{-\lambda t}, \qquad ...(2.20)$$

where λ is decay constant of ${}_6C^{14}$ nuclei. From equations (2.19) and (2.20), we have

$$N_{14} = (N_{12} + N_{14})\, e^{-\lambda t}$$

or
$$e^{\lambda t} = \frac{N_{12}}{N_{14}} + 1 \quad \text{or} \quad t = \frac{1}{\lambda}\log_e\left(\frac{N_{12}}{N_{14}} + 1\right)$$

or
$$t = \frac{2\cdot 303}{\lambda}\log_{10}\left(\frac{N_{12}}{N_{14}} + 1\right)$$

Now, value of decay Constant ${}_6C^{14}$,

$$\lambda = \frac{0\cdot 993}{5568} = 1\cdot 245 \times 10^{-4}\ \text{year}^{-1}$$

and by measuring the ratio of ${}_6C^{14}$ and ${}_6C^{12}$ in the remains of the once living organisms, one can calculate how long ago it died. The ratio carbon technique has been of great value in the research and dating of archeological specimens.

Now, value of decay constant for ${}_6C^{14}$,

$$\lambda = \frac{0\cdot 693}{5568} = 1\cdot 245 \times 10^{-4}\ \text{year}^{-1}$$

and by measuring the ratio of ${}_6C^{14}$ and ${}_6C^{12}$ in the remains of the once living organism, one can calculate how long ago it died. The radio carbon technique has been of great value in the research and dating of archeological specimens.

Self-test Question

Carbon-14 dating is only accurate for organic objects between 200 and 10,000 years old. Why ?

SOLVED NUMERICAL PROBLEMS

Type A. On Radioactive decay law and Half life

Problem 2.01. After a certain lapse of time, the fraction of radioactive polonium undecayed is found to be 12·5 % of the initial quantity. What is the duration of this time lapse, if the half life of polonium is 138 days ?

(C.B.S.E. Sample Q. Paper)

Sol. Here, half life, T = 138 days

Therefore,
$$\lambda = \frac{0\cdot 693}{T} = \frac{0\cdot 693}{138} = 5\cdot 022 \times 10^{-3}\ \text{day}^{-1}$$

Also
$$\frac{N}{N_0} = 12\cdot 5\% = \frac{12\cdot 5}{100} = 0\cdot 125$$

Let t be the required time. Then,

$$\frac{N}{N_0} = e^{-\lambda t}$$

or
$$0\cdot 125 = e^{-5\cdot 022 \times 10^{-3} t}$$

or
$$e^{5\cdot 022 \times 10^{-3} t} = \frac{1}{0\cdot 125} = 8$$

or
$$5\cdot 022 \times 10^{-3}\, t = \log_e 8 = 2\cdot 303 \times 0\cdot 9031 = 2\cdot 08$$

or
$$t = \frac{2\cdot 08}{5\cdot 022 \times 10^{-3}} = \mathbf{414\cdot 18\ days}$$

Problem 2.02. The half-life of U^{238} against alpha decay is $1\cdot 42 \times 10^{17}$ s. How many disintegrations per second occur in 1 g of U^{238} ? Given, Avogadro number = $6\cdot 02 \times 10^{23}$ mol^{-1}.

(I.S.C.E. 1993)

Sol. Here, $T = 1\cdot 42 \times 10^{-17}$ s ;

Avogadro number = $6\cdot 02 \times 10^{23}$ mol^{-1}

Number of U^{238} atoms in 1 g,

$$N = \frac{6\cdot 02 \times 10^{23}}{238} = 2\cdot 53 \times 10^{21}$$

Now,
$$T = \frac{0\cdot 693}{\lambda}$$

or
$$\lambda = \frac{0\cdot 693}{T} = \frac{0\cdot 693}{1\cdot 42 \times 10^{17}} = 4\cdot 88 \times 10^{-18}\,\text{s}^{-1}$$

Now,
$$\frac{dN}{dt} = \lambda N = 4\cdot 88 \times 10^{-18} \times 2\cdot 53 \times 10^{21}$$
$$= \mathbf{1\cdot 235 \times 10^{4}\ s^{-1}}$$

Type B. On Activity of a Radioactive Substance

Problem 2.03. In an experiment, the activity of 1·2 milligram (mg) of radioactive potassium chloride (chloride of isotope K-40) was found to be 170 s^{-1}. Taking molar mass of K-40Cl to be 0·075 kg mole^{-1}, find the number of K-40 atoms in the sample and hence find the half-life of K-40. Given, Avogadro number = $6\cdot 0 \times 10^{23}$ mole^{-1}.

(I.S.C.E. 1997)

Sol. Here, molar mass of K-40Cl, M = 0·075 kg mole^{-1}

mass of K-40Cl , $m = 1\cdot 2$ mg $= 1\cdot 2 \times 10^{-6}$ kg

Since 1 mole of a substance contains molecules equal to Avogadro number (= $6\cdot 0 \times 10^{23}$),

the number of atoms in the K-40Cl sample,

$$N = \frac{6\cdot 0 \times 10^{23}}{0\cdot 075} \times 1\cdot 2 \times 10^{-6} = \mathbf{9\cdot 6 \times 10^{18}}$$

Also, activity of the given sample, $\frac{dN}{dt} = 170\ s^{-1}$

If λ is disintegration constant, then

$$\frac{dN}{dt} = \lambda\ N \quad \text{or} \quad 170 = \lambda \times 9{\cdot}6 \times 10^{18}$$

$$\text{or} \qquad \lambda = \frac{170}{9{\cdot}6 \times 10^{18}} = 1{\cdot}77 \times 10^{-17}\,s^{-1}$$

Therefore, half life of the sample,

$$T = \frac{0{\cdot}693}{\lambda} = \frac{0{\cdot}693}{1{\cdot}77 \times 10^{-17}} = 3{\cdot}915 \times 10^{16}\,s$$

$$= \mathbf{1{\cdot}241 \times 10^{9}\,years}$$

Problem 2.04. Biologically useful technetium nuclei (with atomic weight 99) have a half life of 6 hours. A solution containing 10^{-12} gram of this is injected into the bladder of a patient. Find its activity at the beginning and after one hour. *(Text Problem)*

Sol. Here, half life, T = 6 h

$$\text{Therefore,} \qquad \lambda = \frac{0{\cdot}693}{T} = \frac{0{\cdot}693}{6}\ h^{-1} = 0{\cdot}1155\ h^{-1}$$

Now, number of technetium atoms in 10^{-12} g in the beginning,

$$N_0 = \frac{\text{Avogadro number}}{\text{atomic weight}} \times 10^{-12}$$

$$= \frac{6{\cdot}025 \times 10^{23} \times 10^{-12}}{99} = 6{\cdot}086 \times 10^{9}$$

Activity of technetium nuclei in the beginning ($t = 0$),

$$R_0 = \lambda\ N_0 = 0{\cdot}1155 \times 6{\cdot}086 \times 10^{-9}$$

$$= \mathbf{7{\cdot}03 \times 10^{8}\ h^{-1}}$$

Now, the number of atoms left after time t,

$$N = N_0\, e^{-\lambda t}$$

Here, $t = 1$ h

$$\therefore \qquad N = 6{\cdot}086 \times 10^{9}\, e^{-0{\cdot}1155 \times 1}$$

$$= 6{\cdot}086 \times 10^{9} \times 0{\cdot}891 = 5{\cdot}423 \times 10^{9}$$

Therefore, activity of technetium after 1 hour,

$$R = \lambda\ N = 0{\cdot}1155 \times 5{\cdot}423 \times 10^{9}$$

$$= \mathbf{6{\cdot}264 \times 10^{8}\ h^{-1}}$$

Problem 2.05. Calculate the half life period of a radio-active substance, if its activity drops to $\frac{1}{16}$ th of its initial value in 30 years. *(C.B.S.E. 1998)*

Sol. Let R_0 and R be the values of initial (t = 0) and present (t = 30 years) activities of the radio-active sample.

$$\text{Here,} \qquad R = \frac{1}{16}\, R_0\,;\ t = 30 \text{ years}$$

$$\text{Now,} \qquad R = R_0\, e^{-\lambda t}$$

$$\therefore \qquad \frac{1}{16} R_0 = R_0\, e^{-\lambda \times 30} \quad \text{or} \quad e^{30\lambda} = 16$$

$$\text{or} \qquad \lambda = \frac{\log_e 16}{30} = \frac{2{\cdot}3026 \log 16}{30}$$

$$= \frac{2{\cdot}3026 \times 1{\cdot}2041}{30} = 0{\cdot}0924\ \text{year}^{-1}$$

If T is half life period, then

$$T = \frac{0{\cdot}693}{\lambda} = \frac{0{\cdot}693}{0{\cdot}0924} = 7{\cdot}5 \text{ years}$$

Type C. On α, β and γ-decays

Problem 2.06. The nucleus of $_{92}U^{238}$ is unstable against α-decay with a half life of about $4{\cdot}5 \times 10^{9}$ years. Write down the equation of the decay and estimate the kinetic energy of the α-particle from the following data : mass of $_{92}U^{238}$ atom = 238·05081 a.m.u. ; mass of $_2He^4$ atom = 4·00260 a.m.u. ; mass of $_{90}Th^{234}$ atom = 234·04363 a.m.u. *(Text Problem)*

Sol. The α-decay of $_{92}U^{238}$ may be represented as

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4 + E_\alpha,$$

where E_α is the kinetic energy of the emittted α-particle.

If $m_N(_{92}U^{238})$, $m_N(_{90}Th^{234})$ and $m_N(_2He^4)$ are masses of $_{92}U^{238}$, $_{90}Th^{234}$ and $_2He^4$ nuclei respectively, then

$$E_\alpha = [m_N(_{92}U^{238}) - m_N(_{90}Th^{234}) - m_N(_2He^4)] \times 931{\cdot}5 \text{ MeV} \qquad ...(i)$$

If $m(_{92}U^{238})$, $m(_{90}Th^{234})$ and $m(_2He^4)$ represent masses of $_{92}U^{238}$, $_{90}Th^{234}$ and $_2He^4$ atoms, then equation (*i*) may be expressed as

$$E_\alpha = [\{m(_{92}U^{238}) - 92\, m_e\} - \{m(_{90}Th^{234}) - 90\, m_e\} - \{\, m(_2He^4) - 2\, m_e\}] \times 931{\cdot}5$$

$$= [m(_{92}U^{238}) - m(_{90}Th^{234}) - m(_2He^4)] \times 931{\cdot}5$$

$$= [238{\cdot}05081 - 234{\cdot}04363 - 4{\cdot}00260] \times 931{\cdot}5$$

$$= 0{\cdot}00458 \times 931{\cdot}5 = \mathbf{4{\cdot}27\ MeV}$$

This energy is shared by $_{90}Th^{234}$ and α-particle. Since α-particle is quite lighter, most of the energy released is carried by the α-particle.

Problem 2.07. The nucleus Ne^{23} decays by β-emission. Write down the β-decay equation and determine the maximum kinetic energy of the electron emitted from the following data : mass of $_{10}Ne^{23}$ atom = 22·994466 a.m.u. ; mass of $_{11}Na^{23}$ atom = 22·989770 a.m.u. *(Text Problem)*

Sol. The β-decay of Ne^{23} may be represented as

$$_{10}Ne^{23} \longrightarrow {}_{11}Na^{23} + {}_{-1}e^0 + \overline{\nu} + E_\beta,$$

where E_β is the maximum kinetic energy of the emitted β-particle.

Suppose the $m_N(_{10}Ne^{23})$ and $m_N(_{11}Na^{23})$ represent the masses of $_{10}Ne^{23}$ and $_{11}Na^{23}$ nuclei respectively. Then ignoring the mass of $\overline{\nu}$, we have

$$E_\beta = [m_N(_{10}Ne^{23}) - m_N(_{11}Na^{23}) - m_e] \times 931{\cdot}5 \text{ MeV} \qquad ...(i)$$

If $m(_{10}Ne^{23})$ and $m(_{11}Na^{23})$ represent the masses of $_{10}Ne^{23}$ and $_{11}Na^{23}$ atoms respectively, then equation (*i*) may be expressed as

$$E_\beta = [\{m(_{10}Ne^{23}) - 10\, m_e\} - \{m(_{11}Na^{23}) - 11\, m_e\} - m_e] \times 931{\cdot}5$$

$$= [m(_{10}Ne^{23}) - m(_{11}Na^{23})] \times 931{\cdot}5$$

$$= [22{\cdot}994466 - 22{\cdot}989770] \times 931{\cdot}5 = \mathbf{4{\cdot}374\ MeV}$$

The energy released is shared by $_{11}Na^{23}$ nucleus and the electron and neutrino pair released. As electron-neutrino pair is much lighter than the $_{11}Na^{23}$ nucleus, practically whole of the energy released is carried by electron-neutrino pair. When neutrino gets zero energy, the electron will carry the maximum energy. Therefore, maximum kinetic energy of electron emitted is 4·374 MeV.

Problem 2.08. Obtain the maximum kinetic energy of β-particles and the radiation frequencies corresponding to γ-decays in the decay scheme shown in Fig. 2.08 given below. You are given that

mass of Au^{198} atom = 197·968233 a.m.u.,
mass of Hg^{198} atom = 197·966760 a.m.u. *(Text Problem)*

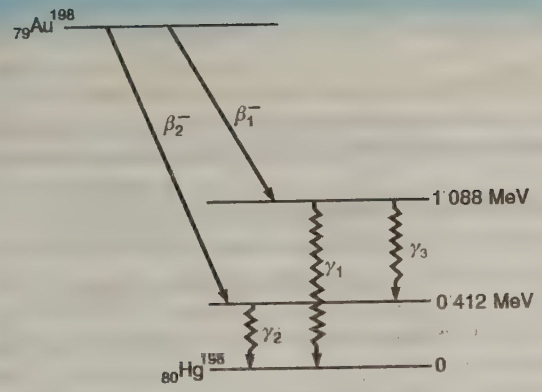


Fig. 2.08

Sol. Here, $E_1 = 0$;

$E_2 = 0{\cdot}412$ MeV $= 0{\cdot}412 \times 1{\cdot}6 \times 10^{-13}$ J ;

$E_3 = 1{\cdot}088$ MeV $= 1{\cdot}088 \times 1{\cdot}6 \times 10^{-13}$ J

If ν_1, ν_2 and ν_3 are frequencies of the γ-ray photons γ_1, γ_2 and γ_3 respectively, then

$$\nu_1 = \frac{E_3 - E_1}{h} = \frac{1{\cdot}088 \times 1{\cdot}6 \times 10^{-13} - 0}{6{\cdot}62 \times 10^{-34}}$$

$$= \mathbf{2{\cdot}63 \times 10^{20}\ s^{-1}}$$

$$\nu_2 = \frac{E_2 - E_1}{h} = \frac{0{\cdot}412 \times 1{\cdot}6 \times 10^{-13} - 0}{6{\cdot}62 \times 10^{-34}}$$

$$= \mathbf{9{\cdot}96 \times 10^{19}\ s^{-1}}$$

and $\nu_3 = \dfrac{E_3 - E_2}{h}$

$$= \frac{1{\cdot}088 \times 1{\cdot}6 \times 10^{-13} - 0{\cdot}412 \times 1{\cdot}6 \times 10^{-13}}{6{\cdot}62 \times 10^{-34}}$$

$$= \mathbf{1{\cdot}63 \times 10^{20}\ s^{-1}}$$

The emission of β_1^- decay may be represented as :

$$_{79}Au^{198} \longrightarrow {}_{80}Hg^{198} + {}_{-1}e^0 + E(\beta_1^-) + E(\gamma_1),$$

where $E(\gamma_1) = E_3 - E_1 = 1{\cdot}088 - 0 = 1{\cdot}088$ MeV

If $m_N({}_{79}Au^{198})$ and $m_N({}_{80}Hg^{198})$ represent masses of $_{79}Au^{198}$ and $_{80}Hg^{198}$ nuclei respectively, then

$$E(\beta_1^-) = [m_N({}_{79}Au^{198}) - m_N({}_{80}Hg^{198}) - m_e] \times 931{\cdot}5 - E(\gamma_1), \quad ...(i)$$

If $m({}_{79}Au^{198})$ and $m({}_{80}Hg^{198})$ represent masses of $_{79}Au^{198}$ and $_{80}Hg^{198}$ atoms respectively, then equation (*i*) may be expressed as

$$E(\beta_1^-) = [\{m({}_{79}Au^{198}) - 79\, m_e\} - \{m({}_{80}Hg^{198}) - 80\, m_e\} - m_e] \times 931{\cdot}5 - 1{\cdot}088$$

$$= [m({}_{79}Au^{198}) - m({}_{80}Hg^{198})] \times 931{\cdot}5 - 1{\cdot}088$$

$$= (197{\cdot}968233 - 197{\cdot}966760) \times 931{\cdot}5 - 1{\cdot}088$$

$$= 1{\cdot}372 - 1{\cdot}088 = \mathbf{0{\cdot}284\ MeV}$$

The emission of β_2^- decay may be represented as :

$$_{79}Au^{198} \longrightarrow {}_{80}Hg^{198} + {}_{-1}e^0 + E(\beta_2^-) + E(\gamma_2)$$

As in cse of β_1^- decay, it can be found that

$$E(\beta_2^-) = [m({}_{79}Au^{198}) - m({}_{80}Hg^{198})] \times 931{\cdot}5 - E(\gamma_2)$$

$$= 1{\cdot}372 - 0{\cdot}412 = \mathbf{0{\cdot}960\ MeV}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 2.09. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive C^{14} present with the ordinary carbon isotope C^{12}. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half life (= 5730 years) of C^{14} and the measured activity, the age of the specimen can be approximately estimated. This is the principle of C^{14} dating used in archaeology.*

Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus valley civilization.

(*Text Problem*)

Sol. Suppose that the number of C^{14} atoms per gram were N_0 at $t = 0$ (initially), when its activity was 15 decays per minute per gram. Today, *i.e.* after time t (age of the once living organism), the number of C^{14} atom per gram left is N and it shows an activity of 9 decays per minute per gram. Now, activity of a radioactive substance is given by

$$R = R_0\, e^{-\lambda t}$$

Here, $R_0 = 15$ decays min^{-1} ; $R = 9$ decays min^{-1}

$$\therefore \quad 9 = 15\, e^{-\lambda t}$$

$$\text{or} \quad e^{\lambda t} = \frac{15}{9} = 1{\cdot}6667$$

$$\text{or} \quad \lambda t = \log_e 1{\cdot}6667 = 2{\cdot}303 \log_{10} 1{\cdot}6667$$

$$= 2{\cdot}303 \times 0{\cdot}2219 = 0{\cdot}5110$$

$$\text{or} \quad t = \frac{0{\cdot}5110}{\lambda}$$

Now, decay constant, $\lambda = \dfrac{0{\cdot}693}{T} = \dfrac{0{\cdot}693}{5730}$ year^{-1}

$$\therefore \quad t = \frac{0{\cdot}5110 \times 5730}{0{\cdot}693} = \mathbf{4225{\cdot}15\ years}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. What do you mean by radioactive substance ?

Ans. A radioactive substance is one, whose atoms have unstable nuclei. The nuclei of radioactive substances emit α, β and γ-rays.

Q. 2.02. Name two radioactive elements, which are not found in observable quantities in nature. Why is it so ?

(*C.B.S.E. 1996 S*)

---

* The example is artificial, meant only to give you practice with the principle of the C^{14} dating method.

Ans. Plutonium and tritium are two radioactive elements, which are not found in observable quantities in nature. It is because of their very small values of half lives.

Q. 2.03. Among alpha, beta and gamma radiations, which are the ones affected by a magnetic field ? *(I.S.C.E. 1996)*

Ans. Alpha and beta radiations are affected by the magnetic field.

Q. 2.04. Which has greater ionising power : alpha particle or beta particle ? *(C.B.S.E. 1996)*

Ans. Alpha particle.

Q. 2.05. Why do alpha particles have a high ionising power ? *(C.B.S.E. 1998 S)*

Ans. Being massive and of large nuclear cross-section, α-particle possesses high ionising power.

Q. 2.06. "All radioactive substances seem to be identical." Comment.

Ans. A radioactive sample requires infinite time to disintegrate completely and in this sense, all the radioactive substances are identical.

Q. 2.07. Define radioactive decay constant. *(P.S.S.C.E. 1999 S)*

Ans. The decay constant of a radioactive substance is defined as the reciprocal of that time in which the number of atoms of the substance become $\frac{1}{e}$th of the atoms present initially.

Q. 2.08. Explain, why it is not possible to define the total life of a radioactive substance.

Ans. The radioactivity is a spontaneous phenomenon and obeys the law of chance only. Therefore, as the nucleus of an atom can have any value of total life (between zero to infinity), it is not possible to define the total life of a radioactive substance.

Q. 2.09. Define half life of a radioactive material.
(H.P.S.S.C.E. 2001 ; P.S.S.C.E. 1999 ; H.S.S.C.E. 1998 S ; C.B.S.E. 2002, 1992, 1991)

Ans. Refer to section 2.17.

Q. 210. The half life of ${}_6C^{14}$ is 5700 years. What does it mean ?

Ans. The nuclei of one half of the ${}_6C^{14}$ atoms will undergo disintegration in 5700 years

Q. 2.11. Give the relation between 'half life' and disintegration constant of a radioactive material. *(I.S.C.E. 1998)*

Ans. Relation between half life and decay constant :

$$T = \frac{0{\cdot}693}{\lambda}$$

Q. 2.12. Plutonium decays with a half life of 24,000 years. If plutonium is stored for 72,000 years, what fraction of it remains ? *(C.B.S.E. 1994)*

Ans. Here, T = 24,000 years ; t = 72,000 years

$$\text{Now,} \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{\frac{72000}{24000}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Q. 2.13. A radioisotope of silver has half-life of 20 minutes. What fraction of the original mass would remain after one hour ? *(I.S.C.E. 1993)*

Ans. Here, T = 20 min ; t = 1 hour = 60 min

$$\text{Now,} \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{\frac{60}{20}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Q. 2.14. A certain radioactive substance has a half-life period of 30 days. What is the disintegration constant ? *(H.P.S.S.C.E. 1997)*

Ans. Here, T = 30 days

Now, disintegration constant,

$$\lambda = \frac{0{\cdot}693}{T} = \frac{0{\cdot}693}{30} = \mathbf{0{\cdot}0231\ day^{-1}}$$

Q. 2.15. Define average life of a radioactive substance. *(P.S.S.C.E. 1999 S)*

Ans. Refer to section 2.18.

Q. 2.16. Write down expression relating half life of a radioactive element to its average life.

Ans. Half life = 0·693 × average life

Q. 2.17. What is meant by the activity of a radioactive material ? *(I.S.C.E. 1997)*

Ans. *The activity of a radioactive substance is defined as the rate at which the nuclei of its atoms in the sample disintegrate.* Thus,

$$\text{activity of a substance, } R = -\frac{dN}{dt}$$

Q. 2.18. Does the activity of a radioactive element depend on external physical conditions ?

Ans. No.

Q. 2.19. Write down the value of one millicurie in terms of rutherford.

Ans. 1 millicurie = $3{\cdot}7 \times 10^7$ disintegrations s^{-1} and 1 rutherford = 10^6 disintegrations s^{-1}

∴ 1 millicurie = 37 rutherford

Q. 2.20. A radioactive substance having N nuclei has activity A. Write down an expression for its half-life in terms of A and N. *(I.S.C.E. 1997)*

Ans. Half life, $T = \frac{0{\cdot}693}{\lambda}$

But $\quad R = N\lambda \quad$ or $\quad \lambda = \frac{R}{N}$

∴ $\quad T = 0{\cdot}693 \times \frac{N}{R}$

Q. 2.21. Write a general equation that represents beta emission. *(P.S.S.C.E. 1998)*

Ans. General equation for β-decay :

$${}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu} + E_\beta$$

Q. 2.22. What is the difference between an electron and a β-particle ? *(P.S.S.C.E. 1999 S)*

Ans. An electron and a β-particle are essentially the same. The electron of nuclear origin is called a β-particle.

Q. 2.23. Why is the energy variation of the electrons emitted during a beta decay continuous ? *(C.B.S.E. 1998 S)*

Or

Why is the energy distribution of beta rays continuous ? *(C.B.S.E. 1993)*

Ans. β-decay is the result of conversion of a neutron into a proton, electron and an antineutrino $(\bar{\nu})$. Since the energy available in β-decay is shared by the electron and the antineutrino in all possible proportions as they come out of nucleus ; the β-ray spectrum is continuous in nature.

Q. 2.24. Why do all electrons emitted during beta decay ot have the same energy ? (C.B.S.E. 1996)

Ans. Refer to VSQ 2.24.

Q. 2.25. Why does a nucleus emit a γ-ray photon ?

Ans. During α or β-decay, the residual nucleus is somemes left in the excited state. When the nucleus de-excites, it emits energy. The energy emitted is found to be in γ-ray region.

Q. 2.26. What is a radioisotope ?

Ans. The isotope of an element capable of emitting radiations like a radioactive substance is called a radioisotope. Usually, radioisotopes are artificially produced.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. State the difference between X-rays and γ-rays..

Ans. Both X-rays and γ-rays are electromagnetic waves. Iowever, they differ in the following aspects :

1. The wavelength of γ-rays is very small as compared o that of X-rays.
2. The penetrating power of γ-rays is very high as ompared to that of X-rays.
3. The origin of X-rays is due to atomic de-excitation, vhle the origin of γ-rays is due to nuclear de-excitation.

Q. 2.02. State laws of radioactivity.
(H.S.S.C.E. 1996 ; Pre-degree Kerala, 1994)

Ans. Refer to section 2.03.

Q. 2.03. What are the characteristics of radioactivity ?

Ans. Following are a few characteristics of radioactivity.

1. It is a spontaneous phenomenon.
2. It obeys exponential decay laws.
3. The rate of radioactive disintegration of a substance annot be altered by changing the external factors, such as ressure, temperature, etc.

Q. 2.04. Natural radioactive nuclei are nuclei of high nass number. Why ?

Ans. The B.E./A of the nuclei of high mass number is mall as compared to that of the stable nuclei. Also, neutron o proton ratio of the nuclei of high mass number is different rom that for stable nuclei. Such nuclei undergo radioactive ecay so as to attain greater value of B.E./A and also to cquire the proper neutron to proton ratio.

Q. 2.05. Why heavy stable nuclei must contain more neutrons than protons ?

Ans. The heavy nuclei tend to become unstable due to large number of protons inside it. For heavy nuclei to be stable, the electrostatic repulsive force between protons must be dominated by attractive nuclear force resulting from neutron-neutron, neutron-proton and proton-proton interactions. So that it can happen, number of neutrons must be greater than number of protons.

Q. 2.06. Define the term `decay constant' for a radioactive substance. Deduce the relation between decay constant and half life period. (C.B.S.E. 1993)

Ans. Decay constant. Refer to section 2.04.

Relation between T and λ. Refer to section 2.05.

Q. 2.07. All the radioactive series terminate at lead as their final product. Explain, why.

Ans. It is found that the half lives of the intermediate radioactive elements formed in any of the three radioactive series (*uranium series, actinium series* and *thorium series*) are very small as compared to the element at the head of the series. For this reason, all the radioactive series ultimately end into lead, the stable element.

Q. 2.08. Two radioactive nuclei X and Y initially contain an equal number of atoms. Their half life is 1 hour and 2 hours respectively. Calculate the ratio of their rates of disintegration after two hours. (C.B.S.E. 2002)

Ans. Here, half life of sample X, $T_1 = 1$ h

and half life of sample Y, $T_2 = 2$ h

Let N be the number of atoms of each of the two samples initially. Thus number of atoms of sampel X after 2 h, $N_1 = \frac{N}{4}$

and number of atoms of sample Y after 2h, $N_2 = \frac{N_2}{2}$

Let R_1 and R_2 be their rates of disintegration after 2 h. If λ_1 and λ_2 are their respective decay constant, then

$$\frac{R_1}{R_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \frac{T_2 N_1}{T_1 N_2} \qquad \left(\because T = \frac{0\cdot693}{\lambda}\right)$$

$$= \frac{2 \times N/4}{1 \times N/2} = 1$$

Q. 2.09. The half life period of a radioactive substance is 30 days. What is the time taken for $\frac{3}{4}$th of its original mass to disintegrate ? (C.B.S.E. 1994)

Ans. Here, T = 30 days ; $\frac{N_0 - N}{N_0} = \frac{3}{4}$ or $\frac{N}{N_0} = \frac{1}{4}$

If *t* is the required time, then

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \quad \text{or} \quad \left(\frac{1}{2}\right)^{t/30} = \frac{1}{4} \quad \text{or} \quad \left(\frac{1}{2}\right)^{t/30} = \left(\frac{1}{2}\right)^2$$

$$\therefore \quad \frac{t}{30} = \frac{1}{2} \quad \text{or} \quad t = 60 \text{ days}$$

Q. 2.10. A radioactive substance decays to $\frac{1}{32}$th its initial activity in 25 days. Calculate its half life. (C.B.S.E. 1992)

Ans. If T is the half life of the radioactive substance, then

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T}$$

Here, $\frac{R}{R_0} = \frac{1}{32}$; $t = 25$ days

$$\therefore \quad \frac{1}{32} = \left(\frac{1}{2}\right)^{25/T} \quad \text{or} \quad \left(\frac{1}{2}\right)^{25/T} = \left(\frac{1}{2}\right)^5$$

$$\text{or} \quad \frac{25}{T} = 5 \quad \text{or} \quad T = 25/5 = 5 \text{ days}$$

Q. 2.11. What fraction of tritium will remain undecayed after 25 years ? Given, half life is 12·5 years. (H.P.S.S.C.E. 1993)

Ans. Now, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$, where T is half life and $\frac{N}{N_0}$ is fraction of atoms left after time *t*.

Here, T = 12·5 years ; t = 25 years

$$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{25}{12\cdot5}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Q. 2.12. Define `disintegration constant' and `mean life' of a radioactive substance. Give the unit for each. *(C.B.S.E. 1997)*

Ans. Disintegration constant. For definition, refer to section 2.04.

Its unit is reciprocal of time *i.e.* s^{-1} or $year^{-1}$.

Mean life. For definition, refer to section 2.06. Its unit is that of time *i.e.* s or year.

Q. 2.13. Distinguish between half and average life of a radioactive substance. *(P.S.S.C.E. 1998)*

Ans. Refer to sections 2.05 and 2.06.

Q. 2.14. The half life period of radioactive element A is the same as the mean life time of another radioactive element B. Initially both of them have same number of atoms. The radioactive element B decays faster than A. Explain, why. *(P.S.S.C.E. 2001)*

Ans. Let λ_1 and λ_2 be decay constants of element A and B respectively.

$$\text{Then,} \quad \frac{0\cdot693}{\lambda_1} = \frac{1}{\lambda_2} \quad \text{or} \quad \frac{\lambda_1}{\lambda_2} = 0\cdot693$$

Let N be the number of atoms of each of the two samples initially. If R_1 and R_2 are their rates of disintegration, then

$$\frac{R_1}{R_2} = \frac{\lambda_1 N}{\lambda_2 N} = \frac{\lambda_1}{\lambda_2} = 0\cdot693$$

Hence, R_2 is greater than R_1.

Q. 2.15. Define two units of radioactivity. How are they related ? *(P.S.S.C.E. 1993)*

Ans. In radioactivity, curie and rutherford are two units used to measure activity of a radioactive substance. For definitions, refer to section 2.07.

The two units are related to each other as below :

$$1 \text{ curie } = 3{\cdot}7 \times 10^4 \text{ rutherford}$$

Q. 2.16. In a radioactive decay, the nucleus emits an α-particle followed by two β-particles. Show that the final nucleus is an isotope of the original one.

Ans. Let the original nucleus and final nucleus be represented as ${}_{Z_1}X^{A_1}$ and ${}_{Z_2}Y^{A_2}$ respectively. Then the given radioactive decay process may be represented as

$${}_{Z_1}X^{A_1} \longrightarrow {}_{Z_2}Y^{A_2} + {}_2He^4 + 2\,{}_{-1}e^0$$

Obviously, $Z_2 = Z_1 - (2 - 2) = Z_1$

Hence, the final nucleus is an isotope of the original nucleus.

Q. 2.17. A radioactive nucleus undergoes a series of decays according to the sequence

$$A \xrightarrow{\beta} A_1 \xrightarrow{\alpha} A_2 \xrightarrow{\alpha} A_3$$

If the mass number and atomic number of A_3 are 172 and 69 respectively, what is the mass number and atomic number of A ? *(C.B.S.E. 1994)*

Ans. Since the nucleus A_3 is ${}_{69}A_3{}^{172}$ and β-particle is an electron (${}_{-1}e^0$) and α-particle is ${}_2He^4$ (nucleus of helium), the given decay series may be represented as

$${}_{72}A^{180} \xrightarrow{\beta} {}_{73}A_1{}^{180} \xrightarrow{\alpha} {}_{71}A_2{}^{176} \xrightarrow{\alpha} {}_{69}A_3{}^{172}$$

i.e. the mass number and atomic number of radioactive nucleus A are 180 and 72 respectively.

Q. 2.18. A radioactive isotope D decays according to the sequence

$$D \xrightarrow{{}_0n^1} D_1 \xrightarrow{\alpha} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, find the mass number and atomic number of D. Amongst D, D_1 and D_2, do we have any isobars or isotopes ? *(C.B.S.E. 1998)*

Ans. α-particle is nucleus of helium (${}_2He^4$).

Therefore, the above mentioned radioactive decays will proceed as below :

$${}_{73}D^{181} \xrightarrow{{}_0n^1} {}_{73}D_1{}^{180} \xrightarrow{{}_2He^4} {}_{71}D_2{}^{176}$$

Therefore, mass number and atomic number of D are 181 and 73 respectively.

Since atomic number of D_1 and D are same, D and D_1 are isotopes.

Q. 2.19. A radioactive nucleus A decays as follows :

$$A \xrightarrow{{}_1e^0} A_1 \xrightarrow{\alpha} A_2$$

If the mass number and atomic number of A_2 are 176 and 71 respectively, what are the mass numbers and atomic numbers of A_1 and A ? Which of these three elements are isobars ? *(C.B.S.E. 1999)*

Ans. Proceed as in SAQ 2.32.

Mass number of A_1 = 180, mass number of A = 180

Atomic number of A_1 = 73, atomic number of A = 74

The elements A_1 and A are isobars.

Q. 2.20. The sequence of stepwise decays of radioactive nucleus is

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\alpha} D_4$$

If the nucleon number and atomic number for D_2 are respectively 176 and 71, what are the corresponding values for D and D_4 nuclei. Justify your answers. *(C.B.S.E. 1998)*

Ans. An α-particle is the nucleus of helium (${}_2He^4$) and a β-particle is an electron (${}_{-1}e^0$). Therefore, the above mentioned radioactive decays will proceed as below :

$${}_{72}D^{180} \xrightarrow{{}_2He^4} {}_{70}D_1{}^{176} \xrightarrow{{}_{-1}e^0} {}_{71}D_2{}^{172} \xrightarrow{{}_2He^4}$$
$${}_{69}D_3{}^{172} \xrightarrow{{}_2He^4} {}_{67}D_4{}^{168}$$

The nucleon number and atomic number corresponding to D are 180 and 72 ; while those corresponding to D_4 are 168 and 67.

Q. 2.21. The isotope uranium ${}_{92}U^{238}$ decays successively to form ${}_{90}Th^{234}$, ${}_{91}Pa^{234}$, ${}_{92}U^{234}$, ${}_{90}Th^{230}$ and ${}_{88}Ra^{226}$. What are the radiations emitted in these five steps ? *(C.B.S.E. 1981)*

Ans. In the first step (${}_{92}U^{238}$ to ${}_{90}Th^{234}$) : When ${}_{92}U^{238}$ decys to ${}_{90}Th^{234}$, mass number decreases by 4 and atomic number decreases by 2. Therefore, α-particle (${}_2He^4$) is emitted. The decay takes place as below :

$${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4$$

In the second step (${}_{90}Th^{234}$ to ${}_{91}Pa^{234}$) : When ${}_{90}Th^{234}$ decays to ${}_{91}Pa^{234}$, mass number remains unchanged and the atomic number increases by 1. Therefore, β-particle (${}_{-1}e^0$) is emitted. The decay takes place as below :

$${}_{90}Th^{234} \longrightarrow {}_{91}Pa^{234} + {}_{-1}e^0$$

In the third step ($_{91}Pa^{234}$ to $_{92}U^{234}$) : Here also, the mass mber remains same and the atomic number increases by Therefore, in the third step, the β-decay takes place as resented below :

$$_{91}Pa^{234} \longrightarrow {}_{92}U^{234} + {}_{-1}e^0$$

In fourth step ($_{92}U^{234}$ to $_{90}Th^{230}$) and fifth step ($_{90}Th^{230}$ $_{8}Ra^{226}$) : In each of these two steps, α-decays takes place.

Q. 2.22. Does the ratio of neutrons to protons in a cleus increase, decrease or remain the same after the ission of an α-particle ? (H.P.S.S.C.E. 1994)

Ans. The number of neutrons (N = A – Z) in a nucleus either equal to or greater than the number of protons (Z). e emission of an α-particle involves the emission of two otons and two neutrons. It can be easily seen that $\frac{N-2}{Z-2}$ ll always be greater than $\frac{N}{Z}$ *i.e.* the ratio of neutrons to otons in a nucleus always increases after the emission of α-rticle.

Q. 2.23. A nucleus contains no electrons, but can eject em. Why ?

Ans. The emission of electron from a radioactive nucleus is termed as β-decay. At the time of β-decay, a neutron converts into a proton according to the following scheme :

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu}$$

In this manner, the nucleus emits electron (along with the antineutrons), although it does not contain it.

Q. 2.24. Explain, with an example, whether the neutron-proton ratio increases or decreases during beta decay. (C.B.S.E. 1998 S)

Ans. When β-decay takes place, a neutron inside the nucleus converts into proton. Therefore, on beta decay, whereas the number of neutrons decreases by one, the number of protons increases by one. Obviously, the neutron-proton ratio decreases on β-decay.

Consider the β-decay of Bi^{210} :

$$_{83}Bi^{210} \longrightarrow {}_{84}Po^{210} + {}_{-1}e^0$$

Before β-decay : $\frac{A-Z}{Z} = \frac{210-83}{83} = \frac{127}{83}$

After β-decay : $\frac{A-Z}{Z} = \frac{210-84}{84} = \frac{126}{84}$

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 2.01. Why all the three types of rays *i.e.* α, β and γ, come out from a radioactive sample though a single dioactive sample obeys a particular decay mode ?

Ans. The decay of a radioactive element is not a single age process. The daughter element formed further decays to some other element and so on, till stable lead is formed. the complete decay scheme (from parent element to the able lead), a few radioactive elements undergo α-decay, hile others β-decay. The γ-ray is emitted in cases, when the ucleus after an α or a β-decay is left in the excited state.

Q. 2.02. The half-life period of a radioactive element X is same as the mean-life time of another radioactive ement Y. Initially both of them have the same number of oms. Which of the radioactive element decays faster than e other ? (I.I.T. 1999)

Ans. Let λ and λ′ be decay constants of radioactive ements X and Y respectively. If T is half-life period of X and $_a'$, the mean life time of element Y, then

$$T = \frac{0\cdot693}{\lambda} \quad \text{and} \quad T_a' = \frac{1}{\lambda'}$$

Since $T = T_a'$, we have

$$\frac{0\cdot693}{\lambda} = \frac{1}{\lambda'} \quad \text{or} \quad \lambda = 0\cdot693\,\lambda' \quad \text{or} \quad \lambda' > \lambda$$

The decay rate of a radioactive sample is given by

$$\frac{dN}{dt} = -\lambda N$$

As both the elements have the same number of atoms nitially,

$$\frac{dN}{dt} \propto \lambda$$

Since decay constant of element Y is greater than that of , the element Y decays faster than X.

Q. 2.03. At a given instant, there are 25% undecayed radioactive nuclei in a sample. After 10 s, the number of undecayed nuclei reduces to 12·5%. Calculate (*a*) mean life of the nuclei and (*b*) the time in which the number of undecayed nuclei will further reduce to 6·25% of the reduced number. (I.I.T. 1996)

Ans. (*a*) Since the number of undecayed nuclei is reduced from 25% to 12·5% (*i.e.* to one half) in 10 s, the half life of the radioactive sample,

$$T = 10\text{ s}$$

Now, $T = \frac{0\cdot693}{\lambda}$ or $\lambda = \frac{0\cdot693}{T}$

Therefore, mean life, $T_a = \frac{1}{\lambda} = \frac{T}{0\cdot693} = \frac{10}{0\cdot693} = 14\cdot43$ s

(*b*) Suppose that the number of undecayed nuclei are N_0 and they will reduce to 6·25% of this number (say N) in time *t*. Then,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$$

Here, $N = \frac{6\cdot25}{100} N_0 = \frac{N_0}{16}$

$$\therefore \quad \frac{N_0/16}{N_0} = \left(\frac{1}{2}\right)^n \quad \text{or} \quad \left(\frac{1}{2}\right)^{t/T} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$$

or $\frac{t}{T} = 4$ or $t = 4\,T = 4 \times 10 = 40$ s

Q. 2.04. The α-particle faces a coulomb barrier. A neutron being uncharged faces no such barrier. Why does the nucleus $_{92}U^{238}$ not decay spontaneously by emitting a neutron ? Given, $m_N(_{92}U^{237}) = 237\cdot04874$ a.m.u., $m_n = 1\cdot00867$ a.m.u. and $m_N(_{92}U^{238}) = 238\cdot05081$ a.m.u.

Ans. The possible decay would be

$$_{92}U^{238} \longrightarrow {}_{92}U^{237} + {}_0n^1$$

Since $m_N(_{92}U^{238})$ *i.e.* 238·05081 a.m.u. is less than the mass of the products *i.e.* $m_N(_{92}U^{237}) + m_n$ *i.e.* 237·04874 + 1·00867 *i.e.* 238·05741 a.m.u., the decay is not energetically allowed.

Q. 2.05. If the α-decay of U^{238} is energetically allowed (*i.e.* the decay products have a total mass

less than the mass of U^{238}), what prevents U^{238} from decaying all at once ? *(Text Question)*

Ans. The α-decay is caused by the quantum mechanical tunnelling of the nucleus by the α-particles. The rate of tunnelling is determined by the height of the nuclear potential barrier, its width, etc.

Q. 2.06. Why are α-particles emitted rather than say protons or $_2He^3$ nuclei ?

Ans. It is because of the high value of binding energy of the α-particle. Therefore, on emission of α-particle, the binding energy per nucleon of the residual nucleus increas appreciably. On the other hand, the emission of a proton $_2He^3$ nuclei may not be energetically possible.

Q. 2.07. Why electron capture is common in case heavy nuclei ?

Ans. A heavy nucleus has large positive charg Moreover, the radius of K-shell in heavy nuclei is muc smaller than that in case of a light nucleus. Due to both thes factors, the K- shell electron experiences a large force attraction due to positive nucleus and is captured by it.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is radioactivity ? Give the properties of different types of rays emitted by radioactive substances.
2. Write down the laws of radioactive decay and deduce expression for the decay law. *(H.P.S.S.C.E. 1997)*
3. What is readioactivity ? State the laws of radioactive disintegration. Show that radioactive decay is expontential in nature. *(P.S.S.C.E. 1999, 1998 S)*
4. State the radioactive decay law. Show by derivation that the number of unchanged nuclei left (*i.e.* the number of parent nuclei) decreases exponentially with time. *(I.S.C.E. 1993)*
5. State the law of radioactive decay. Deduce the relation $N = N_0 e^{-\lambda t}$, where symbols have their usual meanings. Sketch a graph to illustrate radioactive decay. *(H.P.S.S.C.E. 2000, 1999 S ; C.B.S.E. 1999 S)*
6. $_{92}U^{238}$ decays successively to form $_{90}Th^{234}$, $_{91}Pa^{234}$, $_{82}U^{234}$, $_{90}Th^{230}$, $_{88}Ra^{236}$ and $_{86}Rn^{222}$. What are the radioactive radiations emitted in each decay process ? *(C.B.S.E. 1996 S)*
7. Define decay constant of a radioactive sample. Which of the following radiations, α-rays, β-rays, γ-rays. (*i*) are similar to X-rays ? (*ii*) are easily absorbed by matter ? (*iii*) travel with greatest speed ? (*iv*) are similar in nature to cathode rays ? *(C.B.S.E. 2001)*
8. State the law of radioactive decay. Define half life period and obtain an expression for it. *(H.S.S.C.E. 2002)*
9. Define the term decay constant and half life of a radioactive sample. Derive the relation connecting the two. *(C.B.S.E. 2001, P.S.S.C.E. 2001)*
10. Write down the laws of radioactivity and define half li period and find out an expression for it. *(H.S.S.C.E. 1997*
11. Derive the radioactive decay law. Define half-life (T) of radioactive substance and show that
$$T = \frac{0 \cdot 693}{\lambda},$$
where λ is the decay constant. *(I.S.C.E. 199*
12. Define the term `half-life $T_{1/2}$ of a radioactive substanc and establish its relation with decay constant (λ). *(H.S.S.C.E. 2002 ; C.B.S.E. 1999 S ; I.S.C.E. 199*
13. Define half-life time and average life time of a radioactiv element. Derive expression for half life time. *(P.S.S.C.E. 199*
14. What is α-decay ? Explain in brief.
15. Explain β-decay. *(H.S.S.C.E. 200*
16. Why do we say that β-rays emitted from a radioactiv substance have a continuous energy spectrum ? How d this fact lead to the discovery of neutrino ?
17. Explain the phenomenon of γ-decay.
18. What is alpha decay, beta decay and gamma decay ? *(H.P.S.S.C.E. 199*
19. Define half life of a radioactive substance. Derive a expression for it. *(H.S.S.C.E. 2002, 1995, 1994 ; H.P.S.S.C.E. 1995, 199*
20. Explain radio-carbon dating. *(I.S.C.E. 199*

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What is radioactivity ? State the laws of radioactive decay. Show that radioactive decay is exponential in nature. *(P.S.S.C.E. 2000, 2002)*
2. State the laws of radioactive disintegration. What is half life period of a radioactive substance ? Find an expression for it. *(H.S.S.C.E. 2001)*
3. State and explain the laws of radioactive disintegration. Hence define disintegration constant and half life period. *(H.S.S.C.E. 2001)*
4. Explain what is meant by radioactive decay. A radioactive nucleus is represented by the symbol $_bX^a$. How is the new nucleus represented after the emission of (*i*) an alph particle, (*ii*) a beta particle and (*iii*) a gamma ray ? Th activity of a source undergoing a single type of decay is at time $t = 0$. Obtain an expression in terms of the half-li $T_{1/2}$ for the activity I at any subsequent time *t*. *(C.B.S.E. 2001*
5. What do you mean by half-life and mean life of radioactive substance ? Deduce the relation between ther *(H.S.S.C.E. 1999*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Radioactive decay law and Half life

1. How many disintegrations per second will occur in one gram of $_{92}U^{238}$, if its half life against alpha decay is $1{\cdot}42 \times 10^{17}$ s ? *(C.B.S.E. 2001)* **[Ans.** $1{\cdot}235 \times 10^4\ s^{-1}$]
2. The half life of $_{92}U^{238}$ against α-decay is $4{\cdot}5 \times 10^9$ years. How many disintegrations per second occur in 1 g $_{92}U^{238}$? **[Ans.** $1{\cdot}235 \times 10^4$ s
3. The atomic weight of radium is 226. It is observed tha $3{\cdot}67 \times 10^{10}$ α-particles are emitted per second from or gram of radium. Calculate in years the half life period radium. **[Ans.** 1548·7 years

4. Find the half life of U^{238}, if one gram of it emits $1{\cdot}24\times10^4$ α- particles per second. Given that Avogadro number $= 6{\cdot}025 \times 10^{23}$. **[Ans.** $4{\cdot}484 \times 10^9$ **years]**

5. 4 g of radioactive material of half life 10 years is kept in a store for 15 years. How much material is disintegrated ? **[Ans.** 2·5856 g**]**

6. One gram of radium is reduced by 2 milligram in 5 years by α-decay. Calculate the half period of radium. (*P.S.S.C.E. 1993*) **[Ans.** 1730·3 years**]**

7. The half-life of a radioactive substance is 1672 years. If the initial mass of the substance is 1 g, after how many years will only 1 mg of it to be left behind ? (*P.S.S.C.E. 1999 S*) **[Ans. 16669·5 years]**

8. The half life of radium is 1590 years. In how many years will one gram of the pure element lose 1 centigram ? **[Ans. 23·06 years]**

9. A nucleus of UX_1 has a half life of 24·1 days. How long a sample UX_1 will take to change 90% of it to UX_2 ? **[Ans. 79·97 days]**

ype B. On Activity of a radioactive substance

10. The half-life time of a radioactive substance is 100 years. Calculate in how many years the activity will decay to 1/10th of its initial value. (*P.S.S.C.E. 2000, 1999*) **[Ans. 332·3 years]**

11. The half-life of a radioactive substance is 5×10^3 years. In how many years, its activity will decay to 0·2 times of its initial value ? Given $\log_{10} 5 = 0{\cdot}6990$. (*P.S.S.C.E. 2000*) **[Ans.** $1{\cdot}16 \times 10^4$ **years]**

12. One gram of $_{88}Ra^{226}$ has activity of one curie. Find its half life period. Given that a kilo mole of $_{88}Ra^{226}$ contains $6{\cdot}025 \times 10^{26}$ atoms. **[Ans. 1583·2 years]**

13. Determine the amount of $_{84}Po^{210}$ necessary to provide a source of α-particles of 5 mci strength. The half-life of polonium is 138 days. **[Ans.** $1{\cdot}11 \times 10^{-6}$ **g]**

MISCELLANEOUS PROBLEMS

14. The count rate from a radioactive sample falls from $4{\cdot}0 \times 10^6\ s^{-1}$ to $1{\cdot}0 \times 10^6\ s^{-1}$ in 20 hours. What will be the count rate 100 hours after the beginning. **[Ans.** $3{\cdot}91 \times 10^3\ s^{-1}$**]**

15. A radioactive sample can decay by two different processes. The half-life for the first process is T_1 and that for the second process is T_2. Show that the effective half-life T of the nucleus is given by

$$\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$

16. One milligram of thorium emits 22 α-particles per minute per unit solid angle. Calculate average-life of thorium. Atomic weight of thorium = 232.**[Ans.** $1{\cdot}786 \times 10^{10}$ **years]**

17. A point source of γ-radiation has a half-life of 30 minutes. The initial count rate recorded by Geiger counter placed 2 m from the source is $360\ s^{-1}$. The distance between the counter and the source is altered. After 1·5 hours, the count rate recorded is $5\ s^{-1}$. What is the new distance between the counter and the source ? **[Ans. 6 m]**

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Same as solved problem no. 2.02.

2. $$\lambda = \frac{0{\cdot}693}{4{\cdot}5\times10^9}\ \text{year}^{-1}$$
$$= \frac{0{\cdot}693}{4{\cdot}5\times10^9\times365\times24\times60\times60}\ s^{-1}$$

Number of atoms in 1g of U^{238}, $N = \dfrac{6{\cdot}02\times10^{23}}{238}$

Using the relation : $\dfrac{dN}{dt} = \lambda N$,

it can be obtained that

$$\frac{dN}{dt} = \mathbf{1{\cdot}235 \times 10^4\ s^{-1}}$$

3. Number of atoms in 1 g of Ra^{236}, $N = \dfrac{6{\cdot}02\times10^{23}}{226}$

$$\frac{dN}{dt} = 3{\cdot}67\times10^{10}\ s^{-1}$$

Using the relation : $\dfrac{dN}{dt} = \lambda N$,

it can be found that

$$\lambda = 1{\cdot}378 \times 10^{-11}\ s^{-1}$$

$$\therefore\ T = \frac{0{\cdot}693}{\lambda} = \frac{0{\cdot}693}{1{\cdot}378\times10^{-11}}\ s$$
$$= \frac{0{\cdot}693}{1{\cdot}378\times10^{-11}\times60\times60\times24\times365}$$
$$= \mathbf{1548{\cdot}7\ years}$$

4. Number of U^{238} atoms in 1 g,

$$N = \frac{6{\cdot}025\times10^{23}}{238} = 2{\cdot}53\times10^{21}$$

Now, $\dfrac{dN}{dt} = \lambda N$ or $1{\cdot}24 \times 10^4 = \lambda \times 2{\cdot}53 \times 10^{21}$

or $$\lambda = \frac{1{\cdot}24\times10^4}{2{\cdot}53\times10^{21}} = 4{\cdot}9\times10^{-18}\ s^{-1}$$

$$\therefore\ T = \frac{0{\cdot}693}{\lambda} = \frac{0{\cdot}693}{4{\cdot}9\times10^{-18}} = 1{\cdot}414\times10^{17}\ s$$
$$= \mathbf{4{\cdot}484 \times 10^9\ years}$$

5. $$\lambda = \frac{0{\cdot}693}{T} = \frac{0{\cdot}693}{10} = 0{\cdot}0693\ \text{year}^{-1}$$

Now, $m = m_0 e^{-\lambda t} = 4e^{-0{\cdot}0693\times15}$
$= 4 \times 0{\cdot}3536 = 1{\cdot}4144$ g

Therefore, material disintegrated
$= 4 - 1{\cdot}4144 = \mathbf{2{\cdot}5856\ g}$

6. $m_0 = 1$ g, $m_0 - m = 2$ mg $= 0{\cdot}002$ g
$\therefore m = m_0 - 0{\cdot}002 = 1 - 0{\cdot}002 = 0{\cdot}998$ g

Now, $m = m_0 e^{-\lambda t}$ or $e^{\lambda t} = \dfrac{m_0}{m}$

or $$\lambda t = \log_e \frac{m_0}{m}$$

or $$\lambda = \frac{2{\cdot}303}{t}\log_{10}\frac{m_0}{m} = \frac{2{\cdot}303}{5}\log_{10}\frac{1}{0{\cdot}998}$$
$$= \frac{2{\cdot}303}{5}\log_{10}1{\cdot}002 = \frac{2{\cdot}303}{5}\times0{\cdot}0009$$
$$= 4{\cdot}005\times10^{-4}\ \text{year}^{-1}$$

$$\therefore\ T = \frac{0{\cdot}693}{\lambda} = \frac{0{\cdot}693}{4{\cdot}005\times10^{-4}} = \mathbf{1730{\cdot}3\ years}$$

7. $m_0 = 1$ g ; $m = 1$ mg $= 10^{-3}$ g ;

$$\lambda = \frac{0{\cdot}693}{1672} = 4{\cdot}1447\times10^{-4}\ \text{year}^{-1}$$

As obtained in problem no. 6,

$$t=\frac{2\cdot303}{\lambda}\log_{10}\frac{m_0}{m}=\frac{2\cdot303}{4\cdot1447\times10^{-4}}\log_{10}\frac{1}{10^{-3}}$$

$$=\frac{2\cdot303\times3}{4\cdot1447\times10^{-4}}=\mathbf{16669{\cdot}5\ years}$$

8. $\lambda=\frac{0\cdot693}{1590}=4\cdot3585\times10^{-4}\ \text{year}^{-1};$

$$m_0=1\,g\,;m_0-m=0\cdot01\ g$$

$$\therefore\ m=m_0-0{\cdot}01=1-0{\cdot}01=0{\cdot}99\ g$$

Using the relation : $t=\frac{2\cdot303}{\lambda}\log_{10}\frac{m_0}{m}$,

it can be obtained that

$$t=\mathbf{23{\cdot}06\ years}$$

9. $\lambda=\frac{0\cdot693}{24\cdot1}=0\cdot0288\ \text{day}^{-1}$

Let t be the time in which 90% of the sample of UX_1 changes to UX_2.

Therefore, sample of UX_1 left behind after time t = 10%

or $\frac{m}{m_0}=10\%=0\cdot1$ or $\frac{m_0}{m}=10$

Using the relation : $t=\frac{2\cdot303}{\lambda}\log_{10}\frac{m_0}{m}$,

it can be obtained that

$$t=\mathbf{79{\cdot}97\ days}$$

10. $R=\frac{10}{100}\times R_0=0\cdot1\,R_0,$

$$\lambda=\frac{0\cdot693}{100}=6\cdot93\times10^{-3}\ \text{year}^{-1}$$

Now, $R=R_0\,e^{-\lambda t}$ or $\lambda t=\log_e\frac{R_0}{R}$

or $t=\frac{2\cdot303}{\lambda}\log_{10}\frac{R_0}{R}$

$$=\frac{2\cdot303}{6\cdot93\times10^{-3}}\log_{10}\frac{R_0}{0\cdot1\,R_0}=\frac{2\cdot303\times1}{6\cdot93\times10^{-3}}$$

or $t=\mathbf{332{\cdot}3\ years}$

11. Proceed as in problem no. 10.

12. $R=\frac{dN}{dt}=1\,\text{curie}=3\cdot7\times10^{10}\ s^{-1}$

Number of atoms in 1 g of Ra^{226}, $N=\frac{6\cdot025\times10^{26}}{1000\times226}$

Using the relation : $\frac{dN}{dt}=\lambda N$,

it can be obtained that

$$\lambda=1{\cdot}388\times10^{-11}\ s^{-1}$$

$$\therefore\quad T=\frac{0\cdot693}{\lambda}=\frac{0\cdot693}{1\cdot388\times10^{-11}}\ s$$

$$=\frac{0\cdot693}{1\cdot388\times10^{-11}\times60\times60\times24\times365}$$

$$=\mathbf{1583{\cdot}2\ years}$$

13. $R=\frac{dN}{dt}=5\,\text{mci}=5\times3\cdot7\times10^{7}\ s^{-1},$

$$\lambda=\frac{0\cdot693}{T}=\frac{0\cdot693}{138}\ \text{day}^{-1}=\frac{0\cdot693}{138\times24\times60\times60}$$

$$=5\cdot812\times10^{-8}\ s^{-1}$$

Using the relation : $\frac{dN}{dt}=\lambda N$,

it can be obtained that

$$N=3{\cdot}183\times10^{15}$$

Therefore, mass of Po^{210},

$$m=\frac{210\times N}{\text{Avogadro number}}=\frac{210\times3\cdot183\times10^{15}}{6\cdot02\times10^{23}}$$

$$=\mathbf{1{\cdot}11\times10^{-6}\ g}$$

14. $R_0=4{\cdot}0\times10^6\ s^{-1}$; $R=1{\cdot}0\times10^6\ s^{-1}$ and $t=20$ hours

Now, $\frac{R}{R_0}=\left(\frac{1}{2}\right)^{t/T}$ or $\frac{1\cdot0\times10^6}{4\cdot0\times10^6}=\left(\frac{1}{2}\right)^{20/T}$

or $\left(\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^{20/T}$ or $\frac{20}{T}=2$

or $\therefore$ T = 10 hours

Let R′ be count rate, 100 hours after beginning. Then,

$$\frac{R'}{R_0}=\left(\frac{1}{2}\right)^{t/T}\quad\text{or}\quad R'=R_0\left(\frac{1}{2}\right)^{t/T}$$

or $R'=4\cdot0\times10^6\times\left(\frac{1}{2}\right)^{100/10}=4\cdot0\times10^6\times\left(\frac{1}{2}\right)^{10}$

$$=\frac{4\cdot0\times10^6}{1024}=\mathbf{3{\cdot}91\times10^3\ s^{-1}}$$

15. Let N be the total number of atoms of the radioacti

sample present initially. If $\frac{dN_1}{dt}$ and $\frac{dN_2}{dt}$ are decay rat

by the first and second process, then

$$\frac{dN_1}{dt}=\lambda_1 N\quad\text{and}\quad\frac{dN_2}{dt}=\lambda_2 N$$

Here, λ_1 and λ_2 are decay constants for the first and seco

process respectively.

Therefore, decay rate of sample by both the processes,

$$\frac{dN_1}{dt}+\frac{dN_2}{dt}=\lambda_1 N+\lambda_2 N$$

or $\frac{dN_1}{dt}+\frac{dN_2}{dt}=(\lambda_1+\lambda_2)N$ ...

If λ is effective decay constant for the radioactive samp

then

$$\frac{dN_1}{dt}+\frac{dN_2}{dt}=\lambda N$$ ...(

From equations (*i*) and (*ii*), we have

$$\lambda N=(\lambda_1+\lambda_2)N\quad\text{or}\quad\lambda=\lambda_1+\lambda_2$$

or $\frac{\lambda}{0\cdot693}=\frac{\lambda_1}{0\cdot693}+\frac{\lambda_2}{0\cdot693}$ ...(i

Now, $\frac{0\cdot693}{\lambda_1}=T_1,\ \frac{0\cdot693}{\lambda_2}=T_2$

and $\frac{0\cdot693}{\lambda_1}=T$, effective half-life of the sample.

Therefore, equation (*iii*) becomes

$$\frac{1}{T}=\frac{1}{T_1}+\frac{1}{T_2}$$

16. Number of atoms in 1 mg of Th^{232},

$$N=\frac{6\cdot02\times10^{23}}{232}\times10^{-3}=2\cdot595\times10^{18}$$

α-particles are emitted by the thorium source in all th directions *i.e.* in a solid angle of 4 π steradian. Therefor

$$\frac{dN}{dt}=22\times4\pi\ \ \alpha\text{-particles minute}^{-1}$$

Using the relation : $\frac{dN}{dt}=\lambda N$, it can be obtained that

$$T_a = \frac{1}{\lambda} = 9\cdot3865\times10^{15}\ \text{minutes}$$

$$= \frac{9\cdot3865\times10^{15}}{60\times24\times365} = \mathbf{1\cdot786\times10^{10}\ years}$$

17. Initial count rate, $R_0 = 360\ s^{-1}$,
$T = 30$ minutes $= 0\cdot5$ hour
Initial distance between source and counter, $r = 2$ m
Suppose that distance between the source and counter is altered to r' so that after time t (= 1·5 hour) the count rate becomes, $R' = 5\ s^{-1}$
The change in count rate is both due to radioactive disintegration and due to change in intensity of γ-radiation at the counter due to change of its distance from the source.

Suppose that count rate changes from R_0 to R due to radioactive disintegration only. Then,

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T} \quad \text{or} \quad \frac{R}{360} = \left(\frac{1}{2}\right)^{1\cdot5/0\cdot5} \quad \text{or} \quad \frac{R}{360} = \left(\frac{1}{2}\right)^{3}$$

$$\text{or} \quad R = \frac{360}{8} = 45\ s^{-1}$$

The change in count rate from R to R′ is due to change in intensity of γ-radiation at the counter. Since intensity of γ-radiation varies inversely as the square of distance,

$$\frac{R}{R'} = \left(\frac{r'}{r}\right)^2 \quad \text{or} \quad r' = r\times\left(\frac{R}{R'}\right)^{1/2}$$

$$\text{or} \qquad r' = 2\times\left(\frac{45}{5}\right)^{1/2} = 2\times3 = \mathbf{6\ m}$$

CHAPTER

3

Nuclear Reactions

3.01. NUCLEAR REACTIONS

The heavy nuclei such as $_{92}U^{238}$ undergo disintegration, by their own, resulting into the formation of the nuclei of new elements. Such nuclear disintegrations are called **natural nuclear transmutations.**

Quotable Quote

"I do no know with what weapons Worl War 3 will be fought, but World War will be fought with sticks and stones."

– Albert Einstei

The natural nuclear transmutations led Rutherford to think that it might be possible to disintegrate atomic nuclei by artificial means. In 1919, he was able to bring about the first artificial transmutation by bombarding nitrogen nuclei with α-particles which produced an isotope of oxygen and proton. The reaction taking place may be represented by the equation :

$$_7N^{14} + {_2He^4} \longrightarrow {_8O^{17}} + {_1H^1}$$

Such nuclear transmutation carried out by artificial means is called a nuclear reaction.

We know that in a chemical reaction, the rearrangement of atoms takes place. Similarly, in a nuclear reaction, the rearrangement of nucleons takes place, when the bombarding particle strikes the target nucleus.

After the successful attempt made by Rutherford, many other such nuclear reactions were carried out. The bombarding particles called *projectiles*, generally used were α-particle ($_2He^4$), proton ($_1H^1$), deutron ($_1H^2$), neutron ($_0n^1$) and gamma-ray photon (γ). This led to the formation of more than 500 artificial radioactive isotopes. The nuclear reaction

$$_7N^{14} + {_2He^4} \longrightarrow {_8O^{17}} + {_1H^1}$$

is referred as (α, p) *reaction.* A few important nuclear reactions are given below :

1. α-particle as projectile :

(i) (α-p) reactions

$$_{12}Mg^{24} + {_2He^4} \longrightarrow {_{13}Al^{27}} + {_1H^1}$$
$$_{13}Al^{27} + {_2He^4} \longrightarrow {_{14}Si^{30}} + {_1H^1}$$

(ii) (α-n) reactions

$$_3Li^7 + {_2He^4} \longrightarrow {_5B^{10}} + {_0n^1}$$
$$_{11}Na^{23} + {_2He^4} \longrightarrow {_{13}Al^{26}} + {_0n^1}$$

2. Proton as projectile :

(i) (p-α) reactions

$$_5B^{11} + {_1H^1} \longrightarrow {_4Be^8} + {_2He^4}$$
$$_{13}Al^{27} + {_1H^1} \longrightarrow {_{12}Mg^{24}} + {_2He^4}$$

(ii) (p-n) reactions

$$_3Li^7 + {_1H^1} \longrightarrow {_4Be^7} + {_0n^1}$$
$$_8O^{18} + {_1H^1} \longrightarrow {_9F^{18}} + {_0n^1}$$

(iii) (p-d) reactions

$$_3Li^7 + {_1H^1} \longrightarrow {_3Li^6} + {_1H^2}$$
$$_4Be^9 + {_1H^1} \longrightarrow {_4Be^8} + {_1H^2}$$

(iv) (p-γ) reactions

$$_6C^{12} + {_1H^1} \longrightarrow {_7N^{13}} + \gamma$$
$$_{13}Al^{27} + {_1H^1} \longrightarrow {_{14}Si^{28}} + \gamma$$

3. Neutron as projectile :

(i) (n-α) reactions

$$_7N^{14} + {_0n^1} \longrightarrow {_5B^{11}} + {_2He^4}$$
$$_{11}Na^{23} + {_0n^1} \longrightarrow {_9F^{20}} + {_2He^4}$$

(ii) (n-p) reactions

$$_7N^{14} + {_0n^1} \longrightarrow {_6C^{14}} + {_1H^1}$$
$$_{30}Zn^{64} + {_0n^1} \longrightarrow {_{29}Cu^{64}} + {_1H^1}$$

(iii) (n-γ) reactions

$${}_{1}H^{1} + {}_{0}n^{1} \longrightarrow {}_{1}H^{2} + \gamma$$
$${}_{29}Cu^{65} + {}_{0}n^{1} \longrightarrow {}_{29}Cu^{66} + \gamma$$

4. Deutron as projectile :

(i) (d-α) reactions

$${}_{8}O^{16} + {}_{1}H^{2} \longrightarrow {}_{7}N^{14} + {}_{2}He^{4}$$
$${}_{20}Ca^{40} + {}_{1}H^{2} \longrightarrow {}_{19}K^{38} + {}_{2}He^{4}$$

(ii) (d-p) reactions

$${}_{3}Li^{7} + {}_{1}H^{2} \longrightarrow {}_{3}Li^{8} + {}_{1}H^{1}$$
$${}_{11}Na^{23} + {}_{1}H^{2} \longrightarrow {}_{11}Li^{24} + {}_{1}H^{1}$$

(iii) (d-n) reactions

$${}_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{3} + {}_{0}n^{1}$$
$${}_{1}H^{2} + {}_{1}H^{3} \longrightarrow {}_{2}He^{4} + {}_{0}n^{1}$$

5. γ-ray photon as projectile. A nuclear reaction in which γ-ray or photon is ne projectile is called *photo-nuclear reaction* or *photo disintegration.*

$${}_{1}H^{2} + \gamma \longrightarrow {}_{1}H^{1} + {}_{0}n^{1}$$
$${}_{4}Be^{9} + \gamma \longrightarrow {}_{3}Li^{8} + {}_{1}H^{1}$$

3.02. SYMBOLIC REPRESENTATION OF NUCLEAR REACTION

In general, a nuclear reaction may be represented by the equation :

$${}_{Z_1}X^{A_1} + {}_{Z_2}P^{A_2} \longrightarrow {}_{Z_3}Y^{A_3} + {}_{Z_4}O^{A_4} + Q,$$

vhere X, P, Y and O represent the *target nucleus, bombarding, particle, residual nucleus* nd *outgoing* particle (or group of particles) respectively and Q represents the energy equired to carry out the nuclear reaction or the energy liberated in the reaction.

Further, all the nuclear reactions obey the following conservation laws :

1. ***Law of conservation of nucleon number.*** According to this law,

$$A_1 + A_2 = A_3 + A_4$$

2. ***Law of conservation of charge number.*** According to this law,

$$Z_1 + Z_2 = Z_3 + Z_4$$

3. ***Law of conservation of mass and energy.*** According to this law, the energy equired to carry out the nuclear reaction or the energy liberated is equal to the nergy equivalent of the masses of the nuclei on the two sides of the above equation.

4. ***Law of conservation of momentum.*** The linear momenta of residual nucleus nd outgoing particle is always equal to the linear momenta of target nucleus and ombarding particle.

3.03. COMPOUND NUCLEUS

From the discussion made in last sections, it appears as if a nuclear reaction is *one stage process i.e.* bombarding particle strikes the target nucleus resulting into he formation of a new nucleus and a new particle. According to Bohr, many nuclear eactions proceed in two stages.

In the *first stage,* the bombarding particle strikes the target nucleus to form a ew nucleus called ***compound nucleus.*** The atomic and the mass numbers of the ompound nucleus are respectively the sum of the atomic and mass numbers of the arget nucleus and bombarding particle. A compound nucleus may be formed in a ifferent way but from the compound nucleus one cannot know the manner in which t was formed.

In the *second stage,* the compound nucleus decays into a new nucleus by emitting a particle or a group of particles. This is because the compound nucleus is unstable.

Therefore, a nuclear reaction may be represented as

$$X + P \longrightarrow [C] \longrightarrow Y + O$$

Here, [C] represents the compound nucleus. Further compound nucleus may decay by a number of different modes. That the formation of compound nucleus is a reality, is hinted by the fact that it has a life time of the order of 10^{-16} s. It is very large as compared to the time 10^{-21} s, which a bombarding particle (on an average) requires to cross through the nucleus.

3.04. Q-VALUE OF A NUCLEAR REACTION

In a nuclear reaction, sum of the masses of the target nucleus (X) and the bombarding particle (P) may be greater or less than the sum of the masses of the product nucleus (Y) and the outgoing particle (O). To account for the law of conservation of mass-energy*, the nuclear reaction is represented as :

$$X + P \longrightarrow Y + O + Q,$$

where Q is called ***Q-value of the nuclear reaction.***

If the mass of the product nucleus and outgoing particle is less than the mass of the target nucleus and bombarding particle, then Q-value of nuclear reaction will be positive. Such nuclear reaction is called ***exoergic reaction.***

On the other hand, *if the mass of the product nucleus and outgoing particle is greater than the mass of the target nucleus and bombarding particle, the Q-value of nuclear reaction will be negative.* Such nuclear reaction is called ***endoergic reaction.***

The liberation or absorption of energy in the nuclear reaction takes place in accordance with Einstein's mass-energy equivalence formula.

Key point

The changes in nuclei during nuclear reactions alter the potential energies of the nuclei and particle involved so that reactions are a source of nuclear energy.

3.05. NUCLEAR FISSION

The process of splitting of a heavy nucleus into two nuclei of nearly comparable masses with liberation of energy is called nuclear fission.

Fission was first observed, when Fermi and his coworkers were trying to produce *transuranic element* by bombarding $_{92}U^{235}$ with slow neutrons. As a result, compound nucleus $_{92}U^{236}$ was formed. It disintegrated into the two nuclei $_{56}Ba^{141}$ and $_{36}Kr^{92}$ with the release of three neutrons and a lot of thermal energy. The fission reaction of $_{92}U^{235}$ may be represented as given below :

$$_{92}U^{235} + {}_0n^1 \longrightarrow [_{92}U^{235}] \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q$$

The energy (Q) released was estimated to be 200 MeV per fission (or about 0·9 MeV per nucleon) and is equivalent to the difference in masses of the nuclei before and after the fission. Each of the three neutrons carries an energy of about 2 MeV. As such, these fast neutrons will escape and cannot cause the fission of other U^{235} nuclei. In order to utilise them to cause fission of three other U^{235} nuclei and so on, they have to be slowed down.

Key point

Fission is breaking of a heavy nucleus into two nuclei of comparable sizes.

Theory. Neil Bohr and J.A. Wheeler explained the nuclear fission on the basis of ***liquid drop model*** of the nucleus. The $_{92}U^{235}$ nucleus behaves like a liquid drop and owing to surface tension is perfectly spherical in shape. When the neutron strikes the nucleus, some energy called the excitation energy is imparted to the nucleus. The phenomenon of surface tension tries to keep the nucleus spherical in shape, whereas the excitation energy tries to deform it. Due to the struggle between the surface tension and the excitation energy, the oscillations are set up inside the compound nucleus [Fig. 3.01]. As a result of this, the nucleus continuously gets deformed from spherical shape to ellipsoidal and then to the shape of a dumb-bell, till the Coulomb's repulsive force between protons succeeds in tearing the two bells apart in a manner that the products are again spherical in shape. This is illustrated in Fig. 3.01.

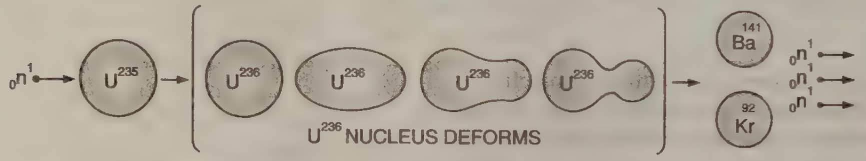


Fig. 3.01

The jargon

The liquid drop model of the nucleus : like a drop of water – a nucleus has a limited size ; a nucleus cannot be compressed ; a nucleus can split into smaller parts.

Further, it has been found that the fission is most favourable energetically, when the two fragments are of unequal sizes. Fig. 3.02 shows the distribution of the mass numbers of the fragments from the fission of $_{92}U^{235}$ by slow moving neutrons. The curve is symmetrical about $A = \frac{235}{2}$ but strong peaks occur in the vicinity of A = 92 and another around A=141. It indicates that when the fission of U^{235} takes place, its splitting into the nuclei Ba^{141} and Kr^{92} is most favourable.

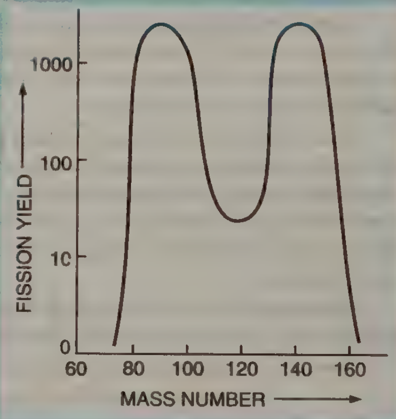


Fig. 3.02

3.06. NUCLEAR CHAIN REACTION

When U^{235} nucleus is bombarded with a thermal neutron, it splits into Ba^{141} and Kr^{92} along with the liberation of three neutrons and an energy of 200 MeV per fission reaction. The fission reaction may be represented as

---

*In nuclear reaction, bombarding particle strikes the target nucleus with some energy. Further, product nucleus and outgoing particle may also appear with some energy. In the present discussion, the mass will include the rest mass and the mass equivalent to energy possessed by the nucleus or the particle.

$$_{92}U^{235} + {_0n^1} \longrightarrow {_{56}Ba^{141}} + {_{36}Kr^{92}} + 3\,{_0n^1} + Q$$

The above fission reaction has two important characteristics :

(i) The energy liberated per fission reaction is very large.

(ii) In each fission reaction, three more neutrons are produced. The three eutrons may, then, cause the fission of three other U^{235} nuclei producing 9 neutrons nd so on. Due to the multiplication of neutrons in this manner, the whole of the J^{235} block may undergo fission in a very short time. Such a fission reaction is called ***ncontrolled chain reaction.*** It is called so because of the reason that once the fission eaction gets initiated, then it proceeds at such a fast pace that it cannot be shut-ff. An atom bomb is an example of an uncontrolled fission reaction.

Controlled chain reaction. To achieve a controlled chain reaction, arrange-nent is made for the partial absorption of neutrons available from each U^{235} ission. Fig. 3.03 shows the controlled chain reaction in which one neutron is bsorbed out of the three neutrons from one U^{235} fission. The neutrons are bsorbed by using cadmium rods, which act as ***control rods.*** The system in which he nuclear chain reaction may proceed in a controlled manner is called ***nuclear eactor.***

Watch out !

One should clearly understand the distinction between *absorbing neutrons* and *absorbing their energy.* The moderator should absorb only the energy ; the control rod must do away with the neutrons themselves !

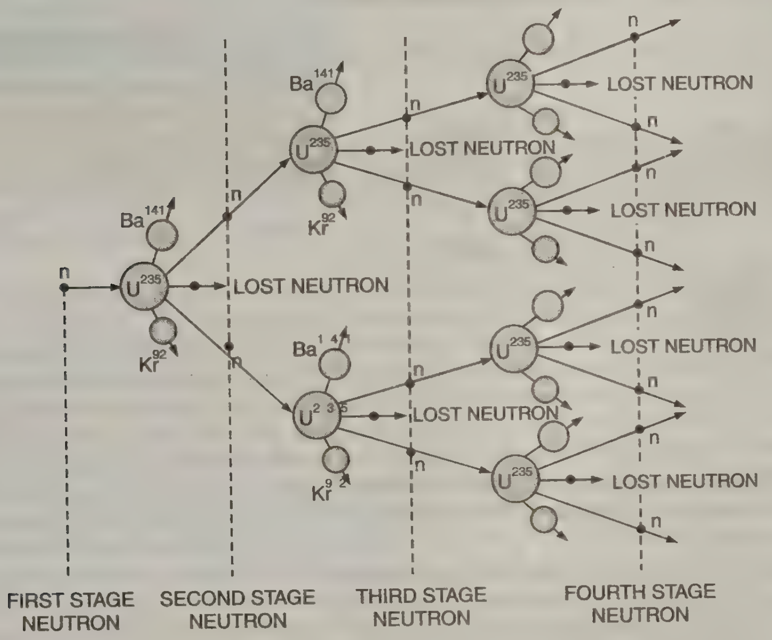


Fig. 3.03

A fission chain reaction may die out because of the following reasons :

1. The energy of the thermal neutrons required for the fission of U^{235} nuclei is of the order of 0·025 eV. However, the neutrons produced in the fission of U^{235} possess quite large energy as compared to the energy of thermal neutrons. As such, the neutrons escape the U^{235} block without causing the fission of U^{235} nuclei and due to this, the fission reaction may come to a stop.

So that the fission reaction does not stop, the neutrons produced in fission of U^{235} nuclei are slowed down by using graphite rods or heavy water. Such substances are rich in hydrogen nuclei. When fast moving neutrons collide against them, neutrons almost come to rest. Such substances used for slowing down the neutrons are called ***moderator.***

2. The neutrons produced in the U^{235} block leak from the block, if the size of the block is smaller than the ***critical size.***

The U^{235} block is said to be of critical size, if the number of neutrons lost per second is just equal to the number of neutrons produced per second in the block.

The mass of the U^{235} block of critical size is called ***critical mass.*** Some neutrons are also lost due to their absorption by the impurities present in the U^{235} block.

So that the fission reaction proceeds smoothly, the number of neutrons produced per second should be greater than the number of neutrons lost per second from the block. It is achieved by having U^{235} block of size bigger than the critical size. The loss of neutrons from the U^{235} block is also minimised by providing a layer of non-fissionable material over the U^{235} block. In fact, whether a chain reaction will remain steady or accelerate or retard is determined by a factor, called ***neutron reproduction factor (k).***

Neutron reproduction factor is defined as the ratio of the rate of production of neutrons to the rate of loss of neutrons. Thus,

$$k = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}} \qquad ...(3.01)$$

A fission reaction will be steady, in case $k = 1$. In case $k > 1$, the fission reaction will accelerate and it will retard, in case $k < 1$.

3.07. NUCLEAR REACTOR

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. By making use of uranium as fuel, the products such as neutrons (useful for causing fission of uranium), radioisotopes and heat energy (to run turbines) are produced.

Construction. A nuclear reactor consists of thick blocks of carbon surrounded by thick absorbing walls of concrete as shown in Fig. 3.04. The other parts of the nuclear reactor are as explained below :

Watch out !

99·3% of uranium is the isotope U^{238} ; only 0·7% of mixed uranium is of the isotope U^{235} which provides the power in a normal fission reactor !

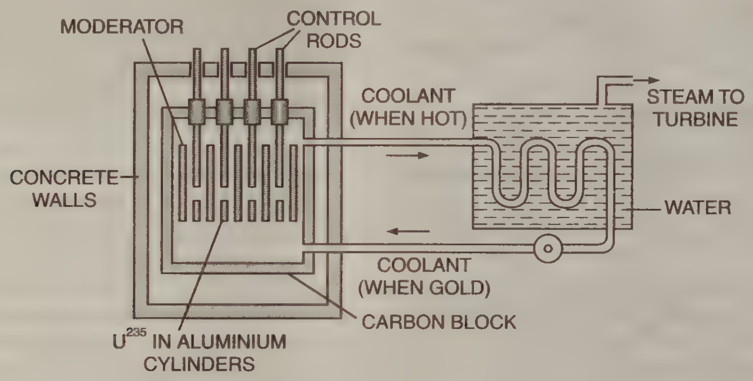


Fig. 3.04

1. ***Nuclear fuel.*** The fissionable material used in the reactor is called nuclear fuel. It generally consists of ${}_{92}U^{235}$ sealed in aluminium cylinders. These cylinders are inserted in the holes drilled in the carbon blocks.

2. ***Moderator.*** The material used to slow down the fast moving neutrons produced as a result of nuclear fission is called moderator. The neutrons released in the fission of uranium possess energy of the order of 2 MeV. Moderator reduces their energy to 0·0235 eV, which corresponds to the thermal motion of the neutron. For this reason, the neutrons slowed down to the energy of 0·0235 eV are called ***thermal neutrons.*** In a nuclear reactor, the moderator is either graphite (carbon) or water or heavy water (deuterium oxide).

The Apsra reactor at Bhabha Atomic Research Centre, Trombay uses water as moderator. It was commissioned in the year 1956. It is a swimming pool type nuclear reactor. It has rendered a great service in the field of genetic studies in agricultural crops. The nuclear reactors at Tarapur, Kalpakkam, Kota and Narora are used for power production and in these reactors, heavy water is used as moderator.

3. ***Control rods.*** The materials that can absorb the neutrons are used to control the nuclear chain reaction. Cadmium* or boron rods are used for this purpose. They can be moved in or out of the holes drilled in the carbon blocks. When the control rods are completely pushed into the carbon blocks, they absorb the neutrons to such an extent that the chain reaction comes to a halt. As the rods are slowly withdrawn, the chain reaction proceeds and more we withdraw the rods, stronger is the intensity of the chain reaction.

4. ***Coolant.*** The material used to absorb the heat generated as a consequence of chain reaction is called coolant. The coolant releases the heat energy to the water and the water is thus converted into super heated steam, which is used to run the turbines. These turbines in turn are used to operate the machines, say electric

---

*Cadmium has high cross-section for neutron absorption.

generator. Liquid sodium may also be used as coolant. Sometimes heavy water is used as coolant.

5. ***Protective shield.*** To prevent the spreading of the radioactive effect to the space around the nuclear reactor, it is enclosed in thick concrete walls called protective shield. The protective shield may be 10 m thick.

Working. A single slow neutron causes the fission of ${}_{92}U^{235}$ nucleus, with the release of 200 MeV energy and three fast neutrons. The cadmium rods partially absorb the neutrons and the moderator slows down the remaining neutrons. The slow neutrons carry out the fission of other ${}_{92}U^{235}$ nuclei and so on. In order to step up the reaction, the cadmium rods are slowly withdrawn, while to step down the reaction, the cadmium rods are slowly introduced. In addition to control rods, the reactors are provided with safety rods. For the fair functioning of a nuclear reactor, its k-factor is kept close to unity by moving the cadmium rods in or out.

The k-factor is defined as *the ratio of the number of fissions produced by a given generation of neutrons to the number of fissions of the preceeding generation of neutrons.* It is also called ***reproduction factor.***

Key point

Fission powers nuclear power stations.

Uses. 1. Nuclear reactors are used in electric power generation.

2. They are used to produce radioactive isotopes for their use in medical science, agriculture and industry.

3. They are also used to produce neutron beams of very high intensity for their use in nuclear research.

3.08. NUCLEAR FUSION

When two or more than two light nuclei fuse together to form heavy nucleus with the liberation of energy, the process is called nuclear fusion. For example, two deutrons can fuse together to form a helium nucleus releasing 24 MeV of energy. The fusion reaction may be expressed as below :

$$ {}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + 24 \text{ MeV} $$

Key point

Fusion is sticking together of two light nuclei.

The above nuclear fusion reaction is energetically possible, only if the mass of the ${}_2He^4$ nucleus is less than the sum of the masses of the two deutron nuclei. The difference in the initial mass and the final mass of the product nuclei is liberated as the energy of fusion reaction. It may be pointed out that the fusion of two light nuclei takes place in an attempt to achieve greater stability. It is because, the high value of the binding energy per nucleon is one of the factors responsible for the greater stability of a nucleus. It follows from the binding energy curve that in the low mass number region, the binding energy per nucleon increases with the mass number. When two lighter nuclei fuse together to form a heavier nucleus, the net mass defect and hence the binding energy of the nucleus so formed will be more.

To carry out the fusion of two nuclei, they must be brought so much close to each other that they overcome the electrostatic repulsion and come within the attractive range of the nuclear forces. This is possible only, when they approach each other with kinetic energy of the order of 0·1 MeV or more. This is best obtained by raising the temperature of the two nuclei to about 10^7 K. At this temperature, the thermal motion of the atoms is with kinetic energy of the order of 0·1 MeV. For this reason, nuclear fusion reaction is also termed as ***thermonuclear reaction***.

Temperature of the order of 10^7 K is very difficult to obtain. However, this can be triggered by atomic explosion involving fission. Once the fusion begins, the energy liberated can maintain the minimum required temperature and the fusion continues. The thermonuclear fusion is thought to be the source of energy in the sun and the stars.

We have already seen that the fission of one ${}_{92}U^{235}$ nucleus releases 200 MeV of energy. Thus, the energy liberated in the above mentioned fusion reaction is smaller (24 MeV) than that released in case of U^{235} fission reaction. However, if we work out the energy released for same mass in the two cases, we find that the energy released in the fusion reaction is about seven times as large as in fission reaction.

3.09. STELLAR ENERGY (ENERGY GENERATION IN THE SUN AND STARS)

The sun radiates energy at a tremendous rate of 4×10^{16} J s^{-1}. The stars are also radiating a huge amount of energy. Further, they have been radiating energy for several billions of years. It has been estimated that the chemical processes like burning or nuclear fission cannot account for such a large burst of radiation for several billions of years. Whereas no known chemical process can be source of such

Key point

Fusion powers the sun and the stars.

a large amount of energy, possibility of fission as the cause of this energy is ruled out on the basis of the fact that sun does not have the required abundance of heavy nuclei of fissionable matter. On the other hand, about 90% of the mass of the sun consists of hydrogen and helium.

Thus, we conclude that thermonuclear reactions are the main source of energy in the sun as well as in the stars. Two types of cyclic processes are proposed to be occurring in the sun and the stars as explained below :

1. Carbon nitrogen cycle. The process was proposed by Bethe in 1938. A proton reacts with carbon-12 producing nitrogen-13 and releasing energy. The nitrogen-13 is unstable and emits a positron producing carbon-13, which further reacts with a proton, producing nitrogen-14 and releasing energy. The nitrogen-14 further reacts with a proton producing oxygen-15 and energy. The oxygen-15 is unstable, which emits a positron and some energy converting itself into nitrogen-15. The nitrogen-15 again reacts with a proton producing carbon-12 and helium. Here, carbon acts as a nuclear catalytic agent. The process can be symbolically represented as follows :

$$\begin{array}{rcl}
{}_1H^1 + {}_6C^{12} & \longrightarrow & {}_7N^{13} + Q_1 \\
{}_7N^{13} & \longrightarrow & {}_6C^{13} + {}_1e^0 \\
{}_1H^1 + {}_6C^{13} & \longrightarrow & {}_7N^{13} + Q_2 \\
{}_1H^1 + {}_7N^{14} & \longrightarrow & {}_8O^{15} + Q_3 \\
{}_8O^{15} & \longrightarrow & {}_7N^{15} + {}_1e^0 + Q_4 \\
{}_1H^1 + {}_7N^{15} & \longrightarrow & {}_6C^{12} + {}_2He^4
\end{array}$$

All the above nuclear reactions, when summed up, give the following net nuclear reaction :

$$4\,{}_1H^1 \longrightarrow {}_2He^4 + 2\,{}_1e^0 + Q$$

i.e. four protons combine together to yield helium and two positrons along with the release of 24·7 MeV of energy (Q). The carbon atom with which the reaction commenced, is returned at the end of the complete cycle. In other words, it acts merely as a catalytic agent.

2. Proton-proton cycle. Another nuclear fusion cyclic process, which can account for the release of same amount of energy is as below :

$$\begin{array}{rcl}
{}_1H^1 + {}_1H^1 & \longrightarrow & {}_1H^2 + {}_1e^0 + Q_1 \\
{}_1H^1 + {}_1H^2 & \longrightarrow & {}_2He^3 + Q_2 \\
{}_1H^1 + {}_2He^3 & \longrightarrow & {}_2He^4 + {}_1e^0 + Q_3,
\end{array}$$

These nuclear reactions can be summed up as

$$4\,{}_1H^1 \longrightarrow {}_2He^4 + 2\,{}_1e^0 + Q$$

The net reaction and the release of energy Q = 24·7 MeV are same in two cases. It is found that the carbon-nitrogen cycle occurs at rather late stage in the life of a star. The proton-proton cycle takes place at comparatively low temperatures and the carbon-nitrogen cycle takes place at predominantly high temperatures. However, in the interior of our sun, the temperature is of the order of 2×10^6 K, at which both of the above mentioned processes are equally probable. Stars having temperature more than that of the sun obtain their energy from carbon-nitrogen cycle and those at lower temperature from proton-proton cycle.

3.10. RADIATION HAZARDS

Radioactive elements and radioisotopes are continuously emitting radiations in the form of α, β and γ–rays. The exposure of skin to X-rays causes severe burns. But a person exposed to radiations like α, β and γ–rays may get damaged to an extent which may ultimately lead to his death and yet he may not be aware of the damage. Madame Curie and Enrico Fermi lost their lives, because of the damage caused by these radiations.

When the radiation such as β-ray enters a living system, the cells and tissues get damaged due to interaction with radiation. The damaged cells and tissues hamper the normal functioning of the living system leading ultimately to the death of the organism. The experiments conducted on persons (already suffering from radiation hazards) reveal that the following types of damages can be caused by the radiations.

1. The exposure to radiation induces deleterious genetic effects.
2. The strong α-ray exposure can cause lung cancer.
3. The exposure to fast and slow neutrons can cause blindness.

4. The exposure to neutrons, protons and α-particles can cause damage to red blood cells.
5. The exposure to α-particles can cause disastrous effects.
6. The strong exposures to protons and neutrons can cause serious damage to productive organs.

Safety measures from radiation hazards. Following precautions are observed y the workers engaged in this field :

1. The radioisotopes are transferred in thick walled lead containers and are kept in rooms with thick walls of lead.
2. The radioisotopes are handled with the help of remote control devices.
3. The workers are asked to wear lead aprons.
4. The radioactive contamination of the work area is avoided at all costs.

3.11. NUCLEAR HOLOCAUST

The energy released on the fission of one U^{235} nucleus is about 200 MeV or $200 \times 1{\cdot}6 \times 10^{-13}$ J. Therefore, energy released, when say 50 kg of U^{235} undergoes ission will be

$$Q = 200 \times 1{\cdot}6 \times 10^{-13} \text{ number of } U^{235} \text{ nuclei in 50 kg}$$

$$= 200 \times 1{\cdot}6 \times 10^{-13} \times \frac{50 \times 6{\cdot}025 \times 10^{26}}{235} \approx 4 \times 10^{15} \text{ J}$$

The same amount of energy can be made available from 20,000 tons of TNT. This simple comparison reveals that nuclear fission is a great source of energy and if 50 kg of U235 in made to undergo fission, it will result in a super explosion. In fact, in 1945, USA dropped an atom bomb (fission bomb) on Hiroshima, (Japan) which produced an explosion equivalent to 20,000 tons of TNT. The radioactive fallout immediately engulfed 10 km² of the city and more than 67% of the structure of the city was destroyed and about 1,35,000 people were either killed or injured.

The fusion bombs are still more devastating. A super explosion of 10^7 tons of TNT was first tested in 1954 by causing the fusion of deuterium and tritium (isotopes of hydrogen).

It is estimated that superpowers have so much of nuclear weapons in their store, which are sufficient to destroy several times the every type of life on the earth. Such a nuclear holocaust will not only destroy the existing life, but the radioactive fallout on this account will render the earth unfit for life for all times.

SOLVED NUMERICAL PROBLEMS

Type A. On Nuclear Reactions

Problem 3.01. Complete the following nuclear reactions :

(*i*) $_{13}Al^{27} + {}_0n^1 \longrightarrow ? + {}_2He^4$

(*ii*) $_{15}P^{31} + \gamma \longrightarrow ? + {}_0n^1$

(*iii*) $_9F^{19} + {}_1H^1 \longrightarrow {}_8O^{16} + ?$

Sol. (*i*) $_{13}Al^{27} + {}_0n^1 \longrightarrow ? + {}_2He^4$

Let the residual nucleus be $_ZY^A$.

Then, $A = (27 + 1) - 4 = 24$

and $Z = (13 + 0) - 2 = 11$

Therefore, residual nucleus is $_{11}Na^{24}$ and the complete nuclear reaction is

$$_{13}Al^{27} + {}_0n^1 \longrightarrow {}_{11}Na^{24} + {}_2He^4$$

(*ii*) $_{15}P^{31} + \gamma \longrightarrow ? + {}_0n^1$

Let the residual nucleus be $_ZY^A$.

Then, $A = (31 + 0) - 1 = 30$

and $Z = (15 + 0) - 0 = 15$

Therefore, residual nucleus is $_{15}P^{30}$ and the complete nuclear reaction is

$$_{15}P^{31} + \gamma \longrightarrow {}_{15}P^{30} + {}_0n^1$$

(*iii*) $_9F^{19} + {}_1H^1 \longrightarrow {}_8O^{16} + ?$

Let the outgoing particle be $_ZO^A$.

Then, $A = (19 + 1) - 16 = 4$

and $Z = (9 + 1) - 8 = 2$

Therefore, outgoing particle is $_2He^4$ (α-particle) and the complete nuclear reaction is

$$_9F^{19} + {}_1H^1 \longrightarrow {}_8O^{16} + {}_2He^4$$

Problem 3.02. Write down the nuclear reaction equation :

(*a*) When neutrons bombard $_{16}S^{32}$ nuclei, radioactive phosphorus $_{15}P^{32}$ is produced.

(*b*) When $_{24}Si^{28}$ nuclei are bombarded by neutrons, protons issue out from the target.

Sol. (*a*) Let the outgoing particle be $_ZO^A$.

Then, nuclear reaction becomes

$$_{16}S^{32} + {}_0n^1 \longrightarrow {}_{15}P^{32} + {}_ZO^A$$

Now, $A = (32 + 1) - 32 = 1$

and $Z = (16 + 0) - 15 = 1$

Therefore, the outgoing particle is $_1H^1$ (proton) and the complete nuclear reaction is

$$_{14}S^{32} + {}_0n^1 \longrightarrow {}_{15}P^{32} + {}_1H^1$$

(*b*) Let the residual nucleus be $_ZY^A$. Then, nuclear reaction becomes $_{16}Si^{28} + {}_0n^1 \longrightarrow {}_ZY^A + {}_1H^1$

Now, $A = (28 + 1) - 1 = 28$

and $Z = (14 + 0) - 1 = 13$

Therefore, the residual nucleus is $_{13}Al^{28}$. The complete nuclear reaction is

$$_{14}Si^{28} + {}_0n^1 \longrightarrow {}_{13}Al^{28} + {}_1H^1$$

Type B. On Q-value of a Nuclear reaction

Problem 3.03. Calculate the energy released in the reaction :

$$_3Li^6 + {}_0n^1 \longrightarrow {}_2He^4 + {}_1H^3$$

Given that mass of $_3Li^6$ = 6·015126 a.m.u. ; mass of $_2He^4$ = 4·002603 a.m.u. mass of $_1H^3$ = 3·016049 a.m.u. ; mass of $_0n^1$ = 1·008665 a.m.u. and 1 a.m.u. = 931 MeV.

(*C.B.S.E. 2002*)

Sol. The nuclear reaction is

$$_3Li^6 + {}_0n^1 \longrightarrow {}_2He^4 + {}_1H^3$$

Total mass of reactants ($_3Li^6$ and $_0n^1$) = 6·015126 + 1·008665 = 7·023791 a.m.u.

Total mass of products ($_2He^4$ and $_1H^3$) = 4·002603 + 3·016049 = 7·018652 a.m.u.

Decrease in mass = 7·023791 – 7·018652 = 0·005139 a.m.u.

Therefore, energy released = 0·005139 × 931 = **4·78 MeV**

Type C. On Nuclear fission

Problem. 3.04. How much U^{235} is consumed in a day in an atomic power house operating at 400 MW, provided the whole of the mass of U^{235} is converted into energy ?

(*P.S.S.C.E. 1998 S*)

Sol. Here, rate of production of energy at atomic power house,

$$P = 400 \text{ MW} = 400 \times 10^6 \text{ J s}^{-1}$$

Therefore, total energy produced in a day *i.e.* 24 × 60 × 60 s,

$$E = P \times 24 \times 60 \times 60 = 400 \times 10^6 \times 24 \times 60 \times 60$$
$$= 3\cdot456 \times 10^{13} \text{ J}$$

If mass of U^{235} consumed per day is *m* (kg) so as to produce the required amount of energy, then

$$E = m\,c^2$$

or $3\cdot456 \times 10^{13} = m\,c^2$

or $$m = \frac{3\cdot456\times10^{13}}{c^2} = \frac{3\cdot456\times10^{13}}{(3\times10^8)^2}$$

$$= 0\cdot384\times10^{-3} \text{ kg} = 0\cdot384 \text{ g}$$

Problem 3.05. The energy released by fission of one U^{235} atom is 200 MeV. Calculate the energy released in kWh, when 1 gram of U^{235} undergoes fission.

Sol. Now, number of atoms in 1 g of U^{235}

$$= \frac{\text{Avogadro number}}{\text{atomic weight}} = \frac{6\cdot023\times10^{23}}{235}$$

Energy released per fission = 200 MeV

Therefore, energy released on fission of 1 g of U^{235}

$$= \frac{6\cdot023\times10^{23}\times200}{235} = 5\cdot126\times10^{23} \text{ MeV}$$

$$= 5\cdot126 \times 10^{23} \times 1\cdot6 \times 10^{-13} \text{ J} = 8\cdot2 \times 10^{10} \text{ W s}$$

$$(\because 1 \text{ J} = 1 \text{ W s})$$

$$= \frac{8\cdot2\times10^{10}}{1000} = 8\cdot2\times10^7 \text{ kW s}$$

$$= \frac{8\cdot2\times10^7}{3600} = \mathbf{2\cdot278 \times 10^4 \text{ kW h}}$$

Problem 3.06. It is estimated that the energy released in the atomic bomb explosion at Hiroshima was 7·6 × 10^{13} J. If on the average 200 MeV energy was released on fission of one $_{92}U^{235}$ atom, calculate (*i*) the number of uranium atoms fissioned, (*ii*) the mass of uranium used for the bomb.

Sol. Here, total energy released, E = 7·6 × 10^{13} J

Energy released per fission = 200 MeV = 200 × 1·6 × 10^{-13}

$$= 3\cdot2 \times 10^{-11} \text{ J}$$

(*i*) Number of U^{235} atoms fissioned,

$$n = \frac{7\cdot6\times10^{13}}{3\cdot2\times10^{-11}} = 2\cdot375 \times 10^{24}$$

(*ii*) Mass of U^{235} used $= \frac{235}{\text{Avogadro number}} \times n$

$$= \frac{235\times2\cdot375\times10^{24}}{6\cdot023\times10^{23}} = \mathbf{926\cdot66 \text{ g}}$$

Problem 3.07. A slow neutron strikes a nucleus of $_{92}U^{235}$ splitting it into lighter nuclei of barium and krypton and releasing three neutrons. Write the corresponding nuclear reaction. Also calculate the energy released in this reaction. Given, $m(_{92}U^{235})$ = 235·043933 a.m.u., $m(_0n^1)$ = 1·008665 a.m.u., $m(_{56}Ba^{141})$ = 140·917700 a.m.u. $m(_{36}Kr^{92})$ = 91·895400 a.m.u.

(*C.B.S.E. 1995*)

Sol. The nuclear reaction can be represented as

$$_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q$$

If $m_N(_{92}U^{235})$, $m_N(_{56}Ba^{141})$, $m_N(_{36}Kr^{92})$ represent masses of $_{92}U^{235}$, $_{56}Ba^{141}$ and $_{36}Kr^{92}$ nuclei respectively and $m(_0n^1)$, the mass of neutron, then energy released is given by

$$Q = [m_N(_{92}U^{235}) + m(_0n^1) - m_N(_{56}Ba^{141}) - m_N(_{36}Kr^{92}) - 3\,m(_0n^1)] \times 931\cdot5$$

$$= [m_N(_{92}U^{235}) - m_N(_{56}Ba^{141}) - m_N(_{36}Kr^{92}) - 2\,m(_0n^1)] \times 931\cdot5 \quad ...(i)$$

If $m(_{92}U^{235})$, $m(_{56}Ba^{141})$ and $m(_{36}Kr^{92})$ represent masses of $_{92}U^{235}$, $_{56}Ba^{141}$ and $_{36}Kr^{92}$ atoms respectively, then equation (*i*) may be expressed as

$$Q = [\{m(_{92}U^{235}) - 92m_e\} - \{m(_{56}Ba^{141}) - 56m_e\} - \{m(_{36}Kr^{92}) - 36\,m_e\} - 2\,m(_0n^1)] \times 931\cdot5$$

$$= [m(_{92}U^{235}) - m(_{56}Ba^{141}) - m(_{36}Kr^{92}) - 2\,m(_0n^1)] \times 931\cdot5$$

$$= [235\cdot043933 - 140\cdot917700 - 91\cdot895400 - 2 \times 1\cdot008665] \times 931\cdot5 = 0\cdot2135 \times 931\cdot5$$

$$= \mathbf{198\cdot88 \text{ MeV}}$$

Note. It is not mentioned in the problem as to whether the given masses are atomic masses or nuclear masses. In case, the given masses are nuclear masses, the solution is quite simple. The problem has been solved by considering the given masses as atomic masses.

Type D. On Nuclear fusion

Problem 3.08. Assuming that four protons combine to form a helium nucleus and two positrons each of mass 0·000549 a.m.u., calculate the energy released. Given, mass of $_1H^1$ = 1· 007825 a.m.u. and mass of $_2He^4$ = 4·002603 a.m.u.

Sol. The nuclear fusion reaction can be written as :

$$_1H^1 + {}_1H^1 + {}_1H^1 + {}_1H^1 \longrightarrow {}_2He^4 + 2\,{}_1e^0 + Q$$

If $m_N(_1H^1)$ and $m_N(_2He^4)$ represent the masses of $_1H^1$ and $_2He^4$ nuclei respectively, then

$$Q = [4\,m_N(_1H^1) - m_N(_2He^4) - 2\,m_e] \times 931\cdot5 \text{ MeV} \quad ...(i)$$

Here, m_e represents the mass of the positron ($_1e^0$).

Here, $m_N(_1H^1)$ = 1·007825 a.m.u. ;

$m_N(_2He^4)$ 4·002603 a.m.u. ; m_e = 0·000549 a.m.u.

Substituting for $m_N({}_1He^1)$, $m_N({}_2He^4)$ and m_e in equation
we have

$$Q = [4 \times 1{\cdot}007825 - 4{\cdot}002603 - 2 \times 0{\cdot}000549] \times 931{\cdot}5$$
$$= 0{\cdot}027599 \times 931{\cdot}5 = \mathbf{25{\cdot}71\ MeV}$$

Problem 3.09. Bombardment of lithium with protons es rise to the following reaction :

$${}_3Li^7 + {}_1H^1 \longrightarrow {}_2He^4 + {}_2He^4 + \text{energy}$$

The atomic masses of lithium, hydrogen and helium 7·016 a.m.u., 1·008 a.m.u. and 4·004 a.m.u. respectively. d the initial energy of each α-particle. Given, 1 a.m.u. 31 MeV. *(I.S.C.E. 1995)*

Sol. If $m_N({}_3Li^7)$, $m_N({}_1Li^1)$ and $m_N({}_2He^4)$ are masses of ^{7}, ${}_1H^1$ and ${}_2He^4$ nuclei respectively, then Q-value of the reaction is given by

$$Q = [m_N({}_3Li^7) + m_N({}_1H^1) - 2\,m_N({}_2He^4)] \times 931$$

If $m\,({}_3Li^7)$, $m\,({}_1H^1)$ and $m\,({}_2He^4)$ are atomic masses of lithium, hydrogen and helium and m_e, the mass of an electron, then

$$Q = [\{m({}_3Li^7) - 3\,m_e\} + \{m({}_1H^1) - m_e\} - 2\,\{m({}_2He^4) - 2\,m_e\}] \times 931$$
$$= [m({}_3Li^7) + m({}_1H^1) - 2\,m({}_2He^4)\,] \times 931$$

Here, $m({}_3Li^7) = 7{\cdot}016$ a.m.u. ; $m({}_1H^1) = 1{\cdot}008$ a.m.u. ;
$m({}_2He^4) = 4{\cdot}004$ a.m.u.

Also, 1 a.m.u. = 931 MeV

$$\therefore \quad Q = [7{\cdot}016 + 1{\cdot}008 - 2 \times 4{\cdot}004] \times 931$$
$$= 0{\cdot}016 \times 931 = \mathbf{14{\cdot}896\ MeV}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 3.10. Consider one of the fission reactions of U^{235} by thermal neutrons :

$${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{38}Sr^{94} + {}_{54}Xe^{140} + 2\,{}_0n^1$$

e fission fragments are, however, not stable. They dergo successive β-decays, until ${}_{38}Sr^{94}$ becomes ${}_{40}Zr^{94}$ d ${}_{54}Xe^{140}$ becomes ${}_{58}Ce^{140}$. Estimate the total energy eased in the process. Is all that energy available as etic energy of the fission products (Zr and Ce) ? u are given that : mass of U^{235} atom = 235·0439 a.m.u. ; ass of neutron, m_n = 1·00866 a.m.u. ; mass of Zr^{94} atom = 9065 a.m.u. ; mass of Ce^{140} atom = 139·9055 a.m.u. *(Text Problem)*

Sol. The ultimate fission of ${}_{92}U^{235}$ may be represented

$${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{40}Zr^{94} + {}_{58}Ce^{140} + 6\,{}_1e^0 + 2\,{}_0n^1 + Q$$

If $m_N({}_{92}U^{235})$, $m_N({}_{40}Zr^{94})$ and $m_N({}_{58}Ce^{140})$ represent clear masses, then

$$Q = [\{m_N({}_{92}U^{235}) + m_n\} - \{m_N({}_{40}Zr^{94}) - m_N({}_{58}Ce^{140}) + 6\,m_e + 2\,m_n\}] \times 931{\cdot}5$$

$$\text{or } Q = [m_N({}_{92}U^{235}) + m_N({}_{40}Zr^{94}) - m_N({}_{58}Ce^{140}) - 6\,m_e - m_n] \times 931{\cdot}5 \quad ...(i)$$

Here, instead of nuclear masses, the mass of ${}_{92}U^{235}$, Zr^{94} and ${}_{58}Ce^{140}$, atoms have been given.

Therefore, nuclear masses of ${}_{92}U^{235}$, ${}_{40}Zr^{94}$ and ${}_{58}Ce^{140}$ oms may be found by subtracting mass of 92 electrons, electrons and 58 electrons respectively from their atomic asses.

If $m({}_{92}U^{235})$, $m({}_{40}Zr^{94})$ and $m({}_{58}Ce^{140})$ represent atomic asses, then equation (*i*) may be expressed as

$$Q = [\{m({}_{92}U^{235}) - 92\,m_e\} - \{m({}_{40}Zr^{94}) - 40\,m_e\} - \{m({}_{58}Ce^{140}) - 58\,m_e\} - 6\,m_e - m_n] \times 931{\cdot}5$$
$$= [m({}_{92}U^{235}) - m({}_{40}Zr^{94}) - m({}_{58}Ce^{140}) - m_n] \times 931{\cdot}5$$
$$= [235{\cdot}0439 - 93{\cdot}9065 - 139{\cdot}9055 - 1{\cdot}00866] \times 931{\cdot}5$$
$$= 0{\cdot}22324 \times 931{\cdot}5 = \mathbf{207{\cdot}95\ MeV}$$

All the available energy does not appear as the kinetic nergy of fission products. It is partly carried by neutrons and ectrons produced.

Problem 3.11. Consider the so called D-T reaction (deuterium-tritium fusion reaction), which may be the basic reaction of a future thermonuclear fusion reactor :

$${}_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + {}_0n^1$$

(*a*) Determine the amount of energy in MeV released in the reaction from the data :

mass of ${}_1H^2$ atom = 2·014102 a.m.u. ; mass of ${}_1H^3$ atom = 3·016049 a.m.u. ; mass of ${}_2He^4$ atom = 4·002603 a.m.u. ; mass of neutron, m_n = 1·008665 a.m.u.

(*b*) Consider the radius of both deuterium and tritium to be approximately 1·5 × 10^{-15} m. What is the kinetic energy needed to overcome coulomb repulsion ? To what temperature must the gases be heated to initiate the reaction ? *(Text Problem)*

Sol. (*a*) The D-T reaction is

$${}_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + {}_0n^1 + Q$$

If $m_N({}_1H^2)$, $m_N({}_1H^3)$ and $m_N({}_2He^4)$ represent nuclear masses, then

$$Q = [m_N({}_1H^2) + m_N({}_1H^3) - m_N({}_2He^4) - m_n] \times 931{\cdot}5 \text{ MeV} \quad ...(i)$$

Here, instead of nuclear masses, the masses of ${}_1H^2$, ${}_1H^3$ and ${}_2He^4$ atoms have been given. Therefore, nuclear masses of ${}_1H^2$, ${}_1H^3$ and ${}_2He^4$ atoms may be found by subtracting mass of 1 electron, 1 electron and 2 electrons respectively from their atomic masses.

If $m({}_1H^2)$, $m({}_1H^3)$ and $m({}_2He^4)$ represent atomic masses, then equation (*i*) may be expressed as

$$Q = [\{m({}_1H^2) - m_e\} + \{m({}_1H^3) - m_e\} - \{m({}_2He^4) - 2m_e\} - m_n] \times 931{\cdot}5$$
$$= [m({}_1H^2) + m({}_1H^2) - m({}_2He^4) - m_n] \times 931{\cdot}5$$
$$= [2{\cdot}014102 + 3{\cdot}016049 - 4{\cdot}002603 - 1{\cdot}008665] \times 931{\cdot}5$$
$$= 0{\cdot}018883 \times 931{\cdot}5 = \mathbf{17{\cdot}59\ MeV}$$

(*b*) The P.E. of the D-T system, when the two nuclei almost come in contact with each other is given by

$$P.E. = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1\,q_2}{r}$$

Here, $q_1 = 1{\cdot}6 \times 10^{-19}$ C (charge on deutrium nucleus)
$q_2 = 1{\cdot}6 \times 10^{-19}$ C (charge on tritium nucleus)
$r = 2 \times 1{\cdot}5 \times 10^{-15}$
(sum of the radii of two nuclei)

$$\text{Therefore, P.E.} = 9 \times 10^9 \times \frac{(1{\cdot}6 \times 10^{-19})^2}{2 \times 1{\cdot}5 \times 10^{-15}}$$
$$= 7{\cdot}68 \times 10^{-14}\ J$$

The kinetic energy needed to overcome Coulomb's repulsion is equal to the potential energy.

Therefore, the required kinetic energy = 7·68 × 10^{-14} J

At temperature T, the system will possess kinetic energy equal to $\frac{3}{2}$ k T, where k is Boltzmann's constant.

Therefore, required temperature is given by

$$\frac{3}{2}kT = 7\cdot68\times10^{-14}\,\text{J}$$

or $$T = \frac{2}{3}\times\frac{7\cdot68\times10^{-14}}{k} = \frac{2\times7\cdot68\times10^{-14}}{3\times1\cdot38\times10^{-23}}$$

$$= 3\cdot71\times10^{9}\,\text{K}$$

FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. In the nuclear decay reaction

$$_1H^1 \longrightarrow {}_0n^1 + {}_QX^P,$$ (*I.S.C.E. 1995*)

find P, Q and hence identify X.

Ans. Now, $P + 1 = 1$ or $P = 0$

Also, $Q + 0 = 1$ or $Q = 1$

Therefore, $_QX^P$ is positron ($_1e^0$).

Q. 3.02. U^{238} has atomic mass number of 92. It decays by emitting an alpha particle to form an isotope of thorium (Th). Write down an equation to represent this. (*I.S.C.E. 1997*)

Ans. $_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4$

Q. 3.03. Complete the following nuclear reaction :

$$_5B^{10} + (?) \longrightarrow {}_3Li^7 + {}_2He^4$$ (*H.P.S.S.C.E. 1998 S*)

Ans. Let $_ZX^A$ be the unknown bombarding particle.

Now, $5 + Z = 3 + 2$ or $Z = 0$

Also, $10 + A = 7 + 4$ or $A = 1$

Therefore, the unknown bombarding particle is $_0n^1$ **(neutron).**

Q. 3.04. When $_3Li^7$ is bombarded with a certain particle, two alpha aprticles are produced. Identify the bombarding particle. (*I.S.C.E. 1998*)

Ans. Let $_ZX^A$ be unknown particle. Then,

$$_3Li^7 + {}_ZX^A \longrightarrow {}_2He^4 + {}_2He^4$$

By proceeding as in VSQ 3.01 or 3.02, it follows that the unknown particle is $_1H^1$ **(proton).**

Q. 3.05. What do you mean by Q-value of a nuclear reaction ? (*P.S.S.C.E. 2002, 2000, 1999*)

Ans. Q-value of a nuclear reaction is equal to the energy equivalent of the difference in masses of the reacting nuclei and the product nuclei. Mathematically,

$$Q = [\text{mass of reacting nuclei} - \text{mass of product nuclei}] \times 931\text{ MeV}$$

Q. 3.06. What do you mean by fissile material ?

Ans. The material which can undergo nuclear fission easily is called fissile material.

Q. 3.07. What do you mean by transuranic elements ?

Ans. The elements having atomic number greater than that of uranium *i.e.* 92 are called transuranic elements.

Q. 3.08. What is the condition for a nuclear fission reaction to act as a real source of energy ?

Ans. It should be a sustained chain reaction.

Q. 3.09. What is a nuclear chain reaction ?

Ans. A nuclear reaction in which the projectile used to carry out the nuclear fission reaction gets multiplied as more and more such fission reactions take place, is called a nuclear chain reaction.

Q. 3.10. A chain reaction dies out. State the reasons, why this could happen.

Ans. A nuclear chain reaction dies out, when the neutrons produced are so fast that they escape the uranium block without interacting or when the neutrons are lost du to the extremely small size of the block.

Q. 3.11. Define critical mass for nuclear chain reactio (*P.S.S.C.E. 2002, 2000, 199*

Ans. Refer to section 3.06.

Q. 3.12. What are the essential parts of a nuclea reactor ? (*Pre-degree Kerala, 1991*

Ans. Nuclear fuel, moderator, control rods, protectiv shield and the coolant.

Q. 3.13. Why is heavy water used as a moderator in nuclear reactor ? (*C.B.S.E. 2001 ; H.P.S.S.C.E. 199*

Ans. In case, the neutrons produced are fast, they ma escape the uranium block without causing fission. Whe heavy water is used, the neutrons lose energy on collisio with the water molecules, get slowed down and becom suitable for carrying out fission reaction.

Q. 3.14. Write any one equation representing nuclea fusion reaction. (*C.B.S.E. 199*

Ans. Example of fusion reaction :

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + 24\text{ MeV}$$

Q. 3.15. A fusion reaction is more energetic than fission reaction. Comment.

Ans. In a nuclear fusion reaction, the energy liberate per unit mass of the nuclei taking part in the fusion reactio is many times larger than the energy liberated in a fissio reaction.

Q. 3.16. Nuclear fusion reactions are also known a thermonuclear reactions. Why ?

Ans. Refer to VSQ 3.16.

Q. 3.17. Explain one similarity and one dissimilarit between nuclear fission and fusion. (*P.S.S.C.E. 199*

Ans. Nuclear fission and fusion are similar in the sens that both are tremendous sources of energy. On the othe hand, the two processes are dissimilar in the sense that i fission, a heavy nucleus splits into two lighter fragments while in fusion, two light nuclei unite to form a heavier on

Q. 3.18. What is the source of solar energy ? (*H.P.S.S.C.E. 199*

Or

Name the principal source of the sun's energy. (*I.S.C.E. 199*

Or

Name the phenomenon by which energy is produce in a star. (*P.S.S.C.E. 1997 S*

Ans. Nuclear fusion reactions taking place in the sun.

Q. 3.19. The sun is constantly losing mass due t thermonuclear fusion. Comment.

Ans. In each fusion reaction taking place in the sun, ther is a small decrease in mass, which then appears as therma energy. For this reason, the sun is constantly losing mass du to thermonuclear fusion reactions.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS With Answers/Hints

Q. 3.01. What are nuclear reactions ? State the laws eyed by these reactions. Give one example of nuclear ction. (*P.S.S.C.E. 1999*)

Ans. Nuclear reaction. The reaction taking place inside nucleus is called a nuclear reaction.

In a nuclear reaction, following laws are obeyed :

1. Mass number is conserved. **2.** Charge number is served. **3.** Spin quantum number is conserved.

Example. $_4Be^9 + {}_2He^4 \longrightarrow {}_6C^{12} + {}_0n^1$

Q. 3.02. A neutron strikes a $_5B^{10}$ nucleus with the sub- uent emission of an alpha particle. Write the corres- nding nuclear reaction. Find the atomic number, mass mber and the chemical name of the remaining nucleus. (*C.B.S.E. 1995 S*)

Sol. Let $_ZX^A$ be the residual nucleus. Then, the nuclear ction may be expressed as

$$_0n^1 + {}_5B^{10} \longrightarrow {}_ZX^A + {}_2He^4$$

Since the nucleon number is conserved, we have

$$A + 4 = 1 + 10 \quad \text{or} \quad A = 7$$

Also, the proton number is separately conserved. There- e,

$$Z + 2 = 0 + 5 \quad \text{or} \quad Z = 3$$

The element having Z = 3 and A = 7 is **lithium (Li)**. ere-fore, the nuclear reaction may be expressed as

$$_0n^1 + {}_5B^{10} \longrightarrow {}_3Li^7 + {}_2He^4$$

Q. 3.03. What are alpha particles ? In the reaction

$$_ZX^A \longrightarrow Y + {}_2He^4,$$

give the atomic number and mass number of Y. (*H.S.S.C.E. 2002*)

Ans. Alpha particle. It is the nucleus of helium atom *i.e.* ntains two protons and two neutrons.

Atomic number of Y = Z – 2

and mass number of Y = A – 4

Q. 3.04. What is nuclear fission ? Give one represen- ive equation. (*H.S.S.C.E. 2002*)

Ans. Nuclear fission. Refer to section 3.05.

Example :

$$_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{141} + {}_{56}Kr^{92} + 3\,{}_0n^1 + \text{energy}$$

Q. 3.05. Explain the term chain reaction with a suitable ample. (*C.B.S.E. 1992*)

Ans. Refer to section 3.06.

Q. 3.06. Uranium $_{92}U^{238}$ is not suitable for chain action. Why ? (*H.P.S.S.C.E. 2001, 1995*)

Ans. The natural uranium consists of three isotopes mely U^{233}, U^{235} and U^{238}. The three isotopes have relative undances of 0·006%, 0·714% and 99·28% respectively. The ow moving neutron having energy of the order of 0·025 eV n cause the fission of U^{235} nuclei but the fission of U^{238} clei can be carried out with fast neutrons having energy ore than 1 MeV.

The natural uranium contains practically U^{238} nuclei, as e percentage of U^{235} nuclei is very small. Therefore, when eutrons of energy even greater than 1 MeV are used to carry ut the fission of U^{238} nuclei, the secondary neutrons get owed down on colliding against U^{238} nuclei and they are longer capable of causing further fission of U^{238} nuclei. For is reason, U^{238} is not suitable for chain reaction.

Q. 3.07. What is critical size and critical mass as regards to nuclear fission ?

Ans. During nuclear fission of U^{235}, three neutrons are produced in every fission reaction. These neutrons carry out the fission of the other U^{235} nuclei in the U^{235} block. In case, the size of U^{235} block is small, a few neutron may be lost from the block without causing fission. In case, the number of neutrons lost per second is just equal to the number of neutrons produced per second, then the size of the U^{235} block is called the critical size and the corresponding mass of the block is called critical mass.

Q. 3.08. What are thermal neutrons ?
(*P.S.S.C.E. 1999 S ; H.S.S.C.E. 1999 S ; C.B.S.E. 1993*)

Ans. The neutrons which have been slowed down as a result of the collisions against the hydrogen nuclei of the moderator are called thermal neutrons. Such neutrons possess energy of about 0·025 eV, which corresponds to the energy of the neutron at room temperature $\left(\frac{3}{2}kT\right)$. Such neutrons can cause the fission of $_{92}U^{235}$ nuclei.

Q. 3.09. What is heavy water ?

Ans. Two deutron atoms and an oxygen atom combine to form heavy water. Symbollically, heavy water is represented as D_2O. In a nuclear reactor, heavy water is used as moderator to slow down the neutrons.

Q. 3.10. Explain the role played by the moderator in a nuclear reactor. (*H.P.S.S.C.E. 2000*)

Ans. In a nuclear reactor, the fast neutrons may escape. So that a sustained nuclear chain reaction can take place, neutrons are slowed down by using graphite rods or heavy water. Such materials used to slow down the neutrons are called moderator.

Q. 3.11. Why are control rods made of cadmium in a nuclear reactor ?

Ans. Cadmium can easily absorb neutrons. In a chain reaction, if all the neutrons produced are available to cause the fission of Uranium atoms, then the fission reaction may take place at an uncontrolled rate. The cadmium rods are used to absorb a suitable fraction of the neutrons produced and a controlled chain reaction is obtained.

Q. 3.12. What is the purpose of (*i*) moderator and (*ii*) controller in a nuclear reactor ? (*I.S.C.E. 1998*)

Ans. Moderator. Refer to SAQ 3.10.

Controller. Refer to SAQ 3.11.

Q. 3.13. What is neutron reproduction factor ? Explain.

Ans. Refer to section 3.06.

Q. 3.14. Name a few safety problems that a nuclear engineer must cope with in designing a nuclear reactor.

Ans. The protective shield of the nuclear reactor should be strong enough to prevent the nuclear radiation from spreading to nearby space. Further, the nuclear waste in the reactor may still contain radioisotopes capable of emitting harmful radiation. Therefore, nuclear waste should be disposed off very carefully.

Q. 3.15. What is a nuclear fusion reaction ? Why is nuclear fusion difficult to carry out ?

Ans. Nuclear fusion reaction. Refer to section 3.08.

The fusion reaction requires very high temperature ($10^6 - 10^7$ K) condition and it has to be obtained by causing explosion due to the fission process. Further, due to very high temperature, no material wall can sustain in the region, where fusion is to be carried out.

Q. 3.16. A neutron is absorbed by a $_3Li^6$ nucleus with subsequent emission of an alpha particle. Write the corresponding nuclear reaction. Calculate the energy released in this reaction.

Given that $m\ (_3Li^6)$ = 6·015126 a.m.u. ; $m\ (_2He^4)$ = 4·002604 a.m.u. ; m_n = 1·008665 a.m.u. ; $m\ (_1H^3)$ = 3·016049 a.m.u. (*C.B.S.E. 1995*)

Sol. The nuclear reaction may be expressed as

$$_0n^1 + {_3Li^6} \longrightarrow {_1H^3} + {_2He^4} + Q,$$

where Q is the energy released.

If $m_N(_3Li^6)$, $m_N(_1H^3)$ and $m_N(_2He^4)$ represent nuclear masses, then

$$Q = [m_n + m_N(_3Li^6) - m_N(_1H^3) - m_N(_2He^4)] \times 931{\cdot}5$$
$$= [m_N + \{m(_3Li^6) - 3\ m_e\} - \{m(_1H^3) - m_e\} - \{m(_2He^4) - 2\ m_e\}] \times 931{\cdot}5$$
$$= [m_n + m(_3Li^6) - m(_1H^3) - m(_2He^4)] \times 931{\cdot}5$$
$$= [1{\cdot}008665 + 6{\cdot}015126 - 3{\cdot}016049 - 4{\cdot}002604] \times 931{\cdot}5$$
$$= 4{\cdot}786 \text{ MeV}$$

Q. 3.17. Explain, why nuclear fission should precede nuclear fusion.

Ans. The nuclear fusion reactions can take place under very high temperature condition, which is very difficult to obtain, However, the fusion reaction can be triggered atomic explosion involving nuclear fission. Once the fusi begins, the energy liberated can maintain the minimu required temperature and the fusion reaction continues. It for this reason that nuclear fission should precede nucle fusion.

Q. 3.18. Differentiate between nuclear fission a fusion.

Ans. Nuclear fission and fusion have the followi points of differences :

1. In a nuclear fission reaction, a heavy nucleus splits in two lighter nuclei, whereas in a nuclear fusion reaction, tw light nuclei unite to form a heavier nucleus.

2. The energy released per nucleon is very high in ca of nuclear fusion reaction in comparison to that in nucle fusion reaction.

3. The nuclear fusion reaction can be carried out o under very high temperature condition, while in case nuclear fission reaction, there is no such requirement.

Q. 3.19. Explain the source of energy in stars.

(*P.S.S.C.E. 20*

Or

Explain the production of energy in stars.

(*P.S.S.C.E. 1997, 20*

Ans. Refer to section 3.09.

Q. 3.20. How is the enormous amount of energy in s produced ? (*Pre-degree Kerala, 19*

Ans. Refer to section 3.09.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 3.01. If both the number of protons and the number of neutrons are conserved in each nuclear reaction, in what way is mass converted into energy (or vice-versa) in a nuclear reaction ? (*Text Question*)

Ans. The total rest mass of protons and neutrons is same on the two sides of the nuclear reaction as their number is conserved. But the total binding energy of nuclei on the two sides need not be equal. The difference in these energies appears as the energy released or absorbed (Q-value of the reaction). Since binding energy contributes to mass, we say that the difference in the total mass of the nuclei on the two sides gets converted into energy and vice-versa.

Q. 3.02. After a series of alpha and beta decays, plutonium-239 ($_{94}Pu^{239}$) becomes lead-207 ($_{82}Pb^{207}$). How many alpha and beta particles are emitted in the complete decay scheme ? State the reason in support of your answer.

Sol. Here, parent nucleus = $_{94}Pu^{239}$

daughter nucleus = $_{82}Pb^{207}$

When α-decay takes place, mass number decreases by 4 units, while atomic number by 2 units. On the other hand, when β-decay takes place, mass number remains unchanged and atomic number increases by one unit.

Thus, mass number changes (decreases) only in α-dec

In the given decay, mass number has decreased by 2 – 207 *i.e.* 32.

Therefore, number of α-particle emitted = 32/4 = 8

Due to emission of 8 α-particles, the atomic numb should decrease by 8 × 2 = 16 *i.e.* atomic number should ha become 94 – 16 = 78. Since, atomic number of daugh nucleus is 82, the plutonium must have emitted β-particl As in a β-decay, atomic number increases by one unit, number of β-particles emitted = 82 – 78 = 4.

Q. 3.03. Why is the neutron so effective as a bo barding particle ?

Ans. If we use positively charged particles such proton, deutron or α-particle, they experience large repulsi due to atomic nuclei. On the other hand, the negativ charged particles such as electron will be repelled by electron cloud around the nucleus.

However, neutrons having no charge can penetrate target without being repelled by the atomic nucleus or electron cloud. The neutron increases the neutron to prot ratio of the nucleus and tends to make it unstable and he causes the nuclear reaction.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is a nuclear reaction ? State the conservation laws obeyed by a nuclear reaction.
2. What is a nuclear reaction ? Why do you mean by Q-value of a nuclear reaction ?
3. Explain the phenomenon of fission. Give one repres tative equation. (*H.S.S.C.E. 1998 ; C.B.S.E. 19*
4. What is nuclear fission ? What is the source of energy nuclear fission ? (*Pre-degree Kerala, 19*

5. What is nuclear fission ? Describe a chain reaction.
6. Explain fission and state what is meant by fusion. *(I.S.C.E. 1997)*
7. In a controlled thermal fission reactor, what is the function of (*i*) the moderator, (*ii*) the control rods, (*iii*) the coolant and (*iv*) the heavy water ? *(C.B.S.E. 2001 S)*
8. What is 'nuclear fission' ? Give an example to illustrate it. What is the importance of fission phenomenon ? *(H.S.S.C.E. 2002)*
9. Draw a labelled diagram of a nuclear reactor and also write the functions of each part. *(P.S.S.C.E. 2001)*
10. What is nuclear fusion ? In what sense, is it different from nuclear fission ?
11. Distinguish between nuclear fusion and nuclear fission reactions. *(P.S.S.C.E. 2002)*
12. Explain the source of energy in sun. *(C.B.S.E. 2000 ; H.P.S.S.C.E. 2000)*
13. Explain energy generation in stars. *(H.S.S.C.E. 1999 ; C.B.S.E. 1995)*
14. Explain briefly about nuclear holocaust. *(P.S.S.C.E. 2001)*

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Explain the principle, construction and working of a nuclear reactor. *(P.S.S.C.E. 2002)*
2. With the help of a labelled diagram, describe the construction and working of a nuclear reactor. *(H.S.S.C.E. 2001)*
3. Explain the principle involved in a nuclear reactor. Name the main components of a nuclear reactor and state the function of each component. *(H.S.S.C.E. 2001)*
4. Discuss the basic principle of a nuclear reactor as an example of controlled chain reaction. *(H.S.S.C.E. 2000)*
5. Explain the process of release of energy in a nuclear reactor. Draw a labelled diagram of a nuclear reactor and write the function of each part. *(C.B.S.E. 1997)*
6. What is nuclear fission and nuclear fusion ? Give one example, each of fission and fusion. What is the cause of fusion ? *(H.P.S.S.C.E. 1998 S)*

FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

pe A. On Nuclear reaction

1. The nucleus of a polonium atom is represented as $_{84}Po^{218}$. What do you mean by this ? Due to radioactivity, an alpha particle and two beta particles are ejected from it in succession. State the nuclear structure of the atoms formed at each stage.
   [Ans. Z = 84 and A = 218 ; $_{82}Pb^{214}$, $_{83}Bi^{214}$, $_{84}Po^{214}$]
2. An aluminium nucleus ($_{13}Al^{27}$) when bombarded by a proton, emits an α-particle and converts to the nucleus of another element. Write down the equation of this nuclear reaction. Of what element is the changed nucleus of this reaction ? What are the number of protons and that of neutrons in the changed nucleus ?
   [**Ans.** $_{13}Al^{27} + {_1H^1} \longrightarrow {_{12}Mg^{24}} + {_2He^4}$; Magnesium ; 12, 12]
3. Calculate the energy generated in kilowatt hours, when 100 g of $_3Li^7$ are converted into $_2He^4$ by proton bombardment. Given : mass of $_3Li^7$ atom = 7·0183 a.m.u., mass of $_2He^4$ atom = 4·0040 a.m.u., mass of $_1H^1$ atom = 1·0081 a.m.u.
   [**Ans.** $6{\cdot}547 \times 10^6$ kWh]

pe B. On Nuclear fission

4. In the hypothetical fission reaction :
   $$_{92}X^{236} \longrightarrow {_aY^{141}} + {_{36}Z^b} + 3\,{_0n^1}$$
   What are the values of the numbers *a* and *b* ? Calculate the total energy released per nuclear fission in MeV units, when the masses in a.m.u. units are of neutron = 1·009, of X-nucleus = 235·891, of Y-nucleus = 140·673 and of Z nucleus 91·791. *(C.B.S.E. 2001 S)*
   [**Ans.** 56, 92, 372·4 MeV]
5. If 200 MeV energy is released in the fission of a single nucleus of $_{92}U^{235}$, how many fissions must occur per second to produce a power of 1 kW ?
   (P.S.S.C.E. 1999, 1996 S) [**Ans.** $3{\cdot}125 \times 10^{13}\ s^{-1}$]
6. Calculate the number of fission of U^{235} taking place per hour in a nuclear reactor, which develops a power of 10^6 kW. Given that the average energy released per fission is 200 MeV. [**Ans.** $1{\cdot}125 \times 10^{22}$]
7. Calculate the energy released by the fission of 2 g of $_{92}U^{235}$ in joule. Given, that energy released per fission is 200 MeV.
   [**Ans.** $1{\cdot}64 \times 10^{11}$ J]
8. What is the power output of $_{92}U^{235}$ reactor, if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy ? *(Roorkee 1988)*
   [**Ans.** $5{\cdot}85 \times 10^4$ kW]
9. Suppose that India has a target of producing by 2000 A.D. 100,000 MW of electric power, 10% of which is to be obtained from nuclear power plants. Suppose, we are given that on an average, the efficiency of utilization (*i.e.* conversion of electric energy produced in a reactor) is 25%. How much amount of fissionable uranium would our country need per year at the turn of this century ? Take the heat energy per fission of U^{235} to be about 200 MeV. Given, Avogadro number = $6{\cdot}023 \times 10^{23}\ mol^{-1}$.
   (Text Problem) [**Ans.** $1{\cdot}538 \times 10^4$ kg]

Type C. On Nuclear fusion

10. Consider the following reaction :
    $$_1H^2 + {_1H^2} \longrightarrow {_2He^4} + Q$$
    If mass of the deuterium atom = 2·0141 a.m.u. and mass of helium atom = 4·0024 a.m.u., find the energy released in the reaction. *(I.I.T. 1996)* [**Ans.** 24·02 MeV]
11. In a star, three α-particles join in succession to form $_6C^{12}$ nucleus. How much energy is evolved in this reaction ? Take mass of $_6C^{12}$ nucleus as 12·000000 a.m.u. and that of α-particle as 4·002603 a.m.u. [**Ans.** 7·27 MeV]
12. The sun is believed to be getting its energy from the fusion of four protons to form a helium nucleus and a pair of positrons. Calculate the release of energy per fusion in MeV. Mass of a proton = 1·007825 a.m.u. ; mass of positron = 0·000549 a.m.u. ; mass of helium nucleus = 4·002603 a.m.u. ; 1 a.m.u. is equivalent to 931 MeV.
    (Pre-degree Kerala, 1994) [**Ans.** 25·695 MeV]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. $_{84}Po^{218} \xrightarrow{\alpha-\text{decay}} {}_{82}Po^{214} \xrightarrow{\beta-\text{decay}}$

$$_{83}Bi^{214} \xrightarrow{\beta-\text{decay}} {}_{84}Po^{214}$$

2. Proceed as in problem no. 1.
3. The nuclear reaction takes place as below :

$$_3Li^7 + {}_1H^1 \longrightarrow 2\ {}_2He^4 + Q$$

$$\therefore \quad Q = [m({}_3Li^7) + m({}_1H^1) - 2\,m({}_2He^4)] \times 931$$
$$= [7{\cdot}0183 + 1{\cdot}0081 - 2 \times 4{\cdot}0040] \times 931$$
$$= 17{\cdot}13 \text{ MeV}$$

Now, number of $_3Li^7$ nuclei in 100 g

$$n = \frac{6 \cdot 02 \times 10^{23}}{7} \times 100 = 8 \cdot 6 \times 10^{24}$$

Therefore, total energy released, when 100 g of Li^7 is converted into He^4 nuclei

$$= n\,Q = 8{\cdot}6 \times 10^{24} \times 17{\cdot}13 \text{ MeV}$$
$$= 8{\cdot}6 \times 10^{24} \times 17{\cdot}13 \times 1{\cdot}6 \times 10^{-13} \text{ J (or Ws)}$$
$$= \frac{8 \cdot 6 \times 10^{24} \times 17 \cdot 13 \times 1 \cdot 6 \times 10^{-13}}{1000 \times 60 \times 60}$$
$$= \mathbf{6{\cdot}547 \times 10^6 \text{ kWh}}$$

4. $_{92}X^{236} \longrightarrow {}_aY^{141} + {}_{36}Z^b + 3\ {}_0n^1$

Obviously, $92 = a + 36$ or $a = 56$

Also, $236 = 141 + b + 3 \times 1$ or $b = 92$

$$Q = [m_X - (m_Y + m_Z + 3\,m_n)] \times 931$$
$$= [235{\cdot}891 - (140{\cdot}673 + 91{\cdot}791 + 3 \times 1{\cdot}009)] \times 931$$
$$= \mathbf{372{\cdot}4 \text{ MeV}}$$

5. Here, $P = 1 \text{ kW} = 10^3 \text{ W} = 10^3 \text{ J s}^{-1}$

Energy released in fission of one U^{235} nucleus,

$$Q = 200 \text{ MeV} = 200 \times 1{\cdot}6 \times 10^{-13} \text{ J}$$

Therefore, number of fission that should occur per second,

$$n = \frac{P}{Q} = \frac{10^3}{200 \times 1 \cdot 6 \times 10^{-13}} = \mathbf{3{\cdot}125 \times 10^{13} \text{ s}^{-1}}$$

6. Proceed as in problem no. 5.
7. Number of U^{235} nuclei in 2 g,

$$N = \frac{6 \cdot 02 \times 10^{23} \times 2}{235}$$

Therefore, energy released,

$$E = N \times 200 = \frac{6 \cdot 02 \times 10^{23} \times 2}{235} \times 200 \text{ MeV}$$
$$= \frac{6 \cdot 02 \times 10^{23} \times 2 \times 200 \times 1 \cdot 6 \times 10^{-13}}{235}$$
$$= \mathbf{1{\cdot}64 \times 10^{11} \text{ J}}$$

8. Mass of U^{235} used per second in the reactor,

$$m = \frac{2 \times 10^3}{30 \times 24 \times 60 \times 60} = 7 \cdot 716 \times 10^{-14} \text{ g}$$

Therefore, number of fission reactions per second

$$= \frac{6 \cdot 02 \times 10^{23}}{235} \times m$$
$$= \frac{6 \cdot 02 \times 10^{23} \times 7 \cdot 716 \times 10^{-4}}{235} = 1 \cdot 9766 \times 10^{18}$$

Therefore, usable energy released per second or power of nuclear reactor

$$= 1{\cdot}9766 \times 10^{18} \times 185 \text{ MeV s}^{-1}$$
$$= 1{\cdot}9766 \times 10^{18} \times 185 \times 1{\cdot}6 \times 10^{-13} \text{ J s}^{-1}$$
$$= 5{\cdot}85 \times 10^7 \text{ W} = \mathbf{5{\cdot}85 \times 10^4 \text{ kW}}$$

9. Energy released per fission = 200 MeV

Efficiency of utilization = 25%

Therefore, electrical energy produced per fission

$$= \frac{25}{100} \times 200 = 50 \text{ MeV}$$
$$= 50 \times 1{\cdot}6 \times 10^{-13} = 8{\cdot}0 \times 10^{-12} \text{ J}$$

Target of electric power = 100,000 MW

Electric power to be obtained from nuclear plants

$$= \frac{10}{100} \times 100{,}000 = 10{,}000 \text{ MW} = 10^{10} \text{ J s}^{-1}$$

Electric energy required in one year

$$= 10^{10} \times 60 \times 60 \times 24 \times 365 \text{ J}$$

Therefore, number of fission reactions required per ye

$$= \frac{10^{10} \times 60 \times 60 \times 24 \times 365}{8 \cdot 0 \times 10^{-12}} = 3 \cdot 942 \times 10^{28}$$

Hence, mass of U^{235} needed per year

$$= \frac{235}{6 \cdot 023 \times 10^{23}} \times 3 \cdot 942 \times 10^{28}$$
$$= 1{\cdot}538 \times 10^7 \text{ g} = \mathbf{1{\cdot}538 \times 10^4 \text{ kg}}$$

10. $Q = [2\,m({}_1H^2) - m({}_2He^4)] \times 931$

$$= (2 \times 2{\cdot}0141 - 4{\cdot}0024) \times 931 = \mathbf{24{\cdot}02 \text{ MeV}}$$

11. Proceed as in problem no. 10.
12. Proceed as in problem no. 10.

Competitive Examination File (Unit VIII)

REVISION AT A GLANCE

Chapter 1. Structure of Nucleus

☐ **Thomson's atom model.** In a sphere, the positive charge is uniformly distributed over the entire sphere and the electrons e embedded in it. The oscillations of electrons about their equilibrium positions give rise to radiations of definite frequency.

☐ **Rutherford's atom model.** Practically, entire mass of the atom and all its positive charge are concentrated in a small ntral core, while the electrons revolve around it. The central positive and massive core is called nucleus.

☐ **Isotopes.** *The atoms of an element (same atomic number) having different mass number are called isotopes.*

☐ **Isobars.** *The atoms of different elements (different atomic number) having the same mass number are called isobars.*

☐ **Atomic mass unit (a.m.u.).** *It is defined as* $\frac{1}{12}$ *th of the mass of one* $_6C^{12}$ *atom.* Its value is given by

$$\mathbf{1\ am.u. = 1{\cdot}660565 \times 10^{-27}\ kg = 931{\cdot}5\ MeV}$$

☐ **Nuclear size.** From scattering experiments, it is found that volume of a nucleus is always directly proportional to its ass number. It leads to the expression for nuclear radius as

$$R = R_0 A^{1/3},$$

here $R_0 = 1{\cdot}1 \times 10^{-15}$ m is known as ***nuclear unit radius.***

☐ **Nuclear density.** The density of nuclear matter is independent of the mass number of the nucleus and it is found to e of the order of 10^{17} kg m^{-3}.

☐ **Nuclear forces.** *The forces holding the nucleons together inside the nucleus are called nuclear forces.* These forces are short nge, basically very strong, charge independent, charge symmetric, spin dependent and non- central forces. These forces ist between nucleons due to exhcange of π-mesons (π^0, π^+ and π^-).

☐ **Mass defect.** *The difference between the sum of the masses of nucleons constituting a nucleus and the rest mass of the nucleus called mass defect.*

Mathematically : $\Delta m = [Z\, m_p + (A - Z)\, m_n] - m_N(_ZX^A)$

$= [Z\, m(_1H^1) + (A - Z)\, m_n] - m(_ZX^A)$

Here, $m_N(_ZX^A)$, $m(_ZX^A)$ and $m(_1H^1)$ represent mass of the nucleus of atom $_ZX^A$, mass of atom $_ZX^A$ and mass of hydrogen om respectively.

☐ **Binding energy.** *The energy equivalent to mass defect of the nucleus is called its binding energy.*

Mathematically : B.E. = $\Delta m \times 931{\cdot}5$ (in MeV)

☐ **Binding energy per nucleon.** *The average energy required to extract one nucleon from the nucleus is called its binding energy r nucleon.*

Mathematically : B.E. per nucleon $= \dfrac{\text{B.E.}}{A}$

☐ **Packing fraction.** *It is defined as mass defect per nucleon of the nucleus.*

Mathematically : Packing fraction $= \dfrac{\Delta m}{A}$

ome Useful Facts :

1. Rutherford α-scattering experiment established the existence of nucleus.
2. Neutron is unstable particle outside the nucleus.
3. Nuclear density is same for all nuclei *i.e.* independent of mass number of the nucleus.

Chapter 2. Radioactivity

☐ **Radioactivity.** *The spontaneous transformation of an element into another with the emission of radiations, such as α, β and rays is called radioactivity.*

☐ **Radioactive decay law.** *The number of atoms of a radioactive sample disintegrating per second at any time is directly proportional the number of atoms present at that time.*

Mathematically : $\dfrac{dN}{dt} = -\lambda N,$

here λ is called ***decay constant*** of the radioactive substance.

The above law may be expressed as

$$N = N_0\, e^{-\lambda t}$$

☐ **Radioactive decay constant.** *It is defined as the reciprocal of the time during which the number of atoms of a radioactive substance creases to* $\frac{1}{e}$ *th of their original number.*

☐ **Activity of a radioactive substance.** *It is defined as the rate at which the nuclei of the atoms in the sample disintegrate.*

Mathematically : $R = -\frac{dN}{dt}$

The radioactive decay law can be expressed as

$$R = R_0 e^{-\lambda t}$$

Units of activity : **1 curie (ci) = 3·7 × 10^{10} disintegrations s^{-1}**

1 rutherford (rd) = 10^6 disintegrations s^{-1}

☐ **Half-life.** *The half-life of a radioactive substance is defined as the time during which half of the atoms of a radioactive substan will disintegrate.*

Mathematically : $T = \frac{0 \cdot 693}{\lambda}$

☐ **Average-life.** *It is defined as the average time for which the atoms of the radioactive substance exist.*

Mathematically : $T_a = \frac{1}{\lambda}$

It follows that $T = 0 \cdot 693\, T_a$

☐ **α-decay.** It is represented by

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q$$

☐ **β-decay.** It is represented by

$$_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + \overline{\nu} + Q$$

☐ **γ-decay.** When on α or β-decay, the nucleus is left in excited state ; the nucleus goes to lower state or ground state b the emission of γ-ray (radiation of very short wavelength).

Some Useful Facts :

1. The total life of a radioactive sample is infinite.
2. After a time t (= n T), the fraction of the radioactive atoms left behind,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

3. After a time t (= n T), the fractional activity of a radioactive sample.

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

4. β-ray spectrum is continuous in nature, while in α-ray and γ-ray spectrum, the emitted energy has only discrete value

Chapter 3. Nuclear Reactions

☐ **Nuclear fission.** *The process of splitting of a heavy nucleus into two nuclei of nearly comparable masses with liberation of energ is called nuclear fission.*

☐ **Nuclear chain reaction.** *A nuclear reaction in which the projectile used to carry out the fission reaction (such as neutron U^{235} fission reaction) gets multiplied as more and more such fission reactions take place, is called nuclear chain reaction.*

☐ **Critical size.** *The fissionable material is said to be of critical size, if the number of neutrons lost per second is just equal to t number of neutrons produced per second in the block.*

The mass of the fissionable material having critical size is called ***critical mass.***

☐ **Nuclear reactor.** *The practical arrangement for carrying out controlled fission reaction so as to continuously obtain nucle energy is called a nuclear reactor.*

☐ **Nuclear fusion.** *When two or more light nuclei fuse together to form a heavy nucleus with the liberation of energy, the proce is called nuclear fusion.*

Some Useful Facts :

1. When nuclear fission and fusion reactions occur, in both cases, there is a decrease in nuclear mass.
2. Thermonuclear fusion reactions are source of stellar energy.

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Structure of Nucleus

Problem 1.01. The binding energies per nucleon for deutron ($_1H^2$) and helium ($_2He^4$) are 1·1 MeV and 7·0 MeV respectively. Calculate the energy released, when two deutrons fuse to form a helium nucleus ($_2He^4$).

(I.I.T. 1988)

Thought Process

The energy released in the fusion reaction is equal to t difference of final and initial binding energies.

Sol. Here, $\frac{\text{B.E.}}{\text{A}}$ of ${}_1\text{H}^2$ nucleus $= 1\cdot1$ MeV ;

$\frac{\text{B.E.}}{\text{A}}$ of ${}_2\text{He}^4$ nucleus $= 7\cdot0$ MeV

The fusion reaction may be represented as

$${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_2\text{He}^4 + \text{Q}$$

Therefore, total B.E. of ${}_1\text{H}^2$ nucleus

$$= 1\cdot1 \times 2 = 2\cdot2 \text{ MeV}$$

Hence, total initial B.E.

$$= 2\cdot2 + 2\cdot2 = 4\cdot4 \text{ MeV}$$

Also, total B.E. of ${}_2\text{He}^4$ nucleus

$$= 7\cdot0 \times 4 = 28\cdot0 \text{ MeV}$$

Therefore, energy released,

$$\text{Q} = 2830 - 4\cdot4 = \mathbf{23\cdot6\ MeV}$$

Chapter 2. Radioactivity

Problem 2.01. How many alpha and beta-particles are emitted, when uranium nucleus (${}_{92}\text{U}^{238}$) decays to lead (${}_{82}\text{Pb}^{206}$) ? *(M.N.R. 1987 ; I.I.T. 1985)*

Thought Process

The mass number in radioactive decay decreases only due to α-decay. For one α-particle emitted, mass number decreases by 4. On the other hand, the atomic number decreases by 2 on emission of an α-particle and increases by 1 on emission of a β-particle.

Sol. Let n_1 and n_2 be the number of α-and β-particles respectively, which transform a ${}_{92}\text{U}^{238}$ nucleus into ${}_{82}\text{Pb}^{206}$.

From principle of conservation of mass number :

$$238 - n_1 \times 4 = 206 \qquad \text{or} \qquad n_1 = 8$$

From principle of conservation of atomic number :

$$92 - n_1 \times 2 + n_2 \times 1 = 82 \qquad \text{or} \qquad 92 - 8 + n_2 = 82$$

or $\qquad n_2 = 6$

Problem 2.02. There is a stream of neutrons with a kinetic energy of 0·0327 eV. If the half life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 m ?

Given, mass of neutron = $1\cdot675 \times 10^{-27}$ kg . *(I.I.T. 1986)*

Sol. Here, half life of neutrons, T = 700 s ;

mass of neutron, $m = 1\cdot675 \times 10^{-27}$ kg ;

and kinetic energy of a neutron,

$$\frac{1}{2} m v^2 = 0\cdot0327 \text{ eV} = 0\cdot0327 \times 1\cdot6 \times 10^{-19} \text{ J}$$

or $\quad \frac{1}{2} \times 1\cdot675 \times 10^{-27}\, v^2 = 0\cdot0327 \times 1\cdot6 \times 10^{-19}$

or $\quad v = \sqrt{\frac{2 \times 0\cdot327 \times 1\cdot6 \times 10^{-19}}{1\cdot675 \times 10^{-27}}} = 2\cdot5 \times 10^3 \text{ m s}^{-1}$

Therefore, time taken by neutron to travel 10 m,

$$t = \frac{10}{2\cdot5 \times 10^3} = 4 \times 10^{-3} \text{ s}$$

Fraction of neutrons, that will be left after time t, is given by

$$\frac{\text{N}}{\text{N}_0} = \left(\frac{1}{2}\right)^{t/\text{T}} 1/2 = \left(\frac{1}{2}\right)^{\frac{4 \times 10^{-3}}{700}} = (0\cdot5)^{5\cdot7 \times 10^{-6}}$$

$$= 0\cdot999952$$

Therefore, fraction of neutrons that will decay in time t,

$$1 - \frac{\text{N}}{\text{N}_0} = 1 - 05999952 = 0\cdot000048 = \mathbf{4\cdot8 \times 10^{-5}}$$

Problem 2.03. Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently, the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days, the room will be safe for occupation ? *(M.N.R. 1992)*

Thought Process

The time t after which the level of radiation will become $\frac{1}{50}$ th of the present level is given by (in terms of half-life T)

$$\frac{\text{N}}{\text{N}_0} = \left(\frac{1}{2}\right)^{t/\text{T}}$$

Sol. Here, half-life of the radioactive substance,

$$\text{T} = 10 \text{ days}$$

Consider that the total number of radioactive atoms spread in the room are N_0 in number and due to their presence, the level of radiation is 50 times the permissible level. Suppose that the room becomes worth normal occupancy in time t. Obviously, in time t, the number of radioactive atoms in the room will become

$$\text{N} = \frac{\text{N}_0}{50}$$

or $$\frac{\text{N}}{\text{N}_0} = \frac{1}{50} \qquad ...(i)$$

If T is half-life of the radiation sample, then

$$\left(\frac{\text{N}}{\text{N}_0}\right) = \left(\frac{1}{2}\right)^{t/\text{T}} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\left(\frac{1}{2}\right)^{t/\text{T}} = \frac{1}{50} \text{ or } (0\cdot5)^{t/\text{T}} = 0\cdot02$$

Taking logarithm of both sides, we have

$$\frac{t}{\text{T}} \log 0\cdot5 = \log 0\cdot02 \text{ or } t = \frac{\log 0\cdot02}{\log 0\cdot5} \times \text{T}$$

or $\quad t = \frac{\bar{2}\cdot3010}{\bar{1}\cdot6910} \times \text{T} = \frac{-1\cdot6990}{-0\cdot3010} \times 10 = \mathbf{56\cdot45\ days}$

Problem 2.04. A count rate meter is used to measure the activity of a given sample. At one instant, the meter shows 4750 counts min^{-1}. Five minutes later, it shows 2700 counts min^{-1}. Find (*a*) the decay constant and (*b*) the half-life of the sample. (Given, $\log_{10} 1\cdot760 = 0\cdot2455$). *(Roorkee, 1986 ; M.N.R. 1985)*

Sol. (*a*) Let N_0 and N be the number of the nuclei present in the sample at $t = 0$ and at $t = 5$ min respectively.

Here, activity at $t = 0$, $R_0 = 4750$ counts min^{-1}

and activity at $t = 5$ min, $R = 2700$ counts min^{-1},

According to radioactive decay law,

activity at $t = 0$, $R_0 = \lambda N_0$

$$\therefore \quad 4750 = \lambda N_0 \qquad ...(i)$$

Also, activity at $t = 5$ min, $R = \lambda N$

$$\therefore \quad 2700 = \lambda N \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*), we have

$$\frac{4750}{2700} = \frac{\lambda N_0}{\lambda N}$$

$$\text{or} \quad \frac{N_0}{N} = 1{\cdot}760 \qquad ...(iii)$$

But the number of nuclei left after $t = 5$ min is given by

$$N = N_0 e^{-\lambda \times 5}$$

$$\text{or} \quad \frac{N_0}{N} = e^{5\lambda} \qquad ...(iv)$$

From equations (*iii*) and (*iv*), we have

$$e^{5\lambda} = 1{\cdot}760$$

$$\text{or } \lambda = \frac{1}{5}\log_e 1{\cdot}760 = \frac{1}{5} \times 2{\cdot}3036 \log_{10} 1{\cdot}760$$

$$= \frac{2{\cdot}3026 \times 0{\cdot}2455}{5} = \mathbf{0{\cdot}113\ min^{-1}}$$

(*b*) Half-life, $T = \dfrac{0{\cdot}693}{\lambda} = \dfrac{0{\cdot}693}{0{\cdot}113} = \mathbf{6{\cdot}13\ min}$

Problem 2.05. A small quantity of solution containing Na^{24} radionuclide (half-life = 15 hours) of activity 1·0 μci is injected into the blood of a person. A sample of blood of volume 1 cm^3 taken after 5 hours shows an activity of 296 disintegrations min^{-1}. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.

Given, 1 curie = 3·7 × 10^{10}disintegrations s^{-1}.

(*I.I.T. 1994*)

Sol. Here, half-life of Na^{24} ;

$$T = 15 \text{ hours} = 15 \times 60 \times 60 \text{ s}$$

Consider that the small quantity of Na^{24} solution injected into the blood of the person contains total nuclei of Na^{24} equal to N_0 at the time $t = 0$.

Initial activity of Na^{24} solution,

$$R_0 = 1\ \mu ci = 10^{-6} \times 3{\cdot}7 \times 10^{10}$$

$$= 3{\cdot}7 \times 10^4 \text{ disintegration s}^{-1}$$

$$\text{Now,} \quad R_0 = \lambda N_0 = \frac{0{\cdot}693}{T} N_0$$

$$\text{or} \quad N_0 = \frac{T}{0{\cdot}693} \times R_0 = \frac{15 \times 60 \times 60}{0{\cdot}693} \times 3{\cdot}7 \times 10^4$$

$$= 2{\cdot}833 \times 10^9$$

Suppose that at the time $t = 0$, 1 cm^3 sample of blood contained Na^{24} nuclei equal to N_0'. Further suppose that the number of Na^{24} nuclei in 1 cm^3 sample reduces to N′ in time $t = 5$ hours. Then, activity of 1 cm^3 sample of the blood at $t = 5$ h is given by

$$R' = \lambda N' = \frac{0{\cdot}693}{T} N'$$

$$\text{or} \quad N' = \frac{T}{0{\cdot}693} \times R'$$

But $R' = 296$ disintegrations min^{-1}

$$= \frac{296}{60} = 4{\cdot}933 \text{ disintegration s}^{-1}$$

$$\therefore \quad N' = \frac{T}{0{\cdot}693} \times R' = \frac{15 \times 60 \times 60}{0{\cdot}693} \times 4{\cdot}993 = 3{\cdot}844 \times 10^5$$

$$\text{Now,} \quad \frac{N'}{N_0} = \left(\frac{1}{2}\right)^{t/T}$$

$$\text{or} \quad N_0' = N' \times (2)^{t/T} = 3{\cdot}844 \times 10^5 \times (2)^{5/15}$$

$$= 3{\cdot}844 \times 10^5 \times 2^{1/3}$$

$$= 3{\cdot}844 \times 10^4 \times 1{\cdot}26 = 4{\cdot}843 \times 10^5$$

Therefore, volume of blood in the body of the person,

$$V = \frac{N_0}{N_0'} = \frac{2{\cdot}883 \times 10^9}{4{\cdot}843 \times 10^5} = \mathbf{5{\cdot}95 \times 10^3\ cm^3}$$

Problem 2.06. A radioactive nucleus X decays to a nucleus Y with decay constant $\lambda_X = 0{\cdot}1\ s^{-1}$. Y further decays to a stable nucleus Z with a decay constant $\lambda_Y = 1/30\ s^{-1}$. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$. Set up the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function time is given by

$$N_Y = \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y}\left(e^{-\lambda_Y t} - e^{-\lambda_X t}\right)$$

Find the time at which N_Y is maximum and determine the populations X and Y at that instant. (*I.I.T. 2001*)

Sol. Here, number of X-nuclei at $t = 0$, $N_0 = 10^{20}$

decay constant of X-nuclei, $\lambda_X = 0{\cdot}1$ s^{-1}

and decay constant of Y-nuclei, $\lambda_Y = 1/30$ s^{-1}

At $t = 0$, number of both Y and Z-nuclei are zero.

According to law of radioactivity, rate equation for the population of X-nuclei,

$$\frac{dN_X}{dt} = -\lambda_X N_X \qquad ...(i)$$

The number of Y-nuclei decreases because of its decay but at the same time increases due to decay of X-nuclei into it.

$$\therefore \quad \frac{dN_Y}{dt} = \lambda_X N_X - \lambda_Y N_Y \qquad ...(ii)$$

Since Z-nuclei formed due to the decay of Y-nuclei are stable, their number goes on increasing with the passage of time.

$$\therefore \quad \frac{dN_Z}{dt} = \lambda_Y N_Y \qquad ...(iii)$$

Now, population of Y-nuclei as a function of time is given by

$$N_Y = \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y}\left(e^{-\lambda_Y t} - e^{-\lambda_X t}\right) \qquad ...(iv)$$

For N_Y to be maximum,

$$\frac{dN_Y}{dt} = 0 \qquad ...(v)$$

From equations (*ii*) from (*v*), we have

$$\lambda_X N_X = \lambda_Y N_Y \qquad ...(vi)$$

$$\text{Now,} \quad N_X = N_0 e^{-\lambda_X t} \qquad ...(vii)$$

Using equations (*iv*) and (*vii*), equation (*vi*) becomes

$$\lambda_X N_0 e^{-\lambda_X t} = \lambda_Y \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y}\left[e^{-\lambda_Y t} - e^{-\lambda_X t}\right]$$

$$\text{or} \quad \frac{\lambda_X - \lambda_Y}{\lambda_Y} e^{-\lambda_X t} = \left[e^{-\lambda_Y t} - e^{-\lambda_X t}\right]$$

$$\text{or} \quad \left[\frac{\lambda_X}{\lambda_Y} - 1\right] e^{-\lambda_X t} = e^{-\lambda_X t}\left[\frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1\right]$$

$$\text{or} \quad \frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

$$\text{or} \quad (\lambda_X - \lambda_Y)t = \log_e \frac{\lambda_X}{\lambda_Y}$$

$$\text{or} \quad t = \frac{1}{\lambda_X - \lambda_Y} \log_e \frac{\lambda_X}{\lambda_Y} = \frac{1}{0{\cdot}1 - 1/30} \log_e \frac{0{\cdot}1}{1/30}$$

$$= 15 \log_e 3$$

$$= 15 \times 2{\cdot}303 \log_{10} 3 = 15 \times 2{\cdot}303 \times 0{\cdot}4771$$

$$= \mathbf{16{\cdot}48\ s}$$

To find populations of X and Y at t = 16·48 s :

Again, $N_X = N_0 e^{-\lambda_X t}$

$$\therefore \quad N_X (\text{at } t = 16{\cdot}48 \text{ s}) = 10^{20} \times e^{-0{\cdot}1 \times 16{\cdot}48}$$

$$= 10^{20} \times e^{-16{\cdot}48} = \mathbf{1{\cdot}924 \times 10^{19}}$$

From equation (*vi*), we have

$$N_Y = \frac{\lambda_X}{\lambda_Y} N_X$$

$$\therefore N_Y (\text{at } t = 16{\cdot}48 \text{ s}) = \frac{\lambda_X}{\lambda_Y} N_X (\text{at } t = 16{\cdot}48 \text{ s})$$

$$= \frac{0{\cdot}1}{1/30} \times 1{\cdot}924 \times 10^{19} = 5{\cdot}572 \times 10^{19}$$

Problem 2.07. Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element.

(*a*) Calculate the number N of nuclei of A at time *t*.

(*b*) If $= 2 N_0 \lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of N as $t \to \infty$. (*I.I.T. 1998*)

Sol. (*a*) Let N_0 and N be the number of nuclei of radioactive element A at times $t = 0$ and $t = t$ respectively. Then, according to radioactive decay law,

$$\frac{dN}{dt} = -\lambda N$$

Since the rate at which nuclei of radioactive element A are produced is α, the net rate of decay of the nuclei of radioactive element A,

$$\frac{dN}{dt} = -\lambda N + \alpha = \alpha - \lambda N$$

$$\text{or} \quad \frac{dN}{\alpha - N} = dt$$

Integrating both sides within proper limits, we have

$$\int_{N_0}^{N} \frac{dN}{\alpha - N} = \int_0^t dt \quad \text{or} \quad \left| -\frac{1}{\lambda} \log_e (\alpha - \lambda N) \right|_{N_0}^{N} = \left| t \right|_0^t$$

$$\text{or} \quad -\frac{1}{\lambda} \log_e (\alpha - \lambda N) - \left[-\frac{1}{\lambda} \log_e (\alpha - \lambda N_0) \right] = t - 0$$

$$\text{or} \quad \frac{1}{\lambda} \log_e \frac{\alpha - \lambda N_0}{\alpha - \lambda N} = t \quad \text{or} \quad \frac{\alpha - \lambda N_0}{\alpha - \lambda N} = e^{\lambda t}$$

$$\text{or } \alpha = \lambda N = (\alpha - \lambda N_0) e^{-\lambda t}$$

$$\text{or} \quad N = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0) e^{-\lambda t}\right] \qquad \text{...}(i)$$

It gives the number of nuclei of the radioactive element A at time *t*.

(*b*) Now, $\alpha = 2 N_0 \lambda$

Therefore, from equation (*i*), number of nuclei of element at time *t*,

$$N = \frac{1}{\lambda}\left[2 N_0 \lambda - \lambda N_0) e^{-\lambda t}\right]$$

$$\text{or} \quad N = N_0 (2 - e^{-\lambda t}) \qquad \text{...}(ii)$$

Therefore, number of nuclei of element A after one half-life of A can be found by setting $t = T = \frac{\log_e 2}{\lambda}$ *i.e.*

$$N = N_0 (2 - e^{-\lambda \times \log_e 2/\lambda}) = N_0 (2 - e^{-\log_e 2})$$

$$= N_0 \left(2 - \frac{1}{2}\right) = \frac{\mathbf{3}}{\mathbf{2}} \mathbf{N_0}$$

Also, number of nuclei of element A, when $t \to \infty$ is given by

$$N = \underset{t \to \infty}{\text{Lt}} [N_0 (2 - e^{-\lambda t})] = N_0 (2 - 0) = \mathbf{2 N_0}$$

Problem 2.08. A nucleus at rest undergoes a decay emitting an α-particle having de-Broglie wavelength $\lambda = 5{\cdot}76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223·60 a.m.u. and that of the α-particle is 4·002 a.m.u., determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in a.m.u. (1 a.m.u. = 931·470 MeV) (*I.I.T. 2001*)

Sol. Suppose that mass of parent nucleus X is M and it decays into daughter nucleus Y of mass M_1 by emitting α-particle of mass M_2. The decay takes place as shown in Fig. 2.01.

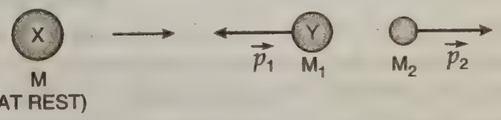


Fig. 2.01

Let $\vec{p}_1$ and $\vec{p}_2$ be linear momentum of daughter nucleus and α-particle respectively. Since parent nucleus is at rest, from the conservation of linear momentum,

$$\vec{p}_1 + \vec{p}_2 = 0 \quad \text{or} \quad |\vec{p}_1| = |\vec{p}_2|$$

Here, $M_1 = 223{\cdot}610$ a.m.u.

$$= 223{\cdot}610 \times 1{\cdot}67 \times 10^{-27} = 3{\cdot}734 \times 10^{-25} \text{ kg}$$

$M_2 = 4{\cdot}002$ a.m.u.

$$= 4{\cdot}002 \times 1{\cdot}67 \times 10^{-27} = 6{\cdot}683 \times 10^{-27} \text{ kg}$$

Now, de-Broglie wavelength of α-particle,

$$\lambda = 5{\cdot}76 \times 10^{-15} \text{ m}$$

$$\therefore \quad p_2 = \frac{h}{\lambda} = \frac{6{\cdot}62 \times 10^{-34}}{5{\cdot}76 \times 10^{-15}} = 1{\cdot}149 \times 10^{-19} \text{ kg m s}^{-1}$$

Also, $p_1 = 1{\cdot}149 \times 10^{-19}$ kg m s^{-1}

Let T_1 and T_2 be kinetic energy of daughter nucleus and α-particle respectively. Then,

$$T_1 = \frac{p_1^2}{2M_1} = \frac{(1{\cdot}149 \times 10^{-19})^2}{2 \times 3{\cdot}734 \times 10^{-25}} = \mathbf{0{\cdot}177 \times 10^{-13}\ J}$$

$$\text{and} \quad T_2 = \frac{p_2^2}{2M_2} = \frac{(1{\cdot}149 \times 10^{-19})^2}{2 \times 6{\cdot}683 \times 10^{-27}} = 9{\cdot}877 \times 10^{-13}\text{ J}$$

Therefore, total kinetic energy of the final state,

$$T = T_1 + T_2 = 0{\cdot}177 \times 10^{-13} + 9{\cdot}877 \times 10^{-13}$$
$$= 10{\cdot}054 \times 10^{-13}\text{ J}$$
$$= \frac{10{\cdot}054 \times 10^{-13}}{1{\cdot}6 \times 10^{-13}} = \mathbf{6{\cdot}284\ MeV}$$

If Δm is mass defect, then

$$M = (M_1 + M_2) - \Delta m$$

Now, Δm = mass equivalent to total kinetic energy of

the final state $= \dfrac{6{\cdot}284}{931{\cdot}470} = 0{\cdot}0067$ a.m.u.

Hence, M = (223·610 + 4·002) – 0·0067 = **227·605 a.m.u.**

Problem 2.09. In the chemical analysis of a rock, the mass ratio of two radioactive isotopes is found to be 100 : 1. The mean lives of the two isotopes are 4 × 10⁹ years and 2 × 10⁹ years respectively. If it is assumed that at the time of formation, the atoms of both the two isotopes were in equal proportion, calculate the age of the rock. The ratio of the atomic weights of two isotopes is 1·02 : 1.

(Roorkee, 1987)

Sol. Let M_A and M_B be the atomic weights of the two radioisotopes A and B present in the rock. Let mass of an atom of the two isotopes be m_A and m_B respectively.

Since mass of an atom is directly proportional to the atomic weight of the element, we have

$$\frac{m_A}{m_B} = \frac{M_A}{M_B} = \frac{1{\cdot}02}{1} \qquad \ldots(i)$$

If λ_A and λ_B are decay constant of the two radioactive isotopes, then

$$\lambda_A = \frac{1}{\text{mean life of A}} = \frac{1}{4 \times 10^9} = 2{\cdot}5 \times 10^{-10}\text{ year}^{-1}$$

$$\text{and } \lambda_B = \frac{1}{2 \times 10^9} = 5\ 10^{-10}\text{ year}^{-1}$$

Let at the time of formation of the rock *i.e.* at $t = 0$, the number of atoms of the two isotopes in the rock are N_{0A} and N_{0B} respectively. Since at the time of formation of the rock, the atoms of the two isotopes are in equal proportions,

$$\frac{N_{0A}}{N_{0B}} = 1 \qquad \ldots(ii)$$

Today, after time t (the age of the rock), suppose that the number of atoms of the two radioisotopes in the rock become N_A and N_B respectively. Since the mass ratio of the two isotopes is found to be 100 : 1,

$$\frac{\text{mass of radioisotope A at time } t}{\text{mass of radioisotope B at time } t} = \frac{m_A \times N_A}{m_B \times N_B}$$
$$= \frac{100}{1} = 100$$

$$\text{or} \quad \frac{N_A}{N_B} = 100 \times \frac{m_B}{m_A} = 100 \times \frac{1}{1{\cdot}02} = 98{\cdot}04$$

$$\text{But} \quad \frac{N_A}{N_B} = \frac{N_{0A}\, e^{-\lambda_A t}}{N_{0B}\, e^{-\lambda_B t}} = 1 \times e^{(\lambda_B - \lambda_A)t} \quad \left(\because \frac{N_{0A}}{N_{0B}} = 1\right)$$

$$\therefore \quad e^{(\lambda_B - \lambda_A)t} = \frac{N_A}{N_B}$$

$$\text{or} \quad e^{(5 \times 10^{-10} - 2{\cdot}5 \times 10^{-10})t} = 98{\cdot}04$$

$$\text{or} \quad e^{2{\cdot}5 \times 10^{-10}\, t} = 98{\cdot}04$$

$$\text{or} \quad 2{\cdot}5 \times 10^{-10}\, t = \log_e 98{\cdot}04 = 2{\cdot}3026 \log_{10} 98{\cdot}4$$
$$= 4{\cdot}5890$$

$$\text{or} \quad t = \frac{4{\cdot}5890}{2{\cdot}5 \times 10^{-10}} = \mathbf{18{\cdot}36 \times 10^9\ years}$$

Chapter 3. Nuclear Reactions

Problem 3.01. What is the power output of a $_{92}U^{235}$ reactor, if it takes 30 days to use up 2 kg of fuel and of each fission gives 185 MeV of usable energy ? *(Roorkee, 1988)*

Sol. Since 235 g *i.e.* 235 × 10⁻³ kg of U^{235} contains nuclei equal to Avogadro number (= 6·02 × 10²³),

the number of U^{235} nuclei used in the reactor in 30 days

$$= \frac{6{\cdot}023 \times 10^{23}}{235 \times 10^{-3}} \times 2 = 5{\cdot}1234 \times 10^{24}$$

Usable energy released during each fission = 185 MeV

Therefore, energy output of the reactor in 30 days

$$= 5{\cdot}1234 \times 10^{24} \times 185 = 9{\cdot}48 \times 10^{26}\text{ MeV}$$
$$= 9{\cdot}48 \times 10^{26} \times 1{\cdot}6 \times 10^{-13} = 1{\cdot}517 \times 10^{14}\text{ J}$$

Hence, power output of the reactor = energy released per second

$$= \frac{1{\cdot}517 \times 10^{14}}{30 \times 24 \times 60 \times 60}\text{ W} = 58{\cdot}53 \times 10^6\text{ W} = \mathbf{58{\cdot}53\ MW}$$

Problem 3.02. In a nuclear reactor U^{235} undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required. *(I.I.T. 2001)*

Sol. Here, power of the nuclear reactor,

$$P = 1000\text{ MW} = 10^9\text{ W} = 10^9\text{ J s}^{-1}$$

Time for which reactor is to function,

$$t = 10\text{ years} = 10 \times 365 \times 24 \times 60 \times 60 = 3{\cdot}1536 \times 10^8\text{ s}$$

Therefore, energy to be delivered by reactor in 10 years,

$$W = P \times t = 10^9 \times 3{\cdot}15\,36 \times 10^8 = 3{\cdot}1536 \times 10^{17}\text{ J}$$

$$\text{Efficiency of reactor, } \eta = 10\% = \frac{1}{10}$$

Hence, actual energy, the reactor is required to produce,

$$W' = \frac{W}{\eta} = 3{\cdot}1536 \times 10^{17} \times 10 = 3{\cdot}1536 \times 10^{18}\text{ J}$$

Energy liberated on fission of one U^{235} atom,

$$E = 200 \text{ MeV} = 200 \times 1{\cdot}6 \times 10^{-13} = 3{\cdot}2 \times 10^{-11} \text{ J}$$

Therefore, number of U^{235} atoms required to produce the needed energy,

$$n = \frac{W'}{E} = \frac{3{\cdot}1536 \times 10^{18}}{3{\cdot}2 \times 10^{-11}} = 9{\cdot}855 \times 10^{28}$$

Number of moles of U^{235} needed,

$$\mu = \frac{n}{\text{Avogadro number}}$$

$$= \frac{9{\cdot}855 \times 10^{28}}{6{\cdot}02 \times 10^{23}} = 1{\cdot}637 \times 10^{5}$$

Hence, mass of U^{235} needed,

$$m = \mu \times 235 = 1{\cdot}637 \times 10^{5} \times 235 \text{ g}$$

$$= \frac{1{\cdot}637 \times 10^{5} \times 235}{10^{3}} = \mathbf{3{\cdot}847 \times 10^{4}\ kg}$$

Problem 3.03. It is proposed to use the nuclear reaction :

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$$

in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium fuel will be needed per day ? The masses of $_1H^2$ and $_2He^4$ nuclei are 2·0141 a.m.u. and 4·0026 a.m.u. respectively. *(I.I.T. 1990)*

Sol. The fusion reaction used in reactor is

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$$

Therefore, energy released per fusion reaction,

$$E = \left[m_N\left({}_1H^2\right) + m_N\left({}_1H^2\right) - m_N\left({}_2He^4\right)\right] \times 931{\cdot}5$$

$$= (2{\cdot}0141 + 2{\cdot}0141 - 4{\cdot}0026) \times 931{\cdot}5$$

or $\quad E = 23{\cdot}846$ MeV

Power rating of the reactor,

$$P = 200 \text{ MW} = 200 \times 10^{6} \text{ J s}^{-1}$$

Energy to be produced per day,

$$W = P \times t = 200 \times 10^{6} \times 24 \times 60 \times 60 = 1{\cdot}728 \times 10^{13} \text{ J}$$

Now, efficiency of the reactor, $\eta = 25\% = \frac{25}{100} = \frac{1}{4}$

Hence, actual energy, the reactor is required to produce per day,

$$W' = \frac{W}{\eta} = \frac{1{\cdot}728 \times 10^{13}}{1/4} = 6{\cdot}912 \times 10^{13} \text{ J}$$

$$= \frac{6{\cdot}912 \times 10^{13}}{1{\cdot}6 \times 10^{-13}} = 4{\cdot}32 \times 10^{26} \text{ MeV}$$

Therefore, number of fusion reactions needed per day,

$$n = \frac{W'}{E} = \frac{4{\cdot}32 \times 10^{26}}{23{\cdot}846} = 1{\cdot}81 \times 10^{25}$$

Since in one fusion reaction, two deutrons are required, the number of deutrons needed per day,

$$N = n \times 2 = 1{\cdot}81 \times 10^{25} \times 2 = 3{\cdot}62 \times 10^{25}$$

Since a mass of 2 g of $_1H^2$ contains number of deuterium nuclei equal to Avogadro number *i.e.* $6{\cdot}02 \times 10^{23}$, mass of deuterium needed per day,

$$m = \frac{N}{\text{Avogadro number}} = \frac{3{\cdot}62 \times 10^{25}}{6{\cdot}02 \times 10^{23}} \times 2$$

$$= \mathbf{120{\cdot}266\ g}$$

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Structure of Nucleus

PART I. TEXT-BASED QUESTIONS

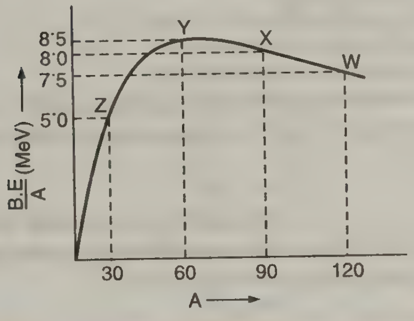
1. **Rutherford α-scattering experiment concludes that**
   (A) electrons are revolving around the nucleus.
   (B) there is heavy mass at centre.
   (C) electrons are scattered in space.
   (D) velocity of all electrons is same. *(A.F.M.C. 1997)*
2. **Rutherford's model of the atom was unstable, because**
   (A) nuclei will break down.
   (B) electrons move in circular orbits.
   (C) orbitting electrons radiate energy.
   (D) electrons are repelled by the nucleus. *(C.E.T. 1999)*
3. **Outside a nucleus,**
   (A) neutron is stable.
   (B) proton and neutron are stable.
   (C) neutron is unstable.
   (D) neither neutron nor proton is stable.
4. **The energy equivalent of one atomic mass unit is**
   (A) $1\cdot6 \times 10^{-19}$ J (B) $6\cdot02 \times 10^{23}$ J
   (C) 931 MeV (D) 9·31 MeV *(C.B.S.E. 1992)*
5. **Nuclei having equal difference in the number of neutrons and protons are called**
   (A) isotopes (B) isobars
   (C) isotones (D) isodiaspheres. *(A.F.M.C. 1998)*
6. **The mass number of He is 4 and that of sulphur is 32. The radius of sulphur nucleus is larger than that of helium by the factor of**
   (A) 8 (B) $\sqrt{8}$
   (C) 4 (D) 2 *(C.B.S.E. 1995)*
7. **The mass density of a nucleus varies with mass number A as**
   (A) A^2 (B) A
   (C) constant (D) 1/A *(C.B.S.E. 1992)*
8. **Order of the magnitude of the density of uranium nucleus is [$m_p = 1\cdot67 \times 10^{-27}$ kg]**
   (A) 10^{20} kg m^{-3} (B) 10^{17} kg m^{-3}
   (C) 10^{14} kg m^{-3} (D) 10^{11} kg m^{-3} *(I.I.T. 1999)*
9. **The origin of nuclear force is ascribed to the exchange of the following particle between a pair of nucleons**
   (A) mu-meson (B) π-meson
   (C) electron (D) photon
10. **The binding energy per nucleon is almost constant for many nuclei. It shows that nuclear forces are**
    (A) short range in nature.
    (B) attractive in nature.
    (C) charge independent.
    (D) saturated in nature.
11. **Which of the following nuclei is most stable ?**
    (A) odd-even (B) even-odd
    (C) even-even (D) odd-odd.
12. **m_p and m_n are masses of proton and neutron respectively. An element of mass M has Z protons and N neutrons. Then,**
    (A) $M > Z\,m_p + N\,m_n$ (B) $M = Z\,m_p + N\,m_n$
    (C) $M < Z\,m_p + N\,m_n$
    (D) The mass M may be greater than, less than or equal to $Z\,m_p + N\,m_n$, depending on nature of element. *(C.B.S.E. 2001)*
13. **The mass defect of an atom of mass M, atomic number Z and mass number A is given by**
    (A) $\frac{M}{A}$ (B) $\frac{M}{ZA}$
    (C) $(A-Z)\,m_p$
    (D) $[Z\,m_p + (A-Z)\,m_n + Z\,m_e] - M$ *(Karnataka Entrance, 1992, 1988)*
14. **The packing fraction for $_7N^{14}$ isotope, whose mass is 14·003 a.m.u., is**
    (A) $7\cdot8 \times 10^{-3}$ (B) $2\cdot1 \times 10^{-4}$
    (C) 1·0002 (D 0·9 *(Karnataka Entrance, 1986)*
15. **The average binding energy of a nucleon inside an atomic nucleus is about**
    (A) 8 MeV (B) 8 eV
    (C) 8 joule (D) 8 erg *(C.B.S.E. 1992)*
16. **The binding energy per nucleon is maximum in the case of**
    (A) $_2He^4$ (B) $_{26}Fe^{56}$
    (C) $_{56}Ba^{141}$ (D) $_{92}U^{235}$ *(C.B.S.E. 1993)*
17. **The mass defect for the nucleus of helium is 0·0303 a.m.u. What is the binding energy per nucleon for helium in MeV ?**
    (A) 28 (B) 7
    (C) 4 (D) 1 *(N.C.E.R.T. 1990)*
18. **The binding energies per nucleon for a deutron and an α-particle are x_1 and x_2 respectively. The energy Q released in reaction**
    $_1H^2 + {_1H^2} \longrightarrow {_2He^4} + Q$ is
    (A) $2\,(x_1 - x_2)$ (B) $2\,(x_1 + x_2)$
    (C) $4\,(x_1 + x_2)$ (D) $4\,(x_1 - x_2)$ *(C.B.S.E. 1995)*
19. **The binding energy of hydrogen atom is**
    (A) 1·0 eV (B) infinite
    (C) – 13·6 eV (D) zero *(Karnataka Entrance, 1988)*

PART II. THOUGHT-BASED QUESTIONS

20. **The atomic weight of boron is 10·81 and it has two isotopes $_5B^{10}$ and $_5B^{11}$. The ratio of $_5B^{10}$ to $_5B^{11}$ in nature would be**
    (A) 19 : 81 (B) 81 : 19
    (C) 15 : 16 (D) 10 : 11 *(Karnataka Entrance, 1994)*
21. **Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a $_{10}Ne^{20}$ nucleus and M_2 the mass of a $_{20}Ca^{40}$ nucleus. Then,**
    (A) $M_2 = 2\,M_1$ (B) $M_2 > 2\,M_1$
    (C) $M_2 < 2\,M_1$ (D) $M_1 < 10\,(m_n + m_p)$ *(I.I.T. 1998)*
22. **If the elements with principal quantum number $n > 4$, were not allowed in nature, the number of possible elements would be**
    (A) 60 (B) 32
    (C) 4 (D) 64 *(I.I.T. 1983)*
23. **The mass number of a nucleus is**
    (A) always less than its atomic number.
    (B) always more than its atomic number.
    (C) sometimes equal to its atomic number.
    (D) sometimes more than and sometimes equal to its atomic number. *(I.I.T. 1986)*

24. **The atomic mass of an element is fractional, because**
(A) it contains electrons.
(B) of uncertainty principle.
(C) it may have isotopes.
(D) it may have isobars.

25. **Binding energy per nucleon versus mass number curve for nuclei is shown in the figure. W, X, Y, Z are four nuclei indicated on the curve. The energy, the process would release, is**



(A) $Y \rightarrow 2Z$ (B) $W \rightarrow X + Z$
(C) $W \rightarrow 2Y$ (D) $X \rightarrow Y + Z$ *(I.I.T. 1999)*

26. **The mass of an α-particle is**
(A) less than the sum of the masses of two protons and two neutrons.
(B) equal to mass of four protons.
(C) equal to mass of four neutrons.
(D) equal to sum of masses of two protons and two neutrons.   *(C.B.S.E. 1992)*

Chapter 2. Radioactivity

PART I. TEXT-BASED QUESTIONS

1. **Which of the following is a correct statement ?**
(A) Beta rays are same as cathode rays.
(B) Gamma rays are high energy neutrons.
(C) Alpha particles are singly ionised helium atoms.
(D) protons and neutrons have exactly the same mass.   *(I.I.T. 1999)*

2. **Which of the following is positively charged ?**
(A) α-particle   (B) β-particle
(C) γ-rays   (D) X-rays   *(C.B.S.E. 2001)*

3. **If I_1, I_2 and I_3 are ionising powers of α, β and γ-rays, then**
(A) $I_1 > I_2 > I_3$ (B) $I_1 < I_2 < I_3$
(C) $I_1 > I_2$ but $I_2 < I_3$ (D) $I_1 < I_2$ but $I_2 > I_3$.

4. **In a given reaction :**

$${}_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} \longrightarrow {}_{Z-1}K^{A-4} \dashrightarrow {}_{Z-1}K^{A-4},$$

the radioactive radiations are emitted in the sequence of
(A) α, β, γ (B) γ, α, β
(C) β, α, γ (D) γ, β, α
(C.B.S.E. 1993 ; similar C.P.M.T. 1990)

5. **Ne^{22} nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is**
(A) nitrogen   (B) carbon
(C) boron   (D) oxygen.   *(I.I.T. 1999)*

6. **A nucleus ${}_{n}Y^{m}$ emits one α and two β-particles. The resulting nucleus is**
(A) ${}_{n-2}X^{m-4}$ (B) ${}_{n-1}X^{m-4}$
(C) ${}_{n-4}Z^{m-4}$ (D) ${}_{n}X^{m-4}$ *(C.B.S.E. 1998)*

7. **The radioactive nucleus ${}_{92}U^{238}$ emits an α-particle and the daughter nucleus a β-particle. The charge number and the mass of the final nucleus are**
(A) 91, 234   (B) 92, 234
(C) 90, 234   (D) 93, 234

8. **After one α and two β-emissions :**
(A) mass number reduces by 2.
(B) mass number reduces by 4.
(C) mass number reduces by 6.
(D) atomic number remains unchanged.   *(C.B.S.E. 1999)*

9. **What is the respective number of α and β particles emitted in the following radioactive decay ?**

$${}_{92}X^{200} \longrightarrow {}_{80}Y^{168}$$

(A) 6 and 8   (B) 6 and 6
(C) 8 and 8   (D) 8 and 6.   *(C.B.S.E. 1995)*

10. **The half life of I^{131} is 8 days. Given a sample of I^{131} at time $t = 0$, we can assert that**
(A) no nucleus will decay before $t = 4$ days.
(B) no nucleus will decay before $t = 8$ days.
(C) all nuclei will decay before $t = 16$ days.
(D) a given nucleus may decay at any time after $t = 0$.
(I.I.T. 1998)

11. **In which radioactive disintegration, a neutron is dissociated into a proton and an electron ?**
(A) He^{++} emission.   (B) β-emission.
(C) γ-emission.   (D) positron emission.
(A.F.M.C. 1995)

12. **Half-life of a radioactive element depends upon**
(A) the amount of element present.
(B) temperature.
(C) pressure.
(D) nature of material.   *(A.F.M.C. 1996 ; N.C.E.R.T. 1978)*

13. **A radioactive sample with a half life of 1 month has the label : `Activity = 2 μci on 1.8.1991'. What was its activity two months earlier ?**
(A) 1·0 μci   (B) 0·5 μci
(C) 4 μci   (D) 8 μci   *(C.B.S.E. 1992)*

14. **The decay constant of radioactive element radium is $4{\cdot}28 \times 10^{-4}$ year^{-1}. Its half-life will be**
(A) 2000 years   (B) 1240 years
(C) 63 years   (D) 1620 years
(A.F.M.C. 1998)

15. **If 10% of a radioactive material decays in 5 days, then the amount of original material left after 20 days is approximately :**
(A) 60%   (B) 65%
(C) 70%   (D) 75%   *(A.F.M.C. 1998)*

16. **If the half-life period of a radioactive substance is 225·8 min, then the value of the decay constant is**
(A) $3{\cdot}1 \times 10^{-3}$ min^{-1}.   (B) $3{\cdot}1 \times 10^{-4}$ min^{-1}.
(C) $3{\cdot}1 \times 10^{-6}$ min^{-1}.   (D) $3{\cdot}1 \times 10^{-9}$ min^{-1}.
(C.B.S.E. 1999 ; A.F.M.C. 1997)

17. **Half-lives of two radioactive substances A and B are 20 min and 40 min respectively. Initially, the samples of A and B have equal number of nuclei. After 80 min, the ratio of remaining number of A and B nuclei is**
(A) 1 : 16   (B) 1 : 4
(C) 1 : 1   (D) 4 : 1   *(C.B.S.E. 1998)*

18. **A radioactive element has half-life period 1600 years. After 6400 years, what amount will remain ?**

(A) $\frac{1}{2}$ (B) $\frac{1}{16}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$ *(C.B.S.E. 1993)*

19. **After a time equal to five half-lives, the amount of radioactive material remaining undecayed is :**

(A) 20% (B) 10%

(C) 5% (D) 3%

(A.F.M.C. 1996 ; N.C.E.R.T. 1990)

20. **A sample of radioactive element contains 4×10^{10} active nuclei. If half-life of element is 10 days, then the number of decayed nuclei after 30 days is**

(A) $0{\cdot}5 \times 10^{10}$ (B) 2×10^{10}

(C) $3{\cdot}5 \times 10^{10}$ (D) 1×10^{10} *(C.B.S.E. 2002)*

21. **Half-life period of a radioactive substance is 6 h. After 24 h, activity is 0·01 μci. What was the initial activity ?**

(A) 0·04 μci (B) 0·08 μci

(C) 0·24 μci (D) 0·16 μci *(C.B.S.E. 2001)*

22. **The half life of At^{215} is 100 μs. The time taken for the radioactivity of a sample of At^{215} to decay to 1/16th of its initial value is**

(A) 400 μs (B) 6·3 μs

(C) 40 μs (D) 300 μs *(I.I.T. 2002)*

23. **Half-life of a radioactive substance is 12·5 h and its mass is 256 g. After what time, the amount of remaining substance is 1 g ?**

(A) 75 h (B) 100 h

(C) 125 h (D) 150 h *(C.B.S.E. 2001)*

24. **The half-life of a radioactive material is 3 h. If the initial amount is 300 g, then after 18h, it will remain**

(A) 4·68 g (B) 46·8 g

(C) 9·375 g (D) 93·75 g *(C.B.S.E. 2000)*

25. **The half-life of radioactive radon is 3·8 days. The time at the end of which $\frac{1}{20}$ th of the radon sample will remain undecayed is (given $\log_{10} e = 0{\cdot}4343$)**

(A) 13·8 days (B) 16·4 days

(C) 33 days (D) 76 days. *(I.I.T. 1981)*

26. **The relationship between desintegration constant (λ) and half-life (T) will be :**

(A) $\lambda = \frac{\log_{10} 2}{T}$ (B) $\lambda = \frac{\log_e 2}{T}$

(C) $\lambda = \frac{T}{\log_e 2}$ (D) $\lambda = \frac{\log_e 2}{T}$ *(C.B.S.E. 2000)*

27. **If the decay constant of a radioactive substance is λ, then its half-life and mean-life are respectively**

(A) $\frac{1}{\lambda}$ and $\frac{\log_e 2}{\lambda}$ (B) $\frac{\log_e 2}{\lambda}$ and $\frac{1}{\lambda}$

(C) $\lambda \log_e 2$ and $\frac{1}{\lambda}$ (D) $\frac{\lambda}{\log_e 2}$ and $\frac{1}{\lambda}$

(M.N.R., 1990 ; I.I.T. 1989)

28. **The half life of a radioactive substance as compared to its mean life is**

(A) smaller and 69·3%.

(B) smaller and 30·7%.

(C) larger and 30·7%.

(D) larger and 69·3%. *(C.E.T. 1998)*

29. **The electron emitted in beta radiation originates from**

(A) inner orbits of atoms

(B) free electrons existing in nuclei

(C) decay of a neutron in a nucleus

(D) photon escaping from the nucleus. *(I.I.T. 2001)*

30. **Beta rays emitted by a radioactive material are**

(A) electromagnetic radiations.

(B) the electrons orbiting around the nucleus.

(C) charged particles emitted by nucleus.

(D) neutral particles. *(I.I.T. 1983)*

31. **Neutrino is a particle with**

(A) chargeless property and has no spin.

(B) chargeless property and has spin.

(C) charged like electron and has spin.

(D) the same as neutron. *(Karnataka Entrance, 1993)*

32. **The nucleus of $_{48}Cd^{115}$, after two successive β^-- decays will give**

(A) $_{46}Pa^{115}$ (B) $_{49}In^{114}$

(C) $_{50}Sn^{113}$ (D) $_{50}Sn^{115}$ *(C.B.S.E. 1992)*

33. **The nucleus $_6C^{12}$ absorbs an energetic neutron and emits a β-particle. The resulting nucleus is**

(A) $_7N^{14}$ (B) $_5B^{13}$

(C) $_7N^{13}$ (D) $_6C^{13}$ *(C.B.S.E. 1990)*

34. **A positron has the same mass as**

(A) proton (B) α-particle

(C) neutron (D) electron. *(A.F.M.C. 1993)*

35. **Which one of the following represents a γ-decay ?**

(A) ${}_ZX^A + \gamma \longrightarrow {}_{Z-1}Y^A + a + b$

(B) ${}_ZX^A + {}_0n^1 \longrightarrow {}_{Z-2}Y^{A-3} + c$

(C) ${}_ZX^A \longrightarrow {}_ZX^A + f$

(D) ${}_ZX^A + {}_{-1}e^0 \longrightarrow {}_{Z-1}Y^A + g$ *(I.I.T. 2002)*

36. **The radioactive isotope used to study thyroid gland is**

(A) carbon. (B) cobalt.

(C) iron. (D) iodine.

(Karnataka Entrance, 1993)

PART II. THOUGHT-BASED QUESTIONS

37. **The same radioactive nucleus may emit**

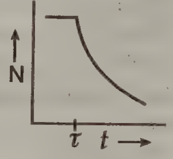
(A) either α or β and γ at a time.

(B) all the three α, β and γ one after another.

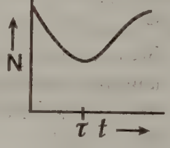
(C) all the three α, β and γ simultaneously.

(D) only α and β simultaneously. *(I.I.T. 1986)*

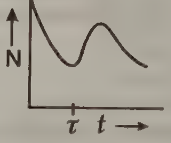
38. **A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life of one species is τ and that of the other is 5 τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represents the form of the plot ?**



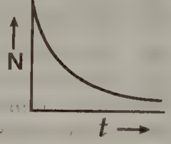
(A)



(B)



(C)



(D)

(I.I.T. 2001)

39. **The counting rate observed from a radioactive source at $t = 0$ was 1600 counts s^{-1} and at $t = 8$ s, it was 100 counts**

s^{-1}. The counting rate (counts s^{-1}) observed at $t = 6$ s will be

(A) 400
(B) 300
(C) 200
(D) 150 *(M.N.R. 1991)*

40. The count rate of a Geiger Muller counter for the radiation of the radioactive material of half-life 30 min decreases to 5 s^{-1} after 2 hours. The initial count rate was

(A) 20 s^{-1}
(B) 25 s^{-1}
(C) 80 s^{-1}
(D) 625 s^{-1}

(A.F.M.C. 1997 ; C.B.S.E. 1995)

41. What is the mass of one curie of U^{234} ?

(A) $1{\cdot}438 \times 10^{-11}$ g
(B) $2{\cdot}348 \times 10^{-11}$ g
(C) $3{\cdot}7 \times 10^{-10}$ g
(D) $6{\cdot}25 \times 10^{-34}$ g

(M.N.R. 1985)

42. Radon –220 will eventually decay to bismuth – 212 as

$$_{86}Rn^{210} \rightarrow {}_{84}Po^{216} + {}_{2}He^{4} \text{ ; half- life = 55 s}$$
$$_{84}Po^{216} \rightarrow {}_{82}Pb^{212} + {}_{2}He^{4} \text{ ; half-life = 0·16 s}$$
$$_{82}Pb^{212} \rightarrow {}_{83}Bi^{212} + {}_{-1}e^{0} \text{ ; half-life = 10·6 h}$$

If a certain mass of radon-220 is allowed to decay in a certain container, after five minutes, the element with the greatest mass will be

(A) radon
(B) polonium
(C) lead
(D) bismuth

(Karnataka Entrance, 1992, 1989)

43. A freshly prepared radioactive source of half- life 2 hours emits radiation of intensity, which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with the source is

(A) 6 hours
(B) 12 hours
(C) 24 hours
(D) 128 hours *(I.I.T. 1988)*

44. The half-life period of a radioactive element X is same as the mean-life time of another radioactive element Y. Initially, both of them have the same number of atoms. Then,

(A) X and Y have the same decay rate initially.
(B) X and Y decay at the same rate always.
(C) Y will decay at a faster rate than X.
(D) X will decay at a faster rate than Y. *(I.I.T. 1999)*

45. Two radioactive materials X_1 and X_2 have decay constants $10\,\lambda$ and λ respectively. If initially, they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time

(A) $1/10\,\lambda$
(B) $1/11\,\lambda$
(C) $11/10\,\lambda$
(D) $1/9\,\lambda$ *(I.I.T. 2000)*

46. Carbon-14 decays with a half-life of about 5,800 years. In a sample of bone, the ratio of carbon-14 to carbon-12 is found to be 1/4 of what it is in free air. This bone may belong to a period about *x* centuries ago, where *x* is nearest to :

(A) 2 × 58
(B) 58
(C) 58/2
(D) 3 × 58

(Karnataka Entrance, 1994)

Chapter 3. Nuclear Reactions

PART I. TEXT-BASED QUESTIONS

1. In the nuclear reaction :

$$_{6}C^{11} \longrightarrow {}_{5}B^{11} + \beta^{+} + X,$$

what does X stand for ?

(A) An electron
(B) A proton
(C) A neutron
(D) A neutrino *(I.I.T. 1992)*

2. When a deuterium is bombarded on $_{8}O^{16}$ nucleus, an α-particle is emitted. Then the product nucleus is

(A) $_{7}N^{13}$
(B) $_{5}B^{10}$
(C) $_{4}Be^{9}$
(D) $_{7}N^{14}$ *(C.B.S.E. 2002)*

3. A nuclear decay is expressed as :

$$_{6}C^{11} \longrightarrow {}_{5}B^{11} + \beta^{+} + X$$

Then the unknown particle X is :

(A) neutron
(B) antineutrino
(C) proton
(D) neutrino *(C.B.S.E. 2000)*

4. The element X, when bombarded by neutron produces an α-particle and $_{3}Li^{7}$. The element X is

(A) $_{2}He^{4}$
(B) $_{5}B^{10}$
(C) $_{5}B^{9}$
(D) $_{4}Be^{11}$ *(C.B.S.E. 2001)*

5. Which of the following is not a mode of radioactive decay ?

(A) Position emission
(B) Electron capture
(C) Fusion
(D) Alpha decay *(N.C.E.R.T. 1978)*

6. Complete the equation for the following fission process :

$$_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{38}Sr^{90} + \ldots\ldots$$

(A) $_{54}Xe^{146}$
(B) $_{54}Xe^{145} + {}_{0}n^{1}$
(C) $_{54}Xe^{143} + 3\,{}_{0}n^{1}$
(D) $_{54}Xe^{142} + {}_{0}n^{1}$

(C.B.S.E. 1998)

7. Nuclear fission can be explained by :

(A) proton-proton cycle
(B) shell model of nucleus
(C) liquid drop model of nucleus
(D) independent of nuclear model of nucleus

(C.B.S.E. 20

8. In nuclear fission process, energy is released, because

(A) mass of particles is more than mass of nucleus
(B) total binding energy of products formed due to nuclear fission is more than the parent fissionable material
(C) total binding energy of products formed due to nucl fission is less than parent fissionable material.
(D) mass of some particles is converted into energy.

(C.B.S.E. 2001)

9. Energy released in the fission of a single U^{235} nucleus is 200 MeV. The fission rate of a U^{235} fuelled reactor operating at a power level of 5 W is

(A) $1{\cdot}56 \times 10^{10}\ s^{-1}$
(B) $1{\cdot}56 \times 10^{11}\ s^{-1}$
(C) $1{\cdot}56 \times 10^{16}\ s^{-1}$
(D) $1{\cdot}56 \times 10^{17}\ s^{-1}$

(C.B.S.E. 1993)

10. The most abundant isotope of natural uranium is

(A) $_{92}U^{238}$
(B) $_{92}U^{235}$
(C) $_{92}U^{234}$
(D) none of the above.

11. The critical mass of a fission reaction is

(A) the mass to start a nuclear fission reaction.
(B) the minimum mass for the chain reaction.
(C) the size of the reactor core.
(D) the size of the fuel + the size of the moderator.

(Karnataka Entrance, 1990, 1989)

12. Which of the following arc suitable for the fusion process ?

(A) light nuclei
(B) heavy nuclei
(C) elements lying in the middle of periodic table
(D) elements lying in the middle of binding energy curve.

(C.B.S.E. 2002)

13. In the fusion reaction

$$_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{1}H^{3} + {}_{1}H^{1} + Q,$$

about 4 MeV of energy is released. How much energy will approximately be released, if all the atoms of 1 kg of deuterium could undergo fusion ?

(A) 2×10^7 kWh (B) $9{\cdot}6 \times 10^{13}$ J
(C) 6×10^{27} calories (D) 8×10^{23} MeV *(M.N.R. 1985)*

14. Which of the following is used as moderator in nuclear reaction ?
(A) Uranium (B) Heavy water
(C) Cadmium (D) Plutonium *(C.B.S.E. 1997)*

15. In a nuclear reactor, cadmium rods are used to
(A) speed up neutrons.
(B) slow down neutrons.
(C) absorb some neutrons.
(D) take out nuclear energy.
(Karnataka Entrance, 1993, 1988)

16. Sharavathy hydroelectric project is expected to produce about 1000 MW of power. This is equivalent to the conversion of a certain mass of matter into energy completely, say in a reactor. This amount is
(A) 4 kg per second
(B) 4 tons per hour
(C) 0·01 mg per second
(D) 10·2 mg per second. *(Karnataka Entrance, 1992, 1988)*

17. Fusion reactions take place at hight temperature, because
(A) atoms are ionised at high temperature.
(B) molecules break up at high temperature.
(C) nuclei break up at high temperature.
(D) kinetic energy is high enough to overcome repulsion between nuclei. *(S.C.R.A. 1996)*

18. Hydrogen bomb is based upon
(A) fission (B) fusion
(C) fission and fusion (D) none of the above.

19. The energy released in a typical nuclear fusion reaction is approximately
(A) 24 MeV (B) 200 MeV
(C) 800 MeV (D) 1050 MeV *(I.I.T. 1992)*

20. Energy generation in stars in mainly due to
(A) nuclear fusion. (B) nuclear fission.
(C) gravitational pull. (D) chemical reactions.
(C.E.T. 1998)

21. Solar energy is due to
(A) fusion reactions. (B) fission reactions.
(C) combustion reactions.
(D) chemical reactions. *(C.B.S.E. 1992)*

22. In case of an artificial radioactive transformation as give by

$$_{15}P^{30} \rightarrow {}_{14}Si^{30} + X,$$

the emitted particle X is
(A) neutron. (B) proton.
(C) electron. (D) positron.
(Karnataka ,1990

PART II. THOUGHT-BASED QUESTIONS

23. Consider the fission reaction :

$$_{92}U^{236} \longrightarrow X^{117} + Y^{117} + 2\,{}_0n^1$$

***i.e.* two nuclei of same mass number 117 are formed alon with two neutrons. The binding energy per nucleon of and Y is 8·5 MeV, whereas that of U^{236} is 7·6 MeV. The tot energy liberated will be about :**
(A) 2 MeV (B) 20 MeV
(C) 200 MeV (D) 2000 MeV *(C.P.M.T. 197*

24. Which of the following undergo fission reaction easily b slow moving neutrons ?
(A) U^{235}, Pu^{239} (B) Pu^{239}, Th^{234}
(C) U^{238}, Rn^{232} (D) Th^{234}, U^{238}

25. From the following equations, pick out the possibl nuclear fusion reactions :
(A) $_6C^{13} + {}_1H^1 \longrightarrow {}_6C^{14} + 4{\cdot}3$ MeV
(B) $_6C^{12} + {}_1H^1 \longrightarrow {}_7N^{13} + 2$ MeV
(C) $_7N^{14} + {}_1H^1 \longrightarrow {}_8O^{15} + 7{\cdot}3$ MeV
(D) $_{92}U^{235} + {}_0n^1 \longrightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 + \gamma + 200$ Me
(I.I.T. 198

26. A star initially has 10^{40} deutrons. It produces energy via th process

$$_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 \text{ and}$$
$$_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + {}_0n^1$$

The masses of the nuclei are as follows : $M(H_2) = 2{\cdot}01$ a.m.u., $m_p = 1{\cdot}007$ a.m.u., and $m_n = 1{\cdot}008$ a.m.u., $M(He^4)$ 4·001 a.m.u. If the average power radiated by the star i 10^{16} W, the deutron supply of the star is exhausted in a tim of the order of
(A) 10^6 s (B) 10^8 s
(C) 10^{12} s (D) 10^{16} s *(I.I.T. 199*

ANSWERS TO MULTIPLE CHOICE QUESTIONS

Chapter 1. Structure of Nucleus

1. (B) 2. (C) 3. (C) 4. (C) 5. (D) 6. (D) 7. (C) 8. (B) 9. (B) 10. (D)
11. (C) 12. (C) 13. (D) 14. (A) 15. (A) 16. (B) 17. (B) 18. (D) 19. (D) 20. (A)
21. (C & D) 22. (A) 23. (D) 24. (C) 25. (C) 26. (A)

Chapter 2. Radioactivity

1. (A) 2. (A) 3. (A) 4. (C) 5. (B) 6. (D) 7. (A) 8. (B & D) 9. (D) 10. (D)
11. (B) 12. (D) 13. (D) 14. (D) 15. (B) 16. (A) 17. (B) 18. (B) 19. (D) 20. (C)
21. (D) 22. (A) 23. (B) 24. (A) 25. (B) 26. (B) 27. (B) 28. (A) 29. (C) 30. (C)
31. (B) 32. (D) 33. (C) 34. (D) 35. (C) 36. (D) 37. (A) 38. (D) 39. (C) 40. (C)
41. (A) 42. (C) 43. (B) 44. (C) 45. (D) 46. (A)

Chapter 3. Nuclear Reactions

1. (D) 2. (D) 3. (D) 4. (B) 5. (C) 6. (C) 7. (C) 8. (B) 9. (B) 10. (A)
11. (B) 12. (A) 13. (B) 14. (B) 15. (C) 16. (C) 17. (D) 18. (B) 19. (A) 20. (A)
21. (A) 22. (D) 23. (C) 24. (A) 25. (B & C) 26. (C)

UNIT
IX

SOLIDS & SEMICONDUCTORS DEVICES

1. **Solids**
2. **Semiconductor Devices**

> "Great discoveries and improvements invariably involve the cooperation of many minds. I may be given credit for having blazed the trail but when I look at the subsequent developments I feel the credit is due to others rather than to myself." – **Alexander Grahm Bell**

CHAPTER 1

Solids

1.01. SOLIDS

All solids have the property of elasticity and by virtue of this property, the lids behave as incompressible substances and exhibit rigidity and the mechanical rength. Basically, all solids are made of atoms and molecules but due to the fferent internal arrangement of the molecules inside them, they are divided into o classes ; namely ***crystalline*** and ***amorphous materials***.

1.02. CRYSTALLINE MATERIALS

The crystalline materials are those in which atoms or molecules are arranged in a finite and regular way throughout the body of the crystal and possess a definite external ometrical shape.

A few examples of crystalline substances are quartz, mica, sugar, copper lphate, sodium chloride, etc.

The crystalline substances have the following characteristics :

1. The atoms or molecules constituting a crystalline solid are arranged in a efinite and regular manner inside the crystal. Due to this, crystalline materials have ***efinite external geometrical shape***.

2. The orderly arrangement of atoms and molecules in a crystal extends over large volume of the crystal *i.e.* crystals exhibit a ***long range order*** of the atoms and olecules.

3. Crystalline substances are bounded by ***flat surfaces***.

4. Crystalline substances are ***anisotropic*** *i.e.* the physical properties like *thermal nductivity, electrical conductivity, compressibility*, etc have different values in different irections.

5. Crystalline materials possess uniform chemical composition *i.e.* bonds etween all the ions, atoms or molecules are of equal strength.

6. A crystalline material has a ***sharp melting point***. It is because, all the bonds etween its constituents are of equal strength ; and on heating, all the bonds get uptured suddenly at a fixed temperature. Due to this, the change from solid state the liquid state occurs suddenly.

7. Below the temperature of crystallisation, the crystalline materials are in stable tate. Since a stable state is the state of minimum energy, such a material acquires state of minimum energy on crystallisation.

Watch out !

Majority of solids in nature have ordered internal structure.

1.03. AMORPHOUS MATERIALS

The *amorphous materials are those in which atoms or molecules are arranged in an rregular manner and do not have definite external geometrical shape.* A few examples of morphous materials are glass, rubber, sulphur, etc.

The jargon . . . . .

The word amorphous means shapeless.

The amorphous substances have the following characteristics :

1. The atoms or molecules constituting an amorphous solid are arranged in an rregular manner within the solid. Due to this, the amorphous solids do not have definite external geometrical shape.

2. Since the distribution of atoms or molecules in such solids is random and haotic, such solids are *not bounded by flat surfaces*.

3. The amorphous materials are ***isotropic*** *i.e.* they have same physical properties n all directions.

4. Amorphous solids do not possess uniform chemical composition. Due to this, onds between all the atoms and molecules are not of equal strength.

5. Amorphous solids do not have *sharp melting point*. It is because, bonds between all the atoms and molecules are not of equal strength. On heating, the weaker bonds get ruptured first and the stronger ones later on. Due to this, change of the state does not occur at a fixed temperature.

1.04. CRYSTAL STRUCTURE

In order to make a study of the structure of different types of crystals, it is necessary to acquaint ourselves with the following terms :

Crystal lattice. *A crystal lattice is defined as the infinite array of atoms and molecules in space (three dimensions), such that at every point, an atom or the molecule has got the identical surroundings.*

The Jargon

Lattice means jangla or a frame of criss-cross of wood or iron commonly seen at windows and doors in houses.

Fig. 1.01 shows a two dimensional crystal lattice. Taking P as some arbitrary point representing the position of some atom, we find that along PA, the distance between adjecent points (atoms) is one unit (assuming the interatomic distance as to be one unit). Along PB, the points lie at a fixed distance $\sqrt{2}$ units apart ; while along PC, the points lie at distances of $\sqrt{5}$ units. The same is true for the separation between the points in any other direction and when extended infinitely. It, therefore, satisfies the definition of a two dimensional lattice.

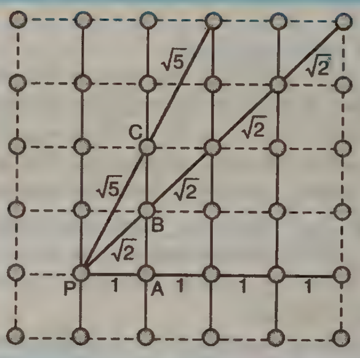


Fig. 1.01.

The actual crystal lattice is three dimensional. The arrangement of atoms in a three dimensional crystal lattice is similar to that in two dimensional crystal lattice explained above.

Unit cells. The entire lattice structure of a crystal is found to consist of identical blocks, called unit cells.

The unit cell is the smallest block or geometrical figure from which the crystal is built up by repetition in three dimensions.

Thus, a unit cell is a *fundamental elementary pattern*, as it is repeated again and again in the three dimensions to form lattice structure of the crystal. Fig. 1.02 represents a unit cell in general.

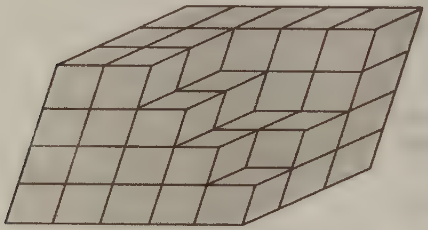


Fig. 1.02.

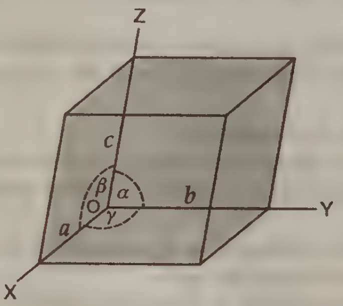


Fig. 1.03.

Fig. 1.03 shows how the entire three-dimensional crystal can be obtained by identically and periodically repeating the same unit cell again and again in three different directions in space.

The following terms are connected with the study of the unit cell of a crystal :

(i) *Crystallographic axes.* *These are the lines drawn parallel to the lines of intersection of any three faces of the unit cell, which do not lie in the same plane.* In Fig. 1.02, OX, OY and OZ are the crystallographic axes.

(ii) *Primitives.* *The three sides of the unit cell are called primitives.* They are denoted by a, b and c and they are also known as ***lattice constants.***

(iii) *Interfacial angles.* *The angles between three crystallographic axes of the unit cell are interfacial angles.* They are denoted by α, β and γ.

The structure of a crystal is fully defined by lattice constants a, b, c and the interfacial angles α, β, γ.

1.05. CRYSTAL SYSTEMS

From the point of view of the shape of the unit cell, all the crystals have been divided into seven crystal systems. In the order of their growth, the seven crystal systems are as given below :

1. Triclinic system. For this system, $a \neq b \neq c$; $\alpha \neq \beta \neq \gamma \neq 90°$. An elementary cell has the form of an oblique parallelopiped. Examples : $CuSO_4$, $K_2Cr_2O_7$, etc.

2. Monoclinic system. For this system, $a \neq b \neq c$; $\alpha = \beta = 90°$; $\gamma \neq 90°$. An elementary cell has the form of a right prism with a parallelogram as its base. Examples : Na_2SO_4, $KClO_3$, $FeSO_4$, etc.

3. Rhombic or Orthorhombic system. For this system, $a \neq b \neq c$; $\alpha = \beta = \gamma =$ 0°; $\gamma \neq 90°$. An elementary cell has the form of a rectangular parallelopiped. xamples : KNO_3, gallanium, mercury chloride, etc.

4. Tetragonal system. For this system, $a = b \neq c$, $\alpha = \beta = \gamma = 90°$. An elementary ell has the form of a right prism with a square base. Examples : $NiSO_4$, white tin, tc.

5. Trigonal or Rhomohedral system. For this system, $a = b = c$; $\alpha = \beta = \gamma \neq 90°$. n elementary cell has the form of a cube deformed by a compression or tension long the diagonal. Examples : As, Sb, Bi, calcite, etc.

6. Hexagonal system. For this system, $a = b \neq c$; $\alpha = \beta = 90°$, $\gamma = 120°$. Three lementary cells brought together form a regular hexagonal prism. Examples ; Zn, nO, Cd, Ni, etc.

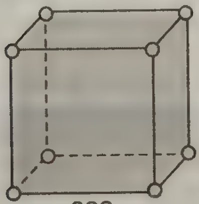
7. Cubic system. For this crystal, $a = b = c$; $\alpha = \beta = \gamma = 90°$. An elementary cell as the form of a cube. Examples : NaCl, KI, Cu, etc.

According to Bravais, who introduced the space lattice idea as early as 1880, here are 14 possible types of space lattices in the above seven crystal systems. It ollows that even for the same crystal system, there may be more than one ***Bravais attices.***

1.06. CUBIC CRYSTAL SYSTEM

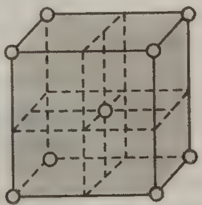
The cubic crystal system is the simplest of all the seven crystal systems. In the ubic crystal system, there are three types of cubic crystals. They are ***Simple Cubic Crystal (SCC), Body Centered Crystal (BCC)*** and ***Face Centered Crystal (FCC).***

(*i*) Simple cubic crystal. It is a crystal, whose unit cell is a cube with one atom r molecule at each of the eight corners of the cube as shown in Fig. 1.04 (*a*). NaCl is n example of simple cubic crystal.



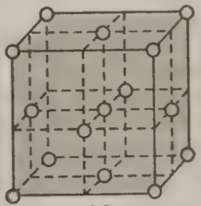
SCC

Fig. 1.04 (*a*)



BCC

Fig. 1.04 (*b*)



FCC

Fig. 1.04 (*c*)

The atom at a corner of the unit cell of SCC is shared by the eight different unit ells. Therefore,

$$\text{effective number of atoms per unit cell of SCC} = 8 \times \frac{1}{8} = 1 \text{ atom}$$

(*ii*) Body-centered cubic crystal. It is a crystal, whose unit cell is a cube with ne atom (or molecule) at each of the eight corners and an additional atom (or molecule) at the centre of the cube [Fig. 1.04 (*b*)]. Sodium, barium, iron, tungsten and uranium belong to this type of structure.

The atom at the centre of the unit cell of BCC completely belongs to that unit ell. Therefore,

$$\text{effective number of atoms per unit cell of BCC} = 8 \times \frac{1}{8} + 1 = 1 + 1 = 2 \text{ atoms}$$

(*iii*) Face-centered cubic crystal. It is a crystal, whose cell is a cube with one atom or molecule at each of the eight corners and one atom at the centre of each of the six faces. Hence, the name face centered cubic crystal. It is shown in Fig. 1.04 (c). Silver, copper, gold, lead, nickel and platinum belong to this type of structure.

The atom at the centre of the face of a cell can be shared by only two unit cells. Therefore,

$$\text{effective number of atoms per unit cell of FCC} = 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 1 + 3 = 4 \text{ atoms}$$

Since, all the three types of cubic crystals are made of cubic unit cells, we have

$$a = b = c \qquad \text{and} \qquad \alpha = \beta = \gamma = 90°$$

The different Bravais lattices in a crystal system are distinguished from each other due to their different values of ***atomic radius, co-ordination number*** and ***atomic packing factor.***

Atomic radius. *It is defined as half of the distance between the centres of two neighbouring atoms.* It is also called ***lattice constant*** or ***lattice parameter.***

Assuming that atoms in a crystal are spheres of equal sizes and touching each other, it can be shown that atomic radius is equal to $\frac{a}{2}$, $\frac{\sqrt{3}}{4}a$ and $\frac{a}{\sqrt{2}}$ for SCC, BCC and FCC respectively.

Co-ordination number. *The number of atoms in a crystal which surround a particular atom as its nearest neighbours is called co-ordination number.*

It can be proved that co-ordination number for SCC, BCC and FCC are 6, 8 and 12 respectively.

Atomic packing factor. *It is defined as the ratio of the volume occupied by all the atoms in a unit cell to the total volume of the unit cell.*

The calculations show that atomic packing factor for SCC, BCC and FCC are 0·52, 0·68 and 0·74 respectively.

1.07. SINGLE CRYSTALS

The crystal in which, the ordered arrangement of atoms (or molecules) extends throughout the piece of solid, is called a single crystal.

The single crystals are anisotropic in nature *i.e.* their mechanical, thermal, electrical and optical properties are different in different directions. Crystals possess plane facets making angles with one another, depending on the system to which crystal belongs. Such crystals can be easily split along some definite planes, known as ***cleavage planes***. It is found difficult to tear off the crystal in a direction perpendicular to the cleavage planes. It shows that the mechanical strength of single crystal is different in different directions.

Further, the size of a single crystal is usually very small*. It may vary from a fraction of a centimetre to a few centimetres. A very small sized crystal, whose inner features cannot be seen with a naked eye is called a ***monocrystal.***

1.08. POLY CRYSTALS

The crystalline solids, which are made of a large number of monocrystals with well developed faces joined together, are called polycrystals.

The anisotropy and the ordered arrangement of atoms (or molecules) is not observed in polycrystals. It is because, in practice, we come across big crystals, which are a collection of intergrown and randomly oriented fine crystals. The irregular and haphazard form is due to the fact that its normal growth has been restricted by neighbouring crystals. Anisotropy is observed in polycrystals only within the confines of each separately taken minute crystal. By providing special conditions of crystallisation from a solution or a melt, it is possible to obtain a large single crystal.

Watch out !

Single crystals are anisotropic whereas polycrystals are isotropic.

The polycrystal ceramic made from PbO, ZrO_2 and TiO_2 are used in gas lighters and telephone receivers. Ceramic oxides are usually bad conductors but ceramics made from materials like Y_2O_3, BaO and CuO_2 exhibit superconductivity.

1.09. LIQUID CRYSTALS

The organic crystalline solids, which on heating become fluid like, but retain their anisotropic properties are called liquid crystals.

Some organic crystalline solids in certain temperature region acquire a phase between solid and liquid and retain their anisotropic property and long range orientational order of its molecules.

Liquid crystal such as alkyl cyanobiphenyl can change the plane of polarisation of light and such *liquid crystal displays* (LCD) are used in watches and micro calculators. Some materials in liquid crystal phase such as cholesterol esters show a change in colour with temperature. This property is made use of in liquid crystal thermometers.

1.10. ENERGY BANDS IN SOLIDS

In a single free atom, electrons have well defined energy levels. But in a crystal, the atoms do not have well defined energy levels. It is because, in a crystal, there are many atoms and each atom in the crystal is surrounded by neighbouring atoms. Due to this, energy levels of the electrons in the atom get modified. However, the so called modification in the energy levels is not appreciable in case of the electrons in the inner shells but it is considerable in case of electrons in the outermost shell. It is because, electrons in the outermost shell are shared by more than one atom in the crystal.

---

*Considering that a unit cell is of size 5×10^{-8} cm and a single crystal has such 10^4 unit cells, the size of the crystal will be $5 \times 10^{-8} \times 10^4$ *i.e.* 5×10^{-4} cm. Such a small crystal can not be seen with a naked eye.

In order to understand as to how the energy levels get modified, consider a ngle small crystal of silicon made of N atom. An atom of silicon has electronic onfiguration 1 s^2, 2 s^2, 2 p^6, 3 s^2, 3 p^6. The various energy levels of silicon atom can e represented as shown in Fig. 1.05.

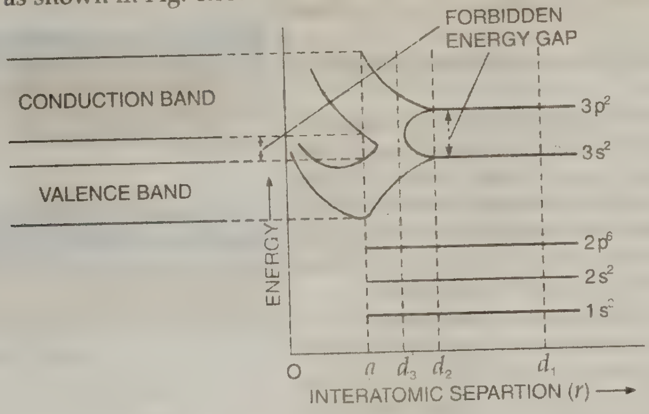


Fig. 1.05.

The outermost shell contains 2 electrons in 3 *s* subshell and 2 electrons in 3 *p* subshell. The 3 *s*-levels (all of the same energy) in the outermost shells of all the N atoms, which are 2 N in number are completely filled by 2 N electrons, while out of total 6 N *p*-levels (all of the same energy) in the outermost shells of N atoms, only 2 N are filled.

Suppose that the interatomic separation between the silicon atoms in the crystal lattice is a and these atoms are brought from the locations $r >> a$ to their actual location $r = a$ by gradually moving the atoms towards each other.

(*i*) When $r = d_1$. At this interatomic separation, electrons in the outermost shell of one atom do not interact with those of the other. Therefore, $r = d_1$, each of N atoms has its own typical energy levels *i.e.* energy levels 1 s^2, 2 s^2, 2 p^6, 3 s^2, 3 p^2 of each atom remain unmodified.

(*ii*) When $r = d_2$. When the interatomic separation is decreased from $r = d_1$, to $r = d_2$, the interaction between the electrons of different atoms in the outermost shells (3 s^2 and 3 p^2) begins. Due to this, the energy of 3 *s* and 3 *p* levels of each atom gets slightly modified, when they are at distance $r = d_2$. Therefore, instead of a single 3 *s* or 3 *p*-level, there occur a large number of closely spaced energy levels. Now, the 2 N *s*-levels will not have same energy but will be spread in small energy band. Similarly, 6 N *p*-levels will also be spread into a small energy band. Due to this, the energy gap between *s* and *p*-levels of an isolated atom reduces.

Watch out !

Energy levels and energy bonds are different terms. A single isolated atom has various energy levels. In a crystal, atoms are packed quite close to each other. Due to interaction between them, energy levels of individual atoms split and form energy bonds.

(*iii*) When $r = d_3$. When the interatomic separation is further decreased, then at $r = d_3$, the energy gap between 3 *s* and 3 *p*-levels completely disappears and all the 8 N levels (2 N of 3 *s*-levels and 6 N of 3 *p*-levels) are distributed in a continuous manner forming an energy band. Now, it is not possible to distinguish between 3 *s* and 3 *p* electrons. One can only say that there are 8 N energy levels, out of which 4 N levels are filled and 4 N levels are empty.

(*iv*) When $r = a$. When the silicon atoms are at the actual interatomic separation a, the energy of the band of 8 N levels splits into two bands, band of 4 N energy levels completely filled with electrons and another band of 4 N empty levels. These two energy bands are separated from each other with a small energy gap. The lower energy band of 4 N filled levels is called the ***valence band***, while the upper energy band of 4 N empty levels is called the ***conduction band***. The energy gap between the valence band and the conduction band is called the ***forbidden energy gap***.

1.11. DISTINCTION BETWEEN METALS, INSULATORS AND SEMI-CONDUCTORS

Metals are good conductors of electricity, insulators do not conduct electricity, while the semiconductors have conductivity inbetween those of metals and insulators. Let us make distinction between condutors, insulators and semiconductors on the basis of band theory of solids.

A solid is a large collection of atoms. The energy levels of an atom get modified due to the presence of other surrounding atoms and the energy levels in the outermost shells of all the atoms form valence band and the conduction band separated by a forbidden energy gap.

The energy band formed by a series of energy levels containing valence electrons is called ***valence band.***

At 0 K, the electrons start filling the energy levels in valence band starting from the lowest one. *The highest energy level, which an electron can occupy in the valence band at 0 K is called* ***Fermi level.***

The lowest unfilled energy band formed just above the valence band is called ***conduction band.***

At 0 K, the Fermi level as well as all the lower energy levels are completely occupied by the electrons. As the temperature rises, the electrons absorb energy and get excited. The excited electrons jump to the higher energy levels. These electrons in the higher energy levels are comparatively at larger distances from the nucleus and are more free as compared to the electrons in the lower energy levels. Depending upon the energy gap between valence band and the conduction band, the solids behave as conductors, insulators and semiconductors as explained below :

Key point

The resistance offered by a semiconductor decreases (in contrast to conducters) with the rise of temperature.

Key point

The conduction band of an insulator is practically empty.

(*a*) Metals. The energy band structure in solids have two possibilities :

(*i*) *The valence band may be completely filled and the conduction band partially filled with an extremely small energy gap between them* [Fig. 1.06 (*a*)]. For example, in sodium, the conduction band is partially filled, while the valence band is completely filled.

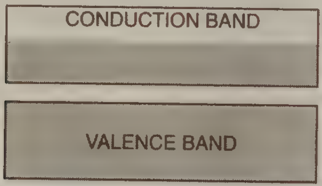


Fig. 1.06 (*a*)

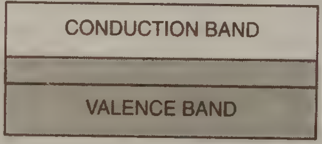


Fig. 1.06 (*b*)

(*ii*) *The valence band is completely filled and the conduction band is empty but the two overlap each other* [Fig. 1.06 (*b*)]. Zinc is an example of band overlap metals.

In both the situations, it can be assumed that there is a single energy band, which is partially filled. Therefore, on applying even a small electric field, the metals conduct electricity.

(*b*) Insulators. In insulators, the forbidden energy gap is quite large [Fig. 1.07]. For example, the forbidden energy gap for diamond is 6 eV, which means that a minimum of 6 eV energy is required to make the electron jump from the completely filled valence band to the conduction band. When electric field is applied across such a solid, the electrons find it difficult to acquire such a large amount of energy and so the conduction band continues to be almost empty. No electron flow occurs *i.e.* no current flows through such solids. So they behave as insulators.

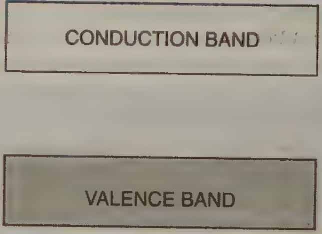


Fig. 1.07.

(*c*) Semiconductors. The energy band structure of the semiconductors is similar to the insulators but in their case ; the size of the forbidden energy gap is much smaller than that for the insulators [Fig. 1.08]. For example, the forbidden band for silicon is 1·1 eV. The electronic structure of the silicon (Si) is similar to that of the diamond, but due to the smaller width of the forbidden energy gap, the electrons in the valence band find it comparatively easier to shift to the conduction band. So, the conductivity of the silicon is inbetween the conductors and the insulators and it is termed as a semiconductor.

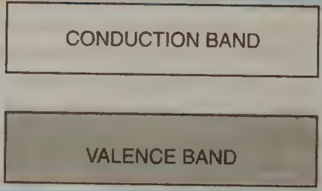


Fig. 1.08.

1.12. CURRENT CARRIERS IN SEMICONDUCTORS

In a pure semiconductor, each atom behaves as if there are 8 electrons in its valence shell (due to formation of covalent bonds) and therefore the entire material behaves as an insulator at low temperatures.

A semiconductor atom needs energy of the order of 1·1 eV to shake off the valence electron. This energy becomes available to the semiconductor even at room temperature. Due to thermal agitation of the crystal structure, electrons from a few covalent bonds come out. The bond from which electron is freed, a vacancy is created there [Fig. 1.09]. The vacancy in the covalent bond (where there should have been an electron) is called a ***hole*.**

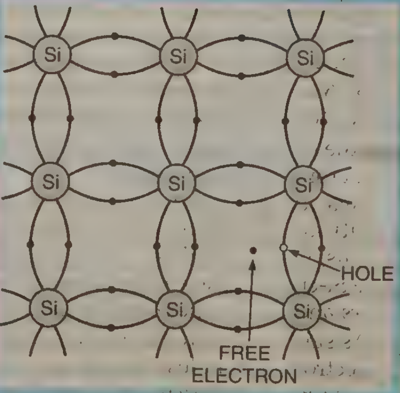


Fig. 1.09.

This hole can be filled by some other electron in a covalent bond. As an electron from a covalent bond moves to fill the hole, the hole is created in the covalent bond from which the electron has moved. In other words, one can say that the hole shifts from one covalent bond to another in a similar way as an electron does in an attempt to fill the hole. Since the direction of movement of the hole is opposite to that of negative electron, a hole behaves as a positive charge carrier. Thus, at room nperature, a pure semiconductor will have electrons and holes wandering in dom directions. These electrons and holes are called ***intrinsic carriers*** and such emiconductor is called ***intrinsic semiconductor.***

Key point

The width of forbidden energy gap between the valence and conduction band distinguishes conductors, semiconductors and insulators from each other.

As the crystal is electrically neutral, the number of free electrons will be equal the number of holes. If we apply potential difference across the semiconductor, e electrons will move towards positive terminal and the holes towards the negative minal of the battery. It may be clearly understood that *electrons and holes are not rrent in themselves but act only as the negative and positive carriers of the current pectively.*

In an intrinsic semiconductor, if $n_e^{(0)}$ denotes the electron number density in nduction band, $n_h^{(0)}$, the hole number density in valence band and n_i, the number nsity or concentration of intrinsic carriers, then

$$n_e^{(0)} = n_h^{(0)} = n_i \qquad \text{...(1.01)}$$

The jargon . . . .

A pure semiconductor is called intrinsic semiconductor. Its small conductivity is due to free electrons and holes created in it due to thermal agitation.

It can be proved that the fraction of the electrons of valence band present in nduction band is given by

$$f \propto e^{-E_g/kT}, \qquad \text{...(1.02)}$$

here k is Boltzmann's constant and E_g is forbidden energy gap. It follows that if e temperature (T) increases, the value of the fraction f will also increase *i.e.* when e temperature of a semiconductor increases, more electrons are raised to nduction band. In other words, the conductivity of the semiconductor increases ith rise in temperature of the semiconductor.

Key point

In an intrinsic semiconductor, the electron and hole concentrations are equal.

Further, when an electron is raised from the valence band to the conduction and, a vacancy is created in the valence band. This vacancy created in the valence and (where electron was present before moving to conduction band) acts as the ole.

1.13. INCREASING THE CONDUCTIVITY OF A SEMICONDUCTOR (Doping)

It follows that a pure semiconductor at room temperature possesses free lectrons and holes but their number is so small that conductivity offered by the ure semiconductor cannot be made of any practical use.

By the addition of impurities to the pure semiconductor in a very small ratio 1 : 10^6), the conductivity of a Si-crystal (or Ge-crystal) can be remarkably improved. *he process of adding impurity to a pure semi conductor crystal (Si or Ge-crystal) so as to nprove its conductivity, is called* ***doping.***

The jargon . . . .

A semiconductor doped with pentavalent or trivalent impurity atoms is called extrinsic semiconductor. Its conductivity is quite large as compared to that of intrinsic semiconductor.

The impurity atoms are of two types :

(i) Pentavalent impurity atoms *i.e.* atoms having 5 valence electrons such as ntimony (Sb) or arsenic (As). Such atoms, when added to a pure semiconductor, roduce excess of free electrons *i.e.* donate electrons to the semiconductor. For this eason, pentavalent impurity atoms are called ***donor impurity*** atoms. The emiconductor so produced is called ***n-type extrinsic semiconductor.***

(ii) Trivalent impurity atoms *i.e.* atoms having 3 valence electrons such as ndium (In) or gallium (Ga). Such atoms on being added to a pure semiconductor, nstead of producing free electrons, accept electrons from the semiconductor. For this reason, trivalent impurity atoms are called ***acceptor impurity*** atoms. The semiconductor so produced is called ***p-type extrinsic semiconductor.***

1.14. n-TYPE (EXTRINSIC) SEMICONDUCTOR

Fig. 1.10 shows the effect of adding pentavalent impurity arsenic to silicon crystal. When the arsenic impurity atoms are added to the silicon crystal in a small ratio (1 : 10^6), its atoms replace the silicon atoms here and there. The four electrons out of the five valence electrons of As-atom take part in covalent bonding with four silicon atoms surrounding it. The fifth electron is set free. Obviously, the extra free electrons created in the crystal will be as many as the number of the pentavalent impurity atoms added. As the pentavalent impurity increases the number of free electrons, it is called ***donor impurity***. The silicon crystal so obtained is termed as ***n-type*** Si-crystal. The electrons so set free in the silicon crystal are called ***extrinsic carriers*** and the *n*-type Si-crystal is called ***n-type extrinsic semiconductor.***

As said earlier, due to thermal agitation, even the pure Si-crystal possesses a few electrons and holes. Therefore, *n- type Si-crystal will have a large number of free electrons (majority carriers) and a small number of holes (minority carriers).*

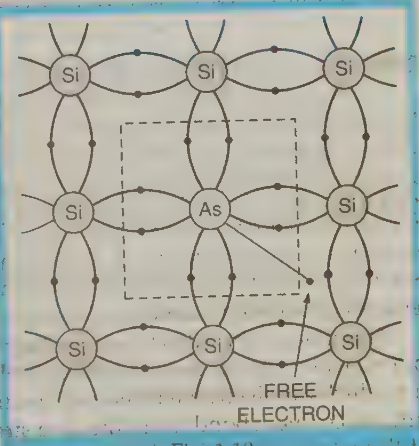


Fig. 1.10.

In the extrinsic *n*-type semiconductor, the fifth electron of the As-atom revolves around the donor atom inside the Si-crystal. As dielectric constant of silicon is very high, it is bound to the donor atom with a very small amount of energy, which is of the order of 0·045 eV. In terms of valence and conduction band, one can think that all such electrons (extrinsic carriers) create a ***donor energy level*** just below (0·045 eV) the conduction band as shown in Fig. 1.11. As the energy gap between donor energy level and the conduction band is very small, the electrons can easily raise themselves to conduction band even at room temperature. Hence, the conductivity of *n*-type extrinsic semiconductor is markedly increased.

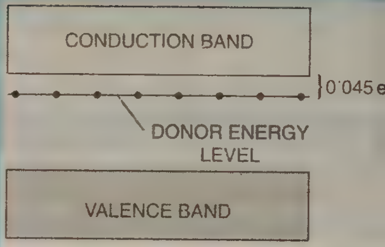


Fig. 1.11.

Key point

Fermi level is *n*-type semi-conducto (also knows as donor energy level) lies i forbidden energy gap and is very close t conduction band.

In a doped or extrinsic semiconductor, the number density of the conduction band (n_e) and the number density of holes in the valence band (n_h) differ from that in a pure semiconductor. If n_i is the number density of electrons in conduction band or the number density of holes in valence band in a pure semiconductor, then it can be proved that

$$n_e \, n_h = n_i^2 \qquad \text{...(1.03)}$$

In *n*-type (extrinsic) semiconductor, the number density of electrons in conduction band is approximately equal to that of donor atoms and very large as compared to number density of holes in valence band. Thus,

$$n_e \approx \mathrm{N}_d >> n_h, \qquad \text{...(1.04)}$$

where N_d represents the number density of donor atoms.

1.15. p-TYPE (EXTRINSIC) SEMICONDUCTOR

Fig. 1.12 shows the effect of adding trivalent impurity indium to silicon crystal. The four silicon atoms surrounding the In-atom, can share one electron each with the In-atom which has got three valence electrons. In an attempt to have 8 electrons in valence shell, the In-atom robs one of the nearby covalent bonds of one electron. Thus, the valence shell of the In-atom possesses 8 electrons but a hole is created in the covalent bond from which electron has been robbed. Thus, for every trivalent impurity atoms added, an extra hole will be created. As the trivalent impurity atoms accept electrons from the silicon crystal, it is called ***acceptor impurity.*** The Si-crystal so obtained is called ***p-type*** as it contains free holes. Each hole is equivalent to positive charge. The holes so created are ***extrinsic carriers*** and the ***p-type*** Si-crystal so obtained is called ***p-type extrinsic semiconductor.***

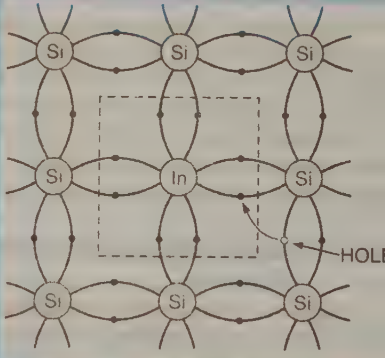


Fig. 1.12.

Again, as the pure Si-crystal also possesses a few electrons and holes, therefore, *the p-type Si-crystal will have a large number of holes (majority carriers) and a small number of electrons (minority carriers).*

In the extrinsic *p*-type Si-crystal, the hole produced revolves round the nucleus of the In-atom. As the hole may be treated as a particle of same mass as electron but having an equal positive charge, it possesses a small positive energy of the order of 0·04 eV. Such holes create an ***acceptor energy level*** just above (≈ 0·04 eV) the top of the valence band [Fig. 1.13]. The electrons from valence band can raise themselves to the acceptor energy level by absorbing thermal energy at room temperature and in turn create holes in the valence band.

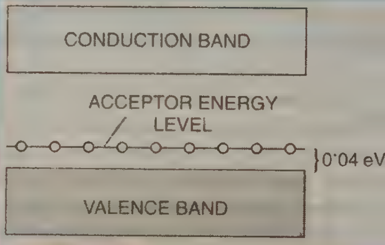


Fig. 1.13.

Key point

Fermi level in *p*-type semi-conductor (also known as acceptor energy level lie in forbidden energy gap and is very clos to valence band.

Number density of valence band holes (n_h) in *p*-type semiconductor is approximately equal to that of the acceptor atoms (N_a) and is very large as compared to the number density of conduction band electrons (N_e). Thus,

$$n_h \approx \mathrm{N}_a >> n_e \qquad \text{...(1.05)}$$

1.16. ELECTRICAL RESISTIVITY OF SEMICONDUCTORS

Consider a block of semiconductor of length *l*, area of cross-section A and having number density of electrons and holes as n_e and n_h respectively. Suppose that on applying a potential difference, say V, a current I flows through it as shown in Fig. 1.14. The electron current (I_e) and the hole current (I_h) constitute the current I flowing through the semiconductor *i.e.*

$$\mathrm{I} = \mathrm{I}_e + \mathrm{I}_h \qquad \text{...(1.06)}$$

If n_e is the number density of conduction band electrons in the semiconductor and v_e, the drift velocity of electrons, then electron current is given by

$$\mathrm{I}_e = e \, n_e \, \mathrm{A} \, v_e \qquad \text{...(1.07)}$$

Also, the hole current ,

$$\mathrm{I}_h = e \, n_h \, \mathrm{A} \, v_h \qquad \text{...(1.08)}$$

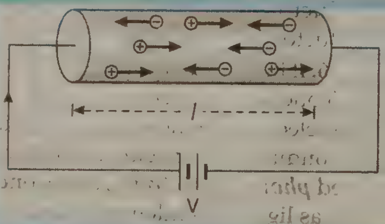


Fig. 1.14.

Using equations (1.07) and (1.08), the equation (1.06) becomes

$$I = e\, n_e\, A\, v_e + e\, n_h\, A\, v_h$$

or $$I = e\, A\,(n_e\, v_e + n_h\, v_h) \qquad \text{...(1.09)}$$

If ρ is the resistivity of the material of the semiconductor, then the resistance fered by the semiconductor to the flow of current is given by

$$R = \rho \frac{l}{A} \qquad \text{...(1.10)}$$

Since V = R I, from equations (1.09) and (1.10), we have

$$V = R\, I = \rho \frac{l}{A} \times e\, A\,(n_e\, v_e + n_h\, v_h)$$

or $$V = \rho\, l\, e\,(n_e\, v_e + n_h\, v_h) \qquad \text{...(1.11)}$$

If E is the electric field set up across the semiconductor, then

$$E = \frac{V}{l} \qquad \text{...(1.12)}$$

From equations (1.11) and (1.12), we have

$$E = \rho\, e\,(n_e\, v_e + n_h\, v_h)$$

or $$\frac{1}{\rho} = e\left(n_e \frac{v_e}{E} + n_h \frac{v_h}{E}\right) \qquad \text{...(1.13)}$$

Key point

For a given applied voltage, conduction current in *n*-type semiconductor is more than that in *p*-type semiconductor. It is because, mobility of electrons is greater than that of holes.

On applying electric field, the drift velocity acquired by the electrons (or holes) per unit trength of electric field is called ***mobility of electrons (or holes)***. Therefore, mobility f electrons and holes is given by

$$\mu_e = \frac{v_e}{E} \quad \text{and} \quad \mu_h = \frac{v_h}{E}$$

$$\therefore \qquad \frac{1}{\rho} = e\,(n_e\, \mu_e + n_h\, \mu_h) \qquad \text{...(1.14)}$$

Also, $\sigma = \frac{1}{\rho}$ is called conductivity of the material of semiconductor.

$$\therefore \qquad \sigma = e\,(n_e\, \mu_e + n_h\, \mu_h) \qquad \text{...(1.15)}$$

It may be pointed out that electron mobility is higher than hole mobility. The mobility of electrons and holes depend weakly on temperature and is independent of their number density. The change in conductivity of a semiconductor is mainly due to changes in carrier concentration.

1.17. SOME IMPORTANT POINTS ABOUT SEMICONDUCTORS

Following points may be noted about the behaviour of semiconductors :

1. At low temperature, a pure semiconductor behaves as an insulator. It becomes slightly conducting at room temperature. It is because, a few electrons in the valence band acquire energy greater than the forbidden energy gap and move to the conduction band.

2. It is very difficult to obtain an extremely pure semiconductor. Therefore, in practice, an intrinsic semiconductor is that in which the concentration of impurity atom is less than the concentration of intrinsic carriers.

3. Very small doping can drastically change the conductivity of intrinsic semiconductors.

4. Electron mobility is greater than the hole mobility in semiconductors.

5. The semiconductors are insulators at low temperature but become slightly conducting at room temperature. Thus, unlike metals, the resistance of semiconductors decreases with increase of temperature.

6. If light of energy greater than forbidden energy gap is incident on an intrinsic semiconductor, the electrons from the valence band move to the conduction band. Thus, electron and hole pairs are created. Due to increase in concentration of carriers, the conductivity of semiconductor increases. This property of semiconductors is called ***photoconductivity***. Semiconductors possessing large photoconductivity are used as light dependent resistors (LDR). Such resistors are used in automatic light control circuits.

Problem 1.01. Find the maximum wavelength of electromagnetic radiation, which can create a hole-electron pair in germanium. Given that forbidden energy gap in germanium is 0·72 eV.

Sol. Here, $E_g = 0\cdot72 \text{ eV} = 0\cdot72 \times 1\cdot6 \times 10^{-19}$ J

The maximum wavelength of radiation, which can create a hole-electron pair in germanium is given by

$$E_g = \frac{hc}{\lambda}$$

or $$\lambda = \frac{hc}{E_g} = \frac{6\cdot62 \times 10^{-34} \times 3 \times 10^{8}}{0\cdot72 \times 1\cdot6 \times 10^{-19}} = \mathbf{1\cdot724 \times 10^{-6}\ m}$$

Problem 1.02. Pure Si at 300 K has equal electron (n_e) and hole (n_h) concentration of 1·5 × 10^{16} m $^{-3}$. Doping by indium increases n_h to 4·5 × 10^{22} m^{-3}. Calculate n_e in the doped silicon. *(C.B.S.E 1990)*

Sol. Here, $n_i = 1\cdot5 \times 10^{16}$ m^{-3} ; $n_h = 4\cdot5 \times 10^{22}$ m^{-3}

Now, $n_e\, n_h = n_i^2$

$$\therefore \quad n_e = \frac{n_i^2}{n_h} = \frac{(1\cdot5 \times 10^{16})^2}{4\cdot5 \times 10^{22}} = \mathbf{5 \times 10^{9}\ m^{-3}}$$

Problem 1.03. A semiconductor is known to have an electron concentration of 8 × 10^{13} cm^{-3} and a hole concentration of 5 × 10^{12} cm^{-3}. (*a*) Is the semiconductor *n*-type or *p*-type ? (*b*) What is the resistivity of the sample, if the electron mobility is 23,000 cm^2 V^{-1}s^{-1} and hole mobility is 100 cm^2 V^{-1} s^{-1} ? Take charge on electron, *e* = 1·6 × 10^{-19} C. *(Text Problem)*

Sol. Here, $n_e = 8 \times 10^{13}$ m^{-3} = 18×10^{19} m^{-3} ;

$n_h = 5 \times 10^{12}$ cm^{-3} = 5×10^{18} m^{-2} ;

$\mu_e = 23{,}000$ cm^3 V^{-1} s^{-1} = $2\cdot3$ m^2 V^{-1} s^{-1} ;

$\mu_h = 100$ cm^2 V^{-1} s^{-1} = $0\cdot01$ m^2 V^{-1} s^{-1} ;

(*a*) Since the semiconductor has greater electr concentration, it is ***n*-type** semiconductor.

(*b*) Now, $$\frac{1}{\rho} = e\,(n_e\,\mu_e + n_h\,\mu_h)$$

$$= 1\cdot6 \times 10^{-19}\,(8 \times 10^{19} \times 2\cdot3 + 5 \times 10^{18} \times 0\cdot01)$$

$$= 1\cdot6 \times 10^{-19}\,(1\cdot84 \times 10^{20} + 5 \times 10^{16}) \approx 29\cdot45 \text{ mho m}$$

or $$\rho = \frac{1}{29\cdot45} = \mathbf{3\cdot396 \times 10^{-2}\ \Omega\ m}$$

Problem 1.04. Determine the number density donor atoms which have to be added to an intrins germanium semiconductor to produce an *n*-type semico ductor of conductivity 5 Ω^{-1} cm^{-1}, given that the mobili of electrons in *n*-type Ge is 3900 cm^2 V^{-1}s^{-1}. Neglect th contribution of holes to conductivity. Take charge o electron, *e* = 1·6 × 10^{-19} C .

Sol. Here, $e = 1\cdot6 \times 10^{-19}$ C ;

$\sigma = 5\ \Omega^{-1}$ cm^{-1} = $500\ \Omega^{-1}$ m^{-1} ;

$\mu_e = 3900$ cm^2 V^{-1} s^{-1} = $0\cdot39$ m^2 V^{-1} s^{-1}

Now, $\sigma = e\, n_e\, \mu_e$ (neglecting contribution of hole

$$\therefore \quad n_e = \frac{\sigma}{e\,\mu_e} = \frac{500}{1\cdot6 \times 10^{-19} \times 0\cdot39}$$

$$= \mathbf{8\cdot013 \times 10^{21}\ m^{-3}}$$

abc TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 1.05. The mean free path of conduction electrons in copper is about 4·2 × 10^{-8} m. Find the electric field which can give 2·4 eV energy (on the average) to a conduction electron in a copper block.

Sol. Let E be the required electric field. Then, force on electron due to electric field,

$$F = e\,E = 1\cdot6 \times 10^{-19}\ E$$

The work done by electric field on electron, before it collides with a copper ion/atom,

$$W = F \times \text{mean free path} = 1\cdot6 \times 10^{-19}\ E \times 4\cdot2 \times 10^{8}$$

$$= 6\cdot72 \times 10^{-27}\ E \text{ (in joule)} = \frac{6\cdot72 \times 10^{-27}\ E}{1\cdot6 \times 10^{-19}}$$

$$= 4\cdot2 \times 10^{-19}\ E \text{ (in eV)}$$

$$\therefore \quad 4\cdot2 \times 10^{-8}\ E = 2\cdot4 \quad \text{or} \quad \mathbf{E = 5\cdot71 \times 10^{7}\ V\ m^{-1}}$$

Problem 1.06. Suppose that the energy liberated in the combination of a hole-electron pair is converted into electromagnetic radiation. If the maximum wavelength of the radiation emitted is 630 m, what is the width of forbidden energy gap ?

Sol. The width of forbidden energy gap is equal to energy of the radiation emitted on the combination of a hole and an electron. Therefore,

$$E_g = \frac{hc}{\lambda} = \frac{6\cdot62 \times 10^{-34} \times 3 \times 10^{8}}{630 \times 10^{-9}} = 3\cdot152 \times 10^{-19}\ \text{J}$$

$$= \frac{3\cdot152 \times 10^{-19}}{1\cdot6 \times 10^{-19}} = \mathbf{1\cdot97\ eV}$$

Problem 1.07. In Fig. 1.14, a battery of e.m.f. 2 V is used The length of the block is 0·1 m and the area is 10^{-4}m^2 If the block is of intrinsic silicon at 300 K, find the electro and hole currents. What will be the magnitude of the tota current ? What will be the magnitude of the total current if germanium is used instead of silicon ?

Given that for Si : $\mu_e = 0\cdot135$ m^2 V^{-1}s^{-1} ;

$\mu_h = 0\cdot048$ m^2 V^{-1} s^{-1} ;

intrinsic carrier concentration, $n_i = 1\cdot5 \times 10^{16}$ m^{-3}

and for Ge : $\mu_e = 0\cdot39$ m^2 V^{-1} s^{-1} ;

$\mu_h = 0\cdot19$ m^2 V^{-1} s^{-1} ;

$n_i = 2\cdot4 \times 10^{19}$ m^{-3}. *(Text Problem)*

Sol. Here, A = 10^{-4} m^2 ; $l = 0\cdot1$ m ; V = 2 volt ; T = 300 K ;

For Si : $\mu_e = 0\cdot135$ m^2 V^{-1} s^{-1} ;

$n_i = n_e = n_h = 1\cdot5 \times 10^{15}$ m^{-3} ;

$\mu_h = 0\cdot048$ m^2 V^{-1} s^{-1}

Now, $$E = \frac{V}{l} = \frac{2}{0\cdot1} = 20 \text{ V m}^{-1}$$

Now, $v_e = \mu_e\, E = 0\cdot135 \times 20 = 2\cdot7$ m s^{-1}

and $v_h = \mu_h\, E = 0\cdot048 \times 20 = 0\cdot96$ m s^{-1}

Now, electron current.

$$I_e = e\,A\,n_e\,v_e = 1{\cdot}6\times10^{-19}\times10^{-4}\times1{\cdot}5\times10^{16}\times2{\cdot}7$$
$$= \mathbf{6{\cdot}48\times10^{-7}\ A}$$

and hole current, $I_h = e\,A\,n_h\,v_h$

$$= 1{\cdot}6\times10^{-19}\times10^{-4}\times1{\cdot}5\times10^{16}\times0{\cdot}96$$
$$= \mathbf{2{\cdot}304\times10^{-7}\ A}$$

Therefore, total current, $I = I_e + I_h$

$$= 6{\cdot}48\times10^{-7} + 2{\cdot}304\times10^{-7} = \mathbf{8{\cdot}784\times10^{-7}\ A}$$

For Ge : $\mu_e = 0{\cdot}39\ m^2V^{-1}s^{-1}$; $\mu_h = 0{\cdot}19\ m^2V^{-1}s^{-1}$;

$$n_e = n_h = n_i = 2{\cdot}4\times10^{19}\ m^{-3}$$

$$v_e = \mu_e\,E = 0{\cdot}39\times20 = 7{\cdot}8\ m\,s^{-1},$$
$$v_h = \mu_h\,E = 0{\cdot}19\times20 = 3{\cdot}8\ m\,s^{-1}$$

$\therefore$ $I_e = e\,A\,n_e\,v_e$

$$= 1{\cdot}6\times10^{-19}\times10^{-4}\times2{\cdot}4\times10^{19}\times7{\cdot}8$$
$$= \mathbf{2{\cdot}995\times10^{-3}\ A}$$

$I_h = e\,A\,n_h\,v_h$

$$= 1{\cdot}6\times10^{-19}\times10^{-4}\times2{\cdot}4\times10^{19}\times3{\cdot}8$$
$$= \mathbf{1{\cdot}459\times10^{-3}\ A}$$

$\therefore$ $I = I_e + I_h = 2{\cdot}995\times10^{-3} + 1{\cdot}459\times10^{-3}$

$$= \mathbf{4{\cdot}454\times10^{-3}\ A}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. Crystalline substances have a sharp melting point. Why ?

Ans. It is because all the bonds in a crystal get ruptured at a unique fixed temperature for that crystal. At this fixed temperature, called melting point, the crystal changes into liquid state.

Q. 1.02. Fill in the blanks :

(*i*) Majority of the solids in nature have .... internal structure (ordered, disordered).

(*ii*) All crystals are .... (isotropic, anisotropic).

(*iii*) If the physical properties of a solid do not depend on the direction, the solid is known as .... (isotropic, anisotropic).

(*iv*) The polycrystalline solids show .... properties, while the monocrystals have .... properties. (isotropic, anistropic)

(*v*) Within the liquid crystal phase, some materials show a change in .... with the change in temperature. (colour, shape).

(*vi*) Physical properties of amorphous solids are identical in .... directions and therefore they are of .... nature. (all, some, isotropic, anisotropic).

(*vii*) The liquid crystal displays (LCD) are based on the principle that they .... (emit light, change the plane of polarisation of polarised light).

(*viii*) Ceramics are .... solids (polycrystalline, amorphous).

(*ix*) Ceramic solids are .... insulators (generally, always).

(*x*) The structure of NaCl is .... (cubic, hexagonal). (*Text Question*)

Ans. (*i*) ordered (*ii*) isotropic (*iii*) isotropic (*iv*) isotropic, anisotropic (*v*) colour (*vi*) all, isotropic (*vii*) change the plane of polarisation of polarised light (*viii*) polycrystalline (*ix*) generally (*x*) cubic

Q. 1.03. Name the seven crystal systems.

Ans. (*i*) Triclinic system (*ii*) Monoclinic system (*iii*) Rhombic system (*iv*) Tetragonal system (*v*) Trigonal system (*vi*) Hexagonal system and (*vii*) Cubic system.

Q. 1.04. What is the electron configuration in various orbits of silicon and germanium ?

Ans. *For silicon* : $1s^2\,2\,s^2\,2\,p^6\,3\,s^2\,3\,p^2$

For germanium : $1s^2\,2\,s^2\,2\,p^6\,3\,s^2\,3\,p^6\,3\,d^{10}\,4\,s^2\,4\,p^2$

Q. 1.05. Name the charge carriers in the following at room temperature : (*i*) Conductor (*ii*) Intrinsic semiconductor (*iii*) Insulator. (*I.S.C.E. 1997*)

Ans. (*i*) Electrons (*ii*) Electrons and holes (*iii*) None.

Q. 1.06. What is valence band ?

Ans. The energy band formed by a series of energy levels containing the valence electrons is known as valence band.

Q. 1.07. What is conduction band ?

Ans. The lowest unfilled energy band lying just above the valence band is called conduction band.

Q. 1.08. What is Fermi energy level ? (*P.S.S.C.E. 2002*)

Ans. The highest energy level, which an electron can occupy in the valence band at 0 K is called Fermi energy level.

Q. 1.09. Fill in the blanks :

(*i*) When two isolated atoms are brought close to each other such that the distance between them is .... (comparable to, much larger than) lattice spacing, the .... (energy bands, energy levels) of the outermost electrons is split into .... (very large, very small) number of levels, called .... (energy gap, energy bands).

(*ii*) The energy band in solids is an outcome of .... (Pauli's exclusion principle, Coulomb's law).

(*iii*) The forbidden energy gap in case of a semi-conductor is .... as compared to that of an insulator (smaller, greater).

(*iv*) The conduction band of an insulator is .... empty. (partially, practically). (*Text Question*)

Ans. (*i*) comparable to, energy levels, very large, energy bands (*ii*) Pauli's exclusion principle (*iii*) smaller (*iv*) practically.

Q. 1.10. Give the ratio of the number of holes and the number of conduction electrons in an instrinsic semiconductor. (C.B.S.E 1997)

Ans. It is 1.

Q. 1.11. Draw an energy-band diagram for an intrinsic semiconductor. (C.B.S.E 1998)

Ans. Refer to Fig. 1.08.

Q. 1.12. What is doping ? (*H.P.S.S.C.E. 2001, 1997 ; I.S.C.E. 1995*)

Ans. The process of adding trivalent or pentavalent atoms to a pure semiconductor in a very small ratio (1 : 10^6) is called doping.

Q. 1.13. What type of impurity is added to obtain *n*-type semiconductor ? (*H.P.S.S.C.E. 1999 S*)

Ans. Pentavalent atoms, such as arsenic.

Q. 1.14. Doping of silicon with indium leads to which type of semiconductor ? (*C.B.S.E. Sample Q. Paper*)

Ans. Doping of silicon with indium leads to *p*-type semiconductor.

Q. 1.15. What type of charge carriers are there in an *n*-type semiconductor ? (*C.B.S.E 1997*)

Ans. Both electrons and holes exist as charge carriers in an *n*-type semiconductor. However, electrons are majority charge carriers, while holes are minority charge carriers.

Q. 1.16. What type of charge carriers are there in a *p*-type semi-conductor ? (*H.S.S.C.E. 1999, 1998*)

Ans. Both holes (majority carriers) and electrons (minority carriers).

Q. 1.17. What is electron mobility ?

Ans. It is defined as the drift velocity acquired by the electrons per unit applied electric field.

Q. 1.18. Fill in the blanks :

(*i*) In an intrinsic semiconductor, the electron and hole concentrations are .... (equal, unequal)

(*ii*) When a block of semiconductor is connected to a battery by a metallic wire, the current flow in the wire is due to the motion of .... in the wire. (electrons, electrons and holes)

(*iii*) The doping of an intrinsic semiconductor with certain type of impurity atoms causes an .... in its electric conductivity. (decrease, increase)

(*iv*) When a battery is connected to a *p*-type semi-conductor with metallic wire, the current in the semi-conductor is predominantly due to .... (electrons, holes)

(*v*) The movement of charge carriers from a region of higher concentration is called .... (drift, diffusion)

(*vi*) The drift velocity of electrons is expected to be .... (equal to, greater than) holes. (*Text Question*)

Ans. (*i*) equal (*ii*) electrons (*iii*) increase (*iv*) holes (*v*) diffusion (*vi*) greater than.

Q. 1.19. What is an extrinsic semiconductor ? (*P.S.S.C.E. 2002, 1998, 1997*)

Ans. A semiconductor doped with a suitable impurit (donor or acceptor), so that it possesses conductivity muc higher than that of pure semiconductor, is called an extrinsi semiconductor.

Q. 1.20. Is the ratio of number of holes and the numbe of conduction electrons in an *n*-type extrinsic semicon ductor more than, less than or equal to 1 ? (*C.B.S.E 1997*

Ans. It is less than 1.

Q. 1.21. Draw energy-band diagram for an *n*-typ extrinsic semiconductor. (*C.B.S.E 1998*

Ans. Refer to Fig. 1.11.

Q. 1.22. Draw the energy band diagram for a *p*-typ seminconductor. (*C.B.S.E. 2001*

Ans. Refer to Fig. 1.13.

Q. 1.23. How does the energy gap of an intrinsi seminconductor vary, when doped with a trivalen impurity ? (*C.B.S.E. 2002*

Ans. When a trivalent impurity is added to an intrinsi semiconductor, an acceptor energy level is created in th forbidden energy gap just above the valence bond. Due t this, electrons from the valence bond are easily transferre to the acceptor energy level.

Q. 1.24. How does conductivity of a semiconducto change with rise in temperature ? (*C.B.S.E 1996 S*

Ans. The conductivity of a semiconductor increases wit temperature.

Q. 1.25. Fill in the blanks :

(*i*) As the temperature rises, the resistance offered b metals .... (increases, decreases)

(*ii*) The semiconductors are .... at absolute zero (conductors, insulators)

(*iii*) The resistance of semiconductors .... with increasing temperature. (decreases, increases) (*Text Question*

Ans. (*i*) increases (*ii*) insulators (*iii*) decreases.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS
With Answers/Hints

Q. 1.01. Distinguish between crystalline and amorphous solids.

Ans. Refer to sections 1.02 and 1.03.

Q. 1.02. What is the shape of the unit cell of a cubic and a hexagonal crystal system ?

Ans. The unit cells of a cubic crystal are cubic in nature *i.e.* $a = b = c$ and $\alpha = \beta = \gamma = 90°$.

The unit cells of a hexagonal crystal are made of three elementary cells, so as to form a hexagonal prism. Therefore, for its unit cell, $a = b \neq c$, $\alpha = \beta = 90°$ and $\gamma = 120°$.

Q. 1.03. What is the difference between a single crystal and a polycrystal ?

Ans. A single crystal is that in which the ordered arrangement of atoms (or molecules) extends throughout the piece of crystal. Its physical properties are different in different directions *i.e.* single cyrstals are ***anisotropic***.

On the other hand, the crystalline solids made of a large number of monocrystals (small sized single crystals) with well developed faces joined together are called polycrystals. They are *isotropic* in nature *i.e.* their physical properties are same in all directions.

Q. 1.04. Distinguish between energy levels and energy bands.

Ans. An isolated atom has various energy levels. In a crystal, atoms are so closely packed that due to interaction between them, splitting of energy levels takes place. The energy levels, then, form energy bands. The energy band formed by a series of energy levels containing the valence electrons is called valence band and the lowest unfilled energy band formed just above the valence band is called conduction band.

Q. 1.05. Distinguish between intrinsic and extrinsic semiconductors ? (*H.S.S.C.E. 2002*)

Ans. A semiconductor free from all types of impurities is called an intrinsic semiconductor. At room temperature, a few covalent bonds break up and the electrons come out. In the bonds, from which electrons come out, vacancies are created. These vacancies in covalent bonds are called *holes*.

n an intrinsic semiconductor, holes and electrons are equal n number and they are free to move about in the emiconductor. On the other hand, a semiconductor doped vith a suitable impurity (donor or acceptor) so that it ossesses conductivity much higher than that of pure emiconductor is called an extrinsic semiconductor. The xtrinsic semiconductor may be of n-type or p-type.

Q. 1.06. Distinguish between insulators, conductors nd semi-conductors in terms of their energy band diagram. *(I.S.C.E. 1998)*

Or

Give the energy band diagram for (*i*) insulators, (*ii*) netals and (*iii*) semi-conductors. *(H.S.S.C.E. 1998)*

Or

Draw a labelled energy band diagram for an insulator, conductor and a semi-conductor. *(I.S.C.E. 1994)*

Ans. Refer to Figs. 1.07, 1.06 and 1.08 respectively.

Q. 1.07. What is meant by doping ? Why is it done ? *(I.S.C.E. 1997)*

Ans. Doping. Refer to VSQ 1.12.

At room temperature, the number of free electrons and holes in a pure semi-conductor is so small that the conductivity offered by it is practically of no use. Doping is done so as to improve the conductivity of a semi-conductor.

Q. 1.08. What is meant by doping ? Draw energy band diagram of a n-type semiconductor. *(H.P.S.S.C.E. 1998 S)*

Ans. Doping. Refer to VSQ 1.12

For energy band diagram, refer to Fig. 1.11.

Q. 1.09. From consideration of energy, show that doping of n-type semiconductor can easily produce transport electrons. *(H.S.S.C.E. 1994)*

Ans. Refer to section 1.14.

Q. 1.10. Distinguish between n-type and p-type semiconductors. *(H.S.S.C.E. 2002)*

Ans. An n-type semiconductor contains a large number of free electrons and a small number of holes. On the other hand, a p-type semiconductor has a large number of holes and a small number of free electrons.

Q. 1.11. Distinguish between n-type and p-type semi-conductors on the basis of energy band diagram. *(C.B.S.E 1994)*

Ans. Refer to Figs. 1.11. and 1.13.

Q. 1.12. For an extrinsic semiconductor, indicate on the energy band diagram the donor and acceptor levels. *(C.B.S.E 1993)*

Ans. Refer to Figs. 1.11 and 1.13.

Q. 1.13. Draw the energy band diagram of a n-type semiconductor. How does the forbidden energy gap of an intrinsic semiconductor vary with increase in temperature ? *(C.B.S.E. 2002)*

Ans. For energy band diagram, refer to Fig. 1.13.

When temperature of a semiconductor is increased, a greater number of electrons are raised from valence to conduction band. It may be considered, as if the width of forbidden energy gap has decreased.

Q. 1.14. Draw the energy band diagram of a p-type semiconductor. How does the forbidden energy gap of an intrinsic semiconductor vary with increase in temperature ? *(C.B.S.E. 2002)*

Ans. For energy band diagram, refer to Fig. 1.11 and for answer to remaining part of the question, refer to SAQ 1.14.

Q. 1.15. What is the effect of rise in temperature on the resistance of conductors (metals) ?

Ans. The number of free electrons in metals is so large that they may be treated to behave as a dense electron gas in the metals. When the temperature increases, the kinetic energy of electrons also increases. With the increase in kinetic energy, the number of collisions between electrons themselves and between electrons and atoms or ions of the metal increases. This increases the resistance of the metal, due to which the conductivity decreases.

Q. 1.16. What is an intrinsic semiconductor ? Why the conductivity of intrinsic semiconductor increases with rise of temperature ? *(H.S.S.C.E. 1993)*

Ans. For intrinsic semi-conductor-refer to SAQ 1.05.

The fraction of the number of electrons raised from valence band to conduction band at temperature T is given by

$$f \propto e^{-E_g/kT},$$

where E_g is value of the forbidden energy gap. It follows that as T is increased, f also increases. The greater number of electrons in conduction band leads to the greater conductivity of the semiconductor.

Q. 1.17. How does the conductivity change with rise in temperature in case of semi-conductors ?

Ans. At 0 K, all semiconductors are insulators. The valence band at 0 K is completely filled and there are no free electrons in conduction band. At room temperature, due to their thermal energy, the electrons jump to the conduction band. When the temperature increases, a large number of electrons cross over the forbidden gap and jump from valence band to conduction band. Hence, conductivity of semiconductor increases with temperature.

Q. 1.18. Explain the effect of temperature variation on the resistivity of pure semiconductors. *(I.S.C.E. 1997)*

Ans. The variation in the resistivity of a pure semi-conductor is mainly due to change in carrier concentration. The fraction of the number of electrons raised from valence band to the conduction band is given by

$$f \propto e^{-E_g/kT}$$

It follows that as temperature (T) increases, the value of fraction f also increases *i.e.* conductivity of the semi-conductor increases. In other words, as temperature of the semi-conductor increases, its resistivity decreases.

Q. 1.19. Why is a semiconductor damaged by a strong current ? *(H.P.S.S.C.E. 1998 ; P.S.S.C.E. 1991)*

Ans. A strong current, when passed through a semiconductor, heats up the semiconductor and the covalent bonds break up. It results in a large number of free electrons. The material, then, behaves just as a conductor. As now the semiconductor no longer possesses the property of low conduction, it is said to be damaged.

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 1.01. The amorphous substances do not melt at a sharp temperature ; rather they have a softening range. Explain this observation.

Ans. The amorphous substances possess short range order and the bonds between all the atoms and molecules are not of equal strength. On heating, the weak bonds get ruptured first and the stronger bonds later on. Owing to this, these substances do not have sharp melting points.

Q. 1.02. An *n*-type semiconductor has a large number of electrons but still it is electrically neutral. Explain.

Ans. An *n*-type semiconductor is obtained by doping pure Si or Ge-crystal with pentavalent impurity. As the impurity atoms enter into the configuration of the Si-crystal, its four electrons take part in covalent bonding, while the fifth electron is left free. Since each atom of the semiconductor as a whole is electrically neutral ; the *n*-type Ge-crystal, though having large number of free electrons, is electrically neutral.

Q. 1.03. What is donor energy level ? Explain.

Ans. In case of *n*-type Si-crystal, the free electron produced by the donor atom (As-atom) revolves round the nucleus of the donor atom. It possesses a very small negative energy (≈ 0·045 eV)as it is revolving in the background of Si-crystal (having a high value of dielectric constant). All such electrons create an energy level just 0·045 eV below the conduction band. The energy level created by such electrons is called donor energy level. As energy gap between donor energy level and conduction band is very small, electrons can easily cross it and the conductivity of the *n*-type semiconductor is greatly improved.

Q. 1.04. What is acceptor energy level ? Explain.

Ans. In case of *p*-type Si-crystal, the hole produced by th acceptor atom (In-atom) revolves around the nucleus o acceptor atom. As the motion of the hole around the nucleu is equivalent to the motion of a positively charged particle bu of the mass same as that of electron, it possesses a smal positive energy (≈ 0·04 eV). All such electrons create a energy level just 0·04 eV above the valence band. The energ level created by such electrons is called acceptor energy leve As the difference of energy between valence band and th acceptor energy level is very small, the electrons from valenc band can easily raise themselves to the acceptor energy leve It leads to an increase in the conductivity of the *p*-typ semiconductor.

Q. 1.05. What is the difference between hole-curren and electron flow ?

Ans. In a *p*-type semiconductor, there are vacancies calle holes. When such a material conducts, an electron from nearby covalent bond jumps into the vacant place in order t fill it and thereby the hole shifts to the covalent bond fron which the electron has jumped. The movement of hole constitutes the hole-current.

In an *n*-type semiconductor, the free electrons constitut the electron-current.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Explain, with neat diagram, formation of energy bands in solids. *(H.S.S.C.E. 1996, 1995)*
2. Draw the energy band diagrams of the following substances :
   (*a*) conductor (*b*) *n*-type semiconductor (*c*) *p*-type semiconductor (*d*) insulator. *(P.S.S.C.E. 1999 S)*
3. What is meant by energy bands in solids ? Draw labelled energy band diagrams to illustrate the behaviour of (*i*) a conductor ; (*ii*) a pure semi-conductor ; and (*iii*) an insulator. *(I.S.C.E. 1995)*
4. With the help of energy band diagrams, distinguish between the conductors, insulators and semiconductors. *(C.B.S.E 1999 S ; P.S.S.C.E. 1996 S)*
5. What is an intrinsic semiconductor ? Give one example. With the help of diagram, explain how you can convert an intrinsic semiconductor into an *n*-type extrinsi semiconductor.
6. What are intrinsic and extrinsic semiconductors ? Explai how an *n*-type semiconductor is obtained.
7. Explain, how an intrinsic semiconductor can be converte into (*i*) *n*-type and (*ii*) *p*-type semiconductor. Give on example of each and their energy band diagrams. *(C.B.S.E. 2001 S*
8. What is meant by doping ? Explain, how it changes th conductivity of the resulting extrinsic semiconductors.
9. What is an extrinsic semiconductor ? Discuss the workin of a donor type semiconductor. *(H.P.S.S.C.E. 1998 S*
10. Compare intrinsic and extrinsic semiconductors. *(P.S.S.C.E. 2002*

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Explain the formation of energy bands in solids and hence distinguish between conductors, semiconductors and insulators. *(H.P.S.S.C.E. 1997)*
2. How are energy bands formed in solid ? Explain the behaviour of conductors, insulators and semiconductors on the basis of energy bands. *(P.S.S.C.E. 2002, 2000, 1994 ; H.S.S.C.E. 2001)*
3. Explain the formation of energy bands in solids. Distinguish between conductors, extrinsic and intrinsic semiconductors and insulators on the basis of band theory. *(C.B.S.E 1994)*
4. Explain the formation of energy bands in solids. On th basis of the energy bands diagrams, distinguish betwee (*i*) a metal, (*ii*) an insulator and (*iii*) a semiconductor. *(C.B.S.E 1996, 1990*
5. What do you mean by doping ? State the necessar conditions for doping ? What are the various methods o doping ? *(P.S.S.C.E. 2002*
6. (*a*) Explain, with examples, the formation of energy band in solids.
   (*b*) For an extrinsic semiconductor, mark the donor an acceptor levels in the energy band diagram. *(C.B.S.E 1993*

7. Derive an expression for current through a semiconductor. Obtain expression for electrical resistivity and conductivity of a semiconductor. *(H.S.S.C.E. 2001, 1998 S)*
8. Using the concept of electron and hole current, derive expression for the conductivity of the semiconductor. *(H.P.S.S.C.E. 1999 S ; H.S.S.C.E. 1994)*
9. Derive expression for current through a semiconductor in terms of concentration and mobility of intrinsic carriers. Hence, obtain expression for electrical resistivity of a semiconductor.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Conductivity of a semiconductor

1. Energy band gap of gallium arsenide phosphate is 1·98 eV. Calculate the wavelength of electromagnetic radiation emitted, when electrons and holes combine in this alloy semiconductor directly. **[Ans.** 6269 Å**]**
2. A semiconductor has equal electron and hole concentration of 6×10^8 m^{-3}. On doping with a certain impurity, electron concentration increases to 8×10^{12} m^{-3}. *(i)* Identify the new semiconductor obtained after doping. *(ii)* Calculate the new hole concentration. *(iii)* How does the energy gap vary with doping ? *(C.B.S.E. 2001)*
   [Ans. *(i) n*-type *(ii)* $4{\cdot}5 \times 10^4$ m^{-3} *(iii)* decreases**]**
3. A semiconductor has equal electron and hole concentrations of 2×10^8 m^{-3}. On doping with a certain impurity, the hole concentration increases to 4×10^{10} m^{-3}. *(i)* What type of semiconductor is obtained on doping ? *(ii)* Calculate the new electron concentration of the semiconductor. *(iii)* How does the energy gap vary with doping ? *(C.B.S.E. 2001)*
   [Ans. *(i) p*-type *(ii)* 10^6 m^{-3} *(iii)* decreases**]**
4. A semiconductor has the electron concentration of 8×10^{13} cm^{-3} and hole concentration 4×10^{12} cm^{-3}. Is this semiconductor *p*-type or *n*-type ? Also calculate the resistivity of this semiconductor. Given, electron mobility = 24000 cm^2 V^{-1} s^{-1} and hole mobility = 200 cm^2 V^{-1} s^{-1}. *(P.S.S.C.E. 2001)*
   [Ans. *n*-type, $3{\cdot}07 \times 10^{-11}$ S m^{-1}**]**
5. A semiconductor has the electron concentration 4×10^{12} cm^{-3} and hole concentration 7×10^{13} cm^{-3}. Is the semiconductor *n*-type or *p*-type ? Also calculate the conductivity of this semiconductor. Given, electron mobility = 22000 cm^2 V^{-1} s^{-1} and hole mobility = 150 cm^2 V^{-1} s^{-1}. *(P.S.S.C.E 2001)*
   [Ans. *p*-type, $1{\cdot}576 \times 10^{-12}$ S m^{-1}**]**
6. A semiconductor has the electron concentration $0{\cdot}45 \times 10^{12}$ m^{-3} and hole concentration 5×10^{20} m^{-3}. Find its conductivity. Given, electron mobility = 0·135 m^2 V^{-1} s^{-1} ; hole mobility = 0·048 m^2 V^{-1} s^{-1} ; $e = 1{\cdot}6 \times 10^{-19}$ C. *(P.S.S.C.E. 1995)* **[Ans.** 3·84 S m^{-1}**]**
7. Predict the effect on the electrical properties of a silicon crystal at room temperature, if every millionth silicon atom is replaced by an atom of indium. Given : concentration in Si-atoms = 5×10^{28} m^{-3} ; intrinsic carrier concentration, $n_i = 1{\cdot}5 \times 10^{16}$ m^{-3} ; μ_e for Si-crystal = 0·135 m^2 V^{-1} s^{-1} ; μ_h for Si-crystal = 0·048 m^2 V^{-1} s^{-1} ; intrinsic conductivity of Si = $4{\cdot}4 \times 10^{-4}$ S m^{-1} ; intrinsic resistivity of Si = 2300 Ω m.
   [Ans. σ = 384 S m^{-1} (becomes very large), ρ = 0·0026 Ω m (becomes very low)**]**
8. Estimate the fraction of electrons, which are excited from the energy band through an energy gap of 1·1 eV at a temperature of 300 K. What is the fraction at 600 K ? Given, Boltzmann's constant, $k = 1{\cdot}38 \times 10^{-23}$ J.
9. In an intrinsic semiconductor, the energy gap E$_g$ of an intrinsic semiconductor is 1·2 eV. Its hole mobility is very much smaller than electron mobility and is independent of temperature. What is the ratio between conductivity at 600 K and that at 300 K ? Assume that the temperature dependence of intrinsic carrier concentration n_i is expressed as

$$n_i = n_0\, e^{-E_g'/kT},$$

where n_0 is a constant and E$_g{}'$ is an energy equal to E$_g$/2. Given, Boltzmann's constant, $k = 8{\cdot}62 \times 10^{-5}$ eV K^{-1}.
[Ans. 10^5**]**

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Proceed as in solved problem no. 1.01.
2. *(i) n*-type
   (ii) Proceed as in solved problem no. 1.02.
   (iii) Decreases.
3. *(i) p*-type
   (ii) Proceed as in solved problem no. 1.02.
   (iii) Decreases.
4. Proceed as in solved problem no. 1.03.
5. Proceed as in solved problem no. 1.03.
6. $\sigma = e\,(n_e \mu_e + n_h \mu_h)$

$$= 1{\cdot}6 \times 10^{-19}\,(0{\cdot}45 \times 10^{12} \times 0{\cdot}135 + 5 \times 10^{20} \times 0{\cdot}048)$$
$$\approx 1{\cdot}6 \times 10^{-19}\,(5 \times 10^{20} \times 0{\cdot}048) = \mathbf{3{\cdot}84\ S\ m^{-1}}$$

7. Number of indium atoms added to silicon crystal

$$= \frac{5 \times 10^{28}}{10^6} = 5 \times 10^{22}\ \text{m}^{-3}$$

Thus, hole concentration, $n_h = 5 \times 10^{22}$ m^{-3}

Now, electron concentration,

$$n_e = \frac{n_i^2}{n_h} = \frac{(1{\cdot}5 \times 10^{16})^2}{5 \times 10^{22}} = 4{\cdot}5 \times 10^9\ \text{m}^{-3}$$

Hence, conductivity of silicon crystal,

$$\sigma = e\,(n_e \mu_e + n_h \mu_h)$$
$$\doteq 1{\cdot}6 \times 10^{-19}\,(4{\cdot}5 \times 10^9 \times 0{\cdot}135 + 5 \times 10^{22} \times 0{\cdot}048)$$
$$= \mathbf{384\ S\ m^{-1}}\ \textbf{(becomes very large)}$$

Also, resistivity of silicon crystal,

$$\rho = \frac{1}{\sigma} = \frac{1}{384} = \mathbf{0{\cdot}0026\ \Omega\ m}\ \textbf{(becomes very low)}$$

8. Fraction of electrons, which are excited from the energy band through an energy gap E$_g$,

$$\frac{n_i}{n_0} = e^{-E_g/kT}$$

Here, E$_g$ = 1·1 eV = $1{\cdot}1 \times 1{\cdot}6 \times 10^{-19}$ J

Boltzmann's constant, $k = 1{\cdot}38 \times 10^{-23}$ J K^{-1}

At T = 300 K :

$$\therefore \frac{n_i}{n_0} = e^{-\frac{1\cdot1 \times 1\cdot6 \times 10^{-19}}{1\cdot38 \times 10^{-23} \times 300}} = e^{-42\cdot5} = \mathbf{3\cdot49 \times 10^{-19}}$$

At T = 600 K :

$$\therefore \frac{n_i}{n_0} = e^{-\frac{1\cdot1 \times 1\cdot6 \times 10^{-19}}{1\cdot38 \times 10^{-23} \times 600}} = e^{-21\cdot26} = \mathbf{5\cdot85 \times 10^{-10}}$$

9. $E_g' = E_g/2 = 1\cdot2/2 = 0\cdot6$ eV ; $k = 8\cdot62 \times 10^{-5}$ eV K^{-1}

$$\frac{n_i \text{ (at 600 K)}}{n_i \text{ (at 300 K)}} = \frac{n_0\, e^{-E_g'/k \times 600}}{n_0\, e^{-E_g'/k \times 300}} = e^{E_g'/k \times 600}$$

$$= e^{0\cdot6/8\cdot62 \times 10^{-5} \times 600} = e^{11\cdot6}$$

$$= 1{,}09{,}098 \approx \mathbf{10^5}$$

CHAPTER 2

Semiconductor Devices

2.01. FORMATION OF p-n JUNCTION

A *p-n* juntion is a basic semiconductor device.

When a *p*-type crystal is placed in contact with *n*-type crystal so as to form one piece, the assembly so obtained is called ***p-n junction*** or ***junction diode*** or ***crystal diode***. The surface of contact of *p* and *n*-type crystals is called ***junction***. In the *p*-section, holes are the majority carriers ; while in *n*-section, the majority carriers are electrons. Due to the high concentration of different types of charge carriers in the two sections, holes from *p*-region diffuse into *n*-region and electrons from *n*-region diffuse into *p*-region. In both cases, when an electron meets a hole, the two cancel the effect of each other and as a result, a thin layer at the junction becomes devoid of charge carriers. This is called ***depletion layer*** [Fig. 2.01]. The thickness of the depletion layer is of the order of 10^{-6} m.

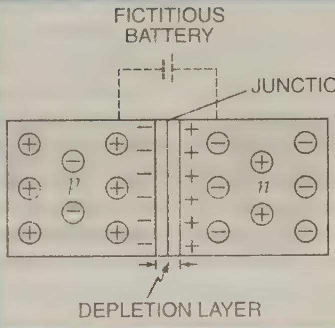


Fig. 2.01.

Key point

An *n*-type semiconductor has a large number of electrons and a *p*-type semiconductor, a large number of holes. But still both are electrically neutral.

Due to the diffusion of holes and electrons, the two sections of the junction diode no longer remain neutral. The *p*-section of the junction diode becomes slightly negative, while the *n*-section is rendered positive. It appears as if some fictitious battery is connected across the junction with its negative pole connected to *p*-region and positive pole connected to *n*-region. The potential difference developed across the junction due to migration of majority charge carriers is called ***potential barrier***. It opposes the further diffusion of charge carriers. The magnitude of the potential barrier is about 0·3 volt for germanium junction diode and about 0·7 volt for silicon junction diode. However, the value of potential barrier depends on the magnitude of doping of the semiconductor crystal.

It may be pointed out that across the junction, a very large electric field is set up due to potential difference developed across it. Taking width of the depletion layer of a silicon junction diode as 10^{-6} and a potential difference of 0·7 V set up across it, the strength of electric field is of the order of

$$E = \frac{0 \cdot 7}{10^{-6}} = 7 \times 10^{-5}\ \text{V m}^{-1}$$

Thus, the formation of *p-n* junction results in a very strong electric field (or potential gradient) across the junction.

2.02. FORWARD AND REVERSE BIASING ON A JUNCTION DIODE

A junction diode can be biased in the following two ways :

1. Forward Bias. *When an external d.c. source is connected to the diode with p-section connected to positive pole and n-section to the negative pole, the junction diode is said to be forward biased* [Fig. 2.02].

Action of p-n junction. When the *p-n* junction is forward biased, the positive holes in the *p*-section are repelled by positive pole of the battery towards the *p-n* junction. Simultaneously, the negative electrons in the *n*-section are repelled by negative pole of the battery towards the junction. However, the movement of electrons and holes across the junction is opposed by the fictious battery voltage (≈ 0·3 V to 0·7 V) developed across the junction. Just near the *p-n* junction, electrons and holes combine and cease to exist as mobile charge carriers after the potential barrier is overcome by the applied potential.

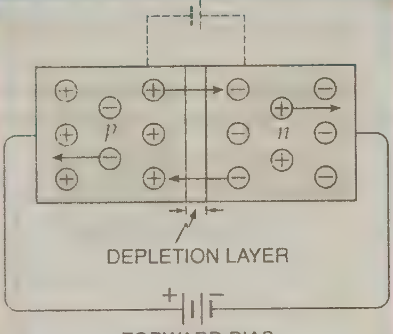


Fig. 2.02.

For each electron-hole combination that takes place near the junction, a covalent bond breaks in the *p*-section near the positive pole of the battery. Of the electron and the hole produced, the electron is captured by the positive terminal, while the hole moves towards the junction. On the other hand, as soon as the hole is created in the *p*-section due to the breaking of a covalent bond, an electron is released from the negative terminal of the battery into the *n*-section to replace the electron lost by the combination with a hole at the junction. These electrons move towards the junction, where they again get neutralised on meeting the new holes coming from left. As a consequence, a relatively large current, called ***forward current*** flows through the junction. The current in the external circuit is due to the electrons and is from negative terminal of battery to positive terminal through the junction diode.

As discussed above, during the forward bias, the applied *d.c.* voltage ***opposes*** the fictitious battery developed across the *p-n* junction. Due to this, the potential drop across the junction decreases and as a result, the diffusion of holes and electrons across the junction increases. It makes the ***depletion layer thin*** and as such, the junction diode offers ***low resistance*** during forward bias.

2. Reverse Bias. *When a battery is connected to junction diode with p-section connected to negative pole and n-section connected to the positive pole, the junction diode is said to be reverse biased* [Fig. 2.03].

Action of p-n junction. When the *p-n* junction is reverse biased, the holes (majority carriers) in the *p*-section get attracted towards the negative terminal of battery and therefore, the holes move away from the junction. At the same time, the electrons (majority carriers) in the *n*-section get attracted towards the positive terminal and move away from the junction. As a very small number of holes and electrons (minority carriers) are left in the vicinity of the junction, practically no flow of current takes place. However, due to thermally generated electron-hole pairs within *p*-region as well as *n*-region, a small current (≈ a few microamperes) still flows. Some covalent bonds always break because of the normal heat energy of the crystal molecules. Electrons liberated by this process in the *p*-region move to the left across the junction, while holes generated in the *n*-region move to the right under the electric field produced by the battery. Thus, a small electron-hole combination current, called ***reverse current*** is maintained by the ***minority carriers.*** If the reverse bias is made very high, all, the covalent bonds near the junction break and a large number of electron-hole pairs are liberated and the reverse current increases abruptly to a relatively high value. The maximum reverse potential difference, which a diode can tolerate without breakdown is called ***reverse break down voltage*** or ***zener voltage***. It is greater for Si than for the Ge. Its value depends upon the density of impurity atoms and may have a value of 1 or 2 volt to several hundred volt.

It may be pointed out that during the reverse bias, the applied d.c. voltage ***aids*** the fictitious battery developed across the junction. Due to this, the potential drop across the junction increases and as a result, the diffusion of holes and electrons across the junction decreases. It makes the ***depletion layer thick*** and the junction diode offers ***high resistance*** during reverse bias.

It may be noted that *the potential barrier opposes the forward current, while it aids the reverse current.*

Key point

When a *p-n* junction is forward biased, the width of depletion layer decreases. As a result, it offers low resistance during forward bias.

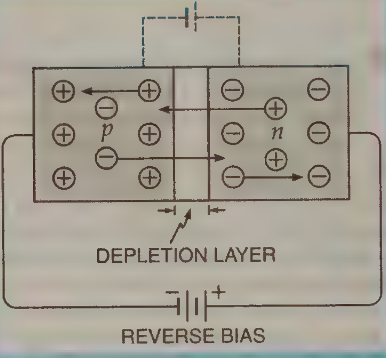


Fig. 2.03.

Key point

When a *p-n* junction is reverse biased, width of depletion layer increases. As a result, it offers high resistance during reverse bias.

2.03. CHARACTERISTICS OF A p-n JUNCTION

Let us now investigate as to how the current flows during forward and reverse bias of a junction diode and then draw the characteristics in the two cases.

Forward bias characteristic. The forward-bias connections of a *p-n* junction are as shown in Fig. 2.04 (*a*). The positive pole of the battery is connected to the *p*-section

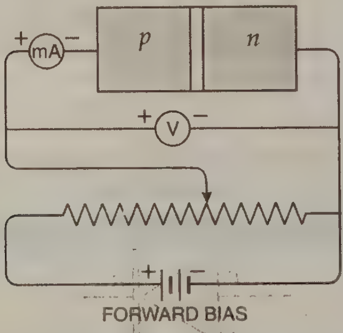


Fig. 2.04 (*a*)

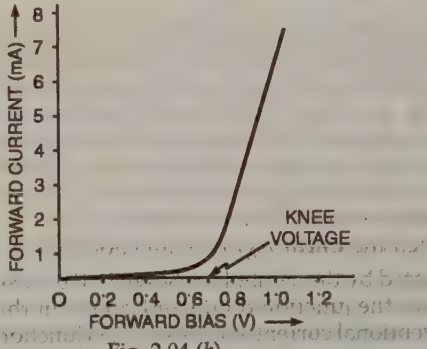


Fig. 2.04 (*b*)

and the negative pole to the *n*-section. In the beginning, when the applied forward bias is low, practically no current flows through the junction diode. It is because, the potential barrier (which is about 0·3V in case of Ge *p-n* junction and 0·7 V in case of Si *p-n* junction) opposes the applied voltage. Therefore, a small forward urrent flows, till the applied forward bias does not become greater than the otential barrier voltage. It is represented by portion OA of the graph between orward bias and the forward current [Fig. 2.04 (*b*)]. As soon as the forward bias ecomes greater than the potential barrier, the forward current increases almost nearly. The point A in the forward characteristic corresponding to the potential arrier appears like a ***knee*** and the forward voltage corresponding to knee point is called ***knee voltage***.

Thus, when a *p-n* junction is forward biased, the current increases linearly and apidly above the knee voltage and the variation in current with increase of forward ias is extremely slow below the knee voltage.

Further, when the *p-n* junction is forward biased, the depletion layer becomes ***hin.*** It is because, the polarity of the external d.c. source *opposes* the fictitious battery eveloped across the junction. As a result, the potential drop across the junction ecreases making the depletion layer thin. It leads to the ***low resistance*** of the junction iode during forward bias.

Key point

In a *p-n* junction, when forward bias is increased even slightly beyond the safe limit, it may get damaged. It is because, number of charge carriers in forward current is very large. Although charge carriers acquire very small kinetic energy and can produce a very small amount of heat, the total heat generated by all the carriers is good enough to rupture the covalent bonds.

Reverse bias characteristic. If the *p*-section is connected to negative terminal nd *n*-section to the positive terminal of the battery, the junction diode is said to be everse biased [Fig. 2.05 (*a*)]. When the *p-n* junction is reverse biased, the current ows due to minority charge carriers and hence a microammeter is used to measure he small current that flows during reverse bias. The reverse bias opposes the ajority carriers but makes the minority carriers to cross the *p-n* junction. Therefore, current of about 1-2 μA is obtained, on applying reverse bias to the *p-n* junction. t remains almost constant till the Zener voltage is reached and then it increases uddenly as shown in Fig. 2.05 (*b*).

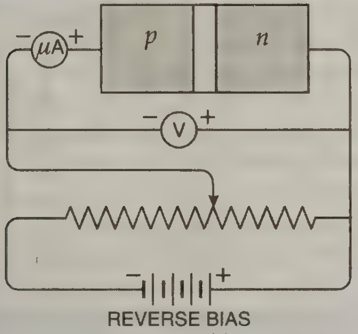


Fig. 2.05 (*a*)

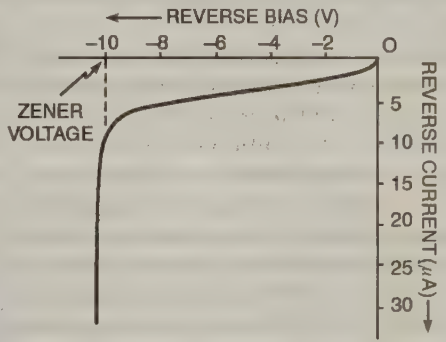


Fig. 2.05 (*b*)

When the *p-n* junction is reverse biased, the depletion layer becomes *thick*. It s because, the external d.c. source in this case ***aids*** the fictitious battery. It results n the increase of potential drop across the junction and the depletion layer appears hick. Because of the increased thickness of the depletion layer, the *p-n* junction offers ***igh resistance*** during reverse bias.

Key point

Static and dynamic resistances mean entirely different terms.

Dynamic resistance. Both the forward bias and reverse bias characteristics of he *p-n* junction have knees and hence Ohm's law is not obeyed. Therefore, the esistance offered by the junction diode depends upon the applied voltage.

The dynamic resistance of a junction diode is defined as the ratio of small change in oltage to the small change in current produced. It is also called ***a.c. resistance*** of the unction diode and is denoted by r_d.

Thus, $$r_d = \frac{\Delta V}{\Delta I} \qquad \text{...(2.01)}$$

The region of the characteristic curve, where dynamic resistance is almost ndependent of the applied voltage, is called the ***linear region*** of junction diode.

Note. 1. The junction diode is represented by the symbol as shown in Fig. 2.06. he arrow-head represents the *p*-section of the junction diode and points in the irection in which the hole current or conventional current will flow, when junction iode is forward biased. The electron current or the electronic current will flow in pposite direction.

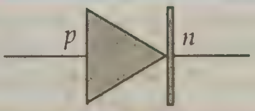


Fig. 2.06.

2. In practice, a Ge junction diode is preferred to Si junction diode. It is because, n case of Ge junction diode, the knee voltage is low (≈ 0·3 V) in comparison to that f Si junction diode (≈ 0·7 V).

2.04. JUNCTION DIODE AS RECTIFIER

An electronic device which converts a.c. power into d.c. power is called a rectifier.

The study of the junction diode characteristics reveals that the junction diode offers a low resistance path, when forward biased ; and a high resistance path, when reverse biased. This feature of the junction diode enables it to be used as a rectifier. The two half cycles of alternating input e.m.f. provide opposite kinds of bias to the junction diode. If the junction diode gets forward biased during first half cycle, it will get reverse biased during the second half cycle and vice-versa. In other words, when an alternating e.m.f. signal is applied across a junction diode, it will conduct only during those alternate half cycles, which bias it in forward direction.

2.05. HALF WAVE RECTIFIER

A rectifier, which rectifies only one half of each a.c. supply cycle, is called a half wave rectifier.

Principle. It is based on the principle that junction diode offers low resistance path, when forward biased ; and high resistance, when reverse biased. When a.c. input is applied to a junction diode, it gets forward biased during one half cycle and reverse biased during the next opposite half cycle. Thus, output is obtained during alternate half cycles of the a.c. input.

Arrangement. The a.c. supply is fed across the primary coil P of a step-down transformer. The secondary coil S of the transformer is connected to the junction diode and a load resistance R_L as shown in Fig. 2.07. The output d.c. voltage is obtained across the load resistance R_L.

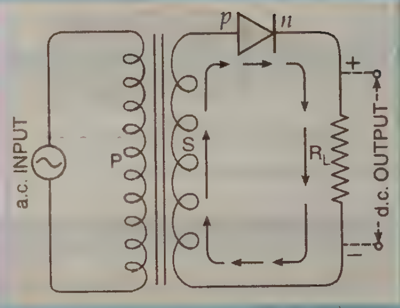


Fig. 2.07.

Theory. Suppose that during the first half of the input cycle, the junction diode gets forward biased. The conventional current will flow in the direction of the arrow-heads. The upper end of R_L will be at positive potential *w.r.t.* the lower end. The magnitude of output across R_L during first half cycle at any time will be proportional to the magnitude of current through R_L *i.e.* proportional to the number of majority carriers crossing the junction, which in turn will be proportional to the magnitude of forward bias and which ultimately depends upon the value of a.c. input at that time. Hence, during the first half of the input cycle, when junction diode conducts, output across R_L will vary in accordance with a.c. input.

During the second half cycle, junction diode will get reverse biased and hence no output will be obtained across R_L. Critically, a small current will flow due to minority carriers and a negligible output will be obtained during this half cycle also. During the next half cycle, output is again obtained as the junction diode gets forward biased. Thus, a half wave rectifier gives discontinuous and pulsating d.c. output across the load resistance as shown in Fig. 2.08. Thus, half wave rectification involves a lot of wastage of energy and hence it is not preferred.

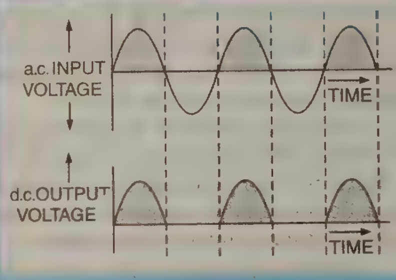


Fig. 2.08.

Note. A small current (≈ a few μA) flows during reverse bias due to minority carriers. As the output across R_L is negligible, the waveform corresponding to small current due to minority carriers has not been shown in Fig. 2.08.

2.06. FULL WAVE RECTIFIER

A rectifier which rectifies both halves of the a.c. input is called a full wave rectifier. To make use of both the halves of input cycle, two junction diodes are used.

Principle. It also works on the principle that a junction diode offers low resistance during forward bias and high resistance, when reverse biased. Here, two diodes are connected in such a manner that if one diode gets forward biased during first half cycle of a.c. input, the other gets reverse biased but when the next opposite half cycle comes, the first diode gets reverse biased and the second forward biased. Thus, output is obtained during both the half cycles of the a.c. input.

Arrangement. The a.c. supply is fed across the primary coil P of a step-down transformer. The two ends of the secondary coil S of the transformer are connected to the *p*-sections of the junction diodes D_1 and D_2. A load resistance R_L is connected across the *n*-sections of the two diodes and the central tapping of the secondary coil [Fig. 2.09]. The d.c. output will be obtained across the load resistance R_L.

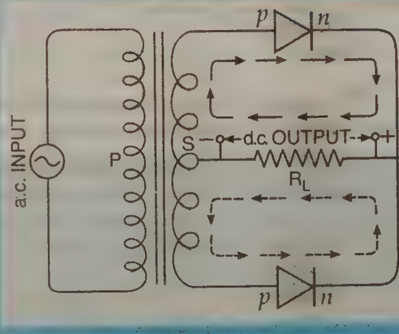


Fig. 2.09.

Theory. Suppose that during first half of the input cycle, upper end of S coil s at positive potential and the lower end is at negative potential. The junction diode D_1 will get forward biased, while the diode D_2 reverse biased. The conventional current due to the diode D_1 will flow along the path of full arrows.

When the second half of the input cycle comes, the situation will be exactly reverse. Now, the junction diode D_2 will conduct and the conventional current will flow along the path of the dotted arrows. Since, current during both the half cycles flows from right to left through the load resistance R_L, the output during both the half cycles will be of the same nature. The right end of the load resistance R_L will be at positive potential *w.r.t.* the left end. As discussed in case of half wave rectifier, the magnitude of output across R_L at any time will vary in accordance with the a.c. input as shown in Fig. 2.10.

Thus, in a full wave rectifier, the output is continuous but pulsating in nature. However, it can be made smooth by using a filter circuit.

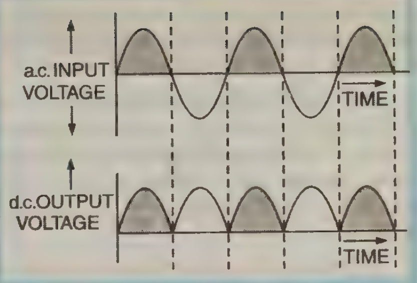


Fig. 2.10

2.07. DIFFERENTTYPES OF JUNCTION DIODES

Junction diodes are of many types and they have a wide range of applications in electronics. A few of them are discussed as below :

1. Zener diode. In the usual junction diodes, when the applied reverse bias increases a particular large value, the reverse current increases suddenly on increasing the reverse bias even slightly. This particular large value of reverse bias is called ***breakdown voltage***. An ordinary junction diode of low power rating will get destroyed on applying reverse bias above its break-down voltage.

*The specially designed junction diodes, which can operate in the reverse breakdown voltage region continuously without being damaged, are called **zener diodes**.*

A zener diode is represented by the symbol as shown in Fig. 2.11.

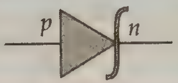


Fig. 2.11

Fig. 2.12 shows the use of zener diode in providing a constant voltage supply. This use of zener diode is based on the fact that in reverse break down (or zener) region, a very small change in voltage across the zener diode produces a very large change in the current through the circuit. If voltage is increased beyond zener voltage; the resistance of the zener diode drops considerably. Consider that the zener diode and a resistor R, called ***dropping resistor*** are connected to a fluctuating d.c. supply (say of a rectifier), such that the zener diode is reverse biased. Whenever voltage across the diode tends to increase, the current through the diode rises out of proportion and causes a sufficient increase in voltage drop across the dropping resistor. As a result, the output voltage lowers back to the normal value. Similarly, when the voltage across the diode tends to decrease, the current through the diode goes down out of proportion, so that the voltage drop across the dropping resistor is much less and now the output voltage is raised to normal.

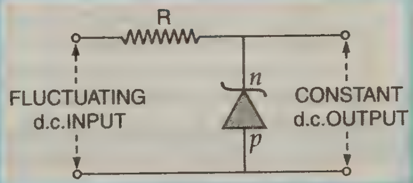


Fig. 2.12

2. Photodiode. In semiconductors, electrons are released from the valence band, when energy is supplied to such electrons. This energy can be supplied in the form of heat or through an accelerating voltage. Light energy can also be used to produce the current in semiconductors.

*A junction diode made from light (or photo) sensitive semiconductor is called **a photodiode**.* It is represented by the symbol as shown in Fig. 2.13 (*a*).

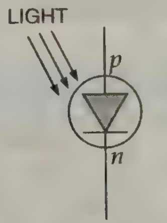


Fig. 2.13. (*a*)

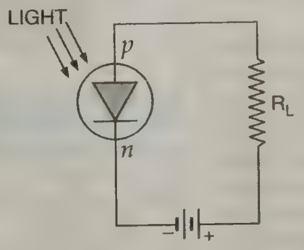


Fig. 2.13. (*b*)

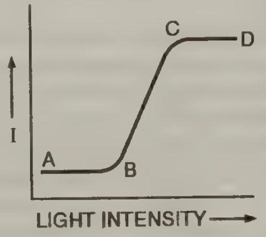


Fig. 2.13. (*c*)

If the photodiode is forward biased [Fig. 2.13 (*b*)], a certain current flows in the circuit as depicted by portion AB of the graph in Fig. 2.13 (*c*). The current AB, that flows when no light is incident, is called ***dark current***. When light of energy *hv* falls on the photodiode, more electrons from valence band move to conduction band, provided $h\nu$ is greater than forbidden energy gap. Due to this, current in the circuit increases. As the intensity of light is increased, the current goes on increasing as indicated by part BC of the graph. A stage comes, when current does not increase with increase in intensity of light and the photodiode is said to be saturated. Portion CD of the graph represents ***saturated current.***

3. Light Emitting Diodes (LED). When a junction diode is forward biased [Fig. 2.14], energy is released at the junction due to recombination of electrons and holes. In case of silicon and germanium diodes, the energy released is in infra-red region. In the junction diode made of gallium arsenide or indium phosphide, the energy is released in visible region. Such a junction diode is called a ***light emitting diode*** or **LED.**

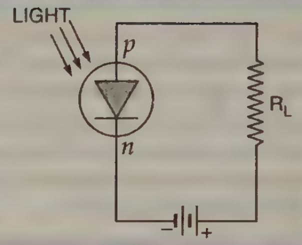


Fig. 2.14.

4. Solar cell. A junction diode, in which one of the p or n-section is made very thin (so that the light energy falling on the diode is not greatly absorbed before reaching the junction), can be used to convert light energy into electric energy. Such junction diodes are called ***solar cells.***

An important application of a solar cell is that a set of solar cells can be used to charge batteries in day time and then to use them during night as a source of electrical energy.

2.08. ADVANTAGES OF SEMICONDUCTOR DIODES OVER VACUUM DIODES

The properties of semiconductor diodes are similar to that of vacuum tube diodes. Following are the main advantages of semiconductor diodes over vacuum tube diodes :

Advantages. 1. There are no filaments in semiconductor diodes and hence no power is needed to heat them so as to cause the emission of electrons.

2. Since no heating is required, the semiconductor diodes are set into operation as soon as the circuit is switched on.

3. During operation, the semiconductor diodes do not produce any humming noise.

4. Semiconductor diodes require low voltage for their operation as compared to vacuum tubes. Thus, the power consumption in semiconductor diodes is very low.

5. Being very small in size, they are very compact.

6. They are shock proof.

7. They much less as compared to vacuum tubes.

8. They have unlimited life.

9. As no vacuum has to be created in semiconductor diodes, they have no vacuum deterioration trouble.

Drawbacks. The main drawbacks of semiconductor diodes is possibility of its breakdown due to rise of temperature and application of high voltage.

2.09. TRANSISTORS

A junction diode can not be used for amplifying a signal. For amplification, another type of semiconductor device called ***transistor*** is used. It is a ***three-section semiconductor.*** The three sections are combined, so that the two at extreme ends have the same type of majority carriers ; while the section that separates them, has the majority carriers of opposite nature. Therefore, a transistor can be n-p-n or p-n-p type. In other words, in an n-p-n transistor, the p-section is sandwitched between two n-sections [Fig. 2.15]. On the other hand, in a p-n-p transistor, the n-section is sandwitched between two p-sections [Fig. 2.16].

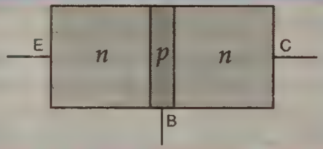


Fig. 2.15.

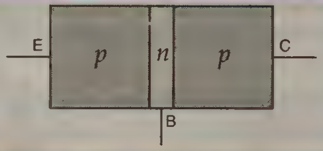


Fig. 2.16

The three sections of the transistor are called ***emitter*** **(E),** ***base*** **(B)** and ***collector*** **(C).** The base of a transistor is made thin and as it is comparatively lightly doped, the number density of majority carriers in the base is always lesser as compared to that in emitter or collector. The emitter supplies the majority carriers for current flow and the collector collects them. The base provides the junctions for proper interaction between the emitter and the collector. The emitter can be, thus, compared to cathode and collector with the plate in a triode valve. The base of the transistor can be compared with the grid of the triode valve, as it performs a function similar to it.

Symbol for transistors. In the symbol for a transistor, the arrow points hole current *i.e.* conventional current. Therefore, the emitter in n-p-n transistor is represented by an arrow pointing away from the base, while the emitter in p-n-p transistor is represented by an arrow pointing towards the base. The symbols for n-p-n and p-n-p transistors are respectively shown in Fig. 2.17 and Fig. 2.18.

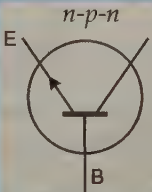


Fig. 2.17.

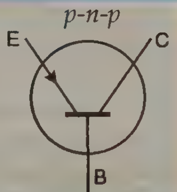


Fig. 2.18.

When a transistor is used in a circuit, the base-emitter junction is always forward biased and the base-collector junction is always reverse biased.

Key point

In a transistor, base is always thin and lightly doped.

2.10. ACTION OFTRANSISTOR

The action of both the types of transistors *i.e.* *n-p-n* and *p-n-p* is similar, except that the majority and minority carriers in the two cases are of opposite nature.

(*a*) Action of *n-p-n* transistor. Fig. 2.19 shows the proper biasing of an *n-p-n* transistor. The *n-type* emitter is forward biased by connecting it to negative pole of the battery V_{ee} (emitter-base battery) and *n*-type collector is reverse biased by connecting it to the positive pole of the battery V_{cc} (collector-base battery).

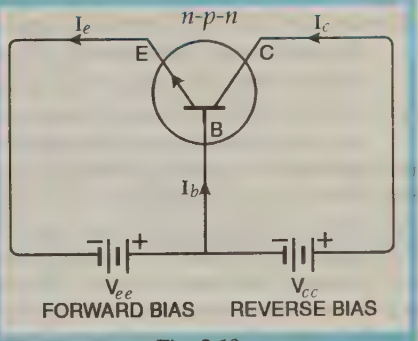


Fig. 2.19.

The majority carriers (which are electrons) in the emitter are repelled towards the base due to the forward bias. The base contains holes as majority carriers but their number density is small as it is doped lightly as compared to emitter or collector. Due to this, the probability of electron-hole combination in base region is very small (≈ 5%). Most of the electrons (≈ 95%) cross into collector region, where they are swept away by the positive terminal of the battery V_{cc} connected to the collector. Corresponding to each *electron that is swept by the collector and that enters the positive pole of the collector-base battery* V_{cc} *, an electron enters the emitter from the negative pole of the emitter-base battery* V_{ee} . Thus, in *n-p-n* transistors, the current is carried inside the transistor as well as external circuit by the electrons. If I_e , I_b and I_c are respectively the ***emitter current, base current*** and ***collector current,*** then

$$I_e = I_b + I_c$$

It may be pointed out that the arrows point in the direction of conventional current or hole current in spite of the fact that in the *n-p-n* transistor, the current is carried by electrons.

(*b*) Action of *p-n-p* transistor. The *p*-type emitter of a *p-n-p* transistor is forward biased by connecting it to positive pole of emitter-base battery V_{ee} and the *p*-type collector is reverse biased by connecting it to the negative pole of the collector-base battery V_{cc} as shown in Fig. 2.20.

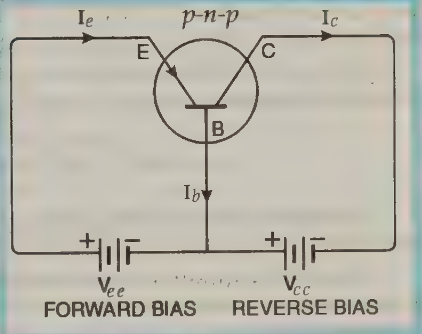


Fig. 2.20

In this case, majority carriers in emitter are holes and they are repelled towards the base due to the forward bias. As base is thin and lightly doped (very small as compared to collector and emitter), it has a low number density of electrons. When holes enter the base region, then only about 5% electron-hole combination takes place. Most of the holes (≈ 95%) reach the collector under the influence of reverse bias. As one hole reaches the collector, an electron leaves the negative pole of collector-base battery V_{cc} and combines with it. At the same time, an electron is released from some covalent bond in the emitter, creating a hole in the emitter. The electron so released, enters the positive pole of the emitter-base battery V_{ee}. Thus, current in *p-n-p* transistor is carried by holes and at the same time their concentration is maintained as explained above.

In this case also,

$$I_e = I_b + I_c$$

2.11. COMMON EMITTER CHARACTERISTICS OF A TRANSISTOR

Common emitter characteristics of a transistor are graphs that are obtained between voltage and current, when emitter is earthed, base is used as input terminal and collector as the output terminal.

Fig. 2.21 shows the experimental arrangement to study the common emitter

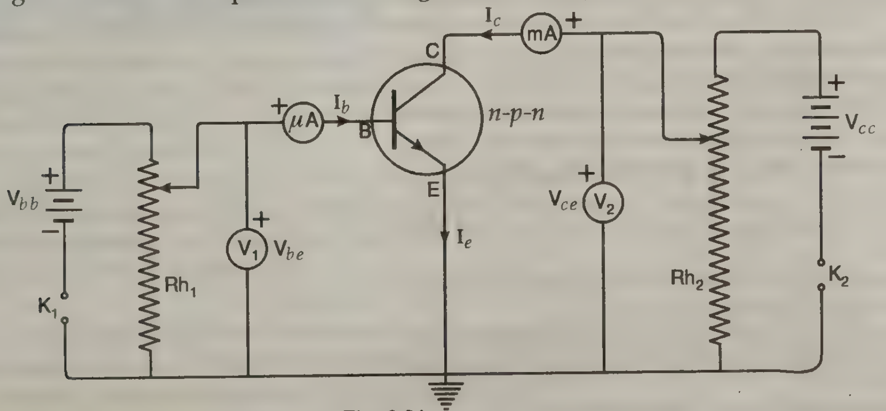


Fig. 2.21.

characteristics of an *n-p-n* transistor. The base-emitter junction is forward biased by using base-emitter battery V_{bb}, while the base-collector junction is reverse biased by using collector-emitter battery V_{cc}. The microammeter and voltmeter in base-emitter circuit measure base current (I_b) and the voltage between base and emitter (V_{be}). Similarly, milliammeter and voltmeter included in collector-emitter circuit measure the collector current (I_c) and voltage between collector and emitter (V_{ce}).

We shall study the following common emitter characteristics :

1. Input characteristics. *These are graphs between base voltage (V_{be}) and the base current (I_b) at different constant values of collector voltages (V_{ce}).*

Let us first plot input characteristic at $V_{ce} = 2$ V. For this, potential divider arrangement in emitter-collector circuit is adjusted, till the voltmeter connected in collector-emitter circuit reads 2 V. It will mean that the voltage between collector and emitter is 2 V *i.e.* collector is at + 2 V *w.r.t.* emitter of the transistor. Now, by making use of potential divider in emitter-base circuit, make base voltage zero. It will be noted that base current is also zero. Now, keeping collector voltage constant at 2 V, start increasing base voltage gradually. It will be found that the base current remains zero as long as the base voltage does not exceed barrier voltage ($\approx 0{\cdot}3$ V or so). As soon as the base voltage becomes greater than the barrier voltage, the current increases slowly and then suddenly to a large value as shown in Fig. 2.22. It is input characteristic at $V_{ce} = 2$ V.

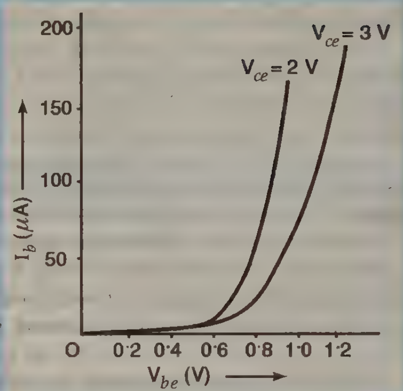


Fig. 2.22.

Similarly, we can draw input characteristic of the transistor at $V_{ce} = 3$ V and so on.

It may be noted that the nature of input characteristics is similar to the forward characteristics of a junction diode.

a.c. input resistance.* *The a.c. input resistance of the transistor in common emitter configuration is defined as the ratio of small change in base voltage to the small change produced in base current at constant collector voltage.* It is denoted by R_{in}.

If ΔV_{be} is small change in base voltage and ΔI_b, small change produced in base current at constant collector voltage V_{ce}, then

$$R_{in} = \left(\frac{\Delta V_{be}}{\Delta I_b}\right)_{V_{ce}} \qquad ...(2.02)$$

2. Output characteristics. *These are graphs between collector voltage (V_{ce}) and the collector current (I_c) at different constant values of base current (I_b).*

Let us first plot output characteristic at say $I_b = 5\,\mu$A. To do so, collector voltage is made zero and voltage between base and emitter is adjusted, till the microammeter in base-emitter circuit reads 50 μA. Now collector voltage is increased gradually and the corresponding collector current is noted. Take care that base current always remains steady at 50 μA. Then, graph between V_{ce} and corresponding values of I_c gives output characteristics at $I_b = 50\,\mu$A. Similarly, output characteristics can be obtained at $I_b = 100\,\mu$A, 200 μA, 300 μA..... . The output characteristic at different constant values of I_b will be as shown in Fig. 2.23.

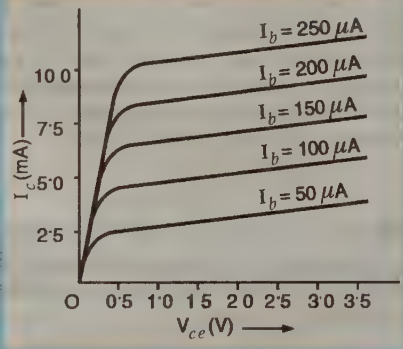


Fig. 2.23.

Following points may be noted about output characteristics :

(*i*) The collector current changes rapidly in beginning, but soon it becomes more or less independent of applied collector-emitter voltage.

(*ii*) For a given value of collector voltage, larger the base current, larger is the collector current.

(*iii*) In audiofrequency amplier circuits, the linear part of the output characteristics is used in order to obtain undistorted output.

a.c. output resistance. *The a.c. output resistance of the transistor in common emitter configuration is defined as the ratio of small change in collector voltage to the small change produced in collector current at constant base current. It is denoted by R_{out}.* Thus,

$$R_{out} = \left(\frac{\Delta V_{ce}}{\Delta I_c}\right)_{I_b} \qquad ...(2.05)$$

3. Transfer characteristics. *These are graphs between the collector current (I_c) and the base current (I_b) at different constant values of collector voltages (V_{ce}).*

Let us plot transfer characteristics at say $V_{ce} = 3$ V. For this, the potential divider arrangement in emitter-collector circuit is adjusted, so that the voltmeter connected in emitter-collector circuit reads 3 V. Now, by making use of potential divider arrangement in emitter-base circuit, the base-emitter voltage (V_{be}) is varied. Due to this, base current will change and consequently the collector current will change. Each time, the base current (I_b) and the corresponding value of the collector current (I_c) is noted. It is ensured that the collector-emitter voltage always remains 3 V. Then, the graph plotted between I_b and I_c is the transfer characteristic of the transistor at $V_{ce} = 3$ V [Fig. 2.24]. It will be found that the graph will be a straight line. It implies that the output current (I_c) varies linearly with the input current (I_b).

---

* a.c. input resistance should be distinguished from d.c. input resistance, $r_{in} = I_{be}/I_b$. It is never used in the analysis of amplifier circuit.

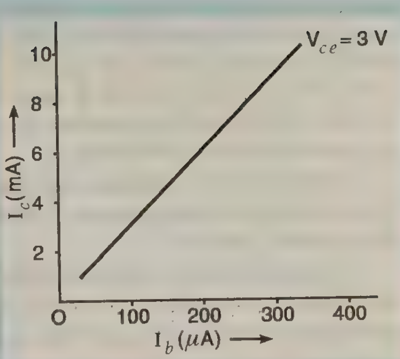


Fig. 2.24.

a.c. current gain. *It is defined as the ratio of change in collector current to the change in base current at constant collector voltage.* It is also called ***current transfer ratio*** and is denoted by β. Therefore,

$$\beta = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{ce}} \qquad ...(2.04)$$

2.12. TRANSISTOR AS AN AMPLIFIER

Like a triode valve, a transistor can also be used as an amplifier, oscillator, detector, etc. As already described, the emitter can be compared with the cathode, the base with the control grid and the collector with the plate of the triode valve. In a triode valve, whereas cathode is always common to input and output circuits ; in a transistor, any of the three sections can be made common to the input and output circuits. Thus, when a transistor is to be used as an amplifier, one may have following three types of the amplifier circuits :

1. ***Common base amplifier,***
2. ***Common emitter amplifier*** and
3. ***Common collector amplifier.***

We shall study only the common base and common emitter amplifiers.

Key point

Irrespective of the configuration (common base, common emitter or common collector), the emitter of a transistor is always forward biased and collector reverse biased.

2.13. COMMON BASE AMPLIFIER

In a common base amplifier, base of the transistor is common to emitter and collector. Let us now discuss the common base amplifier circuits using an *n-p-n* and a *p-n-p* transistor.

(*a*) Amplifier circuit using an *n-p-n* transistor. Fig. 2.25 shows the common base amplifier circuit using an *n-p-n* transistor. The base is made common to both the input and the output circuits.

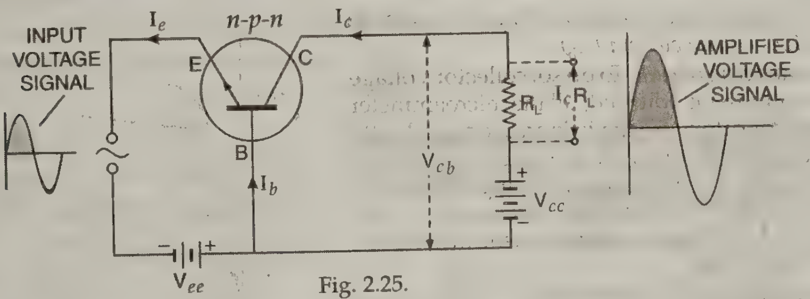


Fig. 2.25.

The emitter is forward biased by using emitter-bias battery V_{ee} and due to this, the resistance of input circuit (R_{in}) is small. The collector is reverse biased by using collector-bias battery V_{cc} and as a result, the resistance of output circuit (R_{out}) is quite large. Thus, $R_{out} > R_{in}$. The low input voltage signal is applied across emitter-base circuit (input circuit) and the amplified output is obtained across the collector and base (output circuit).

Let us assume that I_e, I_b and I_c represent emitter current, base current and collector current respectively, when the applied input signal voltage is zero. The arrows represent the direction of flow of hole current *i.e.* conventional current. According to Kirchhoff's law,

$$I_e = I_b + I_c$$

When current I_c flows in collector circuit, a potential drop equal to R_L will occur across the resistance connected in collector-base circuit and therefore, collector voltage *i.e.* voltage across the collector and base will be

$$V_{cb} = V_{cc} - I_c R_L \qquad ...(2.05)$$

When the input signal is applied in emitter-base circuit, it will change the emitter voltage. Due to this, emitter current will change, which in turn will change collector current and then in accordance with the above equation, the collector voltage will vary. The variation in collector voltage, when input signal is applied, appears as an amplified output.

Phase relation between input and output voltage signal. The output voltage signal obtained across base and collector* is in phase with the input voltage signal as explained below :

Suppose that the first half cycle of input voltage is positive. As the emitter is connected to negative pole of the battery V_{ee}, the first half cycle will reduce the emitter forward voltage. This will make emitter current to decrease and consequently the collector current will also decrease. The decrease in collector current will decrease potential drop across R_L and hence according to equation (2.05), the collector voltage will increase. Since collector is connected to positive pole of the battery V_{cc}, the increase in collector voltage means that collector will become more positive *i.e.* positive output half cycle will be obtained. Thus, *corresponding to positive half cycle of a.c. input, positive output half cycle will be obtained.*

Now, the second half cycle of a.c. input is negative. Therefore, it will make emitter more negative, *i.e.* forward bias will increase. Due to increase in forward bias, emitter current and hence collector current will increase. The increase in collector current will increase the potential drop across R_L. Then, according to equation (2.05), the collector voltage will decrease. As collector is connected to positive pole of the battery V_{cc}, the decrease in collector voltage will make collector less positive *i.e.* negative output half cycle will be produced. Thus, *corresponding to the negative half cycle of a.c. input, negative output half cycle will be produced.* Therefore, input and output voltage signals are in phase with each other as shown in Fig. 2.25, when in a common base amplifier, output is obtained across collector and base.

Self-test Question

The output signal obtained across R_L is out of phase with input signal in a common base amplifier. Explain, why.

a.c. current gain. *It is defined as the ratio of change in collector current to the change in emitter current at constant collector voltage.* It is denoted by α_{ac}.

$$\therefore \qquad \alpha_{cc} = \left(\frac{\Delta I_c}{\Delta I_e}\right)_{V_{cb}} \qquad ...(2.06)$$

a.c. voltage gain. *It is defined as the ratio of change in output voltage to the change in input voltage.* It is denoted by A_v.

Suppose that on applying an a.c. input signal, the input current changes by ΔI_e and correspondingly, the output current changes by ΔI_c. If R_{out} and R_{in} are the resistances of output and input circuits respectively, then

$$A_v = \frac{(\Delta I_c) R_{out}}{(\Delta V_e) R_{in}} = \frac{\Delta I_c}{\Delta I_e} \times \frac{R_{out}}{R_{in}}$$

$$\text{or} \qquad A_v = \alpha_{ac} \times \text{resistance gain,} \qquad ...(2.07)$$

where $\frac{R_{out}}{R_{in}}$ is called ***resistance gain.***

a.c. power gain. *It is defined as the ratio of change in output power to the change in input power.*

Therefore,

$$\text{a.c. power gain} = \frac{\text{change in output power}}{\text{change in input power}} = \frac{(\Delta I_c^2) R_{out}}{(\Delta I_e^2) R_{in}} = \frac{\Delta I_c^2}{\Delta I_e^2} \times \frac{R_{out}}{R_{in}}$$

$$\text{or a.c. power gain} = \alpha_{ac}^2 \times \text{resistance gain} \qquad ...(2.08)$$

(b) Amplifier circuit using *p-n-p* transistor. Fig. 2.26 shows the common base amplifier circuit using a *p-n-p* transistor. The polarities of emitter-bias battery V_{ee} and collector-bias battery V_{cc} are opposite to that in previous case so as to forward bias the emitter and to reverse bias the collector of the *p-n-p* transistor. The basic theory of the circuit making use of a *p-n-p* transistor remains the same as in case of an *n-p-n* transistor circuit. In this case also, the output voltage signal obtained across the collector and the base is in phase with the input voltage signal as explained below :

---

* The output voltage signal may be obtained across R_L. However, in that case, the input and output voltage signals will be out of phase with each other.

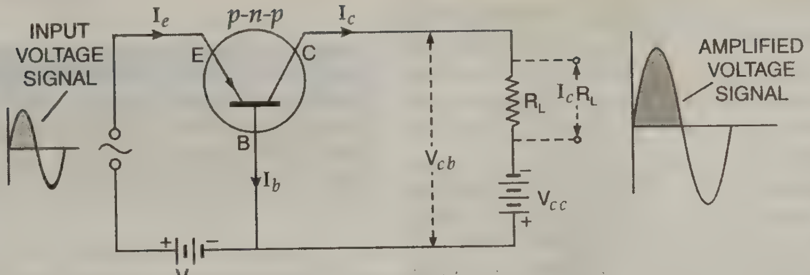


Fig. 2.26.

The first positive half cycle of the input signal increases the forward bias of the emitter. This will increase emitter current and hence the collector current. The increase in collector current will increase the potential drop across R_L and hence according to equation (2.05), the collector voltage will decrease. As collector is connected to negative pole of the battery V_{cc}, the decrease in collector voltage means that it has become less negative *i.e.* positive output signal will be produced. Thus, *corresponding to the positive output half cycle of a.c. input, positive output half cycle will be produced.*

Similarly, we can prove that *corresponding to the negative half cycle of a.c. input, the negative output half cycle will be produced.* Hence, input and output voltage signals are in phase with each other as shown in Fig. 2.26, when output is obtained across collector and base in a common base amplifier.

Note. For higher current and power gain, common emitter amplifier is used.

2.14. COMMON EMITTER AMPLIFIER

In a common emitter amplifier, the emitter of the transistor is common to base and collector. Let us now discuss common emitter amplifier using an *n-p-n* and a *p-n-p* transistor.

(*a*) Amplifier circuit using *n-p-n* transistor. Fig. 2.27 shows the common emitter amplifier circuit using an *n-p-n* transistor.

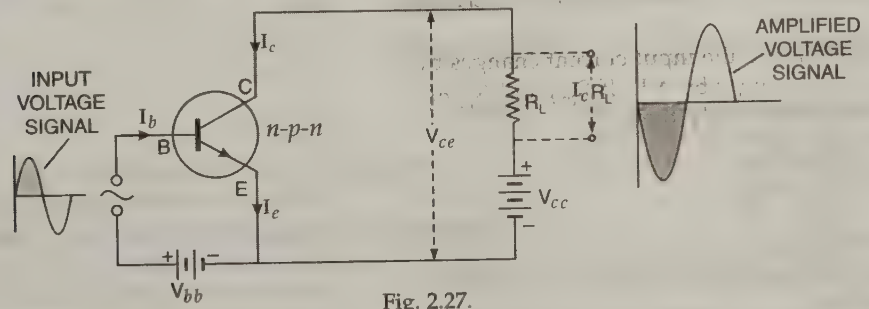


Fig. 2.27.

The emitter is common to both the input and output circuits. The emitter is forward biased by using base bias battery V_{bb} and due to the forward bias, the resistance of input circuit is low. The collector is reverse biased by using collector bias battery V_{cc}. The low input voltage signal is applied in base-emitter circuit (input-circuit) and the amplified output is obtained across the collector and emitter. The arrows point in the direction of the hole current or the conventional current.

The emitter current, base current and collector current are related to each other by the equation,

$$I_e = I_b + I_c \qquad \text{(according to Kirchhoff's first law)}$$

Also, corresponding to collector current I_{cc}, the collector voltage *i.e.* voltage across the collector and emitter will be

$$V_{ce} = V_{cc} - I_c R_L \qquad \text{...(2.09)}$$

The variation in input signal voltage causes variation in emitter current, which in turn produces change in collector current and hence in collector voltage. These variations in collector voltage appear as amplified output voltage.

Phase relation between input and output signals. The input voltage signal and output voltage signal* obtained across collector and emitter are out of phase with each other in common emitter amplifier as explained below :

Suppose that first half cycle of a.c. input voltage is positive. Referring to Fig. 2.27, as base is connected to the positive pole of the battery V_{bb}, it will make base more positive. Thus, negative forward bias of emitter will increase. It will increase the emitter current and hence the collector current. The increase in collector current will increase potential drop across R_L and then according to equation (2.09), the collector voltage V_{ce} will decrease. As collector is connected to positive pole of the battery V_{cc}, the decrease in collector voltage means that collector will become less positive *i.e.* negative output signal will be obtained. Thus, corresponding to positive half cycle of a.c. input, negative output half cycle will be obtained.

Similarly, we can prove that corresponding to negative half cycle of a.c. input, positive output half cycle will be obtained. Therefore, input and output voltage signals are out of phase with each other as shown in Fig. 2.27. From the operation of common emitter amplifier, it follows that a large collector current (output current) flows corresponding to a small base current (input current) in the circuit.

Self-test Question

In a common emitter amplifier, the output voltage signal obtained across R_L is in phase with input voltage signal. Explain, why.

a.c. current gain. *It is defined as the ratio of the change in collector current to the change in base current at constant collector voltage.* It is denoted by β_{ac}.

Therefore, $$\beta_{ac} = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{ce}} \quad \text{...(2.10)}$$

Its value is quite large as compared to 1 and lies between 15 to 50.

Transconductance. *It is defined as the ratio of change in collector current (output current) to the change in base voltage (input voltage) at constant collector voltage.* It is denoted by g_m.

Therefore, $$g_m = \left(\frac{\Delta I_c}{\Delta V_{be}}\right)_{V_{ce}} \quad \text{...(2.11)}$$

It can be expressed in another form as given below :

$$g_m = \frac{\Delta I_c}{\Delta V_{be}} \times \frac{\Delta I_b}{\Delta I_b} = \frac{\Delta I_c}{\Delta I_b} \times \frac{\Delta I_b}{\Delta V_{be}}$$

or $$g_m = \beta_{ac} \times \frac{1}{R_{in}} \quad \text{...(2.12)}$$

a.c. voltage gain. *It is the ratio of the change in output voltage to the change in input voltage.* It is denoted by A_v.

Suppose that on applying an a.c. input signal, the input current changes by ΔI_b and correpondingly, the output current changes by ΔI_c. Then,

$$A_v = \frac{(\Delta I_c) \times R_{out}}{(\Delta I_b) R_{in}} = \frac{\Delta I_c}{\Delta I_b} \times \frac{R_{out}}{R_{in}} \quad \text{...(2.13)}$$

or $$A_v = \beta_{ac} \times \text{resistance gain}$$

Since $\beta_{ac} > \alpha_{ac}$ a.c. voltage gain in common emitter amplifier is very large as compared to that in common-base amplifier.

From equations (2.12) and (2.13), it follows that

$$A_v = g_m R_{out} \quad \text{...(2.14)}$$

a.c. power gain. *It is the ratio of the change in output power to the change in input power.*

$$\text{a.c. power gain} = \frac{\text{change in output power}}{\text{change in input power}} = \frac{(\Delta I_c)^2 R_{out}}{(\Delta I_b)^2 R_{in}}$$

or $$\text{a.c. power gain} = \beta_{ac}^2 \times \text{resistance gain} \quad \text{...(2.15)}$$

As $\beta_{ac}^2 > \alpha_{ac}^2$, a.c. power gain in common emitter amplifier is extremely large as compared to that in case of common base amplifier.

(b) Amplifier circuit using *p-n-p* transistor. Fig. 2.28 shows the common emitter amplifier circuit using a *p-n-p* transistor. The polarities of base-bias battery

---

*The output voltage signal may be obtained across R_c. However in that case, the input and output voltage signals will be in phase with one other.

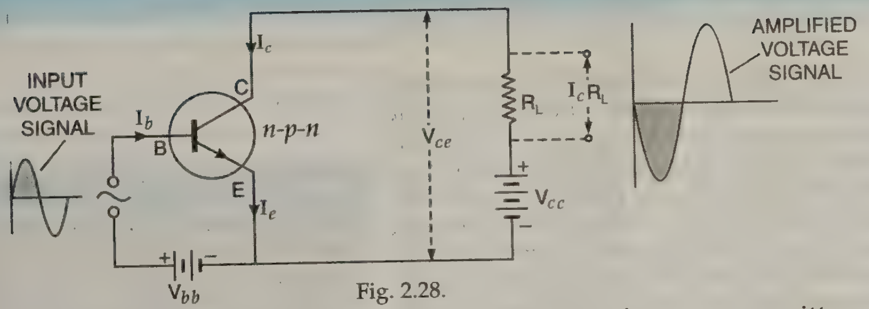


Fig. 2.28.

V_{bb} and collector-bias battery V_{cc} are opposite to those of a common emitter amplifier using *n-p-n* transistor. In this case also, the output signal obtained across the collector and emitter is out of phase with input signal applied across the base and emitter as explained below :

The first positive half cycle of a.c. input will make base less negative as the base is connected to negative pole of the battery V_{bb}. It is equivalent to say that emitter will become less positive. As the forward bias of emitter decreases, the emitter current will decrease, which in turn will decrease the collector current. As the collector current decreases, the potential drop across R_L will decrease and hence according to equation (2.09), the collector voltage V_{ce} will increase. As collector is connected to negative pole of the battery V_{cc}, the increase in collector voltage means that the collector will become negative *i.e.* negative output half cycle will be obtained.

Similarly, we can prove that corresponding to negative half cycle of a.c. input, positive output half cycle will be obtained.

Thus, we conclude that in a common emitter amplifier using *p-n-p* transistor also, *the output and input voltage will be out of phase by* 180° as shown in Fig. 2.28.

2.15. RELATION BETWEEN α AND β

For both the types of amplifier, we have

$$I_e = I_b + I_c$$

Dividing both sides of the above equation by I_c, we get

$$\frac{I_e}{I_c} = \frac{I_b}{I_c} + 1$$

But $\quad \alpha = I_c/I_e \quad$ and $\quad \beta = I_c/I_b$.

$$\therefore \quad \frac{1}{\alpha} = \frac{1}{\beta} + 1 \quad \text{or} \quad \frac{1}{\beta} = \frac{1}{\alpha} - 1 = \frac{1-\alpha}{\alpha}$$

$$\text{or} \quad \beta = \frac{\alpha}{1-\alpha} \qquad \text{...(2.16)}$$

The equation (2.16) is the relation between the current gains of common base and common emitter amplifiers.

Key point

The current gain in a common emmiter amplifier is very large as compared to that in case of a common base amplifier.

2.16. TRANSISTOR AS AN OSCILLATOR

Like a triode valve, a transistor can also be used to produce undamped electromagnetic oscillations. Fig. 2.29 shows the circuit for *p-n-p* transistor as an oscillator in common emitter configuration. The battery is connected in series with the high impedance coil L′ in collector-emitter circuit and a low impedance coil L

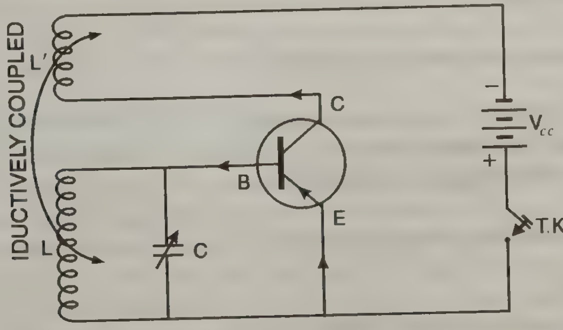


Fig. 2.29.

is connected in the emitter-base circuit. A variable capacitor C of suitable range connected in parallel to the coil L may be used to vary the frequency of the oscillations produced.

Working. When the tapping key (T.K.) is pressed, a very small collector current flows through the coil L′. As the coil L is inductively coupled to the coil L′, due to the change of current and hence change of magnetic flux through the coil L′, a small voltage is induced in the coil L. This small voltage produces a small current to flow in the emitter-base circuit. If due to the induced voltage so produced, the emitter gets *forward biased*, then the small emitter current causes a corresponding increase in the collector current (in a transistor, $I_e = I_b + I_c$). As the collector current rises, the growing magnetic flux linked with the coil L′ increases the induced voltage across the coil L, which in turn will further increase the forward bias of the emitter. This increases the emitter current and hence the collector current increases still further. In this way, the collector current through the coil L′ goes on increasing, till the induced e.m.f. across the coil L attains a saturation value.

Watch out !

The LC-oscillations are produced by the parallel combination of inductor and capacitor. The transistor only keeps amplitude of the oscillations constant by providing right amount of energy at right moment !

As the current reaches saturation value and stops changing, the magnetic flux also stops changing. The emitter current which was being sustained by the induced voltage in coil L begins to decrease, causing collector current to decrease as well. The decreasing collector current through coil L′ will induce voltage in the coil L in the reverse direction, which will further decrease the emitter current. This results in further decrease of the collector current. The 'inertia' of the collapsing flux carries the collector current below its normal value (in the absence of any emitter current).

As the current becomes minimum and stops changing, the magnetic flux also stops changing. In the absence of induced voltage, the collector current starts increasing to its normal value again, causing the induced emitter current once again. This marks the beginning of the next cycle and the process is repeated giving oscillations of constant amplitude because of the *feedback energy* to the L – C circuit proper phase. The frequency of the oscillations produced is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad \text{...(2.17)}$$

The frequency can be varied by changing capacitance of the variable capacitor.

2.17. ADVANTAGES OFTRANSISTORS

Transistors, because of their many merits over vacuum tubes, have practically completely replaced them in all the fields of electronics. Some of the advantages of the transistors over the vacuum tubes are as given below :

Advantages. 1. As transistors have no filaments, hence no power is needed to heat them to cause the emission of electrons.

2. Since no heating is required, transistors are set into operation as soon as the circuit is switched on.

3. During operation, transistors do not produce any humming noise.

4. Transistors require low voltages for their operation as compared to vacuum tubes.

5. Owing to their small sizes, the circuits involving transistors are very compact.

6. Transistors are shock proof.

7. Transistors are cheaper as compared to vacuum tubes.

8. Transistors have almost unlimited life.

9. As no vacuum has to be created in transistors, they have no vacuum deterioration trouble.

Whereas transistors enjoy a number of advantages over the vacuum tubes, they have following drawbacks which put limit on their use in electronic circuits :

Drawbacks. 1. Noise level is higher in transistors as compared to that in the vacuum-tubes.

2. Ordinary semiconductor devices cannot handle as much power as ordinary vacuum tubes can do.

3. In high frequency range, they have poor response.

4. The transistors are temperature-sensitive. The maximum temperature the transistors can withstand, is very low (≈ 50° C). Even a small over-heating spoils the transistor. This is because, at a higher temperature, the covalent bonds break up and the semiconductor piece forming the transistor becomes conducting.

2.18. INTEGRATED CIRCUITS (IC)

Microelectronics is the branch of electronics, which describes the technology that is used to produce electronic circuits of extremely small size, called ***microelectronic circuits*** or ***integrated circuits.*** Integrated circuits consist of whole systems rather than separate electronic components.

Fig. 2.30 (*a*) shows a thin slice of silicon crystal about 0·5 cm in thickness. It is called a ***silicon wafer.*** It may have diameter ranging between 0·25 cm to 10 cm.

Fig. 2.30 (*b*) shows a small part of such a silicon wafer of dimensions 50 mil × 50 mil.* This small portion of the wafer is called a ***silicon chip***. The various components such as resistors, inductors, capacitors, diodes, transistors, logic-gates, etc can be grown over such one silicon chip. For example, a transistor can be constructed in space A of dimensions 6·5 mil × 4 mil, a diode can be constructed in space B of dimensions 4·5 mil × 3 mil and a resistor can be constructed in the portion C of dimensions 12 mil × 2 mil. The components are connected internally to produce a desired circuit.

Fig. 2.30 (*c*) shows the mounting of the silicon chip into a casing. The pins are connected internally to the integrated circuits and the other ends of the pin are used to make external connections.

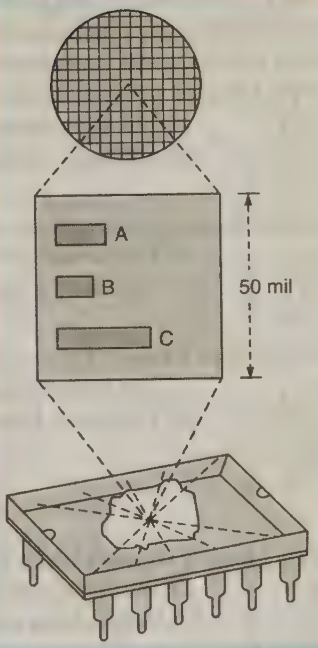


Fig. 2.30

Depending on the number of components fabricated on a chip, the integrated circuits are classified as below :

(i) Medium scale integrated circuits (MSI). Such integrated circuits have about 100 circuit components.

(ii) Large scale integrated circuits (LSI). They have more than 1000 circuit components.

Integrated circuits have following advantages over the conventional electronic circuits :

1. They are highly reliable.
2. They require very small space as compared to conventional electronic circuits.
3. They have lower total cost.

Today, integrated circuits are widely used in televisions, video cassette recorders and computers. Large scale availability of computers in the market has been made possible by the IC technology.

2.19. DIGITAL ELECTRONICS

So far, we have studied analog electronic circuits. In such circuits, the voltage signals vary continuously with time and such signals are called ***continuous*** or ***analog voltage signals***. Fig. 2.31 shows a typical voltage signal varying sinusoidally between 0 and 5 V.

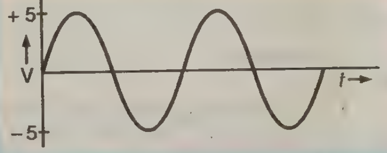


Fig. 2.31

The digital electronic circuits make use of entirely different type of voltage signals. In such signals, the pulse wave form does not vary continuously with time as in analogne voltage signal. Instead, it has only two voltage levels *i.e.* either zero or some constant value of voltage. Such voltage signals are called ***digital voltage signals.*** Fig. 2.32 shows a digital voltage signal, which at any instant will either be equal to 0 V or 5 V.

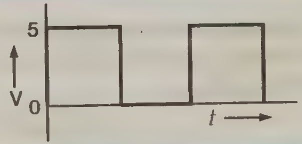


Fig. 2.32.

By representing these two voltage levels by binary numbers 0 and 1, the digital electronics has been developed. The counters, computers, etc are the outcome of digital electronics. We now first learn some elementary ideas about binary number system and then the logic gates, which are the basic building blocks of digital electronics.

2.20. BINARY NUMBER SYSTEM

The binary number system is a code that uses only two digits 0 and 1.

Just as the number next to 9 in decimal number system (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9) is obtained by writing second digit (*i.e.* 1) followed by the first (*i.e.* 0) and it is called 10 ; the number next to 1 in binary number system (0, 1) is obtained by writing second digit (*i.e.* 1) followed by the first (*i.e.* 0) and it is called 10. In other words, the binary representation of decimal digit 2 is 10. Similarly, the decimal digit 3 is represented in binary number system as 11 and so on.

The binary representation of any decimal digit or number can be easily obtained by ***double dabble method***.

---

*The symbol mil stands for milli-inch. Thus, 1 mil = $2{\cdot}54 \times 10^{-3}$ cm.

Double dabble method. To represent a decimal number in binary number system by double dabble method, the following steps are involved :

Step I. Go on dividing the given decimal number by 2, until the quotient is zero.

Step II. Then, the remainders taken in reverse order (obtained in successive divisions) give the binary representation of the given decimal number.

To find binary representation of decimal number 2 :

$$\begin{array}{r|l} 2 & 2 \\ \hline 2 & 1-0 \\ \hline & 0-1 \end{array}$$

(*i*) The remainders in reverse order are 1 and 0.

(*ii*) Therefore, ***binary representation*** of the decimal number 2 is **10.**

To find binary representation of decimal number 3 :

$$\begin{array}{r|l} 2 & 3 \\ \hline 2 & 1-1 \\ \hline & 0-1 \end{array}$$

(*i*) The remainders in reverse order are 1 and 1.

(*ii*) Therefore, ***binary representation*** of the decimal number 3 is **11.**

To find binary representation of decimal number 4 :

$$\begin{array}{r|l} 2 & 4 \\ \hline 2 & 2-0 \\ \hline 2 & 1-0 \\ \hline & 0-1 \end{array}$$

(*i*) The remainders in reverse order are 1, 0 and 0.

(*ii*) Therefore, ***binary representation*** of decimal number 4 is **100.**

……………………………………………………………………………………

……………………………………………………………………………………

To find binary representation of decimal number 9 :

$$\begin{array}{r|l} 2 & 9 \\ \hline 2 & 4-1 \\ \hline 2 & 2-0 \\ \hline 2 & 1-0 \\ \hline & 0-1 \end{array}$$

(*i*) The remainders in reverse order are 1, 0, 0 and 1.

(*ii*) Hence, ***binary representation*** of decimal number 9 is **1001.**

To find binary representation of decimal number say, 37 :

$$\begin{array}{r|l} 2 & 37 \\ \hline 2 & 18-1 \\ \hline 2 & 9-0 \\ \hline 2 & 4-1 \\ \hline 2 & 2-0 \\ \hline 2 & 1-0 \\ \hline & 0-1 \end{array}$$

(*i*) The remainders in reverse order are 1, 0, 0, 1, 0 and i.

(*ii*) Hence, ***binary representation*** of decimal number 37 is **100101.**

The following points may be noted about the binary number system :

1. Each digit of binary number is called a ***bit.***

2. The first digit is called ***least significant bit*** **(LSB),** while the last digit is called ***most significant bit*** **(MSB).** For example, in binary number 110 ; 0 is LSB and 1 is MSB.

3. The binary representation of the decimal number 37 was found to be 10010. The decimal number 37 is represented $(37)_{10}$ and its binary representation is $(100101)_2$. Thus,

$$(37)_{10} = (100101)_2$$

4. In decimal number system, the multiplier of digit in unit place is 10^0, the multiplier of digit in tenth place is 10^1, the multiplier of digit in hundredth place is 10^2, and so on. In binary number system, base is 2. Therefore, in a binary number system, the multipliers of the bits are 2^0, 2^1, 2^2, ... starting from LSB, till MSB is reached. With the help of these multipliers, the decimal representation of a binary number can be easily found as explained in the following two examples :

To find decimal representation of binary number 110 :

$$\begin{pmatrix} 1 & 1 & 0 \\ \text{MSB} & & \text{LSB} \end{pmatrix}_2 = 1\times 2^2 + 1\times 2^1 + 0\times 2^0$$

$$= 4 + 2 + 0 = 6$$

Thus, ***decimal representation*** of binary number 110 is **6.**

To find decimal representation of binary number 10100 :

$$\left(\underset{\text{MSB}}{1}\ \ 0\ \ 1\ \ 0\ \ \underset{\text{LSB}}{0}\right)_2 = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 \times 2^0$$

$$= 16 + 0 + 4 + 0 + 0 = 20$$

Thus, ***decimal representation*** of binary number 10100 is **20.**

2.21. LOGIC GATES

The logic gates are building blocks of digital electronics. They are used in digital lectronics to change one voltage level (input voltage) into another (output voltage) ccording to some logical statement relating them.

Thus, logic gate is a digital circuit, which works according to some logical relationship etween input and output voltages.

A logic gate may have one or more inputs, but it has only one output. The elation between the possible values of input and output voltages are expressed in he form of a table called ***truth table*** or ***table of combinations.***

Truth table of a logic gate is a table that shows all the input and output possibilities or the logic gate.

George Boole invented a different kind of algebra based on the binary nature f the logic. A logical statement can have only two values, such as **HIGH** or **LOW ; ON** or **OFF ; CLOSED** or **OPEN ; YES** or **NO ; TRUE** or **FALSE** ; etc. The two values f the logical statements are denoted by the binary numbers 1 and 0. The binary umber 1 is used to denote the HIGH value and the number 0 to denote the LOW alue. The logical statements that logic gates follow are called ***Boolean expressions.*** The three basic logic gates of digital electronics, called **OR gate, AND gate** and **NOT gate** follow the following Boolean expressions :

1. In Boolean algebra, the addition sign (+) is referred to as **OR.** The Boolean expression :

$$y = A + B \qquad \text{...(2.18)}$$

is read as y equals A **OR** B.

The OR gate is designed on the basis of Boolean expression (2.18).

2. The multiplication sign (.) is referred to as **AND** in Boolean algebra. The Boolean expression :

$$y = A\,.\,B \qquad \text{...(2.19)}$$

is read as y equals A **AND** B.

The AND gate is designed on the basis of Boolean expression (2.19),

3. The bar sign (–) is referred to as **NOT** in Boolean algebra. The Boolean expression :

$$y = \overline{A} \qquad \text{...(2.20)}$$

is read as y equals **NOT** A.

The NOT operation is also called ***negation*** or ***inversion.*** The NOT gate is designed on the basis of Boolean expression (2.20).

2.22. THE 'OR GATE'

The OR gate is a two inputs and one output logic gate. It combines the inputs A and B with the output y following the Boolean expression

$$y = A + B$$

i.e. y equals A OR B. The OR gate is represented by the symbol shown in Fig. 2.33 (*a*).

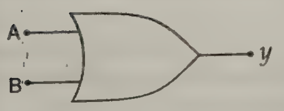


Fig. 2.33. (*a*)

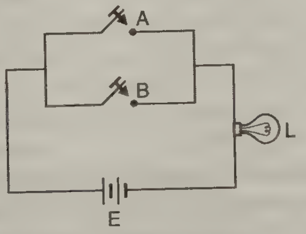


Fig. 2.33. (*b*)

A	B	y
0	0	0
1	0	1
0	1	1
1	1	1

Fig. 2.33. (*c*)

The various possible combinations of the inputs and output of the OR gate can be easily understood with the help of the electrical circuit shown in Fig. 2.33 (*b*). In this electrical circuit, a parallel combination of two switches A and B is connected to a battery and a lamp L.

The following inferences can be easily drawn from the working of the above electrical circuit :

(*a*) If both the switches A and B are **OPEN** (*i.e.* A = 0, B = 0), then the lamp will not glow (*i.e.* $y = 0$).

(*b*) If the switch A is **CLOSED** and B is **OPEN** (*i.e.* A = 1 and B = 0), then the lamp will glow (*i.e.* $y=1$).

(*c*) If the switch A is **OPEN** and B is **CLOSED** (*i.e.* A = 0 and B = 1), then the lamp will glow (*i.e.* $y=1$).

(*d*) If both the switches are **CLOSED** (*i.e.* A = 1, B = 1), then the lamp will glow (*i.e.* $y = 1$).

It follows that in the given electrical circuit, the lamps glows (or output is obtained) even when one of two switches is closed *i.e.* when switch A OR switch B is closed. For this reason, the electrical circuit shown in Fig. 2.33 (*b*) is called OR gate. The various possible input-output combinations for the given electrical circuit *i.e.* OR gate can be written the form of the table given in Fig. 2.33 (*c*). It is called ***truth table of OR gate.***

In practice, the OR gate is realised with the help of the electronic circuit using two diodes D_1 and D_2 as shown in Fig. 2.34. The two diodes play the roles of the switches A and B as explained below :

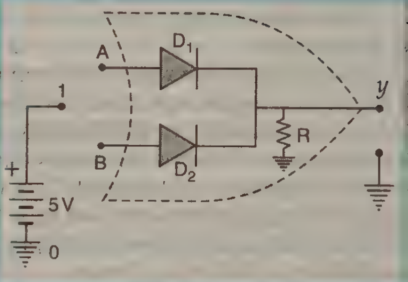


Fig. 2.34.

The input voltage that can be applied at A or B is either 0 or 5 V. There are following possible cases :

(*a*) **A = 0 and B = 0.** When the inputs A and B are connected to earth (0 V), both the diodes D_1 and D_2 do not conduct and hence no voltage develops across resistor and accordingly output y is 0.

(*b*) **A = 1 and B = 0.** When the input A is connected to the positive terminal of battery (5 V) and B is connected to earth (0 V), the junction diode D_1 conducts (as it gets forward biased) while the diode D_2 does not conduct (as it gets reverse biased). If the diodes are ideal ones, output y (due to conduction of diode D_1) will also be 5 V or 1. (In binary number system, the high level of voltage is represented as 1).

(*c*) **A = 0 and B = 1.** In this case, reverse of above will happen *i.e.* the diode D_1 will not conduct and D_2 will conduct. Obviously, the output will again be 5 V or 1.

(*d*) **A = 1 and B = 1.** When A and B are connected to positive terminal of the battery (5 V), both the diodes will conduct. Because the outputs of the two diodes obtained across resistor R are in parallel, the net output y will still be 5 V or 1.

Since input-output combinations of the electronic circuit shown in Fig. 2.34 are same as those of electric circuit shown in Fig. 2.33 (*b*), Fig. 2.33 (*c*) also represents the truth table of the OR gate realised with the help of the electronic circuit shown in Fig. 2.34.

2.23. THE 'AND GATE'

The AND gate is also a two inputs and one output logic gate. It combines the inputs A and B with the output y following the Boolean expression

$$y = A.B$$

i.e. y equals A AND B. The AND gate is represented by the symbol shown in Fig. 2.35 (*a*).

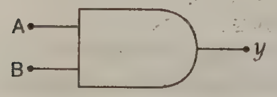


Fig. 2.35. (*a*)

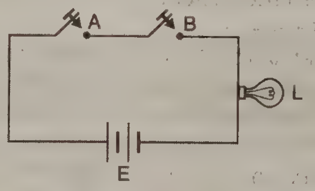


Fig. 2.35. (*b*)

A	B	y
0	0	0
1	0	0
0	1	0
1	1	1

Fig. 2.35. (*c*)

The various possible combinations of the inputs and the outputs of the AND gate can be easily found with the help of the electrical circuit shown in Fig. 2.35 (*b*). Here, a series combination of the switches A and B is connected to a battery and a lamp L. The following conclusions can be easily drawn from the working of the above electrical circuit :

(*a*) If both the switches A and B are **OPEN** (*i.e.* A=0, B=0), then the lamp will not glow (*i.e.* $y = 0$).

(*b*) If the switch A is **CLOSED** and B is **OPEN** (*i.e.* A=1, and B=0), then the lamp ill not glow (*i.e.* $y = 0$).

(*c*) If the switch A is **OPEN** and B is **CLOSED** (*i.e.* A = 0 and B = 1), then also ne lamp will not glow (*i.e* $y = 0$).

(*d*) If both the switches A and B are **CLOSED** (*i.e.* A = 1 and B = 1), then only ne lamp will glow (*i.e.* $y = 1$).

It follows that in the given electrical circuit, the lamp glows (or output is btained), only when both the switches are closed *i.e.* when switches A AND B are losed. For this reason, the electrical circuit shown in Fig. 2.35 (*b*) is called AND gate. he various possible input-output combinations for the given electric circuit *i.e.* AND ate can be written in the form of the table given in Fig. 2.35 (*c*). It is called ***truth able of AND gate.***

The AND gate can be realised by the electronic circuit making use of two ideal unction diodes D_1 and D_2 as shown in Fig. 2.36. The resistor R is connected to ositive terminal of a 5 V battery. The operation of AND gate can be discussed in ollowing four cases :

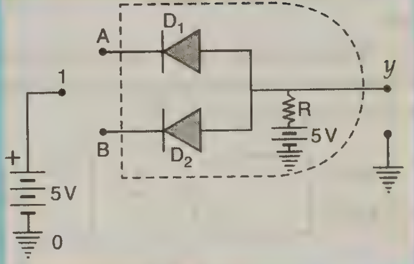


Fig. 2.36.

(*a*) **A = 0 and B = 0.** Both the diodes D_1 and D_2 get forward biased and hence onduct. The output y will be the voltage drop across D_1 or D_2. Since diodes are deal, no voltage drop occurs across the diodes. Hence, the output y is 0.

(*b*) **A = 1 and B = 0.** The diode D_2 will conduct and diode D_1 will remain idle. Now, output y is voltage across D_2 and hence it is 0.

(*c*) **A = 0 and B = 1.** The diode D_1 will conduct, while D_2 will not conduct. Now, utput y is voltage across D_1, which is 0.

(*d*) **A = 1 and B = 1.** Both the diodes will not conduct. The output y will equal attery voltage 5 V *i.e.* equal to 1.

Obviously Fig. 2.35 (*c*) represents the truth table of the AND gate realised with he electronic circuit as shown in Fig. 2.36.

2.24. THE 'NOT GATE'

The NOT gate is a one input and one output logic gate. It combines the input A with the output y following the Boolean expression

$$y = \overline{A}$$

.e. y NOT equals A. The way, the NOT gate gives the output, it is also called ***invertor.*** It is represented by the symbol as shown in Fig. 2.37 (*a*).

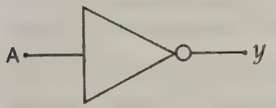


Fig. 2.37. (*a*)

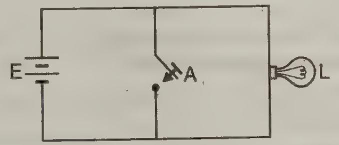


Fig. 2.37. (*b*)

A	y
0	1
1	0

Fig. 2.37. (*c*)

The possible input and output combinations of a NOT gate can be easily discussed with the help of the electrical circuit as shown in Fig. 2.37 (*b*). Here, the switch is connected in parallel to the lamp and the battery. The following conclusions can be easily drawn from the working of the above electrical circuit :

(*a*) If the switch A is **OPEN** (*i.e.* A = 0), the lamp will glow (*i.e.* $y = 1$).

(*b*) If the switch A is **CLOSED** (*i.e.* A = 1), the lamp will not glow (*i.e.* $y = 0$).

It follows that in the given electrical circuit, the lamp glows (or output is obtained), when the switch A is NOT closed. For this reason, the electrical circuit shown in Fig. 2.37 (*b*) is called NOT gate. The two possible input-output combinations can be written in the form of the table as shown in Fig. 2.37 (*c*). It is called ***truth table of the NOT gate.***

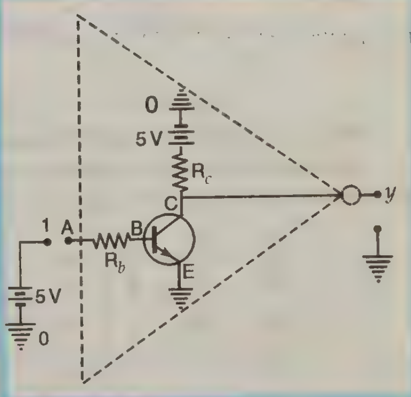


Fig. 2.38.

The NOT gate can be realised by making use of a transistor [Fig. 2.38]. The base (B) of the transistor is connected to A through a resistor R_b, while the emitter (E) is earthed. The collector is earthed through a resistor R_c and a 5 V battery. The operation of NOT gate can be discussed in following two cases :

(*a*) **A = 0.** The base of the transistor also gets earthed. Therefore, base-emitter junction is not forward biased. Since the base current is zero, the collector current is also zero. The transistor is said to be in ***cut-off mode.*** The output y will be equal to the voltage of battery connected to collector *i.e.* 5 V. In other words, when A = 0, $y = 1$.

(*b*) **A = 1.** The base-emitter junction gets forward biased and it leads to a large collector current.* The transistor is said to have gone to ***saturation***. The voltage drop across the resistor R_c is just equal to 5 V and the output y is very nearly 0.

Obviously, Fig. 2.37. (*c*) represents the truth table of the NOT gate achieved by making use of the electronic circuit shown in Fig. 2.38.

2.25. COMBINATION OF GATES

Various combinations of three basic gates *i.e.* OR, AND and NOT give rise to complicated digital circuits. We now discuss a few combinations of these basic gates using their symbols :

1. The NAND gate. *It is a logic circuit in which AND gate is followed by a NOT gate.*

If the ouput (y') of AND gate is connected to the input of NOT gate [Fig. 2.39 (*a*)], the gate so obtained is called NAND gate. The logic symbol of the NAND gate is as shown in Fig. 2.39 (*b*).

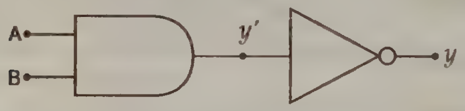


Fig. 2.39. (*a*)

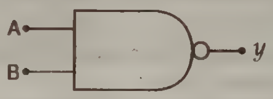


Fig. 2.39. (*b*)

The truth table of NAND gate can be obtained by combining the truth tables of AND and NOT gates. It will be as given in Fig. 2.40 (*a*).

A	B	y'	y
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

Fig. 2.40. (*a*)

A	B	y
0	0	1
1	0	1
0	1	1
1	1	0

Fig. 2.40. (*b*)

The same is equivalent to the truth table given in Fig. 2.40 (*b*).

Boolean expression for the NAND gate is

$$y = \overline{A . B} \qquad ...(2.21)$$

2. The NOR gate. *It is a logic circuit in which OR gate is followed by a NOT gate.* If the output (y') of OR gate is connected to the input of a NOT gate [Fig. 2.41 (*a*)], the gate so obtained is called the NOR gate.

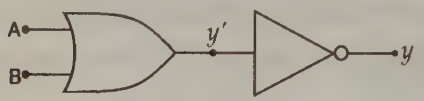


Fig. 2.41. (*a*)

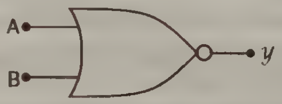


Fig. 2.41. (*b*)

The logic symbol of the NOR gate is shown in Fig. 2.41 (*b*).

The truth table of NOR gate can be obtained by combining the truth tables of OR and NOT gates given in Fig. 2.42 (*a*).

A	B	y'	y
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0

Fig. 2.42. (*a*)

A	B	y
0	0	1
1	0	0
0	1	0
1	1	0

Fig. 2.42. (*b*)

---

*The values of resistors R_b and R_c are so selected.

It is equivalent to the truth table given in Fig. 2.42 (*b*). Boolean expression for the NOR gate is

$$y = \overline{A + B} \qquad ...(2.22)$$

2.26. THE NAND GATE–BUILDING BLOCK IN DIGITAL CIRCUITS

The different gates can be obtained by the repeated use of the three basic gates *i.e.* OR, AND and NOT gates. The repeated use of the NAND gate* can produce all the three basic gates. Thus, in digital circuits, NAND gate serves as a building block. Now, we proceed to show how the repeated use of the NAND gate can produce OR, AND and NOT gates.

Self-test Question ?

Explain, how the repeated use of NOR gate can produce all the three basic gates.

1. To produce NOT gate by using NAND gate. Suppose that the two inputs of the NAND gate are joined, so that it has one input [Fig. 2.43 (*a*)].

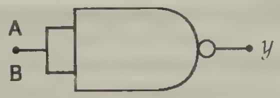


Fig. 2.43. (*a*)

A	B	y
0	0	1
1	1	0

Fig. 2.43. (*b*)

Setting B = A in the truth table of NAND gate, we obtain the truth table as given in Fig. 2.43 (*b*). Since it is the truth table of the NOT gate, it follows that *a NAND gate functions as the NOT gate, when its two inputs are joined.*

2. To produce the AND gate by using the NAND gate. Suppose that the output of the NAND gate is connected to the NOT gate (made from NAND gate by joining the two inputs) as shown in Fig. 2.44 (*a*)

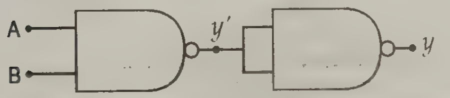


Fig. 2.44. (*a*)

A	B	y'	y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

Fig. 2.44. (*b*)

The truth table of the combination will be as shown in Fig. 2.44 (*b*). It follows that it is the truth table (read columns A, B and y only) of the AND gate. Hence, when a NAND gate is connected to a NOT gate (made from NAND gate), it functions as the AND gate.

3. To produce the OR gate by using the NAND gates. Suppose that the inputs A and B are connected to the two NOT gates (obtained from the NAND gates). The inputs A and B will get inverted *i.e.* will appear as $\bar{A}$ and $\bar{B}$. These inputs are, then, fed to the NAND gate as shown in Fig. 2.45 (*a*).

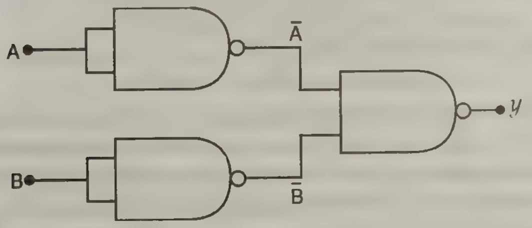


Fig. 2.45. (*a*)

A	B	$\bar{A}$	$\bar{B}$	y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

Fig. 2.45. (*b*)

The truth table of the arrangement will be as shown in Fig. 2.45 (*b*). It follows that it is the truth table of the OR gate (read columns A, B and y only). Hence, *if the inputs A and B after being inverted by using two NOT gates (obtained from the NAND gates) are then fed to the NAND gate, the arrangement functions as the OR gate.*

2.27. THE 'XOR GATE'

An XOR gate is obtained by using OR, AND and NOT gates as shown in Fig. 2.46 (*a*). It is also called **exclusive OR gate** and its logic symbol is as shown in Fig. 2.46 (*b*).

---

*The repeated use of the NOR gate can also produce all the three basic gates and therefore the NOR gate is also the building block of digitial circuits.

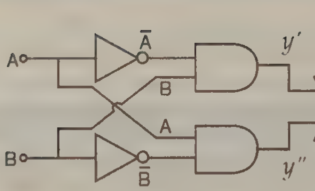


Fig. 2.46. (*a*)

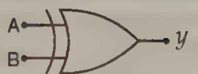


Fig. 2.46. (*b*)

A	B	y
0	0	0
0	1	1
1	0	1
1	1	0

Fig. 2.46. (*c*)

Its truth table is given in Fig. 2.46 (*c*). It follows that output in case of XOR gate is 1, only when inputs are different.

SOLVED NUMERICAL PROBLEMS

Type A. On Junction diode and Zener diode

Problem 2.01. Fig. 2.47 shows a junction diode connected to an external resistance of 100 Ω and a source of e.m.f. of 3·0 V. Assuming that the barrier potential developed in the junction diode is 0·7 V, obtain the current in the circuit.

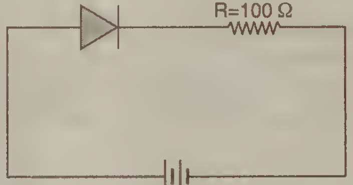


Fig. 2.47.

Sol. Here, e.m.f of source, $E = 3{\cdot}0$ V

Barrier potential developed in the diode, $V_d = 0{\cdot}7$ V

Therefore, potential drop across external resistance,

$$V = E - V_d = 3{\cdot}0 - 0{\cdot}7 = 2{\cdot}3 \text{ V}$$

Hence, current in the circuit,

$$I = \frac{V}{R} = \frac{2\cdot 3}{100} = 2\cdot 3 \times 10^{-2}\,\text{A} = \mathbf{23\ mA}$$

Problem 2.02. A diode used in the circuit shown in Fig. 2.48 has a constant voltage drop of 0·5 V at all currents and a maximum power rating of 100 mW. What should be the value of the resistance R connected in series with diode for obtaining maximum current ? (*C.B.S.E. 1997*)

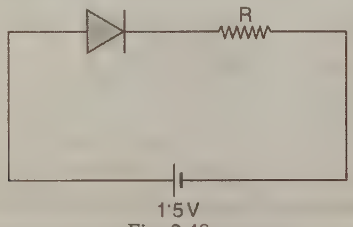


Fig. 2.48.

Sol. Here, e.m.f. of source, $E = 1{\cdot}5$ V

Voltage drop across diode, $V_d = 0{\cdot}1$ V

Maximum power rating of diode, $P = 100$ mW $= 0{\cdot}1$ W

Therefore, maximum current through the diode,

$$I = \frac{P}{V_d} = \frac{0\cdot 1}{0\cdot 5} = 0\cdot 2\text{ A}$$

Potential drop across resistance R,

$$V = E - V_d = 1{\cdot}5 - 0{\cdot}5 = 1{\cdot}0 \text{ V}$$

$$\therefore \qquad R = \frac{V}{I} = \frac{1\cdot 0}{0\cdot 2} = 5\,\Omega$$

Problem 2.03. In the circuit shown in Fig. 2.49, a Zener diode of voltage V_Z (= 6 V) is used to maintain a constant voltage across a load resistance R_L(= 1000 Ω)by using a series resistance R (= 100 Ω). If the e.m.f. of source (E) is 9 V, calculate the value of current through series resistance, Zener diode and load resistance. What is the power being dissipated in Zener diode ?

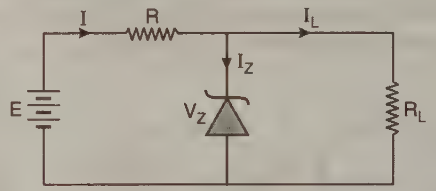


Fig. 2.49.

Sol. Here, $E = 9$ V ; $V_Z = 6$ V ; $R_L = 1000\,\Omega$ and $R = 100\,\Omega$

Potential drop across series resistor,

$$V = E - V_Z = 9 - 6 = 3 \text{ V}$$

Therefore, current through series resistance R,

$$I = \frac{V}{R} = \frac{3}{100} = 0\cdot 03\text{ A}$$

Current through load resistance R_L,

$$I_L = \frac{V_Z}{R_L} = \frac{6}{1000} = 0\cdot 006\text{ A}$$

Therefore, current through Zener diode,

$$I_Z = I - I_L = 0{\cdot}03 - 0{\cdot}006 = 0{\cdot}024 \text{ A}$$

Power dissipated in Zener diode,

$$P_Z = V_Z I_Z = 6 \times 0{\cdot}024 = \mathbf{0{\cdot}144\ W}$$

Problem 2.04. A junction diode of negligible forward resistance is used as a half-wave rectifier. A sinusoidal voltage supply having peak value 50 V is fed to the junction diode and output is obtained across the load resistance R_L. Calculate (*a*) the r.m.s. voltage across R_L and (*b*) reading of a d.c. voltmeter connected across R_L.

Sol. Here, $E_0 = 50$ V

$$\therefore \qquad E_{r.m.s.} = \frac{50}{\sqrt{2}}\text{ V}$$

(*a*) A half-wave rectifier gives output corresponding to alternate half cycles of the a.c. input. Since junction diode is of negligible forward resistance, the r.m.s. value of voltage across R_L during half cycle of conduction is $50/\sqrt{2}$ V. Therefore, r.m.s. value of voltage for a complete cycle (including the non-conducting half cycle) is

$$V_{r.m.s.} = \sqrt{\frac{(50/\sqrt{2})^2 + (0)^2}{2}} = \sqrt{\frac{1250 + 0}{2}} = \sqrt{625}$$
$$= \mathbf{25\ V}$$

(*b*) The d.c. voltmeter connected across R_L would read the average value of voltage across R_L during a complete cycle. Therefore reading of d.c. voltmeter connected across R_L,

$$V_{d.c.} = \frac{2\,E_0/\pi + 0}{2} = \frac{E_0}{\pi} = \frac{50}{\pi} = \mathbf{15{\cdot}92\ V}$$

Type B. On Transistor as an Amplifier

Problem 2.05. For a transistor working as common base amplifier, current gain is 0·96. If the emitter current is 7·2 mA, calculate the base current.

Sol. Here, $\alpha = 0{\cdot}96$; $I_e = 7{\cdot}2$ mA

Now, $\alpha = \dfrac{I_c}{I_e}$ or $I_c = \alpha I_e = 0{\cdot}96 \times 7{\cdot}2 = 6{\cdot}91$ mA

Also, $I_e = I_c + I_b$ or $I_b = I_e - I_c = 7{\cdot}2 - 6{\cdot}91 = \mathbf{0{\cdot}29\ mA}$

Problem 2.06. For a common emitter amplifier, current gain is 70. If the emitter current is 8·8 mA, calculate the collector and base current. Also calculate current gain, when transistor is working as common base amplifier.

Sol. Here, $\beta = 70$; $I_e = 8{\cdot}8$ mA

Now, $\beta = \dfrac{I_c}{I_b}$

or $I_c = \beta I_b = 70\,I_b$

Also, $I_e = I_c + I_b$

or $8{\cdot}8 = 70\,I_b + I_b$

or $I_b = \dfrac{8{\cdot}8}{71} = \mathbf{0{\cdot}124\ mA}$

Again $I_c = 70\,I_b = 70 \times 0{\cdot}124 = \mathbf{8{\cdot}68\ mA}$

We know, $\beta = \dfrac{\alpha}{1-\alpha}$

$\therefore$ $70 = \dfrac{\alpha}{1-\alpha}$ or $\alpha = \dfrac{70}{71} = \mathbf{0{\cdot}986}$

Problem 2.07. The base current of a transistor is 105 μA and collector current is 2·05 mA.

(*a*) Determine the value of β, I_e and α.

(*b*) A change of 27 μA in the base current produces a change of 0·65 mA in the collector current. Find β_{ac}.

Sol. Here, $I_b = 105\ \mu A = 105 \times 10^{-6}$ A ;

$I_c = 2{\cdot}05$ mA $= 2{\cdot}05 \times 10^{-3}$ A ;

(*a*) Now, $\beta = \dfrac{I_c}{I_b} = \dfrac{2{\cdot}05 \times 10^{-3}}{105 \times 10^{-6}} = \mathbf{19{\cdot}5}$

Also, $I_e = I_b + I_c = 105 \times 10^{-6} + 2{\cdot}05 \times 10^{-3}$
$= \mathbf{2{\cdot}155 \times 10^{-3}\ A}$

Further, $\alpha = \dfrac{I_c}{I_e} = \dfrac{2{\cdot}05 \times 10^{-3}}{2{\cdot}05 \times 10^{-3}} = \mathbf{0{\cdot}95}$

(*b*) Here, $\Delta I_b = 27\ \mu A = 27 \times 10^{-6}$ A ;

$\Delta I_c = 0{\cdot}65$ mA $= 650 \times 10^{-6}$ A

Now, $\beta_{ac} = \dfrac{\Delta I_c}{\Delta I_b} = \dfrac{650 \times 10^{-6}}{27 \times 10^{-6}} = \mathbf{24{\cdot}07}$

Problem 2.08. In a silicon transistor, a change of 7·89 mA in the emitter current produces a change of 7·8 mA in the collector current. What change in the base current is necessary to produce an equivalent change in the collector current ?

Sol. Here, $\Delta I_e = 7{\cdot}89$ mA $= 7{\cdot}89 \times 10^{-3}$ A ;

$\Delta I_c = 7{\cdot}8$ mA $= 7{\cdot}8 \times 10^{-3}$ A ;

Now, $\alpha_{ac} = \dfrac{\Delta I_c}{\Delta I_e} = \dfrac{7{\cdot}8 \times 10^{-3}}{7{\cdot}89\ 10^{-3}} = 0{\cdot}9886$

We have, $\beta_{ac} = \dfrac{\alpha_{ac}}{1-\alpha_{ac}} = \dfrac{0{\cdot}9886}{1-0{\cdot}9886} = 86{\cdot}72$

Also, $\beta_{ac} = \dfrac{\Delta I_c}{\Delta I_b}$

or $\Delta I_b = \dfrac{\Delta I_c}{\beta_{ac}}$

Here, $\Delta I_c = 7{\cdot}8 \times 10^{-3}$ A ; $\beta_{ac} = 86{\cdot}72$

$\therefore$ $\Delta I_b = \dfrac{7{\cdot}8 \times 10^{-3}}{86{\cdot}72} = \mathbf{89{\cdot}94 \times 10^{-6}\ A}$

Problem 2.09. In a silicon transistor, the base current is changed by 20 μA. This results in a change of 0·02 V in base to emitter voltage and a change of 2 mA in the collector current.

(*a*) Find the input resistance, β_{ac} and transconductance of the transistor.

(*b*) This transistor is used as an amplifier with the load resistance 5 kΩ. What is the voltage gain of the amplifier ? *(C.B.S.E. 2001)*

Sol. For $\Delta I_b = 20\ \mu A = 20 \times 10^{-6}$ A ;

$\Delta V_{be} = 0{\cdot}02$ V and $\Delta I_c = 2$ mA $= 2 \times 10^{-3}$ A

(*a*) Input resistance

$$= \frac{\Delta V_{be}}{\Delta I_b} = \frac{0{\cdot}02}{20 \times 10^{-6}} = \mathbf{1000\ \Omega}$$

$$\beta = \frac{\Delta I_c}{\Delta I_b} = \frac{2 \times 10^{-3}}{20 \times 10^{-6}} = 100$$

$$g_m = \frac{\Delta I_c}{\Delta V_{be}} = \frac{2 \times 10^{-3}}{0{\cdot}02} = \mathbf{0{\cdot}1\ \Omega^{-1}}$$

(*b*) $R_L = 5\ k\ \Omega = 5000\ \Omega$

Now, voltage gain

$$= \frac{\Delta V_{ce}}{\Delta V_{be}} = \frac{R_L \times \Delta I_c}{\Delta V_{be}} = \frac{5000 \times 2 \times 10^{-3}}{0{\cdot}02} = 500$$

Problem 2.10. The input resistance of a silicon transistor is 665 Ω. Its base current is changed by 15 μA which results in the change in collector current by 2 mA. This transistor is used as a common emitter amplifier with a load resistance of 5 kΩ. Calculate (*i*) current gain '$\beta_{a.c.}$' (*ii*) transconductance 'g_m' and (*iii*) voltage gain 'A_v' of the amplifier. *(C.B.S.E. 2001 S ; C.B.S.E. Sample Paper)*

Sol. Here, $\Delta I_b = 15\ \mu A = 15 \times 10^{-6}$ A ;

$\Delta I_c = 2$ mA $= 2 \times 10^{-3}$ A ;

$R_{in} = 665\ \Omega$; RL $= 5\ k\Omega = 5 \times 10^3\ \Omega$

(*i*) Now, $\beta_{ac} = \dfrac{\Delta I_c}{\Delta I_b} = \dfrac{2 \times 10^{-3}}{15 \times 10^{-6}} = \mathbf{133{\cdot}3}$

(*ii*) Transconductance, $g_m = \dfrac{\beta_{ac}}{R_{in}} = \dfrac{133{\cdot}3}{665} = \mathbf{0{\cdot}2\ \Omega^{-1}}$

(*iii*) Voltage gain, $A_v = g_m R_L = 0{\cdot}2 \times 5 \times 10^3 = 1000$

Problem 2.11. In the circuit shown in Fig. 2.50, the value of β is 100. Find I_b, V_{ce}, V_{be} and V_{bc}, when $I_c = 1{\cdot}5$ mA. Is the transistor in active, cut off or a saturation state ?

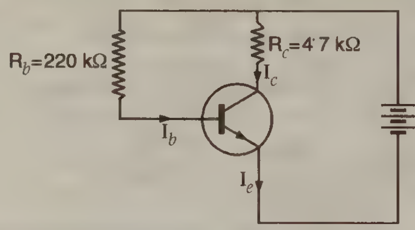


Fig. 2.50.

Sol. Here, $\beta = 100$, $V_{cc} = 24$ V ; $I_c = 1{\cdot}5\text{ mA} = 1{\cdot}5 \times 10^{-3}$ A ;

Now, $\beta = \dfrac{I_c}{I_b}$

or $I_b = \dfrac{I_c}{\beta} = \dfrac{1{\cdot}5 \times 10^{-3}}{100} = \mathbf{1{\cdot}5 \times 10^{-5}\ A}$

Let V_{ce}, V_{be} and V_{bc} be the respective potential drops across collector-emitter ; base-emitter and base-collector junctions. Applying Kirchhoff's second law* to the closed part of the circuit containing R_c and collector-emitter junction, we have

$$V_{ce} = V_{cc} - I_c R_c$$
$$= 24 - 4{\cdot}7 \times 10^3 \times 1{\cdot}5 \times 10^{-3}$$
$$= 24 - 7{\cdot}05 = \mathbf{16{\cdot}95\ V}$$

Applying Kirchhoff's second law to the closed part of the circuit containing R_b and emitter-base junction, we have

$$V_{cc} = V_{be} + I_b R_b$$

or $$V_{be} = V_{cc} - I_b R_b = 24 - 1{\cdot}5 \times 10^{-5} \times 220 \times 10^3$$
$$= 24 - 3{\cdot}3 = \mathbf{20{\cdot}7\ V}$$

Applying Kirchhoff's second law to the closed part of the circuit containing R_b , R_c and base-collector junction, we have

$$V_{be} + I_b R_b - I_c R_c = 0$$

or $$V_{be} = I_c R_c - I_b R_b$$
$$= 1{\cdot}5 \times 10^{-3} \times 4{\cdot}7 \times 10^3 - 1{\cdot}5 \times 10^{-5} \times 220 \times 10^3$$
$$= 7{\cdot}05 - 3{\cdot}3 = \mathbf{3{\cdot}75\ V}$$

The transistor is in ***saturation*** state.

Type C. On Binary algebra

Problem. 2.12. Perform binary addition on the following sets of numbers :

(*a*) 110010 and 111101 **(*b*) 101010 and 010101**

(*Text Problem*)

Sol. (***a***) To find the sum of the given binary numbers 110010 and 111101, we first find their values in decimal number system.

$$(110010)_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$= 32 + 16 + 0 + 0 + 2 + 0 = 50$$

and $$(111101)_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 32 + 16 + 8 + 4 + 0 + 1 = 61$$

Therefore, sum of the given two numbers in decimal system is 50 + 61 *i.e.* 111.

The binary representation of the number 111 in decimal system can be found by double dabble method as below :

2	111
2	55 – 1
2	27 – 1
2	13 – 1
2	6 – 1
2	3 – 0
2	1 – 1
	0 – 1

Writing the remainders in inverse order, it follows that $(111)_{10}$ = 110111 *i.e.* addition of binary numbers 110010 and 111101 is equal to **1101111.**

(***b***) $(101010)_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 2^0$

$$= 32 + 0 + 8 + 0 + 2 = 42$$

$$(010101)_2 = 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$$
$$= 0 + 16 + 0 + 4 + 0 + 1 = 21$$

The sum of the given two numbers in decimal number system is 42 + 21 *i.e.* 63. By proceeding as above, it can be shown that binary representation of 63 *i.e.* $(63)_{10}$ = 111111. Therefore, addition of binary numbers 101010 and 010101 is equal to **111111.**

Type D. On Logic gates

Problem 2.13. Express by a truth table, the output *y* for all possible inputs A and B for the combination of gates shown in Fig. 2.51. Hence, show that it behaves as a NAND gate.

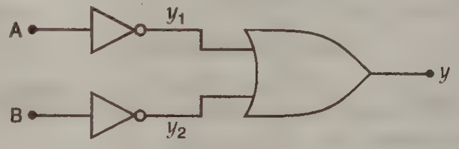


Fig. 2.51.

Sol. Here, outputs y_1 and y_2 of two NOT gates form the two inputs of the OR gate. Let us consider in turn the possibilities for A, B of 0, 0 then 0, 1, then 1, 0 and finally 1, 1. Because of the NOT gates, the column *y*, contains ones, wherever A = 0 and zeroes, where A = 1. Similarly, y_2 is opposite to B. The output *y* of OR gate is 1, if y_1 or y_2 is 1 or both are 1. Hence, the truth table for the given combination of gates is as shown in Fig. 2.52.

A	B	y_1	y_2	y
0	0	1	1	1
1	0	0	1	1
0	1	1	0	1
1	1	0	0	0

≡

A	B	y
0	0	1
1	0	1
0	1	1
1	1	0

Fig. 2.52.

From the completed table showing the relationship between output *y* and the inputs A and B, it follows that it is the truth table for a single NAND gate [refer to Fig. 2.40 (*b*)]. Hence, the combination of gates shown in Fig. 2.51 behaves as NAND gate.

Problem 2.14. Produce the truth table for the combination of gates shown in Fig. 2.53 and hence show that it behaves as XOR gate.

---

*One may use the equation (2.09)

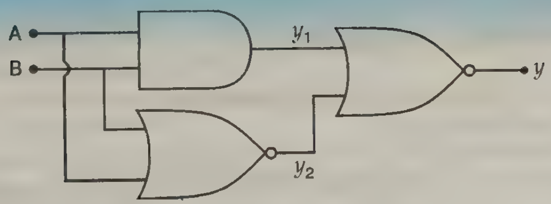


Fig. 2.53.

Sol. Here, the outputs y_1 and y_2 of AND and NOR gates respectively form the two inputs of another NOR gate. Because of NAND gate, the column y_1 contains 1, when both A and B are 1 and the column y_2 contains 1, when both A and B are zero. Hence, the truth table for the given combination of gates is as shown in Fig. 2.54.

A	B	y_1	y_2	y
0	0	0	1	0
1	0	0	0	1
0	1	0	0	1
1	1	1	0	0

≡

A	B	y
0	0	0
1	0	1
0	1	1
1	1	1

Fig. 2.54.

From the completed table showing the relationship between output y and the inputs A and B, it follows that it is the truth table for a single XOR gate [refer to Fig. 2.46 (*c*)]. Hence, the combination of gates shown in Fig. 2.53 behaves as XOR gate.

TECHIE STUFF – CONCEPTUAL NUMERICAL PROBLEMS

For ambitious, brilliant & curious Students

Problem 2.15. Assume that the silicon diode in the circuit shown in Fig. 2.55 requires a minimum current of 1 mA to be above the knee point 0·7 V) of its I-V characteristics. Also assume that the voltage across the diode is independent of current above the knee point.

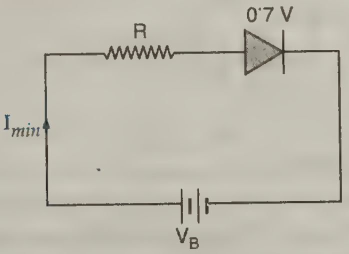


Fig. 2.55.

(*a*) If $V_B = 5$ V, what should be the maximum value of R so that the voltage is above the knee point ?

(*b*) If $V_B = 5$ V, what should be the value of R to establish the current of 5 mA in the circuit ?

(*c*) What is the power dissipated in the resistance R and in the diode, when a current of 5 mA flows in the circuit at $V_B = 6$ V ?

(*d*) If R=1 kΩ, what is the minimum voltage V_B required to keep the diode above the knee point ?

Sol. When current of 1 mA (minimum value) flows through the circuit, the potential drop across the diode becomes 0·7 V for a forward bias of 5 V.

Thus, $V_B = 5$ V ; $I_{min} = 1$ mA 10^{-3} A ;

(*a*) Applying Kirchhoff's second law, we have

$$I_{min} \times R + 0{\cdot}7 = 5$$

or $$10^{-3} \times R = 5 - 0{\cdot}7 = 4{\cdot}3$$

or $$R = \mathbf{4{\cdot}3 \times 10^3\ \Omega}$$

(*b*) Now, $I = 5\text{ mA} = 5 \times 10^{-3}$ A

Again, according to Kirchhoff's second law, we have

$$I \times R + 0{\cdot}7 = 5$$

or $$5 \times 10^{-3} \times R = 5 - 0{\cdot}7 = 4{\cdot}3$$

or $$R = \frac{4 \cdot 3}{5 \times 10^{-3}} = \mathbf{860\ \Omega}$$

(*c*) Now, $V_B = 6$ V ; $I = 5$ mA $= 5 \times 10^{-3}$ A ;

Applying Kirchhoff's second law, we have

$$I \times R + 0{\cdot}7 = 6$$

or $$R = \frac{6 - 0{\cdot}7}{I} = \frac{5{\cdot}3}{5 \times 10^{-3}} = 1060\ \Omega$$

Power dissipated across R = $I^2 R = (5 \times 10^{-3})^2 \times 1060$
$= \mathbf{26{\cdot}5 \times 10^{-3}\ W}$

Power dissipated across diode = $VI = 0{\cdot}7 \times 5 \times 10^{-3}$
$= \mathbf{3{\cdot}5 \times 10^{-3}\ W}$

(*d*) Now, R = 1 kΩ = 1000 Ω ; I = 1 mA = 10^{-3} A ;

P.D. across diode = 0·7 V

$\therefore$ $V_B = IR + 0{\cdot}7 = 10^{-3} \times 1000 + 0{\cdot}7 = \mathbf{1{\cdot}7\ V}$

Problem 2.16. In a *p-n*-junction, the current I can be expressed as

$$I = I_0\left(e^{\frac{eV}{kT}} - 1\right),$$

where I_0 is called the reverse saturation current, V the voltage across the diode and is positive for forward bias and negative for reverse bias and I is the current through the diode, *k* is the Boltzmann's constant ($8{\cdot}6 \times 10^{-5}$ eV K^{-1}) and T is the absolute temperature. If for a diode, $I_0 = 5 \times 10^{-12}$ A and T = 300 K, then

(*a*) what will be the forward current at a forward voltage of 0·6 V ?

(*b*) what will be the increase in the current, if the voltage across the diode is increased to 0·7 V ?

(*c*) what is the dynamic resistance ?

(*d*) what will be the current, if the reverse bias voltage changes from 1 V to 2 V ?

Sol. Here, $I_0 = \times 5\ 10^{-12}$ A ; $k = 8{\cdot}6 \times 10^{-5}$ eV K^{-1}
$= 8{\cdot}6 \times 10^{-5} \times 1{\cdot}6 \times 10^{-19}$ J K^{-1} ;

(*a*) Now, $$I = I_0\left(e^{\frac{eV}{kT}} - 1\right)$$

For V = 0·6 volt, we have

$$I = 5 \times 10^{-12}\left[e^{\frac{1{\cdot}6 \times 10^{-19} \times 0{\cdot}6}{8{\cdot}6 \times 10^{-5} \times 1{\cdot}6 \times 10^{-19} \times 300}} - 1\right]$$

$$= 5 \times 10^{-12} \left[e^{23 \cdot 256} - 1 \right]$$

$$= 5 \times 10^{-12} \; [1 \cdot 256 \times 10^{10} - 1] = \mathbf{0 \cdot 0628 \text{ A}}$$

(*b*) For V = 0·7 volt, we have

$$I = 5 \times 10^{-12} \left[e^{\frac{1 \cdot 6 \times 10^{-19} \times 0 \cdot 7}{8 \cdot 6 \times 10^{-6} \times 1 \cdot 6 \times 10^{-19} \times 300}} - 1 \right]$$

$$= 5 \times 10^{-12} \left[e^{27 \cdot 132} - 1 \right]$$

$$= 5 \times 10^{-12} \; [6 \cdot 054 \times 10^{11} - 1] = \mathbf{3 \cdot 0271 \text{ A}}$$

$$\therefore \qquad \Delta I = 3 \cdot 271 - 0 \cdot 0628 = \mathbf{2 \cdot 9643 \text{ A}}$$

(*c*) Now, $\Delta I = 2 \cdot 9643$ A ; $\Delta V = 0 \cdot 7 - 0 \cdot 6 = 0 \cdot 1$ V

Therefore, dynamic resistance

$$= \frac{\Delta V}{\Delta I} = \frac{0 \cdot 1}{2 \cdot 9643} = \mathbf{0 \cdot 0337 \; \Omega}$$

(*d*) For change in voltage from 1 to 2 V, the current will remain equal to $I_0 = 5 \times 10^{-12}$ A. It shows that the diode possesses practically infinite resistance in reverse biasing.

Problem 2.17. A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 24 mV is added to the base-emitter voltage, the base current changes by 32 μA and collector current by 3·6 mA. The load resistance is 4·8 kΩ. Calculate (*i*) the current gain β, (*ii*) the input resistance R_{be}, (*iii*) the transconductance g_m and (*iv*) voltage gain A_v.

Sol. Here, when $\Delta V_{be} = 23$ mV, $\Delta I_b = 32\ \mu$A and $\Delta I_c = 3 \cdot 6$ mA

(*i*) Now, $$\beta = \frac{\Delta I_c}{\Delta I_b} = \frac{3 \cdot 6 \times 10^{-3}}{32 \times 10^{-6}} = \mathbf{112 \cdot 5}$$

(*ii*) The input resistance R_{be} is given by

$$R_{be} = \frac{\Delta V_{be}}{\Delta I_b} = \frac{24 \times 10^{-3}}{32 \times 10^{-6}} = \mathbf{750 \; \Omega}$$

(*iii*) Transconductance, $$g_m = \frac{\Delta I_c}{\Delta V_{be}} = \frac{3 \cdot 6 \times 10^{-3}}{24 \times 10^{-3}} = \mathbf{0 \cdot 15 \text{ S}}$$

(*iv*) Voltage gain,

$$A_v = \frac{R_L \times \Delta I_c}{\Delta V_{be}} = \frac{4 \cdot 8 \times 10^3 \times 3 \cdot 6 \times 10^{-3}}{24 \times 10^{-3}} = \mathbf{720}$$

Problem 2.18. The Boolean expression for a logic circuit is given by

$$y = A . \overline{B} + \overline{A} . B$$

(*a*) Find the output *y*, if the inputs are A = 0, B = 0 ; A = 1 ; B = 0 ; A = 0, B = 1 and A = 1, B = 1.

(*b*) Identify, which basic gate is represented by the given Boolean expression.

Sol. (*a*) $y = A . B + \overline{A} . B$

For A = 0, B = 0 :

$$y = 0 . \overline{0} + \overline{0} . 0 = 0 . 1 + 1 . 0 = 0 + 0 = \mathbf{0}$$

For A = 1, B = 0 :

$$y = 1 . \overline{0} + \overline{0} . 0 = 1 . 1 + 0 . 0 = 1 + 0 = \mathbf{1}$$

For A = 0, B = 1 :

$$y = 0 . \overline{1} + \overline{0} . 1 = 0 . 0 + 1 . 1 = 0 + 1 = \mathbf{1}$$

For A = 1, B = 1 :

$$y = 1 . \overline{1} + \overline{1} . 1 = 1 . 0 + 0 . 1 = 0 + 0 = \mathbf{0}$$

(*b*) The truth table of the logic circuit represented by the given Boolean expression is

A	B	*y*
0	0	0
1	0	1
0	1	1
1	0	0

It is the truth table of **exclusive OR gate (XOR gate).**

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. What is depletion region in a *p-n*-junction ? (*I.S.S.C.E. 1997 ; C.B.S.E. 1992*)

Ans. A thin layer between *p* and *n* sections of the junction diode devoid of free electrons and holes is called depletion layer.

Q. 2.02. Can we measure the potential barrier of a *p-n* junction by putting a sensitive voltmeter across its terminals ?

Ans. In the depletion layer, there are no free electrons or holes and in the absence of forward bias, it offers infinite resistance. Therefore, potential barrier in the *p-n* junction can not be measured with a voltmeter.

Q. 2.03. Which type of biasing gives a semiconductor diode very high resistance ? (*C.B.S.E. 1996*)

Ans. Reverse biasing.

Q. 2.04. Fill in the blanks :

(*i*) When a *p-n* junction is forward biased, then the motion of charge carriers across the barrier is due to .... (drift, diffusion) and when it is reverse biased, then the motion of charge carriers is due to .... (drift, diffusion)

(*ii*) An ideal *p-n* junction diode conducts, when .... biased and does not conduct, when .... biased. (forward, reverse ; forward, reverse) (*Text Question*)

Ans. (*i*) diffusion, drift (*ii*) forward, reverse.

Q. 2.05. In the Fig. 2.56, is the diode D forward or reverse biased ? (*C.B.S.E. 1997*)

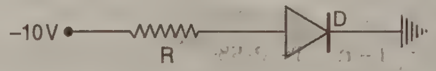


Fig. 2.56

Ans. Since *n*-section is connected to earth and *p*-section is at − 10 V, the diode is ***reverse biased.***

For details, refer to conceptual SAQ 2.02 and 2.03.

Q. 2.06. Draw a *p-n* junction with reverse bias. (*C.B.S.E. 1996*)

Ans. Refer to Fig. 2.05 (*a*).

Q. 2.07. How is an *n-p-n* transistor represented symbolically ? (*H.P.S.S.C.E. 1999 ; C.B.S.E. 1991*)

Ans. Refer to Fig. 2.17.

Q. 2.08. Can two *p-n* junction diodes back to back work as a *p-n-p* transistor ?

Ans. No. It is because, the *n*-section acting as base will become very thick. In a transistor, base is always made thin.

Q. 2.09. Fill in the blanks :

(*i*) The base of a transistor is always .... and .... doped, compared to the emitter and collector. (thinner, thicker ; lightly, heavily)

(*ii*) For using a transistor as an amplifier, the base-emitter junction is .... biased and the base collector junction is .... biased. (forward, reverse ; forward, reverse) (*Text Question*)

Ans. (*i*) thinner, lightly (*ii*) forward, reverse.

Q. 2.10. In the Fig. 2.57, is (*i*) the emitter and (*ii*) the collector forward or reverse biased ? (*C.B.S.E. 2001 S*)

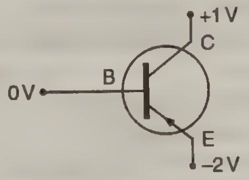


Fig. 2.57

Ans. Of the *p-n-p* transistor shown in Fig. 2.57,

(*i*) emitter is reverse biased and

(*ii*) collector is forward biased.

Q. 2.11. What is the relation between emitter current (I_e), base current (I_b) and collector current (I_c) at any instant in a properly biased common emitter transistor circuit ? Which one of the three has the smallest magnitude ? (*I.S.C.E. 1998*)

Ans. The relation between I_e, I_b and I_c : $I_e = I_b + I_e$

Of the three, I_b has the smallest magnitude.

Q. 2.12. Define current amplification factor in a common emitter mode of transistor. (*H.S.S.C.E. 1994*)

Ans. It is defined as the ratio of change in collector current to the change in base current at constant collector voltage. It is denoted by β.

$$\text{Mathematically, } \beta = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{ce}}$$

Q. 2.13. Write the relation between current gains α and β.

$$\textbf{Ans. } \beta = \frac{\alpha}{1-\alpha}$$

Q. 2.14. Calculate the current gain β of a transistor, if the current gain $\alpha = 0{\cdot}98$. (*I.S.C.E. 1995*)

Ans. Here, $\alpha = 0{\cdot}98$

$$\text{Now, } \beta = \frac{\alpha}{1-\alpha} = \frac{0{\cdot}98}{1-0{\cdot}98} = 49$$

Q. 2.15. Define the transconductance of a transistor. (*C.B.S.E. 1996*)

Ans. It is defined as the ratio of change in collector current to the change in base-emitter voltage. It is denoted by g_m.

$$\text{Mathematically, } g_m = \frac{\Delta I_c}{\Delta V_{be}}$$

Q. 2.16. What kinds of biasing are required to the collector and base of a transistor in a common emitter amplifier ? (*I.S.C.E. 1998*)

Ans. A collector is reverse biased, while a base is forward biased.

Q. 2.17. Would you prefer to use a transistor as a common base or a common emitter amplifier ?

Ans. A transistor as a common emitter amplifier is preferred for the reason that current gain in this case is more as compared to that in case of a common base amplifier.

Q. 2.18. Portable radio receiver sets are generally not made with electron tube circuits. Why ?

Ans. A portable radio receiver should be small in size and should work on d.c. A receiver set made of electron tube is bound to be large in size and works on a.c. Hence, radio receirs made with electron tube circuits are not portable.

Q. 2.19. What is a digital signal ?

Ans. The voltage signal that does not vary continuously but has only two levels of voltage.

Q. 2.20. A square wave (– 1 V to +1 V) is applied to *p-n* junction diode as shown below [Fig. 2.58]. Draw the output wave form. (*I.S.C.E. 1994*)

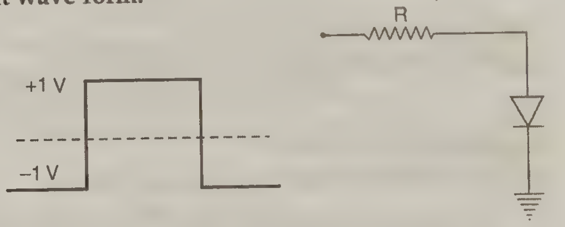


Fig. 2.58

Ans. When input is – 1 V, the diode gets reverse biased and no output is obtained. On the other hand, when input is +1 V, the diode gets forward biased and output obtained is +1 V. Therefore, corresponding to given input cycle, the output is of the shape as shown in Fig. 2.59.

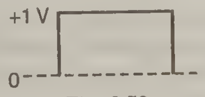


Fig. 2.59

Q. 2.21. Convert number 39 into binary system. (*H.S.S.C.E. 1997*)

Ans.

$$\begin{array}{r|l} 2 & 39 \\ \hline 2 & 19-1 \\ \hline 2 & 9-1 \\ \hline 2 & 4-1 \\ \hline 2 & 2-0 \\ \hline 2 & 1-0 \\ \hline & 0-1 \end{array}$$

The remainders in reverse order are 1, 0, 0, 1, 1 and 1. Hence, binary representation of decimal number 39 is **100111.**

Q. 2.22. What is a logic gate ? (*P.S.S.C.E. 1999, 1998 S ; H.S.S.C.E. 1997 S, 1993*)

Ans. A digital circuit which works according to some logical relationship between input and output voltages.

Q. 2.23. What do you understand by truth table of a logic gate ?

Ans. The truth table of a logic gate shows all possible inputs combination and the corresponding output combinations for a logic gate. It is also called ***table of combination.***

Q. 2.24. What is Boolean expression ?

Ans. The logical statement having only two values (either yes or no) is called Boolean expression. The functioning of a logic gate is based on its Boolean expression.

Q. 2.25. Name the logic gate realised using *p-n* junction diodes as shown in Fig. 2.60. Give its logic symbol. *(C.B.S.E. 2001)*

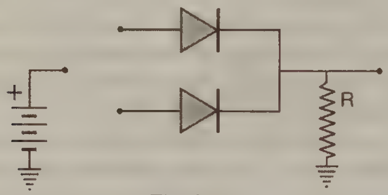


Fig. 2.60

Ans. It is OR gate.

For logic symbol of OR gate, refer to Fig. 2.33 (*a*).

Q. 2.26. Draw the logic symbol of OR gate. *(H.P.S.S.C.E. 2002 ; H.S.S.C.E. 1995)*

Ans. Refer to Fig. 2.33 (*a*).

Q. 2.27. Write the truth table for an OR gate. *(H.S.S.C.E. 2002 ; C.B.S.E. 1995)*

Ans. Refer to table in Fig. 2.33 (*c*).

Q. 2.28. Draw the logic symbol of AND gate. *(H.P.S.S.C.E. 2002 ; H.S.S.C.E. 1995 ; C.B.S.E. 1994, 1990)*

Ans. Refer to Fig. 2.35 (*a*).

Q. 2.29. Write the truth table for an AND gate. *(H.S.S.C.E. 2002 ; C.B.S.E. 1995)*

Ans. Refer to table in Fig. 2.35 (*c*).

Q. 2.30. Draw the logic symbol of NOT gate. *(H.P.S.S.C.E. 2002)*

Ans. Refer to Fig. 2.37 (*a*).

Q. 2.31. Write down the truth table for a NOT gate. *(H.S.S.C.E. 2002)*

Ans. Refer to table in Fig. 2.37 (*c*).

Q. 2.32. In the circuits shown, a switch which is opened represents the logic state 0 and the switch which is closed represents the logic state 1. The lamp L is lit, when output is logic state 1. What types of gates are represented by the circuits shown in (*i*) Fig. 2.61 (*a*) and (*ii*) Fig. 2.61 (*b*) ? *(C.B.S.E. 1995)*

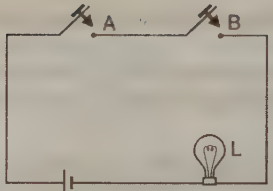


Fig. 2.61. (*a*)

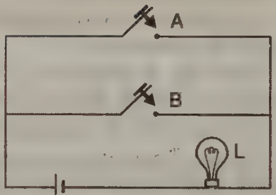


Fig. 2.61. (*b*)

Ans. (*i*) AND gate (*ii*) OR gate.

Q. 2.33. Draw the logic symbol for a NAND gate. *(C.B.S.E. 1993)*

Ans. Refer to Fig. 2.39 (*b*).

Q. 2.34. Write truth table of NAND gate. *(H.S.S.C.E. 2002 ; H.P.S.S.C.E. 1996 ; C.B.S.E. 1991)*

Ans. Refer to table in Fig. 2.40 (*b*).

Q. 2.35. Write down the truth table for a NOR gate. *(H.S.S.C.E. 2002)*

Ans. Refer to table in Fig. 2.42 (*b*).

Q. 2.36. Fill in the blanks :

(*i*) In a digital circuit, diodes and transistors are operated by a 5 V supply. The 1 state will correspond to .... (0 V, 5 V) and the 0 state to .... (0 V, 5 V).

(*ii*) Digital circuits can be made by repetitive use of .... gates. (AND, NAND)

(*iii*) An output 1 is obtained in a AND gate, when .... of the input terminals are at 1 state. (either, both)

(*iv*) An output 1 is obtained in a NOR gate, when .... input terminals are at 0 state. (either, both)

(*v*) If one terminal is an XOR gate is kept at 1; the output will be 0, when the other terminal is at .... (1, 0). *(Text Question)*

Ans. (*i*) 5 V, 0 V (*ii*) NAND (*iii*) both (*iv*) both (*v*) 1.

Q. 2.37. NAND or NOR gate may be called as digital building blocks. Why ?

Ans. The repeated use of NAND or NOR gates can produce all the three basic gates *i.e.* OR, AND and NOT gates. For this reason, these are called digital building blocks.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. Discuss the formation of a *p-n* junction.

Ans. Refer to section 2.01.

Q. 2.02. How is the depletion region formed in a *p-n* junction diode ? *(C.B.S.E. 2000, 1993)*

Ans. Refer to section 2.01.

Q. 2.03. How is potential barrier formed in a *p-n* junction diode ? *(H.S.S.C.E. 2002)*

Ans. Refer to section 2.01.

Q. 2.04. How does the width of the depletion region of a *p-n* junction vary, if the reverse bias applied to it increases ? *(C.B.S.E. 2002)*

Or

Why does the thickness of the depletion layer in a *p-n* diode vary with increase in reverse bias ? *(C.B.S.E. 1999)*

Ans. When a *p-n* junction is formed, a small potential difference (fictitious battery) is set up across the depletion layer. When the junction diode is reverse biased, the polarity of the applied d.c. source aids the fictitious battery. Due to this, potential drop across the junction increases and diffusion of holes and electrons across the junction decreases. It makes the width of the depletion layer larger.

Q. 2.05. In the following diagrams, indicate which of the diodes are forward biased and which are reverse biased. *(C.B.S.E. 1996 S)*

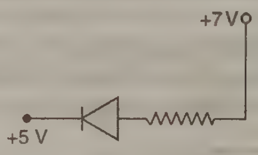


Fig. 2.62 (*a*)

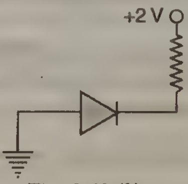
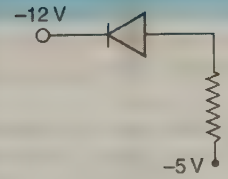


Fig. 2.62 (*b*)



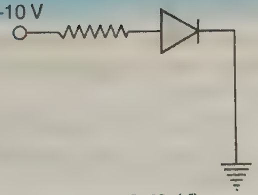


Fig. 2.62 (*c*) Fig. 2.62 (*d*)

Ans. Fig. 2.62 (*a*) : Forward biased.

Fig. 2.62 (*b*) : Reverse biased.

Fig. 2.62 (*c*) : Forward biased.

Fig. 2.62 (*d*) : Reverse biased.

For details, refer to conceptual SAQ 2.02 or 2.03.

Q. 2.06. Fig. 2.63 shows two *p-n* junctions diodes along-h a resistance R and a d.c. battery E. Indicate the path d direction of flow of appreciable current in the circuit. (*I.S.C.E. 1998*)

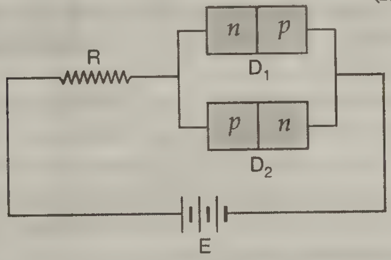


Fig. 2.63

Ans. In the circuit containing two junction diodes D_1 and , the diode D_2 is forward biased and D_1 is reverse biased. erefore, the path of appreciable current will be from sitive pole of the battery to its negative pole through istance R and junction diode D_2.

Q. 2.07. Draw the graph showing the variation of rrent with voltage for a *p-n* junction diode. (*C.B.S.E. 1993*)

Ans. Refer to Fig. 2.04 (*b*).

Q. 2.08. Which type of biasing results in very high sistance of a *p-n*-junction diode ? Draw a diagram owing this bias. (*C.B.S.E. Sample Q. Paper*)

Ans. The reverse biasing results in very high resistance a *p-n* junction diode.

For diagram showing reverse bias, refer Fig 2.05 (*a*).

Q. 2.09. Draw the V-I characteristics of a *p-n* junction ode. Show the Zener voltage on the characteristics curve. (*I.S.C.E. 1994*)

Ans. For V-I characteristics of *p-n* junction diode, fer to Figs 2.04 (*b*) and 2.05 (*b*). Zener voltage corres-onds to the straight portion of the graph [Fig. 2.05 (*b*)], owing the sudden rise in current corresponding to verse bias of − 10 V.

Q. 2.10. A diode is connected to 220 V (r.m.s.) a.c. in ries with a capacitor as shown in Fig. 2.64. What is the ltage V across the capacitor ? (*I.S.C.E. 1996*)

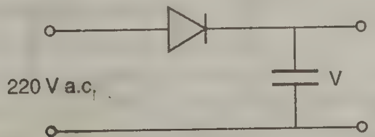


Fig. 2.64

Ans. A junction diode conducts during alternate half cycles of a.c. input supply. During a half cycle of conduction, the capacitor will charge itself to peak value of supply voltage. Therefore, voltage across the capacitor,

$$V = E_0 = E_{r.m.s.} \times \sqrt{2} = 220 \times \sqrt{2} = \mathbf{311{\cdot}1\ V}$$

Q. 2.11. Draw the circuit diagram of a half wave rectifier using a junction diode. (*H.S.S.C.E. 2002 ; I.S.C.E. 1995 ; Pre-degree Kerala, 1994*)

Ans. Refer to Fig. 2.07.

Q. 2.12. Draw a labelled circuit diagram of full wave rectifier using *p-n* junction diodes. (*C.B.S.E. 1995*)

Ans. Refer to Fig 2.09.

Q. 2.13. Draw a labelled circuit diagram showing use of *p-n* junctions as full wave rectifier. (*H.S.S.C.E. 2001, 1998, 1997*)

Ans. Refer to Fig. 2.09.

Q. 2.14. State the funciton of a Zener diode in a circuit. (*I.S.C.E. 1993*)

Ans. Refer to section 2.07.

Q. 2.15. What is a light emitting diode (LED) ? Draw a circuit diagram and explain its action. (*C.B.S.E. 1996 S*)

Ans. Refer to section 2.07 and Fig. 2.14.

Q. 2.16. Represent symbolically Zener diode, photodiode, *n-p-n* and *p-n-p* transistors. (*P.S.S.C.E. 1993*)

Ans. Refer to Figs. 2.11, 2.13 (*a*), 2.17 and 2.18.

Q. 2.17. Why is the base region of transistor usually made thin ? (*C.B.S.E. 2002*)

Or

In a transistor, base is made very thin. Explain. (*H.P.S.S.C.E. 1995*)

Or

Write the function of base region of a transistor. Why is this region made thin and slightly doped ? (*C.B.S.E. 1998 S*)

Ans. Due to the fact that base is doped lightly, the number density of majority carriers (electrons in *p-n-p* and holes in *n-p-n* transistors) is low. When emitter is forward biased, electron-hole combination takes place in base region. Since it is thin and lightly doped, only a small amount (about 5%) of electron-hole recombination will take place.

Q. 2.18. How does the collector current change in a junction transistor, if the base region has larger width ? (*C.B.S.E. 2000, 1999*)

Ans. The collector current will become smaller.

Also refer to SAQ 2.17.

Q. 2.19. Explain the conditions that must prevail in the base for a large current to flow in the collector circuit in spite of reverse bias.

Ans. Base should be made thin and it should be lightly doped. For details, refer to SAQ 2.17.

Q. 2.20. In a transistor, forward bias is always small as compared to the reverse bias. Explain the reason.

Ans. In a transistor, emitter is forward biased and collector is reverse biased. In case, forward bias is made large, majority carriers in the emitter drift towards the collector through the base region with large velocity. Since, their number is also very large, heat produced is so large that it breaks up the covalent bonds and the transistor gets spoiled.

However, if the reverse bias applied to collector is comparatively larger, no such effect is produced as the charge carriers drift only in collector region.

Q. 2.21. How will you test in a simple way whether the transistor is spoiled or in working order ?(*P.S.S.C.E. 2002*)

Ans. The two parts (part between the emitter and base and the part between base and collector) of a transistor in working order possesses low resistance in one direction and high resistance in the opposite direction. In a spoiled transistor, the resistance is low in both the directions. It can be checked with the help of a AVO-meter or by taking a battery cell and a milliammeter.

Q. 2.22. What is a *p-n-p* transistor ? How does it differ from a *n-p-n* transistor ? (*H.S.S.C.E. 2002*)

Ans. In a *p-n-p* transistor, a thin *p*-section is sandwitched between two *n*-sections. The holes are charge carriers in a *p-n-p* transistor.

In a *n-p-n* transistor, a thin *p*-section is sandwitched between two *n*-sections. In a *n-p-n* transistor, electrons are the charge carriers.

Q. 2.23. Give the working of a *p-n-p* transistor. (*H.S.S.C.E. 1995*)

Ans. Refer to section 2.10.

Q. 2.24. Why a transistor cannot be used as a rectifier ? (*P.S.S.C.E. 1991*)

Ans. To use a transistor as a rectifier, either its emitter-base portion or the collector-base portion has to be used. Since base is thin and lightly doped, either of the two portions will not work as a *p-n* junction diode. Hence, a transistor cannot be used as a rectifier.

Q. 2.25. In a transistor, emitter is always forward-biased, while the collector is reverse biased. Explain the answer.

Ans. In a transistor, the function of emitter and collector is same as that of cathode and plate respectively in a triode valve. The emitter will send charge carriers (electrons or holes) towards the base only, when it is forward-biased. On the other hand, the collector will collect them, when it is reverse-biased.

Q. 2.26. Draw a circuit diagram of a *p-n-p* transistor as an amplifier in a common emitter configuration. (*C.B.S.E. 1998 S*)

Ans. Refer to Fig. 2.28.

Q. 2.27. Draw the circuit diagram of an *n-p-n* transistor as an amplifier in common emitter configuration. (*C.B.S.E. 1998 S*)

Ans. Refer to Fig. 2.27.

Q. 2.28. Why the output voltage is out of phase with the input voltage in a common emitter transistor amplifier ? (*C.B.S.E. 1999*)

Ans. Refer to section 2.14.

Q. 2.29. Draw the circuit of a transistor as an amplifier. (*P.S.S.C.E. 1994*)

Ans. Refer to Fig. 2.27.

Q. 2.30. Draw the circuit diagram of an *n-p-n* transistor amplifier and show various potential drops. (*H.S.S.C.E. 1994*)

Ans. Refer to Fig. 2.27.

Q. 2.31. Define current gains, α and β of a transistor. (*I.S.C.E. 1994*)

Ans. α and β are known as current gains of a commo base amplifier and common emitter amplifier respectivel

To define α : It is defined as the ratio of change collector current to the change in emitter current at consta collector voltage.

To define β : It is defined as the ratio of change collector current to the change in base current at consta emitter current.

Q. 2.32. A transistor has a current gain of 30. If t collector resistance is 6 kilo-ohm, input resistance is 1 kil ohm, calculate its voltage gain. (*C.B.S.E. 1998*

Ans. Here, input resistance, $R_{in} = 1\ k\Omega$; outp (collector) resistance, $R_{out} = 6\ k\Omega$

Therefore, resistance gain, $\frac{R_{out}}{R_{in}} = \frac{6}{1} = 6$

Now, voltage gain = current gain × resistance gain
$= 30 \times 6 = 180$

Q. 2.33. Draw the circuit diagram of a transist oscillator. (*P.S.S.C.E. 1999 S, 1992 ; Pre-degree Kerala, 199*

Ans. Refer to Fig. 2.28.

Q. 2.34. What is the logic gate ? Mention the symb and truth table of AND logic gate. (*P.S.S.C.E. 2000, 199*

Ans. A logic gate is a digital circuit, which wor according to some logical relationship between input a output voltages. For symbol and truth table of AND ga refer to Figs. 2.35 (*a*) and 2.35 (*c*).

Q. 2.35. What is a 'logic gate' ? Mention the truth tab of AND gate. (*H.S.S.C.E. 199*

Ans. Logic gate. Refer to SAQ 2.34.

Truth table of AND gate. Refer to the table given in Fi 2.32 (*c*).

Q. 2.36. Give the logic symbol and truth table of AN gate. (*P.S.S.C.E. 1999 ; H.S.S.C.E. 1999*

Ans. Refer to Figs. 2.35 (*a*) and 2.35 (*c*).

Q. 2.37. Give logic symbol, Boolean expression a truth table of AND gate. (*P.S.S.C.E. 2002 ; H.P.S.S.C.E. 199*

Ans. Refer to section 2.23.

Q. 2.38. Describe in brief an OR gate. (*H.S.S.C.E. 199*

Ans. Refer to section 2.22.

Q. 2.39. Give logic symbol, Boolean expression a truth table of an OR gate. (*P.S.S.C.E. 2002 ; H.P.S.S.C.E. 199*

Ans. Refer to section 2.22.

Q. 2.40. Draw circuit diagram of OR gate. (*H.S.S.C.E. 1998 S, 199*

Ans. Refer to Fig. 2.34.

Q. 2.41. Give logic symbol, Boolean expression a truth table of a NOT gate ? (*P.S.S.C.E. 200*

Ans. Refer to section 2.24.

Q. 2.42. Name the logic gate shown in Fig. 2.65 a write its truth table. (*C.B.S.E. 199*

Ans. It is OR gate. For its truth table, refer to the tab given in Fig. 2.33 (*c*).

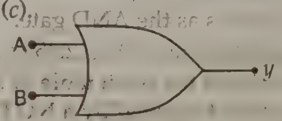


Fig. 2.65

Q. 2.43. Name the logic gate shown in Fig. 2.66 and e its truth table. (*C.B.S.E. 1996*)

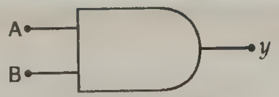


Fig. 2.66

Ans. It is AND gate. For its truth table, refer to table n in Fig. 2.32 (*c*).

Q. 2.44. Write the truth table for the combination of s shown in Fig. 2.67. (*H.S.S.C.E. 2000, C.B.S.E. 1997*)

Ans. The combination of the gates shown in Fig. 2.67 esents NAND gate. Its truth table is as given in Fig. 2.40 (*b*).

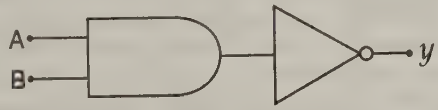


Fig. 2.67

Q. 2.45. Name the logic gate shown in Fig. 2.68 and te its truth table. (*C.B.S.E. 1996*)

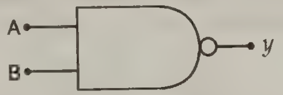


Fig. 2.68

Ans. It is NAND gate. For its truth table refer to table en in Fig. 2.40 (*b*).

Q. 2.46. Draw a logic circuit diagram showing how a ND gate can converted into a NOT gate. (*C.B.S.E. 1999*)

Ans. Refer to section 2.26.

Q. 2.47. Using a logic circuit, explain briefly how a nput NAND gate can be converted into a NOT gate. (*C.B.S.E. 1998 S*)

Ans. When the two inputs of NAND gate are joined, so t it has one input, it works as a NOT gate.

For details, refer to section 2.26 and Fig. 2.43 (*a*).

Q. 2.48. The output of a two-input NAND gate is fed input to a NOT gate. Write down the truth table for the al output of the combination. (*C.B.S.E. 1999*)

Ans. Let A and B be two inputs of NAND gate and y' its output. When its output y' is fed to the NOT gate, the al output will be as given in the adjoining truth table.

A	B	y'	y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

Q. 2.49. The output of a 2-input NAND gate is fed to a OT gate. Write down the truth table for the output of the mbination for all possible inputs of A and B. (*C.B.S.E. 2002*)

Or

If the output of a 2-input NAND gate is fed as the input a NOT gate, (*i*) name the new logic gate obtained and (*ii*) rite down its truth table. (*C.B.S.E. 2001*)

Ans. (*i*) It functions as the AND gate.

(*ii*) For truth table, refer to Fig. 2.35 (*c*).

Q. 2.50. Write down the truth table of a logic circuit in hich an AND gate is followed by a NOT gate.

Ans. AND gate followed by a NOT gate results in NAND gate.

For its truth table, refer to the table given in Fig. 2.40 (*b*).

Q. 2.51. Explain, how a NAND gate can be used to obtain OR gate. (*H.P.S.S.C.E. 1999 S*)

Ans. Refer to section 2.26.

Q. 2.52. Discuss how the NOT gate is realised from a NAND gate.

Ans. Refer to section 2.26 and Fig. 2.43 (*a*).

Q. 2.53. Discuss how the AND gate is realised from the NAND gate.

Ans. Refer to section 2.26 and Fig. 2.44 (*a*).

Q. 2.54. Give the logic symbol and truth table of NOR gate. (*H.S.S.C.E. 2001*)

Ans. Refer to section 2.25.

Q. 2.55. Write truth table and Boolean expression of NOR gate. (*H.P.S.S.C.E. 2001*)

Ans. Refer to section 2.25.

Q. 2.56. If the output of a 2-input NOR gate is fed as the input to a NOT gate, (*i*) name the new logic gate obtained and (*ii*) write down its truth table. (*C.B.S.E. 2001*)

Ans. (*i*) If functions as the OR gate.

(*ii*) For truth table, refer to Fig. 2.33 (*c*).

Q. 2.57. Write down the truth table of a logic circuit in which an OR gate is followed by a NOT gate.

Ans. An OR gate followed by a NOT gate results in NOR gate.

For its truth table, refer to the table given in Fig. 2.42 (*b*).

Q. 2.58. Write the truth table for the following combination of gates shown in Fig. 2.69. (*C.B.S.E. 1998*)

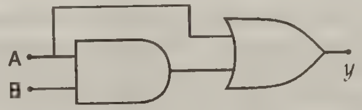


Fig. 2.69

Ans. In the combination of gates shown in Fig. 2.69, input (A) and output (say y') of the AND gate form the two inputs of the OR gate. The following truth table gives the final out put (y) of the combination of two gates.

A	B	y'	y
0	0	0	0
1	0	0	1
0	1	0	0
1	1	1	1

Q. 2.59. Write the truth table for the combination of gates shown in Fig. 2.70. (*C.B.S.E. 1998*)

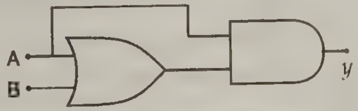


Fig. 2.70

Ans. In the combination of gates shown in Fig. 2.70, the input (A) and output (say y') of the OR gate form the two inputs of the AND gate. The truth table gives the final output (y) of the combination of two gates.

A	B	y'	y
0	0	0	0
1	0	1	1
0	1	1	1
1	1	1	1

Q. 2.60. Identify the logic gates marked, X, Y in Fig. 2.71. Write down the output at y, when A = 1, B = 1 and A = 0 and B = 1. *(C.B.S.E. 2001)*

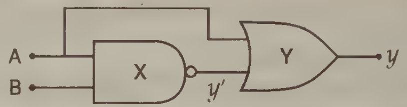


Fig. 2.71

Ans. The logic gate marked X is NAND gate, while the gate marked as Y is OR gate.

Corresponding to input B, let y' be output of NAND gate X. Then, inputs of OR gate Y are A and y'. The output at y will be as given in Fig. 2.72.

A	B	y'	y
1	1	0	1
0	1	1	1

Fig. 2.72

Q. 2.61. Identify the logic gates marked X, Y in Fig. 2.73. Write down the output at y, when A = 1, B = 1 and A = 0 and B = 0. *(C.B.S.E. 2001)*

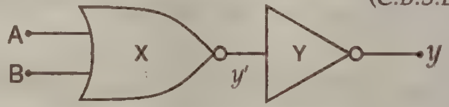


Fig. 2.73

Ans. The logic gate marked as X is NOR gate, while t gate marked as Y is NOT gate.

Let y' be output of NOR gate X. Then, output at y w be as given in Fig. 2.74.

A	B	y'	y
1	1	0	1
0	0	1	0

Fig. 2.74

Q. 2.62. Identify the logic gates marked X, Y in Fig. 2. Write down the output at y, when A = 1, B = 1 and A = B = 0. *(C.B.S.E. 200*

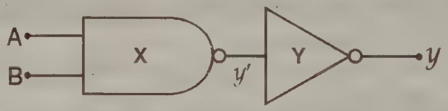


Fig. 2.75

Ans. The logic gate marked as X is NAND gate, wh the gate marked as Y is NOT gate.

Let y' be output of NAND gate X. Then, output at y w be as given in Fig. 2.76.

A	B	y'	y
1	1	0	1
0	0	1	0

Fig. 2.76

TECHIE STUFF – CONCEPTUAL SHORT ANSWER QUESTIONS

For ambitious, brilliant & curious Students

Q. 2.01. An n-type semiconductor is the one which has excess of free electrons and p-type semiconductor is deficient in these. When a p-n junction is formed, the electrons should flow from n to p region. But all the electrons do not do so. Explain, Why.

Ans. Though the semiconductors are called n-type or p-type, yet electrically they are neutral. When p-n junction is formed, electrons flow from n to p region. But as they do so, p-type becomes increasingly negative, while n-type becomes positive. This increasing potential difference across the p-n junction set up due to the electron flow attains a value, which then prevents the further flow of electrons. That is why all the electrons do not flow from n to p-type semiconductor.

Q. 2.02. In the following circuits, which of the diodes is forward-biased and which is reverse-biased and why ?

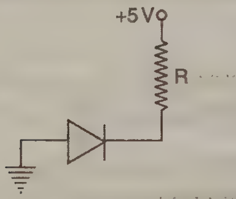


Fig. 2.77 (*a*)

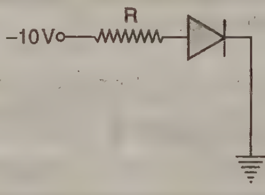


Fig. 2.77 (*b*)

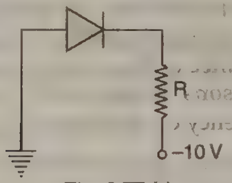


Fig. 2.77 (*c*)

Ans. The circuit shown in Fig. 2.77 (*a*) can be redrawn shown in Fig. 2.78.

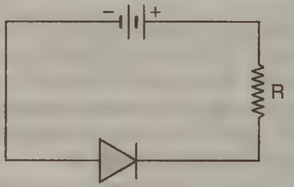


Fig. 2.78

As the p-section is connected to negative terminal of t battery, the diode has been *reverse biased.*

By redrawing the circuit diagrams, it can be shown th diode in circuit shown in Fig. 2.77 (*b*) is *reverse biased* and the circuit shown in Fig. 2.77 (*c*), the diode is *forward biase*

Q. 2.03. In the following circuits, which of the diod is forward biased and which is reverse biased ? *(C.B.S.E. 200*

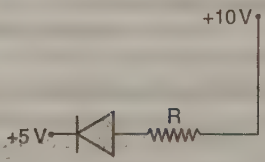


Fig. 2.79 (*a*)

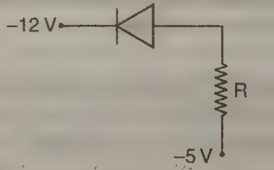


Fig. 2.79 (*b*)

Ans. The circuit shown in Fig. 2.79 (*a*) can be redrawn shown in Fig. 2.80.

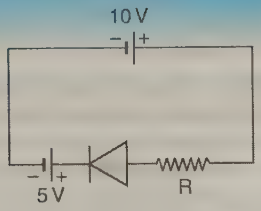


Fig. 2.80

The battery of 10 V makes *p*-section at +10 V, while the tery of 5 V makes *n*-section at + 5 V. Therefore, *p*-section at net + 5 V *w.r.t.* *n*-section. Hence, the diode in the circuit own in Fig. 2.79 (*a*) is *forward biased.*

Similarly, by redrawing the circuit shown in Fig. 2.79 (*b*), an be shown that the *p*-section is at net 7 V *w.r.t.* *n*-section d hence the diode is *forward biased.*

Q. 2.04. The circuit shown in Fig. 2.81 contains two diodes, each with a forward resistance of 50 Ω and with inite backward resistance. If the battery voltage is 6 V, d the current through the 100 Ω resistance.

(*I.I.T. 1997*)

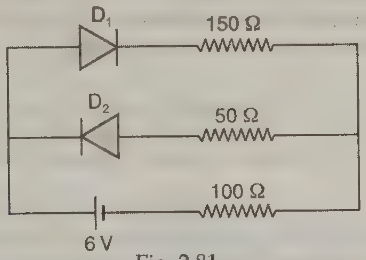


Fig. 2.81

Ans. Since the diode D_2 is reverse biased and it has finite backward resistance, no current will flow through ode D_2. Only the diode D_1 will allow current to flow in the rcuit, as it is forward biased.

Total resistance of the circuit

$$= 150 + 100 + \text{forward resistance of diode } D_1$$
$$= 150 + 100 + 50 = 300\ \Omega$$

Therefore, current in the circuit (or through 100 Ω sistance),

$$I = \frac{E}{R} = \frac{6}{300} = 0{\cdot}02\ A$$

Q. 2.05. Explain the following :

(*i*) A transistor is a temperature-sensitive device.

(*ii*) A transistorised circuit starts working immediately ter the circuit is switched on, whereas a vacuum tube rcuit takes some time to start working.

Ans. (*i*) A transistor contains free electrons and holes as arge carriers. When properly biased (emitter forward biased nd collector reverse biased), they carry current through the ansistor in external circuit. In case, temperature increases, ovalent bonds break up, resulting in an additional large umber of free electrons and holes. If transistor (when hot) is perated now, the current will be very strong, which may esult in excessive heat and ultimately in a complete break own of semiconductor device.

(*ii*) A vacuum tube is based upon thermionic emission. vacuum tube circuit will require some time, so that cathode ets heated and starts emitting electrons. On the other hand, a transistor, as there is no filament and hence no heating s required, the transistorised circuit starts working mmediately, when the circuit is switched on.

Q. 2.06. In only one of the circuits given below, the lamp L lights. Which circuit is it ? Give reason for your answer.

(*C.B.S.E. 2001 S*)

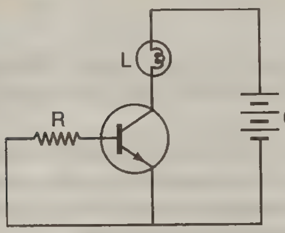


Fig.2.82

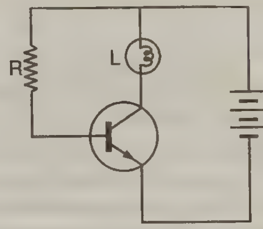


Fig. 2.83

Ans. In the circuit shown in Fig. 2.83, the lamp will light. It is because, the *n-p-n* transistor in the circuit can conduct only when its base is positive.

Q. 2.07. In the circuit shown in Fig. 2.84, a voltmeter V is connected across lamp L. What changes would occur at lamp L and the voltmeter V, if the resistor R is reduced in value ? Give reason for your answer.

(*C.B.S.E. 2001 S, 1999 S*)

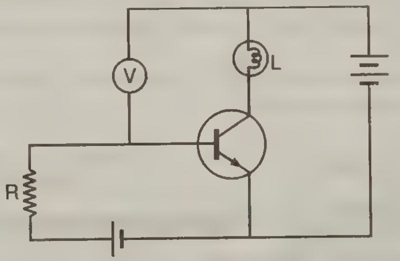


Fig.2.84

Ans. In the circuit shown in Fig. 2.67, the *n-p-n* transitor is forward biased. If resistor R is reduced, emitter current I_e will increase. The collector current flowing through the lamp is given by

$$I_c = I_e - I_b$$

It follows that due to increase in I_e, I_c will also increase. It will lead to make the lamp glow *more brilliantly*.

The voltmeter V measures potential difference across L. Due to increase in current through lamp, potential difference across lamp and hence reading of the voltmeter will **increase.**

Q. 2.08. The gain of a common emitter amplifier is given by $A_v = -g_m R_L$. Does it mean that if we keep on increasing indefinitely R_L, the gain of the amplifier will also increase indefinitely ? Explain your answer.

(*C.B.S.E. Sample Q. Paper*)

Ans. No, the gain of amplifier will not increase indefinitely on increasing R_L indefinitely. We know that

$$V_{ce} = V_{cc} - I_c R_L$$

Therefore, as R_L is increased, V_{ce} decreases and when V_{ce} becomes less than V_{be} (base voltage), both the junctions get forward biased and the saturation starts.

Q. 2.09. All India Radio broadcasts at 710 kcps, whereas BBC broadcasts at 1380 kcps on the same medium wave band. For changing over from All India Radio to BBC, the capacitance of the variable capacitor has to be decreased. Give reasons in support of your answer.

Ans. The frequency of oscillations is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

As both the stations are tuned on the same band (medium wave band), the value of L is same in both the cases and therefore,

$$f \propto \frac{1}{\sqrt{C}}$$

Since in changing over from All India Radio (710 kcps) to BBC (1380 kcps), frequency increases, the capacitance of variable capacitor has to be decreased.

Q. 2.10. In the following circuits, if the input wave form is as shown in the Fig. 2.85, what will be the output wave form across the diode in circuits shown in Fig. 2.86 (*a*) and Fig. 2.86 (*b*) ? Assume that the diode is ideal.

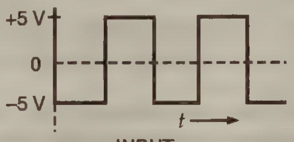


Fig. 2.85

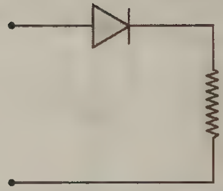


Fig. 2.86 (*a*)

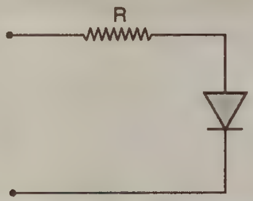


Fig. 2.86 (*b*)

Ans. ***The input cycle shown in Fig. 2.85*** : The inp digital nature *i.e.* it is either – 5 V or + 5 V.

Circuit shown in Fig 2.86 (a) : When the input l – 5 V, the diode in the circuit shown in Fig. 2.86 (*a*) be reverse biased. No output is obtained across resistanc long it remains – 5 V. When the input level becomes the diode gets forward biased and current flows throu resistance R. As diode is ideal, the output across R exactly 5 V. Thus, for the given waveform of the inp output will vary between 0 and 5 V and will be as sh Fig. 2.87.

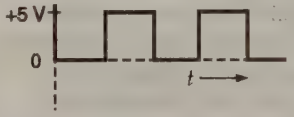


Fig. 2.87

Circuit shown in Fig. 2.86 (b) : In this case, the conducts, when the input level is – 5 V and it do conduct, when input level is + 5 V. Due to ideal na diode, the output obtained across R will be between – 5 0 and will be as shown in the Fig. 2.88.

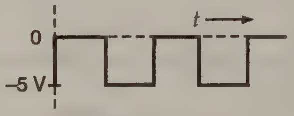


Fig. 2.88

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. Show diagrammatically a forward biased and a reverse biased *p-n* junction. (*C.B.S.E. 1990*)
2. Distinguish between forward biasing and reverse biasing in *p-n* junction. Discuss its use. (*H.P.S.S.C.E. 1995*)
3. Draw a circuit diagram for *p-n* junction diode in forward bias. Sketch the voltage versus current graph for the same. (*C.B.S.E. 1995*)
4. What is a *p-n* junction diode ? Define the term dynamic resistance for the junction diode. (*H.S.S.C.E. 1999*)
5. What is rectification ? Explain with the help of a circuit diagram, how a junction diode acts as a half-wave rectifier. (*I.S.C.E. 1994*)
6. How a *p-n* junction diode can be used as half-wave rectifier ? (*H.P.S.S.C.E. 1998 S*)
7. Explain the forward and reverse bias characteristic curve of a *p-n* junction. Draw the circuit diagram of *p-n* diode as a full-wave rectifier. (*P.S.S.C.E. 1996 S*)
8. Draw the circuit diagram of a full-wave rectifier and briefly explain its working principle. (*C.B.S.E. 2001*)
9. With the help of circuit diagram explain, the working of *p-n* junction diode as a full-wave rectifier. (*C.B.S.E. 2000 ; H.S.S.C.E. 1999, 1998 S*)
10. What do you mean by rectification ? With the help of a circuit diagram, explain the use of a *p-n* junction diode as a full-wave rectifier, giving its input and output wave-forms. (*C.B.S.E. 1999 S*)
11. Explain the function of a *p-n* junction diode as full-wave rectifier. (*P.S.S.C.E. 2002, 1998 S ; H.S.S.C.E. 2001, 1999 S ; H.P.S.S.C.E. 2000, 1999*)
12. Draw the current versus voltage characteristic curve of a Zener diode. Indicate the position of Zener voltage. Draw a labelled circuit diagram of a d.c. voltage regulator using a Zener diode. (*I.S.C.E. 1998*)
13. What is a Zener diode ? Explain its action in reg supply voltage. (*H.S.S.C.E*
14. What is a Zener diode ? How is it symbol represented ? With the help of a circuit diagram, the use of Zener diode as a voltage stabilizer. (*C.B.S.E.*
15. What is a zener diode and a photo diode ? Give the use of such diodes.
16. Distinguish between the light emitting diode an diode. Draw their biasing circuits. (*C.B.S.E. Sample Q*
17. Give the symbol of *n-p-n* and *p-n-p* transistors. Sh biasing of an *n-p-n* transistor and explain transistor (*C.B.S.*
18. With the help of a circuit diagram, explain the act *n-p-n* transistor. (*C.B.S.E.*
19. What is an *n-p-n* transistor ? How does it diffe *p-n-p* transistor ? Give their symbols. Explain tra action. (*C.B.S.E*
20. Describe the working of a *n-p-n* transistor as comm amplifier. Define current amplification. (*H.S.S.C.E.*
21. Explain through a labelled diagram the working transistor (common-base configuration) as an am (*H.P.S.S.C.*
22. Explain the working of an *n-p-n* transistor as an a and determine its voltage gain. (*C.B.S.E. 1995 ; H.S.S.C.*
23. Discuss the working of *p-n-p* transistor as amplifi common-base circuit. (*H.P.S.S.C.E*
24. Write the construction of *p-n-p* transistor. Ho represented symbolically ? Explain its operation

type of emitter base and collector base biasing is required for the use of transistor as common-base amplifier ? (C.B.S.E. 1996 S)

5. Discuss the working of a *p-n-p* transistor as a common-base amplifier and determine its voltage gain. (H.S.S.C.E. 2001 ; P.S.S.C.E. 1996)

6. Explain with the help of a labelled circuit diagram the use of a transistor as an amplifier. (C.B.S.E. 1996, 1995)

7. Explain the working and amplifying action of a transistor. (P.S.S.C.E. 1996)

8. Describe a transistor as an amplifier. (H.S.S.C. E. 1996)

9. Describe the principle, construction and working of a transistor. (H.S.S.C.E. 2002, 1996)

0. Explain the amplifying action of a transistor. (P.S.S.C.E. 1997 S)

1. With the help of a circuit diagram, explain the use of an *n-p-n* transistor as an amplifier. (H.S.S.C.E. 1997 S)

2. Explain through a labelled diagram, the working of an *n-p-n* transistor as an amplifer (common-emitter configuration). (H.P.S.S.C.E. 2002 ; 1996)

3. Draw a labelled circuit diagram for a common-emitter amplifier using *n-p-n* transistor. Write down the expression for its voltage gain. What is the phase difference between input and output signals ? (C.B.S.E. 2002)

4. Draw the circuit diagram of a common-emitter amplifier. What is the phase difference between the input and output signals State two reasons, why a common-emitter amplifier is preferred to a common-base amplifier. (C.B.S.E. 2001)

5. Fig. 2.89 shows the circuit diagram of a common-emitter amplifier. Explain the function of (*i*) resistor R_1, (*ii*) resistor R_2, (*iii*) resistor R_3 and capacitor C_3. (*iv*) capacitors C_1 and C_2.

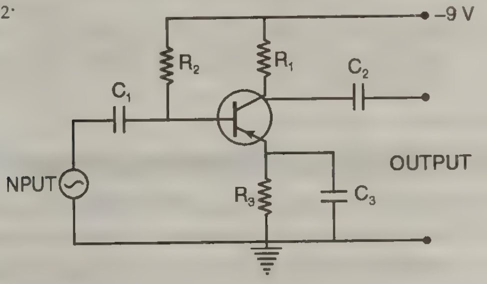


Fig. 2.89.

In the circuit, which junction is forward biased and which one is reverse biased ? (I.S.C.E. 1993)

36. Explain through a labelled diagram, the working of *p-n-p* transistor as an oscillator. (H.P.S.S.C.E. 2002 ; H.S.S.C.E. 2001)

37. With the help of a labelled circuit diagram, explain the use of a transitor as an oscillator. (C.B.S.E. 2000, 1998)

38. Explain through a diagram, the working of transistor as an oscillator. (H.S.S.C.E. 2001 ; H.P.S.S.C.E. 1998 S)

39. Describe a junction transistor and explain its use as an oscillator. (H.S.S.C.E. 1995)

40. Describe a junction transistor. Show with the help of a circuit diagram, how it can be used as an oscillator. (H.S.S.C.E. 1995 ; C.B.S.E. 1991)

41. Explain transistor as an oscillator. (H.S.S.C.E. 2001, 2000, 1993, 1992)

42. Explain the advantages and disadvantages of semiconducting devices compared to vacuum tube devices. (P.S.S.C.E. 2002)

43. Write a short note of integrated circuits. Give their uses.

44. Give symbol, truth table and logic statement of OR gate. (H.S.S.C.E. 1996)

45. What is a logic gate and its truth table ? Draw the logic symbol of the OR gate. Explain its working and obtain the truth table for it.

46. Give the logic, truth table and Boolean expression for OR gate. How is it realised in practice ? (H.S.S.C.E. 1993)

47. Draw the symbol of an AND gate. What type of truth table does it follow ? How can one realise such a gate in actual practice by using a junction diode for two-input feed ? (P.S.S.C.E. 1996)

48. Give the logic symbol, truth table and Boolean expression for AND gate. How is it obtained in practice ? (H.S.S.C.E. 1992)

49. Discuss the working of the NOT and AND gates. Write down the truth table in each case.

50. Give the truth tables and symbols of AND, NAND and NOR logic gates. (P.S.S.C.E. 1993)

51. Give the truth tables and logic symbols of OR, NOR and NAND logic gates. (P.S.S.C.E. 1999 S)

52. What is a digital circuit, a logic gate, truth table and Boolean expression ? Draw the logic symbols of the OR, AND and NOT gates.

53. Explain how a NAND gate is obtained from AND and NOT gates. Give its logic symbol and truth table.

54. Explain how a NOR gate is formed. Give its logic symbol and truth table.

55. Explain how a NAND gate can be used to obtain all the three basic logic gates.

56. Explain, how AND logic gate can be realised from NAND logic gate. (P.S.S.C.E. 1999 S)

57. Explain, how XOR gate is obtained from OR, AND and NOT logic gates. Also give the logic symbol and truth table of XOR gate. (P.S.S.C.E. 1999 S)

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. Define the terms 'potential barrier' and 'depletion region' for a *p-n* junction diode. State how the thickness of depletion region will change when the *p-n* junction diode is (*i*) forward biased, (*ii*) reverse biased. (C.B.S.E. 1998)

2. What is a *p-n* junction ? Explain with the help of a diagram, how depletion layer is formed near the junction. Also explain what happens to this layer, when the junction is (*i*) forward biased and (*ii*) reverse biased. (H.S.S.C.E 2001)

3. What is a *p-n* junction ? How is potential barrier caused in it ? Discuss the characteristics of a *p-n* junction diode. (H.S.S.C.E. 2002)

4. What is a *p-n* junction ? Discuss its action in forward bias and reverse bias arrangement. (H.S.S.C.E. 2002)

5. For a *p-n* junction, draw (*i*) the circuit diagram for forward biasing (*ii*) circuit diagram for reverse biasing (*iii*) the current-voltage characteristics. (*Pre-degree Kerala, 1991*)

6. Why does a *p-n* junction diode offer low resistance in forward bias and a high resistance in reverse bias ? Explain the characteristic curves of this diode by drawing suitable circuit diagrams. (P.S.S.C.E. 1998)

7. Explain the use of *p-n* junction diode as rectifier. Draw the circuit diagram of a full-wave rectifier and explain its working. Draw the input and output waveforms. (C.B.S.E. 1990)

8. What is a rectifier ? Explain the working of a full-wave rectifier giving a neat circuit diagram as well as the input and output waveforms. (H.S.S.C.E. 2002)

9. Describe the characteristics of a semiconductor diode and how it can be used as a full-wave rectifier. *(H.S.S.C.E. 2000)*
10. Explain briefly, why a junction diode acts as a rectifier. Draw a labelled diagram of a full-wave rectifier, explaining the functions of its various components. *(I.S.C.E. 1996, 1993)*
11. Define the terms `potential barrier' and `depletion region' for a *p-n* junction. Explain, with the help of a circuit diagram, the use of a *p-n* diode as a full-wave rectifier. Draw the input and output wave-forms. *(C.B.S.E. 1998)*
12. Give a circuit diagram for common-base characteristics of *p-n-p* transistor. How are input and output characteristics drawn ? Explain them. *(H.S.S.C.E. 1993)*
13. Draw the circuit diagram for *p-n-p* common-base transistor characteristics. Discuss the characteristics. *(H.S.S.C.E. 1997 S)*
14. Draw a circuit diagram to obtain the characteristics of a *n-p-n* transistor in common-emitter configuration. Describe how you will obtain input and output characteristics. Give shape of the curves. *(C.B.S.E. 2001)*
15. Describe the consutrction of a transistor. Explain its working as a common-base amplifier. *(P.S.S.C.E. 2000)*
16. With the help of a labelled circuit diagram, explain the working of *n-p-n* transistor as a common-base amplifier and derive an expression for its voltage gain. *(P.S.S.C.E. 2001)*
17. What is an amplifier ? With the help of a circuit diagram, explain the function of a *p-n-p* transistor as an amplifier. *(H.S.S.C.E. 2002)*
18. Drawing a labelled circuit diagram, explain the working principle of a common-emitter transistor amplifier. State the phase relation between input and out signals. *(C.B.S.E. 2000)*
19. Discuss common emitter amplifier using *n-p-n* transistor. Find its current gain, voltage gain and power gain. *(P.S.S.C.E. 2002, 2000)*
20. With the help of a labelled circuit diagram, explain working of a *p-n-p* transistor as a common-emit amplifier and derive an expression for its voltage gai *(P.S.S.C.E. 20*
21. Draw a practical single stage circuit for a common-emi audio frequency amplifier using an *n-p-n* transistor an single battery. Describe, how (*i*) temperature stabilisat is achieved, (*ii*) base-bias is provided and (*iii*) undesira feed-back of amplified signal into the base-emitter circ is avoided. *(I.S.C.E. 19*
22. Draw a circuit diagram to show the connections for operation of a *p-n-p* transistor as an amplifier. Explain different biasing. What is the phase relationship betw the input and output voltages in this case ? Obtain expression for its current gain. *(C.B.S.E. 1996*
23. Explain through a labelled circuit diagram, the work of a transistor as an amplifier (common-emitter confi ration). Obtain an expression for the current gain, volt gain and power gain. *(H.S.S.C.E. 1999 S ; H.P.S.S.C.E. 1999 ; C.B.S.E. 1996, 19*
24. What is an oscillator ? Discuss the use of a junct transistor as an oscillator. *(H.S.S.C.E. 20*
25. Give the logic symbol, truth table and Boolean express for AND gate. How is it realised in practice ? *(H.S.S.C.E. 20*
26. Draw the logic symbol of OR gate. Write its truth table a How can one realise such a gate in actual practice by us junction diodes ? *(P.S.S.C.E. 20*
27. What do you understand by logic gate ? Why is it called ? Discuss AND gate ? *(H.S.S.C.E. 20*
28. Give the logic symbol, truth table and Boolean express for AND gate. How is it realised in practice ? *(H.S.S.C.E. 20*
29. Draw the logic symbol of AND gate. Write its truth ta also. How can one realise such a gate in actual practice using junction diodes ? *(P.S.S.C.E. 20*

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On p-n junctions

1. In a forward biased silicon diode, current occurs at close to 0·7 V and increases rapidly with very small further potential difference increase. What current will flow, when there is a 200 Ω resistor in series with the diode with a 10 V battery across the series combination ? **[Ans.** 46·5 mA]
2. A *p-n* junction diode when forward biased has a potential drop of 0·5 V, which is assumed to be independent of current. The current in excess of 10 mA through the diode produces a large Joule heating, which damages (burns) the diode. If we want to use a 1·5 V battery to forward bias the diode, what should be the value of the resistor used in series with the diode so that the maximum current does not exceed 5 mA ? **[Ans.** 200 Ω]
3. Refer to the Zener diode circuit shown in Fig. 2.49. If E = 12 V, V_Z = 9 V, R = 200 Ω and R_L = 1·5 kΩ, calculate the power dissipated in (*a*) resistance R (*b*) load resistance R_L and (*c*) the Zener diode. **[Ans.** (*a*) 0·045 W, (*b*) 0·054 W, (*c*) 0·081 W]
4. In the circuit using a Zener diode voltage stabiliser [Fig. 2.49], what is the smallest value of load resistance for stabilisation to be possible. Given, R = 100 Ω, E = 5 V and Zener diode is of 3 V. **[Ans.** 150 Ω]
5. A full-wave recitifier makes use of two identical junction diodes having negligible forward resistance. What is the r.m.s. value of voltage across R_L, when a sinusoidal volt supply of peak value 100 V is fed to the rectifier. **[Ans.** 70·7

Type B. On Characteristics of a transistor

6. Fig. 2.90 gives the variation of I_b as a function of V_{be} at fi V_{ce} = 20 V. (*a*) Find the voltage corresponding to I_b = 40 and 80 μA ; (*b*) Find the resistance R= V_{be}/I_b at these points and the dynamic resistance. **[Ans.** (*a*) 0·7 V, 0·8 V ; (*b*) 17·5 kΩ, 10 kΩ, 2·5

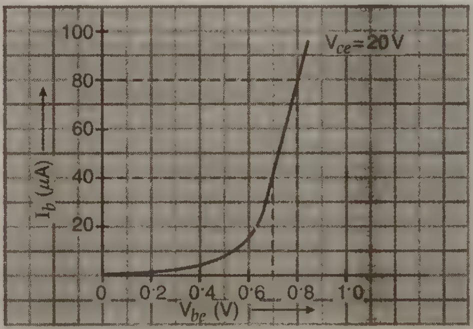


Fig. 2.90.

7. A certain *n-p-n* transistor has the common-emitter output characteristics as shown in Fig. 2.91.
(*a*) Find the emitter current at $V_{ce} = 10$ V and $I_b = 60\ \mu A$.
(*b*) Find α at that point. **[Ans.** (*a*) 6·06 mA ; (*b*) 0·99]

8. A certain transistor has common-emitter input characteristics as shown in Fig. 2.90 and the common emitter output characteristics as shown in Fig. 2.91.

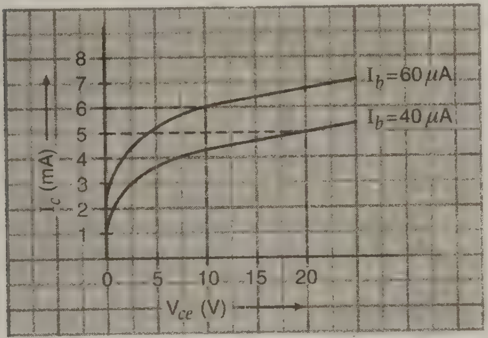


Fig. 2.91.

(*a*) Find I_c, when $V_{be} = 0{\cdot}7$ V and $V_{ce} = 20$ V. (*b*) Find β of the transistor at that point. **[Ans.** (*a*) 5 mA ; (*b*) 125]

Type C. On Transistor as an amplifier

9. For a common base amplifier, emitter current is 5·2 mA and base current is 0·22 mA. Find the current gain of the amplifier. **[Ans.** 0·958]

10. For a transistor working as common base amplifier, current gain is 0·97 and base current is 0·12 mA. Calculate the collector and emitter current. **[Ans.** 3·88 mA, 4 mA]

11. In a common base transistor amplifier, if the input resistance is 200 ohm and load resistance is 20 kiloohm, find (*i*) voltage gain (*ii*) power gain. Given, current gain of common base transistor is 0·95. (*H.P.S.S.C.E. 1993*)
[Ans. (*i*) 95 ; (*ii*) 90·25]

12. In the *n-n-p* transistor circuit shown in Fig. 2.92, what is the potential difference between base and collector ? What is the biasing (forward or reverse) between emitter-base and collector-base junctions ?

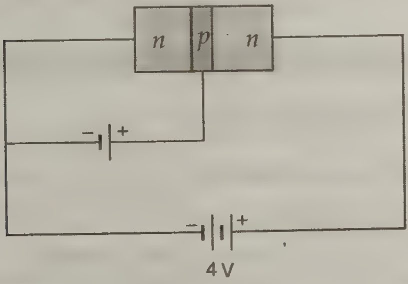


Fig. 2.92.

[Ans. 2 V, 2 V (forward), 2 V (reverse)]

13. Calculate emitter current for which $\beta = 100$ and base current $I_b = 20\ \mu A$. (*H.P.S.S.C.E. 2002*) **[Ans.** 2·02 mA]

14. For a common-emitter amplifier, current gain is 60. If the emitter current is 6·6 mA, calculate the collector and base current. **[Ans.** 6·49 mA, 0·108 mA]

15. In a common-emitter mode of transistor, d.c. current gain is 20 and the emitter current is 7 mA. Calculate (*i*) base current and (*ii*) collector current. (*C.B.S.E. 2002*)
[Ans. (*i*) 0·333 mA, (*ii*) 6·667 mA]

16. For a common-emitter amplifier, current gain is 60. If the emitter current is 7·7 mA, calculate the base current and collector current. Also calculate current gain, when the same transistor is working as common-base amplifier.
(*P.S.S.C.E. 2001*) **[Ans.** 0·126 mA, 7·574 mA, 0·984]

17. The input resistance of a common-emitter amplifier is 2 kΩ and a.c. current gain is 20. If the load resistor used is 5 kΩ, calculate (*i*) the voltage gain of the amplifier and (*ii*) the transconductance of the transistor used. (*C.B.S.E. 1999*)
[Ans. (*i*) 50 ; (*ii*) 0·01 S]

18. A change of 8·0 mA in the emitter current causes a change of 7·9 mA in the collector current. How much change in base current is required to have the same change of 7·9 mA in the collector current ? Find the values of α and β.
[Ans. 0·1 mA, 0·99, 79]

19. In the circuit shown in Fig. 2.93, the base current I_b is 10 μA and the collector current is 5·2 mA. Calculate V_{be} and V_{ce}. Can this transistor circuit be used as an amplifier ?

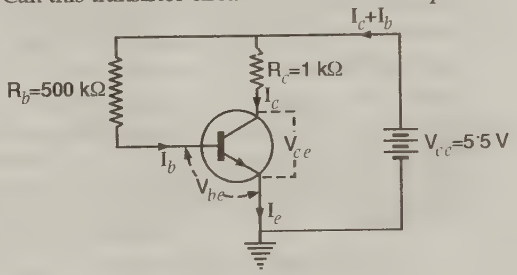
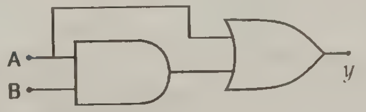


Fig. 2.93.

[Ans. $V_{be} = 0{\cdot}5$ V, $V_{ce} = 0{\cdot}3$ V. It cannot work as an amplifier as both the emitter-base and collector base junctions are forward biased]

Type D. On Logic gate and its Truth table

20. Express by a truth table, the output *y* for all possible inputs A and B in the logic circuit shown in Fig. 2.94.

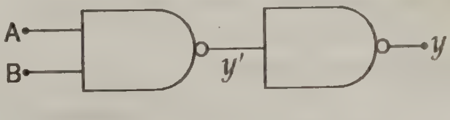


Ans.

A	B	*y*
0	0	0
1	0	1
0	1	0
1	1	1

Fig. 2.94

21. Produce the truth table of the combination of two NAND gates arranged as shown in Fig. 2.95. Hence, show that it behaves as AND gate.

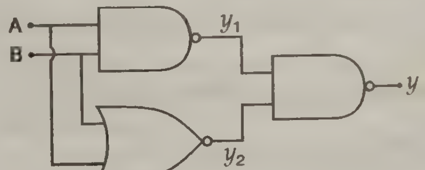


Ans.

A	B	*y*
0	0	0
1	0	0
0	1	0
1	1	1

Fig. 2.95

22. Produce the truth table of the combination of gates shown in Fig. 2.96.

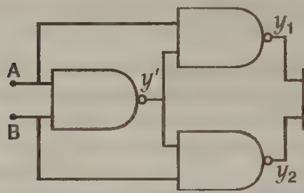


Ans.

A	B	*y*
0	0	1
1	0	0
0	1	0
1	1	1

Fig. 2.96

23. Produce the truth table of the combination of four NAND gates arranged as shown in Fig. 2.97. Hence, show that it behaves as XOR gate.



Ans.

A	B	y
0	0	0
1	0	1
0	1	1
1	1	0

Fig. 2.97

MISCELLANEOUS PROBLEMS

24. A junction diode of negligible forward resistance is used as a half-wave rectifier to rectify a sinusoidal voltage supply 30 V (r.m.s.), 50 Hz. The output is obtained across a load resistance R_L of 10 kΩ. Calculate (*a*) the peak value of output current, (*b*) the reading of d.c. milliammeter connected in series to R_L and (*c*) the reading of the d.c. voltmeter connected across a capacitor of 16 μF connected in parallel to R_L.

[**Ans.** (*a*) 4·24 mA (*b*) 1·35 mA (*c*) $30\sqrt{2}$ V]

25. The circuit shown in Fig. 2.98 uses a Zener diode of 12 V as a voltage regulator to run a 12 V, 6 W tape recorder on 110 V d.c. line. What should be the value of the resistance R, if the d.c. supply varies from 102-115 V ?

(*I.S.C.E. 1996*) [**Ans.** 180 Ω]

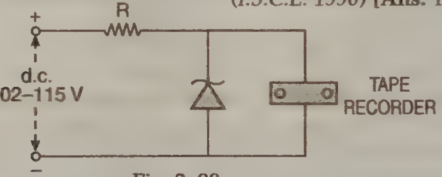


Fig. 2. 98

26. In Fig. 2.99, circuit symbol of logic gate and input wave forms are given. Name the logic gate. Write its truth table and give the output wave form. (*C.B.S.E. 2002*)

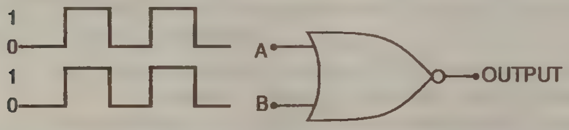
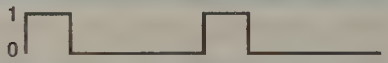


Fig. 2. 99

[**Ans.** NOR gate, truth table as given in Fig. 2.42 (*b*),



27. Identify which basic gate, OR, AND and NOT is represented by the circuits in the dotted line boxes, *a*, *b* and *c* shown in Fig. 2.100.

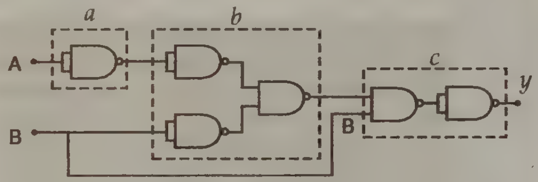


Fig. 2.100

Give the truth table for the entire circuit for all possible values of A and B. [**Ans.** (*a*) NOT ; (*b*) OR ; (*c*) AND]

A	B	y
0	0	0
1	0	0
0	1	1
1	1	1

28. In the circuit shown in Fig. 2.101, if we assume that when the input voltage at the base resistance is 5 V, V_{be} is zero and V_{ce} is also zero, what is I_b, I_c and β.

[**Ans.** $I_b = 25\ \mu$A, $I_c = 3{\cdot}33$ mA, $\beta = 133{\cdot}2$]

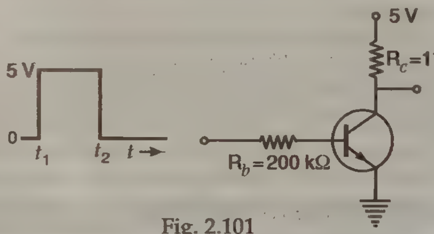


Fig. 2.101

29. The Boolean expression for the logic circuits as given below :

(*a*) $y = A.\overline{B} + A.B$ (*b*) $y = A.B + \overline{A.B}$

Find the output y in each case, if the inputs are A = 0, B = 0 ; A = 1, B = 0 ; A = 0, B = 1 and A = 1, B = 1.

[**Ans.** (*a*) 0, 1, 0, 1 ; (*b*) 1, 1, 1, 1]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. E.M.F. of battery, E = 10 V

Voltage drop across junction diode, $V_d = 0{\cdot}7$ V

Therefore, voltage drop across series resistor,

$$V = E - V_d = 10 - 0{\cdot}7 = 9{\cdot}3 \text{ V}$$

Hence, current in the circuit,

$$I = \frac{V}{R} = \frac{9{\cdot}3}{200} = 4{\cdot}65 \times 10^{-2} \text{ A} = \mathbf{46{\cdot}5\ mA}$$

2. Effective forward bias, V = 1·5 – 0·5 = 1 V

Current desired through *p-n* junction,

$$I = 5 \text{ mA} = 5 \times 10^{-3} \text{ A}$$

Let R be resistance of the resistor used in series with the diode. Then,

$$R = \frac{V}{I} = \frac{1}{5 \times 10^{-3}} = \mathbf{200\ \Omega}$$

3. Refer to Fig. 2.49 :

(*a*) $V = E - V_Z = 12 - 9 = 3$ V

$$\therefore \quad I = \frac{V}{R} = \frac{3}{200} = 0{\cdot}015 \text{ A}$$

Power dissipated in resistance R,

$$P_R = V\,I = 3 \times 0{\cdot}015 = \mathbf{0{\cdot}045\ W}$$

(*b*) Current through R_L, $I_L = \dfrac{9}{1{\cdot}5 \times 10^3} = 0{\cdot}006$ A

Power dissipated in R_L,

$$P_L = V_Z\, I_L = 9 \times 0{\cdot}006 = \mathbf{0{\cdot}054\ W}$$

(*c*) Current through Zener diode,

$$I_Z = I - I_L = 0{\cdot}015 - 0{\cdot}006 = 0{\cdot}009 \text{ A}$$

Power dissipated in Zener diode,

$$P_Z = V_Z\, I_Z = 9 \times 0{\cdot}009 = \mathbf{0{\cdot}081\ W}$$

4. Voltage drop across series resistor,

$$V = E - V_Z = 5 - 3 = 2 \text{ V}$$

Current through series resistor,

$$I = \frac{V}{R} = \frac{2}{100} = 0{\cdot}02 \text{ A}$$

The smallest value of load resistance for which stabilisation occurrs is that which allows a negligible (almost zero) Zener current.

Therefore, current through load resistance,

$$I_L = I = 0{\cdot}02 \text{ A}$$

Hence, smallest value of load resistance,

$$R_L = \frac{V_Z}{I_L} = \frac{3}{0{\cdot}02} = 150\ \Omega$$

5. Here, $E_0 = 100$ V

$$\therefore \quad E_{r.m.s.} = \frac{100}{\sqrt{2}} \text{ V}$$

A full-wave rectifier gives output corresponding to both the half cycles. Therefore, r.m.s. value of voltage for a complete cycle will be same as that over a half cycle. Hence,

$$V_{r.m.s.} = \sqrt{(100/\sqrt{2})^2} = \frac{100}{\sqrt{2}} = \mathbf{70{\cdot}7\ V}$$

6. (*a*) From V_{be} - I_b characteristic at $V_{cc} = 20$ V [Fig. 2.90], corresponding to $I_b = 40\ \mu$A, $V_{be} = \mathbf{0{\cdot}7\ V}$

and corresponding to $I_b = 80\ \mu$A, $V_{be} = \mathbf{0{\cdot}8\ V}$

(*b*) $$R \text{ (at } V_{be} = 0{\cdot}7 \text{ V)} = \frac{0{\cdot}7}{40 \times 10^{-6}} = 1750\ \Omega = \mathbf{17{\cdot}5\ k\Omega}$$

$$R \text{ (at } V_{be} = 0{\cdot}8\text{V)} = \frac{0{\cdot}8}{80 \times 10^{-6}} = 10{,}000\ \Omega = \mathbf{10\ k\Omega}$$

$$\text{Dynamic resistance} = \frac{\Delta V_{be}}{\Delta I_b} = \frac{0{\cdot}8 - 0{\cdot}7}{80 \times 10^{-6} - 40 \times 10^{-6}}$$

$$= \frac{0{\cdot}1}{40 \times 10^{6}} = 2500\ \Omega = \mathbf{2{\cdot}5\ k\Omega}$$

7. (*a*) From V_{be} - I_b characteristic at $V_{ce} = 20$ V [Fig. 2,90], corresponding to $V_{be} = 0{\cdot}7$ V, $I_b = 40\ \mu$A

Now, from V_{ce} - I_c characteristic at $I_b = 40$ mA [Fig. 2.91], corresponding to $V_{ce} = 20$ V, $I_c = \mathbf{5\ mA}$

(*b*) Now, $$\beta = \frac{I_c}{I_b} = \frac{5 \times 10^{-3}}{40 \times 10^{-6}} = \mathbf{125}$$

8. (*a*) From V_{ce} - I_c characteristic at $I_b = 60\ \mu$A [Fig. 2.91], corresponding to $V_{ce} = 10$ V, $I_c = 6$ mA

Now, $I_e = I_c + I_b = 6 \text{ mA} + 60\ \mu\text{A} = \mathbf{6{\cdot}06\ mA}$

(*b*) Also, $$\alpha = \frac{I_c}{I_e} \frac{6}{6{\cdot}06} = \mathbf{0{\cdot}99}$$

9. $$\alpha = \frac{I_c}{I_e} \frac{I_e - I_b}{I_e} = \frac{5{\cdot}2 - 0{\cdot}22}{5{\cdot}2} = \mathbf{0{\cdot}958}$$

10. $$\frac{I_c}{I_e} = 0{\cdot}97 \quad \text{or} \quad I_c = 0{\cdot}97\, I_e$$

or $I_c = 0{\cdot}97\,(I_c + I_b)$ or $0{\cdot}03\, I_c = 0{\cdot}97\, I_b$

$$\text{or} \quad I_c = \frac{0{\cdot}97 \times 0{\cdot}12}{0{\cdot}03} = \mathbf{3{\cdot}88\ mA}$$

Now, $I_e = I_c + I_b = 3{\cdot}88 + 0{\cdot}12 = \mathbf{4{\cdot}0\ mA}$

11. $$\text{Resistance gain} = \frac{R_{out}}{R_{in}} = \frac{20 \times 10^3}{200} = 100$$

(*i*) Voltage gain = current gain × resistance gain
= 0·95 × 100 = **95**

(*ii*) Power gain = voltage gain × current gain
= 95 × 0·95 = **90·25**

12. Here, V_{ce} = **2 V (forward)**

and V_{cc} = **4 V (reverse)**

$\therefore \quad V_{cb} = 4 - 2 = \mathbf{2\ V}$

13. Now, $$\beta = \frac{I_c}{I_b} = \frac{I_e - I_b}{I_b} \quad \text{or} \quad 100 = \frac{I_e - 20}{20}$$

or $I_e = 2020\ \mu\text{A} = \mathbf{2{\cdot}02\ mA}$

14. Here, $$\frac{I_c}{I_b} = 60 \quad \text{or} \quad I_b = \frac{I_c}{60}$$

Also, $I_e = I_c + I_b$ or $6{\cdot}6 = I_c + I_c/60$

or $I_c = \mathbf{6{\cdot}49\ mA}$

$$\therefore \quad I_b = \frac{6{\cdot}49}{60} = \mathbf{0{\cdot}108\ mA}$$

15. Proceed as in problem no. 14.

16. Now, $$\beta = \frac{I_c}{I_b} = \frac{I_e - I_b}{I_b} \quad \text{or} \quad 60 = \frac{7{\cdot}7 \times I_b}{I_b}$$

or $I_b = \mathbf{0{\cdot}126\ mA}$

Now, $I_c = I_e - I_b = 7{\cdot}7 - 0{\cdot}126 = \mathbf{7{\cdot}574\ mA}$

Also, $$\beta = \frac{\alpha}{1-\alpha} \quad \text{or} \quad 60 = \frac{\alpha}{1-\alpha}$$

or $\alpha = \mathbf{0{\cdot}984}$

17. $$\text{Resistance gain} = \frac{R_{out}}{R_{in}} = \frac{5}{2} = 2{\cdot}5$$

(*i*) Voltage gain = current gain × resistance gain
= 20 × 2·5 = **50**

(*ii*) Transconductance, g_m = current gain/R_{input}
= 20/2 × 10^3 = **0·01 S**

18. From the relation : $I_e = I_c + I_b$, we have

$$\Delta I_e = \Delta I_c + \Delta I_b$$

When $\Delta I_e = 8{\cdot}0$ mA, $\Delta I_c = 7{\cdot}9$ mA

$$\therefore \quad \Delta I_b = \Delta I_e - \Delta I_c = 8{\cdot}0 - 7{\cdot}9 = \mathbf{0{\cdot}1\ mA}$$

Also, $$\alpha = \frac{\Delta I_c}{\Delta I_e} = \frac{7{\cdot}9}{8{\cdot}0} = \mathbf{0{\cdot}99}$$

Further, $$\beta = \frac{\Delta I_c}{\Delta I_e} = \frac{7{\cdot}9}{0{\cdot}1} = \mathbf{79}$$

19. Proceed as in solved problem no. 2.11.

20. Proceed as in SAQ 2.58.

21. The truth table of the combination of gates shown in Fig. 2.95 is produced as below :

A	B	y'	y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

≡

A	B	y
0	0	0
1	0	0
0	1	0
1	1	1

Since it is the truth table of AND gate, the given combination of gates represents **AND gate.**

22. The truth table of the combination of gates shown in Fig. 2.96 is produced as below :

A	B	y_1	y_2	y
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0
1	1	0	1	1

≡

A	B	y
0	0	1
1	0	0
0	1	0
1	1	1

23. The truth table of combination of gates shown in Fig. 2.97 is produced as below :

A	B	y'	y_1	y_2	y
0	0	1	1	1	0
1	0	1	0	1	1
0	1	1	1	0	1
1	1	0	1	1	0

≡

A	B	y
0	0	0
1	0	1
0	1	1
1	1	0

Since it is the truth table of XOR gate, the given combination of gates represents **XOR gate.**

24. Here, $E_{r.m.s.} = 30$ V

Therefore, peak value of supply voltage,

$$E_0 = E_{r.m.s.} \times \sqrt{2} = 30\sqrt{2} \text{ V}$$

(*a*) Since the forward resistance of diode is negligible, peak value of current through R_L,

$$I_0 = \frac{E_0}{R_L} = \frac{30\sqrt{2}}{10 \times 10^3} = 3\sqrt{2} \times 10^{-3}\ \text{A} = \mathbf{4{\cdot}24\ mA}$$

(*b*) The d.c. milliammeter connected in series to load resistance R_L measures the average value of current over the complete cycle. Therefore,

$$\text{reading of d.c. milliammeter} = \frac{(2I_0/\pi)+0}{2} = \frac{I_0}{\pi}$$

$$= \frac{4{\cdot}24}{\pi} = \mathbf{1{\cdot}35\ mA}$$

(*c*) When a capacitor is connected across R_L, the capacitor charges to peak supply and when diode is not conducting, it discharges through R_L.

Now, time period of supply voltage, $T = \frac{1}{f} = \frac{1}{50} = 0{\cdot}02$ s

Time constant of CR-circuit,

$$CR = 16 \times 10^{-6} \times 10 \times 10^3 = 0{\cdot}16\ \text{s}$$

Since CR >> T, the output voltage across capacitor will practically remain equal to the peak value of supply voltage and hence output will be as shown in Fig. 2.102 given below.

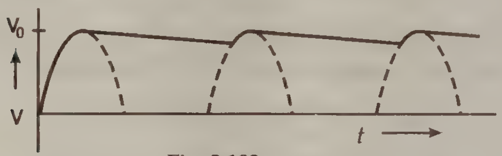


Fig. 2.102

Hence, reading of d.c. voltmeter connected across C

$$= \mathbf{30\sqrt{2}\ V}$$

25. Operating voltage of the tape recorder = 12 V
Power rating of the tape recorder = 6 W
Therefore, current drawn by the tape recorder (load resistance),

$$I_L = \frac{6}{12} = 0{\cdot}5\ \text{A}$$

Now, Zener diode should provide stabilised supply to the tape recorder, when supply voltage varies between 102–115 V. For this, the value of R should be chosen so that a negligible current flows through Zener diode, when supply voltage is minimum *i.e.* 102 V.

Now, $R = \frac{E - V_Z}{I}$

When E = 102 V, $I_Z = 0$ or $I = I_L$

$$\therefore \quad R = \frac{102 - 12}{0{\cdot}5} = \mathbf{180\ \Omega}$$

It may be pointed out that when supply voltage rises to 115 V, the current through series resistor R and the Zener diode increases. The increase in potential drop across R leaves the supply voltage of 12 V to the tape recorder unchanged.

26. Logic gate shown in Fig. 2.99 is **NOR gate**. The truth table of the logic gate will be as given in Fig. 2.42 (*b*).

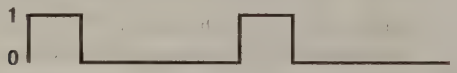


Fig. 2.103

The output wave form will be as shown in the Fig. 2.103

27. ***In dotted line box a*** **:** Refer to section 2.26. When the tw inputs of NAND gate are joined, it acts as **NOT gate.**

In dotted line box b **:** Refer to section 2.26, when output of two NOT gates are fed to a NAND gate, th arrangement acts as **OR gate.**

In dotted line box c **:** Refer to section 2.26, when the input of NAND gate are fed to a NOT gate, the arrangement act as **AND gate.**

Let *y′* be output of logic gate in dotted line box *a* and *y′′* be output of logic gate in box *b*.

The truth table of the whole arrangement will be as give below :

A	B	*y′*	*y′′*	*y*
0	0	1	1	0
1	0	0	0	0
0	1	1	1	1
1	1	0	1	1

≡

A	B	*y*
0	0	0
1	0	0
0	1	1
1	1	1

28. Refer to solved problem no. 2.11 :
When input voltage at the base resistance is 5 V,

$$V_{be} = V_{ce} = 0$$

Also, $\quad V_{cc} = 5\ \text{V}$

Applying Kirchhoff's second law to the closed part of th circuit containing R_c and collector-emitter junction, w have

$$V_{ce} = V_{cc} - I_c R_c$$

$$\therefore \quad 0 = 5 - I_c \times 1{\cdot}5 \times 10^3$$

$$\text{or} \quad I_c = 3{\cdot}33 \times 10^{-3}\ \text{A} = \mathbf{3{\cdot}33\ mA}$$

Applying Kirchhoff's second law to the closed part of th circuit containing R_b and emitter-base junction, we have

$$V_{cc} = V_{be} + I_b R_b$$

$$\therefore \quad 5 = 0 + I_b \times 200 \times 10^3$$

$$\text{or} \quad I_b = 25 \times 10^{-6}\ \text{A} = \mathbf{25\ \mu A}$$

$$\text{Now,} \quad \beta = \frac{I_c}{I_b} = \frac{3{\cdot}33 \times 10^{-3}}{25 \times 10^{-6}} = \mathbf{133{\cdot}2}$$

29. (*a*) Here, $y = A.\overline{B} + A.B$

For A = 0, B = 0 :

$$y = 0.\overline{0} + 0.0 = 0.1 + 0.0 = 0 + 0 = \mathbf{0}$$

For A = 1, B = 0 :

$$y = 1.\overline{0} + 1.0 = 1.1 + 1.0 = 1 + 0 = \mathbf{1}$$

For A = 0, B = 1 :

$$y = 0.\overline{1} + 0.1 = 0.0 + 0.1 = 0 + 0 = \mathbf{0}$$

For A = 1, B = 1 :

$$y = 1.\overline{1} + 1.1 = 1.0 + 1.1 = 0 + 1 = \mathbf{1}$$

(*b*) Here, $y = A.B + \overline{A.B}$

For A = 0, B = 0 :

$$y = 0.0 + \overline{0.0} = 0.0 + \overline{0} = 0 + 1 = \mathbf{1}$$

For A = 1, B = 0 :

$$y = 1.0 + \overline{1.0} = 1.0 + \overline{0} = 0 + 1 = \mathbf{1}$$

For A = 0, B = 1 :

$$y = 0.1 + \overline{0.1} = 0.1 + \overline{0} = 0 + 1 = \mathbf{1}$$

For A = 1, B = 1 :

$$y = 1.1 + \overline{1.1} = 1.1 + \overline{1} = 1 + 0 = \mathbf{1}$$

Competitive Examination File (Unit IX)

REVISION AT A GLANCE

Chapter 1. Solids

☐ **Crystalline materials.** *The materials, in which atoms (or molecules) are arranged in a definite and regular way throughout he body of the crystal.*

☐ **Amorphous materials.** *The materials, in which atoms (or molecules) are arranged in an irregular manner.*

☐ **Crystal lattice.** *It is an infinite array of atoms (or molecules) in space, such that at every point, an atom or a molecule has got he identical surroundings.*

☐ **Unit cell.** *It is the smallest block, whose repetition in three dimensions builds up the whole crystal.*

The three sides a, b and c of the unit cell are called ***primitives*** or ***lattice constants.***

The set of three lines drawn parallel to the lines of intersection of the faces of the unit cell are called ***crystallographic axis.***

The angles α, β and γ between the three crystallographic axes are called ***interfacial angles.***

☐ **Single crystal.** *The crystal in which the ordered arrangement of atoms (or molecules) extends throughout the piece of the crystal.*

The physical properties, such as mechanical, thermal, electrical and optical of a single crystal are different in different directions. In other words, a single crystal is ***anisotropic.***

The small sized single crystal is called ***monocrystal.***

☐ **Polycrystal.** *The crystalline solid, which is made of a large number of monocrystals with well developed faces joined together.* Polycrystals are ***isotropic.***

☐ **Liquid crystal.** *The organic crystalline solid, which on heating becomes fluid like but retains its anisotropic property.*

Liquid crystals change the plane of polarisation of light and are used in ***liquid crystal displays (L.C.D.).***

☐ **Energy bands in solids.** Due to interaction between closed packed atoms in solids, the splitting of energy levels take place and it gives rise to formation of energy bands. The energy band formed by a series of levels containing valence electrons is called ***valence band*** and the lowest unfilled energy band formed just above the valence band is called ***covalent band.***

In some solids, there is a energy gap between valence and conduction band. This energy gap is called ***forbidden energy gap.***

The behaviour of conductors, semiconductors and insulators can be explained on the basis of energy bands.

☐ **Conductors.** The conduction and valence bands partly overlap each other in case of conductors. In other words, there is no forbidden energy gap in conductors.

☐ **Semiconductors.** The conduction and valence bands are separated by the small width ($\approx$ 1 eV) of forbidden energy gap. The valence band is completely filled, while the conduction band is empty. The electrons cross from valence band to conduction band even when a small amount of energy is supplied.

☐ **Insulators.** The width of forbidden energy gap between valence and conduction band is quite large ($\approx$ 10 eV). Ordinarily, electrons cannot jump from valence to conduction band even on applying a strong electric field.

☐ **Intrinsic semiconductors.** A semiconductor free from all types of impurities is called ***intrinsic semiconductor.*** At 0 K, a semiconductor is an insulator *i.e.* it possesses zero conductivity. When temperature is increased, a few covalent bonds break up and release the electrons. These electrons move to conduction band leaving behind equal number of holes in valence band. The conductivity of an intrinsic semiconductor is due to both electrons and holes.

☐ **Doping.** *The process of adding impurity atoms (pentavalent or trivalent) to a pure semiconductor, so as to increase its conductivity in a controlled manner is called doping.* The impurity atoms added are very small ($\approx$ 1 in 10^6 semiconductor atoms). The pentavalent impurity atoms are called ***donor atoms,*** while trivalent impurity atoms are called ***acceptor atoms.***

☐ **Extrinsic semiconductor.** A semiconductor doped with a suitable impurity, so as to possess conductivity much higher than the semiconductor in pure form is called extrinsic semiconductor.

***n*-type semiconductor.** When a pentavalent impurity, such as arsenic or antimony or phosphorus is added to a pure semiconductor, the number of free electrons become more than the holes in the semiconductor and such an extrinsic semiconductor is called n-type semiconductor. In other words, in a n-type semiconductor, electrons are majority carriers and holes are minority carriers.

***p*-type semiconductor.** When a trivalent impurity, such as indium or galium or boron is added to a pure semiconductor, the semiconductor becomes deficient in electrons *i.e.* number of holes become more than the number of electrons. Such a semiconductor is called p-type semiconductor. It has holes as majority carriers and electrons as minority carriers.

☐ **Electrical conductivity of a semiconductor.** The conductivity of a semiconductor is determined by the mobility (μ) of both electrons and holes and their concentration.

Mathematically : $\sigma = e\,(n_e \mu_e + n_h \mu_h)$

Here, n_e and n_h represent number density, while μ_e and μ_h represent mobility of electrons and holes respectively.

Some Useful Facts :

1. Valence band is formed of the outermost energy levels having electrons, while conduction band is formed of lowest unfilled energy levels.
2. The highest energy level, which an electron can occupy in the valence band at 0 K is called ***Fermi level.***
3. Both *n*-type and *p*-type semiconductors are electrically neutral.
4. In semiconductors, electron mobility is greater than hole mobility.
5. The conductivity of a semiconductor increases with temperature.

Chapter 2. Semiconductor devices

☐ ***p-n* junction.** *The device obtained by growing a p-type semiconductor over a n-type semiconductor or vice-versa is called a p-n junction.* It conducts in one direction only. It is also called a ***junction diode.***

☐ **Depletion layer.** *It is a thin layer formed between the p and n-sections and devoid of holes and electrons.* Its width is about 10^{-6} m. A potential difference of about 0·7 V is produced across the junction, which gives rise to a very high electric field ($\approx 10^6$ V m^{-1}).

☐ **Forward biasing.** The *p-n* junction is said to be forward biased, when positive terminal of the external battery in the circuit is connected to *p*-section and negative terminal to *n*-section of the junction diode.

Flow of majority carriers across the junction from both the sections of the junction diode is responsible for the ***forward current.***

☐ **Reverse biasing.** The *p-n* junction is said to be reverse biased, when positive terminal of the external battery in the circuit is connected to *n*-section and negative terminal to *p*-section of the junction diode.

The flow of minority carriers across the junction from both the sections of the junction diode is responsible for the ***reverse current.***

☐ **Transistor.** *A three section semiconductor device formed by growing a thin layer of p-type semiconductor between two comparatively thick layers of n-type semiconductor and vice-versa.*

The thin layer at the centre is lightly doped and is called ***base (B).*** The other two layers serve as ***emitter (E)*** and ***collector (C).*** When the emitter is forward biased, the majority carriers move across the base so as to reach the collector biased in reverse manner. A small recombination of holes and electrons takes place in the base region.

If I_e, I_b and I_c are emitter current, base current and collector current respectively, then

$$I_e = I_b + I_c$$

☐ **Current gain.** (*a*) ***In common base configuration :***

d.c. current gain, $$\alpha = \frac{I_c}{I_e}$$

and a.c. current gain, $$\alpha_{a.c.} = \frac{\Delta I_c}{\Delta I_e}$$

(*b*) ***In common emitter configuration :***

d.c. current gain, $$\beta = \frac{I_c}{I_b}$$

and a.c. current gain, $$\beta_{a.c.} = \frac{\Delta I_c}{\Delta I_b}$$

The values of α and β are related to each other as :

$$\beta = \frac{\alpha}{1-\alpha}$$

☐ **Voltage gain.** *It is defined as the ratio of output voltage to the input voltage.*

☐ **Resistance gain.** *It is defined as the ratio of resistance in output circuit to the resistance in input circuit.* It can be proved that

voltage gain = current gain × resistance gain

☐ **Power gain.** *It is defined as the ratio of output power to input power.* It can be proved that

power gain = current gain × voltage gain = (current gain)^2 × resistance gain

☐ **Digital signal.** A signal which has only two levels of voltage. A digital signal does not vary continuously with time as in case of a analogue signal. The two levels of a digital signal are represented as 0 and 1.

☐ **Logic gate.** *A digital circuit, which works according to some logical relationship between input and output voltages is called a logic gate.*

☐ **Truth table.** *It is a table, which shows all possible input combinations and the corresponding output combination for a logic gate.* Truth table is also called ***table of combinations.***

☐ **Boolean algebra.** The algebra based on the binary nature of logic (either true or false) is called Boolean algebra.

In Boolean algebra :

(*a*) The addition sign (+) implies OR. The Boolean expression,

$y = A + B$ implies y equals A OR B.

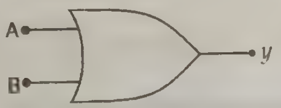
(*b*) The multiplication sign (.) implies AND. The Boolean expression,

$y = A.B$ implies y equals A AND B.

(*c*) The bar sign (–) implies NOT. The Boolean expression,

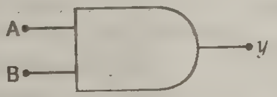
$y = \overline{A}$ implies y equals NOT A

☐ **The OR gate.** It is a digital circuit having two inputs and one output.
Its Boolean expression is $y = A + B$. The symbol and truth table of **OR** gate are as given below :



A	B	y
0	0	0
1	0	1
0	1	1
1	1	1

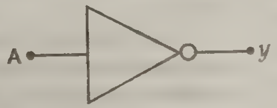
☐ **The AND gate.** It is also a digital circuit having two inputs and one output.
Its Boolean expression is $y = A.B$. The symbol and truth table of AND gate are as given below :



A	B	y
0	0	0
1	0	0
0	1	0
1	1	1

☐ **The NOT gate.** It is a digital circuit having one input and one output.

Its Boolean expression is $y = \overline{A}$. The symbol and truth table of NOT gate are as given below :



A	y
0	1
1	0

Some Useful Facts :

1. A potential difference of about 0·7 V is produced across the *p*-*n* junction due to diffusion of holes and electrons across it. It results in a very strong electric field of the order of 10^6 V m^{-1}.

2. During forward bias, the resistance of the junction diode is low and during reverse bias, the junction diode possesses very high resistance.

3. When reverse bias is made sufficiently high, the reverse current increases abruptly due to breaking of covalent bonds near the junction. This phenomenon is called ***avalanche breakdown*** and is used in Zener diodes.

4. In a transistor, emitter is always forward biased and collector is always reverse biased.

5. The logic gates are used to perform switching action *i.e.* either on (or yes) or off (or no).

NUMERICAL PROBLEMS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Solids

Problem 1.01. A doped semiconductor has impurity levels 40 meV (millielectron volt) below the conduction band. (*a*) Is the material *n*-type or *p*-type ? (*b*) In a thermal collision, an amount of energy *k* T is given to the extra electron loosely bound to the impurity ion and this electron is just able to jump into the conduction band. Calculate the temperature T.

Given, Boltzmann's constant, $k = 8{\cdot}62 \times 10^{-5}$ eV K^{-1}.

Sol. (*a*) In a semiconductor, the forbidden energy gap between valence and conduction bands is of the order of 1 eV. Since in the doped semiconductor, energy gap is much smaller (40 meV), the impurity levels in the doped semiconductor corresponds to donor energy level. Hence, the doped semiconductor is of ***n*-type**.

(*b*) Here, $kT = 40$ meV $= 40 \times 10^{-3}$ eV

$$\therefore T = \frac{40 \times 10^{-3}}{k} = \frac{40 \times 10^{-3}}{8{\cdot}62 \times 10^{-5}} = \mathbf{464\ K}$$

Problem 1.02. A *p*-type semiconductor has acceptor energy levels 54 meV (millielectron volt) above the valence band. Find the maximum wavelength of the radiation, which can create a hole ?

Sol. Energy required to create a hole

= energy required to transfer an electron from valence to conduction band

$= 54$ meV $= 54 \times 10^{-3}$ eV $= 54 \times 10^{-3} \times 1{\cdot}6 \times 10^{-19}$ J

If λ is the wavelength of the radiation, which can create a hole, then

$$\frac{hc}{\lambda} = 54 \times 10^{-3} \times 1{\cdot}6 \times 10^{-19}$$

$$\text{or } \lambda = \frac{hc}{54 \times 10^{-3} \times 1{\cdot}6 \times 10^{-19}}$$

$$\text{or } \lambda = \frac{6{\cdot}62 \times 10^{-34} \times 3 \times 10^{8}}{54 \times 10^{-3} \times 1{\cdot}6 \times 10^{-19}} = \mathbf{2{\cdot}3 \times 10^{-5}\ m}$$

Problem 1.03. Energy of the photon of sodium light of wavelength 5890 Å equals the energy gap between valence and conduction band of a semiconductor.

(*a*) Find the minimum energy E required to create a hole-electron pair.

(*b*) Find the value of E/*k* T at a temperature of 400 K. Given, Boltzmann's constant, $k = 1{\cdot}38 \times 10^{-23}$ J K^{-1}.

Sol. (*a*) The minimum energy required to create a hole-electron pair,

E = energy gap between valence and conduction band
= energy of sodium light of wavelength 5890 Å

$$= \frac{hc}{\lambda} = \frac{6{\cdot}62 \times 10^{-34} \times 3 \times 10^{8}}{5890 \times 10^{-10}} = 3{\cdot}372 \times 10^{-19}\ \text{J}$$

$$= \frac{3{\cdot}372 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = \mathbf{2{\cdot}11\ eV}$$

$$(b)\ \text{Now, } \frac{E}{kT} = \frac{3{\cdot}372 \times 10^{-19}}{1{\cdot}38 \times 10^{-23} \times 400} = \mathbf{61{\cdot}1}$$

Chapter 2. Semiconductor devices

Problem 2.01. A potential barrier of 0·6 V exists across a *p-n* junction. (*a*) If the depletion region is 6 × 10^{-7} m wide, what is the intensity of the electric field in this region ? (*b*) An electron with speed 5 × 10^5 m s^{-1} approaches the *p-n* junction from the *n*-region. With what speed will it enter the *p*-region ?

Sol. (*a*) Here, barrier voltage, V = 0·6 V

and width of depletion region, $d = 6 \times 10^{-7}$ m

Therefore, electric field in the depletion region,

$$E = \frac{V}{d} = \frac{0{\cdot}6}{6 \times 10^{-7}} = \mathbf{10^{8}\ V\ m^{-1}}$$

(*b*) Let *u* be the velocity of electron, with which it enters the depletion region and *v* be the velocity, with which it leaves the depletion region. The barrier voltage across the depletion region acts on the electon, so as to decrease its energy, say by an amount V *e*. Therefore,

$$\frac{1}{2} m u^2 - V e = \frac{1}{2} m v^2 \quad \text{or} \quad v^2 = u^2 - \frac{2 V e}{m}$$

$$\text{or } v^2 = (5 \times 10^5)^2 - \frac{2 \times 0{\cdot}6 \times 1{\cdot}6 \times 10^{-19}}{9{\cdot}1 \times 10^{-31}}$$

$$= 2{\cdot}5 \times 10^{11} - 2{\cdot}1 \times 10^{11} = 0{\cdot}4 \times 10^{11}$$

$$\text{or } v = \mathbf{2 \times 10^{5}\ m\ s^{-1}}$$

Problem 2.02. What is the input voltage needed to maintain 15 V across the load resistance R_L of 2 kΩ, assuming that the series resistance R is 200 Ω and the Zener diode requires a minimum current of 10 mA to work satisfactorily [Fig. 2.01]. What is the Zener rating requried ?

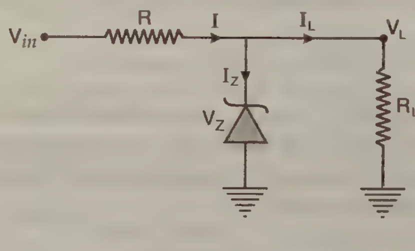


Fig. 2.01

Sol. Here, R = 200 Ω, R_L = 2 kΩ = 2 × 10^3 Ω,

I_Z = 10 mA = 10 × 10^{-3} A and V_L = 15 V

Let I be current through the series resistance. Current through the load resistance is given by

$$I_L = \frac{V_L}{R_L} = \frac{15}{2 \times 10^3} = 7{\cdot}5 \times 10^{-3}\ \text{A} = 7{\cdot}5\ \text{mA}$$

If I_Z is current through Zener diode, then current through series resistance,

$$I = I_Z + I_L = 10 + 7{\cdot}5 = 17{\cdot}5\ \text{mA}$$

Now, V_{in} = potential drop across R + V_L

$= I \times R + V_L = 17{\cdot}5 \times 10^{-3} \times 200 + 15$

$= 3{\cdot}5 + 15 =$ **18·5 V**

Since V_Z is always equal to V_L, break down voltage of Zener diode should be **15 V** and its current rating should be **17·5 mA.**

Problem 2.03. In a *p-n-p* transistor circuit, the collector current is 10 mA. If 90% of the holes reach the collector, find emitter and base currents. (*I.I.T. 1992*)

Sol. Since 90% of the holes reach the collector,

$$I_c = \frac{90}{100} \times I_e = 0{\cdot}9\, I_e$$

$$\text{or} \quad I_e = \frac{I_c}{0{\cdot}9} = \frac{10}{0{\cdot}9} = \mathbf{11{\cdot}1\ mA}$$

Also, $I_b = I_e - I_c = 11{\cdot}1 - 10 =$ **1·1 mA**

Problem 2.04. A *n-p-n* transistor is connected in common emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800 Ω connected in the collector circuit is 0·8 V. If current amplification factor α is 25/26, determine collector-emitter voltage and base current. If the input resistance of the transistor is 200 Ω, calculate the voltage gain and power gain. (*Roorkee, 1994*)

Sol. Fig. 2.02 shows an *n-p-n* transistor in common-emitter configuration.

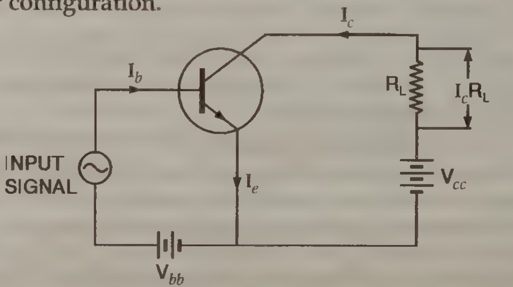


Fig. 2.02.

Here, collector supply, $V_{cc} = 8\text{ V}$;
load resistance, $R_L = 800\ \Omega$
and voltage drop across load resistance, $I_c R_L = 0{\cdot}8\text{ V}$
Now, collector-emitter voltage is given by

$$V_{ce} = V_{cc} - I_c R_L = 8{\cdot}0 - 0{\cdot}8 = \mathbf{7{\cdot}2\ V}$$

Now, $I_c R_L = 0{\cdot}8\text{ V}$

$$\therefore \quad I_c = \frac{0\cdot8}{R_L} = \frac{0\cdot8}{800} = 10^{-3}\text{ A} = 1\text{ mA}$$

Now, current amplification factor,

$$\alpha = \frac{I_c}{I_e} = \frac{25}{26},$$

$$\therefore \quad I_e = \frac{26}{25} \times I_c = \frac{26}{25} \times 1 = 1\cdot04\text{ mA}$$

Also, base current,

$$I_b = I_e - I_c = 1{\cdot}04 - 1 = \mathbf{0{\cdot}04\ mA}$$

The current gain of the transistor in common-emitter nfiguration,

$$\beta = \frac{\alpha}{1-\alpha} = \frac{25/26}{1-25/26} = 25$$

Now, input voltage, $= I_b \times$ input resistance
$= 0{\cdot}04 \times 10^{-3} \times 200 = 8 \times 10^{-3}\text{ V}$

Since output voltage $= I_c R_L = 0{\cdot}8\text{ V}$,

$$\text{voltage gain} = \frac{\text{output voltage}}{\text{input voltage}} = \frac{0\cdot8}{8\times10^{-3}} = \mathbf{100}$$

Also, power gain $= \beta \times$ voltage gain
$= 25 \times 100 = \mathbf{2500}$

Problem 2.05. An amplifier is represented by the ircuit shown in Fig. 2.03. Here, R_{in} is the input resistance of the amplifier and V_{in}, the voltage appearing across it. This voltage is amplified by a factor of A_v and appears across the load as voltage V_o. An external voltage V_s is applied at the input terminals of the amplifier via series resistor R_s. What will be the apparent gain $A_v' = \frac{V_o}{V_s}$ of the amplifier ? Express the gain in terms of A_v, R_s and R_{in}.

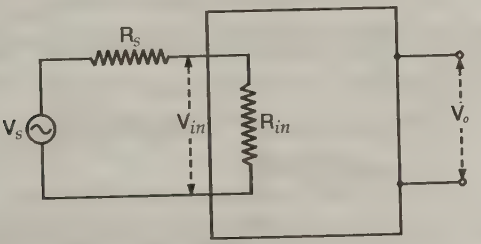


Fig. 2.03

Sol. Let I_{in} be the input current. Then,

$$I_{in} = \frac{V_s}{R_s + R_{in}}$$

$$\therefore \ V_{in} = I_{in} \times R_{in} = \frac{V_s}{R_s + R_{in}} \times R_{in}$$

Since input voltage is amplified by a factor A_v, the voltage appearing across load,

$$V_o = A_v \times V_{in} = A_v \times \frac{V_s R_{in}}{R_s + R_{in}}$$

Hence, apparent voltage gain,

$$A_v' = \frac{V_o}{V_s} = A_v \times \frac{V_s R_{in}}{R_s + R_{in}} \Big/ V_s = \mathbf{A_v \times \frac{R_{in}}{R_s + R_{in}}}$$

Problem 2.06. The Boolean expression for a logic circuit is given by

$$y = \overline{(\overline{A} + \overline{B})}\,.\,A\,.\,B$$

(*a*) Find the output *y*, if the inputs are A = 0, B = 0 ; A = 1, B = 0 ; A = 0, B = 1 and A = 1, B = 1.

(*b*) Identify, which basic gate is represented by the given Boolean expression.

Sol. (*a*) Here, $y = \overline{(\overline{A} + \overline{B})}\,.\,A\,.\,B$

For A = 0, B = 0 :

$$y = \overline{(\overline{0} + \overline{0})}\,.\,0\,.\,0 = \overline{(1+1)}\,.\,0\,.\,0 = \overline{1}\,.\,0\,.\,0 = 0\,.\,0\,.\,0 = \mathbf{0}$$

For A = 1, B = 0 :

$$y = \overline{(\overline{1} + \overline{0})}\,.\,1\,.\,0 = \overline{(0+1)}\,.\,1\,.\,0 = \overline{1}\,.\,0\,.\,0 = 0\,.\,1\,.\,0 = 0$$

For A = 0, B = 1 :

$$y = \overline{(\overline{0} + \overline{1})}\,.\,0\,.\,1 = \overline{(1+0)}\,.\,0\,.\,1 = \overline{1}\,.\,0\,.\,1 = 0\,.\,0\,.\,1 = \mathbf{0}$$

For A = 1, B = 1 :

$$y = \overline{(\overline{1} + \overline{1})}\,.\,1\,.\,1 = \overline{(0+0)}\,.\,1\,.\,1 = \overline{0}\,.\,1\,.\,1 = 1\,.\,1\,.\,1 = \mathbf{1}$$

(*b*) The truth table of the logic circuit represented by the given Boolean expression is

A	B	*y*
0	0	0
1	0	0
0	1	0
1	1	1

It is the truth table of AND gate.

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Solids

PART I. TEXT BASED QUESTIONS

1. **Which of the following is not a property of a crystalline substance ?**
(A) Sharp melting point.
(B) Bounded by flat surfaces.
(C) Isotropic.
(D) Long range order.

2. **Which property makes the crystalline solids to have sharp melting points ?**
(A) Equal strength of all the interatomic bonds.
(B) Anisotropic.
(C) Long range order.
(D) none of the above.

3. **Polycrystals**
(A) are in fact amorphous in nature.
(B) have orderly arrangement throughout their bodies.
(C) donot have orderly arrangement throughout their bodies.
(D) none of the above.

4. **If there are 4 atoms per unit cell in a solid, the crystal is**
(A) tetragonal (B) simple cubic
(C) body-centred cubic (D) face-centred cubic
(*C.E.T. 1999*)

5. **Co-ordination number means**
(A) the co-ordinates of an atom in crystal lattice.
(B) atoms per unit cell.
(C) atoms per unit volume.
(D) closest neighbouring atoms to a particular atom.

6. **For an FCC unit cell, the ratio of the radius of atoms to lattice parameter is**
(A) $1:2$ (B) $1:2\sqrt{2}$
(C) $2:1$ (D) $1:4$ (*C.E.T. 1998*)

7. **The number of atoms per unit cell in BCC lattice is**
(A) 1 (B) 2
(C) 4 (D) 9 (*C.B.S.E. 2002*)

8. **In BCC structure of lattice constant a, the minimum distance between atoms is :**
(A) $\sqrt{3}\,a/2$ (B) $\sqrt{2}\,a$
(C) $a/\sqrt{2}$ (D) $a/2$ (*C.B.S.E. 2001*)

9. **Sodium has body centred packing. If the distance between two nearest atoms is 3·7 Å , then lattice parameter is**
(A) 4·8 Å (B) 4·3 Å
(C) 3·9 Å (D) 3·3 Å (*C.B.S.E. 1999*)

10. **The type of bonds in a silicon crystal is**
(A) ionic (B) metallic
(C) covalent (D) vander Waals
(*Karnataka Entrance, 1991*)

11. **Which of the following is not a semiconductor ?**
(A) germanium (B) silicon
(C) arsenic (D) all of these. (*A.F.M.C. 1997*)

12. **In a semiconductor, the forbidden energy gap between the valence band and the conduction band is of the order of**
(A) 1 MeV (B) 0·1 MeV
(C) 1 eV (D) 5 eV (*C.B.S.E. 1988*)

13. **The main difference between conductors, semiconductors and insulators is because of**
(A) work function.
(B) binding energy of electrons.
(C) width of forbidden energy gap.
(D) mobality of electrons.

14. **In an insulator, the forbidden energy gap between th valence band and conduction band is of the order of**
(A) 5 eV (B) 1 eV
(C) 2 MeV (D) 10^{-3} eV (*C.E.T. 1999*

15. **When the conductivity of a semiconductor is only due t breaking of the covalent bands, the semiconductor is calle**
(A) donor. (B) acceptor.
(C) intrinsic. (D) extrinsic.

16. **If N is Avagadro number, M is atomic weight and ρ, th density, the number of free electrons per unit volume of monovalent metallic crystal can be expressed as**
(A) $\rho\,N/M$ (B) $N/\rho\,M$
(C) $M\,N/\rho$ (D) $\rho\,M/N$ (*C.B.S.E. 1990*

17. **A small impurity is added to germanium to get *p*-typ semiconductor. This impurity is a**
(A) trivalent substance.
(B) pentavalent substance.
(C) bivalent substance.
(D) monovalent substance. (*A.I.I.M.S. 1992*

18. **To obtain a *p*-type germanium semiconductor, it must b doped with**
(A) arsenic (B) antimony
(C) indium (D) phosphorus.
(*C.B.S.E. 1997*

19. **The impurity atoms with which pure silicon should b doped to make a *p*-type semiconductor are those of**
(A) phosphorus (B) boron
(C) antimony (D) aluminium
(*C.E.T. 1998 ; I.I.T. 1988*

20. **When arsenic is added as an impurity to silicon, th resulting material is**
(A) *n*-type semiconductor.
(B) *p*-type semiconductor.
(C) *n*-type conductor. (D) none of these.
(*C.B.S.E. 1996*

21. **When Ge crystal is doped with phosphorus atoms, then i becomes**
(A) insulator (B) *p*-type
(C) *n*-type (D) superconductor.
(*A.F.M.C. 1995*

22. **A *p*-type semiconductor is**
(A) a silicon crystal doped with arsenic impurity.
(B) a silicon crystal doped with aluminium impurity.
(C) a germanium crystal doped with boron impurity.
(D) a germanium crystal doped with phosphorus impurity
(*N.C.E.R.T. 1979*

23. **An *n*-type and a *p*-type silicon can be obtained by doping pure silicon with**
(A) sodium and magnesium respectively.
(B) phosphorus and boron respectively.
(C) boron and phosphorus respectively.
(D) indium and sodium respectively. (*C.E.T. 1991*)

24. **Holes are charge carriers in**
(A) intrinsic semiconductors.
(B) ionic solids.
(C) *p*-type semiconductors.
(D) metals. (*I.I.T. 1996*)

25. **In a *p*-type semiconductor, the majority carriers of current are :**
(A) holes (B) electrons
(C) protons (D) neutrons. (*C.B.S.E. 1999*)

26. **An *n*-type semiconductor is**
(A) negatively charged. (B) positively charged.

(C) neutral.
(D) negatively or positively charged depending upon the amount of impurity. *(A.F.M.C. 1988)*

7. A semiconductor is known to have an electron concentration of 8×10^{13} cm^{-3} and hole concentration of 5×10^{12} cm^{-3}. The semiconductor is
(A) *n*-type. (B) *p*-type.
(C) intrinsic. (D) insulator.

8. The temperature coefficient of resistance of a semiconductor
(A) is always positive. (B) is always negative.
(C) is zero.
(D) may be positive or negative or zero.

ART II. THOUGHT-BASED QUESTIONS

29. The volume of atoms to the volume of unit cell is called
(A) packing fraction.
(B) packing coefficient.
(C) co-ordination number.
(D) lattice parameter. *(A.F.M.C. 1995)*

30. The valence electron in alkali metal is a
(A) *f*-electron (B) *p*-electron
(C) *s*-electron (D) *d*-electron *(C.B.S.E. 1990)*

31. The electronic configuration for a noble gas is
(A) $1s^2\,2s^2\,2p^6$ (B) $1s^2\,2s^2\,2p^5$
(C) $1s^2\,2s^2\,2p^6\,3p^6\,4s^2$ (D) $1s^2\,2s^2$ *(C.B.S.E. 1993)*

32. In a good conductor, the energy gap between the conduction band and the valence band is
(A) infinity. (B) wide.
(C) narrow. (D) zero.
(Karnataka Entrance, 1993)

33. When a semiconductor is heated,
(A) number of electrons increases, while that of holes decreases.
(B) number of holes increases, while that of electrons decreases.
(C) number of electrons and holes remains same.
(D) number of electrons and holes increases equally.
(Pre-Medical/Dental 1993)

34. Doping of a semiconductor (with small traces of impurity atoms) generally changes the resistivity as follows :
(A) Decreases.
(B) Does not alter.
(C) May increase or decrease depending on the dopant.
(D) Increases. *(Karnataka Entrance, 1994)*

35. The mobility of conduction electrons is greater than that of holes. It is because,
(A) electrons are negatively charged.
(B) electrons are lighter
(C) electrons experience a lesser number of collisions.
(D) drift velocity acquired per unit electric field is greater for electrons.

Chapter 2. Semiconductor devices

ART I. TEXT-BASED QUESTIONS

1. The depletion layer in the *p-n* junction region is caused by
(A) diffusion of carriers.
(B) drift of electrons.
(C) drift of holes.
(D) migration of impurity ions. *(Pre-Medical/Dental 1994)*

2. The cause of the potential barrier in a *p-n* junction diode is :
(A) depletion of positive charges near the junction.
(B) concentration of positive charges near the junction.
(C) depletion of negative charges near the junction.
(D) concentration of positive and negative charges near the junction. *(C.B.S.E. 1998)*

3. The potential barrier in the depletion layer is due to
(A) ions (B) holes
(C) electrons (D) forbidden gap.
(B.H.U. 1999)

4. The barrier potential of a silicon diode at room temperature is
(A) 0·3 V (B) 0·7 V
(C) 1 V (D) 2 mV *(C.E.T. 1998)*

5. A *p-n* junction has a thickness of the order of
(A) 1 cm (B) 1 mm
(C) 10^{-6} cm (D) 10^{-12} cm *(B.I.T. 1990)*

6. Diffusion of free electrons across the junction of an unbiased diode produces
(A) forward bias (B) reverse bias
(C) breakdown (D) depletion layer.
(C.E.T. 1998)

7. Which of the following statements is not true ?
(A) The resistance of intrinsic semiconductors decreases with increase of temperature.
(B) Doping pure Si with trivalent impurities gives *p*-type semiconductors.
(C) The majority carriers in *n*-type semiconductors are holes.
(D) A *p-n* junction can act as a semiconductor diode.
(I.I.T. 1997)

8. A semiconducting device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. The polarity of the battery is reversed, then the current drops to almost zero. The device may be a / an
(A) *p-n* junction.
(B) intrinsic semiconductor.
(C) *p*-type semiconductor.
(D) *n*-type semiconductor. *(C.B.S.E. 1998)*

9. In forward bias, the width of depletion layer in a *p-n* junction
(A) decreases. (B) remains unchanged.
(C) increases. (D) first (A) and then (C)
(C.B.S.E. 1999)

10. In a *p-n* junction, the hole current due to drift of charges is from
(A) *n*-section to *p*-section.
(B) *p*-section to *n*-section.
(C) *n*-section to *p*-section, if the junction is forward biased and in the opposite direction, if it is reverse biased.
(D) *p*-section to *n*-section, if the junction is forward biased and in the opposite direction, if it is reverse biased.

11. In a *p-n* junction, the hole current due to diffusion of charges is from
(A) *n*-section to *p*-section.
(B) *p*-section to *n*-section.
(C) *n*-section to *p*-section, if the junction is forward biased and in the opposite direction, if it is reverse biased.
(D) *p*-section to *n*-section, if the junction is forward biased and in the opposite direction, if it is reverse biased.

12. In a *p-n* junction, diffusion current is greater than the drift current (in magnitude),
(A) if the junction is forward biased.
(B) if the junction is reverse biased.
(C) if the junction is unbiased.
(D) cannot be predicted.

13. For conduction of a *p-n* junction, the biasing is
(A) high potential on *n*-side and low potential on *p*-side
(B) high potential on *p*-side and low potential on *n*-side

(C) same potential on p and n sides

(D) undetermined. (C.B.S.E. 2002)

14. When we apply reverse bias to a junction diode, it

(A) lowers the potential barrier.

(B) raises the potential barrier.

(C) increases the majority carrier current.

(D) decreases the majority carrier current. (C.E.T. 1991)

15. In case of p-n junction diode at high value of reverse bias, the current rises sharply. The value of reverse bias is known as

(A) cut off voltage. (B) Zener voltage.

(C) inverse voltage. (D) critical voltage.

(C.E.T. 1991)

16. In a p-n junction diode not connected to any circuit,

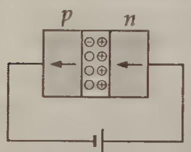
(A) the potential is same everywhere.

(B) the p-type side is at a higher potential than the n-type side.

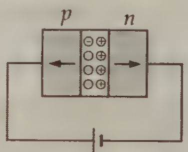
(C) there is an electric field at the junction directed from the n-type side to the p-type side.

(D) there is an electric field at the junction directed from the p-type side to the n-type side. *(I.I.T. 1998)*

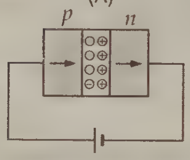
17. In the case of forward biasing of p-n junction, which one of the following figures correctly depicts the direction of flow of carriers ?



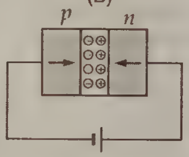
(A)



(B)



(C)



(D)

(C.B.S.E. 1995 ; A.F.M.C. 1995)

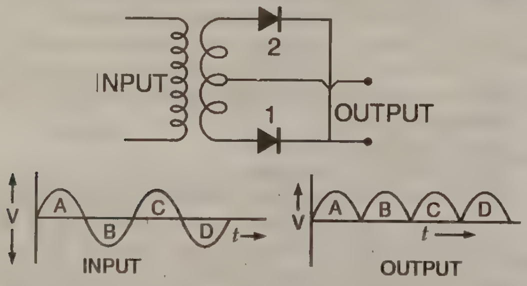
18. A p-n-junction diode can be used as :

(A) rectifier. (B) modulator

(C) demodulator (D) amplifier.

(C.B.S.E. 1999 ; A.F.M.C. 1997)

19. A full-wave rectifier circuit along with the output is shown in the figure. The contribution from the diode 1 is (are)



(A) C (B) A, C

(C) B, D (D) A, B, C, D *(I.I.T. 1996)*

20. The part of a transistor which is heavily doped to produce a large number of majority is called

(A) emitter. (B) base.

(C) collector.

(D) any one out of emitter, base and collector.

(Pre-Medical/Dental, 1993)

21. When n-p-n transistor is used as an amplifier, then

(A) electrons move from base to collector.

(B) holes move from emitter to base.

(C) electrons move from collector to base.

(D) holes move from base to emitter. (C.B.S.E. 199

22. To use a transistor as an amplifier (common-bas configuration),

(A) both junctions are forward biased.

(B) both junctions are reverse biased.

(C) the emitter-base junction is forward biased and bas collector junction is reverse biased.

(D) no biasing voltages are required.

(Pre-Medical/Dental ,199

23. In a common-base amplifier, the phase difference betwee the input signal voltage and the output voltage (acros collector and base) is

(A) 0 (B) $\pi/4$

(C) $\pi/2$ (D) π (C.B.S.E. 199

24. For a transistor, the current amplification factor is 0·8. Th transistor is connected in common-emitter configuratio The change in collector current, when the base curren changes by 6 mA is

(A) 4·8 mA (B) 6 mA

(C) 8 mA (D) 24 mA (C.E.T. 199

25. For a transistor I_c / I_e = 0·96. The current gain in commo emitter configuration is

(A) 6 (B) 12

(C) 24 (D) 48 (C.B.S.E. 200

26. In a common-base configuration of a transistorm $\Delta I_c / \Delta I$ = 0·98. Then current gain in common-emitter configuratio of transistor will be

(A) 49 (B) 98

(C) 4·9 (D) 24·5 (C.B.S.E. 200

27. In n-p-n transistor circuit, the collector current is 10 mA If 90% of the electrons emitted reach the collector,

(A) the emitter current will be 9 mA.

(B) the emitter current will be 11 mA.

(C) the base current will be 1 mA.

(D) the base current will be 0·1 mA. (I.I.T. 199

28. If α and β are current gains in common-base and common emitter configurations of a transistor, then β is equal to

(A) $1/\alpha$ (B) $\alpha/(1+\alpha)$

(C) $\alpha/(1-\alpha)$ (D) $\alpha - 1/\alpha$ (C.B.S.E. 200

29. The current relationship between the two current gain α and β in a transistor is

(A) $\beta = \dfrac{\alpha}{1+\alpha}$ (B) $\beta = \dfrac{\alpha}{1-\alpha}$

(C) $\alpha = \dfrac{\beta}{1+\beta}$ (D) $\alpha = \dfrac{1+\beta}{\beta}$ (C.B.S.E. 1997

30. The current gain β of a transistor is 50. The input resistanc of the transistor, when used in the common emitte configuration, is 1 kΩ. The peak value of the collector a.c current for an a.c. peak input voltage of 0·01 V is

(A) 100 μA (B) 250 μA

(C) 500 μA (D) 800 μA (C.B.S.E. 1999

31. A transistor is preferred to a triode valve, because it

(A) can withstand larger changes in temperature.

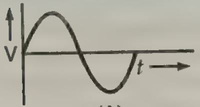
(B) has a higher input impedance.

(C) can handle large powers.

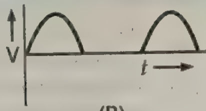
(D) does not require a heater.

(C.P.M.T. 1986 ; N.C.E.R.T. 1978

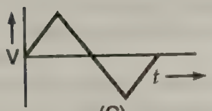
32. Which of the following represents the digital signal :



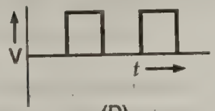
(A)



(B)



(C)



(D)

33. In the binary number system, 111 represents

(A) one. (B) three.

(C) seven. (D) one hundred and eleven.

34. Boolean algebra is essentially based on

(A) logic (B) truth

(C) numbers (D) symbol *(A.I.I.M.S. 1999)*

35. In Boolean algebra, $y = A + B$ implies :

(A) y equals A or B.

(B) y equals sum of A and B.

(C) y equals neither A nor B.

(D) y equals A and B.

36. In Boolean algebra, $y = A.B$ implies

(A) y equals A and B. (B) y equal A or B.

(C) y equals product of A and B .

(D) y equals neither A nor B.

37. In Boolean algebra, $y = \overline{A}$ implies

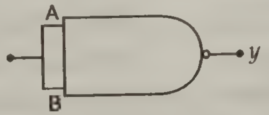
(A) y equals A.

(B) y is not equal to A.

(C) y is equal to – A.

(D) none of the above.

38. The symbol shown in the adjoining figure represents



(A) NOT gate (B) OR gate

(C) AND gate (D) NOR gate *(C.B.S.E. 1996)*

39. A truth table is given below. Which of the following has this type of truth table ?

A	B	y
0	0	1
1	0	0
0	1	0
1	1	0

(A) XOR gate

(B) NOR gate

(C) AND gate

(D) OR gate

(C.B.S.E. 1996)

40. In the truth table given below, A and B are inputs and Y, the output. Which of the following has this type of truth table ?

A	B	y
0	0	1
1	0	0
0	1	0
1	1	0

(A) AND gate.

(B) NOR gate.

(C) OR gate.

(D) XOR gate.

(C.E.T. 1999 ; C.B.S.E. 1997)

41. Which of the following gates corresponds to the truth table given below :

A	B	y
0	0	1
1	0	1
0	1	1
1	1	0

(A) NAND

(B) AND

(C) XOR

(D) OR

(C.B.S.E. 2002, 2001, 1998, 1994 ; A.F.M.C. 1995)

42. The adjoining truth table represents

(A) AND gate

(B) NOR gate

(C) OR gate

(D) NAND gate

(C.B.S.E. 2000)

A	B	y
0	0	0
1	0	0
0	1	0
1	1	1

PART II. THOUGHT-BASED QUESTIONS

43. When a potential difference is applied across, the current passing through

(A) an insulator at 0 K is zero.

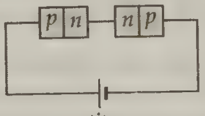
(B) a semiconductor at 0 K is zero.

(C) a metal at 0 K is finite.

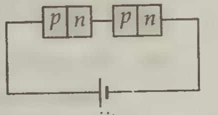
(D) a p-n diode at 300 K is finite, if it is reverse biased.

(I.I.T. 1999)

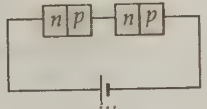
44. The identical p-n junction may be connected in series with a battery in three ways as shown in the circuits given below. The potential drops across the two p-n junctions are equal in



(*i*)



(*ii*)



(*iii*)

(A) circuit (*i*) only.

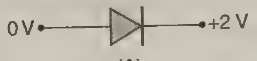
(B) circuits (*i*) and (*ii*)

(C) circuits (*ii*) and (*iii*)

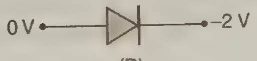
(D) circuits (*i*) and (*iii*).

(A.F.M.C. 1997)

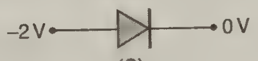
45. In which of the following figures, junction diode is forward biased ?



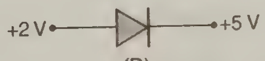
(A)



(B)



(C)



(D)

(C.B.S.E. 2000)

46. When a p-n junction diode is forward-biased, the flow of current across the junction is mainly due to

(A) drift of charges.

(B) diffusion of charges.

(C) both drift and diffusion of charges.

(D) depends on the nature of the material.

(Pre-Medical/Dental, 1994)

47. When a p-n junction diodie is reverse-biased, the flow of current across the junction is mainly due to

(A) diffusion of charges.

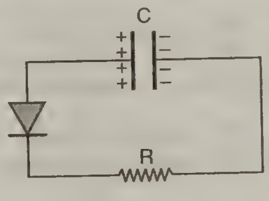
(B) drift of charges.

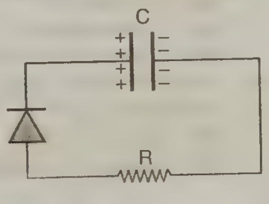
(C) both drift and diffusion of charges.

(D) depends upon the nature of the material.

(C.E.T. 1999)

48. Two identical capacitors each of capacitance C are charged to the same potential V and are used to forward and reverse bias two identical junction diodes as shown in figures.





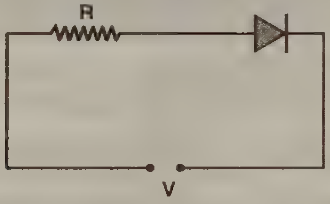
The charges on the two capacitors at a time t = CR are

(A) (*i*) C V (*ii*) C V **(B)** (*i*) C V/*e* (*ii*) C V

(C) (*i*) C V (*ii*) C V/*e* **(D)** (*i*) C V/*e* (*ii*) C V/*e*

Here, *e* = 2·718 is mathematical constant.

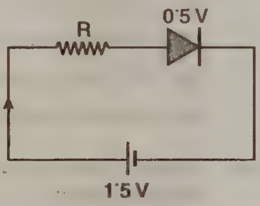
49. For a given circuit of ideal *p-n* junction diode, which of the following is correct ?



(A) In forward biasibng, the voltage across R is V.
(B) In reverse biasing, the voltage across R is V.
(C) In forward biasing, the voltage across R is 2 V.
(D) In reverse biasing , the voltage across R is 2 V.

(C.B.S.E. 2002)

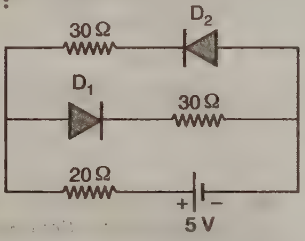
50. The diode used in the circuit shown in figure has a constant voltage drop at 0·5 volt at all currents and a maximum power rating of 100 mW. What should be the value of the resistor R connected in series to the dioide for obtaining maximum current ?



(A) 5 Ω
(B) 5·6 Ω
(C) 6·76 Ω
(D) 10 Ω

(C.B.S.E. 1997)

51. If internal resistance of cell is negligible, then current flowing through the cell is :



(A) 0·06 A
(B) 0·1 A
(C) 0·08 A
(D) 0·02 A

(C.B.S.E. 2001)

52. The circuit shown in the figure contains two diodes each with a forward resistance of 50 Ω and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 Ω resistance (in ampere) is

(A) zero
(B) 0·02
(C) 0·03
(D) 0·036.

(I.I.T. 1996)

53. In which of the configurations of a transistor, the voltage gain is highest ?

(A) Common-base. **(B)** Common-emitter.
(C) Common-collector. **(D)** Same in all the three.

54. In which of the configurations of a transistor, the power gain is highest ?

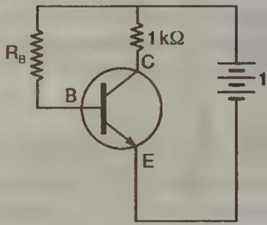
(A) Common-base. **(B)** Common-emitter.
(C) Common-collector. **(D)** Same in all the three.

55. A transistor is used in the common-emitter mode as amplifier. Then,

(A) the base-emitter junction is forward-biased.
(B) the base-emitter junction is reverse-biased.
(C) the input signal is connected in series with the volta applied to bias the base-emitter junction.
(D) the input signal is connected in series with the volta applied to bias the base-collector junction.

(I.I.T. 19

56. In the circuit shown in the figure, the transistor used h a current gain $\beta = 100$. What should be the bias resistor R so that $V_{ce} = 5$ V. (Neglect V_{be}).



(A) 200 × 10^{-3} Ω
(B) 10^6 Ω
(C) 500 Ω
(D) 2 × 10^3 Ω

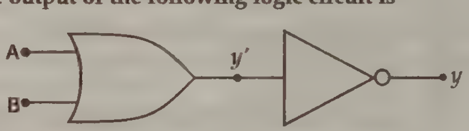
(Pre-Medical/Dental, 19

57. An oscillator is an amplifier with

(A) a large gain. **(B)** negative feedback.
(C) positive feedback. **(D)** no feedback.

(C.B.S.E. 1995 ; Pre-Medical/Dental, 19

58. The output of the following logic circuit is

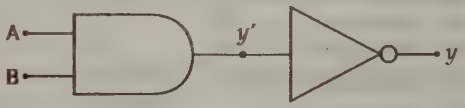


(A) $y = \overline{A + B}$ **(B)** $y = \overline{A} + \overline{B}$
(C) $y = \overline{A . B}$ **(D)** $y = \overline{A} . \overline{B}$

59. The combination of the two basic gates shown in MCQ . represents

(A) NAND gate. **(B)** NOR gate.
(C) NOT gate. **(D)** XOR gate.

60. The output of the following logic circuit is

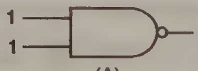


(A) $y = \overline{A + B}$ **(B)** $y = \overline{A} + \overline{B}$
(C) $y = \overline{A . B}$ **(D)** $y = \overline{A} . \overline{B}$

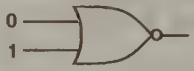
61. The combination of the two basic gates shown in MCQ. represents

(A) NAND gate. **(B)** NOR gate.
(C) NOT gate. **(D)** XOR gate.

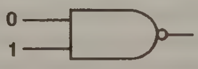
62. Which of the following gates will have an output of 1



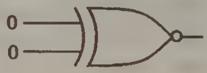
(A)



(B)



(C)

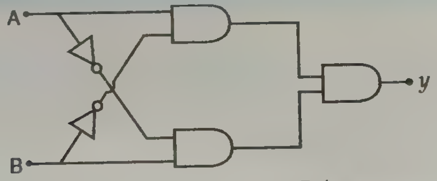


(D)

(A) D **(B)** A
(C) B **(D)** C

(C.B.S.E. 199

3. Which of the following truth table is true ?



(A)

A	B	y
0	0	0
1	0	1
0	1	1
1	1	0

(B)

A	B	y
0	0	0
1	0	0
0	1	1
1	1	1

(C)

A	B	y
0	0	0
1	0	1
0	1	1
1	1	1

(D)

A	B	y
0	0	1
1	0	1
0	1	0
1	1	0

64. How many AND gates are required to form NAND gate ?
(A) 1 (B) 2
(C) 3 (D) 4 *(B.H.U. 1997)*

ANSWERS TO MULTIPLE CHOICE QUESTIONS

hapter 1. Solids

1. (C) 2. (A) 3. (C) 4. (D) 5. (D) 6. (B) 7. (B) 8. (A) 9. (B) 10. (C)
11. (C) 12. (C) 13. (C) 14. (A) 15. (C) 16. (A) 17. (A) 18. (C) 19. (B & D) 20. (A)
21. (C) 22. (B & C) 23. (B) 24. (A & C) 25. (A) 26. (C) 27. (A) 28. (B) 29. (A) 30. (C)
31. (A) 32. (D) 33. (D) 34. (A) 35. (D)

hapter 2. Semiconductor devices

1. (A) 2. (D) 3. (A) 4. (B) 5. (C) 6. (D) 7. (C) 8. (A) 9. (A) 10. (A)
11. (B) 12. (A) 13. (B) 14. (B) 15. (B) 16. (C) 17. (D) 18. (A) 19. (B or C) 20. (A)
21. (A) 22. (C) 23. (A) 24. (D) 25. (C) 26. (A) 27. (B & C) 28. (C) 29. (B & C) 30. (C)
31. (D) 32. (D) 33. (C) 34. (A) 35. (A) 36. (A) 37. (B) 38. (A) 39. (B) 40. (B)
41. (A) 42. (A) 43. (A, B & D) 44. (C) 45. (B) 46. (B) 47. (B) 48. (B) 49. (A)
50. (A) 51. (B) 52. (B) 53. (B) 54. (B) 55. (A & C) 56. (A) 57. (C) 58. (A) 59. (B)
60. (C) 61. (A) 62. (D) 63. (A) 64. (A)

UNIT
X

PRINCIPLES OF COMMUNICATION

1. **Analog and Digital Communication**
2. **Space Communication**
3. **Line Communication**

I think there's a world market for about five computers."

– Thomas Watson (Founder of IBM)

CHAPTER 1

Analog and Digital Communication

1.01. INTRODUCTION

In the early days, people used to send messages through trained pigeons. However, this mean of communication was not reliable. So that general masses could ommunicate with one and another, a messanger called *harkara* used to pass through various towns and people used to receive the messages sent by their friends and elatives from him or could deliver the messages to him. One will appreciate that as such, a messanger would require a lot of time to cover the distance between two destinations and will get fatigued. Therefore, this method also proved to be quite mpracticable. As a remedy, the messanger was provided a horse so that he could move between two destinations quite swiftly. Thus, providing a horse to the messanger, the task of communication became much easier and quicker. As a matter of fact, this method overcame the problem of sending the messages from one place o other only to some extent. However, it laid the foundation of modern methods of communication. With the advent of elctronics, the messages in the form of electric ignals are made to sit on high frequency carrier waves (in place of putting a man on the back of the horse) and the person at the other end can pick the message with he help of some electronic gadget. This mean of communication is known as adioacommunication and is used to send and receive the messages in the form of analog signals. With the advancement of technology, analog communication systems are being replaced by digital communication systems. Before making a study (in principle) of analog and digital communication systems, it is worthwhile to have ust an overview of them.

Analog communication systems. *The communication systems, which make use of analog signals, are called analog communication systems.*

A few of them are as given below :

1. ***Telegraphy.*** It was used for sending electric signals in the form of a code (dots and dashes). Now, optical and audible coded signals can be sent over large distances. The telegraph messages can also be sent through radiowaves, microwaves and artificial satellites.

2. ***Telephony.*** It enables to send voice signals from one place to another by means of wires. In the recent past, it was made more reliable by making use of microwaves and communication satellites. The use of optical fibres in this field has totally evolutionised it.

3. ***Radio network.*** This system makes use of radiowaves for broadcasting speech and music. The long distance transmission of audio signal is achieved by mixing (modulating) it with radiowaves of high frequency.

4. ***Radar.*** It is contraction of *Ra*dio *d*etection *a*nd *r*anging. It is an electronic system used for determining the distance (range) and direction of objects using microwaves.

5. ***Television network.*** It is an electronic system that transmits and receives visual images and sound using very-high frequency (VHF) or ultra-high frequency (UHF) radiowaves.

6. ***Teleprinting.*** It is a system, which makes use of a machine similar to a ypewriter, on which messages can be typed and telegraphed to distant receivers. It can print out the incoming messages also.

7. ***Telex.*** It is contraction of ***Tele***type writer ***ex***change service. Telex* is a telegraphic system using telephone lines through which direct current pulses representing characters typed on a typewriter are sent.

---

* It has been superseded by the FAX machine.

Digital communication systems. *The communication systems, which make use of digital signals, are called digital communication systems.*

The following are a few such latest systems :

1. ***Fax.*** It is used for facsimile transmission and receiving of texts (documents), photographs, etc.

2. ***Mobile telephone network.*** It enables to make use of a portable (mobile) telephone. Such telephones are also called cellular phones, because they operate within a network of ***radiocells***. The first generation mobile phones operated with analog signals. The mobile phones, which are presently in use, operate with digital signals and are called second generation mobile phones.

3. ***Electronic mail (e-mail).*** The messages sent via a computer network are called e-mails. The messages produced using word processing programs are transmitted over the world-wide network, called internet and they get stored in a computer, called a mail server. Anyone connected to the internet can contact the mail server to check, whether it is holding mail for him/her.

4. ***Teleconferencing.*** It is a system in which persons sitting at coloured television screens see and talk to each other via a computer-communication network. It saves travelling cost as well as the time that would otherwise be lost in journey.

5. ***Telemetry.*** It is a system of transmitting data (usually measurements) from a spacecraft to earth using radiowaves.

6. ***Communication satellites.*** They are used to relay radio and television programs. They have proved of immense use in the field of telecommunication and for sending data signals from one part of the earth to another.

7. ***Global positioning system*** **(*GPS*).** It is a navigation system based on a network of earth-orbiting satellites. The users can find their positions within an accuracy of 100 m by receiving signal from the satellite. A user needs to be within the range of four satellites (out of 24 satellites orbiting the earth).

1.02. COMMUNICATION SYSTEM

The exchange of information between a sender and a receiver is called ***communication*** *and the setup used to achieve this task is called a* ***communication system.***

In principle, a communication system consists of following three main parts :

1. Transmitter
2. Communication channel
3. Receiver

In order to familarise ourselves with the three parts of a communication system, consider that two persons A and B are communicating with each other in the following three situations :

(*a*) At first instance, suppose that the two persons are so close that they can communicate directly with each other. Then, the speaker (say person A) is acting as ***transmitter*** and the listener (the other person B) as the ***receiver.*** The air, through which sound waves travel from the speaker to the listener plays the role of ***communication channel.*** The communication channel is also called ***transmission medium*** or ***link.***

(*b*) Now, assume that the two persons are separated by a distance of several kilometres and it is not possible for them to communicate directly. One of the possible ways is to connect the transmitter and the receiver with the help of wires as is done in telephone as a communication system. The signal in the form of sound waves is converted into electrical pulses with a microphone, which then travel along the long wires. The signal in the form of electrical pulses is converted back into sound waves with the help of a loud speaker. Both the microphone and the loudspeaker are said to act as ***transducers*** (device that converts energy in one form to another). Thus, in this mode of communication, information is not exchanged in its original form between the speaker and the listener. The transmitter performs the job of transducer before sending the information to the receiver. Similarly, the receiver also accomplishes the job of transducer before making the information reach the listener.

(*c*) Let us now consider that the two persons are separated by a very large distance, say more than several thousand kilometres so that it is not possible to connect the transmitter and receiver with long wires. In such a situation, the two persons can exchange information between them through radio communication (wireless communication). The sound signal is converted into electrical signal with the help of a transducer as before. The electrical signal corresponding to the sound signal is called audio signal. Its frequency range is from 20 Hz to 20 kHz and is associated with a small amount of energy. If it is radiated in free space as such, it will die out soon. So that the audio signal can travel up to several thousand kilometres, it is mixed with high frequency radiowaves, called ***carrier waves***. This process is calld ***modulation*** and the signal so obtained is called ***modulated signal***. The power of the signal is further boosted up by using a suitable amplifier and then the signal is radiated into the space with the aid of an antenna, called ***transmitting antenna***. Therefore, in this mode of communication, transmitter constitutes a transducer, modulator, amplifier and a transmitting antenna. The free space, through which the modulated signal travels acts as the communication channel. The receiver of this type of communication system consists of a ***pick-up antenna*** (to pick the signal), ***demodulator*** (to separate out the audio signal from the modulated signal), ***amplifier*** (to boost up the audio signal, which may have got weakened) and the ***transducer*** (to convert back audio signal in the form of electrical pulses into sound waves).

The radiocommunication system is schematically as shown in Fig. 1.01.

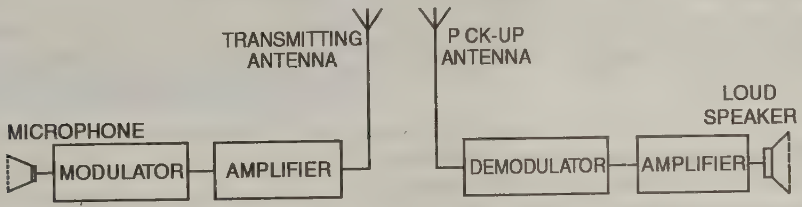


Fig. 1.01

1.03. ANTENNA

In a communication system, antenna plays a vital role. It is employed for both transmitting and receiving the radio frequency signals.

A Hertz antenna is a straight conductor of length equal to half wavelength $\left(\frac{\lambda}{2}\right)$ of the radio signal to be transmitted or received. The transmitting antenna converts electrical energy into electromagnetic waves, whereas the pick-up antenna converts electromagnetic waves into electrical energy. When the transmitting antenna is placed vertically, it produces vertically polarized waves and when placed horizontally, it produces horizontally polarised waves.

A Marconi antenna is quarter wave $\left(\frac{\lambda}{4}\right)$ long. It is placed vertically with its lower end touching the ground. The ground provides a reflection of the voltage and current distributions set up in the antenna. It is fed near the ground. The emitted electromagnetic waves from antenna-ground system are same as that from a Hertz antenna.

There is a large variety of antennas with each type having its own preferred range of applications. A microwave antenna is used in radar systems and for receiving broadcasts directly from the satellites. Such an antenna has a parabolic reflector with a dipole (also called horn feed) at the focus of the reflector. A microwave antenna can transmit waves in a particular direction and can receive waves, which are directed towards it.

1.04. ANALOG AND DIGITAL SIGNAL

In communication systems, a signal means a time varying electrical signal containing information or intelligence. The different signals have their own characteristics, such as amplitude and frequency.

The signals are of the following two types :

1. Analog signal. An analog signal is a continuous function of time. In other words, amplitude of an analog signal varies continuously with time. Such signals can be easily generated from the source of information by using an appropriate transducer. For example, pressure variations in the sound waves can be converted into corresponding current or voltage pulses with the help of a microphone. Similarly, a photocell may be used to convert light intensity variations into current and voltage pulses.

In the simplest form of an analog signal, the amplitude of the signal varies sinusoidally with time and may be represented by the equation

$$E = E_0 \sin \omega t \qquad \text{...(1.01)}$$

It represents an analog signal of a single frequency $f \left(= \dfrac{\omega}{2\pi}\right)$. Such a signal is graphically as shown in Fig. 1.02.

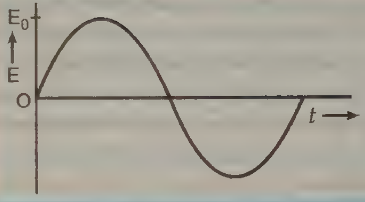


Fig. 1.02

The frequency of analog signals associated with speech or music varies over a range between 20 Hz to 20 kHz. The range over which the frequencies of a signal vary is called its ***bandwidth***. This information bearing signal is also called ***baseband signal***. The term baseband designates the band of frequencies representing the signal supplied by the source of information.

2. Digital Signal. A digital signal is a discontinuous function of time. The pulses in a digital signal do not vary continuously with time as in an analog voltage signal. Instead, it has only two voltage levels *i.e.* either low (zero) or high (some constant value of voltage). The two levels in a digital signal are coded usually in binary code. When voltage is low, it is represented by binary digit 0 and when it is high, it is represented by the binary digit 1 as shown in Fig. 1.03.

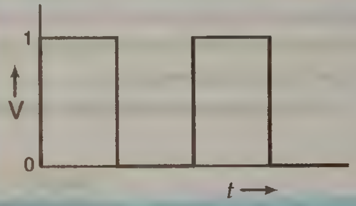


Fig. 1.03

1.05. RADIO COMMUNICATION

Sound offers a flexible means of communication between us. Nature has equipped a human being with its own transmitter and receiver in the form of mouth and ears. However, sound as a means of communication has some disadvantages. Firstly, as such, it cannot travel to a far off listener and secondly, if several persons are speaking at the same time, the listener may find it difficult to sort out the different messages from different persons.

In 1844, telegraphy was developed as a means of communication by Samuel Morse. The information could be sent from one place to another along wires through ***Morse Code***, which consisted of dots and dashes. In 1876, Alexander Graham Bell invented telephone, which could send voice message from one person to another. As such, if a person wanted to talk to other persons, their telephones had to be connected through wires leading to a wild jumble of wires. It became obvious that this technique of communication *i.e.* connecting every telephone to every other telephone was not going to work and it led to the development of wireless radio-communication.

The transmitting and receiving a message in the form of a radiowave signal between two stations without connecting them with wires is known as wireless radio-communication.

In 1865, James Clerk Maxwell predicted the existence of electromagnetic waves, which can propagate through the free space (vacuum) with the speed of light. These waves were first produced and detected by Heinrich Hertz in 1887 using a spark gap transmitter and detector. In 1901, Guglielmo Marconi and his associates succeeded in sending radio signal across the Atlantic by making use of antenna. The transmitting antenna helps the radio oscillator (which generates radio waves) to radiate the energy of the radiowaves into the space. The function of the antenna in transmission of radiowaves is same as that of the resonance tube or the sonometer used with a tuning fork. When the length of the antenna is equal to a quarter or one half of the wavelength of the radio signal, the standing electrical waves are set up along it, which are then strongly radiated into the space. The receiving antenna connected to the detector works in a similar fashion. When the transmitted signal is intercepted by the receiving antenna, the induced electrical waves are set up in it.

1.06. ANALOG MODULATION

The process of producing a continuous high frequency waves, some characteristic of which varies as a function of the instantaneous value of the audio signal, is called analog modulation.

In the process of modulation, the audio signal, called ***modulating wave*** (or ***baseband signal***), is mixed with a high frequency wave, called ***carrier wave*** and the wave so produced is called ***modulated wave***. In the analog modulation, carrier wave is continuous in nature and is usually a sinusoidal waveform. Fig. 1.04 is the block diagram illustrating the process of modulation.

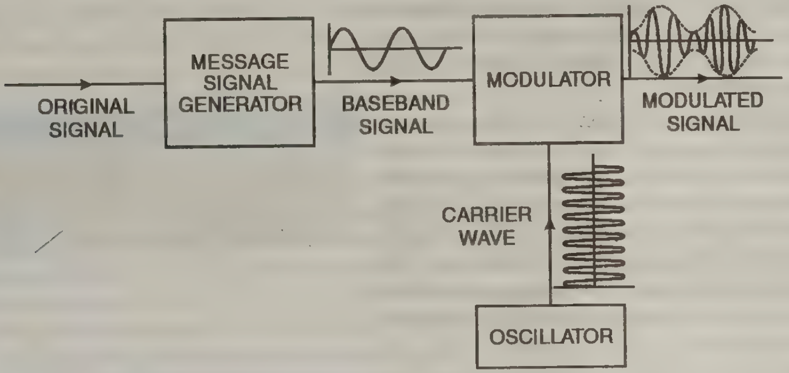


Fig. 1.04

The original signal (in the form of speech or music) in fed to the message signal generator (such as a microphone), which converts it into an electrical signal. The signal so obtained is called baseband or modulating signal. The oscillator generates the high frequency waves, called carrier waves. The modulator, then, carries out the superposition of the baseband signal and the carrier waves. The nature of the modulator depends upon the type of modulation to be carried out, which may be one of the following three types :

1. Amplitude modulation **2.** Frequency modulation **3.** Phase modulation.

In order to understand the three methods of modulation in principle, consider that the instantaneous voltage of the modulated wave is given by

$$e = A \sin (\omega t + \phi),$$

where A, ω and ϕ represent the amplitude, frequency and phase of the modulated wave.

Amplitude modulation. *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the frequency of the modulated wave is same as that of the carrier wave but its amplitude is modified in accordance with that of the modulating wave, the process is called amplitude modulation.*

In amplitude modulation, the amplitude A of the modulated wave is varied in accordance with the modulating wave, while ω and ϕ remain constant.

Frequency modulation. *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the amplitude of the modulated wave is same as that of the carrier wave but its frequency is modified in accordance with the amplitude of the modulating wave, the process is called frequency modulation.*

In frequency modulation, the frequency ω of the modulated wave is varied in accordance with the modulating wave, while A and ϕ remain constant.

Phase modulation. *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the magnitude of the phase angle of the modulated signal varies in accordance with the amplitude of the modulating wave, the process is called phase modulation.*

In phase modulation, the phase ϕ of the modulated wave is varied in accordance with the modulating wave, while A and ω remain constant. It may be noted that phase modulation is not of much practical importance in itself, but it is sometimes used as an intermediate step in achieving frequency modulation. However, we shall restrict ourselves to the study of amplitude and frequency modulation processes only.

1.07. NEED FOR MODULATION

The sound waves within the range of human hearing have frequency from 20 Hz to 20 kHz. The sound waves cannot be transmitted from a radio transmitter by converting them into electrical waves (audio signal) directly for the following reasons :

1. For efficient transmission and reception, the transmitting and receiving antennae must have a length equal to quarter wavelength of the audio signal. For a frequency of 15 kHz of audio signal, the length of the antenna* comes out to be of the order of 5000 m. To set up a vertical antenna of this size is practically impossible.

2. The energy radiated from an antenna is practically zero, when the frequency of the signal to be transmitted is below 15 kHz. It also makes the direct transmission of audio signal as impracticable.

3. Due to the fact that all audio signals from different sources possess frequencies in the same range *i.e.* 20 Hz to 20 kHz, an audio signal can not be transmitted directly. It is because, the audio signal from different transmitting stations will get hopelessely and inseparably mixed up.

The above said difficulties faced during the transmission of audio signal, if transmitted directly, are overcome by the process of modulation. For example, when a carrier wave of frequency 1 MHz is used for transmission of audio signal, the antenna required will be of length 75 m only, which can be easily set up. Further, the energy radiated from the antenna for a radio signal of frequency 1 MHz is quite appreciable and the signal can travel upto a large distance, before it dies out. The problem of mixing up of the signals from the different transmitting stations is overcome by allocating different frequencies of the carrier waves to different transmitting stations.

Self-test Question

Show that the length of antenna required to transmit a radio signal of frequency 1 MHz is about 75 m.

1.08. AMPLITUDE MODULATION

The amplitude modulation is produced by varying the amplitude of the carrier waves in accordance with the amplitude of the modulating wave (audio signal). Let the instantaneous values of the voltage of the carrier waves and the modulating signal be represented by

$$e_c = E_c \cos \omega_c t \quad \text{...(1.01)}$$

and

$$e_m = E_m \cos \omega_m t \quad \text{...(1.02)}$$

respectively. The frequency of carrier waves ω_c is usually much greater than the frequency of audio signal ω_m. Figs. 1.05 (*a*) and 1.05 (*b*) show the graphical representations of the voltages of the carrier waves and the audio signal with time.

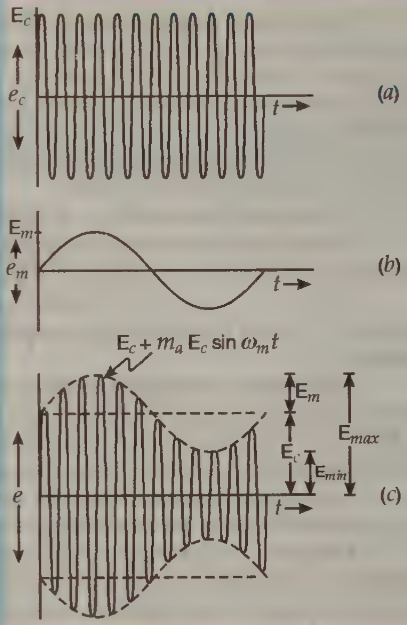


Fig. 1.05

In amplitude modulation, the amplitude of the modulated signal is varied in accordance with the amplitude of the modulating signal so that the frequency of the modulated wave is equal to the frequency of the carrier waves. Accordingly, let the instantaneous voltage of the modulated signal be given by

$$e = A \cos \omega_c t, \quad \text{...(1.03)}$$

where A is amplitude of the modulated signal. Since amplitude of modulated signal should vary in accordance with the audio signal, we assume that

$$A = E_c + k_a e_m, \quad \text{...(1.04)}$$

where the proportionality constant k_a determines the maximum variation in amplitude of the modulated wave for a given modulating signal. In amplitude modulation, the degree of modulation is defined by a term called ***modulation index***, which is given by

$$m_a = \frac{k_a E_m}{E_c} \quad \text{...(1.05)}$$

The modulation index is also called ***modulation factor*** or ***depth of modulation***.

Fig. 1.05 (*c*) gives the sketch of amplitude modulated signal with time. The amplitude of modulated signal varies between E_{min} $(= E_c - E_m)$ and E_{max} $(= E_c + E_m)$. It can be proved that

$$m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \quad \text{...(1.06)}$$

From the detailed study of amplitude modulation, it is found that the expression for instantaneous value of the voltage (or amplitude) of the modulated signal consists of three terms. The first term represents a wave form of carrier frequency ω_c and the second term of frequency $(\omega_c + \omega_m)$, called the ***upper sideband***. The third term represents a wave form of frequency $(\omega_c - \omega_m)$ and is known as ***lower sideband***. Since the frequency of the modulating signal is much smaller than the frequency of the carrier waves, all the three frequencies in the modulated wave are roughly the same and lie in radio frequency range.

---

*Length of antenna $= \frac{\lambda}{4} = \frac{1}{4}\left(\frac{c}{f}\right) = \frac{1}{4} \times \frac{3 \times 10^8}{15 \times 10^3} = 5000$ m

Bandwidth. The modulated signal lies in the frequency range from $\omega_c - \omega_m$ to $\omega_c + \omega_m$ *i.e.* $2\,\omega_m$. It is called the bandwidth of the transmitted signal. Thus,

$$\text{band width} = 2 \times \text{frequency of audio signal} \qquad \text{...(1.07)}$$

In practice, the audio signal is always complex rather than consisting of a single pure frequency ω_m. For example, the voice of a person or a piece of music always consists of a band of frequencies of different amplitudes and phases. If such an audio signal has frequencies ranging from ω_1 to ω_2 $(\omega_2 > \omega_1)$, then each frequency in the audio signal produces a pair of sidebands on modulation. The entire signal, then, produces two sidebands symmetrically disposed about the carrier. The bandwidth in case of a complex audio signal is equal to twice the maximum frequency (ω_2) of the audio signal *i.e.*

$$\text{band width} = 2 \times \text{maximum frequency of audio signal} \qquad \text{...(1.08)}$$

Importance of modulation index. The modulation index determines the strength and quality of the transmitted signal. If the modulation index is small, the amount of variation in the carrier amplitude will be small. Consequently, the audio signal being transmitted will not be strong. The greater the degree of modulation, the stronger and clearer will be the audio signal during reception.

Amplitude modulation is most basic and is widely used for broadcasting speech and music. It is because, the equipments required in AM modulation are simple and cheaper. Further, carrier waves used are of low frequency (0·5 to 20 MHz). However, atmospheric and man made electrical discharges, called noise, also get amplitude modulated along with the message signal. Due to this, reception of signal is not clear particularly in case of weak radio signals.

1.09. FREQUENCY MODULATION

The frequency modulation is produced by varying the frequency of the carrier waves in accordance with the amplitude of the modulating wave. Let the instantaneous values of the voltage of the carrier waves and the modulating signal be represented as

$$e_c = E_c \cos \omega_c t \qquad \text{...(1.09)}$$

and

$$e_m = E_m \cos \omega_m t \qquad \text{...(1.10)}$$

respectively. Fig. 1.06 (*a*) and 1.06 (*b*) show the graphical representations of the voltages of the carrier waves and the audio signal with time.

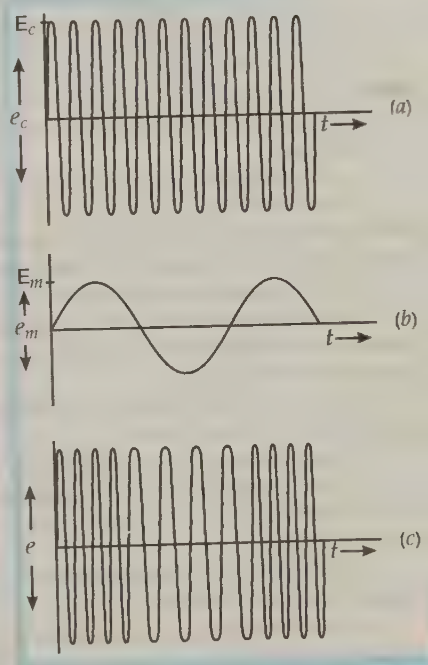


Fig. 1.06

In frequency modulation, the frequency of the modulated signal is varied in accordance with the amplitude of the modulated signal so that the amplitude of the modulated wave is same as that of the carrier waves. Accordingly, let the instantaneous voltage of the modulated signal be given by

$$e = E_c \sin \theta, \qquad \text{...(1.11)}$$

where θ is some function of carrier and modulating frequencies. Since the instantaneous angular frequency $\omega(t)$ of the modulated wave should vary in accordance with the amplitude of the modulating wave, we assume that

$$\omega(t) = \omega_c + k_f e_m, \qquad \text{...(1.12)}$$

where the proportionality constant k_f determines the maximum variation in frequency of the modulated wave for a given modulating signal.

Fig. 1.06 (*c*) gives the sketch of frequency modulated signal with time. It can be proved that the frequency of modulated signal varies between $f_{min} \left(= f_c - \dfrac{k_f E_m}{2\pi} \right)$ and $f_{max} \left(= f_c + \dfrac{k_f E_m}{2\pi} \right)$. The maximum swing of the frequency of modulated wave from the carrier frequency is called ***frequency deviation*** (δ). Thus,

$$\delta = f_{max} - f_c = f_c - f_{min} \qquad \text{...(1.13)}$$

The ratio of frequency swing to the frequency of modulated signal is called ***modulation index*** for frequency modulation. It is denoted by m_f.

$$\therefore \qquad m_f = \frac{\delta}{f_m} = \frac{f_{max} - f_c}{f_m} = \frac{f_c - f_{min}}{f_m} \qquad \text{...(1.14)}$$

As said earlier, in frequency modulation, the amplitude of the modulated signal is same as that of the carrier waves but its frequency changes in accordance with the modulating signal. From the detailed study of frequency modulation it is found that the modulated signal consists of carrier and an apparently infinite number of sidebands, whose frequencies are $(f_c \pm f_m)$, $(f_c \pm 2f_m)$, $(f_c \pm 3f_m)$, ... The number of sidebands depends on the modulation index m_f . The number of sidebands increases, when frequency deviation is increased by keeping frequency of the modulating signal constant. On the other hand, the number of sidebands decreases, when the frequency of modulating signal is increased by keeping frequency deviation constant. It may be pointed out that the sidebands are separated from the carrier by f_m, $2f_m$, $3f_m$, ... *i.e.* they have a recurrence frequency of f_m. The sidebands are disposed symmetrically about the carrier and the sidebands at equal distances from the carrier have equal amplitudes. The amplitude of the sideband separated largely from the carrier becomes negligible. Due to this, the number of significant sideband pairs is limited.

Bandwidth. In frequency modulated signal, the information (audio signal) is contained in the sidebands. Since the sidebands are separated from each other by the frequency of modulating signal (f_m),

$$\text{bandwidth} = 2\,n \times \text{frequency of modulating signal,} \qquad \text{...(1.15)}$$

where n is the number of significant sideband pairs.

Frequency modulation is now fast replacing amplitude modulation for broadcasting speech and music. Frequency modulated signals have larger bandwidth. So that FM signals on the adjacent signals donot interfere, carrier waves of higher frequency have to be used. In FM transmission, the message signal is in the form of frequency variations of carrier waves. Therefore, noise which gets amplitude modulated does not harm the message signal. As a result, in FM broadcasts the reception is of much better quality. The following table gives the range of frequency allotted for FM radio and TV broadcasts :

Types of broadcast	Frequency band
FM radio	88 to 108 MHz
VHF TV	47 to 230 MHz
UHF TV	470 to 960 MHz

1.10. ADVANTAGES OF FREQUENCY MODULATION.

The following are a few advantages of frequency modulation over amplitude lulation :

1. FM reception is quite immune to noise as compared to AM reception.

Noise is a form of amplitude variations in the transmitted signal due to atmosphere, industries, etc. In FM receivers, the noise can be reduced by increasing the frequency deviation or by making use of amplitude limiters.

2. FM transmission is highly efficient as compared to AM transmission.

In FM transmission, all the transmitted power is useful ; whereas in AM transmission, most of the power goes waste in the transmitted carrier, which contains no useful information.

3. Due to a large number of sidebands, FM transmission can be used for the stereo sound transmission.

Disadvantages. The following are a few disadvantages of FM transmission :

1. The bandwidth in FM transmission is about 10 times as large as that needed in AM transmission. As a result, much wider frequency channel is required in FM transmission.

2. FM reception is limited to line-of-sight. Due to this, area of reception for FM is much smaller than that for AM.

3. FM transmitting and receiving equipments are very complex as compared to those employed in AM transmission.

1.11. DEMODULATION

The process of detection or demodulation is the inverse of modulation. In the process of detection, audio signal is separated from the modulated signal, which on being fed to the loud speaker or headphone, reproduces the original speech or music. The action of a diode as detector is basically the same as that of a half-wave rectifier.

Fig. 1.07 shows the basic circuit for using a junction diode as a detector. The input circuit consists of a parallel combination of an inductor L and a variable capacitor C, called *tuned circuit*. It helps to select the desired modulated radio frequency signal from the different signals picked up by the antenna. Due to its rectifying action, the output of the diode is a series of positive half cycles of radio frequency current pulses, whose peaks vary in accordance with the modulating audio signal. If this rectified output is fed to the headphone directly, its diaphram will vibrate with the high frequency of the carrier waves and therefore nothing would be heard. In order to extract the audio signal, the rectified output is fed to the parallel combination of the capacitor C′ and the resistor R. The value of the capacitance of the capacitor C′ is so chosen that its reactance to the high frequency carrier waves is low but is appreciable to the low frequency audio waves. As such, the capacitor C′ acts as a by-pass for carrier waves and the audio frequency voltage appears across the resistor R, which then sends the current through the head phone so as to produce original speech or music.

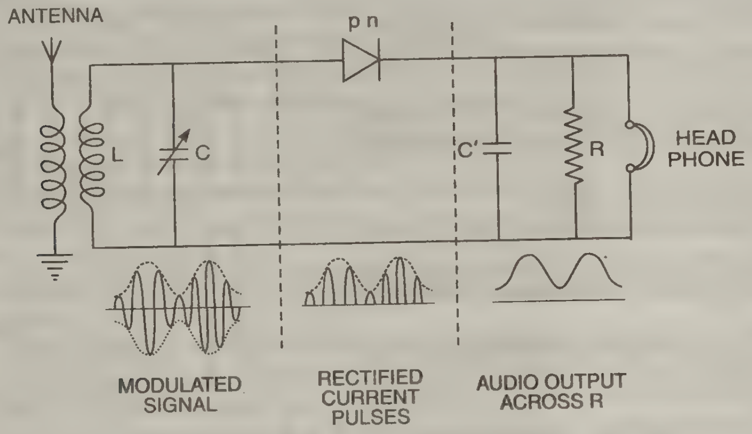


Fig. 1.07

1.12. BASIC ELEMENTS OF MODULATION AND DETECTION OF AN AUDIO SIGNAL

Fig. 1.08 illustrates the basic elements required to transmit and receive back an audio signal.

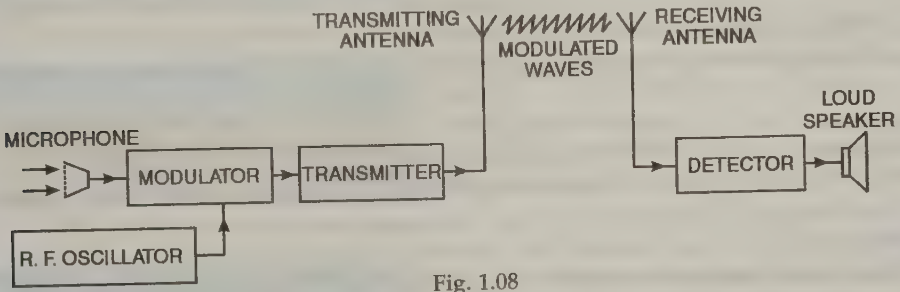


Fig. 1.08

A brief description of the various elements is as given below :

1. An oscillator generates radio frequency waves *i.e.* carrier waves.

2. A microphone converts the sound waves into electrical waves *i.e.* audio signal.

3. The modulator mixes up the audio signal with the carrier waves and feeds the modulated waves to the transmitter.

4. The transmitting antenna radiates the modulated waves into the space.

5. The receiving antenna intercepts the transmitted signal.

6. The detector demodulates (separates out) the audio signal from the modulated waves.

7. The loud speaker converts the demodulated audio signal back into the sound waves.

1.13. PULSE MODULATION

Analog modulation (both AM and FM modulations) are extensively employed for radio and TV transmission. As already said, in analog modulation, some parameter of sinusoidal carrier is varied in accordance with the instantaneous value of the modulating signal. In pulse modulation, the carrier is no longer a continuous sinusoidal wave but consists of pulse train. In this type of modulation also, some parameter of the carrier (a pulse train) is varied in accordance with the instantaneous value of modulating signal.

The pulse modulation is of the following two types :

1. **Pulse Amplitude Modulation (PAM).** In pulse amplitude modulation, amplitude of the pulses of the carrier pulse train is varied in accordance with the instantaneous value of modulating signal.

2. **Pulse Time Modulation (PTM).** In pulse time modulation, timing of the pulses of the carrier pulse train is varied in accordance with the instantaneous value of the modulating signal.

Pulse time modulation is, further, of the following two types :

(a) *Pulse Width Modulation* (PWM). In pulse width modulation, the width of the pulses of carrier pulse train is varied in accordance with the instantaneous value of the modulating signal. It is also called **Pulse Duration Modulation (PDM)** or **Pulse Length Modulation (PLM).**

(b) *Pulse Position Modulation* (PPM). In pulse position modulation, the position of the pulses of the carrier wave train is varied in accordance with the instantaneous value of the modulating signal.

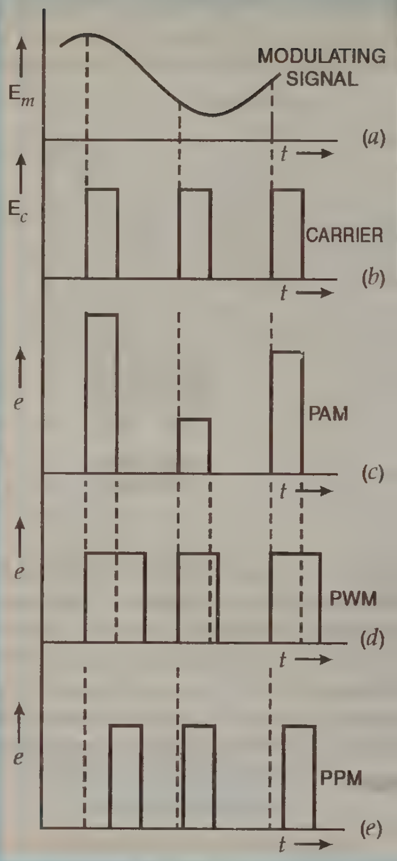


Fig. 1.09

Fig. 1.09 (*a*) shows a modulating signal, whose amplitude E_m is function of time. For the sake of simplicity, it is shown to have only a positive amplitude at different instants of time. In practice, however, the amplitude of the modulating signal may be positive as well as negative. The carrier is a train of pulses of fixed amplitude and spaced uniformly in time [Fig. 1.09 (*b*)].

The amplitude of the modulating signal is measured several times during each cycle of the signal. It is called ***sampling*** of the signal. From Fig. 1.09 (*a*), it follows that the amplitudes of the modulating signal at different sampling instants are different.

In PAM, the variation in the amplitude of each pulse of the carrier pulse train is according to the amplitude of the modulating signal at the sampling instant. Likewise, the PAM signal appears as shown in Fig. 1.09 (*c*). In PAM, the information about the baseband or modulating signal lies in the amplitude of a pulse.

Fig. 1.09 (*d*) shows the PWM signal, in which the width of each pulse changes in accordance with the amplitude of the modulating signal at the sampling instant. In PWM, the information about the baseband signal lies in the trailing edge of the pulse.

Finally, in PPM, the shift in the position of each pulse is according to the amplitude of the modulating signal at the sampling instant. The PPM signal obtained is as shown in Fig. 1.09 (*e*). In PPM, the information about the baseband signal lies in the leading edge of the pulse.

Key point

To achieve Pulse Code Modulation, an analog signal is sampled, quantised and then coded.

1.14. PULSE CODE MODULATION

The pulse amplitude, pulse width and pulse position modulations are not completely digital, as the amplitude, width or position of the pulses transmitted may vary continuously in accordance with variations in baseband signal. A completely digital modulation is obtained by **Pulse Code Modulation (PCM).** Since pulse amplitude modulation (PAM) is the simplest pulse modulation process, the PAM signals are used to obtain PCM signals. An analog signal is pulse code modulated by following the three operations explained as below :

Watch out !

In the sampling process (for the conversion of analog signal into digital signal), the analog signal is sampled at intervals of about 125 μs *i.e.* 8000 samples are taken every second !

1 ***Sampling.*** *It is the process of generating pulses of zero width (duration) and of amplitude equal to the instantaneous amplitude of the analog signal.*

Thus, in sampling process, the amplitude of analog signal is measured several times during each cycle of the signal. According to sampling theorem (known as Nyquist theorem), if a signal is bandlimited to B Hz (*i.e.* there are no frequency components beyond B Hz in the frequency spectrum of the signal), the sampling frequency must be atleast 2 B Hz. The number of samples taken per second is called ***sampling rate.***

For example, consider that voltage amplitude of an analog signal varies between 0 to 7 V as shown in Fig. 1.10. Suppose that when the signal is sampled,

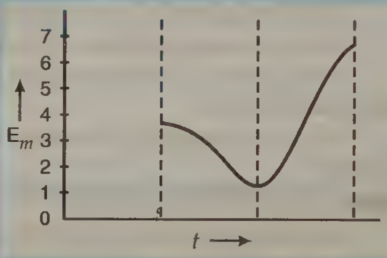


Fig. 1.10

its amplitude at three discrete times is measured to be 3·7 V, 1·3 V and 6·8 V respectively. The three sampled pulses of zero width having amplitudes 3·7 V, 1·3 V and 6·8 V have been shown separately in Fig. 1.11. As such, the analog signal is said to have become **pulse amplitude modulated (PAM).**

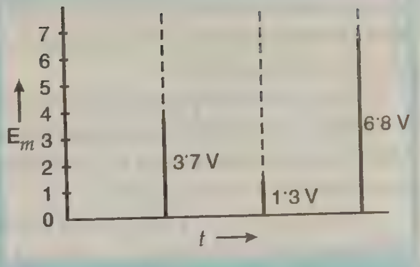


Fig. 1.11

2. ***Quantisation.*** *The process of dividing the maximum amplitude of the analog voltage signal into a fixed number of levels is called quantisation.*

Let us divide the amplitude (7 V) of the analog voltage signal into eight levels viz 0, 1, 2, 3, 4, 5, 6 and 7 V. These levels are called ***quantisation levels.*** Pulses having amplitude between – 0·5 V to 0·5 V are approximated (quantised) to a value 0 V, pulses having amplitudes between 0·5 to 1·5 V are approximated to a value 1 V and so on. Therefore, the amplitude of the three pulses will be approximated to values 4 V, 1 V and 7 V respectively. The pulses so obtained are called ***quantised pulses*** and are as shown in Fig. 1.12.

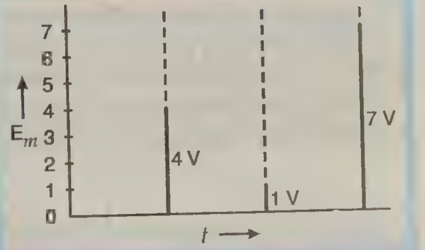


Fig. 1.12

3. Coding. *The process of digitising the quantised pulses according to some code is called coding.*

The most popular code is the binary code. Therefore, each quantised pulse is given a digital code, which is a binary number. The size of the binary number *i.e.* how many bits it has, depends on the number of quantisation levels. Since there are eight levels, a three-bit binary code will suffice. The three-bit binary code corresponding to various quantisation levels is as given in the adjoining table. Zeros have been added to the left of any number that is less than three bits.

Quantisation level	Binary code
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Therefore, The three quantised pulses having amplitudes 4 V, 1 V and 7 V will be coded as 100, 001 and 111 respectively. These coded values are combination of binary digits (bits) 0 and 1. As shown in Fig. 1.13, all the pulses have the same amplitude but possess different lengths. Each binary digit is allocated a fixed period of time. The length of the pulse is the time, for which it is high or low. Binary zeros (0) are represented by 0 volt and binary ones (1) by a fixed voltage (oftenly +5 volt is used). A series of several consecutive 1's or 0's represents a longer pulse. The signal is now said to be **pulse code modulated (PCM).**

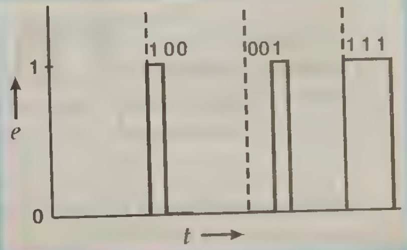


Fig. 1.13

Key point

In case the amplitude of analog signal is divided into 16 quantisation levels, then a 4-bit binary code is used to code the quantised pulses.

An analog signal is described by its `frequency range'. This term has no meaning for a digital signal. Instead, a digital signal is described by the ***bit rate*** *i.e.* number of bits per second (binary digits per second).

Key point

A digital signal is described by the bit rate *i.e.* number of binary digits (bits) per second.

To calculate the bit rate, one should know the sampling rate and the number of quantisation levels. The sampling rate is the number of samples per second. The levels determine the binary code and the number of bits per sample required. Therefore,

$$\text{bit rate} = \text{sampling rate} \times \text{number of bits per sample} \qquad \text{...(1.16)}$$

1.15 DATA TRANSMISSION AND RETRIEVAL

The pulse code modulated signal is a series of 1's and 0's. If they are to be transmitted over copper wires, they can be directly transmitted as two voltage levels, +V corresponding to 1 and – V corresponding to 0. But if they are to be transmitted through space, then some form of modulation has to be employed. Just as in case of analog modulation, the following three modulation techniques are used to transmit a PCM signal :

1. Amplitude Shift Keying (ASK). In amplitude shift keying, two different amplitudes of the carrier represent the two binary values of the PCM signal. The binary value 1 is represented by the presence of carrier of some constant amplitude, while the binary value 0 is represented by the absence of carrier *i.e.* by carrier of zero amplitude. For this reason, this method is also known as **On-Off Keying (OOK).**

2. Frequency Shift Keying (FSK). In frequency shift keying, the binary values of the PCM signal are represented by two frequencies, which are offset from the carrier frequency f_c by equal and opposite amounts f and $-f$. The frequency $f_c + f$ represents the binary value 1, while the frequency $f_c - f$ represents the binary value 0.

3. Phase Shift Keying (PSK). In phase shift keying, the binary value of 0 corresponds to zero phase shift and the binary value 1 corresponds to a phase shift of π.

Fig. 1.14 (*a*) shows a typical PCM signal 110100100 and Fig. 1.14 (*b*) represents the waveform of modulated signal due to ASK method. From the figure, it follows that carrier is present, when binary value of the signal is 1 and is absent, when binary value of the signal is 0.

Fig. 1.14 (*c*) represents the waveform of modulated signal achieved by FSK method. It follows that frequency of the carrier increases by a certain amount, when binary value of the signal is 1 and decreases by an equal amount corresponding to binary value 0 of the signal.

Finally, Fig. 1.14 (*d*) shows the waveform of modulated signal obtained by PSK method. The carrier is without any phase change corresponding to binary value 0 of the signal and its phase changes by π, whenever the binary value of the signal is 1.

It may be pointed out that FSK method is preferred as it gives optimum performance in the presence of noise. The ASK method is rarely used as it does not give a satisfactory value for probability of error.

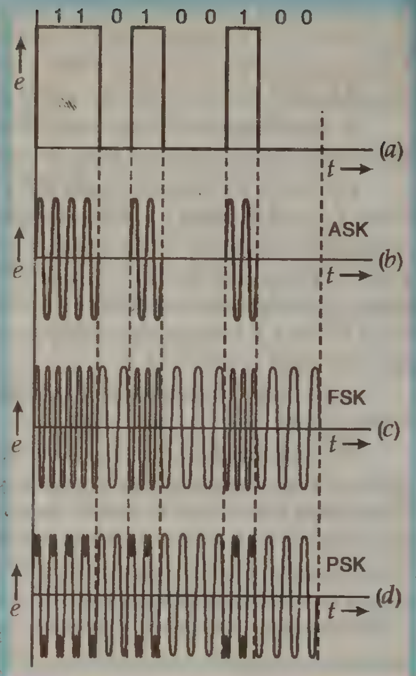


Fig. 1.14

Fig. 1.15 shows the block diagram of the trasmitter of a PCM communication system for the transmission of digital data.

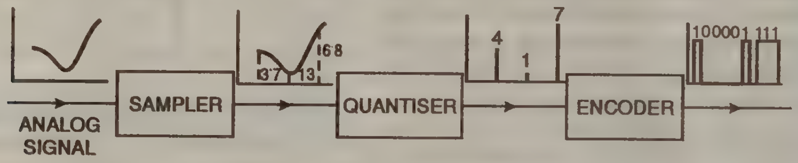


Fig. 1.15

The analog signal is sampled by the sampler. The sampled pulses are then quantised by the quantiser. The encoder codes the quantised pulses according to the binary code. After modulating the PCM signal by one of the three methods viz ASK, FSK or PSK method, the modulated signal is, then, transmitted into free space in the form of bits.

The Jargon

The encoder is also called Analog to Digital (A/D) Converter.

Fig. 1.16 shows the block diagram for the receiver of a PCM communication system for retreiving the digital data.

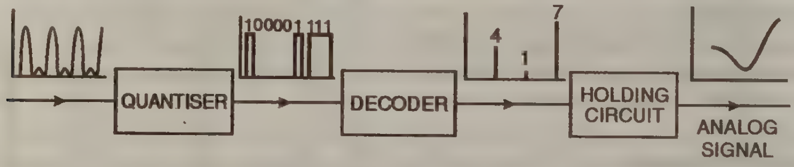


Fig. 1.16

The received signal is first quantised by the quantiser of the receiver. This quantiser is differnt from the quantiser of the transmitter. It is because, it has to take a decision about the presence or absence of a pulse. Likewise, there are only two quantisation levels. The output of the quantiser is PCM signal, which then goes to decoder. It performs the inverse operation of the encoder. The output of the decoder is a sequence of quantised pulses. To reconstruct the original analog signal, a typical circuit, called holding circuit is used.

The Jargon

The decoder is also known as Digital to Analog (D/A) Converter.

1.16 MERITS OF DIGITAL COMMUNICATION

The following are a few merits of digital communication :

1. Digital signals are easy to receive. All the receiver needs to do is to detect, whether a pulse is high or low.

2. AM and FM signals become corrupted over much short distances as compared to digital signals. From a corrupted AM or FM signal, the audio signal cannot be reproduced faithfully. However, in case of digital signals, as long as the receiver can distinguish between `high' and `low', the original signal can be reproduced accurately.

3. The signals lose power as they travel. This is called ***attenuation***. When AM and FM signals are amplified to restore power, the noise is amplified as well and the quality of the signal is lowered. However, a digital signal can be `cleaned up' to restore quality and amplified at different stages by ***regenerators*** to restore the power.

Key point

In a digital signal, the information is carried by the pattern of pulses and not by the shape of the pulses..

4. The digital signals do not get distorted by the noise. The noise may change the shape of the pulses, but not the pattern of the pulses. It is the pattern of the pulses that carries information in the digital signals.

5. Once an AM or FM signal is transmitted, anyone with a suitable receiver can listen in. However, the digital signals can be coded so that only the person, they are intended for, can receive them.

6. Both AM and FM transmissions constitute `real time systems', which means that the signal can be received only at the time, it is transmitted. However, digital signals can be stored in the absence of the person at the receiving end.

7. The digital signals can be stored as digital data or used to produce a display on a computer monitor or converted back into analog signal to drive a loud speaker.

1.17. MODEM

The name ***modem*** is a contraction of the term ***mo***dulator and ***dem***odulator. As the name suggests, a modem performs functions of both modulator and demodulator. In the transmitting mode, modem acts as a modulator and in the receiving mode, it acts as a demodulator.

Fig. 1.17 shows the communication circuit between a transmitting digital device (such as a business machine or a terminal) and a receiving digital device (such as a computer) through a telephone line. When telephone line (analog medium) is used for data communication between the two digital devices, two modems are required, one near each digital device as shown in the figure.

Key point

A modem acts as a modulator in transmitting mode and as a demodulator in receiving anode.

The jargon

An input/output device, which allows a user to communicate directly with a computer system is called a terminal.

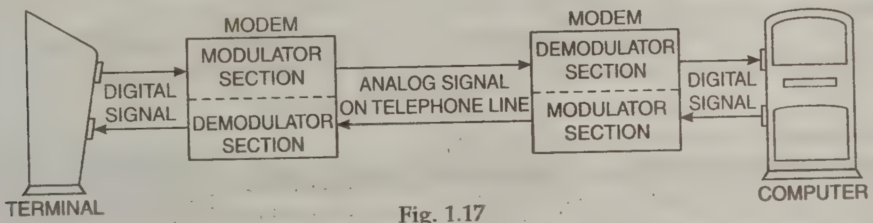


Fig. 1.17

Since digital signals can not be transmitted along the telephone line, the digital data has to be converted into analog signal. The modem placed near the transmitting device (terminal) converts the digital data from the terminal into analog form and then transmits the audio signal after modulating it with carrier along the telephone line. The demodulator section of the modem placed near the receiving device (computer) demodulates the carrier to retrieve the data. This digital data is processed by the computer and the processed data is modulated to the analog form by the modem and returned via the telephone line to the terminal, where the analog signals are demodulated by the modem to digital form.

Modems are described depending upon their mode of operation. The modems can be used to transmit and retrieve the data in the following three modes :

1. **Simplex mode.** *It is mode of using a modem, in which data signals can be transmitted between the transmitter and the receiver in only one direction.*

In this mode, the modem operates as either a send-only or a receive-only device. As in this mode, the modem uses only one transmission channel, no signalling can be made from the receiver to the transmitter so as to send acknowledgement or request for retransmission of data.

2. **Half-duplex.** *It is the mode of using a modem, in which data signals can be transmitted between the transmitter and the receiver in both directions, but only in one direction at a time.*

In this mode, the modem makes use of one transmission channel, but the channel must be bidirectional. Further, in this mode, the line must be `turned around' each time the direction of transmission of data is reversed. This involves a special switching circuit and consumes a small amount of time. As a result, the speed of data transfer reduces. It is because, the same circuit has to be shared for transmission of data in the opposite direction.

3. **Full-duplex.** *It is the mode of using a modem, in which data signals can be transmitted between the transmitter and the receiver in both directions at the same time.*

In this mode, the efficiency improves as the line turn-around time required in the half-duplex arrangement is eliminated. In this mode, two circuits are required, either two 2-wire circuits or one 4-wire circuit. As shown in Fig. 1.17, modems are placed at each end of the circuit to provide modulation and demodulation.

It is worthwhile to note that whereas the use of modems allows higher speeds of transmission of digital data on a given analog line, it also reduces the effects of noise and distortion.

1.18. FAX (FACSIMILE)

The electronic transmission of a document at a distant place via telephone line is known as facsimile or FAX.

Fig. 1.18 is the block diagram showing the function of various parts of a FAX machine to produce a duplicate copy of the document at the receiver end.

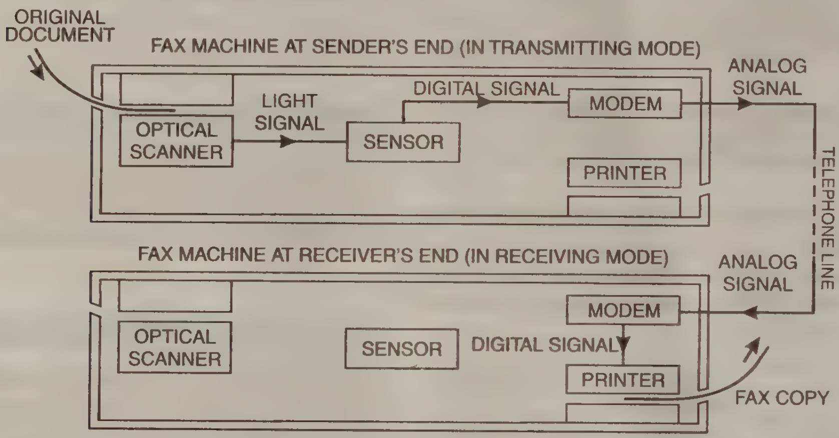


Fig. 1.18

In order to send FAX copy of some document, the sender dials in the phone number of the FAX machine at the receiver's end. The FAX machine at the sender's end is, then, in ***transmitting mode*** and that at the receiver's end in ***receiving mode***. When a FAX machine is in receiving mode, only its printer is active so as to accept the incoming signal through the modem inside it. When the document is fed into the machine, the optical scanner scans the document in the form of tiny parallel strips in turn. When an intense beam of light from a source (not shown in the figure) falls on the scanned document, the amount of light that bounces back is detected by the sensors. Where ink is present, little light is reflected creating an electrical pulse of low voltage. A pulse of high voltage results, when light is reflected from white paper. A modem inside the machine converts these digital pulses into analog signals which is then transmitted to the receiving FAX machine via the telephone line. The modem in the FAX machine at the receiving end converts the analog signals back into digital pulses. The printer of the FAX machine prints these digital pulses faithfully in the form of dots to build up a facsimile copy of the original document. Each FAX machine has its own number, which is dialed in via the sender's FAX machine.

SOLVED NUMERICAL QUESTIONS

Type A. On Analog modulation

Problem 1.01. An audio signal of amplitude one-half the carrier amplitude is employed in amplitude modulation. What is the modulation index ?

Sol. Here, $E_m = 0{\cdot}5\,E_c$

$\therefore \quad E_{max} = E_c + 0{\cdot}5\,E_c = 1{\cdot}5\,E_c$

and $\quad E_{min} = E_c - 0{\cdot}5\,E_c = 0{\cdot}5\,E_c$

$$\text{Now, } m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{1{\cdot}5\,E_c - 0{\cdot}5\,E_c}{1{\cdot}5\,E_c + 0{\cdot}5\,E_c} = \frac{E_c}{2\,E_c} = \mathbf{0{\cdot}5}$$

Problem 1.02. In an amplitude modulator, the tank circuit consists of a coil of inductance 3·6 mH and a capacitor of capacitance 2·5 pF. If an audio signal of frequency 15 kHz modulates the carrier generated by the tank circuit, find the frequencies of the sidebands.

Sol. Here, $L = 3{\cdot}6$ mH $= 3{\cdot}6 \times 10^{-3}$ H ;

$C = 25$ pF $= 2{\cdot}5 \times 10^{-12}$ F ; $f_m = 15$ kHz

The frequency of carrier generated by the tank circuit,

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{3{\cdot}6 \times 10^{-3} \times 2{\cdot}5 \times 10^{-12}}}$$

$$= 1{\cdot}678 \times 10^6 \text{ Hz} = 1678 \text{ kHz}$$

Frequency of upper sideband,

$$f_c + f_m = 1678 + 15 = \mathbf{1693\ kHz}$$

and frequency of lower sideband,

$$f_c - f_m = 1678 - 15 = \mathbf{1663\ kHz}$$

Problem 1.03. An audio signal of 3·2 kHz modulates a carrier of frequency 84 MHz and produces a frequency deviation of 96 kHz. Find (*a*) frequency modulation index and (*b*) frequency range of the frequency modulated wave.

Sol. Here $f_m = 3{\cdot}2$ kHz ; $f_c = 84$ MHz $= 84 \times 10^3$ kHz

frequency deviation, $\delta = 96$ kHz

(*a*) Now, frequency modulation index,

$$m_f = \frac{\delta}{f_m} = \frac{96}{3\cdot 2} = 30$$

(*b*) Frequency range of the modulated wave

$$= f_c \pm f_m = 84 \times 10^3 \pm 3\cdot 2 \text{ kHz}$$
$$= 83\cdot 997 \times 10^3 \text{ kHz to } 84\cdot 003 \times 10^3 \text{ kHz}$$
$$= \mathbf{83\cdot 997 \text{ MHz to } 84\cdot 003 \text{ MHz}}$$

Type B. On Pulse modualtion

Problem 1.04. For a 4-bit D/A converter, work out the output voltage for each of the following input voltage combinations : (*i*) 0001 (*ii*) 0010 (*iii*) 0100 (*iv*) 1000 (*v*) 1001 and (*vi*) 1101. (*Text Problem*)

Sol. Suppose that binary values 0 and 1 of the digital signal correspond to 0 V and 1 V respectively.

(*i*) For input voltage 0001 :

$$\text{Output voltage} = 1 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 0 \times 2^3$$
$$= 1 + 0 + 0 + 0 = \mathbf{1 \text{ V}}$$

(*ii*) For input voltage 0010 :

$$\text{Output voltage} = 0 \times 2^0 + 1 \times 2^1 + 0 \times 2^2 + 0 \times 2^3$$
$$= 0 + 2 + 0 + 0 = \mathbf{2 \text{ V}}$$

(*iii*) For input voltage = 0100 :

$$\text{Output voltage} = 0 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 0 \times 2^3$$
$$= 0 + 0 + 4 + 0 = \mathbf{4 \text{ V}}$$

(*iv*) For input voltage 1000 :

$$\text{Output voltage} = 0 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3$$
$$= 0 + 0 + 0 + 8 = \mathbf{8 \text{ V}}$$

(*v*) For input voltage 1001 :

$$\text{Output voltage} = 1 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3$$
$$= 1 + 0 + 0 + 8 = \mathbf{9 \text{ V}}$$

(*vi*) For input voltage 1101 :

$$\text{Output voltage} = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3$$
$$= 1 + 0 + 4 + 8 = \mathbf{13 \text{ V}}$$

Problem 1.05. Pulse code modulation is also used for digital audio recording. Some compact discs used a 32-bit code. How many quantisation levels would this give ?

Sol. The number of quantisation levels = 2^n, where n is the number of bits per sample.

Since compact discs used 32-bit code, $n = 32$

Therefore, number of quantisation levels

$$= 2^{32} = \mathbf{4\cdot 3 \times 10^9}$$

Problem 1.06. Calculate the bit rate for a signal, which has a sampling rate of 8 kHz and where 16 quantisation levels have been used.

Sol. If n is the number of bits per sample, then number of quantisation level = 2^n

Since the number of quantisation levels is 16,

$$2^n = 16 \quad \text{or} \quad n = 4$$

Now, bit rate = sampling rate × number of bits per sample

$$= 8000 \times 4 = \mathbf{32{,}000 \text{ bits s}^{-1}}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. What do you mean by communication ?

Ans. *The communication is the processing, sending and receiving of information with the help of a suitable communication system and transmission medium.*

Q. 1.02. What is a transducer ? Give one example.

Ans. *A device which converts energy in one form to another is called a transducer.*

In radiocommunication, microphone is used to convert sound signal into electrical signal. Here, the microphone acts as a transducer.

Q. 1.03. What is an analog communication system ?

Ans. *The communication system, which makes use of analog signals (the signal which varies continuously with time), is called analog communiction system.*

Q. 10.4. What is a digital communication system ?

Ans. *The communication system, which makes use of digital signal (the signal, which has only two values-either high or low), is called digital communication system.*

Q. 1.05. What is modulation ?

Ans. *The process of producing a high frequency wave, some characteristic of which varies as a function of the instantaneous value of the audio signal, is called modulation.*

Q. 1.06. What is modulated wave ?

Ans. *The signal wave that results on superimposing the modulating wave on carrier wave is called modulated wave.*

Q. 1.07. Define amplitude modulation.

Ans. *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the frequency of the modulated wave is same as that of the carrier wave but its amplitude is modified in accordance with that of the modulating wave, the process is called amplitude modulation.*

Q. 1.08. What is modulation index ?

Ans. *The degree, to which the carrier wave is modulated, is called modulation index.*

Q. 1.09. What is the importance of modulation index ?

Ans. The modulation index determines the strength and quality of the transmitted signal. When modulation index is high, the reception will be strong and clear.

Q. 1.10. What do you mean by bandwidth ?

Ans. *The frequency range, in which a transmitting system makes transmission, is called bandwidth.*

Q. 1.11. Give the expression for band width in AM transmission.

Ans. In AM transmission,

band width = 2 × maximum frequency of the audio signal

Q. 1.12 What is frequency modulation ? (*Text Question*)

Ans. *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the amplitude of the modulated wave is same as that of the carrier wave but its frequency is modified in accordance with the amplitude of the modulating wave, the process is called frequency modulation.*

Q. 1.13. What do you mean by frequency deviation ?

Ans. *The maximum swing of the frequency of modulated wave from the carrier frequency is called frequency deviation.*

Q. 1.14. Give the expression for bandwidth in FM modulation.

Ans. In FM transmission,

bandwidth = 2 n × frequency of modulating signal, where n is the number of significant sidebands.

Q. 1.15. Which of the two–AM or FM is preferred for high fidelity reception ?

Ans. Frequency modulation is preferred for high fidelity reception.

Q. 1.16. What is noise ?

Ans. *The undesired electrical signals are termed as noise.*

When a signal is transmitted, the undesired signals get mixed with it leading to distortion of the signal.

Q. 1.17. How can you decrease noise in FM receivers ?

Ans. Noise can be decreased in FM receivers by increasing the frequency deviation.

Q. 1.18. What is demodulation ?

Ans. *The process of separating out the audio signal from the modulated signal is known as demodulation.* The process of demodulation is the inverse of modulation.

Q. 1.19. Define pulse amplitude modulation.

Ans. *The process of modulation, in which amplitude of the pulses is varied in according with the modulating signal, is called pulse amplitude modulation.*

Q. 1.20. What do you mean by quantisation ?

Ans. *The process of dividing the maximum value of the analog voltage signal into a fixed number of levels in order to convert a pulse amplitude modulated signal into a binary code is calle quantisation.*

Q. 1.21. What do you mean by sampling of an analo signal ?

Ans. *The proces of generating pulses of zero width (duration and of amplitude equal to the instantaneous amplitude of the analo signal is called sampling.*

Q. 1.22. How is bit rate related to the sampling rate

Ans. Bit rate = sampling rate × number of bits per sampl

Q. 1.23. AM and FM transmission are said to be `rea time systems'. What does it imply ?

Ans. It implies that the AM and FM signals can b received only at the time it is transmitted.

Q. 1.24. What is a modem ?

Ans. A modem performs functions of both modulato and demodulator. In the transmitting mode, modem acts a a modulator, while in the receiving mode, it acts as demodulator.

Q. 1.25. What is FAX ?

Ans. *The electronic transmission of a document at a distan place via telephone line is called FAX.*

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 1.01. Do the telegraphy and telemetry mean the same communication system ? Explain.

Ans. Refer to section 1.01.

Q. 1.02. The audio signal can not be transmitted directly into the space. Why ?

Ans. 1. The length of the antenna required is so large (≈ 500 m) that it is practically impossible to set up it.

2. The energy radiated from the antenna in audio frequency range is practically zero.

3. The audio signals transmitted from the different broadcasting stations will get inseparably mixed up.

Q. 1.03. A radio broadcast is transmitted using *amplitude modulation* at a *carrier frequency* of 680 kHz. Explain the meaning of each of the italicised words.

(*Text Question*)

Ans. Amplitude modulation. Refer to VSQ 1.07.

Carrier frequency. The message signals in low frequency range cannot be efficiently radiated into space and die out over a small distance. This difficulty is overcome by modulating the message signal with waves of high frequency. Such waves of high frequency are called carrier waves and their frequency as carrier frequency.

Q. 1.04. What is a carrier wave ? Why high frequency carrier waves are employed for transmission ?

(*Text Question*)

Ans. Carrier wave. Refer to SAQ. 1.03.

As the message signal propagates through the space, it gets attenuated with distance, so that the transmission can be done over a large distance, the message signal is modulated with high frequency waves, called carrier waves.

Q. 1.05. What is the need for modulation ?

Ans. Refer to section 1.07.

Q. 1.06. Why should transmitters broadcasting programmes use different carrier frequencies ?

(*Text Question*)

Ans. If all the transmitters broadcast programmes b using same carrier frequency, then it will not be possible t tune the receiver to a particular transmission. Further, as th frequency spectrum of all the message signals are identica they will get inseparably mixed up.

Q. 1.07. What are the limitations of amplitude modula tion ?

Ans. 1. The quality of audio signal is not good.

2. The atmospheric and industrial noise affect the audi signal.

3. The efficiency of amplitude modulation is low.

Q. 1.08. What are the advantages of FM transmission

Ans. 1. FM reception is quite immune to noise a compared to AM reception.

2. It gives high fidelity reception due to large number o sidebands.

3. The efficiency of FM transmission is high.

Q. 1.09. What are the advantages of AM over FM transmission ?

Ans. 1. FM transmission requires much wider channe as compared to AM transmission.

2. The reception of FM signal is limited to line-of-sight As a result, the area in which FM transmission can be receive is much smaller than in case of AM transmission.

Q. 1.10. What is a noise ? What are their causes ?

Ans. *The undesired electrical signals are termed as noise.*

The noises are of two types :

Internal noise. The undesired electrical signals arisin due to thermal motion of electrons within the system (due t their random motion, recombination or diffusion) are terme as internal noise.

External noise. The undesired electrical signals arisin due to nearby radio frequency transmitters, electrical circuits sparks, fluroscent lights, etc are termed as external noise.

Q. 1.11. Why is an FM signal less susceptible to noise than an AM signal ? *(Text Question)*

Ans. In FM transmission, the message signal is in the form of frequency variations of carrier waves. During modulation process, the noise gets amplitude modulated. Since it brings about variations only in the amplitude of carrier waves, it does no harm to the message signal. For this reason, FM signal is less susceptible to noise than an AM signal is.

Q. 1.12. What do you mean by pulse amplitude modulation and pulse code modulation ?

Ans. Refer to sections 1.13 and 1.14.

Q. 1.13. What is the difference between bit rate and sampling rate ? Give the relation between the two.

Ans. Refer to section 1.14.

Q. 1.14. Give three main merits of digital communication.

Ans. 1. The digital signals do not get distorted by the noise. It is because, noise may change the shape of the pulse, but not the pattern of the pulses.

2. Digital signals are very easy to receive.

3. The digital signals can be coded.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is an antenna ? What role does it play in a communication system ? What should be the length of a dipole antenna ? *(Text Question)*
2. What is an analog signal ? Explain the terms band-width and baseband. *(Text Question)*
3. What is the need for modulation ?
4. Explain the term modulation ? *(Text Question)*
5. What is amplitude modulation ? Discuss its advantages and disadvantages. *(Text Question)*
6. What is frequency modulation ? Give its advantages and disadvantages ?
7. Distinguish between amplitude and frequency modulation.
8. Give the advantages of FM transmission over AM transmission.
9. Explain the term demodulation. *(Text Question)*
10. Describe, how a diode may be used as a detector.
11. What is pulse modulation ? Describe in brief.
12. What are the merits of digital communication ?
13. With the help of a block diagram, explain how digital data is transmitted and retrieved.
14. Explain the principle of a modem.
15. What is a FAX machine ? Explain its principle.
16. Explain the term FAX ? Distinguish between a FAX and an e-mail. *(Text Question)*

FREQUENTLY ASKED LONG ANSWER QUESTIONS

Carrying 5 Marks

1. What is a communication system ? Describe briefly the major constituents of a communication system. *(Text Question)*
2. What is modulation ? What is the need for modulation ?
3. What is meant by modulation of carrier wave ? Explain amplitude modulation. *(H.P.S.S.C.E. 2001)*
4. Explain frequency modulation. Discuss its advantages over amplitude modulation.
5. What is a discrete signal ? Explain briefly how an analog signal can be converted into a digital signal. Enumerate some of the advantages of digital communication. *(Text Question)*
6. What is pulse code modulation ? Explain, how an analog signal may be converted into a digital signal.
7. What is meant by pulse code modulation ? What are its advantages of digital amplitude modulation ? *(Text Question)*
8. What is a modem ? Explain its role in data communication. *(Text Question)*

FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Antenna length

1. Calculate the length of half wave dipole at (*a*) 30 MHz, (*b*) 300 MHz and (*c*) 3000 MHz. What inference do you draw from these results ? *(Text Problem)*
   [Ans. (*a*) 5 m, (*b*) 50 cm, (c) 5 cm, length of dipole decreases as frequency increases)]

Type B. On Analog modulation

2. An audio signal of amplitude (*a*) equal to (*b*) 1·5 times the carrier amplitude is employed in amplitude modulation. Find the modulation index in each case.
   [Ans. (*a*) 1 (*b*) 1·5]
3. An audio signal in frequency range 370 Hz to 2·5 kHz modulates a carrier of frequency 80 kHz. Find the range of (*a*) upper sideband and (*b*) lower sideband.
   [Ans. (*a*) 80·37 to 82·5 kHz (*b*) 77·5 to 79·63 kHz]
4. An audio signal of 2·8 kHz modulates a carrier of frequency 90 MHz producing a frequency deviation of 84 kHz. Find (*a*) frequency modulation index and (*b*) frequency range of the modulated wave.
   [Ans. (*a*) 30 (*b*) 89·916 to 90·084 MHz]

Type C. On Pulse modulation

5. (*a*) How many bits are needed to represent the following decimal numbers as binary number ? (*i*) 16, (*ii*) 32 and (*iii*) 64.
   (*b*) Each of the decimal numbers in (*a*) can be written as 2^n, where *n* is a whole number. Show that the answers to (*a*) can all be written as $(n + 1)$.
   [Ans. (*a*) (*i*) 5, (*ii*) 6, (*iii*) 7]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. *(a) When f = 30 MHz :*

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{30 \times 10^6} = 10 \text{ m}$$

Therefore, length of half wave dipole,

$$l = \frac{\lambda}{2} = \frac{10}{2} = \mathbf{5\ m}$$

(b) When f = 300 MHz :

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{300 \times 10^6} = 1 \text{ m}$$

Therefore, length of half wave dipole,

$$l = \frac{\lambda}{2} = \frac{1}{2} = 0 \cdot 5 \text{ m } = \mathbf{50\ cm}$$

(c) When f = 3000 MHz :

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3000 \times 10^6} = 0 \cdot 1 \text{ m}$$

Therefore, length of half wave dipole,

$$l = \frac{\lambda}{2} = \frac{0 \cdot 1}{2} = 0 \cdot 05 \text{ m } = \mathbf{5\ cm}$$

It follows that as the frequency of the carrier increases, the length of the half wave dipole decreases.

2. (*a*) Here, $E_m = E_c$

$$\therefore \quad E_{max} = E_c + E_c = 2\,E_c \text{ and } E_{min} = E_c - E_c = 0$$

$$\text{Now, } m_a = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{2\,E_c - 0}{2\,E_c + 0} = \mathbf{1}$$

(*b*) $E_m = 1{\cdot}5\ E_c$

$$\therefore \quad E_{max} = E_c + 1{\cdot}5\ E_c = 2{\cdot}5\ E_c$$

$$\text{and } \quad E_{min} = E_c - 1{\cdot}5\ E_c = -\,0{\cdot}5\ E_c$$

$$\text{Now, } m_a = \frac{2 \cdot 5\,E_c - (-\,0 \cdot 5\,E_c)}{2 \cdot 5\,E_c - 0 \cdot 5\,E_c} = \mathbf{1{\cdot}5}$$

3. Frequency range of audio signal,

f_m = 370 Hz to 2·5 kHz
= 0·37 kHz to 2·5 kHz

Carrier frequency, f_c = 80 kHz

(*a*) Frequency range of upper side band
= 80 + 0·37 to 80 + 2·5
= **80·37 kHz to 82·5 kHz**

(*b*) Frequency range of lower side band
= 80 – 2·5 to 80 – 0·37
= **77·5 kHz to 79·63 kHz**

4. Here, $f_m = 2{\cdot}8$ kHz, $f_c = 90$ MHz $= 90 \times 10^3$ kHz, $\delta = 84$ kH

(*a*) Here, $m_f = \dfrac{\delta}{f_m} = \dfrac{84}{2 \cdot 8} = \mathbf{30}$

(*b*) Frequency range of modulated wave
$= f_c + f_m = 90 \times 10^3 \pm 2{\cdot}8$ kHz
$= 89{\cdot}997 \times 10^3$ kHz to $90{\cdot}003 \times 10^3$ kHz
= **89·997 MHz to 90·003 MHz**

5. (*a*) (*i*) ***To find bits in decimal number 16 :***

$$\begin{array}{c|l} 2 & 16 \\ \hline 2 & 8-0 \\ \hline 2 & 4-0 \\ \hline 2 & 2-0 \\ \hline 2 & 1-0 \\ \hline & 0-1 \end{array}$$

The remainders in reverse order are
1, 0, 0, 0, 0.

Therefore, number of bits in decimal numbe 16 are **5**.

(*ii*) ***To find bits in decimal number 32 :***

Proceeding as above, it can be obtained that remainders in reverse order are 1, 0, 0, 0, 0, 0,.

Therefore, number of bits in decimal number 32 is **6**.

(*iii*) ***To find bits in decimal number 64 :***

Proceeding as above, it can be obtained that remainders in reverse order are 1, 0, 0, 0, 0, 0, 0.

Therefore, number of bits in decimal number 64 is **7**.

(*b*) The each of the three given decimal numbers, say N can be represented as

$$N = 2^n$$

(*i*) Now, $16 = 2^n$ or $n = 4$

Therefore, number of bits in decimal number 16, which is 5, is equal to 4 + 1 *i.e.* $\boldsymbol{n + 1}$.

(*ii*) Now, $32 = 2^n$ or $n = 5$

Therefore, number of bits in decimal number 32, which is 6, is equal to 5 + 1 *i.e.* $\boldsymbol{n + 1}$.

(*iii*) Now, $64 = 2^n$ or $n = 6$

Therefore, number of bits in decimal number 64, which is 7, is equal to 6 + 1 *i.e.* $\boldsymbol{n + 1}$.

CHAPTER 2

Space Communication

2.01. EARTH'S ATMOSPHERE

The gaseous envelope surrounding the earth is called earth's atmosphere.

At sea level, by volume, dry air contains 78·08% nitrogen, 20·95% oxygen, ·93% argon, 0·03% carbon dioxide, 0·0018% neon and traces of gases, such as elium, kyrpton and xenon. In addition to the above, air contains water vapours, ydro-carbons, hydrogen peroxide, sulphur compounds and dust particles. The ompo-sition of earth's atmosphere varies very slightly with altitude. It remains ame up to a height of 100 km, but its density goes on decreasing as we go up. The arth's atmosphere has no sharp boundary. It has been divided into a number of egions (or layers) as shown in Fig. 2.01. They are as explained below :

1. Troposphere. This region extends up to 12 km from the surface of the earth. ts density varies from 1 kg m^{-3} at the surface of earth to 0·1 kg m^{-3} at the top of his layer. All the water vapours of the atmosphere are contained in this layer. In his part of atmosphere, temperature decreases with height from 290 K to 220 K.

2. Stratosphere. This region of atmosphere extends from 12 km from the surface of earth upto 50 km and it forms practically clear sky. The density of atmosphere in this region varies from 0·1 kg m^{-3} to 10^{-3} kg m^{-3} and the temperature varies from 220 K to 280 K. At the upper extreme of this region, there is a layer of ozone between 30 to 50 km from the surface of earth, in which most of the atmospheric ozone is concentrated. Ozone layer absorbs a large proportion of ultra-violet radiation radiated by the sun and therefore it protects the life on earth from its harmful effects. The ozone layer is also called ***ozonosphere***.

3. Mesosphere. The region of earth's atmosphere between 50 to 80 km from the surface of earth is called mesosphere. The density of the atmosphere in this region varies from 10^{-3} kg m^{-3} to 10^{-5} kg m^{-3} and the temperature varies from 290 K to 180 K.

4. Ionosphere. This region of earth's atmosphere extends from 80 km to 400 km from the earth's surface. This is the outermost part of the atmosphere and it receives maximum energy from the sun. As we go up, the temperature increases with altitude from 180 K to 700 K and the density of this region varies from 10^{-5} kg m^{-3} to 10^{-10} kg m^{-3}.

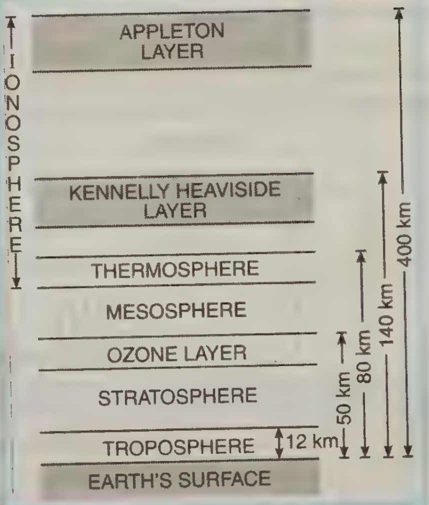


Fig. 2.01

Due to low pressure in this part of atmosphere, ionisation is produced by ultra-violet radiation and X-rays from the sun. As a result, the concentration of electrons and positive ions is very large in this region and for this reason, it is called *ionosphere*. At about 90 km from earth's surface, the electron concentration becomes very large and extends vertically up to 140 kilometres approximately. This layer of electrons is known as ***Kennelly Heaviside layer***. The electron density, then, decreases considerably ; until at a height of about 250 km, a belt of electrons is again met. It is known as ***Appleton layer***. It extends up to 400 km.

Key point ✓

The electromagnetic waves of radio frequencies are reflected by ionosphere and this effect is used for transmission of radio signals round the curved surface of the earth.

The ionosphere plays an important role in the transmission of radiowaves round the curved surface of earth. It is because of its property of reflecting electromagnetic waves of radio frequencies. However, it proves an obstacle in radioastronomy, as it reflects a large part of radiowaves from the stars.

2.02. PROPAGATION OF ELECTROMAGNETIC WAVES

A stationary electric charge produces a static electric field around it. A steady current implies the uniform flow of electric charges. Since a steady electric current produces a steady magnetic field around it, it follows that an electric charge in uniform motion produces a steady or stationary electric field. On the other hand, an accelerated charge produces a magnetic field, which varies with time and space. Since an electromagnetic wave is associated with a magnetic field, which is dependent on time and space, it follows that an accelerated charge is the source of electromagnetic waves. The most common way of possessing accelerated motion is to possess simple harmonic motion. Therefore, a charge oscillating harmonically with a frequency f produces electric and magnetic fields at that point, which vary sinusoidally with the frequency f and then produce electromagnetic waves of the same frequency.

The electromagnetic waves propogate in the form of varying electric and magnetic fields, which are mutually perpendicular to each other.

The variations in electric and magnetic fields in an electromagnetic wave are perpendicular to each other and to the direction of propagation of the wave. As said earlier, electromagnetic waves do not require any material medium and they propagate in free space with the velocity of light.

In 1865, Maxwell had predicted the propagation of electromagnetic waves in the form of varying electric and magnetic fields, which produce each other. It was concluded that the accelerated charge is the source of electromagnetic waves. In 1887, Hertz experimentally demonstrated the production of electromagnetic waves by using a spark oscillator and then succeeded in detecting them upto a distance of several hundred metres.

Fig. 2.02 shows the experimental arrangement (in principle) to transmit electromagnetic waves produced by an LC-circuit (tank circuit) using a half-wave antenna. Due to the oscillating charge on capacitor, the two ends of the antenna become alternately positive and negative, when the LC-circuit produces LC-oscillations. As a result, the electric field is always parallel to the plane of the antenna, while the magnetic field is at right angles to it. The orientation of the electric field with respect to the surface of earth is known as ***polarisation*** of the electromagnetic wave. If the plane of the electromagnetic wave is horizontal in respect to earth, the wave is said to be ***horizontally polarised***. On the other hand, if the electric field is oriented vertically, the wave is said to be ***vertically polarised***. For the vertical orientation of the antenna, electric and magnetic fields constituting the electromagnetic waves propagate in space as shown in the figure. Since the intensity of the two fields varies sinusoidally with the amplitude of the current oscillations, the fields are represented by travelling sine-waves at right angles to each other. Further, as the antenna is vertical, the electric field is also vertically oriented and hence the waves are therefore, vertically polarised. If the antenna is turned parallel to horizontal, the electromagnetic waves would be horizontally polarised. Thus, wave polarisation is primarily a function of the antenna orientation.

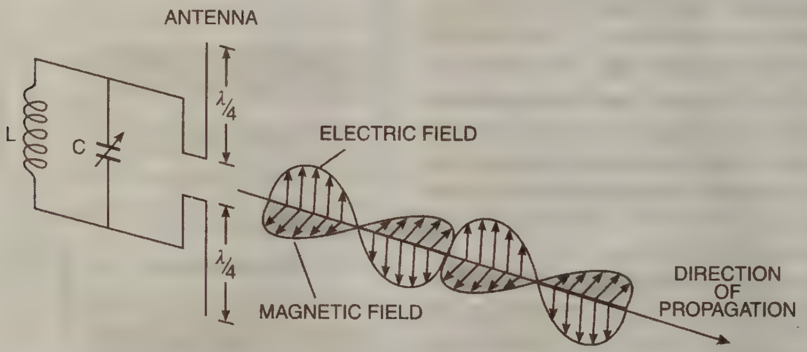


Fig. 2.02

It may be pointed out that the velocity of electromagnetic waves in free space is equal to the ratio of the amplitudes of electric and magnetic fields *i.e.*

$$c = \frac{E}{B} \qquad \text{...(2.01)}$$

2.03. CLASSIFICATION OF RADIOWAVES

In the field of radiocommunication, radiowaves are used. The radiowaves range from 3 kHz to about 300 GHz (1 GHz = 10^3 MHz) in the electromagnetic spectrum. The radiowaves have been subdivided into the following classes :

Frequency Band	Frequency Range	Wavelength Range	Typical use
Very-Low Frequency (VLF)	3 kHz to 30 kHz	10 km to 100 km	Long distance point-to-point communication.
Low Frequency (LF)	30 kHz to 300 kHz	1 km to 10 km	Marine and navigational aids.
Medium Frequency (MF)	300 MHz to 3 MHz	100 m to 1 km	Broadcasting and marine.
High Frequency (HF)	3 MHz to 30 MHz	10 m to 100 m	Communication of all types.
Very-High Frequency (VHF)	30 MHz to 300 MHz	1 m to 10 m	TV, Radar, air navigation, FM and SW transmission.
Ultra-High Frequency (UHF)	300 MHz to 3 GHz	10 cm to 1 m	Radar, microwave reyals and short-distance communication.
Super-High Frequency (SHF)	3 GHz to 30 GHz	1 cm to 10 cm	Radar, radio relays, navigation and experimental
Extermely-High Frequency (EHF)	30 GHz to 300 GHz	1 mm to 1 cm	Experimental

2.04. SPACE COMMUNICATION (RADIOWAVE PROPAGATION)

Depending on the type of transmission, following are the three types of space communication :

1. Amplitude modulated communication
2. Frequency modulated communication
3. Microwave communication

2.05. AMPLITUDE MODULATED COMMUNICATION

In amplitude modulated communication, propagation of radiowaves from the transmitting antenna to the receiving antenna takes place in the following two important ways :

1. Ground wave propagation　　　2. Sky wave propagation

1. Ground wave propagation. The ground wave propagation may be further subdivided into two categories :

(a) Surface wave propagation. *When the radiowaves from the transmitting antenna propagate along the surface of the earth so as to reach the receiving antenna, the wave propagation is called surface wave propagation.*

The transmitted radiowave is supported at the lower edge by the ground. The radiowaves have to be vertically polarised so as to prevent the short circuiting of the electric field component of the wave. The radiowave induces current in the ground over which it passes. It attenuates to some extent due to partial energy absorption by the ground.

The jargon ......

The loss of power in a signal, as it travels, is called attenuation.

The surface wave gets attenuated due to one other reason also. As the wave propagates over the earth, it gradually tilts over. It is called diffraction effect. At some distance from the transmitting antenna, the wave 'lies down and dies'. The maximum range of such a transmitter depends on its frequency as well as its power. Surface wave propagation is of importance only for medium and long wave signals. All amplitude modulated radio signals received in day time propagate by means of surface waves. For TV and FM signals (signals of very high frequency), the ground wave propagation is not used. It is because, at high frequency, the ground wave signal is reduced in strength by absorption to such an extent that it virtually becomes useless beyond an area of a few kilometres around the transmitting antenna.

(b) Space wave propagation. *When the radiowaves from the transmitting antenna reach the receiving antenna either directly or after reflection from the ground or in troposphere, the wave propagation is called space wave propagation.*

The troposphere is the portion of the atmosphere closest to the earth. It extends upto 12 km from the surface of the earth. The radiowaves reflected from the troposphere are also called ***tropospheric waves***. From the transmitting antenna, the radiowaves follow path 1, so as to reach the receiving antenna directly, path 2 so as to reach after reflection from ground and path 3 in order to reach after reflection from the troposphere [Fig. 2.03]. Since in none of the three cases, the radiowaves

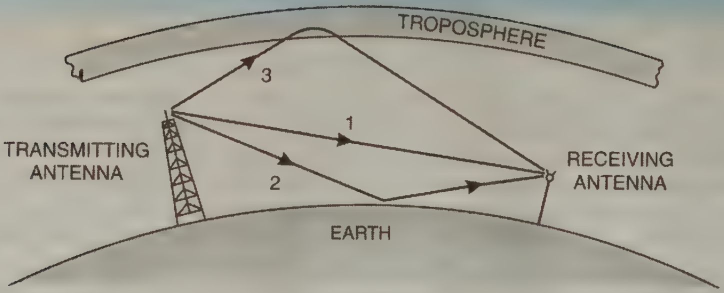


Fig. 2.03

are subjected to continuous absorption by the earth, the signal in this case can reach considerably farther than in case of ground wave propagation. Therefore, space wave propagation can be utilised for transmitting high frequency TV and FM signals. However, undesirable effects occur in space wave propagation due to phase reversal (by 180°) of the ground- reflected wave. For example, if the ground-reflected and direct waves reach the antenna after travelling the same length of the path (or differing by some integral multiple of the wavelength), they will arrive at the antenna 180° out of phase with each other and thus almost completely cancel out. However, if the path lengths of the two waves differs by one half wavelength (or phase difference 180°), the effective path difference will become integral multiple of wavelength. Then, the two waves will reinforce each other, resulting in increased signal strength. It follows that if the height of the receiving antenna is increased, the respective path lengths of the of the waves will alter and alternate points of maximum and minimum signal strengths will be encountered. This phenomenon is called ***selective fading.***

2. Sky wave propagation. *When the radiowaves from the transmitting antenna reach the receiving antenna after reflection in the ionosphere, the wave propagation is called sky wave propagation.*

The upper part of the atmosphere absorbs large amounts of radiant energy from the sun. The intensive heating of the atmosphere during day time causes ionisation, resulting in free electrons, positive ions and negative ions. However, due to different chemical compositions and physical properties of the atmosphere at different heights, the ionisation in the atmosphere is not uniform. The electron concentration is found to be large first in a layer extending from 90 km to 140 km from the surface of earth, called ***Kennelly Heaviside layer*** (or ***E-layer***) and then in a layer extending from 250 km to 400 km, called ***Appleton layer*** (or ***F-layer***). During the night, E-layer disappears almost completely. Therefore, it cannot be used for high frequency radio communication via sky wave during night time. The F-layer proves most useful for long distance transmission of high frequency radiowaves (2 to 30 MHz).

Fig. 2.04 shows the sky wave propagation due to reflection of radiowaves from E and F-layers of the ionosphere. The radiowaves which are radiated at a small vertical angle along the path 1 are reflected from the F-layer and return to earth at a great distance from the transmitting antenna. On the other hand, the radiowaves which are radiated at a somewhat greater vertical angle are reflected from the E-layer. Since it enters the ionosphere sooner (E-layer is closer to the earth), it is reflected back more quickly and at a shorter distance from the transmitting antenna.

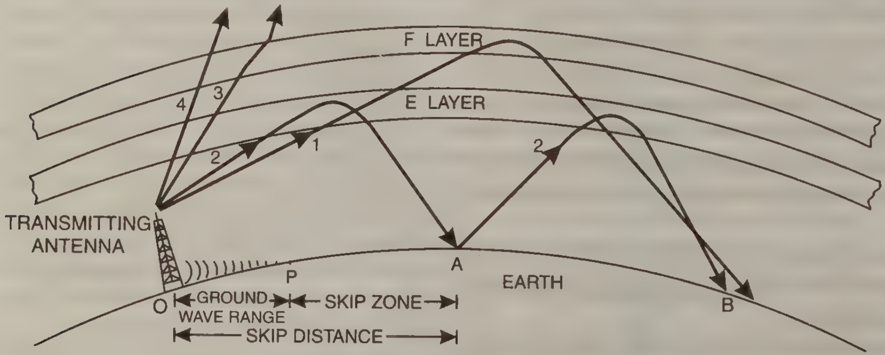


Fig. 2.04

The distance between the transmitting antenna and the point A, where the sky wave s first received after returning to earth is called ***skip distance***. However, the signal can be heard up to some point, say P via ground wave. The distance OP is called ***ground wave range***. Therefore, over the distance between points P and A, no signal will be heard. It is called ***skip zone***. In case the sky wave signal remains sufficiently strong after returning to the earth at point A, it may be reflected by the earth and again by E-layer, thus returning to the earth in a series of hops, separated by the skip distance. In this manner, the signal can be transmitted all over the earth.

As the vertical angle of the radiowaves w.r.t. earth is increased, the ionospheric layers become uncapable of reflecting them back to the earth. As shown in Fig. 2.04, the radiowaves transmitted along the paths 3 and 4 pass through the ionosphere after undergoing refraction (bending). The angle above which the radiowaves are no longer reflected by the ionospheric layer is called ***critical angle of radiation***. The radiowaves above this angle may be refracted by the ionosphere, but they are not reflected back to the earth. The value of the critical angle depends on the density of ionisation (or electron concentration) and on the frequency of the radiowaves. As the frequency increases, the ionosphere becomes progressively less effective. Therefore, with the increase in frequency, the value of the vertical angle should also be decreased so that the waves can reach the earth after reflection. At frequencies above 30 MHz (approximately), the signal is no longer reflected back, even when the vertical angle is smallest possible. The exact frequency at which this occurs depends on the density of ionisation, which is affected, in turn, by sun spot cycles and by daily and seasonal variations. In fact, the height of the ionospheric layers and their density of ionisation vary from hour to hour, day to day, month to month and season to season. It makes the sky wave propagation least reliable as sudden fadeouts and disappearances of signals occur so often.

2.06. FREQUENCY MODULATED COMMUNICATION

The TV signals are frequency modulated. The transmission of TV signals cannot be obtained through ground wave propagation, as such signals get absorbed by ground due to their high frequency. On the other hand, TV signals cannot be transmitted via sky wave propagation, as ionosphere is unable to reflect radio waves of frequencies greater than 40 MHz. Therefore, the only way out for the transmission of TV signals is that the receiving antenna should directly intercept the signal from the transmitting antenna. To achieve larger TV coverage, transmission of TV signal is done from a tall antenna.

Fig. 2.05 shows a TV antenna AB of height h. Due to curvature of the earth, the transmitted signal can be received upto a maximum distance AR_1 (or AR_2) from the antenna. Let $AR_1 = AR_2 = d$ (say). Therefore, the TV signal will be received in a circle of radius d. As said earlier, the distance d is limited by the curvature of earth. Let us derive the relation between height (h) of the TV antenna and the maximum distance (d), up to which the TV signal can be received.

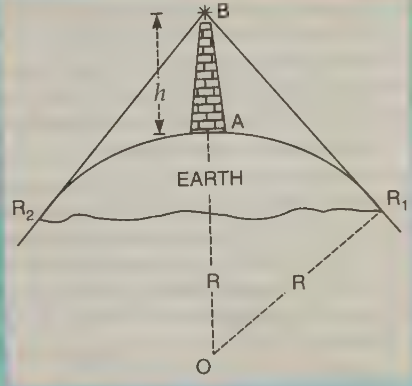


Fig. 2.05

From right angled triangle OBR_1, we have

$$OB^2 = BR_1^2 + OR_1^2$$

Taking $BR_1 \approx AR_1 = d$, we have

$$(R + h)^2 = d^2 + R^2$$

or $$d^2 = h^2 + 2\,h\,R$$

Since h is small as compared to the radius of the earth, h^2 can be neglected as compared to $2\,h\,R$. Therefore,

$$d^2 = 2\,h\,R$$

or $$d = \sqrt{2\,h\,R} \qquad ...(2.02)$$

The above equation gives the estimate of the distance up to which TV signals will be received, when a transmitting antenna of height h is used.

2.07. MICROWAVE COMMUNICATION

The radiowaves of frequencies greater than TV signals are called ***microwaves***. These waves have wavelengths of the order of a few millimetres. Because of their small wavelengths, microwaves can be transmitted as a beam in a particular direction. The microwaves are used in radars to site the aeroplanes in space and to know their distances from the radar station. The distance is calculated by noting the time delay in receiving the reflected signal. In recent times, microwaves have totally changed the concept of telecommunication. Now, with the help of geostationary satellites, it is possible to send signal from one point on earth to practically any other point on the earth.

[redacted]

An artificial object placed in orbit around the earth (or any other planet) is called a satellite.

A satellite moves in an orbit around the earth under the action of gravitational pull. It can be easily deduced that period of revolution of the satellite in an orbit at a height x above the surface of earth,

$$T = 2\pi\sqrt{\frac{(R+x)^3}{g}} \qquad \text{...(2.03)}$$

where R is radius of the earth.

There are hundreds of artificial satellites revolving around the earth, which are used for different purposes. Depending on the task to be performed by the satellite, it is put in a suitable orbit. A geostationary satellite always stays over the same place above the earth *i.e.* it appears stationary relative to the earth.

For a satellite to appear stationary, the following conditions should be satisfied :

1. *It should revolve in an orbit concentric and coplanar with the equitorial plane.*

2. *Its sense of revolution should be same as that of the rotation of earth about its own axis i.e. anticlockwise direction (from west to east).*

3. *Its period of revolution around the earth should be the same as that of earth about its own axis i.e. exactly 24 hours.*

The equation (2.03) can be used to find the height of the geostationary satellite. From equation (2.03), we have

$$x = \left(\frac{T^2\, R^2\, g}{4\pi^2}\right)^{1/3} - R$$

Since period of revolution earth, T = 24 hour = 24 × 3600 s ; radius of earth, R = 6400 km and $g = 9{\cdot}8\ \text{m s}^{-2} = 0{\cdot}0098\ \text{km s}^{-2}$, we have

$$x = \left(\frac{(24 \times 3600)^2 \times 6400^2 \times 0{\cdot}0098}{4\pi^2}\right)^{1/3} - 6400 = 35{,}930\ \text{km}$$

Thus, a satellite will appear stationary, if it revolves around the earth from west to east in an orbit coplanar with equitorial plane and at a height of about 36000 km above the surface of the earth. Such an orbit is known as ***synchronous*** or ***geostationary orbit*** or ***parking orbit*** and the satellite revolving in this orbit is called ***geostationary satellite*** or simply a ***stationary satellite***. The orbital velocity of the geostationary satellite comes out to be about $3{\cdot}08\ \text{km s}^{-1}$. Such a satellite will always appear to be over the same place relative to an observer on the earth. The communication satellites are normally put into a ***geostationary orbit***.

Microwave communication systems are used for long distance communication. Since at microwave frequencies, electromagnetic waves cannot bend across the obstacles, such as the top of the buildings, mountains, etc, it is therefore necessary that microwave transmission is in line-of-sight. Due to curvature in the surface of earth, the range of microwave transmission is very small (≈ 50 km). The range of microwave transmission is also limited by the fact that signals gets weaker and weaker as it propagates. However, these problems are overcome by using ***repeaters*** at intervals between the transmitter and the receiver. Due to this, the cost of transmission of signal between the two stations increases.

The problems faced in a microwave communication system are solved to a large extent by using a geostationary satellite as a communication satellite. A communication satellite is basically a microwave relay station placed in geostationary orbit around the earth. When the signal from the transmitting station on earth is beamed to the satellite, it reflects the signal back to the receiving station on earth. As such, the satellite provides point-to-point link between two earth stations. A geostationary satellite can establish communication link over a large area of the earth, but a single satellite cannot be used to connect people all around the earth. The communication link all around the earth is established by putting three geostationary satellites in synchronous orbit, 120° apart from each other. It is done by transmitting the signal from one such satellite to the other.

In case two communication satellites operate in the same frequency band or close frequency bands, the signals undergo distortion due to interference effects. his problem is overcome by making use of frequency bands of proper spacing for ifferent satellites. In India, the satellite communication is achieved through its two ommunication satellites INSAT-2 B and INSAT-2 C.

Merits of satellite communication. The satellite communication has the ollowing points of merit over other terrestrial means of communication :

1. The satellite relays are wide area broadcasts, whereas the terrestrial relays re point-to-point.

2. The satellite communication can be used for establishing mobile ommunication with great ease.

3. The satellite communication proves much economical as compared to errestrial communication, more so when distances involved are very long. In fact, he cost of satellite communication is independent of the distance.

4. In remote and hilly areas, such as Himachal Pradesh, Ladakh, etc, satellite ommunication is most cost effective.

5. For search, rescue and navigation, satellite communication is far accurate nd economical as compared to other systems.

The communication satellites are now being used in ***Global Positioning System*** GPS). The ordinary users can find their positions within an accuracy of 100 m.

Drawbacks 1. From security and privacy point of view, the communication satellites are complete disaster. It is because, signalling from the communication satellite is open to all *i.e.* everybody can hear everything.

2. The satellite equipment is subject to extreme environmental stresses. If a system on the satellite goes bad, it is almost impossible to repair it.

3. Due to extremely large communication path length (about 2 × 36,000 km), there is a time delay between transmission and reception. This delay causes a time gap during talking, which proves quite annoying.

9.09. REMOTE SENSING

Any method of obtaining and recording information from a distance is called remote sensing.

In remote sensing, information about an object or area under investigation is acquired through sensors, which are not in direct contact with the target of investigation. Areal photography is one kind of remote sensing. Areal photographs of the target are taken from a large distance. This technique was earlier used for military purposes.Now, the areal photography is used in archaeology, survey of earth resources and in town and country planning. Such sensors are placed on the satellite revolving round the earth, which takes photographs or collects the desired data and transmits them back to earth.

Acquiring information about an object under investigation through sensors, which are not in direct contact with the target, is called remote sensing.

The following two types of satellites are used for long distance communication :

1. Passive satellites. *The satellites, which are used only for reflecting the signal back to the earth, are called passive satellites.*

A passive satellite is not equipped with electronic devices, which may be used for processing or amplifying the signal. A reflector on the satellite merely reflects the signal back to the earth. So that signal of suitable strength reaches the earth station back after reflection, the signal from the transmitting earth station has to beamed with good enough power. It, therefore, requires a very powerful transmitter.

2. Active satellites. *The satellites, which carry various equipments so as to process the signal, amplify it and then retransmit it back to the earth, are called active satellites.*

The electronic devices used for these purposes are usually high resolution cameras, microwave scanners, infra-red scanners, etc. Such devices are called ***active devices*** or ***sensors*** and can be commanded from the earth stations. Therefore, remote sensing is possible only through active satellites. The power required for operating such devices is derived from the solar panels of the satellite. These devices can record the information gathered by them. The information collected by the sensors is beamed down to earth in digital form, which is then converted into maps and pictures by the computers.

Uses of various types of sensors. 1. The photographs taken by the camera provide a variety of information including archaeological evidence and weather data.

2. Microwave sensors make use of radar waves and enable us to see through clouds.

3. Infra-red sensors can measure temperature differences over an area.

Now a days, remote sensing is used in geological surveys, water resources surveys, meteorology (weather forecasting), climatology (monitoring climate changes), oceanography (temperature of sea surface and monitoring oil pollution) and coastal studies (sewage and industrial waste monitoring). Apart from this, remote sensing is employed in land surveys, forestry and natural disasters.

SOLVED NUMERICAL PROBLEMS

Problem 2.01. Calculate the time delay between the reception of the ground wave and the sky wave for a receiver at a distance of 200 km from the transmitter, given that the reflecting ionosphere layer is effectively at a height of 100 km.

Sol. Let A and B be respectively transmitter and receiver on the surface surface of earth [Fig. 2.06].

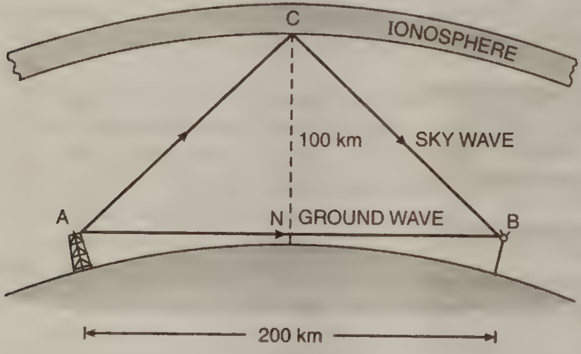


Fig. 2.06

Here, distance between transmitter and receiver

$$AB = 200 \text{ km} = 2 \times 10^5 \text{ m}$$

Height of ionoshpere layer,

$$CN = 100 \text{ km} = 10^5 \text{ m}$$

Let t_g be time of travel of ground wave. Then,

$$t_g = \frac{AB}{c} = \frac{2 \times 10^5}{3 \times 10^8} = 6{\cdot}67 \times 10^{-4} \text{ s}$$

The sky wave follows that path AC and then CB to reach the receiver. Therefore, distance transversed by the sky wave,

$$AC + CB = \sqrt{AN^2 + CN^2} + \sqrt{BN^2 + CN^2}$$

$$= \sqrt{100^2 + 100^2} + \sqrt{100^2 + 100^2} = 283 \text{ km}$$

$$= 2{\cdot}83 \times 10^5 \text{ m}$$

Let t_s be time of travel of sky wave. Then,

$$t_s = \frac{AC + CB}{c} = \frac{2{\cdot}83 \times 10^5}{3 \times 10^8} = 9{\cdot}43 \times 10^{-4} \text{ s}$$

Hence, time delay, $t_s - t_g = 9{\cdot}43 \times 10^{-4} - 6{\cdot}67 \times 10^{-4}$

$$= \mathbf{2{\cdot}76 \times 10^{-4} \text{ s}}$$

Problem 2.02. (*a*) A TV tower has height of 80 m. Find the radius of the circle within which the transmission can be observed, if radius of earth is 6·4 × 10⁶ m.

(*b*) How much population is covered by the TV broadcast, if the average population density around tower is 800 km⁻² ? (*H.S.S.C.E. 1996*)

Sol. (*a*) Here, $h = 80$ m ; $R = 6{\cdot}4 \times 10^6$ m

If d is the radius of the circle within which the transmission can be observed, then

$$d = \sqrt{2hR} = \sqrt{2 \times 80 \times 6{\cdot}4 \times 10^6} = \mathbf{3{\cdot}2 \times 10^4 \text{ m}}$$

(*b*) Area in which transmission can be viewed,

$$A = \pi d^2 = \pi \times (3{\cdot}2 \times 10^4)^2$$

$$= 3{\cdot}217 \times 10^9 \text{ m}^2 = 3{\cdot}217 \times 10^3 \text{ km}^2$$

Since average population density, $\sigma = 800 \text{ km}^{-2}$, the population covered by the transmission

$$= A \times \sigma = 3{\cdot}217 \times 10^3 \times 800 = \mathbf{2{\cdot}574 \times 10^6}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. What is a ground wave ? (*HS.S.C.E. 1998 S*)

Ans. A radiowave signal propagating from one point to another following the surface of earth is called a surface or ground wave.

Q. 2.02. Why are short waves used in long distance broadcasts ? *Or* (*C.B.S.E. 1993*)

Long distance radio broadcasts use short wavebands. Why ? (*Text Question*)

Ans. The short waves (wavelength less than 200 m or frequencies greater than 1500 kHz) are absorbed by the earth due to their high frequency but are effectively reflected by F-layer in ionosphere. After reflection from ionosphere, the short waves reach the surface of earth back only at a large distance from the transmitter. For this reason, short waves are used in long distance transmission.

Q. 2.03. State the reason why ground waves cannot be transmitted above the frequency range of 1500 kHz. (*C.B.S.E. 2001*)

Or

Why is the transmission of signals using ground waves restricted upto a frequency of 1500 kHz ? (*C.B.S.E. 1999, 1998 S*)

Ans. It is because, on further increasing the frequency of the signal, the absorption of the signal from the ground increases.

Q. 2.04. Why ground wave propagation is not suitable for high frequency ? (*Text Question*)

Ans. Refer to VSQ 2.03.

Q. 2.05. It is necessary to use satellites for long distance TV transmission. Why ? (*H.P.S.S.C.E. 1998 S ; H.S.S.C.E. 1992, Text Question*)

Ans. Television signals are not reflected by ionosphere. The TV signals from an earth station are reflected back to earth by making use of artificial satellite.

Q. 2.06. State two factors by which the range of nsmission of signals by a TV tower can be increased. (*C.B.S.E. 1998 S*)

Ans. 1. By increasing the height of the tower.

2. By increasing the height of the receiving antenna, so t it may directly intercept the signal from the transmitting tenna.

Q. 2.07. "Greater the height of a TV transmitting tenna, greater is its coverge." Explain. (*P.S.S.C.E. 2001, 1993*)

Ans. Here, $d=\sqrt{2hR}$

In case, height of tower (h) is increased, the distance (d) up to which TV coverage can be done will also increase.

Q. 2.08. What is a passive satellite ?

Ans. *The satellite, which is used only for reflecting the signals back to earth stations, is called a passive satellite.*

A passive satellite does not have equipments on it, so as to amplify the signal received from the earth.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 2.01. In a radio receiver, the short-wave and edium-wave stations are tuned by using the same pacitor but coils of different inductance. Explain, why the ductance in the former case is less than that in the latter se. (*Roorkee, 1986*)

Ans. Suppose that the short-wave station receives gnals of short wavelengths(λ_s), while the medium-wave ation receives that of medium wavelengths (λ_m).

Let f_m and f_s be corresponding frequencies of the edium-wave and short-wave signals. If c is speed of the diowaves, then

$$f_m=\frac{c}{\lambda_m} \quad \text{and} \quad f_s=\frac{c}{\lambda_s}$$

Since $\lambda_m>\lambda_s, f_m<f_s$.

Let L_m and L_s be the inductances of the medium-wave nd short-wave coils respectively and C be the capacitance f the capacitor of the tuning circuit (LC- circuit used for ning the signal). Then,

$$f_m=\frac{1}{2\pi\sqrt{L_m C}} \qquad ...(i)$$

and

$$f_s=\frac{1}{2\pi\sqrt{L_s C}} \qquad ...(ii)$$

Dividing the equation (*i*) by (*ii*), we have

$$\frac{f_m}{f_s}=\sqrt{\frac{L_s}{L_m}}$$

Since $f_m<f_s$, it follows that

$$L_s<L_m$$

e.the inductance of short-wave coil is less than that of the nedium-wave coil.

Q. 2.02. Explain the two ways, by which electromagnetic waves are propagated in atmosphere. (*P.S.S.C.E. 1998*)

Ans. Refer to section 2.05.

Q. 2.03. Why is short wave communication over long distance not possible via ground waves ? (*H.S.S.C.E. 1998 S*)

Ans. Refer to section 2.05 and VSQ 2.02.

Q. 2.04. Explain the terms (*i*) ground waves and (*ii*) sky waves. (*H.S.S.C.E. 1999 S ; H.P.S.S.C.E. 1996, 1991*)

Ans. Refer to section 2.05.

Q. 2.05. Why sky waves are not used in the transmission of television signals ? (*C.B.S.E. 2002, 2001, 1993 ; P.S.S.C.E. 1998 S*)

Or

Why is the transmission of signals using sky waves restricted to frequencies greater than 30 MHz ? (*C.B.S.E. 1999*)

Ans. The television signals have frequencies in 100-200 MHz range. As ionosphere cannot reflect radio waves of frequency greater than 40 MHz back to earth, the sky waves cannot be used in the transmission of TV signals.

Q. 2.06. What is role of sky wave propagation in transmission of high frequency signals ? Explain. (*H.S.S.C.E. 1999 S*)

Ans. Refer to section 2.05.

Q. 2.07. Optical and radiotelescopes are built on the ground, but X-ray astronomy is possible only from satellites orbiting the earth. Why ? (*Text Question*)

Ans. The earth's atmosphere is transparent to visible light and radiowaves, but absorbs X-rays. Therefore, X-ray astronomy is possible only from satellites orbiting the earth.

Q. 2.08. What is an active satellite ? How is it different from a passive satellite ?

Ans. *The satellite that is equipped with electronic devices so a receive the signal from the earth station, process it, amplify it and then retransmit it, is called an active satellite.*

In addition, an active satellite carries different types of sensors (camera, infra-red scanner, microwave scanner, etc), which transmit information in the form of data and pictures to the earth stations. A passive satellite does not have any such equipement on it. It is used only for reflecting the signals back to the earth stations.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS

Carrying 3 Marks

1. What is earth's atmosphere ? Classify it into different layers. (*H.P.S.S.C.E. 2002*)
2. Name the different layers of earth's atmosphere. Explain them in brief.
3. Discuss briefly the behaviour of earth's atmosphere towards the propagation of electromagnetic waves. (*H.P.S.S.C.E. 2000*)
4. What are different modes of propagation of radiowaves ? (*Text Question*)
5. What is a ground wave ? Why short wave communication over long distances is not possible via ground waves ?
6. Explain surface wave (ground wave) and sky wave propagations of radiowaves. Why is short wave communication over long distances not possible by surface wave propagation ? (*C.B.S.E. 1997*)

7. Differentiate between ground wave and sky wave propagation. (*H.P.S.S.C.E. 2000*)
8. What is ionosphere ? Explain its importance in communication. (*Text Question*)
9. Why sky wave transmission of electromagnetic waves cannot be used for TV transmission ? Suggest two methods by which the range of TV transmission can be increased. (*C.B.S.E. 2001 S*)
10. A TV tower has a height h. Derive the expression for the maximum distance upto which signals be received on earth. (*H.S.S.C.E. 2001*)
11. Deduce an expression for the distance at which TV signals can directly be recieved from a TV tower of height h. (*H.P.S.S.C.E. 2000, 1997*)
12. Derive the expression for the maximum range up to w TV signals can be received on earth's surface. (*C.B.S.E. 2000, 1999, 1996,*
13. "Greater is the height of a TV transmiting antenna, gr is its range." Prove. (*H.P.S.S.C.E.*
14. What mode of communication is employed fo transmission of TV signals ? Explain why TV transmi towers are usually made very high. (*Text Ques*
15. What is a geostationary satellite ? How can it be usec communication satellite ?
16. What are the merits of satellite communication ove terrestrial means of communications ? Give two d backs of satellite communication.
17. What is remote sensing ? Explain in brief.

REQUENTLY SKED LONG ANSWER UESTIONS

Carrying 5 Marks

1. Explain the ground wave and sky wave propagation of electromagnetic waves with the help of necessary diagrams.
2. Explain, how a geostationary satellite may be used as communication satellite. What do you mean by re sensing ? Explain in brief.
3. What is remote sensing ? Explain briefly how it is ca out. Mention some of its applications. (*Text Ques*

REQUENTLY SKED NUMERICAL ROBLEMS

For Practice

1. A ratio can tune to any station in the 7·5 MHz to 12 MHz band. What is the corresponding wavelength band ? (*Text Problem*) **[Ans. 25 m to 40 m band]**
2. A TV tower has height of 100 m. How much population is covered by the TV broadcast, if the average population density around the tower is 1000 km^{-2} ? Given, radi earth = 6·37 × 10^6 m. **[Ans. 4 ×**
3. A TV tower has a height of 150 km. How much po tion the TV broadcast covers, if the average popul density around the tower is 1000 km^{-2} ? Radius of is 6·4 × 10^6 m. (*Text Problem*) **[Ans. 6·032 ×**

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Let λ_1 and λ_2 be wavelengths corresponding to frequencies 7·5 MHz and 12 MHz respectively. Then,

$$\lambda_1 = \frac{c}{f_1} = \frac{3 \times 10^8}{7{\cdot}5 \times 10^6} = 40 \text{ m}$$

and $$\lambda_2 = \frac{c}{f_2} = \frac{3 \times 10^8}{12 \times 10^6} = 25 \text{ m}$$

Therefore, wavelength band corresponding to the given frequency band is **25 m to 40 m.**

2. Here, $h = 100$ m, $R = 6{\cdot}37 \times 10^6$ m

Radius of circle within which transmission can be vie

$$d = \sqrt{2hR} = \sqrt{2 \times 100 \times 6{\cdot}37 \times 10^6} = 3{\cdot}57 \times 10^4 \text{ m}$$

Area in which transmission can be viewed,

$A = \pi d^2 = \pi \times (3{\cdot}57 \times 10^4)^2 = 4{\cdot}004 \times 10^9 \text{ m}^2 = 4 \times 10^3$ k

Population density = 1000 km^{-2}

Therefore, population covered by transmission

$= 4 \times 10^3 \times 1000 = 4 \times 10^6$

3. Proceed as in problem no. 2.

APT

3

Line Communication

3.01. TRANSMISSION MEDIUM

The link, which transfers information (message signal) from the information source to the destination, is called transmission medium or link.

In any communication system, transmission medium plays an important role. In fact, transmission media are the electronic roadways along which the signal is transferred from information source to the destination. So that the transmitted signal is received faithfully at the destination, it should not get distorted due to the transmission medium or pick up any extraneous noise.

The transmission media are of the following two types :

1. Guided transmission medium. *The transmission medium, which enables the message signal to reach the destination by making it to follow a guided path, is called a guided transmission medium.*

A guided transmission medium is used for point-to-point communication. For example, in telephone as a communication system, the signal from the source reaches the destination by travelling along the copper wires. Thus, copper wires in telephone as the communication system act as guided transmission medium. Apart from copper parallel-wire lines ; twisted-wire lines (called twisted pairs) coaxial cables and optical fibres are other examples of guided transmission medium.

2. Unguided transmission medium. *The transmission medium, which enables the message signal to reach the destination along an unguided path, is called an unguided transmission medium.*

In space communication, there is no point-to-point contact between the transmitter and the receiver. Therefore, free space acts as an unguided transmission medium in space communication.

However, in any transmission medium, the signal gets attenuated or suffers energy loss and is subject to degradations by random signals and noise. Therefore, in any communication system, there is a maximum permitted distance between the transmitter and the receiver, beyond which the system ceases to give intelligible communication. For trouble free transmission over a long distance, these factors necessitate the installation of ***repeaters*** at intervals. A repeater is a line receiver, which amplifies and equilizes the attenuated signal and then retransmits it for its continued transmission down the link. It also helps to remove signal distortion and to increase signal level.

3.02. LINE COMMUNICATION

In line communication, the response of transmission medium towards the signal to be transmitted plays an important role. The transmission characteristics of line communication depends upon the nature of the transmission medium and the nature of the signal.

Following guided transmission media are used in line communication :

1. Parallel-wire line (Ribbon). In a parallel-wire line, two metallic wires run parallel to each other inside a protective insulation coating (usually PVC insulation) as shown in Fig. 3.01. The metallic wires may be hard or flexible depending on the power to be handled. For high power transmission, hard wires are used.

The parallel-wire line is employed, where balanced properties of the communication system are the main consideration factors. For this reason, such a wire line is also called ***balanced line***. For example, TV antenna is a folded-dipole antenna. For connecting the TV to its antenna, the impedance of the parallel-wire line must match that of the TV. As such, parallel-wire line is found to be suitable to be used as TV antenna wire.

A system of conductors is found to radiate energy in the form of radio frequency waves, when the separation between the two wires approaches a half-wavelength

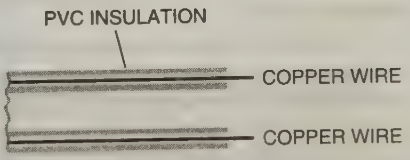


Fig. 3.01

Parallel wire lines are not used for transmission of a microwave signal. It is because, at microwave frequencies, the energy loss becomes quite appreciable.

at the operating frequency. Likewise, when a parallel line wire is used as a transmission line, it also radiates energy. Since the energy loss becomes of serious magnitude at the frequency of microwaves, parallel-wire lines are never used for microwaves. As the attenuation of signal increases with the length of the wire, a parallel-wire line is used for transmission of signal only up to a small distance.

2. Twisted pair. Such a wire line consists of two insulated copper wires twisted around each other along their lengths as shown in Fig. 3.02. In a parallel-wire line, the current due to signal produces magnetic field around each of the two wires. The interference between the two magnetic fields may cause unacceptable variations in the level of signal current, which may result in a distorted signal at the receiving end. In order to minimise this effect, the two metallic wires are twisted around each other. The twisted-wire line is found to be suitable for narrow band transmission. When used in a computer network, such wires can transmit data up to a speed of 1500 bits per second. They are commonly used in the installation of telephone systems.

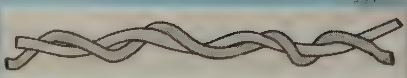


Fig. 3.02

Let us estimate how many message signals can be transmitted over a twisted pair. An analog voice signal has a bandwidth of 3 kHz. In case it is to be transmitted as a modulated signal, it requires a bandwidth of 6 kHz. Therefore, if 100 such signals are to be transmitted over the twisted pair without their interfering with each other, one will require a bandwidth of 6 × 100 *i.e.* 600 kHz.

In case the transmission of message signal is required to a large distance, the length of the pair of wires also becomes large. As a result, there is loss of power along the wires due to their finite resistance. Further, at high frequencies, a copper wire begins to act as antenna and start radiating energy in the surrounding medium. As a result, the message signal gets attenuated.

The attenuated signal is usually boosted by using repeaters (amplifiers). Keeping all the factors in view, the transmission of signal over twisted pair is limited to a bandwidth of about 2 MHz (about 300 speech signals) and to a distance upto 10 km.

Merits. (*i*) They are quite cheap.

(*ii*) They are very easy to install.

(*iii*) They can be easily tapped.

(*iv*) They are less prone to electrical disturbances (noise) as compared to parallel-wire line.

(*v*) They can be used for transmitting both analog and digital signals.

Demerits. (*i*) They can be used only up to a very small distance.

(*ii*) They get easily broken during strong winds.

3. Coaxial cables. It consists of a central copper wire surrounded by a PVC insulation over which a sleeve of copper mesh is placed. The copper mesh is covered with an outer shield of thick PVC material as shown in Fig. 3.03. The signal is transmitted by the inner copper wire, which is electrically shielded by the outer copper mesh. The outer copper mesh is always grounded (connected to earth). The use of coaxial cables does not necessitate the matching of the impendance of the different parts of a communication system. For this reason, it is also called an ***unbalanced line.***

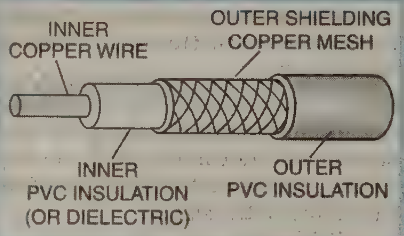

INNER COPPER WIRE
OUTER SHIELDING COPPER MESH
INNER PVC INSULATION (OR DIELECTRIC)
OUTER PVC INSULATION

Fig. 3.03

Coaxial cables have a distinct advantage over the twisted pair. Since the copper mesh surrounds the central copper wire carrying the signal, the coaxial cables prove to be immune to external electrical disturbances. It is because, the magnetic fields produced by the electrical disturbances are unable to affect the signal[*]. Further, the loss of energy in coaxial cables is quite low in comparison to twisted pair, as the outer conductor (copper mesh) is invariably grounded. In some coaxial cables, the inner PVC insulation is replaced by a dielectric.

Key point ✓

The loss of energy is quite low, when transmission of even high frequency signal is done with a coaxial cable. Further, coaxial cables are immune to external electrical disturbances.

Impedance of a coaxial cable. As said earlier, a coaxial cable consists of two conductors. Since each conductor has a certain length and diameter, it will have a resistance (R) and an inductance (L). Further, as the two conductors are very close to each other, they will possess some capacitance (C) between them. In case the coaxial cable has dielectric in place of the inner PVC insulation, the current leakage[†] through dielectric can be represented by a shunt resistance (G). Fig. 3.04 (*a*) shows the

---

[*] The magnetic field due to an electric current cannot penetrate inside a hollow conductor.

[†] A dielectric is never perfect in its insulation.

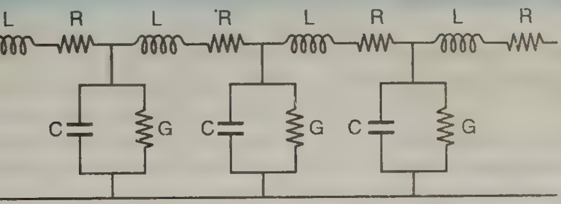


Fig 3.04 (*a*)

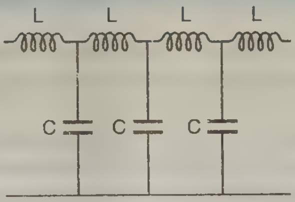


Fig. 3.04 (*b*)

ulting equivalent circuit for a coaxial cable. It may be pointed out that L, R, C l G represent the respective quantities per unit length of the coaxial cable. It is ause, they are distributed throughout the length of the cable.

When the coaxial cable is used to transmit a radio frequency signal (frequency y high), the inductive reactance ($X_L = 2\pi f L$) is very large as compared to R. On other hand, capacitive susceptance $\left(\frac{1}{X_C} = 2\pi f L\right)$ is also very large as npared to the shunt conductance $\left(\frac{1}{G}\right)$. In view of this, R and G can be ignored the equivalent circuit of the coaxial cable. As such, the equivalent circuit is nplified to as shown in Fig. 3.04 (*b*). Since the cable does not offer any ohmic istance at radio frequency, no loss of energy and hence no attenuation of radio quency signal occurs, when transmitted along it. Because of this reason, the use coaxial cables in cable TV network becomes specially important. The attenuation signal along a coaxial cable is quite low ($\approx$ 5 to 10 dB km^{-1}).

Reduced attenuation in coaxial cables increases the repeater spacing to about km and permits much larger bandwidth ($\approx$ 20 MHz). In other words, coaxial bles can be used to transmitt about 3300 speech signals without their interfering th each other.

Merits. **(*i*)** Although coaxial cables are costlier than parallel-wire line or twisted ir, yet they are quite inexpensive keeping in view their performance.

(*ii*) They are easily available.

(*iii*) They are immune to noise upto a good level.

(*iv*) They can be used to transmit data upto a speed of 10 megabits per second.

(*v*) They are extensively used in long distance telephone lines and as cables for closed circuit TV.

(*vi*) The attenuation of signal is very low along coaxial cables.

Demerits. **(*i*)** They are more expensive than twisted pairs.

(*ii*) It is more difficult to tap them as compared to twisted pairs.

3.03. TELEPHONE LINKS

Telephone is the most common means of communication. Individual telephone s in a locality are connected to telephone exchange box using a twisted pair. Such ephone exchange boxes in different localities in a city are connected by a thick ble, which carries various twisted pairs to the main telephone exchange. So that umber of persons can communicate with each other, direct connections by means a pair of copper wires have to be made between them. One can easily visualise at this method may work only when the number of persons involved is small. It because, if this method is adoped to connect a large number of persons, it will ad to a wild jumble of wires passing over houses and trees all over the city. The lution to this problem was found through *multiplexing.*

Multiplexing. *Simultaneous transmission of number of messages over a single annel is known as multiplexing.*

Key note

Simultaneous transmission of a number of messages over a single channel without their interfering with each other is called multiplexing.

The channel may be a pair of wires or free space. If the message is transmitted such (without modulation), the different massage signals over a single channel ll interfere with one another. It is because, the frequency spectrum (or bandwidth) all the message signals are identical. However, different message signals can be ansmitted over a single channel without interference using multiplexing chniques. There are following two types of multiplexing techniques :

1. *Frequency division multiplexing.* It makes used of analog modulation of essage signals.

(*ii*) ***Time division multiplexing.*** It makes use of pulse modulation of message signals.

As said earlier, twisted pair can provide a bandwidth of 2 MHz, while coaxial cable, a bandwidth of 20 MHz. Microwave communication has helped in increasing the bandwidth further.

Microwave link. It is quite a popular method of telephone link as it does not incur the expense of laying wire lines and cables. In microwave system, high frequency radio waves are used to transmit the voice signal. However, at microwave frequencies, the electromagnetic waves cannot bend across the obstacles, such as the top of the buildings and mountains. Therefore, it is necessary that microwave transmission is in the ***line-of-sight*** *i.e.* the transmitter and receiver of a microwave system, which are mounted on high towers, should be in a line. Due to curvature in the surface of earth, the range of microwave transmission is limited to just 50 km. The range of microwave transmission is also limited by the fact that signals become weak after covering a certain distance and require power amplification. To transmit over longer distances, it is necessary to bounce the microwave signal through a chain of towers to negotiate the curvature in the surface of the earth [Fig. 3.05]. To overcome the problem of power amplification of weak signals in microwave link system, repeaters are used at intervals of about 25 to 30 km between the transmitting and receiving stations. The first repeater is placed in line-of-sight of the transmitting station and the last repeater is placed in line-of-sight of the receiving station. Two consecutive repeaters are also placed in line-of-sight of each other. By making use of multiplexing technique, microwave transmission system can send thousands of voice signals at the same time.

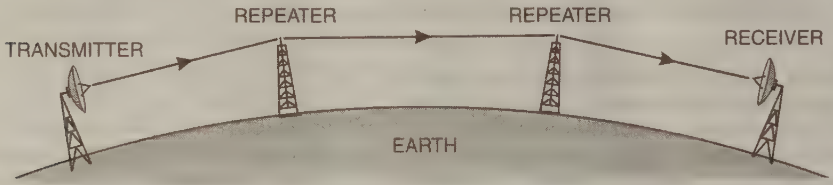


Fig. 3.05

Key point

In microwave transmission syste
transmitter and receiver must be in
line-of-sight. It is because, at microw
frequencies electromagnetic wa
cannot bend around the obstac
(mountains and other structures on
surface of the earth) in their path. Furt
curvature in the surface of earth also po
a problem.

Communication satellite link. The main problem with microwave link system is that the curvature in the surface of earth, mountains and other structures often block the line-of-sight. Due to this, several repeaters are normally required for long distance transmission, which increases the cost of signal transmission between the two stations. This problem has been overcome by using communication satellites.

A communication satellite is basically a microwave relay station placed in geostationary orbit around the earth. Such a satellite is stationary relative to earth and always stays over the same point on the ground. The Indian communication satellites INSAT-2 B and INSAT-2 C are positioned in such a way in the outer space that they are accessible from any place in India.

Key point

The communication satellites are used
transmission of signals to overcome
problems faced in microwave tra
mission due to curvature in surface of
earth and blocking of line-of-sight by
obstacles.

Fig. 3.06 shows the block diagram as to how a typical long distance call from a calling subscriber to a called subscriber is made by making use of communication satellite as a telephone link. The call from the calling subscriber is routed to the local exchange through a 2-wire line, which is then transmitted to the main exchange through coaxial cable. From the main exchange, the call is routed to earth satellite

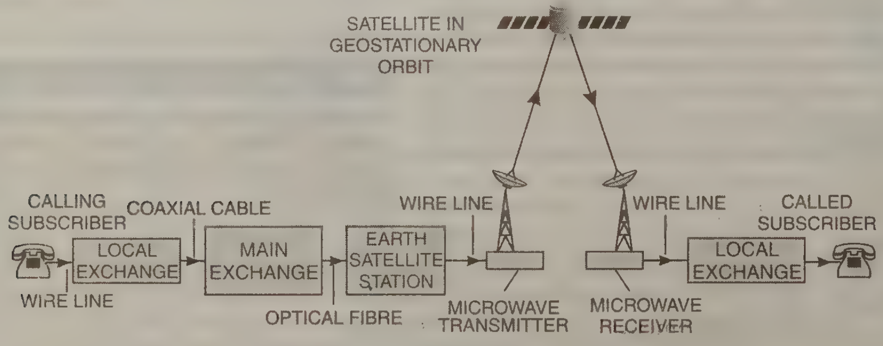


Fig. 3.06

ation (which acts as a transmitter) by means of optical fibre. The earth satellite ation, then transmits the call to the microwave system by simply 2-wire line. The gnal is finally transmitted to the communication satellite, which beams it back to e microwave system at the far receiving end. The signal travels from the receiver the called subscriber in a similar manner, as it does from calling subscriber to the ansmitter. It may be pointed out that the signal gets quite weak while traversing s long onward and return path (≈ 2 × 36,000 km) between the earth and the satellite. his aspect is taken care of by the electronic devices on the satellite. Such devices nplify the signal before sending it back to the earth. To avoid the interference of e incoming weak signal and the powerful retransmitted signal, the transmission equency of the microwave system on earth is kept different from the retransmission equency of the system in the satellite.

Advantages. 1. The satellite link serves as a single microwave relay station isible from any point of a very very large area on the surface of the earth.

2. The transmission and reception costs are independent of the distance between ie caller and the receiver.

3. The quality of the received signal is much the same as that of the transmitted gnal.

4. Three communication satellities (120° apart from each other) can be used to rovide the communication link over whole of the earth.

5. The transmission station can receive back its own transmission and check, vhether the satellite has transmitted the signal correctly or not.

Drawbacks. 1. The high cost of placing the satellite into its orbit is one of the najor drawbacks of this system.

2. The signal delay caused by the extreme length of the transmission path etween sender and receiver produces echo, which proves quite annoying.

However, this problem is negated with electronic echo cancellers and upressors.

3. This system also suffers from the point of view of security and privacy. The ignal transmitted from the satellite can be heard by any individual.

However, this problem has also been overcome by coding the transmitted ignal.

3.04. OPTICAL COMMUNICATION

The use of optical carrier waves for transmission of information from one place to nother is called optical communication.

Historically, as early as in 1880, Alexander Graham Bell was successful in ransmitting a speech signal using a light beam. Some investigation of optical communication continued in the early part of the twentieth century but its use was limited to low capacity communication links. This was due to both the lack of suitable optical sources and the problem that light transmission in the atmosphere is restricted to line-of-sight. Furthermore, light transmission is severely affected by the disturbances, such as rain, dust, fog, snow, etc. It led to the use of radiowaves for the transmission of information signal, as the radiowaves are far less affected by these atmospheric conditions and can be transmitted over considerably large distances. Since information carrying capacity of carrier waves is directly related to the band width (the greater the carrier frequency, the larger the available transmission bandwidth and hence greater the information-carrying capacity of the communication system), radio communication was developed so as to use radiowaves of even higher frequencies (VHF and UHF).

A renewed interest in optical communication began in 1960 with the invention of laser. It provided a powerful optical source, together with the possibility of modulation at high frequency. In addition, the long distance transmission of signals looked a practical possibility due to low beam divergence characteristic of the laser. However, the atmospheric disturbances (rain, dust, fog, snow, etc) tended to restrict the transmission of signal with lasers to short distance only. To avoid degradation of optical signal by the atmosphere, in 1966, optical fibres were developed as waveguides. Such systems were viewed as a replacement for coaxial cables in transmission systems. The optical fibres developed initially exhibited very high signal attenuation (≈ 1000 dB km^{-1}) in comparison to coaxial cables (5 to 10 dB km^{-1}). In addition to it, there were serious problems involved in jointing the fibre cables in a satisfactory manner. However, within 10 years, optical fibre losses were reduced to 5 dB km^{-1} and suitable low loss jointing techniques were perfected.

In optical communication, light is transmitted in the form of light pulses along an optical fibre as transmission medium. Since wavelength of light is very small as compared to the radio frequency waves, it provides a very large bandwidth and can carry a huge amount of information. In order to have an estimate, consider that light emitted from a He–Ne laser is used for optical communication. The wavelength of light emitted from He–Ne laser is 6328 Å, which corresponds to the frequency $4{\cdot}74 \times 10^{14}$ Hz. Suppose that just 1% of this frequency bandwidth *i.e.* $4{\cdot}74 \times 10^{12}$ Hz is used to transmit TV signals, which require a bandwidth of about 4·7 MHz. Thus, number of TV channels, which can be accommodated in a bandwidth of $4{\cdot}74 \times 10^{12}$ Hz is given by

$$n = \frac{4{\cdot}74 \times 10^{12}}{4{\cdot}7 \times 10^{6}} = 10^{6}$$

Thus, optical communication makes it possible to transmit a tremendously large number of channels simultaneously.

Before taking up the study of an optical communication system, it is necessary to learn about optical fibre and a few optical sources, such as LED, LASER and LASER diode.

3.05. OPTICAL FIBRE

The optical fibres are used to transmit light signals from one place to another without any practical loss in the intensity of the light signal.

An optical fibre consists of a very thin ***glass core*** (about 5 μm to 50 μm in diameter) surrounded by a glass coating, called ***cladding***, as shown in Fig. 3.07. The glass core and cladding are enclosed in a protective jacket made of plastic. The refractive index of the glass used for making core (μ_1) is a little more than the refractive index of the glass used for making the cladding (μ_2) *i.e.* $\mu_1 > \mu_2$. In a typical optical fibre, the values of the refractive indices of the core and the cladding materials may be of the order of 1·52 and 1·48 respectively.

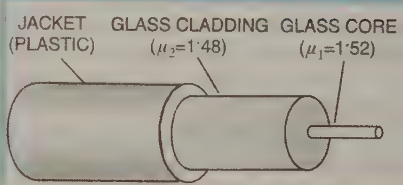


Fig. 3.07

Transmission of light by optical fibre. Consider that a ray of light is incident along AO (at a large angle of incidence) on the glass core from air. It will get refracted along the path OP inside the core and then after suffering refraction at the core-cladding interface, it may follow the path PQ inside the cladding. However, if the angle of incidence is decreased to a value, so that the ray of light inside the core meets the core-cladding interface at an angle equal to or greater that the critical angle for it, then the ray of light will undergo total internal reflection. Fig. 3.08 shows the path of the ray of light, when it comes to incident on the core-cladding interface at critical angle C.

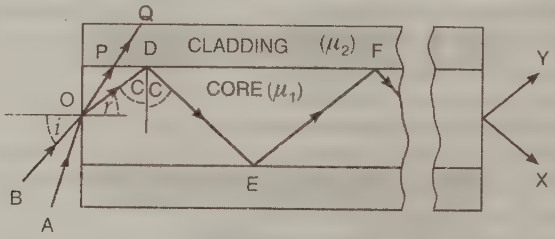


Fig. 3.08

We know that the value of the critical angle at the interface of a pair of media having refractive indices μ_1 and μ_2 is given by

$$\sin C = \frac{\mu_2}{\mu_1} \qquad (\mu_1 > \mu_2)$$

The optical fibre, for which $\mu_1 = 1{\cdot}52$ and $\mu_2 = 1{\cdot}48$, the calculations show that

$$C = 77° \text{ (approx)}$$

It can be easily deduced that the maximum angle of incidence (i) in air, for which all the incident light is totally reflected at the interface of a pair of media having refractive indices μ_1 and μ_2, is given by

$$\sin i = \sqrt{\mu_1^{\,2} - \mu_2^{\,2}} \qquad \ldots(3.01)$$

This value of the maximum angle of incidence is known as *acceptance angle*.

Again, the optical fibre, for which $\mu_1 = 1{\cdot}52$ and $\mu_2 = 1{\cdot}48$, the calculations show hat

$$i = 20°$$

So, for glass core of refractive index 1·52 and cladding of refractive index 1·48, beam of light incident on the glass core from air, making an angle of incidence not nore than 20°, will be totally reflected at the core-cladding interface. Since the angle f incidence at the interface is greater than the critical angle, the ray of light will ndergo total internal reflections again and again inside the optical fibre. As a result, he light is transmitted from one end of the optical fibre to its other end. The glass ised for making the core is of highest quality, so that there is a negligible absorption f light inside the core. Therefore, the light signal comes out almost as bright as it oes in – even if the fibre is several kilometres long. The optical fibres can be as long s 200 km.

Watch Out !

The glass used to make an optical fibre is so much transparent that if the seas were made of it, one could see the sea-beds easily.

From Fig. 3.08, it follows that the ray of light will emerge from the fibre in the irection X after odd number of multiple reflections and in the direction Y after even umber of multiple reflections.

Fibre attenuation. In practice, a very small part of light energy is lost from an ptical fibre due to imperfect transparency (absorption) or due to light leaving the ibre as a result of scattering of light sideways by impurities in the glass fibre. This eduction in energy of the light is called ***attenuation*** and is described by the relation,

$$I = I_0 e^{-\alpha x} \qquad \ldots(3.02)$$

vhere I_0 is intensity of light, when it enters the fibre and I, the intensity of the light t a distance x along the fibre. The constant α is characteristics of the fibre and is alled ***absorption coefficient*** or ***attenuation coefficient.*** It is usually measured as **er kilometre** of the optical fibre. In terms of the power of the light pulses, the elation (3.02) may be written as

$$P = P_0 e^{-\alpha x} \qquad \ldots(3.03)$$

The attenuation of light signal can also be expressed as the ratio $\dfrac{I}{I_0}\left(\text{or } \dfrac{P}{P_0}\right)$.

Thus, attenuation $= \dfrac{I}{I_0}$

In practice, the attenuation is measured on logarithmic scale in the units of **decibel (dB)** using the relation,

$$\text{attenuation (in } dB) = 10 \log_{10} \frac{I}{I_0} \qquad \ldots(3.04)$$

Since $I < I_0$, the attenuation is always negative.

3.06. DIFFERENT TYPES OF OPTICAL FIBRES

The optical fibres are of the following two types :

1. Monomode optical fibre. A monomode optical fibre has a very narrow core of diameter about 5 μm or less, so that the cladding is relatively big ($\approx$125 μm in diameter) as shown in Fig. 3.09 (a).

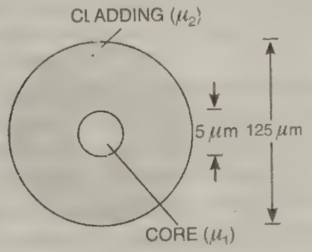


Fig. 3.09 (a)

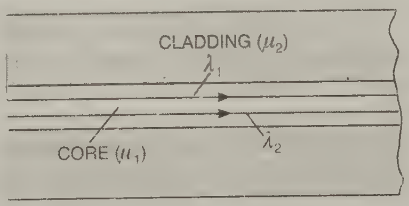


Fig. 3.09 (b)

Fig. 3.09 (b) shows the light paths followed by two wavelengths* λ_1 and λ_2 inside a monomode fibre. In such optical fibres, there is practically no loss in the intensity of the output light signal.

2. Multimode optical fibre. The multimode optical fibres may be subclassified as below :

(i) Step-index multimode fibre. The diameter of the core in step-index multimode fibre is about 50 μm *i.e.* very large as compared to that of a monomode fibre [Fig. 3.10 (*a*)]. It has a constant refractive index μ_1 from its centre to the boundary of the core. The refractive index, then, changes to a lower value μ_2, which remains constant throughout the cladding. Since the refractive index steps down from μ_1 to μ_2 at the core-cladding interface, it is called step-index optical fibre.

---

* Since no light singal is perfectly monochromatic, a narrow band of wavlengths is always present in the signal.

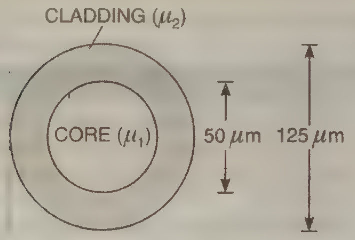


Fig. 3.10 (*a*)

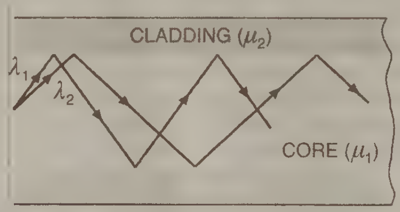


Fig. 3.10 (*b*)

Fig. 3.10 (*b*) shows the light paths followed by two wavelengths λ_1 and λ_2 ($\lambda_1 > \lambda_2$) inside the core of the optical fibre. Since refractive index of a material depends on the wavelength of the light, the two wavelengths follow different paths. As such, the two wavelengths will reach the other end of the optical fibre after covering unequal paths (the wavelength λ_1 travels a longer path than λ_2) and hence at different times. It results in a faulty and distorted signal of lesser intensity. Such an optical fibre is called a multimode fibre, as the different wavelengths in the light signal follow multi light paths inside the fibre. In a step-index multimode fibre, the overall time difference between two wavelengths reaching the other end is of the order of 33×10^{-9}s per kilometre length of the fibre.

(ii) Graded-index multimode fibre. A multimode fibre, whose refractive index decreases smoothly from its centre to the outer surface of the fibre (cladding) is called a graded-index multimode fibre. As such, there is no noticeable boundary between the core and the cladding as shown in Fig. 3.11 (*a*).

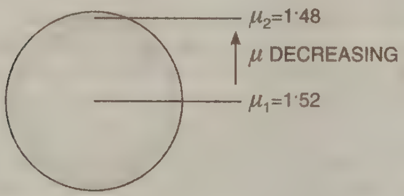


Fig. 3.11 (*a*)

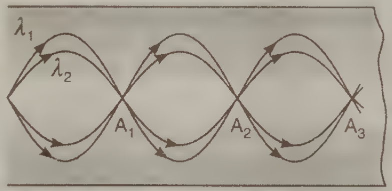


Fig. 3.11 (*b*)

Fig. 3.11 (*b*) shows the light paths followed by two wavelengths λ_1 and λ_2 inside a graded-index multimode fibre. The two wavelengths still follow different light paths inside the fibre. At some layer in the fibre, different for each, the rays are totally reflected. But unlike the step-index fibre, the two wavelengths come to focus at the same point A_1 and then again at A_2 and so on. It is possible, because the refractive index of the material of fibre is lesser for wavelength λ_1 than that of λ_2 and hence speed of wavelength λ_1 is greater than that of λ_2. So the wavelength λ_1 travels a longer path than λ_2 but at a greater speed, so that time of travel is same for both the wavelengths. Thus, in a graded-index multimode fibre, all the wavelengths arrive at the other end of the fibre at the same time. However, there is a small overall time difference ($\approx 10^{-9}$ s per kilometre) between the two wavelengths reaching the other end of the graded-index multimode fibre. Thus, the disadvantage of time delay is considerably reduced in such a fibre in comparison to step-index multimode fibre.

3.07. LIGHT EMITTING DIODE (LED)

A Light Emitting Diode (LED) is a semiconductor *p*-*n* junction diode that emits light, when forward biased.

As studied in Unit IX, a junction diode is formed, when a *p*-type semiconducting material is placed in contact with *n*-type semiconducting material. A thin layer at the junction is devoid of electrons and holes and is called depletion layer. When the junction diode is forward biased, the barrier potential and the width of depletion layer reduces. Due to this, holes from the *p*-section and electrons from the *n*-section of the junction diode cross through the depletion layer. As the holes and electrons encounter, they recombine. The each recombination of a hole and an ctron results in the release of energy. The energy of the emitted photon is equal he amount of energy required to liberate an electron from the valence bond. Thus, velength of the photon of light emitted is determined by the energy gap ΔE tween the valence and conduction band of the semiconductor. The wavelength the light photon is given by the empirical formula,

$$\lambda \text{ (in nm)} = \frac{hc}{\Delta E}$$

In case of silicon and germanium diodes, the energy released is in infra-red gion. In the junction diode made of gallium arsenide or indium phosphide, the ergy is released in visible region. A light emitting diode serves as a good optical urce and is used in optical communication.

Watch Out !

A bright red LED lights up 0·25 s faster than a filament lamp.

A car stoplight becomes visible some time after the brakes have been applied. Therefore, a car stoplight equipped with LED will have a distinct advantage over the one having conventional filament lamp.

3.08. LASER

The term **LASER** stands for **L**ight **A**mplification by **S**timulated **E**mission adiation.

Key point

The word LASER is abbreviated form of the term **L**ight **A**mplification by **S**timulated **E**mission **R**adiation.

We know that when atom jumps from a higher energy state to a lower energy ate, it emits a photon of energy. In any ordinary source of light, atoms emit such hotons independently of each other. As such, the different photons emitted are not phase with each other and as a result, the light is emitted as a whole in the form an incoherent beam. In addition to this, the light emitted is not monochromatic. is because, energy of transition differs by a small amount from photon to photon. urthermore, the light obtained is not in the form of a strictly parallel beam.

Laser is a process by which we can obtain a beam of light, which is highly oherent, highly monochromatic and perfectly parallel. Such a beam is called laser*. laser beam can be sent to a far off place and return back without any practical ss of intensity.

3.09. VARIOUS ATOMIC CONCEPTS RELATED TO PRODUCTION OF LASER

To understand the production of a laser beam, one must know the following tomic processes :

Induced absorption. Consider that two energy states i (lower state) and j higher state) of an atom are associated with energies E_i and E_j respectively. ccording to Boltzmann, the ratio of the atoms in the energy states j and i at a emperature T is given by

$$\frac{N_j}{N_i} = \frac{e^{-E_j/kT}}{e^{-E_i/kT}} = e^{-(E_j/E_i)/kT} \qquad \text{...(3.05)}$$

Since E_j is greater than E_i, it follows that N_j is less than N_i. Therefore, the number f atoms present in the energy state i will be greater than the number of atoms in tate j as shown in Fig. 3.12 (*a*). However, if we shine radiation of frequency

$$\nu = \frac{E_j - E_i}{h}$$

on the atoms, more atoms will be raised to the higher energy state j on absorbing photons of energy $h\nu$ [Fig. 3.12 (*b*)]. This process is called ***induced*** or ***stimulated absorption.***

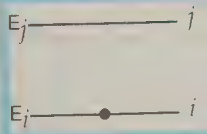


Fig. 3.12 (*a*)

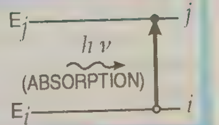


Fig. 3.12 (*b*)

Spontaneous emission. If an atom is present in the higher energy state, it tends to return to the lower energy state within a time of 10^{-8} s by emitting a photon of energy $E_j - E_i$. Fig. 3.13 (*a*) shows one such atom in the higher energy state j, which by emitting a photon of energy $E_j - E_i$ (= $h\nu$) at its own comes down to the lower energy state i as shown in Fig. 3.13 (*b*). This process is called ***spontaneous emission.***

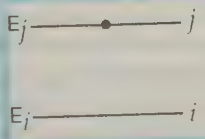


Fig. 3.13 (*a*)

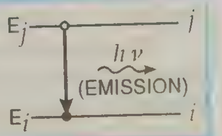


Fig. 3.13 (*b*)

It may be pointed out that during spontaneous emission, photons are emitted in a random manner. As a result of this, the emitted photons will not be in phase with each other. In other words, the beam of light obtained due to spontaneous emission is not a coherent beam.

Stimulated emission. In an atomic system, the higher energy states of the atoms are always populated to some extent, though the population is considerably small as compared to the lower energy state. If the photons of energy $h\nu$ (= $E_j - E_i$) are incident, the chief event that will take place is the induced absorption *i.e.* atoms will be raised from lower to higher energy state. But as the higher energy state is also populated a little [Fig. 3.14 (*a*)], the transition from higher to lower energy state also takes place. As such, the photons of energy $h\nu$ along with the incident photons are emitted [Fig. 3.14 (*b*)]. This process is called *stimulated* or *induced* **emission.**

---

*The word 'laser' is used both for the process and the beam of light obtained by this process.

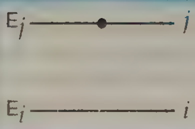


Fig. 3.14 (*a*)

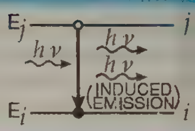


Fig. 3.14 (*b*)

Population inversion. When the higher energy state in an atomic system gets occupied due to induced absorption to a greater extent than the lower energy state, the phenomenon is called ***inverted population of energy levels*** or simply the ***population inversion***. However, the population inversion does not sustain in normal atomic systems, as the atoms tend to return to lower energy state by spontaneous emission.

The process of induced absorption for producing the inverted population of energy levels is also known as ***pumping***.

Metastable states. A metastable state is one, which has a mean life time of the order of 10^{-3} s or more *i.e.* much larger than 10^{-8} s, the life time of a higher energy state. Some atomic systems, such as chromium, neon, etc possess metastable states. The atom of such an atomic system, when in higher energy state, does not come down to lower energy state directly. It first returns to metastable state and then after a finite lapse of time of the order of 10^{-3} s, returns to the lower energy state. Since such atom stays in metastable state for a sufficiently large time, the population inversion can sustain in such atomic systems.

3.10. PRINCIPLE OF PRODUCTION OF LASER BEAM

In the last section, the fundamental atomic concepts involved in the production of laser were discussed. Let us now apply these concepts to understand the basic principle of producing a laser beam.

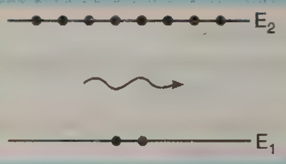


Fig. 3.15

For producing a laser beam, an atomic system is chosen, which has a metastable state. Fig. 3.15 shows an atomic system, which has a lower energy state associated with energy E_1 and a metastable state having energy E_2. Suppose that by some technique, the number of atoms in the metastable state E_2 is made much larger than those in energy state E_1. When photon of energy $E_2 - E_1$ is incident on one of the atoms present in the metastable state, the atom will drop to lower energy state E_1 emitting a photon of same energy as that of the incident photon, which is in phase with it and is emitted in the same direction. The two photons, then, interact with two more atoms present in the metastable state and so on and thus the photons go on multiplying. All these photons possess the same energy, phase and direction. This process is termed as ***amplification of light***. So that this process may take place smoothly, the following two requirements must be fulfilled :

1. *The metastable state should all the time have larger number of atoms than the number of atoms in lower energy state.*

In case it is not so, then a majority of the incident photons, instead of causing stimulated emission, will be used up in raising atoms from lower energy state to the metastable state, unless such a transition is forbidden in the atomic system. For sustained production of a laser beam, the atoms in the metastable state must always be greater than that in the lower energy state *i.e.* population inversion between the two states is must. In other words, the atoms must be kept continuously removed from the lower energy state and raised to the metastable stable to ensure population inversion. It is achieved through the process, called pumping.

2. *The photons emitted due to stimulated emission should stimulate other atoms to multiply the photons inside the system.*

In a way, it requires that the emitted photons should remain inside the system for enough time so that they can interact with the other atoms, before they come out of the system in the form of a beam. It may be achieved by using two reflectors (mirrors) at the two ends of the system so as to reflect the photons back and forth. However, one mirror should be partially transmitting, so as to allow the laser beam to emerge out of the system.

3.11. PRODUCTION OF LASER BEAM

The first successful laser was constructed by T.H. Maiman in 1960 using a small ruby rod and is known as ***ruby laser***. Within a span of 40 years, a variety of lasers have been developed. A few others are known as ***helium-neon laser, carbon dioxide laser, chemical laser, dye laser, junction laser***, etc. Of these, a few laser systems are three energy level systems, while the other are four energy level systems.

1. Ruby laser. Ruby is a crystal of aluminium oxide (Al_2O_3) doped with 0·05% chromium oxide. The adding of chromium oxide imparts red colour to ruby stal. Ruby laser consists of a ruby rod about 5 cm in length and 0·5 cm in diameter. e two faces of the rod are made optically plane and coated with silver. One face he rod is made reflector by thick coating, while the other face is partially coated allow the laser beam to emerge out of the ruby rod. A xenon flash lamp is wound ound the ruby rod. The necessary pumping energy is supplied to the ruby rod the high intensity flash produced by the xenon lamp. The ruby rod is kept cool circulating liquid nitrogen around it [Fig. 3.16].

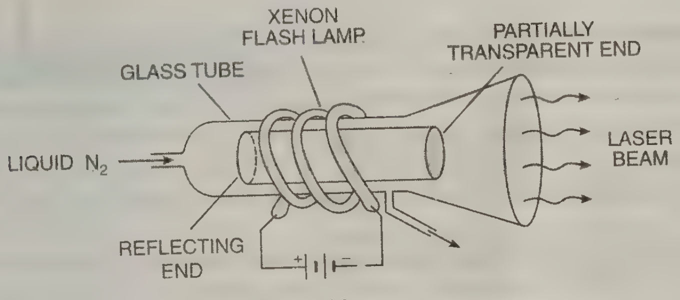


Fig. 3.16

Theory. Ruby laser is a three energy level system. The three energy levels in a romium ion are ground energy level E_0, a metastable level E_1(= 1·79 eV) and a ird higher energy level E_2 (= 2·26 eV) as shown in Fig. 3.17. Initially, the atoms e distributed in different energy levels following Boltzmann's law*. However, hen xenon lamp is flashed, it produces bursts of very intense light (yellowish reen) of wavelength 5500 Å. This light acts as pumping radiation for chromium ons and the atoms are raised from ground state to higher energy state E_2. Within a me of 10^{-8} s, the electrons rapidly drop to either energy level E_1 or directly to round energy level E_0. The energy released during the decay process is lost to the uby rod and before it raises its temperature, it is cooled by the liquid nitrogen used s the coolant. In a short time, there is population inversion *i.e.* metastable state E_1 ecomes more populated than the ground state.

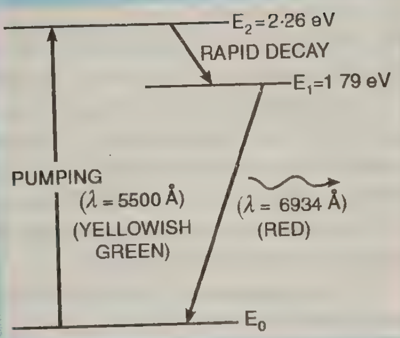


Fig. 3.17

At this stage, once a photon is produced due to spontaneous transition from netastable state E_1 to ground state E_0, another photon of same energy, phase and lirection is produced. The emitted photons suffer multiple reflection between the wo polished ends of the ruby rod. The process of amplification continues, till a quite ntense beam of laser emerges out from the partially transparent end of the rod. The vavelength (λ) of the laser beam obtained can be found from the relation

$$\frac{hc}{\lambda} = E_1 - E_0 \qquad \text{or} \qquad \lambda = \frac{hc}{E_1 - E_0}$$

Setting E_1 = 1·79 eV = $1·79 \times 1·6 \times 10^{-19}$J, $E_0 = 0$, $h = 6·62 \times 10^{-34}$J s and $c = 3 \times 10^8$ m s^{-1} we have

$$\lambda = \frac{6·62 \times 10^{-34} \times 3 \times 10^8}{1·79 \times 1·6 \times 10^{-19} - 0} = 6·943 \times 10^{-7} \text{ m} = 6943 \text{ Å}$$

2. Helium-neon laser. A drawback with the three level system (such as the ruby laser) is that it generally requires very high pumping power. It is because, the terminal state of the laser transition in ruby laser is the ground state. It necessitates that half the ground state atoms must be pumped into the metastable state to achieve population inversion. He-Ne laser is a four energy level system and in constrast to ruby laser, much less pumping power is required in this case.

The experimental arrangement consists of a cylindrical glass tube containing a mixture of helium (≈ 90%) and neon (≈10%) at low pressure. Two parallel mirrors M_1 and M_2 are fixed at the two ends of the glass tube. The mirror M_1 is thickly silvered, while the mirror M_2 is made slightly transmitting. To establish a strong electric field, a high voltage is applied across the electrodes A and C [Fig. 3.18].

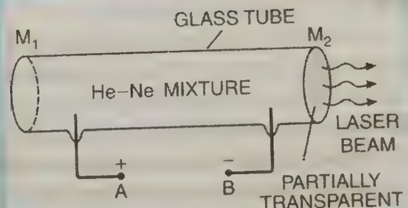


Fig. 3.18

---

*Refer to section 3.09

Theory. Fig. 3.19 shows the four energy levels in neon. They are ground energy level E_0, higher energy level $E_1 (= 18{\cdot}70 \text{ eV})$, metastable energy level $E_1 \ (= 20{\cdot}66 \text{ eV})$ and another high energy level E_3. Helium has a metastable energy level at 20·61 eV, which happens to be close to the metastable level E_2 of neon. It allows helium to be used for pumping neon atoms to the metastable state E_2. When atoms come down from metastable state E_2 to lower energy state E_1 after population inversion, an intense laser beam emerges out of the mirror M_2. The wavelength of the He-Ne laser beam is given by

$$\lambda = \frac{hc}{E_2 - E_1}$$

Here, $E_1 = 18{\cdot}70$ eV and $E_2 = 20{\cdot}66$ eV. It can be calculated that in case of He-Ne laser, the wavelength of the laser beam is equal to 6328 Å.

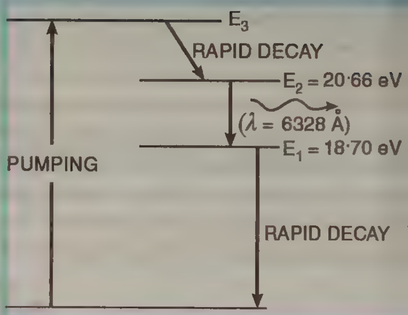


Fig. 3.19

3.12. APPLICATIONS OF LASER

Lasers have proved a boon to the growth of modern scientific world. Lasers have widespread applications in the fields like industry, medical science, telecommunication, scientific research, etc. A few such applications are as below :

1. In communication. Lasers are used to send signals over long distances through optical fibres. They allow to send thousands of signals at the same time due to narrow bandwidth.

2. In measuring long distances. Being highly coherent, laser beams can travel very large distances without any loss in their intensity. As such, a laser beam can be used to measure the distance of a distant object, such as moon.

3. In surgery. Lasers are used to perform practically bloodless surgery. As the laser beam cuts through tissue, it `heat seals' blood vessels and this technique is used in removing tumour from the body, the diseased part of the liver, etc. In the eye, a detached retina can be `welded' back to the wall of the eyeball by laser beam.

Laser beams are used to decolour skin blemishes. They are also used by dental surgeons for drilling of teeth.

4. In engineering. The laser beams are used by surveyors and engineers to ensure that roads, railway lines, pipes, oil tankers, buildings, etc are built accurately.

5. In industry. Laser beams are used to drill holes and cut the metal sheets. They do this job more quickly and accurately than metal drills and saws. In industries, laser beams are used to weld pieces of metal together.

6. In scientific research. Laser beams are used to study molecular structure and nature of chemical reactions.

7. In nuclear power production. A day is not far, when laser beams will be used in carrying out nuclear fusion reactions with much ease for power production.

8. In photography and entertainment. Laser beams can be used to photograph fast moving objects. They are also used in `compact disc' players. Laser TVs have been developed to produce big TV pictures on walls.

9. For national defence. Laser beams are used for controlling and guiding rockets, missiles, satellites and locating planes. Intense laser beams can be used to destroy tanks and planes.

10. In weather forecasting. Laser beams are used to obtain clear pictures of clouds and wind movements for the forecasting of weather. They are also used to measure variations in weather and detection of earthquakes, etc.

3.13. LASER DIODE

A LASER diode is obtained by polishing the two sides of a junction diode made of gallium arsenide. It has the advantage that the material can readily be made *n*-type or *p*-type by varying the relative amounts of the elements Ga and As in it. The two sides of the junction diode are polished so that one side acts as a full mirror and the other side as a partially reflecting surface.

When junction diode is forward biased, the holes and electrons recombine. The photons of light emitted due to recombination of holes and electrons get reflected back and forth between the two sides of the junction diode, till population inversion ccurs. The photons of light falling at this stage trigger the stimulated emission and n intense beam of light is emitted.

3.14. ELEMENTARY PRINCIPLES OF LIGHT MODULATION

The information carrying capacity of carrier waves is directly related to the andwidth. The greater the carrier frequency, the larger the available bandwidth and greater the information carrying capacity of the communication system. Since requency of light is very large as compared to that of the radiowaves, the use of ight as carrier of signal will be more advantageous. Further, a light signal can be efficiently transmitted by using optical fibres without any loss of energy (for practical purposes).

In order to transmit information signal via an optical fibre communication system, it is necessary to modulate light with the information signal. For the information signal, light acts as the carrier. To modulate light, some of its properties such as intensity, frequency, phase or polarisation (direction) are varied in accordance with the information signal. The choice of variation of any parameter is decided by the characteristics of the optical fibre, the available sources and detectors.

Principle of light modulation. There are several methods of carrying out light modulation, namely intensity modulation, frequency modulation, phase modulation, etc. The intensity modulation in itself can be carried out in the following two ways :

1. ***Direct optical modulation.*** It is the process of varying the intensity of light directly in accordance with the information signal to be transmitted. It is achieved by varying current through the optical source (LED or LASER) by biasing it corresponding to the information signal. As a result, when the light emerges out of the optical source, its intensity varies in accordance with the information signal.

However, this method causes a change in the wavelength of light and this defect is called system degradation. This problem is overcome in frequency modulation of the light. In this process, the frequency dependent current is passed through the source. The direct modulation of the optical source is quite satisfactory for the modulation bandwidths currently under investigation.

2. **External optical modulation.** External optical modulators are used, when frequency or phase of the light is to be modulated. They make use of non-semiconductor sources. However, such sources cannot be directly modulated at high frequency.

3.15. OPTICAL COMMUNICATION SYSTEM

Fig. 3.20 shows the block diagram for an optical fibre communication system.

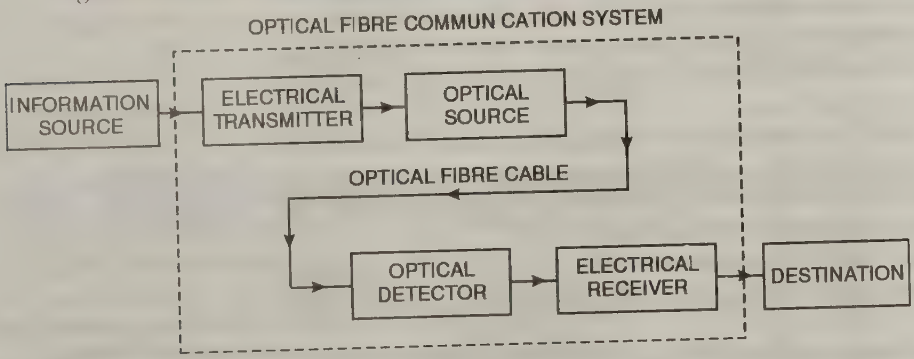


Fig. 3.20

The information source provides an electrical signal to an electrical transmitter, which drives an optical source to give modulation of the light waves. The optical source, which provides the electrical-optical conversion, may be either a semiconductor laser or light emitting diode (LED). The optical fibre cable is used as the transmission medium for the optical signal. At the receiver's end, the optical signal is detected by optical detector, which converts the optical signal into an electrical signal. The optical detector may be a photodiode or a phototransistor or a photoconductor. The electrical signal is further demodulated by the electrical receiver so as to reproduce the original information signal, which is then received at the destination.

3.16. ADVANTAGES OF OPTICAL FIBRE COMMUNICATION

The optical fibre communication is far more advantageous as compared to conventional electrical communication. A few advantages are as given below :

1. Enormous transmission bandwidth. The optical carrier frequency lies in the frequency range 10^{13} to 10^{16} Hz. Due to extremely high frequency of opical carrier waves, the transmission bandwidth and hence information-carrying capacity of an optical communication system is very large as compared to a coaxial cable system.

2. Small size and weight. Optical fibres have very small diameters, which are often no greater than the diameter of a human hair. Even on being covered with protective coatings, they are far smaller and much lighter than coaxial cables.

3. Immunity to electrical disturbances. Optical fibres are made from glass or sometimes from a plastic polymer. Since such materials are electrical insulators, the optical fibres are free from spark hazards and short circuits. Furthermore, unlike metallic transmission lines, they do not exhibit earth loop and interface problems. As such, they are very ideal for transmission of information in electrically hazardous environments.

4. Immunity to interference. Optical fibres serve as dielectric waveguides and are therefore free from electromagnetic interference and radio frequency interference. The optical fibre cables are also not susceptible to lightning strikes, if used overheads rather than underground. Hence, optical fibre communication is unaffected by electrically noisy environment.

5. Immunity to crosstalk. In optical fibres, there is no optical interference between fibres. Therefore, even when many fibres are cabled together, unlike communication using electrical wires, crosstalk is negligible.

6. Signal security. The light from optical fibres does not radiate significantly and hence optical fibre communication provides a high degree of signal security. Further, any attempt to tap a message signal transmitted optically can be detected easily. This feature makes it extremely useful in military, banking and computer network applications.

7. Low transmission loss. The recent developments in optical fibre communication systems have resulted in fabricating optical fibres, which exhibit very low attenuation ($\approx 0{\cdot}2$ dB km^{-1}).

8. Ruggedness and flexibility. Although optical fibres are made from glassy substances, yet they can be bent or twisted without any damage. They prove superior in terms of storage, transportation, handling and installation in comparison to metallic cables.

9. System reliability. Due to low intensity loss property of optical fibre cables, the requirement of intermediate repeaters or line amplifiers to boost the transmitted signal strength is much reduced. Hence, with fewer repeaters, the reliability of an optical communication system is much enhanced. An optical fibre communication system possesses a predicted life time of the order of 20 to 30 years.

10. Potential low cost. The glass which is generally used to fabricate an optical fibre is made from sand. So, in comparison with copper wire lines, optical fibres prove quite inexpensive.

11. Ideal for multiplexing. Optical fibres can carry a large number of signals at the same time much more efficiently than that can be done in radio communication.

12. No signal jamming. Radio signals usually get jammed, when the signals are transmitted at the same frequency of radiowaves. Optical communication is free from jamming of signal for all practical purposes.

Drawbacks. The optical communication systems have a few drawbacks also, which are listed as below :

1. If there are any scratches on the optical fibre cable, the light signal can escape at such points. It is because, a ray of light can meet the surface of the scratch at an angle of incidence less than the critical angle. To ensure that the core of the cable stays smooth, the outer coating of the fibre cable has to be made tougher.

2. The optical fibre technology can be handled only by the experts in this field.

SOLVED NUMERICAL PROBLEMS

Problem 3.01. Work out the critical angle for light at the oundary between the core and the cladding of a step index bre, if the refractive of the core is 1·48 and of the cladding s 1·45. *(Text Problem)*

Sol. Here, $\mu_1 = 1{\cdot}48$ and $\mu_2 = 1{\cdot}45$

Now, $$\sin C = \frac{\mu_2}{\mu_1} = \frac{1{\cdot}45}{1{\cdot}48} = 0{\cdot}9797$$

or $$C = 78{\cdot}4°$$

Problem 3.02. Calculate the critical angle and the angle f acceptance, if the refractive indices of the glass core and he cladding are 1·7 and 1·5 respectively.

Sol. Here, $\mu_1 = 1{\cdot}7$ and $\mu_2 = 1{\cdot}5$

The critical angle for glass core and cladding as the two nedia is given by

$$\sin C = \frac{\mu_2}{\mu_1} = \frac{1{\cdot}5}{1{\cdot}7} = 0{\cdot}8824$$

or $$C = \mathbf{61{\cdot}9°}$$

The acceptance angle for the optical fibre is given by

$$\sin i = \sqrt{\mu_1^{\,2} - \mu_2^{\,2}} = \sqrt{(1{\cdot}7)^2 - (1{\cdot}5)^2} = 0{\cdot}8$$

or $$i = \mathbf{53{\cdot}1°}$$

Problem 3.03. Calculate the time taken for light to ravel through a glass optical fibre 30 m long (*a*) in axial mode (*b*) in highest order mode, given that the refractive index is 1·5 for glass and 1·3 for the cladding.

Sol. Here, $\mu_1 = 1{\cdot}5$, $\mu_2 = 1{\cdot}3$ and $l = 30$ m

(*a*) Speed of light in optical fibre,

$$v = \frac{c}{\mu_1} = \frac{3 \times 10^8}{1{\cdot}5} = 2 \times 10^8 \text{ m s}^{-1}$$

Therefore, time taken for light to travel through optical fibre in axial mode,

$$t = \frac{l}{v} = \frac{30}{2 \times 10^8} = 1{\cdot}5 \times 10^{-7} \text{ s} = \mathbf{150 \text{ ns}}$$

(*b*) Light in the highest order mode meets the fibre surface at the critical angle, which is given by

$$\sin C = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \sin C = \frac{1{\cdot}3}{1{\cdot}5} = 0{\cdot}8667$$

or $$C = 60°$$

Refer to Fig. 3.08. If l is length of the optic fibre, then actual path length DE = x (say) is given by

$$x = \frac{l}{\sin C} = \frac{30}{\sin 60°} = \frac{30}{0{\cdot}8667} = 34{\cdot}6 \text{ m}$$

Therefore, time taken for light to travel through optical fibre in the highest order mode,

$$t = \frac{x}{v} = \frac{34{\cdot}6}{2 \times 10^8} = 1{\cdot}73 \times 10^{-7} \text{ s} = \mathbf{173 \text{ ns}}$$

Problem 3.04. The intensity of a light pulse travelling along a fibre decreases exponentially with distance according to the equation $I = I_0 e^{-\alpha x}$, where I_0 is the intensity at $x = 0$ and α is the attenuation constant. Show that the intensity reduces by 50% over a distance euqal to $\frac{\log_e 2}{\alpha}$. *(Text Problem)*

Sol. The intensity of light pulse over a distance x,

$$I = I_0 e^{-\alpha x} \qquad \ldots(i)$$

Suppose that intensity of light pulse reduces by 50% over a distance x'. Thus,

when $x = x'$, $I = \frac{50}{100} \times I_0 = 0{\cdot}5\, I_0$

Setting the above condition in equation (*i*), we have

$$0{\cdot}5\, I_0 = I_0 e^{-\alpha x'} \quad \text{or} \quad e^{-\alpha x'} = \frac{1}{0{\cdot}5} = 2$$

$$\text{or} \quad \alpha x' = \log_e 2 \quad \text{or} \quad x' = \frac{\log_e 2}{\alpha}$$

Problem 3.05. Attenuation can be expressed in decibels (*dB*) according to the equation $10 \log_{10} \frac{I}{I_0}$. Work out the attenuation in *dB* per km for a fibre in which the intensity falls by 50% over a distance of 50 km. *(Text Problem)*

Sol. Now, attenuation (in *dB*) = $10 \log_{10} \frac{I}{I_0}$

Therefore, when light falls by 50% *i.e.* when $I = 0{\cdot}5\, I_0$,

$$\text{attenuation} = 10 \log_{10} \frac{0{\cdot}5\, I_0}{I_0} = 10 \log_{10} 0{\cdot}5$$
$$= 10 \times (-0{\cdot}3010) = -3{\cdot}01 \; dB$$

This attenuation in signal occurs over a distance of 50 km. Therefore

$$\text{attenuation per km} = -\frac{3{\cdot}01}{50} = \mathbf{-0{\cdot}06 \; dB \; km^{-1}}$$

FAQs FREQUENTLY ASKED VERY SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. What is a transmission medium ?

Ans. *The link, which transfers information from the information source to the destination, is called transmission medium.*

Q. 3.02. What are the various transmission media used in communication systems ?

Ans. 1. 2-wire lines **2.** Coaxial cables **3.** Radio links **4.** Fibre links.

Q. 3.03. What is the main drawback of a 2-wire line ?

Ans. When a 2-wire line is used at microwave frequencies, the signal becomes weak due to energy radiated by the two conducting wires.

Q. 3.04. What is a coaxial cable ?

Ans. It consists of a central copper wire surrounded by a PVC insulation, over which a sleeve of copper mesh is placed. The copper mesh is further covered with an outer shield of thick PVC material.

Q. 3.05. What is the main advantage of using coaxial cable for transmitting a signal in place of 2-wire line ?

Ans. The copper mesh surrounding the central copper wire makes it immune to the external electrical disturbances.

Q. 3.06. What is the function of copper mesh in a coaxial cable ?

Ans. Due to electric current, the magnetic field inside a hollow conductor is always zero. When the central copper wire carries the signal, the copper mesh shields it electrically from the external electrical disturbances.

Q. 3.07. What do you mean by `switching circuit' in a telephone system ?

Ans. All the telephones in a locality are connected to the telephone exchange. When a calling subscriber dials in the called subscriber, the switching circuit interconnects the two.

Q. 3.08. What is an optical fibre ?

Ans. An optical fibre consists of a glass core (very thin thread of glass) surrounded by a glass coating of comparitively lesser refractive index. It is used to transmit optical signals from one place to another place.

Q. 3.09. What is `cladding' ?

Ans. A glass coating surrounding the glass core in an optical fibre is called cladding. The refractive index of cladding is a littile less than that of the glass core.

Q. 3.10. What does LASER stand for ?

Ans. The word LASER stands for Light Amplification by Stimulated Emission Radiation.

Q. 3.11. What are the main characteristics of a laser beam ?

Ans. A laser beam is highly coherent, highly monochromatic and perfectly parallel.

Q. 3.12. What is stimulated emission ?

Ans. *The emission of photons in an atomic system by shining photons of energy equal to the difference in energies associated with the higher and lower energy states alongwith the incident photons is called stimulated emission.*

Q. 3.13. What is a `metastable stable' ?

Ans. *The energy state in an atomic system, which has a mean life time much larger than the life time of a normal higher energy state, is called metastable state.*

Q. 3.14. What is `population inversion' ?

Ans. Normally, the number of atoms in a higher energy state is always less than those in the lower energy state.

The process of making number of atoms in higher energy state more than that in the lower energy state by induced absorption is called population inversion.

Q. 3.15. What do you mean by pumping ?

Ans. *The process of raising atoms from lower energy state to higher energy state by induced absorption is known as pumping.*

Q. 3.16. What do you mean by an `optical source' used in optical communication ?

Ans. In optical communication, an optical source is one, which can convert electrical energy into optical energy, which may be suitable for communication purposes.

Q. 3.17. Name two optical sources used in optical communication.

Ans. 1. Laser **2.** Light emitting diode (LED)

Q. 3.18. What is light modulation ?

Ans. *The process of modulating a light beam from an optical source in accordance with the information signal is called light modulation.*

Q. 3.19. What is direct intensity modulation ?

Ans. The process of light modulation, in which the intensity of light is varied in accordance with the message signal in a direct manner by biasing the optical source corresponding to the message signal, is known as direct intensity modulation.

FREQUENTLY ASKED SHORT ANSWER QUESTIONS

With Answers/Hints

Q. 3.01. Why do we prefer a twisted pair in comparison to parallel-wire line in communication systems ?

Ans. In a parallel-wire line, the interference between the magnetic fields produced by the signal current in the two wires causes distortion in the signal. This effect gets quite minimised in a twisted pair. In addition, the twisted pair is less prone to electrical disturbances. For these reasons, twisted pair is preferred to parallel-wire line.

Q. 3.02. Why does a parallel-wire line radiate energy ? How is this problem overcome in a twisted pair ?

Ans. A system of conductors always radiates energy in the form of radiowaves, when the separation between the two wires approaches a half-wavelength at the operating frequency. This effect becomes more pronounced at the frequency of microwaves. So that the two conductors donot remain at a finite constant distance of the order of wavelength of the transmitted signal, the two insulated copper wires are twisted around each other.

Q. 3.03. What is a coaxial cable ? Why is it superior to other wire-line systems ?

Ans. A coaxial cable consists of a central copper wire surrounded by a PVC insulation over which a sleeve of copper mesh is covered. The copper mesh is covered with an outer shield of a thick PVC material. The signal is transmitted by the inner copper wire, which is electrically shielded by the outer copper mesh. The other types of wire-lines pick up external electrical disturbances leading to distortion in the signal.

Q. 3.04. What are the merits of coaxial cables ?

Ans. Refer to section 3.02.

Q. 3.05. What are the advantages of using a communication satellite as a telephone link ?

Ans. Refer to section 3.03.

Q. 3.06. What is the principle of an optical fibre ? What is the necessary condition that should be obeyed for transmitting a signal with the help of it ?

Ans. An optical fibre is based on the principle of total internal reflection.

It states that *when a ray of light travelling from an optically denser medium to an optically rarer medium is incident at an angle greater than the critical angle for the two media, the ray of light is totally reflected back into the same medium.*

Necessary condition. The ray of light incident on the optical fibre in air should not be incident at an angle greater than the angle given by

$$\sin i = \sqrt{\mu_1^2 - \mu_2^2},$$

where the letters have their usual meanings.

Q. 3.07. What do you mean by `angle of acceptance' in optical fibres ? Give expression for it.

Ans. *The maximum angle of incidence in air, for which all the incident light is totally reflected at the core-cladding interface in an optical fibre, is called the angle of acceptance.*

It is given by

$$\sin i = \sqrt{\mu_1^2 - \mu_2^2},$$

where μ_1 and μ_2 are refractive indices of the glass core and cladding respectively.

Q. 3.08. What is the difference between monomode and multimode optical fibres ?

Ans. A monomode optical fibre has a very narrow core ($\approx 3 \mu m$ in diameter) ; while in a step-index multimode optical fibre, the diameter of the core is compartively very large ($\approx 50 \mu m$). In another type of multimode optical fibre, called graded-index multimode fibre, there is no noticeable boundary between the core and the cladding. Further, in a monomode optical fibre, different wavelengths travel along parallel light paths of equal lengths, whereas in a multimode optical fibres, the light paths of different wavelengths are not equal.

Q. 3.09. What is difference between step-index multimode and graded-index multimode optical fibres ?

Ans. A step-index multimode optical fibre has a constant refractive index from its centre to the boundary of the core and then refractive index changes to a lower value, which remains constant throughout the cladding. The different wavelengths reach the other end of the optical fibre following light paths of different lengths.

In graded-index multimode optical fibre, there is no noticeable boundary between the core and the cladding. The different wavelengths follow paths of different lengths, but reach the other end of the optical fibre, practically at the same time.

Q. 3.10. Why do we prefer a graded-index multimode optical fibre in comparison to step-index multimode optical fibre ?

Ans. In a multimode optical fibre, different wavelengths follow different light paths and therefore there is a time difference between the wavelengths reaching the other end of the optical fibre. In case of step-index multimode optical fibre, the overall time difference is of the order of 33 ns. However, in case of graded-index multimode optical fibre, the overall time difference between different wavelengths is much reduced ($\approx$ 1 ns).

Q. 3.11. What is the principle of LASER ? Name the basic atomic concepts involved in the production of a laser beam.

Ans. Principle. It is based on the principle that in the atomic systems possessing metastable states, the population inversion can be caused and then the process of stimulated emission can be used to produce highly coherent, highly monochromatic and perfectly parallel beam of light.

The various atomic concepts involved in the production of a laser beam are pumping, population inversion and stimulated emission.

Q. 3.12. Why do we prefer He-Ne laser in comparison to ruby laser ?

Ans. A ruby laser is a three level system and it generally requires very high pumping power. It is because, the terminal state of the laser transition in ruby laser is the ground state.

On the other hand, a He-Ne laser is a four level system and in contrast to ruby laser, much less pumping power is needed in this case.

Q. 3.13. Name a few applications of LASER.

Ans. Refer to section 3.12.

Q. 3.14. Name a few advanges of optical fibre communication.

Ans. Refer to section 3.16.

FAQs FREQUENTLY ASKED SHORT ANSWER QUESTIONS Carrying 3 Marks

1. What are parallel-wire and twisted pair ? Explain in brief. Give merits of twisted pair.
2. What is a coaxial cable ? What is its advantage over parallel-wire line ? Why does no attenuation of radio frequency signal occur, when transmitted along it ?
3. Why do we require line communication ? *(Text Question)*
4. Explain, how a coaxial cable can carry many telephone calls at the same time. *(Text Question)*
5. What is a communication channel ? Describe briefly the various communication channels employed in communication. *(Text Question)*
6. Explain microwave link in brief. How is the problem of line-of-sight transmission overcome ?
7. What is communication satellite link ? Explain in brief. Give its advantages and disadvantages.
8. What is an optical fibre ? Explain the various types of optical fibres.
9. What is the principle of optical fibre ? How is a light signal transmitted along it ? Show it with the help of a diagram.
10. Explain, why an optical fibre transmits light round bends. *(Text Question)*
11. Give the advantages of optical communication system.
12. Explain the terms induced absorption, spontaneous emission, stimulated emission and population inversion.
13. Explain the action of a ruby laser with the help of energy level diagram.
14. Explain, how He-Ne laser works. Give the necessary energy level diagram.
15. Give the various applications of LASER.
16. What is an LED ? Briefly describe its operational principle. *(Text Question)*
17. What is light modulation ? Explain, how the direct intensity modulation is carried out.

FAQs FREQUENTLY ASKED LONG ANSWER QUESTIONS Carrying 5 Marks

1. What is a 2-wire line ? Discuss the different types of 2-wire line systems giving the advantages of one over the other.
2. What is twisted pair and how is it employed for line communication ? Discuss its advantages and limitations. *(Text Question)*
3. Describe the construction of a coaxial cable ? How does its construction help in increasing the bandwidth ? Discuss source of its advantages over the twisted pair. *(Text Question)*
4. What is a microwave link ? What are its drawbacks ? How are the problems encountered in microwave link overcome in satellite link ?
5. What is remote sensing ? Explain briefly how it is carried out. Mention some of its applications. *(Text Question)*
6. What is an optical fibre ? Give its construction. With the help of a suitable diagram, explain as to how a signal may be transmitted from its one end to the other. What makes it efficient for transmitting a signal ?

7. What is the principle of an optical fibre ? What are the different types of optical fibres ? Explain them in brief giving a suitable diagram in each case.
8. What does LASER stand for ? Explain the various atomic concepts related to the production of a laser beam.
9. What is the principle of laser ? Explain the experimental arrangement and theory of ruby laser with the help of energy level diagram.
10. What is the merit of He-Ne laser over ruby laser ? Explain the experimental arrangement made in a He-Ne laser. Give the necessary theory of production of laser in such a system.
11. Explain the working of a semiconductor laser. It is an ideal light source for optical communication. Justify. *(Text Question)*
12. What is optical communication ? Give the main advantages of optical communication system.

FAQs FREQUENTLY ASKED NUMERICAL PROBLEMS

For Practice

Type A. On Optical fibre

1. An optical fibre is made from core glass of refractive index 1·62 and cladding of refractive index 1·53. Find the angle of acceptance. **[Ans.** 32·2°]
2. Calculate the critical angle and the angle of acceptance for an optical fibre, whose core and cladding are of refractive indices 1·68 and 1·55 respectively. **[Ans.** 67·3° ; 40·4°]
3. (*a*) Calculate the critical angle for a boundary between a glass fibre for which the refractive index is 1·6 and cladding for which the refractive index is 1·5
   (*b*) Calculate the ratio of the path lengths for the highest order mode and axial mode in the fibre described in (*a*). **[Ans.** (*a*) 69·6° (*b*) 1·067]
4. Calculate the difference in time taken to travel through a 40 m fibre by red light and blue light for which the fibre has refractive indices of 1·45 and 1·47 respectively. Take the velocity of light in vacuum to be 3×10^8 m s^{-1} and consider the axial mode only.
5. Calculate the attenuation in *d*B expected for a fibre-optic cable 300 m long with an absorption coefficient of 0·2 *d*B km^{-1}. **[Ans.** 0·26 *d*B]
6. In a particular digital communication system, the signal to noise ratio must not be less than 21 *d*B.
   (*a*) If the noise power is 10^{-20} W, what is the minimum power that the transmitted signal can have ?
   (*b*) If the attenuation is 2 *d*B km^{-1} and the initial power is 10^{-3} W, what is the maximum distance the signal can travel before it is regenerated ?
   [Ans. (*a*) $1{\cdot}259 \times 10^{-18}$ W (*b*) 74·5 km]

SOLUTIONS/HINTS TO NUMERICAL PROBLEMS FOR PRACTICE

1. Here, $\mu_1 = 1{\cdot}62$, $\mu_2 = 1{\cdot}53$

The acceptance angle for optical fibre is given by

$$\sin i = \sqrt{\mu_1^2 - \mu_2^2} = \sqrt{(1\cdot 62)^2 - (1\cdot 53)^2} = 0\cdot 5324$$

or $i = 32{\cdot}2°$

2. Proceed as in solved problem no. 3.02.
3. (**a**) Proceed as in solved problem no. 3.01.
   (**b**) Proceed as in solved problem no. 3.03.
4. Here, $\mu_r = 1{\cdot}45$, $\mu_b = 1{\cdot}47$

$c = 3 \times 10^8$ m s^{-1} and $l = 40$ m

Now, $$v_r = \frac{c}{\mu_r} = \frac{3 \times 10^8}{1{\cdot}45} = 2{\cdot}069 \times 10^8 \text{ m s}^{-1}$$

and $$v_b = \frac{c}{\mu_b} = \frac{3 \times 10^8}{1{\cdot}47} = 2{\cdot}047 \times 10^8 \text{ m s}^{-1}$$

Time of travel for red light through the fibre,

$$t_r = \frac{l}{v_r} = \frac{40}{2{\cdot}069 \times 10^8} = 1{\cdot}933 \times 10^{-7} \text{ s} = 193{\cdot}3 \text{ ns}$$

Also time of travel for blue light through the fibre

$$t_b = \frac{l}{v_b} = \frac{40}{2{\cdot}041 \times 10^8} = 1{\cdot}960 \times 10^{-7} \text{ s} = 196{\cdot}0 \text{ ns}$$

$\therefore$ $t_b - t_r = 196{\cdot}0 - 193{\cdot}3 = \mathbf{2{\cdot}7 \text{ ns}}$

5. Here, $\alpha = 0{\cdot}2$ *d*B km^{-1}, $x = 300$ m $= 0{\cdot}3$ km

The intensity of light pulse over a distance x is given by

$$\mathrm{I} = \mathrm{I}_0 e^{-\alpha x}$$

$$\therefore \quad \frac{\mathrm{I}}{\mathrm{I}_0} = e^{-\alpha x} = e^{-0{\cdot}2 \times 0{\cdot}3} = e^{-0{\cdot}06} = 0{\cdot}942$$

Therefore, attenuation (in *d*B)

$$= 10 \log_{10} \frac{\mathrm{I}}{\mathrm{I}_0} = 10 \log 0{\cdot}942$$

$$= 10 \times (-0{\cdot}026) = \mathbf{-0{\cdot}26\ dB}$$

6. (**a**) Here, ratio of signal to noise power (in *d*B) = 21 *d*B

noise power, $\mathrm{P}' = 10^{-20}$ W

Now, ratio of power (in *d*B) $= 10 \log_{10} \frac{\mathrm{P}}{\mathrm{P}'}$

$$\therefore \quad 21 = 10 \log_{10} \frac{\mathrm{P}}{10^{-20}} \quad \text{or} \quad \log_{10} \frac{\mathrm{P}}{10^{-20}} = 2{\cdot}1$$

$$\text{or} \quad \frac{\mathrm{P}}{10^{-20}} = 125{\cdot}9 \quad \text{or} \quad \mathrm{P} = \mathbf{1{\cdot}259 \times 10^{-18}\ W}$$

(*b*) Now, attenuation (in *d*B) $= 10 \log_{10} \frac{\mathrm{P}}{\mathrm{P}_0}$

Here, $\mathrm{P} = 1{\cdot}259 \times 10^{-18}$ W, $\mathrm{P}_0 = 10^{-3}$ W

Therefore, attenuation $= 10 \log_{10} \frac{1{\cdot}259 \times 10^{-18}}{10^{-3}}$

$$= \log 1{\cdot}259 \times 10^{-15} = 10 [\log 1{\cdot}259 + \log 10^{-15}]$$

$$= 10 [0{\cdot}1 + (-15)] = -149 \text{ dB}$$

Since attenuation per km is 2 *d*B km^{-1}, the maximum distance the signal can travel is given by

$$x = \frac{149}{2} = \mathbf{74{\cdot}5\ km}$$

Competitive Examination File (Unit X)

REVISION AT A GLANCE

Chapter 1. Analog and Digital Communication

☐ **Communication system.** *The setup used for exchaging information between a sender and a receiver is called communication em.*

☐ **Transmitter.** *The part of the communication system, which sends out the information is called transmitter.*

☐ **Receiver.** *The part of the communication system, which picks up the information sent out by the transmitter is called receiver.*

☐ **Communication channel.** *The medium or the link, which transfers message signal from the transmitter to the receiver of a munication system is called communication channel.*

☐ **Transducer.** *The device which converts energy in one form to another is called a transducer.*

☐ **Analog signal.** *A singal, which is a continuous function (usually a sinusoidal function) of time, is called an analog signal.*

☐ **Bandwidth.** *The frequency range of a signal is called its bandwidth.*

☐ **Baseband signal.** *The information bearing signal is called a baseband signal.*

☐ **Digital signal.** *A discrete signal (discontinuous function of time), which has only two levels (either low or high) is called a ital signal.*

☐ **Analog modulation.** *The process of producing a continuous high frequency wave, some characteristic of which varies as a function he instantaneous value of the audio signal, is called analog modulation.*

☐ **Amplitude modulation.** *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the uency of the modulated wave is same as that of the carrier wave but its amplitude varies in accordance with the amplitude of the dulating wave, the process is called amplitude modulation.*

☐ **Frequency modulation.** *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the plitude of the modulated wave is same as that of the carrier wave but its frequency varies in accordance with the amplitude of the dulating wave, the process is called frequency modulation.*

☐ **Phase modulation.** *When the modulating wave is superimposed on a high frequency carrier wave in a manner that the magnitude he phase angle of the modulated signal varies in accordance with the amplitude of the modulating wave, the process is called phase dulation.*

☐ **Modulation index.** *The degree, to which the carrier wave is modulated, is called modulation index.*

☐ **Demodulation.** *The process of separating out the audio signal from the modulated signal is called demodulation.*

☐ **Pulse modulation.** *The process of producing a train of pulses of the carrier, some characteristic of which varies as a function the instantaneous value of the message singal, is called pulse modulation.*

☐ **Pulse Amplitude Modulation (PAM).** *The process of pulse modulation, in which amplitude of the pulses of the carrier pulse in varies in accordance with the instantaneous value of the message signal.*

☐ **Pulse Width Modulation (PWM).** *The process of pulse modulation, in which the width of the pulses of the carrier pulse train ries in accordance with the instantaneous value of the message signal.*

☐ **Pulse Position Modulation (PPM).** *The process of pulse modulation, in which the position of the pulses of the carrier wave in varies with the instantaneous value of the message signal.*

☐ **Pulse Code Modulation (PCM).** *The process of converting an analog signal into a digital signal by sampling it in time, then antising and coding it, is called pulse code modulation.*

☐ **Sampling.** *It is the process of generating pulses of zero width (duration) and of amplitude equal to the instantaneous amplitude the analog signal.*

The number of samples of analog signal taken per second is called **sampling rate.**

☐ **Quantisation.** *The process of dividing the maximum amplitude of the analog voltage signal into a fixed number of levels is lled quantisation.*

☐ **Coding.** *The process of digitising the quantised pulses according to some code (usually binary code) is called coding.*

☐ **Bit rate.** *The number of binary digits (bits) per second, which describe a digital signal is called its bit rate.*

Mathematically : Bit rate = sampling rate × number of bits per sample

☐ **Fax.** *The electronic transmission of a document at a distant place via telephone line is known as FAX.*

Chapter 2. Space Communication

☐ **Earth's atmosphere.** *The gaseous envelope surrounding the earth is called earth's atmosphere.*

☐ **Ground wave propagation.** It is of the following two types :

1. Surface wave propagation. *When the radiowaves from the transmitting antenna propagate along the surface of the earth so s to reach the receiving antenna, the wave propagation is called surface wave propagation.*

2. Space wave propagation. *When the radiowaves from the transmitting antenna reach the receiving antenna either directly or after reflection from the ground or in the troposphere, the wave propagation is called space wave propagation.*

☐ **Sky wave propagation.** *When the radiowaves from the transmitting antenna reach the receiving antenna after reflection in the ionosphere, the wave propagation is called sky wave propagation.*

☐ **Transmission of TV signals.** The TV signals are frequency modulated. The maximum distance upto which TV signals can be received is given by

$$d = \sqrt{2hR},$$

where h is height of the TV antenna and R, the radius of earth.

☐ **Communication satellite.** *An artificial satellite used for communication purposes is called communication satellite.*

☐ **Remote sensing.** *Any method of obtaining and recovering information from a distance is called remote sensing.*

Chapter 3. Line Communication

☐ **Transmission medium.** *The link, which transfers information (message signal) from the information source to the destination, is called transmission medium.*

☐ **Guided transmission medium.** *The transmission medium, which enables the message signal reach the destination by making it to follow a guided path is called a guided transmission medium.*

☐ **Unguided transmission medium.** *The transmission medium, which enables the message signal to reach the destination along an unguided path is called an unguided transmission medium.*

☐ **Multiplexing.** *Simultaneous transmission of a number of messages over a single channel is known as multiplexing.*

☐ **Optical Communication.** *The use of optical carrier waves for the transmission of information from one place to another is called optical communication.*

MULTIPLE CHOICE QUESTIONS FROM COMPETITIVE EXAMINATIONS

Chapter 1. Solids

RT I. TEXT BASED QUESTIONS

1. **An antenna is a device**
   (A) that converts electromagnetic energy into readio frequency signal.
   (B) that converts radio frequency signal into elecromagnetic energy.
   (C) that converts guided electromagnetic waves into free space electromagnetic waves nad vice-versa.
   (D) none of the above.
2. **An antenna behaves as a resonant circuit only when its length**
   (A) equals $\lambda/4$ (B) equals $\lambda/2$
   (C) equals $\lambda/2$ or its integral multiple
   (D) none of the above.
3. **In modulation process, radio frequency wave is termed as**
   (A) modulating wave (B) modulated wave
   (C) carrier wave (D) none of the above.
4. **Modulation is used to**
   (A) reduce the bandwidth used.
   (B) separate different transmissions from each other.
   (C) ensure that intelligence may be transmitted over long distances.
   (D) allow access to the intelligence at the same time to different persons.
5. **In amlitude modulation**
   (A) the amplitude of the carrier varies in accordance with the amplitude of the modulating signal.
   (B) the amplitude of the carrier remains constant.
   (C) the amplitude of the carrier varies in accordance with the frequency of the modulating signal.
   (D) none of the above.
6. **Amplitude modulation is used for broadcasting, because**
   (A) it is more noise immune than other modulation systems.
   (B) compared with other systems, it requires less transmitting power.
   (C) its circuitry is simple.
   (D) no other modulation system can provide the ncessary bandwidth for high fidelity.
7. **The purpose of oscillator in the AM transmitter is to**
   (A) provide modulating signal.
   (B) provide carrier.
   (C) provide enough power to meet transmission system requirement.
   (D) none of the above.
8. **Modulation index of an AM signal**
   (A) depends upon peak amplitude of modulating signal.
   (B) depends upon peak amplitudes of carrier and the modulating signal.
   (C) is a function of carrier frequency.
   (D) none of the above.
9. **An audio signal in the frequency range of 20 Hz to 10 kHz modulates the carrier. The bandwidth requirement is**
   (A) 20 kHz (B) 40 kHz
   (C) cannot be determined from the given data
   (D) none of the above.
10. **In frequency modulation**
    (A) frequency of the carrier remains constant.
    (B) carrier frequency varies in accordance with the modulating signal frequency.
    (C) carrier frequency varies in accordance with the modulating signal amplitude.
    (D) none of the above.
11. **Modulation index in an FM signal**
    (A) varies inversely as the frequency deviation.
    (B) varies directly as the modulating frequency.
    (C) varies directly as frequency deviation and inversely as the modulating frequency.
    (D) none of the above.
12. **Indicate which one of the following is not an advantage of FM over AM :**
    (A) Better noise immunity is provided.
    (B) Lower bandwidth is required.
    (C) The transmitted power is more useful.
    (D) Less modulating power is required.
13. **In a communication system, noise is most likely to affect the signal**
    (A) at the transmitter (B) in the transmission medium
    (C) in the information source
    (D) at the destination.
14. **The difference between phase and frequency modulation**
    (A) is purely theoretical, but practically they are same.
    (B) lies in the poorer audio response of phase modulator.
    (C) lies in the poorer audio response of frequency modulation.
    (D) lies in the definitions of their modulation index.
15. **While tuning in a certain broadcast station with a receiver, we are actually**
    (A) varying the local oscillator frequency.
    (B) varying the frequency of the radio signal to be picked up.
    (C) tuning the antenna.
    (D) none of the above.
16. **Digital signals**
    (A) donot provide a continuous set of values.
    (B) represent values as discrete steps.
    (C) utilise binary code system.
    (D) All of the above.
17. **In all pulse communication systems, carrier**
    (A) is necessarily a high repetition rate train of pulses.
    (B) is necessarily a high frequency continuous wave.
    (C) is either (A) or (B).
    (D) none of the above.
18. **Indicate which one of the following system is digital :**
    (A) Pulse Position Modulation
    (B) Pulse Code Modulation
    (C) Pulse Width Modulation
    (D) Pulse Amplitude Modulation.
19. **If a 10 kHz audio signal is to be sent accurately using 8-bit PCM digital signal how often should the signal be sampled ?**
    (A) 25 times every μs (B) twice every 25 μs
    (C) once every 50 μs (D) 10 times every μs.
20. **In a PCM system, the number of quantisation levels are 16 and the maximum signal, frequency is 4 kHz. The bit transmission rate is**
    (A) 64,000 bits s^{-1} (B) 16,000 bits s^{-1}
    (C) 32,000 bits s^{-1} (D) 32 bits s^{-1}.
21. **The pulse communication system that is higher immune to noise is**
    (A) PAM (B) PCM
    (C) PWM (D) PPM

22. **The most commonly used system of modulation for telegraphy is :**
(A) on-off keying (B) frequency shift keying
(C) analog modulation (D) pulse code modulation.

Chapter 2. Space Communication

PART I. TEXT BASED QUESTIONS

1. **High frequency waves are**
(A) absorbed in F layer (B) reflected by the E layer
(C) capable of use for long-distance transmission
(D) affected by the solar cycle.

2. **As electromagnetic waves travels in free space, only one of the following can happen to them :**
(A) Absorption (B) Attenuation
(C) Refraction (D) Reflection.

3. **The ground wave eventually disappears, as one moves away from the transmitter, because of**
(A) interference from the sky wave.
(B) loss of line-of-sight condition.
(C) maximum single-hop distance limitation.
(D) tilting.

4. **Frequencies in the UHF range normally propagate by means of**
(A) ground waves (B) sky waves
(C) surface waves (D) space waves.

5. **For television transmission, the frequency employed is normally in the range**
(A) 30 – 300 MHz (B) 30 – 300 GHz
(C) 300 – 300 kHz (D) 30 – 300 Hz
(Karnataka Entrance, 1990, 1989)

6. **The electromagnetic waves used in the telecommunication are**
(A) ultra-violet (B) infra-red
(C) visible (D) microwaves.

7. **The number of geostationary satellites needed fo uninterrupted global coverage is**
(A) 3 (B) 4
(C) 1 (D) 2

8. **The most commonly employed analog modulatio technique in satellite communication is the**
(A) amplitude modulation
(B) frequency modulation
(C) phase modulation (D) All the above.

9. **A payload that is invariably found on all communicatio satellites is the**
(A) photographic camera
(B) optical telescope
(C) transponder (D) none of the above.

10. **Satellite transponders**
(A) use a single frequency for reception and retransmissio of information to and from the earth.
(B) use a lower frequency for reception and highe frequency for retransmission.
(C) use a higher frequency for reception and a lowe frequency for retransmission.
(D) none of the above.

Chapter 3. Line Communication

PART I. TEXT BASED QUESTIONS

1. **Microwave link repeaters are typically 50 km apart,**
(A) because of atmospheric attenuation.
(B) because of the earth's curvature.
(C) to ensure that signal voltage may not harm the repeater.
(D) none of the above.

2. **Optical communication uses the frequency range of**
(A) 10^9 to 10^{10} Hz (B) 10^5 to 10^6 Hz
(C) 10^{14} to 10^{15} Hz (D) 10^{16} to 10^{17} Hz.

3. **Optical fibres rely for their operation on the phenomenon of**
(A) reflection (B) refraction
(C) dispersion (D) total internal reflection.

4. **An optical fibre has μ_{core} = 1·45 and μ_{clad} = 1·48. Is this statement true ?**
(A) Yes (B) No
(C) It may or may not be true.
(D) None of the above.

5. **One bel equals**
(A) 0·1 *d*B (B) 10 *d*B
(C) 100 *d*B (D) 1000 *d*B

6. **Losses is optical fibres is caused by**
(A) impurities in the glass.
(B) inperfect transparency of the glass.
(C) its extremely small area of cross-section.
(D) stepped index structure.

7. **Attenuation in optical fibre is mainly due to**
(A) scattering (B) absorption and scattering
(C) dispersion (D) none of the above.

8. **If the output power is reduced to 25% of its original value after passing through a fibre, the loss in *d*B is**
(A) 0·3 *d*B (B) 0·25 *d*B
(C) 3 *d*B (D) 6 *d*B

9. **A light beam after emerging an optical fibre makes a angle of 10° with the fibre core-fibre clad boundar surface. If the fibre core and fibre clad refractive indice are 1·5 and 1·49 respectively, can this beam propagat along the fibre ?**
(A) Yes (B) No
(C) The given data is insufficient to decide
(D) None of the above.

10. **If the core/cladding dimensions of an optical fibre ar given by 50/125 μm, the fibre type would be**
(A) single mode step index.
(B) single mode graded index.
(C) multimode step index.
(D) multimode graded index.

11. **The light emitting diode is usually made from**
(A) germanium (B) silicon
(C) gallium arsenide (D) none of the above.

12. **LEDs fabricated from gallium arsenide emit radiations i the**
(A) infra-red region (B) ultra-violet region
(C) visible region (D) none of the above.

13. **The output from a LASER is monochromatic. It means tha it is**
(A) directional (B) polarised
(C) narrow beam (D) single frequency.

14. **The type of source modulation required for digita communication using a fibre optic link set up is**
(A) frequency modulation
(B) amplitude modulation
(C) intensity modulation
(D) phase modulation.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

hapter 1. Analog and Digital Communication

1. (C) 2. (C) 3. (C) 4. (B, C & D)5. (A) 6. (C) 7. (B) 8. (B) 9. (C) 10. (C)
11. (C) 12. (B) 13. (B) 14. (D) 15. (A) 16. (D) 17. (A) 18. (B) 19. (C) 20. (C)
21. (B) 22. (A)

hapter 2. Space Communication

1. (B & D) 2. (B) 3. (D) 4. (D) 5. (A) 6. (D) 7. (A) 8. (B) 9. (C) 10. (C)

hapter 3. Line Communication

1. (B) 2. (C) 3. (D) 4. (B) 5. (B) 6. (A & B) 7. (B) 8. (D) 9. (B) 10. (C)
11. (C) 12. (C) 13. (D) 14. (C)

Revision Techniques

WHY SHOULD YOU REVISE ?

You cannot expect to remember all the Physics that you have studied unless you revise. It is important to review all your course, so that you can answer the examination questions.

WHERE SHOULD YOU REVISE ?

In a quiet room, with a table and a clock. The room should be brightly lighted. A reading lamp on the table helps you to concentrate on your work and reduces eye-strain.

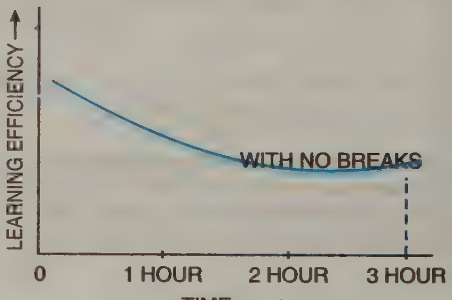
WHEN SHOULD YOU REVISE ?

Start your revision early each evening, before your brain gets tired.

HOW SHOULD YOU REVISE ?

If you sit down to revise without thinking of a definite finishing time, you will find that your learning efficiency falls lower and lower and lower.

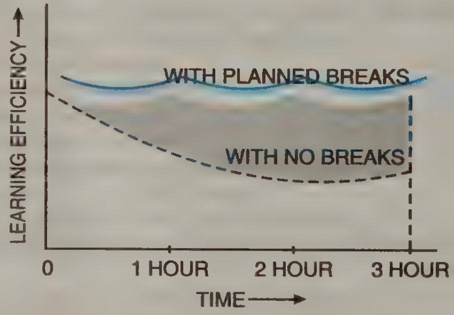
If you sit down to revise, saying to yourself that you will definitely stop work after 3 hours, then your learning efficiency falls at the beginning but *rises towards the end* as your brain realises it is coming to the end of the session (see Graph I).



Graph I

We can use this U-shaped curve to help us work more efficiently by splitting a 3 hour session into 3 shorter sessions, each of about 50 minutes with short, *planned* breaks between them.

The breaks *must* be planned beforehand so that the graph rises near the end of each short session [Graph II].



Graph II

The shaded area on the graph shows how much you gain :

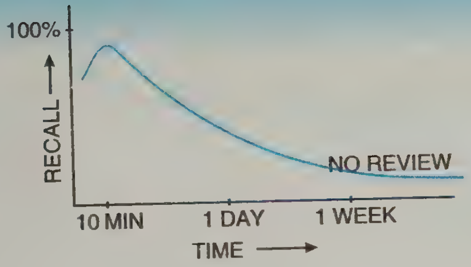
For example, if you start your revision at 6.00 p.m., you should look at your clock or watch and say to yourself, `I will work untill 7.00 p.m. and then stop – no earlier and not later.'

At 7.00 p.m. you should leave the table for a relaxation break of 10 minutes (or less), returning by 7.10 p.m. when you should say to yourself, `I will work untill 8.10 p.m. and then stop – not earlier and not later.'

Continuing in this way is more efficient and causes less strain on you. You get through more work and you feel less tired.

HOW OFTEN SHOULD YOU REVISE ?

The adjoining graphs show the amount of information that your memory can recall at different times after you have finished a revision session :

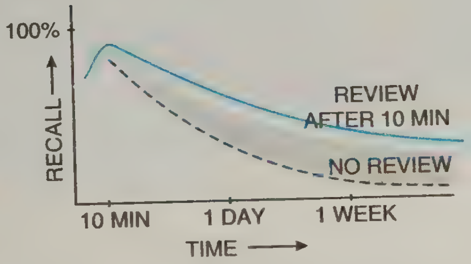


Surprisingly, the graph rises at the beginning. This is because your brain is still sorting out the information that you have been learning.

The graph soon falls rapidly so that after 1 day you may remember only about a quarter of what you have had learned [Graph III].

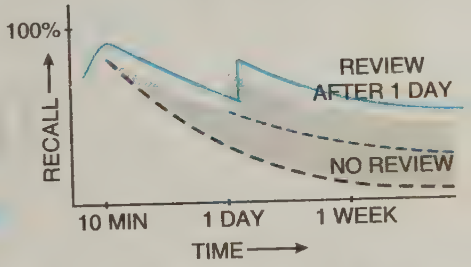
There are two ways of improving your recall and raising this graph

- 1. If you briefly *revise the same work again after 10 minutes* (at the high point of the graph) then the graph falls much more slowly [Graph IV].



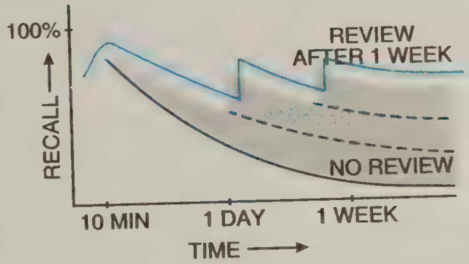
This fits in with your 10-minute break between revision sessions.

Using the example on the last page, when you return to your table at 7.10 p.m., the first thing you should do is ***review***, briefly, the work you learned before 7.00 p.m.



The graph can be lifted again by briefly reviewing the work *after 1 day* and then again *after 1 week*. That is, on Tuesday night you should look through the work you learned on Monday night and the work you learned on the previous Tuesday night, so that it is fixed quite firmly in your long-term memory [Graphs V and VI].

- 2. Another method of improving your memory is by taking care to try to ***understand*** all parts of your work. This makes all the graphs higher.



If you learn your work in a parrot-fashion (as you have to do with telephone numbers), all these graphs will be lower. On the occassions when you have to learn facts by heart, try to picture them as exaggerated, colourful images in your mind.

REMEMBER

The most important points about revision are that it must occur often and be repeated at the right intervals.

NOTES

LOGARITHMIC
AND
OTHER MATHEMATICAL TABLES

LOGARITHMS

Log$_{10}$N

N	0	1	2	3	4	5	6	7	8	9	MEAN DIFFERENCES								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	5 4	9 8	13 12	17 16	21 20	26 24	30 28	34 32	38 36
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 4	8 7	12 11	16 15	20 18	23 22	27 26	31 29	35 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 3	7 7	11 10	14 14	18 17	21 20	25 24	28 27	32 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 3	6 7	10 10	13 13	16 16	19 19	23 22	26 25	29 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 3	6 6	9 9	12 12	15 14	19 17	22 20	25 23	28 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 3	6 6	9 8	11 11	14 14	17 17	20 19	23 22	26 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 3	6 5	8 8	11 10	14 13	16 16	19 18	22 21	24 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 3	5 5	8 8	10 10	13 12	15 15	18 17	20 20	23 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 2	5 4	7 7	9 9	12 11	14 14	17 16	19 18	21 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 2	4 4	7 6	9 8	11 11	13 13	16 15	18 17	20 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											MEAN DIFFERENCES								

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

Example : Log 374·5 = 2·5729 (for 374)
+6 (mean diff. for 5)
2·5735
Mantissa
Characteristic

LOGARITHMS

Log$_{10}$N

N	0	1	2	3	4	5	6	7	8	9	MEAN DIFFERENCES								
											1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											MEAN DIFFERENCES								

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

Example : Log 0·008516 = $\bar{3}$·9299 (for 851)
+3 (mean diff. for 6)
$\bar{3}$·9302

ANTILOGARITHMS

10^x

x	0	1	2	3	4	5	6	7	8	9	MEAN DIFFERENCES								
											1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											MEAN DIFFERENCES								

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

Example : *Antilog 1·3377 = 2173 (for ·337)*
+3 (mean diff. for 7)
21·76 (decimal after 2 s.f.)
--- Indicates that the decimal point must be placed after the first 2 figures.

ANTILOGARITHMS

10^x

x	0	1	2	3	4	5	6	7	8	9	MEAN DIFFERENCES								
											1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											MEAN DIFFERENCES								

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

Example : *Antilog 2·6459 = 4416 (for ·645)*
+9 (mean diff for 9)
0 04425

A simple rule :
(i) If the characteristic is positive, then put the decimal after (n+1) digits from the left
(ii) If the characteristic is negative, then put (n−1) zeros before the first significant digit

NATURAL SINES

Sin θ

Degrees	0' 0°·0	6' 0°·1	12' 0°·2	18' 0°·3	24' 0°·4	30' 0°·5	36' 0°·6	42' 0°·7	48' 0°·8	54' 0°·9		Mean Differences 1'	2'	3'	4'	5'
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	ADD NUMBERS IN MEAN DIFFERENCE COLUMNS	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332		3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506		3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680		3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854		3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028		3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201		3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374		3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547		3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719		3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891		3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062		3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232		3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402		3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571		3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740		3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907		3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074		3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239		3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404		3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567		3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730		3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891		3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051		3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210		3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368		3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524		3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679		3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833		3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985		3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135		3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284		2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432		2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577		2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721		2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864		2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004		2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143		2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280		2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414		2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547		2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678		2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807		2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934		2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059		2	4	6	8	10
Degrees	0' 0°·0	6' 0°·1	12' 0°·2	18' 0°·3	24' 0°·4	30' 0°·5	36' 0°·6	42' 0°·7	48' 0°·8	54' 0°·9		1'	2'	3'	4'	5' Mean Differences

Example : Sin 28°11' = 0·4710 (⇒ for ·28°6')
+ 13 (⇒ mean diff. for 5')
0·4723
[Sin 28°11' = Sin 28°6' + 5']

NATURAL SINES

Sin θ

Degrees	0' 0°·0	6' 0°·1	12' 0°·2	18' 0°·3	24' 0°·4	30' 0°·5	36' 0°·6	42' 0°·7	48' 0°·8	54' 0°·9		Mean Differences 1'	2'	3'	4'
45°	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	ADD NUMBERS IN MEAN DIFFERENCE COLUMNS	2	4	6	8
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302		2	4	6	8
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420		2	4	6	8
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536		2	4	6	8
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649		2	4	6	8
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760		2	4	6	7
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869		2	4	5	7
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976		2	4	5	7
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080		2	3	5	7
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181		2	3	5	7
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281		2	3	5	7
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377		2	3	5	6
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471		2	3	5	6
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563		2	3	5	6
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652		1	3	4	6
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738		1	3	4	6
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821		1	3	4	6
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902		1	3	4	5
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980		1	3	4	5
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056		1	3	4	5
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128		1	2	4	5
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198		1	2	3	5
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265		1	2	3	4
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330		1	2	3	4
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391		1	2	3	4
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449		1	2	3	4
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505		1	2	3	4
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558		1	2	3	3
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608		1	2	2	3
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655		1	2	2	3
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699		1	1	2	3
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740		1	1	2	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778		1	1	2	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813		1	1	2	2
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845		1	1	2	2
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874		0	1	1	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900		0	1	1	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923		0	1	1	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943		0	1	1	1
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960		0	1	1	1
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974		0	0	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985		0	0	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993		0	0	0	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998		0	0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000		0	0	0	0
90	1·000														
Degrees	0' 0°·0	6' 0°·1	12' 0°·2	18' 0°·3	24' 0°·4	30' 0°·5	36' 0°·6	42 0°·7	48' 0°·8	54' 0°·9		1'	2'	3'	4' Mean Differences

Example : Sin 81°23' = 0·9885 (⇒ for 81°18')
+ 2 (⇒ mean diff. for 5')
0·9887
[Sin 81°23' = Sin 81°18' + 5']

NATURAL COSINES

Cos θ

DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	MEAN DIFFERENCES				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
0°	1·000	1·000	1·000	1·000	1·000	1·000	·9999	9999	9999	9999	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	'880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	/771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10
DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	MEAN DIFFERENCES				

SUBTRACT NUMBERS IN MEAN DIFFERENCE COLUMNS

Example : Cos 14°16′ = 0·9694 (⇒for 14°12′
− 3 (⇒mean diff. for 4′)
0·9691

[Cos 14°16′ = Cos 14°12′ + 4′]

Caution : In all co-functions such as Cosine, Cot, Cosec we have to subtract the mean differences

NATURAL COSINES

Cos θ

DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	MEAN DIFFERENCES				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
45°	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	·6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5946	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	·0017	3	6	9	12	15
90	·0000														
DEGREES	0′	6′	12	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	MEAN DIFFERENCES				

SUBTRACT NUMBERS IN MEAN DIFFERENCE COLUMNS

Example : Cos 53°40′ = 0·5934 (⇒ for 53°36′)
− 9 (⇒ mean diff. for 4′)
0·5925

[Cos 53°40′ = Cos 53°36′ + 4′]

NATURAL TANGENTS

Tan θ

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	MEAN DIFFERENCES				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29
DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	MEAN DIFFERENCES				

Example : Tan 27°16′ = 0·5139 (→ for 27°12′)
+15 (→ mean diff. for 4′)
0·5154
[Tan 27°16′ = Tan 27°12′ + 4′]

NATURAL TANGENTS

Tan θ

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	MEAN DIFFERENCES				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95	DIFFERENCE VALUES ARE UNRELIABLE				
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞														
DEGREES	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	MEAN DIFFERENCES				

Example : Tan 80°·7 = 6·1066
[Tan 80°·7 = Tan 80°42′ = 6·1066]

DEGREES TO RADIANS

x° → rad.

DEGREES	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
	0°·0	0°·1	0°·2	0° 3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	·0349	0367	0384	0401	0419	0436	0454	0471	0489	0506
3	·0524	0541	0559	0576	0593	0611	0628	0646	0663	0681
4	·0698	0716	0733	0750	0768	0785	0803	0820	0838	0855
5	·0873	0890	0908	0925	0942	0960	0977	0995	1012	1030
6	·1047	1065	1082	1100	1117	1134	1152	1169	1187	1204
7	·1222	1239	1257	1274	1292	1309	1326	1344	1361	1379
8	·1396	1414	1431	1449	1466	1484	1501	1518	1536	1553
9	·1571	1588	1606	1623	1641	1658	1676	1693	1710	1728
10	·1745	1763	1780	1798	1815	1833	1850	1868	1885	1902
11	·1920	1937	1955	1972	1990	2007	2025	2042	2060	2077
12	·2094	2112	2129	2147	2164	2182	2199	2217	2234	2251
13	·2269	2286	2304	2321	2339	2356	2374	2391	2409	2426
14	·2443	2461	2478	2496	2513	2531	2548	2566	2583	2601
15	·2618	2635	2653	2670	2688	2705	2723	2740	2758	2775
16	·2793	2810	2827	2845	2862	2880	2897	2915	2932	2950
17	·2967	2985	3002	3019	3037	3054	3072	3089	3107	3124
18	3142	3159	3176	3194	3211	3229	3246	3264	3281	3299
19	3316	3334	3351	3368	3386	3403	3421	3438	3456	3473
20	·3491	3508	3526	3543	3560	3578	3595	3613	3630	3648
21	·3665	3683	3700	3718	3735	3752	3770	3787	3805	3822
22	·3840	3857	3875	3892	3910	3927	3944	3962	3979	3997
23	·4014	4032	4049	4067	4084	4102	4119	4136	4154	4171
24	·4189	4206	4224	4241	4259	4276	4294	4311	4328	4346
25	·4363	4381	4398	4416	4433	4451	4468	4485	4503	4520
26	·4538	4555	4573	4590	4608	4625	4643	4660	4677	4695
27	·4712	4730	4747	4765	4782	4800	4817	4835	4852	4869
28	·4887	4904	4922	4939	4957	4974	4992	5009	5027	5044
29	·5061	5079	5096	5114	5131	5149	5166	5184	5201	5219
30	·5236	5253	5271	5288	5306	5323	5341	5358	5376	5393
31	·5411	5428	5445	5463	5480	5498	5515	5533	5550	5568
32	·5585	5603	5620	5637	5655	5672	5690	5707	5725	5742
33	·5760	5777	5794	5812	5829	5847	5864	5882	5899	5917
34	·5934	5952	5969	5986	6004	6021	6039	6056	6074	6091
35	·6109	6126	6144	6161	6178	6196	6213	6231	6248	6266
36	·6283	6301	6318	6336	6353	6370	6388	6405	6423	6440
37	·6458	6475	6493	6510	6528	6545	6562	6580	6597	6615
38	·6632	6650	6667	6685	6702	6720	6737	6754	6772	6789
39	·6807	6824	6842	6859	6877	6894	6912	6929	6946	6964
40	·6981	6999	7016	7034	7051	7069	7086	7103	7121	7138
41	·7156	7173	7191	7208	7226	7243	7261	7278	7295	7313
42	·7330	7348	7365	7383	7400	7418	7435	7453	7470	7487
43	·7505	7522	7540	7557	7575	7592	7610	7627	7645	7662
44	·7679	7697	7714	7732	7749	7767	7784	7802	7819	7837
45	·7854	7871	7889	7906	7924	7941	7959	7976	7994	8011
DEGREES	0'	6'	12'	18	24'	30'	36'	42'	48'	54'
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

MEAN DIFFERENCES

1'	2'	3'	4'	5'
3	6	9	12	15

DIFFERENCE VALUES ARE THE SAME FOR ALL ROWS

3	6	9	12	15
1'	2'	3'	4'	5'

MEAN DIFFERENCES

Note : 180° = π rad.

$\theta^\circ = \frac{\pi \times \theta^\circ}{180^\circ}$ radians

Example : 23°20' = 0·4067 (⇒ for 23°18')
+ 6 (⇒ mean diff for 2')
0·4073

[23°20' = 23°18' + 2' = 0·4073 rad.]

DEGREES TO RADIANS

x° → rad.

DEGREES	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9
45	·7854	7871	7889	7906	7924	7941	7959	7976	7994	8011
46	·8029	8046	8063	8081	8098	8116	8133	8151	8168	8186
47	·8203	8221	8238	8255	8273	8290	8308	8325	8343	8360
48	·8378	8395	8412	8430	8447	8465	8482	8500	8517	8535
49	·8552	8570	8587	8604	8622	8639	8657	8674	8692	8709
50	·8727	8744	8762	8779	8796	8814	8831	8849	8866	8884
51	·8901	8919	8936	8954	8971	8988	9006	9023	9041	9058
52	·9076	9093	9111	9128	9146	9163	9180	9198	9215	9233
53	·9250	9268	9285	9303	9320	9338	9355	9372	9390	9407
54	·9425	9442	9460	9477	9495	9512	9529	9547	9564	9582
55	·9599	9617	9634	9652	9669	9687	9704	9721	9739	9756
56	·9774	9791	9809	9826	9844	9861	9879	9896	9913	9931
57	·9948	9966	9983	1·0001	1·0018	1·0036	1·0053	1·0071	1·0088	1·0105
58	1·0123	0140	0158	0175	0193	0210	0228	0245	0263	0280
59	1·0297	0315	0332	0350	0367	0385	0402	0420	0437	0455
60	1·0472	0489	0507	0524	0542	0559	0577	0594	0612	0629
61	1·0647	0664	0681	0699	0716	0734	0751	0769	0786	0804
62	1·0821	0838	0856	0873	0891	0908	0926	0943	0961	0978
63	1·0996	1013	1030	1048	1065	1083	1100	1118	1135	1153
64	1·1170	1188	1205	1222	1240	1257	1275	1292	1310	1327
65 66	1·1345	1362	1380	1397	1414	1432	1449	1467	1484	1502
67	1·1694	1537	1554	1572	1589	1606	1624	1641	1659	1676
68	1·1868	1886	1729	1746	1764	1781	1798	1816	1833	1851
69	1·2043	2060	2076	1921	1938	1956	1973	1990	2008	2025
					2113	2130	2147	2165	2182	2200
70	1·2217	2235	2252	2270	2287	2479	[illegible]	2339	2357	2374
71	1·2392	2409	2427	2444	2462	2654	2671	[illegible]	2531	2549
72	1·2566	2584	2601	2619	2636	2828	2846	2863	[illegible]	2723
73	1·2741	2758	2776	2793	2811	3003	3020	3038	3055	[illegible]
74	1·2915	2933	2950	2968	2985					
75	1·3090	3107	3125	3142	3160	3177	3195	3212	3230	3247
76	1·3265	3282	3299	3317	3334	3352	3369	3387	3404	3422
77	1·3439	3456	3474	3491	3509	3526	3544	3561	3579	3596
78	1·3614	3631	3648	3666	3683	3701	3718	3736	3753	3771
79	1·3788	3806	3823	3840	3858	3875	3893	3910	3928	3945
80	1·3963	3980	3998	4015	4032	4050	4067	4085	4102	4120
81	1·4137	4155	4172	4190	4207	4224	4242	4259	4277	4294
82	1·4312	4329	4347	4364	4382	4399	4416	4434	4451	4469
83	1·4486	4504	4521	4539	4556	4573	4591	4608	4626	4643
84	1·4661	4678	4696	4713	4731	4748	4765	4783	4800	4818
85	1·4835	4853	4870	4888	4905	4923	4940	4957	4975	4992
86	1·5010	5027	5045	5062	5080	5097	5115	5132	5149	5157
87	1·5184	5202	5219	5237	5254	5272	5289	5307	5324	5341
88	1·5359	5376	5394	5411	5429	5446	5464	5481	5499	5516
89	1·5533	5551	5568	5586	5603	5621	5638	5656	5673	5691
DEGREES	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9

ADD NUMBERS IN MEAN DIFFERENCE COLUMNS

MEAN DIFFERENCES

1'	2'	3'	4'	5'
3	6	9	12	15

DIFFERENCE VALUES ARE THE SAME FOR ALL ROWS

3	6	9	12	15
1'	2'	3'	4'	5'

MEAN DIFFERENCES

Note : 1° = 0·0175 rad.
1 rad = 57°·3

Example : (i) 61°4' = 1·0647 (⇒ for 61°)
+ 12 (⇒ mean diff. for 4',
1·0659

(ii) 1·962 rad = 68°30' + 2' = 68°32' observe for the values at the intersection of 68° and 30' and add 2' for the mean diff value of 6

POWERS ROOTS AND RECIPROCALS

N^2, N^3, $\sqrt[3]{N}$, $\sqrt{10N}$ etc.

N	$\frac{1}{N}$	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$\sqrt{10N}$	$\sqrt[3]{10N}$	$\sqrt[3]{100N}$	Log N!
1	1	1	1	1	1	3·162	2·154	4·642	0·0000
2	·5000	4	8	1·414	1·260	4·472	2·714	5·848	0·3010
3	·3333	9	27	1·732	1·442	5·477	3·107	6·694	0·7782
4	·2500	16	64	2	1·587	6·325	3·420	7·368	1·3802
5	·2000	25	125	2·236	1·710	7·071	3·684	7·937	2·0792
6	·1667	36	216	2·449	1·817	7·746	3·915	8·434	2·8573
7	·1429	49	343	2·646	1·913	8·367	4·121	8·879	3·7024
8	·1250	64	512	2·828	2·000	8·944	4·309	9·283	4·6055
9	·1111	81	729	3·000	2·080	9·487	4·481	9·655	5·5598
10	·1000	100	1000	3·162	2·154	10·0	4·642	10·000	6·5598
11	·09091	121	1331	3·317	2·224	10·488	4·791	10·323	7·6012
12	·08333	144	1728	3·464	2·289	10·954	4·932	10·627	8·6803
13	·07692	169	2197	3·606	2·351	11·402	5·066	10·914	9·7943
14	·07143	196	2744	3·742	2·410	11·832	5·192	11·187	10·9404
15	·06667	225	3375	3·873	2·466	12·247	5·313	11·447	12·1165
16	·06250	256	4096	4·000	2·520	12·649	5·429	11·696	13·3206
17	·05882	289	4913	4·123	2·571	13·038	5·540	11·935	14·5511
18	·05556	324	5832	4·243	2·621	13·416	5·646	12·164	15·8063
19	·05263	361	6859	4·359	2·668	13·784	5·749	12·386	17·0851
20	·0500	400	8000	4·472	2·714	14·142	5·848	12·599	18·3861
21	·04762	441	9261	4·583	2·759	14·491	5·944	12·806	19·7083
22	·04545	484	10648	4·690	2·802	14·832	6·037	13·006	21·0508
23	·04348	529	12167	4·796	2·844	15·166	6·127	13·200	22 4125
24	·04167	576	13824	4·899	2·884	15·492	6·214	13·389	23·7927
25	·0400	625	15625	5·000	2·924	15·811	6·300	13·572	25·1906
26	·03846	676	17576	5·099	2·962	16·125	6·383	13·751	26·6056
27	·03704	729	19683	5·196	3·000	16·432	6·463	13·925	28·0370
28	·03571	784	21952	5·292	3·037	16·733	6·542	14·095	29·4841
29	·03448	841	24389	5·385	3·072	17·029	6·619	14·260	30·9465
30	·03333	900	27000	5·477	3·107	17·321	6·694	14·422	32·4237
31	·03226	961	29791	5·568	3·141	17·607	6·768	14·5	33·9150
32	·03125	1024	32768	5·657	3·175	17·889	6·840	14·736	35·4202
33	·03030	1089	35937	5·745	3·208	18·166	6·91	14·888	36·9387
34	·02941	1156	39304	5·831	3·240	13·439	6·9	15·037	38·4702
35	·02857	1225	42875	5·916	3·271	18·708	7·047	15·183	40·0142
36	·02778	1296	46656	6·000	3·302	18·974	7·114	15 326	41·5705
37	·02703	1369	50653	6·083	3·332	19·235	7·179	15·467	43·1387
38	·02632	1444	54872	6·164	3·362	19·494	7·243	15·605	44·7185
39	·02564	1521	59319	6·245	3·391	19·748	7·306	15·741	46·3096
40	·0250	1600	64000	6·325	3·420	20·00	7·368	15·874	47·9116
41	·02439	1681	68921	6·403	3·448	20·248	7·429	16·005	49·5244
42	·02381	1764	74088	6·481	3·476	20·494	7·489	16·134	51·1477
43	·02326	1849	79507	6·557	3·503	20·736	7·548	16·261	52·7811
44	·02273	1936	85184	6·633	3·530	20·976	7·606	16·386	54·4246
45	·02222	2025	91125	6·708	3·557	21·213	7·663	16·510	56·0778
46	·02174	2116	97336	6·782	3·583	21·448	7·719	16·631	57·7406
47	·02128	2209	103823	6·856	3·609	21·679	7·775	16·751	59·4127
48	·02083	2304	110592	6·928	3·634	21·909	7·830	16·869	61·0939
49	·02041	2401	117649	7·000	3·659	22·136	7·884	16·985	62·7841
50	·020	2500	125000	7·071	3·684	22·361	7·937	17·100	64·4831
N	$\frac{1}{N}$	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$\sqrt{10N}$	$\sqrt[3]{10N}$	$\sqrt[3]{100N}$	Log N!

Example : 39^3 = 59319, $\sqrt[3]{39}$ = 3·391 , $\sqrt{390}$ = 19·748

POWERS, ROOTS AND RECIPROCALS

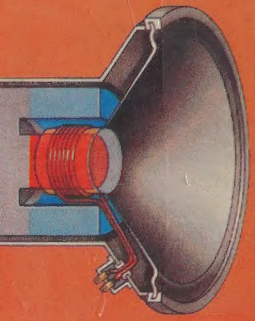
N^2, N^3, $\sqrt[3]{N}$, $\sqrt{10N}$ e c

N	$\frac{1}{N}$	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$\sqrt{10N}$	$\sqrt[3]{10N}$	$\sqrt[3]{100N}$	Log N!
51	·01961	2601	132651	7·141	3·708	22·583	7·990	17·213	66·1906
52	·01923	2704	140608	7·211	3·733	22·804	8·041	17·325	67·9066
53	·01887	2809	148877	7·280	3·756	23·022	8·093	17·435	69·6309
54	·01852	2916	157464	7·348	3·780	23·238	8·143	17·544	71·3633
55	·01818	3025	166375	7·416	3·803	23·452	8·193	17·652	73·1037
56	·01786	3136	175616	7·483	3·826	23·664	8·243	17·758	74·8519
57	·01754	3249	185193	7·550	3·849	23·875	8 291	17·863	76·6077
58	·01724	3364	195112	7·616	3·871	24·083	8·340	17·967	78·3712
59	·01695	3481	205379	7·681	3·893	24·290	8·387	18·070	80·1420
60	·01667	3600	216000	7·746	3·915	24·495	8·434	18·171	81·9202
61	·01639	3721	226981	7·810	3·936	24·698	8·481	18·272	83·7055
62	·01613	3844	238328	7·874	3·958	24·900	8·527	18·371	85·4979
63	·01587	3969	250047	7·937	3·979	25·100	8·573	18·469	87·2972
64	·01562	4096	262144	8·000	4·000	25·298	8·618	18·566	89·1034
65	·01538	4225	274625	8·062	4·021	25·495	8·662	18·663	90·9163
66	·01515	4356	287496	8·124	4·041	25·690	8·707	18·758	92·7359
67	·01493	4489	300763	8·185	4·062	25·884	8·750	18·852	94·5619
68	·01471	4624	314432	8·246	4·082	26·077	8·794	18·945	96·3945
69	·01449	4761	328509	8·307	4·102	26·268	8·837	19·038	98·2333
70	·01429	4900	343000	8·367	4·121	26·458	8·879	19·129	100·0
71	·01408	5041	357911	8·426	4·141	26·646	8·921	19·310	101·9297
72	·01389	5184	373248	8·485	4·160	26·833	9·004	19·399	103·7870
73	·01370	5329	389017	8·544	4·179	27·0	9·045	19·487	105·6503
74	·01351	5476	405224	8·602	4·19	27·386	9·086	19·574	107·5196
75	·01333	5625	421875	8·660					109·3946
76	·01316	5776	438976	8·718	4·236	27·568	9·126	19·661	111·2754
77	·01299	5929	474552	8·775	4·254	27·749	9·166	19·747	113·1619
78	·01282	6241	493039	8·832	4·273	27·928	9·205	19·832	115·0540
79	·01250	6400	512000	8·888	4·291	28·107	9·244	19·916	116·9516
				8·944	4·309	28·284	9·283	20·000	118·8547
81	·01235	6561	531441	9·000	4·327	28·460	9·322	20·083	120·7632
82	·01220	6724	551368	9·055	4·344	28·636	9·360	20·165	122·6770
83	·01205	6889	571787	9·110	4·362	28·810	9·398	20·247	124·5961
84	·01190	7056	592704	9·165	4·380	28·983	9·435	20·328	126·5204
85	·01176	7225	614125	9·220	4·397	29·155	9·473	20·408	128·4498
86	·01163	7396	636056	9·274	4·414	29·326	9·510	20·488	130·3843
87	·01149	7569	658503	9·327	4·431	29·496	9·546	20·567	132·3238
88	·01136	7744	681472	9·381	4·448	29·665	9·583	20·646	134·2683
89	·01124	7921	704969	9·434	4·465	29·833	9·619	20·724	136·2177
90	·01111	8100	729000	9·487	4·481	30·000	9·655	20·801	138·1719
91	·01099	8281	753571	9·539	4·498	30·166	9·691	20·878	140·1310
92	·01087	8464	778688	9·592	4·514	30·332	9·726	20·954	142·0948
93	·01075	8649	804357	9·644	4·531	30·496	9·761	21·029	144·0632
94	·01064	8836	830584	9·695	4·547	30·659	9·796	21·105	146·0364
95	·01053	9025	857375	9·747	4·563	30·822	9·830	21 179	148·0141
96	·01042	9216	884736	9·798	4·579	30·984	9·865	21·253	149·9964
97	·01031	9409	912673	9·849	4·595	31·145	9·899	21·327	151·9831
98	·01020	9604	941192	9·899	4·610	31·305	9·933	21·400	153·9744
99	·01010	9801	970299	9·950	4·626	31·464	9·967	21·472	155·9700
100	·0100	10000	1000000	10·000	4·642	31·623	10·000	21·544	157·9700
N	$\frac{1}{N}$	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$\sqrt{10N}$	$\sqrt[3]{10N}$	$\sqrt[3]{100N}$	Log N!

Example $\sqrt{84}$ = 9·165, $\sqrt{840}$ = 28·983 , $\sqrt[3]{8400}$ = 20·328

MODERN'S abc OF PHYSICS

The book presents the subject matter in full conformity with the syllabi prescribed by C.B.S.E., New Delhi and Education Boards of other Indian states. To keep pace with changing trends in education at national level, the whole text has been arranged strictly according to N.C.E.R.T. pattern. The main stress has been laid on SI. The symbols and signs used for various physical quantities are also in keeping with the recommendations at national and international levels.



The book provides a result-oriented training to young students. The whole text of the book is embedded with short notes in the form of **The jargon** (introducing apparently a new physical term with a proper definition), Key point (highlighting an important point in the text) and **Watch out** (bringing out the difference between the physical and apparent meaning of a physical term). Further, the text has been studded with simple **Self-Study Questions,** so as to provide an insight and a proper grip over the topic, as one learns it. The article work in each chapter of a unit is coupled with well graded and carefully selected **Solved Numerical Problems** for easy comprehension of the beginners. So that the students can prepare for the Annual Examinations in an independent manner, a large number of **Very Short Answer Questions** and **Short Answer Questions** have been incorporated in the book with proper Answers/Hints. **Unsolved Numerical Problems** for self-practice have been categorised into various types, so as to enable the students to choose the appropriate formula with ease. Further, detailed Hints/Solutions have been provided to Unsolved Numerical Problems. **Techie-Stuff** offers a special feature of the book. It contains real **Conceptual Numerical Problems** and **Conceptual Short Answer Questions.** It is aimed to provide intensive understanding and deep insight of the subject to the students, so that they get the feel of the type of questions asked in competitive examinations, such as I.I.T.

The Competitive Examination File in each unit forms another special feature of the book and consists of three parts. **Revision at a Glance** of the contents of a unit is for easy and handy reference of various physical laws, principles, terms and formulae in that unit and for its quick revision. **Numerical Problems from Competitive Examinations,** such as **I.I.T., Roorkee** and **I.S.M., Dhanbad** have been provided with solutions by adopting a novel technique in the form of **Thought Process.** Armed with this technique, the students will be able to attack the otherwise brainteasing and seemingly incomprehensible numerical problems with great ease. **Multiple Choice Questions** set in various competitive examinations, such as **C.B.S.E., A.I.I.M.S., A.F.M.C., M.N.R., C.P.M.T., I.I.T.,** etc have been thoroughly covered in the book. For the sake of easy preparation, these questions have been categorised into **Text-Based** and **Thought-Based** Multiple Choice Questions. The author is of the firm opinion that the learning is a continual and gradual process. With the Competitive Examination Files on different units at their disposal, the students would be able to master them steadily all through the academic year, while preparing for admission to professional courses.

MODERN PUBLISHERS

ISBN 81-245-0041-X